Titre: Title:	Conception et mise à jour des réseaux de télécommunication
Auteur: Author:	Steven Chamberland
Date:	1998
Type:	Mémoire ou thèse / Dissertation or Thesis
Référence: Citation:	Chamberland, S. (1998). Conception et mise à jour des réseaux de télécommunication [Thèse de doctorat, École Polytechnique de Montréal]. PolyPublie. https://publications.polymtl.ca/8724/

URL de PolyPublie: PolyPublie URL:	https://publications.polymtl.ca/8724/
Directeurs de recherche: Advisors:	Brunilde Sanso
Programme: Program:	Non spécifié

UNIVERSITÉ DE MONTRÉAL

CONCEPTION ET MISE À JOUR DES RÉSEAUX DE TÉLÉCOMMUNICATION

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THÈSE PRÉSENTÉE EN VUE DE L'OBTENTION
DU DIPLÔME DE PHILOSOPHIAE DOCTOR (Ph.D.)
(MATHÉMATIQUES DE L'INGÉNIEUR)
MARS 1998

0-612-37374-6

UNIVERSITÉ DE MONTRÉAL

ÉCOLE POLYTECHNIQUE DE MONTRÉAL

Cette thèse intitulée:

CONCEPTION ET MISE À JOUR DES RÉSEAUX DE TÉLÉCOMMUNICATION

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en vue de l'obtention du diplôme de: Philosophiae Doctor
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À mes parents

REMERCIEMENTS

Je tiens tout d'abord à remercier ma directrice de thèse, Madame Brunilde Sansó, professeure à l'École Polytechnique de Montréal, pour sa disponibilité et la confiance qu'elle m'a témoignée tout au long de ces années de travail conjoint. En outre, ses conseils et critiques pertinentes m'ont permis d'avancer rapidement et d'apprendre énormément. J'espère sincèrement que cette thèse est le gage de collaborations futures toutes aussi fructueuses.

Je remercie grandement Madame Odile Marcotte, professeure à l'Université du Québec à Montréal, pour sa collaboration, ses conseils et critiques pertinentes pour la première partie de ce travail.

Je remercie les professeurs, chercheurs et étudiants que j'ai connus lors de mon passage au Département de mathématiques et génie industriel, au Centre de recherche sur les transports (CRT) et au Groupe d'études et de recherche en analyse des décisions (GERAD), pour avoir contribué à ma formation de chercheur. Je tiens à remercier plus particulièrement Benoît Bourbeau, Bernard Gendron et Luís Lopes, pour les nombreuses heures passées ensemble, et Gilbert Laporte pour son implantation de l'heuristique GENIUS que j'ai si souvent utilisée.

Je remercie Paul-André Raymond de NORTEL pour les informations concernant les coûts des bases, ports, multiplexeurs et autres composantes utilisées dans les réseaux à large bande.

Je remercie Marie-Hélène Dupuis, bibliothécaire à l'École Polytechnique de Montréal, pour l'aide qu'elle m'a apportée à la recherche d'articles et documents parfois difficiles à trouver.

Je remercie également Daniel Charbonneau, Guy Bordeleau et Pierre Girard, responsables des ressources informatiques au CRT et au GERAD, pour l'aide technique qu'ils m'ont apportée. Je tiens aussi à remercier les membres des personnels administratifs et des secrétariats du Département de mathématiques et génie industriel, du CRT et du GERAD, plus particulièrement, Anne-Marie Goulet, Suzanne Guindon, Lucie L'Heureux, Claudine St-Pierre, Lucie-Nathalie Cournoyer et Francine Benoît, pour les nombreux services rendus. Je remercie aussi Nicole Paradis, responsable de l'édition au GERAD, pour son aide concernant le logiciel de traitement de texte LaTeX.

Je tiens à remercier les organismes qui, directement ou indirectement, m'ont soutenu financièrement, et particulièrement les suivants: le Conseil de recherches en sciences et en génie du Canada (CRSNG), le Fonds pour la formation des chercheurs et l'aide à la recherche (FCAR), l'École Polytechnique de Montréal, le CRT, le GERAD, l'Association canadienne de technologie de pointe (ACTP) et la Société canadienne de recherche opérationnelle (SCRO).

Je remercie les membres du jury pour avoir accepté de réviser cette thèse.

Finalement, je tiens à remercier vivement mes parents et ma soeur pour leur confiance et encouragement durant toutes ces années d'étude.

RÉSUMÉ

Cette thèse est constituée de quatre articles portant sur la planification des réseaux de télécommunication à deux niveaux. Le terme «deux niveaux» réfère au fait que le réseau est constitué de deux sous-réseaux: le réseau d'accès et le réseau de transmission. Le réseau d'accès relie les clients aux commutateurs tandis que le réseau de transmission relie les commutateurs ensemble. De plus, nous supposons que les commutateurs sont modulaires, avec différents modèles de ports et bases, comme le sont les commutateurs ATM fabriqués par plusieurs compagnies. En fait, un des traits saillants de cette thèse est la considération des commutateurs modulaires dans les modèles de planification.

Le premier article traite du problème de la conception des réseaux d'entreprises. Nous définissons un réseau d'entreprises comme un réseau à deux niveaux contenant des commutateurs modulaires et utilisant la technologie SONET OC-3 dans le réseau d'accès et OC-192 dans le réseau de transmission. De plus, dans les réseaux d'entreprises, des multiplexeurs peuvent être utilisés pour brancher plusieurs liens à un seul port. Dans le premier article nous présentons, premièrement, une revue de la littérature concernant la planification des réseaux à deux niveaux et proposons un modèle de programmation mathématique pour le problème de la conception des réseaux d'entreprises. Parmi toutes les topologies possibles pour le réseau de transmission, nous étudions plus particulièrement les topologies en arbre et en anneau. Par la suite, dans le but de trouver une bonne solution au modèle, nous proposons une heuristique gloutonne et une heuristique basée sur le principe de la recherche avec tabous. Une analyse de la performance des heuristiques montre que la

méthode de recherche avec tabous trouve d'excellentes solutions qui sont en moyenne à 0.86% d'une borne inférieure trouvée par la résolution d'une version relaxée du modèle.

Le deuxième article porte sur le problème de la conception des réseaux multitechnologies. Nous définissions un réseau multitechnologies comme un réseau à deux niveaux contenant des commutateurs modulaires et dont plusieurs technologies et vitesses de transmission peuvent être utilisées dans le réseau d'accès, pour satisfaire la demande spécifique de chaque client (par exemple, une maison, une entreprise, un fournisseur de services ou un réseau). De plus, dans les réseaux multitechnologies, la technologie SONET OC-192 est utilisée dans le réseau de transmission. Dans le deuxième article nous proposons, dans un premier temps, un modèle de programmation mathématique pour le problème de la conception des réseaux multitechnologies et étudions, par la suite, les topologies en arbre, en anneau et en maille pour le réseau de transmission. Comme la structure du modèle est semblable à celle du modèle proposé dans le premier article, des heuristiques similaires sont considérées. La meilleure heuristique, basée sur le principe de la recherche avec tabous, trouve des solutions qui sont en moyenne à moins de 2.87% de l'optimalité.

Le troisième article traite du problème de l'expansion des réseaux multitechnologies. Dans un premier temps, nous proposons un modèle de programmation mathématique pour ce problème et étudions la topologie en anneau multiple pour le réseau de transmission. Cette topologie est très utilisée en pratique car elle a l'avantage d'être à la fois peu dense et biconnexe ainsi que plus performante que la topologie en anneau. Pour trouver une bonne solution au modèle, nous proposons une heuristique initiale qui trouve une solution de départ et une heuristique basée sur le principe de la recherche avec tabous pour améliorer cette solution. Enfin, nous présentons un exemple détaillé de la conception et de l'expansion d'un réseau multitechnologies suivi d'une analyse de la performance des algorithmes proposés, cela en utilisant deux bornes inférieures trouvées par la résolution de deux versions relaxées du modèle. L'approche de résolution proposée permet de trouver des solutions qui sont en moyenne à 0.59% de la meilleure borne inférieure.

Dans le dernier article nous présentons un modèle de programmation mathématique pour le problème de la mise à jour des réseaux d'entreprises et proposons une heuristique initiale, une heuristique basée sur le principe de la recherche avec tabous et un algorithme de post-optimisation. Par la suite, nous présentons un exemple détaillé de la conception et la mise à jour d'un réseau d'entreprises suivi d'une analyse de la performance des heuristiques proposées, cela en utilisant une borne inférieure. L'approche de résolution proposée trouve des solutions qui sont en moyenne à moins de 0.74% de l'optimalité.

ABSTRACT

This thesis is composed of four articles on the planning of two-level telecommunication networks. The term "two-level" refers to the fact that the network is composed of two major subnetworks: the access and the backbone network. The access network links the users to switches whereas the backbone network links the switches to one another. Moreover, we suppose that the switches used are modular, with different types of ports and bases, such as ATM switches. In fact, an important feature of our work is the consideration of modular switches in the planning models.

The first article deals with the design problem of private networks. We define a private network as a two-level network containing modular switches, OC-3 SONET links in the access networks and OC-192 SONET links in the backbone network. Moreover, in such a network, multiplexers may be used to connect several links to a single port. In the first article, we propose a literature review on the two-level network planning problems and a mathematical programming model for the design problem of private networks. This model is specialized to backbone networks having a ring or a tree topology. Next, in order to obtain a good solution, we propose a greedy heuristic that provides a starting solution and a tabu search heuristic to improve that starting solution. The solutions obtained are, on average, at 0.86% from a lower bound found by solving a relaxed version of the model.

The second article is dedicated to the design of multitechnology networks. We define a multitechnology network as a two-level network containing modular switches and several access technologies and rates in order to satisfy the specific demand

of each client. Moreover, in such a network, OC-192 SONET links are used in the backbone network. In this article, we propose a mathematical programming model for the multitechnology network design problem and study three backbone topologies (tree, ring and full-mesh). Next, we propose a greedy heuristic and a more sophisticated heuristic based on the tabu search principle. The solutions obtained with the tabu-based approach are, on average, within 2.87% of the optimal solution.

The third article deals with the problem of how to expand multitechnology networks in a cost-effective way. We propose a mathematical programming model for this problem for the specific case in which the backbone has a multiple ring topology. The choice was made because many telecommunication backbones use this topology. Next, an initial heuristic is designed to provide a starting solution, and a tabu-based heuristic is proposed to improve the solution. We present an illustrative example of a multitechnology network design with its successive expansions followed by a systematic set of experiments designed to assess the performance of the proposed algorithms (using two lower bounds). The solutions obtained are, on average, at 0.59% from the best lower bound.

The last article is devoted to the update problem of private networks. Again, an initial heuristic, a tabu-based heuristic and a post-optimization algorithm are proposed. We present an illustrative example followed by a performance analysis of the proposed heuristics (using a lower bound). The solutions obtained are, on average, within 0.74% of the optimal solution.

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LISTE DES ABRÉVIATIONS

ATM *Asynchronous Transfer Mode*

DCMST *Degree Constraints Minimum Spanning Tree*

DP *Design Problem*

EP *Expansion Problem*

FDDI *Fiber Distributed Data Interface*

FR *Frame Relay*

GH *Greedy Heuristic*

ITU *International Telecommunication Union*

LAN *Local Area Network*

MAN *Metropolitan Area Network*

OC *Optical Carrier*

PCTSP *Prize-Collecting Traveling Salesman Problem*

PO *Post-Optimization algorithm*

SONET *Synchronous OPTical Network*

STS *Steiner Tree-Star problem*

TR *Token Ring*

TS *Tabu Search algorithm*

TSP *Traveling Salesman Problem*

UP *Update Problem*

CHAPITRE 1

Introduction

La déréglementation dans le domaine des télécommunications a produit une augmentation de la compétition dans l'industrie et par le fait même une diminution de tarif pour plusieurs services, par exemple, les appels interurbains et l'Internet. Cependant, pour certains services qui ne sont pas encore vraiment touchés par la déréglementation, les tarifs ont augmenté, par exemple, les services de base de téléphone et de télévision par câble. De plus, l'introduction des nouvelles technologies à large bande, en particulier l'ATM (*Asynchronous Transfer Mode*) et le FR (*Frame Relay*), permet aux compagnies d'offrir de nouveaux services jusqu'alors impossible à offrir avec les équipements et la technologie traditionnelle. L'ATM est une technologie de commutation rapide de paquets de longueur fixe, appelés cellules, et orientée connexion, choisie par l'ITU (*International Telecommunication Union*) comme standard pour les réseaux multiservices. Contrairement à la commutation de paquets standard, la commutation rapide de paquets permet à un commutateur d'acheminer un paquet à un autre sans avoir reçu tout le paquet. Le FR est aussi une technologie de commutation rapide de paquets, sauf que ceux-ci sont de longueur variables, appelés trames. Pour plus de détails concernant l'ATM voir Händel, Huber et Schröder (1994) et concernant le FR voir Stallings (1995). Tout ces changements (variation des tarifs et nouveaux services) font fluctuer constamment le nombre de clients, ce qui oblige les compagnies de télécommunications à faire régulièrement des modifications à leurs réseaux. Ainsi, l'optimisation des coûts lors de la conception et la mise à jour des réseaux devient pratiquement indispensable pour devenir, ou rester, compétitif.

Cette thèse traite d'un sujet de recherche qui nous a été proposé par NORTEL, concernant la planification des réseaux à deux niveaux à large bande. Le terme «deux niveaux» fait référence au fait que le réseau est constitué de deux sous-réseaux: le réseau d'accès reliant les clients aux commutateurs et le réseau de transmission reliant les commutateurs ensembles. De plus, nous considérons que les commutateurs sont modulaires, comme le sont par exemple les commutateurs ATM et FR fabriqués par plusieurs manufacturiers. Ainsi, tout ce qui est proposé dans cette thèse s'applique à tout les réseaux utilisant ce type de commutateurs, les réseaux ATM et FR ne sont que des exemples. Un commutateur modulaire comprends une base et des ports. Une base est constituée d'un contrôleur et d'un réseau de connexion avec des fentes d'entrées et de sorties pour y insérer des ports. Les ports sont utilisés pour brancher des liens ou multiplexeurs aux bases et les multiplexeurs pour brancher plusieurs liens à un port.

Il y a deux principaux problèmes concernant la planification d'un réseau: le problème de la conception et celui de la mise à jour, aussi appelé problème d'évolution des réseaux. Le problème de conception consiste à trouver un premier réseau pour satisfaire la demande des clients avec une certaine performance et fiabilité (qualité et taux de service) tandis que le problème de mise à jour consiste à modifier un réseau existant pour satisfaire la demande des nouveaux et anciens clients avec une certaine performance et fiabilité.

Ce chapitre est divisé comme suit. Dans la section 1.1, nous présentons les types de réseaux et les problèmes de planification traités dans cette thèse tandis que dans la section 1.2, nous présentons une revue de la littérature concernant la planification des réseaux à deux niveaux avec des commutateurs modulaires. Dans la section 1.3, nous présentons les objectifs de cette thèse tandis que dans la section 1.4, nous présentons sa structure.

1.1 Définitions préliminaires

1.1.1 Réseaux d'entreprises et multitechnologies

Dans ce travail, nous considérons deux types de réseaux que nous appelons les réseaux d'entreprises et multitechnologies. Il est important de mentionner que cette appellation est propre à ce travail, à savoir qu'elle ne fait référence à aucune définition de la littérature.

1.1.1.1 Réseaux d'entreprises

Dans les réseaux d'entreprises, les clients (entreprises) sont branchés aux commutateurs avec des liens de technologie SONET (*Synchronous Optical NETwork*) de débit fixe, par exemple OC-3 (155.520 Mbits/sec), formant le réseau d'accès tandis que les commutateurs sont branchés ensembles avec des liens SONET aussi de débit fixe, par exemple OC-192 (9.953 Gbits/sec), formant le réseau de transmission. OC (*Optical Carrier*) est l'unité fondamentale de débit au niveau SONET correspondant à 51.84 Mbits/sec. SONET est la technologie au niveau physique (niveau 1) utilisé dans les réseaux ATM (niveau 2). Pour plus de détails concernant la technologie SONET voir, par exemple, Ballert et Ching (1989). Rappelons que les débits OC-3 et OC-192 ne sont que des exemples et peuvent être remplacés par n'importe quel autre débit.

Les hypothèses suivantes définissent l'organisation d'un réseau d'entreprises:

- chaque client est branché à exactement un commutateur avec un lien OC-3;
- les commutateurs sont reliés par des liens OC-192;
- le nombre de liens branchés à un multiplexeur est inférieur ou égal au nombre d'entrées (capacité) du multiplexeur;
- le nombre de ports installés dans les fentes d'une base est inférieur ou égal au nombre de fentes (capacité) de la base;
- le nombre d'entrées d'un multiplexeur branché à un port OC-3-n est n, ce qui permet de brancher n liens OC-3 à ce port avec un multiplexeur à n entrées;
- le nombre d'entrées d'un multiplexeur branché à un port OC-192-h est h, ce qui permet de brancher h liens OC-192 à ce port avec un multiplexeur à h entrées.

La figure 1.1 illustre un commutateur modulaire où le client C1 est branché à la fente 1 avec un port OC-3-1 tandis que les clients C2 à C5 sont branchés à la fente 2 avec un port OC-3-4 et un multiplexeur à quatre entrées. De plus, le commutateur S1 est branché avec un port OC-192-1 à la fente 3. Nous considérons dans ce travail que les commutateurs sont symétriques, à savoir que l'équipement installé dans les fentes d'entrées est identique à celui installé dans les fentes de sorties.

La figure 1.2 présente un exemple de réseau d'entreprises avec 200 clients et 5 bases contenant chacune 16 fentes. Cette figure illustre la topologie du réseau ainsi que la configuration de chaque commutateur en terme des modèles de ports. Remarquons que le réseau d'accès forme des étoiles et le réseau de transmission un

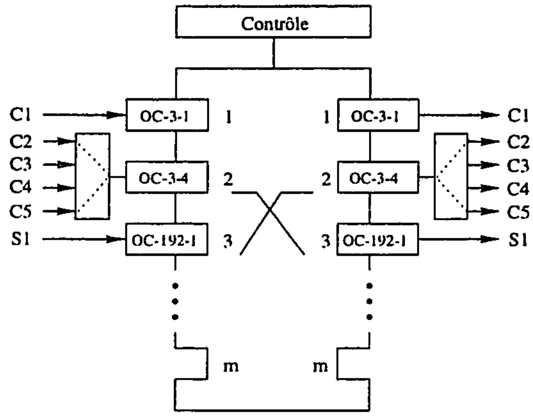


Figure 1.1: Exemple d'un commutateur modulaire utilisé dans un réseau d'entreprises

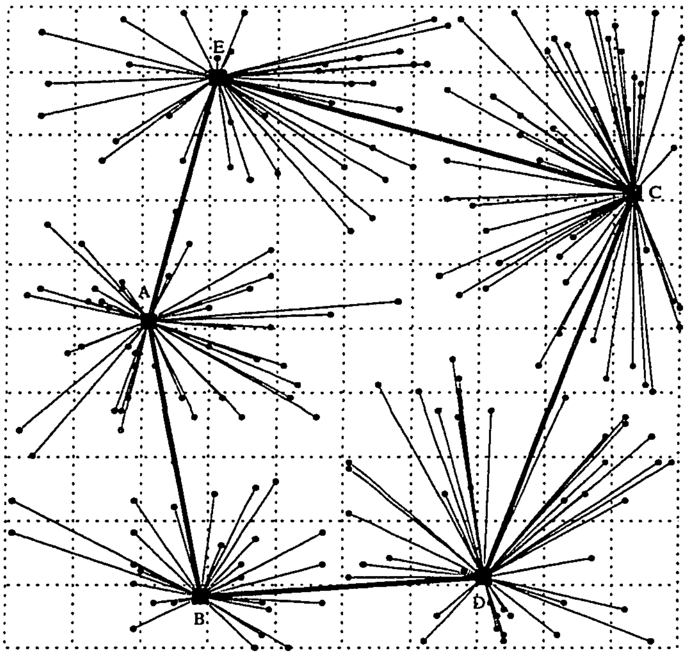
anneau. Nous disons alors que le réseau d'accès a une topologie en étoile et le réseau de transmission a une topologie en anneau.

1.1.1.2 Réseaux multitechnologies

Dans les réseaux multitechnologies, les clients peuvent être branchés au commutateurs avec plusieurs technologies et débits tandis que les commutateurs sont branchés ensembles avec des liens SONET de débit fixe, par exemple OC-192. Ce type de réseau est très intéressant lorsque la demande des clients varie beaucoup d'un à l'autre. Rappelons que le débit OC-192 n'est qu'un exemple et peut être remplacé par n'importe quel autre débit.

Les hypothèses suivantes définissent l'organisation d'un réseau multi-technologies:

- chaque client est branché directement à un commutateur avec un port et un lien de technologie et de débit demandé par le client;



Commutateur A

Port	Nombre
OC-3-1	0
OC-3-4	11
OC-192-1	0
OC-192-2	1

Commutateur B

Port	Nombre
OC-3-1	0
OC-3-4	7
OC-192-1	0
OC-192-2	1

Commutateur C

Port	Nombre
OC-3-1	0
OC-3-4	13
OC-192-1	0
OC-192-2	1

Commutateur D

Port	Nombre
OC-3-1	2
OC-3-4	10
OC-192-1	0
OC-192-2	1

Commutateur E

Port	Nombre
OC-3-1	0
OC-3-4	9
OC-192-1	0
OC-192-2	1

- Client
- Lien OC-3
- Lien OC-192
- Base

Figure 1.2: Exemple d'un réseau d'entreprises

- les commutateurs sont reliés avec des ports et des liens OC-192;
- le nombre de ports installés dans les fentes d'une base est inférieur ou égal au nombre de fentes (capacité) de la base;
- un seul lien peut être branché à un port.

La figure 1.3 présente un exemple de réseau multitechnologies avec 190 clients et 10 bases. Chaque client est branché au réseau avec un lien de technologie SONET de vitesse OC-3 ou OC-12 et deux modèles de bases sont utilisés: le modèle A contenant 16 fentes et le modèle B contenant 32 fentes. Comme le montre la figure, le réseau d'accès forme des étoiles et le réseau de transmission des anneaux contigus sans point d'articulation. Nous disons alors que le réseau de transmission a une topologie en anneau multiple.

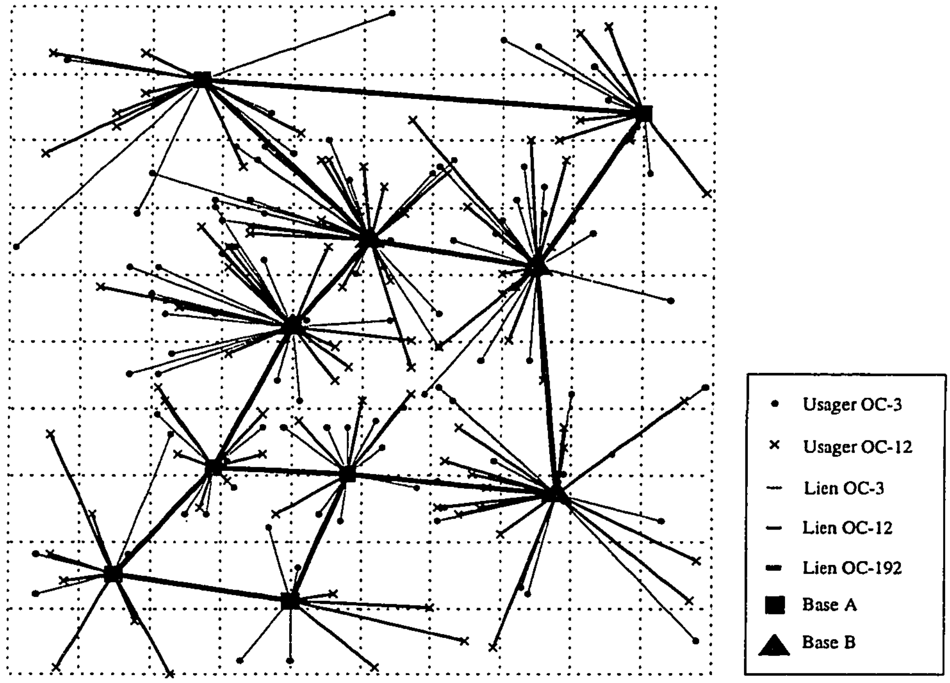


Figure 1.3: Exemple d'un réseau multitechnologies

Plusieurs autres technologies peuvent être utilisées dans l'accès, par exemple, TR (*Token Ring*), FDDI (*Fiber Distributed Data Network*) et Ethernet. TR est une technique et méthode d'accès des réseaux locaux fonctionnant sur le principe du passage de jeton sur une liaison en forme d'anneau. Un jeton est une suite de bits particulière qui circule en permanence d'une station à l'autre, toujours dans le même sens. Si la station n'a rien à émettre, elle retransmet le jeton. FDDI est une norme de transmission pour des réseaux locaux ou des interconnexions de réseaux locaux en fibre optique. On peut aussi l'utiliser comme technologie d'accès. Le fonctionnement de FDDI est similaire à celui de TR sauf que FDDI utilise un double anneau fonctionnant en sens opposé à 100 Mbits/sec. Ethernet est une norme de transmission à 10 Mbits/sec pour des réseaux locaux en câble coaxial avec une topologie en arbre. Une version à 100 Mbits/sec existe, appelée Fast Ethernet. Les normes Ethernet et Fast Ethernet sont surtout utilisées pour transporter le trafic Internet. Pour plus de détails concernant ces technologies voir Walrand (1991).

1.1.2 Topologies du réseau

Plusieurs topologies sont possibles pour le réseau de transmission. La figure 1.4 illustre les topologies en arbre, en anneau, en anneau multiple et en maille. La topologie en arbre est la topologie connexe la moins coûteuse, mais ne procure aucune protection contre les pannes. Ainsi, les réseaux en arbre sont souvent utilisés lorsque l'aspect économique est beaucoup plus important que celui de la fiabilité, comme c'est souvent le cas pour les réseaux privés. Contrairement à la topologie en arbre, la topologie en anneau procure une protection contre les pannes simples d'un lien ou commutateur si une technique de réacheminement du trafic est utilisée. En fait, la topologie en anneau est la topologie biconnexe la moins coûteuse. La topologie en anneau multiple est plus fiable et plus performante que la topologie en anneau. Ainsi, cette topologie est souvent utilisée lorsque le réseau doit couvrir une grande région et que l'aspect fiabilité est assez important, comme c'est le cas pour la majorité des réseaux commerciaux. La topologie en maille, c'est-à-dire un graphe complet, est la topologie la plus fiable, la plus performante, mais aussi la plus coûteuse. Ainsi, cette topologie utilisée lorsque la fiabilité et/ou la performance du réseau doit être maximale, comme c'est le cas pour certains réseaux militaires.

1.1.3 Problèmes de planification des réseaux

Dans cette thèse, nous considérons les problèmes de la conception et la mise à jour des réseaux d'entreprises et multitechnologies.

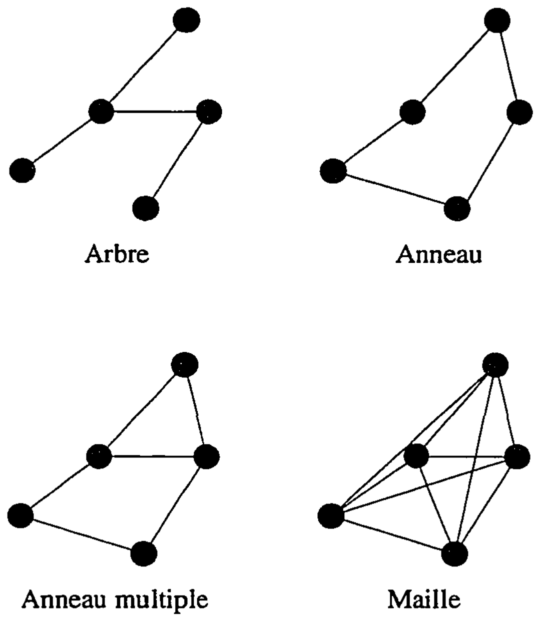


Figure 1.4: Topologies pour le réseau de transmission

1.1.3.1 Conception des réseaux

Pour le problème de la conception des réseaux, nous supposons que nous connaissons l'emplacement des clients et un ensemble de sites potentiels pour installer les commutateurs. Le problème consiste à choisir un sous-ensemble de sites pour y installer les commutateurs, choisir la configuration de chaque commutateur en terme des modèles des bases et ports, trouver le réseau d'accès et le réseau de transmission. L'objectif est de minimiser les coûts tout en respectant les hypothèses concernant l'organisation du réseau.

1.1.3.2 Mise à jour des réseaux

Pour le problème de la mise à jour des réseaux, nous supposons que nous connaissons l'emplacement des nouveaux clients, les clients actuels désirant quitter le réseau et un ensemble de sites potentiels pour installer des commutateurs. Le problème consiste à choisir un sous-ensemble de sites pour y installer des commutateurs, mettre à jour la configuration de chaque commutateur en terme des modèles des bases et ports, mettre à jour le réseau d'accès et le réseau de transmission. L'objectif est de minimiser les coûts tout en respectant les hypothèses concernant l'organisation du réseau.

Il est important de mentionner que lorsque nous supposons que chaque site de commutation actuel doit être utilisé dans le réseau mis à jour, le problème de mise à jour est appelé problème d'expansion. Plus de détails concernant ces problèmes seront donnés dans les chapitres concernés.

1.2 Revue de la littérature

La planification des réseaux de télécommunication a été traité par plusieurs articles et livres dans le domaine. Le lecteur intéressé par le sujet est invité à consulter les articles de synthèse (incluant les citations) de Magnanti et Wong (1984), Minoux (1989) et le livre de Kershenbaum (1993) qui traite de l'évaluation de la performance, de l'optimisation de la topologie, de l'acheminement du trafic, du dimensionnement des liens et de la fiabilité. Le livre de Sharma (1990) traite plus particulièrement de l'optimisation de la topologie et de l'utilisation de logiciels de conception assistée et le livre de Streenstrup (1995) traite des méthodes d'acheminement pour les réseaux d'ordinateurs, téléphoniques et mobiles. Chamberland et Sansó (1995, 1997d) proposent une bonne revue de la littérature concernant le problème conjoint de l'acheminement et dimensionnement (avec ou sans contrainte de fiabilité) pour les réseaux d'ordinateurs (commutation de paquets) et le livre de Girard (1990) traite de ce problème pour les réseaux à commutation de circuits. Les articles de synthèse de Gavish (1991) et Balakrishnan, Magnanti, Shulman et Wong (1991) explorent plus en détail les problèmes de la conception du réseau d'accès. Concernant la mise à jour des réseaux, voir les livres de Freidenfelds (1981) et de Luss (1982) et les articles cités dans l'introduction du chapitre 4.

Dans la pratique, les ingénieurs des compagnies de télécommunications utilisent des logiciels de conception assistée et de simulation pour faire la planification des réseaux. Plusieurs logiciels existent sur le marché, par exemple, CANE par ImageNet (1997), AUTONET par NDA (1997) et ECONETS par HN Telecom (1997). Le logiciel ECONETS est aussi traité en détail dans Sharma (1990). Plusieurs manufacturiers développent aussi leurs outils de conception ou offre des services de consultation à cet effet. Un des principaux désavantages des logiciels et outils disponibles, c'est que la mise à jour du réseau au niveau physique n'est pas considérée. Par contre, ils peuvent être d'une assez grande utilité pour la planification des niveaux logiques du réseau, par exemple, au niveau des VC (*Virtual Connection*) dans le cas des réseaux ATM.

Le principal désavantage des logiciels et outils disponibles c'est que la conception ou mise à jour du réseau au niveau physique n'est pas considérée.

Les problèmes de planification présentés à la section précédente pour les réseaux d'entreprises et multitechnologies sont nouveaux dans la littérature et ne peuvent être résolu par les logiciels de conception assistée que l'on peut trouver présentement sur le marché. En fait, outre les articles constituant cette thèse, seulement Chamberland, Marcotte et Sansó (1996) ont traité de la planification des réseaux à deux niveaux avec commutateurs modulaires. Cet article propose un premier modèle pour le problème de la conception des réseaux d'entreprises, qui est le problème original proposé par NORTEL. Ce premier modèle inclut la localisation des commutateurs, la conception du réseau d'accès avec une topologie en étoile, la conception du réseau de transmission, le dimensionnement des liens et l'acheminement du trafic. Les points faibles de ce modèle sont les suivants. Premièrement, le modèle ne contient pas de contraintes de topologie du réseau de transmission; la connectivité du réseau est assurée par un problème de multiflot. Ainsi, le réseau de transmission est presque toujours un arbre, ce qui n'est pas nécessairement désiré en pratique. Deuxièmement, le modèle de flot utilisé ne peut représenter le trafic en rafale comme, par exemple, celui retrouvé dans les réseaux d'ordinateurs. En fait, il serait même préférable, dans un premier temps, de fixer la capacité des liens dans le modèle, ce qui permettrait de trouver la localisation des commutateurs, la topologie du réseau ainsi que la configuration des commutateurs modulaires. Par la suite, la capacité des liens serait trouvée en utilisant des procédures d'acheminement et dimensionnement sophistiquées, tenant compte d'un trafic plus réaliste, ou bien à l'aide de la simulation. De plus, dans l'article de Chamberland, Marcotte et Sansó (1996), deux exemples résolus par séparation et évaluation progressive sont présentés. Cette approche permet de résoudre des problèmes avec au plus 25 clients et 5 sites potentiels pour installer les commutateurs. Étant donné que nous désirons traiter des problèmes de plus grande taille, contenant jusqu'à 500 clients et 50 sites, cette approche de résolution n'est pas appropriée.

Plusieurs articles traitent de la planification des réseaux à deux niveaux considérant simultanément le réseau d'accès et le réseau de transmission. Une excellente revue de ces articles est présentée par Klincewicz (1997). Nous ajoutons les trois articles suivants qui ne sont pas cités par Klincewicz (1997). Gavish et Pirkul (1986) proposent un modèle de programmation mathématique pour la localisation des ordinateurs et des bases de données dans un réseau distribué comme, par exemple, un réseau de succursales bancaires. Ce modèle peut être utilisé pour la conception des réseaux de communication incluant la localisation des commutateurs, la conception du réseau d'accès avec une topologie en étoile et du réseau de transmission aussi avec une topologie en étoile. Les auteurs proposent une heuristique utilisant l'information obtenue d'une relaxation lagrangienne pour générer une solution réalisable à chaque itération d'un algorithme de sous-gradient. Un algorithme de séparation et évaluation progressive, utilisant des bornes trouvées par une relaxation lagrangienne, est aussi proposé. Des problèmes avec jusqu'à 100 succursales et 20 sites pour installer les ordinateurs sont résolus exactement et les solutions obtenues avec l'heuristique sont en moyenne à moins de 1.22% de l'optimalité.

Vernekar, Anandalingam et Dorny (1990) proposent un problème de localisation des ressources (par exemple, des bases de données, des contrôleurs ou des ordinateurs) dans les réseaux de communication hiérarchiques. Un modèle de programmation mathématique est proposé pour le problème. Ce modèle peut être utilisé pour la conception des réseaux de communication à deux niveaux incluant la localisation des commutateurs, la conception du réseau d'accès avec une topologie en étoile et du réseau de transmission aussi avec une topologie en étoile. Une heuristique gloutonne ainsi qu'une méthode de recuit simulé sont proposées. Des résultats numériques pour des réseaux avec au plus 40 noeuds et trois niveaux sont présentés et les solutions obtenues avec la méthode de recuit simulé sont à moins de 7.9% de l'optimalité.

Chamberland, Sansó et Marcotte (1997b) traitent du problème de la conception des réseaux de télécommunication qui consiste à trouver conjointement la localisation des commutateurs, les modèles des commutateurs, le réseau d'accès avec une topologie en étoile et le réseau de transmission avec une topologie en anneau. Un modèle de commutateur est caractérisé par sa capacité et son coût. Les auteurs proposent un modèle de programmation mathématique pour le problème. Une heuristique gloutonne est proposée ainsi qu'un algorithme de recherche tabou et une borne inférieure. Ces heuristiques sont à la base de plusieurs heuristiques présentées dans cette thèse. Des résultats numériques avec jusqu'à 200 clients et 20 sites potentiels pour installer les commutateurs sont présentés et les résultats obtenus sont en moyenne à 0.18% de l'optimalité.

Parmi les articles traitant de la planification des réseaux à deux niveaux cités dans Klincewicz (1997), aucun ne considère le problème de la mise à jour des réseaux ou même propose un modèle ou une méthode de résolution pouvant être adaptée à plusieurs topologies du réseau de transmission. Cependant Klincewicz, dans la conclusion de son article, fait une série de suggestions pour des travaux futurs qui concordent avec certains développement présentés dans cette thèse. Par exemple, il suggère de considérer le problème de la mise à jour des réseaux à deux niveaux, ce que nous faisons dans les chapitres 4 et 5 de cette thèse. Une autre des suggestion de Klincewicz (1997) est de considérer une topologie en anneau multiple pour le réseaux de transmission, ce que nous faisons dans le chapitre 4 de cette thèse. Il est important de mentionner que nous avons obtenu une copie de l'article de Klincewicz (1997) seulement après l'écriture de la première version de cette thèse. Ainsi, c'est tout à fait de façon involontaire que nous répondons à quelques suggestions proposées par Klincewicz (1997).

1.3 Objectifs de la thèse

Le premier objectif de cette thèse est de modéliser les problèmes de la conception et la mise à jour des réseaux d'entreprises et multitechnologies avec les différentes topologies du réseau de transmission présentées à la section 1.1. Le second objectif est de proposer des méthodes de résolution efficaces en terme du temps d'exécution et de la qualité de la solution trouvée.

1.4 Structure de la thèse

Cette thèse est divisé en deux parties. La première, constituée des chapitres 2 et 3, traite de la conception des réseaux d'entreprises et multitechnologies tandis que la seconde partie, constituée des chapitres 4 et 5, traite de la mise à jour des réseaux. Plus précisément, le chapitre 2 est un article intitulé *Topological Design of Two-Level Telecommunication Networks with Modular Switches* soumis à la revue *Operations Research* et primé par la Société canadienne de recherche opérationnelle[1]. Dans cet article, nous proposons un modèle ainsi que des heuristiques pour le problème de la conception des réseaux d'entreprises. Dans le chapitre 3, nous présentons l'article intitulé *Heuristics for the Topological Design Problem of Two-Level Multitechnology Telecommunication Networks with Modular Switches* que nous avons soumis à la revue *INFORMS Journal of Computing*. Dans cet article, nous proposons un modèle ainsi que des heuristiques pour le problème de la conception des réseaux multitechnologies. Le chapitre 4 présente l'article intitulé *Topological Expansion of Multiple Ring Metropolitan Area Networks*, soumis à la revue *Networks*, dans lequel nous traitons du problème de l'expansion des réseaux multitechnologies.

[1]Concours du meilleur article écrit par un étudiant, 1997.

Nous proposons un modèle ainsi que des heuristiques pour ce problème. Dans le chapitre 5, nous présentons l'article intitulé *Tabu Search and Post-Optimization Algorithms for the Topological Update of Two-Level Networks with Modular Switches* que nous avons soumis à la revue *IEEE/ACM Journal on Networking*. Dans cet article nous proposons un modèle ainsi que des heuristiques et une procédure de post-optimization pour le problème de la mise à jour des réseaux d'entreprises. Finalement, le chapitre 6 présente les principales contributions de cette thèse et propose des avenues de recherche liées à ce travail.

Il est important de dire à ce point que le lecteur trouvera une certaine redondance dans les articles, plus particulièrement dans les sections concernant l'introduction et la définition du problème, l'avantage étant que chaque article peut être lu indépendemment des autres.

PARTIE I

Conception des réseaux de télécommunication

CHAPITRE 2

Topological Design of Two-Level Telecommunication Networks with Modular Switches

by

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Abstract

In this article we propose a mixed 0-1 linear programming model for the topological network design problem with modular switches such as the ones that will be used in future ATM (Asynchronous Transfer Mode) networks. The model includes the location of switches, their configuration with respect to ports and multiplexers, the design of an access network with a star topology and a backbone network with a fixed topology (ring or tree). In order to obtain a solution, we propose a greedy heuristic that provides a good starting solution and a tabu search

heuristic to improve the solution. Finally, we present an example of the application of the heuristics and results for a set of randomly generated problems with up to 500 users and 30 potential switch sites. The solutions obtained are, on average, at 0.86% from a lower bound found by solving a relaxed version of the model.

Key words: Topological design, modular switch, ATM, greedy heuristic, tabu search, local access network, backbone network, ring and tree topologies.

Status: This article has been submitted to the journal *Operation Research*, 1997.

2.1 Introduction

Deregulation and the introduction of new technologies have increased competition and drastically changed the way telecommunication networks are designed and managed. The need for efficiency has increased recently because of new technological advances, in particular, ATM (Asynchronous Transfer Mode) networks. ATM is a connection-oriented fast packet-switch method using fixed-size packets, called cells, which has been established as the standard for broadband multiservice networks (see Minzner, 1989; Händel, Huber and Schröder, 1994 and Stallings, 1995). Thanks to this new technology, companies are investing heavily in modernizing their systems. From the Operations Research standpoint, this means that network design problems are becoming important.

Network planning in the telecommunication industry is a long and complex process that can be visualized in the iterative fashion described in Figure 2.1. The figure shows that the process of designing the main network begins after a set of requirements has been defined. Typically, it consists of designing the network topology, routing the traffic and allocating capacity and spare capacity. The iterative process begins after a set of network performance evaluation tests are carried out to ensure that the resulting network will meet all the requirements. It should be mentionned that this process could be failed, for example, if the requirements are too high or conflict.

Each step of the above procedure is a complex process in itself. Clearly, it is extremely difficult to model the planning process or even just the topology design module as a single problem. Despite its difficulty, however, this is the exact method we propose in order to solve the topological design of two-level networks. The term "two-level" refers to the fact that the network is composed of two major subnetworks that, for historical or administrative reasons within telecommunication companies, were often designed separately. The two subnetworks are the access and backbone networks. The access network links the users to switches whereas the backbone network links the switches to one another. The access network has traditionally taken the form of a star or tree topology whereas the backbone network can have any topology. Recently, for reasons of survivability and due to the introduction of SONET (Synchronous Optical NETwork) technology (see Wu, 1992), much interest has been paid to backbones with a ring topology.

The object of this paper is to present a framework, a methodology and a resolution approach for large network design problems that include the location of switches, the design of an access network with a star topology and the design of a backbone network with a fixed topology (ring or tree). Note that although the use of a tree backbone could be questioned on the basis of reliability, we have chosen to consider it in the study because it provides the least expensive connected topology and can be used for comparative purposes. On the other hand, in certain cases

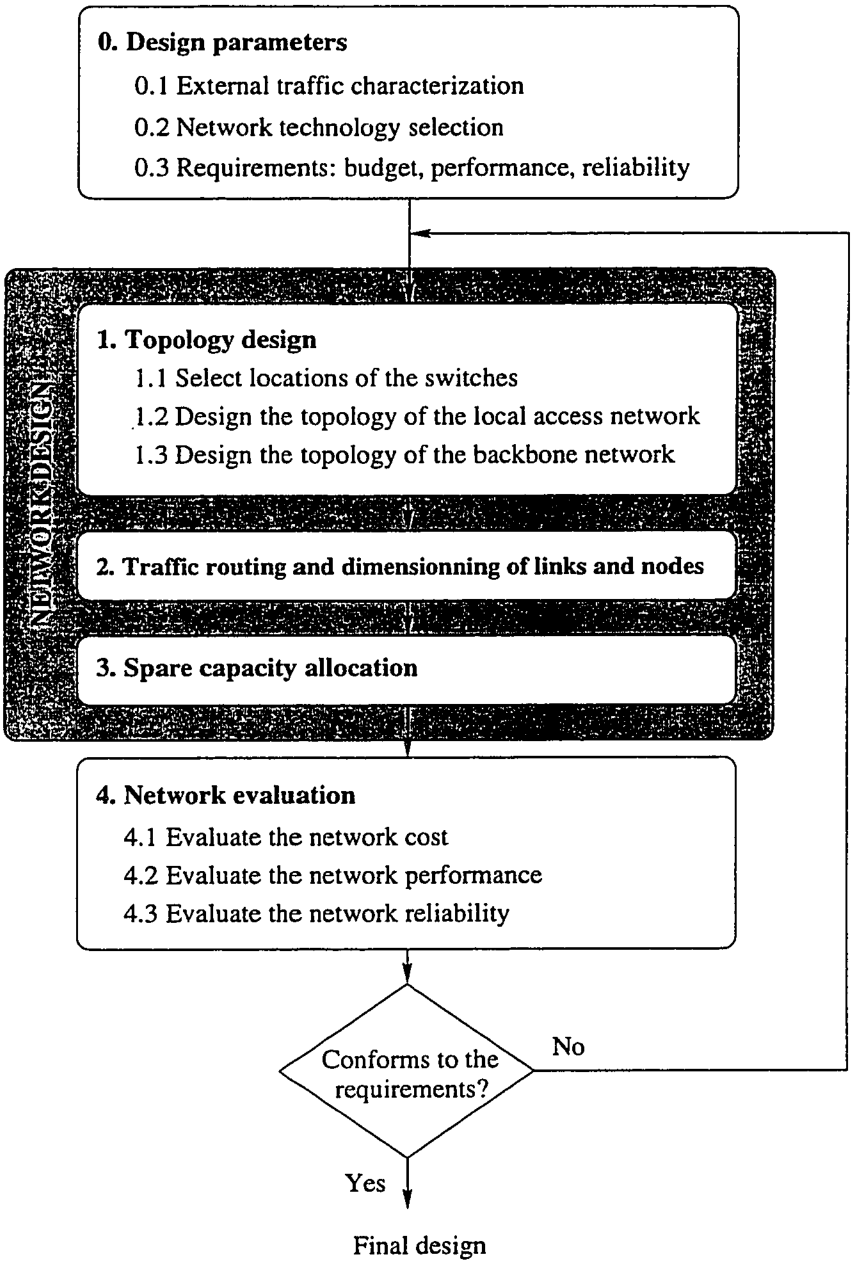


Figure 2.1: Overview of the network planning procedure

such as private networks, a tree is a cost-effective option. An important feature of our work is the inclusion of modular switches similar to those that will be used in future high-capacity networks. The introduction of such switches greatly increases the complexity of the problem and makes its resolution a challenge.

The paper is organized as follows. Section 2.2 contains a review of the literature addressing the problem presented in this paper. Note that the introduction of modular switches and the global approach we take are relatively new in the literature. Section 2.3 discusses the problem and the technical features that make it interesting and timely. It also deals with a general formulation of the problem that is specialized (in the subsections) to backbone networks having a ring or tree structure. In Section 2.4, a greedy heuristic is proposed. This heuristic will be used for comparative purposes and for computing a starting feasible solution used by the tabu search algorithm of Section 2.5. In Section 2.6, the results of an extensive experimental evaluation are presented. The first subsection of Section 2.6 illustrates the solution method on an example and contains a breakdown of the solution cost into its components. The second subsection is dedicated to the evaluation of the heuristic performance, and contains a discussion of results for a set of randomly generated instances with 20 different network sizes. Conclusions and directions for further work are presented in Section 2.7.

2.2 Literature Review

The complex process of network planning illustrated in Figure 2.1 has been explained (in different ways) in review articles and books dealing with the topic. The reader interested in the general planning problem may wish to consult the articles by Magnanti and Wong (1984) and Minoux (1989) and the book by Kershenbaum (1993) on the design of telecommunication networks. The book by Sharma (1990) discusses topology optimization whereas Streenstrup (1995) deals with the routing for computer, telephone and mobile networks. On the other hand, Gavish (1991) and Balakrishnan, Magnanti, Shulman and Wong (1991) explore the design details of the access network.

As previously mentioned, the approach of designing the two network levels simultaneously has been taken by a few authors only. Mirzaian (1985) explores the network design problem that includes the location of concentrators, the assignment of terminals to concentrators in a star topology and the connection of concentrators to the main computer (in a star topology as well). The concentrator capacity is included in the problem. The problem is modeled as a linear 0-1 programming problem and three Lagrangian relaxations are proposed. The author proposes a heuristic that uses information provided by a Lagrangian relaxation and attempts to generate a feasible solution at every iteration of a subgradient optimization algorithm. Problems with up to 50 terminals and 20 concentrator sites are considered. The solutions obtained are within 2.8% of optimality. Several resolution approaches have since been considered for this problem, for instance, see Lo and Kershenbaum (1989).

Gavish and Pirkul (1986) propose a linear 0-1 programming model to find the location of computers and data bases in a distributed network. Such a model can be used for the design of telecommunication networks including the location of switches, the design of an access network with a star topology and a backbone network (with a star topology also). The processing, storage and communication capabilities of computers are considered in the model. The authors propose a heuristic that uses information provided by a Lagrangian relaxation. A branch-and-bound approach using the bounds obtained by the Lagrangian relaxation is also proposed. Problems with up to 100 users and 20 computer sites are solved.

Helme and Magnanti (1989) propose a quadratic 0-1 programming model as well as an equivalent linearized version for the satellite network design problem (including the location of earth stations and the design of an access network with a star topology). The user demand (in circuits) is considered for the evaluation of the cost of the earth stations and the links of the access network. They propose a branch-and-bound algorithm using the bounds obtained by the linear relaxation, as well as two greedy heuristic methods. Problems with up to 40 users and 20 potential earth station sites are solved; the best heuristic provides solutions within 3.3% of optimality.

Vernekar, Anandalingam and Dorny (1990) propose a 0-1 linear programming model for a resource location problem (such as databases, controllers and computers) in a multilevel hierarchical computer network. Such a model can be used for the design of two-level telecommunication networks including the location of switches, the design of a star access network and a backbone network (with a star topology also). The authors propose a greedy heuristic and a simulated annealing algorithm. Results for networks with up to 40 nodes and a three-level hierarchical structure are presented; the solutions obtained are within 7.9% of optimality.

Gavish (1992) explores the computer network design problem including the switch location, the design of an access network with a star topology, the design of a backbone network and the message routing evaluated according to a $M/M/1$ delay model. The problem is modeled as a nonlinear mixed 0-1 program. The author proposes a Lagrangian relaxation as well as a subgradient optimization method. A greedy type heuristic and heuristics based on partial enumeration of the solution space (one for small networks with up to 10 sites and another for larger networks) are considered. The solutions obtained are within 10% of optimality for problems with up to 200 users and 30 switch sites.

Sriram and Garfinkel (1990) deal with the simultaneous determination of facility location and flow routing in the context of communication networks. This problem can be used for the topological design of two-level telecommunication networks including the location of switches, the design of a star access network and a full-meshed backbone network. The authors propose a 0-1 linear programming model, two heuristics (one for problems with euclidean distance and another with given distance) and a lower bounding procedure.

Chung, Myung and Tcha (1992) explore the problem of locating the switches and designing a star access network and a full-meshed backbone network. The authors propose a quadratic 0-1 programming model as well as an equivalent linearized version. A branch-and-bound algorithm that uses the bounds obtained by a dual ascent method is examined. Results for networks with up to 50 users and 50 sites are reported. Kim, Chung and Tcha (1995) propose a generalized version of this model by taking into account dual homing local connections (i.e., each user is connected to two different backbone switches). A branch-and-bound procedure is proposed. Problems with up to 50 users and 20 sites are solved.

Kim and Tcha (1992) propose a branch-and-bound algorithm for the topological network design problem including the location of switches and the design of a star access network and a tree backbone network. A dual ascent procedure and a method for constructing a primal feasible solution are proposed. Problems with up to 200 users and 50 sites are solved exactly and the heuristic produces starting solutions that are within 18.6% of the optimum. Lee, Qiu and Ryan (1994) deal with the same problem and treat it as a variation of the Steiner tree problem (in graphs), called the Steiner Tree-Star (STS) problem. A mixed 0-1 programming model and a set of valid inequalities are proposed. The resolution approach includes a greedy heuristic and a branch-and-cut algorithm that uses the bounds obtained by the linear relaxation and a separation procedure for generating cycle elimination inequalities. Problems with up to 150 users and 70 sites for installing the switches are solved. The greedy heuristic finds starting solutions that are within 0.7% of optimality. Lee, Lu, Qiu and Glover (1994) present several alternative models and cutting planes for the STS problem and Lee, Lim and Park (1995) develop a new model and a Lagrangian relaxation procedure in order to solve large-scale problems. Results with up to 200 customer sites and 200 switch sites are within 10% of optimality.

Vob (1990) deals with the network design problem including the location of switches (the location of a "central" switch has been assumed to be given), the design of a star access network and a ring backbone network. The author show that the problem may be view as a generalized Traveling Salesman Problem (TSP) known as the Traveling Purchaser Problem (TPP) (for further details concerning the TPP see Ramesh, 1981; Golden, Levi and Dahl, 1981 and Ong, 1982). Lee, Ro and Tcha (1993) explore a generalized version of this model by considering the location of all switches. A dual ascent procedure and a method for finding a feasible solution are proposed. Problems with up to 50 users and 20 potential switch sites are solved and the solutions obtained are within 13.4% of optimality.

Altinkemer (1994) considers a linear 0-1 program to model the two-level ring network design problem in which the access network consists of local rings of limited capacity (to connect users), and each local ring begins and terminates at a bridge. Bridges are connected to a high-level ring that constitutes the backbone network. The author propose an adaptation of the parallel saving algorithm to solve the problem.

Finally, Chamberland, Marcotte and Sansó (1996) propose a global approach for the design of broadband networks including the location of switches, the design of a star access network, the design of a backbone network, the link dimensioning and traffic routing. The authors introduce modular switches in the network design process and they also consider the maximum switch capacity. To illustrate the difficulty of the problem, an example with six users and two switch sites is solved by means of a branch-and-bound algorithm.

In this article, we consider a network design model that is slightly less general than the one presented in Chamberland, Marcotte and Sansó (1996) with regard to the selection of link capacities and traffic routing, but captures the main features of the two-level topological network design with modular switches. Nevertheless, the proposed solution algorithms may be used jointly with a sophisticated link dimensioning and traffic routing procedure (with more realistic assumptions than the ones considered in Chamberland, Marcotte and Sansó, 1996) within the iterative process presented in Figure 2.1. Moreover, in the presentation of the model and algorithms, the topology of the backbone network is not specified. Then, the network planner may adapt it for different backbone topologies.

