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CALIBRATION AND MEASUREMENT OF SIX-PORT REFLECTOMETER USING A MATRIX APPROACH

PART III

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ABSTRACT

A new explicit six-port calibration method using four standard terminations based on matrix formalism is developed. The new method has a number of advantages: There is no limitation on the measurements to include a reference port for setting the power level during calibration and testing. The computation method is an explicit one and there is no need to use iterative procedures. Six-port calibrations using this new formalism has shown no singularities in the test results. This finding was contrary to other calibration procedures using the same six-port where singularities were found.

I. INTRODUCTION

During the past decade, microwave measurement instrumentation has seen the emergence of the six-port as an alternative in some applications to conventional network analyzers [1,2]. Using this technique, the magnitude and the phase of an unknown reflection coefficient are determined entirely by four power readings and the calibration constants at the measurement frequency. Moreover six-port reflectometer calibrations have seen the emergence of several methods [3,7] in the process of developing new calibration procedures. However the number of standard terminations as well as the computational effort needs to be further reduced. First Woods used seven standards and a linear procedure [3]. Li and Bosisio used four offset short circuits and matched load [5], Hunter and Somlo introduced an explicit six-port calibration method using five standards [6]. Later Quian used four standards but an iterative method to calculate the reflection coefficient; moreover the above method is limited to reflectometers which include a reference port [7].

In this paper an alternative method using four standards and an explicit non iterative procedure based on a matrix formalism to calibrate the six-port reflectometer is presented. In order to prove the method, a computer simulation was first used to calibrate 2 ideal six-port reflectometers over a frequency range. Secondly an experimental six-port reflectometer was calibrated over 2-4 GHz in 10 MHz steps using a junction previously reported [8] in the literature.

Some of the advantages related to this new calibration are: a) fewer standard terminations for explicit computations, b) matrix formalism is rapidly programmed and executed by computers, c) degenerate solutions encountered in non-linear calibration methods are avoided [5], d) singularities previously encountered in linear calibration procedures are eliminated [3], and finally, e) there is no limitation to dedicate one port to incident power level measurements.

II. MATRIX REPRESENTATION OF SIX-PORT REFLECTOMETER

It has been shown by Engen [9] that the voltage at 4 power detector connected to ports 3,4,5 and 6 of a six-port can be represented by

$$b_e = M_e a_2 + N_e b_2 \qquad (1)$$

where

$$M_e = S_{e2} - \frac{S_{e1} S_{22}}{S_{21}} \qquad e = 3, \ldots 6$$

$$N_e = \frac{S_{e1}}{S_{21}} \qquad e = 3, \ldots 6$$

and S_{ij} is the scaterring parameter of a six-port junction.

The reflection coefficient of the device under test connected to port 2 is given by:

$$\Gamma = \frac{a_2}{b_2} \quad . \tag{2}$$

Substituting (1) into (2), we get P_e as a function of Γ:

$$P_e = K\, b_e b_e^* = K|b_2|^2 [\,|\Gamma|^2 + \left|\frac{N_e}{M_e}\right|^2 + \Gamma \frac{N_e^*}{M_e^*} + \Gamma^* \frac{N_e}{M_e}\,] \tag{3}$$

for $\quad e = 3,\ldots 6$

K : is a scalar parameter.

Equation (3) can be represented as a scalar products of vectors as follows:

$$P_e = K|b_2|^2\, \underline{M}_e . \underline{R} \qquad \text{for} \quad e = 3,\ldots 6 \tag{4}$$

where

$$\underline{M}_e = \begin{vmatrix} |N_e|^2 \\ |M_e|^2 \\ N^*_e M_e \\ N_e M^*_e \end{vmatrix} \quad \text{and} \quad \underline{R} = \begin{vmatrix} 1 \\ |\Gamma|^2 \\ \Gamma \\ \Gamma^* \end{vmatrix}$$

By introducing the linear operator T, we can relate the four power vector readings $\underline{P}$ to the reflection coefficient vector $\underline{\Gamma}$ by the following expression:

$$\underline{P} = K|b_2|^2\, M\, \underline{R} = K|b_2|^2\, M\, T^{-1}\, \underline{\Gamma} = K|b_2|^2\, C\, \underline{\Gamma} \tag{5}$$

where

$$\underline{\Gamma} = \begin{vmatrix} 1 \\ |\Gamma|^2 \\ R(\Gamma) \\ I(\Gamma) \end{vmatrix} \quad ; \quad \underline{P} = \begin{vmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix} \quad ; \quad M = \begin{vmatrix} \underline{M}_1^T \\ \underline{M}_2^T \\ \underline{M}_3^T \\ \underline{M}_4^T \end{vmatrix}$$

$$T = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & j/2 & -j/2 \end{vmatrix}$$

and C is a 4x4 real matrix

$R(\Gamma)$ is the real part of Γ, and

$I(\Gamma)$ is the imaginary part of Γ.

