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(A PROBABILISTIC BENEFIT-COST ANALYSIS OF
A ZONING PROGRAM FOR FLOOD PLAINS)

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SUMMARY

The flooding that occurs on flood plains causes considerable damage. Various techniques, one of which is zoning, are used to control the extent of flood damage. The benefit-cost analysis of zoning programs must take into account the random nature of flooding. This note outlines a method for determining the probability of such a zoning program being profitable and presents an application of the method to the assessment of the Outaouais Regional Community zoning program. A hydro-economic model is used to compute the expected value and variance of the benefits of the program. By applying the Central Limit Theorem, the probability of the benefit-cost ratio being greater than 1 is calculated.

In the application under consideration, a discount rate of 5 % was used for the economy as a whole, which resulted in an 83% probability of the program being profitable. When a discount rate of 10 % was applied to reflect the standards of public decision-makers, the probability rate dropped to 51 %.

INTRODUCTION

Flooding occurs on a random and repetitive basis and results in considerable losses for riverside residents as well as for the governments that may be required to provide compensation. Various techniques ranging from civil engineering works to administrative methods have been utilized to control the extend of flood damage. Zoning is one administrative method that can be used to regulate the level of economic activity in high-risk areas. The benefit-cost analysis of such programs must take into account factors such as the random nature of flooding. This note outlines a method for performing an analysis of this type and presents the results obtained when it was used to assess the federal/provincial zoning program that has been in place in the Outaouais Regional Community since 1977. The damage considered in this application was limited to that sustained by the residential sector and the benefit-cost ratios apply to all economic agents. More precisely, we shall calculate the probability of the benefit-cost ratio being greater than 1.

Hydro-economic models link flood damage to the hydrological characteristics of the floods by means of the submersion depth/damage relationship. Through a statistical analysis of the damage series, it is possible to derive a probability distribution of the damages as well as their expected value and variance. Finally, the Central Limit Theorem is used to determine the probability of the project being profitable.

With a discount rate of 5 per cent for the economy as a whole and 10 % for the government, the probability of the program being profitable is 83 and 51 %, respectively.

I. Expected value and variance of the benefit-cost ratio and probability of the program being profitable

The cost of such programs is considered to be a deterministic parameter consisting essentially of mapping and implementation costs. The mapping for the Outaouais Regional Community delineated the high and low-flow channel zones. high-flow zones are affected by floods with a probability of an annual occurrence greater than 5 per cent, whereas low-flow zones are affected by floods with a probability of an annual occurrence greater than 1 per cent. The zoning program prohibits the construction of new primary and secondary residences in the high-flow zones. The program also introduced construction standards designed to protect new dwellings in the low-flow zones against floods with a probability of an annual occurrence greater than 1 per cent.

The benefits consist primarily of the damage avoided as a result of the ban on or regulation of new housing construction.

With $E(B)$ being the expected value of the benefits of the program and C the program cost, the expected value of the benefit-cost ratio is written as follows:

$$E(B/C) = E(B)/C \quad (1)$$

$E(B)$ is equal to:

$$E(B) = E(TD_{NR}) - E(TD_R) \quad (2)$$

where TD_{NR} denotes the total actualized damage sustained under the non-regulated system and TD_R denotes that sustained under the regulated system.

Note the following: n , the number of years the program will be in effect; r , the discount rate; MVD_0 , the mean value of the dwellings for period 0 in the regulated zones and ψ , the annual growth rate of this value; ND_0 , the number of dwelling for period 0 in the regulated zones and τ , the annual growth rate of the number of dwellings.

The rate r , ψ , τ assumed to be constant over the study period.

With this notation we can write

$$E(\bar{D}_{NR}) = \sum_{i=1}^n MVD_0 * ND_0 * \frac{(1 + \tau)^i (1 + \psi)^i}{(1 + r)^i} E(MUD * PD) \quad (3)$$

MUD is the mean unit damage sustained by a dwelling in the flooded area at submersion depth (Z_1). Floods of various depths (Z_1) may occur during a year. PD is the percentage of dwellings affected at submersion depth (Z_1), in regulated zone. $E(MUD * PD)$ is calculated by means of the hydro-economic model, which we will now explain in detail.

