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Addendum

Addendum: Babic, S., et al. Self-Inductance of the Circular Coils of the Rectangular Cross-Section with the Radial and Azimuthal Current Densities. *Physics* 2020, 2, 352–367

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In [1] some ambiguities and imperfections were subsequently identified and are correspondingly modified and simplified below so that they correspond to those given in our software package [2]. Thus, people that have an interest in this domain could easily calculate the self-inductances presented here using the Mathematica code for them.

On page 354, Equation (5) reads:

$$L_R = -\frac{2\mu_0 N^2 R_1}{b^2 \ln^2(\alpha)} \sum_{n=1}^{11} \int_0^{\frac{\pi}{2}} \cos(2\beta)\, T_n d\beta, \quad (5)$$

where

$$T_1 = \frac{b^3}{3\sin(2\beta)}[2\arctan(q) - \arctan(q_1) - \arctan(q_2)],$$

$$T_2 = \frac{8}{3}(\alpha^3+1)\cos^3(\beta) - \frac{4}{3}\cos^2(\beta)(\alpha^2 r_2 + r_1) + \frac{2}{3}[(\alpha^2+1)\cos(2\beta)2\alpha](r - r_0),$$

$$T_3 = 4b\alpha^2\ \cos^2(\beta)\text{arsinh}[\tfrac{b}{2\alpha\cos(\beta)}] + 4b\cos^2(\beta)\text{arsinh}\left[\tfrac{b}{2\cos(\beta)}\right] \\ -2b[(\alpha^2+1)\cos(2\beta) + 2\alpha]\text{arsinh}(\tfrac{b}{r_0}),$$

$$T_4 = 2b^2[\alpha\ \text{arsinh}(v_{22}) + \text{arsinh}(v_{11})],$$

$$T_5 = -2b^2[\alpha\ \text{arsinh}(v_2) + \text{arsinh}(v_1)],$$

$$T_6 = -2b\sin(2\beta)[\alpha^2\arctan(p_{22}) + \arctan(p_{11})],$$

$$T_7 = 2b\sin(2\beta)[\alpha^2\arctan(p_2) + \arctan(p_1)],$$

$$T_8 = \frac{\alpha^3}{3}\sin^2(2\beta)\ln(\frac{m_2+1}{m_2-1}) + \frac{1}{3}\sin^2(2\beta)\ln(\frac{m_1+1}{m_1-1}),$$

$$T_9 = -\frac{\alpha^3}{3}\sin^2(2\beta)\ln(\frac{m_{20}+1}{m_{20}-1}) - \frac{1}{3}\sin^2(2\beta)\ln(\frac{m_{10}+1}{m_{10}-1}),$$

$$T_{10} = -\frac{\alpha^3}{3}\sin^2(2\beta)\ln(\frac{m_{22}+1}{m_{22}-1}) - \frac{1}{3}\sin^2(2\beta)\ln(\frac{m_{11}+1}{m_{11}-1}),$$

$$T_{11} = \frac{2(\alpha^3+1)}{3}\sin^2(2\beta)\ln\left[\frac{\cos(\beta/2)}{\cos(\beta/2)}\right],$$

where

$$r = \sqrt{b^2+\alpha^2+1+2\alpha\cos(2\beta)}, \quad r_0 = \sqrt{\alpha^2+1+2\alpha\cos(2\beta)}, \quad r_1 = \sqrt{b^2+4\cos^2(\beta)},$$

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$$r_2 = \sqrt{b^2 + 4\alpha^2 \cos^2(\beta)}\,, \quad r_{01} = \sqrt{4\cos^2(\beta)} = 2\cos(\beta), \ \ r_{02} = \sqrt{4\alpha^2 \cos^2(\beta)} = 2\alpha\cos(\beta),$$

$$q = \frac{\alpha \sin^2(2\beta) - b^2\cos(2\beta)}{b\sin(2\beta) r}, \qquad q_1 = \frac{\alpha^2\sin^2(2\beta) - b^2\cos(2\beta)}{b\sin(2\beta) r_2},$$

$$q_2 = \frac{\sin^2(2\beta) - b^2\cos(2\beta)}{b\sin(2\beta) r_1}, \ \ v_1 = \frac{\alpha + \cos(2\beta)}{\sqrt{b^2 + \sin^2(2\beta)}}, \ \ v_2 = \frac{1 + \alpha\cos(2\beta)}{\sqrt{b^2 + \alpha^2\sin^2(2\beta)}},$$

