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AN UPPER BOUND ON CODED PERFORMANCE WITH NONINDEPENDENT RICIAN FADING. *

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Abstract

New upper bounds are proposed for the evaluation of the error performance of coded systems over noninterleaved or partially interleaved Rician fading channels. The correlation between successive received symbols is exploited to bound the error performance. The bounds allows useful evaluation of coding gains on realistic communication systems without going into lengthy computer simulations. They may also provide a useful tool for the search of good codes for continuous channels with memory. Defining the maximum energy degradation factors (which are independent of added diversity), compact bounds for partially interleaved channels may be expressed in a similar way as for the fully interleaved channel. These factors give an interesting evaluation of fading conditions and may be used to define the cut-off rates of Rician channels with memory.

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1- INTRODUCTION

For realistic channels suffering from nonindependent fading, the evaluation of upper bounds on the bit error rate (BER) is often discarded as prohibitively involved. For systems using error control coding over the Rician fading channel, the evaluation of the error performance is usually restricted to the fully interleaved channel. For the more realistic cases as for most of the continuous channels with memory, one has to resort to lengthy computer simulations in order to obtain meaningful performance results. In this paper, we develop upper bounds on the error performance for the noninterleaved and partially interleaved Rician fading channels. These bounds may be useful tools in the evaluation of the error performance as well as in the search for good codes over unquantized channels with memory.

The error performances of convolutional coding over fading channels has been analyzed for the completely interleaved channels [1]-[4] or the very slowly fading channel [1]. Between these two extremes, a Fritchman or Gilbert model of the hard quantized fading channel is traditionally used to obtain additional knowledge about the error performance of coded systems [5],[6]. As for any meaningful performance evaluation, it is usually obtained through extensive computer simulations of the channel and decoding processes [1],[3],[4],[7],[8].

In this paper we first present a new upper bound on the error probability of coded systems over Rician fading channels with nonindependent fading between received symbols. This bound is a union bound of all pairwise error probability where the correlation between consecutive received code symbols is exploited. Using results from [9] and [10], the distribution of the sum of correlated energy levels of the received signal is first determined. The average pairwise error probability can then be evaluated for coherent, differential and noncoherent demodulation with perfect channel-state knowledge. Using an union bound argument, tight upper bounds on the bit error probability may then be computed for any coded systems for which the fine structure of the codeword weight distribution is known.

Bounds on the error performance that use the fine structure of the codes are quite useful in evaluating and comparing different coding techniques. Unfortunately these bounds do not provide a priori information about the channel conditions neither do they provide straightforward design rules for communication systems over Rician channels. By defining limits on the degradation factor induced by the correlation between received symbols, further bounds may be expressed independently of the fine weight structure of the code. For convolutional codes, the bit error performance for P_b these channels may be bounded with expressions that are not different from those on memoryless channels, that is, of the form:

$$P_b \leq \frac{1}{b} \sum_{d=d_{free}}^{\infty} C_d \; D^d$$

where D is parameter which is strictly function of the channel characteristics, d_{free} is the free distance of the code, the coefficients C_d are obtained from the transfer function of the code [11], and where b represents the number of information bits encoded into v codesymbols to obtain a rate b/v convolutional code.

Therefore, the bit error performance may be bounded for a wide variety of Rician fading channels. Furthermore, performance of communication systems using space or frequency diversity may also be evaluated. Using short constraint length convolutional codes a preliminary comparison with simulation results from [1] and [8] indicates that these bounds give a good approximation of the BER for partially interleaved and noninterleaved Rayleigh channels. Useful results are thus obtained without going into lengthy computer simulations.

The paper is divided into four principal sections. In the next section, exact union bounds on the coded error performance over Rician channels are developed. Further bounding techniques to simplify the computation of the bounds constitute the main object of section three. The effect of interleaving and added diversity is treated in the fourth section. Finally, the cases of particular channels are studied in section five.

2- EXACT UNION BOUNDS ON THE PERFORMANCE

The communication system under consideration in this paper consists of a coder, a modulator, a fading channel, a demodulator and a decoder. The channel is a Rician channel where received waveforms are composed of a constant specular component, a varying scattered component and additive noise. The coder maps information bits into codesymbols, and the modulator maps these codesymbols into waveforms that are transmitted. The demodulator and decoder pair attempt to determine the most likely sequence of information bits according to the received channel signal.

In this section exact union bounds on coded error performances are developed taking into account the correlation between the energies of subsequent channel symbols. The section is divided into four parts. The Rician channel is first modeled and the joint distribution of the received signal amplitudes is expressed as a function of typical channel parameters. The Laplace transform of the sum of correlated powers of the received signal is then developed in a form similar to the characteristic function of the sum of squared Gaussian variables. This allows to explicit the density function of the sum of correlated powers of the received signal. In the third subsection, the exact pairwise error probabilities between two codewords are expressed for unquantized maximum ratio combining or perfect channel state knowledge [3],[4], for BPSK, DPSK and noncoherent FSK (NC-FSK) modulated signals. Finally, using these results, exact union bounds on coded error performance are established for maximum likelihood decoding.

Although these bounds are not expressed in a compact form and are more elaborate to compute than the corresponding bounds on memoryless channels, they do provide useful information about practical systems performance when no other method existed but lengthy computer simulation. Furthermore these bounds may be particularly useful in comparing different coded systems on Rician fading channels.

2.1 The Rician channel.

On a Rician channel, for a transmitted sinusoïdal waveform, the received signal may be written as [1],[9]:

$$r(t) = \alpha \cos(\omega_c t) + S_c(t) \cos(\omega_c t) + S_s(t) \sin(\omega_c t) + n(t)$$

or in a complex form:

$$r(t) = \mathrm{Re}[(\alpha + S(t)e^{j\Phi(t)})e^{j\omega_c t}] + n(t)$$

where

$$S_c(t) = S(t) \cos(\Phi(t))$$

$$S_s(t) = S(t) \sin(\Phi(t))$$

In these expressions, α represents the amplitude of the specular component, $S_c(t)$ and $S_s(t)$ are zero mean stationary Gaussian random processes or equivalently, $S(t)e^{j\Phi(t)}$ is a complex Gaussian random process and $n(t)$ is the white Gaussian noise.

Let σ^2 be the variance of $S_c(t)$ and $S_s(t)$, the mean squared value of the scattered component is then σ^2. For convenience, we define $\gamma = \frac{\alpha^2}{2\sigma^2}$ as the ratio of the specular to the diffuse received signal energy.

Let us now define a vector $\underline{X} = [X_1,..,X_{2N}]$ consisting of a sequence of the in-phase and quadrature amplitudes of the received signal taken at $t = t_1, t_2, t_3 \ldots, t_N$ (see Figure 1), that is:

$$\underline{X} = [\alpha + S_c(t_1),\ \alpha + S_c(t_2),\ \ldots,\ \alpha + S_c(t_N),\ S_s(t_1)\ \ldots\ S_s(t_N)]$$

The joint Gaussian density function of $\underline{X}$ is thus

$$f_{\underline{X}}(\underline{x}) = \frac{e^{-\frac{1}{2}(\underline{x} - \overline{\underline{X}})\, M^{-1}(\underline{x} - \overline{\underline{X}})'}}{|M|^{1/2}\,(2\pi)^N} \tag{1}$$

where $\overline{\underline{X}}$ is the average value of $\underline{X}$, that is $\overline{\underline{X}} = [\alpha,\alpha,\alpha,..\alpha,0,0,0,..0] = \alpha\,[1,..,1,0,..,0]$, $\underline{X}'$ is the transpose of $\underline{X}$, M is the autocovariance matrix defined below and $|M|$ is its determinant:

$$M = (M_{ij})\ ,\ M_{ij} = E[(X_i - \overline{X}_i)\,(X_j - \overline{X}_j)]$$

where $E[.]$ denotes expectation.

Note that M is positive definite and

$$M_{ii} = E[S_c(t)S_c(t)] = E[S_s(t)S_s(t)] = \sigma^2$$

For any j and k, j, k = 1,2...N, we have:

$$M_{jk} = M_{N+j,N+k} = E[S_c(t_j)S_c(t_k)] = E[S_s(t_j)S_s(t_k)] = R(t_j - t_k) = \sigma^2\rho(t_j - t_k)$$

where $R(\tau)$ and $\rho(\tau)$ are respectively the covariance and normalized autocovariance function associated with the channel.

Furthermore, with little loss in generality, we may restrict the analysis to symmetrical fading spectra and hence [9]:

$$E[S_c(t_j)S_s(t_k)] = M_{j,k+N} = M_{j+N,k} = 0$$

Thus the covariance matrix has the form:

$$M = \sigma^2 \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1N} & 0 & 0 & \cdots & 0 \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2N} & 0 & 0 & \cdots & 0 \\ \rho_{31} & \rho_{32} & 1 & \cdots & \rho_{3N} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \rho_{N1} & \rho_{N2} & \rho_{N3} & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & \rho_{N+1,N+2} & \cdots & \rho_{N+1,2N} \\ 0 & 0 & 0 & \cdots & 0 & \rho_{N+2,N+1} & 1 & \cdots & \rho_{N+2,2N} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \rho_{2N,N+1} & \rho_{2N,N+2} & \cdots & 1 \end{bmatrix}$$

where $\rho_{ij} = \rho(t_i - t_j)$.

Now let $L=(\rho_{ij})$ be the upper left N by N matrix, then we can write:

$$M = \sigma^2 \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix}$$

These equations will be useful to determine the distribution of energy sequences and thus to obtain bounds on the error performance.

2.2 Distribution of the sum of correlated energies of the received signal and its Laplace transform.

To obtain the error performance of coded systems over Rician channels, the bounds on the bit error probabilities must be averaged over all possible received energy sequences. The distribution of these energy sequences and its Laplace transform are needed to perform the averaging.

The distribution of the sum of the received signal energies corresponds to the distribution of the sum of squared Gaussian random variables. This distribution may be determined using a Laplace transform technique [9][10].

In this paper, the fading is considered varying slowly compared to the duration of a channel symbol. Denoting the sum of the received signal energies and powers by E and W respectively, the distribution of the sum of different received symbol energies f(E) is thus similar to the distribution of the corresponding sum of signal powers f(W). If T_c is the duration of a channel symbol, then

$$E=T_c\ W=T_c/2\ \{(\alpha+S_c(t_1))^2 + S_s(t_1)^2 + (\alpha+S_c(t_2))^2 + S_s(\tau_2)^2 + \ldots\} = (T_c/2)\ \underline{X}\ \underline{X}'$$

Denoting by p(z) the Laplace transform of f(W):

$$p(z) = \int_{-\infty}^{+\infty} e^{-zW} f(W)\ dW = E[e^{-\frac{1}{2} z\ \underline{X}\ \underline{X}'}] \qquad (4)$$

The evaluation of p(z) for the distribution of $\underline{X}$ as expressed by (1) has been carried out in [10] and is reproduced in Appendix 1. We have

$$p(z) = \frac{e^{-\frac{1}{2}\,\bar{\underline{X}}\,M^{-1}(I-(Mz+I)^{-1})\,\bar{\underline{X}}'}}{|Mz + I|^{1/2}}$$

The exponent of this expression may be simplified as follows:

$$M^{-1}(I-(Mz+I)^{-1}) = M^{-1}(Mz+I)^{-1}\;((Mz+I)-I)$$

$$= M^{-1}(Mz+I)^{-1}Mz = z(Mz+I)^{-1}$$

since $(Mz+I)^{-1}$ has the same eigenvectors as M, the product $(Mz+I)^{-1}$ M commutes.

Replacing M by $\sigma^2 \begin{pmatrix} L & 0 \\ 0 & L \end{pmatrix}$ we have

$$P(z) = \frac{e^{-\frac{1}{2}\,z\bar{\underline{X}} \begin{pmatrix} (\sigma^2 zL+I)^{-1} & 0 \\ 0 & (\sigma^2 zL+I)^{-1} \end{pmatrix} \bar{\underline{X}}'}}{|\sigma^2 Lz+I|}$$

Finally, since $\bar{\underline{X}} = \alpha[1,1,1,1,\ldots.0,0,0\ldots]$, the quadratic form at the numerator is simply the sum of all the elements of $(\sigma^2 zL+I)^{-1}$ and hence

$$p(z) = \frac{e^{-\frac{1}{2}\,z\,\alpha^2\,\underline{b}(\sigma^2 zL+I)^{-1}\,\underline{b}'}}{|\sigma^2 zL+I|} \qquad (5)$$

where $\underline{b}=[b_1,\ldots,b_N]=[1,1,1,1,..]$.

If consecutive received symbols are completely independent ($L_{ii}=1, L_{ij}=0$ if $i \neq j$) then p(z) reduces to the form associated with the characteristic function of a chi-squared random variable:

$$p(z) = \left[\frac{e^{-\frac{1}{2} z \alpha^2/(\sigma^2 z+1)}}{(\sigma^2 z + 1)} \right]^N \tag{6}$$

Equation (5) will be quite useful in obtaining the performance of different types of modulation systems over Rician channels. For example, the error probability for noncoherent FSK modulation (uncoded, N=1) with received signal energy E_s may be written as:

$$P(e) = \frac{1}{2} E\left[e^{-\frac{1}{2}\frac{Es}{No}}\right] = \frac{1}{2} E\left[e^{-\frac{Tc}{2No} W} \right] = \frac{1}{2} p(z)\Big|_{z = \frac{Tc}{2No}}$$

$$= \frac{e^{-2\frac{Es}{No}(\frac{\gamma}{1+\gamma}) / (\frac{Es}{No}(\frac{1}{1+\gamma}) + 2)}}{(\frac{Es}{No} + 2)}$$

since $\gamma = \frac{\alpha^2}{2\sigma^2}$, $\frac{E_s}{N_o} = (\frac{\alpha^2}{2} + \sigma^2)\frac{T_c}{No}$, $\sigma^2 \frac{T_c}{No} = \frac{E_s}{No}(\frac{1}{1+\gamma})$ and $\frac{\alpha^2 T_c}{2No} = \frac{E_s}{No}(\frac{\gamma}{1+\gamma})$.

Furthermore, f(W) may be obtained by usual inverse transform techniques. In particular, if p(z) has only simple poles, f(W) is a sum of exponentials of the form [9]:

$$f(W) = \sum_{n=1}^{N} F_n \, e^{Z_n W} \tag{7}$$

where Z_N are the real negatives poles of p(z) and where F_N is given by:

$$F_N = \lim_{z \to Z_N} [(z-Z_N)\, P(z)]$$

With this distribution of the sum of correlated signal powers, it is possible to average error probabilities that are functions of the received signal's energy.

2.3 Average pairwise error probability for unquantized demodulation.

In this section, equations (5) and (7) are used to find the pairwise error probability between two codewords for a signal received using unquantized PSK, DPSK and NC-FSK demodulators with perfect channel state knowledge or maximal ratio combining [3], [4]. The evaluation of these probabilities is rather involved but in section 3 we develop bounds that are much easier to compute.

We begin by defining the covariance matrix using the positions in which two codewords c_1 and c_2 differ. If two codewords are different in d positions (specifically positions i_1, i_2,.., i_d), the covariance matrix for this pair of codewords is:

$$L_{c_1,c_2} = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & & \cdots & \rho_{1,d} \\ \rho_{21} & 1 & \rho_{23} & & & \\ \cdot & \cdot & & & & \rho_{d-1,d} \\ \rho_{d,1} & \cdot & & \rho_{d,d-1} & & 1 \end{bmatrix} \tag{8}$$

where $\rho_{jk} = \rho(T_c\ |i_j - i_k|) = \dfrac{1}{\sigma_2} R(T_c\ |i_j - i_N|)$ and where T_c is again the duration of a channel symbol.

