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PROCESS MONITORING AND CONTROL OF MICROALGAE CULTIVATION

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- To all the special people in my life :
My parents who have dedicated their lives for their children
My brother and sisters who support me always

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Résumé

Les bioprocédés jouent un rôle important dans la production de substances à haute valeur ajoutée. L'une des cultures les plus intéressantes parmi les biocultures sont les microalgues. Il s'agit d'organismes microscopiques vivant en milieu aquatique et dont la biomasse est une excellente source d'acide gras et de vitamines. De plus, la culture de microalgues pourrait être utilisée à grande échelle pour produire de l'énergie. Dans ce contexte, l'un des modèles les plus simples pour décrire son comportement dynamique est le modèle de Droop. Ce modèle largement utilisé a été choisi pour cette étude. L'estimation d'état est un domaine de l'ingénierie basé sur l'extraction des informations sur les variables inconnues à partir des informations connues. En génie biochimique, il est nécessaire de connaître les variables qui caractérisent l'état interne du procédé dans le but de produire de grandes quantités des substances d'int érêt. Cependant, l'un des problémes les plus importants dans la conception de l'estimateur est de pouvoir garantir la convergence de l'erreur d'estimation. C'est pourquoi, en se basant sur les propriétés du modéle de Droop, un observateur de Lipschitz est proposé pour estimer les variables d'état à partir de la mesure. Par ailleurs, l'estimation des paramètres à l'aide de l'observateur est discutée en vue d'estimer certains des paramètres du modèle de Droop. Afin d'évaluer les performances de l'observateur dans le contexte de la commande avancée, le contrôle de la concentration de biomasse et de substrat sont introduits. Deux techniques de contrôle sont considérées en couplage avec l'observateur : le contrôle "backstepping" et le contrôle par linéarisation entrée/sortie. Le suivi de la consigne et le rejet de perturbation sont également étudiés pour ces stratégies. Pour terminer, une extension du modèle de Droop est étudiée pour la production de substances lipidiques. Une structure d'estimation de l'ensemble des variables d'état est ainsi démontrée.

Abstract

Bioprocesses play an important role to produce high-value products. One of the most interesting cultures among the biocultures is microalgae. It is a microscopic organism existing in aquatic environment. The biomass from this culture is a great source of fatty acids and vitamins. Large scale microalgae culture can be used to produce energy. One of the simplest models to describe the dynamic behaviour of the culture is the Droop model. This widely used model has been chosen for this study. State estimation is a field of control engineering that extracts information about unknown variables based on known information. In bioprocess engineering, in order to produce high amounts of valued product, it is necessary to know about internal state variables of the process. One of the most important problems in designing the estimator is to guarantee the stability of the error dynamics. Based on the properties of the Droop model, a Lipschitz observer is proposed to estimate the state variables from measurement. Moreover, the parameter estimation using the Lipschitz observer is discussed in order to estimate some of the parameters of the Droop model. In order to see the observer performance with advanced controller, the biomass and the substrate concentration control are introduced. Two observer- based controllers, input-output linearization and backstepping technique, are considered. The setpoint tracking and the load rejection problem are studied for both strategies. Finally, a lipid production model as an extension of the Droop model is introduced. The observability property of the model is explained. At the end, a structure for the estimation of all state variables using measurement is demonstrated.

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EKF	Extended Kalman Filter
ELO	Extended Luenberger Observer
MHE	Moving Horizon Estimator
LMI	Linear Matrix Inequality
KYP	Kalman Yacubovich Popov
NPO	Nonlinear Passivity Observer
SISO	Single Input-Single Output

CHAPITRE 1

Microalgae Culture

1.1 Introduction

Bioprocesses have an important part in the production of added value products in the pharmaceutical industry (proteins, vaccines, etc), in the manufacturing of agro-food products (yeast, beer, wine, etc), and in the treatment of solid organic waste as well as urban and industrial wastewater.

On line monitoring of bioprocesses is highly desirable since it has the potential to produce significant improvements in the process control. Some hardware sensors are already available, but they often have several disadvantages such as cost, discrete-time measurements (instead of continuous ones), processing delay, sterilization, etc. But, there are some techniques which are able to do the same job as hardware. So, by using these techniques, it is possible in certain instances to obtain an on-line estimation of the process states. This kind of tools is called a software sensor [19]. In other words, a software sensor is a mathematical algorithm, which gives an on-line estimation for state variables in a cultivation such as the biomass, the substrate or the products, whose analyses are normally time consuming, labour intensive and costly.

In the following paragraphs, microalgae culture as an interesting culture to produce energy and high-value products is introduced. In order to describe the dynamic behaviour of this culture, the Droop model will be presented. Then, based on mathematics and control theory, two important properties of the Droop model will be described. Finally, lipid production model will be described as an extension of the Droop model in order to produce lipid.

1.2 Microalgae process

A microalgae is a microscopic plant existing in aquatic environment and is the basic level of the feed chain in the ocean. These organisms have an increasing interest due to their capability to fix CO_2 [56] and to produce hydrogen [14, 48]. In the following, the Droop model is introduced to describe the growth of microalgae.

1.2.1 Droop Model

The Droop model [41] is a simple and widely used model which can represent this natural biological phenomenon. It includes three state variables : the biomass concentration X, the internal quota Q, which is defined as the quantity of nitrogen per unit of the biomass and the substrate concentration S.

The time-varying evolution equations of the Droop model are given by

$$\begin{aligned} \dot{X}(t) &= -D(t)X(t) + \mu(Q)X(t), \\ \dot{Q}(t) &= \rho(S) - \mu(Q)Q(t), \\ \dot{S}(t) &= D(t)(S_{in} - S(t)) - \rho(S)X(t), \end{aligned} \tag{1.1}$$

with $\rho(S) = \rho_m \dfrac{S(t)}{S(t)+K_s}$ as the substrate uptake rate and $\mu(Q) = \bar{\mu}\left(1 - \dfrac{K_Q}{Q(t)}\right)$ as the growth rate.

In these relationships, D represents the dilution rate, S_{in} the input substrate concentration, ρ the specific uptake rate and μ the specific growth rate. In the expression of the uptake rate, K_s and ρ_m represent a half-saturation constant for the substrate and the maximum uptake rate respectively. $\bar{\mu}$ is the theoretical maximum growth rate, obtained for an infinite internal quota and K_Q the minimum internal quota allowing growth.
The parameters of the model are given in the following table [29].

Tableau 1.1: Model parameters

Parameter	**Definition**	**Unit**	**Value**
S_{in}	Input substrate concentration	$\mu mol/L$	100
K_s	Half – saturation constant	$\mu mol/L$	0.105
$\bar{\mu}$	Theoretical maximum growth	$1/d$	2
K_Q	Minimum internal quota	$\mu mol/\mu m^3$	1.8
ρ_m	Maximum uptake rate	$\mu mol/\mu m^3/d$	9.3

It is assumed that the biomass concentration is the only measurement variable, Q and S are estimated variables. Therefore, the objective is to estimate the unmeasurable variables (S, Q) from the measurable variable X.

1.3 Properties of Droop Model

1.3.1 Bounded Trajectory

The trajectories of the Droop model are bounded and $K_Q \leq Q \leq Q_{max}$ [7]. This model can be written in the common form of the nonlinear input affine system as below

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x). \end{aligned} \tag{1.2}$$

At the presented format

$$f(x) = \begin{pmatrix} -\mu X \\ \rho - \mu Q \\ \rho X \end{pmatrix}, \; g(x) = \begin{pmatrix} -X \\ 0 \\ S_{in} - S \end{pmatrix}, \text{ and } h(x) = X.$$

1.3.2 Observability Property of Nonlinear Model

Prior to the design of a state observer, system observability has to be assessed in order to determine the conditions under which it is possible to reconstruct unmeasured state variables using a mathematical model and available measurements. It can be considered a system in the ideal case where measurements are assumed available at any time and without noise as follows

$$\begin{aligned} \dot{x} &= f(x, u), \\ y &= h(x). \end{aligned} \tag{1.3}$$

In the following definition and theory, the observability property of a specific kind of system is presented.

Definition 1.3.1 *(SL_2H systems) System (1.3) is said Strictly Linked Lower Hessenberg (SL_2H) if it satisfies the following conditions for any $x = [x_1, x_2, ..., x_n]$ and any u*

1- for any (i,j) such that $j > (i+1)$, $\dfrac{\partial f_i}{\partial x_j} = 0$.

2- for any index i, $\dfrac{\partial f_i}{\partial x_{i+1}} \neq 0$.

3- $h(x) = h(x_1)$ with $\dfrac{\partial h}{\partial x_1} \neq 0$.

Theorem 1.3.1 *SL_2H systems are uniformly input observable [8, 28].*

These definition and theorem allow to conclude that the Droop model satisfies the uniformly input observable property. If it is assumed the following state vector x and nonlinear differential equations

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} X \\ Q \\ S \end{pmatrix},$$

$$\begin{aligned} \dot{x_1} &= f_1(x_1, x_2) = -Dx_1 + \mu(x_2)x_1, \\ \dot{x_2} &= f_2(x_2, x_3) = \rho(x_3) - \mu(x_2)x_2, \\ \dot{x_3} &= f_3(x_1, x_3) = D(S_{in} - x_3) - \rho(x_3)x_1. \end{aligned} \tag{1.4}$$

The Droop model is SL_2H and is uniformly input observable with $y = X$ if $X \neq 0$. The SL_2H conditions give

$$\frac{\partial f_1}{\partial x_2} = \bar{\mu} K_Q \frac{x_1}{x_2^2} > 0,$$

$$\frac{\partial f_2}{\partial x_3} = \rho_m \frac{K_s}{(K_s + x_3)^2} > 0.$$

If one of these conditions is not satisfied, the system is neither strictly linked nor observable.

Another way to study the observability property of the Droop model is to compute the observability matrix. By computing the observability matrix as shown in the following part, it can be proved that the Droop model is always observable.

If vector O and matrix OM are considered as observability vector and observability matrix respectively.

$$O = [y, \dot{y}, \ddot{y}] \Longrightarrow O = (X, -DX + \mu X, X[(-D + \mu)^2 + (\rho - \mu Q)(\bar{\mu} K_Q / Q^2)]),$$

$$OM = \frac{\partial O}{\partial X},$$

$$OM_{11} = 1,\ OM_{12} = 0,\ OM_{13} = 0,$$

$$OM_{21} = -DX + \mu X,\ OM_{22} = \frac{X K_Q \bar{\mu}}{Q^2},\ OM_{23} = 0,$$

$$OM_{31} = (-D + \mu)^2 + (\rho - \mu Q)(\frac{\bar{\mu} K_Q}{Q^2}),$$

$$OM_{32} = \frac{-2\bar{\mu} DX + 2\mu \bar{\mu} K_Q X - (\bar{\mu})^2 K_Q X}{Q^2} + (\rho - \mu Q)(-2\bar{\mu} X / Q^2),$$

$$OM_{33} = \frac{K_s \rho_m \bar{\mu} K_Q X}{(K_s + S)^2 Q^2}.$$

The determinant of OM would be

$$\Delta(OM) = \frac{\rho_m(\bar{\mu})^2(K_Q)^2K_sX^2}{Q^4(K_s+S)^2}.$$

The rank of the observability matrix is three and determinant of the observability matrix is non-zero as long as the concentration of the biomass is non-zero. So, Based on the above results, the Droop model is always observable.

1.4 Lipid Production in Microalgae Process

As mentioned in the introduction part, biofuel can be produced from microalgae. One of the well-known dynamic models of microalgae to describe the lipid production was studied in [46]. Generally speaking, this model is composed of two parts, the Droop model and two extra states related to lipid production. In this model, intracellular carbon is divided between a functional pool and two storage pools (sugar and neutral lipid).

The time-varying evolution equations of the model are given by

$$\begin{aligned}
\dot{X} &= -DX + \mu X, \\
\dot{Q} &= \rho - \mu Q, \\
\dot{S} &= D(S_{in} - S) - \rho X, \\
\dot{L} &= (\beta Q - L)\mu - \gamma\rho, \\
\dot{F} &= -F\mu + (\alpha + \gamma)\rho,
\end{aligned} \tag{1.5}$$

where X, Q, S, L and F are the biomass, internal quota, the substrate, the neutral lipid and the functional carbon concentration respectively.

The definition and value of each parameter in the model are demonstrated in the following table.

Tableau 1.2: Model parameters for lipid production

Parameter	**Definition**	**Unit**	**Value**
K_Q	Minimum nitrogen quota	$mg[N].mg[C]^{-1}$	0.05
$\bar{\mu}$	Maximum growth rate	d^{-1}	1.83
α	Protein synthesis coefficient	$mg[C].mg[N]^{-1}$	3.1
β	Fatty acid synthesis	$mg[C].mg[N]^{-1}$	3.5
γ	Fatty acid mobilisation	$mg[C].mg[N]^{-1}$	1.7
K_s	Half saturation constant	$mg[N]L^{-1}$	0.018
ρ_m	Maximal uptake rate	$mg[N]mg[C]^{-1}d^{-1}$	0.11

As it can be seen, the dynamics of the fractions F and L do not affect on the kinetics of the biomass concentration. Therefore, as it can be concluded that the lipid model has a cascade structure.

CHAPITRE 2

Estimation and Control

2.1 Introduction

Efficient monitoring and good control of a process are only possible, when accurate information on the state variables and parameters of the process are available. For example, concentrations of the materials inside a reactor and temperature can be considered as process state variables. The rate of production in a reactor, the heat transfer coefficient in jacket reactors and the specific growth rate in a bioreactor are examples of process parameters. Practically, the better understanding of the process dynamics as well as the development of an accurate process model are based on information about process parameters. However, some of the important process state variables cannot be measured due to the insufficiency of available sensors or operational limitations. In such situations, continuous estimates of the inaccessible state variables and parameters of the process are generally obtained by using state and parameter estimation methods. Therefore, the knowledge about internal information of the process variables and parameters are critical in modelling (identification), monitoring (fault detection) and driving (control). These three applications are necessary, when the goal is to keep a system under control. This concept is schematically illustrated in Figure 2.1

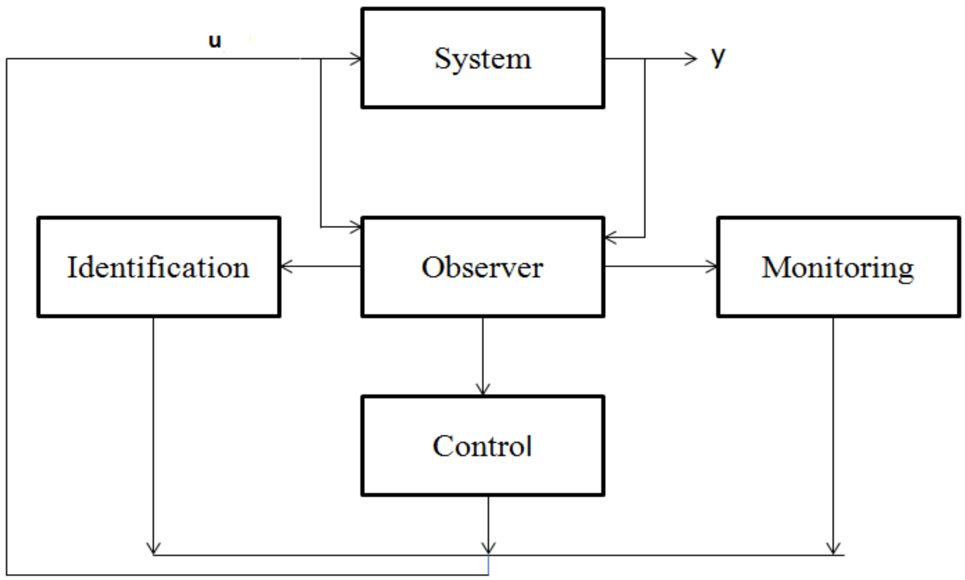

Figure 2.1: Observer as heart of process [9].

2.1.1 Literature Review

The observer is a powerful tool with many applications in the fields of science and engineering, such as, signal processing, economics, medicine and process industries. State estimators are dynamics systems which are used to estimate the important process state variables by means of accessible variables. State estimation problem in chemical and biochemical engineering has been studied since 1970s. The design and the application of state estimators in process control have been an active area over the past decades [22, 36, 55].

One important problem to design the observer is to prove the convergence of error dynamics. The stability analysis of error dynamics can be performed locally or globally based on the method and structure of the model used to design the observer. In the following, a short review of the available observer's methods is presented.

2.1.2 Extension of Linear Observers

The first class of observers is based on the perfect knowledge of the process model. For example, the Extended Kalman Filter (EKF), the Extended Luenberger Observer (ELO) and nonlinear observers belong to this class. The second class of state estimators is the asymptotic observer which is related to bioprocess.

EKF has been widely used for state estimation in chemical processes. For examples, in [36], EKF is used to provide a real time column composition profile from temperature measurements. In [2], Kalman filter was used to reduced the noise of glucose measurements and to estimate the biomass concentration, the substrate concentration as well as the maximum growth rate at cultivations of a recombinant Escherichia coli. Also, in order to understand the effect of the enzymatic deactivation, EKF was applied to identify the enzymatic deactivation in the enzymatic hydrolysis of a pretreated cellulose [13].

EKF was introduced as an approximation of the optimal estimator to develop an estimator for nonlinear systems. The design of EKF is based on a linearization of the nonlinear process model in each sample time. It has a problem of lacking a guaranteed stability except in the works presented in [6, 39], where the authors provided some adequate conditions for stability. Generally, at the best conditions, the guaranteed stability is local. In [61], by using the direct method of Lyapunov, the stability of EKF under certain conditions was obtained. It was proved that the estimation error remains bounded, if the system satisfies the nonlinear observability rank condition and if the initial estimation error, as well as the disturbing noise terms are small enough. The stability properties depend on initial error and process noise, which must be very small values. Although, many applications of EKF prove to be stable and work properly, stability of the general case has shown to be very complex and difficult to be expressed analytically.

Another approach to design an observer, based on local linearization method, is Extended Luenberger Observer (ELO). ELO was used to estimate the concentration profile in order to control the batch distillation column [57].

The objective of this method is to select the observer gain such that, the linearised error dynamics would be asymptotically stable. As same as EKF, there is no global stability for the estimation error dynamics. In [20], the authors showed that the stability of the error dynamics highly depends on value of the eigenvalues, which are chosen for the linearized error dynamics.

2.1.3 Nonlinear Observers

It is necessary to use a nonlinear state estimation method for the nonlinear process since : firstly, most chemical reactors are known to show complex nonlinear behaviors and secondly, several studies have shown that linear estimators are not sufficient for very highly nonlinear processes [67, 10]. Therefore a better state estimation should be based on the nonlinear models which can take into account the complex nonlinear behavior.

The two previous methods work based on the linear version of the model to determine the gain of the estimator. But, there are several kinds of nonlinear observers which are working with the nonlinear model without any local linearization. In the following, some of them will be presented.

Recently, the design of nonlinear observers has been a very active research area. Deza et al. [17] used an exponential observer for a large class of multiple input-multiple output nonlinear system. A simple observer for multiple input-single output nonlinear system with uniform observability was designed. The author proved the robustness of the observer with respect to bounded modelling errors [18]. Hammouri et al. [27] designed a simple nonlinear observer for a bioreactor to estimate the biomass concentration. The application of the nonlinear observer was showed for the nonlinear elastic robot system to estimate the joint position [54].

The design of state observers for general nonlinear systems remains a difficult task. Generally speaking, there are several major approaches to design state observers for nonlinear systems. The first one is to utilize the extension of linear version as a nonlinear observer. Another approach in the literature since 1980s is on the basis of exact linearized error dynamics. These designs have a major drawback that a suitable transformation should be found. However, the existence of such a transformation relies on a set of assumptions which are hard to verify in practice. The model which is used in this approach is

$$\begin{aligned} \dot{x} &= Ax + \gamma(y, u), \\ y &= Cx. \end{aligned} \tag{2.1}$$

Based on this model, the observer and the error dynamics can be written as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \gamma(y, u) + K(y - \hat{y}), \\ \hat{y} &= C\hat{x}, \\ \dot{e} &= (A - KC)e. \end{aligned} \tag{2.2}$$

As can be seen, the error dynamics has a linear structure and it's easy to have global stability. This methodology was applied for nonlinear multi-output systems. Xiao used this observer with some modifications for one class of nonlinear system without any input. Also, the problem of the observer for single-output dynamical systems in the presence of output-dependent time-scaling was studied by Guay [32]. He introduced the alternative algorithm for the solution of the observer linearization problem. He provided a simple procedure for the solution of the observer linearization problem by means of an output dependent time-scale transformation. Alan et al. used the nonlinear error linearizaion observer for the Van der Pol oscillator. They showed that by using this observer, the performance of the estimation was improved in comparison with the extended Luenberger observer [34, 70, 44].

Another approach to design the nonlinear observer is to use an exact linearization structure. The model is used to design the observer described as below

$$\dot{x} = Ax + bu, \tag{2.3}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix},$$

$$u = \alpha(x) + \beta(x)\nu, \tag{2.4}$$

where α and β are scalar functions of the state variables in order to obtain the linear structure. Based on this model, the observer and error dynamics can be written as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + b\hat{u} + K(y - \hat{y}), \\ \hat{y} &= C\hat{x}, \\ \dot{e} &= (A - KC) + b(u - \hat{u}). \end{aligned} \tag{2.5}$$

In order to design the observer using this approach, the relative degree of the nonlinear system should be the same as the order of the system. The stability of the error dynamics depends on its nonlinear part. Therefore, local or global convergence can be achieved. High Gain Observer (HGO) is another nonlinear design methodology, which also has attracted much attention. It is based on the observable canonical form

$$\begin{aligned} \dot{x} &= Ax + \phi(u, x), \\ y &= Cx, \end{aligned} \tag{2.6}$$

$$\phi(x) = \begin{pmatrix} \phi_1(x_1, u) \\ \phi_2(x_1, x_2, u) \\ \vdots \\ \phi_n(x_1, ..., x_n, u) \end{pmatrix}.$$

Based on this model, the observer and error dynamics can be written as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(u, \hat{x}) + \delta\theta K(C\hat{x} - y), \\ \dot{e} &= (A + \delta\theta KC)e + \phi(x, u) - \phi(\hat{x}, u), \end{aligned} \tag{2.7}$$

where δ and θ are tuning parameters in order to control the rate of convergence.

The exponential stability of the error dynamics highly depends on the Lipschitz condition on the nonlinear part. Several articles used this approach to design nonlinear observers.

Biagiola et al. [10] applied HGO with the state feedback controller in order to control the unstable stirred tank reactor.

Another class of systems for the design of the nonlinear observer can be the Lipschitz system described as

$$\begin{aligned} \dot{x} &= Ax + \phi(x, u), \\ y &= Cx. \end{aligned} \tag{2.8}$$

Based on this model, the observer and error dynamics can be written as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(\hat{x}, u) + K(y - \hat{y}), \\ y &= C\hat{x}, \\ \dot{e} &= (A + KC)e + \phi(x, u) - \phi(\hat{x}, u). \end{aligned} \tag{2.9}$$

where ϕ is the Lipschitz nonlinearity with a Lipschitz constant. To guarantee the convergence of the error dynamics, the observer gain should be obtained such that the Lipschitz constant would satisfy the specific condition [68, 58, 43]. In the next chapter, more details about this observer will be presented.

The optimization approach is another way to estimate the process state. Moving Horizon Estimator (MHE) is a famous observer in this kind of approach. The ability to consider constraints distinguishes MHE from other estimation methods. The main drawback of MHE is related to the computational cost since MHE uses nonlinear programming to solve the optimization problem. The basic principle of MHE is to compute estimation of state trajectories using the process model and the initial state vector coming from optimization procedure. Global stability under certain observability condition for a simple case of this approach has been studied in [60, 49]. Sometimes, having a local optimum as well as lack of convergence are two important problems, when we have a large deviation in the initial condition of the estimation from true values. The application of the moving horizon estimator can be seen in high nonlinear batch terpolymerization processes. The author in [1] was able to show a good performance of this observer in presence of measurement noise and up-to ten percent error on the right hand side of the ODE's describing the system's dynamics.

Linear Matrix Inequality (LMI) is another tool to design the observer. The model used to design the estimator is described by the following equations

$$\begin{aligned} \dot{x} &= Ax + \sigma(x, y) + \gamma(u, y), \\ y &= Cx, \end{aligned} \tag{2.10}$$

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \sigma(\hat{x}, y) + \gamma(u, y), \\ \hat{y} &= C\hat{x}, \end{aligned} \tag{2.11}$$

$$\dot{e} = (A + KC)e + \sigma(x, y) - \sigma(\hat{x}, y). \tag{2.12}$$

Moreno [51] used LMI approach to design the nonlinear observer. The main idea is to prove the passivity of nonlinear part of the error dynamics and to satisfy the Kalman-Popov lemma for the linear part by solving an LMI problem. Moreno used this approach to design the observer for some chemical systems such as chemical reactors and simple bioreactor to show the performance in presence of precise model and perfect measurements [62, 50, 52]. To design a LMI observer, it is necessary to transform the model in the above nonlinear structure.

In 2003, Shim et al. used passivity concept for the passivation of the estimation error dynamics directly for the more general class of nonlinear system [65]. They used the concept of output passivation to obtain the observer by using some restricted assumptions. The structure of the model as well as observer based on their approach are given by

$$\begin{aligned} \dot{x_1} &= f_1(x_1, x_2, u), \\ \dot{x_2} &= f_2(x_1, x_2, u), \\ y &= x_1, \end{aligned} \tag{2.13}$$

$$\begin{aligned} \dot{\hat{x_1}} &= f_1(\hat{x_1}, \hat{x_2}, u) + k_1(y - \hat{y}), \\ \dot{\hat{x_2}} &= f_2(\hat{x_1}, \hat{x_2}, u) + k_2(y - \hat{y}), \\ \hat{y} &= \hat{x_1}, \end{aligned} \tag{2.14}$$

$$\begin{aligned} \dot{e_1} &= F_1(\hat{x}, e) + k_1(y - \hat{y}), \\ \dot{e_2} &= F_2(\hat{x}, e) + k_2(y - \hat{y}). \end{aligned} \tag{2.15}$$

They considered the gain of observer as a constant value multiplied by a nonlinear function. They could prove the global stability of the error dynamics. However, they put zero for the constant part of the gain for some state variables. This hypothesis means they ignored the correction term for some state estimation variables in order to satisfy their assumptions (nonlinearity growth conditions related to output feedback passivation conditions [12, 66]).

