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UNIVERSITÉ DE MONTRÉAL

TYPE-2 TAKAGI-SUGENO-KANG FUZZY LOGIC MODELING

USING SUBTRACTIVE CLUSTERING

QUN REN

DÉPARTEMENT DE GÉNIE MÉCANIQUE

ÉCOLE POLYTECHNIQUE DE MONTRÉAL

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Ce mémoire intitulé :

TYPE-2 TAKAGI-SUGENO-KANG FUZZY LOGIC MODELING

USING SUBTRACTIVE CLUSTERING

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a été dûment accepté par le jury d'examen constitué de :

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So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.

– Albert Einstein

As complexity rises, precise statements lose meaning and meaningful statements lose precision.

– Lotfi Zadeh

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ABSTRACT

In this thesis, type-1 and type-2 Takagi-Sugeno-Kang (TSK) inference engines are presented. The uncertainties in type-1 TSK fuzzy logic systems (FLSs) using subtractive clustering is analyzed. A new subtractive clustering based type-2 TSK fuzzy system identification algorithm [1] and fuzzy model evaluation is proposed. Experimental result shows the effectiveness of our method. Furthermore, a comparison of these two TSK FLSs is given. Finally, the importance and limitation of the results are discussed.

Our new subtractive clustering algorithm [1] is designed to identify type-2 TSK FLSs. This algorithm is an extension of the type-1 TSK modeling algorithm proposed by Chiu [2, 3]. In our proposed method, the subtractive clustering method is combined with least-square estimation algorithm to pre-identify a type-1 fuzzy model from input/output data. Then with type-2 TSK fuzzy logic theory [4], considering the type-1 membership functions as principal membership functions (MFs) of type-2 FLS, the antecedent MFs are extended as interval type-2 fuzzy memberships by assigning uncertainty to cluster centers, and the consequent parameters are extended as fuzzy numbers (type-1 fuzzy subsets) by assigning uncertainty to consequent parameter values. Minimum error model is obtained through enumerative search of optimum values for spreading percentage of cluster centers and consequent parameters. By doing so, fuzzy modeling result of type-2 TSK FLS is found to be more appropriate than the one of type-1 TSK FLS.

CONDENSÉ EN FRANÇAIS

0.1 introduction

Développée à partir de l'année 1965 par le professeur Zadeh de l'université de Californie à Berkeley, la logique floue est une branche de la logique qui permet de manipuler, dans des conditions incertaines, des réalités dont la connaissance est imprécise. C'est une méthodologie de contrôle qui tente de simuler la pensée humaine par l'intégration des imprécisions propres à chaque système physique.

L'approche linguistique proposée par Zadeh a les potentialités pour modéliser le comportement de systèmes complexes d'une manière qualitative telle que le modèle sera plus efficace et plus souple pour cerner le comportement des systèmes mal définis avec des approximations réalistes. Plus tard, elle s'est étendue aux systèmes flous en tant que modèles qualitatifs. Le système de logique flou (en anglais: Fuzzy Logic Systems, FLSs) proposé par Takagi, Sugeno et Kang (appelé TSK) a attiré beaucoup d'attention. Il a été proposé dans un effort pour développer une approche systématique pour générer des règles floues d'un ensemble de données. Ce modèle consiste en des règles avec des prémisses floues et une fonction mathématique dans la conclusion.

Dans les TSK FLSs, les prémisses divisent l'espace des entrées en un ensemble de régions floues, alors que les conclusions décrivent le comportement du système dans ces régions. Il y a un besoin de développer des méthodes automatique ou du moins semi-automatique pour obtenir ces modèles basés sur des données mesurées. Des méthodes de regroupement ont été proposées pour identifier les regroupements cohérents d'un grand ensemble de données de telle manière à produire une représentation plus concise du comportement du système. L'approche de Chiu connue sous le nom de regroupement soustractif, fonctionne en trouvant la donnée optimale pour définir un centre du groupement basé sur la densité des données voisines. Cette méthode est une version rapide conçue pour les problèmes de grande dimension avec un nombre modéré de points de données, parce que son calcul a une complexité linéaire de la dimension de l'espace de données. Son utilisation permet facilement de trouver les groupements flous afin d'établir le nombre de règles floues et de prémisses.

Les TSK FLSs sont employés couramment pour le contrôle et le diagnostic par défaut basés sur un modèle. Dû aux propriétés modèles, c'est un approximateur non-linéaire général qui peut approximer tout tracé continu, et aussi un modèle linéaire par morceaux qu'il est relativement facile d'interpréter et dont les sous-modèles linéaires peuvent être exploités pour le contrôle et la détection de défaut.

Les ensembles flous de type-2 ont été initialement définis par Zadeh [28] en 1975. Le concept principal de la logique floue type-2 est que « **les mots signifient différentes choses pour différentes personnes** ».

La définition d'un ensemble floue de type-2 est « **Un ensemble floue de type-2 est caractérisé par une fonction d'appartenance, *i.e.* le degré d'appartenance pour chaque élément de cet ensemble est un ensemble flou dans l'intervalle [0, 1], à la différence d'un ensemble flou de type-1 où le degré d'appartenance est un nombre dans l'intervalle [0, 1]** ». La caractérisation dans cette définition des ensembles flous de type-2 utilise la notion que des ensembles flous de type-1 peuvent être considérés comme une première approximation d'ordre à l'incertitude et, en conséquence, les ensembles flous de type-2 fournissent une approximation du second ordre. Ils jouent un rôle important dans les modèles d'incertitude qui existent dans les systèmes de logique floue, et deviennent de plus en plus importants dans le « Computing with Words » et « Computational Theory of Perceptions ».

Plus tard, Liang et Mendel ont développé une théorie complète pour l'intervalle type-2 FLSs. Les FLSs de type-2 ont été développés pour répondre à l'exigence fondamentale de conception suivante:

Quand toutes les sources d'incertitude disparaissent, un type-2 FLS doit se réduire à un FLS de type-1 comparable.

Les TSK FLSs Type-2 sont présentés en 1999 par Liang et Mendel, et ils ont le potentiel d'être employés pour contrôle et dans d'autres secteurs où un modèle du type 1 TSK ne peut pas avoir de bonnes performances.

Les objectifs de la recherche de ce mémoire sont :

- présenter des moteurs d'inférence différents de TSK type-1 et de TSK type -2;
- décrire le type-1 TSK FLS basé sur le regroupement soustractif, et analyser les incertitudes dans cette méthode;
- proposer un algorithme d'identification de type-2 TSK FLS utilisant le regroupement soustractif; récapituler les similitudes et les différences du type-1 et type-2 TSK FLSs; analyser les influences des différents facteurs d'incertitudes d'un type-2 TSK FLS: RMSE, les fonctions d'appartenance (en

anglais: membership functions, MFs) Gaussiennes, sortie de modèle et erreur de modèle;

- récapituler les limitations du nouvel algorithme d'identification de type-2 TSK FLS et donner les futures directions de recherches.

Dans ce mémoire, on se propose aussi de développer un programme en MATLAB utilisant l'algorithme pour réaliser l'identification de type-2 TSK FLS en utilisant le regroupement soustractif.

0.2 Type-1 TSK FLS

Un modèle du type-1 TSK s'appuie sur des règles de la forme « si … alors » qui convertissent les données d'entrées en sorties. Les règles gouvernant un système flou définissent un diagramme de zones floue, qui relient toutes les données d'entrée possibles à tout les sorties envisagées. Dans le type-1 TSK FLSs, les MFs précédentes sont les ensembles flous de type-1 et la conclusion est une fonction mathématique. Dans son moteur d'inférence, la moyenne pondérée, l'agrégation est employée pour obtenir une sortie unique, aucune défuzzification n'est nécessaire.

Une description détaillée des concepts des MFs, de la méthode et de l'inférence de type-1 TSK FLS sont les principales matières abordées au Chapitre 2 pour les deux différents modèles : type-1 TSK FLS d'ordre zéro et type-1 de premier ordre TSK FLS. Un exemple numérique basé sur le moteur d'inférence de premier ordre FLS est donné.

Bien que l'inférence du type-1 TSK FLS d'ordre zéro est présentée et des modèles plus élevés du type 1 TSK FLS de premier-ordre ont été décrits dans la littérature, ce mémoire traite exclusivement des modèles ayant une seule sortie de premier ordre du type-1 TSK FLS, parce qu'ils sont les plus couramment employés et peuvent être facilement étendus aux modèles de type-2 TSK FLS.

Le TSK FLS de type-1 basé sur le regroupement soustractif est conçu en deux étapes: la méthode de regroupement soustractif (identification du modèle de structure) pour déterminer les MFs précédentes et un algorithme d'évaluation en moindre carré (identification de paramètre) pour estimer les paramètres des fonctions de la conclusion.

Basé seulement sur des données mesurées sans connaissance préalable, il n'y a pas de manière systématique d'obtenir un modèle flou de TSK avec une structure simple et une exactitude suffisante. En employant le regroupement soustractif, il est facile d'obtenir un type-1 TSK FLS efficace. Mais il reste quelques incertitudes dans cet algorithme.

Le TSK FLS de type-1 utilisant le regroupement soustractif a des incertitudes à cause de la nécessité de l'initialisation des paramètres. Ces paramètres ont de l'influence sur l'évaluation des groupes, le nombre de règles et des mesures d'erreur de performance. L'initialisation de paramètres cause des incertitudes dans l'algorithme basé sur le regroupement soustractif du modèle d'identification du type-1 TSK FLS. En d'autres mots, il y a des incertitudes dans l'identification du modèle de structure, comme le nombre de règles et de variables impliquées dans les prémisses, les centres des groupes et la largeur gaussienne de diffusion de MFs. Il y a également des incertitudes dans l'identification du modèle de paramètres, comme les paramètres de conclusion et les coefficients de régression.

0.3 Type-2 TSK FLS

Selon Mendel, pour des modèles de type-2 TSK, il y a trois structures possibles en raison des fonctions d'apparences de type-1 ou type-2 et des paramètres de conclusion pour chaque règle:

Model I (A2-C1) -- les fonctions d'apparences de prémisses sont des ensembles flous de type-2 et les paramètres de conclusion sont des ensembles flous de type-1;

Modèle II (A2-C0) -- les fonctions d'apparences de prémisses sont des ensembles flous de type-2 et les paramètres de conclusion sont des valeurs numériques uniques;

Modèle III (A1-C1) -- les fonctions d'apparences de prémisses et les paramètres de conclusion sont des ensembles flous de type-1;

La plupart des modèles de type-2 TSK FLS sont du type Modèle I, dans lequel les catégories conséquentes de paramètres et des fonctions d'apparences sont des ensembles intervalles. Le principe théorique de type-2 TSK FLS et le calcul détaillé d'inférence sont expliqués au chapitre 3. Un type-2 TSK FLS a plus de degrés de liberté dans la conception qu'un type-1 TSK FLS parce que les ensembles flous de type-2 sont décrits par plus de paramètres que les ensembles flous de type-1. Ceci suggère qu'un type-2 TSK FLS a le potentiel de surpasser un type-1 TSK FLS en raison de son plus grand nombre de degrés de liberté dans la conception.

En outre, la sortie du type-2 TSK FLS fournit plus d'informations, pas seulement la sortie unique qui est comme celle du type-1 TSK FLS, mais aussi l'ensemble intervalle de sortie. Ce dernier contient les informations sur les incertitudes qui sont associées à la sortie unique, et cette information peut seulement être obtenue en travaillant avec le type-2 TSK FLS.

Un type-2 FLS peut manipuler des incertitudes parce qu'il peut les modéliser et potentiellement réduire au minimum leurs effets. Le type-2 TSK FLS est nécessaire pour modéliser directement des incertitudes par la méthode du regroupement soustractif et pour réduire au minimum leurs effets. Comparant un type-1 TSK FLS à un type-2 TSK FLS (le modèle le plus général A2-C1), ceux-ci ont différentes structures de prémisses et de conclusions. Pour prolonger un type-1 TSK FLS en ses contreparties type-2 avec l'accent sur l'ensemble intervalle, les MFs des prémisses doivent être changé d'ensembles flous de type-1 en des ensembles flous de type-2; des paramètres de conclusion doivent aussi être changés à partir d'un certain nombre en un nombre flou.

L'algorithme flou basé sur le regroupement soustractif d'identification de système de type-2 TSK est une prolongation du type-1 TSK modelant l'algorithme proposé par Chiu. Dans la méthode proposée, d'abord un modèle flou de type-1 employant le regroupement soustractif est pré-identifié à partir des données d'entrée-sortie. Puis avec la théorie de logique floue de type-2 TSK, les MFs de type-1 sont considéré comme les MFs primaires de type-2 FLS, les centres des groupements sont prolongés comme nombre intervalle flou de type-1 en assignant l'incertitude aux centres des groupements pour les changer à partir d'un certain nombre en un ensemble flou d'intervalles de largeur constante. Les degrés d'appartenance changent de certaines valeurs à des nombres flous, et les paramètres de conclusion sont prolongés comme des nombres flous (sous-ensembles flou de type-1) en assignant l'incertitude aux valeurs de paramètre de conclusion dans le modèle du type-1 TSK.