The C matrix is a calibration matrix, and it represents the invariant properties of the measurement instrument including the six-port reflectometer. The value of $K|b_2|^2$ is the input power level of the device under test (DUT).

Referring to (5) the reflection coefficient vector can also be expressed as follows:

$$\underline{\Gamma} = \frac{1}{K\left|b_2\right|^2} C^{-1} \underline{P} = \frac{1}{K\left|b_2\right|^2} X.\underline{P} \qquad (6)$$

where $C^{-1} = X$ is the inverse matrix of C.

The above formalism can be used to calibrate the six-port reflectometer and to calculate the reflection coefficient only if the matrix C is non singular. In such a case, a six-port junction with a reference port can be simply represented by a real matrix C; the properties of the elements c_{ij} of this matrix are as follows:

$c_{ij} \neq 0$ for $i = 1,...3$ and for $j = 1,...4$

$c_{11} \neq 0$

and

$c_{12} = c_{13} = c_{14} = 0$

The above values of c_{ij} are related as shown below, to the position of the circle centers of the six-port junction in the Γ plane.

More generally any six-port junction (no reference port need to be specified) has a calibration matrix such that:

$c_{ij} \neq 0$ for $i = 1,...4$ and for $j = 1,...4$

Normalizing all the elements of each row of the C matrix by the second we get:

$C = \mathrm{Diag}(1/c_{12};\ 1/c_{22};\ 1/c_{23};\ 1/c_{24})C'$

where $c'_{ij} = (c_{ij}/c_{i2})$

for $i = 1,...4$ and for $j = 1,...4$.

The exact relation between the elements of the matrix C' and the positions of the circle centers for a specific six-port junction are as follows:

$$L_i = \left(-\frac{c'_{i3}}{2} \quad -\frac{c'_{i4}}{2} \right)$$

and

$$R_i^2 = |\bar{O}\bar{L}_i|^2 = c'_{i1}$$

where L_i is the center of the i^{th} circle

R_i is the distance from the origin to the center L_i.

and $i = 1,2,3$ and 4.

The elements c_{ij} of each row of the calibration matrix C are related together by the following "error function"

$$F_i(\epsilon) = c^2_{i3} + c^2_{i4} - 4c_{i1}c_{i2}.$$

An error in the actual calibration parameters produce a non-zero value for the above equation.

The complex reflection coefficient is deduced by normalizing all the elements of the reflection coefficient $\underline{\Gamma}$ by the first element of the same vector. In this case the second, third, and fourth element of the $\underline{\Gamma}$ vector becomes:

$$|\Gamma|^2 = (X_2.\underline{P})/(X_1\underline{P}) \quad (7.1)$$

$$R(\Gamma) = (X_3.\underline{P})/(X_1\underline{P}) \quad (7.2)$$

and $I(\Gamma) = (X_4 . \underline{P})/(X_1 \underline{P})$ (7.3)

where X_i is the i^{th} row of X matrix, and

$(X_1 \underline{P})$ is equal to the power level exciting the device under test.

We can see from equations (7) that since the first element of $\underline{\Gamma}$ is unity the level in the computation is automatically set. This formalism gives a unified approach for different junction designs (with or without a reference port).

III. CALIBRATION TECHNIQUE USING FOUR STANDARDS

To calibrate a six-port in a given frequency band it is necessary to determine the matrix X at a number of pre-determined frequency points within the chosen bandwidth. Several linear methods have been published to calibrate the six-port reflectometer. Woods used 7 standard terminations [3]; Somlo and Hunter used 6 standard terminations [4]. In our case the determination of matrix X (16 unknowns) based on the above methods needs to be resolved in a linear system with 16 equations for an arbitrary six-port junction. In order to determine the 16 unknowns it is necessary to use 6 standard terminations. For each standard we can write

$$(X_1\underline{P}_k)\ |\Gamma_k|^2 - X_2\underline{P}_k = 0$$

$$(X_1\underline{P}_k)\ R(\Gamma_k) - X_3\underline{P}_k = 0$$

$$(X_1\underline{P}_k)\ I(\Gamma_k) - X_4\underline{P}_k = 0$$

for $k = 1, \ldots 6$

The solution of the above linear system gives the required matrix calibration.