2.3 Problem Description

In this section we present a topological network design problem including modular switches. A modular switch is composed of a base, ports and multiplexers. A base consists of a controller and a switching network with slots used to insert ports. A port is identified by its rate, for instance, OC-3 and OC-12. OC stands for Optical Carrier and is a fundamental unit in the SONET hierarchy. OC-n indicates an optical signal and n represents increments of 51.84 Mbit/s. Thus, OC-3 and OC-12 equal optical signals of 155.520 Mbit/s and 622.080 Mbit/s respectively. Another device, called a multiplexer, is identified by its number of inputs and its output rate and is used for connecting several users to a given switch through a single port. Figure 2.2 illustrates a modular switch where the users are connected to switches with OC-3 links and the switches are connected to one another with OC-192 (9.953 Gbit/s) links. In this figure, user U1 is connected directly to slot 1 through an OC-3 port whereas users U2 to U5 are connected to slot 2 through a multiplexer with four inputs and an OC-12 port. Moreover, switch S1 is connected to slot 3 through an OC-192 port. In this article, we assume that the modular switches are symmetric in the sense that the equipment installed in an ingress slot is identical to that installed in the corresponding egress slot.

Many topologies may be used for the backbone network. In this paper we consider the cases of ring and tree topologies. The problem of designing a network with a ring topology is important, especially for the LANs (Local Area Networks) and MANs (Metropolitan Area Networks). A ring topology (with a restoration procedure) provides protection against failure of a single link or node and many communications protocols such as FDDI (Fiber Distributed Data Interface) and

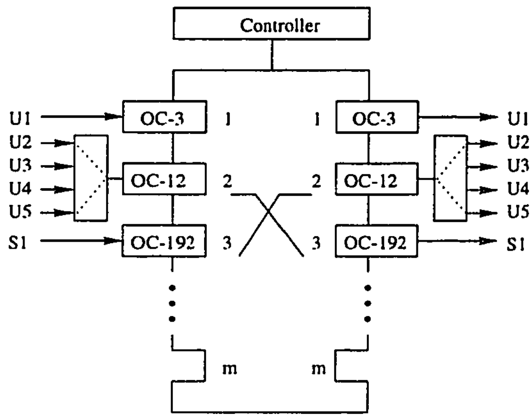


Figure 2.2: Possible connections with a modular switch

SONET make use of this topology (for more details see Ross, 1986, 1989 and Ballert and Ching, 1989). A tree topology, on the other hand, does not provide protection against failure, but is less expensive. In fact, because the tree topology is the least expensive one, it is sometimes used in practice (see Sharma, 1990).

We assume that the following information is known: (I1) the location of users; (I2) the potential switch sites; (I3) the cost of a base; (I4) the cost of installing a switch at a given site; (I5) the costs of the different port types and multiplexers; (I6) the costs of the different link types (OC-3 and OC-192), including the installation cost, in $/km.

We also make the following assumptions about the organization of the network: (A1) each user is connected to a switch through an OC-3 link; (A2) the switches are interconnected through OC-192 links with a specified topology; (A3) the number of users connected to a multiplexer cannot exceed the number of inputs of the multiplexer; (A4) at most one switch may be installed at a given site; (A5) the number of ports installed in a base cannot exceed the number of slots (capacity) of the base; (A6) at most one level of multiplexing is used; (A7) the number of inputs (capacity) of a multiplexer connected to a port of type OC-$3n$ is n. Note that, in the formulation, we assume that a direct connection of a user to an OC-3 port (such as the connection of user U1 in Figure 2.2) is achieved through a multiplexer with one input and zero cost.

The problem involves selecting the switch sites and the types of the ports to be inserted in the slots, selecting the multiplexers, connecting the users to the switches through OC-3 links, and finally interconnecting the switches through OC-192 links with a specified topology. The goal is to minimize the total network cost.

2.3.1 Mathematical Formulation

The following notation is used throughout the paper. Let $M = \{1, \ldots, |M|\}$ be the set of users. A user can be an institution, a LAN or group of LANs. Let $N = \{1, \ldots, |N|\}$ be the set of potential switch sites; $S = \{1, \ldots, |S|\}$ the set of slots in a base, and $R = \{1, \ldots, |R|\}$ the set of port types used to connect the users to the switches. The capacity of a multiplexer connected to a port of type $r \in R$ is denoted by n^r.

Next we define the following parameters. Let c_{ij} denote the cost of connecting the user $i \in M$ to site $j \in N$ through an OC-3 link, and d_{jk} the cost of connecting site $j \in N$ to site $k \in N$ (for $j < k$) through an OC-192 link (including the cost of the two OC-192 ports). We assume that the cost of a link is proportional to the euclidean distance between the sites joined by this link. Let b_j denote the cost of purchasing a base and installing it at site $j \in N$, and p^r the cost of a port of type $r \in R$ (including the cost of a multiplexer of capacity n^r).

We now define the decision variables. Let $x_{ij} \in \mathsf{B}$ ($\mathsf{B} = \{0,1\}$) be a variable such that $x_{ij} = 1$ if and only if user $i \in M$ is connected to site $j \in N$ through an OC-3 link, and $y_{jk} \in \mathsf{B}$ a variable such that $y_{jk} = 1$ if and only if the site $j \in N$ (for $j < k$) is connected to site $k \in N$ through an OC-192 link. Let $u_j \in \mathsf{B}$ be a variable such that $u_j = 1$ if and only if a switch is installed at site $j \in N$; $v_j^{rs} \in \mathsf{B}$ a variable such that $v_j^{rs} = 1$ if and only if a port of type $r \in R$ is installed in slot $s \in S$ at site $j \in N$, and finally, $w_j^s \in \mathsf{B}$ a variable such that $w_j^s = 1$ if and only if a port of type OC-192 is installed in slot $s \in S$ at site $j \in N$ that will be used in the backbone network.

We present below the model, noted DP (Design Problem), for the two-level topological network design problem with modular switches. Observe that the topology of the backbone network has not yet been specified.

DP:

$$\min_{\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{x},\mathbf{y}} \left(\sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} + \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} + \sum_{j \in N} b_j u_j + \sum_{j \in N} \sum_{s \in S} \sum_{r \in R} p^r v_j^{rs} \right) \tag{2.1}$$

s.t.

Assignment constraints

$$\sum_{j \in N} x_{ij} = 1 \quad (i \in M) \tag{2.2}$$

Port type uniqueness constraints

$$\sum_{r \in R} v_j^{rs} + w_j^s \leq u_j \quad (j \in N; s \in S) \tag{2.3}$$

Multiplexer capacity constraints

$$\sum_{i \in M} x_{ij} \leq \sum_{r \in R} \sum_{s \in S} n^r v_j^{rs} \quad (j \in N) \tag{2.4}$$

Backbone topology constraints

$$\boxed{\text{Backbone topology constraints}} \qquad (2.5)$$

Backbone ports capacity constraints

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} \le \sum_{s\in S} w_j^s \quad (j \in N) \qquad (2.6)$$

Integrality and nonnegativity constraints

$$\mathbf{u} \in \mathsf{B}^{|N|}, \mathbf{v} \in \mathsf{B}^{|N||R||S|}, \mathbf{w} \in \mathsf{B}^{|N||S|}, \mathbf{x} \in \mathbf{R}_+^{|M||N|}, \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)}. \qquad (2.7)$$

The objective function (2.1), representing the total cost of the network, is the sum of four terms: the cost of the OC-3 links for the access network, the cost of the OC-192 links (including the cost of the OC-192 ports used in the backbone network), the cost of the bases and the cost of the ports and multiplexers. Constraints (2.2) require that each user be connected to exactly one switch, and constraints (2.3) that at most one port type be installed in each slot and this type be chosen only if a switch has been installed at the site. Constraints (2.4) require that the number of users connected to a switch be at most the sum of the capacities of the multiplexers installed at the switch. Constraints (2.5) require that the topology of the backbone network be the one specified by the user of the model. Constraints (2.6) require that the number of OC-192 links connected to a given switch be at most the number of OC-192 ports (for the backbone network) installed at this switch, and finally, constraints (2.7) are nonegativity and integrality constraints.

In model DP, we need not impose integrality constraints on the x_{ij} variables because the matrix of constraints where these variables appear is totally unimodular. This observation is made precise below.

Proposition 1. Let us assume that all the variables (with the exception of the x_{ij}) have been fixed and have values in $\mathbb{B}$. Then the polyhedron of feasible solutions for the x_{ij} variables has integral extreme points only.

Proof. If we enumerate the x_{ij} variables in the following manner

$$x_{1,1}, x_{1,2}, \ldots, x_{1,|N|}, x_{2,1}, x_{2,2}, \ldots, x_{2,|N|}, \ldots, x_{|M|,1}, x_{|M|,2}, \ldots, x_{|M|,|N|},$$

the constraint matrix for these variables (that is, constraints (2.2) enumerated for $i = 1, \ldots, |M|$ and constraints (2.4) for $j = 1, \ldots, |N|$) is of the form

$$\mathbf{A} = \begin{pmatrix} \mathbb{1}_{|N|} & & & \\ & \mathbb{1}_{|N|} & & \\ & & \ddots & \\ & & & \mathbb{1}_{|N|} \\ \mathrm{I}_{|N|} & \mathrm{I}_{|N|} & \cdots & \mathrm{I}_{|N|} \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} |M| \text{ times} \\ \\ \\ \end{matrix}$$

where $\mathrm{I}_{|N|}$ is the $|N|$-dimensional identity matrix and $\mathbb{1}_{|N|}$ is the $|N|$-dimensional row vector all of whose components are 1. Observe that $\mathbf{A}$ is the node-edge incidence matrix of a complete bipartite graph and thus totally unimodular (for further details concerning integer polyhedra and totally unimodular matrices see Nemhauser and Wolsey, 1988). □

Proposition 2. The following inequalities

$$x_{ij} \le u_j \quad (i \in M;\ j \in N) \tag{2.8}$$

$$y_{jk} \le u_j \quad (j < k,\ \ j, k \in N) \tag{2.9}$$

$$y_{jk} \le u_k \quad (j < k,\ \ j, k \in N) \tag{2.10}$$

are valid for DP.

The following inequality gives a lower bound on the number of ports to be installed in the network.

Proposition 3. The inequality

$$\sum_{j\in N}\sum_{s\in S}\sum_{r\in R} v_j^{rs} \geq \left\lceil \frac{|M|}{\max_{r\in R}\{n^r\}} \right\rceil \tag{2.11}$$

is valid for DP.

Proof. If we sum on $j \in N$ the two sides of inequality (2.4) and use (2.2), we obtain the following inequality.

$$\begin{aligned} |M| &\leq \sum_{j\in N}\sum_{r\in R}\sum_{s\in S} n^r v_j^{rs} \\ &\leq \max_{r\in R}\{n^r\} \sum_{j\in N}\sum_{r\in R}\sum_{s\in S} v_j^{rs}. \end{aligned}$$

Then

$$\sum_{j\in N}\sum_{s\in S}\sum_{r\in R} v_j^{rs} \geq \frac{|M|}{\max_{r\in R}\{n^r\}}. \tag{2.12}$$

The proposition follows because the left-hand side of this inequality is an integer in all feasible solutions of DP. □

The following subsections present the topology constraints for ring and tree backbone networks, respectively.

2.3.2 Ring Topology

The constraints we shall use to model ring topology were proposed by Balas (1989) for the Prize-Collecting Traveling Salesman Problem (PCTSP) and used by

Gendreau, Labbé and Laporte (1994) to model a ring network design problem (for further details concerning the PCTSP see Balas, 1986, 1989, 1995 and Fischetti and Toth, 1988).

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} = 2u_j \quad (j \in N) \tag{2.13}$$

$$\frac{1}{2}\left(\sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j<k}} y_{jk} + \sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j>k}} y_{kj}\right) + (1-u_l) + (1-u_m) \geq 1$$
$$(H \subset N; l \in H; m \in N\setminus H; 3 \leq |H| \leq |N|-3). \tag{2.14}$$

These constraints are valid because $\sum_{j\in N} u_j \geq 3$ in any network with a ring topology. Their number is in $O(|N|^2 2^{|N|})$ and it is not possible to enumerate them all when the problem is large. Constraints (2.13) require that the number of OC-192 links connected to a switch be exactly two. Constraints (2.14) are connectivity constraints and require at least two OC-192 links between switches installed in H and switches in $N\backslash H$. A formulation equivalent to constraints (2.14) is the following.

$$\sum_{j\in H}\sum_{\substack{k\in H\\ j<k}} y_{jk} \leq \sum_{j\in H\setminus\{l\}} u_j + (1-u_m) \quad (H \subset N; l \in H; m \in N\setminus H; 3 \leq |H| \leq |N|-3). \tag{2.15}$$

These constraints are obtained by subtracting from (2.14) a subset of equations (2.13), given below.

$$\frac{1}{2}\left(\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj}\right) = u_j \quad (j \in H). \tag{2.16}$$

Note that it is possible to use other types of constraints to model a ring topology, for instance, flow constraints such as those proposed by Lee, Ro and Tcha

(1993). We prefer constraints (2.14) or (2.15) because they are facets of the PCTSP polyhedron (see Balas, 1989) and may be generated by a separation procedure as the need arises.

In the sequel, DPR denotes the version of DP in which the backbone network has a ring topology, i.e., the version in which the general constraints (2.5) are replaced by constraints (2.13) and (2.14). With these constraints, we can remove the w_j^s variables and constraints (2.6) from model DP by using slots $\{1,2\}$ for the backbone network and slots in $\overline{S} = S \setminus \{1,2\}$ for the access network. The precise formulation of DPR is given below.

DPR:

$$\min_{\mathbf{u},\mathbf{v},\mathbf{x},\mathbf{y}} \left(\sum_{i\in M} \sum_{j\in N} c_{ij}x_{ij} + \sum_{j\in N} \sum_{\substack{k\in N \\ j<k}} d_{jk}y_{jk} + \sum_{j\in N} b_j u_j + \sum_{j\in N} \sum_{s\in\overline{S}} \sum_{r\in R} p^r v_j^{rs} \right) \tag{2.17}$$

s.t. (2.2), (2.13), (2.14) and

$$\sum_{r\in R} v_j^{rs} \le u_j \quad (j \in N; s \in \overline{S}) \tag{2.18}$$

$$\sum_{i\in M} x_{ij} \le \sum_{r\in R} \sum_{s\in\overline{S}} n^r v_j^{rs} \quad (j \in N) \tag{2.19}$$

$$\mathbf{u} \in \mathsf{B}^{|N|}, \mathbf{v} \in \mathsf{B}^{|N||R||\overline{S}|}, \mathbf{x} \in \mathbf{R}_+^{|M||N|}, \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)}. \tag{2.20}$$

Proposition 4. DPR is $\mathcal{NP}$-hard.

Proof. Let M and N denote the same set, $R := \{1\}$ with $n^1 := 1$, $|\overline{S}| := 1$, $c_{ij} := 0$ for all $i \in M$ and $j \in N$, $b_j := 0$ for all $j \in N$, and $p^1 := 0$. Observe that in any feasible solution of DPR, one switch is installed at each site in N and one user is connected to each switch. Hence there is a one-to-one correspondence between the feasible solutions of DPR and those of the Traveling Salesman Problem (TSP) on the set N (where the costs are the d_{jk} for all $j < k$ and $j,\ k \in N$). The proposition follows because the TSP with symmetric costs is polynomially reducible to DPR, and is an $\mathcal{NP}$-hard problem (for further details concerning the TSP, see Garey and Johnson, 1979 and Lawler, Lenstra, Rinnooy Kan and Shmoys, 1985). □

The valid inequalities proposed by Balas (1989) for the PCTSP are also valid for DPR. We shall also use the following inequality, which gives a lower bound on the number of switches to be installed in the network.

Proposition 5. The inequality

$$\sum_{j \in N} u_j \geq \left\lceil \frac{|M|}{|\overline{S}| \max_{r \in R}\{n^r\}} \right\rceil \tag{2.21}$$

is valid for DPR.

Proof. If we sum on $j \in N$ the two sides of (2.19) and use (2.2), we obtain the following inequality.

$$\begin{aligned} |M| &\leq \sum_{j \in N} \sum_{r \in R} \sum_{s \in \overline{S}} n^r v_j^{rs} \\ &\leq \max_{r \in R}\{n^r\} \sum_{j \in N} \sum_{r \in R} \sum_{s \in \overline{S}} v_j^{rs}. \end{aligned}$$

From inequality (2.18), we obtain

$$\begin{aligned} |M| &\leq \max_{r \in R}\{n^r\} \sum_{j \in N} \sum_{s \in \overline{S}} u_j \\ &= |\overline{S}| \max_{r \in R}\{n^r\} \sum_{j \in N} u_j. \end{aligned} \tag{2.22}$$

Then

$$\sum_{j \in N} u_j \geq \frac{|M|}{|\overline{S}| \max_{r \in R}\{n^r\}}. \tag{2.23}$$

The proposition follows because the left-hand side of inequality (2.23) is an integer in all feasible solutions of DPR. □

Proposition 6. DPR is feasible if and only if $|N| \geq 3$ and

$$|M| \leq |N||\overline{S}| \max_{r \in R}\{n^r\}. \tag{2.24}$$

Proof. ($\Rightarrow$) If DPR is feasible, inequality (2.22) is respected for every feasible solution and, since (2.24) corresponds to (2.22) when all sites are used, then inequality (2.24) is respected (for all feasible solutions), and $|N| \geq 3$ because the number of switches in any network with a ring topology is at least three. ($\Leftarrow$) Suppose that inequality (2.24) is respected and $|N| \geq 3$. Then, if we install $|N|$ switches and interconnect them with a ring backbone network, the maximum number of users that can be connected to the network (given by the right-hand side of (2.24)) is larger or equal to $|M|$. Thus, a feasible solution can be constructed using $|N|$ switches, so DPR is feasible. □

2.3.3 Tree Topology

The constraints we shall use to model tree topology were first proposed by Lee, Qiu and Ryan (1994) in the context of the STS problem (see Section 2.2 for a description of the STS problem).

$$\sum_{j \in N} \sum_{\substack{k \in N \\ j < k}} y_{jk} = \sum_{j \in N} u_j - 1 \tag{2.25}$$

$$\sum_{j\in H}\sum_{\substack{k\in H\\ j<k}} y_{jk} \le \sum_{j\in H\setminus\{l\}} u_j \quad (H\subset N; l\in H; 2\le |H|\le |N|-2). \tag{2.26}$$

These constraints are valid because $\sum_{j\in N} u_j \ge 1$ in any network with a tree topology. Their number is in $O(|N|2^{|N|})$ and it is not possible to enumerate them all when the problem is large. Constraints (2.25) require that the number of OC-192 links in the network be exactly the number of switches minus one. Constraints (2.26) are cycle elimination inequalities and require that the number of OC-192 links connecting switches installed in H be at most equal to the number of switches in H minus one (if at least one switch is installed at a site in H).

Note that it is possible to use other constraints to model a tree topology. For instance, one may use flow constraints such as those proposed by Kim and Tcha (1992). We prefer constraints (2.26) because they are facets of the STS polyhedron and may be generated by a separation procedure as the need arises (for further details see Lee, Qiu and Ryan, 1994).

In the sequel, DPT denotes the version of DP in which the backbone network has a tree topology, i.e., the version in which the general constraints (2.5) are replaced by constraints (2.25) and (2.26). With these constraints, it is not possible to remove variables and constraints from DP as we did for the ring topology.

Proposition 7. DPT is $\mathcal{NP}$-hard.

Proof. Let M and N be the same set, $R := \{1\}$ with $n^1 := 1$, $b_j := 0$ for all $j \in N$, $p^1 := 0$, and $c_{ij} := 0$ if $i = j$ and $c_{ij} := +\infty$ if otherwise, for all $i \in M$ and $j \in N$. Observe that in any optimal solution of DPT, one switch is installed at each site in N and each user is connected to the corresponding switch (that is, user $i \in M$ is connected to switch $i \in N$). The tree corresponding to the backbone network is then a solution of an instance of the Degree Constrained Minimum Spanning Tree (DCMST) problem in which the set of nodes is N, the maximum degree is $|S|-1$ and the costs are the d_{jk} for all $j<k$ and $j,\ k \in N$. The DCMST problem with symmetric costs is $\mathcal{NP}$-hard (for further details concerning the DCMST see Garey and Johnson, 1979; Gavish, 1982; Narula and Ho, 1980; Savelsbergh and Volgenant, 1985 and Volgenant, 1989). The proposition follows from the fact that the DCMST problem is polynomially reducible to DPT. □

Many valid inequalities proposed for the STS problem by Lee, Qiu and Ryan (1994) are also valid for DPT. We shall also use the following inequality, which gives a lower bound on the number of switches to be installed in the network.

Proposition 8. The following inequality

$$\sum_{j\in N} u_j \geq \left\lceil \frac{1}{|S|-2}\left(\frac{|M|}{\max_{r\in R}\{n^r\}}-2\right)\right\rceil \tag{2.27}$$

is valid for DPT.

Proof. If we sum on $j \in N$ the two sides of (2.4) and use (2.2), we obtain the following inequality.

$$\begin{aligned} |M| &\leq \sum_{j\in N}\sum_{r\in R}\sum_{s\in S} n^r v_j^{rs} \\ &\leq \max_{r\in R}\{n^r\}\sum_{j\in N}\sum_{r\in R}\sum_{s\in S} v_j^{rs}. \end{aligned}$$

Using inequality (2.3), we obtain

$$|M| \leq \max_{r\in R}\{n^r\}\left(|S|\sum_{j\in N} u_j - \sum_{j\in N}\sum_{s\in S} w_j^s\right). \tag{2.28}$$

If we sum on $j \in N$ the two sides of inequality (2.6) we obtain the following inequality.

$$2\sum_{j\in N}\sum_{\substack{k\in N\\ j<k}} y_{jk} \leq \sum_{j\in N}\sum_{s\in S} w_j^s. \tag{2.29}$$

Using equation (2.25), we obtain

$$2\left(\sum_{j\in N} u_j - 1\right) \leq \sum_{j\in N}\sum_{s\in S} w_j^s. \tag{2.30}$$

Then (2.28) becomes the following inequality if we replace the sum of the w_j^s by the left-hand side of equation (2.30).

$$\begin{aligned} |M| &\leq \max_{r\in R}\{n^r\}\left(|S|\sum_{j\in N} u_j - 2\left(\sum_{j\in N} u_j - 1\right)\right) \\ &= \max_{r\in R}\{n^r\}\left((|S|-2)\sum_{j\in N} u_j + 2\right). \end{aligned} \tag{2.31}$$

Then

$$\sum_{j\in N} u_j \geq \frac{1}{|S|-2}\left(\frac{|M|}{\max_{r\in R}\{n^r\}} - 2\right). \tag{2.32}$$

The proposition follows because the left-hand side of (2.32) is an integer in all feasible solutions of DPT. □

Proposition 9. DPT is feasible if and only if $|N| \geq 1$ and

$$|M| \leq (|N|(|S|-2)+2)\max_{r\in R}\{n^r\}. \tag{2.33}$$

Proof. (⇒) If DPT is feasible, inequality (2.31) is respected for every feasible solution of DPT and since (2.33) corresponds to (2.31) when all sites are used, then (2.33) is respected (for all feasible solutions), and $|N| \geq 1$ because the number of switches in any network with a tree topology is at least one. (⇐) Suppose that inequality (2.33) is respected and $|N| \geq 1$. Then, if we install $|N|$ switches and interconnect them with a tree backbone network (respecting the capacity of the bases), the maximum number of users that can be connected to the network (given by the right-hand side of (2.33)) is larger or equal to $|M|$. Thus, a feasible solution can be constructed using $|N|$ switches, so DPT is feasible. □

Because DPR and DPT are $\mathcal{NP}$-hard and contain large numbers of constraints and binary variables, it is unlikely that large-size instances of these problems can be solved to optimality. Therefore in the rest of this paper we describe efficient heuristic algorithms to solve DPR and DPT.

2.4 A Greedy Heuristic

We now describe a greedy heuristic, called GH (Greedy Heuristic), to find a satisfactory solution for DP presented in Section 2.3. Such a solution will be used as a starting point for the tabu search algorithm described in the next section. Our heuristic incorporates features of the heuristic of Lee, Qiu and Ryan (1994) for the STS problem.

The following notation and definitions will be used in the presentation of GH. A star is a subnetwork that includes a switch (the center of the star) and users connected to this switch. The size of the star is the number of users in the star and the cost of the star is the sum of the cost of the base (including the installation cost) and the cost of the OC-3 links connecting the users to the switch. Let E_N and E_M denote, respectively, the set of switch sites of the current solution and the set of users connected to these switches in the current solution. Let $\Gamma(j)$ denote the set of users connected to site $j \in E_N$, $\Phi(j)$ the maximum number of OC-192 links that can be connected to site $j \in E_N$, and $\overline{\Phi}(j)$ the number of OC-192 links connected to site $j \in E_N$.

If the problem is feasible, the heuristic generates at most m solutions corresponding to stars of size k for $1 \leq k \leq m$, where

$$m = \min\left\{|M|, |S| \max_{r \in R}\{n^r\}\right\}. \tag{2.34}$$

Then m is the maximum number of users that can be connected to a single switch. The best solution found will be returned by the heuristic.

Heuristic GH

Step 1: (Feasibility check)

If the problem is not feasible, stop. Otherwise, set $k := 1$ and go to Step 2.

Step 2: (Star size feasibility check)

If it is possible to generate a solution with stars of size k and $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\}$ switches do Steps 3 and 4. Otherwise go to Step 5.

Step 3: (Generating a solution)

3.1 Set $E_M := \emptyset$, $E_N := \emptyset$, $\Gamma(j) := \emptyset$ for all $j \in N$.

3.2 For $i := 1$ to $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\}$ do

3.2.1 Determine the star of minimum cost among the stars of size $\min\{k, |M \setminus E_M|\}$ containing a switch in $N \setminus E_N$ and users in $M \setminus E_M$. Let j^* denote the chosen switch site and $\Gamma(j^*)$ the set of users of the star of center j^*.

3.2.2 Set $E_N := E_N \cup \{j^*\}$, $E_M := E_M \cup \Gamma(j^*)$ and $\Phi(j^*) := |S| - \left\lceil \frac{|\Gamma(j^*)|}{\max_{r\in R}\{n^r\}} \right\rceil$.

3.3 Connect the switches in E_N with OC-192 links to form a backbone network of minimum cost respecting constraints (2.5) and the degree constraints such that switch j cannot be connected to more than $\Phi(j)$ switches.

3.4 For each site $j \in E_N$, set $\overline{\Phi}(j)$ to the number of OC-192 links connected to site j.

3.5 For each user $i \in M \setminus E_M$ do

3.5.1 Connect user i to the nearest switch with available space, i.e., set j^* to the solution of the following problem

$$\min_{j\in E_N} \left\{ c_{ij} : |\Gamma(j)| < \left(|S| - \overline{\Phi}(j)\right) \max_{r\in R}\{n^r\} \right\}.$$

3.5.2 Set $\Gamma(j^*) := \Gamma(j^*) \cup \{i\}$ and $E_M := E_M \cup \{i\}$.

3.6 For each site $j \in E_N$ do

3.6.1 Install $\overline{\Phi}(j)$ OC-192 ports (for the backbone network) in the slots $\{1, ..., \overline{\Phi}(j)\}$.

3.6.2 Install in each slot of $S \setminus \{1, ..., \overline{\Phi}(j)\}$ a port of maximal access rate with the associated multiplexer.

3.7 Compute the cost of the current solution, given by the objective function of DP.

Step 4: (Best solution update)

If the cost of the current solution is less than that of the best solution obtained so far, update the best solution obtained so far.

Step 5: (Termination test)

If $k = m$, return the best solution found and stop. Otherwise, set $k := k + 1$ and go to Step 2.

We can adapt the heuristic GH to find satisfactory solutions for DPR and DPT.

2.4.1 Ring Topology

The heuristic GH adapted for DPR is called GHR. Step 1 of GHR consists of verifying if DPR is feasible using Proposition 6. Step 2 consists of verifying if $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\} \geq 3$ and

$$|S| \geq 2 + \left\lceil \frac{k}{\max_{r\in R}\{n^r\}} \right\rceil, \tag{2.35}$$

where the left-hand side is the number of slots and the right-hand side is the number of slots required at each switch to construct stars of size k and to connect the switches with a ring, if we use ports of maximal access rate. If $|N| \geq \left\lceil\frac{|M|}{k}\right\rceil$, then all the users are used to form the $\left\lceil\frac{|M|}{k}\right\rceil$ stars in Step 3.2 but, if $|N| < \left\lceil\frac{|M|}{k}\right\rceil$, the users not used to form the $|N|$ stars in Step 3.2 are connected to the switches in Step 3.5. Step 3.3 consists of connecting switches in E_N with OC-192 links to form a minimum cost ring. This is a TSP on the node set E_N with symmetric costs (d_{jk} for $j < k$ and j, $k \in E_N$). To solve this problem, we use the composite heuristic GENIUS proposed by Gendreau, Hertz and Laporte (1992).

2.4.2 Tree Topology

The heuristic GH adapted for DPT is called GHT. Step 1 of GHT consists of verifying if DPT is feasible by using Proposition 9. Step 2 consists of verifying if

$$|S| \geq \begin{cases} \left\lceil \frac{k}{\max_{r \in R}\{n^r\}} \right\rceil & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} = 1 \\ 1 + \left\lceil \frac{k}{\max_{r \in R}\{n^r\}} \right\rceil & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} = 2 \\ 2 + \left\lceil \frac{k}{\max_{r \in R}\{n^r\}} \right\rceil & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} \geq 3, \end{cases} \tag{2.36}$$

where the left-hand side is the number of slots available and the right-hand side is the minimum number of slots required at each switch to construct stars of size k and to connect the switches with a tree, if we use ports of maximal access rate. Step 3.3 consists of connecting switches in E_N with OC-192 links to form a minimum cost tree respecting the degree constraints. This is a DCMST problem on the node set E_N with symmetric costs (d_{jk} for $j < k$ and j, $k \in E_N$) and maximal degree $\Phi(j)$ for $j \in E_N$. To solve this problem, we use the dual heuristic with exchanges proposed by Narula and Ho (1980).

2.5 The Tabu Approach

In this section, we propose a tabu search algorithm, called TS (Tabu Search), for DP. The basic principle of the tabu search is to define a set of possible solutions and, starting from the current solution, to find a better one in its neighborhood. In order for the algorithm to move away from a local minimum, the search allows moves resulting in a degradation of the objective function value and the solutions obtained recently are considered tabu to prevent the algorithm from examining a local minimum more than once. For further details concerning tabu search see Glover (1989, 1990) and Glover, Taillard and de Werra (1993).

Let $s \in S$ be a given slot located at site $j \in N$. The state e_{js} of this slot is defined in the following manner: $e_{js} = 0$ if there is no port installed in the slot, $e_{js} = k$ (for $k \in R$) if a port of type $k \in R$ is installed in the slot, and $e_{js} = |R| + 1$ if an OC-192 port used in the backbone network is installed in the slot. This means that if $e_{js} = 0$, then $v_j^{rs} = 0$ for all $r \in R$ and $w_j^s = 0$, if $e_{js} = k$ (for $k \in R$), then $v_j^{ks} = 1$, $v_j^{rs} = 0$ for all $r \in R \setminus \{k\}$ and $w_j^s = 0$, and if $e_{js} = |R| + 1$, then $v_j^{rs} = 0$ for all $r \in R$ and $w_j^s = 1$. Moreover $u_j = 0$ if and only if $\sum_{s \in S} e_{js} = 0$. Let us define the vector $\mathbf{e} = \{e_{js}\}_{j \in N, s \in S}$ as the state of the network slots. Let $\mathbf{v}(\mathbf{e})$, $\mathbf{w}(\mathbf{e})$ and $\mathbf{u}(\mathbf{e})$ be the vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{u}$, respectively, when the network slots state $\mathbf{e}$ is fixed.

Subsections 5.1 to 5.5 outline the main features of algorithm TS and are followed by a detailed description of this algorithm.

2.5.1 Problem Decomposition

Let DP($\mathbf{e}$) be the model DP when the decision vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{u}$ are equal to the vectors $\mathbf{v}(\mathbf{e})$, $\mathbf{w}(\mathbf{e})$ and $\mathbf{u}(\mathbf{e})$, respectively. When $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{u}$ are fixed, DP can be decomposed into two subproblems. The first one, noted $\overline{\text{DP}}(\mathbf{v})$, is given below.

$$\overline{\text{DP}}(\mathbf{v}) : \min_{\mathbf{x}} \left\{ \sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} : (2.2), (2.4), \mathbf{x} \in \mathbf{R}_+^{|M||N|} \right\}. \qquad (2.37)$$

The purpose of this subproblem is to connect the users to switches by using the ports and multiplexers already chosen (given by vector $\mathbf{v}$). $\overline{\text{DP}}(\mathbf{v})$ is an instance of the linear assignment problem, and to solve it, we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant (1987). The latter is considered the best method for solving linear assignment problems on dense graphs (see Kennington and Wang, 1991 and Ahuja, Magnanti and Orlin, 1989).

The second subproblem, noted $\overline{\overline{\mathrm{DP}}}(\mathbf{u},\mathbf{w})$, is given below.

$$\overline{\overline{\mathrm{DP}}}(\mathbf{u},\mathbf{w}) : \min_{\mathbf{y}} \left\{ \sum_{j\in N} \sum_{\substack{k\in N \\ j<k}} d_{jk} y_{jk} : (2.5), (2.6), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{2.38}$$

The above subproblem consists of connecting the switches to form a backbone network whose topology is given by constraints (2.5).

2.5.2 Solution Space

Let E be the set of all possible states for the slots of the network, including those states $\mathbf{e}$ that do not correspond to feasible solutions of DP($\mathbf{e}$). Note that $|E| = (|R| + 2)^{|N||S|}$. Let $\mathbf{x}(\mathbf{e})$ denote the exact solution of $\overline{\mathrm{DP}}(\mathbf{v}(\mathbf{e}))$, and $\mathbf{y}(\mathbf{e})$ the exact or heuristic solution of $\overline{\overline{\mathrm{DP}}}(\mathbf{u}(\mathbf{e}),\mathbf{w}(\mathbf{e}))$. The solution space is thus $\Omega = \{\mathbf{u}(\mathbf{e}), \mathbf{v}(\mathbf{e}), \mathbf{w}(\mathbf{e}), \mathbf{x}(\mathbf{e}), \mathbf{y}(\mathbf{e})\}_{\mathbf{e}\in E}$.

2.5.3 Neighborhood Structure

Let $\omega \in \Omega$ be a solution. $\mathcal{N}(\omega)$ is called the neighborhood of ω and consists of the solutions obtained by modifying the state of a given slot in the current solution. The total number of possible moves is $|N||S|(|R| + 1)$, but it is not necessary to perform all the moves for exploring the solutions in the neighborhood. Indeed, in the current solution, it suffices to modify the state of a unique slot among all the slots of a site that have the same state. Hence at most $|R| + 2$ slots are considered at each site, and we avoid generating a given neighbor several times. Thus for each site, the number of moves is at most $(|R|+2)(|R|+1)$ and the maximum number of moves needed to explore all the solutions in the neighborhood is $|N|(|R|+2)(|R|+1)$.

2.5.4 Tabu Moves and Aspiration Criterion

Each move of the tabu search consists of modifying the state of a given slot in the current solution. Once the slot is chosen, it is declared tabu for a number of iterations randomly determined according to a uniform discrete distribution on the interval $[1, 5]$.

The aspiration criterion can be described as follows. If the use of a tabu slot allows us to discover a solution that is better than any obtained so far, we remove the tabu from this slot.

2.5.5 Perturbation Method

We consider a perturbation method for visiting several local minima in few iterations (100 iterations) of the search process. Let $\mathbf{e}$ be the state of the network slots of the current solution and U be the set of sites where a switch is installed in the current solution. The proposed perturbation method, performed once every 10 iterations, is describe below.

Perturbation Method

Step 1: (Remove a switch)

For each switch $j \in U$ do

1.1 Set $\mathbf{e}' := \mathbf{e}$ and $e'_{js} := 0$ for all $s \in S$.

1.2 Solve $\mathrm{DP}(\mathbf{e}')$ by solving $\overline{\mathrm{DP}}(\mathbf{v}(\mathbf{e}'))$ exactly by using the algorithm LAPJV and $\overline{\overline{\mathrm{DP}}}(\mathbf{u}(\mathbf{e}'), \mathbf{w}(\mathbf{e}'))$ exactly or heuristically.

1.3 If the current solution cost is less than that of the best solution obtained by the perturbation method, update this best solution.

Step 2: (Add a switch)

For each site $j \in N \setminus U$ do

2.1 Set $\mathbf{e}' := \mathbf{e}$, $e'_{js} := |R| + 1$ for all $s \in \{1, ..., t\}$ and $e'_{js} := \arg\max_{r \in R}\{n^r\}$ for all $s \in S \setminus \{1, ..., t\}$, where t is select according to backbone topology and the number of switches in the current solution.

2.2 Do Steps 1.2 and 1.3.

Step 3: (Move a switch)

For each switch $j \in U$ and site $k \in N \setminus U$ do

3.1 Set $\mathbf{e}' := \mathbf{e}$, $e'_{ks} := e'_{js}$ for all $s \in S$ and $e'_{js} := 0$ for all $s \in S$.

3.2 Do Steps 1.2 and 1.3.

Step 4: (Return the best solution)

Return the best solution obtained by the perturbation method.

Each slot whose state is changed by the perturbation method is declared tabu for a number of iterations randomly determined according to a uniform discrete distribution on the interval $[1, 5]$.

2.5.6 The Algorithm

We will now proceed to the detailed description of algorithm TS.

Algorithm TS

Step 1: (Initial solution)

Find an initial solution using the GH heuristic.

Repeat Steps 2 and 3 for 100 iterations

Step 2: (Exploring the neighborhood)

If the iteration count is a multiple of 10, apply the perturbation method to the current solution. Otherwise do Steps **2.1** and **2.2**.

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \to \mathbf{e}'$ (i.e., modification of the state of a given slot in the current solution), we solve DP($\mathbf{e}'$) by solving $\overline{\text{DP}}(\mathbf{v}(\mathbf{e}'))$ exactly by using the algorithm LAPJV, and $\overline{\overline{\text{DP}}}(\mathbf{u}(\mathbf{e}'), \mathbf{w}(\mathbf{e}'))$ exactly or heuristically. The cost of a solution is given by the objective function (2.1) of model DP.

2.2 Evaluate the tabu for the new transformation.

Step 3: (Best solution update)

If the current solution cost is less than that of the best solution obtained so far, update the best solution obtained so far.

In what follows we show how to adapt TS to find solutions for DPR and DPT.

2.5.6.1 Ring Topology

The heuristic TS adapted for DPR is called TSR. Step 1 of TSR consists of finding an initial solution by using the GHR heuristic. The ring topology simplifies the search algorithm because it suffices to consider only the slots in $\overline{S}$ and the port types in R. Then $|E| = (|R| + 1)^{|N||\overline{S}|}$ and the maximum number of moves which must be performed to explore all the solutions in the neighborhood becomes $|N||R|(|R| + 1)$. With this topology, the second subproblem becomes

$$\overline{\overline{\mathrm{DPR}}}(\mathbf{u}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} : (2.13), (2.14), \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{2.39}$$

The purpose of the above subproblem is to connect the switches through a ring. It is therefore a TSP with symmetric costs. We solve it with the GENIUS composite heuristic proposed by Gendreau, Hertz and Laporte (1992).

Moreover, we set $t := 2$ (in the perturbation method) because constraints (2.13) require that the number of OC-192 links connected to a switch be exactly two.

2.5.6.2 Tree Topology

The heuristic TS adapted for DPT is called TST. Step 1 of TST consists of finding an initial solution by using the GHT heuristic. With this topology, the second subproblem becomes

$$\overline{\overline{\mathrm{DPT}}}(\mathbf{u}, \mathbf{w}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} : (2.6), (2.25), (2.26), \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{2.40}$$

The purpose of the above subproblem is to connect the switches through a tree with degree constraints. It is therefore a DCMST problem with symmetric costs.

To solve it, we use the dual heuristic with exchanges proposed by Narula and Ho (1980).

With this topology, we set $t := 2$ (in the perturbation method) because it is the minimum value of t for which the second subproblem is always feasible in Step 2.2 of the perturbation method.

2.6 Computational Results

In this section, we present two types of results. First we present an instance of our problem and the solutions obtained by our methods in Section 2.6.1. Next we present a systematic study of the performance of the heuristics in Section 2.6.2. We now give some general information concerning the implementation.

All the algorithms were programmed in the C language on a Sun Ultra 1 (model 140). We should mention that the monetary unit used in this paper, denoted \$, is an arbitrary unit. The costs used for all the examples are the following. OC-3 and OC-192 links cost \$1000/km and \$2000/km respectively (including the installation cost). Two port models are used to link the users to the network. The cost of the first model, of rate OC-3, was set at \$1000 whereas the cost of the second model, of rate OC-12, including a multiplexer with four inputs, is \$3000. The cost of an OC-192 port (used for the backbone network) is \$10 000. With respect to the switches, we assume that there are three base models; their main features are presented in Table 2.1. In this table, the capacity of the base model is given by the number of slots and the costs include the installation costs, which we assume to be equal for all sites.

Tableau 2.1: Features of the base models

	Model A	Model B	Model C
Capacity	16	24	32
Cost	\$150 000	\$200 000	\$250 000

All the heuristic results were compared with lower bounds obtained as follows. For DPR, the bound was obtained by using the CPLEX Mixed Integer Optimizer (for more information about CPLEX see the CPLEX user's manual, 1993) to solve DPR without constraints (2.14), relaxing the integrality constraints on $\mathbf{y}$ and $\mathbf{v}$ variables but with valid inequalities (2.8) to (2.11) and (2.21). The lower bound for DPT was obtained by solving DPT without constraints (2.26) and without integrality constraints on $\mathbf{y}$, $\mathbf{v}$ and $\mathbf{w}$ variables but with valid inequalities (2.8) to (2.11) and (2.27).

2.6.1 An Illustrative Example

We have applied both versions of the greedy and tabu heuristics (the tree version and the ring version) to a randomly generated instance of our problem (see Section 2.6.2 for a description of how the instance is generated). In the example, the number of users is 250, the number of potential switch sites is 15 and the base considered is of model A. In Figure 2.3, the geographical location of users and potential switch sites are presented for a square region of side length 100 km.

In the case where the backbone was a ring, the solution obtained with GHR took 4.37 seconds and its value was \$6 570 900. On the other hand, the solution obtained by TSR took 241.16 seconds, but its value was \$5 765 200. This solution is illustrated in Figure 2.4. The value of the lower bound for DPR (obtained, as

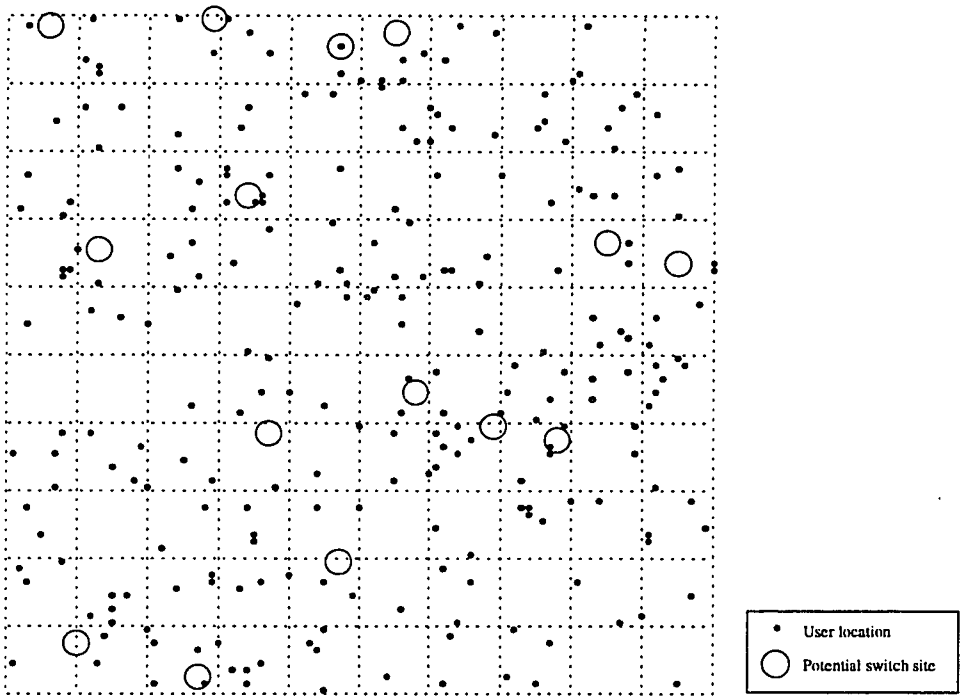
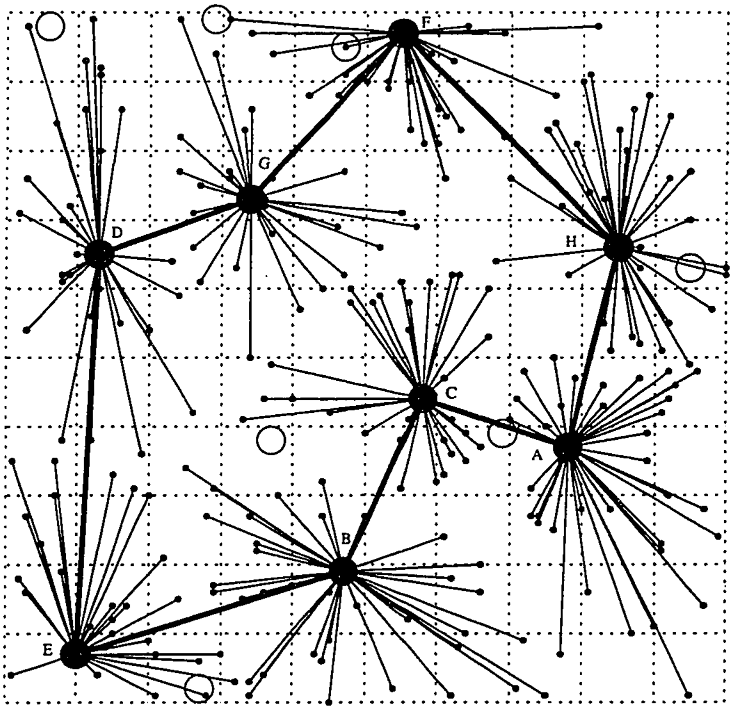


Figure 2.3: Users and potential switch sites location

described previously, by solving exactly a relaxed version of DPR using CPLEX) was $5 763 467 and it was obtained by exploration of 8 branch-and-bound nodes in 10.33 seconds. Hence, for this example, the solution found by TSR is within 0.03% of optimality.



Switch A

Port	Number
OC-3	2
OC-12	9
OC-192	2

Switch E

Port	Number
OC-3	0
OC-12	7
OC-192	2

Switch B

Port	Number
OC-3	0
OC-12	9
OC-192	2

Switch F

Port	Number
OC-3	3
OC-12	6
OC-192	2

Switch C

Port	Number
OC-3	0
OC-12	8
OC-192	2

Switch G

Port	Number
OC-3	0
OC-12	7
OC-192	2

Switch D

Port	Number
OC-3	0
OC-12	7
OC-192	2

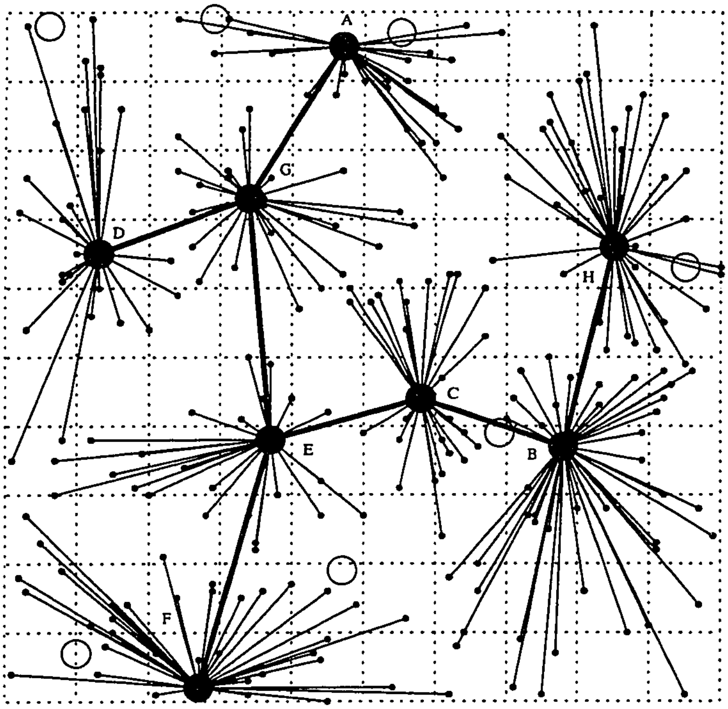
Switch H

Port	Number
OC-3	1
OC-12	8
OC-192	2

- User site
- Unused switch site
- Switch site
- OC-3 link
- OC-192 link

Figure 2.4: Solution obtained by TSR

When the backbone network was a tree, the solution obtained with GHT took 4.41 seconds and its value was $6 420 800, whereas TST took 418.13 seconds to reach a solution valued at $5 669 100. This solution is illustrated in Figure 2.5. The lower bound for DPT was $5 597 867 and was obtained after the exploration of 9 nodes, in 10.57 seconds. Hence, for this example, the solution obtained is within 1.27% of the optimal solution.



Switch A

Port	Number
OC-3	0
OC-12	7
OC-192	1

Switch E

Port	Number
OC-3	0
OC-12	6
OC-192	3

Switch B

Port	Number
OC-3	2
OC-12	11
OC-192	2

Switch F

Port	Number
OC-3	0
OC-12	10
OC-192	1

Switch C

Port	Number
OC-3	0
OC-12	7
OC-192	2

Switch G

Port	Number
OC-3	1
OC-12	6
OC-192	3

Switch D

Port	Number
OC-3	2
OC-12	6
OC-192	1

Switch H

Port	Number
OC-3	0
OC-12	9
OC-192	1

- User site
- Unused switch site
- Switch site
- OC-3 link
- OC-192 link

Figure 2.5: Solution obtained by TST

It is interesting to note that the solution obtained for the ring backbone network is only 1.70% higher than the one obtained for the tree network. Thus the increase in cost necessary to protect the system from the failure of a single backbone edge is relatively small when compared with the minimum investment (i.e., the cost of a tree backbone).

Figures 2.6 and 2.7 picture the distribution of the costs obtained with TSR and TST. It can be seen that in both cases, the highest costs are due to the OC-3 links, which constitute the access network (63.68% for TSR and 66.17% for TST). The base costs were a distant second (20.81% for TSR and 21.17% for TST). The OC-192 links that constitute the backbone network are third in importance (9.46% and 6.82% respectively for TSR and TST), followed by the costs of ports and multiplexers (6.05% and 5.84%). These results confirm a fact known to practitioners in the field (see, for instance, Yan and Beshai, 1995), that is, the access is responsible for most of the network costs, regardless of the topology of the backbone.

We conclude the analysis of our example by the presentation of Figures 2.8 and 2.9, where the values of the best solutions found by TSR and TST (respectively) are plotted as functions of the number of iterations, and where the lower bounds are indicated in dashed lines. The figures show that the proposed heuristics allow us to find satisfactory solutions in a few iterations.