$$v_{11} = \frac{1 + \cos(2\beta)}{\sqrt{b^2 + \sin^2(2\beta)}}, \ \ v_{22} = \frac{\alpha + \alpha\cos(2\beta)}{\sqrt{b^2 + \alpha^2\sin^2(2\beta)}}, \ \ p_1 = \frac{b[\alpha + \cos(2\beta)]}{\sin(2\beta) r},$$

$$p_2 = \frac{b[1 + \alpha\cos(2\beta)]}{\alpha\ \sin(2\beta) r}, \ \ p_{11} = \frac{b[1 + \cos(2\beta)]}{\sin(2\beta) r_1}, \ \ p_{22} = \frac{b[\alpha + \alpha\cos(2\beta)]}{\alpha\sin(2\beta) r_2}\,,$$

$$m_1 = \frac{r}{\alpha + \cos(2\beta)}, \ \ m_2 = \frac{r}{1 + \alpha\cos(2\beta)}, \ \ m_{10} = \frac{r_0}{\alpha + \cos(2\beta)}, \ \ m_{20} = \frac{r_0}{1 + \alpha\cos(2\beta)}\,,$$

$$m_{11} = \frac{r_1}{1 + \cos(2\beta)}\,, \ \ m_{22} = \frac{r_2}{\alpha + \alpha\cos(2\beta)}.$$

Expressions for T_n, $n = 1, 2, \ldots, 11$, are given as in [2] and Appendix A where we calculate the self-inductance for the coil of the rectangular cross-section with radial current in Example 2 [1].

On page 355, Equation (6) reads:

$$L_{R-\mathrm{disk}} = \frac{4\mu_0 N^2 R_1(\alpha + 1)}{\ln^2\alpha}[E(k_0) - 1], \tag{6}$$

where

$$k_0^2 = \frac{4\alpha}{(\alpha + 1)^2}.$$

On page 355, Equation (7) reads:

$$L_A = -\frac{\mu_0 N^2 R_1}{15 b^2(\alpha - 1)^2} \sum_{n=1}^{6} \int_0^{\frac{\pi}{2}} \cos(2\beta)\ S_n d\beta, \tag{7}$$

where

$$S_1 = \frac{b^4}{\sin^2(2\beta)}\left[r_2 - \frac{b\cos(2\beta)}{\sin(2\beta)}\arctan(q_2)\right] + \frac{b^4}{\sin^2(2\beta)}\left[r_1 - \frac{b\cos(2\beta)}{\sin(2\beta)}\arctan(q_1)\right] - \frac{2b^4}{\sin^2(2\beta)}[r - \frac{b\cos(2\beta)}{\sin(2\beta)}\arctan(q)],$$

$$\begin{gathered} S_2 = 9a^2b^2r_2 + 9b^2r_1 - 9(a^2+1)b^2 r + 2\left[6a^4 cos^2(2\beta) - 2a^4\cos(2\beta) - 8a^4\right](r_2 - r_{02}) \\ +2[6\cos^2(2\beta) - 2\cos(2\beta) - 8](r_1 - r_{01}) - 4[3(a^4+1)\cos^2(2\beta) - \alpha(a^2+1)\cos(2\beta) \\ -2(\alpha^2+1)^2](r - r_0)\,, \end{gathered}$$

$$S_3 = 30b\sin(2\beta)\cos(2\beta)[a^4\arctan(p_{22}) + \arctan(p_{11}) - a^4\arctan(p_2) - \arctan(p_1)],$$

$$S_4 = 30b\left\{\alpha^4\sin^2(2\beta)\ln\left(\frac{r_2 + b}{r_2 - b}\right) + \sin^2(2\beta)\ln\left(\frac{r_1 + b}{r_1 - b}\right) - \frac{1}{2}[\left(\alpha^2 + 1\right)^2 - 2(\alpha^4 + 1)\cos^2(2\beta)]\ln\left(\frac{r + b}{r - b}\right)\right\},$$

$$\begin{gathered} S_5 = 12\cos(2\beta)\sin^2(2\beta)\{\alpha^5\ln[r_0 + 1 + \alpha\cos(2\beta)] + \ln[r_0 + \alpha + \cos(2\beta)] \\ -(\alpha^5 + 1)\ln[4\cos(\beta)\cos^2(\beta/2)] - \alpha^5\ln(\alpha)\}, \end{gathered}$$

$$S_6 = 4\cos(2\beta)[5b^2 - 3\sin^2(2\beta)]\ln\left[\frac{r+\alpha+\cos(2\beta)}{r_1+1+\cos(2\beta)}\right] + 4\alpha^3\cos(2\beta)[5b^2 - 3\alpha^2\sin^2(2\beta)]\ln\left[\frac{r+1+\alpha\cos(2\beta)}{r_2+\alpha+\alpha\cos(2\beta)}\right].$$

