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PRICING FOR QOS PROVISIONING ACROSS MULTIPLE INTERNET SERVICE PROVIDER DOMAINS

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Ce mémoire intitulé:

PRICING FOR QOS PROVISIONING ACROSS MULTIPLE INTERNET SERVICE PROVIDER DOMAINS

présenté par: SABERI Soheil

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a été dûment accepté par le jury d'examen constitué de:

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RÉSUMÉ

L'apparition de nouvelles technologies ainsi que les récents progrès dans les vitesses de traitement des ordinateurs et de transmission des données, combinées à l'ubiquité de l'Internet et la déréglementation du marché des télécommunications ont créé de nouvelles conditions de concurrence entre fournisseurs de services d'Internet (ISP, Internet Service Providers). Le succès de nouveaux produits dans ce marché dépend autant de considérations d'ordre technique que d'ordre économique. Dans ce contexte, les ISP désirent pouvoir imposer une tarification spécifique aux applications Internet considérées (e.g. téléphonie IP, vidéo en différé, etc.), et en rapport avec une exigence de qualité de service differenciée. À ce niveau, une problématique double se présente: comment assurer la qualité de service désirée sans pour autant perturber l'ordre établi de l'Internet avec sa grande capacité d'évolution et d'adaptation (protocoles IP) ? Comment gérer le partage des revenus entre opérateurs multiples responsables des différents réseaux à travers lesquels transite le trafic ?

Dans ce mémoire, nous présentons un mécanisme de tarification à utiliser entre un groupe d'ISP et un client désirant faire transiter un trafic important à partir d'une origine fixe et en direction d'une destination donnée (et vice versa). Les ISP ne sont pas directement en contact avec le client qui s'adresse à un tiers (*TP*, Third Party) dont la fonction est de choisir les ISP le long de la route. Le client négocie avec le tiers une courbe dite *courbe de réponse du client* (trafic offert versus prix unitaire payé par le client). Le client paie seulement pour la fraction du trafic qui subit un délai inférieur à un seuil maximal préétabli, et en relation avec les exigences du type de trafic en question. Après avoir prélevé un pourcentage fixe du revenu total, *TP* redistribue la fraction restante entre les ISP selon une formule qui reflète à la fois la performance vis-à-vis une exigence de temps de transit que les ISP individuels déclarent vouloir s'imposer, ainsi que le niveau d'exigence qu'ils se fixent pour eux-mêmes puisqu'il sera une mesure de leur contribution à l'effort global bout à bout. Le niveau de trafic adopté, la performance obtenue,

ainsi que les bénéfices des ISP émergent dans ce schéma comme le résultat d'un jeu de Stackelberg ayant pour leader *TP* et pour suiveurs les différents ISP participants.

ABSTRACT

The emergence of new technologies and the recent advancements in the processing and transmission speeds of computers, combined with the proliferation of the Internet and deregulation of the communication market, have created a new and highly competitive environment for communication service providers. Success in this new market depends almost equally both on economics and technology, making service provider's decision of adopting an efficient pricing scheme conforming to the quickly evolving technologies, among the most important issues of the company. As a result of this, Internet Pricing has been a very attractive area of research during the creation of commercial Internet.

In this thesis, a pricing scheme to be used between a group of Internet service providers (ISPs) and a customer who wishes to initiate a packet flow from a fixed origin to a fixed destination, has been introduced. The ISPs are transparent to the customer who relies on a third party company for both the choice of the relevant ISPs and the unit flow price negotiated. The customer pays only for that portion of the traffic which meets a predefined maximum tolerable total delay within the ISP networks. After taking in a fixed percentage of the total profit, the third party redistributes the remaining benefits to the ISPs according to a sharing mechanism which reflects both the QoS the ISPs declare they will meet, as well as their real performance. The pricing emerges as the result of a Stackelberg game with the third party as the leader and the ISPs as the followers.

CONDENSÉ

0.1 Introduction

Le réseau Internet tel qu'il existe correspond avant tout à un système *meilleur effort* (best effort), c'est-à-dire à un système où la qualité de service est irrégulière et dépend des conditions qui prévalent dans le réseau au moment de son utilisation. Dans un tel contexte, il est difficile de justifier une tarification différenciée des différents services. Cependant, l'ambition des opérateurs est d'offrir précisément une telle facturation pour des services différenciés, à l'image de ce que l'on peut observer depuis fort longtemps déjà dans le contexte des services téléphoniques. Le problème est compliqué d'une part, par le fait que le trafic traverse souvent les domaines de multiples opérateurs indépendants (problèmes de négociation), et d'autre part, par le fait que toute tentative de faire de la réservation de chemin de bout en bout en vue de garantir une qualité de service systématique se heurte en fin de compte à l'obstacle d'explosion de complexité lorsque la taille des réseaux augmente.

L'objectif principal de ce mémoire est le développement d'un schéma de division du revenu entre opérateurs de service Internet sur les réseaux desquels doit transiter le trafic d'un utilisateur donné, dans un contexte d'offre de qualité de service. Nous avons visé un schéma qui soit flexible, et qui encourage les opérateurs à rechercher de bonnes performances pour leurs clients. D'autre part, une contrainte que nous avons cherché à satisfaire est celle d'un schéma le plus décentralisé possible et qui ne requière donc pas une coordination étroite de tous les intervenants de bout en bout, dans le but d'assurer une qualité de service donnée. Ceci, pour pouvoir envisager une implantation à grande échelle sans explosion de complexité. Dans ce contexte, il a fallu opter pour un objectif de qualité de service de type statistique (satisfaction avec un certain niveau de probabilité); en revanche, et dans un souci d'équité, nous avons développé le concept du client qui paie un extra uniquement pour la fraction de son trafic qui est transitée avec la qualité de service requise. Également, dans le contexte de schémas décentralisés recherchés, la théorie des jeux non coopératifs paraissait comme un contexte mathématique naturel. Elle reflète aussi les modes d'opération réels avec des opérateurs qui cherchent à se positionner le mieux possible dans un contexte de concurrence. Enfin, nous avons introduit le concept d'un intermédiaire payant entre le client et les opérateurs. Ce dernier peut être assimilé à un agent de régulation (comme la CRTC au Canada), mais peut également être interprété comme un intermédiaire au sens purement commercial du terme, qui pour une commission consistant en un pourcentage des revenus, va trouver les opérateurs pour un client donné, et définir les conditions de partage des bénéfices entre ces derniers ainsi que le niveau de trafic auquel le système opère. Dans ce sens, il s'agira d'un jeu de Stackelberg avec l'intermédiaire dénoté *TP* (Third Party) comme leader, et les opérateurs candidats le long de la route dénotés ISP (Internet Service Providers) comme suiveurs, recherchant un équilibre de Nash pour chaque niveau de trafic proposé par le leader. Dans ce qui suit, nous résumons le modèle présenté dans le mémoire, ainsi que certains de nos résultats mathématiques et numériques.

0.2 Description du modèle proposé

Le modèle proposé met en jeu trois types d'agents: un client dénoté *C*, un intermédiaire dénoté *TP* et un ensemble d'opérateurs Internet dénotés ISP. Dans notre modèle, *C* est un gros usager Internet en mesure de générer un trafic important (grosse compagnie) à envoyer régulièrement d'une origine donnée A à une destination donnée B, par exemple bureau chef communiquant vers succursale ,et qui fait des démarches auprès d'un intermédiaire spécialisé *TP* à cette fin. *C* spécifie un temps de transit T_{max} bout à bout maximal tolérable pour les paquets qu'il envoie et pour le respect duquel il est prêt à payer une surcharge unitaire. Un exemple de type de trafic illustrant bien notre contexte applicatif est le trafic VoIP. De plus, il existe déjà des régulateurs de marché dans le contexte de VoIP et qui peuvent d'emblée être identifiés à nos intermédiaires *TP*.

Ainsi, la décision Télécom CRTC 2005 - 28, mise de l'avant par la Radio Télévision canadienne et la Commission des télécommunications est un exemple clair d'un ensemble de réglementations qui préconise une organisation pour les services Internet essentiellement identique à celle en place pour les services téléphoniques. Le partage des revenus entre compagnies de téléphone est fondé sur des accords bilatéraux entre couples d'opérateurs. Dans le cas d'un grand nombre de tels opérateurs à différents niveaux hiérarchiques (opérateurs pour réseaux d'accès versus opérateurs trunk par exemple), la tâche du partage des revenus incombe en ce moment à une compagnie intermédiaire. Les échanges de factures et l'information concernant chaque appel téléphonique traversant le système entre différents opérateurs sont basés sur des calculs annuels.

Dans le modèle proposé ici, *TP* est toutefois appelé à jouer un rôle plus étendu que celui joué dans les réseaux téléphoniques en ce qu'il fait circuler de l'information en temps réel et est responsable d'un mécanisme de partage des bénéfices.

TP s'entend au préalable avec *C* sur une courbe de trafic offert versus coût unitaire, appelée ici *courbe de réponse du client*, telle que le niveau de trafic augmente lorsque le prix unitaire décrot. Cette courbe de réponse constitue une forme d'engagement de la part du client à l'effet qu'il est prêt à payer un certain prix unitaire pour l'envoi de trafic autour d'un niveau moyen donné, hormis la fraction de trafic pour laquelle il peut établir que *TP* n'a pas réussi à opérer le transit à l'intérieur de la contrainte temporelle T_{max}. Il est donc dans l'intérêt du client de toujours envoyer du trafic au niveau entendu, ne fut-ce que pour tester la performance du système. Ainsi donc, les revenus de *TP* sont réduits de la fraction de trafic qui ne passe pas le test de qualité définie par le temps de transit maximal toléré. *TP* sélectionne ensuite un nombre de ISP le long d'un chemin allant de la destination A vers la destination B et qui sont prêts à être sollicités pour faire passer le trafic de *C*. À ce stade, dans le version d'information complète du jeu, *TP* collecte les informations sur les paramètres des ISP qui serviront à définir les règles du jeu à l'issue duquel la fraction des revenus venant de *C* après déduction de la commission de *TP* sera connue pour chaque ISP. À moins de se placer dans un contexte expérimental pour lequel les temps de transit des paquets individuels sont mesurés, il est nécessaire de faire des hypothèses probabilistes a priori sur le comportement du trafic, et à partir desquelles, on pourra évaluer la fraction de trafic qui traverse de bout en bout avec la qualité de service requise. Cette fraction, à son tour, servira à calculer les paiements du client à l'ensemble *TP* - ISP.

Dans le contexte pratique, nous supposerons que les temps de transit bout à bout des paquets peuvent être mesurés à des fins d'évaluation de la performance. Cependant, toutes les décisions d'optimisation liées au jeu que nous sommes sur le point de décrire seront fondées sur des analyses théoriques découlant des hypothèses mathématiques du modèle. À ce sujet, nous avons adopté un simple modèle M/M/1 pour chaque réseau traversé. Cette décision est un compromis entre complexité de l'analyse et fidélité à la situation réelle. En résumé, nous supposons que chaque réseau possède un nœud d'engorgement dominant du trafic et c'est ce seul nœud dont nous tenons compte dans les calculs. De plus, nous supposons une longueur constante de paquets, qui sans perte de généralité sera considérée unitaire, de sorte que la probabilité de satisfaire les exigences de temps de transit pourra être directement exprimée en fonction du taux d'émission des paquets et du taux de service à l'intérieur de chaque réseau.

Dans ce qui suit, nous précisons les règles et paramètres du jeu de type Stackelberg dont les décisions à l'équilibre spécifieront les paiements du client et la qualité de service associée à son trafic. Nous débutons par les fonctions utilité censées être une expression mathématique de l'intérêt individuel de chacun des joueurs.

0.3 Fonctions utilité et variables de décisions.

La fonction utilité de *TP* s'exprime comme suit :

$$TP_U(\lambda) = M \, \Pr(t \leq T_{\max}) C_v(\lambda) \lambda \, . \tag{1}$$

où M est la fraction du revenu total prélevée par *TP* comme commission, $\Pr(t \leq T_{\max})$ est la fonction de distribution du temps de transit évaluée à T_{max} (probabilité de succès), $C_v(\lambda)$ est la courbe de réponse du client, avec λ le taux de la source en bits par secondes. La seule décision (cruciale) de *TP* est le niveau λ qu'il choisit en vue de maximiser son revenu. Quant aux ISP, nous supposons qu'ils ont chacun une certaine quantité de bande passante qu'ils mettent a priori à disposition pour le trafic du client. Elle est dénotée par μ_i dans le réseau ISPi. Chaque réseau ISPi peut, de plus, moyennant au coût unitaire c_i, acquérir et allouer au trafic en question, un surplus de bande passante $\Delta\mu_i$, s'il juge la chose payante de son point de vue. La fonction utilité de ISPi est alors définie comme suit :

$$ISP_{U_i} = (1 - M)C_v(\lambda)\Pr(t \leq T_{max})\lambda S_i - c_i\Delta\mu_i\ . \qquad (2)$$

avec S_i la fraction des revenus disponibles après commission de *TP* pour ISPi.

Détaillons à présent la manière de calculer la fraction S_i. Dans un effort de réalisation d'une qualité de service donnée, bout à bout, définie par la grandeur liée au temps de transit des paquets, il est nécessaire de répartir l'effort, ou encore le budget de temps de transit T_{max} entre les ISP. Cette décision est difficile à prendre a priori, et en définitive, ce qui compte, c'est que le temps total de transit reste à l'intérieur de T_{max}. C'est pour cette raison que nous avons créé une variable de décision préliminaire T_i que chaque ISP doit fournir, et qui constitue la borne de temps de transit *déclarée* qu'il se donne. La performance d'ISPi sera alors jugée à la lumière du T_i qu'il s'est fixé (probabilité de succès locale); ainsi, la fraction S_i croîtra proportionnellement à cette probabilité de succès, mais en relation inverse avec la taille de T_i puisque, plus T_i est grand, moins l'effort fourni par ISPi au niveau de la rapidité de service est contraignant. Il est possible de montrer que la valeur optimale de T_i dépend de variables purement locales à chaque réseau, et peut donc être fixée indépendamment par l'ISPi, et ceci, en amont du choix des $\Delta\mu_i$.

0.3.1 Règles de répartition et facteur d'ajustement β

La fraction S_i est donnée par l'expression suivante:

$$S_i = \frac{(1 - e^{-(\mu_i + \Delta\mu_i - \lambda)T_i})}{T_i^{\beta}} \left[\sum_{j=1}^{n} \frac{(1 - e^{-(\mu_j + \Delta\mu_j - \lambda)T_j})}{T_j^{\beta}} \right]^{-1}, \qquad (3)$$

Le terme au numérateur est le rapport de la probabilité de succès local (traverser le réseau en un temps ne dépassant pas T_i) pour la borne T_i que le réseau doit se fixer obtenu par le modèle M/M/1, divisée par la taille de T_i élevée à une puissance β comprise entre 0 et 1, et appelée facteur d'ajustement. Il s'avère que ce facteur d'ajustement permet compenser quelque peu des situations inéquitables dans lesquelles certains ISP le long du parcours auraient accès à de la bande passante, à un coût unitaire beaucoup moins élevé que d'autres. Plus β diminue, moins les tailles relatives des bandes passantes dédiées au trafic du client dans chaque ISP ont d'impact sur le partage du revenu.
Dans la littérature des télécommunications, le rapport $(1-e^{-T_i(\mu_i+\Delta\mu_i-\lambda)})\lambda/T_i$ est appelé facteur de puissance (power factor). Ainsi, pour $\beta = 1$, le mécanisme de répartition ci-dessus correspondrait au rapport relatif des facteurs de puissance. Plus spécifiquement:

$$S_i = P_i / \sum_{j=1}^{n} P_j \quad \text{où } P_i = (1 - e^{-(\mu_i + \Delta\mu_i - \lambda)T_i})\lambda / T_i. \qquad (4)$$

L'hypothèse que chaque joueur est rationnel requiert que chaque ISP choisisse le T_i qui lui ramène la plus grande fraction de revenus possible, et ceci pour chaque choix de $\Delta\mu_i$ possible. On peut montrer que le choix optimal de T_i mène à la règle de répartition suivante:

$$S_i = \frac{x_i^{\beta}}{\sum_{j=1}^{n} x_j^{\beta}} \qquad (5)$$

où $x_i = \Delta\mu_i + \mu_i - \lambda$.

0.3.2 Formulation du jeu de type Stackelberg

Étant donné le rôle proéminent de *TP* comme organisateur principal, nous suggérons que *TP* soit considéré comme le niveau supérieur de la hiérarchie au sein d'un jeu de type Stackelberg, c'est-à-dire qu'il soit le leader. Tous les ISP participants sont alors des suiveurs, et pour chaque valeur donnée du trafic λ qui serait proposée par *TP*, nous serons à la recherche d'un équilibre de Nash. Nous supposerons dans un premier temps, dans les Chapitres 2 et 3, un environnement de jeu avec information complète où les fonctions utilité sont connues et partagées par tous, et parfaite c'est à dire où les décisions individuelles des joueurs sont également connues de tous. Au Chapitre 3, une version répétée du jeu est présentée, et ceci en vue de lever l'hypothèse quelque peu restrictive de partage complet de l'information.

En sa qualité de leader du jeu avec information complète, *TP* est en mesure d'anticiper, pour chaque choix possible du niveau de trafic λ, les réactions des différents ISP et l'équilibre de Nash associé, lui-même dictant chaque fois un niveau de revenu différent. *TP* est alors en mesure d'imposer le niveau de trafic qui maximise ses propres revenus. Ce choix pourrait ne pas être le plus favorable pour chacun des joueurs ISP. Cependant, dans la version du jeu sans achat de bande passante additionnelle possible, ce choix correspondra à la maximisation du bien-être collectif.

0.4 Propriétés mathématiques du jeu des suiveurs

Le théorème suivant constitue la justification théorique de notre recherche d'équilibre de Nash pour chaque valeur de trafic proposée par le leader *TP* dans un jeu avec deux ISP pour $\beta \neq 1$ et un nombre arbitraires d'ISP pour $\beta = 1$.

Théorème: Dans le jeu de Stackelberg défini par la fonction utilité du leader (1), et les fonctions utilité des suiveurs (2), avec $\beta \in (0; 1]$, et $n = 2$, à condition que les conditions suivantes soient satisfaites:

1. $x_i > 2/(5T_{max}) \quad \forall i \in 1, 2, \dots n$

2. $(x_j - x_i)T_{max} < 1 \quad \forall i, j \in 1, 2, \ldots n$

3. $(1 - M)C_v(\lambda)F(x_i, X_{-i}, T_{\max}) > \max_{i=1,\ldots,n}\{c_i\} \sum_{j=1}^{n} x_j,$

alors, pour chaque valeur admissible de λ imposée par le leader, le jeu des suiveurs admettra un équilibre de Nash qui pourrait a priori ne pas être unique. Soulignons cependant que pour le cas $\beta = 1$, la condition 1 à elle seule est suffisante et ceci pour n entier arbitraire. La démonstration du théorème est présentée dans le Chapitre 1 et l'Annexe I.

0.5 Méthodes et calculs numériques

Dans un premier temps, l'environnement d'information complète et parfaite a été considéré. Dans ce contexte, on peut imaginer que *TP* exécute tous les calculs, anticipant ainsi, pour chaque choix admissible d'une grille discrétisée de choix de niveaux de trafic λ, l'équilibre de Nash associé au jeu des suiveurs. En vue de calculer ce dernier, une version virtuelle de jeu répété est simulée où, à partir d'un choix initial de $\Delta\mu_i$ nuls, les joueurs, perturbent leur position à tour de rôle jusqu'à ce que la condition d'équilibre de Nash (optimalité de l'équilibre pour chaque joueur étant donné les positions des autres joueurs) soit satisfaite à un niveau de tolérance ϵ préfixé, près.
La fonction utilité de *TP* est alors tracée en fonction de λ et le choix optimal pour *TP* est arrêté. Pour ce choix, les suiveurs refont alors individuellement la simulation du jeu répété de *TP* en vue de retrouver la position d'équilibre anticipée par *TP*, et ainsi calculer la quantité de bande passante qu'ils doivent acquérir. L'algorithme est présenté au Chapitre 2, ainsi que des résultats de sensibilité par rapport à chacun des paramètres tels que le coût unitaire de bande passante, la durée maximale acceptable du temps de transit, la fraction des revenus prélevée par *TP*,le facteur d'ajustement β, etc. Notons que toute l'implantation des algorithmes a été faite en Matlab®.
Au Chapitre 3, le jeu est repris, mais dans un contexte où l'information n'est pas partagée a priori. En particulier, les coûts d'acquisition de bande passante additionnelle ne sont communiqués ni aux ISP, ni à *TP*. Dans ce cas, il est proposé que le jeu répété simulé par

TP au Chapitre 2 soit repris ici, mais dans un effort d'estimation des quantités inconnues à partir des décisions des joueurs. Ainsi, *TP* annonce un niveau de trafic arbitraire et demande aux joueurs de se positionner séquentiellement à partir de conditions initiales des autres réseaux, et qu'il aura auparavant estimées à partir de mesures actives. La décision de chaque joueur est ensuite estimée et communiquée par *TP* aux autres joueurs dans la séquence. *TP* compare constamment la position observée des joueurs à celle qu'il aurait anticipée sur la base des paramètres qu'il aura estimés jusque là et en particulier, le coût d'acquisition de bande passante. La convergence vers les paramètres exacts se fait assez rapidement. On retombe alors dans le contexte du jeu précédent avec information complète.

0.6 Conclusion

La croissance incontestable de la téléphonie sur IP (VoIP) et d'autres applications sensibles aux temps de transit des paquets, requièrt le développement de schémas assurant une certaine qualité de service en échange de modèles de facturation et de comptabilité appropriés. La complexité du problème est exacerbée, à la fois par la taille des réseaux Internet suggérant la nécessité d'algorithmes simples, et la nécessité de faire transiter les trafics à travers des domaines tombant sous la juridiction d'opérateurs multiples (problèmes de négociation). De nouvelles stratégies sont requises qui puissent refléter le niveau d'effort de chaque opérateur dans la réduction du temps de transit des paquets à l'intérieur de son réseau.