Les incertitudes du centre du groupement, la déviation de MFs et les paramètres de conclusion, qui sont assignés au modèle flou de type-1, influencent les différents facteurs du type-2 TSK FLS : l'erreur en racine de moindre carré (en anglais: Root Mean Square Error, RMSE), la sortie du modèle, MFs Gaussien et l'erreur du modèle. Il est recommandé de faire une recherche énumérative jusqu'à un certain rang pour obtenir les valeurs optimales des pourcentages de propagation des centres de groupements et des paramètres de conclusion pour avoir le meilleur modèle flou de type-2 TSK possible. La performance d'un type-2 TSK FLS est évaluée en utilisant le RMSE. En faisant ainsi, le résultat expérimental par type-2 TSK FLS s'avère meilleur que celui du type-1 TSK FLS.

0.4 Conclusion

Le type-2 TSK FLSs a un plus grand nombre de paramètres de conception pour chaque règle. Ceci suggère qu'un type-2 TSK FLS ait le potentiel de surpasser un type-1 TSK FLS. Quand des données du système sont bruitées, le type-2 FLS est plus à même de les modéliser. En outre, en raison d'un plus grand nombre de paramètres de conception, il est plus difficile d'identifier un type-2 TSK FLS qu'un type 1 TSK FLS.

L'algorithme d'identification de type-2 TSK FLS proposé peut être comparé relativement à la méthode de propagation arrière de Mendel. Ses avantages et ses inconvénients sont récapitulés ci-dessous :

- Avantage :

  1) plus facile à comprendre pour l'expert en matière de modèle du type-1 TSK;
  2) plus facile de manipuler la précision du modéliser de type-2 TSK;
  3) éviter la recherche énumérative de paramètres du modèle type-2;

4) facile de contrôler le nombre de règles et de réduire au minimum RMSE.

- Inconvénients :

1) dans un certain sens, la précision de modéliser de type-2 TSK est souvent basée sur la précision du modèle du type-1 TSK de laquelle elle découle;
2) le pourcentage de croissance des centres de groupements et les paramètres de conclusion dépendent de la précision du modèle du type-1 TSK.

Dans l'algorithme basé sur le regroupement soustractif d'identification de système flou de type-2 TSK proposé, le centre de groupement et le paramètre de conclusion sont étendus en un ensemble flou d'intervalles de largeur constante. Le Type-2 FLS avec un ensemble flou d'intervalles de largeur non constante sera un sujet intéressant pour une recherche future. Par ailleurs, jusqu'à présent, aucun modèle plus élevé que le type-2 TSK FLS de premier-ordre n'a été décrit dans la littérature. Cela peut également être une direction de recherche très intéressante.

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LIST OF ABBREVIATIONS AND SYMBOLES

FBF	Fuzzy Basis Function
FL	Fuzzy Logic
FLS	Fuzzy Logic System
LSE	Least Square Error
MF	Membership Function
MIMO	Multi-Input Multi-Output
MISO	Multi-Input Single-Output
RMSE	Root Mean Square Error
TSK	Takagi-Sugeno-Kang
TV	Television

a_j^k	Spread percentage of the cluster center
b_j^k	Spread percentage of fuzzy numbers
c_j^k	Centre (mean) of fuzzy numbers
f	Mathematical function.
f^k	Total firing strengths interval sets for the *k*th rule in a type-2 TSK FLS
k	Centre (mean) of fuzzy numbers
m	Number of rules
n	Number of antecedents
P_i	Potential value of data vector
p_j^k	Regression parameters for *j*th antecedent in *k*th TSK rule r_k

$\tilde{p}_j^k$	Fuzzy consequent parameters for *j*th antecedent in *k*th TSK rule r_k in a type-2 TSK FLS
Q_{jk}	Type-1 fuzzy sets on universe of discourse for *j*th antecedent in the *kth* TSK rule
$\tilde{Q}_{jk}$	Type-2 fuzzy sets on universe of discourse for *j*th antecedent in the *kth* TSK rule
r	Number of consequents
r_a	Hypersphere *cluster radius* in data space
r_b	Hypersphere *penalty radius* in data space
r_k	kth rule, $k \in [1, m]$
s_j^k	Spread of fuzzy number,
$sumMu_k$	Total membership grade for the *k*th rule
Xk	Universe of discourses for the *k*th rule
w	Number of data points
w^*	Crisp output
$\tilde{w}^k$	Interval value of the consequent of the *k*th rule in a type-2 TSK FLS
$\tilde{w}$	Interval set of total output for all rules of the type-2 TSK FLS
w_k^*	Conclusion for *k*th TSK rule r_k
W_{im}	Fuzzy model output
W_{is}	System output
x_k	*k*th linguistic variable, $k \in [1, n]$
x^i	*i*th normalized data vector $i \in [1, w]$

x_k^0	*k*th crisp input, $k \in [1,n]$
x_i^j	*j*th element in the vector x^i, $j \in [1,n]$
x_{jk}^*	Cluster center
Z	Linguistic variables
α_k	Rule firing strength (weight) for *k*th type-1 TSK rule r_k
β_k	Variable
σ	Deviation of Gaussian MFs
σ_j^k	Deviation of *j*th Gaussian MF in *k*th rule
$\mu_{jk}(x)$	Membership grade for for *j*th antecedent in *kth* TSK rule r_k, $x \in X$
$\tilde{\mu}_{jk}$	Interval membership grade for *j*th antecedent in *kth* TSK rule r_k,
η	Squash factor
$\underline{\varepsilon}$	Reject radio
$\bar{\varepsilon}$	Accept radio

CHAPTER 1

INTRODUCTION

1.1 Fuzzy logic and fuzzy logic system

Fuzzy logic (FL) was introduced by Zadeh in the 1960's [5]. FL is a superset of conventional (Boolean) logic that has been extended to process data by allowing partial set membership rather than crisp set membership or non-membership. FL is a problem-solving control system methodology. FL provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information.

A *Fuzzy logic system* (FLS) is an expert system that uses a collection of fuzzy membership functions (MFs) and rules to reason about data. If-then rule statements are used to formulate the conditional statements that comprise FL. The rules in a fuzzy expert system are usually of a form similar to the following:

IF x_1 *is* Q_1 and x_2 *is* Q_2

THEN Z *is* w (1.1)

where x_1 and x_2 are linguistic input variables (names for know data values), Z is a linguistic output variable (a name for a data value to be computed). Q_1 and Q_2 are linguistic values defined by fuzzy sets on the universes of discourse X_1 and X_2. The **IF**-part of the rule " x_1 *is* Q_1 and x_2 *is* Q_2 " is called the *antecedent* or premise, while the **THEN**-part of the rule " Z *is* w " is called the *consequent* or conclusion. The consequent w can be a fuzzy set or a mathematical function.

The antecedent (the rule's premise) describes to what degree the rule applies, while the conclusion (the rule's consequent) assigns a MF to each of one or more output variables. The set of rules in a FLS is known as the *rulebase* or knowledge base. A rule can be of MIMO type, *i.e.*, multiple inputs multiple outputs, or be of MISO type, *i.e.*, multiple inputs single output.

The two most important classes of FLSs used by today's engineers are Mamdani FLS and Takagi-Sugeno-Kang (TSK) FLS. Table 1.1 summarizes the similarities and differences between these two classes of FLSs. Both systems are characterized by IF-THEN rules and have the same antecedent structures. There are some differences between them. First, the structure of consequents for a TSK rule is a function instead of a fuzzy set in a Mamdani rule. Second, output for a TSK FLS is a crisp value, whereas output for a Mamdani FLS is a fuzzy set, defuzzication is needed to obtain the crisp output by using the composition operator. Third, uncertainty can be accounted for both antecedent and consequent MFs in a Mamdani FLS, but in a TSK FLS, only for the antecedent MFs.

To reiterate, TSK FLSs is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems. A TSK FLS is not applicable to as many kinds of time-series forecasting problems as is a Mamdani FLSs.

Table 1.1 Comparisons of Mamdani FLS and TSK FLS

		Mamdani FLS	TSK FLS
Similarity		• Both are characterized by IF-THEN rules • Both have same antecedent structures, MFs are fuzzy sets • Both are universal approximator	
Difference	structures of consequent	a fuzzy set	a function
	inference	When all of the fuzzy sets are type-1, then its output is a type-1 set that is then defuzzified to obtain a type-0 set.	When all of the fuzzy sets are type-1, then its output is a type-0 set. No defuzzification is needed
	applicability	Uncertainty can be accounted for both antecedent and consequent membership functions	Uncertainty can be accounted for just in the antecedent membership functions
Advantage and Disadvantage	Advantage	• Intuitive. • Widespread acceptance. • Well suited to human input.	• Computationally efficient. • Works well with linear techniques (e.g., PID control). • Works well with optimization and adaptive techniques. • Guaranteed continuity of the output surface. • Well-suited to mathematical analysis. • More number of design parameters for each rule. It is possible to need fewer rules
	Disadvantage		More computationally efficient but lose linguistic interpretability.

Because a MIMO system can be viewed as a set of MISO fuzzy system, we assume in this thesis, without loss of generality, that fuzzy systems are only MISO mappings. It does not seem to be any mention of a non-singleton TSK FLS in the literature; hence this thesis research focuses exclusively on singleton TSK fuzzy MISO system.

1.2 TSK FLS development

In Zadeh's paper in 1968 [6] following the first paper, "Fuzzy Sets" in 1965 [5], he suggested using an idea of fuzzy algorithm such as

a) set y approximately equal to 10 if x is approximately equal to 5;

b) if x is large, increase y by several units.

Those fuzzy algorithms are nothing but qualitative descriptions of a human action, or decision making. As for the necessity of fuzzy algorithms, he notes that "most realistic problems tend to be complex, and many complex problems are either algorithmically unsolvable or, if solvable in principle, are computationally infeasible."

The most remarkable paper related to qualitative modeling is his paper of 1973 [7] on linguistic analysis, where he states "the principle of incompatibility," according to which "as the complexity of system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behavior of humanistic systems are not likely to have much relevance to the real world societal, political, economic, and other types of problems which involve humans either as individuals or in groups."

The proposed linguistic approach by Zadeh [6, 7] has the capability to model complex system behavior in such a qualitative way that the model is effective and versatile in capturing the behavior of ill-defined systems with realistic approximations. Motivated by these ideas of "fuzzy algorithm" and "linguistic analysis," Mamdani first applied fuzzy logic (FL) to control [8]. This topic has come to be known as fuzzy algorithmic control or linguistic control. The main problem with fuzzy control is the design of a fuzzy controller where we usually take an expert-system-like approach. That is, we derive fuzzy control rules from the human operator's experience and/or engineer's knowledge, which are mostly based on their qualitative knowledge of an objective system. A set of fuzzy control rules is a linguistic model of human control actions which is not based on a mathematical description of human control actions but is directly based on a human way of thinking about plant operation.

Zadeh's proposal of linguistic approach is effective and versatile in modeling ill-defined systems with fuzziness or fully defined systems with realistic approximations. Later it expanded into fuzzy systems modeling as a qualitative modeling by Tong [9], Pedrycz [10], Takagi and Sugeno [11, 12], Trojan et al. [13], Sugeno and Kang [14], Sugeno and Tanaka [15], Sugeno and Yasukawa [16].

Takagi-Sugeno-Kang type fuzzy model structure, also being referred to as TSK fuzzy logic systems (FLSs), after Takagi, Sugeno and Kang, has attracted many attentions. It was proposed in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and mathematical function in the consequent part. Usually conclusion function is in form of dynamic linear equation [11, 12]. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviours of the system in those regions. There is a need to develop a semi-automatic method to obtain those models based on sets of input-output data.

In fuzzy system modeling, system identification can be done by using fuzzy clustering techniques. Clustering methods are proposed to identify natural grouping of data from a large data set such that a concise representation of system's behavior is produced. Yager and Filev [17-19] developed the mountain method for estimating cluster centroids. Mamdani and Assilian [20], Bezdek [21] and Bezdek et al. [22] proposed a variety of clustering algorithms, including hierarchical, k-means, and fuzzy c-means algorithms. The initial selection of cluster details has been made automatic by Emami et al. [23]. Chiu' approach [2, 3] known as subtractive clustering reduce the computational complexities. In literature different modeling techniques can be found [24].

TSK FLSs are widely used for model-based control and model-based fault diagnosis. This is due to the model's properties of, on one hand being a general nonlinear approximator that can approximate every continuous mapping, and on the other hand being a piecewise linear model that is relatively easy to interpret [25] and whose linear submodels can be exploited for control and fault detection [26, 27].

1.3 Type-2 FLS

The original fuzzy logic (FL), founded by Zadeh [5], is unable to handle uncertainties.

By "handle," that means "to model and minimize the effect of.". That the original FL, type-1 FL, can not do this sounds paradoxical because the word fuzzy has the connotation of uncertainty. The expanded FL, type-2 FL, is able to handle uncertainties because it can model them and minimize their effects. And, if all uncertainties disappear, type-2 FL reduces to type-1 FL, in much the same way that, if randomness disappears, probability reduces to determinism [4].

A FLS that is described completely in terms of type-1 fuzzy sets is called a *type-1 FLS*, whereas a FLS that is described using at least one type-2 fuzzy set is called a *type-2 FLS*. Type-1 FLSs cannot directly handle rule uncertainties because they use type-1 fuzzy sets that are certain. Type-2 FLSs, on the other hand, are very useful in circumstances in which it is difficult to determine an exact membership function for a fuzzy set; hence, they can be used to handle rule uncertainties and even measurement uncertainties.