For example consider two codewords c_1 = (1101) and c_2 = (0110), which differ in 3 positions: 1,3 and 4. Hence the particular covariance matrix for this pair of codewords is:

$$L_{c_1 c_2} = \begin{bmatrix} 1 & \rho(2Tc) & \rho(3Tc) \\ \rho(2Tc) & 1 & \rho(Tc) \\ \rho(3Tc) & \rho(Tc) & 1 \end{bmatrix}$$

Hence the distribution of the sum of the d=3 received symbol energies and its Laplace transform may be given using equations (5),(6),(7) with the corresponding covariance matrix.

Unquantized PSK

For unquantized PSK demodulation, the pairwise sequence error probability conditioned on the sum of the powers of the different coded symbols with maximal ratio combining is [12]:

$$P_e\ (W_{c_1 c_2}) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2W_{c_1 c_2} T_c}{No}}}^{\infty} e^{-\frac{1}{2} t^2}\, dt = Q\left(\sqrt{\frac{2W_{c_1 c_2} T_c}{N_o}}\right) \qquad (9)$$

where

$$W_{c_1 c_2} = \tfrac{1}{2}\, \underline{X}_{c_1 c_2}\, \underline{X}'_{c_1 c_2}$$

$$\underline{X}_{c_1 c_2} = [\alpha + S_c(T_c i_1), \ \alpha + S_c(T_c i_2) \ \ldots, \ S_S(T_c i_1), \ S_S(T_c i_2) \ \ldots]$$

Following [9], we take the average of (9) over the distribution of W given by (7) with the covariance matrix defined in (8). The pairwise error probability is then given by:

$$P_e = \int_0^\infty Q\left(\sqrt{\frac{2W_{c_1 c_2} T_c}{No}}\right) f(W_{c_1 c_2}) \, d\, W_{c_1 c_2}$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty \int_{\sqrt{\frac{2WST_c}{N_o}}}^\infty e^{-\frac{1}{2} t^2} \sum_{n=1}^{d} F_N \, e^{Z_n W} \, dt \, dW$$

$$= \sum_{n=1}^{d} \frac{Fn}{-2Z_n} \left(1 - \frac{1}{\sqrt{1 - Z_N N_o / T_c}}\right) \qquad (10)$$

if all the poles Z_N are distinct. Here Z_N and F_N are found with the Laplace transform associated with the particular correlation matrix $L_{c_1 c_2}$.

Unquantized DPSK and NC-FSK

It is well known [13] that maximal ratio combining of DPSK demodulated signals provides exactly a 3dB energy gain over maximal ratio combining of noncoherent FSK signals. Hence all the results for NC-FSK may be used to obtain the coded performance of DPSK.

From [9], the uncoded error probability for NC-FSK with maximal ratio combining and M-fold diversity is:

$$P_e = \sum_{m=o}^{M-1} \frac{1}{m!} \zeta_m \left(\frac{-T}{2No}\right)^m \left[\frac{d^{(m)}}{dz^m} P(z)\right]_{z = \frac{T_c}{2No}} \tag{11}$$

where p(z) is associated with the particular covariance matrix L of M order diversity and

$$\zeta_m = \sum_{j=m}^{M-1} \frac{(M-1+j)!\ 2^{-d-j}}{(M-1+m)!\ (j-m)!}$$

To obtain a pairwise coded sequence error probability between codewords c_i and c_j, we simply substitute L by $L_{c_i c_j}$ and M by $d_H(c_i, c_j)$, the Hamming distance between c_i and c_j.

2.4 Union bound on coded error probability

With expressions evaluating pairwise error probabilities, upper bounds on coded performance may be established using a union bound. Specifically for a linear (n,K) block code [12], with a set of codewords $C = \{c_o, \ldots, c_{2^K}\}$, and where c_o denotes the all zero codeword, then the BER is given by

$$P_b \le \frac{1}{K} \sum_{j=1}^{2^K} B_j\ P_e(c_j, c_o)$$

where B_j is the Hamming weight of the binary information sequence corresponding to codeword c_j. For a rate r=b/v convolutional code, if the c_j, $j=1,2,3..\infty$ are all the codewords whose corresponding b first information bits differ from the zero codeword c_0, we obtain a similar bound:

$$P_b \le \frac{1}{b} \sum_{j=1}^{\infty} B_j \; P_e(c_j, c_o) \tag{12}$$

Note that the usual bound on the error performance does not take into account all the individual codewords. For memoryless channels, the pairwise error probability does not depend upon the positions of different symbols. The pairwise error probability is a function of the Hamming distance $d_H(c_j, c_k)$ between codewords, and hence on memoryless channels the BER is bounded by:

$$P_b \le \frac{1}{b} \sum_{j=1}^{\infty} B_j \; P(d_H(c_j, c_o)) \quad = \frac{1}{b} \sum_{d=d_{free}}^{\infty} C_d \quad P(d) \tag{13}$$

where C_d is the sum of all the B_j for which codewords have the same Hamming weight d, and where d_{free} (or d_{min} for block codes) is the smallest of these weights.

For nonfully interleaved Rician channels the exact union bound on the error performance is obtained by combining equations (12) and (10) or (12) and (11) for unquantized PSK, DPSK and NC-FSK demodulation respectively. In the next sections these bounds are refined and a compact form resembling (13) will be introduced. The problem associated with finding a bound like for memoryless channels consists of developing an expression for P(d) that is independent of the fine structure of the code.

3- REFINED BOUNDS TO SIMPLIFY THE COMPUTATION OF THE ERROR PERFORMANCE

In this section, the union bound on coded performance is further refined to simplify the evaluation of the error performance. We first develop several bounds on the Laplace transform and its derivatives. The pairwise error probability, if expressed as a function of the Laplace transform, may then be bounded by expressions that are easier to compute. By defining limits on the degradation factor induced by the correlation between received symbols, new bounds may be expressed independently of the fine weight structure of the code. This implies that the performance of coded systems over Rician fading channels may be bounded just like for memoryless channels.

Bounds that use the fine structure of codes, such as the positions of erroneous coded symbols, are quite suitable for comparing different coding techniques and for obtaining fairly tight upper bounds on the performance, if the correlation between received symbols is not too important. Unfortunately they do not give a priori information about the channel condition and they do not give straightforward design rules for communication systems over Rician channels. Therefore by providing a limit on the degradation due to residual correlation, the effect of interleaver design (see section 4), diversity and transmission rate on overall performances may be evaluated.

This section is divided in two principal subsections. We first develop several bounds on the Laplace transform (5). We then use these bounds to find new expressions for bounding the BER of coded systems for the different modulation techniques of interest.

3.1 Bounds on the Laplace transform and its derivatives.

In this subsection different bounds on equation (5) are developed. The usefulness of each is determined by their computational complexity for particular channels. The main objective is to simplify the expressions of the quadratic form and the determinant in the Laplace transform given by (5):

$$p(z) = \frac{e^{-\frac{1}{2} z \alpha^2 \underline{b} (\sigma^2 zL+I)^{-1} \underline{b}'}}{|\sigma^2 zL+I|}$$

where $\underline{b}=[1,1,...1]$.

Bounding the quadratic form

We now bound the quadratic form by a similar expression containing its inverse, so that the inversion of $(\sigma^2 zL+I)$ will not have to be carried out.

In appendix 2, it is shown that

$$\underline{b}(\sigma^2 zL+I)^{-1} \underline{b}' \geq \frac{N^2}{\underline{b}(\sigma^2 zL+I)\underline{b}'}$$

Using this result, instead of computing the sum of all the elements of the inverse of $(\sigma^2 zL+I)$, we bound this sum and compute only the sum of the elements of L:

$$\underline{b}(\sigma^2 zL+I)^{-1}\underline{b}' \geq \frac{N}{\sigma^2 z(\frac{\underline{b}L\underline{b}'}{N})+1} \geq \frac{N}{\sigma^2 z\lambda_{Max}[L]+1} \tag{15}$$

where $\lambda_{Max}[L]$ is the greatest eigenvalue of L and may upper bound $\frac{\underline{b}L\underline{b}'}{N}$ using fundamental properties of quadratic forms [14].

Depending on the channel both bounds in equation (15) may be useful in the evaluation of bounds for the Laplace transform of the sum of squares. For particular autocovariance functions it may be possible to evaluate directly the sum of the elements in the covariance matrix, or an analysis of this matrix may provide the maximum eigenvalue of this matrix.

We also point out that if it is possible to find the maximum value of $\frac{(\underline{b}L\underline{b}')}{N}$ or $\lambda_{Max}[L]$ for all possible coded sequences, the numerator of p(z) will be upper bounded by a term that is exponentially dependent of N and independent of the symbol positions. Hence the numerator will have a form similar to the one used for memoryless channels.

Bounding the determinant

The denominator of the Laplace transform of the sum of squares (5) is

$$|\sigma^2 zL+I|$$

Denoting by λ_i , i=1,2,...N the eigenvalues of L, this determinant may be written as:

$$|\sigma^2 zL+I| = \prod_{i=1}^{N} (\sigma^2 z\lambda_i+1) = (\sigma^2 z)^N |L| + \ldots + \sigma^2 z \sum_{i=1}^{N} \lambda_i+1$$

Since the trace of a matrix is invariant upon diagonalization, the sum of the eigenvalues is always equal to N and we may write

$$|\sigma^2 zL+I| \geq (\sigma^2 z)^N \ |L| + \sigma^2 zN+1 > (\sigma^2 z \sqrt[N]{|L|})^N \tag{16}$$

We may also use λ_{Min}, the smallest eigenvalue of L, to further bound:

$$|\sigma^2 zL+I| \geq (\sigma^2 z\lambda_{Min}+1)^N \tag{17}$$

Hence the remarks that applied for the numerator apply here also but we now need a lower bound on $\sqrt[N]{|L|}$ or λ_{Min}.

Bounding the derivatives of p(z)

For DPSK and NC-FSK demodulations several derivatives of p(z) have to be computed in order to obtain a union bound on the coded error performance. If p(z) has a small number of poles, this may not be a serious computational problem. But if p(z) has many poles, as is usually the case for powerfull codes, the bound becomes impractical. Hence a bound on the successive derivatives of p(z) may be very usefull.

In Appendix 3 we prove that when α=0 (Rayleigh fading)

$$(-1)^M \frac{d^{(M)}}{dz^M} \; p(z) \;\; \leq \;\; \frac{(N+M-1)!}{(N-1)!} \frac{p(z)}{z^M} \tag{18}$$

furthermore a similar bound may be obtained for a Rician channel, that is $\alpha \neq 0$.

In this subsection, we have obtained bounds on the Laplace transform p(z). These bounds simplify the computation of the coded error performance and are all asymptotically tight for high signal-to-noise ratios. In summary we have

- a bound on the quadratic form at the numerator (15) which does not contain matrix inversions,
- bounds on the determinant at the denominator (16) and (17),
- a bound on the successive derivatives of the Laplace transform (18) which alleviates lengthy symbolic differentation of p(z).

In addition, the values of complex matrix operations become independent of the signal-to-noise ratio. For a given code and symbol duration, these matrix operations need to be computed once for a whole set of performance curves. Using these bounds, we may also obtain energy degradation factors which are independent of specific codewords.

For all fading channels (see section 5) we may define the maximum energy degradation at the numerator of p(z) by:

$$\zeta_n = \frac{1}{\underset{L}{\mathrm{Max}} \left(\frac{\underline{b}L\underline{b}'}{N}\right)} \leq \frac{1}{\lambda_{Max}}$$

and the maximum energy degradation at the denominator by:

$$\zeta_d = \underset{L}{\mathrm{Min}} \left(\sqrt[N]{|L|}\right)$$

and

$$\zeta_\lambda = \mathrm{Min}(\lambda_{Min}) \leq \zeta_d$$

where all the minima and maxima are evaluated for all possible L matrices.

As will be discussed in section 5, ζ_n, ζ_d and ζ_λ should be independent of N. With these new values an upper bound on p(z) may be given by:

$$p(z) \leq \left[\frac{e^{-\frac{1}{2} z\alpha^2 \zeta_n/(\sigma^2 z+\zeta_n)}}{\sigma^2 z \ \zeta_d} \right]^N$$

or by:

$$p(z) \leq \left[\frac{e^{-\frac{1}{2} z\alpha^2 \zeta_n/(\sigma^2 z+\zeta_n)}}{(\sigma^2 z \ \zeta_\lambda + 1)} \right]^N \qquad (19)$$

3.2 Refining the union bound on coded error probability.

We now present two principal bounding techniques on the error performance of coded systems over Rician fading channels. The first technique uses the fine structure of the weight spectrum of the code. This bound is not in a form similar to the one for memoryless channels, but it allows a relatively fast and accurate evaluation of the coded error performance. Furthermore, this bounding technique provides useful information for comparing different codes on a continuous channel with memory.

The second technique does not take into account the position of incorrect channel symbols. It is a somewhat looser bound but it is expressed in a compact form. This bounding technique may therefore be quite useful in determining the maximum degradation of the coding gain associated with residual correlation between coded symbols. These compact bounds also allow straightforward methods to evaluate powerful communication systems on fading channels.

Unquantized PSK

For this type of demodulator, we first bound the error function by its exponential asymptotic expression, using

$$Q(x) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2}\, dt \le \frac{1}{2}\, e^{-x^2/2} \tag{21}$$

Equation (12) gives an exact union bound on the BER. By using equation (9) for unquantized PSK and averaging over all energy sequences, we obtain:

$$P_b \le \frac{1}{b} \sum_{j=1}^{\infty} B_j \; E\left[Q\left(\sqrt{\frac{2T_c W_{c_j c_0}}{N_0}}\right)\right]$$

where $E[.]$ denotes expectation.

Using (21) we can write:

$$P_b \le \frac{1}{b} \sum_{j=1}^{\infty} B_j \; E\left[e^{-\left(\frac{T_c}{N_0}\right) W_{c_j c_0}}\right] = \frac{1}{b} \sum_{j=1}^{\infty} B_j \; p_{c_j}(z) \Big|_{z = T_c/N_0}$$

and finally using (5) and (15)

$$P_b < \frac{1}{2}\frac{1}{b} \sum_{j=1}^{\infty} B_j \; \frac{e^{-\frac{1}{2}(T_c/N_0)\alpha^2 \; d_{cj} \; / \; \left[\sigma^2 T_c/N_0 \left(\frac{\underline{b}L_{cj}\underline{b}'}{d_{c_j}}\right)+1\right]}}{\left|\sigma^2 \left((T_c/N_0)L_{cj} + I\right)\right|}$$

or equivalently

$$P_b < \frac{1}{2}\frac{1}{b} \sum_{j=1}^{\infty} B_j \; \frac{e^{-d_{cj} \frac{E_s}{N_0}\left(\frac{\gamma}{1+\gamma}\right) \Big/ \left[\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)\left(\frac{\underline{b}L_{cj}\underline{b}'}{d_{cj}}\right) + 1\right]}}{\left|\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right) L_{cj} + I\right|} \tag{22}$$

In this expression, $\frac{E_s}{N_0}$ is the channel symbol signal-to-noise ratio, γ is the specular to diffuse power ratio and B_j, d_{cj}, L_{cj} are respectively the information errors, the Hamming weight and the covariance matrix associated with the codeword c_j.

The evaluation of (22) involves computing both the sum of d_{cj}^2 terms and the determinant of a d_{cj} by d_{cj} matrix. For well behaved channels, if the codeword are ordered according to their weights, the sum (22) converges rapidly as in the case of memoryless channels.