The second class of the state estimator is the asymptotic observer. They are open-loop state estimators, which make use of some parts of the process model, and substitute the knowledge about the missing part by some available measurements [11]. This kind of observer can only be applied to bioprocess models. The structure of the model to design the observer is

$$\begin{aligned} &\dot{x} = K\phi(x) + (x_{in} - x)D, \\ &\dot{x} = (k_1, k_2)^T\phi(x) + D((x_{1in}, x_{2in}) - (x_1, x_2)). \end{aligned} \tag{2.16}$$

where x_1 are the measurable variables and x_2 are the estimated variables. The main advantage of the asymptotic observer method is possibility to make a state estimator without any knowledge about the reaction kinetics. The main drawback of this technique is the rate of convergence of the state estimator determined by operating conditions. The necessary condition is that the number of measurements should be equal to the number of unknown kinetic reactions. Therefore, the speed of convergence only depends on the experimental conditions. So, there is no guarantee for convergence in batch reactors. In the following table, the summary of above discussion is presented.

Tableau 2.1: Summary-Critical Analysis

Ref.No	**Observer**	**No of tuning parameters**	**Kind of stability**
1	EKF	Two (R,Q)	Local stability
2	ELO	Number of poles (order of system)	Local stability
3	Error linearization	Depends on the linear technique	Global
4	Exact linearization method	Depends on the linear technique	Local or Global
5	High gain observer	Number of poles	Local or Global
6	MHE	One	Asymptotic
7	LMI	Two	Global stability
8	Passivity-based	Number of state	Global
9	Asymptotic observer	Dilution rate	Asymptotically

2.2 Parameter Estimation

The necessity to achieve an accurate estimation of the important unknown process parameters has been emphasized in Section 2.1.1. The accurate estimation of the unknown parameters of a process can be computed using available measurements and a parameter estimation procedure. The general problem of the parameter estimation is to fit a model to a set of measurements. In the off-line parameter estimation, a model is fitted to the process measurements from one or several process simulation runs [25]. But, in the on-line parameter estimation, a model is fitted optimally to the past and present process measurements until the process is in the operating state [3].
In the following paragraphs, available methods of on-line parameters estimation are presented.

1- **Parameter estimation by state estimation**

In this method, there is no dynamic model for each of the unknown parameters to be estimated (zero dynamics). A state estimator, such as EKF or reduce-order observer [67] is used to estimate the process parameters which appear as a subset of the state vector of the combined process and parameter models. This method has been widely used in chemical and biochemical engineering [24, 53].

2- **Parameter estimation by on-line optimization**

This approach converts the parameter estimation problem to the minimization problem. Parameter values are obtained by solving the on-line minimization problem such as the sum of square errors [59]. This technique is difficult to apply and also computationally is expensive. But, it can handle constraints in estimation.

3- **Gradient method**

The standard least squares estimator and the least squares estimator with exponential forgetting factors are the most common methods for these techniques.

All of the above methods are applicable for a limited class of nonlinear systems or linear processes. Also, some of them have suffered from lack of the proof of convergence of the parameters.

2.3 Nonlinear Control Strategies

The area of biotechnology is characterized by rapid changes in terms of innovation and by complicated process which need advanced methods for design, operate and control. From engineering point of view, the control of bioprocess has a number of challenging problems. The problems come from the presence of living organism, the complexity of interaction between the micro-organisms. In addition, as mentioned in the previous section, for monitoring and control purpose, only a few measurements are available and measuring devices do not give reliable measurement. Also, they are too expensive. Generally speaking, the main difficulties in the control of bioprocess come from two challenging problems :the process complexity and difficulty to have precise measurement of bioprocess variables [63].

In order to figure out the proper solution for these difficulties, several control strategies for control of bioprocess were developed such as optimization based approach [4], adaptive approach [47, 45, 15], sliding mode control [64], exact linearization approach [5], and backstepping approach [21].

2.3.1 Feedback Linearization Technique

Feedback linearization is a technique for nonlinear controller design which is interesting subject in recent years. The main idea of this approach is to use an algebraic transformation to transform a nonlinear system dynamic into a linear one. Feedback linearization has been widely used for some practical control problems [42, 23, 33, 31, 26]. In the following paragraphs, basic definitions and concept related to this algorithm will be presented [37].

It can be assumed that the nonlinear system has the following equation

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x). \end{aligned} \tag{2.17}$$

Definition 2.3.1 *If f and h are considered as a vector field and a scalar field respectively, Lie derivative will be defined as*

$$L_f h = \sum f \frac{\partial h}{\partial x} = < \partial h, f > . \tag{2.18}$$

The result of Lie derivative is a scalar field. The following items refer to properties of Lie derivative, where t is assumed as a scalar vector.

1- $L_f(t + h) = L_f h + L_f t,$

2- $L_f(ht) = hL_f t + tL_f h,$

3- $L_{f+g} = L_f h + L_g h,$

4- $[f, g](x) = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g,$

5- For three vectors field f_1, f_2, f_3, Jacobi's identity is defined as

$$[[f_1, f_2], f_3] + [[f_2, f_3], f_1] + [[f_3, f_1], f_2] = 0,$$

6- Higher order of Lie derivative : $L_f^2 h = L_f L_f h =< dL_f h, h > .$

Definition 2.3.2 *Single input - single output system will have relative order r if $L_g L_f^{r-1} h$ is non-zero. In fact, the relative order is the number of derivative with respect to y in order to appear u in the equation of y derivative.*

Definition 2.3.3 *The linear system is said to be minimum phase if inverse of the system is stable. On the other word, all zeros of the system should be negative.*

Definition 2.3.3 is not valid for nonlinear system, because, they do not have any pole and zero. In the following passage, after some definitions and theorems, the definition of minimum phase for nonlinear system will be expressed.

Definition 2.3.4 *The nonlinear system is said to be input affine if it has linear relationship with respect to input.*

Theorem 2.3.1 *(Hirschorn Inversion) It can be possible to write the inverse of the nonlinear system as the following form*

$$\begin{aligned}\dot{z} &= f(z) + g(z)(\frac{d^r y}{dt^r} - L_f^r h(z))/(L_g L_f^{r-1} h(z)),\\ u &= (\frac{d^r y}{dt^r} - L_f^r h(z))/(L_g L_f^{r-1} h(z)).\end{aligned} \tag{2.19}$$

Definition 2.3.5 *The following dynamic system with scalar vectors* $(F_1, F_2, ...$ $F_{n-r}, \Phi, G)$ *is a normal form for System (2.17) with relative degree* r*.*

$$\begin{aligned}\dot{x}_1 &= F_1(x),\\ \dot{x}_2 &= F_2(x),\\ &\vdots\\ \dot{x}_{n-r} &= F_{n-r}(x),\\ &\vdots\\ \dot{x}_n &= \Phi(x) + G(x)u,\\ y &= x_{n-r+1}.\end{aligned} \tag{2.20}$$

Theorem 2.3.2 *By using a transformation vector field with the following structure, it is possible to transfer the nonlinear System (2.17) to nonlinear System (2.20)*

$$\eta = T(x) = \begin{pmatrix} t_1(x) \\ \vdots \\ t_{n-r}(x) \\ \vdots \\ L_f^{r-1} h \end{pmatrix}, \tag{2.21}$$

$$\begin{aligned}F_i(\eta) &= [< dt_i(x), f(x) >]_{x=T^{-1}(\eta)},\\ \Phi(\eta) &= [L_f^r h(x)]_{x=T^{-1}(\eta)},\\ G(\eta) &= [L_g L_f^{r-1} h(x)]_{x=T^{-1}(\eta)}.\end{aligned} \tag{2.22}$$

Therefore, using Theorem 2.3.2, and Definition 2.3.5, inverse of dynamic System (2.17) can be obtained as

$$
\begin{aligned}
\dot{z}_1 &= f_1(z), \\
&\vdots \\
\dot{z}_r &= f_{n-r}, \\
&\vdots \\
\dot{z}_n &= \frac{d^r y}{dt^r}, \\
\\
u &= (\frac{d^r y}{dt^r} - \Phi(z))/G(z).
\end{aligned} \tag{2.23}
$$

Zero dynamics : If dynamic System (2.23) is considered as an inverse of minimum realization of the nonlinear System (2.17) then (y and it's derivatives), and $(z_1, ..., z_{n-r})$ can be considered as inputs and states for inverse of the system respectively.

Definition 2.3.6 *Forced zero dynamic is defined as*

$$
\begin{aligned}
\dot{z}_1 &= F_1(z_1, ..., z_{n-r}, U_1, ..., U_r), \\
&\vdots \\
\dot{z}_{n-r} &= F_{n-r}(z_1, ..., z_{n-r}, U_1, ..., U_r).
\end{aligned} \tag{2.24}
$$

Definition 2.3.7 *The nonlinear transformed system with stable point equal to zero has the following unforced zero dynamics*

$$
\begin{aligned}
\dot{z}_1 &= F_1(z_1, ..., z_{n-r}, 0, 0, .., 0), \\
&\vdots \\
\dot{z}_{n-r} &= F_{n-r}(z_1, ..., z_{n-r}, 0, 0, ..., 0).
\end{aligned} \tag{2.25}
$$

Unforced zero dynamic can divide the nonlinear systems in two main parts, minimum phase and non-minimum phase.

Definition 2.3.8 *Nonlinear system will be minimum phase if it's zero dynamic is stable, otherwise, it is non-minimum phase.*

Theorem 2.3.3 *The following control law can exactly linearize the system between external input ν and output y*

$$
u = \frac{\nu - L_f^r h - \beta_1 L_f^{r-1} h ... - \beta_r h}{L_g L_f^{r-1} h}, \tag{2.26}
$$

where

$$
L_f h = \sum f_i \frac{\partial h}{\partial x_i}.
$$

After linearization, the input-output behaviour of the system can be written as

$$\frac{d^r y}{dt^r} + \beta_1 \frac{d^{r-1} y}{dt^{r-1}} + ... + \beta_{r-1} \frac{dy}{dt} + \beta_1 y = \nu. \tag{2.27}$$

There are two necessary conditions in order to use Theorem 2.3.3
1- the nonlinear system should be minimum phase,
2- the system should be input affine.

2.3.2 Backstepping Technique

Another advanced nonlinear control strategy is backstepping technique. Backstepping is a recursive methodology to obtain the feedback control law as well as associated Lyapunov function in systematic manner. Feedback linearization techniques cancel all the nonlinearities in the system by using specific static nonlinearity, based on the model of the system. But, in the backstepping technique, the design is more flexible because of choosing the nonlinear damping terms. Therefore, additional robustness is obtained. This is important in industrial control systems, because the cancellation of all nonlinearities require precise model which is difficult to obtain.

Krstic and et al. published the first book describing the backstepping methodology [38]. Recently, the application of backstepping methodology for designing nonlinear controller has been grown. This technique provides a powerful design tool for nonlinear system in the pure feedback and strict feedback form [35]. Robustness is one of the advantage of using backstepping nonlinear control scheme, which cannot be obtained by traditional controller scheme. This technique has been applied for temperature control of CSTR [71]. Another example of backstepping controller is the scheme proposed by Gopaluni et al. [30]. They applied this technique to design a robust adaptive nonlinear controller for a benchmark CSTR in order to control the product concentration. Also, adaptive nonlinear backstepping controller has been applied for a nonminimum phase CSTR in order to control the concentration inside the reactor [69]. Also, the application of backstepping for biological system has grown [21].

In this thesis, a backstepping controller to control the biomass and the substrate concentration for microalgae cultivation is designed. The detailed procedure to design the controller based on the model of process will be presented in the next chapter.

CHAPITRE 3

Observer-Control Development and Results

In this chapter, computer simulations related to observer and controller design based on the Droop model are presented. At first, nonlinear continuous and discrete observer are presented. Then, parameter estimation in the Droop model is described. After that, the observer performance in the presence of model based controllers is illustrated. Finally, the observer design strategy for estimation of the state variables based on a model dedicated to lipid production is described, and some simulation results related to the strategy are presented.

3.1 Representation of the Droop Model

By adding and subtracting extra terms to the Droop model, it is possible to obtain the nonlinear model, combination of a linear and a nonlinear part, as below

$$\begin{aligned} \dot{X} &= X(\bar{\mu} - D) + \bar{\mu}Q + (-\bar{\mu}Q - \tfrac{\bar{\mu}K_Q X}{Q}), \\ \dot{Q} &= -\bar{\mu}Q - \rho_m S + (K_Q\bar{\mu} + S\rho_m + \rho), \\ \dot{S} &= -\rho_m X - DS + (DS_{in} + X(\rho_m - \rho)), \\ y &= Cx. \end{aligned} \tag{3.1}$$

Then, by defining matrix $A(D)$ for the linear part, the following representation is obtained

$$\begin{aligned} \dot{x} &= A(D)x + \psi(x), \\ y &= Cx, \end{aligned} \tag{3.2}$$

where $C = (1, 0, 0)$ and

$$A(D) = \begin{pmatrix} \bar{\mu} - D & \bar{\mu} & 0 \\ 0 & -\bar{\mu} & -\rho_m \\ -\rho_m & 0 & -D \end{pmatrix}, \psi(x) = \begin{pmatrix} -\bar{\mu}Q - \frac{\bar{\mu}K_Q X}{Q} \\ K_Q\bar{\mu} + S\rho_m + \rho \\ DS_{in} + X(\rho_m - \rho) \end{pmatrix}.$$

3.1.1 Steady State Behaviour

In order to study the steady state behaviour of the process, three nonlinear equations are solved. The equilibrium points would be

$$Q_s = \frac{K_Q \bar{\mu}}{\bar{\mu} - D}, \ \rho(S_s) = DQ_s,$$

$$S_s = \frac{K_s \rho(S_s)}{\rho_m - \rho(S_s)}, \ X_s = \frac{D(S - S_s)}{\rho(S_s)}.$$

In the following figures, the steady state behaviour of the state variables are shown. As can be seen, the biomass concentration has a linear behaviour with respect to dilution rate but internal quota and substrate concentration have nonlinear behaviour.

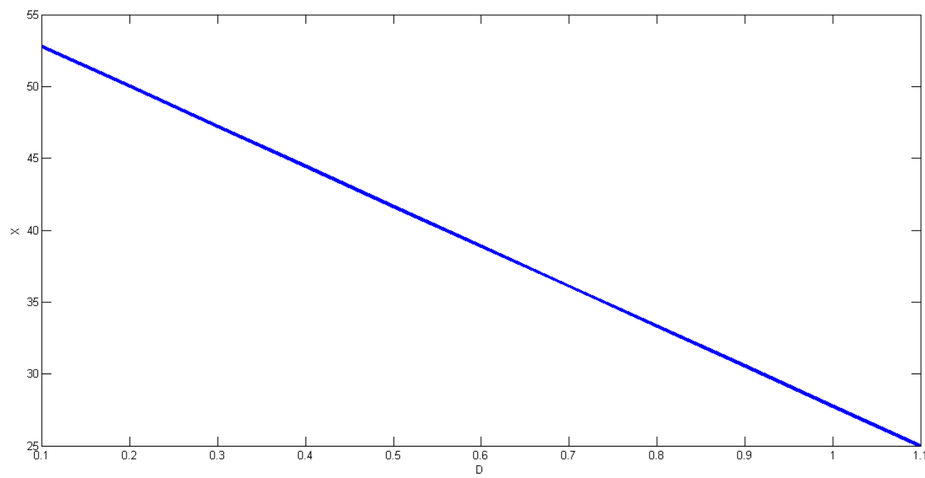

Figure 3.1: Steady state behaviour X-D.

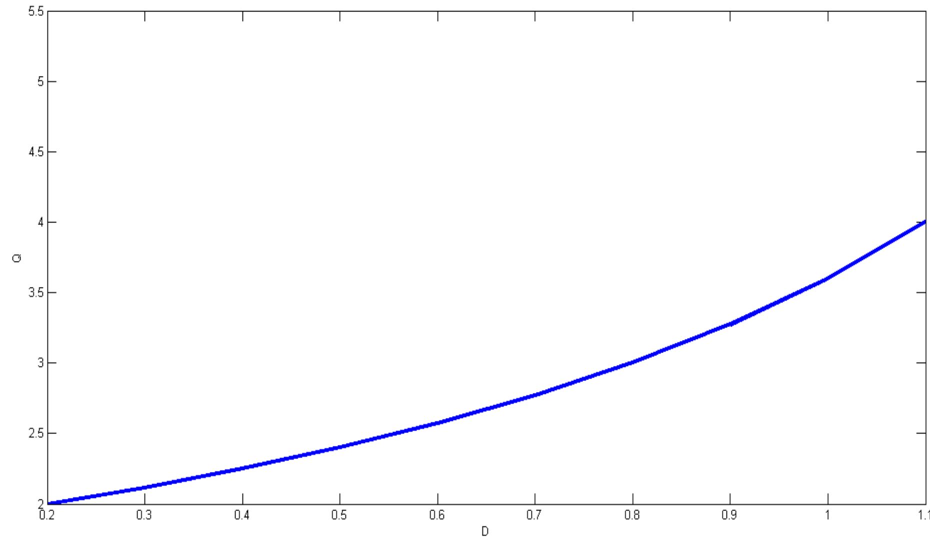

Figure 3.2: Steady state behaviour Q-D.

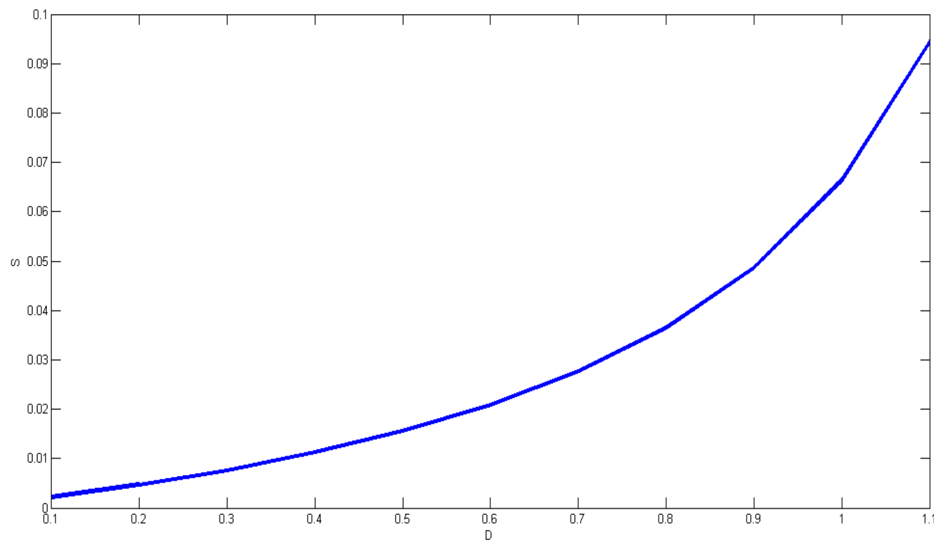

Figure 3.3: Steady state behaviour S-D

3.2 Lipschitz Observer

As can be seen, the nonlinear part of the new representation of the Droop model with respect to the state variables is continuous and differentiable. Also, based on the first property of the Droop model, the nonlinear part is bounded. Therefore, it has Lipschitz property. Therefore, Lipschitz observer can be a good candidate for this process. Based on Equation (3.2), the observer equation would be

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \psi(\hat{x}) + K(y - \hat{y}), \\ \hat{y} &= C\hat{x}. \end{aligned} \tag{3.3}$$

The dynamics of the estimation error $e = x - \hat{x}$ would be

$$\begin{aligned} \dot{e} &= (A - KC)e + (\psi(x) - \psi(\hat{x})), \\ y_d &= Ce. \end{aligned} \tag{3.4}$$

where K refers to the observer gain and it can be determined by using some techniques such as pole placement.

3.2.1 Stability Analysis of the Error Dynamics

If γ is considered a Lipschitz constant for nonlinear part of the model, based on the Lipschitz property, the following inequality should be satisfied

$$\mid \psi(x) - \psi(\hat{x}) \mid \leq \gamma \mid x - \hat{x} \mid . \tag{3.5}$$

The following Lyapunov function is considered for the error dynamics

$$V = e^T Pe, \qquad P > 0.$$

The time derivative of Lyapunov function would be

$$\begin{aligned} \dot{V} &= \dot{e^T} Pe + e^T P\dot{e}, \\ \dot{V} &= [(A - KC)e + \psi(x) - \psi(\hat{x})]Pe + e^T P[(A - KC)e + \psi(x) - \psi(\hat{x})], \\ \dot{V} &= e^T (A - KC)^T Pe + (\psi(x) - \psi(\hat{x}))^T Pe + e^T P(A - KC)e + e^T P(\psi(x) - \psi(\hat{x})), \\ \dot{V} &= e^T ((A - KC)^T P + P(A - KC))e + 2e^T P(\psi(x) - \psi(\hat{x})). \end{aligned}$$

Based on the Lipschitz property of the nonlinear part, it can be written

$$\begin{aligned} 2e^T P(\psi(x) - \psi(\hat{x})) &\leq 2 \parallel Pe \parallel\parallel \psi(x) - \psi(\hat{x}) \parallel \\ &\leq 2 \parallel Pe \parallel\parallel e \parallel . \end{aligned} \tag{3.6}$$

Using the following mathematics inequality, the derivative of Lyapunov function would be

$$2 \parallel Pe \parallel\parallel e \parallel \gamma \leq \gamma^2 e^T PPe + e^T e,$$

$$\begin{aligned} \dot{V} &\leq e^T((A - KC)^T + (A - KC)e + \gamma^2 e^T PPe + e^T Pe) \\ &= e^T((A - KC)^T P + P(A - KC) + \gamma^2 PP + I)e. \end{aligned} \tag{3.7}$$

So, if the inequality (3.8) is satisfied, the error dynamics would be stable.

$$(A - KC)^T P + P(A - KC) + \gamma^2 PP + I < 0. \tag{3.8}$$

The left hand side of the inequality is known as Riccati equation.

Remark 3.2.1 *As can be seen, matrix A in the Droop model is a function of the dilution rate ($A(D)$). So, Equation (3.8) should be solved for each value of the dilution rate.*

In the following theorem , the existence of the solution for Riccati equation is presented.

Theorem 3.2.1 *[72] Consider following Riccati equation*

$$A^T P + PA + PRP + Q = 0 \tag{3.9}$$

suppose that $P \geq 0$ is a solution for Algebraic Riccati Equation (ARE). It is necessary that the following relations are true

$$\lambda_{min}(R)tr(Q) - n\lambda^2(S) < 0, \tag{3.10}$$

$$\lambda(S) < 0,\ S = \frac{(A + A^T)}{2}. \tag{3.11}$$

Lemma 3.2.1 *If A is Hurwitz, then $\lambda_{min}(S) < 0$.*

For our problem, R and Q are equal to identity matrix, and $\tilde{A} = A(D) - KC$ is Hurwitz. So, Equation (3.11) could be satisfied. Equation (3.10) can be written in the following inequality

$$1 < \lambda_{min}^2(S).$$

However, λ_{min} is a function of the dilution rate. Therefore, in order to satisfy Equation (3.10), trial and error is used.

In the following section, the performance of the designed observer with different scenarios is demonstrated.

3.2.2 Observer Design with Perfect Measurement

In this section, it is assumed that the biomass is measured without any uncertainty. Two cases are studied for the designed observer

1- Observer with a constant dilution rate.

2- Observer with variable dilution rate.

1- Observer with a constant dilution rate

At first, it is assumed that the dilution rate has a constant value (D=1.3). The following eigenvalues are considered as closed-loop eigenvalues for the linear part of the error dynamics. By using pole placement technique, the observer gain is computed. After that, by solving the ARE, the stability analysis of the error dynamics with respect to selected eigenvalues is verified.

$$\lambda_{closed-loop} = (-7, -6, -4),\ K = (10, 12, -9.3),\ \text{and}\ \gamma = 19.$$

The performance of observer is illustrated in the following figures.

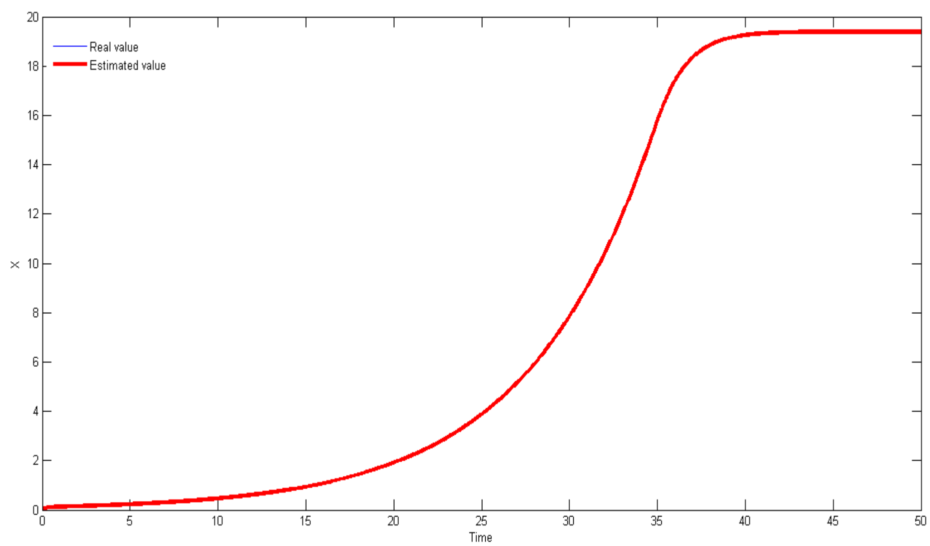

Figure 3.4: Estimation of biomass-Constant D.