Expressions for $S_n, n = 1, 2, \ldots, 6$, are given as in [2] and Appendix B where we calculate the self-inductance for the coil of the rectangular cross-section with azimuthal current in Example 4 (case $\alpha = 3,\ b = 2$) [1].

On page 359, the self-inductance of the thin Bitter disk (pancake) is to be replaced by:

$$L_{R-\text{disk}} = 3.56991288673\ \text{H}.$$

At the request of many interested people, who have contacted us, we give the Mathematica codes for calculating the self-inductances with modified expressions for T_n and S_n [2], which are more friendly for the calculations than those given in [1].

Appendix A

MATHEMATICA CODE (RADIAL CURRENT) - EXAMPLE 2

```
ClearAll["Global'*"]
$Messages = {};
Beep[[]
mu = 4Pi/10000000;
R1 = 25/1000;
R2 = 35/1000;
l = 4/100;
a = R2/R1;
b = l/R1;
n1 = 100;
r = (a^2 + 2 a Cos[2x] + 1 + b^2)^(1/2);
r0 = (a^2 + 2a Cos[2x] + 1)^(1/2);
r2 = (2a^2 + 2 a^2 Cos[2x] + b^2)^(1/2);
r1 = (2 + 2 Cos[2x] + b^2)^(1/2);
f11 = 2b^3/3/Sin[2x] ArcTan[(a Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r)];
f22 = b^3/3/Sin[2x](ArcTan[( Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r1)] +
+ ArcTan[(a ^2Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r2)]);
T1 = f11-f22;
T2 = 8/3(a^3 + 1) Cos[x]^3 -4/3Cos[x]^2(a^2r2 + r1) + 2/3((a^2 + 1)Cos[2x] + 2a)(r-r0);
T3 = 4b a^2 Cos[x]^2 ArcSinh[b/(2a Cos[x])] + 4b Cos[x]^2 ArcSinh[b/(2Cos[x])]-2b((a^2 + 1)Cos[2x] + 2a)ArcSinh[b/r0];
T4 = 2b^2 (a ArcSinh[(a + a Cos[2x])/(a^2Sin[2x]^2 + b^2)^(1/2)] + ArcSinh[(1 + Cos[2x])/(Sin[2x]^2 + b^2)^(1/2)]);
T5 = -2b^2 (a ArcSinh[(1 + a Cos[2x])/(a^2Sin[2x]^2 + b^2)^(1/2)] + ArcSinh[(a + Cos[2x])/(Sin[2x]^2 + b^2)^(1/2)]);
T6 = -2b Sin[2x](a^2ArcTan[b (a + a Cos[2x])/(a Sin[2x]r2)] + ArcTan[b (1 + Cos[2x])/(Sin[2x]r1)]);
T7 = 2b Sin[2x](a^2ArcTan[b (1 + a Cos[2x])/(a Sin[2x]r)] + ArcTan[b (a + Cos[2x])/(Sin[2x]r)]);
T8 = Sin[2x]^2/3(a^3Log[(r + 1 + a Cos[2x])/(r-1-a Cos[2x])] + Log[(r + a + Cos[2x])/(r-a- Cos[2x])]);
T9 = -Sin[2x]^2/3(a^3Log[(r0 + 1 + a Cos[2x])/(r0-1-a Cos[2x])] + Log[(r0 + a + Cos[2x])/(r0-a- Cos[2x])]);
T10 = -Sin[2x]^2/3(a^3Log[(r2 + a + a Cos[2x])/(r2-a-a Cos[2x])] + Log[(r1 + 1 + Cos[2x])/(r1-1- Cos[2x])]);
T11 = 2/3(a^3 + 1) Sin[2x]^2Log[Cos[x/2]/Sin[x/2]];
f = Cos[2x](T1 + T2 + T3 + T4 + T5 + T6 + T7 + T8 + T9 + T10 + T11);
```

```
A = NIntegrate[f,{x,0,Pi/2},WorkingPrecision->30, AccuracyGoal->30];
N[A,16];
B = -2mu n1^2 R1/(b^2Log[a]^2);
N[B,16];
L = A B;
N[L,16]
0.0004383988542717143
L = 0.0004383988542717143 (H) = 0.4383988542717143 (mH)
```

