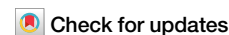




Fully tunable strong spin orbit interactions in light hole germanium quantum channels



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Spin-orbit interaction is a fundamental component for electrically driven spin qubits and hybrid superconducting-semiconducting systems. In particular, Rashba spin-orbit interaction is a key mechanism enabling all-electrical spin manipulation schemes. However, in common planar systems, Rashba interaction is weak because of the small mixing between heavy-holes and light-holes, and instead relies on complex strain and interface phenomena that are hard to reliably harness in experiment. Here, MOS-like epitaxial germanium on relaxed germanium-tin is introduced and shown to exhibit an inherently large, highly gate-tunable Rashba interaction that is compatible with both spin qubits and hybrid devices. This large Rashba interaction is a consequence of the light-hole-like ground state in germanium. Notably, the built-in asymmetry of the device causes the Rashba interaction to completely vanish at specific gate fields, effectively acting as an on/off spin-orbit interaction switch. The light-hole g -tensor is less anisotropic than that of state-of-the-art heavy-hole qubits, alleviating precise magnetic field orientation requirements. The large in-plane g -factor also facilitates the integration of superconductors. Moreover, the out-of-plane g -factor is strongly gate-tunable and completely vanishes at specific gate fields. Thus, this material system combines the large Rashba interaction with the scalability of planar devices, paving the way towards robust spin qubit applications and enabling access to regimes of complex spin physics.

The large spin-orbit interaction (SOI) intrinsic to semiconductor hole systems is a defining characteristic that provides the foundation for practical spin qubit devices. In fact, SOI enables all-electrical manipulation schemes such as electric dipole spin resonance (EDSR), positioning hole-spin qubits as one of the most promising candidates for scalable quantum processors^{1–12}. In this context, Ge has emerged as a leading material for hole-spin qubits¹³, due to its large intrinsic SOI, absence of valley degeneracies, compatibility with Si processing, the inherently weak coupling of holes with surrounding nuclear spins^{14,15}, and the availability of nuclear spin-depleted Ge¹⁶. Therefore, the development of quantum device structures to control SOI in Ge has been an active area of research in the pursuit of reliable and robust qubits.

The type of SOI is directly related to the dimensionality of the quantum device^{17–20}. For example, in quasi-one-dimensional systems, strong two-axis confinement leads to a large mixing of the heavy hole (HH) and light hole (LH) valence bands even at zero longitudinal momentum k_z , giving rise to the so-called direct Rashba SOI effect^{21–24}. This linear-in- k_z SOI is highly tunable with transverse DC gate fields^{25,26}, and plays a significant role in the electrical driving of the spin^{27–41}. Moreover, cold spots where SOI is turned off, at specific gate fields, have been reported^{23,42,43}, which can be leveraged as a switch to change the qubit in an operational state or in an idle state. On the

other hand, quasi-two-dimensional systems, such as the conventional planar Ge/Si_{1–y}Ge_y heterostructure¹⁰, do not show significant HH-LH mixing at zero momentum. As a result, electrical driving in these systems relies on a cubic-in- k component of Rashba SOI (RSOI), in addition to contributions from strain-induced g -tensor tilts and wavefunction deformations⁴⁴. Notwithstanding the tunability with respect to the dimensions of the quantum well and electromagnetic fields^{1,45–49} and enhancements by interface symmetry effects^{50,51} and inhomogeneous strain fields^{51–53}, the component of RSOI compatible with electrical driving is weak and remains a challenge to harness in experiments⁵⁴. These properties of conventional planar strained Ge/Si_{1–y}Ge_y heterostructures limit the prospects of this potentially scalable system.