The main concept of type-2 fuzzy logic is that "**words mean different things to different people**"; thus, there are uncertainties associated words. Based on Extension Principle [5, 28], Algebraic structure of type-2 fuzzy sets were studied by Mizumoto and Tanaka [29, 31], also Niminen [32]. Debois and Prade [33-35] discussed fuzzy valued logic depended on minimum conjunction. Karnik and Mendel [36] extended those works and obtained practical algorithms for conjunction, disjunction and complication operations of type-2 fuzzy sets, they also developed a general formula for the extended composition of type-2 relations which is consider as an extension of the type-1 composition. Based on this formula, Karnik et al [37] established a complete theory of type-2 Mamdani FLSs in 1999. Later, Liang and Mendel developed a complete theory for interval type-2 FLSs [38, 39].

A FLS that is described completely in terms of type-1 fuzzy sets is called a *type-1 FLS*, whereas a FLS that is described using at least one type-2 fuzzy set is called a *type-2 FLS*. Type-1 FLSs cannot directly handle rule uncertainties because they use type-1 fuzzy sets that are certain. Type-2 FLSs, on the other hand, are very useful in circumstances in which it is difficult to determine an exact membership function for a fuzzy set; hence, they can be used to handle rule uncertainties and even measurement uncertainties. Type-2 FLSs have been developed that satisfy the following fundamental design requirement:

When all sources of uncertainty disappear, a type-2 FLS must reduce to a comparable type-1 FLS.

By using the discrete probability theory, John [40-42] used embedded interval valued type-2 fuzzy sets to aid the discussion of interval valued type-2 fuzzy sets and to the join and meet of interval valued type-2 fuzzy sets, and made type-2 fuzzy sets easy to understand and explain.

Type-2 TSK FLSs are presented in 1999 by Liang and Mendel [43], and type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well [4] because of its large numbers of design parameters.

1.4 Objective

The main objective of this thesis is to develop a new identification algorithm to model type-2 TSK FLS using subtractive clustering method. The contributions of this document include:

- Present different type-1 and -2 TSK inference engines. This objective is achieved by explaining the theoretical principles based on articles and books written on this subject. With this intention, a detailed description of concepts of function of membership, method and inference engine are the principal topics approached. Also, examples based on difference TSK inference engines are given.

- Describe subtractive clustering based type-1 TSK modeling, and analyze uncertainties in this method. This objective is achieved by explaining the theoretical principles of subtractive clustering. The results of some Matlab program are presented.

- Propose a new identification algorithm to model type-2 TSK FLS using subtractive clustering. Summarize the similarities and differences of Type-1 and -2 TSK FLSs; analyze influences of uncertainties on different factures of a type-2 TSK FLS: RMSE, Gaussian MFs, model output and model error. Experimental results show the effectiveness of this method. A comparison of results from Type-1 and -2 TSK FLSs is presented and the limitations of this method are discussed.

- Summarize the limitations of the new type-2 TSK FLS identification algorithm and the future research directions are given.

- A complete program for the proposed identification algorithm is developed by Matlab.

1.5 Thesis structures

This thesis is organized as follows: Chapter 1 is the history of development of TSK FLS and type-2 FL, also the objective of the thesis research. Chapter 2 and Chapter 3 review type-1 and type-2 TSK inference engines. Chapter 4 discusses the subtractive clustering based type-1 TSK modeling and its uncertainties. Chapter 5 presents the proposed type-2 TSK FLS identification algorithm. Finally, in Chapter 6, conclusions and future research direction are given.

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CHAPTER 2

TYPE-1 TSK INFERENCE ENGINES

A detailed description of concepts of MF and inference engine of type-1 TSK FLS are the main topics of this chapter for two difference models: zero-order type-1 TSK FLS and first-order type-1 TSK FLS. Also, a numerical example based on type-1 TSK inference engines is given at the end.

2.1 Principles

In a fuzzy and nonfuzzy model proposed by Takagi and Sugeno [11] which is called TSK fuzzy model [19], the consequent of a rule can be expressed as a polynomial function of the inputs. The order of the polynomial also determines the order of the model.

For a first-order TSK model, the consequent part is expressed as a linear combination of antecedents for a MIMO system. Assume r is the total number of consequents; the conclusion for the kth rule can be expressed as:

$$Z\ is\ w_1^k = f_1^k(x_1, x_2, \ldots, x_n) = p_{10}^k + p_{11}^k x_1 + p_{12}^k x_2 + \ldots + p_{1n}^k x_n$$

and

:

and

$$w_r^k = f_r^k(x_1, x_2, \ldots, x_n) = p_{r0}^k + p_{r1}^k x_1 + p_{r2}^k x_2 + \ldots + p_{rn}^k x_n \tag{2.1}$$

where $p_{10}^k, p_{11}^k, ..., p_{1n}^k, ..., p_{r0}^k, p_{r1}^k, ..., p_{rn}^k$ are constant regression parameters. When r =1, this rule base represents a MISO system.

A type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of a system. A MISO type-1 TSK model with a rule base of *m* rules, each having *n* antecedents, $k \in [1, m]$, it can be expressed as

$$
\begin{aligned}
&\textit{Rule } r_1 : \textbf{IF} \quad x_1 \textit{ is } Q_{11} \text{ and } x_2 \textit{ is } Q_{21} \textbf{ and } \ldots \textbf{ and } x_n \textit{ is } Q_{n1}, \\
&\quad \textbf{THEN } Z \textit{ is } w^1 = f^1(x_1, x_2, ..., x_n) \\
&\quad \vdots \\
&\textit{Rule } r_k : \textbf{IF} \quad x_1 \textit{ is } Q_{1k} \text{ and } x_2 \textit{ is } Q_{2k} \textbf{ and } \ldots \textbf{ and } x_n \textit{ is } Q_{nk}, \\
&\quad \textbf{THEN } Z \textit{ is } w^k = f^k(x_1, x_2, ..., x_n) \\
&\quad \vdots \\
&\textit{Rule } r_m : \textbf{IF} \quad x_1 \textit{ is } Q_{1m} \textbf{ and } x_2 \textit{ is } Q_{2m} \textbf{ and } \ldots \textbf{ and } x_n \textit{ is } Q_{nm}, \\
&\quad \textbf{THEN } Z \textit{ is } w^m = f^m(x_1, x_2, ..., x_n)
\end{aligned}
\tag{2.2}
$$

where r_1 , ... , r_k , ..., r_m are the m rules with r_k being the *k*th rule; $x_1, x_2 ... x_n$ and *Z* are linguistic variables; Q_{1k} , Q_{2k} , …, and Q_{nk} are type-1 fuzzy sets on universe of discourses *X1, X2, ..., Xn*; w^1 , ... , w^k , ..., w^m are output of each rule and $f^1(x_1, x_2, ..., x_n)$, ... , $f^k(x_1, x_2, ..., x_n)$, ..., $f^m(x_1, x_2, ..., x_n)$ are a mathematical functions.

As explained previously, when certain input values $x_1^0, x_2^0, \ldots, x_n^0$ are given to $x_1, x_2, \ldots, x_n$, the conclusion from a TSK rule r_k is a crisp value w_k^*:

$$w_k^* = f^k(x_1^0, x_2^0, \ldots, x_n^0) \tag{2.3}$$

having certain rule firing strength (weight) defined as

$$\alpha_k = \mu_{1k}(x_1^0) \cap \mu_{2k}(x_2^0) \cap \ldots \cap \mu_{nk}(x_n^0) \tag{2.4}$$

α_k is the activation value of weight for the antecedent of the rule r_k. Moreover, $\mu_{1k}(x_1^0), \mu_{2k}(x_2^0), \ldots, \mu_{nk}(x_n^0)$ are membership grade for fuzzy sets Q_{1k}, Q_{2k}, …, and Q_{nk} in the rule r_k. The symbol $\cap$ is a conjunction operator, which is a T-norm. In this thesis, the conjunction operator is the minimum operator $\wedge$ or the product operator $*$.

The output of a TSK fuzzy system with m rules in the form shown in eq.(2.1) to (2.3) can be expressed (using *weighted average aggregation*) as

$$w^* = \frac{\sum_{k=1}^{m} \alpha_k w_k^*}{\sum_{k=1}^{m} \alpha_k} \tag{2.5}$$

The MISO system for fuzzy-nonfuzzy modeling in which the output function could be represented in general as high order hypersurface of input variable with even interactive term of inputs. Higher order system modeling could identify a system without any parameter identification with smaller error for the same number of rules [44]. A larger reduction in the number of rules is possible with higher order system identification technique. Although high order type-1 TSK models have been described in the literature, this thesis focuses exclusively on first-order type-1 TSK models MISO system, because they are the most widely used and are easily extended to type-2 TSK models.

The general *k*th rule of a first-order TSK fuzzy MISO system is as below for simplicity, *i.e.*,

$$r_k : \textbf{IF} \quad x_1 \; is \; Q_{1k} \text{ and } x_2 \; is \; Q_{2k} \textbf{ and } \ldots \textbf{ and } x_n \; is \; Q_{nk},$$

$$\textbf{THEN} \;\; Z \; is \; w^k = f^k(x_1, x_2, \ldots, x_n) = p_0^k + p_1^k x_1 + p_2^k x_2 + \ldots + p_n^k x_n \qquad (2.6)$$

where $p_0^k, p_1^k, p_2^k, \ldots, p_n^k$ are constant regression parameters.

We can observe in eq.(2.6) the structure of a MISO TSK system and its associated fuzzy inference method which comprise a set of m **IF-THEN** rules. The MFs are only associated with a rule's antecedents. There is no consequent MF, *i.e.*, the type-1 TSK rule's consequent is an algebraic function of the *n* antecedent values. Hence, the rule also acts as the interface mechanism for a type-1 TSK FLS. Therefore, it is not necessary to use the composition operator to obtain the output of a fired type-1 TSK rule.

2.2 Type-1 TSK fuzzy inference

Sugeno [13] suggested the use of a single spike, *i.e.* a *singleton*, as the membership function of the rule consequent. A *fuzzy singleton* is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else. The rule consequent, instead of a fuzzy set, Sugeno proposed to use a mathematical function of the input variable as the rule consequent, such as the kth rules can be expressed as eq.(2.6) instead of a fuzzy set.

2.2.1 Zero-order Type-1 TSK fuzzy model

In a zero-order type-1 TSK fuzzy model, the output of each fuzzy rule is constant. The expression (2.6) is simplified as

$$r_k: \textbf{IF} \quad x_1 \text{ is } Q_{1k} \text{ and } x_2 \text{ is } Q_{2k} \textbf{ and } \ldots \textbf{ and } x_n \text{ is } Q_{nk},$$

$$\textbf{THEN } Z \text{ is } w^k = p_0^k \tag{2.7}$$

where p_0^k is a constant.

For example, Let us consider the following set of Sugeno fuzzy rules that describe the behavior of a zero-order type-1 TSK fuzzy system with two rules and two antecedents, *i.e.*,

r_1 : **if** x_1 *is* Q_{11} and x_2 *is* Q_{21} **then** $w^1 = p_0^1$

r_2 : **if** x_1 *is* Q_{12} and x_2 *is* Q_{22} **then** $w^2 = p_0^2$

Observation: x_1^0, x_2^0

Conclusion: $$w^* = combine(w_1^*, w_2^*) \tag{2.8}$$

where Q_{11}, Q_{21}, Q_{12}, Q_{22} are defined as in Figure 2.1

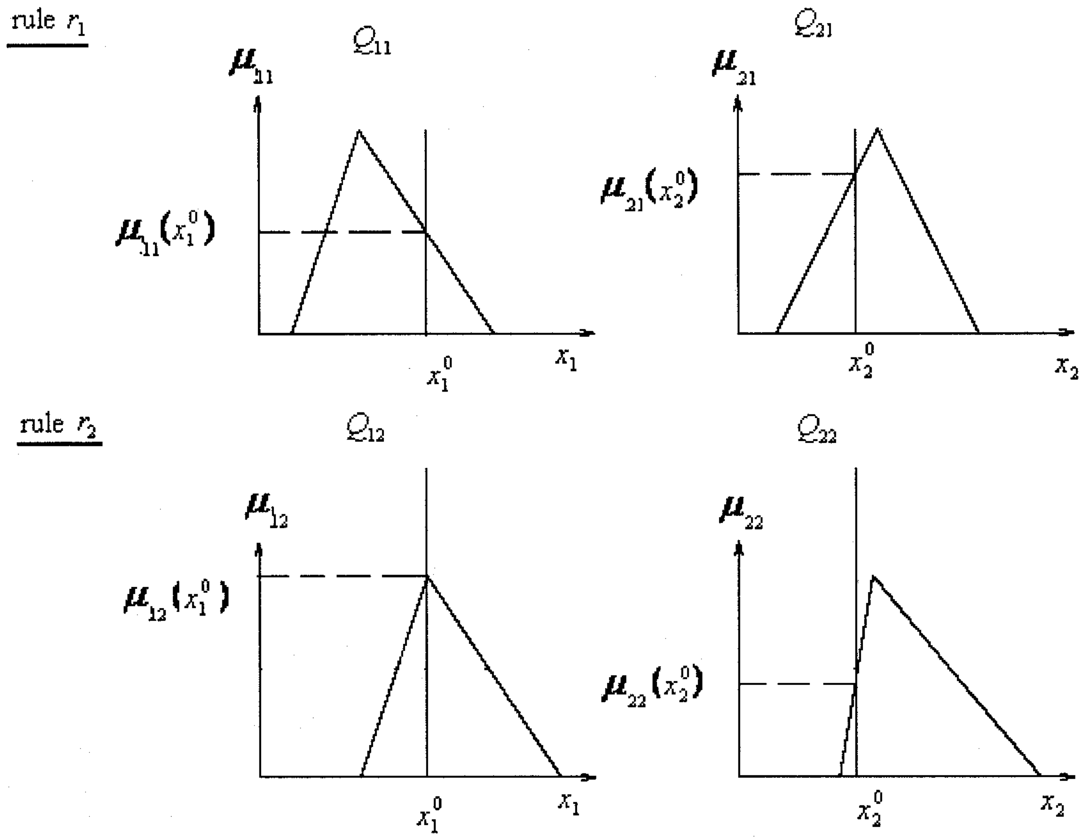


Figure 2.1 TSK FLS with two antecedents

As shown in Figure 2.1, using Sugeno approach with crisp input x_1^0 and x_2^0, the computation of the inference engine is done through the following steps:

1) obtain the firing strengths for the two rules r_1 and r_2;

$$\alpha_1 = \mu_{11}(x_1^0) \cap \mu_{21}(x_2^0)$$

$$\alpha_2 = \mu_{12}(x_1^0) \cap \mu_{22}(x_2^0) \tag{2.9}$$

2) obtain the conclusion for each rule;

$$w_1^* = p_0^1$$

$$w_2^* = p_0^2 \qquad (2.10)$$

3) combine these conclusions by using the equation to get crisp output:

$$w^* = \frac{\alpha_1 w_1^* + \alpha_2 w_2^*}{\alpha_1 + \alpha_2} \qquad (2.11)$$

2.2.2 First-order TSK FLS

A generalized *k*th rule in the first-order Sugeno fuzzy MISO system is expressed as eq. (2.6), the output is obtained by a weighted average aggregation as in eq. (2.4).

For example, let us consider the following set of Sugeno fuzzy rules that describe the behavior of a first-order type-1 TSK fuzzy system with two rules and two antecedents, *i.e.*

r_1 : **if** x_1 *is* Q_{11} and x_2 *is* Q_{21} **then** $w^1 = p_0^1 + p_1^1 x_1 + p_2^1 x_2$

r_2 : **if** x_1 *is* Q_{12} and x_2 *is* Q_{22} **then** $w^2 = p_0^2 + p_1^2 x_1 + p_2^2 x_2$

Observation: x_1^0, x_2^0

Conclusion: $w^* = combine(w_1^*, w_2^*)$ (2.12)

where Q_{11}, Q_{21}, Q_{12}, Q_{22} are defined again as in Figure 2.1

Using Sugeno approach with crisp input x_1^0 and x_2^0, the computation of the inference engine is done through the following these steps:

1) Obtain the firing strengths for the two rules r_1 and r_2;

$$\alpha_1 = \mu_{11}(x_1^0) \cap \mu_{21}(x_2^0)$$

$$\alpha_2 = \mu_{12}(x_1^0) \cap \mu_{22}(x_2^0) \tag{2.13}$$

2) Obtain the conclusion for each rule;

$$w_1^* = p_0^1 + p_1^1 x_1^0 + p_2^1 x_2^0$$

$$w_2^* = p_0^2 + p_1^2 x_1^0 + p_2^2 x_2^0 \tag{2.14}$$

3) Combine these conclusions by using

$$w^* = \frac{\alpha_1 w_1^* + \alpha_2 w_2^*}{\alpha_1 + \alpha_2} \tag{2.15}$$

<u>Numerical Example 2.1</u>: Fuzzy inference using Sugeno approach with two antecedents

Let us assume in the fuzzy system (2.12), Q_{11}, Q_{21}, Q_{12}, Q_{22} are defined as in Figure 2.2, $p_0^1 = p_0^2 = 0$, $p_1^1 = 2$, $p_1^2 = 5$ and $p_2^2 = p_2^1 = 1$. The conclusions for the two rules become $w^1 = 2x_1 + x_2$, $w^2 = 5x_1 + x_2$.

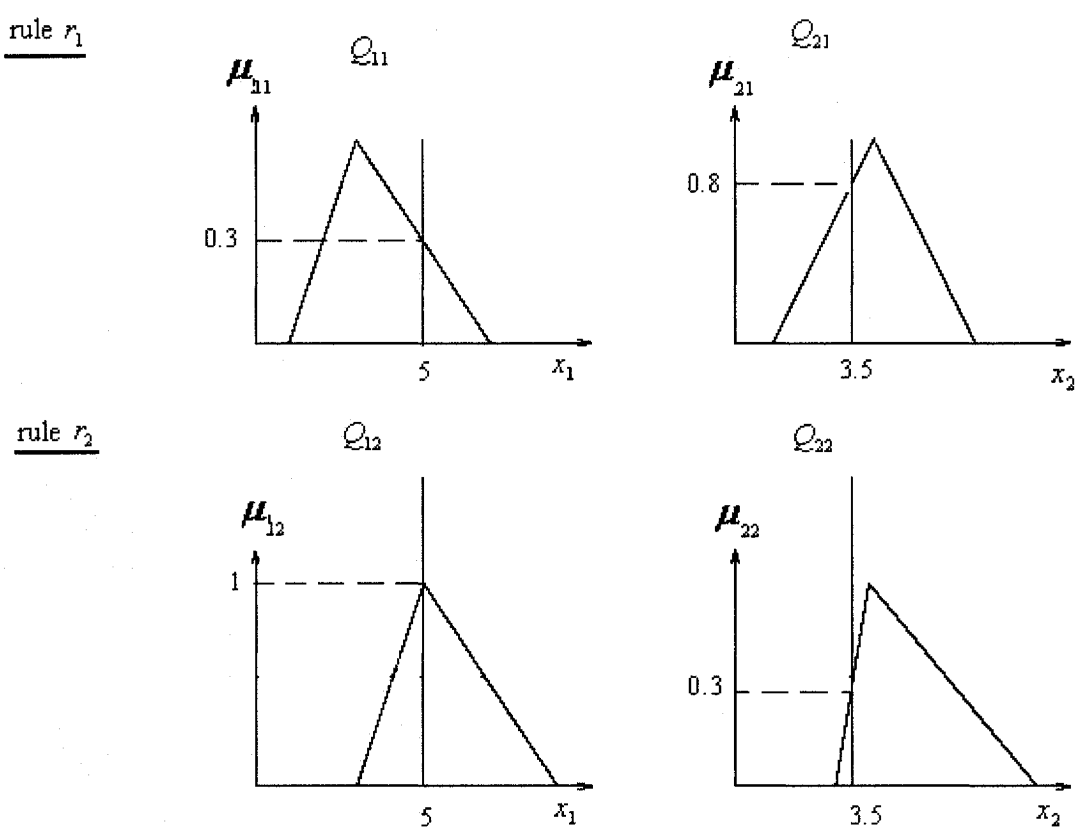


Figure 2.2 Example for TSK FLS with two antecedents

Crisp input $x_1^0 = 5$ and $x_2^0 = 3.5$ are given for this TSK system. By using minimum

($\wedge$) operator, the weights for the two rules are

$$\alpha_1 = 0.3 \wedge 0.8 = 0.3$$

$$\alpha_2 = 1 \wedge 0.3 = 0.3$$

and the crisp outputs for the two rules are

$$w_1^* = 2x_1^0 + x_2^0 = 2 \times 5 + 3.5 = 13.5$$

$$w_2^* = 5x_1^0 + x_2^0 = 5 \times 5 + 3.5 = 28.5$$

So the crisp output is w^* for the system is

$$w^* = \frac{13.5 \times 0.3 + 28.5 \times 0.3}{0.3 + 0.3} = 21$$

In this chapter, we have presented type-1 TSK FLS. Next chapter, type-2 TSK FLS will be described.

CHAPTER 3

TYPE-2 TSK INFERENCE ENGINES

This chapter describes some definition of type-2 FL, structures and inference engines of type-2 FLS. Numerical examples are given to compare type-2 system output relative to their type-1 counterpart.

3.1 Type-2 FL

Karnik and Mendel [36] provide this definition of a type-2 fuzzy set:

"A *type-2 fuzzy set* is characterized by a fuzzy membership function, *i.e.* the membership value (or membership grade) for each element of this set is a fuzzy set in [0, 1], unlike a type-1 fuzzy set where the membership grade is a crisp number in [0, 1]."

The characterization in this definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first order approximation to uncertainty and, therefore, type-2 fuzzy sets provides a second order approximation. They play an important role in modeling uncertainties that exist in fuzzy logic systems [17, 45], and are becoming increasingly important in the goal of "Computing with Words" [46] and "Computational Theory of Perceptions" [47].

An example of a type-2 *principal MF* is the Gaussian MF depicted in Figure 3.1, whose vertices have been assumed to vary over some interval of value. The *footprint of uncertainty* (FOU) associated with this type-2 MF is a bounded shaded region in Figure 3.1. FOU represents the entire interval type-2 fuzzy set $\tilde{Q}$. *Upper MF* and *Lower MF* are two type-1 MFs that are bounds for the FOU of a type-2 set $\tilde{Q}$. The intersections of crisp input x^0 shows there are *lower MF degree* $\underline{\mu}$ and *upper MF degree* $\bar{\mu}$ with respective lower and upper MFs.

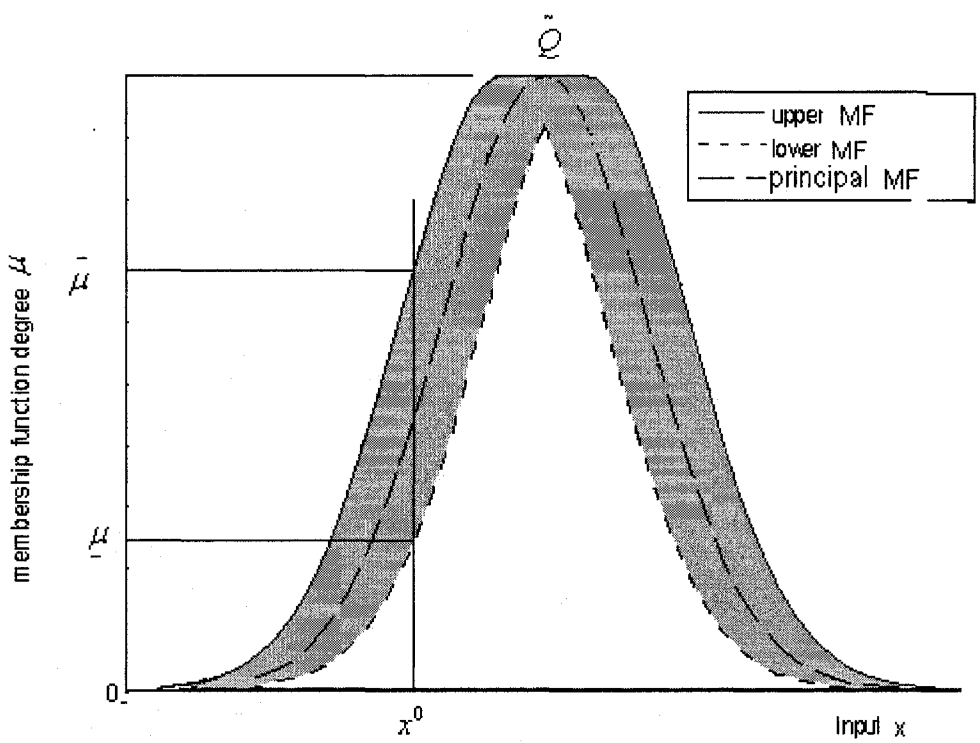


Figure 3.1 Type-2 Gaussian MF

The complete type-2 FL theory is summarized in Mendel's book "Uncertain Rule-Based Fuzzy Logic Systems – Introduction and New Directions" [4].

3.2 Principles

A generalized kth rule in the first-order type-2 TSK fuzzy MISO system [43] with a rule base of m rules, each having n antecedents, the rule r_k can be expressed as eq. (3.1) instead of eq. (2.6) in type-1 TSK fuzzy MISO system, *i.e.*,

$$r_k: \textbf{IF} \quad x_1 \; is \; \tilde{Q}_{1k} \text{ and } x_2 \; is \; \tilde{Q}_{2k} \textbf{ and } \ldots \textbf{ and } x_n \; is \; \tilde{Q}_{nk},$$

$$\textbf{THEN} \quad Z \; is \; w^k = \tilde{p}_0^k + \tilde{p}_1^k x_1 + \tilde{p}_2^k x_2 + \ldots + \tilde{p}_n^k x_n \tag{3.1}$$

where $\tilde{p}_0^k, \tilde{p}_1^k, \ldots, \tilde{p}_n^k$ are consequent type-1 fuzzy sets, while w^k is the consequent of the kth IF-THEN rule. Moreover, $\tilde{Q}_{1k}, \tilde{Q}_{2k}, \ldots, \tilde{Q}_{nk}$ are fuzzy sets on universe of discourses *X1, X2, ..., Xn.* $k \in [1, m]$, where m is the total number of rules.