2.6.2 Evaluation of the Heuristics Performance

In this section, we will present the results of a systematic set of experiments that were designed to assess the performance of the proposed algorithms. Twenty problem sizes were chosen, and for each size, 10 problems were randomly generated

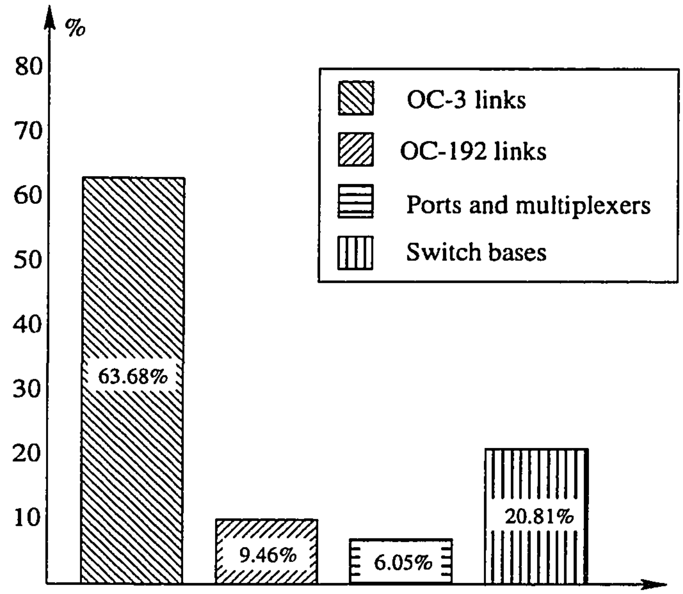


Figure 2.6: Distribution of the costs obtained with TSR

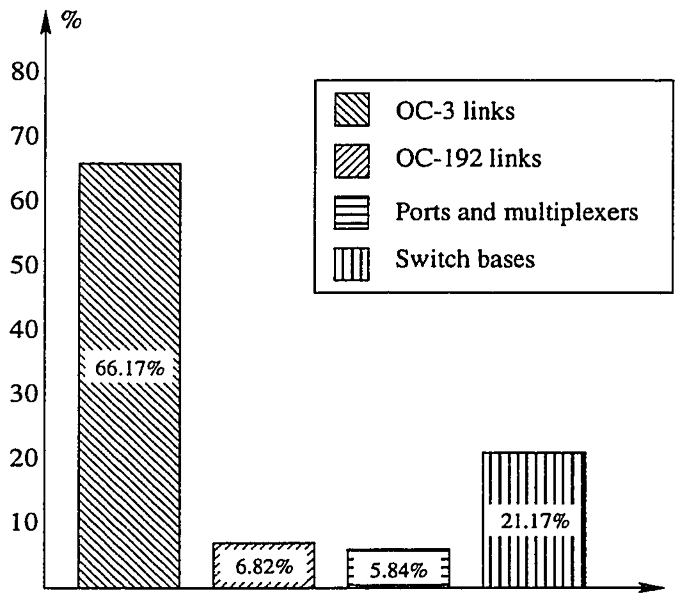


Figure 2.7: Distribution of the costs obtained with TST

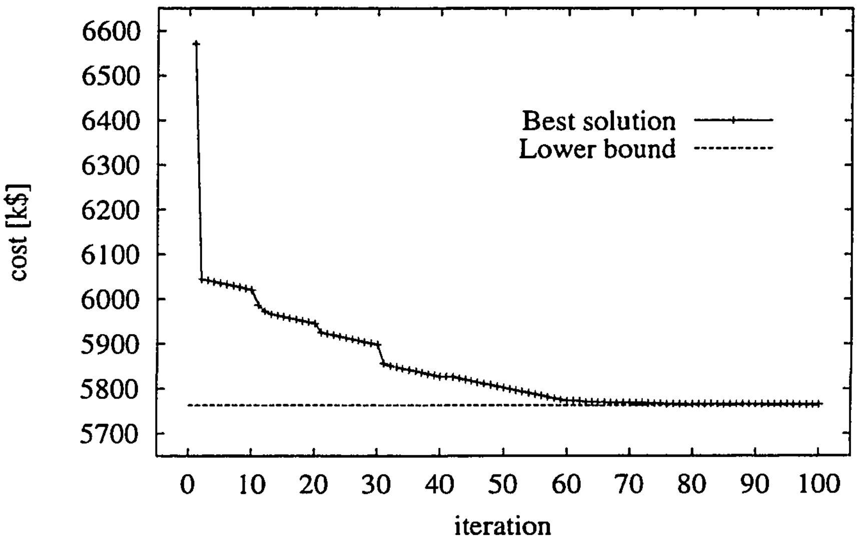


Figure 2.8: Value of the best solution found by heuristic TSR as a function of the number of iterations

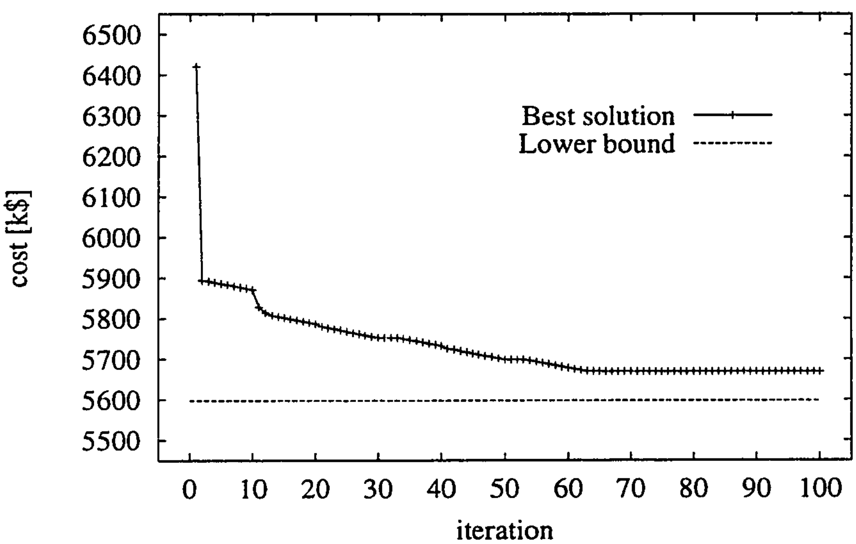


Figure 2.9: Value of the best solution found by heuristic TST as a function of the number of iterations

as follows. $|M|$ points (corresponding to the users' locations) and $|N|$ points (corresponding to the candidate switch sites) were generated in the square of side length 100 km following a uniform distribution law.

The results relating to the problems with a ring backbone are presented in Tables 2.2 to 2.4, whereas those relating to tree backbones are presented in Tables 2.5 to 2.7. For each type of backbone, a different set of results was presented, according to the type of base considered. Tables 2.2 to 2.4 present the results when bases A, B and C, respectively, are taken into account. The same applies for Tables 2.5 to 2.7. In each table, the first column contains the number of users and the second, the number of potential switch sites. For each problem size, the mean, the minimum and the maximum values obtained for the set of random tests are indicated by MEAN, MIN and MAX in the third column. Column 4 presents the value of the lower bound whereas columns 5 to 7 present the results for the greedy heuristics and columns 8 to 10, the results for the tabu search. For each group of heuristic results, OBJ indicates the value of the objective function, CPU indicates the CPU execution time, whereas GAP represents the percentage gap between the heuristic solution and the lower bound with respect to the value of the lower bound.

As can be gathered from the tables, the greedy heuristics GHR and GHT produce in a short time solutions that are far from the lower bound. For all the problems solved with GHR, the mean gap is 11.89% and the mean CPU time is 17.60 seconds; for GHT, the mean gap is 12.28% and the mean CPU time is 18.61 seconds. On the other hand, the improvement method based on the tabu search produces excellent solutions (however, as expected, with an increase in CPU time). In fact, the mean gap for TSR is 0.64% and the mean CPU time is 875.18 seconds; for TST, the mean gap is 1.07% and the mean CPU time is

Tableau 2.2: Computational results for GHR and TSR with model A base

$\|M\|$	$\|N\|$		LOWB	GHR			TSR		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	3710.82	4001.61	0.18	7.84	3725.18	12.26	0.39
		MIN	3401.20	3744.00	0.17	3.53	3402.00	7.00	0.02
		MAX	4163.50	4512.50	0.20	11.87	4260.60	17.07	2.33
150	5	MEAN	5048.15	5289.15	0.39	4.77	5050.31	19.79	0.04
		MIN	4136.77	4288.30	0.38	2.21	4139.20	14.01	0.01
		MAX	5980.67	6333.90	0.40	8.04	5981.70	28.35	0.17
200	5	MEAN	6457.53	6836.02	0.55	5.86	6458.66	36.08	0.02
		MIN	5669.40	5867.00	0.55	2.04	5670.40	24.95	0.00
		MAX	7093.40	7652.20	0.57	10.48	7094.70	47.80	0.04
250	5	MEAN	7567.45	8011.16	0.70	5.86	7568.06	40.08	0.01
		MIN	6734.37	6757.90	0.69	0.35	6735.70	25.17	0.00
		MAX	8759.77	9372.60	0.73	11.96	8760.10	59.16	0.02
150	10	MEAN	4383.87	4871.43	1.04	11.12	4387.60	49.16	0.09
		MIN	4249.57	4635.10	1.02	9.07	4250.90	34.18	0.03
		MAX	4595.67	5077.20	1.06	15.06	4597.20	69.25	0.28
200	10	MEAN	5453.84	6116.43	1.66	12.15	5469.63	88.46	0.29
		MIN	5187.00	5534.10	1.63	6.42	5199.90	58.93	0.02
		MAX	5843.40	6650.10	1.71	14.71	5884.50	129.87	0.91
250	10	MEAN	6388.94	6973.11	2.09	9.14	6421.74	164.00	0.51
		MIN	5736.87	6226.90	2.06	5.93	5761.10	142.25	0.02
		MAX	7296.97	7920.50	2.12	12.77	7354.40	197.91	1.64
300	10	MEAN	7263.74	7807.29	2.97	7.48	7283.05	210.09	0.27
		MIN	6754.60	7165.60	2.92	5.27	6755.90	165.61	0.00
		MAX	8472.70	9296.20	3.09	11.92	8532.70	277.30	0.77
250	15	MEAN	5962.13	6671.20	4.42	11.89	5967.82	193.68	0.10
		MIN	5686.27	6350.30	4.37	8.66	5689.10	167.97	0.02
		MAX	6158.57	7144.40	4.50	16.98	6159.90	241.16	0.36
300	15	MEAN	6879.47	7691.33	5.26	11.80	6896.34	362.77	0.25
		MIN	6575.80	7345.00	5.05	7.03	6606.60	243.48	0.03
		MAX	7357.50	8518.90	5.54	15.79	7359.90	625.14	0.74
350	15	MEAN	7770.95	8740.22	7.20	12.47	7796.40	520.78	0.33
		MIN	7452.57	8163.00	7.10	9.53	7454.90	322.28	0.02
		MAX	8315.37	9624.10	7.33	16.35	8370.00	802.86	1.19
300	20	MEAN	6662.61	7447.69	7.87	11.78	6677.60	410.61	0.22
		MIN	6201.40	7093.30	7.69	8.83	6203.40	356.35	0.03
		MAX	7425.10	8206.30	8.13	14.62	7501.80	559.87	1.03
350	20	MEAN	7432.84	8358.45	10.42	12.45	7458.53	592.56	0.35
		MIN	7142.47	7759.50	10.22	8.19	7144.80	541.20	0.02
		MAX	8311.67	9575.00	10.72	15.24	8391.30	697.52	0.96
400	20	MEAN	8068.22	9069.46	13.46	12.41	8082.15	876.79	0.17
		MIN	7761.60	8623.80	13.13	7.65	7765.70	733.07	0.03
		MAX	8378.30	9833.50	13.97	17.37	8423.60	1036.29	0.54
350	25	MEAN	7252.89	8166.00	14.47	12.59	7267.99	769.80	0.21
		MIN	7054.77	7700.90	14.29	8.28	7088.20	623.55	0.02
		MAX	7568.27	8842.90	14.79	16.84	7624.20	841.08	0.74
400	25	MEAN	8010.33	9121.77	17.94	13.88	8028.05	1091.52	0.22
		MIN	7479.60	8392.80	17.67	11.36	7481.90	972.13	0.03
		MAX	8453.70	9917.70	18.47	17.32	8497.00	1297.11	0.53
450	25	MEAN	8608.06	9763.20	21.41	13.42	8629.52	1464.58	0.25
		MIN	8212.22	9339.50	20.95	7.64	8215.70	1297.42	0.03
		MAX	8834.57	10169.00	22.10	16.93	8861.00	1677.15	0.95
400	30	MEAN	7709.01	8748.28	22.69	13.48	7735.36	1360.49	0.34
		MIN	7358.20	8387.60	22.21	11.46	7361.70	1271.15	0.03
		MAX	8068.20	9268.20	23.96	14.88	8096.70	1591.88	1.52
450	30	MEAN	8366.89	9454.82	27.92	13.00	8402.58	1739.08	0.43
		MIN	8162.47	9124.50	27.51	10.07	8166.90	1623.62	0.02
		MAX	8562.67	9826.00	28.90	17.52	8564.50	1899.73	1.06
500	30	MEAN	9120.75	10325.68	35.12	13.21	9151.09	2378.56	0.33
		MIN	8865.30	9944.20	34.52	8.82	8868.20	2239.63	0.03
		MAX	9499.50	10995.40	35.90	15.75	9546.10	2522.30	0.55

Tableau 2.3: Computational results for GHR and TSR with model B base

$\lvert M \rvert$	$\lvert N \rvert$		LOWB	GHR			TSR		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	3872.57	4287.55	0.18	10.72	3879.01	17.54	0.17
		MIN	3551.20	3968.50	0.17	5.89	3552.00	12.28	0.00
		MAX	4313.50	4743.00	0.19	13.95	4313.50	25.87	1.03
150	5	MEAN	5190.97	5577.49	0.57	7.45	5202.53	34.69	0.22
		MIN	4336.77	4658.30	0.55	4.66	4348.00	19.53	0.01
		MAX	6069.87	6563.00	0.59	10.39	6071.20	57.60	1.83
200	5	MEAN	6433.20	6792.81	1.14	5.59	6434.73	47.76	0.02
		MIN	5804.10	6134.20	1.07	3.40	5806.10	33.95	0.01
		MAX	7101.60	7494.80	1.47	10.01	7103.20	63.91	0.05
250	5	MEAN	7429.11	7665.70	1.49	3.18	7446.89	73.08	0.24
		MIN	6945.37	7126.60	1.47	1.73	6945.70	48.55	0.00
		MAX	8220.47	8457.60	1.51	6.98	8221.10	89.07	0.58
150	10	MEAN	4630.03	5235.25	1.45	13.07	4643.29	78.79	0.29
		MIN	4412.27	4931.10	1.39	9.11	4414.30	53.92	0.01
		MAX	4845.67	5447.20	1.53	19.59	4847.20	138.31	0.95
200	10	MEAN	5721.46	6450.47	2.96	12.74	5728.92	111.16	0.13
		MIN	5471.50	5978.10	2.90	9.26	5473.50	73.49	0.02
		MAX	6118.90	7046.80	3.00	15.65	6120.80	166.75	0.60
250	10	MEAN	6703.59	7576.23	3.99	13.02	6727.20	202.98	0.35
		MIN	6036.87	6808.80	3.84	10.29	6039.50	127.70	0.02
		MAX	7593.67	8379.30	4.95	15.74	7603.60	333.85	0.95
300	10	MEAN	7550.46	8334.67	5.63	10.39	7580.34	338.69	0.40
		MIN	7067.00	7687.70	5.51	8.00	7097.40	207.61	0.03
		MAX	8613.00	9301.70	5.78	14.59	8628.10	453.29	1.46
250	15	MEAN	6324.27	7156.42	8.01	13.16	6350.46	346.85	0.41
		MIN	6036.27	6820.70	7.66	9.30	6042.30	281.77	0.03
		MAX	6507.97	7588.40	9.68	18.69	6529.60	456.39	1.39
300	15	MEAN	7239.70	8123.47	9.48	12.21	7255.96	416.48	0.22
		MIN	6925.80	7527.70	9.23	4.86	6929.00	284.08	0.02
		MAX	7627.90	8533.60	9.61	17.60	7629.40	707.27	0.92
350	15	MEAN	8143.71	9221.35	13.25	13.23	8169.35	568.24	0.31
		MIN	7759.67	8681.00	13.05	9.65	7791.80	408.93	0.01
		MAX	8673.17	9770.10	13.50	17.29	8755.00	733.30	0.94
300	20	MEAN	7063.24	7987.91	13.88	13.09	7102.98	673.40	0.56
		MIN	6601.40	7419.30	13.65	9.37	6701.30	441.40	0.02
		MAX	7750.70	8650.30	14.46	15.42	7819.50	889.49	1.51
350	20	MEAN	7865.17	8846.23	18.57	12.47	7906.26	922.70	0.52
		MIN	7590.87	8351.50	18.32	9.71	7623.60	510.56	0.09
		MAX	8552.07	9541.50	18.97	15.52	8702.20	1209.82	1.76
400	20	MEAN	8532.98	9535.20	24.31	11.75	8566.74	1043.58	0.40
		MIN	8214.80	9108.80	23.95	8.34	8230.20	869.26	0.03
		MAX	8872.50	10002.50	24.87	16.71	8898.30	1303.38	0.90
350	25	MEAN	7711.00	8687.83	25.18	12.67	7764.08	1175.77	0.69
		MIN	7548.77	8291.20	24.99	6.92	7597.50	855.57	0.03
		MAX	7961.57	9402.30	25.66	18.10	7965.50	1697.19	2.16
400	25	MEAN	8485.24	9728.52	31.79	14.65	8542.00	1468.55	0.67
		MIN	8012.60	9172.70	31.42	9.33	8130.40	1150.67	0.04
		MAX	8919.20	10653.50	32.13	19.44	8984.30	1901.89	1.47
450	25	MEAN	9134.56	10311.26	38.22	12.88	9196.34	1709.33	0.68
		MIN	8741.67	10005.50	37.02	9.32	8772.90	1400.22	0.08
		MAX	9341.17	10716.60	39.08	17.74	9428.20	2334.85	1.30
400	30	MEAN	8201.67	9273.50	39.32	13.07	8255.81	1678.22	0.66
		MIN	7858.20	8962.90	38.73	9.62	7935.20	1274.80	0.12
		MAX	8513.70	9860.20	39.83	15.82	8568.60	2156.29	1.00
450	30	MEAN	8907.48	10080.66	49.83	13.17	8987.93	2318.30	0.90
		MIN	8706.27	9833.00	48.27	11.22	8768.80	1772.58	0.19
		MAX	9066.87	10408.60	57.24	15.91	9153.40	3594.25	1.91
500	30	MEAN	9696.34	11076.24	62.53	14.23	9747.63	2864.45	0.53
		MIN	9428.80	10736.30	61.39	10.12	9480.90	2400.36	0.09
		MAX	10045.30	11735.40	64.94	18.27	10054.00	3358.47	1.07

Tableau 2.4: Computational results for GHR and TSR with model C base

|M|	|N|		LOWB	GHR			TSR		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	4029.00	4513.55	0.18	12.03	4033.19	25.23	0.10
		MIN	3701.20	4190.50	0.18	5.78	3703.00	23.73	0.02
		MAX	4463.50	4965.00	0.19	15.86	4464.90	27.68	0.66
150	5	MEAN	5364.87	5915.34	0.57	10.26	5385.94	57.89	0.39
		MIN	4536.77	5028.30	0.55	6.52	4566.00	29.11	0.02
		MAX	6219.87	6913.20	0.59	13.60	6221.20	103.82	2.14
200	5	MEAN	6607.95	7155.71	1.30	8.29	6614.89	76.19	0.11
		MIN	6004.10	6504.20	1.27	6.22	6006.10	54.35	0.02
		MAX	7251.20	7864.80	1.32	12.16	7261.50	108.63	0.54
250	5	MEAN	7628.44	7989.47	2.34	4.73	7662.06	103.50	0.44
		MIN	7106.07	7496.60	2.27	3.43	7107.70	69.58	0.01
		MAX	8379.67	8767.40	2.39	7.40	8380.60	135.79	1.59
150	10	MEAN	4860.27	5539.25	1.49	13.97	4894.63	123.83	0.71
		MIN	4612.27	5227.10	1.40	9.79	4614.30	81.79	0.03
		MAX	5095.67	5796.70	2.04	18.72	5134.80	159.62	2.84
200	10	MEAN	5966.17	6760.82	3.38	13.32	5996.11	186.88	0.50
		MIN	5710.30	6422.10	3.35	10.50	5713.60	109.20	0.01
		MAX	6318.90	7268.80	3.43	15.55	6319.60	282.29	1.00
250	10	MEAN	6984.18	7973.92	5.71	14.17	7022.62	296.82	0.55
		MIN	6336.87	7252.80	5.68	10.06	6348.70	156.41	0.02
		MAX	7834.57	8823.30	5.75	16.85	7842.40	480.91	2.16
300	10	MEAN	7862.16	8857.14	8.50	12.66	7927.19	501.84	0.83
		MIN	7367.00	8057.70	8.32	8.94	7418.20	226.80	0.12
		MAX	8858.80	9671.70	9.67	15.52	8887.70	755.36	1.92
250	15	MEAN	6643.67	7587.66	11.43	14.21	6722.93	552.96	1.19
		MIN	6339.17	7264.70	11.15	9.76	6412.30	383.30	0.25
		MAX	6827.27	8032.40	13.33	20.90	6898.00	990.14	2.58
300	15	MEAN	7567.35	8549.28	13.77	12.98	7637.50	613.92	0.93
		MIN	7275.80	7897.70	13.66	5.60	7410.80	434.70	0.01
		MAX	7927.90	8829.60	13.90	18.89	7986.90	923.88	1.86
350	15	MEAN	8509.88	9703.01	19.53	14.02	8605.30	918.87	1.12
		MIN	8139.07	9199.00	19.46	9.49	8238.30	611.84	0.34
		MAX	9058.67	10196.20	19.71	18.72	9187.70	1136.83	2.60
300	20	MEAN	7437.29	8471.47	19.82	13.91	7563.29	1061.09	1.69
		MIN	6992.30	7789.30	19.64	9.46	7144.40	685.40	0.40
		MAX	8088.70	9094.30	20.18	17.48	8121.00	1626.02	2.45
350	20	MEAN	8259.81	9339.28	27.44	13.07	8390.83	1228.14	1.59
		MIN	7990.87	8916.80	26.45	9.74	8169.30	656.12	0.57
		MAX	8902.07	9911.50	33.31	16.33	8952.70	1752.61	2.40
400	20	MEAN	8953.15	10055.53	35.95	12.31	9087.08	1681.50	1.50
		MIN	8627.30	9478.80	35.13	8.73	8792.30	1119.08	1.07
		MAX	9256.80	10540.50	40.91	16.63	9365.60	2268.77	2.10
350	25	MEAN	8129.46	9212.77	36.22	13.33	8290.02	1799.89	1.98
		MIN	7975.37	8735.20	35.98	7.77	8014.90	1154.21	0.50
		MAX	8361.57	9856.30	36.44	17.88	8528.40	3089.26	3.59
400	25	MEAN	8932.98	10229.32	45.89	14.51	9068.81	1957.33	1.52
		MIN	8487.70	9699.90	45.52	9.54	8655.20	1133.54	0.32
		MAX	9369.20	11055.30	46.22	20.39	9463.30	2958.02	2.85
450	25	MEAN	9613.82	10907.12	55.48	13.45	9776.61	2737.02	1.69
		MIN	9220.27	10601.20	54.96	9.49	9422.60	1938.29	0.25
		MAX	9819.57	11390.10	56.06	17.77	9991.30	3626.34	3.02
400	30	MEAN	8651.25	9781.68	56.12	13.07	8808.90	2503.03	1.82
		MIN	8358.20	9406.90	55.54	9.17	8502.00	1637.97	0.74
		MAX	8963.70	10420.30	56.49	16.36	9143.10	3805.59	3.07
450	30	MEAN	9396.50	10680.86	70.80	13.67	9574.37	3200.31	1.89
		MIN	9206.27	10499.00	70.24	11.88	9411.80	2317.14	0.78
		MAX	9545.77	11000.60	71.31	15.24	9752.10	4082.59	2.88
500	30	MEAN	10207.87	11731.43	90.60	14.93	10411.68	4412.97	2.00
		MIN	9931.50	11462.80	90.22	11.97	10099.60	2570.98	0.99
		MAX	10497.10	12447.00	91.00	18.58	10609.20	5868.11	3.26

Tableau 2.5: Computational results for GHT and TST with model A base

$\|M\|$	$\|N\|$		LOWB	GHT			TST		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	3540.52	3817.41	0.24	7.82	3549.62	25.61	0.26
		MIN	3259.20	3552.80	0.23	3.92	3260.00	13.87	0.02
		MAX	3971.50	4335.00	0.26	12.69	3973.90	39.56	1.59
150	5	MEAN	4860.37	5111.11	0.38	5.16	4873.14	50.02	0.26
		MIN	4012.87	4153.50	0.38	2.43	4018.50	31.85	0.02
		MAX	5838.07	6207.80	0.40	8.26	5840.10	66.57	0.91
200	5	MEAN	6215.57	6573.91	0.55	5.77	6262.66	97.59	0.76
		MIN	5472.90	5631.80	0.54	2.03	5475.70	63.27	0.00
		MAX	6768.90	7371.90	0.57	11.23	6818.90	135.49	4.00
250	5	MEAN	7280.63	7780.72	0.72	6.87	7350.34	160.08	0.96
		MIN	6547.17	6615.20	0.70	0.71	6579.20	82.78	0.00
		MAX	8302.17	9028.00	0.79	14.12	8392.50	251.26	2.24
150	10	MEAN	4213.42	4712.87	0.96	11.85	4229.21	83.72	0.37
		MIN	4061.17	4484.30	0.94	8.87	4077.70	53.97	0.03
		MAX	4389.07	4923.50	0.99	17.11	4390.60	125.21	1.79
200	10	MEAN	5280.42	5937.84	1.70	12.45	5310.63	176.21	0.57
		MIN	5035.60	5384.90	1.67	6.94	5047.20	102.60	0.03
		MAX	5718.50	6464.70	1.74	15.74	5828.50	301.48	1.92
250	10	MEAN	6235.02	6820.35	2.05	9.39	6276.50	370.44	0.67
		MIN	5616.87	6120.80	2.02	6.37	5649.00	324.78	0.03
		MAX	7060.97	7713.80	2.13	12.03	7128.70	445.23	1.40
300	10	MEAN	7044.73	7618.40	3.03	8.14	7094.90	453.48	0.71
		MIN	6559.50	6968.30	2.96	5.29	6560.50	366.48	0.02
		MAX	8166.90	9071.90	3.16	12.27	8315.30	572.81	1.82
250	15	MEAN	5808.85	6532.07	4.42	12.45	5854.30	303.41	0.78
		MIN	5526.07	6222.70	4.37	9.29	5605.40	239.20	0.04
		MAX	6036.17	6993.80	4.64	17.09	6044.40	418.13	1.61
300	15	MEAN	6696.85	7549.87	5.34	12.74	6757.66	712.25	0.91
		MIN	6436.80	7232.60	5.27	8.52	6466.40	448.07	0.46
		MAX	7154.90	8352.10	5.61	16.73	7222.70	1501.48	2.19
350	15	MEAN	7580.71	8568.39	7.37	13.03	7649.62	1071.19	0.91
		MIN	7230.97	8031.90	7.26	10.39	7302.30	571.52	0.40
		MAX	8135.77	9388.80	7.50	16.51	8229.70	1729.45	2.24
300	20	MEAN	6512.19	7297.71	8.05	12.06	6556.38	652.13	0.68
		MIN	6107.40	6962.10	7.91	8.60	6192.90	519.28	0.03
		MAX	7236.20	8056.40	8.40	14.82	7303.90	971.86	1.47
350	20	MEAN	7260.60	8214.49	10.73	13.14	7312.92	931.43	0.72
		MIN	6952.87	7649.20	10.45	8.76	7068.30	755.78	0.26
		MAX	8078.67	9338.30	11.10	15.75	8173.10	1183.57	1.80
400	20	MEAN	7902.87	8934.55	14.07	13.05	7959.22	1385.02	0.71
		MIN	7612.90	8461.10	13.53	8.23	7705.00	1155.56	0.03
		MAX	8249.20	9675.50	14.59	17.29	8287.30	1608.35	1.48
350	25	MEAN	7095.43	8017.33	15.10	12.99	7148.20	1181.12	0.74
		MIN	6905.97	7601.10	14.93	8.81	6966.80	914.05	0.43
		MAX	7387.57	8672.00	15.34	17.39	7443.20	1350.20	1.60
400	25	MEAN	7847.26	8958.10	18.75	14.16	7912.59	1645.72	0.83
		MIN	7369.60	8257.00	18.30	11.94	7435.30	1390.39	0.07
		MAX	8292.50	9679.50	19.29	17.73	8298.60	2031.27	2.42
450	25	MEAN	8451.01	9636.56	22.36	14.03	8531.34	2264.31	0.95
		MIN	8073.07	9269.30	21.93	8.47	8144.20	1774.96	0.05
		MAX	8663.07	10040.10	22.94	17.60	8682.40	2789.39	3.28
400	30	MEAN	7548.16	8605.90	23.69	14.01	7619.57	1997.72	0.95
		MIN	7225.20	8255.00	23.00	11.61	7232.70	1753.37	0.10
		MAX	7902.20	9113.10	25.77	15.37	7982.30	2543.96	2.58
450	30	MEAN	8225.44	9323.25	29.06	13.35	8288.70	2604.98	0.77
		MIN	8042.27	8989.50	28.28	10.20	8081.60	2237.67	0.24
		MAX	8444.47	9705.80	32.76	17.86	8464.80	3521.41	2.04
500	30	MEAN	8971.61	10193.01	37.57	13.61	9074.06	3404.96	1.14
		MIN	8718.40	9799.10	36.45	9.01	8752.30	2991.44	0.39
		MAX	9353.20	10863.80	41.74	16.56	9409.20	3820.35	2.52

Tableau 2.6: Computational results for GHT and TST with model B base

$\|M\|$	$\|N\|$		LOWB	GHT			TST		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	3704.73	4066.56	0.42	9.77	3709.44	34.00	0.13
		MIN	3409.20	3832.50	0.41	6.70	3411.00	22.49	0.02
		MAX	4121.50	4557.00	0.44	12.49	4122.50	52.85	0.74
150	5	MEAN	5020.92	5391.88	0.80	7.39	5031.42	77.40	0.21
		MIN	4212.87	4523.50	0.77	4.96	4215.20	34.51	0.01
		MAX	5847.47	6436.90	0.86	10.67	5849.40	145.11	2.03
200	5	MEAN	6265.80	6622.70	1.15	5.70	6267.78	109.26	0.03
		MIN	5651.50	5969.10	1.12	3.96	5653.50	77.93	0.00
		MAX	6912.70	7305.00	1.21	9.25	6916.10	156.11	0.05
250	5	MEAN	7263.47	7494.54	1.56	3.18	7290.17	166.82	0.37
		MIN	6780.07	6973.00	1.50	1.86	6801.90	111.06	0.02
		MAX	8114.37	8264.90	1.66	7.15	8122.50	216.00	1.22
150	10	MEAN	4467.08	5079.45	1.84	13.71	4487.77	163.64	0.46
		MIN	4284.27	4788.90	1.80	8.98	4286.20	98.99	0.03
		MAX	4639.07	5293.50	1.89	20.59	4640.60	370.63	1.17
200	10	MEAN	5550.46	6281.65	3.23	13.17	5570.49	208.15	0.36
		MIN	5306.70	5828.90	3.09	9.84	5308.70	126.92	0.01
		MAX	5959.20	6910.70	3.99	15.98	5959.90	339.81	1.96
250	10	MEAN	6542.31	7419.16	3.92	13.40	6573.22	447.52	0.47
		MIN	5916.87	6694.80	3.77	9.90	5921.10	228.54	0.04
		MAX	7359.57	8172.60	4.03	16.45	7387.30	927.43	1.37
300	10	MEAN	7363.91	8167.62	5.93	10.91	7417.45	751.04	0.73
		MIN	6909.40	7562.80	5.86	8.57	6965.00	416.50	0.01
		MAX	8344.20	9105.60	6.08	14.90	8413.50	1103.94	2.06
250	15	MEAN	6168.32	7011.56	7.96	13.67	6223.92	588.27	0.90
		MIN	5876.07	6693.10	7.84	9.91	5907.80	409.57	0.21
		MAX	6383.27	7437.80	8.20	18.82	6397.00	849.69	2.65
300	15	MEAN	7080.26	7977.91	9.85	12.68	7131.87	789.54	0.73
		MIN	6786.40	7372.20	9.66	5.17	6920.10	496.75	0.31
		MAX	7426.30	8409.20	10.05	19.02	7458.60	1598.40	1.97
350	15	MEAN	7969.68	9066.46	13.80	13.76	8023.17	1022.18	0.67
		MIN	7604.67	8549.90	13.44	10.59	7654.70	664.21	0.17
		MAX	8479.77	9593.50	14.35	17.14	8526.80	1406.02	1.53
300	20	MEAN	6911.81	7831.62	14.37	13.31	6983.84	1163.64	1.04
		MIN	6507.40	7294.50	13.90	9.10	6587.80	731.77	0.13
		MAX	7568.60	8500.40	14.77	16.20	7642.90	1687.89	1.61
350	20	MEAN	7696.33	8711.68	19.33	13.19	7785.58	1486.91	1.16
		MIN	7402.87	8241.20	18.99	10.40	7500.20	713.28	0.48
		MAX	8380.57	9391.50	19.91	16.66	8544.00	2251.12	2.88
400	20	MEAN	8375.86	9387.94	25.58	12.08	8451.71	1721.46	0.91
		MIN	8095.40	9010.40	25.08	8.88	8140.40	1397.53	0.33
		MAX	8720.30	9903.00	26.44	17.60	8771.50	2386.65	1.87
350	25	MEAN	7553.99	8538.11	26.96	13.03	7643.55	1930.46	1.19
		MIN	7385.77	8165.90	26.27	7.37	7486.10	1182.09	0.14
		MAX	7787.57	9267.60	29.62	19.01	7946.20	3694.09	2.17
400	25	MEAN	8333.07	9566.07	33.52	14.80	8435.11	2520.96	1.22
		MIN	7906.70	9021.70	32.95	9.81	7982.40	1827.33	0.45
		MAX	8736.10	10489.20	33.96	20.07	8780.60	4325.02	2.16
450	25	MEAN	8983.44	10178.26	40.38	13.30	9091.28	3016.81	1.20
		MIN	8621.47	9845.20	39.61	9.83	8663.10	1949.98	0.27
		MAX	9190.97	10611.50	41.39	17.92	9263.80	5192.93	2.27
400.	30	MEAN	8051.62	9141.13	40.97	13.53	8194.77	2507.36	1.78
		MIN	7725.20	8856.40	40.24	10.30	7956.20	1556.57	0.40
		MAX	8365.40	9705.10	42.07	16.01	8585.50	3552.10	3.00
450	30	MEAN	8766.09	9948.83	50.89	13.49	8877.03	3693.79	1.27
		MIN	8586.87	9729.50	49.90	12.02	8704.40	2679.01	0.31
		MAX	8941.17	10267.90	52.40	15.89	9070.20	5912.48	2.03
500	30	MEAN	9552.78	10945.22	66.12	14.58	9646.78	4614.47	0.98
		MIN	9283.30	10585.90	64.81	10.27	9355.40	3802.97	0.33
		MAX	9883.90	11603.80	68.02	18.42	9956.20	5612.55	1.41

Tableau 2.7: Computational results for GHT and TST with model C base

$\|M\|$	$\|N\|$		LOWB	GHT			TST		
				OBJ	CPU	GAP	OBJ	CPU	GAP
			[k$]	[k$]	[sec]	[%]	[k$]	[sec]	[%]
100	5	MEAN	3852.26	4230.16	0.47	9.81	3865.35	42.75	0.34
		MIN	3559.20	4054.50	0.45	6.91	3560.00	19.75	0.00
		MAX	4265.80	4715.30	0.49	13.92	4265.80	59.16	1.68
150	5	MEAN	5192.56	5710.06	1.18	9.97	5215.48	119.45	0.44
		MIN	4412.87	4893.50	1.16	6.43	4483.20	56.65	0.01
		MAX	5997.47	6806.90	1.22	15.39	5999.40	191.40	2.39
200	5	MEAN	6449.93	6992.70	1.90	8.42	6463.78	183.76	0.21
		MIN	5851.50	6339.10	1.86	6.51	5853.50	129.47	0.02
		MAX	7102.00	7675.00	2.01	12.55	7121.30	258.71	0.73
250	5	MEAN	7464.90	7834.49	2.52	4.95	7507.33	250.91	0.57
		MIN	6940.77	7343.00	2.49	3.61	6948.10	156.38	0.02
		MAX	8270.67	8634.90	2.59	7.62	8276.90	332.09	2.06
150	10	MEAN	4697.72	5388.43	2.68	14.70	4741.37	242.56	0.93
		MIN	4484.27	5084.90	2.64	9.75	4485.60	149.46	0.03
		MAX	4889.07	5599.40	2.73	20.22	4948.70	327.11	2.17
200	10	MEAN	5795.37	6599.62	4.90	13.88	5837.57	371.37	0.73
		MIN	5556.70	6272.90	4.80	10.83	5575.00	203.61	0.01
		MAX	6159.20	7132.70	4.99	16.08	6159.90	586.37	1.59
250	10	MEAN	6833.71	7823.26	5.96	14.48	6901.03	624.85	0.99
		MIN	6216.67	7138.80	5.91	9.68	6250.50	270.62	0.05
		MAX	7635.87	8616.60	6.12	17.26	7654.10	1205.28	2.39
300	10	MEAN	7686.53	8699.10	9.12	13.17	7794.77	1226.99	1.41
		MIN	7223.30	7932.80	8.98	9.28	7314.00	268.49	0.76
		MAX	8620.30	9475.60	9.26	16.14	8709.10	2043.76	2.28
250	15	MEAN	6488.04	7440.76	11.74	14.68	6606.86	1071.82	1.83
		MIN	6182.97	7137.10	11.63	9.45	6301.90	688.93	0.84
		MAX	6694.67	7881.80	12.02	21.08	6750.80	1351.04	2.79
300	15	MEAN	7416.51	8402.96	14.60	13.30	7509.04	1103.04	1.25
		MIN	7136.40	7742.20	14.43	5.91	7275.40	731.53	0.09
		MAX	7726.30	8705.20	14.72	19.43	7828.10	2028.96	3.17
350	15	MEAN	8346.50	9548.06	20.64	14.40	8463.61	1651.65	1.40
		MIN	7976.37	9033.20	20.49	10.39	8107.20	1075.05	0.35
		MAX	8879.77	10027.10	20.91	18.60	9034.20	2267.65	2.55
300	20	MEAN	7285.55	8311.52	21.05	14.08	7417.17	1887.28	1.81
		MIN	6877.60	7664.50	20.85	9.69	7019.60	1099.46	0.59
		MAX	7918.60	8944.40	21.66	17.60	7965.40	3685.96	3.53
350	20	MEAN	8100.21	9194.28	28.43	13.51	8268.77	1989.44	2.08
		MIN	7852.87	8784.20	28.17	10.22	8049.60	1038.20	0.73
		MAX	8730.57	9761.50	28.97	16.49	8794.20	2701.06	4.28
400	20	MEAN	8795.92	9902.39	38.27	12.58	8957.60	2939.46	1.84
		MIN	8524.10	9318.20	37.65	8.65	8685.30	1812.20	1.33
		MAX	9099.30	10421.00	40.71	17.58	9261.80	3958.20	2.21
350	25	MEAN	7968.62	9069.78	38.84	13.82	8163.96	3099.24	2.45
		MIN	7806.17	8609.90	38.47	8.13	7915.40	1862.12	0.93
		MAX	8187.57	9721.00	39.17	18.73	8406.20	5579.13	4.07
400	25	MEAN	8776.18	10085.74	49.13	14.92	8951.22	3317.39	1.99
		MIN	8355.20	9539.70	48.51	9.44	8533.60	1513.60	0.71
		MAX	9186.10	10919.10	50.06	20.65	9328.30	6161.98	3.10
450	25	MEAN	9463.87	10772.43	59.48	13.83	9643.53	4092.51	1.90
		MIN	9089.87	10437.20	58.82	10.15	9295.90	3187.44	0.53
		MAX	9634.47	11212.20	60.64	18.05	9860.80	5325.99	3.07
400	30	MEAN	8506.01	9652.50	59.48	13.48	8727.43	3940.43	2.60
		MIN	8223.00	9300.40	59.08	9.80	8447.40	2394.83	1.00
		MAX	8815.40	10297.10	59.76	16.81	9091.20	6317.23	3.60
450	30	MEAN	9248.96	10559.79	74.30	14.17	9467.51	4869.83	2.36
		MIN	9086.87	10396.60	73.82	12.57	9358.90	3506.36	1.04
		MAX	9399.07	10859.90	75.00	15.57	9638.90	7236.86	3.18
500	30	MEAN	10069.87	11600.63	96.94	15.20	10292.09	7064.01	2.21
		MIN	9809.20	11325.90	96.35	12.14	10002.30	3975.03	1.02
		MAX	10363.50	12302.10	99.40	18.71	10511.30	9993.49	3.20

1444.56 seconds. Although it is interesting to assess the mean values, it is also important to consider the maximum and minimum values. For the greedy heuristics GHR and GHT, it is evident that for a given size, the difference between the gap for the minimum and the maximum values can be very large. Such a difference is less obvious for the tabu search heuristics TSR and TST. For instance, for a 20-site, 300-user problem with base type B, the minimum and maximum gaps for GHR are 9.37% and 15.42%, respectively, whereas for TSR they are 0.02% and 1.51%. Therefore we can say that the tabu-based heuristics are more stable than the greedy heuristics.

Tables 2.8 and 2.9 present the mean results as a function of the base model for the ring and tree topologies, respectively. The first column indicates the base model, the second column provides the lower bound and the next set of columns present the objective value, the CPU time and the gap found with the greedy and the tabu search heuristics (respectively).

Tableau 2.8: Mean results for GHR and TSR

Model	LOWB	GHR			TSR		
	[k$]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
A	6905.92	7673.22	9.89	11.11	6922.88	619.06	0.25
B	7241.84	8097.44	17.59	11.81	7274.42	804.53	0.45
C	7580.30	8547.73	25.33	12.76	7673.70	1201.96	1.23

Tableau 2.9: Mean results for GHT and TST

Model	LOWB	GHT			TST		
	[k$]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
A	6728.58	7510.19	10.31	11.62	6780.58	978.57	0.77
B	7081.09	7941.42	18.43	12.15	7141.82	1350.68	0.86
C	7421.96	8390.93	27.08	13.06	7539.77	2004.44	1.59

According to the results provided in the tables, the larger the number of slots, the larger the mean CPU time and the mean gap between our heuristics and the lower bound. Moreover, for the same base model, the mean CPU time and the mean gap for the tree topology are larger than for the ring topology. Finally, as in the case of the example we presented in the last subsection, it can be gathered from the tables that the mean cost for a ring topology is slightly higher than for a tree topology. For example, in the case of base A, the mean cost of a ring topology is only 2.17% higher than the mean cost of a tree topology.

2.7 Conclusions and Directions for Further Work

In this paper, a network design model that includes the location of switches, the design of the local access network and the design of the backbone network with a fixed (ring or tree) topology has been presented. This model assumes that the switches used are modular, because networks will contain such switches. The model is much more realistic than any similar model found in the literature.

We have proposed two different kinds of heuristics (with specialized versions for two topologies of the backbone network, the ring topology and the tree topology). The first kind consists of greedy heuristics whereas the second kind is based on the tabu search principle. We carried out a set of experiments that included the systematic evaluation of randomly generated networks with up to 500 users and 30 potential sites. In order to assess the performance of the approaches, lower bounds were proposed for the case of a ring topology as well as for the case of a tree topology. It was observed that the tabu-based heuristics result in solutions that are, on average, within 0.64% and 1.07% of the optimal solution (for the ring and tree topologies, respectively). These are excellent results considering the difficulty of the problems that we have studied.

There are several avenues of research that are open at this point. First, the only impediment to the application of our method to very large problems is the difficulty in evaluating the lower bound. Such evaluation becomes time-consuming as the problem instances increase in size. Since the tabu approach presents relatively stable behavior in terms of closeness to the optimal solution (as demonstrated by the results of the previous section), it can be assumed that such behavior would be maintained for larger networks. The only way to verify this statement is to develop exact methods or more efficient ways of evaluating the lower bound.

In this paper we considered the model in which the topology of the backbone network is a single ring. However, because of physical limitations, it may be more appropriate to design survivable ring structures of limited sizes. This amounts to choosing for the backbone network a topology of several interconnected rings. A more intricate version of our model is one in which different types of bases coexist in the same network. We are currently examining these extensions of our model.

2.8 Acknowledgements

The authors are grateful to Gilbert Laporte of the CRT for having let them use his implementation of the GENIUS algorithm. This work was funded in part by FCAR grants and strategic grant STR 0166996 from NSERC.

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CHAPITRE 3

Heuristics for the Topological Design Problem of Two-Level Multitechnology Telecommunication Networks with Modular Switches

by

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Abstract

In this article we propose a model for the topological design problem of two-level multitechnology telecommunication networks that includes the optimal location of modular switches, their configuration with respect to ports and bases, the design of an access network with a star topology and a backbone network with a fixed topology. Moreover, we consider that several access technologies and rates may be used. Three backbone topologies are studied (ring, tree and full-mesh) and sets of valid inequalities are proposed. The model is of the integer programming variety, and in order to find a good solution, we propose a greedy heuristic that yields a starting solution and a more sophisticated heuristic based on the tabu search principle. Numerical results for problems with up to 400 clients and 30 potential

switch sites are presented. The solutions obtained with the tabu-based approach are, on average, within 2.87% of the optimal solution.

Key words: Topological design, multitechnology and multiservice networks, modular switch, ATM, tabu search, local access network, backbone network, ring, tree and full-mesh topologies.

Status: This article has been submitted to the journal *INFORMS Journal of Computing*, 1997.

3.1 Introduction

Deregulation and the introduction of new broadband technologies, in particular, ATM (Asynchronous Transfer Mode) networks, have increased competition between telecommunication companies. The ATM, a form of packet transmission with fixed-size packets, called cells, has become the standard for broadband multiservice networks where each client can be either an institution, or a LAN (Local Area Network), or groups of LANs. As the technology and the services needed by each client may differ considerably, it is appropriate to consider using various available transmission technologies and rates (such as ATM/OC-3, ATM/OC-12, FDDI, Token Ring, Ethernet and Frame Relay) to connect the clients to the switches to form the access network. The reader interested in the details of these technologies may wish to consult Walrand (1991) and Bertsekas and Gallager (1992). In this paper, however, we assume that a unique technology is used in the backbone network. We call such networks two-level multitechnology telecommunication networks, and demonstrate their structure in Figure 3.1.

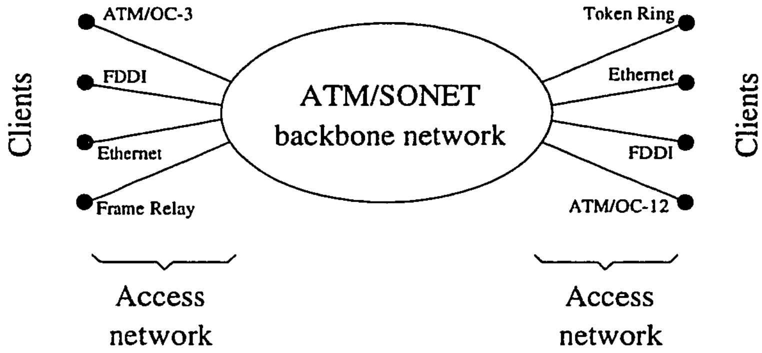

Figure 3.1: Structure of a two-level multitechnology telecommunication network

In the literature of Engineering and Operations Research almost all publications related to network design problems focus only on a portion of the overall network design, often switch location problems, or topological network design problems, or joint routing/capacity assignment problems. For an overview of these types of problems review articles by Magnanti and Wong (1984), Minoux (1989), Gavish (1991), and books by Sharma (1990) and Kershenbaum (1993) are useful. Other publications deal with the overall network design problems as does the article by Chamberland, Sansó and Marcotte (1997a), who review the literature related to the overall network design, and propose a model for a topological design problem for two-level telecommunication networks with modular switches but using a single switch base type and a single access technology. Also, Chamberland, Sansó and Marcotte (1997b) propose two heuristics (a greedy and a tabu-based heuristic) for a ring network design problem when several types of switches are available but using a single access technology.

The object of this paper is to present a model and a resolution approach for the topological design problem of two-level multitechnology telecommunication networks that includes: the location of switches, the design of an access network with a star topology and the design of a backbone network with a fixed topology. The switches used are modular as are ATM switches. A modular switch is composed of a base and ports. A base consists of a controller and a switching network with slots to insert ports, which are used to connect links to the base. We take into account that different types of ports and links may be used, each corresponding to an available transmission technology. We also consider that different types of bases are available, each with a different number of slots (capacity). This is illustrated in Figure 3.2, where client C1 is connected directly to slot 1 through an ATM/OC-3 port, whereas client C2 is connected to slot 2 through a FDDI port. Moreover, switch S1 is connected to slot 3 through an ATM/OC-192 port.

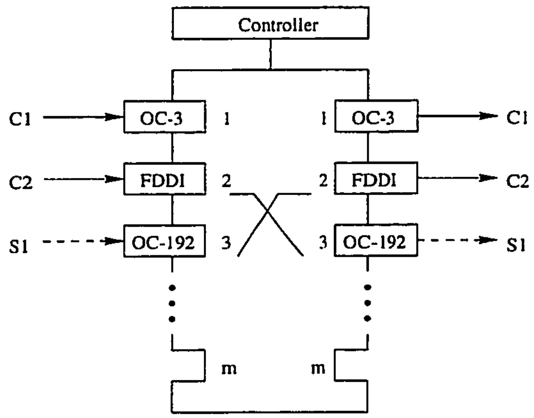

Figure 3.2: A modular switch

The problem we treat in this paper is more general than those in the aforementioned articles (and the references therein) since we consider that clients are connected to switches with different access technologies and rates in order to satisfy the demand of a maximum number of clients. Also, we consider that the bases are of different types, where each type is characterized by its capacity (in terms of the number of slots) and by its cost. Finally, we study in detail the full-mesh topology for the backbone network, in addition to switch capacity constraints.

The paper is organized as follows. In Section 3.2, a general model for the topological design problem of two-level multitechnology telecommunication networks with modular switches is presented. This general model is adapted to take into account specialized backbone topologies (ring, tree and full-mesh). These topologies were chosen for several reasons. A ring topology provides protection against single link and node failure, and many protocols such as SONET and FDDI make use of this topology. Although a tree topology does not provide this kind of protection, it is, nevertheless, the least expansive topology. Consequently, tree networks are sometimes used in practice, often when reliability is not of primary importance, for example with private networks. A full-mesh topology (i.e. a complete graph in the graph-theoretic sense) is the most reliable one but, it is also the most expansive topology. Consequently, full-meshed networks are frequently used when system reliability and fast response is the most important performance criteria, for example with military networks. In Section 3.3 we describe a greedy heuristic able to find a "good" starting solution for the tabu search algorithm presented in Section 3.4. Computational results for a set of randomly generated problems with 20 different network sizes and their analysis are given in Section 3.5. Indications for further research are outlined in Section 3.6.

3.2 Problem Description

For the topological design problem of two-level multitechnology networks (with modular switches), the following information is considered known: (I1) the location of clients and the technology requested by everyone; (I2) the location of potential switch sites; (I3) the different types of bases, their capacities and costs; (I4) the cost of installing a given type of base at a given site; (I5) the costs of the different port types (for the different technologies); (I6) the costs of the different link types (for the different technologies), including the installation cost, in \$/km;

We also make the following assumptions about the organization of the network: (A1) each client is connected to a switch through a link and a port of type corresponding to the technology requested by the client (for instance, if a client requests a FDDI connection, it will be connected to a switch through a link and a port of technology FDDI, such as the connection of client C2 in Figure 3.2); (A2) the switches are interconnected through links and ports of technology ATM/OC-192 with a specified topology; (A3) the number of ports installed in a switch cannot exceed the number of slots of the switch base; (A4) at most one switch may be installed at a given site. Note that, despite the last assumption, the case of several switches per site can be handled by defining fictitious base types constituted by several connected bases.

Solving the problem involves selecting switch sites, types of bases and ports to be inserted in the slots, connecting the clients to the switches, and finally interconnecting the switches through ATM/OC-192 links with a specified topology. The goal is to minimize total network cost, subject to all the information and assumptions described above.

3.2.1 Mathematical Formulation

The following notation is used throughout the paper. Let $M = \{1, \ldots, |M|\}$ denote the set of client sites; $N = \{1, \ldots, |N|\}$ the set of potential switch sites, and

$T = \{1, \ldots, |T|\}$ the set of base types (where m^t denotes the number of slots of a base of type $t \in T$).

Next we define the following parameters. Let c_{ij} denote the cost of connecting client $i \in M$ to site $j \in N$ through a link and a port of a type corresponding to the technology requested by the client, and d_{jk} the cost of connecting site $j \in N$ to site $k \in N$ (for $j < k$) through ATM/OC-192 link (which includes the cost of two ATM/OC-192 ports). Assume that the cost of a link is proportional to the euclidean distance between the sites joined by this link. Finally, let b_j^t denote the cost of purchasing a base of type $t \in T$ and installing it at site $j \in N$.

We now describe the decision variables of the model. Let $x_{ij} \in \mathsf{B}$ ($\mathsf{B} = \{0,1\}$) be a variable such that $x_{ij} = 1$ if and only if client $i \in M$ is connected to site $j \in N$; $y_{jk} \in \mathsf{B}$ a variable such that $y_{jk} = 1$ if and only if the site $j \in N$ is connected to site $k \in N$, and $u_j \in \mathsf{B}$ a variable such that $u_j = 1$ if and only if a switch is installed at site $j \in N$. Finally let $z_j^t \in \mathsf{B}$ be a variable such that $z_j^t = 1$ if and only if the base installed at site $j \in N$ is of type $t \in T$.

Note that in the Design Problem model, noted DP, given below for the two-level multitechnology networks with modular switches, the topology of the backbone network has not yet been specified.

DP:

$$\min_{\mathbf{u},\mathbf{x},\mathbf{y},\mathbf{z}} \left(\sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} + \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} + \sum_{j \in N} \sum_{t \in T} b_j^t z_j^t \right) \tag{3.1}$$

s.t.

Assignment constraints

$$\sum_{j \in N} x_{ij} = 1 \quad (i \in M) \tag{3.2}$$

Base type uniqueness constraints

$$\sum_{t \in T} z_j^t = u_j \quad (j \in N) \tag{3.3}$$

Base capacity constraints

$$\sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq \sum_{t \in T} m^t z_j^t \quad (j \in N) \tag{3.4}$$

Backbone topology constraints

$$\boxed{\text{Backbone topology constraints}} \tag{3.5}$$

Intergrality and nonegativity constraints

$$\mathbf{u} \in \mathsf{B}^{|N|}, \mathbf{x} \in \mathbb{R}_+^{|M||N|}, \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)}, \mathbf{z} \in \mathsf{B}^{|T||N|}. \tag{3.6}$$

In model DP, we suppose that the number of switch base types is at least one, that is $|T| \geq 1$, with $\max_{t \in T}\{m^t\} \geq 3$ and that the number of potential switch sites is at least one, that is $|N| \geq 1$. We also suppose that the number of clients is at least three, that is $|M| \geq 3$, since, if the number of clients is equal to one or two, it is not cost-effective to build a two-level network.

The objective function (3.1), representing the total network cost, is composed of three terms: access, backbone and switch base costs. Assignment constraints (3.2) require that each client be connected to precisely one switch. Base type uniqueness constraints (3.3) require that at most one base type be installed in site $j \in N$, and this type be chosen if and only if a switch has been installed at the site (i.e., $u_j = 1$).

Base capacity constraints (3.4) require that the number of clients connected to a base of type $t \in T$ be at most m^t, whereas backbone topology constraints (3.5) require that the topology of the backbone network be the one specified by the user of the model. Constraints (3.6) are integrality and nonegativity constraints. Note that we need not impose integrality constraints on the x_{ij} variables because the matrix of constraints where these variables appear is totally unimodular. This observation is made precise in Proposition 1 which follows.