To obtain a further compact form by using the degradation factors, equations (18) or (19) may be used to obtain the usual form associated with memoryless channels where, with no loss in generality, we may consider here convolutional codes:

$$P_b \le \frac{1}{2}\frac{1}{b}\sum_{d=d_{free}}^{\infty} C_d\, D^d \tag{23}$$

where

$$D = \frac{e^{-\frac{E_s}{N_0}\left(\frac{\gamma}{1+\gamma}\right)\zeta_n \Big/ \left(\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)+\zeta_n\right)}}{\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)\zeta_d}$$

or by using ζ_λ instead of ζ_d:

$$D = \frac{e^{-\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)\zeta_n \Big/ \left(\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)+\zeta_n\right)}}{\left(\frac{E_s}{N_0}\left(\frac{1}{1+\gamma}\right)\zeta_\lambda + 1\right)}$$

As will be discussed in section 5 these forms are always valid and ζ_n, ζ_λ and ζ_d independent of the size of the L matrix. For specific channels, compact forms for ζ_n, ζ_λ and ζ_d may be found or they may be computed with the use of the normalized autocorrelation function. These factors should be very good indicators of the maximum energy degradation associated with residual correlation between subsequent coded symbols, as for example for partially interleaved and noninterleaved fading channels.

These bounds have been computed for a rate r=1/2, constraint length K=7 convolutional code with unquantized PSK demodulation. Two types of channels have been considered: the exponential correlation channel ($\rho(\tau) = e^{-2\pi F_D \tau}$) and the land mobile channel with a vertical whip antenna ($\rho(\tau) = J_o(2\pi F_D \tau)$), where F_D denotes the Doppler frequency and where $J_0(.)$ is the Bessel function. In figures 2 to 9 the computed results are illustrated for three bounds, two typical specular to diffuse signal energy ratios (0dB and 10dB) and several $F_D T_c$ values. Bound B1 denotes the exact union bound where the Q(x) function is bounded by $\frac{1}{2} e^{-x^2/2}$, bound B2 is given by equation (22) and bound B3 is given by equation (23). For bound B3, the degradation factors have been computed as discussed in section 5. The value of these factors may be taken from figures 10 to 13 for the exponential, land mobile and aeronautical channels.

From these figures many conclusions may be drawn concerning the above bounds for the BER of coding systems over Rician channels. To limit this discussion let us restrict ourselves to these main points:

- The tightness of the bounds improves as the correlations between adjacent symbols decreases.
- The tightness of bounds B2 and B3 as compared to B1 is quite dependent upon the channel's autocovariance function.
- For channels such as the land mobile channel, where $\rho(\tau)$ is not a monotonously decreasing function of τ, the degradation factors may be misleading.

Finally, a preliminary evaluation of the tightness of these bounds was carried out using simulation results presented by Modestino and Mui [1]. In that paper, simulation results are given for a similar communication system but with equal gain combining instead of maximal ratio combining. Maximal ratio combining with our coding strategy over a Rayleigh channel allows an additional gain of at most 2dB as compared to equal gain combining [3]. This maximum additional gain may be reached when the channel is fully interleaved. For a BER of 10^{-3}, on the exponential correlation Raleigh channel, we may thus infer that our bound is tight to within 1.5 to 2 dB for $F_D T_c$ values ranging from ∞ to 0.02 respectively.

Unquantized DPSK and NC-FSK

For unquantized DPSK and NC-FSK, we will limit the bounds to the Rayleigh fading case (α=0). Using the same procedure as with the coherent demodulation, with the added bounds on the derivatives of p(z) (20), a bound for the NC FSK modulation may be given by:

$$P_b < \frac{1}{b} \sum_{j=1}^{\infty} B_j \sum_{m=0}^{d_j-1} \frac{1}{m!} \zeta_m \left(\frac{-T}{2N_0} \right)^m \left[\frac{d^{(m)}}{dz^m} p(z) \right] \Bigg|_{z=T/2N_0}$$

$$< \frac{1}{b} \sum_{j=1}^{\infty} B_j \sum_{m=0}^{d_j-1} \frac{1}{m!} \zeta_m \frac{(d_j+m-1)}{(d_j-1)!} p(z) \Bigg|_{z=T/2N_0}$$

$$< \frac{1}{b} \sum_{j=1}^{\infty} B_j \frac{1}{\left|\sigma^2 \left(\frac{T}{2N_0} \right) L_{c_j} + I\right|} \left(\sum_{m=0}^{d_j-1} \frac{1}{m!} \zeta_m \frac{(d_{j+m-1})!}{(d_{j-1})!} \right)$$

or

$$P_b < \frac{1}{b} \sum_{j=1}^{\infty} \frac{B_j \varphi_{dj}}{\left|\frac{E_s}{N_0} L_{c_j} + 2I\right|} \tag{24}$$

where

$$\varphi_{d_j} = \sum_{m=0}^{d_j-1} \binom{d_j+m-1}{m} \sum_{K=m}^{d_j-1} \binom{d_j-1+k}{k-m} 2^{-K}$$

This bound has been computed for a mobile radio channel with a constraint length 7, rate 1/2 convolutional code and the results are illustrated in figures 14 to 16. Once again the degradation factors may be used by minimizing $\sqrt{|L|}$ and λ_{Min} to obtain:

$$P_b < \frac{1}{b} \sum_{d=d_{free}}^{\infty} C_d \varphi_d D^d \tag{25}$$

where

$$D = \frac{1}{\frac{E_s}{N_0}\zeta_d} \qquad \text{or} \qquad D = \frac{1}{(\frac{E_s}{N_0}\zeta_\lambda + 2)}$$

For DPSK the bounds on the BER are given by (24) and (25) with E_s/N_0 replaced by 2 E_s/N_0.

Hard quantized NC-FSK

For hard quantized NC-FSK demodulation, bounds on the coded error performance may be obtained only if the channel state knowledge is not used at the decoder.

For a given coded sequences the pairwise error probability may be given by

$$P_e(c_j, c_0) \leq E\left[\sum_{i=\frac{d_j+1}{2}}^{d_j} \binom{d_j}{i} \prod_{P_{ij}} \left(\frac{1}{2} e^{-\frac{1}{2}\frac{Es_{ij}}{N_0}}\right) \prod_{P_{ij}'} \left(1 - \frac{1}{2} e^{-\frac{1}{2}\frac{Es_{ij}}{N_0}}\right)\right], \quad d_j \text{ odd}$$

where P_{ij} is the set of i codesymbols of unit weight which are the most correlated and P_{ij}' are all others.

In this expression we have assumed that the worst case error sequence is the one where the erroneous symbols are the most correlated.

We may further bound $P_e(c_j, c_o)$ by deleting the second product $\prod(1-\frac{1}{2} e^{-\frac{1}{2}\frac{Es}{N_0}})$ and finally obtain (with the union bound on the BER):

$$P_b \le \frac{1}{b} \sum_{j=1}^{\infty} B_j \; E[P_e(c_j, c_0)] \tag{26}$$

where

$$E[P_e(c_j,c_0)] \le \begin{cases} \displaystyle\sum_{i=\frac{d_j+1}{2}}^{d_j} \binom{d_j}{j} \left.\frac{P_{c_{j,i}}(z)}{2^j}\right|_{z=\frac{T}{2N_0}} + \binom{d_j}{d_j/2} \left.\frac{P_{c_{j,i}}(z)}{2^{d_i/2}}\right|_{z=\frac{T}{2N_0}} & d_j \text{ even} \\ \displaystyle\sum_{i=\frac{d_j+1}{2}}^{d_j} \binom{d_j}{j} \frac{1}{2^j} \left. P_{c_{j,i}}(z)\right|_{z=\frac{T}{2N_0}} & d_j \text{ odd} \end{cases}$$

here $L_{c_{j,i}}$ is the matrix of dimension i by i of elements of which maximizes p(z). Now assuming that the $L_{c_{j,i}}$ matrix is composed of the i errors which are closest to each other, this bound has been computed for a Rayleigh fading mobile radio channel and its values are compared with simulation results on figures 14 to 16. Notice that soft quantization and perfect channel state knowledge provide more than a 6 dB gain on this channel, thus result in confirming [3] and [4]. Furthermore, with the usual technique seen earlier, maximum degradation factors due to correlation are defined in the same manner.

The numerous bounds presented in this section provide simplified approaches to evaluate the bit error probability of coded systems on Rician channels. Each successive bound is reduced in complexity at the expense of tightness.

Let us note that this analysis may be extended to most modulation strategies. Whenever the pairwise error probability is bounded by exponentials of the sum of received signal energies, the bit error rate may be evaluated using the results of this paper. The error performance of coded systems with many modulation techniques, such as MPSK, MFSK and trellis coding, is thus bounded. Finally, the degradation factors are independent of the modulation used since they appear when bounding the Laplace transform of the sum of received signal energies.

4- EFFECT OF INTERLEAVING AND ADDED DIVERSITY.

On Rician channels, interleaving and diversity are used to improve the error performance of communication strategies. We now show how to bound the coded error performance of communication systems that uses these techniques. This allows to adequately design the interleaving depth and the number of M-diversity branches to be used on a given channel.

4.1 Interleaving

Interleaving may alleviate part of the fading problem caused by the correlation between adjacent received codesymbol energies. Interleaving consists of scrambling the coded symbols before transmission and reordering them after reception. If the scrambling is thorough, then adjacent received symbols appear to be independent at the decoder. The channel is then said to be fully interleaved and may be modeled as a memoryless channel. Partial interleaving refers to an incomplete scrambling where adjacent received symbols are not completely independent.

Here, we assume that block interleaving is used. The encoded symbols are written column by column into a matrix of M_e rows and N_e columns. The channel symbols to be transmitted are then read line by line from the matrix and after transmission, the received coded symbols are reordered in the reverse manner. Thus, two adjacent coded symbols are separated by M_e-1 symbols during transmission.

We formally prove that block interleaving where the depth of the interleaving matrix N_e is an integer multiple of the block length n of the (n,k) block code, has exactly the same affect from an error performance point of view as transmitting at a lower rate $\frac{R_c}{M_e}$, where R_c equals the transmission rate on the channel. This is true for any stationary fading channel. Hence instead of using the $\rho(T_c(i_j - i_K))$ terms in the covariance matrix, we use $\rho(M_e T_c(i_j - i_K))$ to take into account the interleaving effect. Since $\lim_{\tau\to\infty} \rho(\tau)=0$, block interleaving asymptoticaly improves the error performance.

For convolutional coding of coding rate r and constraint length K, similar arguments with semi-infinite interleaving, where the depth of the matrix tends to ∞, yield the same result. But as shown in [1] and [8], this remains true for most values of $M_e>N_e$ or N_e greater than 5 or 6 times the ratio K/r.

Furthermore if $\zeta_n, \zeta_d, \zeta_\lambda$ are monotonous functions of τ, which is usually the case (see section 5), for any interleaving which insures that adjacent coded symbols are separated by at least (M_e-1) others code symbols during the transmission, the use of bounds of the type of (23) will be valid for an equivalent transmission rate $\frac{R_c}{M_e}$ or, in a mobile environment, an equivalent Doppler frequency of $M_e F_d$.

4.2 Diversity

If added diversity is available at the receiver, such as the use of M_d

antennas for space diversity, we may use maximal ratio combining and obtain similar bounds as before by increasing the dimension of the covariance matrix L. But simpler expressions are obtained if the signals from each antenna are independent. In this subsection, the effect of using independent maximal ratio combining diversity on all the bounds of sections 2 and 3 is evaluated for any type of independent diversity. The main result is that the degradation of energy due to residual correlation is independent of diversity. Hence, even if added diversity significantly improves the coded error performance, the degradation in energy remains if the codesymbols are not fully interleaved.

In our derivation, we follow the same steps as in section 2, whereas the received signal vectors were of dimensionality 2N, these vectors are now of dimension $2NM_d$ to take into account all the NM_d symbols received at each of the M_d antenna. Now if the signals from each diversity antenna are independent, the Laplace transform of the distribution of the sum of the NM_d received signal energies is:

$$P_{M_d}(z) = E\left[e^{-\frac{1}{2}\,\underline{X}\,\underline{X}'}\right] = E\left[e^{-\frac{1}{2}\sum_{i=1}^{2NM_d} x_i^2}\right] = \left(E\left[e^{-\frac{1}{2}\sum_{i=1}^{2N} x_i^2}\right]\right)^{M_d} = (p(z))^{M_d} \tag{27}$$

Hence, with diversity, all the previous bounds (22),(23),(24),(25) and (26) are valid if we take into account that p(z) is simply replaced by $(p(z))^{M_d}$.

In particular for unquantized PSK, (23) becomes

$$P_b < \frac{1}{2}\frac{1}{b}\sum_{d_{free}}^{\infty} C_d\, D^{d\,M_d} \tag{28}$$

For example, consider a fully interleaved coding strategy providing a 5 dB coding gain and assume 1 dB degradation due to residual correlation, resulting in a net coding gain of 4 dB. If added independent diversity increases the fully interleaved coding gain to 10 dB, then an effective 9 dB coding gain may be observed.

5- COMPUTING THE DEGRADATION FACTORS.

In this section, the values of maximum energy degradations ζ_d, ζ_n, ζ_λ are further discussed in general and these values are determined for particular channels. These values are important in order to obtain bounds on the performance for Rician channels that are similar to those on memoryless channels. Furthermore, these values allow to define a minimal cut-off rate $R_{0_{Min}}$ which depends only on the channel [15]. The term minimal refers to the fact that for a given code rate smaller than $R_{0_{Min}}$, an increase in the error correction capability of the code will insure an increase of its error performance.

It is also important to set the limiting values of ζ_n, ζ_d, ζ_λ for all channels. Recall that

$$\zeta_n = \frac{1}{\mathrm{Max}(\frac{\underline{b}L\underline{b}'}{N})} = \frac{1}{\mathrm{Max}(\frac{1}{N}\sum_j \sum_k L_{jk})} \geq \frac{1}{\lambda_{Max}[L]}$$

$$\zeta_d = \mathrm{Min}\ \sqrt[N]{|L|}$$

$$\zeta_\lambda = \lambda_{Min}[L]$$

Now since L is always definite positive, then

$$0 \leq \zeta_d \leq 1$$

$$0 \leq \zeta_\lambda \leq 1$$

$$0 \leq \zeta_n \qquad (29)$$

where the two first upper bounds come from the fact that

$$(\lambda_{Min}[L])^N \leq |L| \leq \prod_{j=1}^{N} L_{jj} = 1$$

Whenever ζ_d, ζ_λ or ζ_N are equal to zero, then the channel is said to be degenerate as for example an infinitely slow fading channel where $\rho_{ij} = \rho(\tau) = 1$.

Before attempting to find the bounds on the degradation factors for different channels, we now state two conjectures that are intuitively appealing.

Conjecture 1:

If there exists a covariance function $\rho'(\tau)$ such that $\rho'(\tau) \geq \rho(\tau)$, then the values of ζ_n, ζ_d, ζ_λ are upper bounded by the values of ζ'_n, ζ'_d, ζ'_λ associated with $\rho'(\tau)$.

In other words added correlation degrades the error performance.

Conjecture 2:

The values of ζ_N, ζ_d and ζ_λ are attained for $L = (L_{ij}) = (\rho(T_c|i-j|))$, where $\rho(\tau)$ is the autocovariance function and where T_c is the duration of a channel symbol.

This is slightly less restrictive than saying that, on a fading channel, among all the erroneous codewords at distance d, the most probably decoded codeword will be the one in which all the erroneous symbols appear consecutively.

This conjecture seems particularly obvious if the autocovariance function is monotonously decreasing with τ. For other channels, such as the mobile channel where $\rho(\tau) = J_0(2\pi F_D \tau)$, this is far from being obvious.

In the next subsection the above conjectures will be examined and applied to several channels. Channels for which the autocorrelation is rapidely and slowly descreasing are examined in the two first subsections respectively. The exponential correlation and mobile channels are then treated in the two last subsections. For these channels, values for the maximum energy degradation will be computed.