For the most general model of type-2 TSK FLS, antecedents are type-2 fuzzy sets and consequents are type-1 fuzzy sets, then consequent parameter $\tilde{p}_0^k, \tilde{p}_1^k, \ldots, \tilde{p}_n^k$ are assumed as convex and normal type-1 fuzzy number subsets of the real line, so that they are fuzzy number., *i.e.*,

$$\tilde{p}_j^k = \left[c_j^k - s_j^k, c_j^k + s_j^k \right] \tag{3.2}$$

where c_j^k denotes the centre (mean) of $\tilde{p}_j^k$, and s_j^k denotes the spread of fuzzy number $\tilde{p}_j^k$. $j \in [0, n]$, where j is the total number of rules.

Similar to that shown in Figure 3.1, MF degree $\tilde{\mu}_{1k}, \tilde{\mu}_{2k}, \ldots, \tilde{\mu}_{nk}$ are interval set, *i.e.*

$$\tilde{\mu}_{1k} = \left[\underline{\mu}_{1k}, \bar{\mu}_{1k} \right] \qquad k = 1, \ldots, m$$

$$\tilde{\mu}_{2k} = \left[\underline{\mu}_{2k}, \bar{\mu}_{2k} \right] \qquad k = 1, \ldots, m \tag{3.3}$$

$$\vdots$$

$$\tilde{\mu}_{nk} = \left[\underline{\mu}_{nk}, \bar{\mu}_{nk} \right] \qquad k = 1, \ldots, m$$

Mendel in his book [4] describes the inference computation of type-2 TSK FLS which is as same as type-1 TSK FLS that no defuzzification is needed.

The *firing set* [4] alters the consequent set for a fired rule in a singleton type-2 FLS. It conveys the uncertainties of the antecedents to the consequent set. The total firing set f^k for the rule r_k is interval type-1 set, *i.e.* $f^k = [\underline{f}^k, \bar{f}^k]$. The explicit dependence of f^k can be computed as:

$$\underline{f}^k = \underline{\mu}_{1k}(x_1) \cap \underline{\mu}_{2k}(x_2) \cap \ldots \cap \underline{\mu}_{nk}(x_n)$$

$$\bar{f}^k = \bar{\mu}_{1k}(x_1) \cap \bar{\mu}_{2k}(x_2) \cap \ldots \cap \bar{\mu}_{nk}(x_n) \tag{3.4}$$

The consequent w^k of the rule r_k is a type-1 fuzzy set because it is a linear combination of type-1 fuzzy sets [4]. It is also an interval set, *i.e.*,

$$w^k = \left[w_l^k, w_r^k\right] \tag{3.5}$$

where w_l^k and w_r^k are its two end-points, while w_l^k is the consequent of a type-1 TSK FLS, whose antecedent MF are the lower MFs of the type-2 TSK FLS. Moreover, w_r^k is the consequent of a type-1 TSK FLS whose antecedent MF are the upper MFs of the type-2 TSK FLS and w_l^k and w_r^k can be computed as

$$w_l^k = \sum_{i=1}^{p} c_i^k x_i + c_0^k - \sum_{i=1}^{p} s_i^k |x_i| - s_0^k$$

$$w_r^k = \sum_{i=1}^{p} c_i^k x_i + c_0^k + \sum_{i=1}^{p} s_i^k |x_i| + s_0^k \tag{3.6}$$

$\tilde{w}$ is the *extended output* of a type-2 TSK FLS. It reveals the uncertainty of the output of a type-2 TSK FLS due to antecedent or consequent parameter uncertainties. The interval set of total output $\tilde{w}$ for *m* rules of the system (3.1) is obtained by applying the Extension Principle [5, 28].. When interval type-2 fuzzy sets are used for the antecedents, and interval type-1 fuzzy sets are used for the consequent sets of a type-2 TSK rules [40-42], $\tilde{w}$ can be obtained by following equation:

$$\tilde{w} = [w_l, w_r] = \int_{w^1 \in [w_l^1, w_r^1]} \cdots \int_{w^n \in [w_l^n, w_r^n]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^n \in [\underline{f}^n, \bar{f}^n]} 1 \Big/ \frac{\sum_{k=1}^{m} f^k w^k}{\sum_{k=1}^{m} f^k} \tag{3.7}$$

Hence $\tilde{w} = [w_l, w_r]$ is an interval type-1 set. To compute $\tilde{w}$, its two end-points w_l and w_r must be computed. Let the value of f^k and w^k that are associated with w_l be denoted f_l^k and w_l^k, respectively, and those associated with w_r be denoted f_r^k and w_r^k. The two endpoints w_l and w_r can be obtained as follows:

$$w_l = \frac{\sum_{k=1}^{m} f_l^k w_l^k}{\sum_{k=1}^{m} f_l^k}$$

$$w_r = \frac{\sum_{k=1}^{m} f_r^k w_r^k}{\sum_{k=1}^{m} f_r^k} \tag{3.8}$$

To compute w_l and w_r, f_l^k and f_r^k have to be determined. w_l and w_r can be obtained by using the iterative procedure proposed by Karnik and Mendel [36]. Here, the computation procedure is briefly provided.

Without loss of generality, assume that the pre-computed w_r^k are arranged in ascending order; $w_r^1 \leq w_r^2 \leq \ldots \leq w_r^m$, then,

Step 1: Compute w_r in (3.8) by initially setting $f_r^k = (\underline{f}^k + \bar{f}^k)/2$ for $k = 1, \ldots,$ m, where

$\underline{f}^k$ and $\bar{f}^k$ have been previously computed using (3.4), respectively, and let

$w_r^{'} \equiv w_r$.

Step 2: Find R $(1 \leq R \leq m-1)$ such that $w_r^R \leq w_r^{'} \leq w_r^{R+1}$.

Step 3: Compute w_r in (3.8) with $f_r^k = \underline{f}^k$ for $k \leq R$ and $f_r^k = \bar{f}^k$ for $k > R$, and

let $w_r^{''} \equiv w_r$.

Step 4: If $w_r^{''} \neq w_r^{'}$, then go to Step 5. If $w_r^{''} = w_r^{'}$, then stop and set $w_r^{''} \equiv w_r$.

Step 5: Set $w_r^{'} = w_r^{''}$, and return to Step 2.

The procedure for computing w_l is very similar to the one just given for w_r. Replace w_r^k by w_l^k , and , in Step 2 find L $(1 \leq L \leq m-1)$ such that $w_l^L \leq w_l^{'} \leq w_l^{L+1}$. Additionally, in Step 3, compute w_l in (3.8) with $f_l^k = \underline{f}^k$ for $k \leq L$ and $f_l^k = \bar{f}^k$ for $k > L$.

In an interval type-2 TSK FLS, output $\tilde{w}$ is an interval type-1 fuzzy set, so the crisp output w^* of any interval type-2 TSL FLS can be obtained by using the average value of w_l and w_r. Hence, the crisp output of type-2 TSK FLS is

$$w^* = \frac{w_l + w_r}{2} \tag{3.9}$$

3.3 Structures of type-2 TSK models

We can classify TSK models by using Table 3.1 according to antecedent MFs type and consequent parameters type of rules, where A donates Antecedent, C donates Consequent, 1 donates type-1 and 2 donates type-2. There are four structures for TSK FLS model because of the possible type-1 or type-2 natures of the antecedent memberships and consequent parameters for their rules: one for type-1 TSK model and three for type-2 model.

Table 3.1 Classification of TSK FLSs

TSK model type		Antecedent MF type	
		type-1 fuzzy sets (A1)	type-2 fuzzy sets (A2)
Consequent parameter type	type-1 fuzzy sets (C1)	Type-2 Model III (A1-C1)	Type-2 Model I (A2-C1)
	crisp numbers (C0)	Type-1 (A1-C0)	Type-2 Model II (A2-C0)

The three type-2 TSK structures [43] and their different representative of the symbols in eq.(3.1) are shown is Table 3.2.

Table 3.2 Structures of type-2 TSK FLSs

Type-2 TSK FLS		Model I (A2-C1)	Model II (A2-C0)	Model III (A1-C1)
structure	Antecedent	type-2 fuzzy sets	type-2 fuzzy sets	type-1 fuzzy sets
	consequent	type-1 fuzzy sets	crisp numbers	type-1 fuzzy sets
Representative of the symbols in general expression	$\underset{\sim}{p}_0^k, \underset{\sim}{p}_1^k, \ldots, \underset{\sim}{p}_n^k$	fuzzy numbers	crisp numbers	fuzzy numbers
	$\underset{\sim}{w}^k$	type-1 fuzzy set	crisp number	type-1 fuzzy set
	$\tilde{Q}_{1k}$, $\tilde{Q}_{2k}$, …, $\tilde{Q}_{nk}$	type-2 fuzzy sets	type-2 fuzzy sets	type-1 fuzzy sets

3.3.1 Type-2 TSK FLS - Model I

As shown in Table 3.2, Model I (A2-C1) is the most general case of type-2 TSK FLS. Its generalized *k*th rule is expressed as eq. (3.1), and antecedent MFs $\tilde{Q}_{1k}$, $\tilde{Q}_{2k}$, …, $\tilde{Q}_{nk}$ are type-2 fuzzy sets, consequent parameters $\underset{\sim}{p}_0^k, \underset{\sim}{p}_1^k, \ldots, \underset{\sim}{p}_n^k$ are fuzzy numbers and consequent $\underset{\sim}{w}^k$ is type-1 fuzzy sets.

Let us consider the following set of Mendel fuzzy rules that describe the behavior of a first-order type-2 TSK fuzzy system with two rules and two antecedents:

r_1 : **if** x_1 *is* $\tilde{Q}_{11}$ and x_2 *is* $\tilde{Q}_{21}$, **then** $w^1 = \tilde{p}_0^1 + \tilde{p}_1^1 x_1 + \tilde{p}_2^1 x_2$

r_2 : **if** x_1 *is* $\tilde{Q}_{12}$ and x_2 *is* $\tilde{Q}_{22}$, **then** $w^2 = \tilde{p}_0^2 + \tilde{p}_1^2 x_1 + \tilde{p}_2^2 x_2$

Observation: x_1^0, x_2^0

Conclusion: $w^* = combine(w_1^*, w_2^*)$ (3.10)

where $\tilde{Q}_{11}$, $\tilde{Q}_{21}$, $\tilde{Q}_{12}$, $\tilde{Q}_{22}$ are defined as in Figure 3.2, and $\tilde{p}_1^1 = [a_1 - \alpha_1, a_1 + \alpha_1]$, $\tilde{p}_2^1 = [b_1 - \beta_1, b_1 + \beta_1]$, $\tilde{p}_1^2 = [a_2 - \alpha_2, a_2 + \alpha_2]$, $\tilde{p}_2^2 = [b_2 - \beta_2, b_2 + \beta_2]$, $\tilde{p}_0^1 = [k_1 - \delta_1,$ $k_1 + \delta_1]$, $\tilde{p}_0^2 = [k_2 - \delta_2, k_2 + \delta_2]$, with a_1, b_1, k_1, a_2, b_2, k_2 and α_1, α_2, $\beta_1, \beta_2, \delta_1, \delta_2$ denotes the centre (mean) and the spread of fuzzy numbers $\tilde{p}_1^1$, $\tilde{p}_2^1$, $\tilde{p}_0^1$, $\tilde{p}_1^2$, $\tilde{p}_2^2$, $\tilde{p}_0^2$, respectively.

As shown in Figure 3.2 and using the Mendel approach with crisp input x_1^0 and x_2^0, the computation of the inference engine is done through the following five steps:

1) Determine the explicit dependence of the total firing interval for each rule.