Proposition 1. Let us assume that all variables (with the exception of the x_{ij}) are fixed and have values in $\mathbb{B}$. Then the polyhedron of feasible solutions for the x_{ij} variables has only integral extreme points.

Proof. If we enumerate the x_{ij} variables in the following manner

$$x_{1,1}, x_{1,2}, \ldots, x_{1,|N|}, x_{2,1}, x_{2,2}, \ldots, x_{2,|N|}, \ldots, x_{|M|,1}, x_{|M|,2}, \ldots, x_{|M|,|N|},$$

the constraint matrix for these variables (that is, constraints (3.2) enumerated for $i = 1, \ldots, |M|$ and constraints (3.4) for $j = 1, \ldots, |N|$) is the node-edge incidence matrix of a complete bipartite graph and thus totally unimodular (see Nemhauser and Wolsey, 1988). $\square$

The two following propositions present valid inequalities that will be used in Section 3.5 to find a lower bound for DP.

Proposition 2. The following inequalities

$$x_{ij} \leq u_j \quad (i \in M;\ j \in N) \tag{3.7}$$

$$y_{jk} \leq u_j \quad (j < k,\ \ j,k \in N) \tag{3.8}$$

$$y_{jk} \leq u_k \quad (j < k,\ \ j,k \in N) \tag{3.9}$$

are valid for DP.

Proof. We prove only inequalities (3.7), since (3.8) and (3.9) can be similarly proven. Let $j \in N$ be a given site. If $u_j = 0$ using constraints (3.3) and (3.4) we obtain $x_{ij} = 0$ for all $i \in M$. However, if $u_j = 1$ using constraints (3.2) we obtain $x_{ij} \leq 1$ for all $i \in M$. Thus, inequalities (3.7) are valid for DP. □

Proposition 3. The following inequalities

$$\sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq m^t + \left(\max_{s \in T \setminus \{t\}} \{m^s\} - m^t \right) \left(1 - z_j^t\right) \quad (t \in T, j \in N) \tag{3.10}$$

are valid for DP.

Proof. Let $t \in T$ be a given base type located at site $j \in N$. If $z_j^t = 1$ then, using (3.3), (3.4) and constraint $u_j \in \mathrm{B}$ we obtain

$$\sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq m^t.$$

Alternatively, if $z_j^t = 0$ we obtain

$$\begin{aligned} \sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} &\leq \sum_{s \in T \setminus \{t\}} m^s z_j^s \\ &\leq \max_{s \in T \setminus \{t\}} \{m^s\} \sum_{s \in T \setminus \{t\}} z_j^s. \end{aligned}$$

Using equation (3.3), we have

$$\begin{aligned} \sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} &\leq \max_{s \in T \setminus \{t\}} \{m^s\} u_j \\ &\leq \max_{s \in T \setminus \{t\}} \{m^s\}. \end{aligned}$$

Thus, inequalities (3.10) are valid for DP. □

Subsections 2.2, 2.3, 2.4, which follow, present the topology constraints for ring, tree and full-meshed backbone networks, respectively.

3.2.2 Ring Topology

In what follows DPR denotes the version of DP in which the backbone network has a ring topology, i.e., the version in which the general constraints (3.5) are replaced by the following constraints.

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} = 2u_j \quad (j \in N) \tag{3.11}$$

$$\frac{1}{2}\left(\sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j<k}} y_{jk} + \sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j>k}} y_{kj}\right) + (1-u_l) + (1-u_m) \geq 1$$
$$(H \subset N; l \in H; m \in N\setminus H; 3 \leq |H| \leq |N|-3). \tag{3.12}$$

These constraints were described by Balas (1989), in the context of the Prize-Collecting Traveling Salesman Problem (PCTSP) (for further details concerning the PCTSP see Balas, 1986, 1989, 1995 and Fischetti and Toth, 1988). They were also used in Chamberland, Sansó and Marcotte (1997a) in the context of two-level topological design but with a single base type and a single technology used in the access network. Constraints (3.11) require that the number of ATM/OC-192 links (for the backbone network) connected to a switch be exactly two. Constraints (3.12) are connectivity ones. Substituting constraints (3.11) into (3.4) and using (3.3), we obtain the following inequalities.

$$\sum_{i\in M} x_{ij} \leq \sum_{t\in T}\left(m^t - 2\right) z_j^t \quad (j \in N). \tag{3.13}$$

Proposition 4. DPR is $\mathcal{NP}$-hard.

Proof. Transformation from the Traveling Salesman Problem (TSP) (for further details concerning the TSP see Lawler, Lenstra, Rinnooy Kan and Shmoys, 1985 and Nemhauser and Wolsey, 1988). □

The following inequality, which gives a lower bound on the number of switches to be installed in the network, will be used in Section 3.5 to find a lower bound for DPR.

Proposition 5. The following inequality

$$\sum_{j \in N} u_j \geq \left\lceil \frac{|M|}{\max_{t \in T}\{m^t\} - 2} \right\rceil \tag{3.14}$$

is valid for DPR.

Proof. If we sum on $j \in N$ the two sides of (3.13) and use (3.2), we obtain the following inequality.

$$\begin{aligned} |M| &\leq \sum_{j \in N} \sum_{t \in T} \left(m^t - 2\right) z_j^t \\ &\leq \left(\max_{t \in T}\{m^t\} - 2\right) \sum_{j \in N} \sum_{t \in T} z_j^t. \end{aligned}$$

Using equation (3.3), we obtain

$$|M| \leq \left(\max_{t \in T}\{m^t\} - 2\right) \sum_{j \in N} u_j, \tag{3.15}$$

and finally

$$\sum_{j \in N} u_j \geq \frac{|M|}{\max_{t \in T}\{m^t\} - 2}. \tag{3.16}$$

The proposition follows because $\sum_{j \in N} u_j$ is an integer in all feasible solutions of DPR. □

Proposition 6. DPR is feasible if and only if $|N| \geq 3$ and

$$|M| \leq |N| \left(\max_{t \in T} \{m^t\} - 2 \right). \tag{3.17}$$

Proof. ($\Rightarrow$) If DPR is feasible, inequality (3.15) is respected for every feasible solution of DPR and since

$$\begin{aligned} |M| &\leq \left(\max_{t \in T} \{m^t\} - 2 \right) \sum_{j \in N} u_j \\ &\leq |N| \left(\max_{t \in T} \{m^t\} - 2 \right), \end{aligned}$$

then inequality (3.17) is respected for all feasible solutions and $|N| \geq 3$ because the number of switches in any network with a ring topology is at least three. ($\Leftarrow$) Suppose that inequality (3.17) is respected and $|N| \geq 3$. Then, if we install $|N|$ bases of capacity equal to $\max_{t \in T}\{m^t\}$ and interconnect it with a ring backbone network, the number of slots left to the access networks (given by the right-hand side of (3.17)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Thus, a feasible solution can be constructed, so DPR is feasible. □

3.2.3 Tree Topology

In what follows DPT denotes the version of DP in which the backbone network has a tree topology, i.e., the version in which the general constraints (3.5) are replaced by the following constraints.

$$\sum_{j \in N} \sum_{\substack{k \in N \\ j < k}} y_{jk} = \sum_{j \in N} u_j - 1 \tag{3.18}$$

$$\sum_{j\in H}\sum_{\substack{k\in H\\ j<k}} y_{jk} \leq \sum_{j\in H\setminus\{l\}} u_j \quad (H \subset N; l \in H; 2 \leq |H| \leq |N|-2). \tag{3.19}$$

These constraints were described by Lee, Qiu and Ryan (1994), in the context of the Steiner Tree-Star (STS) problem (for further details concerning the STS problem see Lee, Lu, Qiu and Glover, 1994 and Lee, Lim and Park, 1995). They were also used in Chamberland, Sansó and Marcotte (1997a) in the context of two-level topological design but with a single base type and a single technology used in the access network. Constraints (3.18) require that the number of ATM/OC-192 links (for the backbone network) be exactly the number of switches minus one. Constraints (3.19) are cycle elimination inequalities.

Proposition 7. DPT is $\mathcal{NP}$-hard.

Proof. Transformation from the Degree Constrained Minimum Spanning Tree (DCMST) problem (for further details concerning the DCMST see Garey and Johnson, 1979; Narula and Ho, 1980; Savelsbergh and Volgenant, 1985 and Volgenant, 1989).

The following inequality, which gives a lower bound on the number of switches to be installed in the network, will be used in Section 3.5 to find a lower bound for DPT.

Proposition 8. The following inequality

$$\sum_{j\in N} u_j \geq \left\lceil \frac{|M|-2}{\max_{t\in T}\{m^t\}-2} \right\rceil \tag{3.20}$$

is valid for DPT.

Proof. If we sum on $j \in N$ the two sides of (3.4) and use (3.2) and (3.18), we obtain the following inequality.

$$\begin{aligned} |M| + 2\left(\sum_{j \in N} u_j - 1\right) &\leq \sum_{j \in N}\sum_{t \in T} m^t z_j^t \\ &\leq \max_{t \in T}\{m^t\} \sum_{j \in N}\sum_{t \in T} z_j^t. \end{aligned}$$

Using equation (3.3), we obtain

$$|M| + 2\left(\sum_{j \in N} u_j - 1\right) \leq \max_{t \in T}\{m^t\} \sum_{j \in N} u_j, \tag{3.21}$$

and finally

$$\sum_{j \in N} u_j \geq \frac{|M| - 2}{\max_{t \in T}\{m^t\} - 2}. \tag{3.22}$$

The proposition follows because $\sum_{j \in N} u_j$ is an integer in all feasible solutions of DPT. □

Proposition 9. DPT is feasible if and only if

$$|M| \leq |N|\left(\max_{t \in T}\{m^t\} - 2\right) + 2. \tag{3.23}$$

Proof. (⇒) If DPT is feasible, inequality (3.21) is respected for every feasible solution and since

$$\begin{aligned} |M| &\leq \max_{t \in T}\{m^t\} \sum_{j \in N} u_j - 2\left(\sum_{j \in N} u_j - 1\right) \\ &\leq |N|\left(\max_{t \in T}\{m^t\} - 2\right) + 2 \end{aligned}$$

then inequality (3.23) is respected for all feasible solutions. (⇐) Suppose that inequality (3.23) is respected. Then, if we install $|N|$ bases of capacity equal to

$\max_{t \in T}\{m^t\}$ and interconnect it with a tree backbone network (respecting the base capacity constraints) the number of slots left to the access network (given by the right-hand side of (3.23)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Thus, a feasible solution can be constructed, so DPT is feasible. □

3.2.4 Full-Mesh Topology

In what follows DPF denotes the version of DP in which the backbone network has a full-mesh topology, i.e., the version in which the general constraints (3.5) are replaced by the following constraints.

$$u_j + u_k \leq y_{jk} + 1 \quad (j < k, \;\; j, k \in N). \tag{3.24}$$

These constraints were described by Chung, Myung and Tcha (1992), in the context of the topological network design problem with two-level structure. Constraints (3.24) with (3.3) and (3.4) require an ATM/OC-192 link (for the backbone network) between two sites if and only if switches have been installed at these two sites.

Proposition 10. In DPF, we need not impose integrality constraints on the y_{jk} variables.

Proof. Let j, $k \in N$ (such that $j < k$) be two given sites and suppose $y_{jk} \in [0, 1]$. There are two cases to be considered: u_j and u_k are both equal to one and at least one of them is equal to zero. More precisely, if $u_j = 1$ and $u_k = 1$ then, using (3.24), we obtain $y_{jk} \geq 1$, and since $y_{jk} \in [0, 1]$, then $y_{jk} = 1$. Alternatively, if $u_j = 0$ or

$u_k = 0$, using valid inequalities (3.8) and (3.9), we obtain $y_{jk} = 0$. Thus y_{jk} may be only zero or one. □

Proposition 11. DPF is $\mathcal{NP}$-hard.

Proof. Transformation from the Uncapacitated Facility Location Problem (UFLP) (for further details concerning the UFLP see Nemhauser and Wolsey, 1988). □

The following inequalities, which provide bounds on the number of switches to be installed in the network, will be used in Section 3.5 to find a lower bound for DPF.

Proposition 12. The following inequalities

$$\sum_{j\in N} u_j \geq \left\lceil \frac{1}{2}\left((\max_{t\in T}\{m^t\}+1) - \sqrt{\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M|}\right)\right\rceil \tag{3.25}$$

$$\sum_{j\in N} u_j \leq \left\lfloor \frac{1}{2}\left((\max_{t\in T}\{m^t\}+1) + \sqrt{\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M|}\right)\right\rfloor \tag{3.26}$$

are valid for DPF.

Proof. With this topology, we have $y_{jk} = u_j u_k$ for all $j < k$ and $j,\ k \in N$. Then inequalities (3.4) become

$$\begin{aligned}\sum_{i\in M} x_{ij} &\leq \sum_{t\in T} m^t z_j^t - \left(\sum_{\substack{k\in N\\ j<k}} u_j u_k + \sum_{\substack{k\in N\\ j>k}} u_k u_j \right)\\ &= \sum_{t\in T} m^t z_j^t - \sum_{k\in N\setminus\{j\}} u_k u_j \quad (j \in N).\end{aligned} \tag{3.27}$$

If we sum on $j \in N$ the two sides of (3.27) and use (3.2), we obtain the following inequality.

$$|M| \leq \sum_{j\in N}\sum_{t\in T} m^t z_j^t - \sum_{j\in N}\left(\sum_{k\in N\setminus\{j\}} u_k\right) u_j$$

$$\leq \max_{t\in T}\{m^t\} \sum_{j\in N}\sum_{t\in T} z_j^t - \sum_{j\in N}\left(\sum_{k\in N\setminus\{j\}} u_k\right) u_j$$
$$= \max_{t\in T}\{m^t\} \sum_{j\in N}\sum_{t\in T} z_j^t - \sum_{j\in N} u_j \left(\sum_{j\in N} u_j - 1\right).$$

Using equation (3.3), we obtain

$$|M| \leq \max_{t\in T}\{m^t\} \sum_{j\in N} u_j - \sum_{j\in N} u_j \left(\sum_{j\in N} u_j - 1\right) \tag{3.28}$$

or

$$-\left(\sum_{j\in N} u_j\right)^2 + \left(\max_{t\in T}\{m^t\} + 1\right) \sum_{j\in N} u_j - |M| \geq 0. \tag{3.29}$$

Consider the following equation

$$f(x) = -x^2 + \left(\max_{t\in T}\{m^t\} + 1\right) x - |M| = 0, \tag{3.30}$$

where $x \in \mathbb{R}$. The solutions of (3.30) are

$$x_- = \frac{1}{2}\left((\max_{t\in T}\{m^t\} + 1) - \sqrt{\left(\max_{t\in T}\{m^t\} + 1\right)^2 - 4|M|} \right)$$
$$x_+ = \frac{1}{2}\left((\max_{t\in T}\{m^t\} + 1) + \sqrt{\left(\max_{t\in T}\{m^t\} + 1\right)^2 - 4|M|} \right)$$

and the maximum value of $f(x)$ is obtained for

$$x_{max} = \frac{\max_{t\in T}\{m^t\} + 1}{2}.$$

In Figure 3.3, we depict equation $f(x)$ and indicate the interval of x such that $f(x) \geq 0$. Then, values of $x \in \mathbb{Z}_+$ such that $f(x) \geq 0$ are $\{\lceil x_- \rceil, \ldots, \lfloor x_+ \rfloor\}$ and

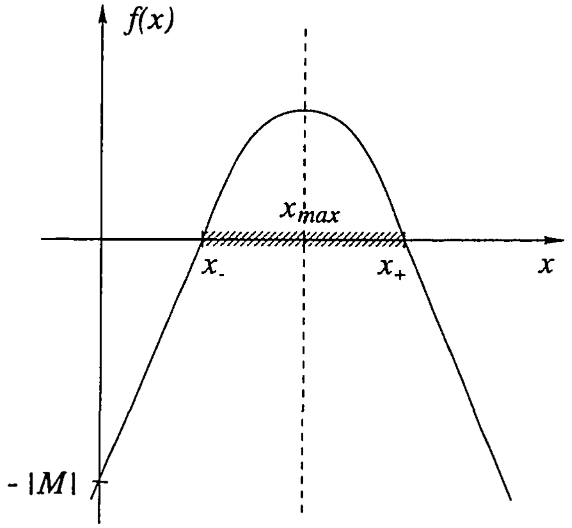

Figure 3.3: Equation $f(x)$

the maximum value of $f(x)$ is obtained when x is equal to $\lceil x_{max} \rceil$ and $\lfloor x_{max} \rfloor$. The proposition follows because $\sum_{j \in N} u_j$ is an integer in all feasible solutions of DPF. □

Proposition 13. The following inequality

$$|M| \leq \left\lceil \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rceil \left\lfloor \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rfloor \tag{3.31}$$

is respected for all feasible solutions of DPF.

Proof. In Proposition 12, we showed that the maximum of the right-hand side of inequality (3.28) is obtained when $\sum_{j \in N} u_j$ is equal to $\left\lceil \frac{\max_{t \in T}\{m^t\}+1}{2} \right\rceil$ and $\left\lfloor \frac{\max_{t \in T}\{m^t\}+1}{2} \right\rfloor$. Then, if we replace $\sum_{j \in N} u_j$ by $\left\lceil \frac{\max_{t \in T}\{m^t\}+1}{2} \right\rceil$ in inequality (3.28), we obtain

$$\begin{aligned} |M| &\leq \left\lceil \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rceil \left(\left(\max_{t \in T}\{m^t\} + 1 \right) - \left\lceil \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rceil \right) \\ &= \left\lceil \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rceil \left\lfloor \frac{\max_{t \in T}\{m^t\} + 1}{2} \right\rfloor . \square \end{aligned}$$

Proposition 14. DPF is feasible if and only if

$$|M| \leq \left\lceil \frac{\max_{t\in T}\{m^t\}+1}{2} \right\rceil \left\lfloor \frac{\max_{t\in T}\{m^t\}+1}{2} \right\rfloor \tag{3.32}$$

and

$$|N| \geq \left\lceil \frac{1}{2}\left((\max_{t\in T}\{m^t\}+1) - \sqrt{\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M|} \right) \right\rceil . \tag{3.33}$$

Proof. ($\Rightarrow$) Because inequalities (3.32) and (3.33) are obtained from constraints of DPF, if DPF is feasible, then (3.32) and (3.33) are respected for all feasible solution. ($\Leftarrow$) Suppose that inequalities (3.32) and (3.33) are respected. If inequality (3.32) is respected, the following inequality

$$\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M| \geq 0 \tag{3.34}$$

is also respected (see Appendix A), and then the right-hand side of (3.33) may be computed since the square root has a nonnegative argument. If we install a number of bases equal to the right-hand side of (3.33), use only bases with capacity equal to $\max_{t\in T}\{m^t\}$, and interconnect it with a full-meshed backbone network, the number of slots left to the access networks (given by the right-hand side of (3.28)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Thus, a feasible solution can be constructed, so DPF is feasible. □

Since DPR, DPT and DPF are $\mathcal{NP}$-hard, it is unlikely that large-size instances of these problems can be optimally solved. Therefore, in the next two sections, we propose efficient heuristic algorithms to solve them.

3.3 A Greedy Heuristic

In this section, we describe a greedy heuristic, called GH (Greedy Heuristic), able to find a solution for DP. This solution will be used as a starting point for the tabu search algorithm proposed in Section 3.4. GH incorporates features of the heuristic of Lee, Qiu and Ryan (1994) for the Steiner Tree-Star (STS) problem.

The following notation and definitions will be used in the presentation of GH. A star is a subnetwork that includes a switch (the center of the star) and clients connected to this switch. The size of the star is the number of clients in the star. The cost of the star is the cost of the links connecting the clients to the center of the star. Let E_M and E_N be two sets defined to contain clients and switch sites of the current solution, respectively. Let $\Gamma(j)$ be the set of clients of the star connected at site $j \in E_N$, $\Phi(j)$ the maximum number of slots available for the backbone network at site $j \in E_N$, and $\overline{\Phi}(j)$ the number of backbone links connected to site $j \in E_N$. Finally, let $t(j)$ be the type of the base installed at switch site $j \in E_N$.

The proposed heuristic generates at most m solutions corresponding to stars of size k for $1 \leq k \leq m$, where m is the maximum number of clients that can be connected to a single switch, i.e.,

$$m = \min\left\{|M|, \max_{t \in T}\{m^t\}\right\}. \tag{3.35}$$

The best solution found will be returned by GH.

Heuristic GH

Step 1: (Feasibility check)

If the problem is not feasible, stop. Otherwise, set $k := 1$ and go to Step 2.

Step 2: (Stars size feasibility)

If it is possible to generate a solution with stars of size k and $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\}$ switches do Steps 3 and 4. Otherwise go to Step 5.

Step 3: (Generating a solution)

3.1 Set $E_M := \emptyset$, $E_N := \emptyset$ and $\Gamma(j) := \emptyset$ for all $j \in N$.

3.2 For $i := 1$ to $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\}$ do

3.2.1 Determine the star of minimum cost among the stars of size $\min\{k, |M\backslash E_M|\}$ containing a switch in $N\backslash E_N$ and clients in $M\backslash E_M$. This is done in the following manner. For all $j \in N\backslash E_N$, we generate a star with a switch installed at site j and the set of clients $\Gamma(j)$ corresponding to the $\min\{k, |M\backslash E_M|\}$ nearest clients to j in $M\backslash E_M$. The best star generated is denoted j^*.

3.2.2 Set $E_N := E_N \cup \{j^*\}$, $E_M := E_M \cup \Gamma(j^*)$ and $\Phi(j^*) := \max_{t\in T}\{m^t\} - |\Gamma(j^*)|$.

3.3 Connect the switches in E_N with ATM/OC-192 links to form a backbone network of minimum cost respecting constraints (3.5) and the degree constraints such that switch $j \in E_N$ cannot be connected to more than $\Phi(j)$ switches.

3.4 For each site $j \in E_N$, set $\overline{\Phi}(j)$ to the number of ATM/OC-192 links connected to site j.

3.5 For each site $j \in E_N$ set $t(j)$ to the solution of the following problem

$$\min_{t\in T}\left\{b_j^t : m^t \geq |\Gamma(j)| + \overline{\Phi}(j)\right\}.$$

3.6 For each client $i \in M \setminus E_M$ do

3.6.1 Set $(j^*, t(j^*))$ to the solution of the following problem

$$\min_{j \in E_N, t \in T} \left\{ c_{ij} + b_j^t - b_j^{t(j)} : m^t > |\Gamma(j)| + \overline{\Phi}(j) \right\}.$$

3.6.2 Set $\Gamma(j^*) := \Gamma(j^*) \cup \{i\}$ and $E_M := E_M \cup \{i\}$.

3.7 Compute the cost of the current solution.

Step 4: (Best solution update)

If the cost of the current solution is less than that of the best solution obtained so far, update the best solution obtained so far.

Step 5: (Termination test)

If $k = m$, return the best solution found and stop. Otherwise, set $k := k + 1$ and go to Step 2.

Step 1 in GH consists of checking if DP is feasible according to the selected topology given by constraints (3.5). If the problem is infeasible, the algorithm stops. Otherwise, Step 2 is performed which consists of verifying if a feasible solution may be generated with stars of size k and $\min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\}$ switches, also according to the selected topology. Step 3 generates a feasible solution. More precisely, in Step 3.2, stars of size k are constructed (the last star constructed may be smaller) and a set of switch sites E_N is also found. Step 3.3 consists of connecting the switches in E_N to form a minimum cost the backbone network. Step 3.5 selects, for each site in E_N, the base type of minimum cost such that the number of slots of the base type selected for site j is greater or equal to $|\Gamma(j)| + \overline{\Phi}(j)$, which is the number of users and switches connected to this site. Step 3.6 consists of connecting each client not yet considered into the network, at minimum cost, while taking into account the capacity of the base types installed in Step 3.5, and the possibility of changing these base types. In Step 3.7, the cost of the current solution, given by the objective function of DP, is computed. Steps 4 and 5 update the best solution, if necessary, and if k is equal to m, the algorithm stops. However, if $k < m$, the algorithm returns to Step 2 with $k := k + 1$.

In what follows we show how to adapt GH to find solutions for DPR, DPT and DPF.

3.3.1 Ring Topology

The heuristic GH adapted for DPR is called GHR. Step 1 of GHR consists of verifying if DPR is feasible using Proposition 6. Step 2 consists of verifying if $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\} \geq 3$ and

$$\max_{t\in T}\{m^t\} \geq 2 + k, \tag{3.36}$$

where the left-hand side is the maximum number of slots available and the right-hand side is the number of slots required at each switch to construct stars of size k and to connect the switches with a ring backbone network. If $|N| \geq \left\lceil\frac{|M|}{k}\right\rceil$, then all the users are used to form the $\left\lceil\frac{|M|}{k}\right\rceil$ stars in Step 3.2, but if $|N| < \left\lceil\frac{|M|}{k}\right\rceil$, the users not used to form the $|N|$ stars in Step 3.2 are connected to the switches in Step 3.6. Step 3.3 consists of connecting switches in E_N with ATM/OC-192 links to form a minimum cost ring. This is a TSP on the node set E_N with symmetric costs (d_{jk} for $j < k$ and j, $k \in E_N$). To solve this problem, we use the composite heuristic GENIUS proposed by Gendreau, Hertz and Laporte (1992).

3.3.2 Tree Topology

The heuristic GH adapted for DPT is called GHT. Step 1 of GHT consists of verifying if DPT is feasible using Proposition 9. Step 2 consists of verifying if

$$\max_{t\in T}\{m^t\} \geq \begin{cases} k & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} = 1 \\ 1+k & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} = 2 \\ 2+k & \text{if } \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} \geq 3, \end{cases} \tag{3.37}$$

where the left-hand side is the maximum number of slots available and the right-hand side is the minimum number of slots required at each switch to construct stars of size k and to connect the switches with a tree backbone network. Step 3.3 consists of connecting switches in E_N with ATM/OC-192 links to form a minimum cost tree respecting the degree constraints. This is a DCMST problem on the node set E_N with symmetric costs (d_{jk} for $j < k$ and $j,\ k \in E_N$) and maximal degree $\Phi(j)$ for $j \in E_N$. To solve this problem, we use the dual heuristic with exchanges proposed by Narula and Ho (1980).

3.3.3 Full-Mesh Topology

The heuristic GH adapted for DPF is called GHF. Step 1 of GHF consists of verifying if DPF is feasible using Proposition 14. Step 2 consists of verifying if

$$\max_{t\in T}\{m^t\} \geq \min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\} - 1 + k, \tag{3.38}$$

where the left-hand side is the maximum number of slots available and the right-hand side is the number of slots required at each switch to construct stars of size k and to connect the switches with a full-meshed backbone network. Step 3.3 consists of connecting switches in E_N with ATM/OC-192 links to form a full-mesh topology.

3.4 The Tabu Algorithm

In this section we propose a tabu search algorithm, called TS (Tabu Search), for DP. An introduction to tabu search can be found in Glover (1989, 1990) and in Glover, Taillard and de Werra (1993).

Let $j \in N$ be a given site. The state e_j for this site is defined in the following manner: $e_j = 0$ if there is no base installed at site j, and $e_j = k$ (for $k \in T$) if a base of type k is installed at site j. This amounts to saying that if $e_j = 0$, then $u_j = 0$ and $z_j^t = 0$ for all $t \in T$, and if $e_j = k$ (for $k \in T$), then $u_j = 1$, $z_j^k = 1$ and $z_j^t = 0$ for all $t \in T \setminus \{k\}$. Note that there are $|T| + 1$ possible states for each site. Let us define vector $\mathbf{e} = \{e_j\}_{j \in N}$ so that e_j is the state of site $j \in N$. Thus $\mathbf{e}$ is the state of the network sites. Let $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$ be the vectors $\mathbf{u}$ and $\mathbf{z}$ respectively when the network sites state $\mathbf{e}$ is fixed. We outline the main features of the algorithm before giving its detailed description.

3.4.1 Problem Decomposition

Let DP($\mathbf{e}$) be the model DP when the decision vectors $\mathbf{u}$ and $\mathbf{z}$ are respectively equal to vectors $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$. When $\mathbf{u}$ and $\mathbf{z}$ are fixed (respecting constraints (3.3)), DP can be decomposed into two subproblems.

Consider the following valid inequalities for DP

$$\sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq \sum_{t \in T} m^t z_j^t \quad (j \in N), \tag{3.39}$$

obtained by using constraints (3.4) and nonnegativity constraints on the x_{ij}

variables. The first subproblem, noted $\overline{\mathrm{DP}}(\mathbf{u},\mathbf{z})$, is given below.

$$\overline{\mathrm{DP}}(\mathbf{u},\mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j\in N} \sum_{\substack{k\in N \\ j<k}} d_{jk} y_{jk} : (3.5), (3.39), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{3.40}$$

The purpose of the first subproblem is to connect the switches to form a backbone network whose topology is given by constraints (3.5).

The second subproblem, noted $\overline{\overline{\mathrm{DP}}}(\mathbf{y},\mathbf{z})$, is given below.

$$\overline{\overline{\mathrm{DP}}}(\mathbf{y},\mathbf{z}) : \min_{\mathbf{x}} \left\{ \sum_{i\in M} \sum_{j\in N} c_{ij} x_{ij} : (3.2), (3.4), \mathbf{x} \in \mathbf{R}_{+}^{|M||N|} \right\}. \tag{3.41}$$

The purpose of this second subproblem is to connect the clients to the switches at minimum cost. It is therefore an assignment problem, and to solve it we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant (1987) considered the best method for solving linear assignment problems for dense graphs (see Kennington and Wang, 1991).

3.4.2 Solution Space

Let E be the set of all possible states for the sites of the network, including those states that do not correspond to feasible solutions of the model. Note that $|E| = (|T|+1)^{|N|}$. Let $\mathbf{y}(\mathbf{e})$ be the exact or heuristic solution of $\overline{\mathrm{DP}}(\mathbf{u}(\mathbf{e}),\mathbf{z}(\mathbf{e}))$, and let $\mathbf{x}(\mathbf{e})$ be the exact solution of $\overline{\overline{\mathrm{DP}}}(\mathbf{y}(\mathbf{e}),\mathbf{z}(\mathbf{e}))$. The solution space is thus $\Omega = \{(\mathbf{u}(\mathbf{e}),\mathbf{x}(\mathbf{e}),\mathbf{y}(\mathbf{e}),\mathbf{z}(\mathbf{e}))\}_{\mathbf{e}\in E}$.

3.4.3 Neighborhood Structure

Let $\omega \in \Omega$ be a solution. $\mathcal{N}(\omega)$ is called the neighborhood of ω and consists of the solutions obtained by modifying the state of a given site in the current solution. The number of possible modifications is then $|N||T|$.

3.4.4 Tabu Moves and Aspiration Criterion

Each move of tabu search consists of modifying the state of a given site in the current solution. At each iteration of the search, we determine the best move while taking into account the tabus and the aspiration criterion (described in the next paragraph). Once the site is determined, it is declared tabu for a number of iterations that is randomly determined according to a uniform discrete distribution on the interval $[5, 10]$.

The aspiration criterion implies that if the use of a tabu site j allows us to discover a solution better than any found up to this point, we should remove the tabu from site j.

3.4.5 The Algorithm

Algorithm TS is now described in detail.

Algorithm TS

Step 1: (Initial solution)

Find an initial solution by using the GH heuristic.

Repeat Steps 2 to 3 for MAXITER iterations

Step 2: (Exploring the neighborhood)

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \rightarrow \mathbf{e}'$ (i.e., modification of the state of a given site in the current solution), we solve DP($\mathbf{e}'$) by solving $\overline{\text{DP}}(\mathbf{u}(\mathbf{e}'), \mathbf{z}(\mathbf{e}'))$ exactly or heuristically, and $\overline{\overline{\text{DP}}}(\mathbf{y}(\mathbf{e}'), \mathbf{z}(\mathbf{e}'))$ exactly by using the algorithm LAPJV. The cost of a solution is given by the objective function (3.1) of model DP.

2.2 Determine the number of iterations (according to a uniform discrete distribution on the interval [5, 10]) for the chosen site is tabu.

Step 3: (Best solution update)

If the current solution cost is less than the best solution found so far, update the best solution found so far.

In what follows we show how to adapt TS so as to find satisfactory solutions for DPR, DPT and DPF.

3.4.5.1 Ring Topology

The heuristic TS adapted for DPR is called TSR. Step 1 of TSR consists of finding an initial solution by using the GHR heuristic. With this topology, the first subproblem becomes

$$\overline{\text{DPR}}(\mathbf{u}, \mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j < k}} d_{jk} y_{jk} : (3.11), (3.12), (3.39), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \quad (3.42)$$

The purpose of the above subproblem is to connect the switches through a ring. It is therefore a TSP with symmetric costs if $\sum_{j \in N} u_j \geq 3$ and $\sum_{t \in T} m^t z_j^t \geq 2$

for all $j \in N$ such that $u_j = 1$. Otherwise, the subproblem is infeasible. We solve it with the GENIUS composite heuristic proposed by Gendreau, Hertz and Laporte (1992).

3.4.5.2 Tree Topology

The heuristic TS adapted for DPT is called TST. Step 1 of TST consists of finding an initial solution by using the GHT heuristic. With this topology, the first subproblem becomes

$$\overline{\mathrm{DPT}}(\mathbf{u},\mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} : (3.18), (3.19), (3.39), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \quad (3.43)$$

The purpose of the above subproblem is to connect the switches through a tree with degree constraints. It is therefore a DCMST problem with symmetric costs. We solve it using the dual heuristic with exchanges proposed by Narula and Ho (1980).

3.4.5.3 Full-Mesh Topology

The heuristic TS adapted for DPF is called TSF. Step 1 of TSF consists of finding an initial solution by using the GHF heuristic. With this topology, the first subproblem becomes

$$\overline{\mathrm{DPF}}(\mathbf{u},\mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} : (3.24), (3.39), \mathbf{y} \in [0,1]^{\frac{|N|}{2}(|N|-1)} \right\}. \quad (3.44)$$

The purpose of the above subproblem is to connect the switches through a full-meshed network. This problem can be solved by hand because, in a full-meshed network, $y_{jk} = 1$ if and only if $u_j = u_k = 1$, for all $j < k$ and $j, k \in N$. If constraints (3.39) are not satisfied by this solution, the subproblem is infeasible.

3.5 Computational Results

In this section, we present the results of a systematic set of experiments that were designed to assess the performance of the proposed algorithms. The algorithms presented in this paper have been implemented in C language and tested on a Sun Ultra 1 (model 140) workstation.

Twenty problem sizes were chosen, and for each size, 10 problems were randomly generated as follows. $|M|$ points corresponding to the clients' locations and $|N|$ points corresponding to the candidate switch sites were generated in the square region of side length 100 km following a uniform distribution law. Moreover, the technology requested by each client is selected randomly from among the technologies ATM/OC-12, ATM/OC-3, FDDI, Token Ring, Ethernet and Frame Relay, following a uniform distribution law.

The costs of the types of links and ports (including the installation costs) considered are given in Table 3.1. We should mention that the monetary unit used in this paper, denoted \$, is an arbitrary unit. The ATM/OC-192 technology is used only in the backbone network. Moreover, Table 3.2 gives the features of the three base types, where the first line contains the number of slots and the second contains the total switch cost which is purchase plus installation costs.

Tableau 3.1: Costs of the types of ports and links

Technology	Port cost [$]	Link cost [$/km]
ATM/OC-192	10000	3000
ATM/OC-12	3000	1700
ATM/OC-3	1000	1000
FDDI	1000	1000
Token Ring	500	1000
Ethernet	500	800
Frame Relay	500	500

Tableau 3.2: Features of the base models

	Type A	Type B	Type C
Capacity	16	32	64
Cost	$150 000	$250 000	$400 000

All the heuristic results were compared with lower bounds. Each lower bound was obtained by solving, using CPLEX Mixed Integer Optimizer a relaxed version of the problem. The choice of the constraints to be relaxed was done with the objective to have a good bound found in a reasonable amount of time. For more information about CPLEX, see the CPLEX user's manual (1993). More precisely, the bound for DPR was obtained by using CPLEX to solve DPR without integrality constraints on $\mathbf{y}$ and $\mathbf{z}$ variables (i.e., the integrality on $\mathbf{u}$ variables were conserved), relaxing connectivity constraints (3.12) but with valid inequalities (3.7) to (3.10) and (3.14). For DPT, the lower bound was obtained by solving DPT (using CPLEX) without integrality constraints on $\mathbf{y}$ and $\mathbf{z}$ variables, relaxing cycle elimination constraints (3.19) but with valid inequalities (3.7) to (3.10) and (3.20). Finally, for DPF, the lower bound was obtained by solving DPF (using CPLEX) without integrality constraints on $\mathbf{z}$ variables but with valid inequalities (3.7) to (3.10), (3.25) and (3.26).

The first results relating to the problems with a ring backbone are presented in Table 3.3, for the tree backbone in Table 3.4, and for the full-meshed backbone in Table 3.5. In each table, the first column contains the number of clients and the second, the number of potential switch sites. For each problem size, the mean, the minimum and the maximum values obtained for the set of random tests (of size equal to 10 problems) are indicated by MEAN, MIN and MAX respectively. The other columns present the CPU execution time and the gap between the heuristic solution and the lower bound (with respect to the value of the lower bound) for different values of the number of iterations (indicated by MAXITER in the tables) of the tabu search method. Moreover, at the bottom of each table we present the mean, the minimum and the maximum of each column.

As can be gathered from the tables, the tabu search heuristics produce, in just a few iterations, solutions that are close to the lower bound. For all problems solved with 10 iterations of TSR the mean gap is 4.09% and the mean CPU time is 62.54 seconds; for TST, the mean gap is 4.55% and the mean CPU time is 58.33 seconds; for TSF, the mean gap is 2.21% and the mean CPU time is 55.67 seconds. The solutions obtained with more iterations are closer to the lower bound, as was expected, with an increase in CPU time. In fact, the mean gap for 100 iterations of TSR is 3.39% (-0.7%) and the mean CPU time is 517.37 seconds (+454.83 seconds); for TST, the mean gap is 3.89% (-0.66%) and the mean CPU time is 471.72 seconds (+413.39 seconds); for TSF, the mean gap is 1.34% (-0.87%) and the mean CPU time is 498.25 seconds (+442.58 seconds). Note that the best gaps are obtained for TSF, and for some problems the gap is even equal to zero. This can be explained by the fact that in TSF, the two subproblems are solved exactly and the lower bounds (for problems with the full-mesh topology) contains just few fractional variables.

Tableau 3.3: Computational results for TSR

$\lvert M\rvert$	$\lvert N\rvert$		MAXITER = 10		MAXITER = 20		MAXITER = 50		MAXITER = 100	
			CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
150	10	MEAN	7.61	3.63	13.59	3.49	32.21	3.33	64.59	3.32
		MIN	6.65	1.22	12.47	1.22	30.28	1.22	58.93	1.22
		MAX	8.37	5.01	15.41	4.85	35.00	3.93	70.60	3.85
200	10	MEAN	11.78	3.89	21.05	3.63	50.66	3.47	101.47	3.45
		MIN	10.56	2.69	19.80	2.18	46.71	2.18	91.28	2.18
		MAX	14.85	6.79	23.99	5.49	57.17	4.88	110.77	4.88
250	10	MEAN	18.90	2.63	31.33	2.52	70.74	2.42	142.95	2.35
		MIN	17.10	1.86	27.46	1.84	59.00	1.84	127.16	1.84
		MAX	22.89	4.54	36.63	4.54	80.78	4.54	163.28	4.43
300	10	MEAN	23.90	2.93	44.99	2.77	103.95	2.50	215.33	2.49
		MIN	19.77	2.04	38.41	2.04	94.25	1.72	196.14	1.72
		MAX	26.47	3.76	52.49	3.58	119.38	3.58	232.83	3.53
150	15	MEAN	12.66	3.91	22.95	3.70	53.11	3.66	104.64	3.62
		MIN	11.52	2.80	21.65	2.80	49.63	2.80	99.54	2.80
		MAX	15.33	4.58	24.79	4.58	56.16	4.58	110.01	4.58
200	15	MEAN	19.47	3.62	34.31	3.38	81.74	3.19	162.47	3.18
		MIN	17.44	2.83	32.15	2.64	76.68	2.64	152.41	2.64
		MAX	24.33	4.93	38.23	3.97	88.72	3.76	173.08	3.76
250	15	MEAN	30.00	3.61	52.04	3.20	120.39	2.92	236.29	2.81
		MIN	25.31	2.44	44.39	2.40	108.34	2.22	218.10	2.22
		MAX	36.12	5.89	59.19	4.00	130.20	4.00	265.56	3.74
300	15	MEAN	42.71	3.83	74.07	3.31	172.73	3.12	339.78	3.06
		MIN	37.61	2.56	63.37	2.19	148.92	2.19	292.79	2.19
		MAX	51.17	5.23	83.94	4.43	188.53	4.34	369.80	4.34
250	20	MEAN	39.33	4.05	71.78	3.67	166.73	3.37	327.67	3.25
		MIN	36.38	2.62	63.90	2.62	154.62	2.42	300.12	2.42
		MAX	45.38	5.25	77.41	4.42	177.98	4.42	346.72	3.77
300	20	MEAN	57.28	4.31	103.05	3.95	241.98	3.74	479.90	3.74
		MIN	46.84	3.15	85.46	3.15	211.73	3.15	427.76	3.15
		MAX	74.37	6.40	119.06	5.53	262.94	4.77	534.56	4.77
350	20	MEAN	73.18	4.28	132.23	4.04	316.69	3.59	630.02	3.53
		MIN	66.50	3.02	124.49	3.02	297.37	3.02	589.33	2.87
		MAX	76.37	5.19	136.63	4.90	344.45	4.34	692.09	4.34
400	20	MEAN	106.80	4.30	185.14	3.85	427.08	3.49	837.22	3.42
		MIN	92.78	2.56	163.62	2.56	362.32	2.56	729.17	2.56
		MAX	149.24	6.59	217.25	6.30	469.89	4.86	929.82	4.24
250	25	MEAN	50.04	4.09	90.77	3.80	213.82	3.33	426.26	3.32
		MIN	44.34	2.74	80.87	2.74	196.26	2.65	396.42	2.65
		MAX	55.29	4.87	99.80	4.87	230.26	4.26	458.24	4.26
300	25	MEAN	75.63	5.48	131.26	4.58	304.64	4.18	614.25	4.13
		MIN	62.97	4.04	114.12	3.74	281.36	2.79	552.24	2.79
		MAX	103.95	7.74	160.95	5.64	339.50	5.56	732.99	5.56
350	25	MEAN	102.59	4.33	181.71	3.97	425.98	3.60	839.27	3.60
		MIN	92.36	3.37	169.58	2.69	399.94	2.69	783.09	2.69
		MAX	122.87	7.07	202.46	5.30	444.45	4.81	900.11	4.81
400	25	MEAN	139.77	4.13	250.08	3.90	585.23	3.75	1159.05	3.65
		MIN	121.20	2.68	220.83	2.19	506.05	2.19	1030.50	2.19
		MAX	157.07	5.86	281.29	5.73	670.39	4.79	1349.51	4.79
250	30	MEAN	62.24	4.40	111.77	3.92	260.25	3.48	512.40	3.37
		MIN	57.01	3.40	103.95	3.24	235.91	2.97	477.14	2.97
		MAX	69.04	5.45	124.54	4.77	291.32	4.32	575.58	3.68
300	30	MEAN	83.54	4.76	152.06	4.20	357.63	3.79	706.70	3.64
		MIN	78.10	3.99	143.03	3.53	323.72	3.25	638.70	3.25
		MAX	90.90	5.79	163.91	5.06	385.16	4.52	764.22	4.13
350	30	MEAN	127.92	4.90	228.31	4.45	536.57	4.02	1056.88	3.98
		MIN	115.26	3.93	210.41	3.77	503.70	3.38	1013.28	3.38
		MAX	160.03	7.20	260.43	5.38	556.25	4.87	1102.58	4.87
400	30	MEAN	165.42	4.29	300.16	4.13	703.42	3.80	1390.25	3.66
		MIN	140.32	3.49	264.00	3.49	624.03	2.98	1261.18	2.98
		MAX	191.15	5.16	345.87	4.78	794.63	4.64	1533.53	4.64
MEAN			62.54	4.09	111.63	3.74	261.28	3.45	517.37	3.39
MIN			6.65	1.22	12.47	1.22	30.28	1.22	58.93	1.22
MAX			191.15	7.74	345.87	6.30	794.63	5.56	1533.53	5.56

Tableau 3.4: Computational results for TST

$\|M\|$	$\|N\|$		MAXITER = 10 CPU [sec]	MAXITER = 10 GAP [%]	MAXITER = 20 CPU [sec]	MAXITER = 20 GAP [%]	MAXITER = 50 CPU [sec]	MAXITER = 50 GAP [%]	MAXITER = 100 CPU [sec]	MAXITER = 100 GAP [%]
150	10	MEAN	6.42	4.26	11.97	3.88	28.94	3.78	57.61	3.75
		MIN	5.83	2.96	10.20	2.74	25.43	2.74	50.56	2.74
		MAX	7.36	6.75	14.61	6.74	33.48	6.57	63.34	6.57
200	10	MEAN	9.99	4.27	18.35	4.09	45.30	3.93	90.20	3.83
		MIN	8.82	3.04	16.44	3.01	41.00	2.52	84.36	2.52
		MAX	12.45	5.96	22.24	5.00	48.69	5.00	97.29	4.93
250	10	MEAN	17.02	3.50	29.23	3.46	65.78	3.33	128.27	3.27
		MIN	14.58	3.05	24.60	2.80	56.21	2.42	109.10	2.42
		MAX	22.38	4.85	40.05	4.85	81.29	4.85	145.94	4.85
300	10	MEAN	22.02	3.36	39.48	3.19	91.87	2.99	188.27	2.98
		MIN	17.60	2.28	30.26	1.85	74.58	1.85	146.35	1.85
		MAX	28.05	5.95	49.81	5.95	101.45	5.47	219.23	5.46
150	15	MEAN	10.54	4.41	18.75	4.38	43.58	4.16	85.83	4.13
		MIN	9.68	3.32	17.77	3.32	41.03	3.32	81.47	3.32
		MAX	12.37	6.09	20.43	6.08	46.93	5.72	90.37	5.72
200	15	MEAN	17.21	4.57	29.90	4.39	70.56	4.13	139.48	4.11
		MIN	14.78	4.06	27.05	4.01	64.64	3.59	127.80	3.59
		MAX	19.86	5.58	32.12	5.58	76.34	5.58	153.11	5.58
250	15	MEAN	27.56	4.73	47.12	4.25	108.03	4.10	210.35	3.99
		MIN	22.55	2.98	40.07	2.98	100.83	2.98	192.05	2.98
		MAX	32.04	5.81	54.54	5.22	123.14	5.22	238.98	5.06
300	15	MEAN	41.05	4.05	70.73	3.67	164.11	3.55	319.79	3.43
		MIN	34.94	3.11	62.68	2.28	137.31	2.16	278.66	2.16
		MAX	46.55	5.51	81.16	4.72	186.25	4.37	352.77	4.37
250	20	MEAN	35.66	4.54	63.58	4.32	145.70	4.10	289.03	4.02
		MIN	31.44	3.45	58.22	3.45	138.68	3.45	273.90	3.09
		MAX	45.00	5.59	75.44	5.59	162.00	4.89	313.88	4.89
300	20	MEAN	57.78	4.65	97.37	4.23	225.71	3.98	442.15	3.94
		MIN	49.51	3.41	88.35	3.41	205.34	3.41	395.33	3.34
		MAX	67.02	6.53	111.97	5.32	254.53	5.32	484.80	5.32
350	20	MEAN	72.14	4.78	130.98	4.54	308.95	3.99	607.04	3.80
		MIN	63.39	4.02	112.01	4.02	266.47	2.87	552.42	2.87
		MAX	85.55	5.31	146.43	5.12	362.86	5.12	672.61	4.74
400	20	MEAN	102.26	5.01	178.04	4.63	403.12	4.50	786.74	4.22
		MIN	93.13	4.02	156.26	3.56	350.96	3.34	696.49	3.28
		MAX	116.81	6.86	195.63	5.87	441.01	5.49	895.06	5.10
250	25	MEAN	45.79	4.83	83.89	4.36	196.87	4.28	387.13	4.17
		MIN	40.21	3.13	75.85	2.87	178.20	2.87	350.95	2.79
		MAX	54.19	7.13	93.63	5.58	211.37	5.58	406.18	5.58
300	25	MEAN	69.37	5.32	122.30	4.41	277.89	4.00	545.79	3.97
		MIN	59.63	3.75	107.44	3.29	251.28	3.29	492.56	3.29
		MAX	87.62	7.83	151.30	5.38	308.00	4.86	592.07	4.86
350	25	MEAN	93.84	4.64	168.09	4.35	387.96	4.14	758.74	4.05
		MIN	81.84	3.88	149.99	3.39	346.79	3.39	697.27	3.39
		MAX	105.52	7.09	191.24	5.45	450.03	5.09	870.96	4.58
400	25	MEAN	134.66	4.56	240.35	4.32	555.79	4.14	1096.91	3.88
		MIN	111.38	3.55	207.50	3.45	498.18	3.45	1006.28	3.42
		MAX	168.41	6.26	304.05	5.21	674.39	5.19	1316.72	5.01
250	30	MEAN	55.37	4.48	98.62	4.21	229.28	3.95	453.12	3.88
		MIN	49.23	2.99	86.15	2.99	205.64	2.65	413.68	2.59
		MAX	58.15	6.18	107.28	5.13	251.60	4.86	487.54	4.54
300	30	MEAN	75.99	4.66	137.30	4.28	319.69	4.03	630.57	3.98
		MIN	68.91	3.78	125.99	3.78	285.74	3.48	587.05	3.48
		MAX	90.11	5.34	165.17	5.20	369.26	4.97	719.37	4.97
350	30	MEAN	115.31	5.30	203.22	4.70	473.98	4.55	936.70	4.43
		MIN	107.16	3.57	182.00	3.57	431.99	3.57	875.59	3.57
		MAX	132.67	9.79	228.83	6.34	541.90	5.80	1011.57	5.80
400	30	MEAN	156.60	4.81	279.47	4.58	647.19	4.09	1280.74	4.03
		MIN	136.90	3.91	245.91	3.85	591.97	3.45	1161.36	3.42
		MAX	187.30	5.99	321.71	5.42	706.56	4.95	1417.72	4.84
MEAN			58.33	4.55	103.44	4.22	239.51	3.99	471.72	3.89
MIN			5.83	2.28	10.20	1.85	25.43	1.85	50.56	1.85
MAX			187.30	9.79	321.71	6.74	706.56	6.57	1417.72	6.57

Tableau 3.5: Computational results for TSF

$\|M\|$	$\|N\|$		MAXITER = 10		MAXITER = 20		MAXITER = 50		MAXITER = 100	
			CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
150	10	MEAN	5.98	3.19	10.98	2.92	27.01	2.11	57.56	2.11
		MIN	5.54	1.78	10.08	1.54	25.20	1.20	53.19	1.20
		MAX	6.61	5.60	12.45	5.60	29.37	3.23	61.62	3.23
200	10	MEAN	9.38	2.66	17.59	2.16	43.51	2.06	91.19	1.86
		MIN	8.54	1.12	15.15	1.12	37.05	1.12	80.45	1.12
		MAX	10.69	4.99	21.77	3.11	46.71	3.11	98.71	2.81
250	10	MEAN	13.32	1.79	25.49	1.79	62.00	1.77	123.77	1.61
		MIN	10.35	0.76	19.92	0.76	42.46	0.76	93.55	0.76
		MAX	17.93	2.53	33.77	2.53	88.70	2.53	163.20	2.35
300	10	MEAN	17.00	1.43	31.99	0.68	80.25	0.68	167.48	0.57
		MIN	12.14	0.00	25.56	0.00	68.21	0.00	138.64	0.00
		MAX	20.94	4.55	40.26	2.54	102.67	2.54	201.96	2.54
150	15	MEAN	9.68	2.99	17.87	2.80	44.82	2.66	93.66	2.36
		MIN	8.71	0.84	15.83	0.84	39.99	0.84	83.42	0.84
		MAX	11.01	4.95	20.12	4.49	48.64	4.49	105.25	4.49
200	15	MEAN	14.84	2.64	28.53	2.34	70.06	2.28	143.81	2.03
		MIN	13.52	1.41	24.92	1.41	62.74	1.41	136.16	1.41
		MAX	16.44	6.68	32.45	4.19	76.37	4.19	152.97	2.94
250	15	MEAN	23.17	3.18	41.76	2.39	103.01	2.28	210.30	2.07
		MIN	20.45	1.52	35.49	1.52	87.93	1.52	177.21	1.48
		MAX	26.16	6.65	47.10	4.32	116.55	4.32	247.74	2.77
300	15	MEAN	35.27	1.10	64.49	1.01	150.01	0.99	300.63	0.67
		MIN	29.12	0.00	50.55	0.00	118.33	0.00	232.20	0.00
		MAX	45.05	2.68	77.75	2.68	186.54	2.68	386.87	2.68
250	20	MEAN	32.91	3.46	61.59	3.09	151.72	2.97	299.93	2.30
		MIN	30.85	1.53	56.10	1.53	128.19	1.53	259.47	1.53
		MAX	35.69	6.55	67.46	6.55	172.53	6.55	341.24	3.40
300	20	MEAN	49.09	2.13	90.36	1.53	213.71	1.01	430.50	0.54
		MIN	41.80	0.00	74.56	0.00	192.18	0.00	382.28	0.00
		MAX	56.03	5.83	101.85	5.83	245.30	3.30	500.11	1.59
350	20	MEAN	70.07	1.45	129.54	1.22	317.15	1.07	638.91	0.69
		MIN	61.62	0.19	115.79	0.19	258.03	0.18	498.24	0.18
		MAX	79.08	3.39	149.19	3.24	372.10	3.24	716.11	2.22
400	20	MEAN	96.54	2.38	181.64	2.18	440.97	1.39	890.28	1.03
		MIN	81.09	0.23	150.83	0.23	374.51	0.23	762.44	0.23
		MAX	118.12	5.45	217.02	3.91	533.18	3.25	983.79	1.65
250	25	MEAN	42.07	3.21	76.83	2.53	180.01	2.40	346.79	2.20
		MIN	38.05	1.28	66.93	1.28	159.92	1.28	309.50	1.28
		MAX	48.10	4.77	89.79	4.58	198.71	4.27	385.86	4.27
300	25	MEAN	63.53	1.65	117.94	1.27	269.02	1.07	535.25	0.88
		MIN	58.96	0.00	106.33	0.00	251.21	0.00	462.43	0.00
		MAX	70.02	4.04	126.40	3.27	277.28	3.27	569.00	2.98
350	25	MEAN	92.96	1.72	173.23	1.38	410.20	0.94	826.74	0.77
		MIN	80.99	0.28	149.47	0.28	346.41	0.28	695.07	0.18
		MAX	102.00	3.93	204.56	3.45	462.87	2.33	946.56	2.33
400	25	MEAN	136.59	1.95	259.49	1.74	612.20	1.43	1229.31	1.26
		MIN	122.23	0.24	223.37	0.24	552.76	0.24	1088.71	0.24
		MAX	178.49	4.94	314.57	3.55	744.23	3.55	1549.21	3.55
250	30	MEAN	52.99	3.40	94.84	2.95	224.15	2.68	443.31	2.41
		MIN	45.34	1.33	81.18	1.33	193.68	1.33	396.72	1.33
		MAX	61.02	5.77	108.70	4.84	249.41	3.62	495.61	3.46
300	30	MEAN	73.57	2.25	133.48	1.85	324.98	1.66	669.76	1.49
		MIN	61.13	0.00	112.82	0.00	279.20	0.00	541.66	0.00
		MAX	81.97	6.18	150.63	5.21	359.41	5.21	735.53	3.51
350	30	MEAN	110.94	2.03	201.50	1.99	509.27	1.90	1043.43	1.50
		MIN	102.61	0.27	187.56	0.27	478.57	0.27	962.50	0.27
		MAX	121.05	4.22	225.84	3.76	571.78	3.76	1127.23	3.31
400	30	MEAN	163.48	1.72	305.95	1.37	732.19	1.03	1422.49	0.82
		MIN	131.37	0.28	252.61	0.28	589.82	0.28	1208.80	0.24
		MAX	196.93	3.63	364.11	3.63	902.94	1.90	1598.53	1.90
MEAN			55.67	2.21	103.26	1.86	248.31	1.61	498.25	1.34
MIN			5.54	0.00	10.08	0.00	25.20	0.00	53.19	0.00
MAX			196.93	6.68	364.11	6.55	902.94	6.55	1598.53	4.49

We conclude that the proposed tabu-based heuristics allow us to find quasi-optimal solutions, and sometimes, when the two subproblems are solved exactly, the optimal solution.