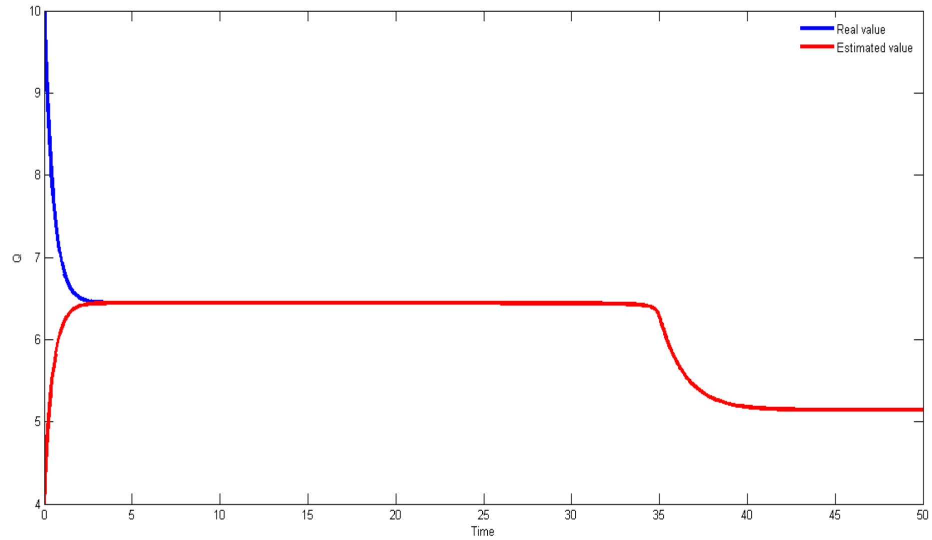

Figure 3.5: Estimation of internal quota-Constant D.

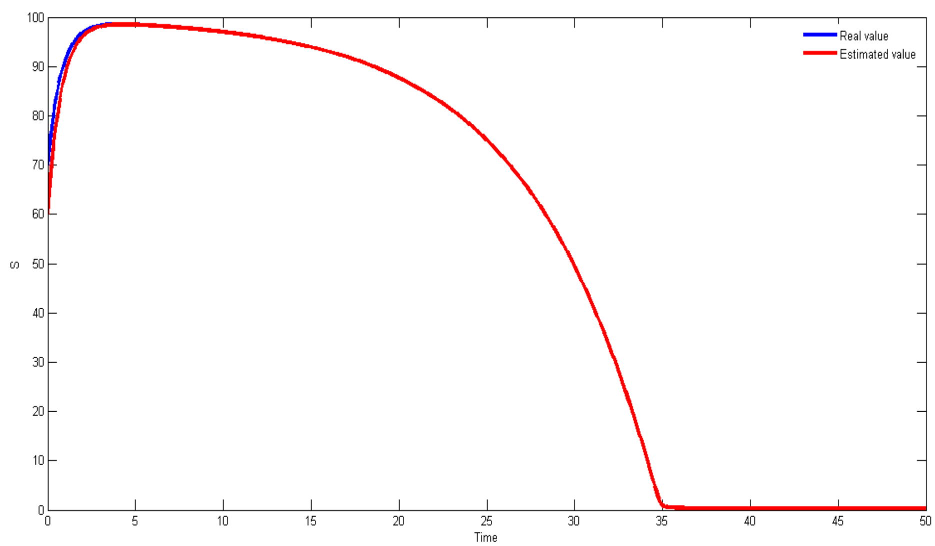

Figure 3.6: Estimation of substrate-Constant D

As can be seen, the designed observer has a good performance in estimation of the state variables of the process. Also, we can conclude that when the concentration of substrate goes to zero, the concentration of internal quota decreases.

2- Observer with variable dilution rate

In order to see the performance of observer in presence of different values of the dilution rate, D changes during the operating time (from 1.3 to 1.1 at time 25). As it can be seen, the designed observer can follow properly the process dynamics.

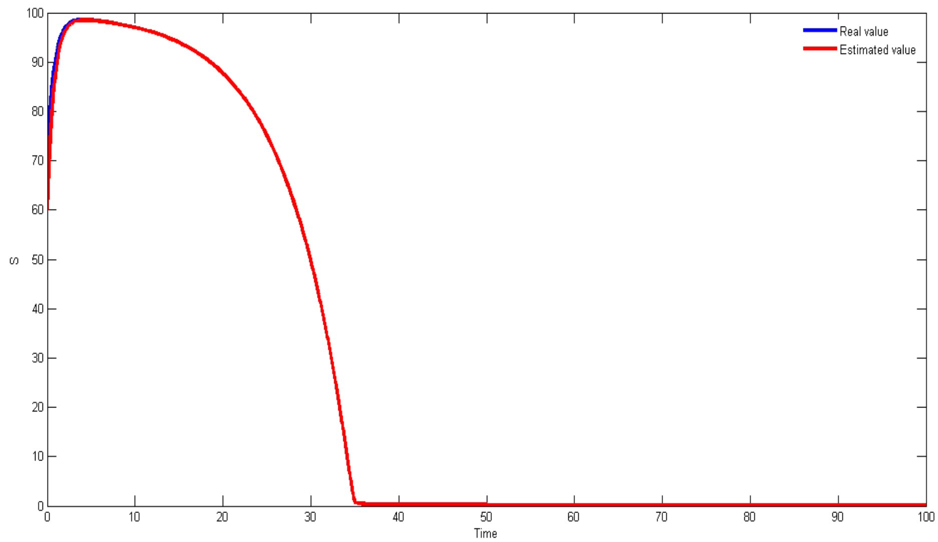

Figure 3.7: Estimation of substrate-Variable D.

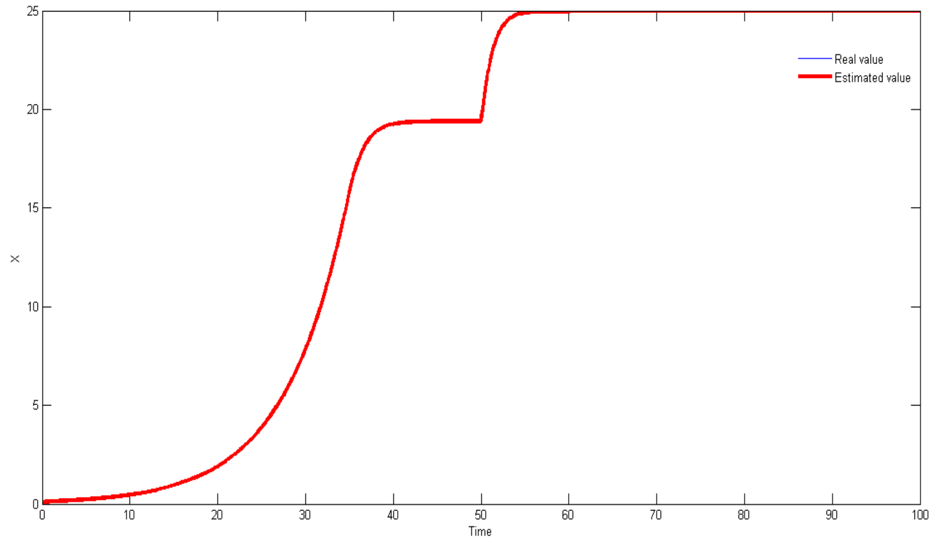

Figure 3.8: Estimation of biomass-Variable D.

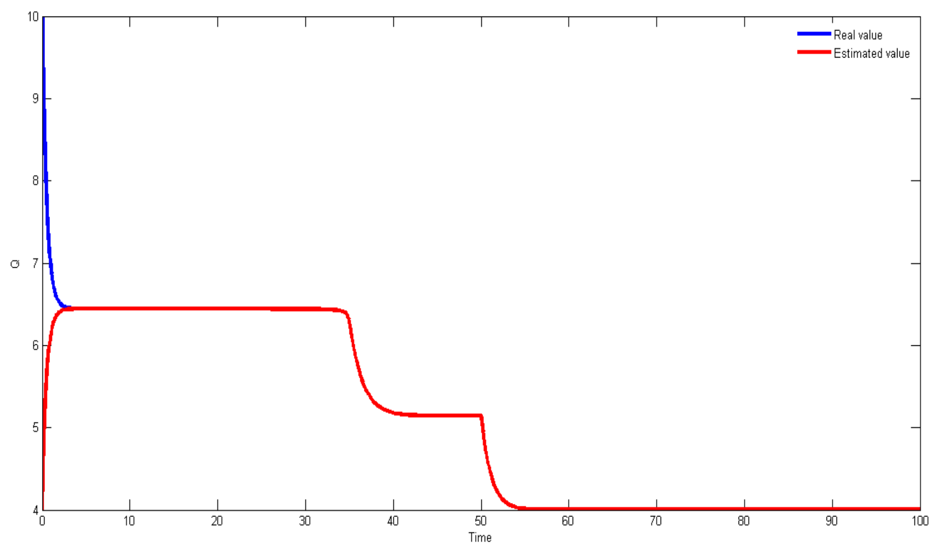

Figure 3.9: Estimation of internal quota-Variable D

3.2.3 Observer Design with Measurement Noise

In this part, it is assumed that there is uncertainty on the measurement variable and bounded measurement noise is considered for the biomass concentration. The nonlinear control theory related to stability of forced dynamic system and it's application are studied at the following sections

1- Stability analysis of forced nonlinear systems.

2- Stability analysis of the error dynamics.

The nonlinear system can be considered as follows

$$\begin{aligned} \dot{x} &= Ax + \psi(x), \\ y &= Cx + \vartheta. \end{aligned} \tag{3.12}$$

The observer dynamics would be

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \psi(\hat{x}) + K(y - \hat{y}), \\ \hat{y} &= C\hat{x}. \end{aligned} \tag{3.13}$$

Therefore, the error dynamics would be

$$\begin{aligned} \dot{e} &= (A - KC)e + (\psi(x) - \psi(\hat{x})) + K\vartheta, \\ y_d &= Ce + \vartheta. \end{aligned} \tag{3.14}$$

Two assumptions are considered for stability analysis of error dynamics

1- The maximum absolute value of the measurement noise is known.

2- The initial value for the error dynamics is known.

1- Stability analysis of forced nonlinear systems [35]

Let us consider the nonlinear system $\dot{x} = f(t, x, u)$ and we assume that $\dot{x} = f(t, x, 0)$ has a uniformly asymptotically stable equilibrium point at the origin $x = 0$.

Definition 3.2.1 *The forced nonlinear system is said to be locally input-to-state stable if there exit a class K_L function β, a class K function γ and positive constant k_1 and k_2 such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any input $u(t)$ with $sup\|u\| < k_2$, the solution $x(t)$ exists and satisfies*

$$\|x\| \leq \beta(\|x_0\|, t - t_0) + \gamma(sup\|u\|).$$

2- Stability analysis of the error dynamics

Based on the previous definition, the following functions are considered in order to satisfy the stability conditions.

$$\gamma(x) = \frac{\lambda_{max}(P)\|x\|}{\lambda_{min}(P)\theta}, \qquad \beta = \|e_0(t_0)\|exp(-t).$$

The initial condition and parameters for stability analysis are chosen as follows

$$(\lambda_{max}(P), \lambda_{min}(P)) = (0.081, 0.045),$$

$$e(0) = (-0.100, 6.00, 50.00),\ \ \theta = 0.8.$$

As can be seen in the Figure 3.12, the magnitude of the real error is less than the magnitude of the error which comes from the theory. Also, from Figures 3.10-3.11, it can be concluded that the observer has an acceptable performance in order to estimation the state variables.

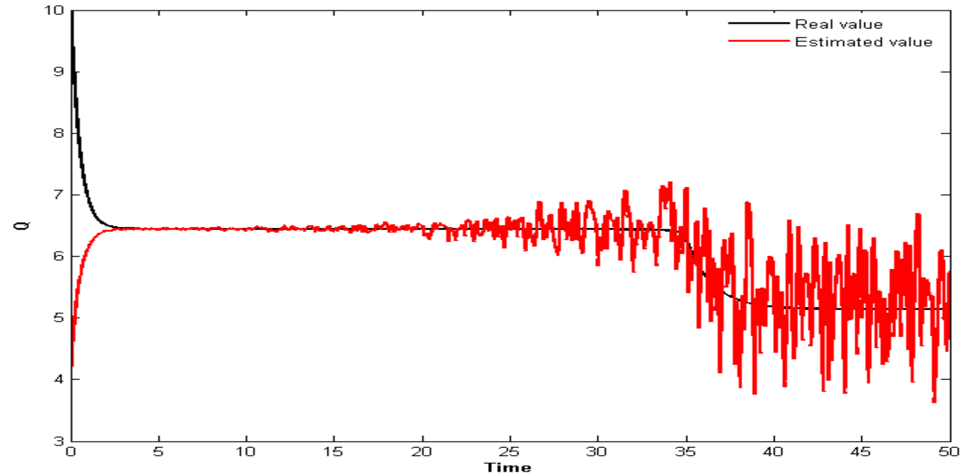

Figure 3.10: Estimation of quota-Measurement noise.

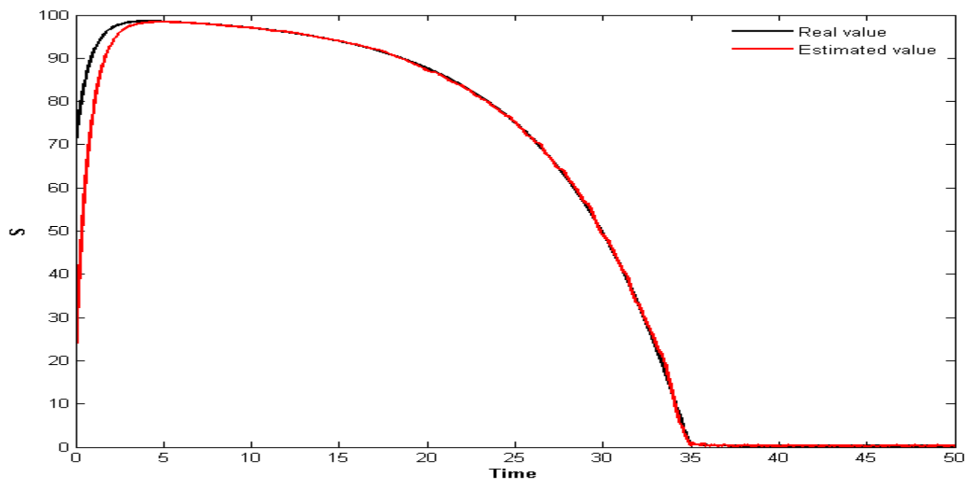

Figure 3.11: Estimation of substrate-Measurement noise.

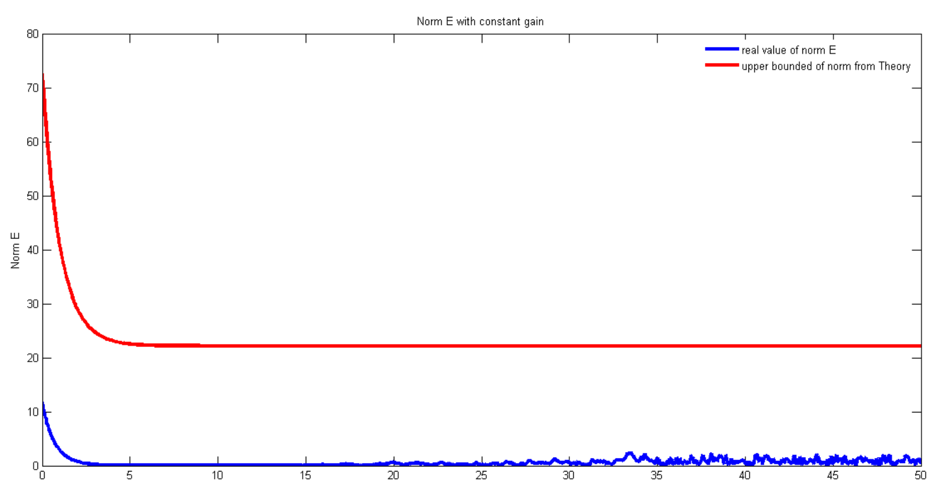

Figure 3.12: Norm of error.

Also, it is important to notice when the magnitude of the measurement noise increases, the observer performance decreases. In order to improve the results, a variable gain is suggested instead of a constant gain (switching approach).

3.2.4 Switching Approach

In this approach, the observer gain is a function of time. To compute the gain of observer, the eigenvalues of the closed-loop matrix $(A - KC)$ is changing with an exponential pattern as follows

$$\lambda_i(t) = -\alpha_i \exp(-b_i t) + \epsilon_i. \tag{3.15}$$

So, in this strategy, fast convergence will be expected at the beginning of the estimation (High gain estimator). Eigenvalue's parameters for switching approach are chosen as follow

$$(\alpha_1, \alpha_2, \alpha_2) = (-8.00, -6.00, -4.00),$$

$$(b_1, b_2, b_3) = (-0.07, -0.05, -0.04), \text{ and}$$

$$(\epsilon_1, \epsilon_2, \epsilon_3) = (-0.6, -0.8, -0.9).$$

In the following figures, the observer performance with the new approach is illustrated.

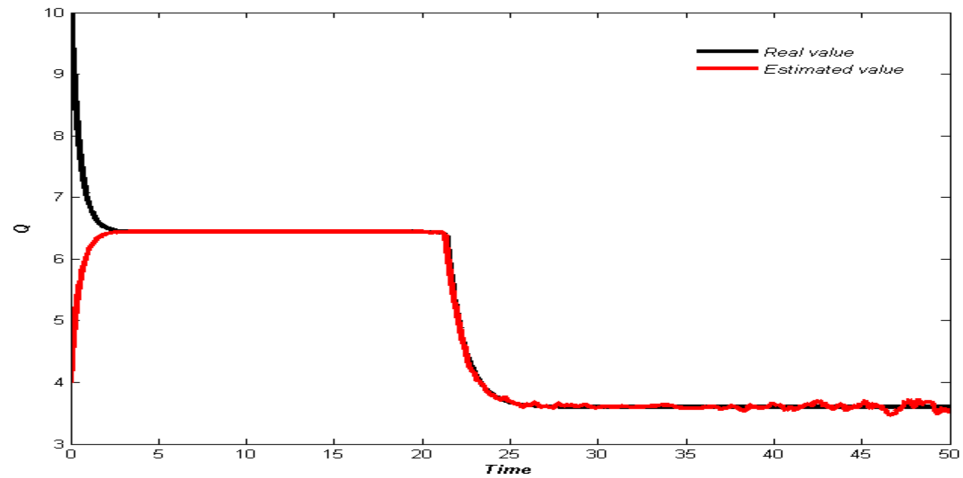

Figure 3.13: Estimation of quota-Switching approach.

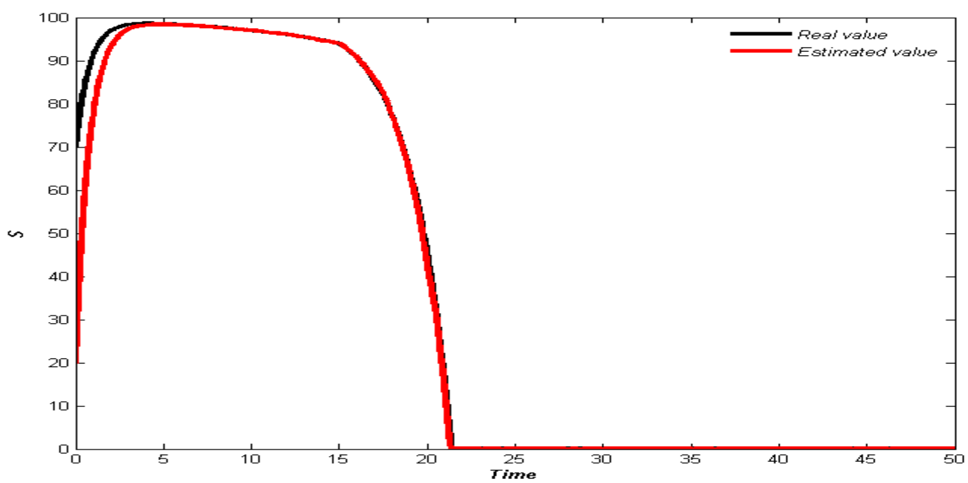

Figure 3.14: Estimation of substrate-Switching approach.

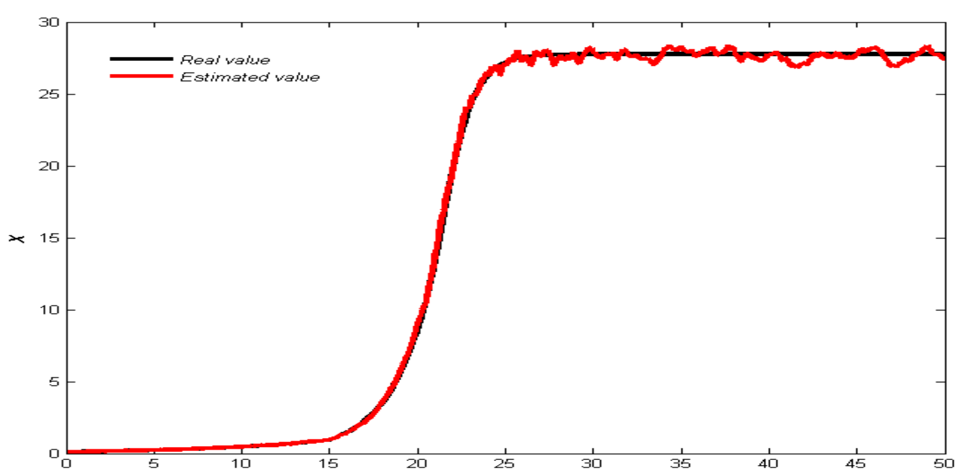

Figure 3.15: Estimation of biomass-Switching approach.

As it can be concluded, the observer performance using this approach is much better than the previous case with a constant gain because at the beginning of operation , the gain of observer has large value. Therefore, the rate of convergence is high but at the steady state operating condition with high amount of biomass, the magnitude of the observer gain decreases.

3.2.5 Performance Analysis

In order to evaluate the performance of the proposed observer, different scenarios are studied as follow

1- Dynamic behaviour of observe.

2- Observer performance in presence of disturbance.

3- Validity of the proposed observer for a wide rang of model's parameter.

4- Observer performance in presence of process noise.

First, a PI controller is coupled with designed observer in order to see how the observer can follow the process dynamics. Then, the performance of observer in presence of process noise and variations in model's parameters is studied.

1- Dynamic behaviour of observer

In order to evaluate the dynamic behaviour of the designed observer, a PI controller is coupled with the observer. To design the PI controller, we linearized the nonlinear model around the operating point as follows

$$G(s) = \frac{K_p(s+a_1)(s+a_2)}{(s+b_1)(s+b_2)(s+b_3)}. \tag{3.16}$$

Equation 3.16 can be simplified as following model, as it can be seen in Figures 3.16-3.17, via the zero-pole cancelation of the linearized model.

$$G(s) = \frac{K_p}{s+\alpha} \tag{3.17}$$

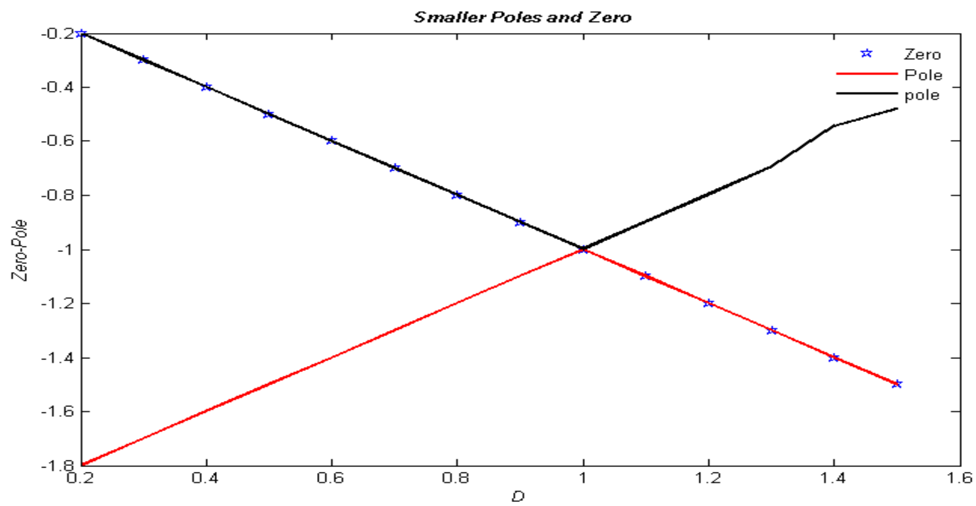

Figure 3.16: Pole-Zero.

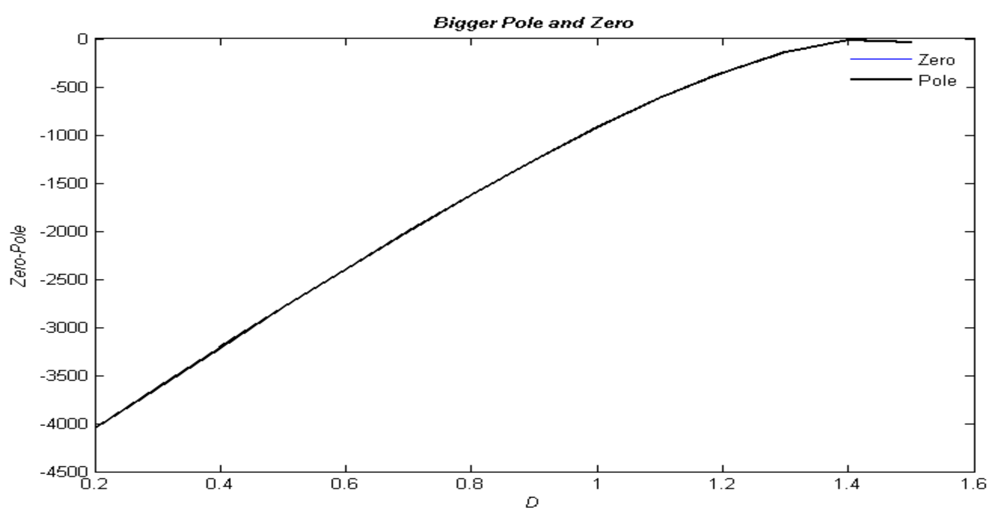

Figure 3.17: Pole-Zero.

PI Controller's parameters are obtained as follow

$$\tau_I = 0.693,\ K_C = -0.037,\ D = 1.300,\ \text{and}\ K_p = -27.970.$$

In Figures 3.18-3.20, the dynamic behaviour of the observer in presence of measurement noise is shown. As it can be concluded, the estimator can properly follow the dynamic of the process.