These limitations can be alleviated in heterostructures with an LH ground state. To this end, recent progress in the growth of high-quality epitaxial Ge_{1–x}Sn_x planar heterostructures offers this possibility, thus bringing another promising low-dimensional system for engineering RSOI. Tensile-strained Ge quantum wells (QW) grown on relaxed Ge_{1–x}Sn_x barriers can host LH ground states⁵⁵ with large linear-in- k RSOI^{56,57}. Consequently, spin-qubit devices based on scalable planar Ge/Ge_{1–x}Sn_x heterostructures (Fig. 1a) would benefit from large RSOI inherited directly

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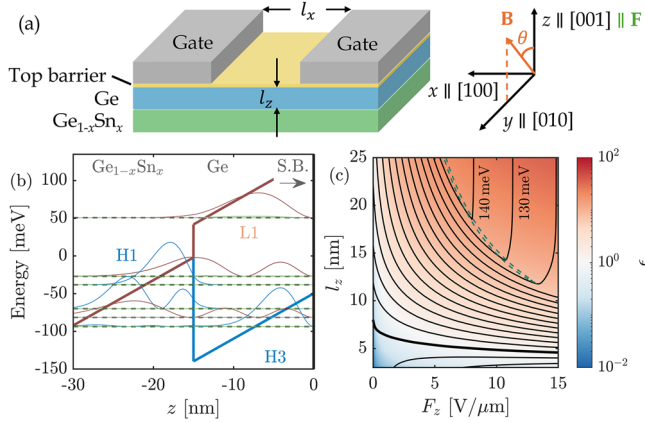


Fig. 1 | Heterostructure and confinement properties. **a** Schematic of the Ge/Ge_{1-x}Sn_x quantum channel device with Cartesian axes and fields. **b** Energy confinement profile along the growth direction. Sn content in the relaxed barrier is fixed to $x = 10\%$ for better visuals, the Ge well thickness is $l_z = 15$ nm, and the static electric field strength $F_z = 6$ V/ μ m. The z -domain covers the Ge_{1-x}Sn_x layer ($z < -l_z$) and the Ge layer ($-l_z < z < 0$). The surface barrier (S.B.) is located at $z = 0$. Blue envelopes correspond to the heavy-hole (HH) part of the hole spinor, while red and green envelopes correspond to the light-hole (LH) and the spin-orbit split-off part of the spinor, respectively. **c** Colormap of the ratio $\epsilon \equiv l_z^2/l_F^2$ as a function of l_z and F_z , along with contour lines of the $\Delta E = E_1^L - E_1^H$ LH-HH energy splitting (10 meV-steps contour lines). Sn content is $x = 16\%$. The thicker black line indicates $\Delta E = 0$ meV. The first and second HH levels H1 and H2, respectively, are 2 meV apart on the dashed green line.

from the planar geometry, rather than from transverse confinements or from hard-to-harness strain/interface effects. Herein, building upon recent theoretical developments on LH spin physics in planar Ge⁵⁷ and in isotropic quantum dots⁵⁶ in symmetrical QW heterostructures, LH spin dynamics in Ge/Ge_{1-x}Sn_x gate-defined quantum channels (QC) and elongated quantum dots (QD) with built-in structural asymmetry are investigated, revealing SOI that is strongly tunable by gate fields and compatible with electrical driving of spin qubits defined in quantum dots and superconducting-semiconducting hybrid devices.

Results and discussion

The proposed heterostructure

The proposed material system consists of a MOS-like Ge epitaxial layer (the quantum well) deposited on a strain-relaxed [001]-oriented Ge_{1-x}Sn_x layer (the barrier). The out-of-plane confinement is provided by the valance band energy offsets between Ge and Ge_{1-x}Sn_x. The lattice mismatch between Ge and relaxed Ge_{1-x}Sn_x generates epitaxial biaxial strain in the Ge QW and lifts the degeneracy between LHs and HHs at the Γ point. The ensuing strain tensor in Ge is diagonal, with components $\epsilon_{xx} = \epsilon_{yy} \equiv \epsilon_{\parallel} = a_x/a_0 - 1$ and $\epsilon_{zz} = -2\epsilon_{\parallel}c_{12}/c_{11}$, where a_0 is the lattice constant of Ge and a_x is that of relaxed Ge_{1-x}Sn_x, and c_{11} , c_{12} are elastic constants for Ge. Since $a_x > a_0$, the epitaxial strain in the QW is tensile and therefore pushes the LH band closer to the bandgap, enabling LH ground states in Ge. The surface barrier on top of Ge is assumed to be a large band offset material such that LHs remain inside the QW when subjected to large electric fields directed towards the surface. For instance, this material could be a Si capping (e.g. 5–10 nm thick), an oxide⁵⁸, or a combination thereof.