La situation qui prévaut actuellement impose aux usagers de VoIP une facturation strictement fonction de la durée des appels, parallèle à celle qui existe pour le téléphone ordinaire. Or, VoIP fonctionne avec le protocole IP qui lui-même est un protocole meilleur effort (best effort), ne fournissant donc aucune garantie quant à la qualité de service fut elle reliée au taux de perte des paquets ou encore les temps de transit de ces derniers. Nous avons donc jugé important de développer de nouveaux schémas où le client paie une surcharge uniquement pour la fraction de trafic transitée à l'intérieur des contraintes temporelles, et où les ISP sont compensés à la mesure de leur contribution à l'effort de réduction des temps de transit. Nous avons alors exploré des règles de partage tel qu'un ISP donné reçoive une fraction des revenus qui puisse croître avec la probabilité de succès local vis-à-vis un objectif de temps de transit T_i qu'il se fixe lui-même, mais qui puisse décroître avec la taille de T_i puisque plus T_i est grand, moins l'ISP contribue à l'effort global. Les règles de partage ont été paramétrées par un facteur d'ajustement β.

Dans notre formulation du problème dans un contexte de théorie des jeux, nous avons privilégié la physique par rapport aux mathématiques, en ce que les fonctions utilité ont reflété avant tout nos objectifs d'équité et d'efficacité. Le résultat est que contrairement à la littérature dominante dans la théorie des jeux où les fonctions utilité sont choisies avant tout pour leurs propriétés mathématiques agréables, nous avons du dans ce cas nous battre plus longtemps avec les mathématiques menant à l'établissement de conditions suffisantes pour l'existence d'équilibres. Des efforts additionnels seront faits en vue de trouver des conditions suffisantes d'existence de ces équilibres pour un nombre de joueurs supérieur à 2 lorsque β est différent de 1. De plus, notons que dans des travaux futurs, nous considérerons le cas de jeux étendus avec choix de routes multiples par exemple, et des joueurs pouvant désirer mentir quant à leurs coûts d'acquisition réels de bande passante. Dans ce dernier cas, *TP* pourrait jouer le rôle d'un régulateur qui compenserait plus favorablement les joueurs qu'il perçoit plus honnêtes à partir de la fraction M des revenus qu'il prélève.

Notons enfin que, dans des travaux futurs, le modèle proposé ici pourrait servir de paradigme dans l'étude du problème de transport de produits par des transporteurs multiples à l'intérieur de contraintes temporelles, ou encore la manufacture de produits donnés à travers des chaînes de production opérées par des compagnies différentes, et devant collaborer en vue de livrer un produit fini à l'intérieur d'un délai maximal.

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LIST OF NOTATIONS AND SYMBOLS

TP:	Third party company
ISP:	Internet Service Provider
C:	Customer
λ:	Rate of packet transfer (packet/ms)
μ_i:	Initial bandwidth of ISP i (packet/ms)
$\Delta\mu_i$:	Additional bandwidth of ISP i (packet/ms)
$x_i = \mu_i + \Delta\mu_i - \lambda$:	Excess bandwidth of ISP i
X_{-i}:	The array of all ISP's excess bandwidths except ISPi
c_i:	The unit cost associated with additional bandwidth in ISPi
T_{max}:	The maximum tolerable delay asked by customer (ms)
$c_v(\lambda)$:	Customer response curve: The variable price paid by the Customer for a unit of bandwidth that reaches destination in time less than T_{max}.
β:	Delay tuning factor

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INTRODUCTION

0.7 Background

The dramatic increase in the processing power of workstations and bandwidth of high speed networks has given rise to new real-time applications such as multimedia, online financial transactions, television broadcasting over network, voice over IP, point-to-point file sharing and recently the new market of online games. These applications have traffic characteristics and performance requirements that are quite different from the previous generation of data-oriented applications. With the advent of these new Internet applications for which more quality guarantees are expected from Internet service providers, existing static pricing schemes have become more and more inappropriate [DaSilva, 2000].

For all its advantages in terms of simplified accounting and scalability, the current practice of flat rate charging which falls under the category of static pricing, presents important disadvantages for customers, in that it is aimed at the average customer. Thus, at the low usage end of the customer spectrum, there is a sense that the high volume users are in effect being subsidized. At the high usage end of it, users have to contend with the irregular QoS resulting from a best effort organization of the Internet, while if given a choice, they may in fact be willing to pay more for a steadier grade of service. Finally, from an ISP point of view, there is some level of uneasiness with regards to the high variance of net benefits resulting from operating under a static pricing paradigm.

The proposed solution to these drawbacks is a dynamic pricing approach, which can account for multiple parameters when setting unit prices and charges. The diversity of parameters and factors that can play a role in pricing decisions, new evaluation tools, different aspects of QoS, as well as the different issues that could be addresses by tarifications, has turned Internet pricing in to a very active area of research.

0.8 Dynamic Pricing Approaches

Various pricing approaches have been proposed, based on the notion of effective bandwidth, a statistically founded tool for the evaluation of quality constrained bandwidth requirements for certain types of traffic in data networks [Courcoubetis and Weber, 2003, Kelly et al., 1998], as well as different results from both cooperative and non cooperative game theory [Basar and Olsder, 1995].

In many schemes, along with the basic objective of pricing, which is to recover the incurred costs, other goals have been considered:

- Congestion control and fair allocation of resource to users [Courcoubetis and Weber, 2003, Srisankar and Kunniyur, 2001]
- Admission control and QoS provisioning [Dziong and Mason, 1996, Li et al., 2004]
- Allocating the resource to users who value it most by selling the service in an online auction, and in a repeated game context [Lazar and Semret, 2000, Semret, 1999, Maillé and Tuffin, 2004]
- Differentiated pricing in a best effort network environment [Odlyzko, 1998, Odlyzko, 1999]
- Pricing schemes associated with DiffServ and IntServ [Semret et al., 2000]
- Pricing scheme based on a non-cooperative environment [Haogang Chen, 1998]

As argued in [He and Walrand, 2006], the profit of ISPs as major players in Internet, has been neglected in many pricing schemes. In this thesis, the interaction between ISPs and the outcome of a non-cooperative game between them is investigated. However, our model differs in a number of ways from that in [He and Walrand, 2006]. We have assumed the case of only one data flow that passes through designated ISPs, and the end user who initiates the process is assumed to be willing to pay only for that portion of the traffic that meets a specific delay bound. Furthermore, an ISP reward structure is defined whereby each ISP obtains a share of customer payments which depends on both its initially declared individual quality of service goals, as well as on a statistical measure of how successful this ISP is in meeting the goals in question. Furthermore, the setup here is not one of guaranteed quality of service, but rather statistical quality of service. Such a choice was made for at least two reasons: firstly, deterministic quality of service guarantees can be quite wasteful in terms of bandwidth requirements. Secondly, when involving multiple ISP domains, guaranteed qualities of service tend to require a high degree of end to end coordination, and thus the complexity and overhead communications requirements of such schemes can quickly reach unmanageable levels as network size increases. Instead here, the setup is such that the enforcement of quality of service is an affair left as entirely internal to each independent network. If a particular network complies with a high degree of success rate relative to its declared goals, it will be rewarded accordingly. If not, it will not. This way, the control scheme for quality enforcement can be left as *decentralized* as possible.

A third party company herein referred to as *TP* has been introduced as a coordinator between the end user and ISPs. In return, it receives a fixed portion of customers payments. We adopt a Stackelberg game environment, in which *TP*, is the leader, and ISPs form the group of followers.

0.9 Game Theory, Oligopolies and Stackelberg Games

An oligopoly is a market dominated by a small number of sellers called oligopolists. The word is derived from the Greek for few sellers. Because of the existence of few participants in this type of market, each oligopolist is aware of the actions of the others. Oligopolistic markets are characterized by interactivity between agents. Oligopolistic competition can give rise to a wide range of different outcomes. In some situations, the firms may collude to raise prices and restrict production in the same way as a monopoly. Where there is a formal agreement for such collusion, this is known as a cartel.

The Stackelberg leadership model is a strategic oligopoly game in economics in which the leader firm moves first and then the follower firms move sequentially. In game theory terms, the players of this game are a leader and a follower and they compete on quantity. The leader moves first, choosing a quantity. The follower observes the leader's choice and then picks a quantity, and the envelope of chosen quantities define the utility of each player of the game. The Stackelberg leader is sometimes referred to as the Market Leader.

One important issues in Stackelberg games is the commitment of the leader (*TP* in this model), to the chosen decision variable; once it is announced it can not be changed.

Overprovisioning of capacities may be the solution for many network operators to deal with delay and congestion issues, but as discussed in [Courcoubetis and Weber, 2003], while this looks like the right choice in backbones of the network, it may not be so for its metropolitan part, and even less so in the access part of the network. This stems from the fact that overdimensioning in the the access network requires large investments. Based on this observation, we have assumed that each ISP involved in our model has at least one congestion node along the chosen route, and the imposed delay caused by this node dominates that of any other route link within the ISP domain. In summary, each ISP is represented by a single bottleneck node along the chosen route.

0.10 Thesis Outline

This thesis is organized in the following manner. In Chapter 1, the structure of the game, as well as the agents or players, their utility functions and reaction functions, revenue sharing mechanism and its fairness, and mathematical analysis are presented. In Chapter 2, algorithms and sequence of the game are presented, and the space of decision variables and discretization is discussed. This is followed by a base example of three competing ISPs. In subsequent sections, the effect of variation of each element of the game is studied numerically. In Chapter 3, a more generalized repeated version of the game in incomplete information environment is presented, where the ISPs unit cost vary across time. Finally, conclusion and future research directions are presented in the last Chapter. A lengthy mathematical proof in chapter 1 is carried to Appendix I, and in Appendix 2 the article presented at *International Conference on Network Control and Optimization*, at Avignon, France, on 6th of June 2007, is attached. This paper, which summarizes most parts of Chapter 1, won the best student paper award in the conference.

CHAPTER 1

GAME THEORETICAL PRICING MODEL

1.1 Introduction

In this chapter the main ideas of the model and its properties are discussed. In Section 1.2, we present our game theoretic based model involving a customer, a collection of candidate ISPs and an intermediary company known as the Third Party. It is a Stackelberg game with the Third Party as leader and the ISPs as followers. In Section 1.3, we give details of the utility functions of the ISPs and the Third party and develop some preliminary structural properties relating to the optimum decisions of players in the ISP game. In Section 1.4.1, we establish sufficient conditions for the existence of a Nash equilibrium in for an arbitrary number of players when $\beta = 1$, and for two players only when $\beta = 0$.

1.2 Model description

The main objective of this thesis is to model the interactions between ISPs and end users to provide them the required Quality of Service QoS. Here the scope is limited to the case of a single end user, which is dealing with a group of service providers.

The proposed model involves three types of agents: a *customer* herein referred to as *C*, *TP*, and a collection of *ISP's* to be selected by *TP*. In our model, *C* is an end user with a potentially large volume of traffic to be sent on a regular basis from a given destination A to a destination B, and who initiates contacts with *TP* for that purpose. However, *C* specifies a maximum end to end tolerable delay for those transmitted packets for which he is willing to pay a per unit premium. We denote the maximum delay tolerated by C as T_{max}. The list of variables used in the model and their definitions is presented on page xxiv.

An example of traffic type particularly relevant to the context here is VoIP. This is because in VoIP one can sustain the high loss probabilities that may occasionally result from the organization scheme to be proposed. Furthermore, there does already exist market regulators in the VoIP context and they can readily be identified as potential TPs in our model. Indeed the Telecom Decision CRTC 2005-28, which has been set by Canadian Radio-Television and Telecommunications Commission is a clear example of a set of regulations, upholding rather identical regulatory framework as extant traditional phone services for VoIP [''CRTC'', 2005].

Division of revenues amongst telephone companies is based on mutual agreements between pairs of service providers. In the case of a large number of such providers of different hierarchical levels e.g. trunk and access network providers, the task of revenue sharing is currently performed by a third party company. Exchanges of balances, and information about each traversing telephone call between service providers are based on annual calculations. In the current model, *TP* plays an enhanced role, as compared to the case of telephone networks, in that a real-time information and revenue sharing mechanism is adopted.

TP and *C*, agree on an *offered traffic versus unit flow price curve*, whereby offered traffic levels increase as unit price decreases.This curve is a form of commitment on the part of the customer that it will pay a fixed bandwidth unit price per unit time for sending a given ultimately agreed to traffic level, unless it can demonstrably establish failure by *TP* to meet the QoS requirements at that traffic level. In the latter case, *C*'s per unit time payment is reduced by the fraction of its total traffic transmitted with a delay larger than

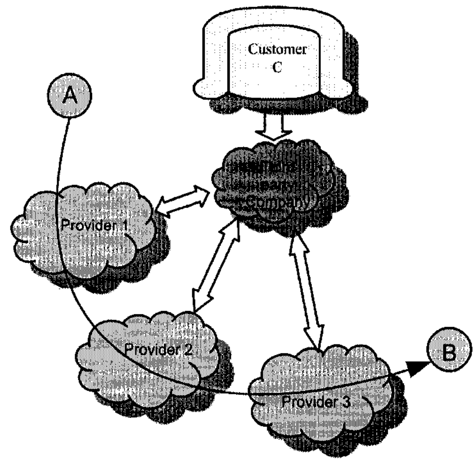


Figure 1.1 A representation of the model for three ISP's

T_{max}. As a consequence of this arrangement, it is in *C*'s best interest to constantly probe performance by sending traffic (useful or otherwise) at the agreed to level.

TP selects a number of ISPs along the route from A to B who are willing to be solicited in offering the service to *C*. At this stage, *TP* gathers from the candidate ISPs the parameters which specify the rules of the game they have to play and whose outcome will be their individual share of the income from *C* after *TP*'s commission is deducted. The parameters of the ISP profit sharing are discussed in Section 1.3. Fig.1.1 shows a schematic view of the model.

Unless one can measure the delay for each packet and provide a cumulative payment for all packets which meet the delay test successfully, it is necessary to carry an a priori probabilistic analysis for estimating mean success rate and the ensuing payments, which will then rely on specific modelling assumptions.

In the practical context, we assume that packet end to end delays can be monitored for performance verification, within ISP domains. However, all optimization decisions are founded on specific modelling assumptions. In the current context, we have settled for a simple M/M/1 queueing model of each network. We have assumed an exponential distribution for packet lengths, so that the probability of meeting the delay requirement can be expressed as a function of service rate of that network and arrival rate. More specifically:

$$P(t \leq T_i) = 1 - e^{(\lambda - \mu_i)T_i} \tag{1.1}$$

where t is the random delay in network i, λ is the rate of transfer, μ_i is the service rate of network i and T_i is the *declared* maximum transit time in network i.

The need to calculate success probabilities in each network stems from the fact that we wish to reflect the customer payment mechanism on the ISPs involved in the negotiation. More specifically, the fraction of total revenue dedicated to an ISP will directly depend on the probability of meeting the declared delay within its network. Moreover, as mentioned earlier, C pays according to the probability that its packet reaches the destination in time; this probability can be derived from the probability distribution of individual network delays.

The per unit time cost for the customer will be:

$$J = Pr(t \leq T_{\max})C_v(\lambda)\lambda \tag{1.2}$$

where $C_v(\lambda)$ is the unit cost versus traffic λ, dependency curve, herein referred to as the *customer response* curve. For convenience, it is taken to be a decaying exponential. Indeed, anticipating a decreasing function of demand versus price is standard (see [He and Walrand, 2006] for example). With all active agents and their declared parameters thus defined, we are ready to formulate the rules of a Stackelberg game whose outcome will be the traffic rate submitted by C to the ISPs, the corresponding premium unit flow price paid by C, and the revenue obtained by each of the candidate ISPs.

1.3 Utility functions: definitions and preliminary structural properties

1.3.1 Third Party *TP*

TP is a company responsible for all negotiations with the ISPs, with the understanding that the negotiation process must remain transparent to the customer. *TP*'s unit time revenue is a fixed fraction of the total unit time payments made by *C*. The utility function of *TP* is considered to be:

$$TP_U(\lambda) = M\ \Pr(t \leq T_{\max})C_v(\lambda)\lambda \tag{1.3}$$

where $M \in [0;1]$ is the fraction of the total benefit reserved for *TP* .

The only decision variable of *TP* is λ, and it is chosen to maximize *TP*'s revenue, or equally the *total* customer payments to the ISPs, so that in a formulation of the game where ISPs cannot acquire more bandwidth, this corresponds to the social welfare optimization problem. We also assume an upper bound λ_{max} for the rate of data transfer.

1.3.2 Service Providers

We assume each network involved in the transaction to have a certain amount of bandwidth μ_i, already available for *C*'s traffic. Furthermore, we assume that this initial bandwidth is sufficient to insure that the maximum possible source rate λ_{max} can be satisfied by any of the μ_i's, ($\lambda < \mu_i \quad \forall i$). The ISPs have the option of increasing the amount of bandwidth they dedicate to *C*'s traffic, via a specified cost of c_i per unit of added bandwidth. Let $\Delta\mu_i$ be the added bandwidth with an upper bound $\Delta\mu_i^{\max}$, so that the actual bandwidth that network i can allocate to the flow becomes: $\mu_i + \Delta\mu_i$. For each potential λ, the fraction of profit, which is not taken by *TP*, is assumed to be available in its entirety to the participating ISPs. However, for each fixed λ, ISPs are pitted against each other in a game, the rules of which will be defined in what follows. The idea is to reflect the payment mechanisms at *TP*'s level all the way down to the ISPs. More specifically, ISPi is asked to provide a (hypothetical) maximum delay T_i that it will try to meet. This $T_i \in [0; T_i^{\max}]$ is very instrumental in determining ISP i's share of total income available after *TP*'s payment, in that it is proposed that the fraction of that total allocated to ISPi be given by:

$$S_i = \frac{\left(1 - e^{-(\mu_i + \Delta\mu_i - \lambda)T_i}\right)}{T_i^{\beta}} \left[\sum_j \frac{\left(1 - e^{-(\mu_j + \Delta\mu_i - \lambda)T_j}\right)}{T_j^{\beta}} \right]^{-1}, \tag{1.4}$$

with $\beta \in (0; 1]$ as a coefficient which we refer to as *delay tuning factor*. Note that the term $1 - e^{-(\mu_i + \Delta\mu_i - \lambda)T_i}$ is simply the probability that the declared delay in ISPi is met.

Also note that the larger the declared time, the less margin is left for other providers to accommodate their own delays along the packet route. From that point of view, fairness would dictate that a large declared T_i should correspondingly penalize the declarer This is why we chose to have the T_i^{β} in the denominator in (1.4). This penalty prevents ISPs from letting their own declared T_i's to go to infinity in an effort to maximize their chances of success. Also, note that for an adequate choice of delay tuning factor β, we shall establish that the optimal choice of declared T_i is the mean delay within the network. In addition, as alluded to earlier, the ISP has the option of either buying for a given unit price extra bandwidth, or equivalently freeing, albeit at the cost of some loss of revenue per unit bandwidth, a given amount of bandwidth, thus modulating its effective service rate μ_i.

As a consequence ISPi, must provide two decision variables: T_i, and the extra amount of bandwidth $\Delta\mu_i$ it wishes to buy. Note that if we fix $\Delta\mu_i = 0$ (no bandwidth buying allowed), it is not difficult to see that, modulo a reward shift by an appropriate constant, the game is equivalent to a *zero sum* game. Using this allocation rule, we define the utility function as:

$$ISP_{U_i} = (1 - M)C_v(\lambda)\Pr(t \leq T_{max})\lambda S_i - c_i\Delta\mu_i \tag{1.5}$$

where $(1 - M)C_v(\lambda)\Pr(t \leq T_{max})\lambda$ represents the revenue after payment of *TP*, and c_i is the purchased unit cost of extra bandwidth.

Considering the expression of ISP_i's utility function in (1.5), we note that except for the share term S_i, the utility does not depend on the choice of declared maximum transit time T_i. In the following, we present a string of results aiming at establishing that for given $(\mu_i + \Delta\mu_i)$, T_i can be selected independently of other decision variables to maximize S_i, leaving $\Delta\mu_i$ as the unique decision variable of ISP_i. Furthermore, for the special case where the delay tuning factor in (1.4) is equal to 1, the optimum choice is $T_i = 0 \quad \forall i$.

Define a new variable x_i, as the surplus bandwidth that each ISP has as follows:
$x_i \triangleq \mu_i + \Delta\mu_i - \lambda \quad \forall i.$

Each ISP, as an independent agent, tries to maximize its share of revenue S_i with respect to T_i regardless of its other decision variable $\Delta\mu_i$, i.e. each ISP faces the following optimization problem :

$$\max_{T_i} \frac{1 - e^{-x_i T_i}}{T_i^{\beta}} \text{ where } x_i = \mu_i + \Delta\mu_i - \lambda = cte. \tag{1.6}$$

The optimization problem in (1.6) will yield the same result for T_i as the following problem:

$$\max_{\alpha_i} G(\alpha_i) \quad \text{where } \alpha_i \triangleq T_i x_i \text{ and } G(\alpha_i) \triangleq \frac{1-e^{-\alpha_i}}{\alpha_i^{\beta}}. \tag{1.7}$$

Notice that $G(\alpha_i) \geq 0 \quad \forall \alpha_i \geq 0$ and $\forall \beta \quad 1 \geq \beta > 0$. Moreover $\lim_{\alpha_i \to +\infty} G(\alpha_i) = 0$. The derivative of $G(\alpha_i)$ yields:

$$\frac{dG(\alpha_i)}{d\alpha_i} = \frac{e^{-\alpha_i}(\alpha_i + \beta) - \beta}{\alpha_i^{\beta+1}} \tag{1.8}$$

We first analyze the maximization problem in (1.6) for $\beta = 1$.

Proposition 1.3.1 *For the case $\beta = 1$, the optimum choice of T_i is zero.*

Proof By studying the sign of the derivative of the numerator with respect to α_i, it is possible to conclude that the derivative will be monotone decreasing for $\alpha_i > 0$. Since the numerator of (1.8) is zero for $\alpha_i = 0$, it will be negative for any $\alpha_i >$, and thus, so will the function $G(\alpha_i)$. As a result, since for stability reasons, x_i is constrained to be positive, one can conclude that for $\beta = 1$, the maximizer of (1.6) is $T_i = 0$.