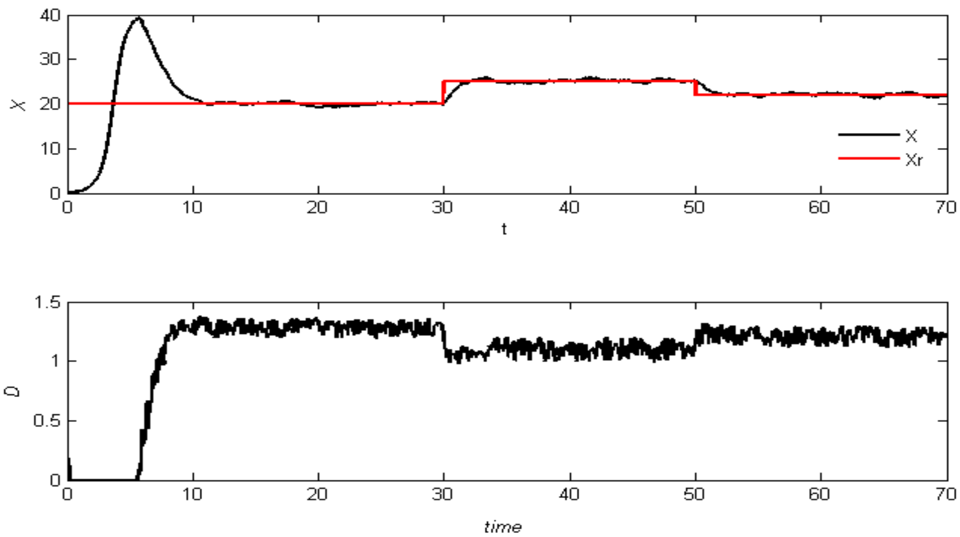

Figure 3.18: Biomass-Dilution rate.

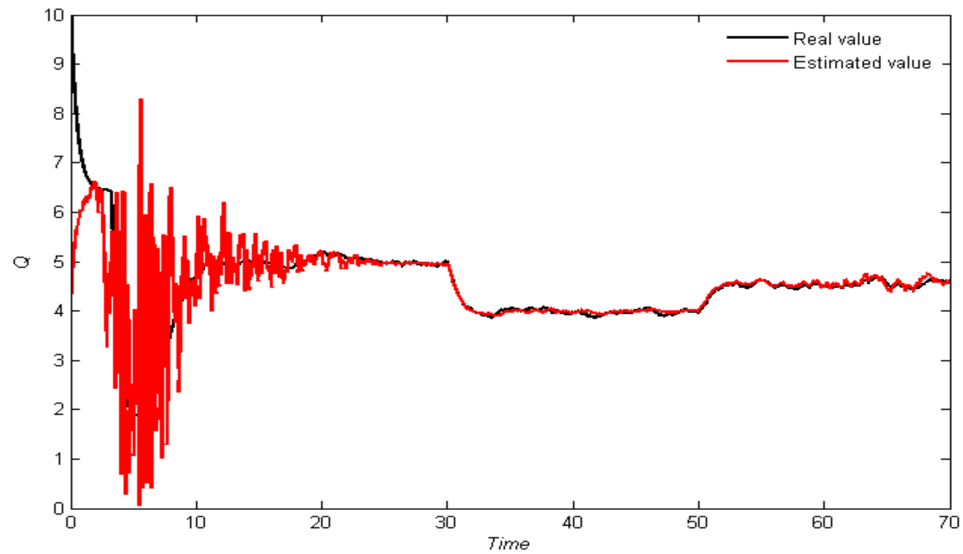

Figure 3.19: Estimation of internal quota.

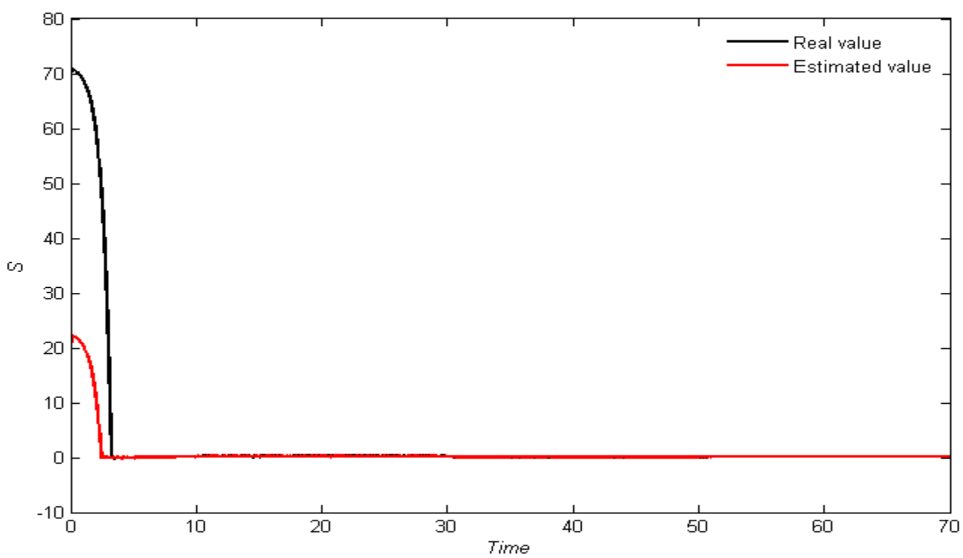

Figure 3.20: Estimation of substrate.

2- Observer performance in presence of disturbance

In order to see the performance of the observer in presence of disturbance, the substrate feed concentration is considered as an unmeasurable disturbance. Ten percent increase and decrease are assumed for this variable. The simulation results show that the observer has good performance in presence of the bounded disturbance.

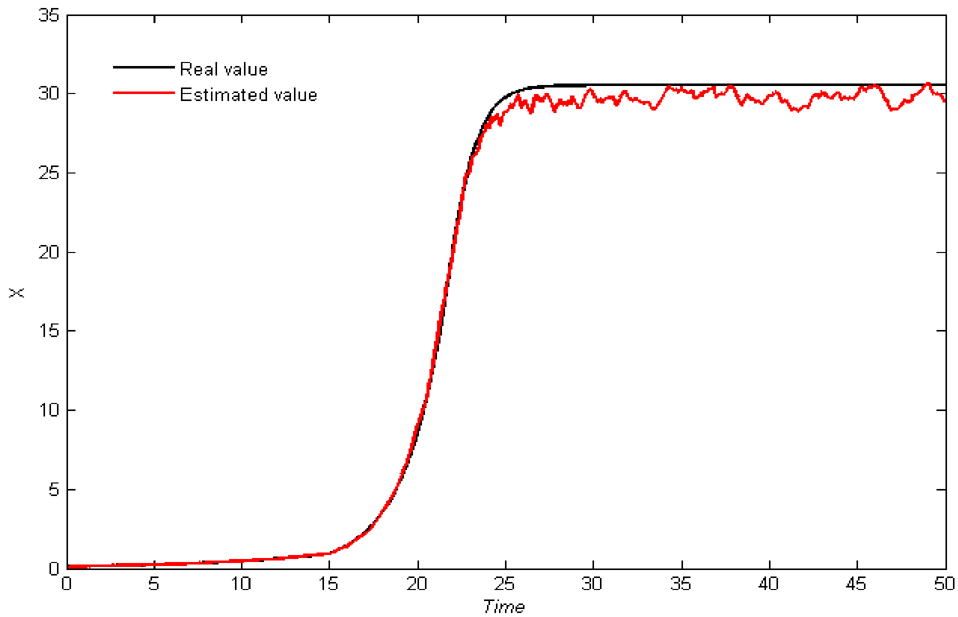

Figure 3.21: Estimation of biomass -Presence of disturbance (%10).

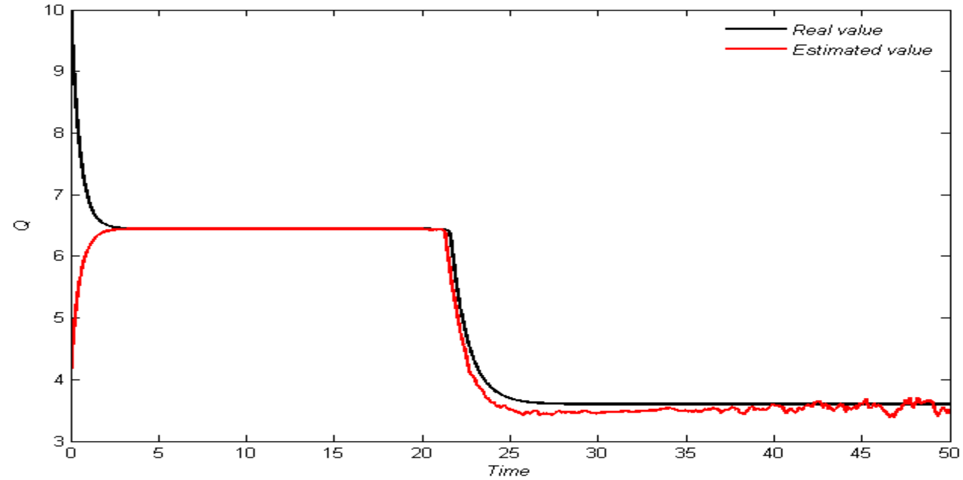

Figure 3.22: Estimation of quota-Presence of disturbance (%10).

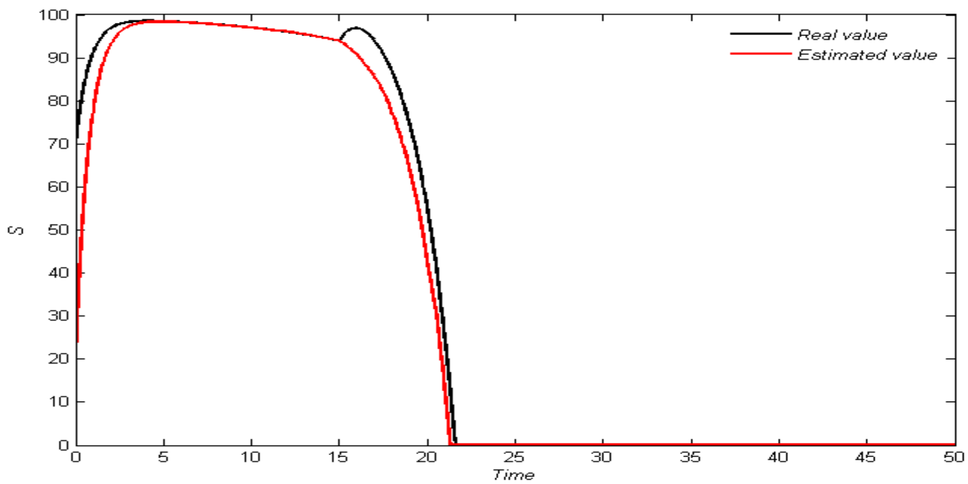

Figure 3.23: Estimation of substrate -Presence of disturbance (%10).

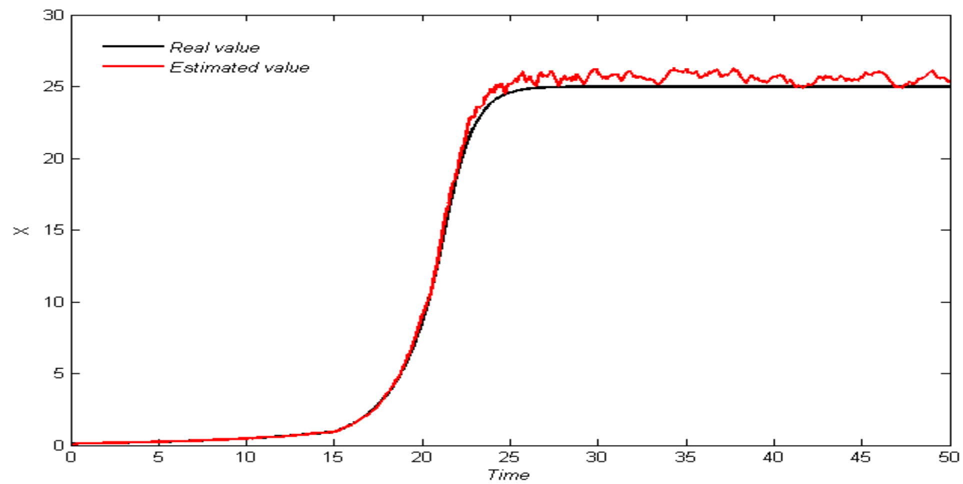

Figure 3.24: Estimation of biomass -Presence of disturbance (-%10).

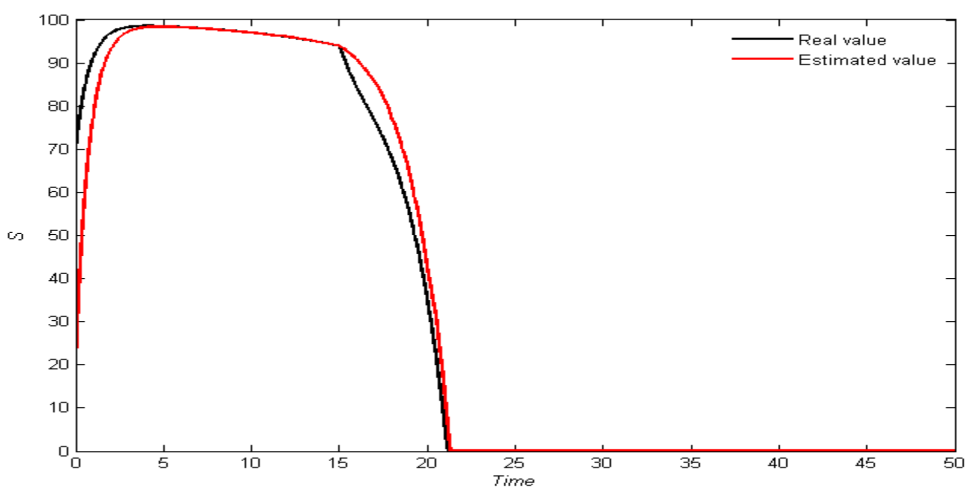

Figure 3.25: Estimation of substrate -Presence of disturbance (-%10).

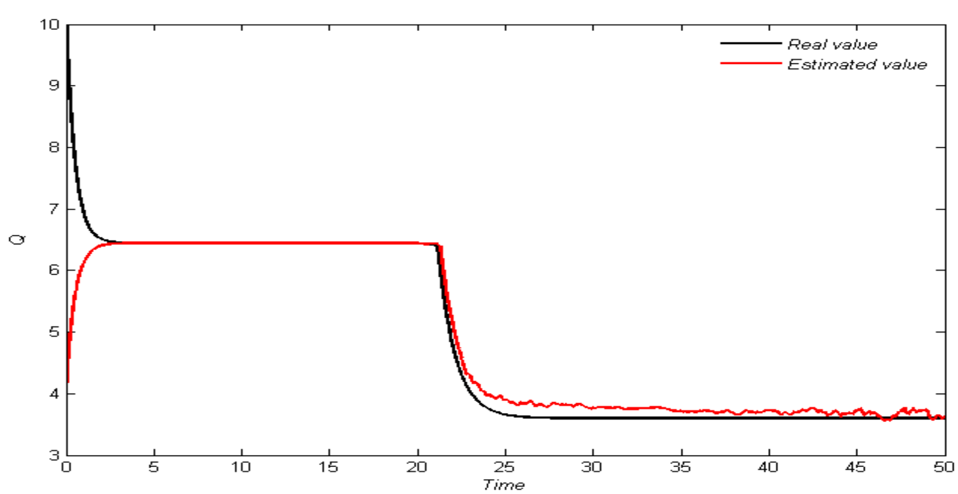

Figure 3.26: Estimation of quota-Presence of disturbance (-%10).

As can be seen, because, the disturbance is not estimated with the state variables, the designed observer has offset in estimation of the state variables.

3- Validity of the proposed observer for a wide rang of model's parameter

In order to study the validity of the designed observer for a wide range of model's parameter, these parameters change during the simulation with the following trajectories.

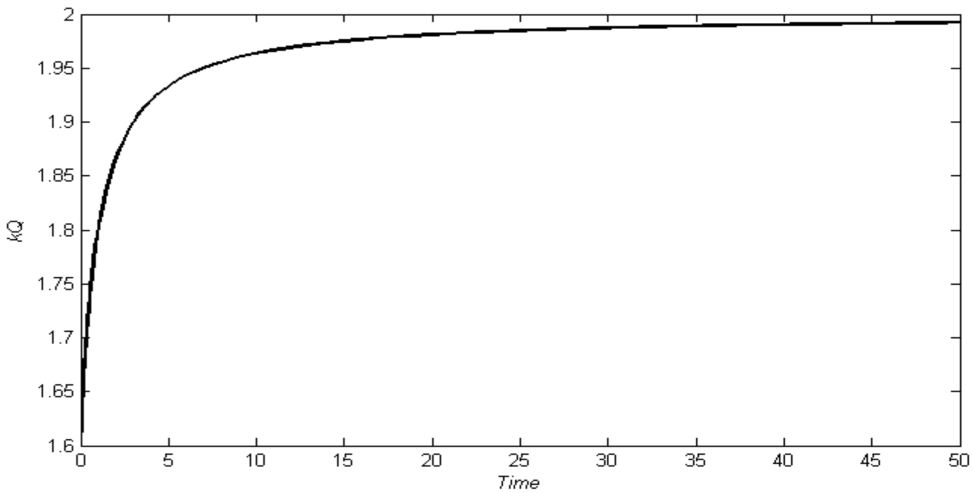

Figure 3.27: Minimum internal quota-K_Q.

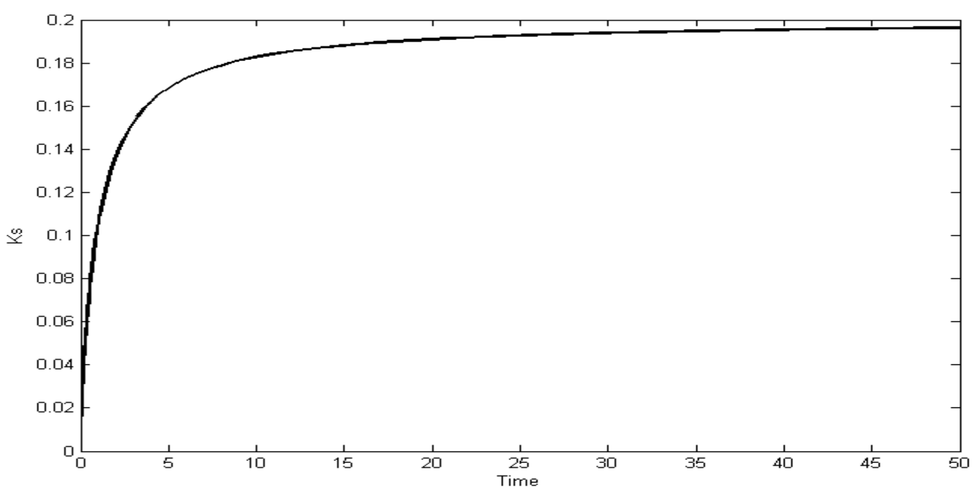

Figure 3.28: Half saturation constant-K_S.

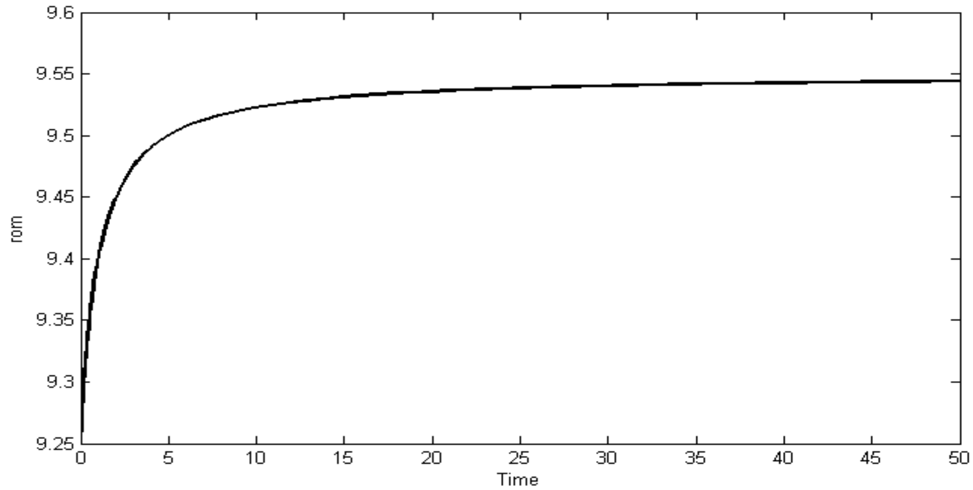

Figure 3.29: Maximum uptaken rate-ρ_m.

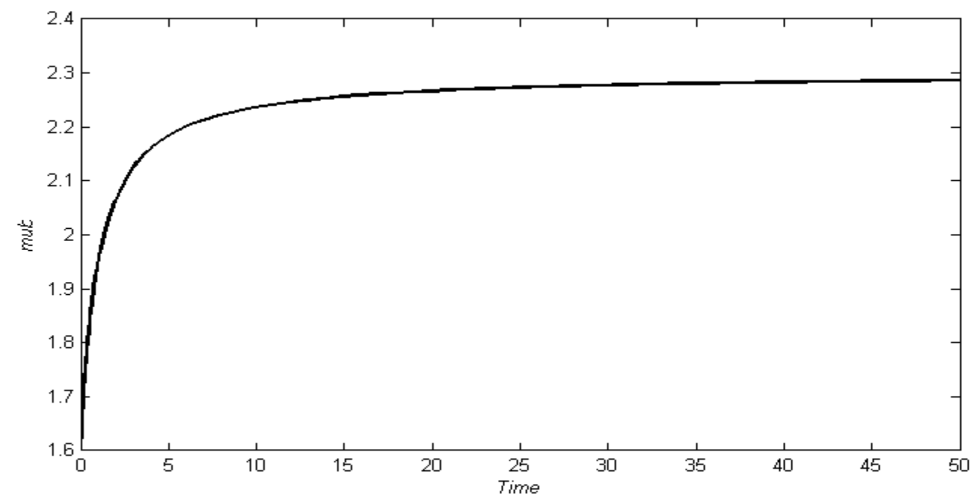

Figure 3.30: Theoritical maximum growth-$\bar{\mu}$.

Simulation results show that the proposed algorithm is valid for a wide range of model's parameters.

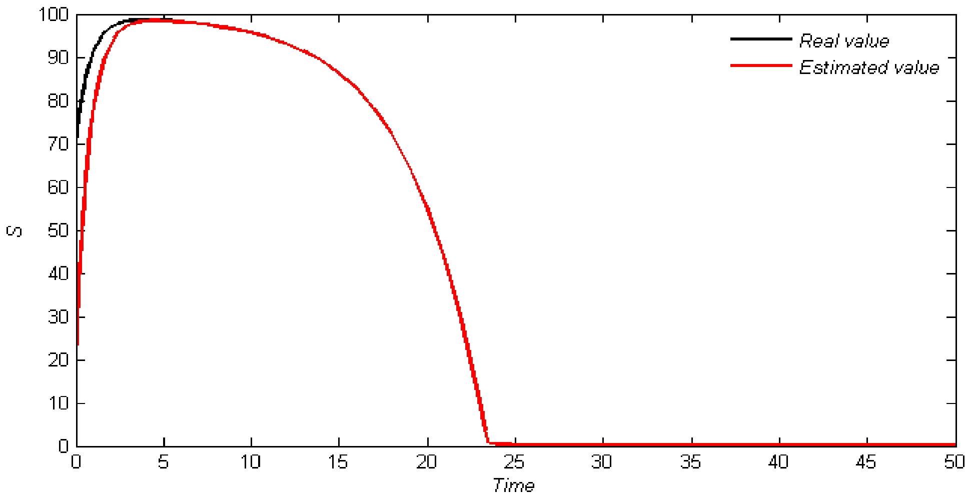

Figure 3.31: Estimation of substrate -Variable parameters.

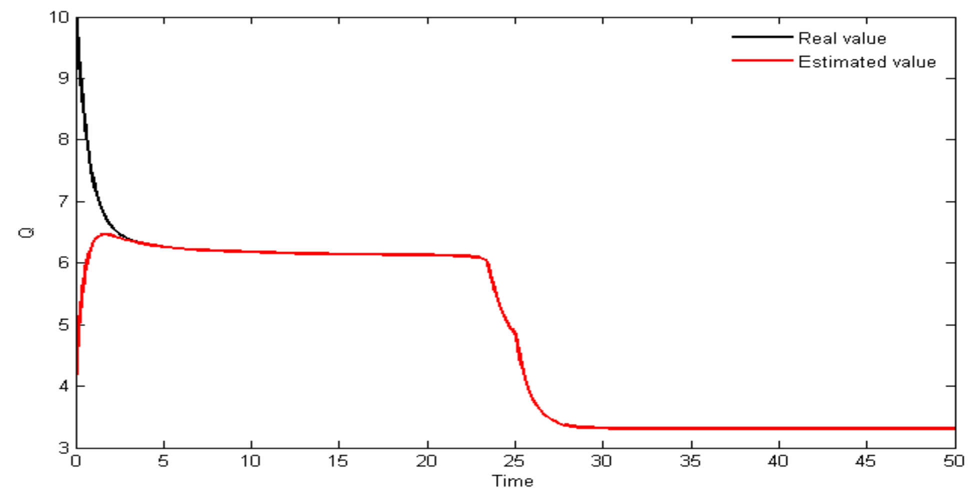

Figure 3.32: Estimation of quota-Variable parameters.