The LH-HH degeneracy at the Γ point is also broken by spatial confinement due to the effective mass difference between LHs and HHs, where the latter are usually energetically favored due to their larger out-of-plane mass⁵⁹. Therefore, an LH ground state in Ge is only attainable with large tensile strains that can compete with the confinement-induced energy splitting⁵⁷. The confinement energy profile of the device is displayed in Fig. 1b, where a static gate electric field $F = F_z \mathbf{e}_z$ pushes the holes towards the surface barrier. The peculiar profile of the HH band creates an additional triangular well at the interface between Ge_{1-x}Sn_x and Ge. Depending on the

well thickness l_z and electric field strength F_z , the HH ground state (H1) may be located either at the interface (as in Fig. 1b) or at the surface barrier. In both cases, the wavefunction overlap and its spread across the Ge/Ge_{1-x}Sn_x interface enables LH-HH mixing between the ground LH subband (L1) and surrounding HHs. Additional LH-HH mixing arising from alloy and atomistic disorder at interfaces are neglected in the following, as most of the LH wavefunction resides within pure Ge, and because non-common atom corrections to $k \cdot p$ theory⁶⁰ at the Ge/Ge_{1-x}Sn_x interface are not yet well documented.

Light hole ground state requirements

Contour lines of the LH-HH energy splitting $\Delta E \equiv E_1^L - E_1^H$ computed from 6-band $k \cdot p$ theory (see Supplemental Material) are shown in Fig. 1c as a function of l_z and F_z . The Sn content is fixed to 16%, which is achievable using current epitaxy processes^{61,62}. Three regions are identified as follows: the first corresponds to a global HH ground state. It is located at small l_z to the bottom of the bold contour line indicating $\Delta E = 0$ meV. As l_z decreases, tensile strain is unable to compensate the increasingly strong confinement from the well, which eventually pushes H1 to the ground state. The second region corresponds to a global LH ground state, with H1 located at the Ge/Ge_{1-x}Sn_x interface (as in Fig. 1b). This region is located directly to the top of the 0 meV contour line, where ΔE is strongly dependent on l_z and/or F_z . The third region also corresponds to an LH ground state, but with H1 located in Ge (in analogy to H3 in Fig. 1b). This region is identifiable by the mostly l_z -independent ΔE , which is expected for electric field-dominated confinements. A sharp boundary between regions 2 and 3 coincides with $E_1^H - E_2^H \ll \Delta E$. This is indicated by the dashed green line, corresponding to $E_1^H - E_2^H = 2$ meV. The abruptness of the transition scales with the hybridization strength between HH levels located on either side of the triangular barrier at the Ge/Ge_{1-x}Sn_x interface. Hybridization increases with lower Sn content in Ge_{1-x}Sn_x, corresponding to a smaller triangular barrier.

The two primary contributions to the ground state energy E_1^L are the band offset at the Ge/Ge_{1-x}Sn_x interface and the force eF_z on L1 from the gate field. The former contribution scales roughly as $1/l_z^2$, whereas the latter scales as $1/l_F^2$, where l_F is a characteristic electric length $l_F^3 \equiv \alpha_0 \gamma_{\perp}^p / (eF_z)$, with $\alpha_0 = \hbar^2 / (2m_0)$ and $\gamma_{\perp}^p$ being the inverse of the dimensionless out-of-plane effective mass of L1 (the p superscript stands for “planar”). The colormap in Fig. 1c shows the ratio of the two lengths, $\epsilon \equiv l_z^2/l_F^2$, indicating whether the ground state energy is dominated by the gate field ($\epsilon \gg 1$) or by the Ge/Ge_{1-x}Sn_x band offset ($\epsilon \ll 1$).

Planar ground state properties

Kinetic and spin properties of the ground state in the Ge/Ge_{1-x}Sn_x planar system are readily quantified by computing an effective two-dimensional Hamiltonian for L1 with all other subbands L_j ($j \geq 2$) and H_l ($l \geq 1$) treated perturbatively. For static out-of-plane magnetic fields $\mathbf{B} = B\mathbf{e}_z$ and up to k^3 , L1 is described by (assuming $E_1^L = 0$)

$$H_{\text{eff}}^p = \alpha_0 \gamma_{\parallel}^p K_{\parallel}^2 + \frac{\alpha_0}{2l_F^p} g_{\perp}^p \sigma_z + [(i\beta_1^p K_- - i\beta_2^p K_+^3 + i\beta_3^p K_- K_+ K_-) \sigma_+ + \text{h.c.}], \quad (1)$$