An alternative method based on matrix formalism and using only four standard terminations is developed below.

Equation (5) can be written for four standards terminations with four reflection coefficients $\Gamma_k (k=1, \ldots 4)$ as follows:

$$\underline{P}_k = \alpha\, C\, \underline{\Gamma}_k \qquad \text{for} \qquad k = 1, \ldots 4$$

where $\alpha = k|b_2|^2$

An explicit expression for the last equation gives:

$$\alpha(c_{11} + c_{12}|\Gamma_k|^2 + c_{13}R_k + c_{14}I_k) = P_{1k} \tag{8.1}$$

$$\alpha(c_{21} + c_{22}|\Gamma_k|^2 + c_{23}R_k + c_{24}I_k) = P_{2k} \tag{8.2}$$

$$\alpha(c_{31} + c_{32}|\Gamma_k|^2 + c_{33}R_k + c_{34}I_k) = P_{3k} \tag{8.3}$$

$$\alpha(c_{41} + c_{42}|\Gamma_k|^2 + c_{43}R_k + c_{44}I_k) = P_{4k} \tag{8.4}$$

The normalization of P_{2k}, P_{3k} and P_{4k} on P_{1k} gives:

$$c_{21}+c_{22}|\Gamma_k|^2+c_{23}R_k+c_{24}I_k = P_{2k}/P_{1k}(c_{11}+c_{12}|\Gamma_k|^2+c_{13}R_k+c_{14}I_{1k}) \tag{9.1}$$

$$c_{31}+c_{32}|\Gamma_k|^2+c_{33}R_k+c_{34}I_k = P_{3k}/P_{1k}(c_{11}+c_{12}|\Gamma_k|^2+c_{13}R_k+c_{14}I_{1k}) \tag{9.2}$$

$$c_{41}+c_{42}|\Gamma_k|^2+c_{43}R_k+c_{44}I_k = P_{4k}/P_{1k}(c_{11}+c_{12}|\Gamma_k|^2+c_{13}R_k+c_{14}I_{1k}) \tag{9.3}$$

⋮

for $k = 1, \ldots 4$.

Representing the vector reflection coefficients as a (4x4) G matrix, we have:

$$g_{k1} = 1$$

$$g_{k2} = |\Gamma_k|^2 \qquad \text{for} \quad k = 1, \ldots 4$$

$$g_{k3} = R_k$$

and

$$g_{k4} = I_k \ .$$

In addition the four power readings related to the four standards used above, in four (4x4) matrices can be given as follows:

$$P_2 = \mathrm{Diag}(P_{21}, P_{22}, P_{23}, P_{24})$$

$$P_3 = \mathrm{Diag}(P_{31}, P_{32}, P_{33}, P_{34})$$

$$P_4 = \mathrm{Diag}(P_{41}, P_{42}, P_{43}, P_{4}\ )$$

and

$$P'_1 = \mathrm{Diag}(1/P_{11},\ 1/P_{12},\ 1/P_{13},\ 1/P_{14}) \ .$$

Using the above matrix notation, the 4 equations (9.1) can be written as:

$$G\ C_2^T = P_2\ P'_1\ G\ C_1^T \ .$$

Therefore

$$C_2^T = G^{-1}\ P_2\ P'_1\ G\ C_1^T \ . \tag{10.1}$$

The same treatment can be done for equations (8.2) and (8.3) to give:

$$C_3^T = G^{-1}\ P_3\ P'_1\ G\ C_1^T \tag{10.2}$$

$$C_4^T = G^{-1}\ P_4\ P'_1\ G\ C_1^T \tag{10.3}$$

where C_i is the i^{th} row of Matrix C.

The determination of the three last rows (C_2,C_3,C_4) of the calibration matrix C requires that C_1 be first known. However, if it is assumed that Matrix P'_1 is equal to matrix pI, then one must have

$$c_{11} = p$$

$$c_{12} = c_{13} = c_{14} = 0$$

where p can be made equal to unity.