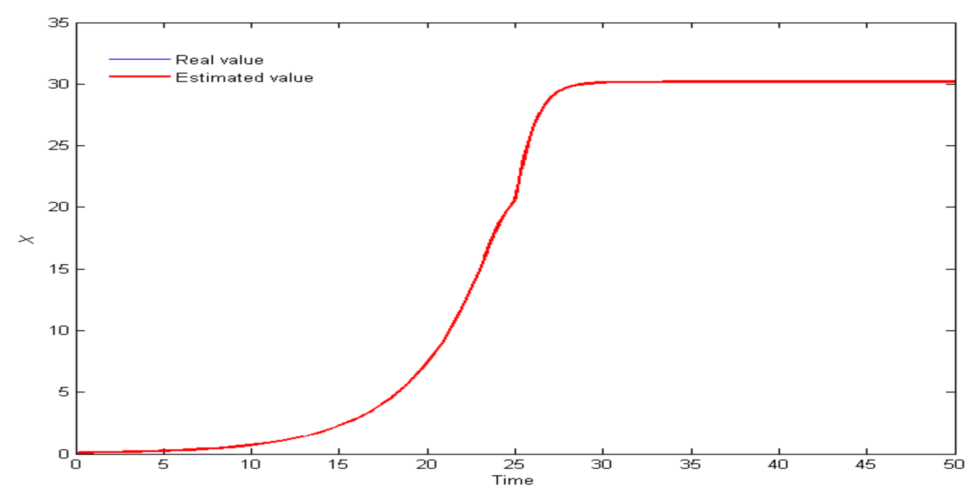

Figure 3.33: Estimation of biomass-Variable parameters.

4- Observer performance in presence of process noise

In order to evaluate the performance of observer in presence of process noise, the bounded value of noise is considered for the process variables. In the following figures, the simulation results are demonstrated.

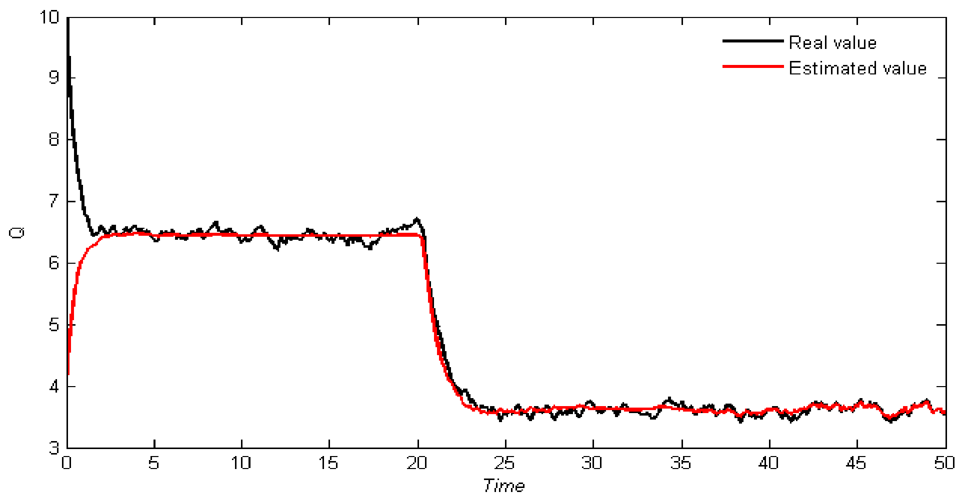

Figure 3.34: Estimation of quota-Process noise.

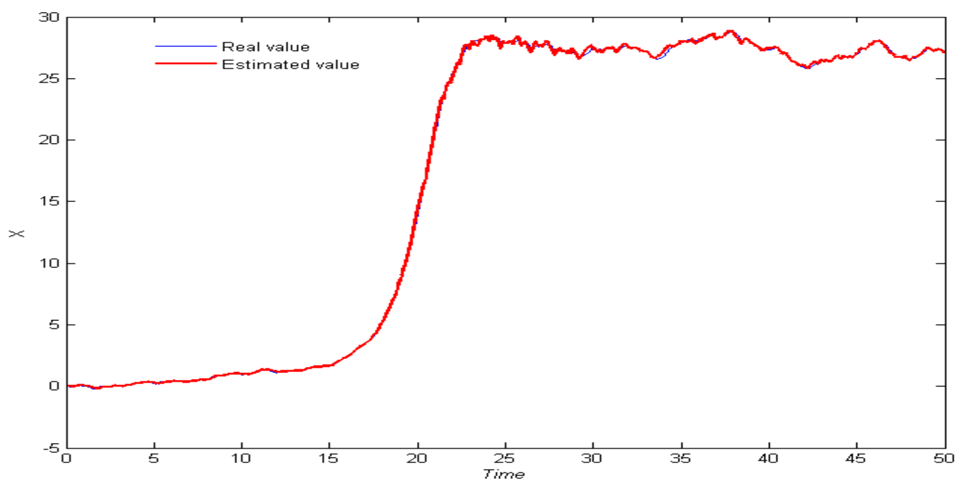

Figure 3.35: Estimation of biomass- Process noise.

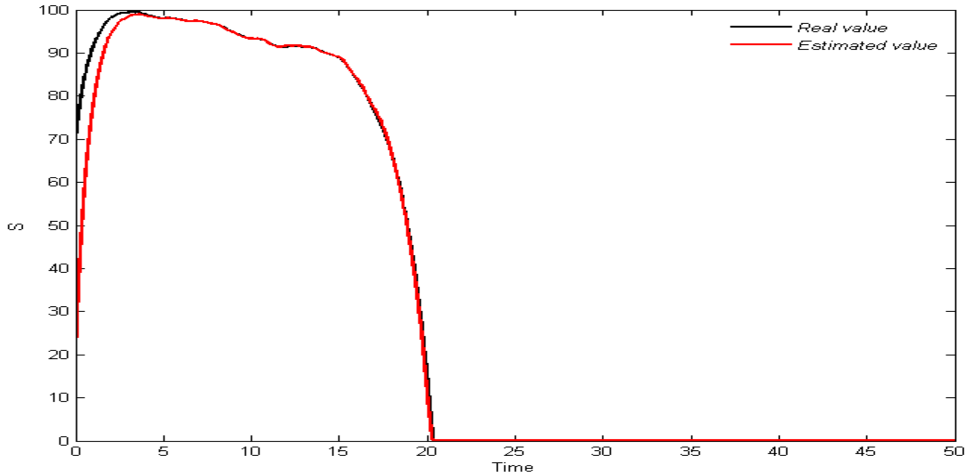

Figure 3.36: Estimation of substrate-Process noise.

As can be noticed, the observe has an acceptable performance in presence of white random noise with magnitude $\sigma = 0.1$.

3.2.6 Observer with Discrete Measurement

In the previous part, a continuous observer with continuous measurements was designed. But, as we know, in many experimental and industrial conditions, we are not able to have the measurements continuously. Because of this, it is necessary to design the software sensor with discrete time measurement. In order to design the observer with discrete measurements, three different scenarios are considered. In the following sections, each scenario is described.

1- First scenario.

In the first scenario, the equation of the observer is solved continuously, but the measurement is measured discretely (sample time=1). The following equation and figures shows the observer dynamics and performance respectively.

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(\hat{x}) + L(t)(y(k) - \hat{y}(t)), \\ \hat{y} &= C\hat{x}. \end{aligned} \tag{3.18}$$

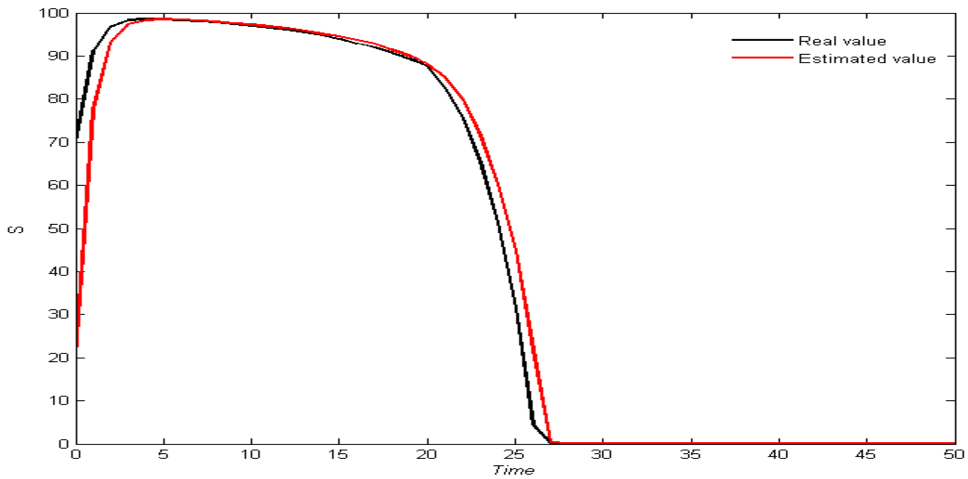

Figure 3.37: Estimation of substrate-First scenario.

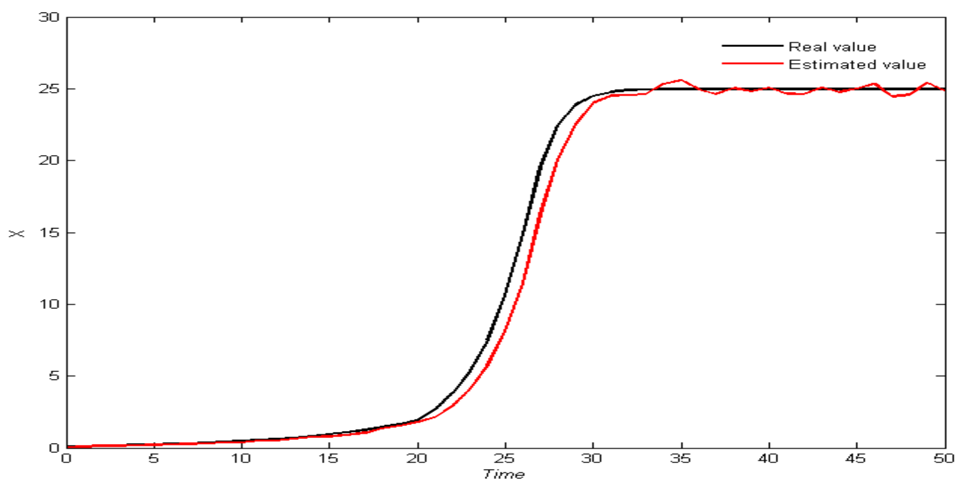

Figure 3.38: Estimation of biomass-First scenario.

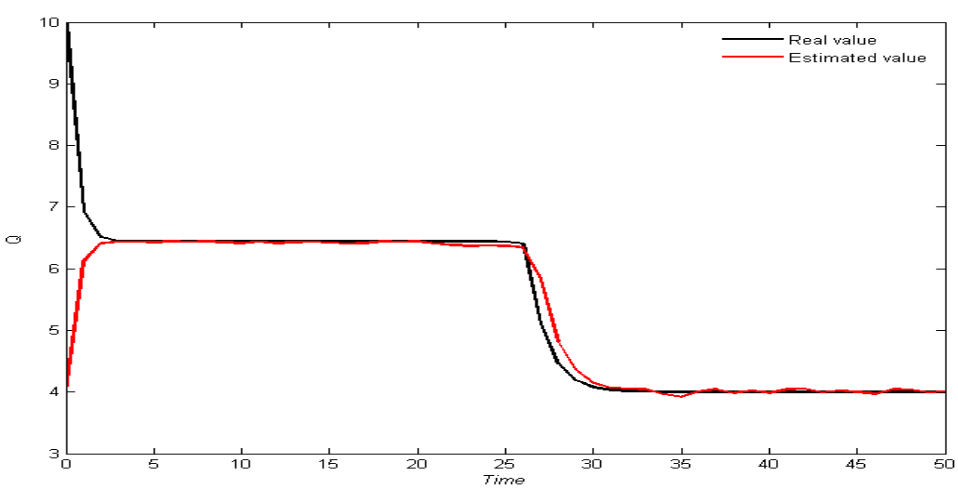

Figure 3.39: Estimation of quota-First scenario.

2- Second scenario.

In this scenario, the observer gain is computed at each sample time. But, the observer dynamics is solved continuously. The following equation shows the observer dynamics.

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(\hat{x}) + L(k)(y(k) - \hat{y}(t)), \\ \hat{y} &= C\hat{x}. \end{aligned} \tag{3.19}$$

The following figures are shown the performance of observer with measurement noise.

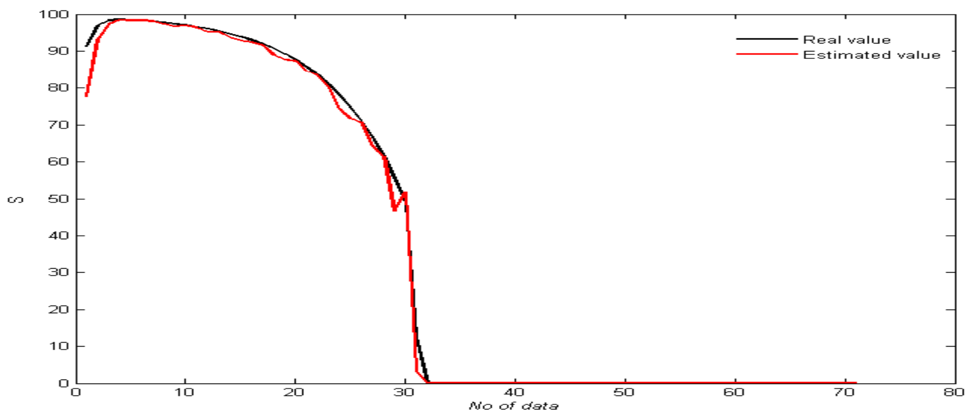

Figure 3.40: Estimation of substrate-Second scenario.

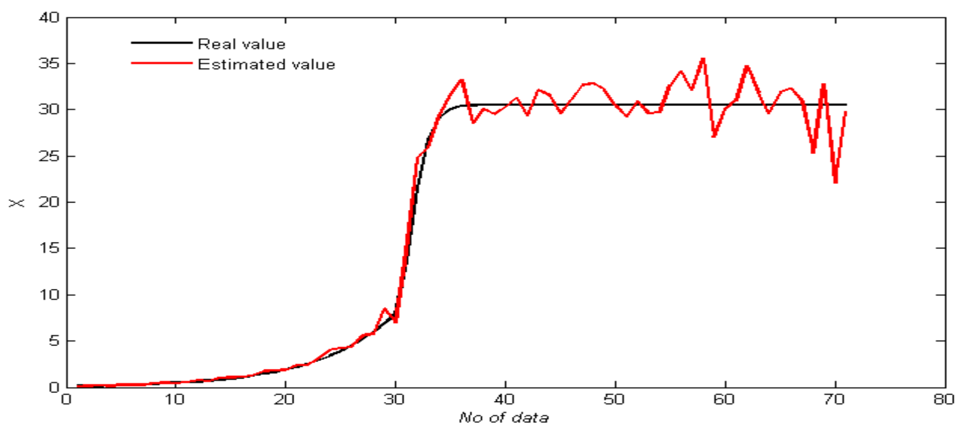

Figure 3.41: Estimation of biomass-Second scenario.

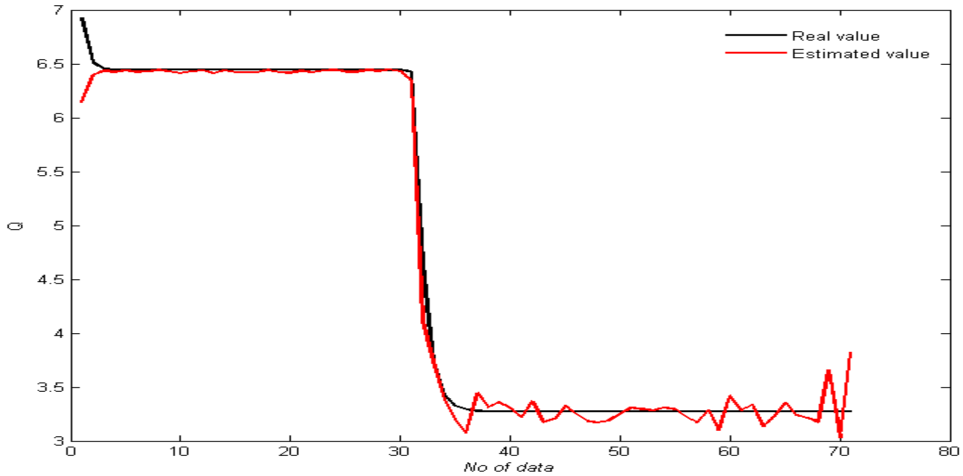

Figure 3.42: Estimation of internal quota-Second scenario.

As can be seen, by increasing the amount of biomass, the performance of observer is decreasing. Because at each sample time, the value of the observer gain would be constant.

3- Third scenario.

In this scenario, the first step is to discretize the dynamics of the Droop model by using proper method. Then, the gain of observer is obtained based on continuous model.

In this scenario, the equation of the Droop model is discretized using standard methods in order to solve differential equations. Four well-known methods for discritization are used as follows [16]

1- First order Euler method,

2- Rung-Kutta,

3- Two step Adams Bashforth method,

4- Implicit one-step method, Backward Euler method.

In order to evaluate the effect of sampling time for each method, the simulation results with three different values of sampling time are considered. Based on the simulation results, the complexity of the method and the required accuracy, first order Euler method with $T = 0.02$ is selected.

By using Euler method, the nonlinear discrete observer for discrete time measurement is designed. The following equation shows the observer dynamics.

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + \phi(\hat{x}(k)) + L(t)(y(k) - \hat{y}(k)), \\ \hat{y}(k) &= C\hat{x}(k). \end{aligned} \tag{3.20}$$

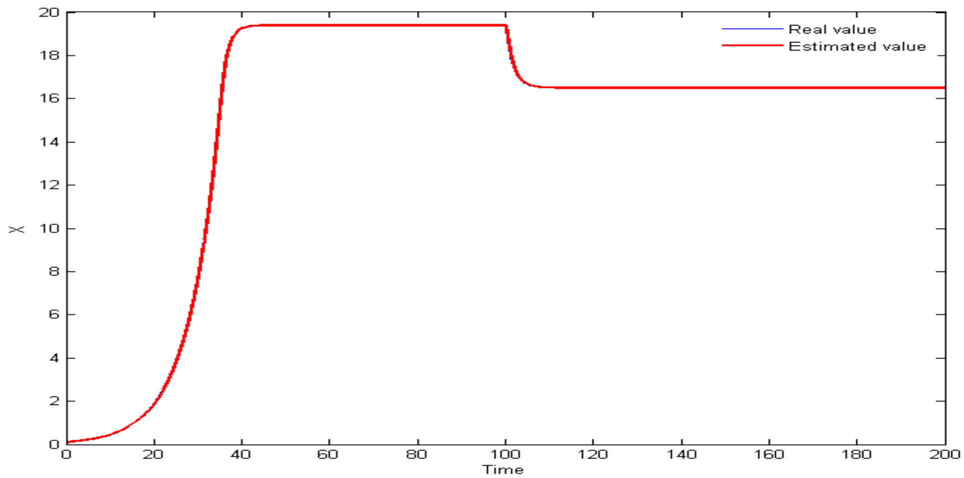

Figure 3.43: Estimation of biomass-Third scenario.

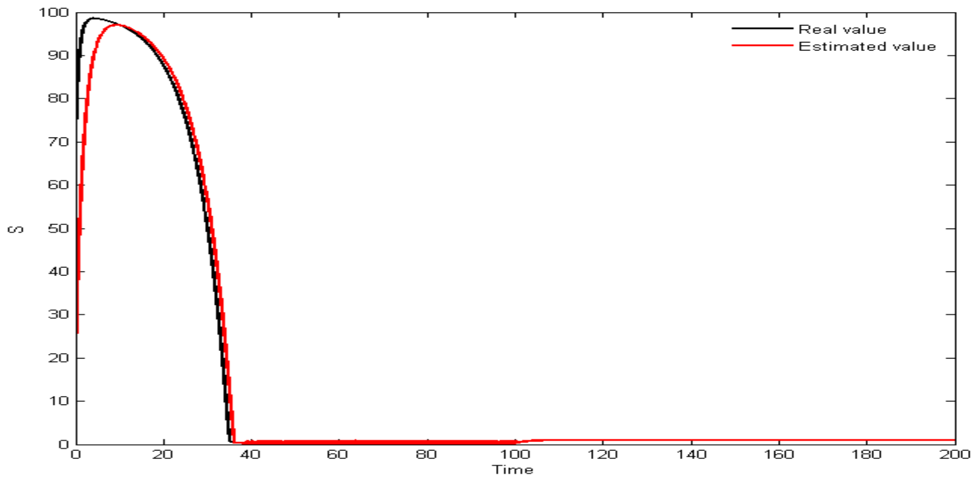

Figure 3.44: Estimation of substrate-Third scenario.

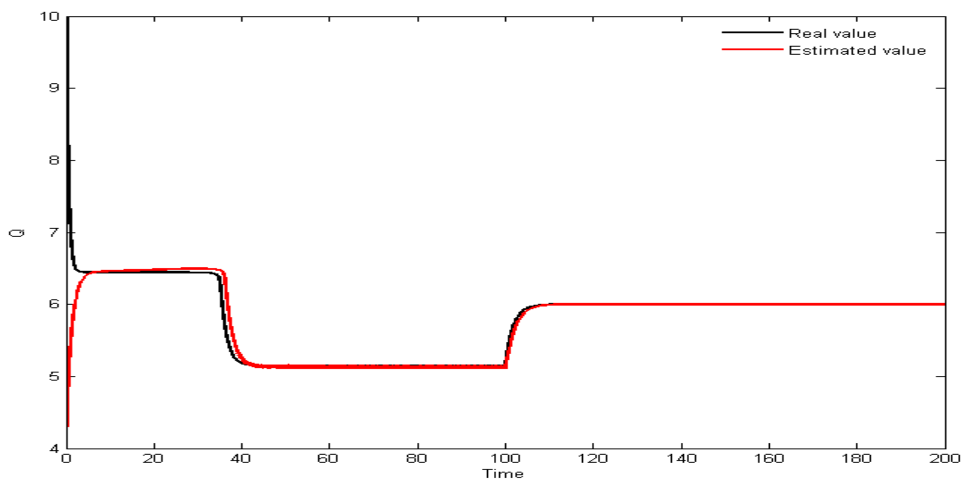

Figure 3.45: Estimation of quota-Third scenario.

The simulation results show that the proposed discrete estimator has a good performance for estimation of the state variables.

3.3 Parameter and State Estimation

In this section, the parameter estimation in the Droop model is investigated. Three important parameters namely, the maximum substrate uptake rate, the theoretical maximum growth rate and the substrate feed concentration, are estimated separately. For each parameter, the Droop model is demonstrated by different representations in terms of a linear and a nonlinear part. In the following sections, the simulation results related to each parameter are depicted.

3.3.1 Estimation of the Maximum Uptake Rate ρ_m

In order to estimate the maximum uptake rate, the following observable augmented model is considered

$$\begin{aligned}
&\dot{X} = X(\bar{\mu} - D) + \bar{\mu}Q + DS + (-\bar{\mu}Q - DS - \frac{\bar{\mu}K_Q X}{Q}),\\
&\dot{Q} = -\bar{\mu}Q + \rho_m + (-\rho_m + K_Q\bar{\mu} + \frac{\rho_m S}{K_s + S}),\\
&\dot{S} = -DX + DS + (DS_{in} - DX + \frac{\rho_m SX}{K_s + S}),\\
&\dot{\rho_m} = 0.
\end{aligned} \tag{3.21}$$

As same as previous method, the equations of dynamic model can be written as

$$\dot{x} = A(D)x + \psi(x), \tag{3.22}$$

$$y = Cx, \tag{3.23}$$

where

$$A(D) = \begin{pmatrix} \bar{\mu} - D & \bar{\mu} & D & 0 \\ 0 & -\bar{\mu} & 0 & 1 \\ -D & 0 & D & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad C = (1, 0, 0, 0).$$

Pole placement technique is used in order to compute the observer gain. In the following figures, the performance of the designed estimator is presented.

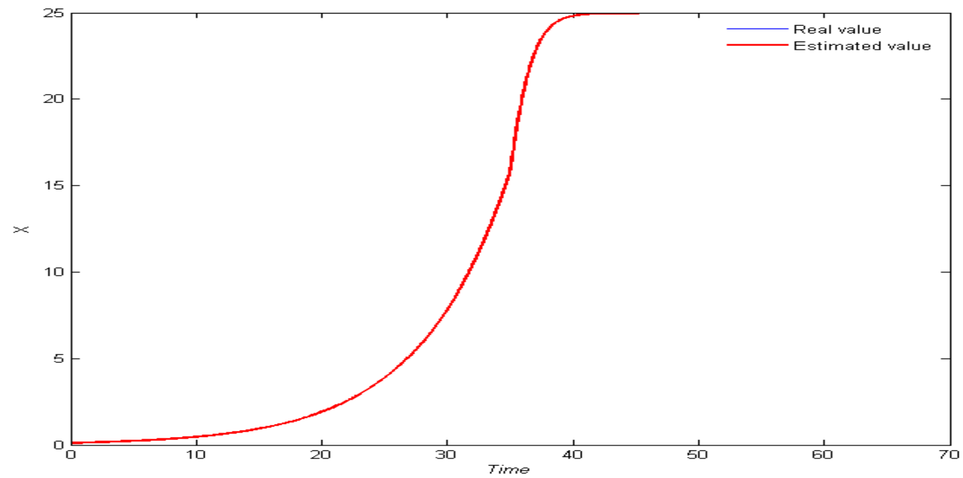

Figure 3.46: Estimation of biomass-Parameter estimation ρ_m.

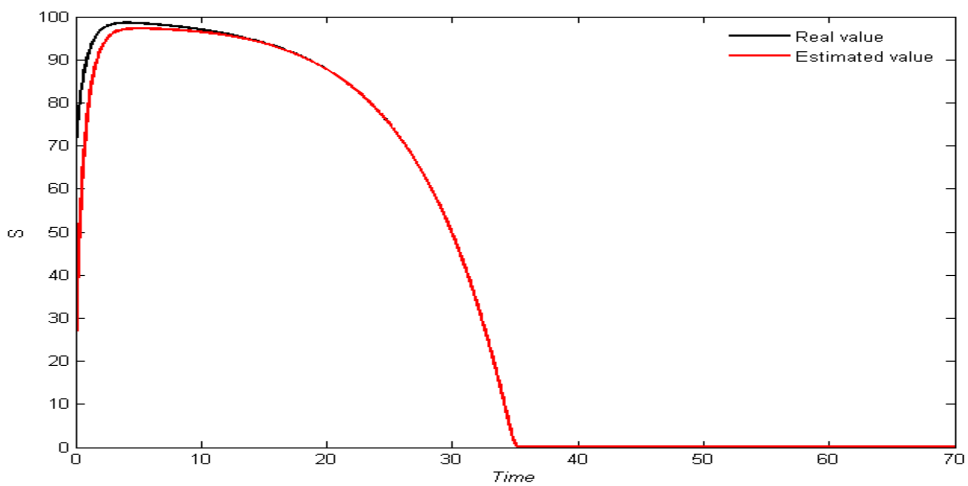

Figure 3.47: Estimation of substrate-Parameter estimation ρ_m.