Suppose the firing strengths interval sets for the two rules are $f^1 = [\underline{f}^1, \bar{f}^1]$

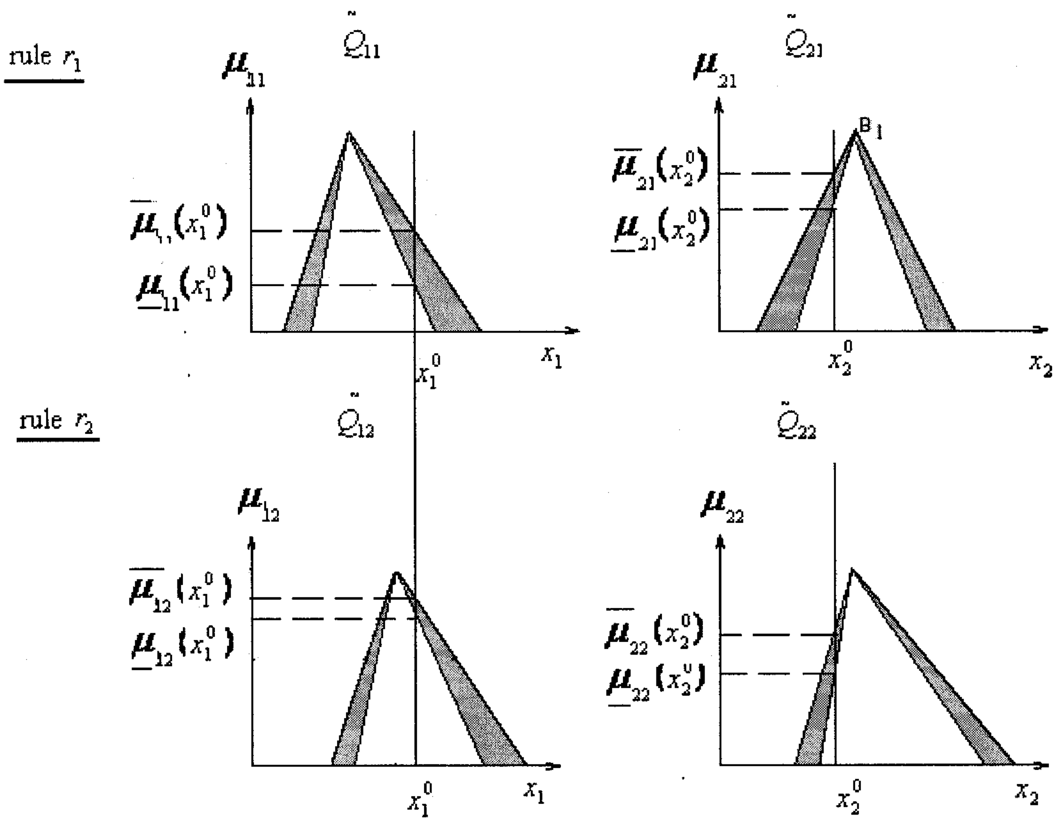


Figure 3.2 Type-2 TSK FLS - Model I with two antecedents

and $f^2 = [\underline{f}^2, \bar{f}^2]$

for r_1:

$$\underline{f}^1 = \underline{\mu}_{11}(x_1^0) \cap \underline{\mu}_{21}(x_2^0)$$

$$\bar{f}^1 = \bar{\mu}_{11}(x_1^0) \cap \bar{\mu}_{21}(x_2^0) \tag{3.11}$$

for r_2:

$$\underline{f}^2 = \underline{\mu}_{12}(x_1^0) \cap \underline{\mu}_{22}(x_2^0)$$

$$\bar{f}^2 = \bar{\mu}_{12}(x_1^0) \cap \bar{\mu}_{22}(x_2^0) \tag{3.12}$$

2) Obtain the interval value of the consequents for each rule; *i.e.*

$$w^1 = \left[w_l^1, w_r^1\right],\ w^2 = \left[w_l^2, w_r^2\right] \tag{3.13}$$

The lower and upper value for the consequents w^1 and w^2 are shown below.

For r_1:

$$w_l^{\,1} = a_1 x_1^0 + b_1 x_2^0 + k_1 - \alpha_1\left|x_1^0\right| - \beta_1\left|x_2^0\right| - \delta_1$$

$$w_r^{\,1} = a_1 x_1^0 + b_1 x_2^0 + k_1 + \alpha_1\left|x_1^0\right| + \beta_1\left|x_2^0\right| + \delta_1 \tag{3.14}$$

For r_2:

$$w_l^{\,2} = a_2 x_1^0 + b_2 x_2^0 + k_2 - \alpha_2\left|x_1^0\right| - \beta_2\left|x_2^0\right| - \delta_2$$

$$w_r^{\,2} = a_2 x_1^0 + b_2 x_2^0 + k_2 + \alpha_2\left|x_1^0\right| + \beta_2\left|x_2^0\right| + \delta_2 \tag{3.15}$$

3) Obtain the two end-points, w_l and w_r, of total output interval set w

For w_l, the interval value for firing strength is

$$f_l^1 = \bar{f}^1 \ , \ f_l^2 = \underline{f}^2 \tag{3.16}$$

For w_r, the interval value for firing strength is

$$f_r^1 = \underline{f}^1 , \ f_r^2 = \bar{f}^2 \tag{3.17}$$

4) the two end-points w_l and w_r, of total output interval set w can be obtained as:

$$w_l = \frac{\bar{f}^1 w_l^1 + \underline{f}^2 w_l^2}{\bar{f}^1 + \underline{f}^2}$$

$$w_r = \frac{\bar{f}^2 w_r^2 + \underline{f}^1 w_r^1}{\bar{f}^2 + \underline{f}^1} \tag{3.18}$$

5) Obtain the conclusion crisp output w^* as average value of w_l and w_r

$$w^* = \frac{w_l + w_r}{2} \tag{3.19}$$

Numerical Example 3.1: Type-2 TSK FLS - Model I with two antecedents

Two fuzzy rules describe the behavior of a fuzzy system with two antecedents, its MFs $\tilde{Q}_{11}$, $\tilde{Q}_{21}$, $\tilde{Q}_{12}$, $\tilde{Q}_{22}$ are defined as in Figure 3.3. The consequents for the two rules are

$w^1 = (2 \pm 0.2)x_1 + (1 \pm 0.1)x_2 + 3 \pm 0.3$ and $w^2 = (5 \pm 0.5)x_1 + (1 \pm 0.1)x_2$.

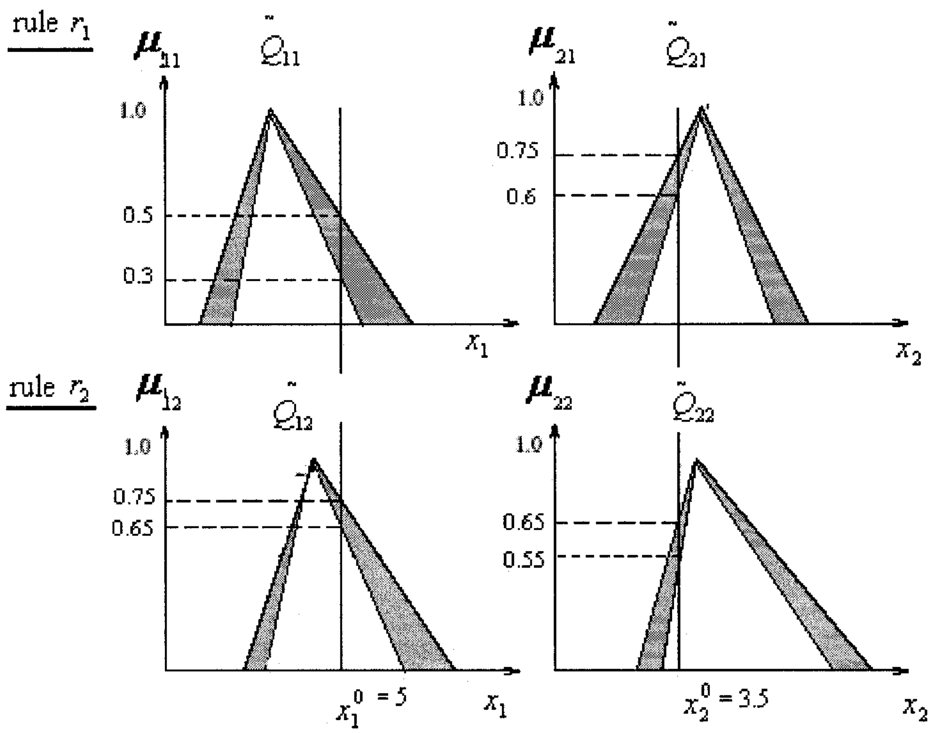


Figure 3.3 - Example for Type-2 TSK FLS - Model I with two antecedents

We assume crisp input as $x_1^0 = 5$ and $x_2^0 = 3.5$. Using product $(*)$ t-norm operator, the endpoints of firing interval for each rule can be obtained as:

$\underline{f}^1 = 0.3 \times 0.6 = 0.18$ $\qquad$ $\bar{f}^1 = 0.5 \times 0.75 = 0.375$

$\underline{f}^2 = 0.65 \times 0.55 = 0.3575$ $\qquad$ $\bar{f}^2 = 0.75 \times 0.65 = 0.4875$

so for r_1:

$$w_l^1 = 2\times5 + 1\times3.5 + 3 - 0.2\times5 - 0.1\times3.5 - 0.3 = 14.85$$

$$w_r^1 = 2\times5 + 1\times3.5 + 3 + 0.2\times5 + 0.1\times3.5 + 0.3 = 18.45$$

and for r_2:

$$w_l^2 = 5\times5 + 1\times3.5 - 0.5\times5 - 0.1\times3.5 = 25.65$$

$$w_r^2 = 5\times5 + 1\times3.5 + 0.5\times5 + 0.1\times3.5 = 31.35$$

So the endpoints for total output are

$$w_l = \frac{0.375\times14.85 + 0.3575\times25.65}{0.375 + 0.3575} = 20.1210$$

$$w_r = \frac{0.4875\times31.35 + 0.18\times18.45}{0.4875 + 0.18} = 27.8713$$

The crisp output becomes

$$w^* = \frac{w_l + w_r}{2} = \frac{20.1210 + 27.8713}{2} = 23.9961$$

3.3.2 Type-2 TSK FLS Model II

Model II (A2-C0) is a special case of Model I. The generalized *k*th rule is expressed as eq.(3.21), and the antecedents $\tilde{Q}_{1k}$, $\tilde{Q}_{2k}$, …, $\tilde{Q}_{nk}$ are type-2 fuzzy sets. Consequent parameters $p_0^k, p_1^k, p_2^k, \ldots, p_n^k$ are certain numbers and consequent w^k is crisp numbers (type-0 sets).

$$r_k: \textbf{IF} \quad x_1 \textit{ is } \tilde{Q}_{1k} \text{ and } x_2 \textit{ is } \tilde{Q}_{2k} \textbf{ and } \ldots \textbf{ and } x_n \textit{ is } \tilde{Q}_{nk},$$

$$\textbf{THEN } Z \textit{ is } w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \ldots + p_n^k x_n \tag{3.20}$$

In this case, the value of the consequent w_k for the *k*th rules

$$w^k = w_l^k = w_r^k \tag{3.21}$$

<u>Numerical Example 3.2</u>: Type-2 TSK FLS Model II

A two fuzzy rules that describe the behavior of a fuzzy system, their antecedent MFs $\tilde{Q}_{11}$, $\tilde{Q}_{21}$, $\tilde{Q}_{12}$, $\tilde{Q}_{22}$ are defined as in Figure 3.3. The consequent of the two rules are $w^1 = 2x_1 + x_2$ and $w^2 = 5x_1 + x_2$

We assume crisp input are $x_1^0 = 5$ and $x_2^0 = 3.5$. Using product $(*)$ t-norm operator, the endpoints of interval value of firing strength for both rules are

$$\underline{f}^1 = 0.3 \times 0.6 = 0.18 \qquad \bar{f}^1 = 0.5 \times 0.75 = 0.375$$

$$\underline{f}^2 = 0.65 \times 0.55 = 0.3575 \qquad \bar{f}^2 = 0.75 \times 0.65 = 0.4875$$

For r_1 and r_2, their outputs are certain number because of consequent parameters are constant numbers.

$$w^1 = w_l^1 = w_r^1 = 2 \times 5 + 1 \times 3.5 = 13.5$$

$$w^2 = w_l^2 = w_r^2 = 5 \times 5 + 1 \times 3.5 = 28.5$$

The total output endpoints are

$$w_l = \frac{0.375 \times 13.5 + 0.3575 \times 28.5}{0.375 + 0.3575} = 20.8208$$

$$w_r = \frac{0.4875 \times 28.5 + 0.18 \times 13.5}{0.4875 + 0.18} = 24.4550$$

The output becomes

$$w^* = \frac{w_l + w_r}{2} = \frac{20.8208 + 24.4550}{2} = 22.6379$$

3.3.3 Type-2 TSK FLS Model III

Model III (A1-C1) is a special case of Model II. The generalized *k*th rule is expressed as eq.(3.24), the antecedents Q_{1k}, Q_{2k}, …, and Q_{nk} are type-1 fuzzy sets. Consequent parameters $\tilde{p}_0^k, \tilde{p}_1^k, \ldots, \tilde{p}_n^k$ are fuzzy numbers and consequent $\tilde{w}^k$ is type-1 fuzzy sets.

$$r_k:\ \textbf{IF}\quad x_1\ is\ Q_{1k}\ \text{and}\ x_2\ is\ Q_{2k}\ \textbf{and} \ldots \textbf{and}\ x_n\ is\ Q_{nk},$$

$$\textbf{THEN}\ \ Z\ is\ \tilde{w}^k = \tilde{p}_0^k + \tilde{p}_1^k x_1 + \tilde{p}_2^k x_2 + \ldots + \tilde{p}_n^k x_n \tag{3.22}$$

Its output is same as the output of a type-1 TSK FLS whose consequent parameters are the centers of the consequent sets of the interval type-2 TSK FLS Model III. They both provide identical results, but Model III also provides additional information about the uncertainties that associated with the output. The interval value of output w^* can be obtained as:

$$w^* = [w_l, w_r] \tag{3.23}$$

Numerical Example 3.3: Type-2 TSK FLS Model III

A two fuzzy rules that describe the behavior of a fuzzy system, their antecedent MFs Q_{11}, Q_{21}, Q_{12} and Q_{22} are defined as in Figure 2.2. The consequents are $w^1 = (2 \pm 0.2)x_1 + (1 \pm 0.1)x_2$ and $w^2 = (5 \pm 0.5)x_1 + (1 \pm 0.1)x_2$.