We now consider the maximization in (1.7) for β different from 1. Since $G(\alpha_i)$ is a generic form for all ISPs, we remove the index i in the following analysis.

Lemma 1.3.2 *For $0 < \beta < 1$, there exists α_1 and α_2 such that:*
$\forall \alpha \in (0, \alpha_1] \quad G(\alpha) \leq G(\alpha_1)$
$\forall \alpha \in [\alpha_2, +\infty) \quad G(\alpha_2) \geq G(\alpha)$

Proof Calculating the value of (1.8) at the point $\alpha = 0$, via of L'Hôpital's rule, yields:

$$\lim_{\alpha \to 0^+} \frac{dG(\alpha_i)}{d\alpha_i} = \lim_{\alpha \to 0^+} \frac{(e^{-\alpha})(1 - \alpha - \beta)}{(\beta + 1)\alpha^{\beta}} = +\infty. \tag{1.9}$$

Since $dG(\alpha_i)/d\alpha_i$ is a continuous function in α_i, and the results in (1.9) hold, there exits a finite nonnegative neighborhood of 0, for which $dG(\alpha_i)/d\alpha_i$ is positive and as a result $\exists \alpha_1$ such that $\forall \alpha \in (0, \alpha_1]$, $G(\alpha)$ is monotone increasing; equivalently:

$$\exists \alpha_1 \quad s.t. \quad \forall \alpha \in (0, \alpha_1] \quad G(\alpha) < G(\alpha_1).$$

The existence of α_2 is shown by using the definition of continuity and the fact that $\lim_{\alpha \to +\infty} G(\alpha) = 0$. Now $\epsilon = G(1) = 1 - e^{(-1)}$, then: $\exists \alpha_2 \quad s.t. \quad \forall \alpha \in [\alpha_2, \infty) \quad |G(\alpha) - 0| < 1 - e^{(-1)}$, and thus $G(\alpha) \leq \alpha_2$.

By using the Weierstrass extreme value Theorem, the existence of at least a maximizer in the compact interval $[\alpha_1, \alpha_2]$ is established. Next we will show that this maximizer is unique and indeed can be computed by using the first order necessary condition for optimality:

Theorem 1.3.3 *The maximizer in(1.7) for $\beta \in (0, 1)$, can be obtained using the first order necessary optimality condition.*

Proof The existence of a maximizer in (1.7) has been shown in Lemma (1.3.2). Furthermore, since $G(\alpha)$ is a smooth continuous function in $(0, +\infty)$ the maximizer should satisfy the first order necessary condition for optimality. In this proof we show uniqueness of the stationary point, which then becomes the unique maximizer of $G(\alpha)$.
The following equation yields all possible stationary points:

$$\frac{\partial G(\alpha_i)}{\partial \alpha_i} = \frac{e^{-\alpha_i}(\alpha_i + \beta) - \beta}{\alpha^{\beta+1}} = 0 \tag{1.10}$$

By re-writing (1.10), we will have:

$$Z_1(\alpha) = Z_2(\alpha) \quad \text{where: } Z_1(\alpha) = \alpha e^{-\alpha}, \quad Z_2(\alpha) = \beta(1 - e^{-\alpha}) \tag{1.11}$$

Next we show that $Z_1(\alpha)$ and $Z_2(\alpha)$ have only one point of intersection other than $\alpha = 0$ (which is excluded when considering (1.10) directly). Note that at $\alpha = 0: Z_1'(\alpha) = 1 > Z_2'(\alpha) = \beta$. Since $Z_2''(\alpha) = -\beta e^{-\alpha} < 0$, $Z_2(\alpha)$ is a strictly concave function, thus $Z_2'(\alpha)$ remains less than one for $\alpha > 0$ and is monotone decreasing. On the other hand $Z_1(\alpha)$ admits a maximizer at $\alpha = 1$, past which it decreases to zero. Given that $Z_2(\alpha)$ is positive and below $Z_1(\alpha)$ for $(\alpha \to 0^+)$, and is a monotone increasing function, and given that $Z_1(\alpha)$ decreases eventually to zero, there will be at least one point of intersection for $\alpha > 0$. Let α^* be such an intersection point. We can write:
$Z_2'(\alpha^*) \geq Z_1'(\alpha^*) \Rightarrow \beta e^{-\alpha^*} \geq (1 - \alpha^*)e^{-\alpha^*} \Rightarrow \beta \geq 1 - \alpha^*$. In the next step we show that for $\alpha > \alpha^*$, we will have $Z_2'(\alpha) > Z_1'(\alpha)$:
$\forall \alpha > \alpha^*, \quad \beta \geq 1 - \alpha^* > 1 - \alpha$ thus $\beta e^{-\alpha} > (1 - \alpha)e^{-\alpha}$, and as a result:

$$\forall \alpha > \alpha^*, \quad Z_2'(\alpha) > Z_1'(\alpha) \tag{1.12}$$

Since $Z_2(\alpha^*) = Z_1(\alpha^*)$, and in view of (1.12), the two functions cannot intersect at a point past α^* again. Besides, α^* is unique, since existence of a non negative intersection point strictly smaller than α^* would, using the same line of arguments, preclude the existence of α^* itself. This completes the proof of existence of a unique maximum in (1.7). Furthermore it can be found using the first order necessary condition. ∎

Corollary 1.3.4 *There exists a unique value for β, for which the declared time by each ISP is equal to the mean delay ($T_i^{avg} = 1/x_i$) that packets receive in that ISP's network. This truth revealing value of β is:*

$$T_i^{avg} = \frac{1}{x_i} \Rightarrow T_i^{avg} x_i = 1 = \alpha \Rightarrow \textit{from (1.11)}, \beta = (e-1)^{-1} \approx 0.58 \tag{1.13}$$

Another approach is to try to express $\Delta\mu_i$ as a function of T_i, and use T_i as the only decision variable of the ISP. However since the $\Delta\mu_i$'s are directly used in the probability function of global success we have adopted $\Delta\mu_i$ as the working decision variable for each ISP. As a result, $\Delta\mu_i$ becomes the unique decision variable of ISP_i.

1.3.3 The sharing mechanism between ISPs

We investigate the sharing rule for the two cases $\beta = 1$ and $0 < \beta < 1$:

Remark 1.3.5 *The fact that, at least for the $\beta = 1$ case, the optimal choices of declared maximum network transit times T_i for the ISPs correspond to the highly unrealistic value of zero, justifies their characterization as declared values. This leads to a reasonable rule for sharing benefits among ISPs. Indeed, for $\beta = 1$ as T_i goes to zero, L'Hôpital's rule yields:*

$$\frac{S_i}{S_j} = \frac{x_i}{x_j}. \tag{1.14}$$

The corresponding general form (n ISPs) of (1.14) is:

$$S_i = \frac{x_i}{\sum_{j=1}^{n} x_j} \tag{1.15}$$

which indicates that customer payments after commission are shared among ISPs in inverse proportion to the mean packet transit time in each of the networks.

Remark 1.3.6 *Having established that (1.7) admits a unique solution α^*, we can consider α^* to be a function of the β, say $P(\beta)$, where the function $P(\beta)$, can be obtained numerically. Since we have decided to designate x_i (or $\Delta\mu_i$ equivalently) as the decision* *variable of each ISP, we try to obtain T_i as a function of the other decision variable:*

$$T_i^* = \frac{\alpha^*}{x_i} = \frac{P(\beta)}{x_i}. \tag{1.16}$$

Substituting (1.16) in the general sharing function (1.4) we obtain:

$$S_i = \frac{\left(1 - e^{-(x_i)\frac{P(\beta)}{x_i}}\right)}{\left(\frac{P(\beta)}{x_i}\right)^\beta} \left[\sum_{j=1}^{n} \frac{\left(1 - e^{-(x_j)\left(\frac{P(\beta)}{x_j}\right)}\right)}{\left(\frac{P(\beta)}{x_j}\right)^\beta}\right]^{-1} = \frac{\left(1 - e^{-P(\beta)}\right)}{\left(\frac{P(\beta)}{x_i}\right)^\beta} \left[\sum_{j=1}^{n} \frac{\left(1 - e^{-P(\beta)}\right)}{\left(\frac{P(\beta)}{x_j}\right)^\beta}\right]^{-1} \tag{1.17}$$

And as a result the sharing mechanism for $\beta \in (0, 1)$ becomes:

$$S_i = \frac{x_i^\beta}{\sum\limits_{j=1}^{n} x_j^\beta} \tag{1.18}$$

Parallel to Remark (1.3.5) above, the sharing mechanism in (1.18) indicates that the revenue of each ISP is proportional to the inverse of mean packet transit time raised to the power $\beta < 1$.

In light of this property we next investigate the effect of β as a tuning factor in the game.

Proposition 1.3.7 *In the case of n competing ISPs, assume that there exist ISP i, and j such that $x_i < x_j$, then as β increases from 0 to 1 the ratio of the shares S_j/S_i increases. In the case of only two ISPs, the share of the ISP with lower x ,ISP x_i, (or equivalently higher average transit time T_i) decreases while that of ISP j increases.*

Proof The ratio S_i/S_j and its derivative as a function of β can be written respectively as:

$$\frac{S_i(\beta)}{S_j(\beta)} = R(\beta) = \frac{x_i^\beta}{x_j^\beta}, \quad R'(\beta) = ln(\frac{x_i}{x_j})\frac{x_i^\beta}{x_j^\beta} \tag{1.19}$$

Since $x_i < x_j$ then $ln(\frac{x_i}{x_j}) < 0$. Thus $R(\beta)$ is a decreasing function and as a result, as β increases (from 0 to 1) the ratio of $\frac{S_i}{S_j}$ decreases. In the case of only two ISPs, since the sum of shares is 1, this would mean that S_i decreases while S_j increases.

In conclusion, a decrease in β tends to de-emphasize the advantage that ISPs may accrue from contributing more bandwidth to the game. This property may in turn be used to create fairer game conditions whenever the packets route traverses ISP domains with widely different congestion and access to bandwidth properties. However, an undesirable side effect is that this can result in a decrease of the customer perceived QoS at a given unit cost.

1.4 Existence of Nash equilibria in the followers game

While we have specified different agents utility functions, we have not thus far specified the sequence in which the game is played. Given the predominant role of *TP* as the main organizer, we suggest that *TP* be considered as the higher level of the hierarchy within a Stackelberg game, i.e. *TP* is the leader. All participating ISPs are followers, and thus, for each fixed value of customer traffic rate λ decided by the leader *TP*, we shall be looking for potential Nash equilibria. We also assume a perfect information environment in chapter 1 and 2, whereby each player knows all buying costs of extra bandwidth unit , initial networks dedicated bandwidths to *C*'s traffic, as well the customer response curve. These strong information availability assumptions will be relaxed in further sections. In Chapter 3 a repeated version of the game is presented, letting us make the complete information assumption less restrictive. Having the position of the leader in this game, *TP* can predict the outcome of the non-cooperative game among the followers, for any specific choice of *TP*'s unique decision variable λ. By exploiting this fact, *TP* can specify the customer traffic level which best suits its interests. In general, this may not be the optimum choice for any specific individual participating ISP, but will in general correspond to the maximization of the envelope paid by C to the ISPs. Thus, in a version of the game where ISPs do not have the possibility of buying more bandwidth, *TP* will be solving the *welfare maximization* problem.

In general however, for n ISPs, *TP* will have to calculate the n-tuple Nash solution of the game $(\Delta\mu_1, ..., \Delta\mu_n)^{Nash}$ for each potential value of λ, provided it exists and is unique, and subsequently optimize its utility function with respect to λ.

The following theorem provides sufficient conditions for the existence of a (possibly non unique) Nash equilibrium in the followers game.

Theorem 1.4.1 *In the Stackelberg game defined by leader utility function (1.3) and followers utility functions (1.5) with* $\beta = 1$, *if:*
$(1 - M)C_v(\lambda)Pr(t \leq T_{\max}) > \max_{i=1,\dots,n} \{c_i\} \sum_{j=1}^{n} x_j,$

Then for every admissible λ *set by the leader, the follower game will admit a possibly non unique Nash equilibrium.*

For the special case $\beta \in (0; 1)$ *and* $n = 2$, *if the following conditions hold:*

a. $x_i > 2/5T_{max} \quad \forall i \in 1, 2$

b. $(x_j - x_i)T_{max} < 1 \quad \forall i, j \in 1, 2$

c. $(1 - M)C_v(\lambda)F(x_i, x_j, T_{\max}) > \max_{k=1,2}\{c_k\}(x_i + x_j),$

then the follower game will admit a Nash equilibrium.

Proof To prove the existence of Nash equilibria, we use a paraphrase of the following Theorem [Basar and Olsder, 1995]:

Theorem 1.4.2 *For each player, assuming the sets of decision variables are closed, bounded and convex, and furthermore that each player's utility function is continuous in all decision variables associated with all players, and strictly concave in the entries associated with its own decision variables, for every admissible combination of decisions of other players, the associated n-person none zero sum game will admit a Nash equilibrium in pure strategies.*

The theorem above can be easily shown to hold if strict concavity is replaced by the assumption of existence of a unique maximizer for each player's utility function for arbitrary decisions made by other players. The existence of a unique maximizer is satisfied provided utility functions can be shown to be strictly *log concave* in their own decision variable. Sufficient conditions for log concavity to hold are established through a string of results discussed below. Since, $\Delta\mu_i \in [0; \Delta\mu_i^{\max}]$, the set of decision variables is both convex and compact. Furthermore, the continuity of utility functions on the admissible decision variable set is clear. Therefore a Nash Equilibrium will exist.

First, we consider the case $\beta = 1$, for n arbitrary. Existence of a unique maximizer for each ISP problem is established in two steps: first, by showing the strict concavity of the global probability of success function in Lemma 1.4.3, and subsequently, by showing that the ISP utility function reduces to the product of two strictly concave functions and is therefore strictly log concave in Lemma 1.4.4.

The proof will cover all possible values of $\beta \in (0, 1]$ for the case of 2 ISPs, while for n ISPs we are able to establish the proof only for $\beta = 1$. We start with the case $\beta = 1$.

1.4.1 *The case of* $\beta = 1$.

Lemma 1.4.3 *The global success probability function $Pr(t \leq T_{max})$ is strictly concave with respect to each ISP decision variable $\Delta\mu_i$, irrespective of $\Delta\mu_j, j \neq i$.*

Proof The probability density function (pdf) of the time in the system t in a simple M/M/1 queue is [Kleinrock, 1975] : $g(t,x) = xe^{-xt}$. where $x = \mu + \Delta\mu - \lambda$. The total delay T that is imposed on each packet, is the sum of individual delays within each ISP's network. Under the assumption of approximate independence of the transit times within the successive ISPs, the pdf of T, $f(T,X)$, will be the convolution of all component pdf's $g(t_i, x_i)$.

$$f(T,X) = g(t_1,x_1) * g(t_2,x_2) * ...... * g(t_n,x_n) \text{ where: } X = [x_1, x_2, ..., x_n]. \quad (1.20)$$

Defining $F(T,X)$ as the probability distribution function (PDF) of T, and $X_{-i} = [x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$, the pdf of the total transit time when the time spent in ISPi is excluded, can be defined as: $h_{-i}(T, X_{-i}) = g(t_1,x_1) * \ldots * g(t_{i-1},x_{i-1}) * g(t_{i+1},x_{i+1}) \ldots * g(t_n,x_n) > 0$

The probability that the total packet delay be less than T_{max} is given by:

$$F(T_{\max}, X) = \int_0^{T_{\max}} h_{-i}(t, X_{-i}) * g(t, x_i)dt = \int_0^{T_{\max}}\int_0^{t} h_{-i}(\tau, X_{-i})g(t-\tau, x_i)d\tau dt. \quad (1.21)$$

Using Fubini's Theorem to change the order of integration in (1.21), we will have:

$$F(T_{\max}, X) = \int_0^{T_{\max}}\int_{\tau}^{T_{\max}} (g(t-\tau, x_i)\,dt \quad)h_{-i}(\tau, X_{-i})\,d\tau \quad (1.22)$$

where $G(x,t) = 1 - e^{-xt}$ is the PDF of $g(x,t)$. Our goal is to show that $\frac{\partial^2 F}{\partial x_i^2} < 0 \forall X_{-i}$. Using Lebesgue's dominated convergence, the differentiation can be carried across the integral:

$$\frac{\partial^2 F}{\partial x_i^2} = \int_0^{T_{\max}} h_{-i}(\tau, X_{-i})\frac{\partial^2}{\partial x_i^2}G(T_{\max} - \tau, x_i)d\tau. \quad (1.23)$$

Note that $\frac{\partial^2}{\partial x_i^2}G(T_{\max} - \tau, x_i) = -(T_{\max} - \tau)^2 e^{-(x_i)(T_{\max}-\tau)} < 0$ and $h_{-i} > 0$; hence

$\partial^2 F/\partial x_i^2$ is always negative, and as a result the global success probability is strictly concave in x_i or equally in $\Delta\mu_i$. ▮

Lemma 1.4.4 *For any admissible values of decision variables X_{-i}, $ISP_{U_i}(x_i, X_{-i})$ has a unique maximizer with respect to x_i for $\beta = 1$.*

Proof Our goal is to show that:

$$ISP_{U_i}(x_i, X_{-i}) = \frac{x_i}{\sum\limits_{j=1}^{n} x_j}(1 - M)C_v(\lambda)F(x_i, X_{-i}, T_{\max})\lambda - c_i(x_i - \mu_i + \lambda) \quad (1.24)$$

always admits a unique maximizer, where $F(x_i, X_{-i}, T_{\max})$ is recalled to be the probability of global success. In Lemma 1.4.3, the strict concavity of $F(x_i, X_{-i}, T_{\max})$, with respect to x_i was established. On the other hand the function $-c_i \sum\limits_{j=1}^{n} x_j$ is a linear function in x_i, thus the function:

$$AF(x_i, X_{-i}, T_{\max}) - c_i \sum_{j=1}^{n} x_j, \text{ where: } A \triangleq (1 - M)C_v(\lambda)\lambda \quad (1.25)$$

is also strictly concave in x_i. Next we will assume the following for the total reward paid by the customer to satisfy the following inequality:

$$AF(x_i, X_{-i}, T_{\max}) > \max_i\{c_i\} \sum_{j=1}^{n} x_j \quad (1.26)$$

The assumption (1.26) will ensure a positive value for (1.25) for all ISPs and has been called assumption c in Theorem (1.4.1). Using Mangasarian's theorem [Avriel et al., 1988], the log of (1.25) is a strictly concave function in x_i, thus (1.25) is said to be strictly *log concave*. Furthermore, $x_i \left[\sum\limits_{j=1}^{n} x_j\right]^{-1}$ is also a strictly log concave function in x_i. Since log concavity is preserved under multiplication, the following function is strictly log concave in x_i:

$$x_i \left[\sum_{j=1}^{n} x_j\right]^{-1} (AF(x_i, X_{-i}, T_{\max}) - c_i \sum_{j=1}^{n} x_j). \quad (1.27)$$

The strictly increasing nature of the log function, and strict log concavity of (1.27) imply that it has a unique maximizer with respect to x_i [Paninski, 2004]. Furthermore $ISP_{U_i}(x_i, X_{-i})$ can be obtained by adding the constant $c_i(\mu_i - \lambda)$, which will not alter the existence of the unique maximizer of (1.27). Thus $ISP_{U_i}(x_i, X_{-i})$ admits a unique maximizer in x_i. ∎

1.4.2 *The case of* $\beta \in (0, 1)$

In this case, while assuming only ISPi and ISPj are present, our goal is to derive the sufficient conditions that:

$$ISP_{U_i}(x_i, x_j) = \frac{x_i^\beta}{x_i^\beta + x_j^\beta}(1 - M)C_v(\lambda)F(x_i, x_j, T_{\max})\lambda - c_i(x_i - \mu_i + \lambda) \quad (1.28)$$

always admits a unique maximizer with respect to x_i. Before proceeding, we introduce a change of variables: $y_i \triangleq x_i^\beta$, $y_j \triangleq x_j^\beta$and $\alpha \triangleq \frac{1}{\beta}$, and further we define the following function:

$$R(y_i) \triangleq AF(T_{max}, y_i^\alpha, y_j^\alpha) - c_i(y_i^\alpha + y_j^\alpha) \quad (1.29)$$

where we recall that $A = (1 - M)C_v(\lambda)\lambda$ and F is the global success probability function, defined in (1.21). We have the following Lemma which is the counterpart of Lemma (1.4.3) for the case of $\beta \in (0; 1)$.

Lemma 1.4.5 *For all* $\beta \in (0, 1)$, *and provided*

$$a) x_k > 2/5 T_{max} \quad \forall k \in i, j$$

$$b)|x_j - x_i|T_{max} < 1 \quad (1.30)$$

the function $R(y_i(x_i))$ *is concave in* x_i.

Proof Since $y_i = x_i^\beta$ and $0 < \beta < 1$, y_i is strictly concave in x_i, using Manganesian's Theorem [Avriel et al., 1988], establishing the concavity of $R(y_i)$ in y_i, is sufficient to prove the Lemma. We start with the term $F(T_{max}, y_i^\alpha, y_j^\alpha)$ and establish that $\frac{\partial^2 F}{\partial y_i^2} < 0$. The proof follows the same procedure as for Lemma (1.4.3). Substituting $y_i = x_i^\beta$ in 1.23 after partial differentiation across the integral, yields:

$$\frac{\partial^2 F}{\partial y_i^2} = \int_0^{T_{\max}} h_j(\tau, x_j) \frac{\partial^2}{\partial y_i^2} G(T_{\max} - \tau, y_i^\alpha) d\tau. \quad (1.31)$$

Furthermore,

$$\frac{\partial^2}{\partial y_i^2} G(T_{\max} - \tau, y_i^\alpha) = \frac{\alpha(T_{max} - \tau) y_i^\alpha e^{-y_i^\alpha (T_{max} - \tau)} (\alpha - 1 - \alpha y_i^\alpha (T_{max} - \tau))}{y_i^2}. \quad (1.32)$$

Note that the second partial derivative in (1.32) goes to zero whenever:

$$y_i^\alpha = \frac{\alpha - 1}{\alpha(T_{max} - \tau)} \quad (1.33)$$

or equivalently $x_i = \frac{1-\beta}{T_{max} - \tau}$.