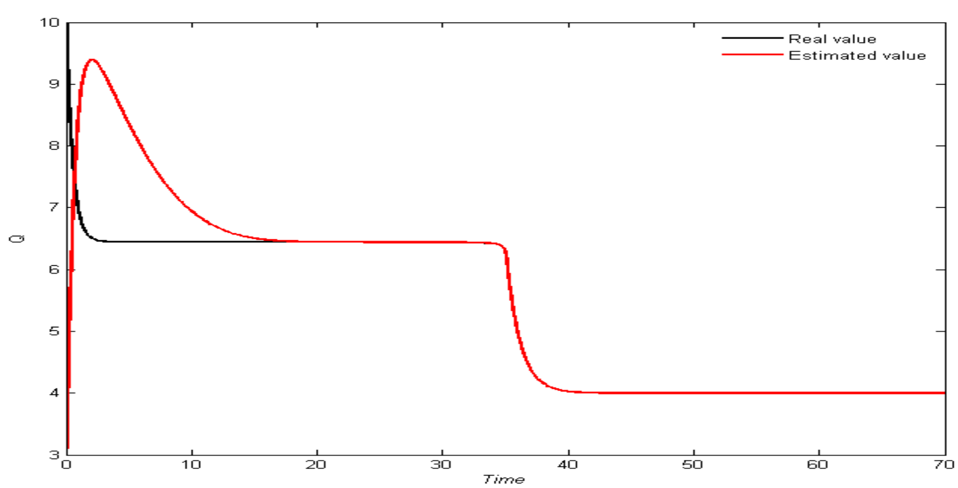

Figure 3.48: Estimation of quota-Parameter estimation ρ_m.

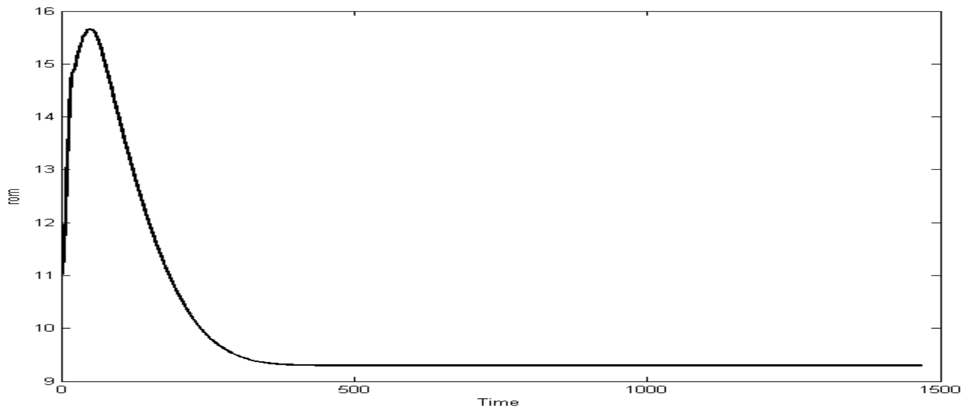

Figure 3.49: Estimation of ρ_m.

3.3.2 Estimation of the Theoretical Maximum Growth Rate $\bar{\mu}$

In order to estimate $\bar{\mu}$, the following augmented observable model is considered

$$\begin{aligned}
\dot{X} &= -DX + Q + DS + (\bar{\mu} - DS - \tfrac{\bar{\mu} K_Q X}{Q} - Q),\\
\dot{Q} &= K_Q \bar{\mu} + \rho_m S + (-\rho_m S - \bar{\mu} + \rho),\\
\dot{S} &= -\rho_m X + DS + (DS_{in} + X(\rho_m - \rho)),\\
\dot{\bar{\mu}} &= 0,
\end{aligned} \tag{3.24}$$

$$A(D) = \begin{pmatrix} -D & 1 & D & 0 \\ 0 & 0 & \rho_m & K_Q \\ -\rho_m & 0 & D & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad C = (1, 0, 0, 0).$$

Using the pole placement technique for the linear part of the error dynamics, the observer gain is computed. The simulation results show that the observer has a good performance. Also, as it can be seen in the figures, the variation of the theoretical maximum growth rate has a big effect on concentration of the biomass inside the bioreactor.

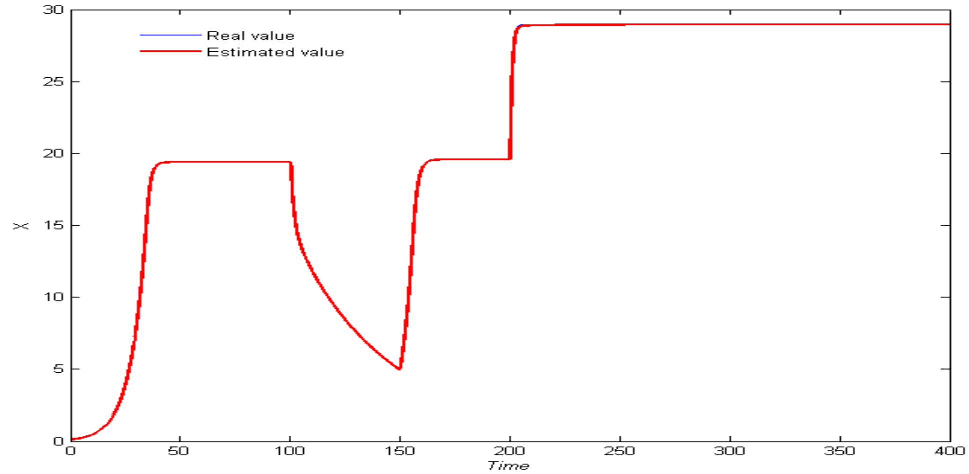

Figure 3.50: Estimation of biomass-Parameter estimation $\bar{\mu}$.

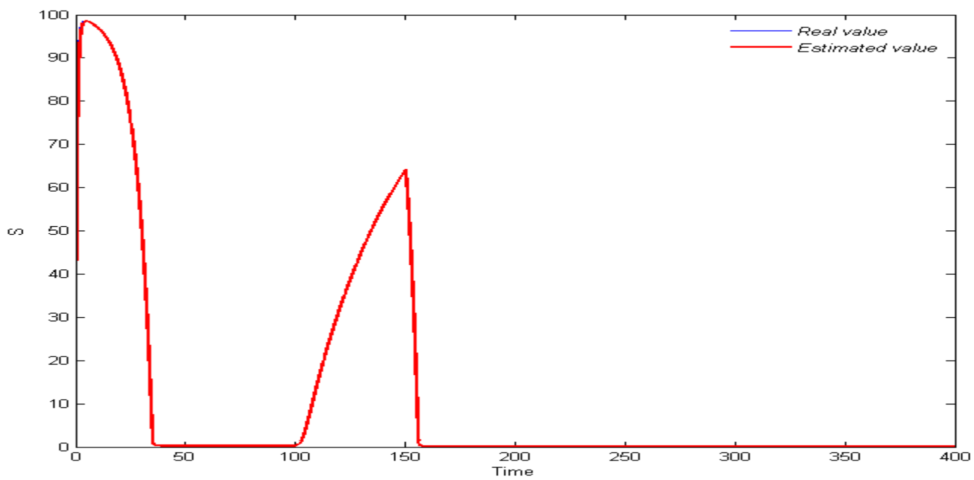

Figure 3.51: Estimation of substrate-Parameter estimation $\bar{\mu}$.

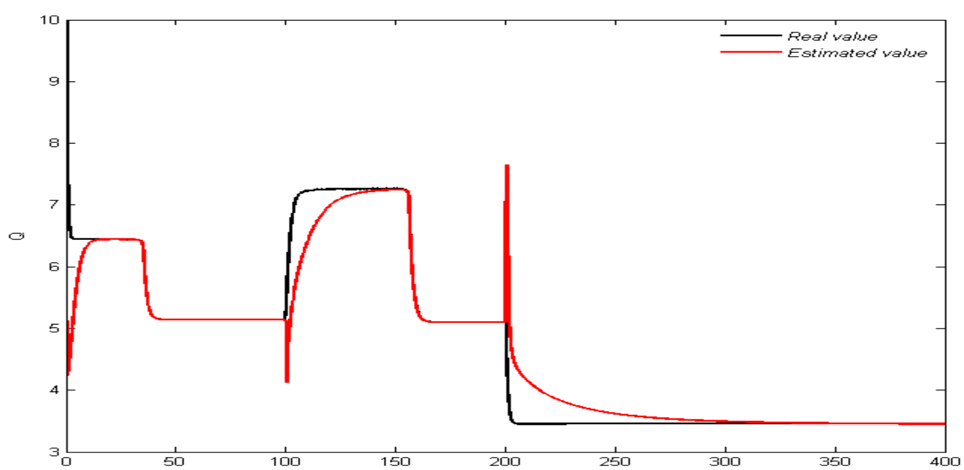

Figure 3.52: Estimation of internal quota-Parameter estimation $\bar{\mu}$.

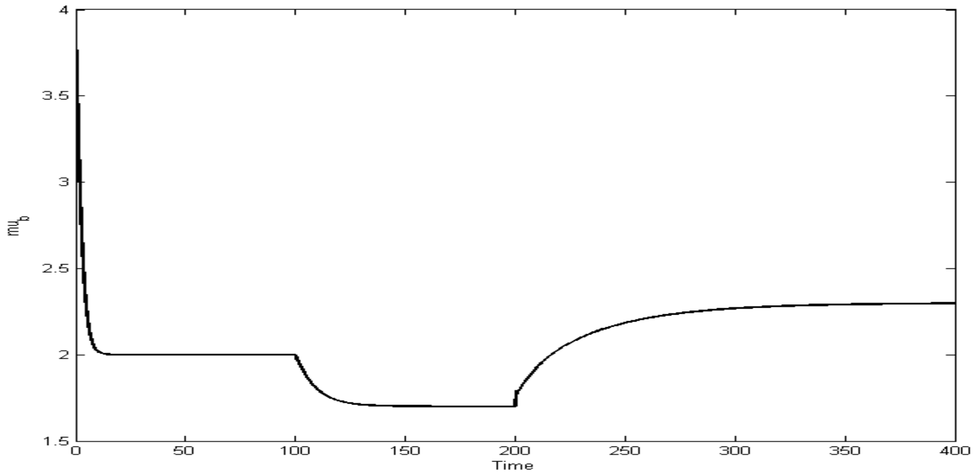

Figure 3.53: Estimation of $\bar{\mu}$.

3.3.3 Estimation of the Substrate Feed Concentration S_{in}

One of the important parameters, which can be considered as a disturbance in the process, is the substrate feed concentration. The following observable augmented model can be considered to estimate the states and feed concentration

$$\begin{aligned}
\dot{X} &= X(\bar{\mu} - D) + \bar{\mu}Q + (-\bar{\mu}Q - \tfrac{\bar{\mu}K_Q X}{Q}),\\
\dot{Q} &= -\bar{\mu}Q + \rho_m S + (-\rho_m S - K_Q\bar{\mu} + \tfrac{\rho_m S}{K_s + S}),\\
\dot{S} &= -\rho_m X - DS + DS_{in} + (\rho_m X - \tfrac{X\rho_m S}{K_s + S}),\\
\dot{S_{in}} &= 0.
\end{aligned} \tag{3.25}$$

By computing the observer gain using the previous procedure, the following simulation results are obtained

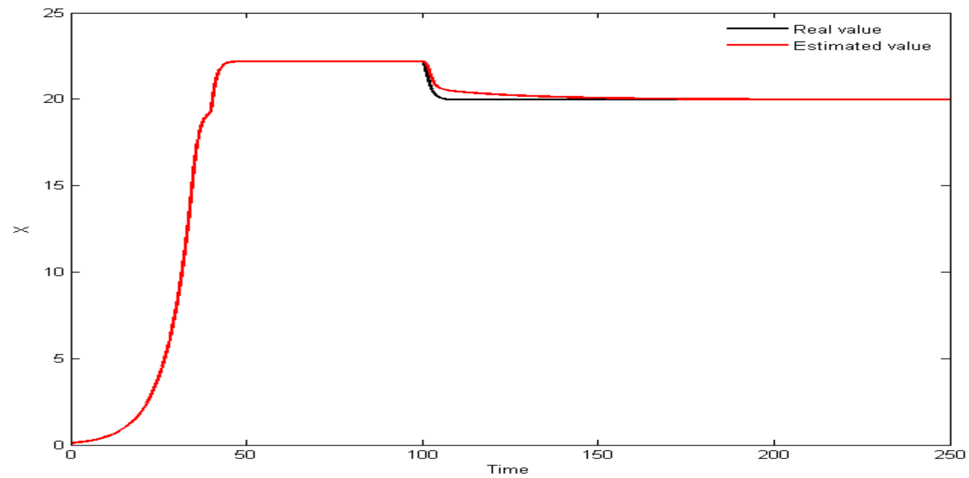

Figure 3.54: Estimation of biomass-Parameter estimation S_{in}.

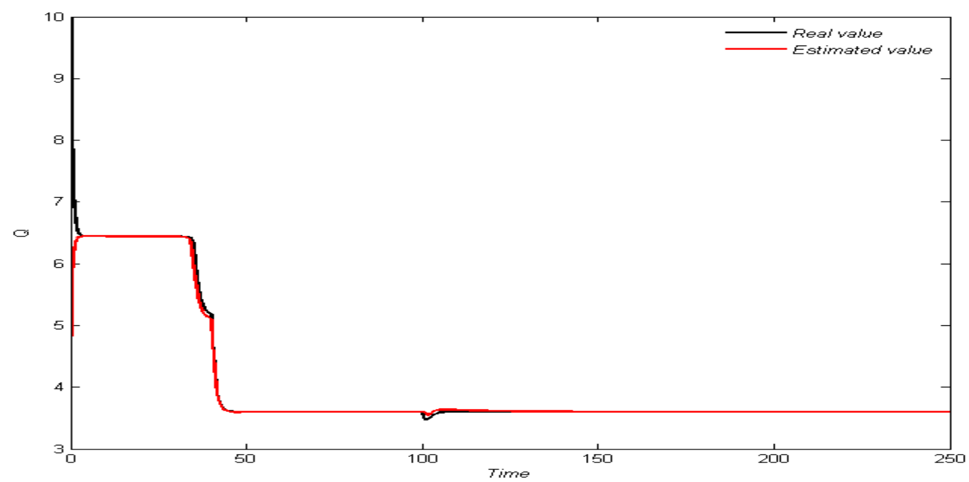

Figure 3.55: Estimation of quota-Parameter estimation S_{in}.

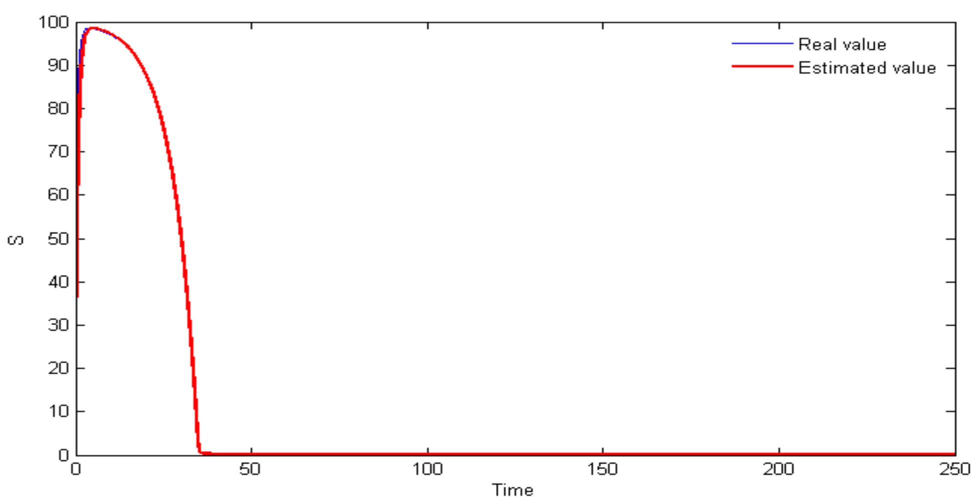

Figure 3.56: Estimation of substrate-Parameter estimation S_{in}.

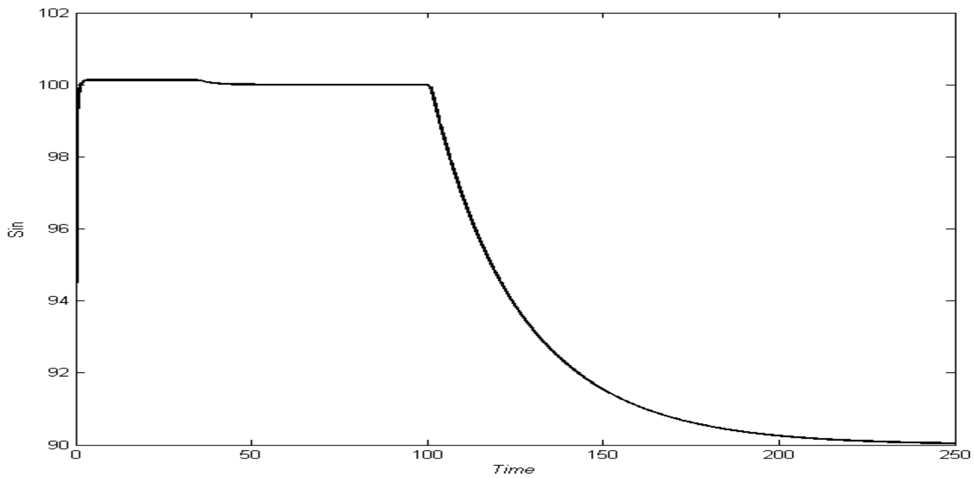

Figure 3.57: Estimaton of S_{in}.

As can be seen, the observer can follow the variations of the substrate concentration.

3.4 Observer-Based Nonlinear Controllers

In order to evaluate the observer performance in the presence of nonlinear controllers, two control objectives are considered. First, the biomass concentration control and second, the substrate concentration control. To achieve these objectives, two well-known observer-based nonlinear control strategies namely input-output linearization and backstepping are developed for the process. In the next sections, simulation results for each control objective and methodology are presented.

3.4.1 Biomass Concentration Control

Microalgae process is controlled to operate at a constant biomass concentration mode, in order to maintain the culture at the optimal population density and to sustain high biomass production levels.

As mentioned in the previous part, two nonlinear control strategies are considered. In the following part, each of them is described.

1- Input-output exact linearization

The objective of this technique is to linearize the relation between input and output of the system using nonlinear static functions. By using input-output linearization technique, it is easy to design the controller for obtaining the control objective, because of the linear behaviour between input and output of the process.

Based on the property of the Droop model, we can show that the model is minimum phase. By using the following transformation, we can transfer the Droop model to the normal form

$$T(x) = \begin{pmatrix} \frac{X(S_{in}-S)}{S_{in}} \\ Q \\ X \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}.$$

and the normal form would be

$$\begin{aligned} \dot{\eta_1} &= \mu X \tfrac{S_{in}-S}{S_{in}} + \tfrac{\rho X^2}{S_{in}}, \\ \dot{\eta_2} &= \rho - \mu Q, \\ \dot{\eta_3} &= \mu X - uX, \\ y &= \eta_3. \end{aligned} \tag{3.26}$$

In order to obtain the zero dynamics from normal form, it is necessary to consider the output equal to zero. By putting $y = 0$, the zero dynamics would be

$$\dot{\eta_2} = \rho - \mu Q. \tag{3.27}$$

Based on the first property of the Droop model described at the first chapter, Equation (3.27) is bounded and stable. So, the zero dynamics of the Droop model is stable. At the following paragraph, the derivative of the input-output controller will be depicted. If the nonlinear system is considered as

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x). \end{aligned} \tag{3.28}$$

Then, the control law, which can exactly linearize the system between external input ν and output y, would be

$$u = \frac{\nu - L_f^r h - \beta_1 L_f^{r-1} h ... - \beta_r h}{L_g L_f^{r-1} h}. \tag{3.29}$$

So, after linearization, the input-output behaviour of the system can be shown as

$$\frac{d^r y}{dt^r} + \beta_1 \frac{d^{r-1} y}{dt^{r-1}} + ... + \beta_{r-1} \frac{dy}{dt} + \beta_1 y = \nu. \tag{3.30}$$

The signal ν comes from external controller which is linear controller such as PID. Based on the Droop model, the control law can be used to linearize the behaviour of input-output of the system

$$u = \frac{\nu - \mu X - \beta_1 X}{-X}. \tag{3.31}$$

After substitution of Equation (3.31) into the model, the relation between input and output can be represented by

$$\frac{y(s)}{\nu(s)} = \frac{1}{s + \beta_1}. \tag{3.32}$$

A PI controller with $K_C = 1, \tau_I = 1$ is used as an external controller for the control loop.

In the following figures, the output feedback controller performance with measurement noise is presented.

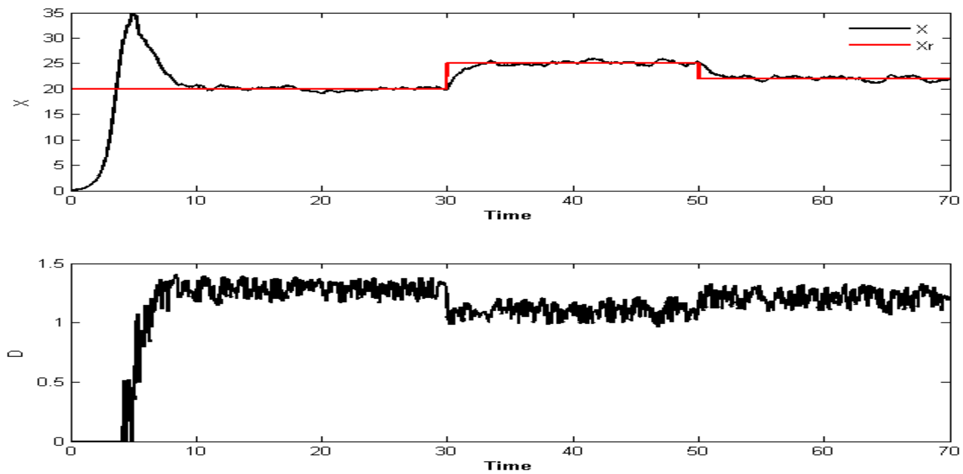

Figure 3.58: Biomass-Dilution rate-Input-output linearization.

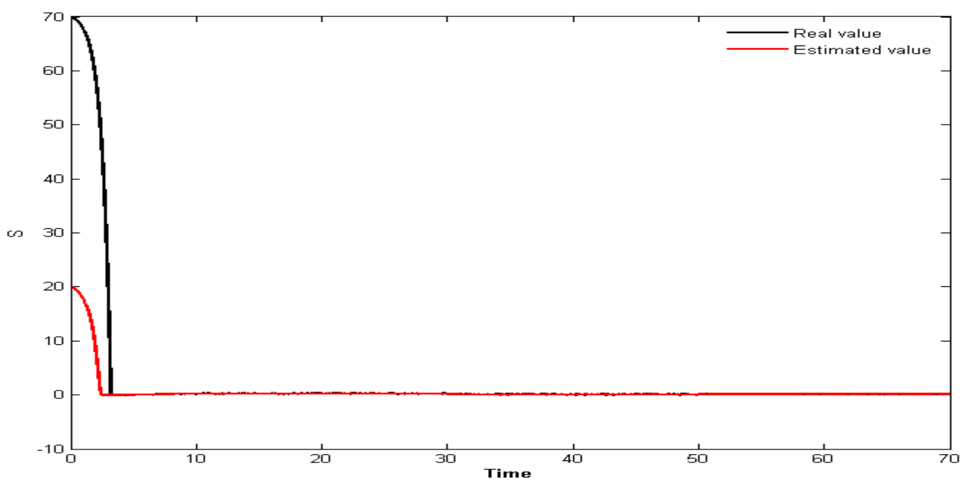

Figure 3.59: Estimation of substrate-Input-output linearization.

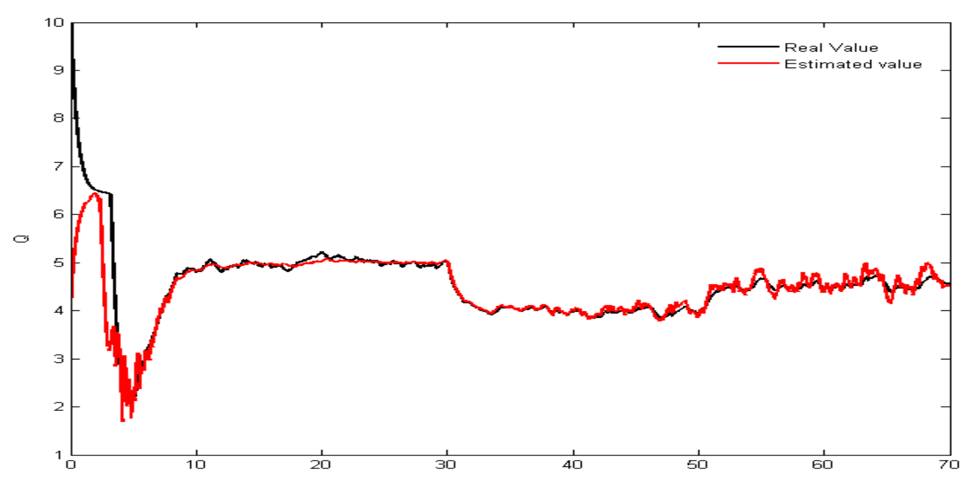

Figure 3.60: Estimation of internal quota-Input-output linearization.

In order to evaluate the performance of the proposed controller-observer scheme in presence of disturbance, ten percent increase in the substrate feed concentration is considered.