where $\mathbf{K} = \mathbf{k} + e\mathbf{A}/\hbar$ is the mechanical wavevector with $\mathbf{A} = Bx\mathbf{e}_y$ being the vector potential, $K_{\pm} = K_x \pm iK_y$, $K_{\parallel}^2 = \{K_-, K_+\}/2$, and $1/l_F^p = [K_-, K_+]/2 = eB/\hbar$. The effective parameters are the following: $\gamma_{\parallel}^p$ is the inverse in-plane effective mass, $g_{\perp}^p$ is the out-of-plane g -tensor component, and β_i^p with $i = 1, 2, 3$ are RSOI parameters that arise from breaking the inversion symmetry of the confining potential and generate a $B = 0$ spin-splitting away from $\mathbf{k}_{\parallel} = 0$ ⁵⁹. The parameter β_1^p contributes linearly in k and scales with the wavefunction overlap between the LH and the split-off hole components of the ground state, while β_2 and β_3 contribute cubically in k and scale with the LH-HH mixing strength (see Supplemental Material). For HHs in Ge, the spin splitting is typically dominated by $\beta_2^{p,63}$, while the term relevant for EDSR, β_3^p , is smaller than β_2^p by a factor $(\gamma_2 + \gamma_3)/(\gamma_2 - \gamma_3)$ (with

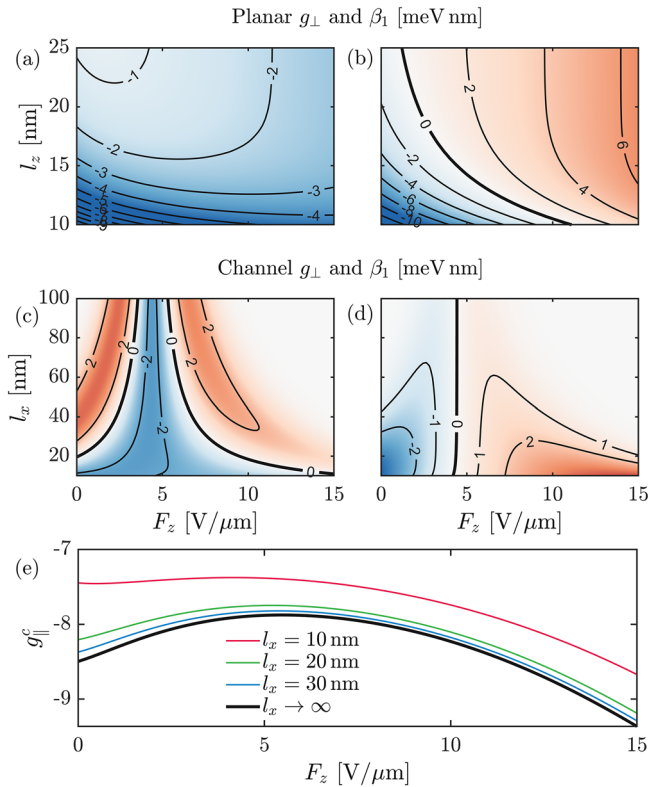


Fig. 2 | g-factor and spin-orbit parameters. **a, b** Planar out-of-plane $g_{\perp}^p$ and linear Rashba coefficient β_1^p from Eq. (1) as a function of the gate electric field F_z and quantum well (QW) thickness l_z with Sn content $x = 0.16$. The thick black line indicates $\beta_1^p = 0$. **c, d** Quantum channel (QC) out-of-plane $g_{\perp}^c$ and linear Rashba coefficient β_1^c of the ground state as a function of F_z and l_x with fixed Sn content $x = 16\%$ and QW thickness $l_z = 15$ nm. Color code in panels (a)–(d): blue (red) for negative (positive) values and white for zero. **e** QC in-plane $g_{\parallel}^c$ as a function of F_z for selected l_x . The thick black line corresponds to the QW limit.