Eq. (1.31) can then be re-written as:

$$\frac{\partial^2 F}{\partial y_i^2} = \int_0^{T_{\max} + \frac{\beta - 1}{x_i}} h_j(\tau, x_j) \frac{\partial^2}{\partial y_i^2} G(T_{\max} - \tau, y_i^\alpha) d\tau +$$

$$\int_{T_{\max}+\frac{\beta-1}{x_i}}^{T_{\max}} h_j(\tau, x_j)\frac{\partial^2}{\partial y_i^2}G(T_{\max}-\tau, y_i^\alpha)d\tau. \quad (1.34)$$

The negative characteristic of 1.34 is preserved for the following two conditions:

$$a) x_k > 2/5T_{max} \quad \forall k \in i, j$$

$$b) |x_j - x_i|T_{max} < 1$$

The proof of (1.34) has been carried to Appendix I.

Multiplying the strictly concave function $F(T_{max}, y_i^\alpha, y_j^\alpha)$ by a constant multiplier: $A = (1-M)\lambda C_v(\lambda)$ will not change its property, neither will adding a strictly concave function: $-c_i(y_i^\alpha + y_j^\alpha)$, change its property, as a result the function:

$$R(y_i) = AF(T_{max}, y_i^\alpha, y_j^\alpha) - c_i(y_i^\alpha + y_j^\alpha)$$

is strictly concave in y_i

Following the same assumption discussed in (1.26), $R(y_i)$ is positive and strictly concave, as a result it is strictly log-concave, multiplying $R(y_i)$ by the strict log-concave function $y_i^\beta[y_i^\alpha + y_j^\alpha]^{-1}$, will result in a strictly log-concave function, so the following function has a unique maximizer in y_i:

$$\frac{y_i}{y_i^\alpha + y_j^\alpha}AF(y_i^\alpha, y_j^\alpha, T_{\max}) - c_i(y_i^\alpha) \quad (1.35)$$

The function $y_i(x_i)$ is a one-to-one, strictly increasing function in x_i, as a result (1.35) has also a unique maximizer in x_i. By replacing y_i^α by x_i, we will arrive at:

$$\frac{x_i^\beta}{y_i^\alpha + y_j^\alpha}(1 - M)C_v(\lambda)F(x_i, x_j, T_{\max})\lambda - c_i(x_i) \tag{1.36}$$

By adding the constant $c_i(\mu_i - \lambda)$, which will not alter the property of having a unique maximizer, we will arrive at the objective function. Thus the function

$$ISP_{U_i}(x_i, x_j) = \frac{x_i^\beta}{x_i^\beta + x_j^\beta}(1 - M)C_v(\lambda)F(x_i, x_j, T_{\max})\lambda - c_i(x_i - \mu_i + \lambda) \tag{1.37}$$

has a unique maximizer in x_i, given the conditions 1.30 are satisfied. ∎

1.5 Conclusion

In this chapter, we have developed our Stackelberg game theoretic model for statistical decentralized QoS assurance for Internet services across multiple operator domains. Some important structural properties of the game were derived. In particular, while on the surface of things, individual ISPs have in principle to provide each two distinct decision variables, namely the packet transit time they aim at not exceeding within their network, and the extra amount of bandwidth they aim to buy, respectively, it turns out that the first decision can be made independently within each network. Thus, once this first decision is settled, the extra bandwidth for each network becomes the only core decision within the followers game. We have also investigated the influence of the bandwidth tuning parameter β on the utility functions and concluded that the smaller β, the lower the overall success rate, and the lesser the sensitivity of utility to bandwidth allocation. Finally, we have derived our main theoretical result consisting of sufficient conditions for the existence of a Nash equilibrium in the followers game, respectively for an arbitrary number of players when $\beta = 1$, and only two players at this stage for

$\beta \in (0;1)$. Further extensions of these results will be considered in future work. In the next chapters, we shall numerically investigate the performance of the proposed QoS assurance scheme.

CHAPTER 2

THE COMPLETE INFORMATION ENVIRONMENT GAME: ALGORITHMS AND NUMERICAL PERFORMANCE INVESTIGATION.

In this chapter, computational algorithms are presented for the proposed game in the case of perfect completed information, and the properties of the results are numerically investigated. In Sections 2.1 and 2.2, the algorithm and the design of the codes are presented; in the subsequent section a reference example is presented and the impact of the different system parameters on the solution of the game is investigated by perturbing these parameters one at a time. Although a mathematical proof of the existence of Nash equilibria for values of β other than 1 has not been established for 3 or more ISPs as yet, we have numerically investigated an example of three ISPs.

The goal in this chapter is to establish the numerical methods to solve the following non-linear equations:

$$\underset{j}{Max}\, TP_U(\Gamma_j), j = 1, \ldots, m \tag{2.1}$$

with m the number of discrete points, and $\lambda_d = \lambda_{max}/m$, while Γ_j is a row vector corresponding to the Nash equilibrium of the ISP game, when traffic rate is set at $j\lambda_d$. More specifically:

$$\Gamma_j = [j\lambda_d \quad \Delta\mu^*_{1,j} \ldots \Delta\mu^*_{n,j}], \quad j = 1 \ldots m \tag{2.2}$$

$$\text{with: } \Delta\mu^*_{i,j} = \arg \underset{\Delta\mu_{i,j}}{Max}\, ISP_{U_i}(\Delta\mu_{i,j}, \Delta\mu^*_{-i,j}, j\lambda_d),$$

$$\text{and: } \Delta\mu_{i,j} \in \{\Delta\mu_d, 2\Delta\mu_d, \ldots, m\Delta\mu_d\}, \text{ while } \Delta\mu_d = \Delta\mu_{max}/m$$

2.1 Infinite Vs. finite space of decision variables

In the game as formulated in chapter 1, ISP's decision variables $\Delta\mu_i$ can take any value between zero and $\Delta\mu_i^{max}$, while the rate of transfer λ can vary between zero and λ_{max} for *TP*. These assumptions categorize the game as an infinite action space game, i.e. an infinite number of possible values exist for each decision variables of each player of such a game. This situation simply cannot be implemented as is on a computer program.

In order to deal with this issue, the decision variable space has to be reduced to a finite number of elements. For the purpose of the simulations presented here, each continuous space of decision variables has been descritized to 1000 equidistant decision variables. This number represents a trade-off between speed of the algorithm and accuracy of the results. Clearly, more precise solutions can be obtained by increasing the number of discrete points.

2.2 Reaction functions, algorithms and updating schemes

In the game of followers, the information available to each ISP includes: $C_v(\lambda)$ the cost per unit of traffic for successfully delivered traffic, $\forall i$, c_i the unit cost for additional bandwidth for all ISPs, μ_i the initially available bandwidth of every ISP, *TP*'s fraction *M* of the customer payments, as well as the maximum tolerable delay (T_{max}), specified by *C*. Note that all these variables are defined in a unit time scale which is assumed to be shared by all players. This defines a *complete information* environment; it requires that every player know the strategies and payoffs of all the other players, but not necessarily their actions.

This would normally be the context in which the game in this chapter is defined. However, in order to carry out the computation of the (hopefully unique) Nash equilibrium in a numerical context, we pretend that the game is a sequential one at the ISP level and that all previous moves of every ISP at a given time step are known to all players. This is a so-called *perfect* information environment in sequential games. However, only the final Nash equilibrium solution is considered as the practically implementable step in a simultaneous game. Note that this virtual repeated game will become a true repeated game in the incomplete perfect information environment to be considered in Chapter 3.

The following algorithm explains the unfolding of a virtual perfect complete information sequential game between n followers (ISPs), in which the rate of transfer λ is announced by *TP* to all ISPs, and where each ISP knows his and every other ISP's initial bandwidth μ_i. Instead of working with $\Delta\mu_i$, we adopt the excess bandwidth x_i of each ISP as its decision variable. Recall that $x_i = \mu_i + \Delta\mu_i - \lambda$ and K is the iteration counter. Also X^K is a $1 \times n$ vector containing each ISP's response at the end of iteration K.

***Algorithm* 2.2.1**

(1) Let $K = 0$

(2) Assign a random initial feasible value to each entry of X^K*, e.g. zero buying bandwidth assignment*

(3) For i from 1 *to* n*, calculate the best response of ISPi:*

$$x_i^{K+1} = \arg\max_{x_i^K} ISP_{U_i}(x_1^{K+1}, \ldots, x_{i-1}^{K+1}, x_i^K, \ldots, x_n^K)$$

(4) If $X^{K+1} = X^K \pm \kappa$ *exit, if not let* $K = K + 1$ *and go to 3.*

The κ value is a tolerance threshold. It can be either increased to increase the speed, or decreased to increase the accuracy of the algorithm. A similar approach has been adopted in [Semret, 1999].

The algorithm (2.2.1) allows players to move in a predetermined numerical order. i.e. in the first round of iterations the first ISP will choose its decision variable x_1^K based on the preset random values of X^K, then the second ISP will base its calculation on the ISP1 decision (x_1^{K+1}) and the randomly assigned values for other ISPs ($x_3^K, x_4^K, \ldots x_n^K$), and this procedure continues until we reach ISP n. The subsequent rounds of the algorithm follow the same routine as the first round, but instead of a random set of values as for the starting point, the result of the previous round is used. The algorithm will stop once it has hopefully converged. i.e. when ISPs decisions all become fixed.

The updating scheme used in algorithm (2.2.1) is similar in sequence to that of the Gauss-Seidel iterative method as relates the finding of solutions to sets of linear equations. In the current context, however, a closed form for best response functions of each ISP is not available, and ISP_{U_i} displays a highly non-linear dependance on ISPi decision variable x_i. Algorithm 2.2.1 in fact solves a set of nonlinear equations of reaction functions, for which an exact closed form of equations does not exist. The sequential unfolding of the optimization for each one of the ISP utility functions can dramatically reduce the observed speed of convergence. Furthermore, no theoretical guarantees of convergence are provided at this stage.

Algorithm 2.2.1, can be run individually by each ISP, in the case of a perfect information environment. In the next chapter numerical analysis and properties of the game are considered in the context of perfect incomplete information.

For convenience, we can consider that *TP* sets the initial random conditions of all ISPs, although this is not essential since this is a virtual computation is carried by *TP* for each set of discretized possible traffic rate, to chose the most favorable one. If the algorithm converges, the solution is a stable unique ϵ-Nash equilibrium with respect to the given updating scheme, and set of initial conditions for the corresponding infinite nonzero-sum game, where ϵ is some function of the threshold value κ. The definition of ϵ-Nash equilibrium is the following: [Basar and Olsder, 1995]

Definition For a given $\epsilon \geq 0$, an n-tuple $\{\Delta\mu_1^\epsilon, \ldots, \Delta\mu_n^\epsilon\}$ with $\Delta\mu_i^\epsilon \in [0; \Delta\mu_i^{max}], \forall i \in \{0, \ldots, n\}$ is called a (pure) ϵ-Nash equilibrium for an n-person nonzero-sum infinite game if:

$$ISP_{U_i}(\Delta\mu_1^\epsilon, \ldots, \Delta\mu_n^\epsilon) \geq \underset{\Delta\mu_i}{Sup}\, ISP_{U_i}(\Delta\mu_1^\epsilon, \ldots, \Delta\mu_{i-1}^\epsilon, \Delta\mu_i, \Delta\mu_{i+1}^\epsilon, \ldots, \Delta\mu_n^\epsilon) - \epsilon$$

For $\epsilon = 0$, one simply speaks of *equilibrium* instead of *0-equilibrium.*

Thus, summarizing the above discussion, the following algorithm is executed by *TP*, in the first stage of the game, while assuming the following m discrete points as the admissible rates of sources: $[\lambda_d, 2\lambda_d, \ldots, m\lambda_d = \lambda_{max}]$:

***Algorithm* 2.2.2**

(1) For j from 1 to m, calculate the Nash equilibrium of the followers game by using Algorithm (2.2.1), when $j\lambda_d$ is taken to be the current source rate.

(2) For j from 1 to m, calculate $TP_U(j\lambda_d)$, from the results obtained in step 1.

(3) The optimal solution of TP is: $j^*\lambda_d$ where: $j^* = \arg \underset{j}{Max}\, TP_U(j\lambda_d)$, and $j \in 1, 2, \ldots, m$.

Thus, using Algorithm 2.2.1 in conjunction with (3) above, our particular Stackelberg problem detailed in (2.1 and 2.2)can be solved.

2.3 Numerical experiments for the case of three ISPs

In this section, we will investigate the case of three ISPs with arbitrary values presented in Table 2.1. A sensitivity analysis is carried out in subsections 2.3.1 to 2.3.5 by altering one input parameter at a time, and a numerical comparison between the original case and the new results as well as an analysis of the solution are undertaken.

Table 2.1 Values used in the base case

	ISP1	ISP2	ISP3
C_i	0.249	0.283	0.175
μ_i (Packet/ms)	1.1	1.2	1.28
M		20%	
$C_v(\lambda)$		$2e^{-\lambda/2}$	
T_{max}		6ms	

2.3.1 Impact of the delay tuning factor β

The model is investigated for six different values of $\beta \geq 0.5$. The results are shown in Table 2.2, and Figures 2.1, 2.2, 2.5 and 2.6. A more detailed analysis is presented for the special case of $\beta = 1$, with Figures 2.3 and 2.4.

In view of (1.18), and for a fixed λ and a fixed choice of bandwidths, decreasing the delay tuning factor β tends to reduce the relative advantage of the ISP with the largest bandwidth, and increase the returns of the lower bandwidth ISPs (see Fig. 2.1 and 2.2). This tends to reduce the amount of bandwidths that ISPs are willing to buy, thus reducing the overall probability of success (see Table 2.2), which drives *TP*'s benefits down and lowers the customer QoS. Although lower β results in less income for *TP*, at the Stackelberg game level, *TP* still chooses to settle for a the same $\lambda_{max} = 1$.

In Fig. 2.3, shares and utilities of ISPs are depicted. In an interval of λ, where no ISP is buying any additional bandwidth, the revenue shares of ISPs remain constant. This

Table 2.2 Experiment results of three competitive ISPs for different values of β

	$\beta = 1$	$\beta = 0.9$	$\beta = 0.8$	$\beta = 0.7$	$\beta = 0.6$	$\beta = 0.5$
Optimal λ(packet/ms)	1	1	1	1	1	1
$\Delta\mu_1$ at NE (packet/ms)	0.795	0.710	0.654	0.597	0.537	0.472
$\Delta\mu_2$ at NE (packet/ms)	0.508	0.462	0.435	0.399	0.356	0.306
$\Delta\mu_3$ at NE (packet/ms)	1	1	0.872	0.747	0.628	0.514
ISP_{U_1} at NE	0.0744	0.0786	0.0897	0.0977	0.1015	0.1003
ISP_{U_2} at NE	0.0717	0.0822	0.0970	0.1086	0.1160	0.1183
ISP_{U_3} at NE	0.2145	0.2106	0.2019	0.1924	0.1810	0.1667
TP_U at NE	0.2181	0.2122	0.2050	0.1953	0.1824	0.1655
$Pr(t \leq T_{max})$ %	90.42	88.01	85.25	81.51	76.56	70.02

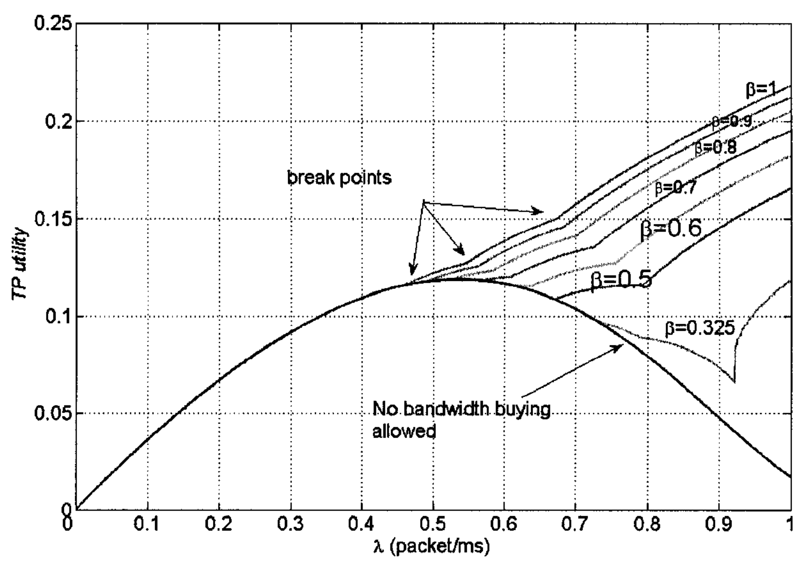


Figure 2.1 *TP*'s utility versus transfer rate λ dependency curves, as pre-computed by *TP* based on knowledge of the reaction functions of the ISPs, and as a function of the tuning factor β. The bottom solid line represents the case of the zero-sum game in which no ISP is allowed to buy any bandwidth

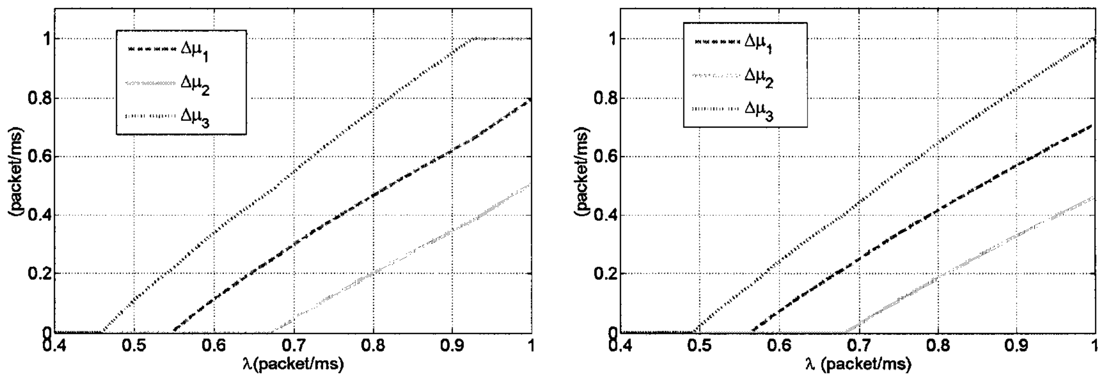


Figure 2.2 $\Delta\mu$ Vs. λ; left: $\beta = 1$; right: $\beta = 0.9$

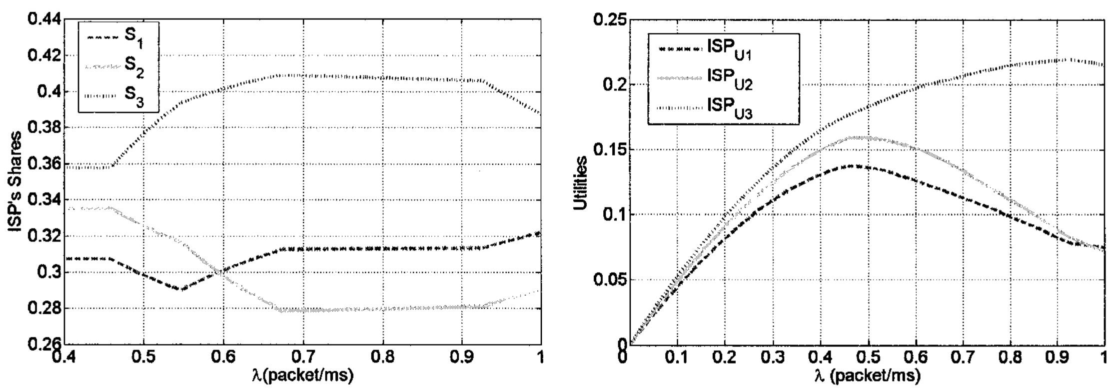


Figure 2.3 Right: ISP's utilities versus λ, for the case of $\beta = 1$, left: shares of ISPs for the case of $\beta = 1$ according to 1.4 in Chapter 1. An almost constant set of shares is observed in the interval at which ISPs are buying bandwidth linearly with respect to λ

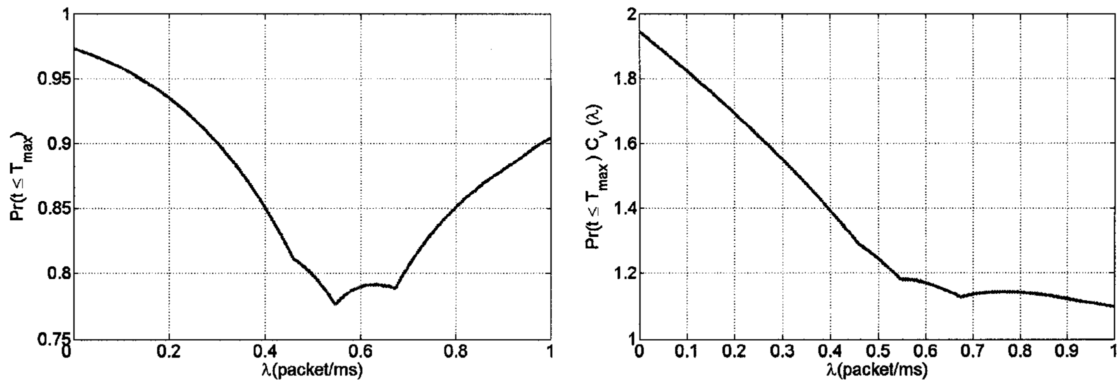


Figure 2.4 Left: The global probability of success versus λ; right: the unit cost of successful traffic paid by customer.

appears to be also the case on trafic rate intervals where the three ISPs choose to increase their decision variable in a linear fashion with respect to λ.