5.1 Channels for which $R(\tau)$ is rapidly decreasing.

We define these channels as the ones for which $\rho(T_c i) = q_i$, $i < p$ and $\rho(T_c i) \approx 0$, $i > p$ for a certain integer p. In practice certain regions of operation of a channel where $\rho(\tau) = e^{-b\tau^2}$ may be treated as such a channel. For these channels, the correlation matrix may be treated as a sparse matrix, that is $L_{ij} = 0$ if $|i-j| > p$. The degradation factors may also easily be found. For example let use take the sparse matrix

$$L = \begin{pmatrix} 1 & q & 0 & 0 & \cdots \\ q & 1 & q & 0 & \cdots \\ 0 & q & 1 & q & \cdots \\ 0 & 0 & q & 1 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \end{pmatrix} \qquad , \; 0 < q < \frac{1}{2}$$

Now ζ_n may immediately be written as:

$$\zeta_n = \frac{1}{\underset{N>1}{\mathrm{Max}}(1 + \frac{2(N-1)q}{N})} = \frac{1}{\lim_{N\to\infty}(1 + \frac{2(N-1)q}{N})} = \frac{1}{1+2q}$$

And since the eigenvalues of this matrix are [9]:

$$\lambda_i = \left[1 - 2q \cos(\frac{i\pi}{(N+1)}) \right] , \; i = 1,2,\ldots N$$

Then $\qquad \lambda_{Min} = 1 - 2q$

and also

$$|L| = \prod_{i=1}^{N} (1 - 2q \cos (\frac{i\pi}{N+1}))$$

let N be odd, then

$$|L| = \prod_{i=1}^{\frac{N-1}{2}} (1-2q \cos\frac{(i\pi)}{(N+1)}) \; (1-2q \cos(\frac{(N+1)\pi}{(N+1)} - \frac{i\pi}{(N+1)}) \quad (1-2q \cos (\frac{\pi}{2}))$$

$$= \prod_{i=1}^{\frac{N-1}{2}} (1-4q^2 \cos^2 (\frac{(i\pi)}{(N+1)}))$$

$$|L| > (1-4q^2)^{\frac{N-1}{2}} > (1-4q^2)^{N/2}$$

For N even $\quad |L| > (1-4q^2)^{N/2}$

and therefore

$$\zeta_d = \min \sqrt[N]{|L|} > \sqrt{1-4q^2}$$

Finally for PSK modulation, a bound on the BER may be obtained:

$$P_b \leq \frac{1}{2}\frac{1}{b}\sum_{d=d_{free}}^{\infty} C_d\, D^d \tag{30}$$

where

$$D = \frac{e^{-\frac{E_s}{N_o}\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{1+2q}\right) \Big/ \left[\frac{E_s}{N_o}\left(\frac{1}{1+\gamma}\right)+\frac{1}{1+2q}\right]}}{\frac{E_s}{N_o}\left(\frac{1}{1+\gamma}\right)\sqrt{1-4q^2}}$$

5.2 <u>Channels for which $\rho(\tau)$ is slowly decreasing</u>

In this subsection, we consider a hypothetical channel for which $\rho(\tau)$=q for $\tau > 0$ and $\rho(0)$=1. Such a channel is unrealizable but may be used to evaluate a "worst case channel". Practically, this channel gives a worst case interpretation when the L matrix is difficult to handle or if the autocorrelation function is not well defined. For this channel the L matrix may be written as

$$L = \begin{bmatrix} 1 & q & q & q & \\ q & 1 & q & q & . \; . \\ q & q & 1 & q & \end{bmatrix}$$

Now

$$\frac{1}{\zeta_n} = \frac{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} L_{ij}}{N} = \frac{N+(N^2-N)q}{N} = (1+(N-1)q) \Rightarrow \zeta_n = 0 \tag{31}$$

We now find the determinant of L. This can be readily accomplished by first using some matrix manipulations. First subtract from each column of L the succeding one,

$$|L| = \begin{vmatrix} 1-q & 0 & 0 & . \; . & q \\ q-1 & 1-q & 0 & . \; . & q \\ 0 & q-1 & 1-q & . \; . & q \\ 0 & 0 & q-1 & . \; . & q \\ 0 & 0 & 0 & & 1 \end{vmatrix}$$

and then adding to each row its succeding one, yields the determinant $|L|$

$$|L| = \begin{vmatrix} (1-q) & 0 & 0 & q \\ 0 & (1-q) & 0 & 2q \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & (1-q) & Nq \\ 0 & 0 & 0 & (1-Nq) \end{vmatrix} = (1-q)^{N-1}\ (1+Nq) = (1-q)^N\ (\frac{1+Nq}{1-q}) \tag{32}$$

The eigenvalues of L are $\lambda_i=1-q$, $i=1,2, \; ..N-1$, and $\lambda_{Max} = 1+Nq$

Now if we have $|\frac{E_s}{N_o}\ (\frac{1}{1+\gamma})\ L+I|$, then we obtain

$$\left|\frac{E_s}{N_o}\left(\frac{1}{1+\gamma}\right) L+I\right| = (1+\sigma(1-q))^N \left(\frac{1+\sigma^2(1+Nq)}{1+\sigma^2(1-q)}\right)$$

For unquantized PSK, bounding equation (22) with (31) and (32) yields

$$P_b < \frac{1}{2}\frac{1}{b}\sum_{d=d_{free}}^{\infty} C_d \frac{e^{-d\frac{E_s}{N_o}\left(\frac{\gamma}{1+\gamma}\right) / \left[\left(\frac{E_s}{N_o}\right)\left(\frac{1}{1+\gamma}\right)(1+(d-1)q)+1\right]}}{\left(\frac{1+\sigma^2(1+Nq)}{1+\sigma^2(1-Nq)}\right)(1+\sigma^2(1-q))^d} \tag{33}$$

Notice that for large values of d, the exponential tends to

$$e^{-\frac{\gamma}{2}\left(\frac{1}{q}\right)}$$

and hence for convolutional codes with a large free distance the importance of the specular component decreases. This is the main difference between such a worst case channel and a usual channel where $\rho(\tau)$ tends to zero as τ increases.

Furthermore, for Rayleigh channels where $\alpha = 0$, an upper bound may be written as

$$P_b < \frac{1}{2}\frac{1}{b}\sum_{d_{free}}^{\infty} C_d \frac{1}{\left(1+\frac{E_s}{N_o}(1-q)\right)^d} \tag{34}$$

where we have used

$$\frac{1+\frac{E_s}{N_o}(1-q)}{1+\frac{E_s}{N_o}(1+q)} < 1$$

The implication of (34) is that the maximum energy degradation is bounded by (1-q) for the worst case Rayleigh fading channel. And thus it may be conjectured that for most channels the maximum degradation of energy due to residual correlation is bounded by

$$\zeta_d > 1-\rho(T_c M_e)$$

where M_e is the interleaving depth. This is surely true if conjecture 1 holds and if $\rho(\tau)$ is a monotonously decreasing function of τ.

5.3 <u>The exponential correlation channel.</u>

In the so called exponential correlation channel, $\rho(\tau) = e^{-B\tau}$. Using conjecture 2, finding ζ_N, ζ_d and ζ_λ is carried out by analyzing the following matrix

$$L = \begin{pmatrix} 1 & q & q^2 & \\ q & 1 & q & \\ q^2 & q & 1 & \ldots \\ & \cdot & & \\ & \cdot & & \end{pmatrix}$$

where

$$q = e^{-BT_c}$$

This matrix has been extensively and thoroughly analyzed in [9]. The eigenvalues of L are bounded by

$$\frac{1-q}{1+q} < \lambda_i < \frac{1+q}{1-q}$$

so

$$\zeta_\lambda = \frac{1-q}{1+q}$$

$$\zeta_N > \frac{1-q}{1+q}$$

Furthermore

$$\frac{\underline{b}L\underline{b}'}{N} = \frac{\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} L_{ij}}{N} = 1 + 2 \sum_{i=1}^{N-1} \frac{(N-i)}{N} q^i$$

$$= 1 + 2q \frac{(1-q^N)}{(1-q)} - \frac{2}{N} (\frac{q}{1-q}) (\frac{1-q^N}{1-q})$$

$$= 1 + (2q - \frac{2q}{N(1-q)}) (\frac{1-q^N}{1-q})$$

$$\frac{1}{\zeta_N} = \lim_{N\to\infty} \frac{\underline{b}L\underline{b}'}{N} = \frac{1+q}{1-q} = \lambda_{max}$$

Now to evaluate the determinant $|L|$ of the matrix, subtract from each column the subsequent one multiplied by q,

$$|L| = \begin{vmatrix} (1-q^2) & q(1-q^2) & q^2(1-q^2) & .. & . \\ 0 & (1-q^2) & 0 & .. & . \\ 0 & 0 & (1-q^2) & .. & . \\ 0 & 0 & 0 & .. & 1 \end{vmatrix} = (1-q^2)^{N-1}$$

yielding

$$\zeta_d = (1-q^2)$$

Typical curves to determine the degradation due to residual correlation are given in figures 10 and 11 for a typical exponential covariance function $e^{-2\pi F_D \tau}$, where F_D is the associated Doppler frequency. Obviously these are asymptotic degradation factors that do not take into account the fine weight structure of the code. Furthermore performance bounds using these degration factors were illustrated in figures 6 to 9.

5.4 Mobile channels

For these channels the degradation factors must be determined numerically. For a land mobile channel with a vertical antenna, $\rho(\tau) = J_o(2\pi F_D \tau)$, and for an aeronautical channel $\rho(\tau) = e^{-(\pi F_D \tau)^2}$ [16]. The degradation factors associated with these channels have been computed numerically and are illustrated in figures 10 to 13. Notice that for these channels, the degradation due to residual correlation become rapidly important.

6- CONCLUSION

In this paper, we have presented several bounds on the error performance of coding over Rician fading channels that take into account the effect of residual correlation between received symbols. With the additional results concerning diversity and interleaving, these bounds on the bit error rate cover most practical cases. The analytical results may thus provide for a step by step design approach for communication strategies over continuous fading channels.

Further work on the subject should extend these results to other types of modulation and trellis coding. Furthermore many other types of channels have yet to be characterized to compute their associated degradation functions.

Appendix 1

We now evaluate the Laplace transform of the distribution of the sum of squared Gaussian random variables.

$$P(z) = \mathbf{E}[e^{-\frac{1}{2} z \underline{X}\underline{X}'}]$$

$$= \int\int_{-\infty}^{\infty}..\int \frac{e^{-\frac{1}{2} z \underline{X}\underline{X}'} e^{-\frac{1}{2} (\underline{X} - \overline{\underline{X}}) M^{-1} (\underline{X} - \overline{\underline{X}})'}}{|M|^{1/2} (2\pi)^N} d x_1 \; d x_2 \; \ldots \; d x_{2N}$$

or equivalently

$$p(z) = \frac{e^{-\frac{1}{2} \overline{\underline{X}} M^{-1} \overline{\underline{X}}'}}{(M)^{1/2} (2\pi)^N} \int\int_{-\infty}^{+\infty}\ldots\int e^{-\frac{1}{2} [\underline{X} (zI+M^{-1})\underline{X}' + 2\underline{X} M^{-1} \overline{\underline{X}}']} d x_1 \; .. \; d x_{2N}$$

but $\underline{X}(zI+M^{-1})\underline{X}'$ is a positive definite quadratic form, so $[zI+M^{-1}]$ may be diagonalized by a substitution of variable $\underline{V} = \underline{X} Q$ where Q is an orthonormal matrix composed of the eigenvectors of $(zI + M^{-1})$.

$$p(z) = \frac{e^{-\frac{1}{2} \overline{\underline{X}} M^{-1} \overline{\underline{X}}'}}{|M|^{1/2} (2\pi)^N} \int\int_{-\infty}^{+\infty}\int e^{-\frac{1}{2} \underline{V} \Lambda \underline{V}' + \underline{V} Q^{-1} M^{-1} \overline{\underline{X}}'} dV_1 dV_2 \ldots$$

Where Λ is the diagonal matrix of the eigenvalues of $(zI+M^{-1})$. If $\underline{R}' = Q^{-1} M^{-1} \overline{\underline{X}}'$, then all the integrals may be separated to form a product

$$p(z) = \frac{e^{-\frac{1}{2}\,\underline{\bar{X}}M^{-1}\underline{\bar{X}}'}}{(M)^{1/2}} \prod_{i=1}^{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\,\lambda_i V_i^2 + R_i V_i}\, dV_i$$

and so

$$p(z) = \frac{e^{-\frac{1}{2}\,\underline{\bar{X}}\,M^{-1}\underline{\bar{X}}'}}{|M|^{1/2}} \prod_{i=1}^{2N} \frac{e^{R_i^2/d\lambda_i}}{\sqrt{\lambda_i}}$$

or equivalently:

$$p(z) = \frac{e^{-\frac{1}{2}\,\underline{\bar{X}}M^{-1}\underline{\bar{X}}'}}{|M|^{1/2}} \quad \frac{e^{-\frac{1}{2}\,\underline{R}\,\Lambda^{-1}\,\underline{R}'}}{|\Lambda|^{1/2}}$$

$$= \frac{e^{-\frac{1}{2}\,\underline{\bar{X}}M^{-1}\,(I-(Mz+I)^{-1})\,\underline{\bar{X}}'}}{|Mz + I|^{1/2}}$$

QED

APPENDIX 2

In this appendix we show that

$$\underline{b}(\sigma^2 zL + I)^{-1}\,\underline{b}' \;\geq\; \frac{N^2}{\underline{b}(\sigma^2 zL + I)^{-1}\underline{b}'}$$

The matrix L is definite positive, hence it may be diagonalized. Denoting by Q the matrix of the eigenvectors of L and Λ the diagonal matrix of the eigenvalues of L, that is $\Lambda_{ii} = \lambda_i$, $\Lambda_{ij} = 0$ if $i \neq j$, the quadratic form may be diagonalized:

$$\underline{b}\;(\sigma^2 zL+I)\underline{b}' = \underline{b}Q'\;(\sigma^2 z\;\Lambda+I)Q\underline{b}' = \underline{r}(\sigma^2 z\Lambda+I)\underline{r}'$$

where

$$\underline{r} = [r_1, r_2\;\ldots r_N] = [\sum_{j=1}^{N} Q_{ij}\;,\;\sum_{j=1}^{N} Q_{2j}\;\ldots\;\sum_{j=1}^{N} Q_{Nj}\;]$$

and hence

$$\underline{r}\;(\sigma^2 z\Lambda+I)^{-1}\;\underline{r}' = \sum_{i=1}^{N}\;[\;(\sum_{j=1}^{N} Q_{ij})^2\;\frac{1}{(\sigma^2 z\lambda_i+1)}\;]$$

Now multiplying this expression by the same development for $\underline{b}\;(\sigma^2 L+Z)\underline{b}'$ we obtain

$$(\underline{b}(\sigma^2 zL+I)^{-1}\underline{b}')\;(\underline{b}(\sigma^2 zL+I)\underline{b}') = \sum_{i=1}^{N}\left[\frac{(\sum_j Q_{ij})^2}{(\sigma^2 z\lambda_i+1)}\right]\sum_i\left[(\sum_j Q_{ij})^2\;(\sigma^2 z\lambda_i+1)\right]$$

by multiplying both sums and regrouping terms

$$(\underline{b}(\sigma^2 zL+I)^{-1}\underline{b}')\ (\underline{b}(\sigma^2 zL+I)\underline{b}') = \sum_{i=1}^{N} \ (\sum_{j=1}^{N} Q_{ij})^4 \ +$$

$$\sum_{i=1}^{N} \ \sum_{k=i+1}^{N} \left[(\sum_{j=1}^{N} Q_{ij})^2 \ (\sum_{j=1}^{N} Q_{1j})^2 \ \left(\frac{(\sigma^2 z\lambda_k+1)}{(\sigma^2 z\lambda_i+1)} + \frac{(\sigma^2 z\lambda_i+1)}{(\sigma^2 z\lambda_k+1)} \right) \right]$$

but for any $x > 0$ and $y > 0$ the value of $(x/y)+(y/x)$ is always larger than 2 so the inner parenthesis containing a similar expression may be lower bounded to obtain:

$$(\underline{b}(\sigma^2 zL+I)^{-1}\ \underline{b}')\ (\underline{b}(\sigma^2 zL+I)\underline{b}') \geq (\sum_{i=1}^{N} (\Sigma Q_{ij})^2)\ (\sum_{i=1}^{N} (\Sigma Q_{ij})^2)$$

$$\geq (\underline{b}Q'IQ\underline{b}')^2 = (\underline{b}\ \underline{b}')^2 = N^2$$

QED

Appendix 3

An upper bound on the derivatives of the Laplace transform of the sum of powers for a Rayleigh channel ($\alpha^2 = 0$).