$\gamma_2 = 4.24$ and $\gamma_3 = 5.69$ being Ge Luttinger parameters) due to its origins from small valence band anisotropies and from coupling with neighboring LH levels^{59,63}. As discussed in the following, LH ground states exhibit a sizable β_1^p parameter compatible with EDSR, along with a β_3^p parameter $(\gamma_2 + \gamma_3)/(\gamma_2 - \gamma_3)$ -times larger than β_2^p . The Schrieffer-Wolff transformation employed to derive Eq. (1) provides exact formulas for these parameters in terms of the envelopes and energies at $k_x = k_y = 0$ and $B = 0$ ^{56,57} (also see Supplemental Material).

$g_{\perp}^p$ and β_1^p are shown in Fig. 2a, b as a function of l_z and F_z with Sn content $x = 16\%$. $g_{\perp}^p$ slowly diverges for $l_z \lesssim 10$ nm, which coincides with ΔE approaching 0 meV and therefore leads to vanishing energy denominators in the perturbative expansion. In this regime, Hamiltonian (1) fails to accurately describe LH because of strong couplings with neighboring subbands. On the other hand, $g_{\perp}^p$ and β_1^p are well behaved for $l_z \gtrsim 10$ nm, and also become l_z -independent as $\epsilon \gg 1$. At a QW thickness $l_z = 15$ nm, $g_{\perp}^p \sim -3$ for the whole range of static electric fields. Most importantly, β_1^p vanishes at finite F_z due to the asymmetric confinement profile even at $F_z = 0$. The same is also true for the two other cubic Rashba parameters (see Supplemental Material). Notably, the $\beta^p = 0$ contour line coincides in (l_z, F_z) -space for all three Rashba parameters, suggesting a fully gate-tunable RSOI. In the following, the Ge layer thickness is fixed to $l_z = 15$ nm, which ensures an LH ground state over the whole range of F_z , while remaining well separated from the lowest HH level by $\Delta E \gtrsim 20$ meV.

Quantum channel ground state properties

The SOI tunability with respect to F is now further investigated for a QC device. A parabolic in-plane confinement $V_{\parallel} = -\alpha_0 x^2 / l_x^2$ is introduced towards the x -axis, such that the hole is free to move along the y -direction.

Here, the characteristic length l_x is interpreted as being relative to a hole sharing the same mass as a bare electron of mass m_0 . The energy eigenstates in this geometry are denoted by $|\tilde{\sigma}; i, N\rangle$ (with corresponding energy $E_{i,N}^c$), where $\tilde{\sigma}$ is the QC pseudo-spin index, $i \geq 1$ is an out-of-plane subband index and $N \geq 0$ is an in-plane orbital index. Spin properties of the QC ground state $|\tilde{\sigma}; 1, 0\rangle$ are readily quantified by writing its two-dimensional effective Hamiltonian for $k_y \neq 0$ and $B > 0$, with the coupling to all other levels $|\tilde{\sigma}; 1, N \geq 1\rangle$ and $|\tilde{\sigma}; i \geq 2, N \geq 0\rangle$ treated perturbatively. The result is

$$H_{\text{eff}}^c = E_{1,0}^c + \alpha_0 \gamma^c k_y^2 + \frac{\alpha_0}{2l_x^2} (g_{\perp}^c \cos \theta \tilde{\sigma}_3 + g_{\parallel}^c \sin \theta \tilde{\sigma}_2) + (\beta_1^c + \beta_3^c k_y^2) k_y \tilde{\sigma}_1. \quad (2)$$

Parameters in (2) are named similarly to those in (1), with a “c” superscript indicating the QC geometry. Hamiltonian (2) is written within the gauge $\mathbf{A}(\mathbf{r}) = Bx(\cos \theta \mathbf{e}_y - \sin \theta \mathbf{e}_z)$, such that the field $\mathbf{B}$ lies within the y - z plane with polar angle θ . Hamiltonian (2) contains a direct RSOI parameter $\beta_1^c = \hbar v_{\text{so}}$ which arises from the strong confinement along z and the weaker confinement along x . It is analogous to the so-called spin-orbit velocity v_{so} introduced in squeezed hole spin qubits⁶⁴.