We assume crisp input are $x_1^0 = 5$ and $x_2^0 = 3.5$. Using minimum $(\wedge)$ t-norm operator, the firing strengths of both two rules are constant, as that of type-1 TSK FLS in example 2.1.

$$f^1 = 0.3 \wedge 0.8 = 0.3$$

$$f^2 = 1 \wedge 0.3 = 0.3$$

The output endpoints for each rule are

$$w_l^1 = 2\times5 + 1\times3.5 - 0.2\times5 - 0.1\times3.5 = 12.15$$

$$w_r^1 = 2\times5 + 1\times3.5 + 0.2\times5 + 0.1\times3.5 = 14.85$$

and

$$w_l^2 = 5\times5 + 1\times3.5 - 0.5\times5 - 0.1\times3.5 = 27.15$$

$$w_r^2 = 5\times5 + 1\times3.5 + 0.5\times5 + 0.1\times3.5 = 29.85$$

The endpoints of the interval set of output $\tilde{w} = [w_l, w_r]$ can be obtained as

$$w_l = \frac{0.3\times12.15 + 0.3\times27.15}{0.3 + 0.3} = 19.65$$

$$w_r = \frac{0.3 \times 29.85 + 0.3 \times 14.85}{0.3 + 0.3} = 22.35$$

The crisp output is the average value of w_l and w_r, *i.e.*,

$$w^* = \frac{w_l + w_r}{2} = \frac{19.65 + 14.85}{2} = 21$$

Comparing example 3.3 with the example 2.1 in which the consequent parameters of the type-1 TSK FLS are the centers of the consequent sets of the interval A1-C1 type-2 TSK FLS, both of them have the same crisp output $w^* = 21$, but example 3.3 provides more information, that is the interval value of output fuzzy set $\tilde{w} = [w_l, w_r] = [19.65, 22.35]$.

Consequently, when the output of an interval Model III is the same as the output of a type-1 TSK FLS, a type-1 TSK FLS can be used directly instead of interval type-2 TSK FLS - Model III if only the defuzzified output of interval type-2 TSK FLS - Model III is interested, because both of them provide identical results. If the extended output is also interested, just Model III will be used because it is the only one that can provide this information.

In this chapter, we have introduced type-2 TSK FLS by means of simple examples. Next, subtractive clustering based type-1 TSK FLS is described in detail in order to analyze the uncertainties in its identification algorithm.

CHAPTER 4

SUBTRACTIVE CLUSTERING BASED TYPE-1 TSK FLS

Modeling of system behaviour has been a challenging problem in various disciples. In this chapter, a type-1 TSK modeling method in which subtractive clustering method is combined with a least-square estimation algorithm is explained in details because we propose an extension of it latter in this thesis.

4.1 Generality

The qualitative model is the one that describes the system behavior through linguistic explications. Even when the model developed by an expert is able to represent the complete behavior of a system, it becomes expensive to develop another model with slight changes in the requirement. Moreover, the model may also lose its versatility and fails to accommodate new situations. Alternatively, it is more appropriate in such situation to evolve the system modeling using input and output data of the real system. The data can be obtained using perfect mathematical modeling or from expert's knowledge or form experimental analysis of the system. The system identification can be done with a clustering technique which does grouping of data into clusters of similar behavior and evaluates the system behaviors from these clusters. Since these data can also contain uncertainty and vagueness, a suitable approach is thus essential in order to not miss any important information about the system.

The generality of TSK models makes the *data driven identification* very complex. A fuzzy model consists of multiple rules, each rule containing a premise part and a consequent part. The premise part specifies a certain input subspace by a conjunction of fuzzy clauses that contain the input variables. The consequent part is a linear regression model. The identification task includes two subtasks: *structure identification* and *parameter identification.* The former is the determination of *the* number of rules and the variables involved in the rule premises, while the latter is the estimation of the membership function parameters and the estimation of the consequent regression coefficients.

4.2 Identification of fuzzy type-1 TSK model

The TSK FLSs is a more compact and computationally efficient representation than a Mamdani FLSs. It lends itself to the use of adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership functions so that the fuzzy system best models the data.

The identification of fuzzy TSK model is usually done in two steps [26, 48]. First, the structure of the model needs to be discovered. During this step, fuzzy sets (membership functions) in the rule antecedents need to be determined. In the second step, parameters of consequent functions are estimated.

4.2.1 Cluster estimation

The structure of fuzzy TSK model can be done manually based on knowledge about process or using some data-driven techniques, e.g., adaptive neuro-fuzzy inference systems

(ANFIS) [49, 50] and fuzzy clustering [26, 51]. Separate method is needed to discover the number of membership functions for each fuzzyfied variable.

The identification of the system using clustering involves formation of clusters in the data space and translation of these clusters into TSK rules such that the model obtained is close to the system to be identified. As part of the structure identification, the reasoning mechanism [23] is optimized in order to get the closest model. The tuning of fuzzy sets after structure identification is done as part of the parameter identification [16]. The fuzzy c-means (FCM) clustering algorithm [21, 22 and 52] has been widely studied and applied. Subtractive clustering algorithm has been proposed by Chiu [2, 3]. It is an extension of the grid-based *mountain clustering method* earlier introduced by Yager and Filev [18]. It is computationally simple and gives better distribution of cluster centers in comparison with the mountain method [18].

Subtractive clustering method is a fast clustering method designed for high dimension problem with moderate number of data points, because its computation grows linearly with the data dimension and as the square of the number of data points. Subtractive clustering method can easily find fuzzy clusters to establish the number of fuzzy rules and the rule premises.

Subtractive clustering [2, 3] operates by finding the optimal data point to define a cluster center based on the density of surrounding data points. Using the symbols displayed in Table 4.1, where r_a, η, $\underline{\varepsilon}$ and $\bar{\varepsilon}$ are positive constants. It is possible to control the subtractive clustering method.

Consider a group of data points $\{x^1, x^2, ..., x^w\}$ for a specific class. The M dimensional

Table 4.1 Symbols used to control the subtractive clustering method

Symbol	Definition
x^i, x^l	*i*th and *l*th normalized data vector of both input and output dimensions in an *n* dimensional feature space, $i \in [1, w]$, $l \in [1, w]$
x_i^j	*j*th element in the vector x^i, $x_j^i = \frac{x_j^i - \min(x_j^i)}{\max(x_j^i) - \min(x_j^i)}$, $j \in [1, n]$
w	Number of data points
P_i	Potential value of data vector x^i
r_a	Hypersphere *cluster radius* in data space -- *defines a neighborhood, data points outside this radius has little influence on the potential*
r_b	Hypersphere *penalty radius* in data space -- *defines the neighborhood which will have the measurable reductions in potential*
η	Squash factor $\eta = \frac{r_b}{r_a}$
$\underline{\varepsilon}$	Reject radio -- *specifies a threshold for the potential above which we will definitely accept the data point as a cluster center*
$\bar{\varepsilon}$	Accept radio -- *specifies a threshold below which we will definitely reject the data point*

feature space is normalized so that all data are bounded by a unit hypercube. Calculate potential P_i for each point as follow:

$$P_i = \sum_{j=1}^{w} e^{-\alpha \left\| x^i - x^j \right\|^2} \tag{4.1}$$

where

$$\alpha = \frac{4}{r_a^2} \tag{4.2}$$

Thus, the measure of potential for a data point is a function of its distance to all other data points. A data point with many neighboring data points will have a high potential value. After the potential of every data computed, cluster counter $k = 1$, the data point x^i with the maximum potential $P_k^* = P_1^*$ is selected as the first cluster center $x_k^* = x_1^*$. Then revise the potential of each data point x^i by the formula

$$P_i = P_i - P_k^* e^{-\beta \left\| x^i - x^j \right\|^2} \tag{4.3}$$

where

$$\beta = \frac{4}{r_b^2} \text{ and } r_b = \eta * r_a$$

Pick up data point x' with the current maximum potential P_t as the candidate for the next cluster center. The process of acquiring new cluster center and revising potentials use the following criteria:

if $P_t > \bar{\varepsilon} P_1^*$

Accept x' as the next cluster center, cluster counter $k = k + 1$, and continue.

else if $P_t < \underline{\varepsilon} P_1^*$

Reject x' and end the clustering process

else

Let $d_{\min}$ =shortest of the distances between x' and all previously found cluster centers.

if $\frac{d_{\min}}{r_a} + \frac{P_t}{P_1^*} \geq 1$

Accept x' as the next cluster center. Cluster counter $k = k + 1$, and continue.

else

Reject x' and set $P_t = 0$.

Select the data point with the next highest potential as the new candidate cluster center and retest.

end if

end if

end if

By using standard Gaussian MF, the premise MF for *k*th rule *j*th variable can be written as:

$$Q_{jk} = \exp\left[-\frac{1}{2}\left(\frac{x_j - x^*_{jk}}{\sigma}\right)^2\right] \tag{4.4}$$

where x^*_{jk} is the *j*th input feature of *k*th cluster center x^*_k, the standard deviation σ of Gaussian MF given as

$$\sigma = \sqrt{1/2\alpha} \tag{4.5}$$

In a TSK FLS, rule premises are represented by exponential membership function. The optimal consequent parameters $p^k_0, p^k_1, p^k_2, \ldots, p^k_n$ (coefficients of the polynomial function) in eq.(2.6) for a given set of clusters are obtained by the least-square estimation method.

4.2.2 Rule parameter optimization by least-squares estimation

As consequent functions usually are linear, the least-square estimation method [53] was used to identify the consequent parameters of one TSK model [11, 12]. In the method of Sugeno and Kang [14], the premise structure, premise parameters, consequent structure, and consequent parameters were identified and adjusted recursively. Appling the least-squares estimation method can optimize the rule consequents.

For a given input u_0, the model output $w^*(u_0)$ which has m rules is computed as:

$$\begin{aligned}\varpi^*(u_0) = \beta_1(p_0^1 + p_1^1 u_{11} + p_2^1 u_{21} + \ldots + p_n^1 u_{n1}) \\ + \beta_2(p_0^2 + p_1^2 u_{12} + p_2^2 u_{22} + \ldots + p_n^2 u_{n2}) + \cdots \\ + \beta_m(p_0^m + p_1^m u_{1m} + p_2^m u_{2m} + \ldots + p_n^m u_{nm}))\end{aligned} \tag{4.6}$$

where for the kth rule with

$$\beta_k = \frac{\mu_k(u_0)}{sumMu_k} \tag{4.7}$$

and the total grade for *k*th cluster center x_k^* is

$$sumMu_k = \sum_{j=1}^{n} \mu_{jk}(u_0) \tag{4.8}$$

Consider of a group of w data points, those equations can be obtained.

$$\begin{aligned}\varpi_1^*(u_0) = \beta_{11}(p_0^1 + p_1^1 u_{11} + p_2^1 u_{21} + \ldots + p_n^1 u_{n1}) \\ + \beta_{21}(p_0^2 + p_1^2 u_{12} + p_2^2 u_{22} + \ldots + p_n^2 u_{n2}) + \cdots \\ + \beta_{m1}(p_0^m + p_1^m u_{1m} + p_2^m u_{2m} + \ldots + p_n^m u_{nm})) \\ \varpi_2^*(u_0) = \beta_{12}(p_0^1 + p_1^1 u_{11} + p_2^1 u_{21} + \ldots + p_n^1 u_{n1}) \\ + \beta_{22}(p_0^2 + p_1^2 u_{12} + p_2^2 u_{22} + \ldots + p_n^2 u_{n2}) + \cdots \\ + \beta_{m2}(p_0^m + p_1^m u_{1m} + p_2^m u_{2m} + \ldots + p_n^m u_{nm}))\end{aligned} \tag{4.9}$$

$\vdots$

$$\varpi_w^*(u_0) = \beta_{1w}(p_0^1 + p_1^1 u_{11} + p_2^1 u_{21} + ... + p_n^1 u_{n1}) \\ + \beta_{2w}(p_0^2 + p_1^2 u_{12} + p_2^2 u_{22} + ... + p_n^2 u_{n2}) + \cdots \\ + \beta_{mw}(p_0^m + p_1^m u_{1m} + p_2^m u_{2m} + ... + p_n^m u_{nm}))$$

So

$$\begin{bmatrix} \beta_{11}\mu_{11} & \beta_{11}\mu_{12}\cdots\beta_{11}\mu_{n1} & \beta_{11} & \cdot & \cdot & \cdot & \beta_{m1}\mu_{11} & \beta_{m1}\mu_{12}\cdots\beta_{m1}\mu_{n1} & \beta_{m1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{1w}\mu_{11} & \beta_{1w}\mu_{12}\cdots\beta_{1w}\mu_{n1} & \beta_{1w} & \cdot & \cdot & \cdot & \beta_{mw}\mu_{1m} & \beta_{mw}\mu_{2m}\cdots\beta_{mw}\mu_{nm} & \beta_{mw} \end{bmatrix} \begin{bmatrix} p_1^1 \\ p_2^1 \\ \vdots \\ p_n^1 \\ p_0^1 \\ \cdot \\ \cdot \\ p_1^m \\ p_2^m \\ \vdots \\ p_n^m \\ p_0^m \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ w_n \end{bmatrix} \tag{4.10}$$

where the parameter β_{ik} for the ith element in the kth cluster center x_k^* is

$$\beta_{ik} = \frac{\mu_{ik}(u_0)}{sumMu_{ik}} \tag{4.11}$$

and the total grade $sumMu_{ik}$ is calculated as

$$sumMu_{ik} = \sum_{i=1}^{w} \mu_i(u_k) \tag{4.12}$$

As pointed out by Takagi and Sugeno [11], given a set of rules with fixed promises, optimizing the parameters in the consequent equations with respect to training data reduces to a linear least-square estimation problem.