Figures 3.4 to 3.6 show the variation of the mean CPU time as a function of the number of clients for TSR, TST and TSF (with MAXITER = 100 iterations) respectively, for different numbers of sites (10, 20, 30 and 40 sites). The figures show that the CPU time increases as a function of the number of clients before decreasing which is observable in the figures with 10 sites. This behavior of the CPU time can be explained as follows. We observe in Figures 3.7 to 3.9 (for TSR, TST and TSF respectively) that the percentage of the number of infeasible moves is in direct ratio to the number of clients. We define an infeasible move by a move of TS for which at least one of the subproblems is infeasible. Also, Figures 3.10 to 3.12 (for TSR, TST and TSF respectively) show that the percentage of the number of infeasible moves is in inverse ratio to the number of sites. Then, the CPU time increases as a function of the number of clients until the search process contains enough infeasible moves such that the CPU execution time starts to decrease.

Figures 3.13 to 3.15 picture the variation of the CPU execution time as a function of the number of sites for TSR, TST and TSF (with MAXITER = 100 iterations) respectively and, for different numbers of clients (100, 200, 300 and 400 clients). The figures show that the CPU execution time seems to increase linearly as a function of the number of sites.

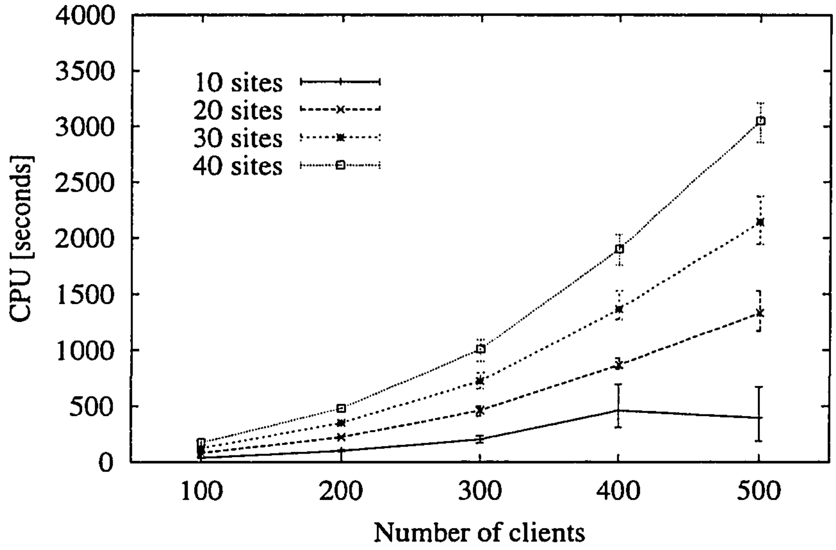

Figure 3.4: Mean CPU execution time of TSR as a function of the number of clients for different numbers of sites

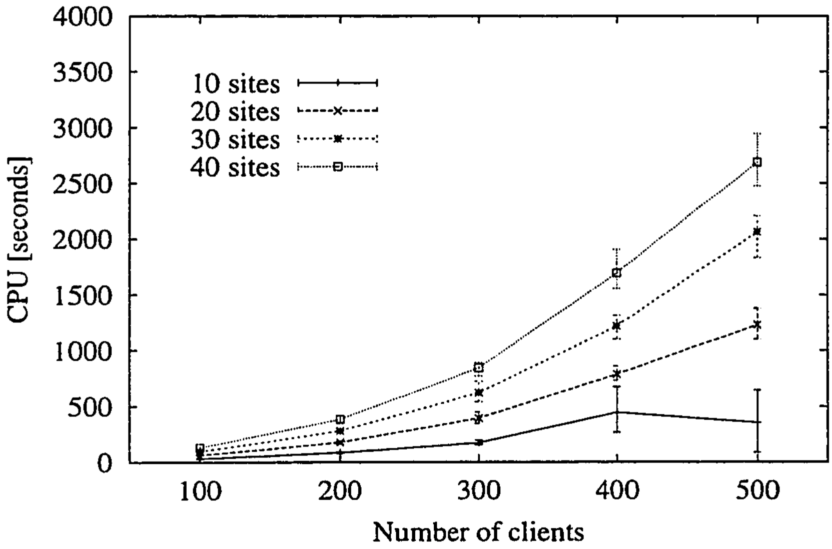

Figure 3.5: Mean CPU execution time of TST as a function of the number of clients for different numbers of sites

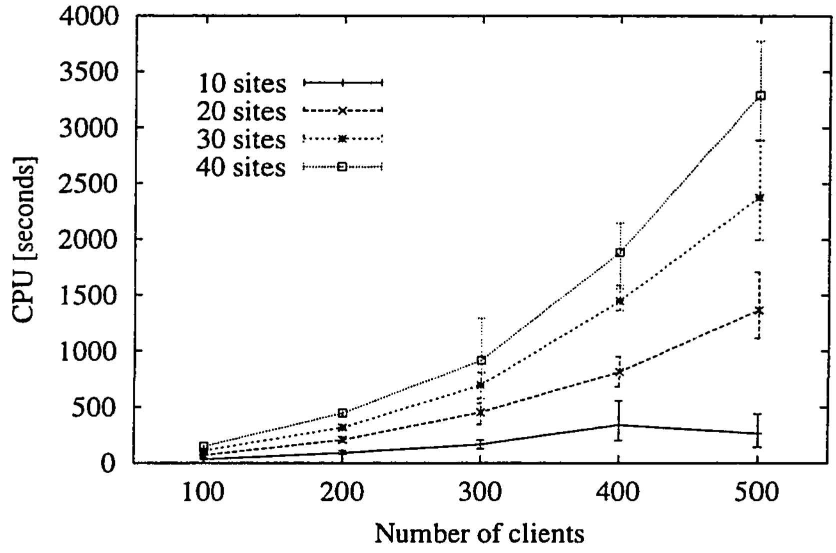

Figure 3.6: Mean CPU execution time of TSF as a function of the number of clients for different numbers of sites

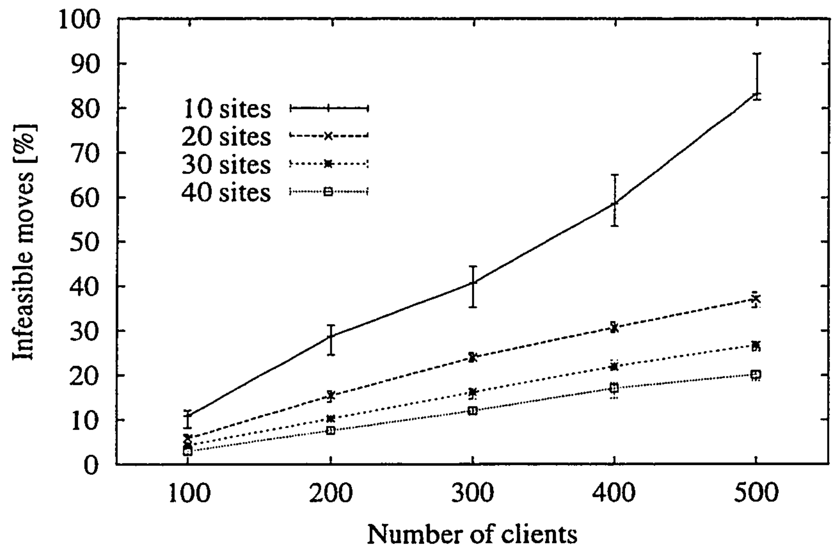

Figure 3.7: Mean percentage of the number of infeasible moves of TSR as a function of the number of clients for different numbers of sites

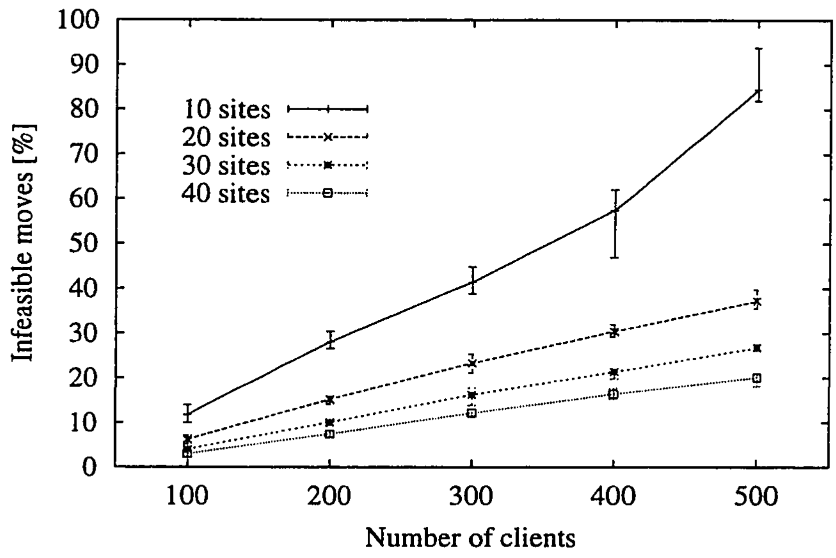

Figure 3.8: Mean percentage of the number of infeasible moves of TST as a function of the number of clients for different numbers of sites

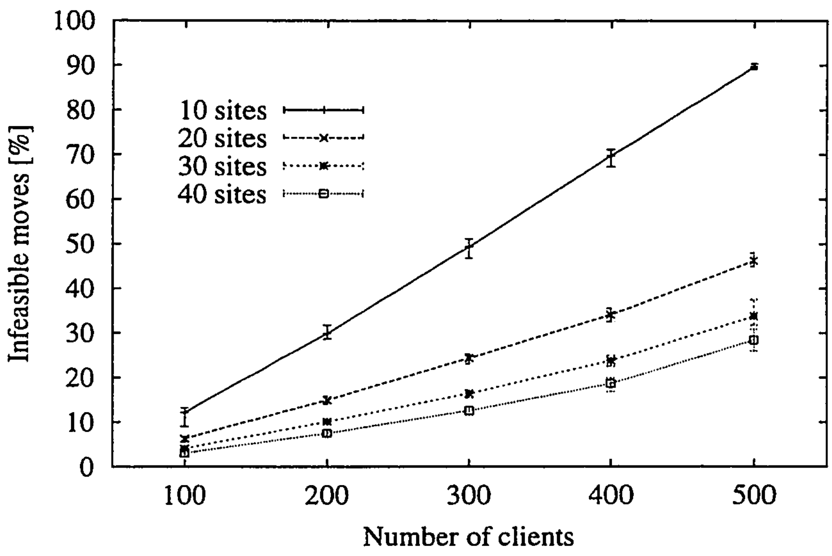

Figure 3.9: Mean percentage of the number of infeasible moves of TSF as a function of the number of clients for different numbers of sites

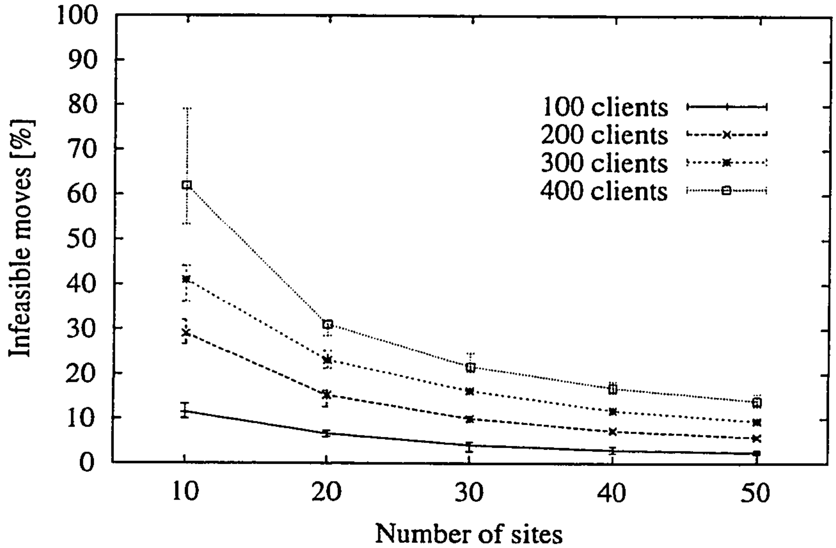

Figure 3.10: Mean percentage of the number of infeasible moves of TSR as a function of the number of sites for different numbers of clients

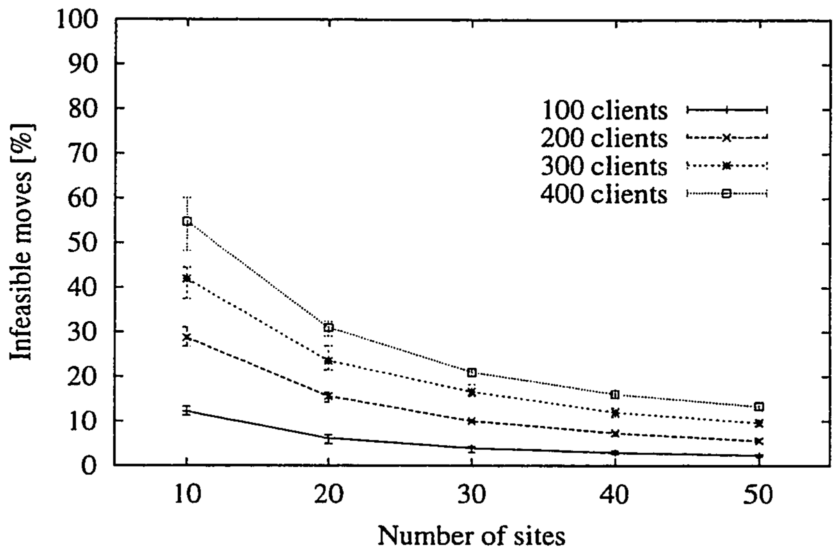

Figure 3.11: Mean percentage of the number of infeasible moves of TST as a function of the number of sites for different numbers of clients

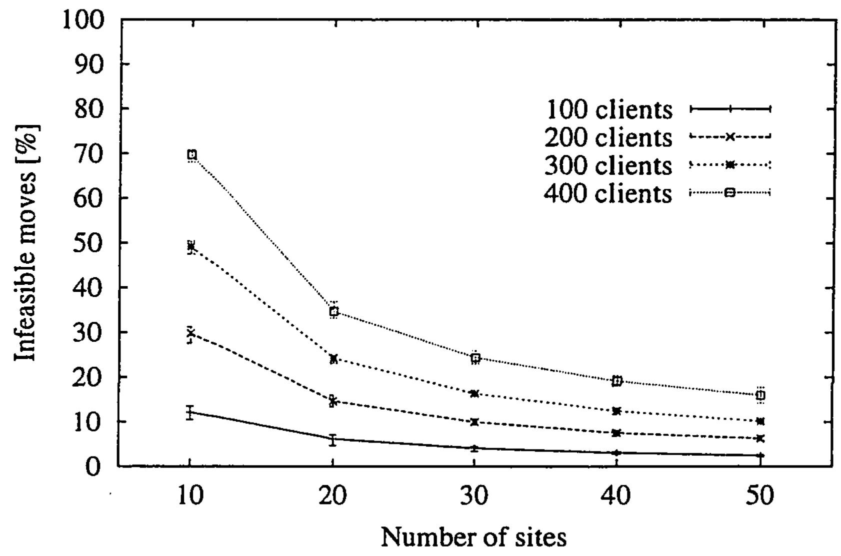

Figure 3.12: Mean percentage of the number of infeasible moves of TSF as a function of the number of sites for different numbers of clients

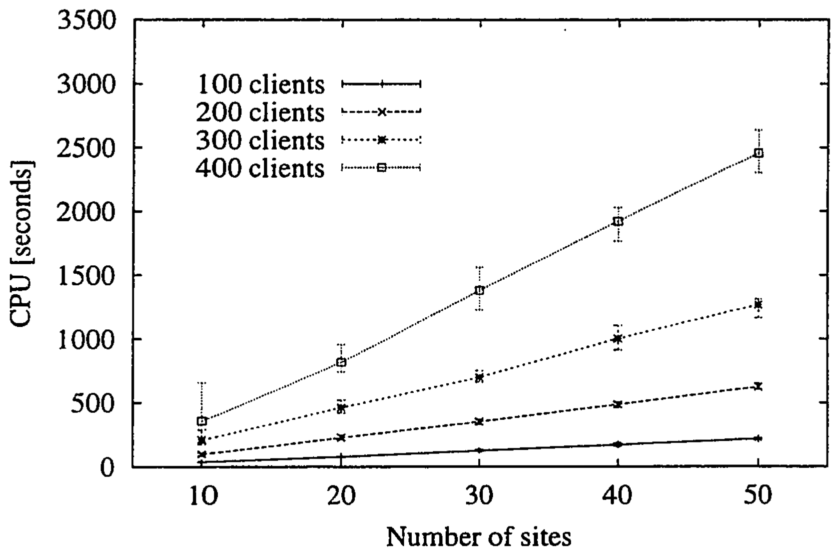

Figure 3.13: Mean CPU execution time of TSR as a function of the number of sites for different numbers of clients

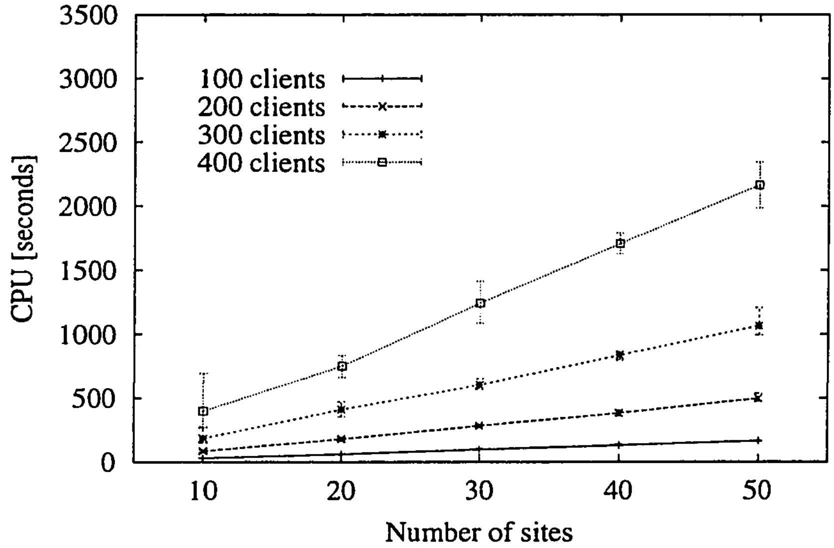

Figure 3.14: Mean CPU execution time of TST as a function of the number of sites for different numbers of clients

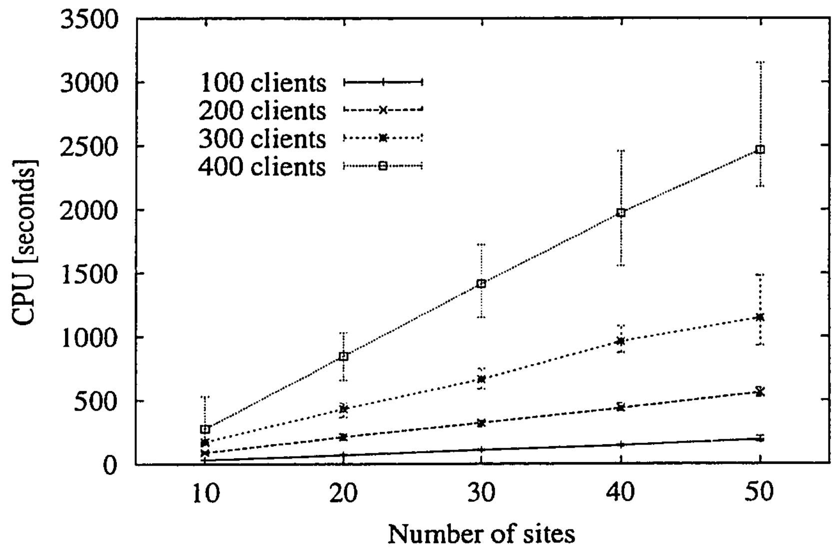

Figure 3.15: Mean CPU execution time of TSF as a function of the number of sites for different numbers of clients

3.6 Concluding Remarks

In this paper, we have presented a model for the problem of topological design problem of two-level multitechnology telecommunication networks with modular switches, such as the ones used in ATM networks. The model includes the optimal location of modular switches, their configuration, the design of an access network with a star topology and a backbone network with a fixed (ring, tree or full-mesh) topology. Our model allows several transmission technologies to be used in the access network.

We have proposed both a general tabu search-based approach and specialized versions, for different backbone topologies. In order to assess the performance of the heuristics, numerical results for problems with up to 400 clients and 30 potential switch sites were performed. The results show that the heuristics yield quasi-optimal solutions, and in some cases the optimal solution itself. If fact, the mean gap for all the problems solved in this paper with 100 iterations of the tabu search process is 2.87%.

There are still several avenues of research open at this point. For example, in this paper we propose a model in which the access network presents a star topology. It would be interesting to consider a more general model in which the topology of the access network is not specified. Also, even if DPR, DPT and DPF are $\mathcal{NP}$-hard, it would be interesting to develop exact algorithms, in order to find the optimal solution for problems of fair size. This approach, moreover, will help to evaluate more accurately the proposed heuristics. We are currently working on network expansion and update problems.

3.7 Appendix A

In this appendix, we prove that if inequality (3.32) is respected, then inequality (3.34) is also respected. First, we have the following equation.

$$\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M| \;=\; 4\left(\left(\frac{\max_{t\in T}\{m^t\}+1}{2}\right)^2 - |M|\right).$$

Using inequality (3.32), we obtain

$$\begin{aligned}
&\left(\max_{t\in T}\{m^t\}+1\right)^2 - 4|M| \\
&\geq\; 4\left(\left(\frac{\max_{t\in T}\{m^t\}+1}{2}\right)^2 - \left\lceil\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rceil\left\lfloor\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rfloor\right) \\
&=\; 4\left(\frac{1}{2}\left\lceil\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rceil - \frac{1}{2}\left\lfloor\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rfloor - \frac{1}{4}\right) \\
&=\; 4\left(\frac{1}{2}\left(\left\lceil\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rceil - \left\lfloor\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rfloor\right) - \frac{1}{4}\right). \qquad (3.45)
\end{aligned}$$

If $\max_{t\in T}\{m^t\}$ is odd, $\left\lceil\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rceil - \left\lfloor\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rfloor = 0$, and the right-hand side of (3.45) is equal to zero. However, if $\max_{t\in T}\{m^t\}$ is even, $\left\lceil\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rceil - \left\lfloor\frac{\max_{t\in T}\{m^t\}+1}{2}\right\rfloor = 1$, and the right-hand side of (3.45) is equal to one. Thus, if inequality (3.32) is respected, inequality (3.34) is also respected.

3.8 Acknowledgments

The authors would like to thank Dr. Gilbert Laporte of the Centre for Research on Transportation for permission to use his implementation of the algorithm GENIUS. This work has been completed thanks to FCAR grants and NSERC strategic grant STR 0166996.

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PARTIE II

Mise à jour des réseaux de télécommunication

CHAPITRE 4

Topological Expansion of Multiple Ring Metropolitan Area Networks

by

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Abstract

In this paper we deal with the problem of how to expand a MAN (Metropolitan Area Network) in a cost-effective way. We first propose a mathematical programming model that includes the location of new switch sites, the update of the configuration of modular switches with respect to ports and bases, the update of the access network with a star topology and the expansion of the backbone network with a fixed topology. Moreover, we consider that several access technologies and rates (such as OC-3 and OC-12) may be used. A multiple ring topology is chosen for the backbone network since it is sparse, therefore not too expensive, while providing protection against single link or switch failure. Also, we propose an initial heuristic that provides a starting solution, and a tabu-based heuristic to improve the solution. Finally, we present an illustrative example of a MAN design with its

successive expansions, followed by a systematic set of experiments designed to assess the performance of the proposed algorithms. The solutions obtained are, on average, at 0.59% from the best lower bound.

Key words: Topological expansion, multiple ring topology, metropolitan area network, modular switch, ATM, tabu search, local access network, backbone network.

Status: This article has been submitted to the journal *Networks*, 1998.

4.1 Introduction

The deregulation of the telecommunication industry has produced price reduction of many telecommunication services, such as internet and long distance telephone calls. Also, the introduction of new technologies, in particular, ATM (Asynchronous Transfer Mode) networks (see Minzner, 1989), has resulted in a diversity of broadband telecommunication services such as videotelephony, videoconference, high-speed data transfer, and high-resolution image transfer. Because of these developments, the number of potential clients grows continuously obliging telecommunication companies to expand, and update their networks regularly in order to stay abreast of their competition.

Network expansion planning in the industry starts, typically, by estimating the number of potential clients and their geographical location, often using sample survey or marketing techniques, for a period of up to five years. Next, a target network is defined, using an interactive decision support system (see Sharma, 1990) in order to satisfy the estimated demand of both current and potential clients by the end of the projected time period. The current network is then updated adaptatively in order to obtain the target network by the end of the chosen time period.

The literature of Engineering and Operations Research contains many articles relating to network expansion problems. One may consult the book by Freidenfelds (1981) and the review article by Luss (1982). Certain network expansion problem studies focus on a portion of the overall network expansion, including local access capacity expansion problems (see Shulman and Vachani, 1990; Balakrishnan, Magnanti, Shulman and Wong, 1991; Bienstock, 1993; Balakrishnan, Magnanti and Wong, 1995 and Flippo, Kolen, Koster and van de Leensel, 1996), backbone capacity expansion problems (see Ahuja, Batra, Gupta and Punnen, 1996), and backbone topological expansion problems (see Leblanc and Narasimhan, 1994). Other studies concern the overall network expansion, including joint topological and capacity expansion problems (see Chang and Gavish, 1993, 1995; Parrish, Cox, Kuehner and Qiu, 1992 and Cox, Kuehner, Parrish and Qiu, 1993).

This paper presents a model and resolution approach for the overall expansion problem of a MAN (Metropolitan Area Network). The proposed model deals with the location of new switch sites, the update of the configuration of modular switches, the update of the access network, and the expansion of the backbone network. We consider the use of several access rates and a backbone network with a multiple ring topology. This topology has the advantage that it is sparse, therefore not too expensive, yet provides protection against single link or switch failure. Moreover, for reasons of survivability, many transmission technologies such as SONET (Synchronous Optical NETwork) make use of this topology (see Wu, 1992).

The problem we deal with here is more general than those currently proposed in the literature since we introduce modular switches with different port and base types in the network expansion process. These switches are similar to those that will be used in future ATM networks. Moreover, we consider simultaneously the expansion of the backbone network and the update of the local access network. Finally, we analyze in detail the multiple ring topology for the backbone network.

The paper is organized as follows. In Section 4.2 we present a model for the topological expansion problem of a MAN with modular switches. The model is specialized for backbone networks having a multiple ring topology; valid inequalities and feasibility conditions are proposed. In Section 4.3 we present a tabu search algorithm. This algorithm needs a starting solution found by an initial heuristic presented in Section 4.4. Section 4.5 gives an illustrative example of the design of a MAN and its successive expansions, followed by a systematic set of experiments designed to assess the performance of the proposed algorithms. Conclusions and directions for further research are discussed in Section 4.6.

4.2 Problem Description

In this section we present the topological expansion problem of a MAN with modular switches. A modular switch is composed of a base and ports. A base consists of a switching network with slots to insert ports, which are used to connect links to the base. Different types of ports and links may be used, corresponding to different rates, such as OC-3, OC-12 (for the access network) and OC-192 (for the backbone network). Moreover, different base types are available, each with a different number of slots.

We make the following assumptions about the organization of the network: (A1) each client is connected to a switch through a link and a port of type corresponding to the access rate requested by the client; (A2) the switches are interconnected through links and ports of rate OC-192 with a specified topology; (A3) the number of ports installed in a switch cannot exceed the number of slots of the base; (A4) at most one switch may be installed at a given site.

We make the following assumptions about the expansion of the network: (A5) each link in the current access network may be kept in place or removed from the network; (A6) each link in the current backbone network is kept in place; (A7) each base and port in the current network may be kept in place, moved to another site or be removed from the network; (A8) each switch site in the current network is also a switch site in the expanded network; (A9) new bases may be installed at new potential switch sites and switch sites in the current network; (A10) new ports may be installed in the slots of both new bases and bases used in the current network; (A11) the topology of both the access and backbone network is preserved in the expanded network.

Finally, we suppose that the following information is known: (I1) the location of new clients and the access rates (such as OC-3 and OC-12) requested by each; (I2) the current clients leaving the network (i.e., cancelling their subscription); (I3) the new potential switch sites; (I4) the different types of bases, their capacities in term of the number of slots, and costs; (I5) the cost of installing a given type of base at a given site; (I6) the cost of removing a given type of base from a given site; (I7) the cost of installing the ground structure used for installing a switch at a given site; (I8) the costs of the different types of ports (for the different rates); (I9) the costs of the different types of links (for the different rates), including the installation cost, in \$/km; (I10) the costs of removing the different types of links in \$/km.

The problem is to find the minimum cost expanded network subject to all of the above assumptions (A1 to A11) and facts (I1 to I10).

4.2.1 Mathematical Formulation

The following notation is used throughout the paper. Let $M = \{1, \ldots, |M|\}$ be the set of all clients such that $M = M_N \bigcup M_C$ (and $M_N \bigcap M_C = \emptyset$), where M_N is the set of new clients, and M_C is the set of the current clients remaining in the network (i.e., renewing their subscription), and let $L = \{1, \ldots, |L|\}$ be the set of all access rates such that $L = L_N \bigcup L_C$ (and $L_N \bigcap L_C = \emptyset$), where L_N is the set of new access rates, and L_C is the set of access rates used in the current network. Let $N = \{1, \ldots, |N|\}$ be the set of all potential switch sites such that $N = N_N \bigcup N_C$ (and $N_N \bigcap N_C = \emptyset$), where N_N is the set of new candidate switch sites, and N_C is the set of switch sites in the current network, and let $T = \{1, \ldots, |T|\}$ be the set of all base types (where m^t denotes the number of slots of a base of type $t \in T$) such that $T = T_N \bigcup T_C$ (and $T_N \bigcap T_C = \emptyset$), where T_N is the set of new base types, and T_C is the set of base types used in the current network. Finally, let M_l be the set of clients that request an access rate $l \in L$ such that $\bigcup_{l \in L} M_l = M$ and $M_k \bigcap M_l = \emptyset$ for all $k \neq l$ and $k, l \in L$.

To define the decision variables, let $x_{ij} \in \mathsf{B}$ ($\mathsf{B} = \{0, 1\}$) be a variable such that $x_{ij} = 1$ if and only if client $i \in M_l$ is connected to site $j \in N$ through a link and a port of access rate $l \in L$; $y_{jk} \in \mathsf{B}$ a variable such that $y_{jk} = 1$ if and only if site $j \in N$ is connected to site $k \in N$ (for $j < k$) through a link and two ports of rate OC-192, and $z_j^t \in \mathsf{B}$ a variable such that $z_j^t = 1$ if and only if the base installed at site $j \in N$ is of type $t \in T$. Also, let $u_j \in \mathbb{B}$ be a variable such that $u_j = 1$ if and only if a switch is installed at site $j \in N$; $v_j^l \in \mathbb{Z}_+$ a variable representing the number of ports of access rate $l \in L$ in the slots of the base installed at site $j \in N$, and finally, $w_j \in \mathbb{Z}_+$ a variable representing the number of ports of rate OC-192 in the slots of the base installed at site $j \in N$ that will be used in the backbone network.

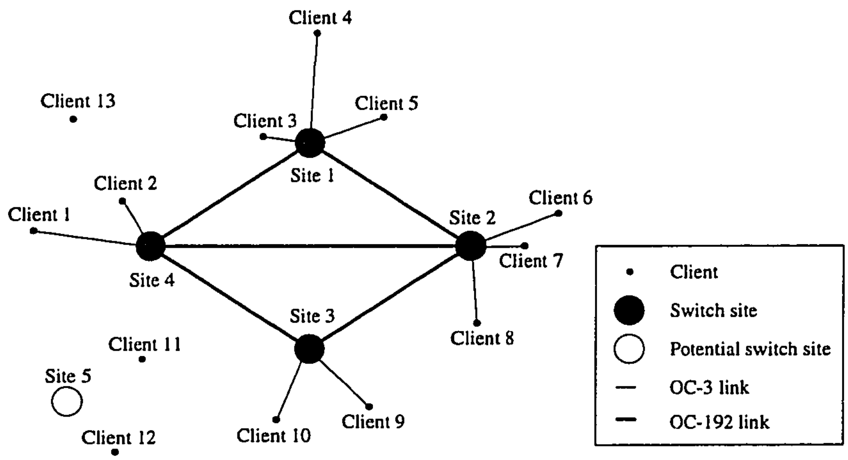

Figure 4.1: A network example

We identify the topology of the current network by the decision variables overlined. Figure 4.1 provides an example where the current clients (clients 1 to 10) are connected to the switches through OC-3 links and ports, and the new clients (clients 11 to 13) request OC-12 access rates. The switch sites (sites 1 to 4) are interconnected through OC-192 links and ports forming a multiple ring topology, and we consider a new candidate switch site (site 5) for the expansion. Thus, $L_C = \{1\}$, $L_N = \{2\}$, $M_C = M_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $M_N = M_2 = \{11, 12, 13\}$, $N_C = \{1, 2, 3, 4\}$ and $N_N = \{5\}$. Note that, a unique base type with 16 slots is used, thus $T_C = \{1\}$ and $T_N = \emptyset$ (with $m^1 = 16$). The current network in Figure 4.1 is identified by the following overlined variables: $\overline{x}_{1,4} = \overline{x}_{2,4} = \overline{x}_{3,1} = \overline{x}_{4,1} = \overline{x}_{5,1} =$

$\overline{x}_{6,2} = \overline{x}_{7,2} = \overline{x}_{8,2} = \overline{x}_{9,3} = \overline{x}_{10,3} = 1$, $\overline{y}_{1,2} = \overline{y}_{1,4} = \overline{y}_{2,3} = \overline{y}_{2,4} = \overline{y}_{3,4} = 1$, $\overline{u}_1 = \overline{u}_2 = \overline{u}_3 = \overline{u}_4 = 1$, $\overline{z}_1^1 = \overline{z}_2^1 = \overline{z}_3^1 = \overline{z}_4^1 = 1$, $\overline{v}_1^1 = \overline{v}_2^1 = 3$, $\overline{v}_3^1 = \overline{v}_4^1 = 2$, $\overline{w}_1 = \overline{w}_3 = 2$, $\overline{w}_2 = \overline{w}_4 = 3$, and all other overlined variables are zero.

We define the following cost parameters. Let c_{ij} be the link cost (including the installation cost) of connecting the client $i \in M_l$ to site $j \in N$ (through a link of access rate $l \in L$) if $\overline{x}_{ij} = 0$ (this link is not in the current network), and $c_{ij} = 0$ if $\overline{x}_{ij} = 1$ (this link is in the current network), and let θ_{ij} be the cost of removing the link connecting the client $i \in M$ to site $j \in N$ if $\overline{x}_{ij} = 1$, and $\theta_{ij} = 0$ if $\overline{x}_{ij} = 0$. Let d_{jk} be the link cost (including the installation cost) of connecting site $j \in N$ to site $k \in N$ (through an OC-192 link) if $\overline{y}_{jk} = 0$, and $d_{jk} = 0$ if $\overline{y}_{jk} = 1$; b^t the cost of purchasing a base of type $t \in T$; α_j^t the cost of installing a base of type $t \in T$ at site $j \in N$ if $\overline{z}_j^t = 0$, and $\alpha_j^t = 0$ if $\overline{z}_j^t = 1$, and let β_j^t be the cost of removing a base of type $t \in T$ from site $j \in N$ if $\overline{z}_j^t = 1$, and $\beta_j^t = 0$ if $\overline{z}_j^t = 0$. Let γ_j be the cost of installing the ground structure, used for installing a switch, at site $j \in N$ if $\overline{u}_j = 0$, and $\gamma_j = 0$ if $\overline{u}_j = 1$; p^l the cost of a port of access rate $l \in L$, and finally, let q be the cost of a port of rate OC-192 used for the backbone network.

4.2.1.1 The Objective Function

The objective function, representing the total expansion cost, is composed of link costs, base costs, ground structure costs, and port costs. Link costs, noted C_L, given by the following equation, include the cost of the new access and backbone links and the cost of removing access links.

$$C_L(\mathbf{x}, \mathbf{y}) = \sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} + \sum_{j \in N} \sum_{\substack{k \in N \\ j < k}} d_{jk} y_{jk} + \sum_{i \in M} \sum_{j \in N} \theta_{ij}(1 - x_{ij}). \qquad (4.1)$$

Base costs, noted C_B, include the cost of the new bases and the cost of installing and removing them. Since the number of new bases of type $t \in T$ in the expanded network is given by $\left(\sum_{j \in N}(z_j^t - \overline{z}_j^t)\right)^+$, C_B is given by the equation

$$C_B(\mathbf{z}) = \sum_{t \in T} \left(\sum_{j \in N} (z_j^t - \overline{z}_j^t) \right)^+ b^t + \sum_{j \in N} \sum_{t \in T} \alpha_j^t z_j^t + \sum_{j \in N} \sum_{t \in T} \beta_j^t (1 - z_j^t). \tag{4.2}$$

Ground structure costs, noted C_S, are determined by

$$C_S(\mathbf{u}) = \sum_{j \in N} \gamma_j u_j. \tag{4.3}$$

Finally, port costs, noted C_P, include the cost of the new ports used both in the access and in the backbone network. Since the number of new ports of access rate $l \in L$ is given by $\left(\sum_{j \in N}(v_j^l - \overline{v}_j^l)\right)^+$, and the number of new OC-192 ports (for the backbone network) by $\left(\sum_{j \in N}(w_j - \overline{w}_j)\right)^+$, C_P is given by the equation

$$C_P(\mathbf{v}, \mathbf{w}) = \sum_{l \in L} \left(\sum_{j \in N} (v_j^l - \overline{v}_j^l) \right)^+ p^l + \left(\sum_{j \in N} (w_j - \overline{w}_j) \right)^+ q. \tag{4.4}$$

Note that $\sum_{j \in N} v_j^l = |M_l|$ for all $l \in L$, that is the number of port of access rate l must be equal the number of clients that request an access rate l, and $\sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} = w_j$ for all $j \in N$, that is the number of OC-192 links connected to switch site j must be equal the number of OC-192 ports in the switch. Consequently, C_P can be rewritten as

$$C_P(\mathbf{y}) = \sum_{l \in L} \left(|M_l| - \sum_{j \in N} \overline{v}_j^l \right)^+ p^l + \left(2 \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} y_{jk} - \sum_{j \in N} \overline{w}_j \right)^+ q. \tag{4.5}$$

4.2.1.2 The Model

The model for the general topological expansion problem of a MAN with modular switches, noted EP (Expansion Problem), can now be given. Note that the topology of the backbone network has not yet been specified.

EP:

$$\min_{\mathbf{u},\mathbf{x},\mathbf{y},\mathbf{z}} (C_L(\mathbf{x},\mathbf{y}) + C_B(\mathbf{z}) + C_S(\mathbf{u}) + C_P(\mathbf{y})) \tag{4.6}$$

s.t.

Assignment constraints

$$\sum_{j \in N} x_{ij} = 1 \quad (i \in M) \tag{4.7}$$

Base type uniqueness constraints

$$\sum_{t \in T} z_j^t = u_j \quad (j \in N) \tag{4.8}$$

Base capacity constraints

$$\sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq \sum_{t \in T} m^t z_j^t \quad (j \in N) \tag{4.9}$$

Backbone topology constraints

$$\boxed{\text{Backbone topology constraints}} \tag{4.10}$$

Backbone expansion constraints

$$\overline{u}_j \leq u_j \quad (j \in N) \tag{4.11}$$

$$\overline{y}_{jk} \leq y_{jk} \quad (j < k,\ jk \in N) \tag{4.12}$$

Intergrality and nonegativity constraints

$$\mathbf{u} \in \mathsf{B}^{|N|}, \mathbf{x} \in \mathbb{R}_{+}^{|M||N|}, \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)}, \mathbf{z} \in \mathsf{B}^{|T||N|}. \tag{4.13}$$

The objective function (4.6), representing the total expansion cost, is composed of link, base, ground structure and port costs. Assignment constraints (4.7) require that each client be connected to exactly one switch, and base type uniqueness constraints (4.8) demand that the base installed at site $j \in N$ have an unique type, and that this type be chosen if and only if a switch has been installed at the site. Base capacity constraints (4.9) require that the number of links connected to a base of type $t \in T$ be at most m^t, and backbone topology constraints (4.10) necessitate that the topology of the backbone network be the one specified by the user of the model. Backbone expansion constraints (4.11) and (4.12) require that a switch site in the current network also be a switch site in the expanded network (assumption A8) and that all backbone links in the current network be used in the expanded network (assumption A6). Constraints (4.13) are integrality and nonnegativity constraints.

In model EP, we suppose that $|T| \geq 1$ with $\max_{t \in T}\{m^t\} \geq 3$, $|N| \geq 1$, and $|M| \geq 3$, because if the number of clients $|M|$ is equal to one or two, it is not cost-effective to build a two-level network. This model can be used for the design problem if we set all overlined variables to zero and if $|M_C| = |N_C| = |L_C| = |T_C| = 0$, that is, if the current network is nonexistent.

Proposition 1. We need not impose integrality constraints on the x_{ij} variables because the matrix of constraints where these variables appear is totally unimodular.

Proof. The proposition follows since the constraint matrix for the x_{ij} variables is the node-edge incidence matrix of a complete bipartite graph (see Chamberland, Marcotte and Sansó, 1997a) and thus totally unimodular. □

The next two propositions present valid inequalities that will be used in Section 4.5 to strengthen the linear relaxation of EP.

Proposition 2. The following inequalities

$$x_{ij} \le u_j \quad (i \in M;\ j \in N) \tag{4.14}$$

$$y_{jk} \le u_j \quad (j < k,\ \ j,k \in N) \tag{4.15}$$

$$y_{jk} \le u_k \quad (j < k,\ \ j,k \in N) \tag{4.16}$$

are valid for EP.

Proof. We prove inequalities (4.14) only, since (4.15) and (4.16) can be similarly proven. Let $j \in N$ be a given site. If $u_j = 0$, no switch is installed at site j, therefore no client can be connected to this site, i.e., $x_{ij} = 0$ for all $i \in M$. However if $u_j = 1$, a switch is installed at site j and each client can be connected to this site, i.e., $x_{ij} \le 1$. Thus, inequalities (4.14) are valid for EP. □

Proposition 3. The following inequalities

$$\sum_{i \in M} x_{ij} + \sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \le m^t + \left(\max_{s \in T \setminus \{t\}} \{m^s\} - m^t \right) (1 - z_j^t) \quad (t \in T; j \in N) \tag{4.17}$$

are valid for EP.

Proof. Let $t \in T$ be a given base type located at site $j \in N$. If $z_j^t = 1$, a base of type t is installed at site j, and the total number of links connected to the base must be at most m^t, i.e.,

$$\sum_{i\in M} x_{ij} + \sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} \leq m^t. \tag{4.18}$$

However if $z_j^t = 0$, a base of type t is not installed at site j, and an upper bound on the number of links connected to site j is given by the maximum number of slots for any other given type of base, i.e.,

$$\sum_{i\in M} x_{ij} + \sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} \leq \max_{s\in T\setminus\{t\}} \{m^s\}. \tag{4.19}$$

Thus, inequalities (4.17) are valid for EP. □

In the next subsection we present the topology constraints, valid inequalities and feasibility conditions for the multiple ring backbone topology.

4.2.2 Multiple Ring Topology

A multiple ring network includes several rings obtained by successive expansions of the network as is shown in Figure 4.2. Note that, the old switch sites are the ones in the current network, whereas the new switch sites are the ones added in the expansion process. Constraints used to model this topology are similar to those proposed by Balas (1989, 1995) for the Prize-Collecting Traveling Salesman Problem (PCTSP) problem. The multiple ring constraints are

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} = 2u_j \quad (j \in N_N) \tag{4.20}$$

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} \geq 2u_j \quad (j \in N_C) \tag{4.21}$$

$$\frac{1}{2}\left(\sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j<k}} y_{jk}+\sum_{j\in H}\sum_{\substack{k\in N\setminus H\\ j>k}} y_{kj}\right)+(1-u_l)+(1-u_m)\geq 1$$

$$(H\subset N; l\in H; m\in N\setminus H; 3\leq |H|\leq |N|-3). \quad (4.22)$$

Constraints (4.20) require that the number of OC-192 links connected to a new switch site be exactly two (i.e., this switch belongs to a unique ring), and constraints (4.21) require that the number of OC-192 links connected to an old switch site be at least two (i.e., this switch belongs to one or more rings). Constraints (4.22) are connectivity constraints and require at least two OC-192 links between switch sites in H and switch sites in $N\setminus H$. Their number is in $O(|N|^2 2^{|N|})$ and it is not possible to enumerate them all when the problem is "large". Note that it is possible to use other connectivity constraints such as flow constraints, but with additional flow variables.

With this topology, $|N_C| = 0$ or $|N_C| \geq 3$, because the number of switches in any network with a multiple ring topology is at least three. If $|N_C| = 0$, which means that the current network is nonexistent, these constraints are exactly those proposed by Balas (1989) for modeling a single ring topology.

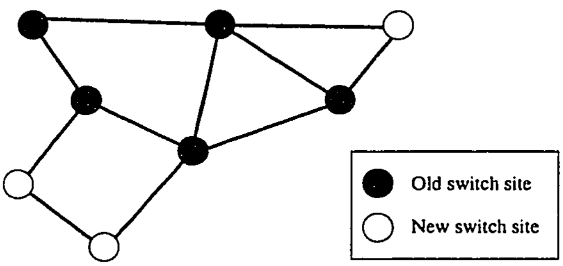

Figure 4.2: A multiple ring backbone network

For reasons of reliability, in a multiple ring topology (see Figure 4.3) we want to avoid articulation points so as to have a biconnected topology. In a biconnected network, there is at least two independant paths between each pair of nodes. Because the proposed multiple ring constraints are respected for the network picture in Figure 4.3, we propose additional constraints in order to eliminate articulation points in the backbone network. These constraints are

$$\sum_{j\in H}\sum_{\substack{k\in(N\setminus\{n\})\setminus H\\ j<k}} y_{jk} + \sum_{j\in H}\sum_{\substack{k\in(N\setminus\{n\})\setminus H\\ j>k}} y_{kj} \geq u_l$$
$$(H \subseteq N\setminus N_C; 1 \leq |H| \leq |N\setminus N_C|; l \in H; n \in N_C). \quad (4.23)$$

Their number is in $O(|N_C||N\setminus N_C|2^{|N\setminus N_C|})$, and it is not possible to enumerate them all when the problem is "large". For the network given in Figure 4.3, if we set $n = 4$ (the articulation point), $H = \{6, 7\}$ and $l = 6$, then $(N \setminus \{n\}) \setminus H = \{1, 2, 3, 5, 8\}$. Inequality (4.23) becomes $0 \geq 1$ and constraints (4.23) are not respected for this network. More precisely, constraints (4.23) require the connectivity of the backbone network when the switch site $n \in N_C$ is removed from the network. Since we suppose that the current backbone network has a multiple ring topology with no articulation point, then it suffices to consider $H \subseteq N \setminus N_C$, rather than $H \subseteq N \setminus \{n\}$.

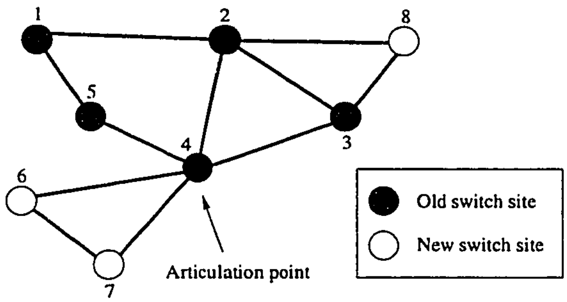

Figure 4.3: A multiple ring backbone network with an articulation point

Proposition 4. A biconnected network has a multiple ring topology.

Proof. In a biconnected network, we can cover all the switches and links with one or more rings (see Luss, Rosenwein and Wong, 1996). □

Since many telecommunication backbone networks are biconnected, it is of primary importance to study the multiple ring topology.

In what follows, EPM denotes the version of EP in which the backbone network has a multiple ring topology, i.e., the version in which the general constraints (4.10) are replaced by constraints (4.20), (4.21), (4.22) and (4.23).

Proposition 5. EPM is $\mathcal{NP}$-hard.

Proof. Transformation from the Traveling Salesman Problem (TSP) (for further details concerning the TSP see Lawler, Lenstra, Rinnooy Kan and Shmoys, 1985 and Nemhauser and Wolsey, 1988). □

Proposition 6. (a) If $|N_C| = 0$, then the following inequality

$$\sum_{j \in N_N} u_j \geq \left\lceil \frac{|M|}{\max_{t \in T}\{m^t\} - 2} \right\rceil \tag{4.24}$$

is valid for EPM.

(b) If $|N_C| \geq 3$ and $|M| > |N_C| \max_{t \in T_C}\{m^t\} - 2\sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk}$, then the following inequality

$$\sum_{j \in N_N} u_j \geq \left\lceil \frac{|M| + 2 - |N_C| \max_{t \in T_C}\{m^t\} + 2\sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk}}{\max_{t \in T}\{m^t\} - 2} \right\rceil \tag{4.25}$$

is valid for EPM.

Proof. If we sum on $j \in N$ the two sides of (4.9) and use (4.7), we the inequality

$$\begin{aligned} |M| &\leq \sum_{j \in N} \sum_{t \in T} m^t z_j^t - 2 \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} y_{jk} \\ &= \sum_{j \in N_C} \sum_{t \in T_C} m^t z_j^t + \sum_{j \in N_N} \sum_{t \in T} m^t z_j^t - 2 \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} y_{jk} \\ &\leq \max_{t \in T_C}\{m^t\}|N_C| + \max_{t \in T}\{m^t\} \sum_{j \in N_N} u_j - 2 \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} y_{jk} \end{aligned}$$

Using equation

$$\sum_{j\in N}\sum_{\substack{k\in N\\ j<k}} y_{jk} = \sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} y_{jk} + \sum_{j\in N_N}\sum_{\substack{k\in N_N\\ j<k}} y_{jk} + \sum_{j\in N_C}\sum_{\substack{k\in N_N\\ j<k}} y_{jk} + \sum_{j\in N_C}\sum_{\substack{k\in N_N\\ j>k}} y_{kj}$$

we obtain

$$\begin{aligned}|M| \;\le\; & \max_{t\in T_C}\{m^t\}|N_C| + \max_{t\in T}\{m^t\}\sum_{j\in N_N} u_j - 2\Bigg(\sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + \sum_{j\in N_N}\sum_{\substack{k\in N_N\\ j<k}} y_{jk} \\ & + \sum_{j\in N_C}\sum_{\substack{k\in N_N\\ j<k}} y_{jk} + \sum_{j\in N_C}\sum_{\substack{k\in N_N\\ j>k}} y_{kj}\Bigg). \qquad (4.26)\end{aligned}$$

(a) If $|N_C| = 0$, then (4.26) becomes the inequality

$$|M| \le \max_{t\in T}\{m^t\}\sum_{j\in N_N} u_j - 2\sum_{j\in N_N}\sum_{\substack{k\in N_N\\ j<k}} y_{jk}.$$

Using equation (4.20), we obtain

$$|M| \;\le\; \max_{t\in T}\{m^t\}\sum_{j\in N_N} u_j - 2\sum_{j\in N_N} u_j. \qquad (4.27)$$

Thus

$$\sum_{j\in N_N} u_j \ge \frac{|M|}{\max_{t\in T}\{m^t\} - 2}. \qquad (4.28)$$

Inequality (4.24) follows because the left-hand side of (4.28) is an integer in all feasible solutions of EPM.