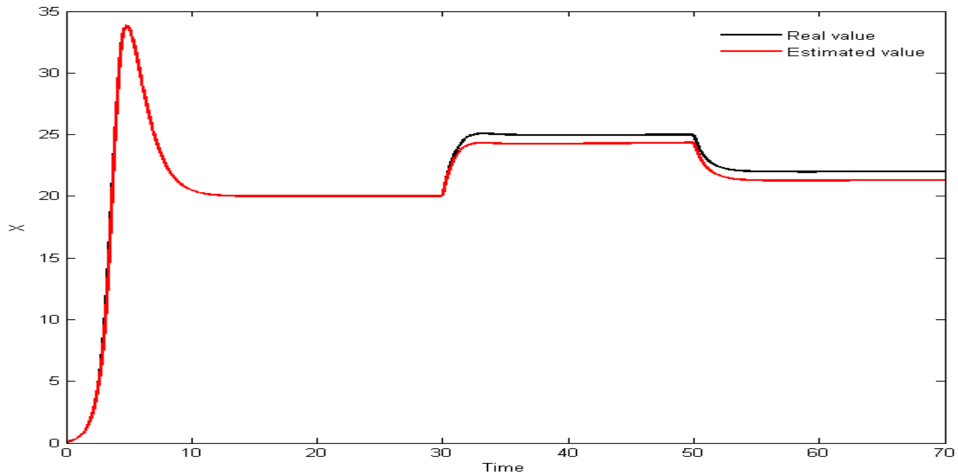

Figure 3.61: Biomass-Dilution rate-Disturbance rejection.

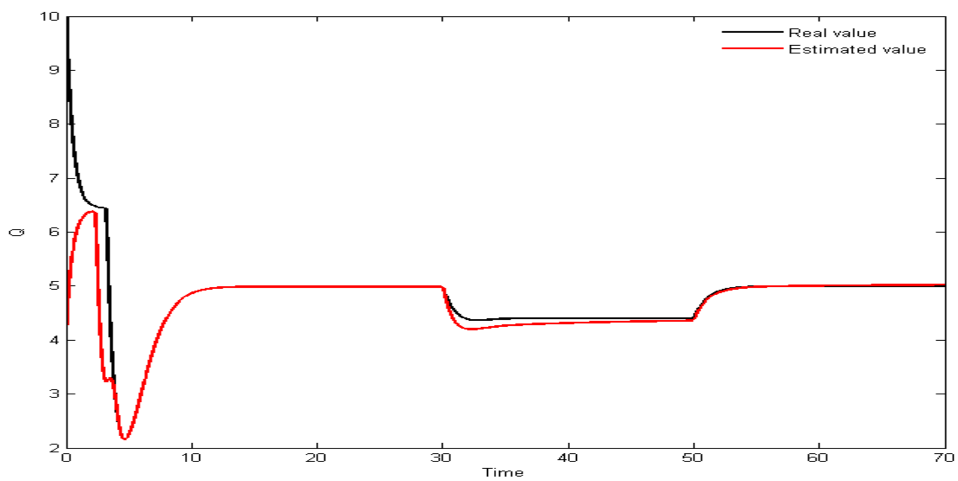

Figure 3.62: Estimation of quota-Disturbance rejection.

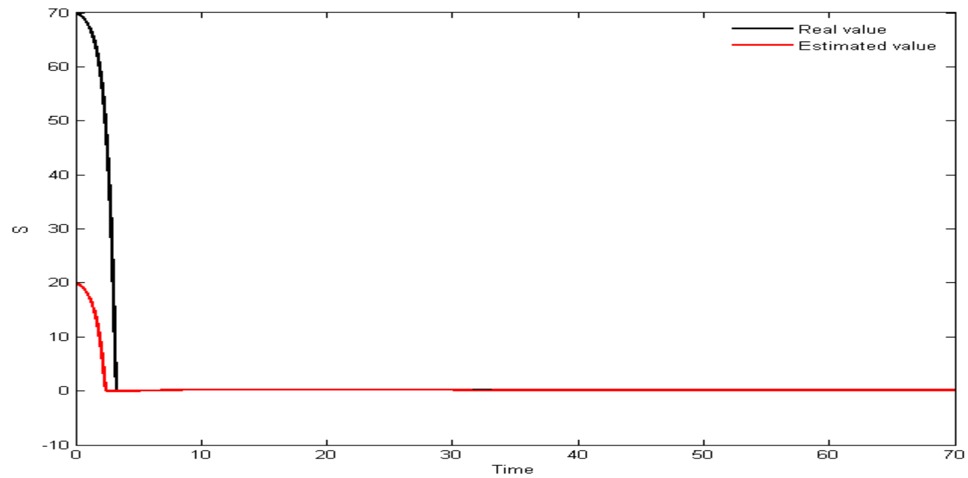

Figure 3.63: Estimation of substrate-Disturbance rejection.

As it can be seen, the output feedback controller has a good performance in setpoint tracking and load rejection.

2- Backstepping controller

The backstepping control suggests a systematic stabilizing procedure. In addition, a wide class of uncertain nonlinear system can be handle by backstepping technique. The detailed design procedure of a general backstepping controller can be found in [38]. The key procedures of the backstepping design for the microalgae process are given in [40]. For the integral control action, we define an error variable Z_1 as

$$Z_1 = \int_0^t (y - y_d)dt \tag{3.33}$$

where y_d is setpoint for y. we define another error variable as follows

$$Z_2 = \frac{1}{\eta}(y - y_d - \alpha). \tag{3.34}$$

where the stabilizing function α is given by

$$\alpha = -c_1 Z_1. \tag{3.35}$$

Using Equations (3.33) and (3.35), $\dot{Z}_1$ is written as

$$\dot{Z}_1 = -c_1 Z_1 + \eta Z_2. \tag{3.36}$$

The first Lyapunov function is defined as $V_1 = \frac{1}{2}Z_1^2$ then derivative of V_1 is

$$\dot{V}_1 = -c_1 Z_1^2 + \eta Z_1 Z_2. \tag{3.37}$$

Now, the second Lyapunov function is defined

$$V_2 = V_1 + \tfrac{1}{2}Z_2^2. \tag{3.38}$$

To compute $\dot{V}_2$, $\dot{Z}_2$ is obtained as

$$\dot{Z}_2 = \tfrac{1}{\eta}(\dot{y} - \dot{y}_d - \tfrac{\partial \alpha}{\partial Z_1}\dot{Z}_1), \tag{3.39}$$

where $\dot{y}$ is $-DX + \mu X$. So, we have the following equation for $\dot{Z}_2$

$$\dot{Z}_2 = \tfrac{1}{\eta}(-DX + \mu X - \dot{y}_d - \tfrac{\partial \alpha}{\partial Z_1}\dot{Z}_1). \tag{3.40}$$

For stabilization of the second Lyapunov function in Equation (3.38), the backstepping controller law should be as follows

$$D = u = \frac{-\eta^2 Z_1 - c_2 Z_2 \eta - \mu X + \dot{y}_d + c_1^2 Z_1 - c_1 \eta Z_2}{X}, \tag{3.41}$$

where c_1, c_2, η refer to the controller's parameters where assumed as positive constants with values

$$(c_1, c_2) = (0.6, 0.1), \text{ and } \eta = 0.1.$$

In order to evaluate the performance of backstepping controller with proposed observer, setpoint tracking and disturbance rejection problems are studied. In the following figures the results are illustrated.

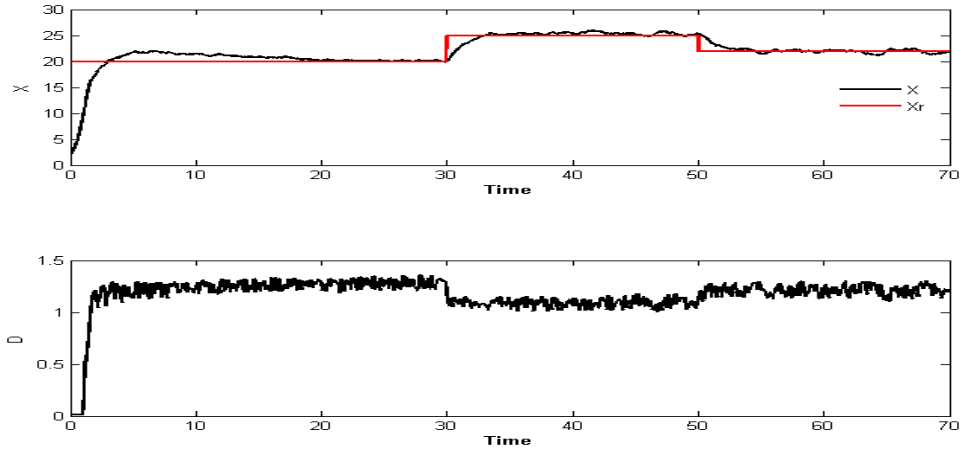

Figure 3.64: Biomass-Dilution rate-Backstepping.

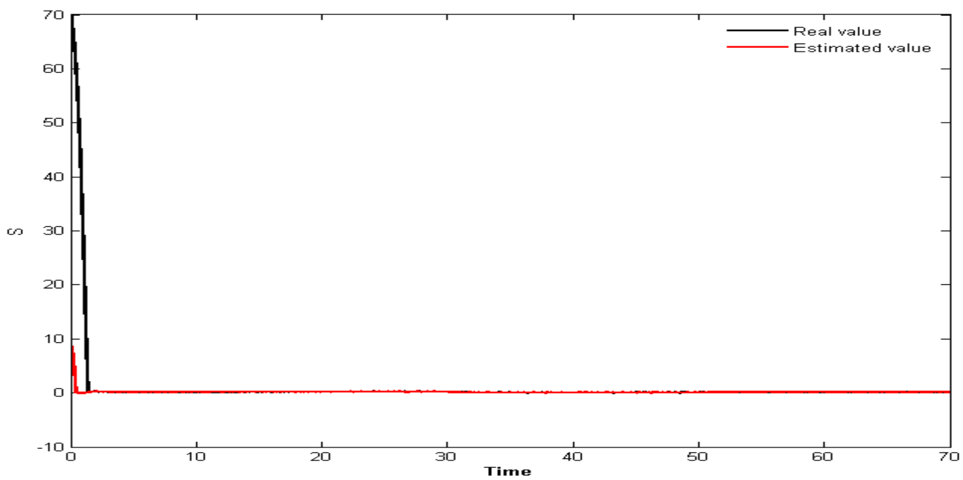

Figure 3.65: Estimation of substrate-Backstepping.

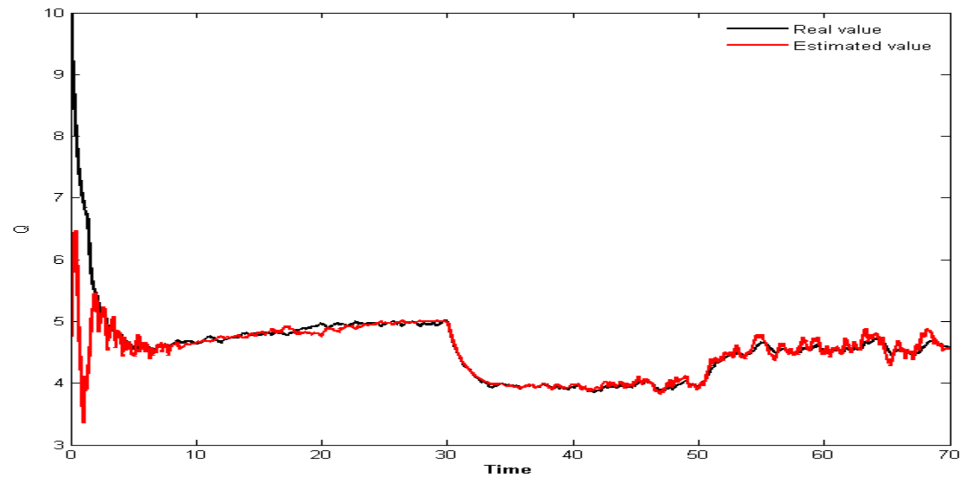

Figure 3.66: Estimation of internal quota-Backstepping.

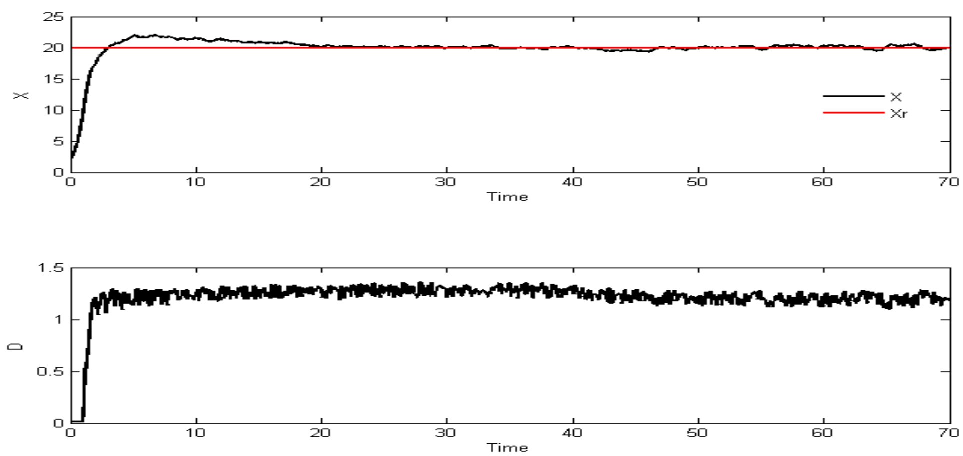

Figure 3.67: Biomass-Dilution rate-Backstepping (Disturbance rejction).

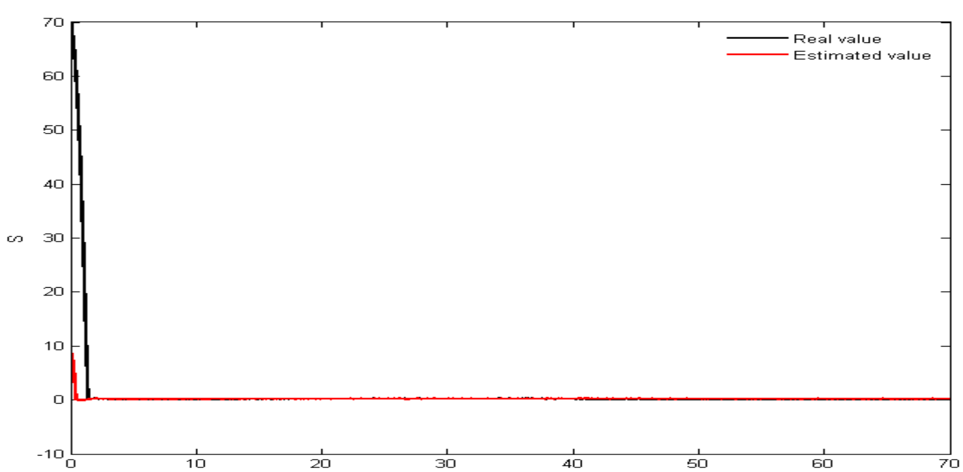

Figure 3.68: Estimation of substrate-Backstepping (Disturbance rejection).

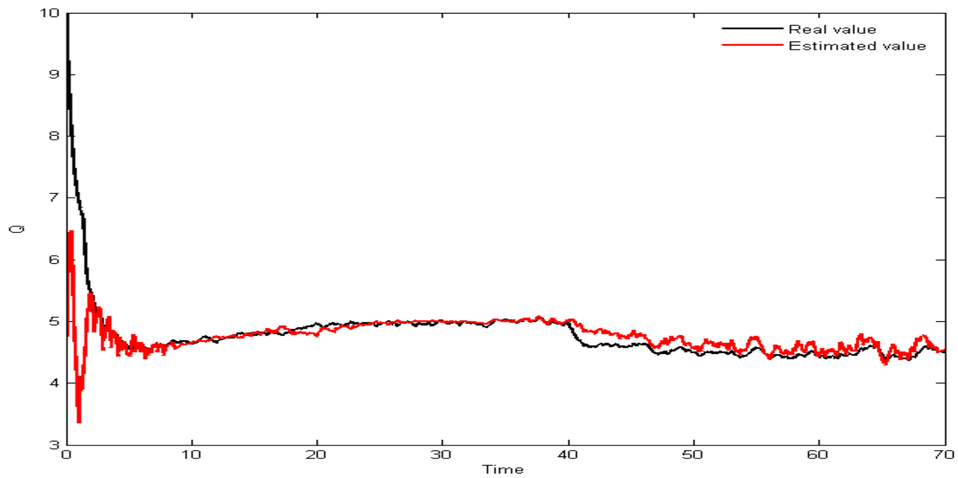

Figure 3.69: Estimation of internal quota-Backstepping(Disturbance rejection).

As it can be seen, the proposed controller in the presence of measurement noise has a good performance to follow the desired trajectory and to reject the disturbance.

3.4.2 Substrate Concentration Control

In this section, the substrate concentration regulation in order to keep the culture in the optimal operating condition, is studied. It is assumed that the biomass concentration is only available measurement. Three different controllers are used . In the following parts, each controller is developed and the simulation results are demonstrated.

1- PI Controller

In order to design PI controller, the Droop model is linearized around the operating. The linearized model is obtained as

$$G(s) = \frac{99.7315(s+2)(s+1.3)}{(s+137)(s+0.6933)(s+1.3)}. \tag{3.42}$$

$K_C = 0.05$ and $\tau_I = 2.00$ are computed based on linearized model as controller's parameters. The performance of the designed PI controller with measurement noise is illustrated in the following figures

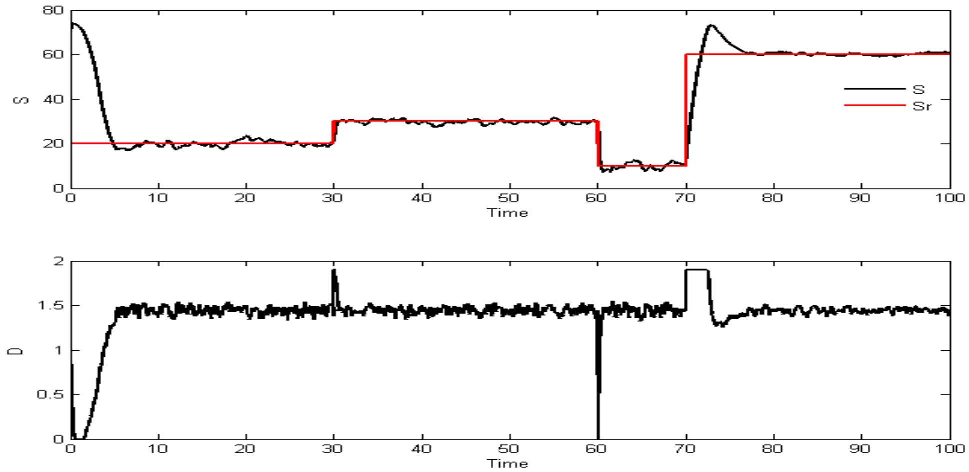

Figure 3.70: Substrate-Dilution-PI controller.

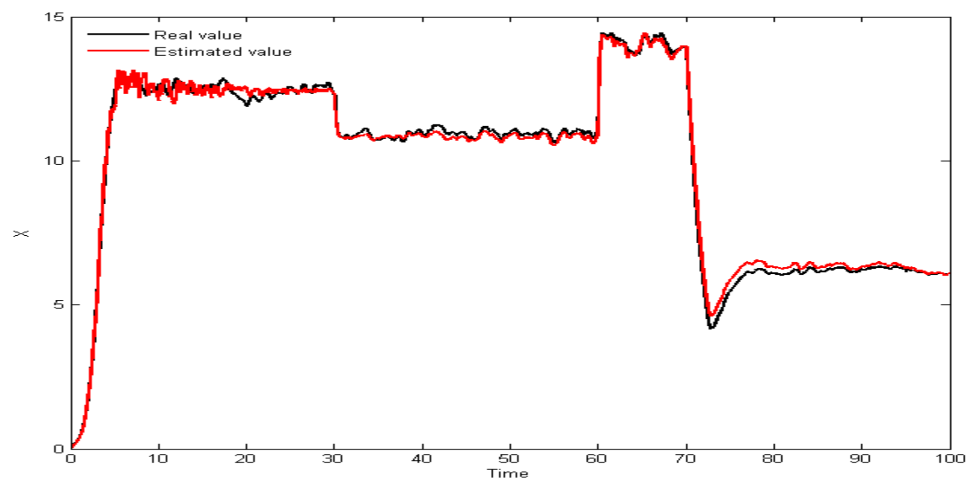

Figure 3.71: Estimation of biomass-PI controller.

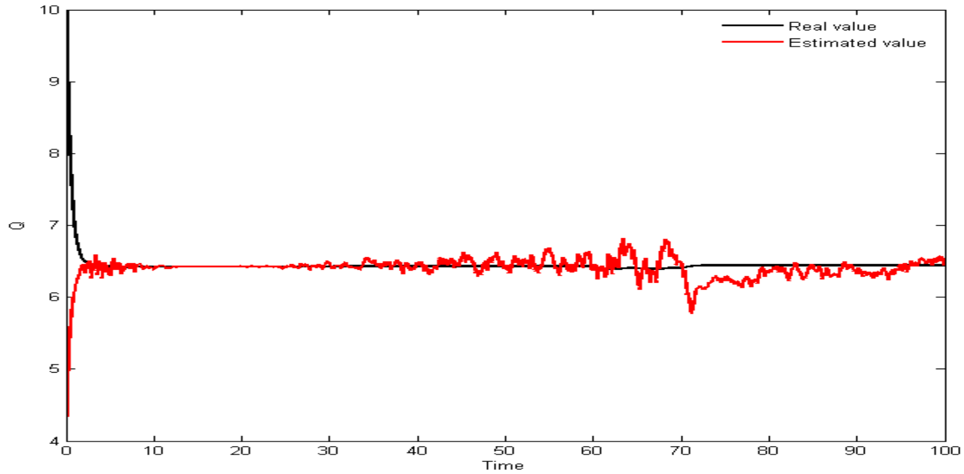

Figure 3.72: Estimation of internal quota-PI controller.

2- Input-output linearization

To derive the control law, Equation (3.29) is used. After computing all terms in the equation, the compensator law would be

$$u = D = \frac{\nu + \rho X - \beta_1 S}{S_{in} - S}. \tag{3.43}$$

A conventional PI controller is applied as an external controller. $K_C = 1, \tau_I = 1$, and $\beta_1 = 1$ are considered as controller's parameters. The performance of controller with measurement noise is illustrated in the following figures.

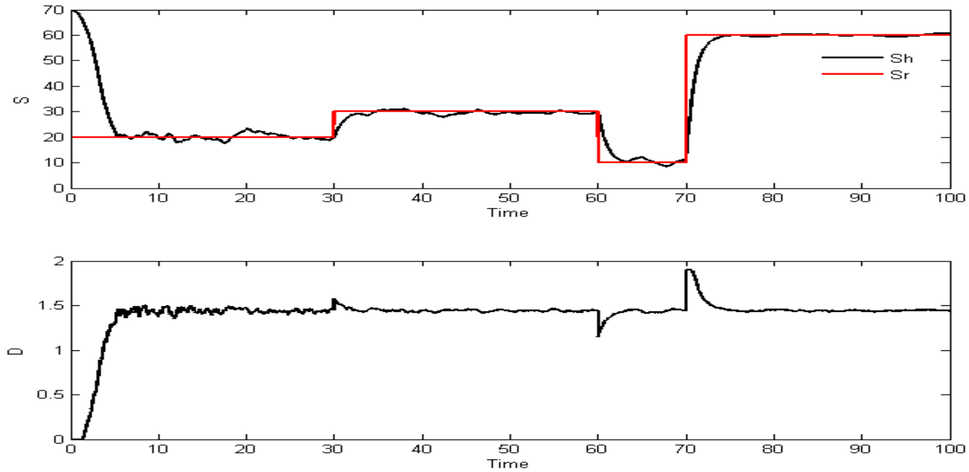

Figure 3.73: Substrate-Dilution-Input-Output linearization.

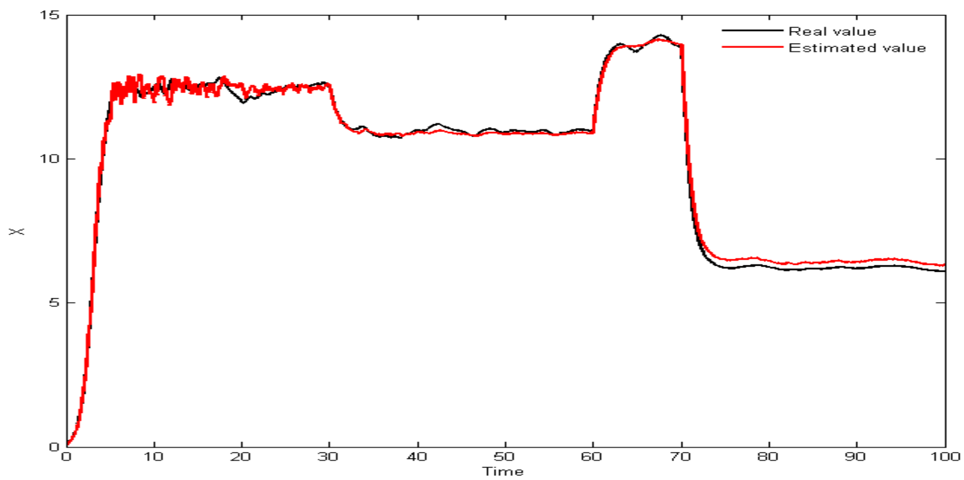

Figure 3.74: Estimation of biomass-Input-Output linearization.