We have

$$p(z) = \frac{1}{|\sigma^2 z \ L+I|}$$

the denominator may be developed into the product of eigenvalues

$$p(z) = \frac{1}{|\sigma^2 z \ L+I|} = \frac{1}{\prod_{i=1}^{N} (\sigma^2 z \ \lambda_i + 1)} = K \prod_{i=1}^{N} \left(\frac{1}{z+r_i}\right)$$

where

$$r_i = \frac{1}{\sigma^2 \ \lambda_i} > 0$$

Now differentiating p(z) with respect to z

$$\frac{d}{dz} p(z) = K \sum_{i=1}^{N} \left[\frac{-1}{(z+r_i)} \prod_{j=1}^{N} \left(\frac{1}{z+r_j}\right)\right] = (-1) \ p(z) \sum \left(\frac{1}{z+r_i}\right)$$

$$p'(z) > \frac{(-1)}{2} N \ (p(z))$$

now by repeating this operation

$$\frac{d^M}{dz^M} p(z) = (-1)^M \left[\sum_{i=1}^{N(N+1)..(N+M)} \prod_{i=1}^{M} \left(\frac{1}{z+q_{i,j}}\right) \right]$$

where $q_{i,j}$ corresponds to a particular r_i. And since

$$\frac{1}{z} < \frac{1}{z+r_i}$$

Then

$$(-1)^M \; p^{(M)}(z) \;\le\; \frac{(N+M-1)!}{(N-1)!} \; \frac{p(z)}{z^M}$$

QED

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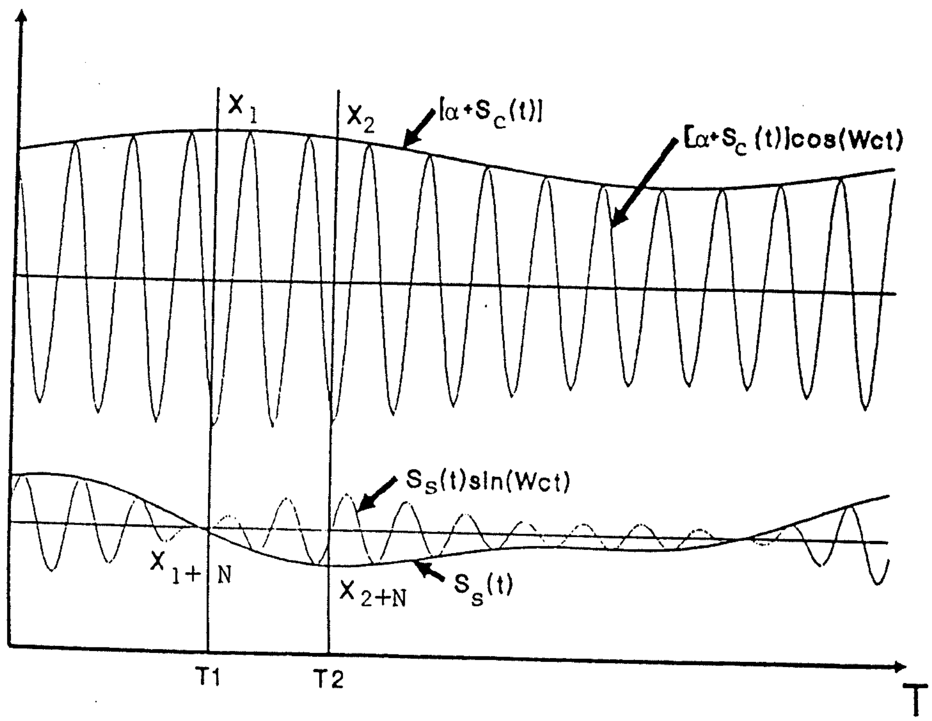

Fig. 1 : Typical sequence of sampled in-phase and quadrature components of $\underline{X}$.

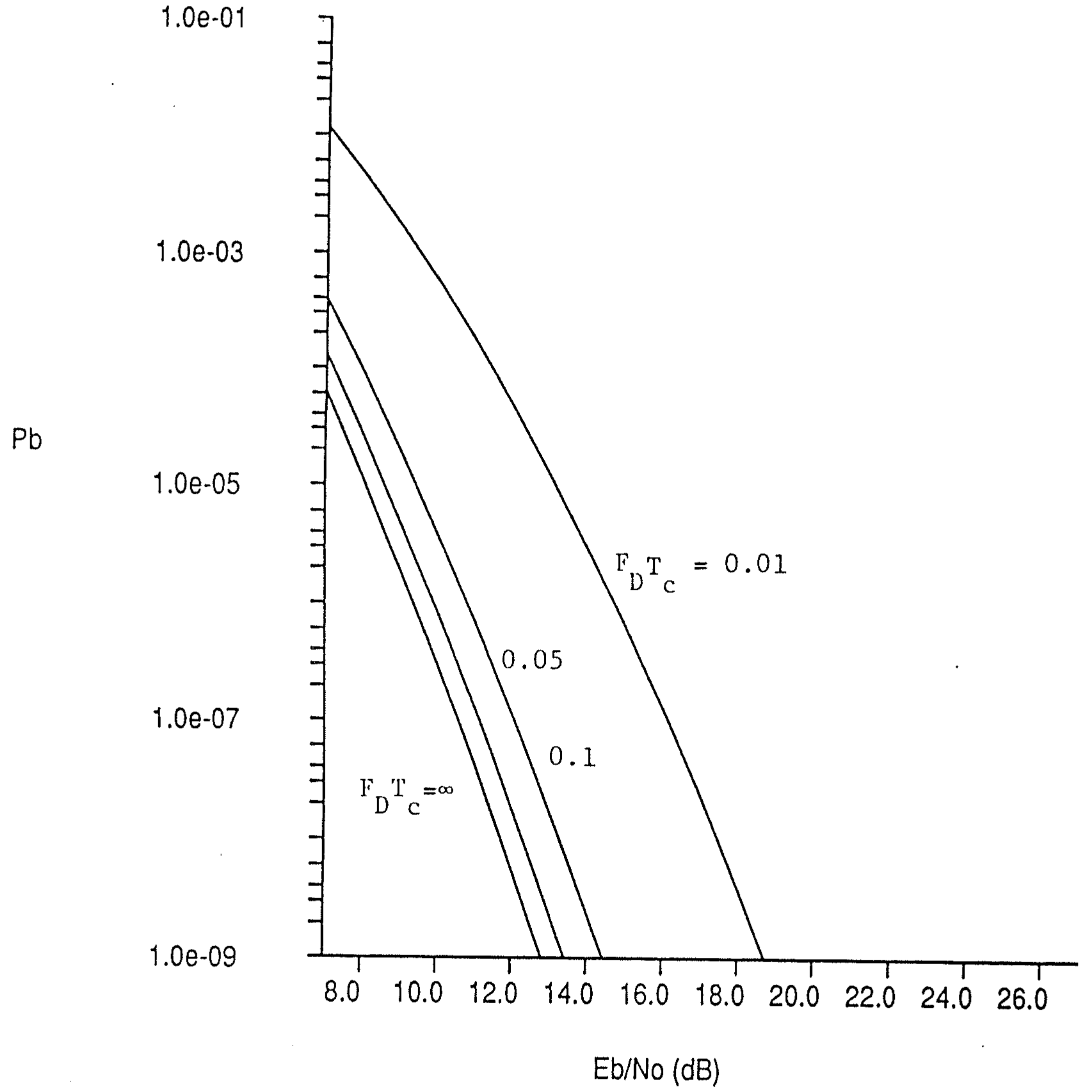

Fig. 2 : Bound B1, bit error probability vs Eb/No on a Rician Channel ($\rho(\tau) = e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=0dB.

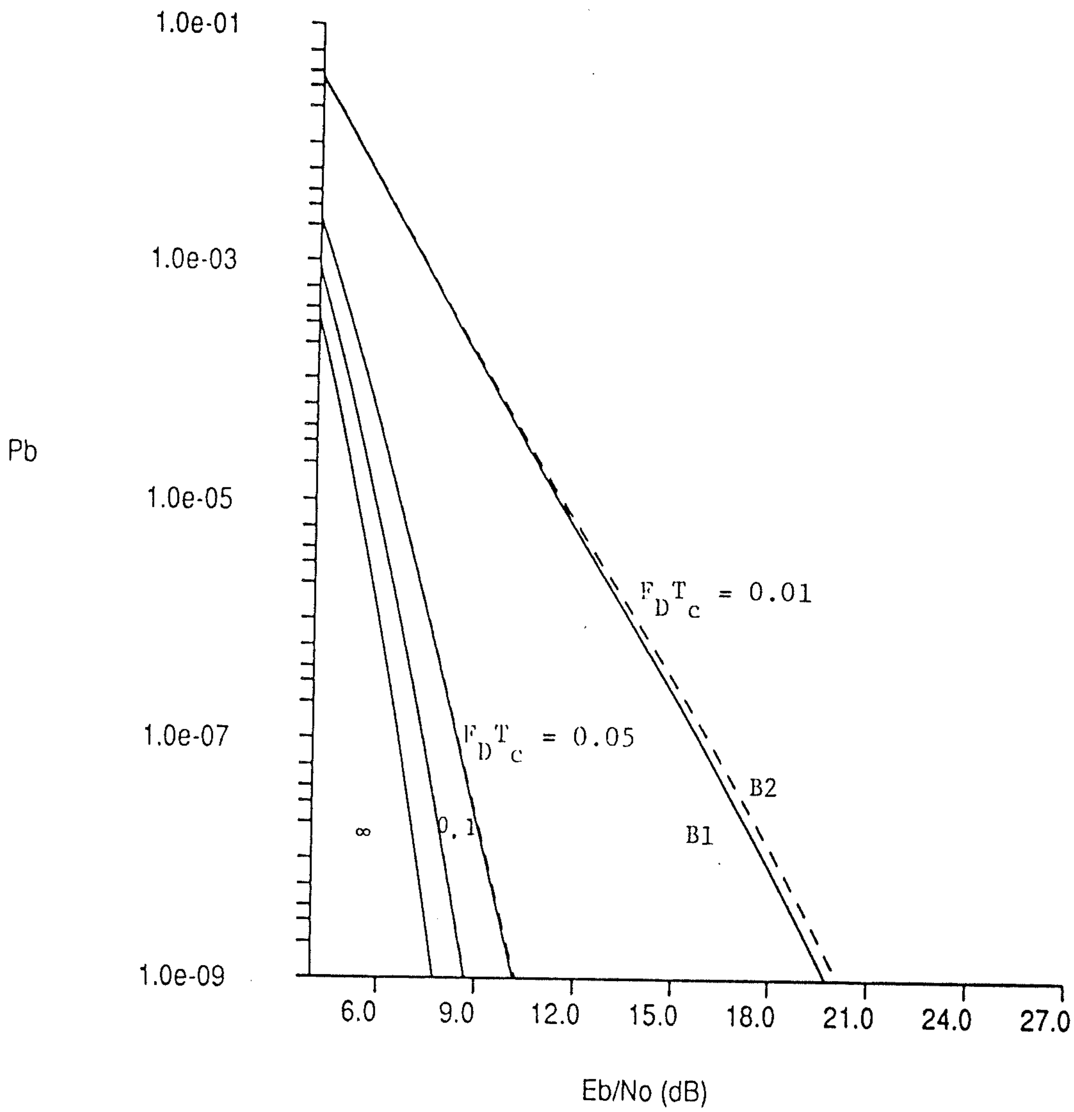

Fig. 3 : Bounds B1 and B2 on the bit error probability vs Eb/No on a Rician channel ($\rho(\tau) = e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=10dB.

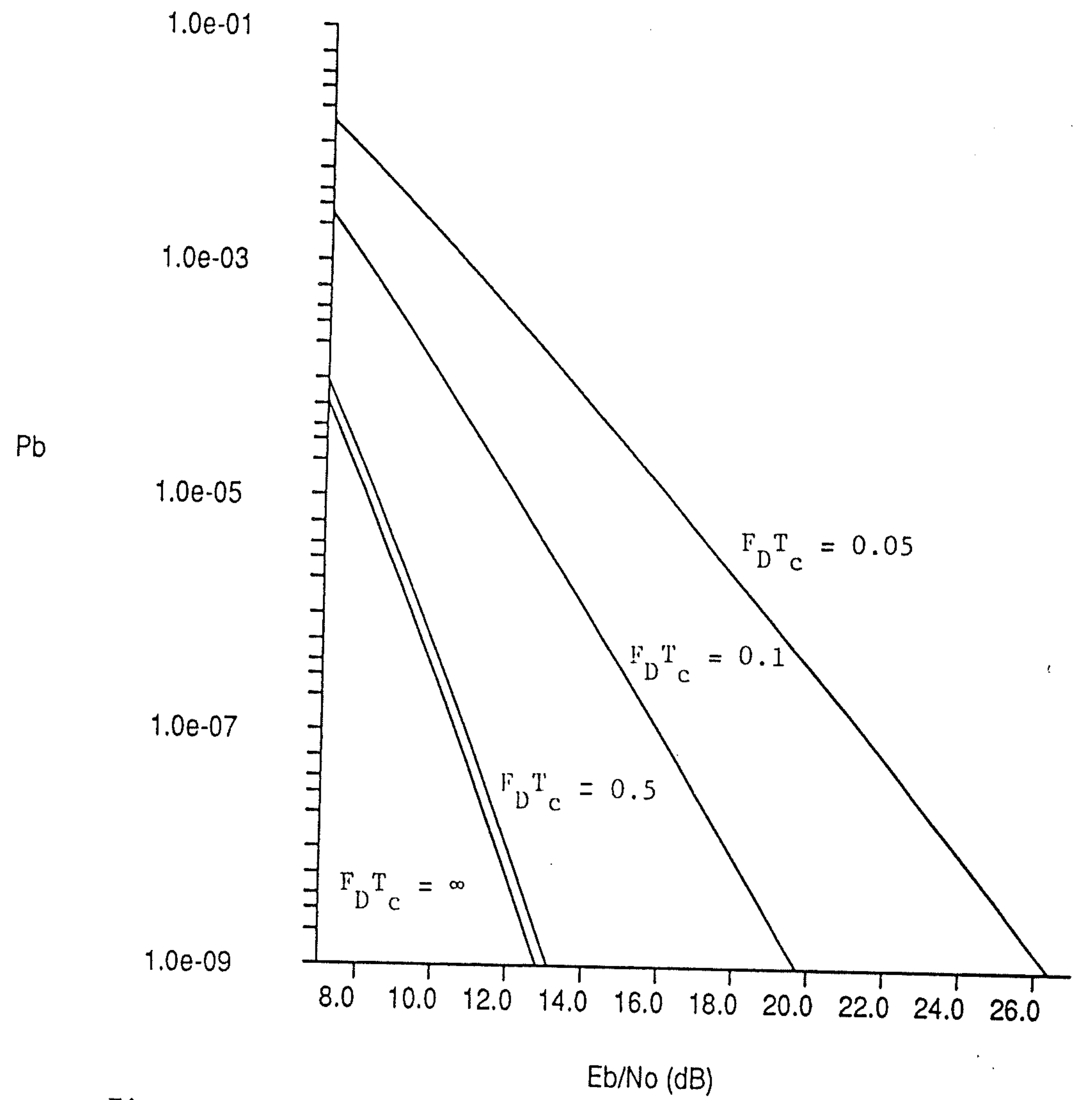

Fig. 4 : Bound B1, bit error probability vs Eb/No on a Rician channel $(\rho(\tau) = J_0(2\pi F_D \tau))$. Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=0dB.