(b) If $|N_C| \ge 3$, let r be the number of new rings in the expanded backbone network. In Figure 4.4 we show an expanded backbone network with $r = 3$. Then,

$r \leq \sum_{j \in N_N} u_j$ and $r \geq 1$ if $\sum_{j \in N_N} u_j \geq 1$. Moreover, we obtain the following equation

$$\sum_{j \in N_N} \sum_{\substack{k \in N_N \\ j<k}} y_{jk} + \sum_{j \in N_C} \sum_{\substack{k \in N_N \\ j<k}} y_{jk} + \sum_{j \in N_C} \sum_{\substack{k \in N_N \\ j>k}} y_{kj} = \sum_{j \in N_N} u_j + r, \tag{4.29}$$

where the left-hand side is the number of links connected to at least one new switch site. Indeed, if $\sum_{j \in N_N} u_j = 0$, then $r = 0$ and the number of links connected to at least one new switch site is zero. Alternatively, if $\sum_{j \in N_N} u_j \geq 1$, then $1 \leq r \leq \sum_{j \in N_N} u_j$ and the number of links between the new and the old switch sites is $2r$, because every new ring contained exactly two links connected to old switch sites. Also, the number of links between the new switch sites is $\sum_{j \in N_N} u_j - r$, because the number of links between the new switch sites is $\sum_{j \in N_N} u_j - 1$ if $r = 1$, and if r increases by one, this number diminishes by one. Thus, the number of links connected to at least one new switch site is $\sum_{j \in N_N} u_j + r$.

With (4.26) and (4.29), we obtain

$$\begin{aligned} |M| &\leq \max_{t \in T_C}\{m^t\}|N_C| + \max_{t \in T}\{m^t\} \sum_{j \in N_N} u_j - 2\left(\sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk} + \sum_{j \in N_N} u_j + r\right) \\ &= \max_{t \in T_C}\{m^t\}|N_C| - 2 \sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk} + (\max_{t \in T}\{m^t\} - 2) \sum_{j \in N_N} u_j - 2r. \end{aligned}$$

There are two cases. If $\sum_{j \in N_N} u_j = 0$, then $r = 0$ and

$$|M| \leq \max_{t \in T_C}\{m^t\}|N_C| - 2 \sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk}, \tag{4.30}$$

or, if $\sum_{j \in N_N} u_j \geq 1$, then $1 \leq r \leq \sum_{j \in N_N} u_j$ and

$$|M| \leq \max_{t \in T_C}\{m^t\}|N_C| - 2 \sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk} + (\max_{t \in T}\{m^t\} - 2) \sum_{j \in N_N} u_j - 2. \tag{4.31}$$

Then, if $|M| > \max_{t\in T_C}\{m^t\}|N_C| - 2\sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk}$, which means that the number of clients is greater than the maximum number of clients that can be connected to an expanded network using only the old switch sites, the following inequality

$$\sum_{j\in N_N} u_j \geq \frac{|M| + 2 - |N_C| \max_{t\in T_C}\{m^t\} + 2\sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk}}{\max_{t\in T}\{m^t\} - 2}, \tag{4.32}$$

gives a lower bound for the number of new switch sites in the expanded network. Inequality (4.25) follows, because the left-hand side of (4.32) is an integer in all feasible solutions of EPM. □

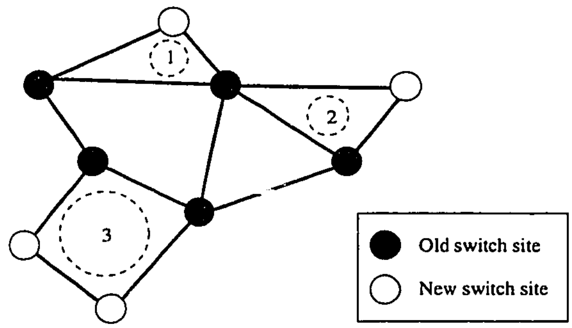

Figure 4.4: Number of new rings in the expanded backbone network

Proposition 7. (a) If $|N_C| = 0$, then EPM is feasible if and only if $|N| \geq 3$ and the following inequality

$$|M| \leq |N| \left(\max_{t\in T}\{m^t\} - 2\right), \tag{4.33}$$

is respected.

(b) If $|N_C| \geq 3$, then EPM is feasible if and only if (i) the following inequality

$$|M| \leq |N_C| \max_{t\in T_C}\{m^t\} - 2 \sum_{j\in N_C} \sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk}, \tag{4.34}$$

is respected or, (ii) the following inequality

$$|M| \leq |N_C| \max_{t \in T_C}\{m^t\} - 2 \sum_{j \in N_C} \sum_{\substack{k \in N_C \\ j<k}} \overline{y}_{jk} + |N_N|(\max_{t \in T}\{m^t\} - 2) - 2, \tag{4.35}$$

is respected and that for at least two switch sites $j \in N_C$, we have

$$\max_{t \in T}\{m^t\} - \left(\sum_{\substack{k \in N \\ j<k}} \overline{y}_{jk} + \sum_{\substack{k \in N \\ j>k}} \overline{y}_{kj} \right) \geq 1. \tag{4.36}$$

Proof. (a) ($\Rightarrow$) If EPM is feasible, inequality (4.27) is respected for every feasible solution of EPM and since

$$\begin{aligned} |M| &\leq \max_{t \in T}\{m^t\} \sum_{j \in N_N} u_j - 2 \sum_{j \in N_N} u_j \\ &\leq |N_N| \left(\max_{t \in T}\{m^t\} - 2 \right) \\ &= |N| \left(\max_{t \in T}\{m^t\} - 2 \right), \end{aligned}$$

inequality (4.33) is then respected for all feasible solutions and $|N| \geq 3$, because the number of switches in any network with a multiple ring topology is at least three. ($\Leftarrow$) Suppose that inequality (4.33) is respected and $|N| \geq 3$. Then, if we install $|N|$ bases of capacity equal to $\max_{t \in T}\{m^t\}$ and interconnect it with a ring backbone network, the number of slots left to the access networks (given by the right-hand side of (4.33)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Consequently, a feasible solution can be constructed, so EPM is feasible.

(b) ($\Rightarrow$) Suppose EPM is feasible. For every feasible solution with $\sum_{j \in N_N} u_j = 0$, inequality (4.30) is respected, and since inequalities (4.30) and (4.34) are identical, inequality (4.34) is also respected. For every feasible solution with $\sum_{j\in N_N} u_j \geq 1$, inequality (4.31) is respected, and since

$$\begin{aligned} |M| &\leq \max_{t\in T_C}\{m^t\}|N_C| - 2\sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + (\max_{t\in T}\{m^t\} - 2)\sum_{j\in N_N} u_j - 2 \\ &\leq |N_C|\max_{t\in T_C}\{m^t\} - 2\sum_{j\in N_C}\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + |N_N|(\max_{t\in T}\{m^t\} - 2) - 2, \end{aligned}$$

inequality (4.35) is respected for all feasible solutions with $\sum_{j\in N_N} u_j \geq 1$. Moreover, since every feasible solution is biconnected, inequality (4.36) is also respected for at least two switch sites $j \in N_C$. ($\Leftarrow$) Suppose that (4.34) is respected. Then, if we install a base of capacity equal to $\max_{t\in T}\{m^t\}$ at each site in N_C and interconnect it with the current backbone network, the number of slots left to the access networks (given by the right-hand side of (4.34)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Consequently, a feasible solution can be constructed, so EPM is feasible. Suppose that (4.35) is respected and that for at least two switch sites in the current network, inequality (4.36) is also respected. Then, if we install $|N|$ bases of capacity equal to $\max_{t\in T}\{m^t\}$, interconnect the switch sites in N_C with the current backbone network, and connect the switch sites in N_N to the current backbone network with one new ring (with $r = 1$ as is shown in Figure 4.5) such that, for each switch site in N_C connected to one another in N_N, inequality (4.36) is respected. As a result, the number of slots left to the access networks (given by the right-hand side of (4.35)) is large enough to connect the $|M|$ clients to the bases in order to form an access network. Consequently, a feasible solution can be constructed, so EPM is feasible. □

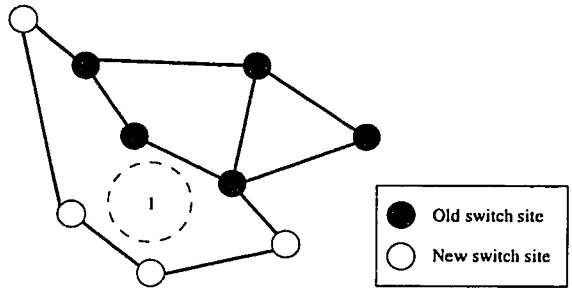

Figure 4.5: Multiple ring backbone network with one new ring

4.3 The Tabu Algorithm

In this section we propose a tabu search algorithm for EP, called TS (Tabu Search). An introduction to tabu search can be found in the articles by Glover (1989, 1990) and Glover, Taillard and de Werra (1993) and in the book by Glover and Laguna (1997).

Let $j \in N$ be a given site. If $j \in N_C$, the state e_j for this site is defined as follows: $e_j = k$ (for $k \in T$) if a base of type k is installed at site j. Otherwise, if $j \in N_N$, the state e_j for this site is defined as follows: $e_j = 0$ if there is no base installed at site j, and $e_j = k$ (for $k \in T$) if a base of type k is installed at site j. This means that if $e_j = 0$, then $u_j = 0$ and $z_j^t = 0$ for all $t \in T$, and if $e_j = k$ (for $k \in T$), then $u_j = 1$, $z_j^k = 1$ and $z_j^t = 0$ for all $t \in T \setminus \{k\}$.

Let us define vector $\mathbf{e} = \{e_j\}_{j \in N}$ so that e_j is the state of site $j \in N$. Thus $\mathbf{e}$ is the state of the network sites. Let $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$ be the vectors $\mathbf{u}$ and $\mathbf{z}$ respectively when the network sites state $\mathbf{e}$ is fixed. We outline the main features of the algorithm before giving its detailed description.

4.3.1 Problem Decomposition

Let EP(e) be the model EP when the decision vectors $\mathbf{u}$ and $\mathbf{z}$ are respectively equal to the vectors $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$. When $\mathbf{u}$ and $\mathbf{z}$ are fixed (respecting constraints (4.8) and (4.11)), EP can be decomposed into two subproblems.

Consider the following valid inequalities for EP

$$\sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq \sum_{t \in T} m^t z_j^t \quad (j \in N), \tag{4.37}$$

obtained by inequalities (4.9) and nonnegativity constraints on the x_{ij} variables. The first subproblem, noted $\overline{\text{EP}}(\mathbf{u}, \mathbf{z})$, is given below.

$$\overline{\text{EP}}(\mathbf{u}, \mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} : (4.10), (4.12), (4.37), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{4.38}$$

The purpose of this first subproblem is to connect the switches at minimum cost in order to form a backbone network whose topology is given by constraints (4.10).

The second subproblem, noted $\overline{\overline{\text{EP}}}(\mathbf{y}, \mathbf{z})$, is given below.

$$\overline{\overline{\text{EP}}}(\mathbf{y}, \mathbf{z}) : \min_{\mathbf{x}} \left\{ \sum_{i \in M} \sum_{j \in N} (c_{ij} - \theta_{ij}) x_{ij} : (4.7), (4.9), \mathbf{x} \in \mathbb{R}_+^{|M||N|} \right\}. \tag{4.39}$$

The purpose of this second subproblem is to connect the clients to the switches at minimum cost. It is, therefore, an assignment problem, and to solve it we use the shortest augmenting path algorithm, called LAPJV, of Jonker and Volgenant (1987), considered the best method for solving linear assignment problems for dense graphs (see Kennington and Wang, 1991).

4.3.2 Solution Space

Let E be the set of all possible states for the sites of the network, including those states that do not correspond to feasible solutions of the model. Note that $|E| = |T|^{|N_C|}(|T|+1)^{|N_N|}$. Let $\mathbf{y}(\mathbf{e})$ be the exact or heuristic solution of $\overline{\mathrm{EP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ and $\mathbf{x}(\mathbf{e})$ the exact solution of $\overline{\overline{\mathrm{EP}}}(\mathbf{y}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$. The solution space is thus $\Omega = \{(\mathbf{u}(\mathbf{e}), \mathbf{x}(\mathbf{e}), \mathbf{y}(\mathbf{e}), \mathbf{z}(\mathbf{e}))\}_{\mathbf{e}\in E}$.

4.3.3 Neighborhood Structure

Let $\omega \in \Omega$ be a solution. $\mathcal{N}(\omega)$ is called the neighborhood of ω and consists of the solutions obtained by modifying the state of a given site in the current solution. The number of possible modifications is $|N_C|(|T|-1) + |N_N||T|$.

4.3.4 Tabu Moves

Each move of tabu search consists of modifying the state of a given site in the current solution. At each iteration of the search, we determine the best move (among the $|N_C|(|T|-1) + |N_N||T|$ moves) while taking into account the tabus as well as the aspiration criterion described in the next subsection. The chosen site is declared tabu for a number of iterations that is randomly determined according to a uniform discrete distribution on the interval $[5, 10]$.

4.3.5 Aspiration Criterion

The aspiration criterion states that if the use of tabu site j allows us to discover a solution better than any other found so far, we may remove the tabu from site j.

4.3.6 The Algorithm

We now proceed to a detailed description of TS.

Algorithm TS

Step 1: (Initial solution)

Find an initial solution using the initial heuristic presented in the next section.

Repeat Steps 2 to 3 for 100 iterations

Step 2: (Exploring the neighborhood)

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \rightarrow \mathbf{e}'$, which modifies the state of a given site in the current solution, we solve $\mathrm{EP}(\mathbf{e}')$ by solving $\overline{\mathrm{EP}}(\mathbf{u}(\mathbf{e}'), \mathbf{z}(\mathbf{e}'))$ exactly or heuristically, and $\overline{\overline{\mathrm{EP}}}(\mathbf{y}(\mathbf{e}'), \mathbf{z}(\mathbf{e}'))$ exactly by using the algorithm LAPJV. The cost of a solution is given by the objective function (4.6) of model EP.

2.2 Determine the number of iterations according to a uniform distribution on the interval $[5, 10]$ for which the chosen site is tabu.

Step 3: (Best solution update)

If the current solution cost is less than the best solution found so far, update this best solution.

In the following subsection, we adapt TS in order to find a satisfactory solution for EPM.

4.3.7 Multiple Ring Topology

The heuristic TS adapted for EPM is called TSM. With this topology, the first subproblem becomes

$$\overline{\mathrm{EPM}}(\mathbf{u},\mathbf{z}) : \min_{\mathbf{y}} \left\{ \sum_{j\in N} \sum_{\substack{k\in N\\ j<k}} d_{jk}y_{jk} : (4.12), (4.20-4.23), (4.37), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{4.40}$$

The purpose of this subproblem is to connect the switches through a minimum cost multiple ring network while respecting the capacity of the bases.

Proposition 8. $\overline{\mathrm{EPM}}(\mathbf{u},\mathbf{z})$ is $\mathcal{NP}$-hard.

Proof. Transformation from the TSP. □

We define the following notation. Let Φ_j be the number of slots of the base installed at site $j \in N$, i.e., $\Phi_j = \sum_{t\in T} m^t z_j^t$. Also, let $N_C^{=0}$ be a subset of N_C such that $j \in N_C^{=0}$ if and only if the number of ports available at site j is equal to zero, i.e., $\Phi_j - \left(\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + \sum_{\substack{k\in N_C\\ j>k}} \overline{y}_{kj}\right) = 0$; $N_C^{=1}$ a subset of N_C such that $j \in N_C^{=1}$ if and only if the number of ports available at site j is equal to one, i.e., $\Phi_j - \left(\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + \sum_{\substack{k\in N_C\\ j>k}} \overline{y}_{kj}\right) = 1$, and let $N_C^{\geq 2}$ be a subset of N_C such that $j \in N_C^{\geq 2}$ if and only if the number of ports available at site j is at least two, i.e., $\Phi_j - \left(\sum_{\substack{k\in N_C\\ j<k}} \overline{y}_{jk} + \sum_{\substack{k\in N_C\\ j>k}} \overline{y}_{kj}\right) \geq 2$. Finally, set $N_C^{<0} = N_C \setminus (N_C^{=0} \cup N_C^{=1} \cup N_C^{\geq 2})$ and let U (such that $N_C \subseteq U \subseteq N$) be the set of sites where a base is installed, i.e., $U = \{j \in N : u_j = 1\}$.

Proposition 9. (a) If $|N_C| \geq 3$ and $N_C \subset U$, then $\overline{\mathrm{EPM}}(\mathbf{u},\mathbf{z})$ is feasible if and only if $|N_C^{<0}| = 0$, $|N_C \setminus N_C^{=0}| \geq 2$ and $\Phi_j \geq 2$ for all $j \in U \setminus N_C$.

(b) If $|N_C| \geq 3$ and $N_C = U$, then the subproblem $\overline{\mathrm{EPM}}(\mathbf{u},\mathbf{z})$ is feasible if and only if $|N_C^{<0}| = 0$. Moreover, if the subproblem is feasible, the vector $\bar{\mathbf{y}}$ (i.e., the current backbone network) is its optimal solution.

(c) If $|N_C| = 0$, then $\overline{\text{EPM}}(\mathbf{u}, \mathbf{z})$ is feasible if and only $|U| \geq 3$ and $\Phi_j \geq 2$ for all $j \in U$.

Proof. (a) ($\Rightarrow$) If $|N_C^{<0}| \geq 1$ or $|N_C \setminus N_C^{=0}| < 2$ or $\Phi_j < 2$ for at least one $j \in U \setminus N_C$, then the subproblem is infeasible. Indeed, if $|N_C^{<0}| \geq 1$ then, for at least one current switch site, the number of current backbone links connected to the site is greater than the capacity of the base (installed at the site), and since each link in the current backbone network is kept in place, the subproblem is infeasible. If $|N_C \setminus N_C^{=0}| < 2$, then at most one current switch site can be used to connect new links, and since we want a biconnected backbone network, the subproblem is infeasible. Finally, if $\Phi_j < 2$ for at least one $j \in U \setminus N_C$, since exactly two OC-192 links are connected to each sites in $U \setminus N_C$ in the expanded network, the subproblem is infeasible. ($\Leftarrow$) Suppose that $|N_C^{<0}| = 0$, $|N_C \setminus N_C^{=0}| \geq 2$ and $\Phi_j \geq 2$ for all $j \in U \setminus N_C$. We can connect the switch sites in U with one new ring (with $r = 1$ as is illustrated in Figure 4.5) such that each of the two switch sites in N_C connected to one another in $U \setminus N_C$ is also in $N_C \setminus N_C^{=0}$. Thus, a feasible solution can be constructed, so the subproblem is feasible.

(b) ($\Rightarrow$) If $|N_C^{<0}| \geq 1$ then, for at least one current switch site, the number of current backbone links connected to the site is greater than the capacity of the base, and since each link in the current backbone network is kept in place, the subproblem is infeasible. ($\Leftarrow$) Suppose that $|N_C^{<0}| = 0$. $\bar{\mathbf{y}}$ is then a feasible solution of zero cost.

(c) ($\Rightarrow$) If $|U| < 3$ or $\Phi_j < 2$ for at least one $j \in U$, the subproblem is infeasible, since every feasible solution have a ring topology with at least three switches and exactly two OC-192 links connected to each switch. ($\Leftarrow$) If $|U| \geq 3$ and $\Phi_j \geq 2$ for $j \in U$, we can connect the switches in U with OC-192 links to form a ring backbone network, so the subproblem is feasible. □

Because $\overline{\text{EPM}}(\mathbf{u}, \mathbf{z})$ is $\mathcal{NP}$-hard and contains a large number of constraints and binary variables, it is unlikely that large-size instances of this subproblem can be solved to optimality. Therefore, we propose an efficient heuristic for this subproblem, called MR (Multiple Ring).

To describe MR, let E_{N_C} and E_{N_N} be two sets containing sites in N_C and N_N respectively, and let a_{jk} be the cost of connecting site j to site k used to find the multiple ring backbone network.

Heuristic MR

Step 1: (Feasibility check)

If the subproblem is not feasible using Proposition 9, stop.

Step 2: If $|N_C| = 0$, then connect the switches in U with OC-192 links to form a ring network by using the GENIUS composite heuristic (proposed by Gendreau, Hertz and Laporte, 1992), where the costs are the d_{jk}. Let $\hat{\mathbf{y}}$ be the solution obtained with GENIUS, and go to Step 8.

Step 3: If $|U \setminus N_C| = 0$ go to Step 7. Otherwise, set $a_{jk} := d_{jk}$ for all j, $k \in U$.

Step 4: If $|U \setminus N_C| = 1$ set $E_{N_C} := N_{\overline{C}}^{=1}$ and go to Step 6.

Step 5: If $|N_{\overline{C}}^{=1}| \geq 1$ and $|U \setminus N_C| \geq 2$ do

5.1 Set $E_{N_C} := \emptyset$ and $E_{N_N} := \emptyset$.

5.2 For $i := 1$ to $\min\{|N_{\overline{C}}^{=1}|, |U \setminus N_C|\}$ do

5.2.1 Set (j^*, k^*) to the solution of the following problem

$$\min_{j,k} \left\{ d_{jk} : j \in N_{\overline{C}}^{=1} \setminus E_{N_C}, k \in U \setminus \{N_C \cup E_{N_N}\} \right\}.$$

5.2.2 Set $a_{j^*k} := +\infty$ for all $k \in U \setminus \{N_C \cup \{k^*\}\}$, $E_{N_C} := E_{N_C} \cup \{j^*\}$ and $E_{N_N} := E_{N_N} \cup \{k^*\}$.

Step 6: Set $a_{jk} := 0$ for all j, $k \in N_C$ and connect the switches in $U \setminus (N_{\overline{C}}^{=0} \cup (N_{\overline{C}}^{=1} \setminus E_{N_C}))$ with OC-192 links to form a ring using the GENIUS composite heuristic where the costs are the a_{jk}. Let $\overline{\mathbf{y}}$ be the solution obtained with GENIUS.

Step 7: Set $\widehat{y}_{jk} := \overline{y}_{jk}$ for all $j < k$ and j, $k \in N_C$.

Step 8: Return the vector $\widehat{\mathbf{y}}$.

Step 1 in MR consists of checking if the subproblem is feasible using Proposition 9. If the subproblem is infeasible the algorithm stops. Otherwise, Step 2 is performed. In Step 2, if the current network is nonexistent (i.e., if $|N_C| = 0$), the algorithm finds a single ring backbone network by using the GENIUS composite algorithm, and returns this solution (Step 8). In Step 3, if only the current switch sites are used (i.e., if $|U \setminus N_C| = 0$), MR returns the vector $\overline{\mathbf{y}}$ (Steps 7 and 8). Otherwise (i.e., if $|U \setminus N_C| \geq 1$), the algorithm sets a_{jk} to d_{jk} for all j, $k \in U$. In Step 4, if the number of new switch sites is exactly one (i.e., if $|U \setminus N_C| = 1$), MR sets $E_{N_C} := N_{\overline{C}}^{=1}$ and goes to Step 6. In Step 5, if $|N_{\overline{C}}^{=1}| \geq 1$ and the number of new switch sites is at least two (i.e., if $|U \setminus N_C| \geq 2$), MR modifies the a_{jk} in order to respect the capacity constraint at each site in $N_{\overline{C}}^{=1}$. Step 6 modifies the a_{jk} again by setting the cost of connecting two switch sites in N_C to zero, since these sites are already connected in the current backbone network. Next, MR connects the switches in $U \setminus (N_{\overline{C}}^{=0} \cup (N_{\overline{C}}^{=1} \setminus E_{N_C}))$ with a ring using GENIUS where the costs are the a_{jk}. Since the subproblem feasible (see Proposition 9), if $|U \setminus N_C| = 1$, then $E_{N_C} = N_{\overline{C}}^{=1}$ (see Step 4) and

$$|N_C \setminus (N_{\overline{C}}^{=0} \cup (N_{\overline{C}}^{=1} \setminus E_{N_C}))| = |(N_C \setminus N_{\overline{C}}^{=0})| \geq 2,$$

if $|U \setminus N_C| \geq 2$ and $|N_{\overline{C}}^{=1}| \leq |U \setminus N_C|$, then $E_{N_C} = N_{\overline{C}}^{=1}$ (see Step 5) and

$$|N_C \setminus (N_{\overline{C}}^{=0} \cup (N_{\overline{C}}^{=1} \setminus E_{N_C}))| = |(N_C \setminus N_{\overline{C}}^{=0})| \geq 2,$$

and if $|U \setminus N_C| \geq 2$ and $|N_{\overline{C}}^{=1}| > |U \setminus N_C|$, then $E_{N_C} \subset N_{\overline{C}}^{=1}$, $|E_{N_C}| \geq 2$ (see Step 5) and

$$|N_C \setminus (N_{\overline{C}}^{=0} \cup (N_{\overline{C}}^{=1} \setminus E_{N_C}))| \geq |E_{N_C}| \geq 2.$$

Thus, at least two current switch sites belong to the ring found by Step 6. Therefore, the solution is always biconnected. Step 7 uses the backbone links in the current network to connect the sites in N_C, and Step 8, returns the solution.

4.4 The Initial Heuristic

We now describe an initial heuristic, called IH (Initial Heuristic), able to find a "good" initial solution for TS.

To describe IH let a star be a subnetwork that includes a switch (the center of the star) and clients connected to this switch. Let $\Gamma(j)$ be the set of users of the star with center j. The size of the star with center j is $|\Gamma(j)|$ and its cost is given by $\sum_{i\in\Gamma(j)}(c_{ij}-\theta_{ij})$. Let E_M and E_N be two sets defined to contain clients and switches respectively.

IH generates at most m solutions corresponding to stars of size k for $1 \leq k \leq m$, where m is the maximum number of clients that can be connected to a single switch (i.e., $m = \min\{|M|, \max_{t\in T}\{m^t\}\}$). For each value of k considered, the new switch sites are found using the costs of the stars of size k (in a greedy fashion), whereas, the backbone network and the access network are found by solving the two subproblems presented in Section 3.1. The best solution found will be returned by the heuristic.

Heuristic IH

Step 1: (Feasibility check)

If the problem is not feasible, stop. Otherwise, set $k := 1$ and go to Step 2.

Step 2: (Generating a solution)

2.1 Set $E_M := \emptyset$, $E_N := \emptyset$ and $\Gamma(j) := \emptyset$ for all $j \in N$.

2.2 For $i := 1$ to $\min\left\{|N|, \left\lceil\frac{|M|}{k}\right\rceil\right\}$ do

2.2.1 Determine the star of minimum cost among the stars of size $\min\{k, |M\backslash E_M|\}$ containing a switch in $N\backslash E_N$ and clients in $M\backslash E_M$. Let j^* be the chosen switch site and $\Gamma(j^*)$ the set of clients of the star of center j^*.

2.2.2 Set $E_N := E_N \cup \{j^*\}$ and $E_M := E_M \cup \Gamma(j^*)$.

2.3 For each site $j \in N$ do

2.3.1 If $j \in E_N \cup N_C$ set $e_j := \arg\max_{t\in T}\{m^t\}$. Otherwise, set $e_j := 0$.

2.4 Solve EP(e) in order to find the backbone network $\mathbf{y}(\mathbf{e})$ by solving $\overline{\text{EP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ exactly or approximatively, and the access network by solving $\overline{\overline{\text{EP}}}(\mathbf{y}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ exactly using the algorithm LAPJV.

2.5 Compute the cost of the current solution given by the objective function (4.6).

Step 3: (Best solution update)

If the cost of the current solution is less than that of the best solution obtained so far, update this best solution.

Step 4: (Termination test)

If $k = m$, return the best solution found and stop. Otherwise, set $k := k + 1$ and go to Step 2.

Step 1 in IH consists of checking if EP is feasible. If the problem is infeasible the algorithm stops. Otherwise, Step 2 is performed. Steps 2.1 and 2.2, select a set of switch sites, E_N, using the costs of the stars in a greedy fashion. Step 2.3 installs a base of capacity equal to $\max_{t \in T}\{m^t\}$ at each site in $E_N \cup N_C$ and finds the backbone network by solving $\overline{\text{EP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ exactly or approximatively, and the access network by solving $\overline{\overline{\text{EP}}}(\mathbf{y}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ by using the algorithm LAPJV. Step 2.5 computes the cost of the solution, given by the objective function (4.6). Steps 3 and 4 update the best solution, if necessary, and if k is equal to m, the algorithm stops. However, if $k < m$, IH returns to Step 2 with $k := k + 1$.

In the next subsection, we show how to adapt IH to find an initial solution for EPM.

4.4.1 Multiple Ring Topology

The heuristic IH adapted for EPM is called IHM. Step 1 in IHM consists of checking if the EPM is feasible using Proposition 7. In Step 2.4, the backbone network is found by solving the first subproblem $\overline{\text{EP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$ using the heuristic MR.

We summarize the general structure of algorithm TSM in Figure 4.6, in which each box corresponds to a task or an algorithm execution and arcs represent dependency and data communication.

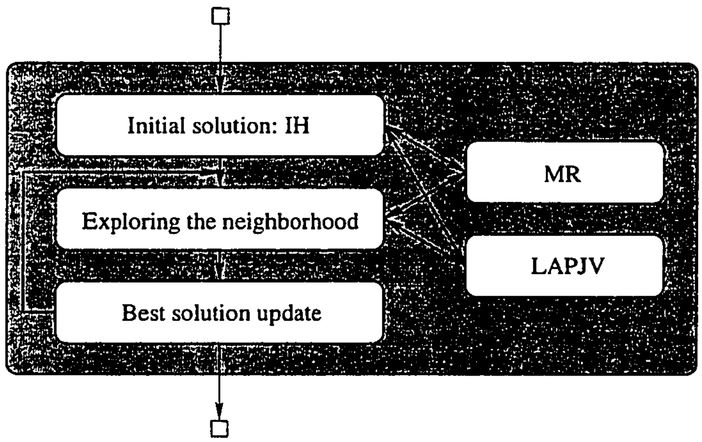

Figure 4.6: General Structure of algorithm TSM

4.5 Experimental Results

In this section, we analyze numerical results obtained by applying the previously described heuristics. We first give an illustrative example in Section 4.5.1. Then in Section 4.5.2, we describe a systematic study of the performance of the heuristics.

The algorithms used were implemented in C language and tested on a Sun Ultra 1 (model 140) workstation. We should mention that the monetary unit used in this paper, denoted \$, is an arbitrary unit. Costs for all the examples include: OC-3, OC-12 and OC-192 links which cost \$1000/km, \$1500/km and \$3000/km respectively, including the installation cost. Two port models are used to link clients to the network. The cost of the first model is \$1000 (rate OC-3) and the cost of the second model is \$3000 (rate OC-12). The cost of an OC-192 port (used for the backbone network) is \$10 000. With respect to the switches, we assume that there are three base models described in Table 4.1 and that installing and removing a base costs, respectively, \$5000 and \$1000 (which we assume to be equal for all sites and base models). Installing a ground structure cost \$5000 (which we assume to be equal for all sites). Finally, the cost of removing the different access link types is 500\$/km.

Tableau 4.1: Features of the base models

	Type A	Type B	Type C
Capacity	16	32	64
Cost	\$140 000	\$240 000	\$390 000

All the heuristic results were compared with two lower bounds obtained as follows. The first bound for EPM, noted LBA (Lower Bound A), was obtained by using the CPLEX Mixed Integer Optimizer (for more information about CPLEX see the CPLEX user's manual, 1993) to solve EPM with the objective function linearized (by using additional variables and constraints) and without constraints (4.22) and (4.23), relaxing the integrality constraints on $\mathbf{y}$ and $\mathbf{z}$ variables (which means that the integrality constraints on $\mathbf{u}$ variables were conserved) but with valid inequalities (4.14) to (4.17). The valid inequalities presented in Proposition 5 were also included.

The other bound, noted LBB (Lower Bound B), was similarly obtained except that integrality constraints on **z** variables were conserved while those on **u** variables were relaxed.

It can be shown for a given problem, that the value of LBA is at most the value of LBB but, unless $|T| = 1$, LBB contains more binary variables than LBA, with $|N|$ binary variables for LBA and $|N||T|$ for LBB. Therefore the procedure to find LBB was more time consuming than the one to find LBA.

4.5.1 An Illustrative Example

In our example, we use initial and tabu heuristics. The initial number of clients is 100, and the number of potential switch sites is 10. Figure 4.7 shows the geographical location of clients and potential switch sites for a square region of side length 100 km.

The solution obtained with IHM took 5.14 seconds with a value of \$3 737 300, whereas the solution obtained by TSM took 43.77 seconds, but its value was \$3 235 000. Thus, the tabu algorithm improves the initial solution by 13.44%, but with a significant increase in CPU time. This solution illustrated in Figure 4.8 has a ring topology. The value of the lower bound LBA is \$3 197 250, obtained by exploring 14 branch-and-bound nodes in 1.69 seconds. The value of the second bound, LBB, is \$3 235 000, obtained by exploring 73 branch-and-bound nodes in 7.81 seconds. For this example, the solution found by TSM is the optimal solution.

Figure 4.9 presents the geographical location of 100 new clients and 10 new potential switch sites with the assumption that 10% of the current clients (10 clients) will leave the network. The solution obtained with IHM took 20.09 seconds with a

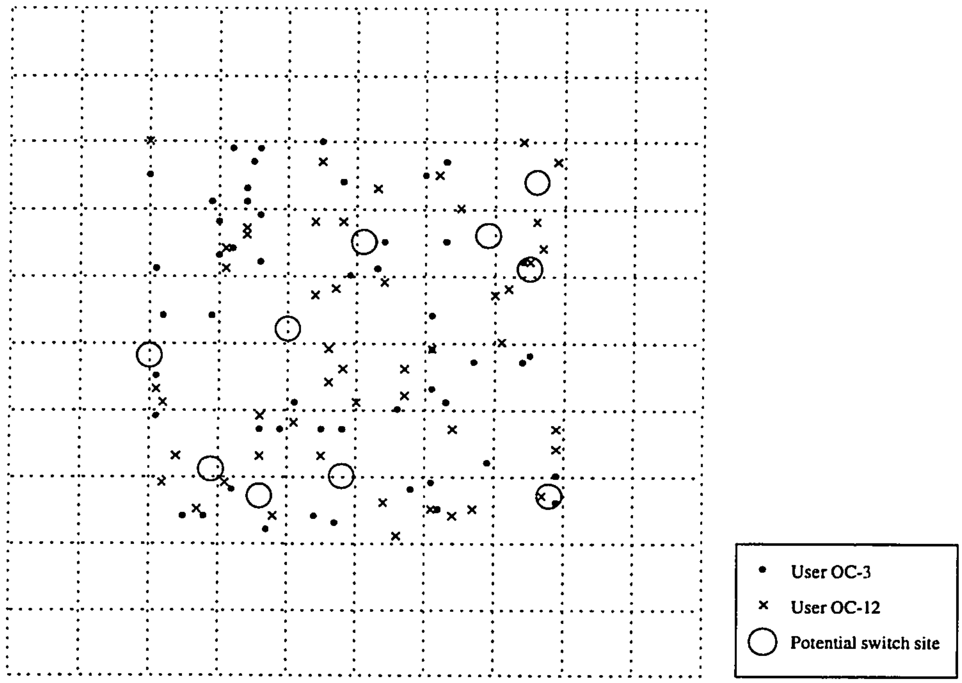

Figure 4.7: Clients and potential switch sites location

value of $5 177 200, whereas the solution obtained by TSM took 211.11 seconds, but its value was $3 605 700. Thus, the tabu algorithm improves the initial solution by 30.35%. This solution illustrated in Figure 4.10 has two new rings, and three bases of model A which were moved from old switch sites (see Figure 4.9) to new switch sites and replaced by bases of model B. The value of the bound LBA is $3 307 983, obtained by exploring 12 branch-and-bound nodes in 2.09 seconds, while the value of the second bound, LBB, is $3 593 000, obtained by exploring 422 branch-and-bound nodes in 78.16 seconds. Here too, the value of the second bound, LBB, was better, but its evaluation is far more time-consuming than that of LBA. In this example, the solution obtained is within 0.35% of the optimal solution.

Figure 4.11 presents the geographical location of 200 new clients and 20 new potential switch sites, again with the assumption that 10% of the current

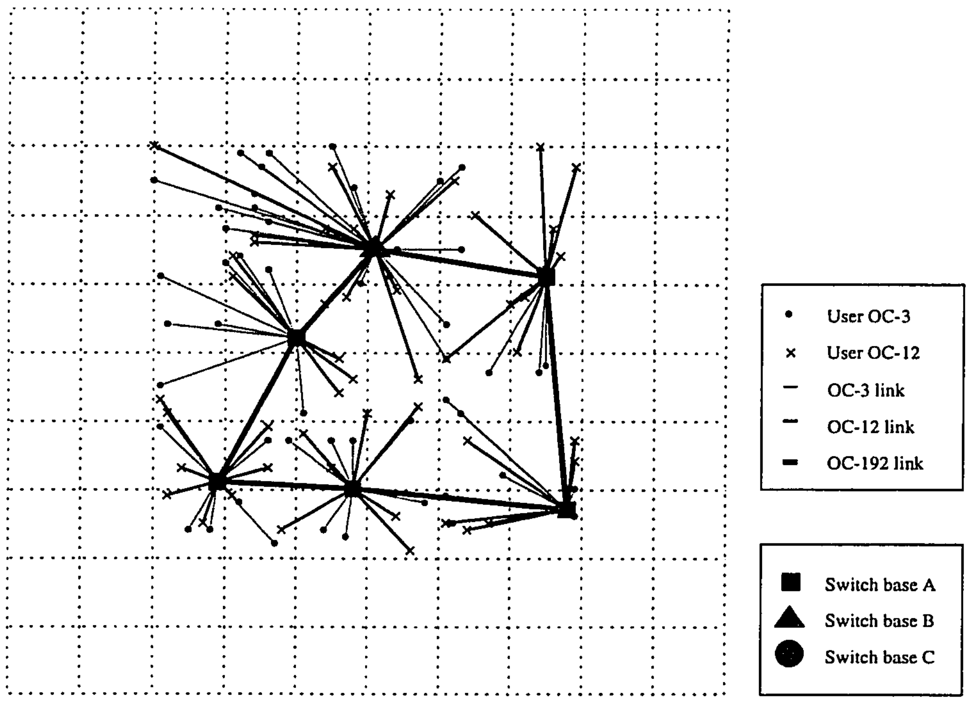

Figure 4.8: Initial network obtained by TSM

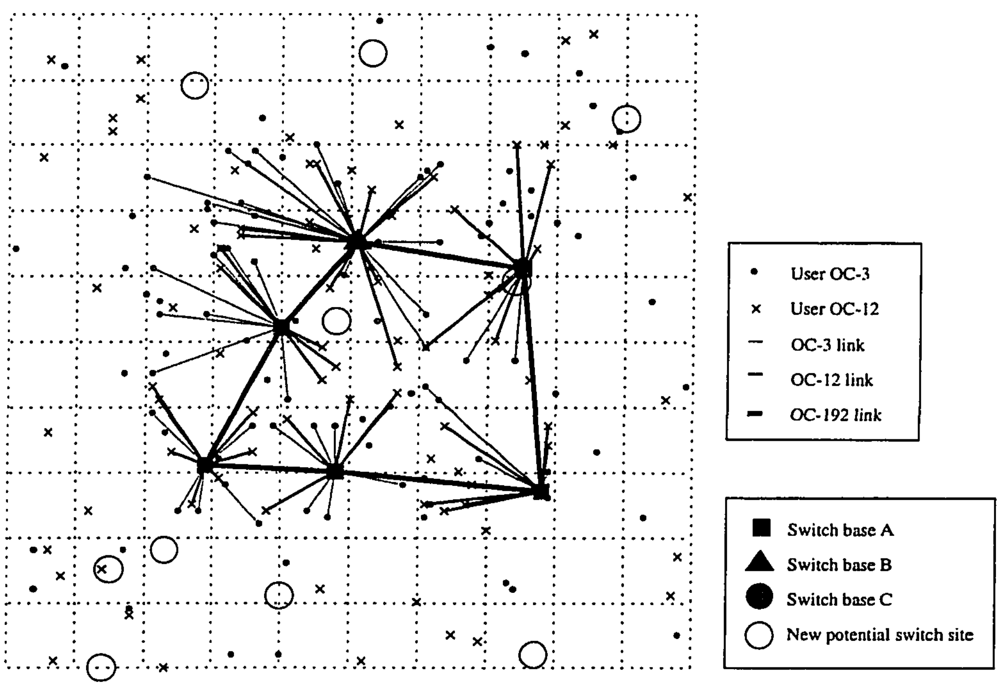

Figure 4.9: New clients and new potential switch sites location for the first expansion

clients (19 clients) will leave the network. The solution obtained with IHM took 95.70 seconds with a value of $7 849 800, whereas the solution obtained by TSM took 1435.89 seconds, but its value was $5 140 000. Thus, the tabu algorithm improves the initial solution by 34.42%. Figure 4.12 shows that this solution has one new ring. The value of the bound LBA is $4 763 504, obtained by exploring 73 branch-and-bound nodes in 81.42 seconds whereas the value of the second bound, LBB, is $5 058 800, obtained by exploring 850 branch-and-bound nodes in 1140.70 seconds. For this example, the solution obtained is within 1.61% of the optimal solution.

In both expansions, the higher costs are due to access links (50.66% of the total cost for the first expansion, 62.85% for the second). This confirms a fact known to practitioners in the field that the access network is responsible for most network costs (see, for instance, Yan and Beshai, 1995). Costs due to the bases were considerably

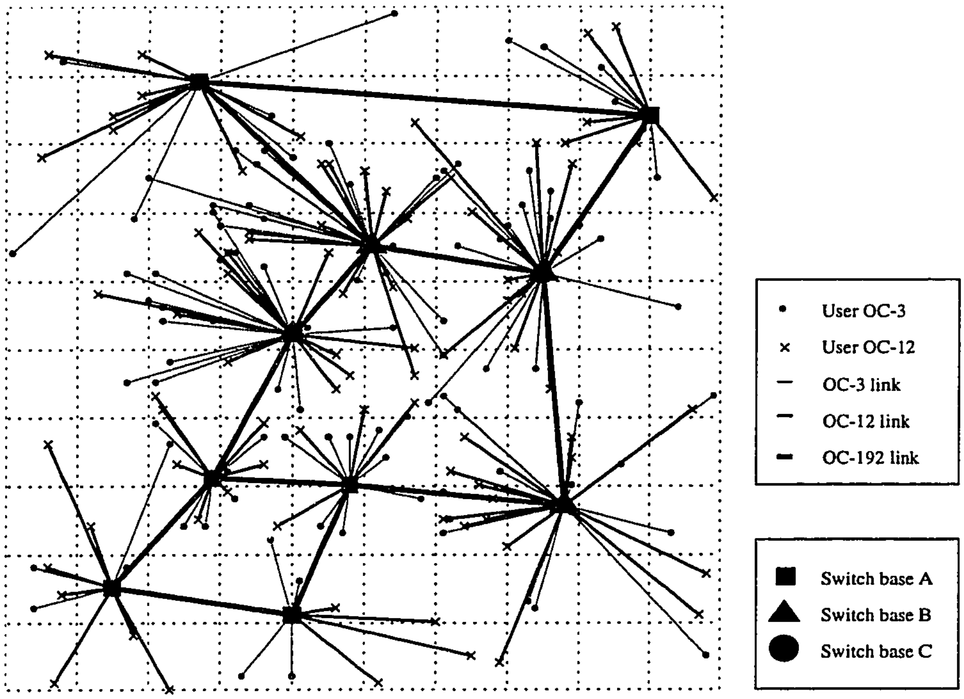

Figure 4.10: First expanded network obtained by TSM

smaller (25.46% for the first expansion, 24.05% for the second), followed by costs due to the OC-192 links, which constitute the backbone network (15.95% for the first expansion, 4.64% for the second), then costs due to ports (7.93% for the first expansion, 8.46% for the second).

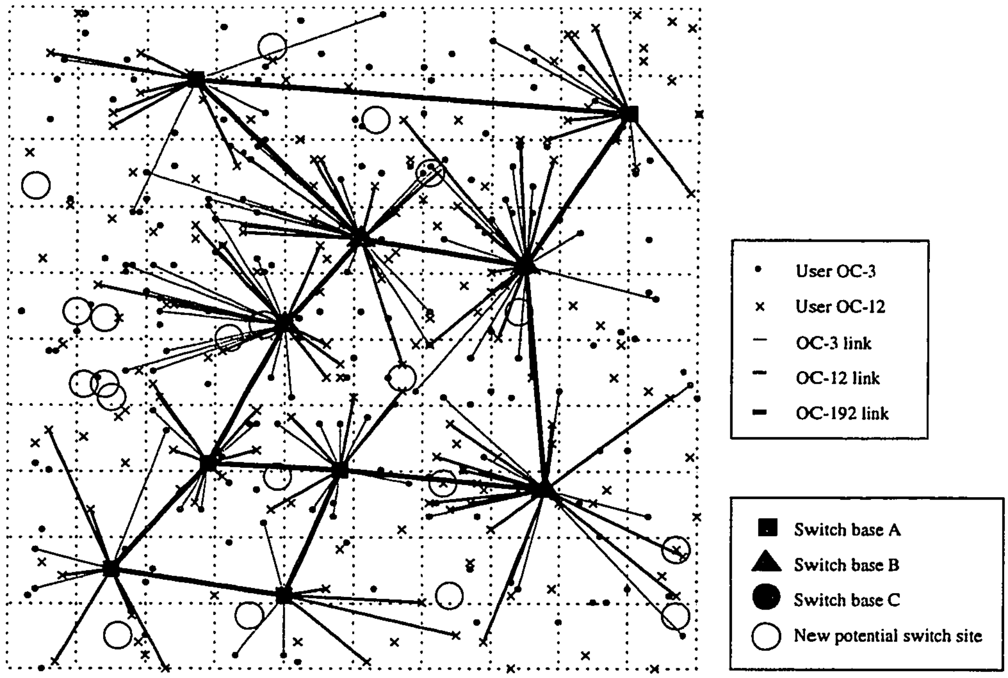

Figure 4.11: New clients and new potential switch sites location for the second expansion

4.5.2 Evaluation of the Heuristics Performance

The results of a systematic set of experiments designed to assess the performance of the proposed algorithms are now given. First, five initial network problem sizes (with up to 250 clients and 10 potential switch sites) were selected. For each size, 10 problems were randomly generated as follows: $|M|$ points (corresponding to clients' locations) and $|N|$ points (corresponding to candidate

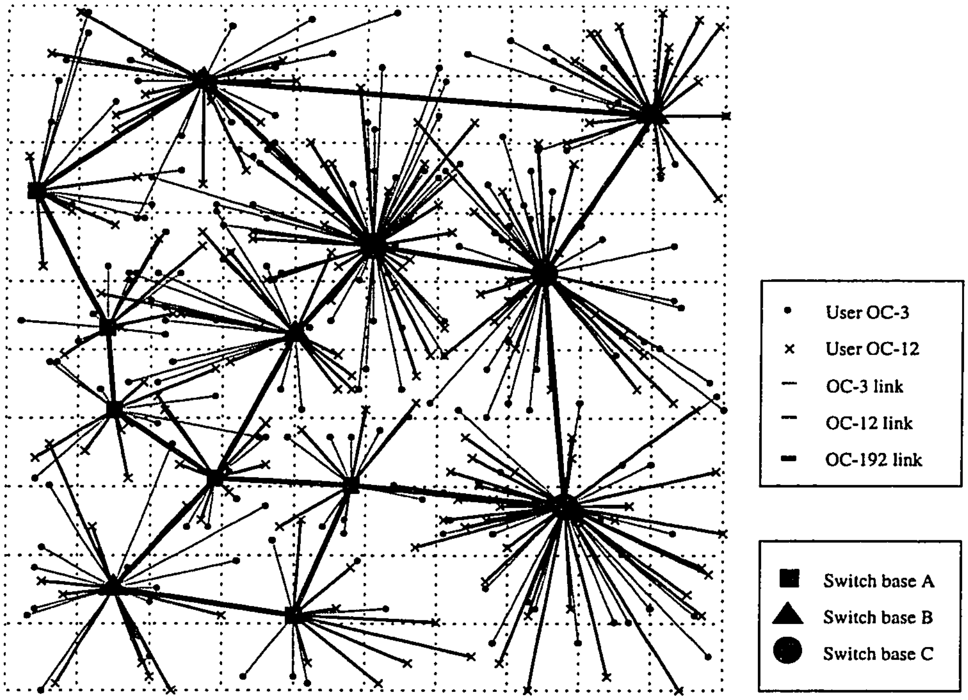

Figure 4.12: Second expanded network obtained by TSM

switch sites) were generated in the square of side length 100 km following a uniform distribution law. For each of these problems generated, an initial network was obtained using TSM (similar to that of Figure 4.8). Next, for each initial network, twenty network expansion problems sizes (with up to 250 new clients and 20 new potential switch sites) were chosen, and each size was randomly generated using $|M_N|$ points (corresponding to new clients' locations) and $|N_N|$ points (corresponding to new candidate switch sites) generated in the square of side length 100 km (also following a uniform distribution law). For each problem generated, an expanded network was obtained using TSM.

Results for each initial network problem size are given in Tables 4.2 to 4.6 respectively, where the first column in each table contains the number of new users, and the second column shows the number of new potential switch sites. Columns 3 and 4 give results for the lower bound LBA, and columns 5 and 6, give results for the lower bound LBB. For each group of lower bound results, OBJ indicates the value of the objective function which is the value of the lower bound, and CPU indicates the CPU execution time. Columns 7 to 9 show results for the initial heuristic, and columns 10 to 12, show results for the tabu search heuristic. For each group of heuristic results, OBJ indicates the value of the objective function which is the cost of the solution found by the heuristic, and CPU indicates the CPU execution time. GAP represents the percentage gap between the heuristic solution and the best lower bound found (with respect to the value of the best lower bound). Each line of these tables presents the mean values obtained for the set of random tests (of size equal to 10 problems), which contain the 10 expansion problems generated (of size determined by $|M_N|$ and $|N_N|$) corresponding to the 10 initial networks problems (of size determined by $|M|$ and $|N|$). At the bottom of each table, is the mean, minimum and maximum values (MEAN, MIN and MAX) obtained for all expansion problems in the table.

Tableau 4.2: Results for TSM with initial networks obtained with $|M| = 50$ and $|N| = 10$

$\lvert M_N \rvert$	$\lvert N_N \rvert$	LBA		LBB		IHM			TSM		
		OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
50	5	1955.61	0.38	2226.02	2.04	3112.48	4.73	39.82	2234.72	42.43	0.39
100	5	3710.21	0.63	4013.02	4.54	4778.83	6.76	19.08	4028.41	70.04	0.38
150	5	5148.18	1.01	5435.33	5.90	6272.83	9.08	15.41	5467.09	102.86	0.58
200	5	6932.86	1.32	7209.94	11.82	7936.19	11.88	10.07	7234.77	129.16	0.34
250	5	8423.63	1.62	8715.11	12.74	9464.77	16.30	8.60	8767.82	149.76	0.60
50	10	1941.92	1.20	2205.40	8.17	3091.69	7.60	40.19	2210.75	74.67	0.24
100	10	3563.46	2.57	3791.80	38.19	4696.53	11.61	23.86	3812.28	120.15	0.54
150	10	5082.00	2.88	5308.48	45.95	6200.54	15.77	16.80	5346.73	182.84	0.72
200	10	6480.47	4.08	6735.50	112.53	7801.09	20.52	15.82	6777.96	276.52	0.63
250	10	7778.01	4.94	8079.00	175.53	9248.53	25.68	14.48	8123.66	351.08	0.55
50	15	1902.95	2.71	2134.47	25.26	3071.07	10.90	43.88	2144.22	106.87	0.46
100	15	3528.06	6.17	3756.76	122.60	4700.78	16.65	25.13	3799.40	173.80	1.14
150	15	4903.28	9.80	5118.00	270.99	6215.97	23.25	21.45	5146.69	262.37	0.56
200	15	6326.30	13.86	6581.61	294.92	7840.49	30.13	19.13	6652.45	388.08	1.08
250	15	7611.30	13.73	7911.48	782.08	9248.17	38.35	16.90	7947.37	537.31	0.45
50	20	1841.62	7.40	2092.36	67.92	2992.49	14.34	43.02	2107.67	137.70	0.73
100	20	3428.59	16.15	3634.43	392.94	4756.11	21.98	30.86	3667.31	222.59	0.90
150	20	4832.15	29.59	5036.84	1522.14	6169.31	31.00	22.48	5062.63	346.63	0.51
200	20	6117.29	34.49	6355.24	1816.71	7632.89	42.78	20.10	6401.90	515.12	0.73
250	20	7226.64	29.84	7511.51	1886.95	9001.83	50.72	19.84	7551.06	728.32	0.53
MEAN		4936.73	9.22	5192.62	379.99	6211.63	20.50	19.62	5224.24	245.92	0.61
MIN		1649.90	0.24	1957.80	0.96	2739.30	4.00	5.65	1969.20	37.12	0.00
MAX		10041.58	57.52	10120.70	7303.72	10959.60	52.09	50.89	10283.50	784.28	2.96

From these tables, it can be seen that the initial heuristic very quickly gives solutions that are far from the lower bound LBB, and that the tabu-based heuristic produces solutions that are very close to LBB with, however, an increase in CPU time. Indeed, for all 1000 expansion problems solved in this subsection, with IHM, the mean gap is 28.92%, and the mean CPU time is 40.79 seconds. With TSM, the mean gap is 0.59%, and the mean CPU time is 633.44 seconds. The values of LBB are much better than those of LBA, but, as was expected, with an increase in CPU time. More precisely, for all problems solved, the mean value of the bound LBA is 4623.77 and the mean CPU time is 16.53 seconds; the mean value of the bound LBB is 4938.21 (+6.80%) and the mean CPU time is 395.95 seconds (+379.42 seconds).