Consequently, we can deduce C_2, C_3 and C_4 as follows:

$$C_2^T = (G^{-1} P_2 G) C_0 \qquad (11.1)$$

$$C_3^T = (G^{-1} P_3 G) C_0 \qquad (11.2)$$

$$C_4^T = (G^{-1} P_4 G) C_0 \qquad (11.3)$$

where $C_0^T = (1,0,0,0)$.

Normalizing P_{1k}, P_{3k}, P_{4k} on P_{2k} and using the above treatment it is easy to show that

$$C_1^T = (G^{-1} P_1 G) C_0 \qquad (11.4)$$

where $P_1 = \mathrm{Diag}(p_{11},p_{12},p_{13},p_{14})$

From equations (11) it is seen that the determination of the calibration matrix needs only an explicit matrix product without any iterative computation procedures.

IV. <u>COMPUTER SIMULATION AND EXPERIMENTAL RESULTS</u>

In order to test the above calibration method, computer simulations were performed. At first, the relative vector power reading $\underline{P}_k$ related to vector reflection coefficient $\underline{\Gamma}_k$ is calculated using (5) with an ideal six-port matrix calibration C. Then according to $\underline{P}_k$ and $\underline{\Gamma}_k$ (k=1,...4) the six-port matrix calibration using the above method is calculated. Two different six-port junctions were used. The first was a six-port junction with a reference port as proposed by Engen [10]; the results are shown in Table I. A second six-port junction, without reference port as part of configuration suggested in reference [11], was then used. Table II shows the results obtained in this second case. By adding a random variation, within 1% for each of the four simulated power readings in equations (11), the resulting deviations are on the average less than 1% in the non-zero values of the matrix calibration elements as shown in Tables I and II. For this case the values of the error function are indicated in Tables I and II.

Computer simulation shows that if the four standard terminations have all the same magnitude or the same argument or if two standard terminations have the same magnitude and two standards (not necessary the same) have the same argument then the matrix G becomes singular and this method cannot be used.

The calibration algorithm has been implemented on a desk-top computer (HP 9816) and applied to a six-port junction [8]. Three positions of a precision Narda 901NF type sliding short (separated by a distance of $\lambda/8$ of a given calibration frequency value) and a Wiltron matched load (type 28A50-1) were used to calibrate the reflectometer over a 2-4 GHz range in 10 MHz steps. Tables III and IV show that the value of this error function $F_i(\epsilon)$ are very small. This is an indication that the actual calibration matrix describes very well the response of six-port reflectometer. Table III shows the measured magnitude and phase of an open circuit, a short circuit and the return loss of a 10 dB attenuator at 4 GHz. The measured phase value of the open circuit is taken as the reference plane. Table IV shows the measured magnitude and phase of a short circuit and a coaxial air line (10.35 cm) terminated by a short circuit at 2.89 GHz. At this frequency a coaxial air line of 10.35 cm length has a 4π phase angle. In this case the measured phase value of a short circuit is taken as the reference plane. The phase values found for open and short circuit conditions in Tables III and IV are within 1% of expected values. In addition it was found that the use of this linear calibration procedure produced no singularities in the test results. This is contrary to other calibration procedures using the above six-port where a number of singularities were found [8].

V. CONCLUSION

A new explicit six-port calibration method using four standards based on matrix formalism is presented and verified by computer simulation and experimental results. It is shown that the new method has the following advantages:

1) The four standards are restricted only in that their impedances must not all have the same magnitude nor the same argument.

2) There is no limitation that the reflectometer must include a reference port to indicate and set the power level during calibration and testing.

3) The calibration method is completly based on matrix formalism; this

allows rapid calculations and reduces the computational effort.

4) A random variation, within 1% of the power reading was added to the four simulated power levels. This random disturbance introduced a deviation of less than 5% in the elements of the calibration matrix.

5) The computation method is an explicit one and there is no need to use iterative procedures.

6) This method avoids degenerate solutions encountered in non-linear calibration procedures. Singularities sometimes encountered in linear calibration methods are also avoided.