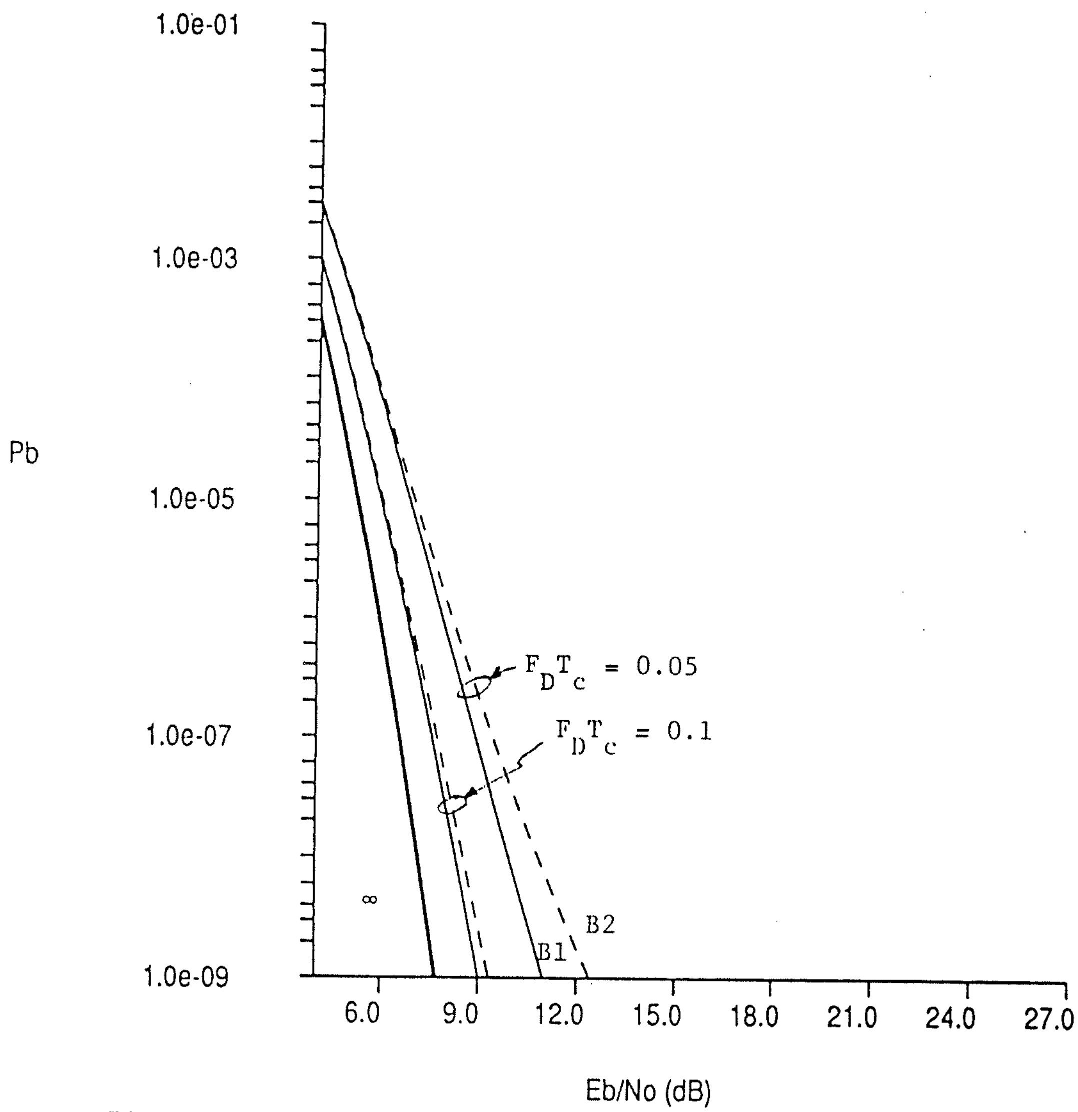

Fig. 5 : Bonds B1 and B2 on the bit error probability vs Eb/No on a Rician channel ($\rho(\tau)=J_0(2\pi F_D \tau)$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=10dB.

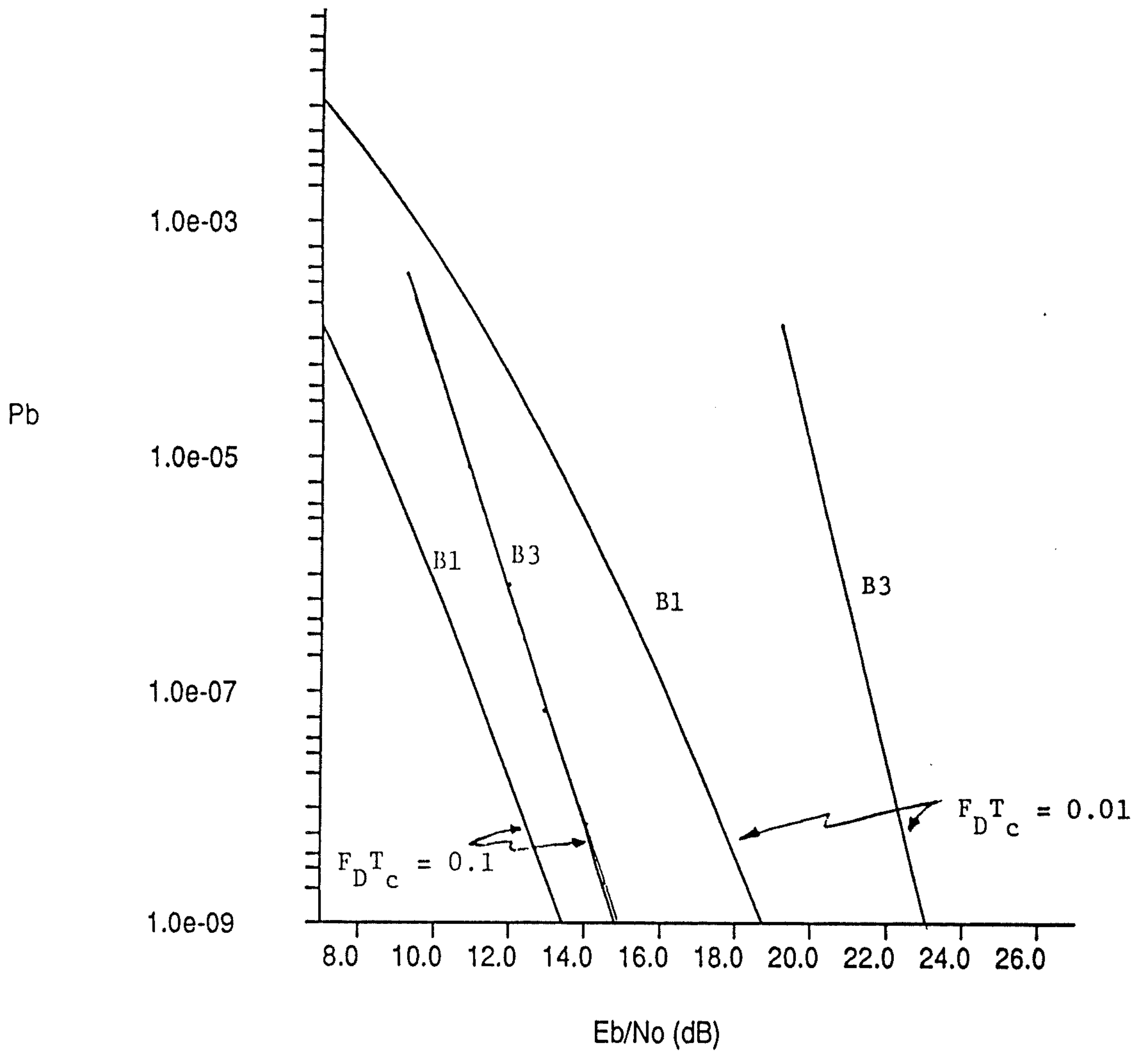

Fig. 6 : The bound using degradation factors (B3) as compared to B1, on a Rician channel ($\rho(\tau)=e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=0dB.

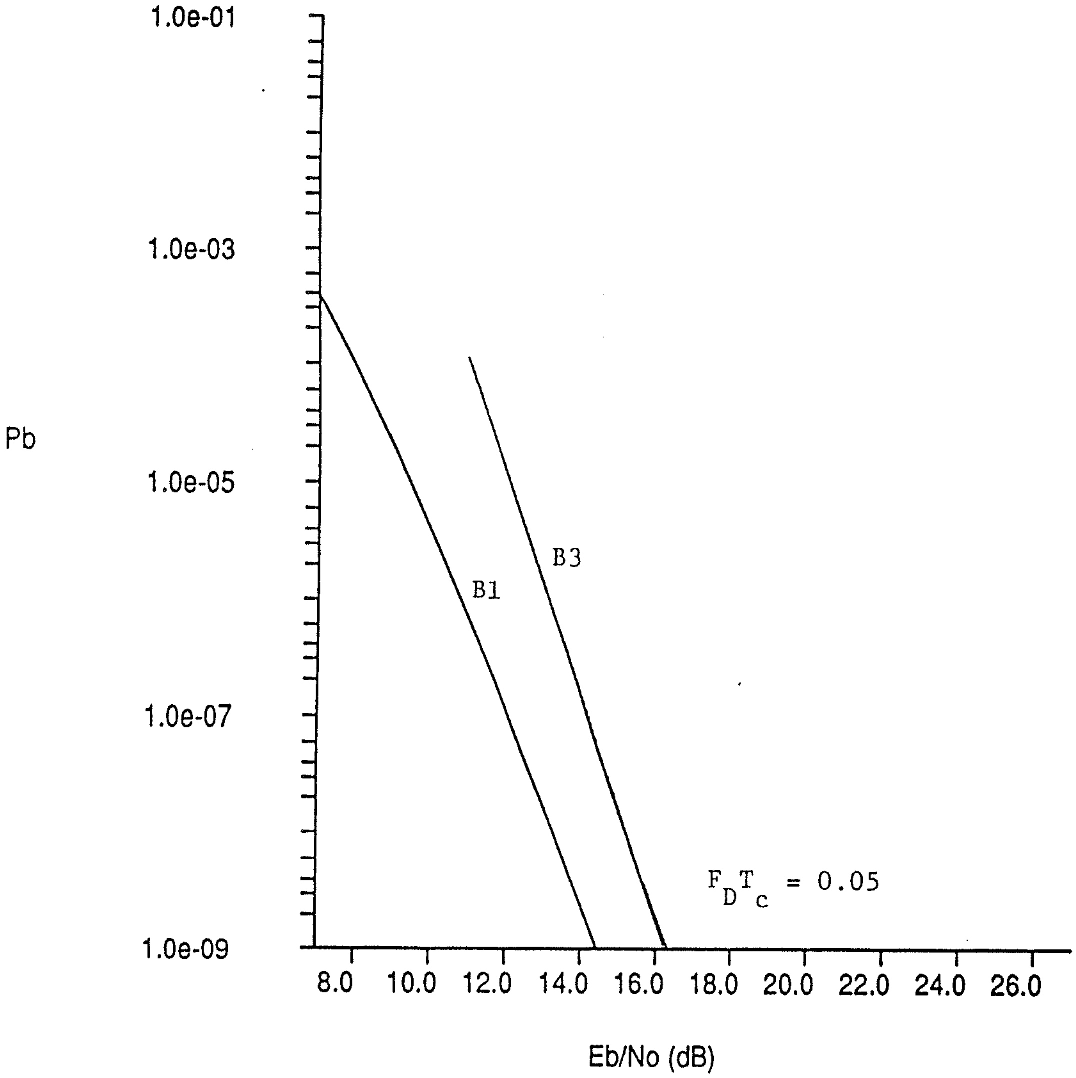

Fig. 7 : The bound using degradation factors (B3) as compared to B1, on a Rician channel ($\rho(\tau)=e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=0dB.

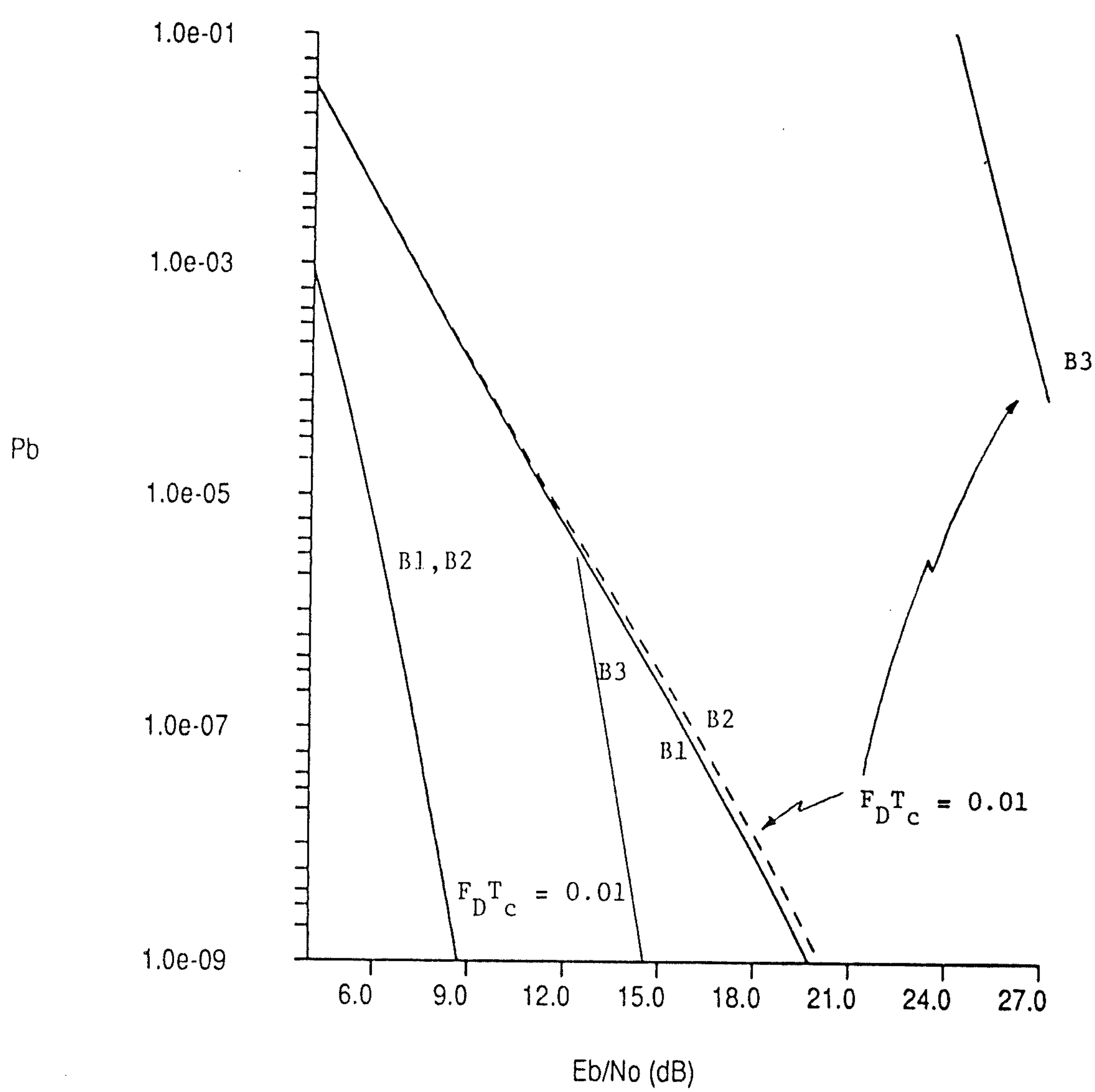

Fig. 8 : The bound using degradation factors (B3) as compared to B1 and B2, on a Rician channel ($\rho(\tau)=e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=10dB.