QC effective parameters $g_{\perp}^c$ and β_1^c are displayed in Fig. 2c, d. A roughly l_x -independent $\beta_1^c = 0$ line is clearly visible in panel (d) near $F_z = F_z^* \approx 4.4$ V/ μm , in agreement with $\beta_1^p = 0$ reported in panel (b) at the corresponding QW thickness. Panels (c) and (d) also reveal that near $F_z = F_z^*$ (or $\beta_1^c = 0$), $g_{\perp}^c \sim g_{\parallel}^c$ for all l_x considered. More generally, if $|\beta_1^c| l_x / \alpha_0 \ll 1$, $g_{\perp}^c$ is well approximated by (see Supplemental Material)

$$g_{\perp}^c \approx g_{\parallel}^c + \frac{1}{\gamma^p} \left(\frac{\beta_1^c l_x}{\alpha_0} \right)^2, \quad (3)$$

where $g_{\perp}^c < 0$. This is in close analogy to the exponential renormalization of the g -tensor reported in quasi-1D systems^{33,65}, but with an additional g -factor sign change for $l_x^2 \geq \alpha_0^2 |g_{\perp}^c| / \sqrt{\gamma^p} / (\beta_1^c)^2$ (see Fig. 2c). Compared to $g_{\perp}^c$, $g_{\parallel}^c \sim -8$ and remains finite for every F_z and l_x (see Fig. 2e). The regime where $g_{\perp}^c \rightarrow 0$ and $\beta_1^c \rightarrow 0$ in panels (c) and (d) corresponds to the linear Rashba term dominating the in-plane confinement energy (see Supplemental Material). It is analogous to the decrease towards zero of the direct RSOI at very strong electric fields reported in Ge/Si core/shell nanowires and squeezed dots^{21,64}.

Quantum dot ground state properties

The effects of RSOI on the Rabi frequency of the LH ground state are investigated by introducing an additional parabolic confinement towards the y -direction, thus corresponding to a quantum dot (QD) system with characteristic lengths l_x and $l_y \geq l_x$ in the x - and y -directions, respectively. An electrical driving $\mathbf{F} = F_{\text{ac}} \mathbf{e}_y \cos \omega t$ is applied along the axis of weak confinement, where $\hbar \omega$ is the QD ground state two-level system energy at finite B . The Rabi frequency Ω is estimated with $\Omega = (eF_{\text{ac}}/\hbar) |\langle 0 | y | 1 \rangle|$, where $|0\rangle$ and $|1\rangle$ are the two-level system states. The magnetic field frequency $\omega_B \equiv \mu_B B_0 / \hbar$ is also introduced, where $B_0 = 0.05$ T is the magnitude of the applied $\mathbf{B}$ -field defining the qubit.

Figure 3 shows the computed Rabi frequency Ω in normalized units as a function of $l_y > l_x = 35$ nm and F_z with a perpendicular-to-plane $\mathbf{B}$ -field. An $\Omega = 0$ line at $F_z = F_z^*$ is clearly visible and corresponds to $\beta_1^c = 0$. This line is referred as the β -line. In addition, two more lines of $\Omega = 0$ appear near $F_z \approx 2$ V/ μm and $F_z \approx 7.5$ V/ μm , and correspond to $g_{\perp}^c = 0$ with $l_x = 35$ nm and $l_y \rightarrow \infty$. These lines are referred as the two g -lines. Figure 3b plots Ω along the dashed red line in panel (a) for various $\mathbf{B}$ -field polar angles θ . Ω increases as the field is tilted in the QW plane, in accordance with $g_{\parallel}^c > g_{\perp}^c$. Moreover, the two g -lines disappear as soon as $\mathbf{B}$ acquires an in-plane component, since the effective g factor g_{θ}^c in the tilted configuration is $(g_{\theta}^c)^2 = (g_{\perp}^c \cos \theta)^2 + (g_{\parallel}^c \sin \theta)^2$, and $g_{\parallel}^c$ never vanishes (see Fig. 2e). On the other hand, the β -line remains unperturbed for all θ . A similar plot of Ω along the dashed red line is shown in panel (c) for $\theta = 0$ and for selected