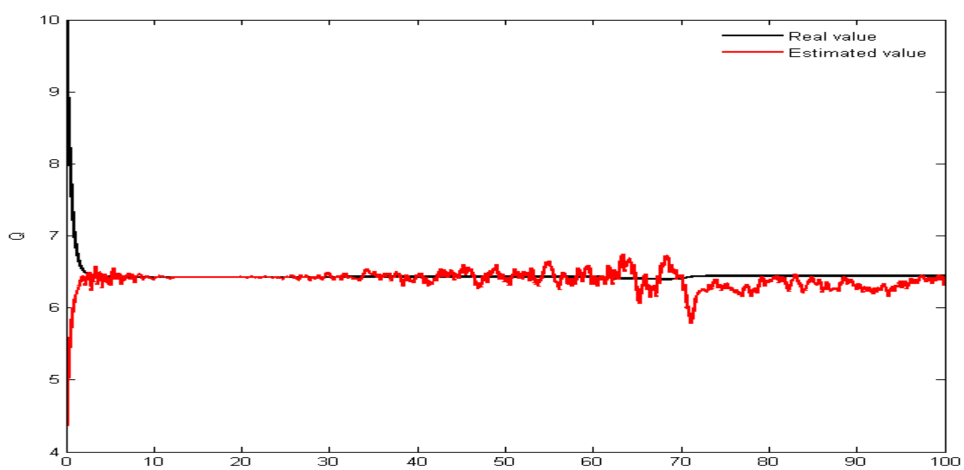

Figure 3.75: Estimation of internal quota-Input-Output linearization.

3- Backstepping Controller

By using the same procedure for designing the backstepping controller, the following control law, in order to regulate the substrate concentration inside the bioreactor , is obtained

$$u = \frac{-\eta^2 Z_1 - c_2 Z_2 \eta + \rho X + \dot{y_d} + c_1^2 Z_1 - c_1 \eta Z_2}{S_{in} - S}. \tag{3.44}$$

The controller's parameters ($c_1 = 2.0, c_2 = 2.0$, and $\eta = 0.5$) are chosen based on simulation results. To evaluate the performance of the observer-controller scheme, the setpoint tracking problem with measurement noise in the biomass concentration is simulated. As it can be seen, the proposed observer-controller scheme, has a good performance.

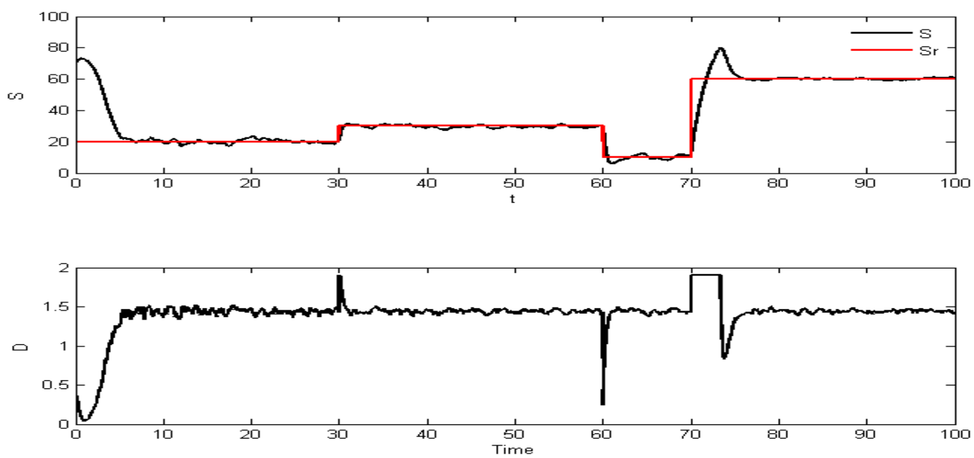

Figure 3.76: Substrate-Dilution rate-Backstepping.

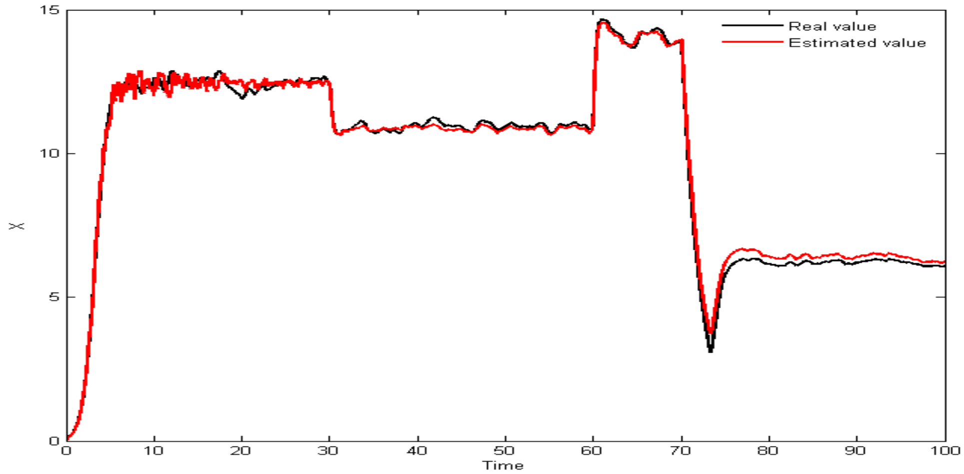

Figure 3.77: Estimation of biomass-Backstepping.

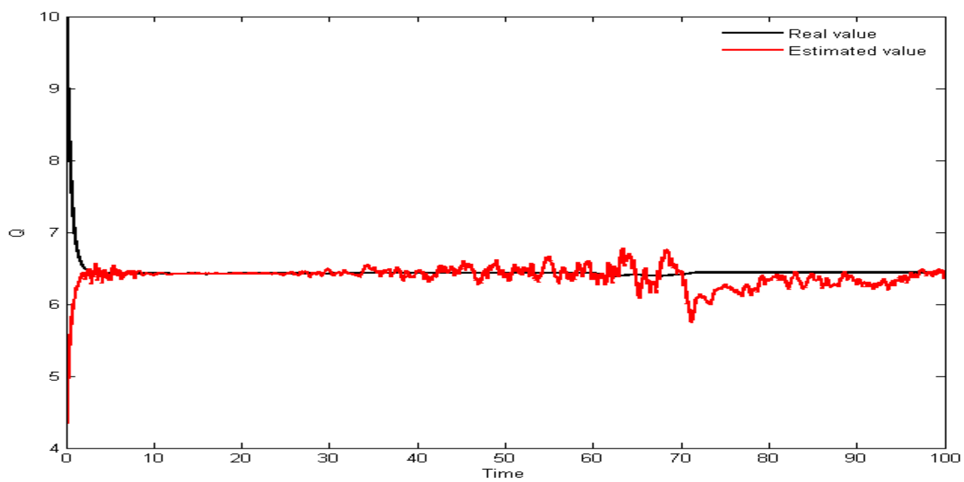

Figure 3.78: Estimation of internal quota-Backstepping.

3.5 Lipid Production in Microalgae

As mentioned in the first chapter, lipid production model includes five state variables. Basically, it includes two parts, the Droop model and lipid production. The model has a cascade structure. The dynamics of the fraction F and L do not affect on the biomass kinetics. Because of this, the cascade structure for designing the observer is chosen. In the following parts, the steady state behaviour and observer design strategy are presented.

3.5.1 Steady State Behaviour

By solving the steady state equations of the model, the following steady state points are obtained

$$Q_s = \frac{K_Q \bar{\mu}}{\bar{\mu} - D},\ S_s = \frac{K_s \rho_s}{\rho_m - \rho_s},\ X_s = \frac{D(S_i n - S)}{\rho_s},$$
$$F_s = (\alpha + \beta) Q_s,\ L_s = (\beta - \alpha) Q_s,\ \text{and}\ \rho_s = D Q_s.$$

In the next figures, the steady state behaviour of the state variables are illustrated.

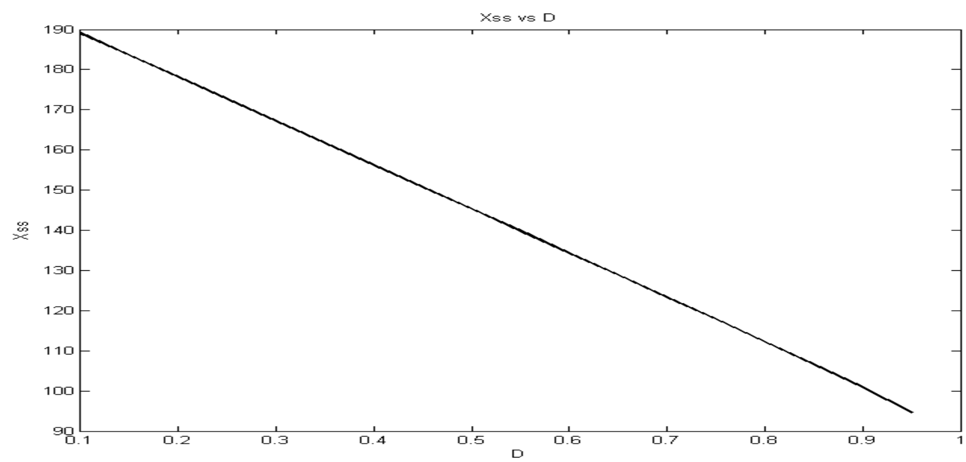

Figure 3.79: Biomass steady state behaviour.

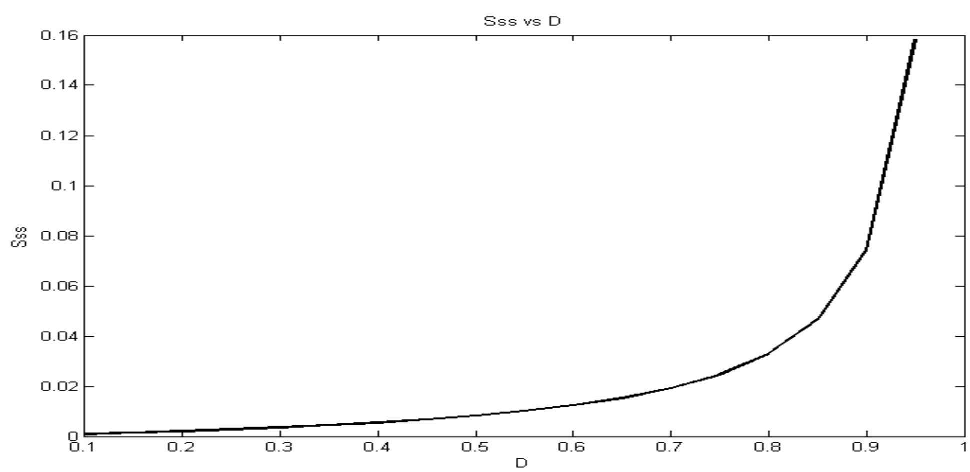

Figure 3.80: Substrate steady state behaviour.

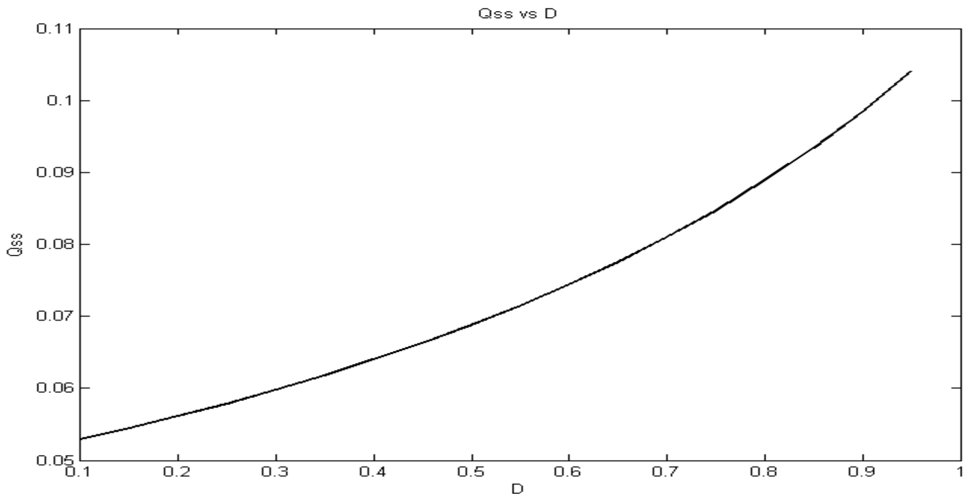

Figure 3.81: Quota steady state behaviour.

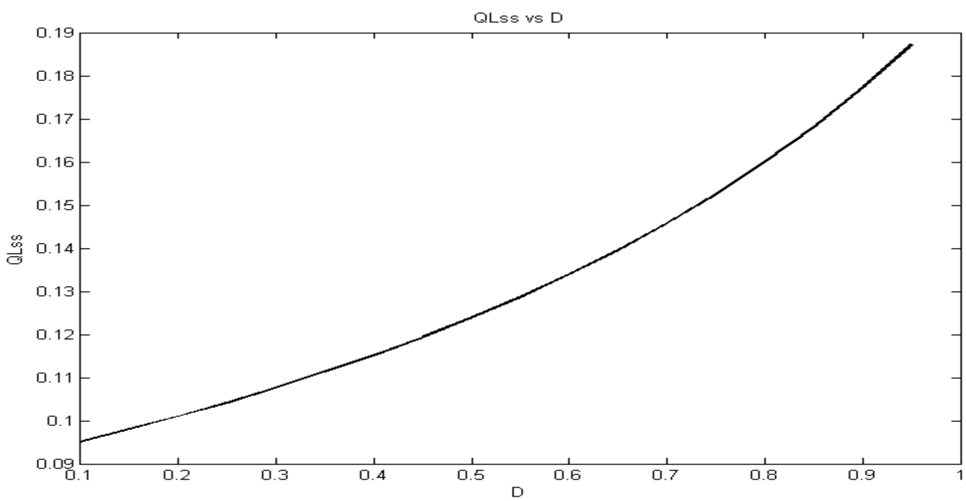

Figure 3.82: Neutral lipid steady state behaviour.

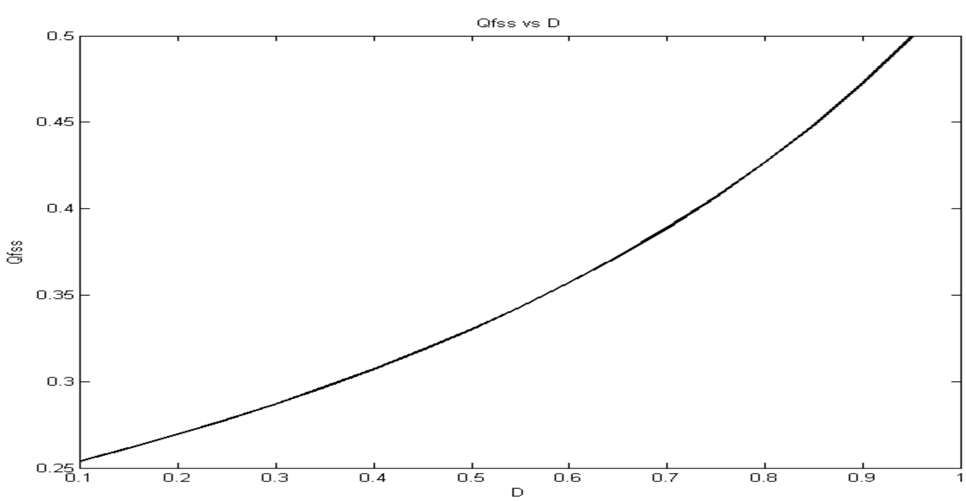

Figure 3.83: Functional carbon steady state behaviour.

As it can be seen, the biomass has a linear relation with respect to dilution rate. But the rest of the variables have nonlinear behaviour. Also, the steady state value of each state variable (except the biomass) increases when the dilution rate increases.

3.5.2 Observer Design

Based on observability property of the Droop model and lipid production model, the following structure is chosen to estimate the internal state of the process from the biomass concentration measurement.

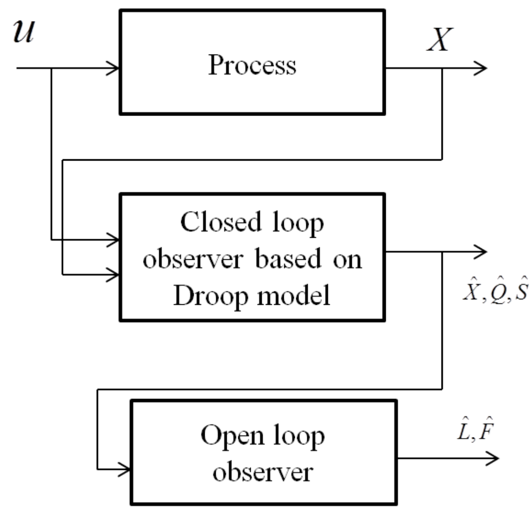

Figure 3.84: Observer loop structure.

Therefore, the dynamic model of the observe would be

$$\begin{aligned} \dot{\hat{X}} &= -D\hat{X} + \hat{\mu}\hat{X} + L_1(y - \hat{y}), \\ \dot{\hat{Q}} &= \hat{\rho} - \hat{\mu}Q + L_2(y - \hat{y}), \\ \dot{\hat{S}} &= D(S_{in} - \hat{S}) - \hat{\rho}\hat{X} + L_3(y - \hat{y}), \\ \dot{\hat{L}} &= (\beta\hat{Q} - \hat{L})\hat{\mu} - \gamma\hat{\rho}, \\ \dot{\hat{F}} &= -\hat{F}\hat{\mu} + (\alpha + \beta)\hat{\rho}, \\ \hat{y} &= \hat{X}. \end{aligned} \tag{3.45}$$

The switching approach is used to compute the closed-loop eigenvalues of the error dynamics. The values of eigenvalues are

$$(\alpha_1, \alpha_2, \alpha_3) = (-4, -2, -1), \ (b_1, b_2, b_3) = (-.075, -.05, -.04), \text{ and}$$

$$(\epsilon_1, \epsilon_2, \epsilon_3) = (-.5, -.6, -.6).$$

In order to evaluate the dynamic behaviour of the proposed observer, a PI controller with $K_C = -0.0078$ and $\tau_I = 1.341$ as controller's parameters, is designed. In the following figures, the performance of designed observer is depicted.

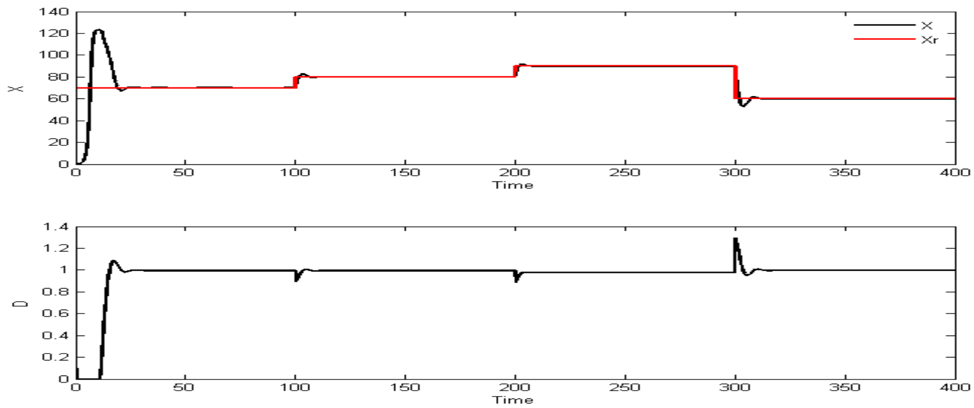

Figure 3.85: Biomass Concentration-Dilution rate.

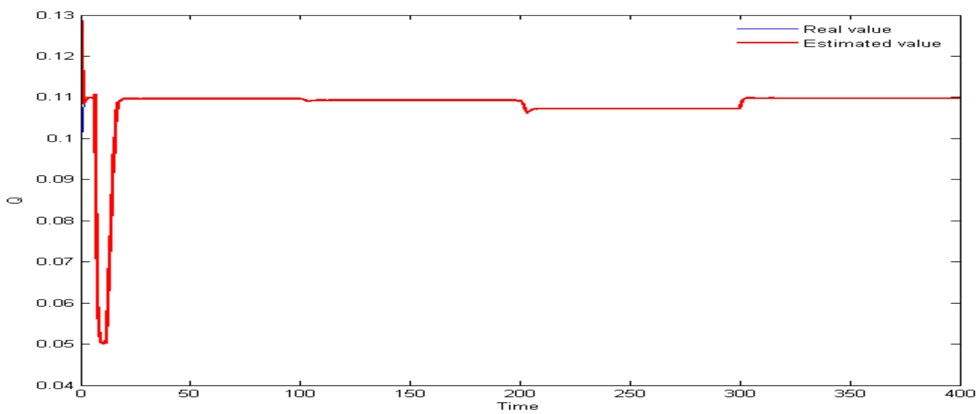

Figure 3.86: Estimation of quota-PI controller.

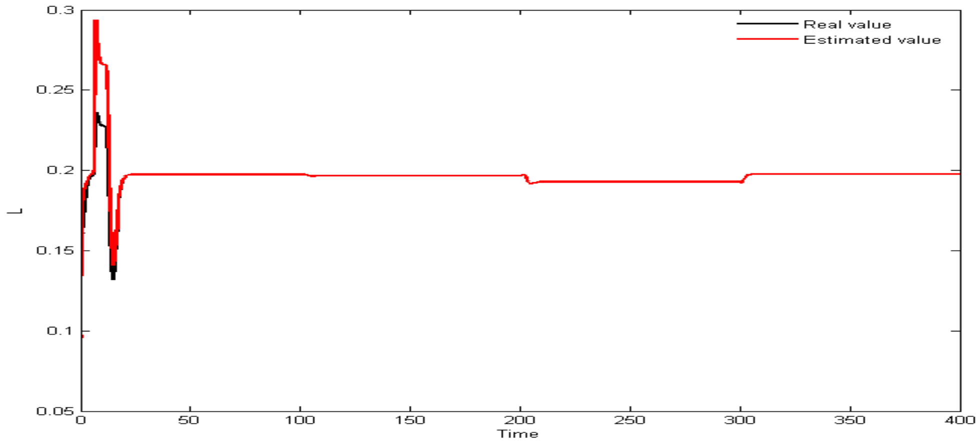

Figure 3.87: Estimation of neutral lipid -PI controller.

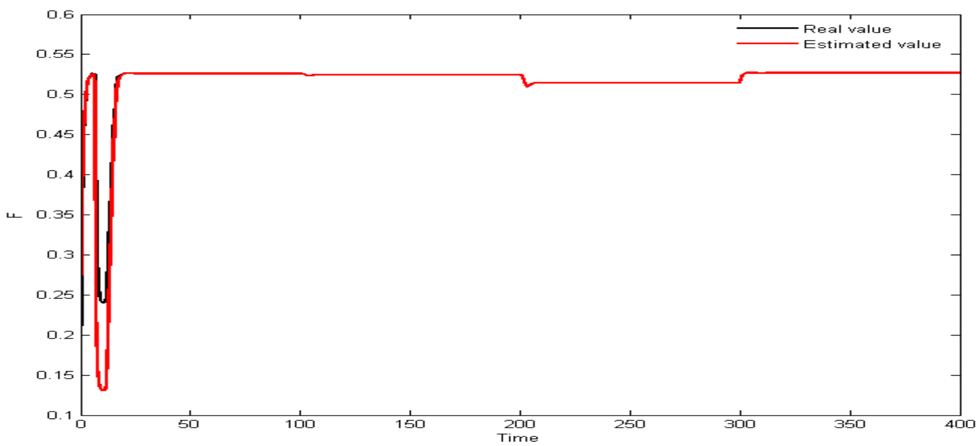

Figure 3.88: Estimation of functional carbon-PI controller.

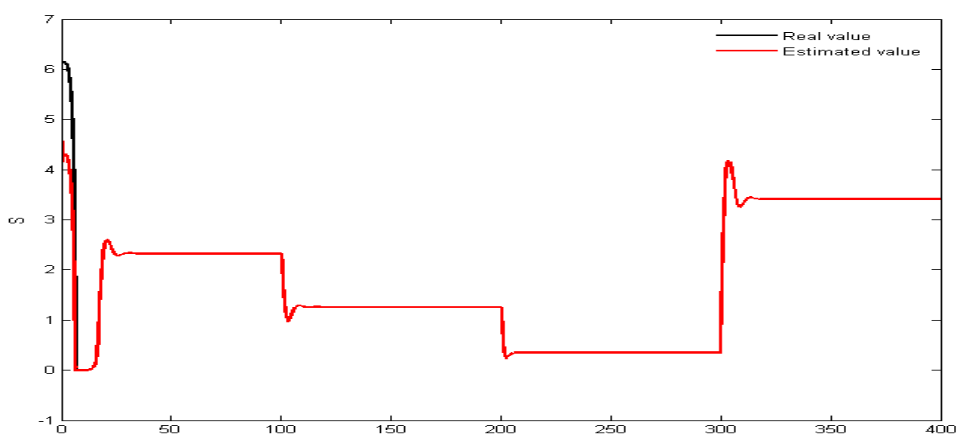

Figure 3.89: Estimation of substrate-PI controller.

As it can be seen, the proposed structure is suitable for estimation of the state variables in the presence of the perfect biomass measurement.

CHAPITRE 4

Conclusion and Future work

The objective of this work was to design of nonlinear observer for microalgae cultivation. In order to design the observer, the Droop model was introduced as the simplest and widely used model to describe the dynamics behaviour of the process. Having the bounded trajectories and observability were two important properties of the model. Based on these properties, the Lipschitz observer with guaranteed stability of the error dynamics was proposed as a good candidate for estimation of the internal state variables of the process. The performance of the designed observer in presence of measurement and process noise was studied and the simulation results showed that it has a good performance. In order to improve the observer performance in presence of the measurement noise, a variable observer gain was proposed instead of constant one. After that, three parameters of the model namely, the maximum substrate uptake rate, the theoretical maximum growth rate and the substrate feed concentration, were chosen to estimate. The results showed that the augmented observer has an acceptable performance. Then, to see the proposed observer performance with advanced process controllers, two observer-based controllers, the input-output linearization and the backstepping approaches, were introduced. The two control strategies were designed in order to control the biomass and the substrate concentration. The simulation results showed that both observer-controller schemes have a good performance in setpoint tracking and load rejection. In addition, the observer performance in order to estimate the internal state variables was very good. Finally, the observer design problem for lipid production in microalgae culture was studied. Because of the structure of the lipid model, the cascade structure was selected in order to estimate the state variables. In this structure, there were two kinds of observers namely, closed-loop and open loop, which are related to the Droop model part and the lipid production part respectively. The simulation results demonstrated that the proposed structure has a good performance in presence of perfect measurements. To this end, the following problems can be considered as future works

1- Implement of designed observer on an experimental set up.

2- Stability analysis of the error dynamics for the discrete time observer.

3- Performance analysis of the discrete time observer with a discrete time controller.

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