Letting $G(Z)$ denote the percentage of damage sustained by a dwelling at submersion depth (Z), El-Jabi, et als (1982) and Ouellette, et als (1985) derived the following relationships ^{*1}:

^{*1} This model assumes uniform distribution of the flooded dwellings in the flooded area. Different versions of the model with various distributions were studied by El-Jabi (1980).

$$G(Z) = \frac{1}{\exp(\exp(-\alpha))-1} \left[\exp((1 - \exp(-\beta Z)) \exp(-\alpha)) - 1 \right] \quad (4)$$

$$MUD = \int_0^{Z_1} \frac{G(Z)}{Z_1} dZ \quad (5)$$

The parameters α and β of the function G are estimated on the basis of surveys of the damage sustained by flooded dwellings. PD, the percentage of dwellings affected at submersion depth (Z_1), is determined with reference to topographical maps. Environment Canada provides the hydrological data on the submersion depths (Z_1) for the study region.

A statistical series of $MUD * PD^{*1}$ is then generated. The frequency and extent of the floods are taken into account to determine the annual distribution of the product ($MUD * PD$). With $\rho(x)$ being the probability of ($MUD * PD$) exceeding a threshold x in the course of a year, Λ being the mean annual number of such events and δ being the average of the ($MUD * PD$) values per event, Todorovic (1970) showed that $\rho(x)$ is written as follows:

$$\rho(x) = 1 - \exp[\Lambda * \exp(x/\delta)] \quad (6)$$

from which we can now calculate the annual expected value of ($MUD * PD$):

$$E(MUD * PD) = \int_0^{\infty} \rho(x) dx \quad (7)$$

*1 This statistical series represents the damage that would have been sustained by the current housing stock based on observations from previous floods.

In the case of the zoned system, since the program prohibits the construction of new dwellings without flood protection, the growth rate of the number of dwellings (τ) is zero and the expected value of total damage is written as follows:

$$E(TD_R) = \sum_{i=1}^n MVD_0 * ND_0 * \frac{(1 + \psi)^i}{(1 + r)^i} E(MUD * PD) \quad (8)$$

The equation can then be simplified (see Canada, White (1980), p. 33-34) to derive the expected value of the benefit-cost ratio:

$$E(B/C) = \frac{1}{C} * MVD_0 * ND_0 * \left[\frac{1 - (1+r)^{-n} (1 + \tau + \psi + \tau\psi)^n}{r - (\tau + \psi + \tau\psi)} - \frac{1 - (1+r)^{-n} (1 + \psi)^n}{r - \psi} \right] * E(DUM * PR) \quad (9)$$

Since the floods in the study region are independent over time, the values for $(MUD * PD)$ are independent random variables. Hence, the variance of the benefit-cost ratio $V(B/C)$ is the sum of the independent random variables and is written as follows:

$$V(B/C) = \frac{1}{C^2} \sum_{i=1}^n MVD_0 * ND_0 \left(\frac{(1 + \tau)^i (1 + \psi)^i}{(1 + r)^i} - \frac{(1 + \psi)^i}{(1 + r)^i} \right)^2 * V(MUD * PD) \quad (10)$$

Equations (6) and (7) give

$$V(MUD * PD) = \int_0^{\infty} (MUD * PD)^2 \left\{ \frac{1}{\delta} \Lambda \exp \left(-\frac{1}{\delta} MUD * PD \right) \exp \left[-\Lambda \exp \left(-\frac{1}{\delta} MUD * PD \right) \right] \right\} * d(MUD * PD) - [E(MUD * PD)]^2 \quad (11)$$

By applying the Central Limit Theorem to the 30 random variables,

$$MVD_0 * ND_0 * \left[\frac{1 + \psi}{1 + r} \right]^i * (MUD * PD)_i, \quad i = 1, \dots, 30, \quad (12)$$

we can approximate the probability distribution of the benefit-cost ratio by means of a normal distribution of parameters $E(B/C)$ and $V(B/C)$.

The probability of the program being profitable can thus be expressed as follows:

$$P(B/C \geq 1) = P\left(\frac{B/C - E(B/C)}{\sqrt{V(B/C)}} \geq \frac{1 - E(B/C)}{\sqrt{V(B/C)}}\right) \quad (13)$$

and is found in the standardized normal distribution function table.

II. APPLICATION OF THE MODEL TO THE OUTAOUAIS REGIONAL COMMUNITY ZONING PROGRAM

The region affected by flooding in the Outaouais Regional Community can be subdivided into six economic units or municipalities: Masson, Gatineau, Aylmer and Pontiac, Hull-ouest, La Pêche-sud and La Pêche-nord.

The compensation paid in connection with the 1974 and 1976 floods was in the order of \$1.8 and \$1.2 million, respectively. A zoning program based on the federal/provincial mapping program for the flood zones was then implemented by the Outaouais Regional Community. As previously stated, the program was aimed at banning new construction in the high-flow channel zones and regulating it in the low-flow channel zones. This section presents the data and assumptions used in the application of the model and the results obtained.