Tableau 4.3: Results for TSM with initial networks obtained with $|M| = 100$ and $|N| = 10$

$\|M_N\|$	$\|N_N\|$	LBA		LBB		IHM			TSM		
		OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
50	5	1732.21	0.59	2209.20	7.66	3453.34	9.03	56.32	2218.55	86.09	0.42
100	5	3358.36	1.00	3683.61	8.15	4968.15	11.95	34.87	3703.87	117.10	0.55
150	5	5026.03	1.64	5395.44	17.13	6500.57	15.03	20.48	5428.11	189.82	0.61
200	5	6387.04	2.12	6785.26	25.69	7831.97	19.96	15.43	6805.82	262.71	0.30
250	5	7817.67	2.75	8192.01	22.79	9144.36	23.36	11.63	8220.81	324.70	0.35
50	10	1745.57	1.67	2160.69	30.71	3498.61	13.71	61.92	2169.45	143.03	0.41
100	10	3255.21	3.13	3606.27	39.72	4829.39	19.02	33.92	3631.91	210.58	0.71
150	10	4568.93	4.76	4880.79	87.23	6216.93	23.78	27.38	4942.08	299.91	1.26
200	10	6085.84	5.90	6402.83	126.82	7688.49	29.44	20.08	6419.15	431.54	0.25
250	10	7295.81	6.62	7581.49	110.80	8983.56	34.67	18.49	7632.11	550.87	0.67
50	15	1739.10	4.13	2147.92	95.97	3477.15	19.01	61.88	2150.64	202.47	0.13
100	15	3194.14	7.60	3516.99	126.20	4806.60	26.01	36.67	3552.94	291.34	1.02
150	15	4548.59	12.47	4872.04	289.64	6209.14	33.55	27.44	4899.70	419.79	0.57
200	15	5869.63	17.98	6162.93	536.63	7615.64	41.60	23.57	6204.69	583.93	0.68
250	15	7085.17	16.49	7376.00	502.31	8933.11	49.30	21.11	7435.14	805.50	0.80
50	20	1751.29	9.27	2124.43	220.87	3497.77	24.67	64.65	2130.89	256.39	0.30
100	20	3195.98	19.86	3506.71	361.22	4874.65	33.98	39.01	3547.23	369.79	1.16
150	20	4563.08	32.34	4815.40	457.60	6324.51	44.69	31.34	4873.94	530.98	1.22
200	20	5793.00	54.35	6062.83	1073.59	7580.21	54.33	25.03	6103.38	736.67	0.67
250	20	7024.81	42.27	7316.73	2068.51	8798.09	65.98	20.25	7368.89	1030.12	0.71
MEAN		4601.87	12.35	4939.98	310.46	6261.61	29.65	26.75	4971.96	392.17	0.65
MIN		1523.50	0.40	1953.80	1.62	2931.80	6.69	7.15	1953.80	30.13	0.00
MAX		9049.77	167.20	9224.20	4056.11	10026.90	71.21	76.60	9224.20	1210.84	3.41

Tableau 4.4: Results for TSM with initial networks obtained with $|M| = 150$ and $|N| = 10$

$\|M_N\|$	$\|N_N\|$	LBA		LBB		IHM			TSM		
		OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
50	5	1640.64	0.88	2048.20	10.82	3782.91	15.15	84.69	2049.24	157.54	0.05
100	5	3223.63	1.50	3581.75	12.37	5111.11	18.61	42.70	3586.94	227.34	0.14
150	5	4689.83	2.62	4985.51	16.85	6516.48	22.24	30.71	5000.08	308.73	0.29
200	5	6037.18	2.98	6373.71	29.16	7787.78	26.13	22.19	6403.98	384.56	0.47
250	5	7519.54	4.21	7786.76	23.31	9125.87	32.47	17.20	7829.21	494.49	0.55
50	10	1557.04	2.02	1936.41	39.27	3724.62	22.05	92.35	1938.50	243.86	0.11
100	10	3121.51	5.45	3451.46	76.36	5057.11	27.49	46.52	3465.30	348.08	0.40
150	10	4462.03	6.37	4754.92	57.30	6429.92	33.29	35.23	4778.51	480.21	0.50
200	10	5840.53	9.36	6120.68	118.95	7711.39	43.05	25.99	6152.89	646.87	0.53
250	10	7105.75	8.50	7394.10	193.81	9009.91	51.20	21.85	7420.92	879.87	0.36
50	15	1566.64	6.06	1923.79	126.48	3706.04	31.14	92.64	1926.29	328.82	0.13
100	15	3045.45	11.50	3381.42	193.60	5028.07	38.05	48.70	3403.89	476.36	0.66
150	15	4427.86	15.78	4717.33	244.03	6489.26	45.80	37.56	4747.67	635.07	0.64
200	15	5584.83	24.27	5937.81	698.54	7636.35	59.03	28.61	5961.36	908.52	0.40
250	15	6799.31	20.20	7115.06	728.81	8886.48	74.42	24.90	7167.03	1212.70	0.73
50	20	1546.14	12.51	1933.99	267.64	3697.14	37.48	91.17	1938.61	414.94	0.24
100	20	2993.44	29.32	3289.81	438.06	5016.50	47.56	52.49	3302.98	583.27	0.40
150	20	4319.09	30.66	4617.72	502.82	6339.08	61.68	37.28	4663.86	830.21	1.00
200	20	5567.16	56.21	5861.69	1992.71	7556.28	81.46	28.91	5908.93	1128.57	0.81
250	20	6696.36	63.79	7021.16	3239.10	8842.29	84.09	25.94	7062.94	1515.81	0.60
MEAN		4387.20	15.71	4711.66	450.50	6372.73	42.62	35.25	4735.46	610.29	0.50
MIN		1311.65	0.61	1673.30	3.20	3029.20	11.80	9.66	1673.30	82.42	0.00
MAX		8585.60	144.80	8737.80	11082.11	9741.30	128.23	147.94	8737.80	1708.04	2.86

Tableau 4.5: Results for TSM with initial networks obtained with $|M| = 200$ and $|N| = 10$

$\|M_N\|$	$\|N_N\|$	LBA OBJ [k$]	LBA CPU [sec]	LBB OBJ [k$]	LBB CPU [sec]	IHM OBJ [k$]	IHM CPU [sec]	IHM GAP [%]	TSM OBJ [k$]	TSM CPU [sec]	TSM GAP [%]
50	5	1730.81	1.49	2151.58	17.37	3588.35	18.55	66.78	2161.42	233.88	0.46
100	5	3289.73	2.44	3606.91	20.04	5085.56	22.96	40.99	3636.73	320.00	0.83
150	5	4858.28	3.15	5185.65	31.99	6512.15	26.85	25.58	5203.63	399.90	0.35
200	5	6343.05	5.04	6674.16	35.49	8097.94	31.80	21.33	6716.79	453.20	0.64
250	5	7930.75	3.78	8207.85	26.67	9580.08	39.34	16.72	8227.09	605.65	0.23
50	10	1733.51	4.10	2094.40	65.71	3579.77	27.92	70.92	2099.93	357.09	0.26
100	10	3295.73	6.03	3601.36	62.47	5159.57	34.46	43.27	3623.88	503.71	0.63
150	10	4585.66	9.07	4854.57	102.75	6423.28	40.75	32.31	4914.66	659.81	1.24
200	10	6072.65	8.91	6370.43	124.84	7949.66	51.25	24.79	6398.14	864.48	0.43
250	10	7332.04	10.10	7625.94	243.91	9295.59	56.67	21.89	7678.62	1122.30	0.69
50	15	1695.77	8.43	2058.54	143.40	3584.81	38.43	74.14	2066.06	496.05	0.37
100	15	3200.43	13.77	3546.61	199.39	5091.87	46.67	43.57	3555.89	673.33	0.26
150	15	4520.60	31.64	4852.67	387.74	6369.28	55.82	31.25	4888.67	913.88	0.74
200	15	5851.88	24.68	6148.46	344.91	7764.66	68.99	26.29	6175.23	1142.56	0.44
250	15	6985.28	29.60	7250.56	989.26	8980.82	75.11	23.86	7311.89	1466.42	0.85
50	20	1694.50	17.83	2075.20	449.15	3600.95	48.69	73.52	2076.89	621.58	0.08
100	20	3198.75	33.50	3541.95	566.01	5169.06	63.19	45.94	3566.45	878.72	0.69
150	20	4524.98	51.06	4831.13	887.63	6505.69	72.76	34.66	4878.21	1135.98	0.97
200	20	5669.02	72.63	5968.23	2422.16	7710.32	92.28	29.19	6008.89	1487.12	0.68
250	20	7002.35	67.61	7267.31	2119.76	9096.85	98.94	25.18	7297.79	1894.02	0.42
MEAN		4575.79	20.24	4895.68	462.03	6457.31	50.57	31.90	4924.34	811.48	0.59
MIN		1446.58	0.84	1828.20	3.52	2521.60	12.42	4.01	1828.20	71.00	0.00
MAX		9052.02	163.83	9148.60	7903.06	10153.70	148.44	134.23	9160.80	2136.38	3.10

Tableau 4.6: Results for TSM with initial networks obtained with $|M| = 250$ and $|N| = 10$

$\|M_N\|$	$\|N_N\|$	LBA		LBB		IHM			TSM		
		OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	GAP [%]	OBJ [k$]	CPU [sec]	GAP [%]
50	5	1799.45	2.33	2187.19	17.54	3667.88	24.07	67.70	2189.94	351.82	0.13
100	5	3401.74	3.45	3759.79	15.71	5168.35	28.59	37.46	3783.87	468.05	0.64
150	5	4922.35	5.23	5261.03	20.18	6636.44	32.83	26.14	5276.01	615.22	0.28
200	5	6265.28	4.60	6567.63	22.03	7917.42	40.42	20.55	6615.37	634.59	0.73
250	5	8152.55	4.84	8396.93	25.94	9691.81	47.07	15.42	8440.46	831.98	0.52
50	10	1777.18	7.24	2130.42	58.81	3659.67	37.96	71.78	2134.14	521.56	0.17
100	10	3327.64	9.43	3689.97	81.87	5116.05	43.09	38.65	3706.41	719.78	0.45
150	10	4634.45	9.72	4988.56	80.23	6551.88	49.62	31.34	5015.43	913.39	0.54
200	10	6093.32	18.38	6406.74	126.08	7844.04	57.16	22.43	6440.23	1151.04	0.52
250	10	7250.61	13.09	7507.49	135.67	9067.00	65.13	20.77	7533.30	1362.06	0.34
50	15	1823.54	12.75	2186.09	197.59	3725.26	48.71	70.41	2187.75	694.55	0.08
100	15	3222.20	17.63	3626.80	334.56	5184.33	58.02	42.95	3657.24	975.26	0.84
150	15	4602.03	32.13	4954.01	346.70	6544.97	68.17	32.11	4998.40	1255.31	0.90
200	15	5789.25	35.54	6113.20	520.16	7718.59	82.53	26.26	6178.01	1591.80	1.06
250	15	6965.90	30.01	7237.88	517.67	9064.97	88.91	25.24	7283.14	1841.02	0.63
50	20	1753.87	25.62	2114.14	489.57	3646.50	61.48	72.48	2125.87	870.73	0.55
100	20	3228.38	39.33	3587.96	648.99	5160.34	74.61	43.82	3619.92	1226.00	0.89
150	20	4556.92	69.65	4910.55	938.19	6508.94	87.58	32.55	4929.46	1603.86	0.39
200	20	5729.74	79.67	6069.49	1400.53	7795.10	101.29	28.43	6125.96	2069.58	0.93
250	20	7048.93	81.52	7326.54	1457.89	9110.61	114.91	24.35	7364.64	2449.35	0.52
MEAN		4617.27	25.11	4951.12	371.79	6489.01	60.61	31.06	4980.28	1107.35	0.59
MIN		1432.20	1.21	1991.20	5.09	2776.80	17.66	3.88	1991.70	138.07	0.00
MAX		11543.02	175.89	11618.40	4659.86	12068.70	118.76	105.77	11706.90	2691.80	4.94

4.6 Concluding Remarks

We have presented, in this article, a model for a problem of topological expansion of metropolitan area network with multiple ring topology and modular switches. The model includes the location of new switch sites, the update of the configuration of modular switches, the update of the access network with a star topology, and the expansion of the backbone network with a fixed (multiple ring) topology. Moreover, we have considered that several access rates may be used in the access network.

Two heuristics have been proposed in order to find a solution. The first heuristic, IH, produces quick solutions that are, unfortunately, far from optimality (within 28.92%, on average, of the optimal solution). However, the second heuristic, based on the tabu search principle, TS, produces solutions that are very close to optimality (within 0.59%, on average, of the optimal solution), yet with an increase in CPU time.

Several directions for further work are open at this point. Since the restoration time of a network in case of failure should be small (see Wu, 1992), it would be interesting to consider additional constraints for the multiple ring topology in order to limit the size of each ring in terms of the number of switches and the length of the circumference. While in this paper we propose a model in which the access network presents a star topology, we are currently working on models with other topologies for the access network.

4.7 Acknowledgments

The authors are grateful to Gilbert Laporte of the Centre for Research on Transportation for having let them use his implementation of the GENIUS algorithm. This work was funded in part by FCAR grants and NSERC strategic grant STR 0166996.

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CHAPITRE 5

Tabu Search and Post-Optimization Algorithms for the Topological Update of Two-Level Networks with Modular Switches

by

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Abstract

This paper deals with the problem of how to update a telecommunication network economically. We first propose a mixed integer programming model that includes the update of the location and configuration (with respect to ports, multiplexers and bases) of switches, the update of the access network with a star topology, and the update of the backbone network with a topology specified by the network planner. Moreover, we suppose the use of multiplexers both in the access and in the backbone network. In order to find a solution, we propose an initial heuristic, a tabu-based heuristic, and a post-optimization algorithm. Among all possible topologies for the backbone network, we consider the ring topology for computational tests. Finally, we present an illustrative example, followed by a performance analysis of the proposed heuristics (using a lower bound). The solutions obtained are, on average, within 0.74% of the optimal solution.

Key words: Topological update, hierarchical two-level network, ATM, modular switch, tabu search, post-optimization algorithm.

Status: This article has been submitted to the journal *IEEE/ACM Journal on Networking*, 1997.

5.1 Introduction

Technological advances, in particular ATM (Asynchronous Transfer Mode) networks, combined with deregulation of the telecommunication industry, have resulted in a diversity of new broadband services such as videotelephony, videoconference, high-speed data transfer, and high-resolution image transfer. Due to these developments, the number of potential clients grows continuously obliging telecommunication companies to update their networks regularly in order to stay abreast of their competition.

The network update process starts, typically, by estimating the number of potential clients and their geographical location (using sample survey or marketing techniques) for a projected time period. Next, a target network is defined (using an interactive decision support system) satisfying the estimated demand of both current and potential clients. The current network is then updated adaptatively in order to obtain the target network by the end of the projected time period.

The object of this paper is to present a model and a resolution approach for the overall update problem of two-level networks with modular switches as are ATM switches. A modular switch is composed of a base with ports and multiplexers. A base consists of a switching network with slots to insert ports. A port is used to connect a link or a multiplexer used to connect several links to a single port. Different types of bases, ports and multiplexers may be used.

The modular switches in network planning models have been considered by a few articles only. Chamberland, Marcotte and Sansó (1996) propose a global approach for the design of broadband networks including the location of switches, the design of a star access network, the design of a backbone network, the link dimensioning, and traffic routing. The authors introduce modular switches in the network design process. An example with six users and two switch sites is solved by means of a branch-and-bound algorithm. Chamberland, Sansó and Marcotte (1997a) propose a mixed 0-1 linear programming model for the topological network design problem with modular switches including the location of switches, their configuration with respect to ports and multiplexers, the design of an access network with a star topology, and a backbone network with a fixed topology. The authors also propose a greedy heuristic that provides a good starting solution and a tabu search heuristic to improve the solution. Results for problems with up to 500 users and 30 potential switch sites are presented. The tabu-based heuristic provides solutions, on average, within 0.86% of optimality. Chamberland and Sansó (1997a) consider a model for the topological design problem of two-level multitechnology telecommunication networks that includes the optimal location of the modular switches, their configuration with respect to ports and bases, the design of an access network with a star topology, and a backbone network with a fixed topology. In order to find a solution, a greedy heuristic and a more sophisticated heuristic are presented. Numerical results for problems with up to 400 users and 30 potential switch sites are reported. The best heuristic finds solutions, on average, within 2.87%, of optimality. Finally, Chamberland and Sansó (1997b) deal with the problem of how to expand a MAN (Metropolitan Area Networks) in a cost-effective way. The proposed model considers the update of the location and configuration of modular switches with respect to ports and base, the update of the access network with a star topology and, the expansion of the backbone network with a fixed topology. Moreover, the model includes that several access technologies and rates may be used. The multiple ring topology (for the backbone network) is studied in detail. A heuristic approach is proposed to solve the model. A systematic set of experiments to assess the performance of the proposed algorithms (using lower bounding procedures) are presented. The solutions obtained are, on average, within 0.59% of optimality.

The problem we treat in this paper is more general than those proposed in the literature concerning the following points since we consider that different port and base types may be used simultaneously and we propose a model including the possibility of using multiplexers in the backbone network. Moreover, we introduce the network update problem of two-level networks with modular switches. The network update problem is more general than the network expansion problem proposed by Chamberland and Sansó (1997b) since, in the update problem, the switch sites and backbone links may be removed from the network.

The paper is organized as follows. Section 5.2 is devoted to the presentation of a general model for the topological update problem of two-level networks with modular switches. In Section 5.3 we present an initial heuristic. A tabu-based heuristic that provides a good solution is proposed in Section 5.4, and a post-optimization algorithm (also based on the tabu search principle) to improve the solution is presented in Section 5.5. In Section 5.6 we adapt the model and the proposed algorithms to backbone networks having a ring topology. This topology has the advantage that it is sparse (therefore not too expensive), but provides protection against single link or switch failure. Moreover, many protocols, such as SONET (Synchronous Optical NETwork) and FDDI (Fiber Distributed Data Interface), make use of this topology (see Bertsekas and Gallager, 1992). Section 5.7 presents an illustrative example of the design and the update of a network, followed by a systematic set of experiments to assess the performance of the proposed algorithms. We summarize our results and outline directions for further research in Section 5.8.

5.2 Problem Formulation

In this section we present the general topological update problem of two-level networks with modular switches. First, we make the following assumptions about the organization of the network: (A1) each user is connected to a switch through an OC-3 (155.520 Mbit/s) link; (A2) the switches are interconnected through OC-192 (9.953 Gbit/s) links with a topology specified by the network planner; (A3) the number of ports installed in a switch cannot exceed the number of slots of the base; (A4) at most one switch may be installed at a given site; (A5) the number of OC-3 (respectively, OC-192) links connected to a multiplexer for the access (respectively, backbone) network cannot exceed the number of inputs of the multiplexer; (A6) at most one level of multiplexing is used; (A7) the number of inputs (capacity) of a multiplexer (for the access networks) connected to a port of type OC-3-n is n; (A8) the number of inputs (capacity) of a multiplexer (for the backbone networks) connected to a port of type OC-192-h is h.

We make the following assumptions about the update of the network: (A9) each link in the current network may be kept in place or removed from the network; (A10) each base and port in the current network may be kept in place, moved to another site or be removed from the network; (A11) new bases may be installed at new potential switch sites and switch sites in the current network; (A12) new ports may be installed in the slots of both new bases and bases used in the current network; (A13) the topology of both the access and backbone network is preserved in the updated network.

Finally, we suppose that the following information is known: (I1) the location of new users; (I2) the current users leaving the network; (I3) the new potential switch sites; (I4) the different types of bases, their capacities in term of the number of slots, and costs; (I5) the cost of installing a given type of base at a given site; (I6) the cost of removing a given type of base from a given site; (I7) the cost of installing the ground structure, used for installing a switch, at a given site; (I8) the cost of removing the ground structure from a given site; (I9) the costs of the different types of ports and multiplexers (for the access and the backbone network); (I10) the costs of OC-3 and OC-192 links, including the installation cost, in \$/km; (I11) the cost of removing OC-3 and OC-192 links in \$/km.

The problem is to find the minimum cost updated network subject to all the above assumptions (A1 to A13) and information (I1 to I11).

5.2.1 Mathematical Formulation

Let $M = \{1, \ldots, |M|\}$ be the set of all users such that $M = M_N \bigcup M_C$ (and $M_N \bigcap M_C = \emptyset$), where M_N is the set of new users, and M_C the set of the current users remaining in the network (i.e., renewing their subscription). A user can be an institution, a LAN (Local Area Network) or group of LANs. Let $N = \{1, \ldots, |N|\}$ be the set of all potential switch sites such that $N = N_N \bigcup N_C$ (and $N_N \bigcap N_C = \emptyset$), where N_N is the set of new candidate switch sites, and N_C the set of switch sites in the current network, and let $T = \{1, \ldots, |T|\}$ be the set of all base types (where m^t denotes the number of slots of a base of type $t \in T$) such that $T = T_N \bigcup T_C$ (and $T_N \bigcap T_C = \emptyset$), where T_N is the set of new base types, and T_C the set of base types used in the current network. Let $R = \{1, \ldots, |R|\}$ be the set of all port types for the access network (where n^r denotes the number of OC-3 inputs of a multiplexer connected to a port of type $r \in R$) such that $R = R_N \bigcup R_C$ (and $R_N \bigcap R_C = \emptyset$), where R_N is the set of new port types, and R_C the set of port types used in the current access network, and let $Q = \{1, \ldots, |Q|\}$ be the set of port types for the backbone network (where h^q denotes the number of OC-192 inputs of a multiplexer connected to a port of type $q \in Q$) such that $Q = Q_N \bigcup Q_C$ (and $Q_N \bigcap Q_C = \emptyset$), where Q_N is the set of new port types, and Q_C the set of port types used in the current backbone network.

We now define the decision variables. Let $x_{ij} \in \mathsf{B}$ ($\mathsf{B} = \{0, 1\}$) be a variable such that $x_{ij} = 1$ if and only if user $i \in M$ is connected to site $j \in N$ through an OC-3 link; $y_{jk} \in \mathsf{B}$ a variable such that $y_{jk} = 1$ if and only if site $j \in N$ is connected to site $k \in N$ (for $j < k$) through an OC-192 link; $z_j^t \in \mathsf{B}$ a variable such that $z_j^t = 1$ if and only if the base installed at site $j \in N$ is of type $t \in T$, and let $u_j \in \mathsf{B}$ be a variable such that $u_j = 1$ if and only if a switch is installed at site $j \in N$. Finally, let $v_j^r \in \mathbf{Z}_+$ be a variable representing the number of ports of type $r \in R$ in the slots of the base installed at site $j \in N$, and let $w_j^q \in \mathbf{Z}_+$ be a variable representing the number of ports of type $q \in Q$ in the slots of the base installed at site $j \in N$.

The topology of the current network is identified by the decision variables overlined. For example, if user $i \in M$ is connected to site $j \in N$ in the current network, then $\overline{x}_{ij} = 1$, and if the number of ports of type $r \in R$ in the slots of the base located at site $j \in N$ in the current network is k, then $\overline{v}_j^r = k$.

We define the following cost parameters. Let c_{ij} be the link cost (including the installation cost) of connecting the user $i \in M$ to site $j \in N$ through an OC-3 link if $\overline{x}_{ij} = 0$ (this link is not in the current network), and $c_{ij} = 0$ if $\overline{x}_{ij} = 1$ (this link is in the current network), and let θ_{ij} be the cost of removing the OC-3 link connecting the user $i \in M$ to site $j \in N$ if $\overline{x}_{ij} = 1$, and $\theta_{ij} = 0$ if $\overline{x}_{ij} = 0$. Let d_{jk} be the link cost (including the installation cost) of connecting site $j \in N$ to site $k \in N$ through an OC-192 link if $\overline{y}_{jk} = 0$, and $d_{jk} = 0$ if $\overline{y}_{jk} = 1$, and let λ_{jk} be the cost of removing the OC-192 link connecting the site $j \in N$ to site $k \in N$ if $\overline{y}_{jk} = 1$, and $\lambda_{jk} = 0$ if $\overline{y}_{jk} = 0$. Let b^t be the cost of purchasing a base of type $t \in T$; α_j^t the cost of installing a base of type $t \in T$ at site $j \in N$ if $\overline{z}_j^t = 0$, and $\alpha_j^t = 0$ if $\overline{z}_j^t = 1$, and let β_j^t be the cost of removing a base of type $t \in T$ from site $j \in N$ if $\overline{z}_j^t = 1$, and $\beta_j^t = 0$ if $\overline{z}_j^t = 0$. Let γ_j be the cost of installing the ground structure at site $j \in N$ if $\overline{u}_j = 0$, and $\gamma_j = 0$ if $\overline{u}_j = 1$, and let δ_j be the cost of removing the ground structure from site $j \in N$ if $\overline{u}_j = 1$, and $\delta_j = 0$ if $\overline{u}_j = 0$. Finally, let p^r be the cost of a port of type $r \in R$ (including the cost of a multiplexer of capacity n^r for the access network), and let o^q be the cost of a port of type $q \in Q$ (including the cost of a multiplexer of capacity h^q for the backbone network).

5.2.1.1 The Objective Function

The objective function, representing the total update cost of the network, is composed of four terms: link costs, base costs, ground structure costs, and port costs. Link costs, noted C_L, given by the following equation, include the cost of the new (access and backbone) links and the cost of removing (access and backbone) links.

$$C_L(\mathbf{x}, \mathbf{y}) = \sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} + \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} d_{jk} y_{jk} + \sum_{i \in M} \sum_{j \in N} \theta_{ij}(1 - x_{ij}) + \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} \lambda_{jk}(1 - y_{jk}). \tag{5.1}$$

Base costs, noted C_B, include the cost of the new bases and the cost of installing and removing them. Since, the number of new bases of type $t \in T$ in the updated network is given by $\left(\sum_{j \in N}(z_j^t - \overline{z}_j^t)\right)^+$, C_B is given by the equation

$$C_B(\mathbf{z}) = \sum_{t \in T} \left(\sum_{j \in N} (z_j^t - \overline{z}_j^t) \right)^+ b^t + \sum_{j \in N} \sum_{t \in T} \alpha_j^t z_j^t + \sum_{j \in N} \sum_{t \in T} \beta_j^t (1 - z_j^t). \tag{5.2}$$

Ground structure costs, noted C_S, given by the following equation, include the cost of installing and removing the ground structures.

$$C_S(\mathbf{u}) = \sum_{j \in N} \gamma_j u_j + \sum_{j \in N} \delta_j (1 - u_j). \tag{5.3}$$

Finally, port and multiplexers costs, noted $C_{P/M}$, include the cost of the new ports and multiplexers used in the access and in the backbone network. Since the number of new ports of type $r \in R$ in the updated network is given by $\left(\sum_{j \in N}(v_j^r - \overline{v}_j^r)\right)^+$, and the number of new ports of type $q \in Q$ by $\left(\sum_{j \in N}(w_j^q - \overline{w}_j^q)\right)^+$, $C_{P/M}$ is given by equation

$$C_{P/M}(\mathbf{v},\mathbf{w}) = \sum_{r\in R}\left(\sum_{j\in N}(v_j^r - \overline{v}_j^r)\right)^+ p^r + \sum_{q\in Q}\left(\sum_{j\in N}(w_j^q - \overline{w}_j^q)\right)^+ o^q. \tag{5.4}$$

5.2.1.2 The Model

Below the model UP (Update Problem) for the topological update problem of two-level networks with modular switches is presented. Note that the topology of the backbone network has not yet been specified.

UP:

$$\min_{\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z}} \Big(C_L(\mathbf{x},\mathbf{y}) + C_B(\mathbf{z}) + C_S(\mathbf{u}) + C_{P/M}(\mathbf{v},\mathbf{w})\Big) \tag{5.5}$$

s.t.

User assignment constraints

$$\sum_{j\in N} x_{ij} = 1 \quad (i\in M) \tag{5.6}$$

Base type uniqueness constraints

$$\sum_{t\in T} z_j^t = u_j \quad (j\in N) \tag{5.7}$$

Base capacity constraints

$$\sum_{r\in R} v_j^r + \sum_{q\in Q} w_j^q \le \sum_{t\in T} m^t z_j^t \quad (j\in N) \tag{5.8}$$

Access multiplexer capacity constraints

$$\sum_{i\in M} x_{ij} \le \sum_{r\in R} n^r v_j^r \quad (j\in N) \tag{5.9}$$

Backbone multiplexer capacity constraints

$$\sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} \leq \sum_{q \in Q} h^q w_j^q \quad (j \in N) \tag{5.10}$$

Backbone topology constraints

$$\boxed{\text{Backbone topology constraints}} \tag{5.11}$$

Additionnal valid inequalities

$$x_{ij} \leq u_j \quad (i \in M;\ j \in N) \tag{5.12}$$

$$y_{jk} \leq u_j \quad (j < k,\ \ j, k \in N) \tag{5.13}$$

$$y_{jk} \leq u_k \quad (j < k,\ \ j, k \in N) \tag{5.14}$$

Intergrality and nonegativity constraints

$$\mathbf{u} \in \mathsf{B}^{|N|}, \mathbf{v} \in \mathbb{Z}_+^{|N||R|}, \mathbf{w} \in \mathbb{Z}_+^{|N||Q|}, \mathbf{x} \in \mathbb{R}_+^{|M||N|}, \mathbf{y} \in \mathsf{B}^{\frac{|N|}{2}(|N|-1)}, \mathbf{z} \in \mathsf{B}^{|N||T|}. \tag{5.15}$$

The objective function of UP represents the total update cost of the network. Constraints (5.6) require that each user be connected to exactly one switch, constraints (5.7) demand that the base installed at site $j \in N$ have an unique type, and that this type be chosen if and only if a switch has been installed at the site. Constraints (5.8) require that the number of ports installed in the slots of a base of type $t \in T$ be at most m^t, and constraints (5.9) demand that the number of OC-3 links connected to a switch be at most the sum of the capacities of the multiplexers for the access network installed at the switch. Constraints (5.10) require that the number of OC-192 links connected to a switch be at most the sum of the capacities of the multiplexers for the backbone network installed at the switch. Constraints (5.11) necessitate that the topology of the backbone network be the one specified by the network planner. For instance, the backbone network may have a ring, a multiple ring, a tree or a full-mesh topology. For further details about these topologies see Chamberland and Sansó (1997a, 1997b). Inequalities (5.12) to (5.14) are optional, but are included in the model to strengthen the linear relaxation. Constraints (5.15) are integrality and nonnegativity constraints.

In model UP, we suppose that $|N| \geq 1$, $|T| \geq 1$ with $\max_{t \in T}\{m^t\} \geq 3$, $|R| \geq 1$, $|Q| \geq 1$ and $|M| \geq 3$, since, if the number of users $|M|$ is equal to one or two, it is not cost-effective to build a two-level network. This model can be used for the design problem if we set all overlined variables to zero and if $|M_C| = |N_C| = |T_C| = |R_C| = |Q_C| = 0$, that is, if the current network is nonexistent.

In the next three sections, we present an initial heuristic, a tabu search heuristic, and a post-optimization algorithm to find a good solution of UP.

5.3 The Initial Heuristic

In this section, we propose an initial heuristic, called IH, to find a solution that will be used as a starting point for the search algorithm proposed in the next section.

To describe IH let a star be a subnetwork that includes a switch (the center of the star) and users connected to this switch. Let $\Gamma(j)$ be the set of users of the star with center j. The size of the star with center j is $|\Gamma(j)|$ and its cost is given by

$\sum_{i \in \Gamma(j)} (c_{ij} - \theta_{ij})$. Let E_M and E_N be two sets defined to contain users and switches respectively. Finally, let $\Phi(j)$ be the maximum number of OC-192 links that can be connected to site $j \in N$, and let $\overline{\Phi}(j)$ be the number of OC-192 links connected to site $j \in E_N$.

IH generates at most m solutions corresponding to stars of size k for $1 \leq k \leq m$, where m is the maximum number of users that can be connected to a single switch (i.e., $m = \min\{|M|, \max_{r \in R}\{n^r\} \max_{t \in T}\{m^t\}\}$). The best solution found will be returned by the heuristic.

Heuristic IH

Step 1: (Feasibility check)

If the problem is not feasible, stop. Otherwise, set $k := 1$ and go to Step 2.

Step 2: (Generating a solution)

2.1 Set $E_M := \emptyset$, $E_N := \emptyset$ and $\Gamma(j) := \emptyset$ for all $j \in N$.

2.2 For $i := 1$ to $\min\left\{|N|, \left\lceil \frac{|M|}{k} \right\rceil\right\}$ do

2.2.1 Determine the star of minimum cost among the stars of size $\min\{k, |M \backslash E_M|\}$ containing a switch in $N \backslash E_N$ and users in $M \backslash E_M$. Let j^* denote the chosen switch site and $\Gamma(j^*)$ the set of users of the star of center j^*.

2.2.2 Set $E_N := E_N \cup \{j^*\}$, $E_M := E_M \cup \Gamma(j^*)$ and $\Phi(j^*) := \left(\max_{t \in T}\{m^t\} - \left\lceil \frac{|\Gamma(j^*)|}{\max_{r \in R}\{n^r\}} \right\rceil\right) \max_{q \in Q}\{h^q\}$.

2.3 For each site $j \in E_N$ install a base of capacity equal to $\max_{t \in T}\{m^t\}$.

2.4 Connect the switches in E_N with OC-192 links at minimum cost (where the costs are the $d_{jk} - \lambda_{jk}$ for all $j < k$ and $j, k \in E_N$) respecting constraints (5.11) and maximal degree $\Phi(j)$ for $j \in E_N$. If a backbone network is found, set $\overline{\Phi}(j)$ to the number of OC-192 links connected to switch $j \in E_N$. Otherwise, go to Step 4.

2.5 For each user $i \in M \setminus E_M$ do

2.5.1 Set j^* to the solution of the following problem

$$\min_{j \in E_N} \left\{ c_{ij} - \theta_{ij} : \left(\max_{t \in T}\{m^t\} - \left\lceil \frac{\overline{\Phi}(j)}{\max_{q \in Q}\{h^q\}} \right\rceil \right) \max_{r \in R}\{n^r\} > |\Gamma(j)| \right\}.$$

2.5.2 Set $\Gamma(j^*) := \Gamma(j^*) \cup \{i\}$ and $E_M := E_M \cup \{i\}$.

2.6 For each site $j \in E_N$ do

2.6.1 Install $\left\lceil \frac{\overline{\Phi}(j)}{\max_{q \in Q}\{h^q\}} \right\rceil$ ports and multiplexers (for the backbone network) of maximal rate.

2.6.2 Install $\left(\max_{t \in T}\{m^t\} - \left\lceil \frac{\overline{\Phi}(j)}{\max_{q \in Q}\{h^q\}} \right\rceil \right)$ ports and multiplexers (for the access network) of maximal rate.

2.7 Compute the cost of the current solution given by the objective function (5.5).

Step 3: (Best solution update)

If the cost of the current solution is less than that of the best solution obtained so far, update this best solution.

Step 4: (Termination test)

If $k = m$, return the best solution found and stop. Otherwise, set $k := k + 1$ and go to Step 2.

In algorithm IH, Step 1 consists of checking if UP is feasible. If the problem is feasible the algorithm performs Step 2 otherwise it stops. Step 2 generates a feasible solution in the following manner. In Step 2.2, stars of size k are constructed (the last star found may be smaller) and a set of switch sites E_N is found. In Step 2.3 a base of maximum capacity is installed at each site in E_N. In Step 2.4, a backbone network of minimum cost connecting the switches in E_N is found. Step 2.5 consists of connecting each user not yet in the network, at minimum cost, while taking into account the capacity of the base types and the use of ports of maximal rate (for both the access and backbone network). In Step 2.6, the ports and multiplexers are installed, and in Step 2.5, the cost of the solution is computed. Steps 3 and 4 update the best solution, if necessary, and if k is equal to m, the algorithm stops. However, if $k < m$, IH returns to Step 2 with $k := k + 1$.

5.4 The Tabu Algorithm

In this section we propose a tabu search algorithm, called TS, for UP. For an introduction to the theory of tabu search see the articles by Glover (1989, 1990) and Glover, Taillard and de Werra (1993) and the book by Glover and Laguna (1997). TS incorporates features of the tabu algorithm proposed by Chamberland and Sansó (1997a) for the topological design problem of multitechnology networks with modular switches (but without multiplexers).

The following notation and definitions will be used to describe TS. Let e_j be the state of site $j \in N$ such that $e_j = 0$ if there is no base installed at site j, and $e_j = k$ (for $k \in T$), if a base of type k is installed at site j. Let $\mathbf{e} = \{e_j\}_{j \in N}$ be the vector of the state of the network sites. Therefore, when the vector $\mathbf{e}$ is fixed, the variables $\mathbf{u}$ and $\mathbf{z}$ are also fixed. Let $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$ be the vectors $\mathbf{u}$ and $\mathbf{z}$ respectively when the network sites state $\mathbf{e}$ is fixed.

In the next subsection we propose a decomposition approach to solve UP(e), i.e., the model UP when the decision vectors $\mathbf{u}$ and $\mathbf{z}$ are respectively equal to the vectors $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$.

5.4.1 Solving UP(e)

When vectors $\mathbf{u}$ and $\mathbf{z}$ are respectively set to $\mathbf{u}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$, UP can be decomposed into two subproblems. Consider the following valid inequalities for UP

$$\sum_{\substack{k\in N\\ j<k}} y_{jk} + \sum_{\substack{k\in N\\ j>k}} y_{kj} \le \max_{q\in Q}\{h^q\} \sum_{t\in T} m^t z_j^t \quad (j \in N), \tag{5.16}$$

obtained using inequalities (5.8), (5.10) and nonegativity constraints. The first subproblem, denoted $\overline{\text{UP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$, is written out below.

$$\overline{\text{UP}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e})) : \min_{\mathbf{y}} \left\{ \sum_{j\in N} \sum_{\substack{k\in N\\ j<k}} (d_{jk} - \lambda_{jk}) y_{jk} : (5.11), (5.16), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{5.17}$$

The purpose of this first subproblem is to connect the switches chosen (given by vector $\mathbf{u}(\mathbf{e})$) at minimum cost in order to form a backbone network, whose topology is given by constraints (5.11), considering the base types chosen (given by vector $\mathbf{z}(\mathbf{e})$). Let $\mathbf{y}(\mathbf{e})$ be the (exact or heuristic) solution of the first subproblem. Set $q^* := \arg\max_{q\in Q}\{h^q\}$, and for each site $j \in N$, set $w_j^{q^*}(\mathbf{e}) := \left\lceil \frac{\sum_{k\in N, j<k} y_{jk}(\mathbf{e}) + \sum_{k\in N, j>k} y_{kj}(\mathbf{e})}{h^{q^*}} \right\rceil$ and $w_j^q(\mathbf{e}) := 0$ for all $q \in Q \setminus \{q^*\}$.

Consider the following valid inequalities for UP

$$\sum_{i\in M} x_{ij} \le \max_{r\in R}\{n^r\} \left(\sum_{t\in T} m^t z_j^t - \sum_{q\in Q} w_j^q \right) \quad (j \in N), \tag{5.18}$$

obtained using inequalities (5.8) and (5.9). The second subproblem, denoted $\overline{\overline{\mathrm{UP}}}(\mathbf{w}(\mathbf{e}), \mathbf{z}(\mathbf{e}))$, is given below.

$$\overline{\overline{\mathrm{UP}}}(\mathbf{w}(\mathbf{e}), \mathbf{z}(\mathbf{e})) : \min_{\mathbf{x}} \left\{ \sum_{i \in M} \sum_{j \in N} (c_{ij} - \theta_{ij}) x_{ij} : (5.6), (5.18), \mathbf{x} \in \mathbf{R}_+^{|M||N|} \right\}. \qquad (5.19)$$

The purpose of this subproblem is to connect the users to switches at minimum cost considering the base types chosen (given by vector $\mathbf{z}(\mathbf{e})$) and the slots used for the backbone network (given by vector $\mathbf{w}(\mathbf{e})$). This subproblem is an instance of the linear assignment problem, and to solve it we use the algorithm LAPJV proposed by Jonker and Volgenant (1987). LAPJV is considered the best method for solving linear assignment problems on dense graphs (see Kennington and Wang, 1991). Let $\mathbf{x}(\mathbf{e})$ be the solution of the second subproblem. The vector $\mathbf{v}(\mathbf{e})$ is found as follows. Set $r^* := \arg\max_{r \in R}\{n^r\}$, and for each site $j \in N$, set $v_j^{r^*}(\mathbf{e}) := \left\lceil \frac{\sum_{i \in M} x_{ij}(\mathbf{e})}{n^{r^*}} \right\rceil$ and $v_j^r(\mathbf{e}) := 0$ for all $r \in R \setminus \{r^*\}$.

5.4.2 Solution Space and Neighborhood Structure

Let E be the set of all possible states for the sites of the network, including those states that do not correspond to feasible solutions of the model. For a given state of the network sites $\mathbf{e} \in E$, let $\mathbf{v}(\mathbf{e})$, $\mathbf{w}(\mathbf{e})$, $\mathbf{x}(\mathbf{e})$ and, $\mathbf{y}(\mathbf{e})$ be the solution of UP($\mathbf{e}$). The solution space is thus $\Omega = \{(\mathbf{u}(\mathbf{e}), \mathbf{v}(\mathbf{e}), \mathbf{w}(\mathbf{e}), \mathbf{x}(\mathbf{e}), \mathbf{y}(\mathbf{e}), \mathbf{z}(\mathbf{e}))\}_{\mathbf{e} \in E}$.

Let $\omega \in \Omega$ be a solution. $\mathcal{N}(\omega)$ is called the neighborhood of ω and consists of the solutions obtained by modifying the state of a given site in the current solution. The number of possible modifications is $|N||T|$.

5.4.3 Tabu Moves and Aspiration Criterion

Each move of tabu search consists of modifying the state of a given site in the current solution. At each iteration of the search, we determine the best move (among the $|N||T|$ moves) while taking into account the tabus as well as the aspiration criterion described in the next paragraph. The chosen site (i.e., the site for which the best move is obtained), is declared tabu for a number of iterations that is randomly determined according to a uniform discrete distribution on the interval $[5, 10]$.

The aspiration criterion states that if the use of tabu site j allows us to discover a solution better than any other found so far, we may remove the tabu from site j.

5.4.4 Algorithm TS

We now proceed to a detailed description of TS.

Algorithm TS

Step 1: (Initial solution)

Find an initial solution using IH.

Repeat Steps 2 to 3 for 20 iterations

Step 2: (Exploring the neighborhood)

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \rightarrow \mathbf{e}'$, which modifies the state of a given site in the current solution, we solve UP($\mathbf{e}'$). The cost of a solution is given by the objective function (5.5) of model UP.

2.2 Determine the number of iterations according to a uniform distribution on the interval $[5, 10]$ for which the chosen site is tabu.

Step 3: (Best solution update)

If the current solution cost is less than the best solution found so far, update this best solution.

5.5 The Post-Optimization Algorithm

The major drawback with algorithm TS is that port and multiplexer costs are not optimized. Therefore, in this section, we propose a post-optimization algorithm, called PO, that optimize these costs. PO incorporates features of the tabu algorithm proposed by Chamberland, Sansó and Marcotte (1997a) for the topological design problem of two-level networks with modular switches (but with a single base type and without multiplexers in the backbone network).

The following notation and definitions will be used to describe PO. Let $\hat{\mathbf{u}}$ and $\hat{\mathbf{z}}$ be respectively the vectors $\mathbf{u}$ and $\mathbf{z}$ found using algorithm TS, and let $S_j = \{1, ..., \sum_{t \in T} m^t \hat{z}_j^t\}$ be the set of slots available at site $j \in N$ if $\hat{u}_j = 1$, and $S_j = \emptyset$ if $\hat{u}_j = 0$. Let f_{js} be the state of slot $s \in S_j$ such that $f_{js} = 0$ if there is no port installed in the slot, $f_{js} = k$ (for $k \in R$), if a port of type $k \in R$ is installed in the slot, and $f_{js} = |R| + k$ (for $k \in Q$), if a port of type $k \in Q$ is installed in the slot. Let $\mathbf{f} = \{f_{js}\}_{s \in S_j, j \in N}$ be the state of the network slots. Therefore, when the vector $\mathbf{f}$ is fixed, the variables $\mathbf{v}$ and $\mathbf{w}$ are also fixed. Let $\mathbf{v}(\mathbf{f})$ and $\mathbf{w}(\mathbf{f})$ be the vectors $\mathbf{v}$ and $\mathbf{w}$ respectively when the network slots state $\mathbf{f}$ is fixed.

In the next subsection we propose a decomposition approach to solve UP($\mathbf{f}$), i.e., the model UP when the decision vectors $\mathbf{v}$, $\mathbf{w}$, $\mathbf{u}$ and $\mathbf{z}$ are respectively set to the vectors $\mathbf{v}(\mathbf{f})$, $\mathbf{w}(\mathbf{f})$, $\widehat{\mathbf{u}}$ and $\widehat{\mathbf{z}}$.

5.5.1 Solving UP(f)

When vectors $\mathbf{v}$, $\mathbf{w}$, $\mathbf{u}$ and $\mathbf{z}$ are respectively equal to the vectors $\mathbf{v}(\mathbf{f})$, $\mathbf{w}(\mathbf{f})$, $\widehat{\mathbf{u}}$ and $\widehat{\mathbf{z}}$, UP can be decomposed into two subproblems. The first subproblem, denoted $\overline{\mathrm{UP}}(\widehat{\mathbf{u}}, \mathbf{w}(\mathbf{f}))$, is given below.

$$\overline{\mathrm{UP}}(\widehat{\mathbf{u}}, \mathbf{w}(\mathbf{f})) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} (d_{jk} - \lambda_{jk}) y_{jk} : (5.10), (5.11), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \quad (5.20)$$

The purpose of this first subproblem is to connect the switches chosen (given by vector $\widehat{\mathbf{u}}$) at minimum cost in order to form a backbone network, whose topology is given by constraints (5.11), considering the port types chosen for the backbone network (given by vector $\mathbf{w}(\mathbf{f})$). Let $\mathbf{y}(\mathbf{f})$ be the (exact or heuristic) solution of this first subproblem.

The second subproblem, denoted $\overline{\overline{\mathrm{UP}}}(\mathbf{v}(\mathbf{f}))$, is given below.

$$\overline{\overline{\mathrm{UP}}}(\mathbf{v}(\mathbf{f})) : \min_{\mathbf{x}} \left\{ \sum_{i \in M} \sum_{j \in N} (c_{ij} - \theta_{ij}) x_{ij} : (5.6), (5.9), \mathbf{x} \in \mathbb{R}_+^{|M||N|} \right\}. \quad (5.21)$$

The purpose of this subproblem is to connect the users to switches at minimum cost considering the port types chosen for the access network (given by vector $\mathbf{v}(\mathbf{f})$). This subproblem is an instance of the linear assignment problem, and to solve it we use the algorithm LAPJV. The solution of this second subproblem is denoted $\mathbf{x}(\mathbf{f})$.

5.5.2 Solution Space and Neighborhood Structure

Let F be the set of all possible states for the slots of the network, including those states that do not correspond to feasible solutions of the model. Let $\mathbf{x}(\mathbf{f})$ and $\mathbf{y}(\mathbf{f})$ be the solution of UP(f). The solution space is thus $\Psi = \{\hat{\mathbf{u}}, \mathbf{v}(\mathbf{f}), \mathbf{w}(\mathbf{f}), \mathbf{x}(\mathbf{f}), \mathbf{y}(\mathbf{f}), \hat{\mathbf{z}}\}_{\mathbf{f}\in F}$.

Let $\psi \in \Psi$ be a solution. $\mathcal{N}(\psi)$ is called the neighborhood of ψ and consists of the solutions obtained by modifying the state of a given slot in the current solution. The total number of possible moves is $(|R| + |Q|) \sum_{j\in N} |S_j|$, but it is not necessary to perform all the moves for exploring the solutions in the neighborhood. Indeed, in the current solution, it suffices to modify the state of a unique slot among all the slots of a site that have the same state. Hence at most $|R| + |Q| + 1$ slots are considered at each site, and we avoid generating a given neighbor several times.

5.5.3 Tabu Moves and Aspiration Criterion

Each move of the tabu search consists of modifying the state of a given slot in the current solution. Once the slot is chosen, it is declared tabu for a number of iterations randomly determined according to a uniform discrete distribution on the interval $[1, 5]$.

The aspiration criterion states that if the use of a tabu slot allows us to discover a solution better than any other found so far, we may remove the tabu from this slot.

5.5.4 Algorithm PO

We now proceed to a detailed description of PO.

Algorithm PO

Step 1: (Initial solution)

Find an initial solution using TS.

Repeat Steps 2 to 3 for 20 iterations

Step 2: (Exploring the neighborhood)

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{f} \rightarrow \mathbf{f}'$, which modifies the state of a given slot in the current solution, we solve UP($\mathbf{f}'$). The cost of a solution is given by the objective function (5.5) of model UP.

2.2 Determine the number of iterations according to a uniform distribution on the interval $[1, 5]$ for which the chosen slot is tabu.

Step 3: (Best solution update)

If the current solution cost is less than the best solution found so far, update this best solution.

5.6 Ring Backbone Topology

Several topologies may be used for the backbone network (see Chamberland and Sansó, 1997a, 1997b). In this section we adapt the model and the proposed algorithms to backbone networks having a ring topology.

In what follows, UPR denotes the version of UP in which the backbone network has a ring topology, i.e., the version in which the general constraints (5.11) are replaced by the following constraints.

$$\sum_{\substack{k \in N \\ j<k}} y_{jk} + \sum_{\substack{k \in N \\ j>k}} y_{kj} = 2u_j \quad (j \in N) \tag{5.22}$$

$$\frac{1}{2}\left(\sum_{j \in H} \sum_{\substack{k \in N \setminus H \\ j<k}} y_{jk} + \sum_{j \in H} \sum_{\substack{k \in N \setminus H \\ j>k}} y_{kj}\right) + (1-u_l) + (1-u_m) \geq 1$$
$$(H \subset N; l \in H; m \in N \setminus H; 3 \leq |H| \leq |N| - 3). \tag{5.23}$$

These constraints were proposed by Balas (1989), in the context of the Prize-Collecting Traveling Salesman Problem (PCTSP) (for further details concerning the PCTSP see Balas, 1986, 1989, 1995 and Fischetti and Toth, 1988). They were also used in Chamberland, Sansó and Marcotte (1997a, 1997b), Chamberland and Sansó (1997a) and Gendreau, Labbé and Laporte (1995). Constraints (5.22) require that the number of OC-192 links (for the backbone network) connected to each switch be exactly two and (5.23) are connectivity constraints.

Proposition 1. UPR is $\mathcal{NP}$-hard.

Proof. Transformation from the Traveling Salesman Problem (TSP) (for further details concerning the TSP see Lawler, Lenstra, Rinnooy Kan and Shmoys, 1985). □

The next proposition gives a lower bound on the number of switches to be installed in the updated network.

Proposition 2. The following inequality

$$\sum_{j\in N} u_j \geq \left\lceil \frac{|M|}{\max_{r\in R}\{n^r\}\left(\max_{t\in T}\{m^t\} - \left\lceil\frac{2}{\max_{q\in Q}\{h^q\}}\right\rceil\right)} \right\rceil \tag{5.24}$$

is valid for UPR.

Proof. If we sum on $j \in N$ the two sides of (5.9) and use (5.6), we obtain the following inequality.

$$\begin{aligned} |M| &\leq \sum_{j\in N}\sum_{r\in R} n^r v_j^r \\ &\leq \max_{r\in R}\{n^r\} \sum_{j\in N}\sum_{r\in R} v_j^r. \end{aligned}$$

Using inequality (5.8), we obtain

$$|M| \leq \max_{r\in R}\{n^r\} \sum_{j\in N}\left(\sum_{t\in T} m^t z_j^t - \sum_{q\in Q} w_j^q\right). \tag{5.25}$$

With (5.10) and (5.22), we obtain

$$\begin{aligned} 2u_j &\leq \sum_{q\in Q} h^q w_j^q \\ &\leq \max_{q\in Q}\{h^q\} \sum_{q\in Q} w_j^q \end{aligned}$$

Then

$$\sum_{q\in Q} w_j^q \geq \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil u_j. \tag{5.26}$$

With (5.26) and (5.25), we obtain

$$|M| \leq \max_{r\in R}\{n^r\} \sum_{j\in N}\left(\sum_{t\in T} m^t z_j^t - \left\lceil\frac{2}{\max_{q\in Q}\{h^q\}}\right\rceil u_j\right)$$

$$\leq \max_{r\in R}\{n^r\} \sum_{j\in N} \left(\max_{t\in T}\{m^t\} \sum_{t\in T} z_j^t - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil u_j\right)$$

$$= \max_{r\in R}\{n^r\} \left(\max_{t\in T}\{m^t\} - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil\right) \sum_{j\in N} u_j. \quad (5.27)$$

Finally

$$\sum_{j\in N} u_j \geq \frac{|M|}{\max_{r\in R}\{n^r\} \left(\max_{t\in T}\{m^t\} - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil\right)}. \quad (5.28)$$

The proposition follows because $\sum_{j\in N} u_j$ is an integer in all feasible solutions of UPR. □

The following proposition gives the feasibility conditions for UPR.

Proposition 3. UPR is feasible if and only if $|N| \geq 3$ and the following inequality

$$|M| \leq \max_{r\in R}\{n^r\} \left(\max_{t\in T}\{m^t\} - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil\right) |N|, \quad (5.29)$$

is respected.

Proof. (⇒) If UPR is feasible, inequality (5.27) is respected for every feasible solution of UPR and since

$$|M| \leq \max_{r\in R}\{n^r\} \left(\max_{t\in T}\{m^t\} - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil\right) \sum_{j\in N} u_j$$

$$\leq \max_{r\in R}\{n^r\} \left(\max_{t\in T}\{m^t\} - \left\lceil \frac{2}{\max_{q\in Q}\{h^q\}} \right\rceil\right) |N|,$$

inequality (5.29) is then respected for all feasible solutions and $|N| \geq 3$, because every ring backbone network contains at least three switches. (⇐) Suppose that inequality (5.29) is respected and $|N| \geq 3$. Then, if we install $|N|$ bases of capacity equal to $\max_{t\in T}\{m^t\}$ and interconnect it with a ring backbone network using ports of maximum rate, the maximum number of users that can be connected to the switches (given by the right-hand side of (5.29)) is greater or equal to $|M|$. Consequently, a feasible solution can be constructed, so UPR is feasible. □

In the following, we adapt the proposed algorithms to find a good solution for UPR.