As ISP3 starts to buy extra bandwidth at $\lambda \approx 0.45$, its share (S_3) starts to increase. This happens at the cost of S_1 and S_2, plunging down. The decision of ISP3 affects ISP_{U_2} and ISP_{U_1}, which see their utility function start to decrease. (See right part of Fig. 2.3). Later when ISP1 enters the stage by buying bandwidth, its share (S_1) starts to grow, thus decreasing the slope of th S_2 versus λ curve even more (see left part of Fig. 2.3), and making the situation less desirable for ISP2. Finally when ISP2 starts kicking in extra bandwidth, it can barely increase its share for higher λ's, since ISP1 and ISP2 are already major players buying more bandwidth, and claiming bigger portions of the total revenue. The outcome of such a linear increase in $\Delta\mu$ of every ISP is that the relative position of each ISP with respect to other in sharing the revenues remains constant.

ISP's behavior as would be perceived by customer C is presented in Fig. 2.4. It is observed that the probability of global success goes through a minimum, while the total unit price paid by C decreases with traffic level.

The so called breakpoints in $TP_U(\lambda)$ are presented in Fig. 2.1. They correspond to the

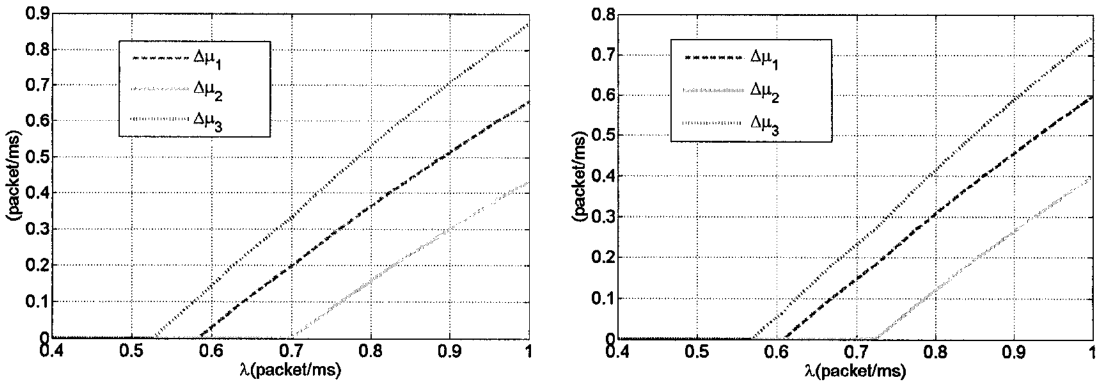


Figure 2.5 $\Delta\mu$ Vs. λ; left: $\beta = 0.8$; right: $\beta = 0.7$

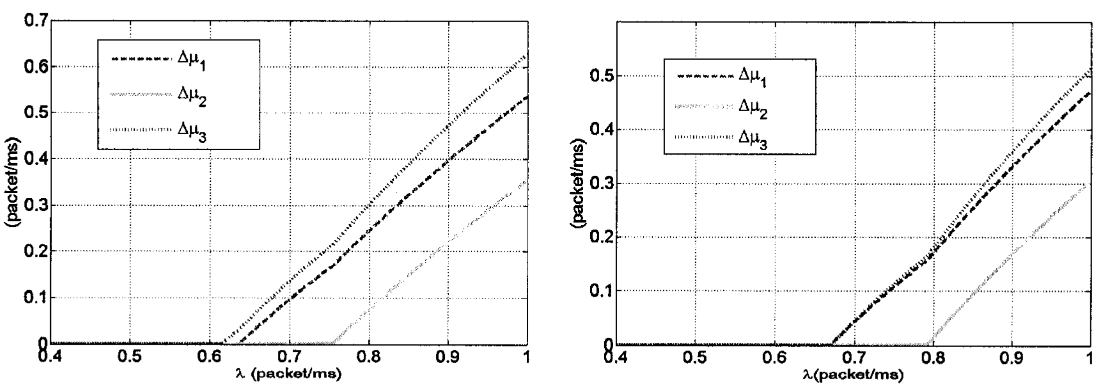


Figure 2.6 $\Delta\mu$ Vs. λ; left: $\beta = 0.6$; right: $\beta = 0.5$

points at which a given ISP starts to buy bandwidth. The first breakpoint can be defined as a rate of transfer λ^*, past which *TP* expects a higher revenue as compared to the game where no ISP is allowed to buy more bandwidth (the zero sum version of the game). This is due to the decision of an ISP to contribute to the data flow by buying more bandwidth.

As other ISPs enter the market (i.e. start to buy bandwidth) when $\lambda > \lambda^*$, new breakpoints in TP_U are observed, which all tend to increase the slope of $TP_U(\lambda)$. As a result, since for our example, the three ISPs eventually buy some bandwidth for every $\beta \geq 0.5$, a total of three breakpoints are observed, while for $\beta = 0.5$ and 0.6, the first and second breakpoints appear to coincide since, ISP1 and ISP2 start to buy almost at the same source rate. This is clearly depicted in Fig. 2.6.

The effects of ISPs on $TP_U(\lambda)$ is large enough in the considered range of β, so that *TP* chooses λ_{max} instead of the its optimal choice in zero sum game, as the announced rate of transfer. Further numerical calculations in Fig. 2.1 show that at $\beta < 0.325$, this is reversed and *TP* will prefer to chose $\lambda < \lambda^0 = 0.536$.

The decision of ISPi, x_i affects two different parameters in the model, as viewed externally; firstly, the share of the ISP from the total benefits received from customer after fixed deduction by *TP*, which was specified as $S_i = x_i^\beta / \sum_{j=1}^{n} x_j^\beta$; secondly, the total benefits itself which is a direct function of every ISPs decision variable, given by $A = (1 - M)C_v(\lambda)Pr(t \leq T_{max})$ in Chapter 1. Thus, the first ISP to kick in more bandwidth has its strategy dominated by the improvement in his share of benefit, rather than the impact that it can have on success rate.

The variation of β will directly affect the share $S_i$that each ISP receives, but also it has an indirect effect on the total benefits received from customer A.

The total benefits that each ISP receives $(S_i(1 - M)C_v(\lambda)Pr(t \leq T_{max})\lambda)$ is the main source of income for ISPi, on the other hand it limits the spending of ISPi on additional

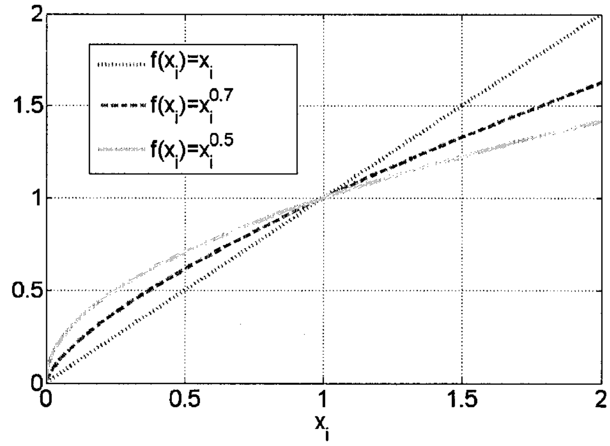


Figure 2.7 The function x_i^{β} for three different values of β

bandwidth, or in other words it plays a major role on the decision that ISPi takes. This decision $\Delta\mu_i$ is a building block of the global probability of success, which regulates the payment of *C*, to *TP*. The less total benefits received by *TP*, the less will go to any agent of the game, including ISPi. This closes the chain of reaction loop caused by decreasing the β factor.

A β value less than one, increases (or decreases) the *bargaining power* of each ISP in the sharing mechanism, in terms of yielding a higher (or lower) value in x_i^{β} than x_i, depending whether $x_i > 1$ or $x_i < 1$. In Fig. 2.7, the simple function of x_i^{β} is presented for three values of β, respectively 1, 0.7 and 0.5. The range of $[0; 2]$ has been chosen for x_i to present feasible values of x_i in the current example ($\Delta\mu_i^{max} + \lambda_{max} = 2$).

This amplifying effect can be indicated as $x_i^{\beta} - x_i$, which has a maximum at $x_i^* = \beta^{\frac{1}{1-\beta}}$.

A lower β, yields lower x_i^*, encouraging ISPs to invest less in their bandwidth, to enjoy the maximum amplifying effect in the sharing mechanism. On the other hand, since lowering β requires a lower additional bandwidth by ISPs, it will also eliminate the economical advantage of ISPs which had cheaper additional bandwidth at their possession. For instance ISP 3 in the current example has considerably cheaper bandwidth; as β

Table 2.3 Experiment results for three different values of $T_{max} = 6, 12, 18$ms.

	$T_{max} = 6$	$T_{max} = 12$	$T_{max} = 18$	$T_{max} = 100$
Optimal λ(packet/ms)	1	1	1	1
$\Delta\mu_1$ at NE (packet/ms)	0.795	0.736	0.733	0.732
$\Delta\mu_2$ at NE (packet/ms)	0.508	0.405	0.381	0.379
$\Delta\mu_3$ at NE (packet/ms)	1	1	1	1
ISP_{U_1} at NE	0.0744	0.1135	0.1174	0.1177
ISP_{U_2} at NE	0.0717	0.1001	0.1014	0.1015
ISP_{U_3} at NE	0.2145	0.2794	0.2860	0.2864
TP_U at NE	0.2181	0.2414	0.2425	0.2426
$Pr(t \leq T_{max})$ %	90.42	99.55	99.98	1

decreases, its total utility decreases at the final *lambda.*

One of the drawbacks of the presented results is the low values for probability of global success as β decreases, which is due to less additional bandwidth that each ISP brings in. However, these low probabilities could be justified given the stringent required delay $T_{max} = 6$ ms, and high bandwidth unit costs of ISPs. In the next subsection we investigate the variation of the total allowed transit time T_{max}.

2.3.2 Impact of T_{max}

VoIP communication in general tolerates loss probabilities of less than 1%. However every VoIP protocol (SIP, H.323 etc.) is equipped with a mechanism to buffer packets to be able to arrange out of order packets. The average tolerable delay by a typical VoIP is about 100 ms; thus considering higher values than a T_{max} of 6 ms is justifiable.

In this section, we study the effect of increasing T_{max} on the global probability of success, and on the behavior of ISPs for the base case ($\beta = 1$) parameters. The outcome of the game is presented in Table 2.3.

Increasing T_{max}, increases the global success probability and as a result, increases the total revenue that each ISP and *TP* get from the customer. Although one may think that this would encourage ISPs to invest more in their additional bandwidth, an inverse effect is observed in Table 2.3 for ISPs 1 and 2. One way to interpret this behavior is that the ISPs can obtain the same utility with lower investments.

For each given λ, as $T_{max} \to \infty$, the probability of global success is replaced by 1, and the game between ISPs can be further simplified. Since each ISP's decision variable is not going to affect the total income, every ISP's optimization problem can be focused on its share of the total S_i and the internal costs $c_i \Delta\mu_i$. As a result, the final solution to the followers game will depend only on the initial bandwidths and unit costs. In the mentioned example, at $T_{max} = 6ms$, ISP1 and ISP2 chose to invest more in their additional bandwidth, to increase the global performance. As T_{max} increases, probability of success increases for equal bandwidths. As a result, ISP1 and ISP2 decide to cut some of the budget that goes to additional bandwidth buying, and do gain more at the end of the day. An overall higher global probability of success is achieved. For the sake of comparison the results of $T_{max} = 100ms$, are also presented in Table 2.3.

2.3.3 Variation of the unit costs

Unit costs are a major factor in defining the feasibility region of decision variables of ISPs, defined by a positive return for each agent of the game. On the other hand, additional bandwidth unit cost defines each ISP's capabilities to compete with others and produce more profit.

In this section four different sets of unit costs, other than the reference example values, are investigated for the case of $\beta = 1$. The costs are presented in Table 2.4 and the results in Table 2.5, Fig. 2.8, and Fig. 2.9.

Table 2.4 Different sets of costs

	ISP1	ISP2	ISP3
Unit costs set A	0.220	0.255	0.162
Unit costs set B	0.310	0.490	0.265
Unit costs set C	0.440	0.244	0.235
Unit costs set D	0.323	0.244	0.235
Unit Cost set of the reference example	0.249	0.284	0.175

Table 2.5 Experiment results for different unit cost sets.

	reference example	Cost set A	Cost set B	Cost set C	Cost set D
Optimal λ(packet/ms)	1	1	0.785	0.711	1
$\Delta\mu_1$ at NE (packet/ms)	0.795	0.940	0.268	0	0.505
$\Delta\mu_2$ at NE (packet/ms)	0.508	0.598	0	0.211	0.776
$\Delta\mu_3$ at NE (packet/ms)	1	1	0.234	0.207	0.747
ISP_{U_1} at NE	0.0744	0.0961	0.0999	0.1099	0.084
ISP_{U_2} at NE	0.0717	0.0799	0.1300	0.1482	0.064
ISP_{U_3} at NE	0.2145	0.2108	0.1668	0.1710	0.1555
TP_U at NE	0.2181	0.2260	0.1312	0.1311	0.2079
$Pr(t \leq T_{max})$ %	90.42	93.59	63.94	67.63	86.64

2.3.3.1 Set A and B

The two cost sets A and B represent a cheaper and a more expensive market respectively. Or equivalently they can also correspond to a low and high priced time periods.

As expected, each ISP's decision to buy more, is a decreasing function of the unit costs. With lower unit cost (set A) ISPs buy more and the global performance,$Pr(t \leq T_{max})$, improves. The inverse effect is observed for cost set B.

As λ increases, for a fixed additional bandwidth of ISPs, the global success probability will decrease exponentially. The cost set A results, depicted in Fig. 2.8, shows a situation where all 3 ISPs will finally start to buy some bandwidth as λ increases, giving the total benefits graph a boost each time they enter the game.

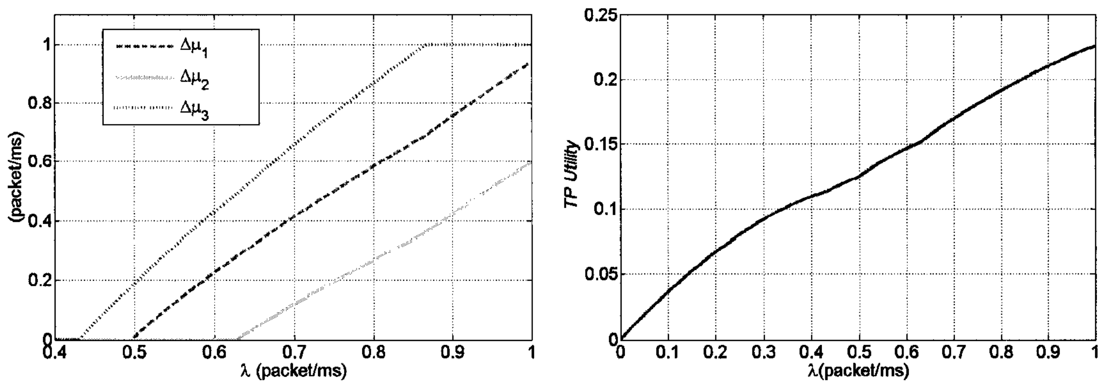


Figure 2.8 Left: $\Delta\mu$ Vs. λ and $\beta = 1$, for cost set A; right: TP_U Vs. λ.

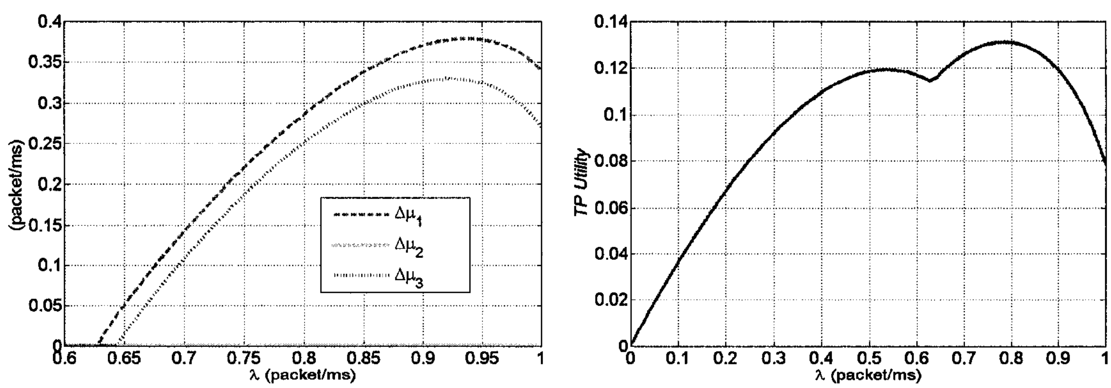


Figure 2.9 Left: $\Delta\mu$ Vs. λ and $\beta = 1$, for cost set B; right: TP_U Vs. λ.

However, in the case of higher unit prices (Cost set B), ISP2 does not find it worthwhile to participate in the bandwidth buying game due to its high unit cost. As a result, the two other ISPs tend to decrease their investments in additional bandwidth as λ increases, probably due to the corresponding fast decrease of probability of success rate, as the the final outcome of the game appears not to yield enough revenue to justify this investment, due to the bottleneck created by ISP2. In other words, the strictly increasing nature of optimal additional bandwidth of ISPs, which has been observed for other unit cost sets, has disappeared in Fig. 2.9. This, in turn, reflects on the behavior of $TP_U(\lambda)$ i.e. a decreasing property toward the end of the range λ, highlighting the fact that the optimal choice of *TP* can be situated between λ^0 and λ_{max} where λ^0 is the maximizer of $TP_U(\lambda)$, in the game where no ISP buys any additional bandwidth.

It remarkable that, although unit costs are increasing from set A to set B, the utilities of ISP1 and ISP2 are increasing too. This comes at the cost of ISP3 and *TP*, which starts to loose some profits. The effect is mainly due to the fact that in the leader optimization part of the game, *TP* is choosing a smaller rate of source λ for cost set B rather than choosing $\lambda_{max} = 1$ (See Fig. 2.8 and 2.9).

With this new low value of λ, ISP1 and ISP2 can still achieve a good performance both in their individual probability of success, and their share of the total benefits, by giving up on almost any additional bandwidth and relying mainly on their initial bandwidths.

2.3.3.2 Sets C and D

These two unit costs are tailored to show the effect of only one ISP's unit cost on the whole system. In cost set C, unit cost of ISP1 is considerably higher than other ISPs. As a result in a similar effect observed for cost set B, ISP2, and ISP3 will start to buy bandwidth, but as λ increases, they limit their additional bandwidth, since ISP1 is not going to contribute more to the flow, and any effort to improve the total delay is not fruitful. This reaction by ISP2, and ISP3 result in a strictly concave set of curves for optimal additional bandwidths of ISP1 and ISP2, as shown in Fig. 2.10.

In comparison to cost set C, cost set D associates a lower unit cost to ISP1, while the rest of the prices remain the same. Substantial differences can be seen in the behavior of the two other ISPs, and *TP*. ISP1 enters the bandwidth buying game at $\lambda = 0.663$, while giving the two other ISPs incentives to invest more, i.e. increasing the slope of $\Delta\mu_i$ versus λ to maintain their shares of revenue. This gives a boost to $TP_U(\lambda)$ in Fig.2.10. The lower unit costs for ISP1 will move the optimal decision of *TP* to λ_{max}, and increase its utility, but has a negative effect on every ISP's utility, since a higher λ puts more pressure on them to meet the delay criteria.

2.3.4 Impact of customer response curve

One important factor that can affect the global performance of ISPs is $C_v(\lambda)$, the unit cost that the customer is willing to pay per successful (in terms of low delay) bandwidth unit. Thus one possible cause for settling at a λ which is far less than λ^{max} might be a sharp decrease in $C_v(\lambda)$, which eliminates the effect of higher λ, in the ISP's utility function.

In this subsection, we investigate the base example for different customer response curves parameterized by K. $C_v(\lambda) = 2e^{(-\lambda/k)}$ for *k*= 1.2, 1.5, and 2.8, where $K = 2$ was presented in the base example. Higher values of K result in a milder slope for $C_v(\lambda)$ versus λ, and thus a higher payment by customer as one moves towards λ_{max}.

The results are shown in Table 2.6. ISP's responses are shown in Fig. 2.11, while TP_U associated with ISP's response is depicted in Fig. 2.12.