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LIST OF TABLES

TABLE I

SIMULATION DATA (IDEAL VALUES)								
CALIBRATION MATRIX					CALIBRATION STANDARDS*			
i \ j	1	2	3	4	standard	$\|\Gamma_i\|^2$	R_i	I_i
1	0.000	1.0000	0.0000	0.0000	1	0	0	0
2	.2500	1.0000	-.7071	-.7071	2	1	1	0
3	.2500	1.0000	.7071	-.7071	3	1	0	1
4	.5000	1.0000	0.0000	1.4142	4	1	-1	0

COMPUTER SIMULATION RESULTS										
CALCULATED CALIBRATION MATRIX						CALCULATED CALIBRATION MATRIX WITH 1% RANDOM DISTURBANCE IN POWER READINGS				
i \ j	1	2	3	4	$F_i(\varepsilon)$	1	2	3	4	$F_i(\varepsilon)$
1	0.000	1.0000	0.0000	0.0000	.0000	.0000	.9943	-.0000	.0134	.0001
2	.2500	1.0000	-.7071	-.7071	.0000	.2506	1.0036	-.7148	-.7023	-.0016
3	.2500	1.0000	.7071	-.7071	.0000	.2501	.9933	.7073	.7253	.0326
4	.5000	1.0000	0.0000	1.4142	.0000	.4986	1.0151	.0095	1.4031	-.0557

* Calibration standards 1,2,3 and 4 are the ideal values of a mached load, and 3 positions of a sliding short separated by $\lambda_g/8$.

TABLE II

SIMULATION DATA (IDEAL VALUES)								
CALIBRATION MATRIX					CALIBRATION STANDARDS*			
i \ j	1	2	3	4	standard	$\lvert\Gamma_i\rvert^2$	R_i	I_i
1	4.0000	1.0000	.0000	-4.0000	1	0	0	0
2	2.0000	1.0000	2.8284	0.0000	2	1	1	0
3	4.0000	1.0000	.0000	4.0000	3	1	0	1
4	2.0000	1.0000	-2.8284	1.4142	4	1	-1	0

COMPUTER SIMULATION RESULTS										
CALCULATED CALIBRATION MATRIX						CALCULATED CALIBRATION MATRIX WITH 1% RANDOM DISTURBANCE IN POWER READINGS				
i \ j	1	2	3	4	$F_i(\varepsilon)$	1	2	3	4	$F_i(\varepsilon)$
1	4.0000	1.0000	.0000	-4.0000	.0000	3.9952	1.0075	-.0104	-3.9643	-.3845
2	2.0000	1.0000	2.8284	0.0000	.0000	2.0044	.9943	2.8031	.0243	-.1138
3	4.0000	1.0000	.0000	4.0000	.0000	4.0018	.9828	.0106	3.8943	-.5671
4	2.0000	1.0000	-2.8284	0.0000	.0000	1.9945	1.0409	-2.8070	-.0250	-.4248

* Calibration standards 1,2,3 and 4 are the ideal values of a matched load, and three positions of sliding short separated by $\lambda_g/8$.

TABLE III

EXPERIMENTAL CALIBRATION MATRIX						MEASURED VALUE / EXPECTED VALUE		
i \ j	1	2	3	4	$F_i(\varepsilon)$	standard	amplitude	phase (degree)
1	.1096	-.0013	-.0060	-.0171	.0009	open circuit	1.000/1.000	reference plane
2	.0679	.0106	-.0663	.0160	.0018	short circuit	1.019/1.000	-180.913 / -180.000
3	.0373	.0174	.0295	.0182	-.0014	—	—	—
4	.0323	.0230	.0114	-.0573	.0004	attenuator* in db	-19.573 / -19.086	—

* The expected value is value as measured with HP 8510 T ANA at Northern Telecom Canada (March 86).

TABLE IV

EXPERIMENTAL CALIBRATION MATRIX						MEASURED VALUE / EXPECTED VALUE		
$i \backslash j$	1	2	3	4	$F_i(\varepsilon)$	standard	amplitude	phase (degree)
1	.3381	.0023	-.0211	.0174	-.0023	—	—	—
2	.2326	.0999	-.0240	-.1299	-.0755	short circuit	$\frac{1.013}{1.000}$	reference plane
3	.1090	.0512	-.0998	.0982	-.0027	coax air line + S.C. (10.35cm)	$\frac{1.003}{1.000}$	$\frac{+179.868}{+180.000}$
4	.0898	.0291	.1161	.0069	.0031	—	—	—