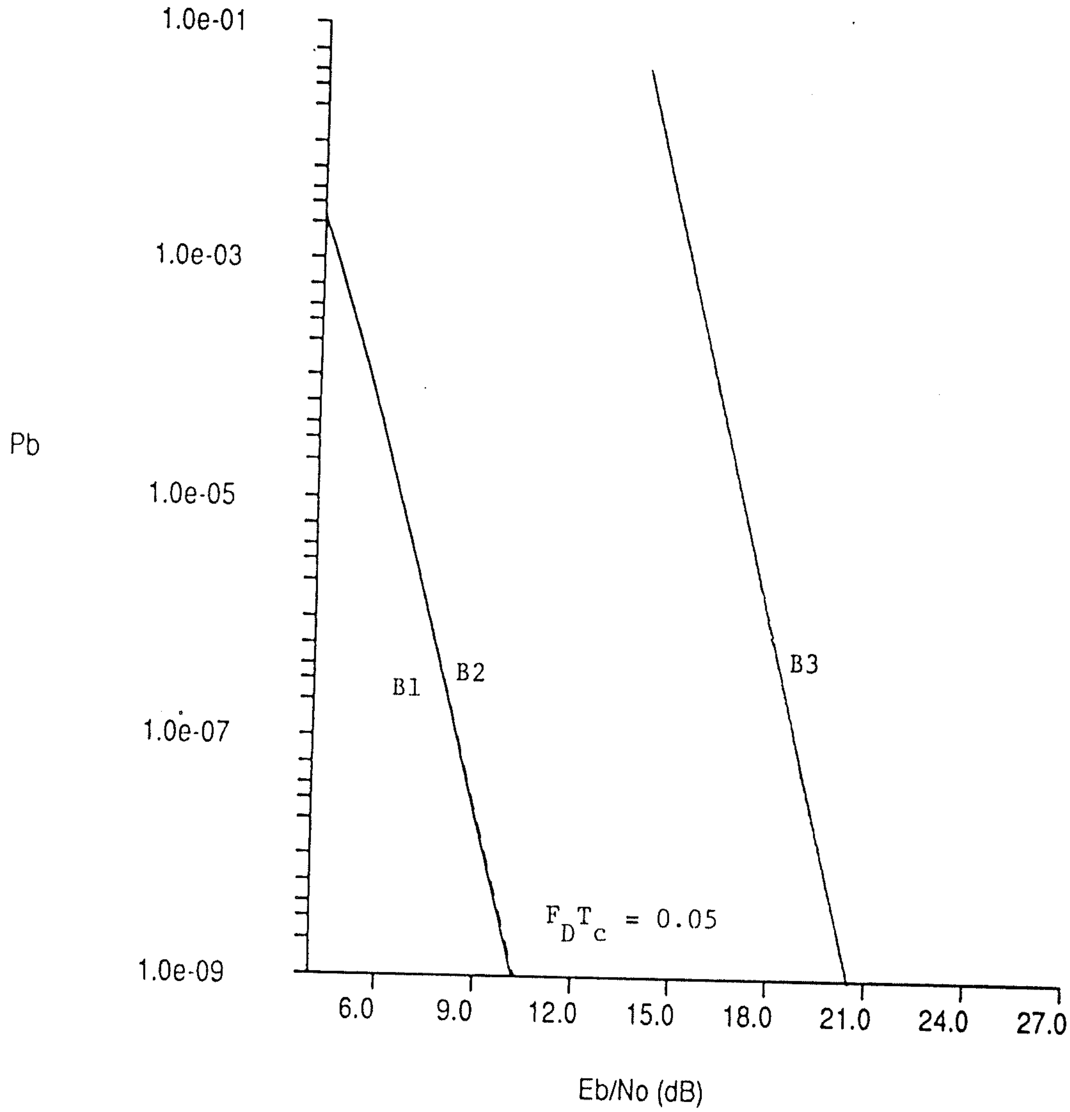

Fig. 9 : The bound using degradation factors (B3) as compared to B1 and B2, on a Rician channel ($\rho(\tau)=e^{-2\pi F_D \tau}$). Convolutional code: K=7, r=1/2; unquantized PSK demodulation; γ=10dB.

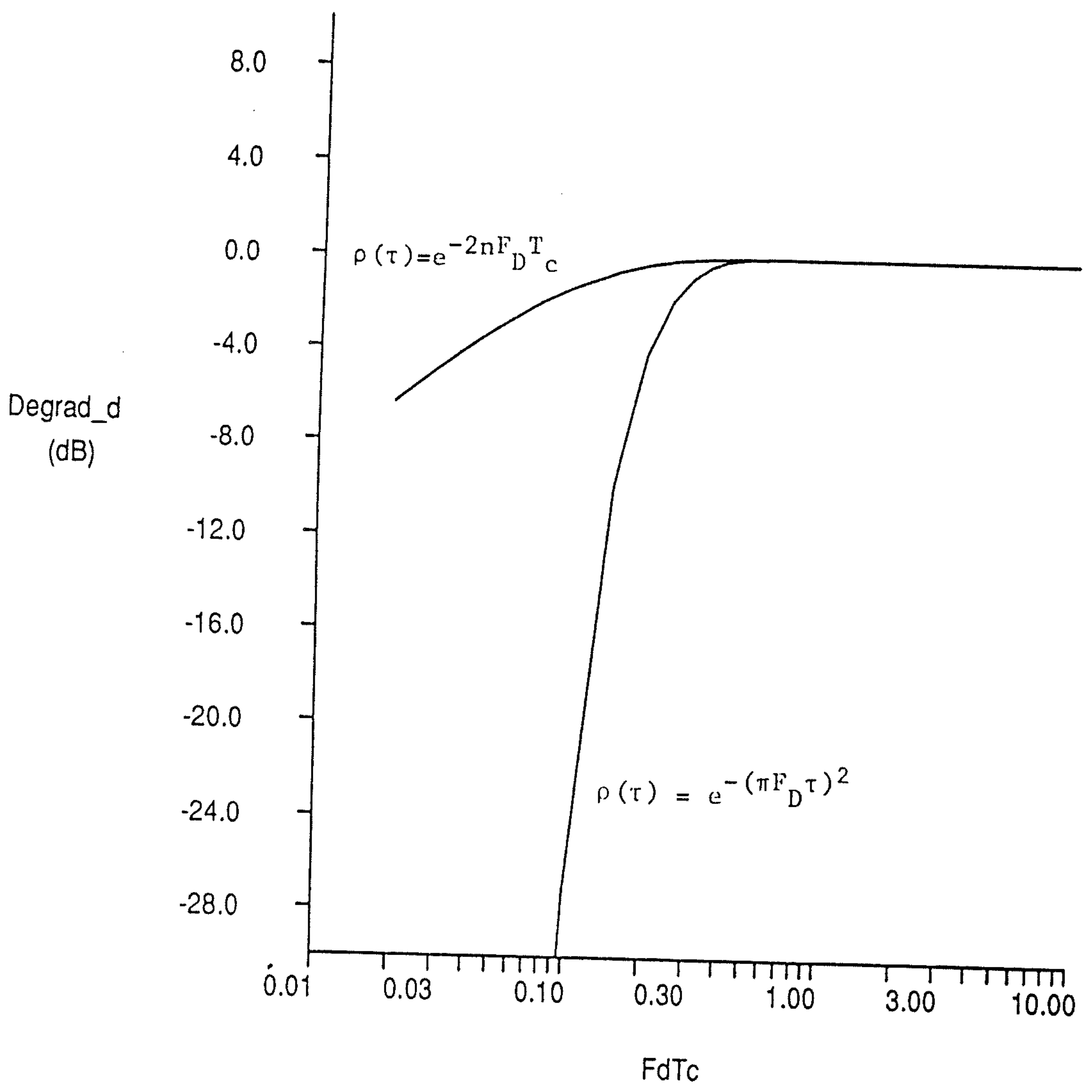

Fig. 10 : Degradation factor ζ_d as a function of $F_D T_c$. Exponential correlation and aeronautical channel.

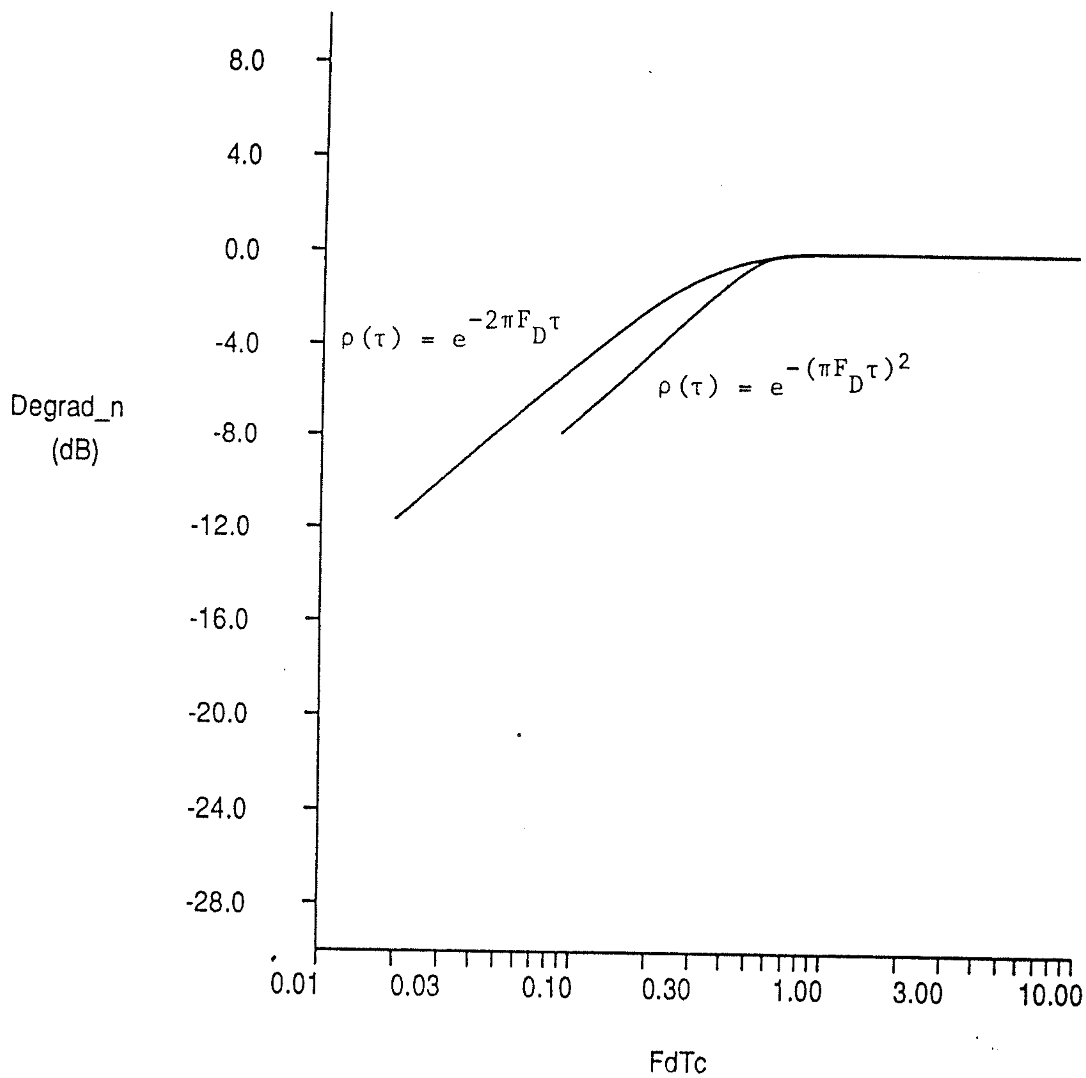

Fig. 11 : Degradation factor ζ_n as a function of $F_D T_c$. Exponential correlation and aeronautical channel.

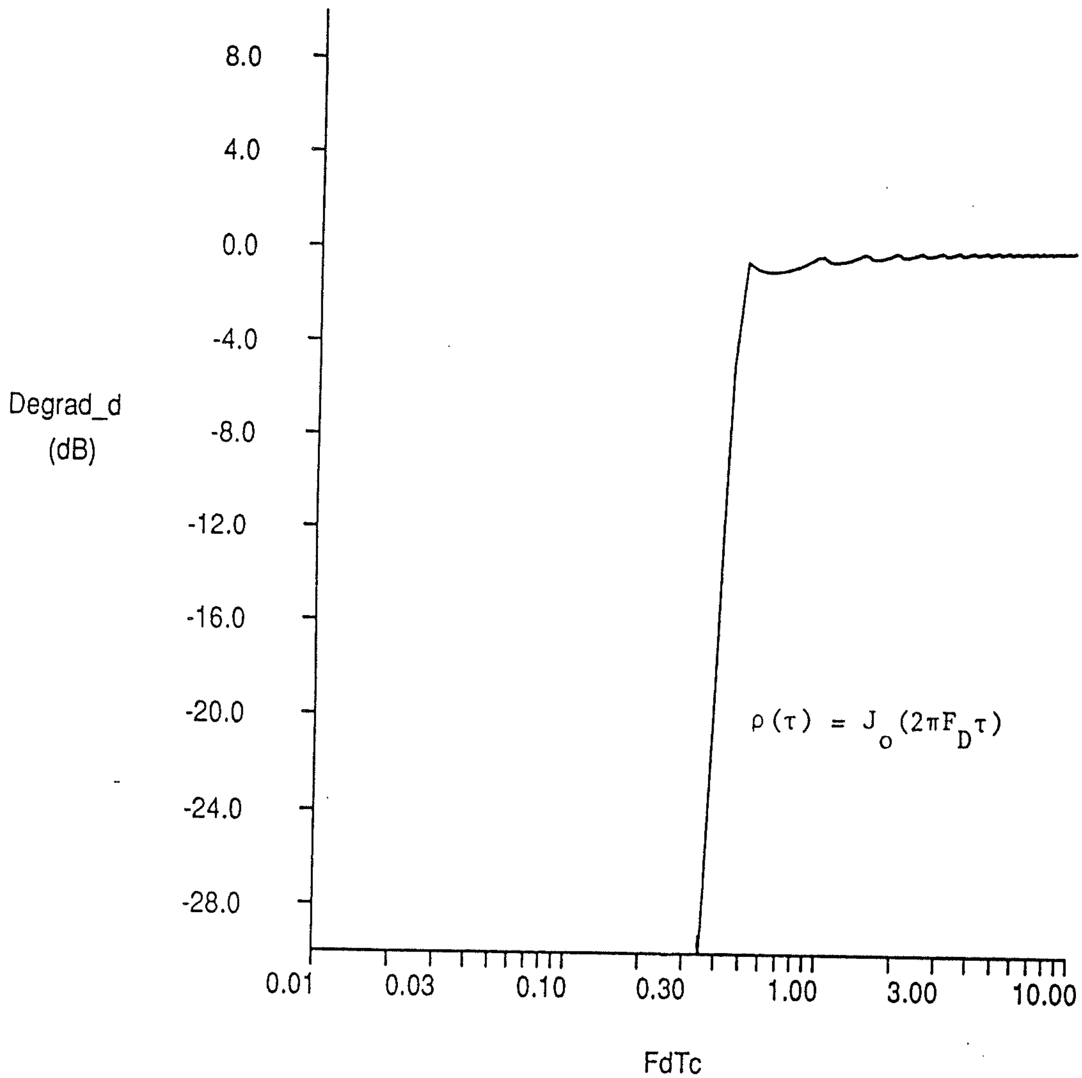

Fig. 12 : Degradation factor ζ_d as a function of $F_D T_c$. Land mobile channel.