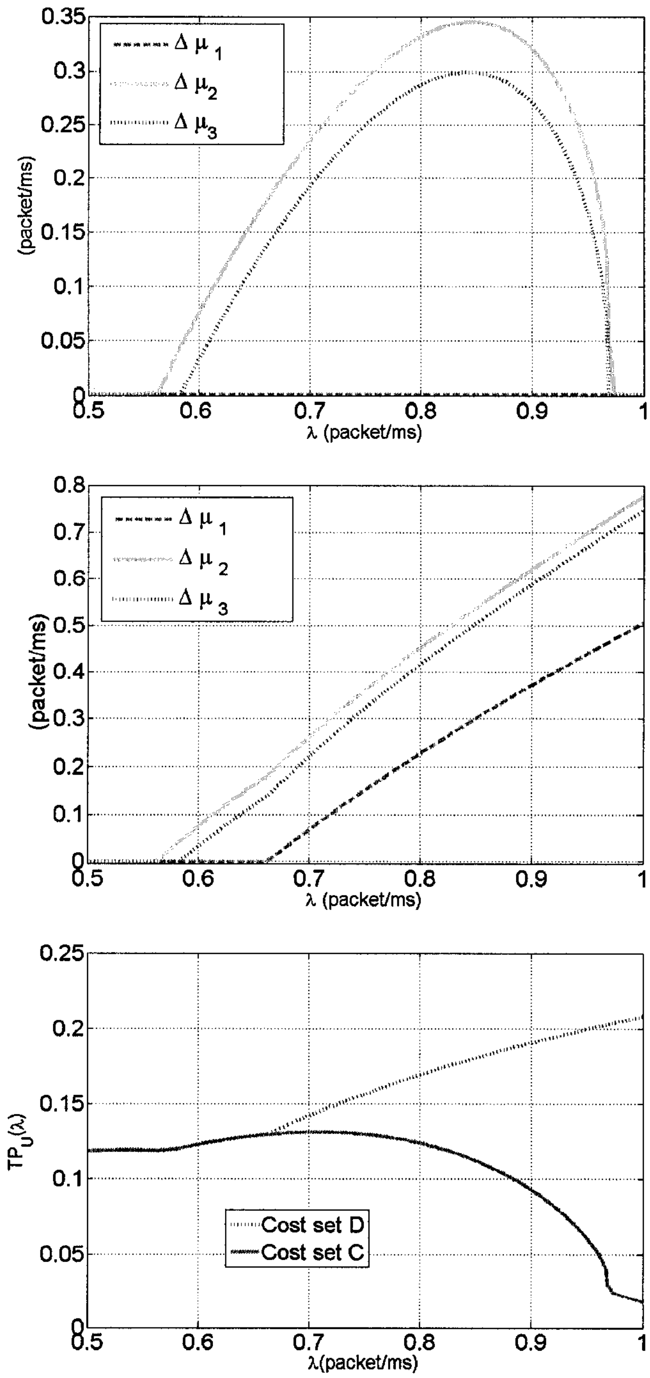
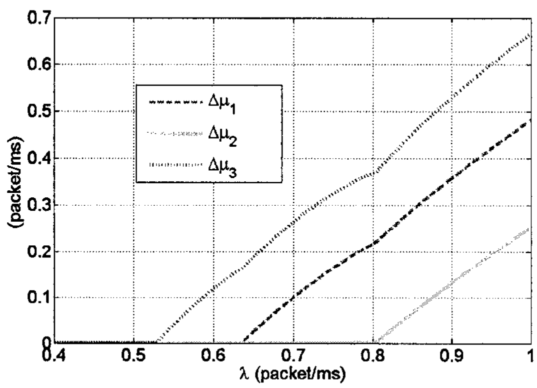


Figure 2.10 Top: $\Delta\mu$ Vs. λ and $\beta = 1$, for cost set C; middle:$\Delta\mu$ Vs. λ and $\beta = 1$, for cost set D; bottom: the corresponding utility of *TP* for cost sets C, and D

Table 2.6 Experiment results for different unit customer response curves.

	$K = 1.2$	$K = 2$	$K = 2.8$
Optimal λ(packet/ms)	1	1	1
$\Delta\mu_1$ at NE (packet/ms)	0.480	0.795	0.958
$\Delta\mu_2$ at NE (packet/ms)	0.251	0.508	0.642
$\Delta\mu_3$ at NE (packet/ms)	0.664	1	1
ISP_{U_1} at NE	0.0237	0.0744	0.1128
ISP_{U_2} at NE	0.0404	0.0717	0.0980
ISP_{U_3} at NE	0.1166	0.2145	0.2501
TP_U at NE	0.1197	0.2181	0.2629
$Pr(t \leq T_{max})$ %	70.14	90.42	94.35



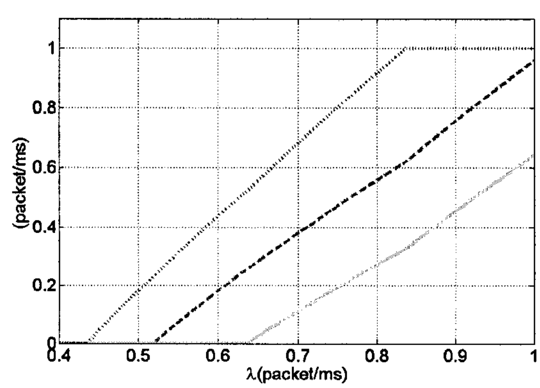


Figure 2.11 $\Delta\mu$ Vs. λ for different customer response curves; left: $K = 1.2$; right: $K = 2.8$

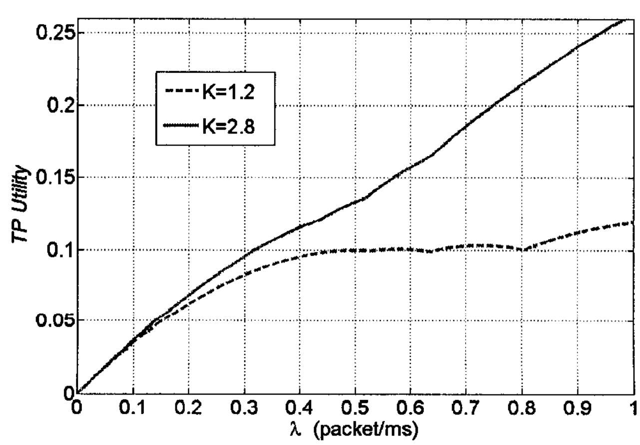


Figure 2.12 The utility of *TP* while predicting ISPs reaction for $\beta = 1$ versus the rate of transfer λ for different customer response curves; $C_v(\lambda) = 2e^{(-\lambda/k)}$

As expected, a higher payment by customer (a milder decreasing slope of $C_v(\lambda)$), increases the performance of the outcome, as well as utilities of all the agents in the game. While a faster decreasing $C_v(\lambda)$, can significantly deteriorate the performance and utilities.

It is important to remember that the total price paid by customer is $\lambda C_v(\lambda)Pr(t \leq T_{max})$, at the announced rate of source λ. For K=1.2 and 2.8 the total prices per unit time paid by the customer are respectively 0.6096 and 1.3202. In other words, the customer is paying more than double the price per unit time to push the probability of success from 70% to 94%, at $\lambda = 1$.

2.3.5 Impact of the share of *TP*

TP as the regulator and leader of the game receives a fixed share of the revenues. This portion of the total, which was indicated by M in all utility functions, will also definitely have an influence on each ISP's net income. As a result, changing the preset variable M will alter ISPs reaction, and the final settling point of the game.

In this subsection, we investigate two different values of $M = 0.1$ and $M = 0.3$, besides the reference example ($M = 0.2$). As expected a higher share for *TP* tends to dampen ISP's enthusiasm for bandwidth buying, and as a result the final probability of success is lower. Table 2.7 and Fig.2.13 summarize the obtained results.

2.4 Conclusion

In this chapter, we have presented results of the implementation of the pricing scheme proposed in Chapter 1. The numerical algorithms and their implementation using Matlab® program have been discussed. The focus here was on a perfect complete information en-

Table 2.7 Experiment results for different *TP*'s share.

	$M = 0.1$	$M = 0.2$ (reference example)	$M = 0.3$
Optimal λ(packet/ms)	1	1	1
$\Delta\mu_1$ at NE (packet/ms)	0.929	0.795	0.658
$\Delta\mu_2$ at NE (packet/ms)	0.618	0.508	0.395
$\Delta\mu_3$ at NE (packet/ms)	1	1	0.946
ISP_{U_1} at NE	0.1056	0.0744	0.0472
ISP_{U_2} at NE	0.0930	0.0717	0.0539
ISP_{U_3} at NE	0.2442	0.2145	0.1759
TP_U at NE	0.1132	0.2181	0.3055
$Pr(t \leq T_{max})$ %	93.80	90.42	84.59

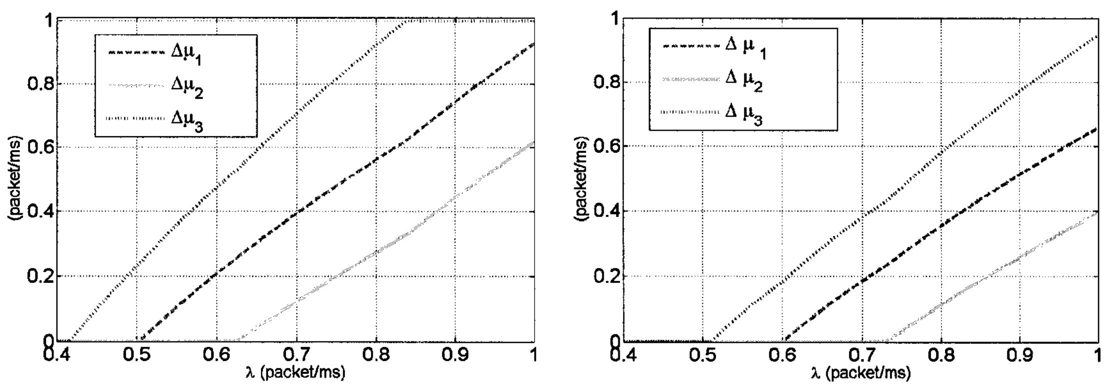


Figure 2.13 $\Delta\mu$ Vs. λ and $\beta = 1$ for different fractions for *TP*'s profit; left: $M = 0.1$; right: $M = 0.3$

vironment, whereby all calculations could be carried out in a decentralized manner by each ISP. As a result the assumption of perfectness does not seem to be substantial in this chapter. In the following chapter the case of an incomplete but perfect information game is discussed.

The numerical results indicate that the pricing schemes leads to overall very reasonable results, and occasionally some interesting, non intuitive behaviors, which can emerge in view of the sheer complexity of players interactions. Under some circumstances, ISPs can end up earning more money even though their bandwidth buying costs have increased. The scheme also tends to force ISPs to increase their bandwidths, even though their individual profits may be decreasing; this ISP action is in an effort to prevent their shares of profit from falling. Furthermore, a tuning parameter β can help offset some of the inequalities that can come from too widely spread bandwidth unit costs or initial bandwidths, along the route traversed by traffic. However, decreasing β can have a negative impact on performance, as measured by the probability of meeting maximum delay constraints.

The numerical analysis appears to indicate that for the given set of values, as long as every ISP participates in the bandwidth buying game, *TP* will chose λ_{max} as the optimal traffic rate. However, if one ISP decides not to participate in the bandwidth buying game, and relies only on its initial bandwidth, other ISPs can lose interest in investing more as λ increases, due to their unit costs: the higher the unit cost is for participating ISPs, the sharper will the decrease in their additional bandwidth buying versus λ be. In this case, *TP* will also choose a rate of transfer that is less than λ_{max}.

CHAPTER 3

NUMERICAL ANALYSIS IN AN INCOMPLETE PERFECT INFORMATION ENVIRONMENT

3.1 Introduction

The complete information assumption in Chapters 1 and 2 can be considered in practice to be a strong assumption. In an incomplete information game, either certain relevant details are withheld from the players, or knowledge is unreliable. In this chapter, the cost of additional bandwidth of each ISP is considered to be concealed from other players including the leader of the game *TP*. The initial available bandwidth is still supposed to be a known variable to every agent of the game, since it can be measured using methods like active probing.

3.2 Sequence and Structure of the Perfect Incomplete Information Game

TP as the leader of the Stackelberg game needs to predict the exact response of each ISP in order to be able to calculate its best decision variable. In this chapter, we consider that the costs of additional bandwidth, which play an important role in the willingness of each ISP to buy more bandwidth, and contribute to the global success probability, are unknown to *TP*. On the other hand, the utility function of *TP* always yields a positive value for every outcome of the game. From the point of view of *TP*, this situation suggests preliminary probing steps, since it can examine ISP's feedbacks by choosing random $lambda$ values.

More specifically, the game is played in two phases: a preliminary learning phase, and a more definitive decision making phase. In the preliminary phase of the game, *TP* picks an arbitrary point in the interval $[\lambda_0, \lambda_{max}]$, and announces it to the ISPs which in turn go through a, hopefully convergent, repeated game phase about to be described, and with *TP* acting as a mediator. λ_0 is reminded to be the optimal choice of *TP*, where no ISP is allowed to buy any additional bandwidth. The preliminary phase of the game, can last several stages until *TP* learns every ISP's unit cost and moves to the decision making phase.

The idea is to implement iterations analogous to Algorithm 2.2.1, but no longer virtually and mediated by *TP* during what we call an arbitration transient period. The iterations in Algorithm 2.2.1 are carried out in a specific sequence. The role of *TP* is to impose the sequence in which the ISPs update their bandwidth assignments.

Thus, in summary, the mediating agent should perform two major tasks:

- Maintain an array X^K containing the estimated current decision variable of every ISP
- For i from 1 to n : Deliver the array $(x_1^{K+1}, x_2^{K+1}, \ldots, x_{i-1}^{K+1}, x_{i+1}^{K} \ldots, x_n^{K})$ to ISPi, and update the array with ISPi estimated response x_i^{K+1}.

Note that K is the iteration number in the repeated game, played among followers.

TP as the leader of the game, is responsible for performing the mediation between ISPs, and is able to estimate the unit cost associated with each ISP through ISP's best responses.

3.3 Unit cost estimation algorithm

As discussed in previous section, the unit cost of additional bandwidth is an essential information for *TP* to predict the responses of followers for each rate of transfer λ it chooses. To do this task an arbitrary λ is announced to ISPs, and an iterative game between followers starts, hopefully converging and thus yielding a solution $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$. Based on that solution, the following algorithm is aimed at estimating the unknown unit bandwidth costs of ISPi from a single response vector X^*.

***Algorithm* 3.3.1** *(1) Compute the maximum possible unit cost of ISPi as:*

$$C_{i_{max}} = \frac{x_i AF(X^*)}{(x_i - \mu_i + \lambda)\sum_{j=1}^{n} x_j^*} \quad \textit{where: } A = (1 - M)C_v(\lambda)\lambda \tag{3.1}$$

(2) Let $C_{High} = C_{i_{max}}$ *and* $C_{Low} = 0$

(3) Let $C_{op} = \frac{C_{High}+C_{Low}}{2}$, *where "op" stands for operational point.*

(4) Calculate the best response of ISPi for C_{op}*:*

$$x_{i,op} = \arg\max_{x_i}(A\Pr(t \leq T_{max})(\frac{x_i}{x_i + \sum_{j=1,j\neq i}^{n} x_j^*}) - C_{op}(x_i - \mu_i + \lambda) \quad)$$

(5) If $|x_i^* - x_{i,op}| \leq \varepsilon'$, *exit the algorithm with* C_{op} *as the estimated cost.*

(6) if $x_{i,op} \geq x_i^*$, *let* $C_{Low} = C_{op}$ *and go to step 3*

(7) if $x_{i,op} < x_i^*$ *let* $C_{High} = C_{op}$, *and go to step 3.*

It is needless to say that a necessary condition for the above algorithm to converge for ISPi, is a *positive* value of $\Delta\mu_i$. In other words, for the given rate of transfer λ, every ISP has to buy a certain amount of additional bandwidth, so that *TP* could estimate the associated unit costs of every ISP. In that sense, *TP* may wish to probe the ISPs through more than one value of λ in the hope to solicit some bandwidth buying. This being said, if a given ISP never buys bandwidth, one can consider the associated cost to be infinite. Moreover, an accurate estimate of ISP's unit cost requires a decision variable $\Delta\mu_i$ less than the allowed maximum bandwidth $\Delta\mu_{max}$ or otherwise *TP* can only estimate the higher limit of the unit cost of the ISP in question.

3.4 An incomplete information numerical example

In this section, a numerical example is presented, in which the unit costs associated with additional bandwidths are not announced. *TP* initiates the learning phase of the game by announcing a random rate of transfer. This learning phase can be broken up into separate sub-learning phases during which some of the ISPs decide to buy bandwidth, and their cost is accordingly estimated via Algorithm. 3.3.1. As long as all ISP's costs have not been estimated, *TP* continues the learning phase, but with a different value of λ. The choice of λ may be chosen to get a reaction from a particular previously unidentified ISP. Once the internal unit costs are all estimated, with possibly some costs set at infinity, *TP* can move to the next phase of the game, i.e. the decision phase, thus anticipating the actions of every ISP, for every possible λ. Given this information, the leader of the game can choose a value for λ that benefits it most, and announce it to the followers who then seek the related Nash equilibrium through a *TP* mediated repeated game.

We present in Table 3.1 the estimated and actual cost values for the case of $\beta = 1$, using the same inputs as the 3 ISPs example of the previous chapter.

Although, the result in Table 2.2 indicate, $\Delta\mu_3 = \Delta\mu_{max}$ at the announced λ, the estimated cost of ISP3 is close to the real one. This is due to the fact that the optimal decision of ISP3 in the case where no upper bound exist for the decision variable is close to the current point, which is $\Delta\mu_{max}$.

Table 3.1 Estimated and real unit costs associated with additional bandwidth for the 3 ISPs example of chapter 2 (the base example).

	ISP1	ISP2	ISP3
Real Costs	0.24900	0.28400	0.17500
Estimated Costs	0.24914	0.28429	0.17513
ε'		10^{-5}	

3.5 Dynamic Perfect Incomplete Information Environment for Repeated Games

> *Most of the past research on the use of game theory in networking problems, has restricted itself to the use of* static *games as models, although in some cases the players clearly interact with each other many times [La and Anantharam, 2002]*

In order to bring the proposed model even closer to a practical situation, a stochastic environment in which game parameters can possibly change at the end of each learning phase is considered in this section.

In the dynamic repeated version of the game, we let the unit cost of each ISP change as we move from a cycle of learning/decision phases to the next, while *TP* keeps on validating/verifying the estimates of the unit costs based on the most recent reactions of the ISPs. We do not expect frequent changes of unit costs of ISPs, so that *TP*'s calculations should remain valid for at least some cycles of the game. The flowchart in Fig. 3.1 presents the dynamic iterative game.

The setup of the dynamic game is such that whenever *TP* sees a difference between the predicted and actual response of ISPs, it will reestimate the unit costs to adapt to any cost changes. Also, in an extension to this version of the game, one could consider some smoothing algorithm for the initial bandwidths estimates as time goes by, e.g., exponential forgetting factors.

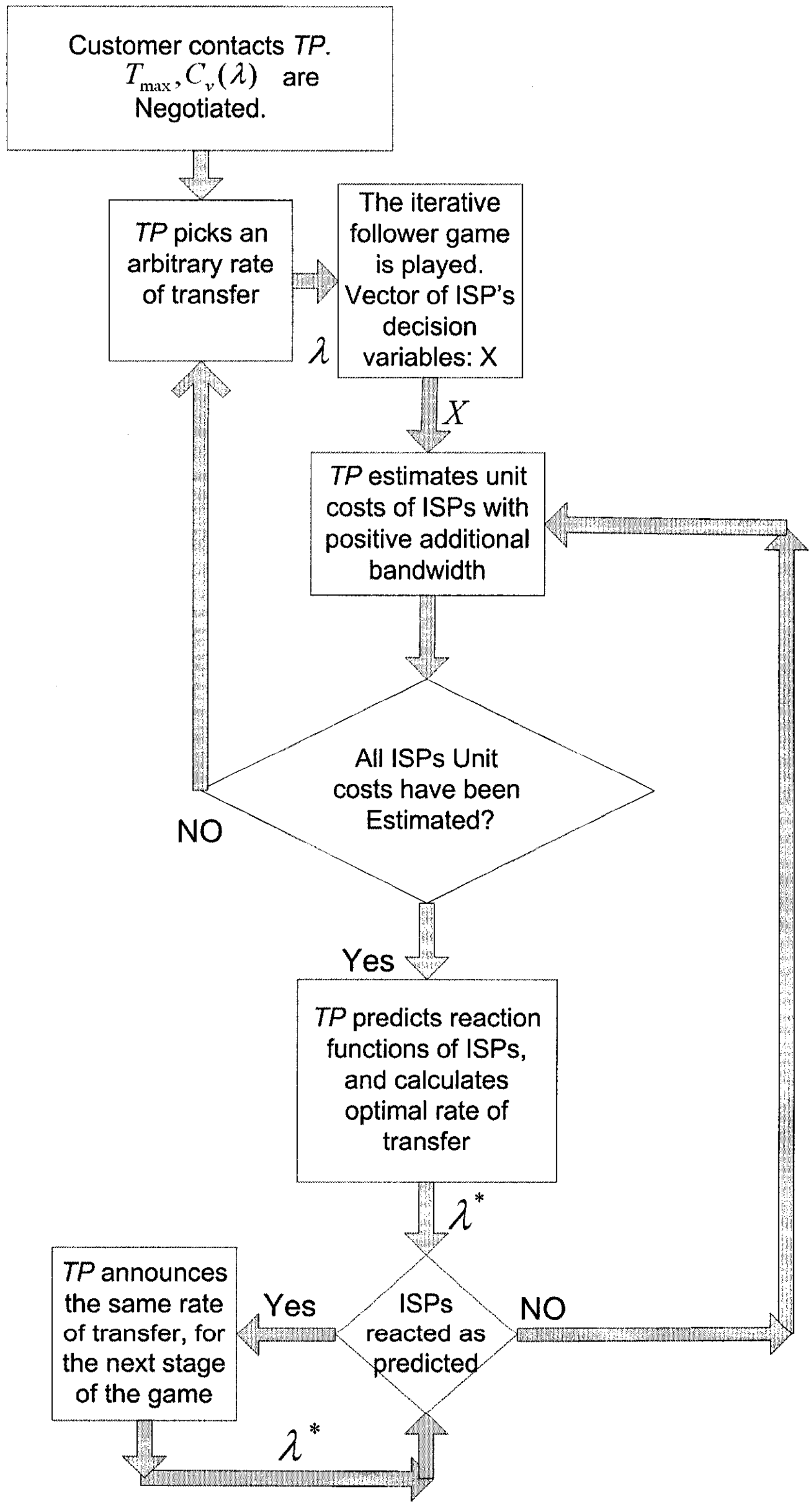


Figure 3.1 The flowchart representing the dynamic version of the game

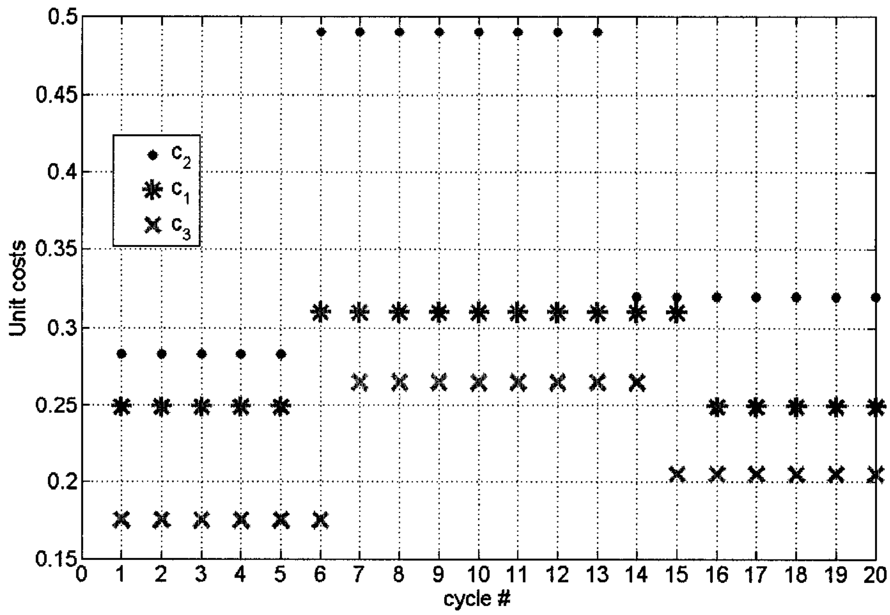


Figure 3.2 Arbitrary unit costs of ISPs in each cycle.