5.6.1 The Initial Heuristic

The heuristic IH adapted for UPR is called IHR. Step 1 of IHR consists of verifying if UPR is feasible using Proposition 3. Step 2.4 consists of connecting switches in E_N with OC-192 links to form a minimum cost ring if $\Phi(j) \geq 2$ for all $j \in E_N$. This is a TSP on the node set E_N with symmetric costs ($d_{jk} - \lambda_{jk}$ for all $j < k$ and $j,\ k \in E_N$). To solve this problem, we use the composite heuristic GENIUS proposed by Gendreau, Hertz and Laporte (1992).

5.6.2 The Tabu Algorithm

The heuristic TS adapted for UPR is called TSR. With the ring topology, the first subproblem of UP(e) becomes

$$\overline{\text{UPR}}(\mathbf{u}(\mathbf{e}), \mathbf{z}(\mathbf{e})) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j<k}} (d_{jk} - \lambda_{jk}) y_{jk} : (5.16), (5.22), (5.23), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{5.30}$$

The purpose of the above subproblem is to connect the switches through a ring. It is therefore a TSP with symmetric costs if $\sum_{j \in N} u_j \geq 3$. Otherwise, the subproblem is infeasible. We solve this subproblem with the GENIUS composite heuristic.

5.6.3 Post-Optimization Algorithm

The heuristic PO adapted for UPR is called POR. With the ring topology, the first subproblem of UP(f) becomes

$$\overline{\text{UPR}}(\hat{\mathbf{u}}, \mathbf{w}(\mathbf{f})) : \min_{\mathbf{y}} \left\{ \sum_{j \in N} \sum_{\substack{k \in N \\ j < k}} (d_{jk} - \lambda_{jk}) y_{jk} : (5.10), (5.22), (5.23), \mathbf{y} \in \mathrm{B}^{\frac{|N|}{2}(|N|-1)} \right\}. \tag{5.31}$$

The purpose of the above subproblem is to connect the switches through a ring. It is therefore a TSP with symmetric costs if $\sum_{j \in N} u_j \geq 3$ and

$$\sum_{\substack{s \in S_j \\ |R|+1 \leq e_{js} \leq |R|+|Q|}} h^{e_{js}-|R|} \geq 2$$

for all $j \in N$ such that $\hat{u}_j = 1$. Otherwise, the subproblem is infeasible. This subproblem is solved with the GENIUS composite heuristic.

5.7 Experimental Results

In this section we present and analyze the results obtained with IHR, TSR and POR. We first present an illustrative example of the design and the update of a network in Section 5.7.1, followed by a systematic study of the performance of the proposed heuristics in Section 5.7.2.

The proposed algorithms have been implemented in C language and tested on a Sun Ultra 1 (model 140) workstation. The costs used for all the examples are the following. We should mention that the monetary unit used in this paper, denoted \$, is an arbitrary unit. OC-3 and OC-192 links cost \$1000/km and \$3000/km respectively (including the installation cost). Two port models are used to link the users to the network. The cost of the first model, of type OC-3-1, was set at $1000 whereas the cost of the second model, of type OC-3-4, including a multiplexer with four OC-3 inputs, is $3000. Two port models are used to interconnect the switches. The cost of the first model, of type OC-192-1, was set at $10 000 whereas the cost of the second model, of type OC-192-2, including a multiplexer with two OC-192 inputs, is $15 000. With respect to the switches, we assume that there are three base models; their main features are presented in Table 5.1. Moreover, installing and removing a base cost, respectively, $5000 and $1000 (which we assume to be equal for all sites and base models). The cost of installing and removing the ground structure were respectively set to $5000 and $1000 (which we assume to be equal for all sites). Finally, the cost of removing the different access link types is 500$/km.

Tableau 5.1: Features of the base models

	Model A	Model B	Model C
Capacity	16	24	32
Cost	$140 000	$190 000	$240 000

All results were compared with lower bounds obtained as follows. The bound for UPR, noted LB (Lower Bound), was obtained by using the CPLEX Mixed Integer Optimizer (for more information about CPLEX see the CPLEX user's manual, 1993) to solve UPR with the objective function linearized by using additional variables and constraints, and without connectivity constraints (5.23) but with valid inequality (5.24) and relaxing the integrality constraints on all binary variables exept for the $\mathbf{z}$ variables.

5.7.1 An Illustrative Example

In our example, the initial number of users is 200, and the number of potential switch sites is 10. Each user and potential switch site was located in the square of side length 100 km following a uniform distribution law. In Figure 5.1 we present the geographical location of users and potential switch sites.

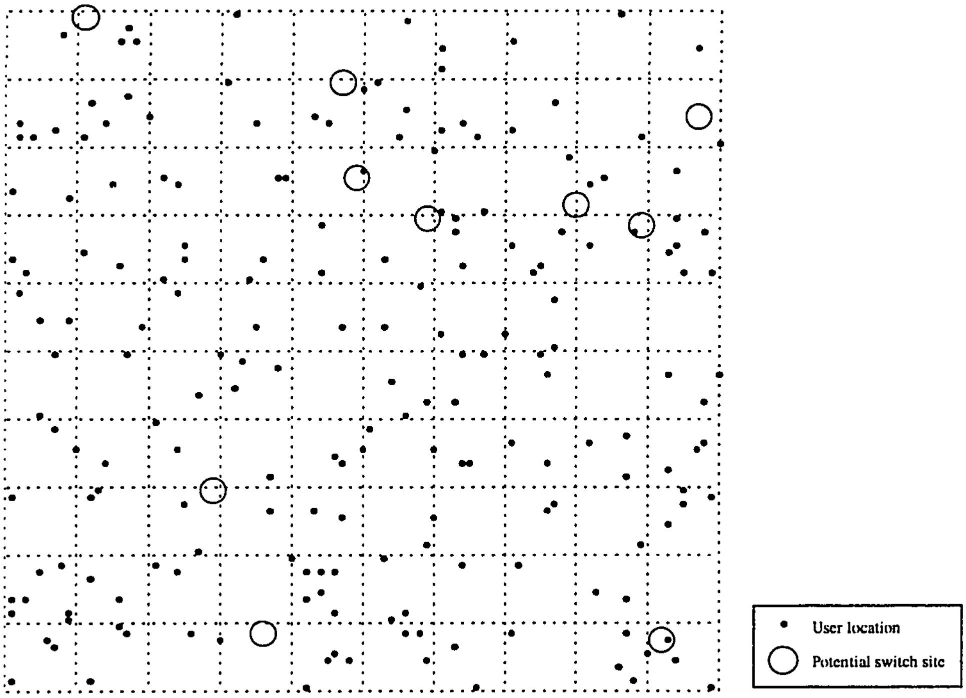
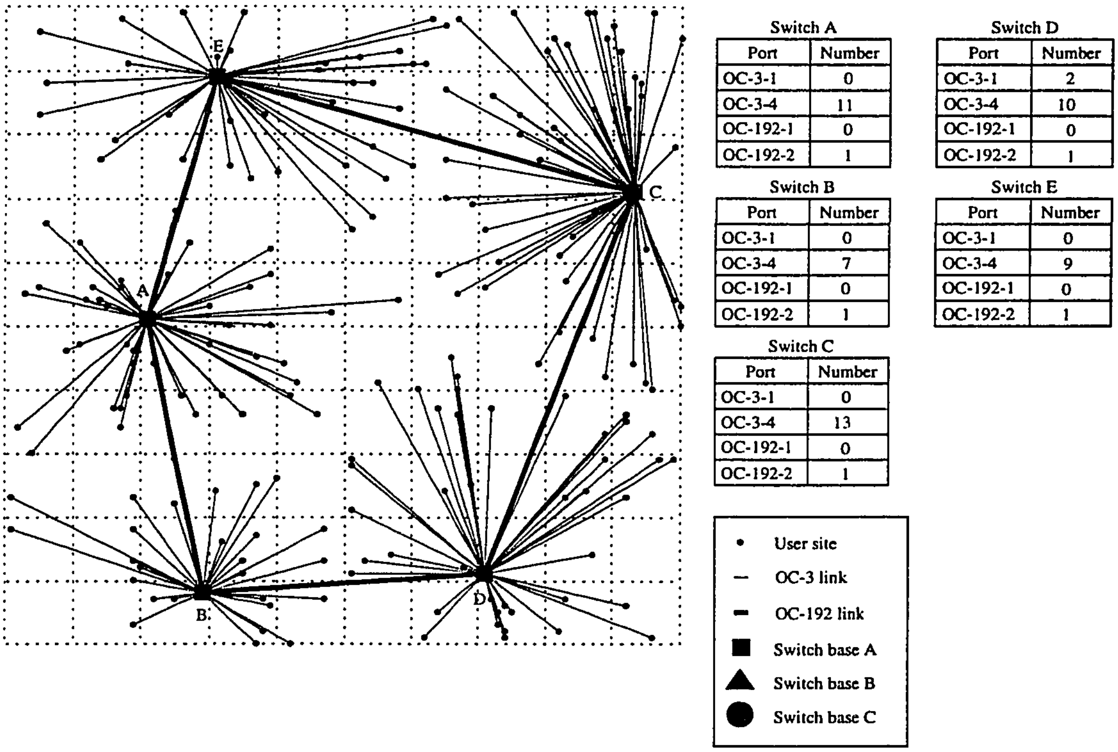


Figure 5.1: Users and potential switch sites location

The value of the lower bound for this example is \$5 524 000, obtained by exploring 4 branch-and-bound nodes in 2.57 seconds. The solution obtained with IHR took 4.56 seconds with a cost of \$6 487 700 (17.45% from the lower bound), the solution obtained by TSR took 78.22 seconds, but with a cost of \$5 530 000 (0.11% from the lower bound) and the post-optimization phase took 18.12 seconds and found a solution of cost equal to \$5 526 300 (0.04% from the lower bound). For this solution, illustrated in Figure 5.2, 68.56% of the total cost is due to the access links, 13.76% due to the backbone links, 13.57% due to the bases (including the ground structures) and 4.11% due to the ports and multiplexers.



Switch A

Port	Number
OC-3-1	0
OC-3-4	11
OC-192-1	0
OC-192-2	1

Switch B

Port	Number
OC-3-1	0
OC-3-4	7
OC-192-1	0
OC-192-2	1

Switch C

Port	Number
OC-3-1	0
OC-3-4	13
OC-192-1	0
OC-192-2	1

Switch D

Port	Number
OC-3-1	2
OC-3-4	10
OC-192-1	0
OC-192-2	1

Switch E

Port	Number
OC-3-1	0
OC-3-4	9
OC-192-1	0
OC-192-2	1

Figure 5.2: Initial network obtained by POR

Figure 5.3 presents the geographical location of 200 new users and 10 new potential switch sites with the assumption that 10% of the current users (20 users) will leave the network. The value of the lower bound for this example is \$4 074 500, obtained by exploring 69 branch-and-bound nodes in 39.11 seconds. The solution obtained with IHR took 25.78 seconds with a cost of \$5 618 500 (37.89% from the lower bound), the solution obtained by TSR took 268.80 seconds, but with a cost of \$4 107 300 (0.81% from the lower bound). We present in Figure 5.4, the value of the best solution found by TSR, as function of the number of iterations, and where the lower bounds are indicated in dashed lines. The figure show that TSR found a good solution in a few iterations. More precisely, TSR found its best solution at iteration seven. The post-optimization phase took 143.25 seconds and found a solution of cost equal to $4 101 700 (0.67% from the lower bound). Therefore, the tabu algorithm improves the initial solution by 26.90%, and the post-optimization phase improves only by 0.14% the solution found by the tabu algorithm. The best solution illustrated in Figure 5.5 shows that the updated network contains two new switch sites. For this solution, 79.34% of the total update cost of the network is due to the access links, 7.96% due to the backbone links, 8.68% due to the bases (including the ground structures) and 4.02% due to the ports and multiplexers. Then, the highest costs are due to the access links in the design and also in the update process. This confirms by practitioners in the field that the access network is responsible for most network costs (see Yan and Beshai, 1995).

5.7.2 Performance Analysis of the Heuristics

The results of a systematic set of experiments designed to assess the performance of the proposed algorithms are now given. Fourty update test problems were randomly generated as follows. First, fourty initial networks, with different number of users and switch sites, were found similar to that of Figure 5.2. Second, for each initial network, we generate an update problem with $|M_N|$ points corresponding to the new clients' locations and $|N_N|$ points corresponding to the candidate switch sites were generated in the square of side length 100 km following a uniform distribution. We display in Table 5.2 the dimensions of the test problems. The first column identifies the problem, the second presents the number of users in the current network, and the third column shows the number of switch sites in the

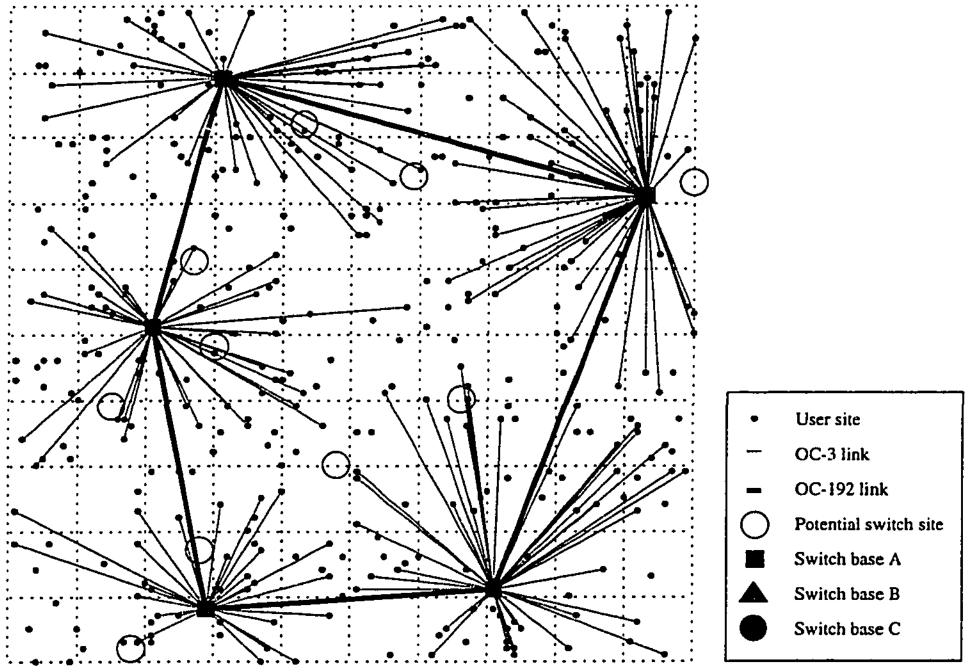


Figure 5.3: New users and new potential switch sites location

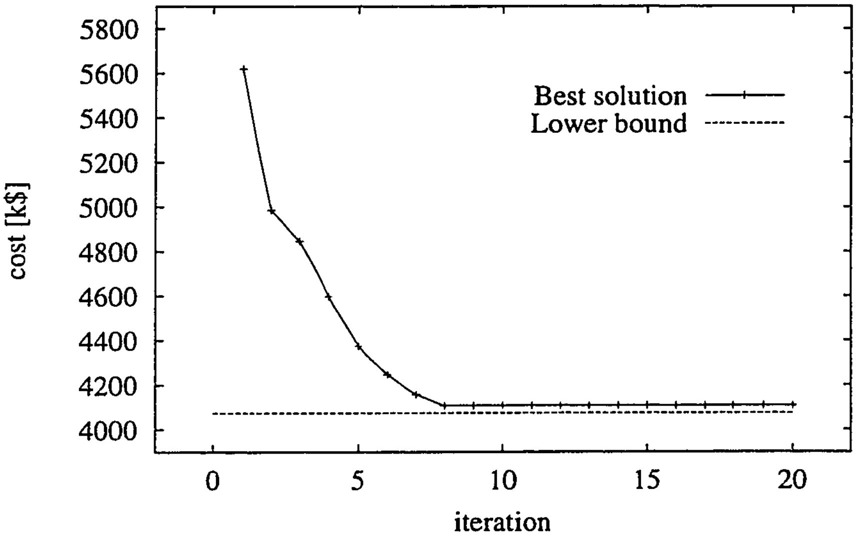
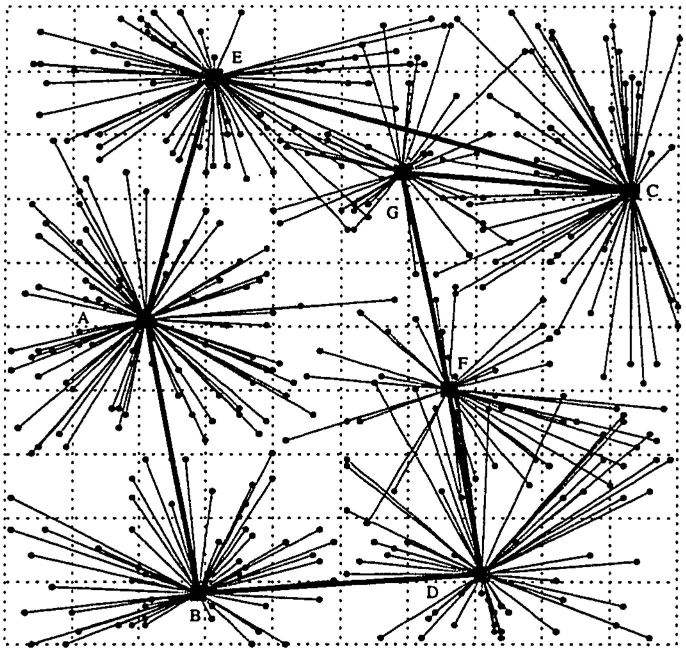


Figure 5.4: Value of the best solution found for the update problem by heuristic TSR as a function of the number of iterations



Switch A

Port	Number
OC-3-1	2
OC-3-4	20
OC-192-1	0
OC-192-2	1

Switch B

Port	Number
OC-3-1	0
OC-3-4	14
OC-192-1	0
OC-192-2	1

Switch C

Port	Number
OC-3-1	1
OC-3-4	14
OC-192-1	0
OC-192-2	1

Switch D

Port	Number
OC-3-1	0
OC-3-4	15
OC-192-1	0
OC-192-2	1

Switch E

Port	Number
OC-3-1	0
OC-3-4	15
OC-192-1	0
OC-192-2	1

Switch F

Port	Number
OC-3-1	0
OC-3-4	9
OC-192-1	0
OC-192-2	1

Switch G

Port	Number
OC-3-1	2
OC-3-4	7
OC-192-1	0
OC-192-2	1

- • User site
- — OC-3 link
- ▬ OC-192 link
- ■ Switch base A
- ▲ Switch base B
- ● Switch base C

Figure 5.5: Updated network obtained by POR

current network. Columns 4 and 5 present the number of new users and the number of new potential switch sites, respectively.

Tableau 5.2: Dimensions of the test problems

Problem	$\|M_C\|$	$\|N_C\|$	$\|M_N\|$	$\|N_N\|$
P_1	50	3	50	10
P_2	50	3	50	20
P_3	50	3	100	10
P_4	50	3	100	20
P_5	50	3	150	10
P_6	50	3	150	20
P_7	50	3	200	10
P_8	50	3	200	20
P_9	100	4	50	10
P_{10}	100	3	50	20
P_{11}	100	4	100	10
P_{12}	100	4	100	20
P_{13}	100	3	150	10
P_{14}	100	3	150	20
P_{15}	100	5	200	10
P_{16}	100	3	200	20
P_{17}	150	5	50	10
P_{18}	150	5	50	20
P_{19}	150	5	100	10
P_{20}	150	4	100	20
P_{21}	150	6	150	10
P_{22}	150	5	150	20
P_{23}	150	4	200	10
P_{24}	150	5	200	20
P_{25}	200	6	50	10
P_{26}	200	6	50	20
P_{27}	200	6	100	10
P_{28}	200	5	100	20
P_{29}	200	5	150	10
P_{30}	200	6	150	20
P_{31}	200	7	200	10
P_{32}	200	8	200	20
P_{33}	250	8	50	10
P_{34}	250	7	50	20
P_{35}	250	6	100	10
P_{36}	250	5	100	20
P_{37}	250	6	150	10
P_{38}	250	7	150	20
P_{39}	250	7	200	10
P_{40}	250	6	200	20

The first results are presented in Table 5.3 where the first column indicates the problems considered, and columns 2 and 3 give results for the lower bound, where OBJ indicates the value of the objective function which is the value of the lower bound, and CPU indicates the CPU execution time. The other columns show results for the algorithms IHR, TSR and POR. For each group of results, CPU indicates the execution time, and GAP indicates the percentage gap between the heuristic solution and the lower bound (with respect to the value of the lower bound). Moreover, at the bottom of the table we present the mean, the minimum and the maximum (MEAN, MIN and MAX) of each column of the table.

Tableau 5.3: Computational results with original base costs

Problem	LB		IHR		TSR		POR	
	OBJ [k$]	CPU [sec]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
P_1	1331.00	1.05	1.30	77.84	74.07	0.56	78.46	0.11
P_2	1368.85	7.24	2.33	90.96	140.38	0.38	144.56	0.09
P_3	2910.50	3.79	4.19	47.40	82.44	0.21	90.52	0.00
P_4	2615.60	22.00	7.41	44.70	174.40	0.29	186.70	0.06
P_5	3997.45	6.26	6.66	39.76	109.01	3.10	142.59	2.97
P_6	3584.90	31.33	12.53	42.94	238.75	1.19	270.11	0.99
P_7	4729.15	3.78	10.37	34.57	164.38	0.17	230.19	0.03
P_8	4612.20	33.02	19.79	37.73	352.20	0.10	412.46	0.05
P_9	1198.10	1.62	4.01	143.66	106.44	0.38	119.15	0.21
P_{10}	1600.60	12.70	7.45	88.27	149.45	0.09	157.56	0.04
P_{11}	2566.20	6.15	7.22	64.18	117.45	0.12	133.76	0.02
P_{12}	2527.80	18.84	12.85	64.85	228.34	0.12	251.04	0.03
P_{13}	3968.50	13.11	10.29	38.77	133.69	0.11	173.65	0.02
P_{14}	3651.80	41.60	19.41	44.30	282.22	0.74	323.19	0.58
P_{15}	4204.25	20.41	17.50	46.66	213.35	0.23	303.97	0.05
P_{16}	4777.90	214.34	28.95	37.43	338.22	1.54	441.33	1.41
P_{17}	1228.45	2.52	9.93	182.16	157.22	0.43	182.58	0.18
P_{18}	1129.55	45.13	13.79	217.56	316.38	0.73	345.73	0.02
P_{19}	2456.85	6.44	12.31	92.95	184.26	0.32	235.50	0.12
P_{20}	2409.85	129.77	20.84	69.45	248.80	0.40	286.52	0.04
P_{21}	2735.15	8.27	18.83	84.45	262.22	0.30	329.95	0.02
P_{22}	3329.80	67.80	32.50	66.38	358.00	0.23	435.51	0.04
P_{23}	4274.65	15.02	21.22	39.76	219.21	0.69	345.01	0.59
P_{24}	4138.20	79.78	41.81	53.55	509.36	0.14	625.11	0.02
P_{25}	907.50	4.18	13.72	295.55	233.38	0.66	294.25	0.44
P_{26}	925.75	12.31	23.24	236.02	363.90	1.22	413.63	0.14
P_{27}	2028.65	14.07	18.81	129.59	248.59	0.33	314.52	0.04
P_{28}	2238.30	57.42	32.49	122.76	364.03	0.27	428.59	0.00
P_{29}	3365.10	19.49	22.48	76.57	254.73	0.13	339.32	0.01
P_{30}	3101.65	176.27	42.89	93.31	679.18	1.45	789.48	1.35
P_{31}	3873.95	35.34	36.18	69.58	415.71	0.33	638.12	0.08
P_{32}	3416.20	120.78	57.06	94.22	814.93	0.26	985.07	0.06
P_{33}	858.80	6.75	21.83	425.69	450.69	0.70	557.57	0.45
P_{34}	886.30	19.21	33.57	266.03	511.15	1.18	602.78	0.21
P_{35}	2530.60	42.38	24.57	113.12	328.45	0.36	473.39	0.05
P_{36}	2400.60	233.30	42.34	72.83	412.83	0.47	530.57	0.20
P_{37}	3439.50	29.06	33.79	82.11	364.97	0.22	540.92	0.01
P_{38}	2844.80	93.28	56.66	110.58	682.56	0.47	913.56	0.18
P_{39}	3836.70	89.49	44.03	65.79	453.95	0.23	680.87	0.00
P_{40}	4048.00	440.13	70.10	69.46	697.79	0.32	933.83	0.16
MEAN	2801.24	54.64	22.93	101.84	310.93	0.53	392.04	0.28
MIN	858.80	1.05	1.30	34.57	74.07	0.09	78.46	0.00
MAX	4777.90	440.13	70.10	425.69	814.93	3.10	985.07	2.97

From this table, the initial heuristic IHR produces, in a short time, solutions that are very far from the lower bound (101.84%, on average) and the tabu-based heuristic TSR produces solutions much closer to the lower bound (0.53%, on average), however, with an increase in CPU time (+288.00 seconds, on average). The algorithm POR produces solutions very close to the lower bound (0.28%, on average) and, for some problems, the gap is even equal to zero. These results show that the proposed heuristics allow us to find quasi-optimal solutions, and sometimes, when the two subproblems are solved exactly, the optimal solution.

Other results are presented in Tables 5.4 and 5.5 where the base costs (presented in Table 5.1) are divided by 10 and 5 respectively, and in Tables 5.6 and 5.7 where the base costs are multiplied by 5 and 10 respectively. It can be gathered from these tables that the mean gap obtained with algorithm POR, decreases when the base costs increased (and the other costs are fixed) before increasing. Without an exact solution method it is difficult to affirm that this behavior is due to the lower bound or to the proposed solution procedure or both. Moreover, in many cases the algorithm IHR finds solutions that are very far from the lower bound, up to 1777.03% for problem P_{33} in Table 5.7. This is due to the fact that IH uses only maximum capacity bases to construct the solution (see Step 2.3 of IH) and this is not, of course, always the better choice. However, IHR provides a good initial solution for the algorithm TSR.

5.8 Concluding Remarks

In this article, we have presented a model for the general two-level network update problem with modular switches, such as are ATM switches. The model includes the update of the access network (with a star topology), the update of the backbone network (with a topology specified by the network planner) and the update of the location and configuration of the switches with respect to ports,

Tableau 5.4: Computational results with base costs divided by 10

Problem	LB		IHR		TSR		POR	
	OBJ [k$]	CPU [sec]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
P_1	1331.00	1.79	1.31	29.15	75.62	0.56	80.02	0.11
P_2	1368.85	68.64	2.30	27.84	158.33	0.38	162.51	0.09
P_3	2696.10	4.19	4.07	19.06	146.32	0.45	187.56	0.16
P_4	2193.70	104.76	7.40	33.15	260.60	7.38	284.04	7.30
P_5	3515.85	3.62	6.70	23.92	205.90	5.12	253.38	5.02
P_6	3043.60	80.87	12.49	25.16	842.86	1.65	935.39	1.34
P_7	4126.75	2.30	10.35	21.24	305.24	0.85	386.67	0.69
P_8	3790.00	67.57	19.76	17.32	1156.76	1.88	1294.03	1.67
P_9	1198.10	1.92	4.02	53.52	103.76	0.38	116.29	0.21
P_{10}	1432.40	8.66	7.43	50.06	179.23	3.76	189.77	3.48
P_{11}	2412.30	2.69	7.26	29.88	135.21	1.76	160.58	1.70
P_{12}	2286.50	98.10	12.71	32.45	409.32	1.60	461.29	1.30
P_{13}	3463.20	3.04	10.26	26.22	271.23	0.22	350.78	0.05
P_{14}	2985.50	80.59	19.31	36.15	623.15	2.97	708.50	2.69
P_{15}	3723.25	5.39	17.38	30.80	434.85	0.74	606.58	0.45
P_{16}	3886.00	93.66	28.74	28.34	969.07	1.37	1183.66	1.11
P_{17}	1228.45	2.70	9.85	87.16	163.82	0.43	189.22	0.18
P_{18}	1106.55	23.11	13.80	87.52	302.47	2.81	331.05	1.99
P_{19}	2179.55	3.65	12.22	58.04	251.70	4.61	327.63	4.21
P_{20}	2036.65	38.91	20.77	42.67	379.42	5.05	452.92	4.51
P_{21}	2657.45	7.40	18.43	32.94	343.05	0.42	438.84	0.06
P_{22}	2771.60	190.26	32.34	47.94	896.74	4.27	1039.67	3.79
P_{23}	3733.75	7.45	21.23	25.29	367.56	1.31	569.82	1.02
P_{24}	3528.40	26.43	41.49	34.76	1246.34	1.47	1533.01	1.16
P_{25}	907.50	2.91	13.48	128.94	220.83	0.66	281.16	0.44
P_{26}	925.75	5.28	23.06	96.02	348.60	1.22	398.05	0.14
P_{27}	1917.75	9.22	18.47	64.02	280.64	0.35	343.09	0.04
P_{28}	2013.70	126.22	32.36	81.19	606.41	0.92	703.39	0.44
P_{29}	3109.30	13.00	22.39	42.47	301.20	0.50	427.68	0.37
P_{30}	2727.05	795.22	42.49	48.58	837.13	2.29	998.18	1.97
P_{31}	3663.05	7.71	35.95	32.17	482.75	1.08	794.28	0.75
P_{32}	3004.40	92.01	56.72	53.71	1520.80	5.42	1917.95	4.96
P_{33}	858.80	5.43	21.80	199.32	435.09	0.70	542.31	0.45
P_{34}	886.30	13.87	32.80	95.43	483.86	1.18	575.46	0.21
P_{35}	2171.20	10.15	24.37	78.76	373.21	3.29	518.19	2.93
P_{36}	1935.20	65.87	42.02	50.70	558.17	6.00	726.04	5.69
P_{37}	3156.00	9.49	33.84	43.72	376.52	0.43	596.28	0.08
P_{38}	2577.70	29.06	56.09	65.37	850.46	0.52	1115.60	0.20
P_{39}	3440.40	29.85	43.35	40.94	618.78	1.15	1137.53	0.63
P_{40}	3504.00	384.87	69.13	49.98	1369.26	1.34	1928.04	0.83
MEAN	2487.34	63.20	22.75	51.80	497.31	1.96	631.16	1.61
MIN	858.80	1.79	1.31	17.32	75.62	0.22	80.02	0.04
MAX	4126.75	795.22	69.13	199.32	1520.80	7.38	1928.04	7.30

Tableau 5.5: Computational results with base costs divided by 5

Problem	LB		IHR		TSR		POR	
	OBJ [k$]	CPU [sec]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
P_1	1331.00	1.55	1.29	34.56	73.65	0.56	78.07	0.11
P_2	1368.85	15.88	2.35	34.85	137.19	0.38	141.37	0.09
P_3	2752.20	3.00	4.08	21.00	138.24	0.33	158.81	0.07
P_4	2277.70	86.20	7.48	32.45	259.75	5.38	279.82	5.34
P_5	3599.85	4.26	6.69	26.02	199.93	4.22	247.45	4.12
P_6	3127.60	62.15	12.51	28.70	723.75	2.39	791.79	2.27
P_7	4210.75	3.10	10.27	22.81	260.73	0.83	342.14	0.68
P_8	3910.90	63.63	19.87	21.06	1121.05	1.49	1227.48	1.31
P_9	1198.10	1.36	4.05	63.53	103.63	0.38	116.10	0.21
P_{10}	1474.40	9.78	7.44	52.29	182.50	1.76	193.01	1.49
P_{11}	2454.30	3.34	7.29	32.55	134.86	0.59	160.38	0.53
P_{12}	2328.50	35.83	12.92	36.25	369.78	1.98	411.29	1.74
P_{13}	3533.20	3.01	10.23	28.48	264.01	0.21	344.54	0.05
P_{14}	3111.50	139.96	19.50	36.03	608.55	1.54	690.04	1.28
P_{15}	3807.25	8.54	17.61	31.70	374.44	0.73	547.71	0.44
P_{16}	4012.00	122.53	29.03	29.09	843.09	0.96	1022.86	0.76
P_{17}	1228.45	2.35	9.96	98.88	165.14	0.43	190.86	0.18
P_{18}	1129.55	24.48	13.91	98.57	301.09	0.73	326.20	0.02
P_{19}	2221.55	4.78	12.33	61.53	245.01	4.27	300.36	3.92
P_{20}	2088.65	52.17	20.80	46.01	337.92	4.38	370.30	3.95
P_{21}	2685.45	7.27	18.56	37.81	284.70	0.42	381.06	0.06
P_{22}	2879.00	80.92	32.80	49.09	849.85	3.84	996.80	3.44
P_{23}	3817.75	9.25	21.65	26.31	334.74	1.16	564.69	0.99
P_{24}	3629.10	37.27	41.86	36.31	1048.80	1.39	1301.67	1.18
P_{25}	907.50	3.40	13.59	147.45	222.61	0.66	283.63	0.44
P_{26}	925.75	6.43	23.31	111.58	351.03	1.22	400.79	0.14
P_{27}	1936.75	9.08	18.53	71.09	254.50	0.35	316.99	0.04
P_{28}	2060.70	153.05	32.67	85.21	473.56	0.62	553.29	0.26
P_{29}	3155.40	13.16	22.58	45.72	305.10	0.24	430.39	0.11
P_{30}	2779.05	168.86	42.62	53.57	756.04	2.07	906.97	1.64
P_{31}	3710.05	8.38	36.18	35.67	437.54	0.66	656.66	0.41
P_{32}	3089.30	165.11	56.62	58.03	1396.06	4.07	1743.59	3.79
P_{33}	858.80	7.32	21.87	224.48	436.26	0.70	542.72	0.45
P_{34}	886.30	12.94	32.79	114.39	483.20	1.18	574.57	0.21
P_{35}	2218.20	11.45	24.59	82.55	372.87	2.59	518.06	2.24
P_{36}	1992.20	100.38	42.04	53.62	447.25	5.34	588.19	5.03
P_{37}	3194.00	15.91	33.24	48.02	381.86	0.42	602.16	0.08
P_{38}	2610.70	34.20	55.17	70.63	787.58	0.52	1053.34	0.20
P_{39}	3528.70	93.38	42.72	42.18	596.86	0.63	1029.18	0.29
P_{40}	3595.90	361.08	67.94	52.15	1205.22	0.42	1690.20	0.08
MEAN	2540.67	48.67	22.77	57.06	456.75	1.55	576.89	1.24
MIN	858.80	1.36	1.29	21.00	73.65	0.21	78.07	0.02
MAX	4210.75	361.08	67.94	224.48	1396.06	5.38	1743.59	5.34

Tableau 5.6: Computational results with base costs multiplied by 5

Problem	LB		IHR		TSR		POR	
	OBJ [k$]	CPU [sec]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
P_1	1331.00	2.60	1.37	294.21	75.10	0.56	79.56	0.11
P_2	1368.85	7.47	2.43	315.74	103.56	0.38	107.79	0.09
P_3	2910.50	5.26	4.13	175.33	70.63	0.21	78.83	0.00
P_4	2625.70	19.93	7.40	157.20	154.81	0.17	161.73	0.02
P_5	4742.05	8.91	6.73	88.01	95.58	0.08	115.18	0.02
P_6	4583.10	58.14	12.42	97.80	142.89	0.16	167.96	0.01
P_7	6123.45	10.59	10.32	81.68	100.22	0.04	130.35	0.02
P_8	6074.80	75.04	19.97	81.08	203.56	0.12	244.76	0.05
P_9	1198.10	1.80	4.16	448.51	101.56	0.38	114.06	0.21
P_{10}	1600.60	30.99	7.39	278.60	155.15	0.09	163.34	0.04
P_{11}	2566.20	5.49	7.27	215.11	108.98	0.12	125.35	0.02
P_{12}	2610.00	56.73	12.66	212.01	191.54	0.11	209.55	0.03
P_{13}	4961.50	61.19	10.41	69.04	125.95	0.03	158.16	0.01
P_{14}	4830.70	96.11	19.12	88.58	228.78	0.09	259.29	0.03
P_{15}	4650.30	68.37	17.33	150.41	174.17	0.00	218.22	0.00
P_{16}	6391.00	496.59	28.64	61.02	269.72	0.46	338.42	0.41
P_{17}	1228.45	2.81	10.02	572.90	152.75	0.43	178.23	0.18
P_{18}	1129.55	27.85	13.63	568.27	223.50	0.73	248.39	0.02
P_{19}	3016.85	18.82	12.23	232.55	162.56	0.26	213.49	0.10
P_{20}	3098.25	140.91	20.53	155.74	243.66	2.63	281.24	2.34
P_{21}	2735.15	15.01	18.27	290.61	200.65	0.30	270.44	0.02
P_{22}	3431.10	97.13	32.09	186.41	292.35	0.00	322.98	0.00
P_{23}	5632.35	46.92	21.07	96.18	179.30	0.12	275.37	0.03
P_{24}	4989.00	205.19	41.07	145.84	387.80	0.12	474.00	0.00
P_{25}	907.50	11.12	13.43	873.50	182.71	0.66	243.58	0.44
P_{26}	925.75	22.07	23.01	809.36	271.93	1.22	321.47	0.14
P_{27}	2028.65	14.58	18.28	389.09	198.93	0.33	275.79	0.04
P_{28}	2998.30	101.08	32.44	207.14	320.62	0.20	385.39	0.00
P_{29}	4125.10	36.29	22.19	169.46	205.17	0.11	289.44	0.01
P_{30}	3834.75	235.97	42.08	185.59	436.66	0.29	578.80	0.03
P_{31}	3880.65	14.13	35.53	244.30	320.99	0.17	502.95	0.08
P_{32}	3614.50	208.89	56.43	274.05	629.42	0.25	824.46	0.03
P_{33}	858.80	10.35	21.65	1099.94	283.67	0.70	390.44	0.45
P_{34}	886.30	36.26	32.57	1024.24	511.60	1.18	603.29	0.21
P_{35}	3256.40	43.56	24.44	238.32	250.00	0.18	352.27	0.00
P_{36}	3277.50	239.05	41.79	173.04	412.51	0.18	501.76	0.00
P_{37}	4199.50	56.51	33.65	185.37	286.18	0.18	456.82	0.01
P_{38}	3104.50	144.59	55.63	299.92	544.20	0.14	704.85	0.05
P_{39}	4596.70	133.67	43.04	184.57	434.55	2.46	678.47	2.30
P_{40}	5186.50	611.13	68.26	141.19	611.50	0.04	773.78	0.04
MEAN	3287.75	86.98	22.63	289.05	251.14	0.40	320.51	0.19
MIN	858.80	1.80	1.37	61.02	70.63	0.00	78.83	0.00
MAX	6391.00	611.13	68.26	1099.94	629.42	2.63	824.46	2.34

Tableau 5.7: Computational results with base costs multiplied by 10

Problem	LB		IHR		TSR		POR	
	OBJ [k$]	CPU [sec]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]	CPU [sec]	GAP [%]
P_1	1331.00	2.78	1.31	564.69	54.77	0.56	59.18	0.11
P_2	1368.85	7.54	2.35	578.74	105.24	0.38	109.48	0.09
P_3	2910.50	6.38	4.10	299.02	47.37	0.21	55.64	0.00
P_4	2625.70	31.53	7.49	294.31	115.63	0.17	122.62	0.02
P_5	5442.05	40.44	6.76	129.98	94.28	0.07	113.77	0.02
P_6	5283.10	245.91	12.58	139.73	175.37	0.14	200.49	0.01
P_7	7073.45	22.66	10.30	111.43	95.39	0.03	125.91	0.02
P_8	7254.80	614.16	20.04	101.25	186.15	0.02	218.04	0.01
P_9	1198.10	7.22	3.94	748.99	67.50	0.38	79.77	0.21
P_{10}	1600.60	21.54	7.40	503.51	117.17	0.09	125.61	0.04
P_{11}	2566.20	6.78	7.23	355.39	81.38	0.12	97.63	0.02
P_{12}	2610.00	36.87	12.70	349.94	146.81	0.11	164.93	0.03
P_{13}	5911.50	79.33	10.25	102.77	95.60	0.74	127.21	0.67
P_{14}	5780.70	312.13	19.39	130.55	190.67	0.08	221.08	0.03
P_{15}	4650.30	47.99	17.36	246.87	136.00	0.00	172.76	0.00
P_{16}	7593.90	313.97	28.76	82.92	267.66	1.28	298.95	1.28
P_{17}	1228.45	3.42	9.85	878.45	100.17	0.43	128.65	0.18
P_{18}	1129.55	50.35	13.79	898.00	165.22	0.73	190.22	0.02
P_{19}	3079.25	35.46	12.26	343.26	134.81	0.12	191.34	0.02
P_{20}	3798.25	456.57	20.67	234.99	243.73	2.72	286.63	2.64
P_{21}	2735.15	20.64	18.48	466.10	170.11	0.30	237.78	0.02
P_{22}	3431.10	124.09	32.43	326.30	277.67	0.00	308.28	0.00
P_{23}	6988.25	92.88	21.19	126.80	179.26	0.08	259.43	0.04
P_{24}	5868.30	571.36	41.61	178.86	339.22	1.09	433.27	1.06
P_{25}	907.50	9.38	13.40	1407.81	141.47	0.66	203.89	0.44
P_{26}	925.75	27.95	23.05	1327.86	239.79	1.22	289.57	0.14
P_{27}	2028.65	28.63	18.45	604.98	175.47	0.33	241.16	0.04
P_{28}	3612.70	224.08	32.49	254.56	285.81	0.00	327.53	0.00
P_{29}	4971.30	76.72	22.45	196.01	201.80	2.18	285.78	2.10
P_{30}	3993.80	490.39	42.49	302.48	391.48	0.10	534.27	0.03
P_{31}	3880.65	47.25	35.74	374.71	300.54	0.17	483.77	0.08
P_{32}	3614.50	156.28	56.58	417.18	511.77	0.25	706.40	0.03
P_{33}	858.80	14.05	21.62	1777.03	210.05	0.70	316.63	0.45
P_{34}	886.30	88.55	32.83	1680.63	294.40	1.18	386.56	0.21
P_{35}	3256.40	26.04	24.33	364.27	224.87	0.18	327.35	0.00
P_{36}	4122.70	376.28	41.99	262.60	396.84	2.69	486.09	2.54
P_{37}	4966.20	73.01	33.48	241.10	287.31	0.21	463.27	0.03
P_{38}	3104.50	180.88	55.84	454.53	493.49	0.14	653.98	0.05
P_{39}	5383.00	309.81	43.34	273.55	409.55	0.11	668.27	0.00
P_{40}	6136.50	550.04	78.05	182.07	633.16	3.06	796.55	3.06
MEAN	3652.71	145.78	22.96	457.86	219.62	0.58	287.49	0.39
MIN	858.80	2.78	1.31	82.92	47.37	0.00	55.64	0.00
MAX	7593.90	614.16	78.05	1777.03	633.16	3.06	796.55	3.06

multiplexers and bases.

Three heuristics have been proposed in order to find a good solution. The first heuristic, IH, produces initial solutions that are very far from optimality (within 131.52%, on average, of the optimal solution for all the problems solved with the different base costs). On the other hand, the second heuristic based on the tabu search principle, TS, produces solutions that are close from optimality (within 1.00%, on average, of the optimal solution). Finally, the third heuristic includes a post-optimization procedure and produces solutions very close to optimality (within 0.74%, on average, of the optimal solution), however, with an increase in CPU time. Therefore, these heursitics allow us to find quasi-optimal solutions, and sometimes, the optimal solution.

The following directions for further research are open. First, in this paper we have proposed a heuristic approach to solve the model. It would be interesting to consider an exact method in order to solve fair size problems and evaluate more accurately the behavior of the proposed heuristics. However, since the proposed model is $\mathcal{NP}$-hard, it is unlikely that large-size problems can be solved, in a short time, to optimality. Second, we plan to consider a more intricate version of our model in which different access rates can be used in the access network. Finally, we plan to use the proposed solution algorithms jointly with a sophisticated link dimensioning and ATM traffic routing procedure (with more realistic assumptions than the ones considered previously in Chamberland, Marcotte and Sansó, 1996).

5.9 Acknowledgments

The authors are grateful to Gilbert Laporte of the Centre for Research on Transportation for having let them use his implementation of the GENIUS algorithm. This work was funded in part by FCAR grants and NSERC strategic grant STR 0166996.

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CHAPITRE 6

Conclusion

Dans ce chapitre, nous présentons les principales contributions de cette thèse et proposons des avenues de recherche liées à ce travail.

6.1 Contributions

Nous avons traité dans cette thèse des problèmes concernant la conception et la mise à jour des réseaux d'entreprises et multitechnologies. Plus précisément, nous avons proposé dans ce travail:

- des modèles de programmation mathématique avec variables mixtes, adaptables pour différentes topologies du réseau de transmission, pour les problèmes de la conception et la mise à jour des réseaux d'entreprises et pour les problèmes de la conception et l'expansion des réseaux multitechnologies;
- des inégalités valides, une analyse de la complexité et des conditions de réalisabilité pour chaque modèle que nous avons adapté à une topologie spécifique pour le le réseau de transmission (par exemple, en arbre, en anneau, en anneau multiple ou en maille);
- des heuristiques initiales (une pour chaque modèle) permettant de trouver des solutions rapidement, mais avec des écarts par rapport à la solution optimale très grands;

- des heuristiques basées sur la recherche avec tabous (une pour chaque modèle) permettant de trouver des solutions avec de faibles écarts par rapport à la solution optimale;
- un algorithme de post-optimisation, basé sur le principe de la recherche avec tabous, pour le problème de la mise à jour des réseaux d'entreprises;
- un modèle (sous formes de contraintes) pour la topologie en anneau multiple;
- une heuristique pour trouver un réseau de transmission en anneau multiple avec contraintes de degré;
- des bornes inférieures (au moins une pour chaque modèle que nous avons adapté à une topologie spécifique), chacune obtenue par la résolution exacte d'une version relaxée du modèle, en utilisant le logiciel d'optimisation CPLEX (1993).

Parmi toutes ces contributions, la plus importante selon nous est le modèle et la méthode de recherche avec tabous pour le problème de la mise à jour des réseaux à deux niveaux. Cela car c'est un problème important, au niveau des applications, qui n'a pas été traité auparavant.

En ce qui concerne les résultats numériques, nous avons montré que les méthodes de résolution approximatives proposées sont très stables et donnent d'excellentes solutions. En fait, parmi tout les problèmes traités dans cette thèse (3000 au total), nous avons trouvé des solutions qui sont en moyenne à moins de 1.16% de l'optimalité et, dans plusieurs cas, nous avons même trouvé la solution optimale.

Selon nous, si les méthodes de recherche avec tabous proposées donnent des bons résultats c'est parce que des déplacements sont faits au niveau du choix de la localisation des commutateurs, ce qui est primordial considérant que la majorité du coût du réseau est pour les liens d'accès. De plus, si la topologie du réseau d'accès serait différente, par exemple, en anneau ou en arbre, les approches de résolution proposées seraient selon nous aussi efficaces en autant que la méthode pour trouver le réseau d'accès soit exacte ou très bonne.

6.2 Recherche future

Plusieurs avenues de recherche s'ouvrent à la suite de cette thèse. Notamment, il serait intéressant de concevoir des algorithmes de résolution exacte pour les modèles proposés dans ce travail, dans le but de résoudre des problèmes de taille raisonnable et pour évaluer plus précisément les heuristiques présentées dans cette thèse. D'autres recherches futures pourraient porter sur le développement de procédures de calcul de bornes inférieures plus performantes (en terme du temps de calcul) que celles proposées. Elles pourraient être utilisées, par exemple, par des algorithmes de résolution exacte.

Une autre avenue de recherche serait de proposer des modèles et des heuristiques adaptables à plusieurs topologies du réseau d'accès. De plus, rappelons que dans cette thèse, la vitesse des liens utilisés dans le réseau de transmission est unique et est choisie par le planificateur. Comme ce choix peut devenir difficile lorsque la topologie du réseau est complexe, nous pourrions utiliser des procédures de dimensionnement et d'acheminement du trafic conjointement avec nos heuristiques pour la conception et la mise à jour des réseaux. Cela ne nécessiterait que des modifications mineures de nos heuristiques.

Suite aux travaux concernant la topologie en anneau multiple, comme le temps de remise en service d'un anneau SONET en cas de panne est proportionnel à sa taille (voir Wu, 1992), il serait intéressant d'ajouter des contraintes supplémentaires limitant la taille des anneaux, soit en terme de la longueur de la circonférence et/ou du nombre de commutateurs dans chaque anneau. De plus, ces contraintes pourraient être utilisées pour augmenter la densité du réseau et, par le fait même, sa fiabilité.

Dans cette thèse, nous avons considéré des réseaux à deux niveaux. Une avenue de recherche intéressante serait de considérer une architecture de réseau à trois niveaux où les usagers serait branchés à des commutateurs d'accès et les commutateurs d'accès serait reliés ensembles avec des commutateurs de plus grande capacité. Ce genre d'architecture est souvent utilisée en pratique dans les réseaux multitechnologies.

Finalement, il serait intéressant de considérer le cas où il y a plusieurs ports sur une carte et une carte par fente, avec certains ports utilisés seulement pour les connexions usagers et d'autres pour les connexions réseaux.

En résumé, la recherche future, faisant suite aux travaux de cette thèse, devrait s'orienter autour des axes suivants:

- la résolution exactes des modèles proposés;
- le développement de procédures de calcul de bornes inférieures plus performantes;
- la généralisation des modèles et heuristiques proposées pour concevoir des réseaux avec différentes topologies pour le réseau d'accès;
- l'utilisation de procédures de dimensionnement et d'acheminement du trafic

conjointement avec nos heuristiques;

- la considération de contraintes supplémentaires pour limiter la taille des anneaux dans la topologie en anneau multiple;
- la considération des réseaux à trois niveaux;
- la considération de plusieurs ports sur une carte et une carte par fente.

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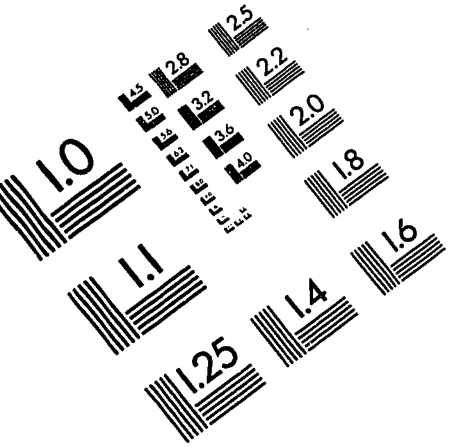
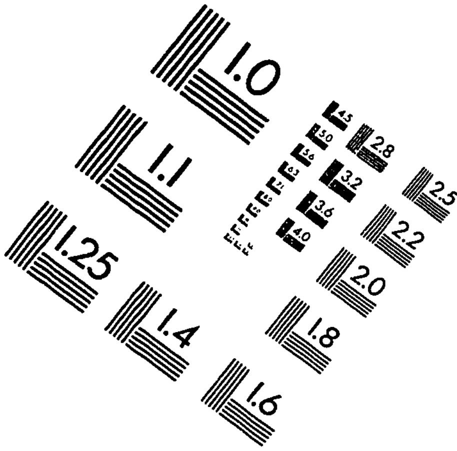
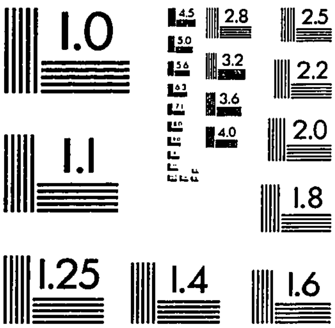
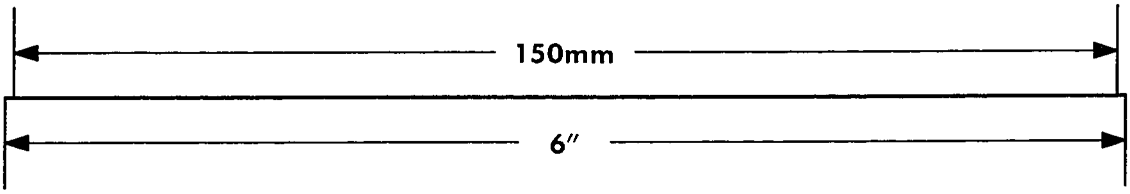
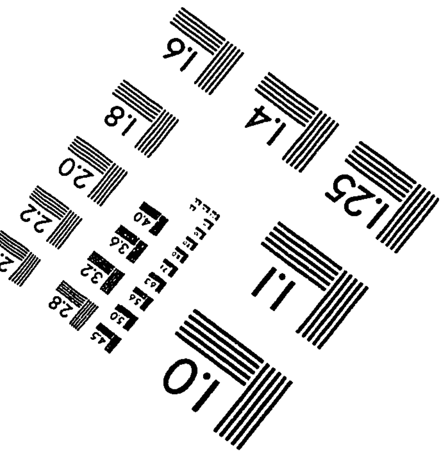
IMAGE EVALUATION TEST TARGET (QA-3)

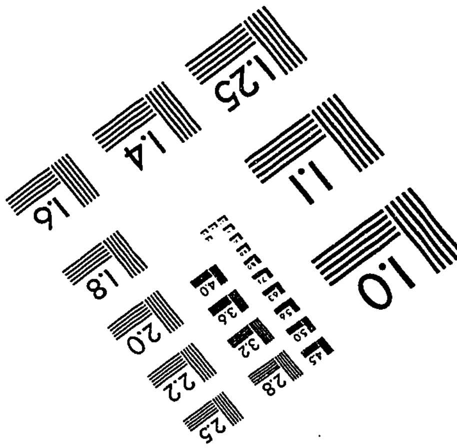












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