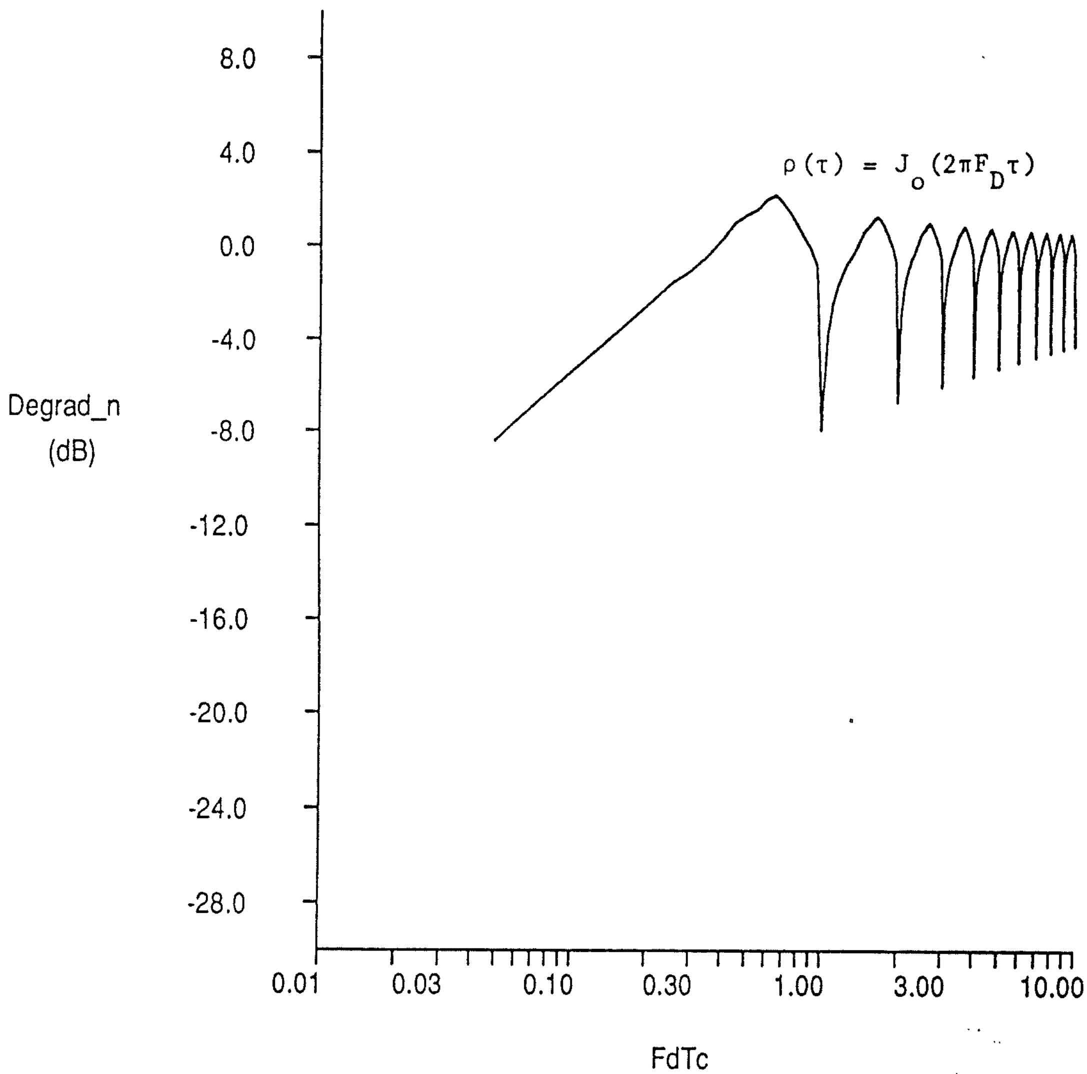

Fig. 13 : Degradation factor ζ_n as a function of $F_D T_c$. Land Mobile channel.

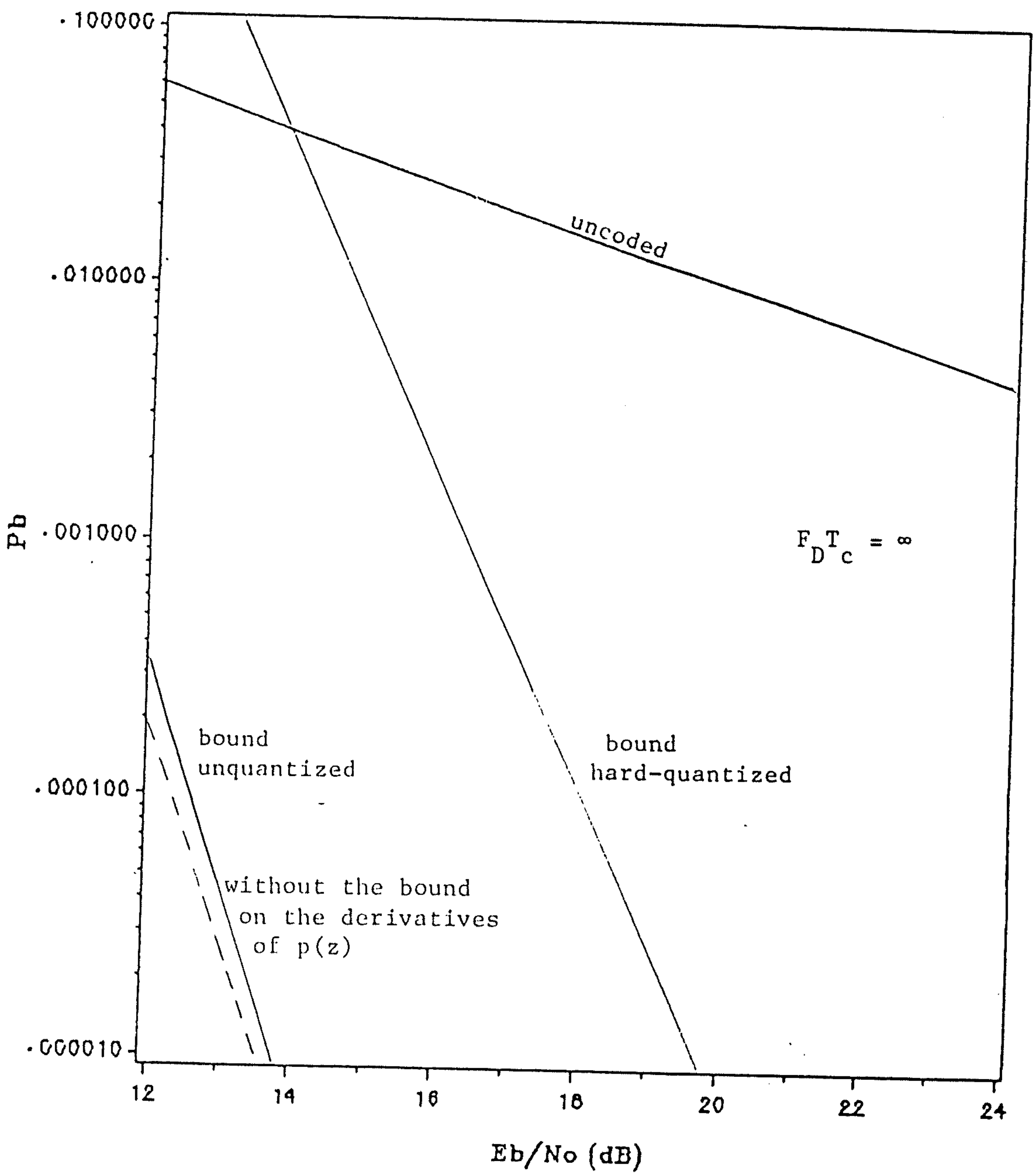

Fig. 14 : Bounds on the bit error probability vs Eb/No on a Rayleigh channel ($\rho(\tau)=J_0(2\pi F_D \tau)$). convolutional code: K=7, r=1/2; unquantized and hard-quantized NC-FSK demodulation.

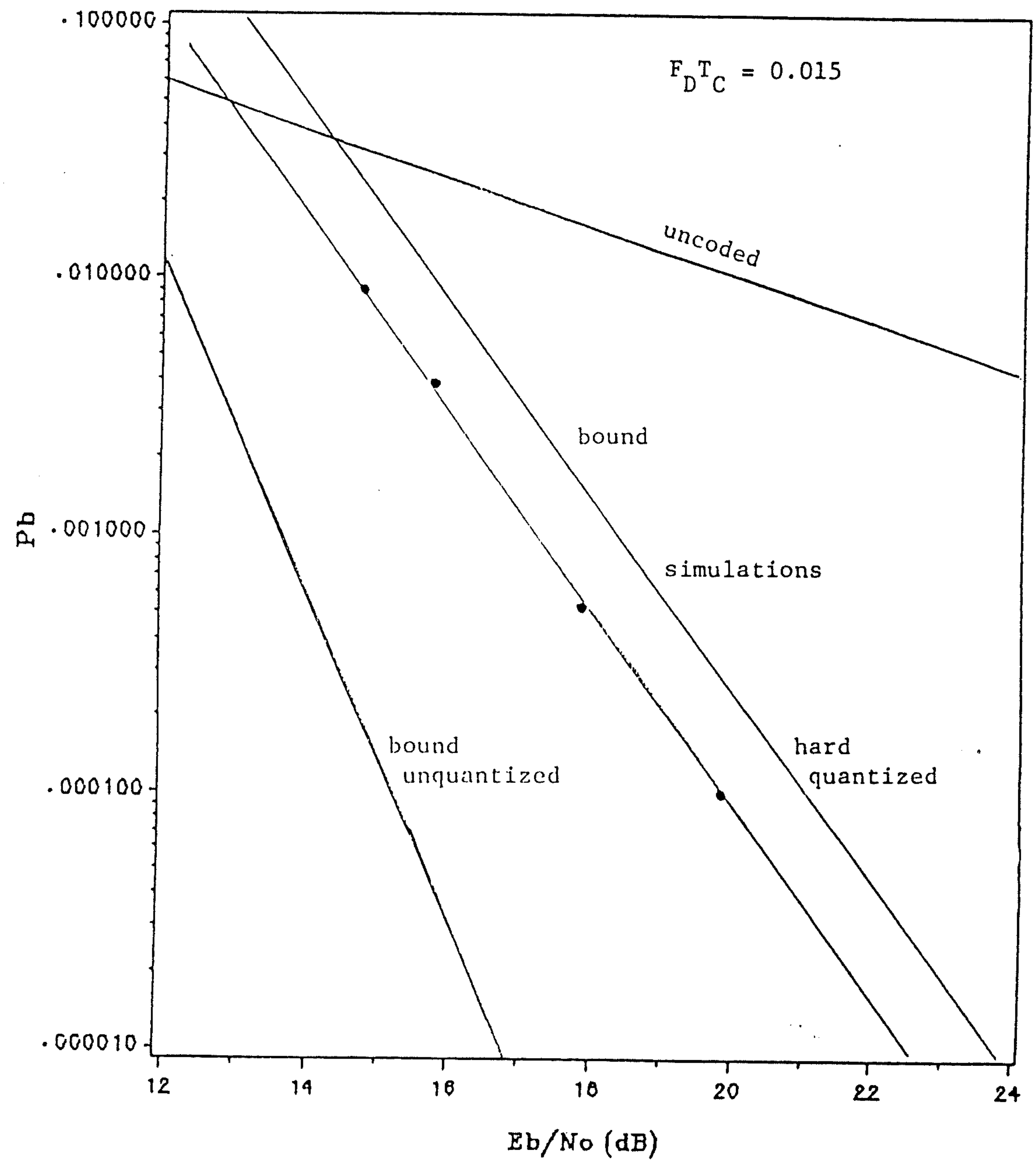

Fig. 15 : Bounds on the bit error probability vs Eb/No on a Rayleigh channel ($\rho(\tau)=J_0(2\pi F_D \tau)$). convolutional code: K=7, r=1/2; unquantized and hard-quantized NC-FSK demodulation.

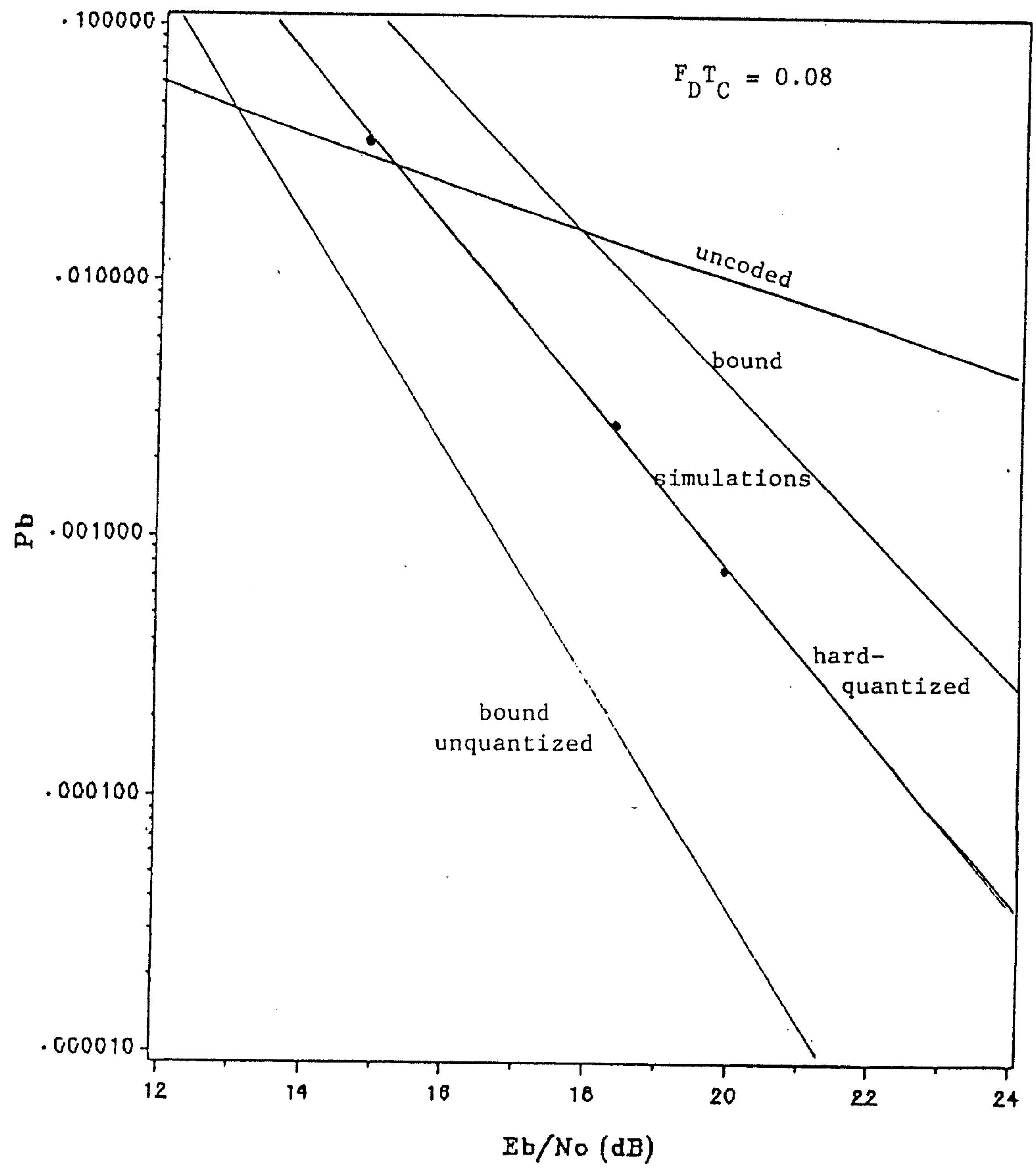

Fig. 16 : Bounds on the bit error probability vs Eb/No on a Rayleigh channel ($\rho(\tau)=J_0(2\pi F_D \tau)$). convolutional code: K=7, r=1/2; unquantized and hard-quantized NC-FSK demodulation.