3.5.1 Numerical experiments

Every time a unit cost associated to an ISP changes, the best response of that ISP will be affected. In order for *TP* to reestimate this new value, the new decision variable of the ISP should be a positive value, otherwise *TP* cannot simulate the interaction among ISPs, and cannot predict their best responses or the corresponding Nash equilibria. When the costs change *TP* moves into a performance monitoring mode where it chooses a random admissible variable as the source rate λ for the next stage of the game.

In the following, a dynamic example of a game detailed in Fig. 3.1 is presented. We run the game for 20 cycles or unit times. The initial unit costs of ISPs are the same as the example in Chapter 2, but changes in ISP's unit costs are happening according to Fig. 3.2.

TP's utility is shown in Fig. 3.3 for each cycle of the game in the two cases of complete

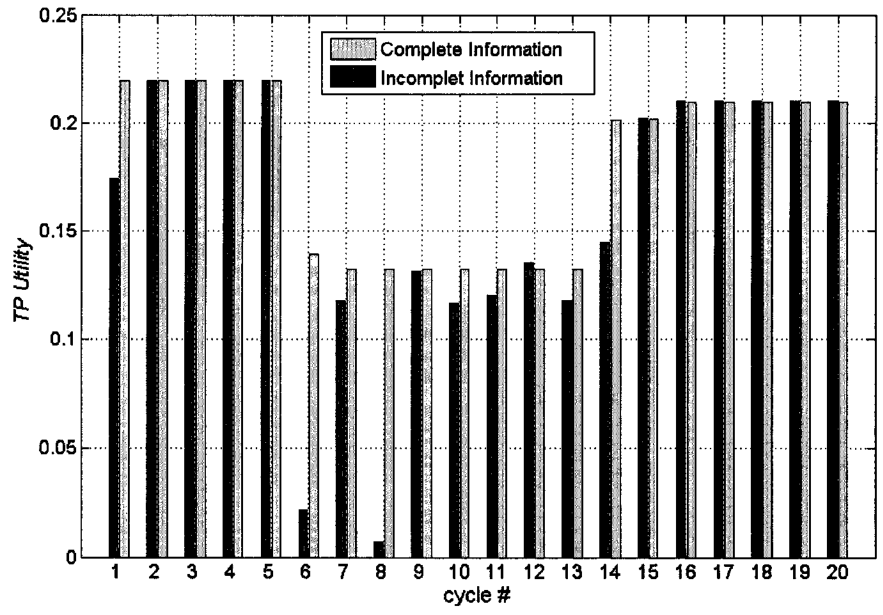


Figure 3.3 TP_U for the two cases of complete and incomplete information game

and incomplete information, while unit costs of ISPs are varying. *TP* is unable to estimate all the ISP's unit costs of ISPs during cycle 6 to 14, and as a result receives a lower utility. This is due to the fact that during these cycles the unit costs of ISPs are increasing and some ISPs are not willing to buy any bandwidth. Because of the random nature of the rate of source λ that *TP* announces during the performance monitoring mode, the results presented in Fig. 3.3 are not unique.

3.6 Conclusion

Among all parameters associated with every ISP's utility function, the unit costs of additional bandwidth and initial bandwidths seem to be the most important factors, in the sense that they can affect each ISP's position with respect to other ISPs in the competing market. Since initial bandwidths can be measured by methods like active probing, and T_is can be obtained from other decision variables, in this chapter, we have chosen to keep unit costs of ISPs as information internal to every ISP, thereby extending the model to an incomplete information environment.

In this new setup, Algorithm 2.2.1, has to be used between ISPs in a centralized mode, whereby *TP* has the responsibility of coordinating between ISPs. Also, *TP* needs to obtain an estimate of ISPs additional bandwidth unit costs to be able to choose its best λ. Algorithm 3.3.1 enables *TP* to estimate unknown quantities essential to its optimization problem.

CHAPTER 4

CONCLUSION

4.1 What was achieved

Along with the growth of VoIP and other delay-sensitive Internet applications, pricing and accounting of the new services demand new techniques and methods to better reflect each provider's performance.

Service providers impose a usage-based billing of VoIP services, in view of the fact that for the typical customer an IP-based telephone has the same functionality as an ordinary one and thus can be expected to be billed in the same manner. Yet in practice, companies are using the best effort delivery infrastructure of Internet to provide such services which are commonly charged according to duration. As most other communications on the Internet, VoIP and multimedia application rely on the IP protocol, which is a best effort service where no fixed guarantee is present either on the packet loss rate or more significantly, on transmission delay.

This situation was the motivation for proposing a pricing scheme in which the end user is charged a premium only for that portion of the traffic that receives the desired packet delay. On the other hand, since the delay in a network is highly dependant on each participating network's performance, a similar reward structure has been adopted to distribute the revenues among ISPs based on their individual contribution and performance in the transit process. The probability of meeting total delay requirements is a result of each agent's effort to reduce transit time in their own networks. This points to the importance of fair revenue sharing rules between ISPs. To deal with this issue, we have investigated a class of sharing rules, parameterized by a tuning factor β, whereby revenues are shared according to both the size of target packet maximum delays, set by individual ISPs for themselves, and how well they can meet them.

In chapter 1, the complete scenario of a sharing leader-follower game is presented. The building blocks of the game are utility functions which have been chosen carefully, to reflect the main ideas of the thesis, and also harmonize with practices in the real world. To reach this, different form of utility functions and even decision variables were initially investigated, including multiple different models in which *TP* matches reactions of ISPs to a response curve, and try to minimize the error through iterations of the game, which is not presented in Chapter 1. Out of all the investigated schemes, the current choice of utility functions appear to be the most adequate in terms of efficiency and ability to tackle more general situations.

In our utility functions, we were first and foremost guided by their ability to correctly reflect both the physics of the transmission process and the sense of fairness to both customer and ISPs which constitutes the initial thrust behind our search for new pricing schemes. In doing so, we did not put mathematical success ahead of the objective. As a result, establishing sufficient reasonable conditions under which a Nash equilibrium exist in the follower's game has been somewhat of a significant mathematical challenge.

In Section 1.4 of Chapter 1, mainly by using ideas of log-concavity, sufficient conditions for the existence of a Nash equilibrium in some instance of the follower's game are given. More specifically, for the special case of $\beta = 1$ conditions are provided for an arbitrary number of players. For $0 < \beta < 1$, only the case of two players has been addressed.

In Chapter 2, an algorithmic implementation of the proposed model is investigated. This is followed by a base example of three competing ISPs with arbitrary input values. A numerical sensitivity study to various parameters, be it tuning factor β, or ISP's bandwidth unit price, maximum transmission delay, or *TP*'s percentage of total benefits, follows. A wide variety of intriguing behaviors can be displayed.

In Chapter 3, in an effort to make the model more realistic, an incomplete but perfect game has been considered. However the scope of investigation has been limited to unknown additional bandwidth unit cost of each ISP. This new assumption for the model entails a repeated version of the game in which multiple consecutive unit times are considered. Furthermore, a dynamic incomplete perfect information version is presented in which ISPs unit costs can change in time. In this case *TP* tries to follow this change and estimate the new unit costs, to better utilize his position of being a leader, by announcing the exact rate of source λ that benefits him the most. However this is not possible until all ISPs unit bandwidth costs are estimated. Until then, *TP* keeps probing ISPs by announcing random values of source rates.

While the ideas developed in the thesis remain at a relatively young stage, the results of both mathematical analysis and numerical testing of the proposed decision making and profit sharing schemes, justify a sense of cautious optimism as to their potential usefulness within practical implementations.

4.2 Future Work

Many extensions of the current thesis work are possible on the mathematical front; in particular developing sufficient conditions for the existence of a Nash equilibrium for more than two ISPs when β differs from one. Also, the whole issue of the uniqueness of Nash equilibria is left open.

From a statistical point of view, one could extend the game to the case of multiple routes. From an estimation point of view, for the incomplete information environment, one could consider the currently ignored aspect of how to convert statistical (on a possibly non stationary process) measurements obtained through active probing of the various networks into reliable estimates of the unknown parameters (bandwidth unit costs, initially available bandwidths, etc...)

Furthermore, the thesis analysis is founded on M/M/1 network models. One possible direction of further research is to investigate the problem under the assumption of M/D/1 queue, and constant packet sizes, i.e., a deterministic pattern for service rate in each queue.

Another potentially fruitful idea, which is left as future work, would be considering *TP* as a company which can distribute revenues among participating ISPs at the end of each transaction, in a context where ISPs can try to cheat on their real unit costs. *TP* would then attempt to use the revenue fraction it earns so as to reward those ISPs which are behaving more honestly.

Finally, the conceptual framework developed in the thesis could be applicable to other research areas facing similar problems be it companies sharing the transportation of goods across origin/destination pairs, or groups of suppliers and manufacturers involved in producing a finished good within a certain lead time.

REFERENCES

Avriel, M., Diewert, W. E., Schaible, S., and Zang, I. (1988). *Generalized Concavity.* Plenum Press, New York.

Basar, T. and Olsder, G. J. (1995). *Dynamic Non cooperative Game.* Academic Press, London and San Diego.

Courcoubetis, C. and Weber, R. (2003). *Pricing Communication Networks: Economics, Technology and Modeling.* Wiley, New York.

"CRTC" (2005). *Telecom Decision CRTC 2005-28 : Regulatory framework for voice communication services using Internet Protocol.*

DaSilva, L. A. (2000). Pricing for QoS-enabled networks: A survey,. *IEEE Communications Review*, **3**(2), 1–4.

Dziong, Z. and Mason, L. G. (1996). Fair-exficient call admission control policies for broadband networks - A game theoretic framework. *IEEE/ACM Transactions on Networking*, **4**(1), Pages 123–130.

Haogang Chen, K. P. (1998). An architecture for noncooperative QoS provision in many-switch systems. In *INFOCOM*, pages Pages 13–18.

He, L. and Walrand, J. (2006). Pricing and revenue sharing strategies for internet service providers. *IEEE/JSAC*, **24**(5), Pages 942–947.

Kelly, F., Muallo, A., and Tan, D. (1998). Rate control for communication networks: Shadow prices, proportional fairness, and stability. *Journal of the Operational Research Society*, **49**(1), Pages 237–252.

Kleinrock, L. (1975). *Queuing Systems*, volume 1. Wiley Interscience, New york.

La, R. J. and Anantharam, V. (2002). Optimal routing control: Repeated game approach. *IEEE Transactions on Automatic Control*, **47**(3), Pages 437–444.

Lazar, A. and Semret, N. (2000). Design and analysis of the progressive second price auction for network bandwidth sharing. *Telecommunication Systems, Special Issue on Network Economics*, **1**(2), Pages 487–498.

Li, T., Iraqi, Y., and Boutaba, R. (2004). Pricing and admission control for QoS enabled internet. *The International Journal of Computer and Telecommunications Networking*, **46**(1), Pages 87–110.

Maillé, P. and Tuffin, B. (2004). Multi-bid auctions for bandwidth allocation in communication networks. *INFOCOM*, pages Pages 2–8.

Mazumdar, R., Mason, L. G., and Douligieris, C. (1991). Fairness in network optimal flow control: Optimality of product forms. *IEEE/ACM Transactions on Networking*, **39**(5), 775 – 782.

Odlyzko, A. (1998). Paris metro pricing for the internet. *ACM Conference on Electronic Commerce*, **1**(1), Pages 140–147.

Odlyzko, A. (1999). Paris metro pricing: The minimalist differentiated services solution. *ACM Conference on Electronic Commerce*, **1**(1), Pages 140–147.

Paninski, L. (2004). Log-concavity results on gaussian process methods for supervised and unsupervised learning. *Neural Information Processing Systems*, **17**(1), Pages 23–30.

Semret, N. (1999). *Market Mechanisms for Network Resource Sharing*. PhD thesis, Columbia University.

Semret, N., Liao, R. R.-F., Campbell, A. T., and Lazar, A. A. (2000). Pricing, provisioning and peering: Dynamic markets for differentiated internet services and implications for network interconnections. *IEEE/JSAC*, **18**(12), pages 2499–2508.

Srisankar, R. and Kunniyur, S. (2001). Analysis and design of an adaptive virtual queue (AVQ) algorithm for active queue management. *Special Interest Group on Data Communications*, **31**(4), Pages 123–134.

APPENDIX I

I.1 Sufficient Conditions for Concavity of Modified Global Probability Function for 2 ISPs and $\beta \in (0;1)$

In this appendix, we establish the sufficient conditions under which the following partial derivative is always negative:

$$\frac{\partial^2 F}{\partial y_i^2} = \int_0^{T_{\max}} x_j e^{-\tau x_j} \frac{\partial^2}{\partial y_i^2} G(T_{\max} - \tau, y_i^\alpha) d\tau \tag{I.1}$$

By replacing the value of $\frac{\partial^2}{\partial y_i^2} G(T_{\max} - \tau, y_i^\alpha)$, from 1.32 into (I.1), we obtain:

$$\frac{\partial^2 F}{\partial y_i^2} = \frac{x_j e^{-x_j T_{max}}}{x_i^{2\beta}\beta} \int_0^{T_{\max}} (T_{max} - \tau) e^{x_j(T_{max}-\tau)} e^{-x_i(T_{max}-\tau)} (1 - \beta - x_i(T_{max} - \tau) d\tau. \tag{I.2}$$

Let $\tau' = T_{max} - \tau$:

$$\frac{\partial^2 F}{\partial y_i^2} = \frac{x_j e^{-x_j T_{max}}}{x_i^{2\beta}\beta} \int_0^{T_{\max}} \tau' e^{(x_j - x_i)\tau'} (1 - \beta - x_i \tau') d\tau' \tag{I.3}$$

which yields the following after integration by parts:

$$\frac{\partial^2 F}{\partial y_i^2} = (1 - \beta + \frac{2x_i}{x_j - x_i}) \int_0^{T_{\max}} \tau' e^{(x_j - x_i)\tau'} d\tau' - \frac{x_i}{x_j - x_i} T_{max}^2 e^{(x_j - x_i)T_{max}}. \tag{I.4}$$

Note that for $\beta = 1$, the expression above is negative. As β decreases for a fixed T_{max}, $\frac{\partial^2 F}{\partial y_i^2}$ is more and more likely to go negative. Let β^* be the value of β, for which our objective function equals zero for the first time. A sufficient condition for $\frac{\partial^2 F}{\partial y_i^2}$ to remain negative is that β^* be outside the range of $[0; 1]$. To find β^*, we have to solve the equation: $\frac{\partial^2 F}{\partial y_i^2} = 0$:

$$(1 - \beta^* + \frac{2x_i}{x_j - x_i}) \int_0^{T_{\max}} \tau' e^{(x_j - x_i)\tau'} d\tau' - \frac{x_i}{x_j - x_i} T_{max}^2 e^{(x_j - x_i)T_{max}} = 0. \quad \text{(I.5)}$$

Solving (I.5) yields:

$$1 - \beta^* = \frac{x_i}{x_j - x_i}\{\frac{T_{max}^2 e^{(x_j - x_i)T_{max}}}{\frac{1}{x_j - x_i} T_{max} e^{(x_j - x_i)T_{max}} - \frac{1}{(x_j - x_i)^2}(e^{(x_j - x_i)T_{max}} - 1)} - 2\}. \quad \text{(I.6)}$$

Let $u = (x_j - x_i)T_{max}$, then (I.6) can be re-written as:

$$\frac{1 - \beta^*}{x_i T_{max}} = \frac{ue^u}{(u-1)e^u + 1} - \frac{2}{u}. \quad \text{(I.7)}$$

The two cases of $u \geq 0$, and $u < 0$ are investigated. Furthermore since we are seeking an interval of variables for which (I.1) is negative, we assume throughout that $|u| < 1$.

If $0 \leq u < 1$, then the following inequality can be achieved from (I.7):

$$1 - \beta^* \leq \frac{x_i T_{max}}{u}(\frac{u^2}{u-1} - 2). \quad \text{(I.8)}$$

We note immediately that for the given range of u the right hand side of (I.8) is negative, thus $1 - \beta^* < 0$. This yields a $\beta^* > 1$, which is outside the allowed range.

Thus for $u > 0$, it is enough that $u < 1$, to insure that $\frac{d^2F}{dy_i^2}$ always remain negative.

Now consider the case of $-1 < u < 0$. The parallel inequality of (I.8) will be:

$$1 - \beta^* > \frac{x_i T_{max}}{u}(\frac{u^2}{u-1} - 2). \tag{I.9}$$

The goal is to show that the right hand side (RHS) of (I.9) is always greater than 1. This way we can claim that $\beta^* < 0$, and thus out of the allowed range.

Since the RHS of (I.9) is always a strictly increasing function for the given range of u, the minimum of RHS of (I.9), is always greater than its value at $u = -1$. This can be verified by considering the sign of the derivative with respect to u. Thus, it is sufficient to satisfy the following inequality:

$$\frac{x_i T_{max}}{u}(\frac{u^2}{u-1} - 2)|_{u=-1} > 1 \tag{I.10}$$

The above inequality is satisfied if $x_i > 2/5T_{max}$.

In summary, the two following conditions will result in a strictly concave function $F(y_i^\alpha, y_j^\alpha)$:

$$\begin{aligned} &a) \quad x_k > 2/5T_{max} \quad \forall k \in i, j \\ &b) \quad |(x_j - x_i)T_{max}| < 1 \end{aligned} \tag{I.11}$$

APPENDIX II

Authors: *Soheil Saberi, Roland P. Malhamé and Lorne G. Mason* .

Presented at the International Conference on Network Control and Optimization, Avignon, France, and received the best student paper award.

II.1 Pricing for QoS Provisioning across Multiple Internet Service Provider Domains

II.1.1 Abstract

In this paper we introduce a pricing scheme to be employed between a group of Internet service providers (ISPs) and a customer who wishes to initiate a packet flow from a fixed origin to a fixed destination. The ISPs are transparent to the customer who relies on a third party company for both the choice of the relevant ISPs and the unit flow price negotiated. The customer pays only for that portion of the traffic, which meets a predefined maximum tolerable total delay within the ISP networks. After taking in a fixed percentage of total profit, the third party redistributes the remaining benefits to the ISPs according to a sharing mechanism, which reflects both, the QoS the ISPs declare they will meet, as well as their real performance. The pricing emerges as the result of a Stackelberg game with the third party as the leader and the ISPs as the followers.[3]

Keywords: Multiple Domain Internet Pricing, Game theory, Statistical Quality of Service, Stackelberg Games.

[3]This research was supported by a strategic grant from NSERC, CANADA, grant: STPGP269449-03

II.1.2 Introduction

With the advent of new Internet applications for which more quality guarantees are expected from Internet service providers, existing flat rate charging schemes have become more and more inappropriate [DaSilva, 2000]. As a result, Internet pricing is currently a very active area of research. Based on the notion of effective bandwidth, a statistically founded tool for the evaluation of quality constrained bandwidth requirements for certain types of traffic in data networks [Courcoubetis and Weber, 2003, Kelly et al., 1998], as well as different results from both cooperative and non cooperative game theory [Basar and Olsder, 1995], various pricing approaches have been proposed.

In many schemes, along with the basic objective of pricing which is to recover the incurred costs, other goals have been considered among which, congestion control and fair allocation of resource to users [Courcoubetis and Weber, 2003, Srisankar and Kunniyur, 2001], admission control and QoS provisioning [Li et al., 2004], allocating the resource to users who value it most by selling the service in an online auction [Lazar and Semret, 2000, Maillé and Tuffin, 2004]. As argued in [He and Walrand, 2006], the profit of ISPs as major players in Internet, has been neglected in many pricing schemes; therefore in this paper we are also interested in the interaction between ISPs and the outcome of the non-cooperative game between them. However, our model differs in a number of ways from that in [He and Walrand, 2006]. We have assumed the case of only one data flow that passes through designated ISPs, and the end user who initiates the process is assumed to be willing to pay only for that portion of the traffic that meets a specific delay bound. On the other hand, an ISP reward structure is defined whereby each ISP obtains a share of customer payments which depends on both its initially declared individual quality of service goals, as well as on a statistical measure of how successful this ISP is in meeting the goals in question. Furthermore, the setup here is not one of guaranteed quality of service, but rather statistical quality of service. Such a choice was made for at least two reasons: firstly, deterministic quality of service guarantees can be quite wasteful in terms of bandwidth requirements. Secondly, when involving multiple ISP domains, guaranteed qualities of service tend to require a high degree of end to end co-ordination, and thus the complexity and overhead communications requirements of such schemes can quickly reach unmanageable levels as network size increases. Instead here, the setup is such that the enforcement of quality of service is an affair left as entirely internal to each independent network. If a particular network complies with a high degree of success rate relative to its declared goals, it will be rewarded accordingly. If not, it will not. This way, the control scheme for quality enforcement can be left as *decentralized* as possible.

A third party company herein referred to as *TP* has been introduced as a coordinator between the end user and ISPs. In return, it receives a fixed portion of customers payments. We adopt a Stackelberg game environment, in which *TP*, is the leader, and ISPs form the group of followers.