Appendix B

MATHEMATICA CODE (AZIMUTHAL CURRENT)-EXAMPLE 4 (case $\alpha = 3$, $b = 2$).

```
ClearAll["Global'*"]
$Messages = {};
Beep[[]
mu = 4Pi/10000000;
n1 = 1;
R1 = 1;
R2 = 3;
l = 2;
a = R2/R1;
b = l/R1;
n1 = 1;
r = (a^2 + 2 a Cos[2x] + 1 + b^2)^(1/2);
r0 = (a^2 + 2 a Cos[2x] + 1)^(1/2);
r2 = (2a^2 + 2 a^2Cos[2x] + b^2)^(1/2);
r02 = (2a^2 + 2 a^2Cos[2x])^(1/2);
r1 = (2 + 2 Cos[2x] + b^2)^(1/2);
r01 = (2 + 2 Cos[2x])^(1/2);
f11 = b^4/Sin[2x]^2( r2-b Cos[2x]/Sin[2x] ArcTan[(a ^2Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r2)]);
f12 = b^4/Sin[2x]^2( r1-b Cos[2x]/Sin[2x] ArcTan[(Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r1)]);
f13 = -2b^4/Sin[2x]^2( r-b Cos[2x]/Sin[2x] ArcTan[(a Sin[2x]^2-b^2 Cos[2x])/(b Sin[2x]r)]);
S1 = f11 + f12 + f13;
f21 = 9a^2b^2r2 + 2(6a^4Cos[2x]^2-2a^4Cos[2x]-8a^4)(r2-r02);
f22 = 9b^2r1 + 2(6Cos[2x]^2-2Cos[2x]-8)(r1-r01);
f23 = -9b^2(a^2 + 1)r -4(3(a^4 + 1)Cos[2x]^2-a(a^2 + 1)Cos[2x]-2(a^2 + 1)^2)(r-r0);
S2 = f21 + f22 + f23;
f31 = 30b Sin[2x]Cos[2x](a^4ArcTan[b (1 + Cos[2x])/( Sin[2x]r2)] + ArcTan[b (1 + Cos[2x])/( Sin[2x]r1)]);
f32 = -30b Sin[2x]Cos[2x](a^4ArcTan[b (1 + a Cos[2x])/( a Sin[2x]r)] + ArcTan[b (a + Cos[2x])/( Sin[2x]r)]);
S3 = f31 + f32;
S4 = 15b (a^4Sin[2x]^2Log[(r2 + b)/(r2-b)] + Sin[2x]^2Log[(r1 + b)/(r1-b)]-((a^2 + 1)^2-2(a^4 + 1)Cos[2x]^2)/2Log[(r + b)/(r-b)]);
S5 = 12Cos[2x]Sin[2x]^2(a^5Log[r0 + 1 + a Cos[2x]] + Log[r0 + a + Cos[2x]]-(a^5 + 1)Log[4Cos[x]Cos[x/2]^2]-a^5Log[a]);
f61 = 4Cos[2x](5b^2-3Sin[2x]^2)Log[(r + a + Cos[2x])/(r1 + 1 + Cos[2x])];
f62 = 4a^3Cos[2x](5b^2-3a^2Sin[2x]^2)Log[(r + 1 + a Cos[2x])/(r2 + a + a Cos[2x])];
S6 = f61 + f62;
f = Cos[2x](S1 + S2 + S3 + S4 + S5 + S6);
A = NIntegrate[f,{x,0,Pi/2},WorkingPrecision->30, AccuracyGoal->30];
N[A,16];
B = -mu n1^2 R1/15/b^2/(a-1)^2;
N[B,16];
```

L = A B;
N[L,16]
$2.533006546891938*10^{-6}$
L = 2.533006546891938 (μH)

References

1. Babic, S.; Akyel, C. Self-Inductance of the Circular Coils of the Rectangular Cross-Section with the Radial and Azimuthal Current Densities. *Physics* **2020**, *2*, 19. [CrossRef]
2. Babic, S. Calculation of the Self, Mutual Inductances, and the Magnetic Forces between Circular Coils-Software Package. Available on the request.