The mean values of the dwellings (MVD) and the number of dwellings (ND) for the various economic units in 1977, the first year of the program and the base-year for the assessment, were obtained from the Quebec Department of Municipal Affairs. The rate of growth of MVD (ψ), was derived from 1977 to 1984 municipal assessments, which were adjusted for inflation and corrected to reflect basic land assessment changes. The rate of growth of the number of dwellings (τ) was computed for the same period. These rates were used for the 30-year study as well. Hence, it was assumed that zoning would have no impact on the overall rate of growth of the individual economic units but would affect the distribution of growth within the units. The values for these variables are given in Table 1.

Tableau 1. Data on housing stock

Municipality	Masson	Gatineau	Aylmer-Pontiac	Hull-Ouest	La Pêche-sud	La Pêche-nord
MVD (\$1977)	15 400	35 200	35 200	25 700	18 300	14 600
NDo	82	879	75	37	101	47
ψ	- 4,5%	- 3,0%	- 2,7%	- 1,7%	- 2,9%	- 2,9%
τ	0,7%	1,5%	1,4%	0,3%	0,6%	0,6%

Two discount rates were used in the application to reflect the focus of the benefit-cost ratio. A rate of 10 per cent was used in accordance with government standards and a rate of 5 per cent was used to represent the real long-term discounted rate for the economy as a whole.

The α et β values in the mean unit damage function (equ. 5) are normally obtained from post-flood surveys of the study region. No such survey exists for the Outaouais Regional Community. But El-Jabi, et als. (1982) and Ouellette, et als. (1985) estimated that $\alpha = 0.953$ and $\beta = 0.108$ for the mean unit damage in the Richelieu region. The use of these values for the Outaouais Regional Community is justified because of the similarity between the type and quality of dwellings in the two regions. Values for PD, the percentage of dwellings affected by flooding, were obtained by counting the dwellings indicated on the topographical maps.

With this data, we can calculate the statistical series (MUD * PD) for each area and estimate the coefficients Λ et δ in equation 6. Integral

(7) is then computed by means of the Continuous System Modelling Program (CSMP) and the following estimates of the expected values (8) and variance (9) of the benefits of the program for each region are derived. The results are presented in Table 2.

Table 2. Expected values and variance of the benefits

Municipality	Masson	Gatineau	Aylmer-Pontiac	Hull-Ouest	La Pêche-sud	La Pêche-nord
$E(B_i)_{r=.05}$	30 075	307 066	38 502	12 020	28 824	7 606
$V(B_i)_{r=.05}$	$1,39 \times 10^8$	$3,38 \times 10^{10}$	$3,51 \times 10^8$	$2,05 \times 10^8$	$1,31 \times 10^8$	$1,82 \times 10^7$
$E(B_i)_{r=.10}$	15 323	148 848	18 542	5 721	14 103	3 722
$V(B_i)_{r=.10}$	$4,37 \times 10^7$	$9,47 \times 10^9$	$9,18 \times 10^7$	$5,14 \times 10^6$	$3,60 \times 10^7$	$5,01 \times 10^6$

According to the Central Limit Theorem, both regional (B/C) ratios, $i = 1, 2, \dots, 6$, are normally distributed random variables. However, these regional variables are perfectly correlated, since the floods affect all economic units. The expected value and variance of the benefit-cost ratio for the region as a whole are formulated in this case (Canada and White, 1980, p. 290):

$$E(B/C) = \sum_{i=1}^6 E(B/C)_i = (1/C) \sum_{i=1}^6 E(B)_i$$

$$V(B/C) = \left\{ \sum_{i=1}^6 [V(B/C)_i]^{1/2} \right\}^2 = (1/C)^2 \left\{ \sum_{i=1}^6 [V(B)_i]^{1/2} \right\}^2$$

Table 3 gives the values for these variables in terms of the discount rate used. The cost of the program amounted to \$202 000 (1977 dollars).

Table 3. Expected values and variance of the mapping and zoning program

	$r = 0,05$	$r = 0,10$
$E(B)/C$	2,10	1,02
$V(B)/C^2$	1,16	0,60

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From these values we can calculate the probability of the project being profitable, $P(B/C \geq 1)$, which amounts to 83% or 51% per cent with a discount of 5% or 10%, respectively.

CONCLUSION

In this note we have developed a method that can be used to calculate the probability of a zoning program being profitable while taking the random nature of flooding into account. The approach combines readily available economic and hydrological data. The results for the application reveal that the program stands a good chance of being profitable from the standpoint of the general economy. But from the point of view of the government, which must take its financial capacity into account, the program would only be marginally profitable.

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