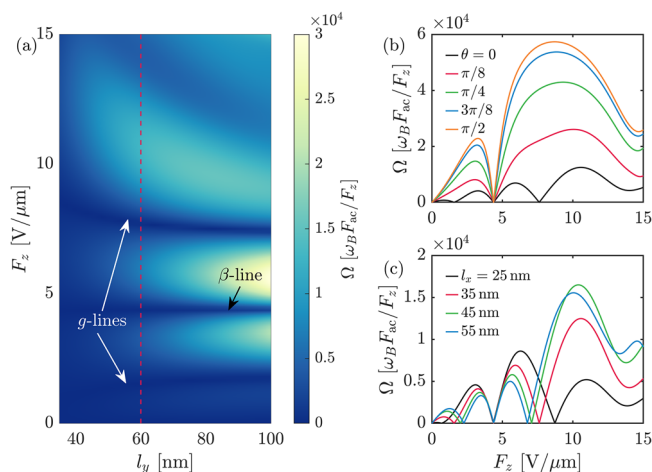


Fig. 3 | Electrical driving. **a** Rabi frequency Ω as a function of the gate electric field F_z and quantum dot (QD) y-axis effective length l_y , with $l_x = 35$ nm and magnetic field polar angle $\theta = 0$. **b** Ω along the dashed line in panel (a) for various θ , with $l_x = 35$ nm, $l_y = 60$ nm and magnetic field azimuthal angle $\phi = \pi/2$. **c** Ω for selected l_{xx} with $l_y = 60$ nm and $\theta = 0$.

values of l_x . Here again, the β -line remains unperturbed by variations of l_x , whereas the two g-lines slightly shift towards the β -line with increasing l_x , in accordance with the $g_{\perp}^c = 0$ behavior shown in Fig. 2c. The stability of the β -line with respect to l_x , l_y and θ in the QD system is attributed to the linear RSOI being inherited directly from the planar geometry, i.e. $\beta_1 = 0$ occurs even without in-plane confinement. The steep incline $\partial\Omega/\partial F_z$ at the β -line enables an efficient modulation from, and to the idle mode, while not necessarily increasing the dephasing rate from charge noise, since the latter is enabled by fluctuations that change the Larmor frequency of the ground state, and those are suppressed to first order in F_z by the small inclines of both components of the g-factor at the corresponding gate field (see Fig. 2c, e). In contrast, the g-lines originate from the combined effects of in-plane confinement and $\beta_1^c \neq 0$ [see Eq. (3)]. The complete tunability of the out-of-plane g-factor, notably to $g_{\perp} = 0$, suggests that the proposed heterostructure may provide an implementation of the recently proposed gapless spin qubits⁶⁶, which presents many advantages for integration by not requiring a global clock and by enabling full electrical baseband control. Large in-plane g factors make the system appealing for superconducting-semiconducting hybrid systems, as they show larger quality factors in in-plane magnetic fields compared to perpendicular fields at equal magnitude⁶⁷.

Conclusion

This work introduced a MOS-like Ge/Ge_{1-x}Sn_x planar heterostructure as a material system for LH spin qubits with fully gate-tunable RSOI. In the *on* state, RSOI allows for efficient EDSR driving of the spin through strong linear-in- k and cubic-in- k terms, whereas in the *off* state, the qubit is put in idle mode. In addition, fully gate-tunable out-of-plane g-factors provide a scalable implementation of qubits that are free of leakage states if $g_{\perp} \approx 0$ ⁶⁶. On the other hand, the large in-plane g-tensor component of the LH spin alleviates precise magnetic field orientation requirements. This heterostructure leverages the potential of LH spins as qubits for hybrid systems and for scalable quantum computing.

Methods

The band structure of the planar heterostructure, of the quantum channel, and of the quantum dot system are computed using 6-band $k \cdot p$ theory within the virtual crystal approximation, and the envelope function approximation. The z -dimension of the heterostructure (along the growth direction) is discretized using a constant mesh size of $\delta z = 0.01$ nm. The Hamiltonian evaluated at $k_x = k_y = 0$ is diagonalized by means of the finite difference method using the MATLAB function `eigs()`. All calculations are performed using MATLAB, version R2020a and/or later.

Data availability

The MATLAB data that was used to generate the figures and that support the findings of this study are available in our repository: <https://doi.org/10.5281/zenodo.20698832>.

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Author contributions

P.D.V. developed the theoretical framework and performed the $k \cdot p$ simulations. O.M., D.L. and S.B. initiated and supervised the work. All authors contributed to the analysis of the results and review of the manuscript.

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Competing interests

The authors declare no competing interests.

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