Overprovisioning of capacities may be the solution for many network operators to deal with delay and congestion issues, but as discussed in [Courcoubetis and Weber, 2003], while this looks like the right choice in backbones of the network, it may not be so for its metropolitan part, and even less so in the access part of the network. This stems from the fact that overdimensioning in the latter parts requires a lot more investment and this would raise the costs as edge nodes are approached. Based on this observation, we have assumed that each ISP involved in our model has at least one congestion node along the chosen route, and the imposed delay caused by this node, dominates that of any other route link within the ISP domain. In summary, each ISP is represented by a single bottleneck node along the chosen route.

The organization of the paper is as follows. In Section II.1.3, we describe our modelling framework. In Section II.1.4 we specify the utility functions associated with all of the active agents. In Section II.1.5 we present our success-rate based pricing scheme, and we establish existence of a unique Nash equilibrium for the ISP part of the Stackelberg game. This is followed by a set of examples in Section II.1.6, while Section II.1.7 summarizes our conclusions and plans for future work.

II.1.3 Model description

The proposed model involves three types of agents: a *customer* herein referred to as *C*, *TP*, and a collection of *ISPs* to be selected by *TP*. In our model, *C* is an end user with a potentially large volume of traffic to be sent on a regular basis from a given destination A to a destination B, and who initiates contacts with *TP* for that purpose. However, *C* specifies a maximum end to end tolerable delay for those transmitted packets for which it is willing to pay a per unit premium. We denote the maximum delay tolerated by *C* as T_{max}. An example of traffic type particularly relevant to the context here is VoIP. This is because in VoIP one can sustain the high loss probabilities that may occasionally result from the organization scheme to be proposed. Furthermore, there does already exist market regulators in the VoIP context and they can readily be identified as potential TPs in our model. Indeed the Telecom Decision CRTC 2005-28, which has been set by Canadian Radio-Television and Telecommunications Commission is a clear example of a set of regulations, upholding rather identical regulatory framework as extant traditional phone services for VoIP [”CRTC”, 2005].

Division of revenues amongst telephone companies is based on mutual agreements between pairs of service providers. In the case of a large number of such providers of different hierarchical levels e.g. trunk providers and access network providers, the task of revenue sharing is currently performed by a third party company. Exchanges of balances, and information about each traversing telephone call between service providers are based on annual calculations. In the current model, *TP* plays an enhanced role, as compared to the case of telephone networks, in that a real-time information and revenue sharing mechanism is adopted.

TP together with *C*, agree on an *offered traffic versus unit flow price curve*, whereby offered traffic levels increase as bandwidth unit price decreases. This curve is a form of commitment on the part of the customer that it will pay a fixed bandwidth unit price per unit time for sending a given ultimately agreed to traffic level, unless it can demonstrably establish failure by *TP* to meet the QoS requirements at that traffic level. In the latter case, *C*'s per unit time payment is reduced by the fraction of its total traffic inadequately transmitted. As a consequence of this arrangement, it is in *C*'s best interest to constantly probe performance by sending traffic (useful or otherwise) at the agreed to level.

TP selects a number of ISPs along the route who are willing to be solicited in offering the service to *C*. At this stage, *TP* gathers from the candidate ISPs the parameters which specify the rules of the game they have to play and whose outcome will be their individual share of the income.

In the practical context, we assume that packet end to end delays, and within ISP domains, can be monitored for performance verification. However, all optimization decisions are founded on specific modelling assumptions. In the current context, we have settled for a simple M/M/1 queueing model of each network. We have assumed an exponentially distributed packet lengths, so that the probability of meeting the delay requirement can be expressed as $P(t \leq T_i) = 1 - e^{(\lambda - \mu_i)T_i}$, where λ is the rate of the source, t, μ_i and T_i are random delay, service rate and *declared* maximum transit time in network$i$, respectively.

The need to calculate success probabilities in each network, stems from the fact that we wish to reflect the customer payment mechanism on the ISPs involved in the negotiation. More specifically, the fraction of total revenue dedicated to an ISP directly depends on the probability of meeting the declared delay within its network. Moreover, as mentioned earlier, *C* pays according to the probability that its packet reaches the destination in time; the latter probability can be derived from the probability distribution of individual network delays.

The per unit time cost for the customer will be: $\Pr(t \leq T_{\max})C_v(\lambda)\lambda$, where $C_v(\lambda)$ is the unit cost versus traffic λ, dependency curve , herein referred to as the *customer response* curve. For convenience here, it is taken to be a decaying exponential. Indeed, anticipating a decreasing function of demand versus price is standard (see [He and Walrand, 2006] for example). With all active agents and their declared parameters thus defined, we are ready to formulate the rules of a Stackelberg game whose outcome is the traffic rate submitted by *C* to the ISPs, the corresponding premium unit flow price paid by *C*, and the revenue obtained by each of the candidate ISPs.

II.1.4 Utility functions and game framework

II.1.4.1 Third Party *TP*

TP, is a company responsible for all negotiations with the ISPs, with the understanding that the negotiation process must remain transparent to the customer. *TP*s unit time revenue is a fixed fraction of the total unit time payments made by *C*. The utility function of *TP* is considered to be:

$$TP_U(\lambda) = M\ \Pr(t \leq T_{\max})C_v(\lambda)\lambda\,. \tag{II.1}$$

where $M \in [0;1]$ is the fraction of total benefit reserved for *TP* . The only decision variable of *TP* is λ, and it is chosen to maximize *TP*'s revenue, or equally *total* customer payments to the ISPs, so that in a formulation of the game where ISPs cannot acquire more bandwidth, this corresponds to the social welfare optimization problem. We also assume an upper bound λ_{max} for the rate of data transfer.

II.1.4.2 Service Providers

We assume each network involved in the transaction to have a certain amount of bandwidth μ_i, naturally available for *C*'s traffic. Furthermore, we assume that this initial bandwidth is sufficient to insure that the maximum possible source rate λ_{max} can be satisfied by any of the μ_i's ($\lambda < \mu_i \quad \forall i$). The ISPs have the option of increasing the amount of bandwidth they dedicate to *C*'s traffic, via a specified cost of c_i per unit of added bandwidth. Let $\Delta\mu_i$ be the added bandwidth with an upper bound $\Delta\mu_i^{\max}$, so that the actual bandwidth that network i can allocate to the flow becomes: $\mu_i + \Delta\mu_i$. For each potential λ, the fraction of profit, which is not taken by *TP*, is assumed to be available in its entirety to the participating ISPs. However, for each fixed λ, ISPs are pitted against each other in a game, the rules of which will be defined in what follows. The idea is to reflect the payment mechanisms at *TP*'s level all the way down to the ISPs. More specifically, ISPi is asked to provide a (hypothetical) maximum delay T_i that it declares itself ready to aim at meeting. This $T_i \in [0; T_i^{\max}]$ is very instrumental in determining ISP i's share of total income available after *TP*'s payment, in that it is proposed that the fraction of that total allocated to ISPi be given by:

$$S_i = \frac{\left(1 - e^{-(\mu_i + \Delta\mu_i - \lambda)T_i}\right)}{T_i^{\beta}} \left[\sum_{j=1}^{n} \frac{\left(1 - e^{-(\mu_j + \Delta\mu_j - \lambda)T_j}\right)}{T_j^{\beta}} \right]^{-1}, \qquad \text{(II.2)}$$

with β as a coefficient between 0 and 1 (inclusively), and n as the number of ISPs.

Also note that, the larger the declared time, the less margin is left for other providers to accommodate their own delays along the packet route. From that point of view, fairness would dictate that a large declared T_i should correspondingly penalize the declarer (this explains the T_i^β in the denominator in (II.2)). The latter penalty prevents ISPs from letting their own declared T_i's go to infinity in an effort to maximize their chances of success. Also, note that for an adequate choice of β the optimal choice of declared T_i may well become the mean delay in the network. In addition, as alluded to earlier, the ISP has the option of either buying for a given unit price extra bandwidth, or equivalently freeing, albeit at the cost of some loss of revenue per unit bandwidth, a given amount of bandwidth, thus modulating its effective service rate μ_i. As a consequence ISPi, must provide two decision variables: T_i, and the extra amount of bandwidth $\Delta\mu_i$ it wishes to buy . Note that if we fix $\Delta\mu_i = 0$ (no bandwidth buying allowed), it is not difficult to see that, modulo a reward shift by an appropriate constant, the game is equivalent to a *zero-sum* game. Using this allocation rule, we define the utility function as:

$$ISP_{U_i} = (1 - M)C_v(\lambda)\Pr(t \leq T)\lambda S_i - c_i\Delta\mu_i\,. \tag{II.3}$$

where $(1 - M)C_v(\lambda)\Pr(t \leq T)\lambda$ represents the revenue after payment of *TP*, and c_i is the extra per unit bandwidth equivalent cost.

II.1.4.3 Formulation of the game

While we have specified different agents utility functions, we have not thus far specified the sequence in which the game is played. Given the predominant role of *TP* as the main organizer, we suggest that *TP* be considered as the higher level of the hierarchy within a Stackelberg game, i.e. *TP* is the leader. All participating ISPs are followers, and thus, for each fixed value of customer traffic rate λ decided by the leader *TP*, we shall be looking for potential Nash equilibria. We also assume a perfect information environment, whereby each player knows all extra bandwidth unit buying costs, initial networks dedicated bandwidths to *C*'s traffic, as well the customer response curve. This strong assumption is made in order to investigate the feasibility of the ideal game. However, more relaxed versions of the game where ISP's costs per unit bandwidth are assumed unknown to TP as well as to other competing ISPs, are possible and indeed workable.

Having the position of the leader in this game, *TP* can predict the outcome of the non-cooperative game among the followers, for any λ. By exploiting this fact, *TP* can specify the customer traffic level which best suits its interests.

Remark II.1.1 *Considering the expression of* ISP_i*'s utility function in (II.3), we note that except for the share term* S_i*, the utility does not depend on the choice of declared maximum transit time* T_i*. Also for given* $(\mu_i + \Delta\mu_i)$*,* T_i *can be selected independently of other decision variables to maximize* S_i*, leaving* $\Delta\mu_i$ *as the unique decision variable of* ISP_i*. Furthermore, for the special case where the coefficient* β *in (II.2) is equal to 1, the optimum choice is* $\forall i, T_i = 0$.

Remark II.1.2 *The fact that, at least for the* $\beta = 1$ *case, the optimal choices of declared maximum network transit times* T_i *for the ISPs correspond to the highly unrealistic value of zero, justifies their characterization as declared values. This leads to a reasonable rule for sharing benefits among ISPs. Indeed, for* $\beta = 1$ *as* T_i *goes to zero, L'Hôpital's rule yields:*

$$S_i/S_j = (\mu_i + \Delta\mu_i - \lambda)/(\mu_j + \Delta\mu_j - \lambda) . \tag{II.4}$$

(II.4) in fact indicates that customer payments after commission are shared among ISPs in inverse proportion to the mean packet transit time in each of the networks. Also, it can be shown that choosing a β *different from 1 is equivalent to a sharing rule where shares become proportional to* $(\mu_i + \Delta\mu_i - \lambda)^\beta$*. Thus as* β *decreases, ISPs could become more reluctant to buy bandwidth.*

However, more relaxed versions of the game where ISP's costs per unit bandwidth are assumed unknown to *TP* as well as to other competing ISPs, are possible and indeed workable. In the next section, analysis is focused on the $\beta = 1$ case. For that special case, we establish the existence of Nash Equilibrium (NE) for the followers game corresponding to any admissible λ.

II.1.5 Properties of the followers game for $\beta = 1$

In the telecommunication literature the throughput of the data stream $(1-e^{-(\mu_i+\Delta\mu_i-\lambda)T_i})\lambda$ over mean delay T_i is defined as the *power* factor. Thus for $\beta = 1$ the sharing mechanism presented in (II.2) can be regarded as a function of each ISP's power factor P_i. More specifically:

$$S_i = P_i / \sum_{j=1}^{n} P_j \quad \text{where: } P_i = (1 - e^{-(\mu_i+\Delta\mu_i-\lambda)T_i})\lambda / T_i. \tag{II.5}$$

In [Mazumdar et al., 1991] an approach based on maximization of product of power factors to allocate a fair division of flows to users, has been introduced. Indeed, this corresponds to a so-called Nash bargaining solution. Instead, in the current model, each ISP tries to maximize its power factor and has an interest in securing a high overall success rate in meeting end to end QoS constraints.

Theorem II.1.3 *Under an inequality detailed in Lemma 2 in Appendix, in the Stackelberg game defined by leader utility function (II.1) and followers utility functions (II.3) with $\beta = 1$, for every admissible λ set by the leader, the follower game admits a Nash Equilibrium.*

Proof To prove the existence of NE's we use a paraphrase of the following theorem [Basar and Olsder, 1995]:

Theorem II.1.4 *For each player, assuming the sets of decision variables are closed, bounded and convex, and assuming that each player's utility function is continuous in all decision variables associated with all players, and strictly concave in the entries associated with its own decision variables, for every admissible combination of decisions of other players, the associated n-person nonzero-sum game admits a Nash Equilibrium in pure strategies.*

The theorem above can be easily shown to hold if strict concavity is replaced by the assumption of existence of a unique maximizer for each player's utility function for arbitrary decisions made by other players. Existence of a unique maximizer is satisfied, provided utility functions can be shown to be strictly *log concave* in their own decision variables. See Appendix for the proof. Since, $\Delta\mu_i \in [0; \Delta\mu_i^{\max}]$, the set of decision variables are both convex and compact. The continuity of utility functions on the admissible decision variable set is also obvious. Therefore a Nash Equilibrium exists. ∎

II.1.6 Numerical Results for a two ISP game

We consider the case of two competing ISPs and associate arbitrary bandwidth unit costs to them. The inputs are: $\mu_1 = 1.1$, $\mu_2 = 1.2$ packet/ms, $C_v(\lambda) = e^{-(\lambda/0.75)}$, $\lambda_{\max} = 1$ packet/ms, $M = 20\%$, $T_{\max} = 6$ ms, $\Delta\mu_1, \Delta\mu_2 \in [0; 1]$ and $c_1 = 0.075, c_2 = 0.055$. Although, a mathematical proof of the existence of Nash equilibria for values of β other than 1, has not been established as yet, we numerically investigate the two cases of $\beta = 1$ and $\beta = 0.5$. Simulation results are shown in Table II.1 and Fig.II.1. From Table II.1, one sees that when β changes from 1 to 0.5, both ISP utilities increase, but more so for the ISP with less initial bandwidth. This comes at the price of decreasing the incentives of ISPs in buying more bandwidth. This in turn lowers the QoS to the customer who has to contend with a lower probability of success.

Table II.1 Simulation results of two competitive ISPs for $\beta = 1$ and $\beta = 0.5$

	$\beta = 1$	$\beta = 0.5$
Optimal λ(packet/ms)	0.750	0.708
$(\Delta\mu_1, \Delta\mu_2)$ at NE (packet/ms)	(0.442,0.603)	(0.275,0.360)
(T_1, T_2) at NE (ms)	(0,0)	(1.88,1.47)
(ISP_{U_1}, ISP_{U_2}) at NE	(0.0588,0.0891)	(0.0701,0.0961)
TP_U at NE	0.0536	0.0517
$Pr(t \leq T_{max})$ %	97.06	93.75

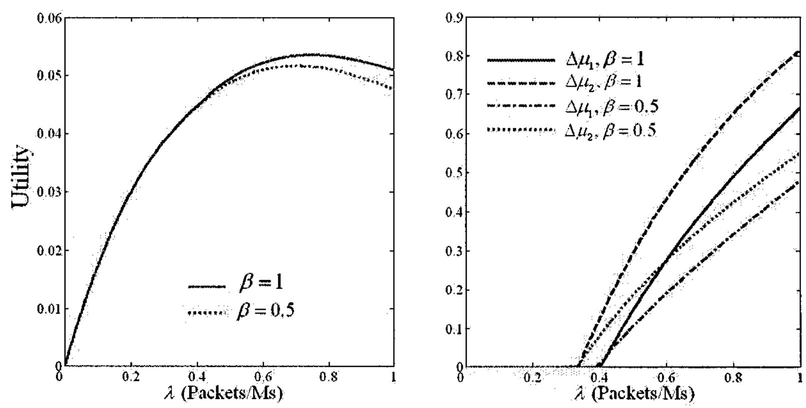


Figure II.1 Left: *TP*'s utility versus the rate of transfer for $\beta = 1$ and $\beta = 0.5$, Right: $\Delta\mu_1$ and $\Delta\mu_2$ at Nash equilibria for $\beta = 1$ and $\beta = 0.5$.

II.1.7 Conclusion and future work

Along with the growth of VoIP and other delay sensitive Internet applications, pricing and accounting of the new services, demand new techniques and methods to better reflect each provider's performance. In this article we have proposed a scheme for rewarding Internet provider companies, which can provide low delay communications. However, no performance guarantees are given.

The global end to end performance (or equivalently the probability of meeting total delay requirements) is a result of all agents efforts to cut transit time in their own networks. This points to the importance of fair revenue sharing rules between ISPs. To deal with this issue we have investigated a class of sharing rules, parameterized by the β variable. Setting β at a value less than 1, tends to reduce the financial advantage that a given ISP gets from an increase in bandwidth relative to other ISP's along the route. While this results in lower QoS, it can help offset unfair competitive advantages enjoyed by some ISP's along the route. Finding the β that makes declared transit times equal to mean transit times, and existence and uniqueness of NE in the followers game for $\beta \neq 0$, are other future areas of investigation. Also in the future we will consider repeated forms of the game to account for the possibility of imperfect information, and online utility parameter estimation. Finally ISPs along the route could be divided into subgroups in which competition is deemed fairer, insofar as the cost of acquiring bandwidth is concerned.

APPENDIX

Lemma II.1.5 *The global success probability function $Pr(t \leq T_{max})$ is strictly concave with respect to each ISP decision variable $\Delta\mu_i$, regardless of $\Delta\mu_j, j \neq i$.*

Proof The probability density function (pdf) of waiting time t in a simple M/M/1 queue is [Kleinrock, 1975] : $g(t, x) = xe^{-xt}$. where $x = \mu + \Delta\mu - \lambda$. The total delay T that is imposed on each packet, is the sum of individual delays within each ISP's network. Thus, the pdf of T ($f(T, X)$), is the result of a convolution of all component pdf's.

$$f(T, X) = g(t_1, x_1) * g(t_2, x_2) * \cdots * g(t_n, x_n) \text{ where: } X = [x_1, x_2, \cdots, x_n]\,. \quad \text{(II.6)}$$

Defining $F(T, X)$ as the probability distribution function (PDF) of delay T, and $X_{-i} = [x_1, \ldots, x_{i-1}, x_{i+1}, \cdots, x_n]$, the pdf of total transit time when the time spent in ISPi is excluded, can be defined as: $h_{-i}(T, X_{-i}) = g(t_1, x_1) * \cdots * g(t_{i-1}, x_{i-1}) * g(t_{i+1}, x_{i+1}) * \cdots * g(t_n, x_n) > 0$

The probability that the total packet delay be less than T_{max} is given by:

$$F(T_{\max}, X) = \int_0^{T_{\max}} h_{-i}(t, X_{-i}) * g(t, x_i)dt = \int_0^{T_{\max}} \int_0^{t} h_{-i}(\tau, X_{-i})g(t-\tau, x_i)d\tau dt\,. \tag{II.7}$$

Using Fubini's theorem to change the order of integration in (II.7), we will have:

$$F(T_{\max}, X) = \int_0^{T_{\max}} \int_{\tau}^{T_{\max}} (g(t-\tau, x_i)\,dt \quad)h_{-i}(\tau, X_{-i})\,d\tau\,, \tag{II.8}$$

where $G(x,t)$ is the PDF of $g(x,t)$. Our goal is to show that $\forall X_{-i},\ \ \frac{\partial^2 F}{\partial x_i^2} < 0$.
Using Lebesgue's dominated convergence, the differentiation can be carried across the integral:

$$\frac{\partial^2 F}{\partial x_i^2} = \int_0^{T_{\max}} h_{-i}(\tau, X_{-i})\frac{\partial^2}{\partial x_i^2}G(T_{\max}-\tau, x_i)d\tau\,. \tag{II.9}$$

Note that $\frac{\partial^2}{\partial x_i^2}G(T_{\max}-\tau, x_i) = -(T_{\max}-\tau)^2 e^{-(x_i)(T_{\max}-\tau)} < 0$ and $h_{-i} > 0$; hence (II.9) is always negative, and as a result the global success probability is strictly concave in x_i or equally in $\Delta\mu_i$. ∎

Lemma II.1.6 *For any admissible values of decision variables X_{-i}, and assuming the following threshold for the total cost paid by the customer:*

$$AF(x_i, X_{-i}, T_{\max}) > max\{c_i\}\sum_{j=1}^{n} x_j,\ \textit{where: } A = (1-M)C_v(\lambda)\lambda, \tag{II.10}$$

$ISP_{U_i}(x_i, X_{-i})$ has a unique maximizer with respect to x_i.

Proof Our goal is to show that:

$$ISP_{U_i}(x_i, X_{-i}) = \frac{x_i}{\sum_{j=1}^{n} x_j} AF(x_i, X_{-i}, T_{\max}) - c_i(x_i - \mu_i + \lambda) , \qquad \text{(II.11)}$$

always admits a unique maximizer. In Lemma II.1.5, the strict concavity of $F(x_i, X_{-i}, T_{\max})$, with respect to x_i was established. On the other hand the function $-c_i \sum_{j=1}^{n} x_j$ is a linear function in x_i, thus the function:

$$AF(x_i, X_{-i}, T_{\max}) - c_i \sum_{j=1}^{n} x_j . \qquad \text{(II.12)}$$

is also strictly concave in x_i. ISP_{U_i} is assumed to have positive value for all ISPs and as a result, (II.12) is always positive. Assumption (II.10) ensures a positive value for (II.12) for all ISPs. Using Mangasarian's theorem [Avriel et al., 1988], the log of (II.12) is a strictly concave function in x_i, and (II.12) will be strictly *log concave*. Furthermore $x_i(\sum_{j=1}^{n} x_j)^{-1}$ is also a strictly log concave function in x_i. Since log concavity is preserved under multiplication, and in view of the strictly increasing nature of the log function, the utility function in (II.11) has a unique maximizer in x_i. ∎