If A, X and B are used to represent the three matrix of eq.(4.10), the lather can be simplified as

$$AX=B \tag{4.13}$$

The sizes of matrix A, X and B are $w(m+1)\times n$, $1\times w(m+1)$ and $1\times n$, respectively, this is a line least-square estimation problem where A is a constant matrix (known), B is a matrix of output values, and X is a matrix of parameters to be estimated.

The well-known pseudo-inverse solution [54] that minimizes $\|AX-B\|^2$ is given by

$$X=\left(A^T A\right)^{-1} A^T B \tag{4.14}$$

The total LSE of w-rule fuzzy (first-order Sugeno) model is

$$LSE=\sum_{i=1}^{n}(W_{is}-W_{im})^2 \tag{4.15}$$

where W_{is} is the system output and W_{im} is the model output.

If LSE in eq.(4.15) is the smallest, it is the one best, so we get its cluster centers and output parameters, which mean we get the rules.

Numerical Example 4.1 : Identification of a type-1 TSK FLS

Consider the following SISO (single input single output) system

$$y = -(x - 2.5)^3 + 3.25 \qquad where \qquad x \in [0,4] \tag{4.16}$$

shown in Figure 4.1, and represented by 33 discrete data points in Table 4.2.

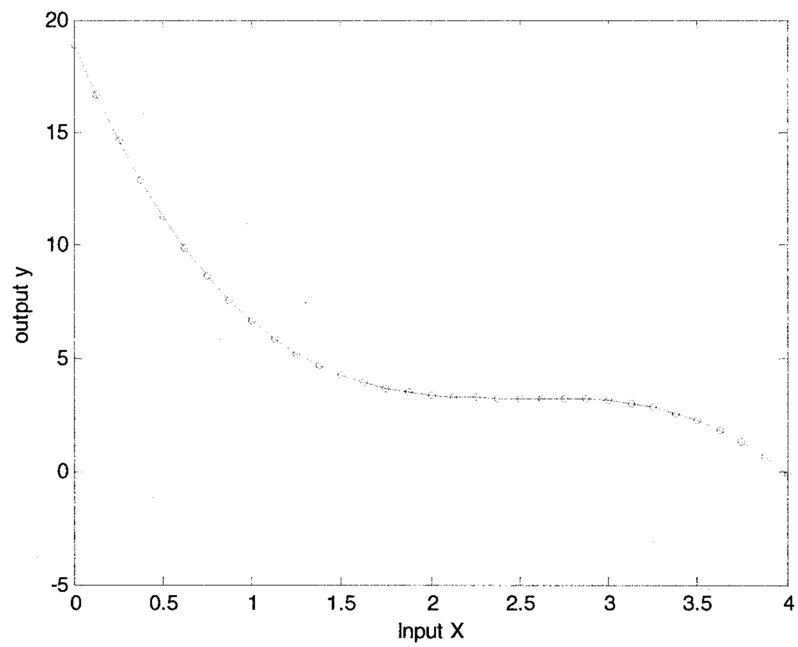


Figure 4.1 System $y = -(x - 2.5)^3 + 3.25$

By using subtractive clustering which is combined with least-square estimation method, four cluster centers can be identified correspond to four fuzzy rules as that in Table 4.3 with

Table 4.2 Data used for modeling system $y = -(x - 2.5)^3 + 3.25$

Data Point	Input	System Output	Model Output
1	0	18.8750	18.865
2	0.1250	16.6465	16.661
3	0.2500	14.6406	14.651
4	0.3750	12.8457	12.843
5	0.5000	11.2500	11.237
6	0.6250	9.8418	9.8275
7	0.7500	8.6094	8.6021
8	0.8750	7.5410	7.5452
9	1.0000	6.6250	6.6393
10	1.1250	5.8496	5.8670
11	1.2500	5.2031	5.2144
12	1.3750	4.6738	4.6721
13	1.5000	4.2500	4.2347
14	1.6250	3.9199	3.8984
15	1.7500	3.6719	3.6558
16	1.8750	3.4941	3.4934
17	2.0000	3.3750	3.3916
18	2.1250	3.3027	3.3290
19	2.2500	3.2656	3.2884
20	2.3750	3.2520	3.2591
21	2.5000	3.2500	3.2365
22	2.6250	3.2480	3.2188
23	2.7500	3.2344	3.2025
24	2.8750	3.1973	3.1783
25	3.0000	3.1250	3.130
26	3.1250	3.0059	3.0348
27	3.2500	2.8281	2.8684
28	3.3750	2.5801	2.6111
29	3.5000	2.2500	2.2533
30	3.6250	1.8262	1.8008
31	3.7500	1.2969	1.2525
32	3.8750	0.6504	0.61893
33	4.0000	-0.1250	-0.064784

r_a=0.5, $\underline{\varepsilon}$= 0.15, $\bar{\varepsilon}$= 0.5, η= 1.25 which are recommended by Chiu [2, 3]. Rule premises are represented by exponential membership functions are shown in Figure 4.2.

Table 4.3 Cluster centers for system $y = -(x-2.5)^3 + 3.25$

Cluster Center	x	y	Membership function
1	2.5	3.25	$\mu_1 = e^{-(x-2.5)^2}$
2	1	6.625	$\mu_2 = e^{-(x-1)^2}$
3	3.875	0.65039	$\mu_3 = e^{-(x-3.875)^2}$
4	0.25	14.641	$\mu_4 = e^{-(x-0.25)^2}$

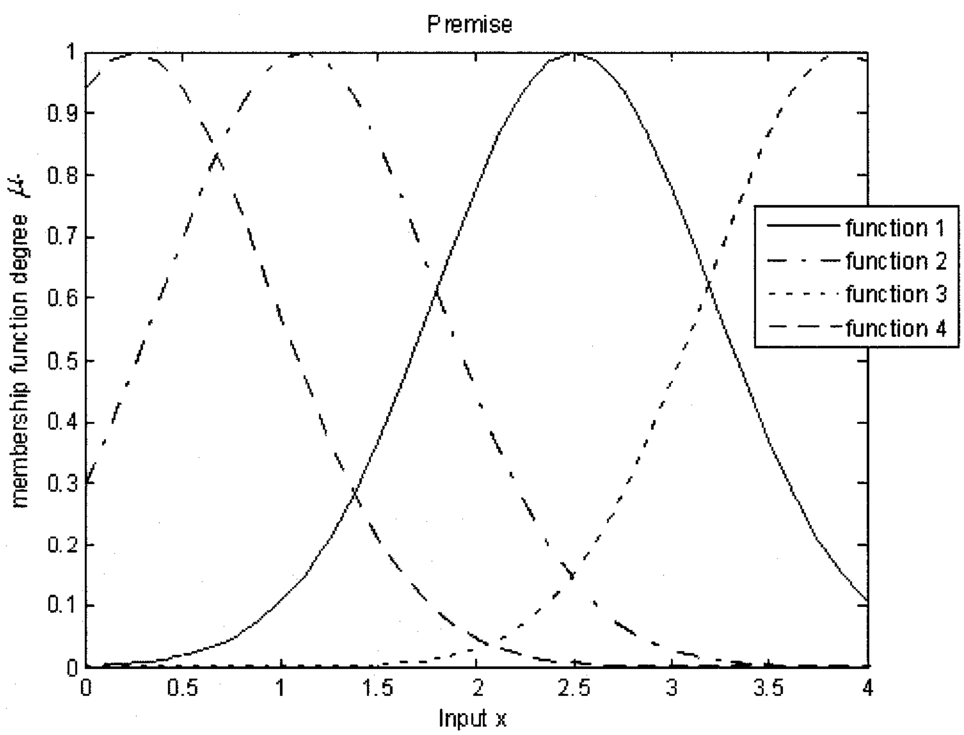


Figure 4.2 Gaussian MFs for premises

Type-1 TSK model rules are shown in Table 4.4 and comparison of the system output and model output are shown in Figure 4.3. Membership values are shown in Table 4.5.

Table 4.4 TSK model rules for system $y=-(x-2.5)^3+3.25$

Rule	If x then $y = p_1x_1 + p_2x_2 + p_0$
1	If $x=e^{-(x-2.5)^2}$, then $y=0.0823x_1+65.7967_2x_2-11.7197$
2	If $x=e^{-(x-1)^2}$, then $y=0.0834_1x_1+34.5726x_2-8.3673$
3	If $x=e^{-(x-3.875)^2}$, then $y=-0.1966x_1+11.1147x_2+22.6022$
4	If $x=e^{-(x-0.25)^2}$, then $y=0.1138x_1+47.7914x_2+47.4714$

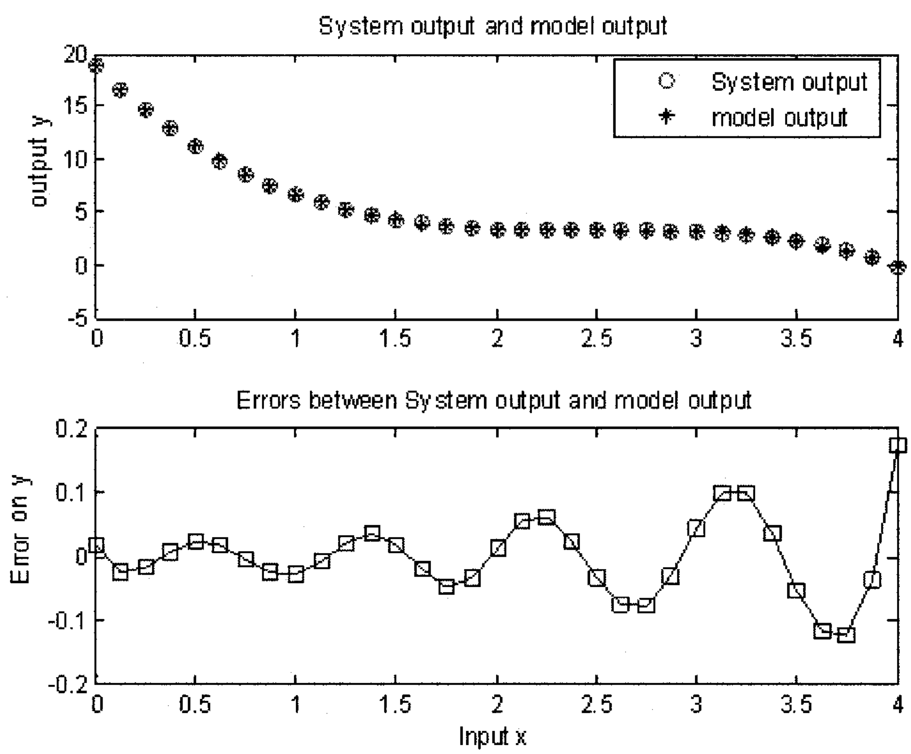


Figure 4.3 Comparison of system output and model output for $y=-(x-2.5)^3+3.25$

Table 4.5 Gaussian membership function values for rule premises

Data point	Gaussian membership function			
	$x = e^{-(x-2.5)^2}$	$x = e^{-(x-1)^2}$	$x = e^{-(x-3.875)^2}$	$x = e^{-(x-0.25)^2}$
1	0.0019302	0.29819	3.0107e-007	0.93941
2	0.0035503	0.38649	7.8094e-007	0.9845
3	0.0063291	0.48553	1.9633e-006	1
4	0.010936	0.59118	4.784e-006	0.9845
5	0.018314	0.69767	1.1298e-005	0.93941
6	0.029727	0.79801	2.5863e-005	0.86881
7	0.046768	0.8847	5.738e-005	0.7788
8	0.071313	0.95063	0.00012339	0.67663
9	0.10539	0.99005	0.00025717	0.56978
10	0.15097	0.99938	0.0005195	0.46504
11	0.20961	0.97775	0.0010171	0.36787
12	0.28206	0.92716	0.0019302	0.28206
13	0.36787	0.85214	0.0035503	0.20961
14	0.46504	0.75909	0.0063291	0.15097
15	0.56978	0.6554	0.010936	0.10539
16	0.67663	0.54846	0.018314	0.071313
17	0.7788	0.44485	0.029727	0.046768
18	0.86881	0.34971	0.046768	0.029727
19	0.93941	0.26646	0.071313	0.018314
20	0.9845	0.19678	0.10539	0.010936
21	1	0.14085	0.15097	0.0063291
22	0.9845	0.097718	0.20961	0.0035503
23	0.93941	0.065707	0.28206	0.0019302
24	0.86881	0.042823	0.36787	0.0010171
25	0.7788	0.02705	0.46504	0.0005195
26	0.67663	0.016561	0.56978	0.00025717
27	0.56978	0.0098273	0.67663	0.00012339
28	0.46504	0.0056521	0.7788	5.738e-005
29	0.36787	0.0031508	0.86881	2.5863e-005
30	0.28206	0.0017023	0.93941	1.1298e-005
31	0.20961	0.00089147	0.9845	4.784e-006
32	0.15097	0.00045248	1	1.9633e-006
33	0.10539	0.00022259	0.9845	7.8094e-007

Figure 4.4 is the type-1 TSK model error bounds for system $y = -(x-2.5)^3 + 3.25$. The LSE of the model is 0.018331.

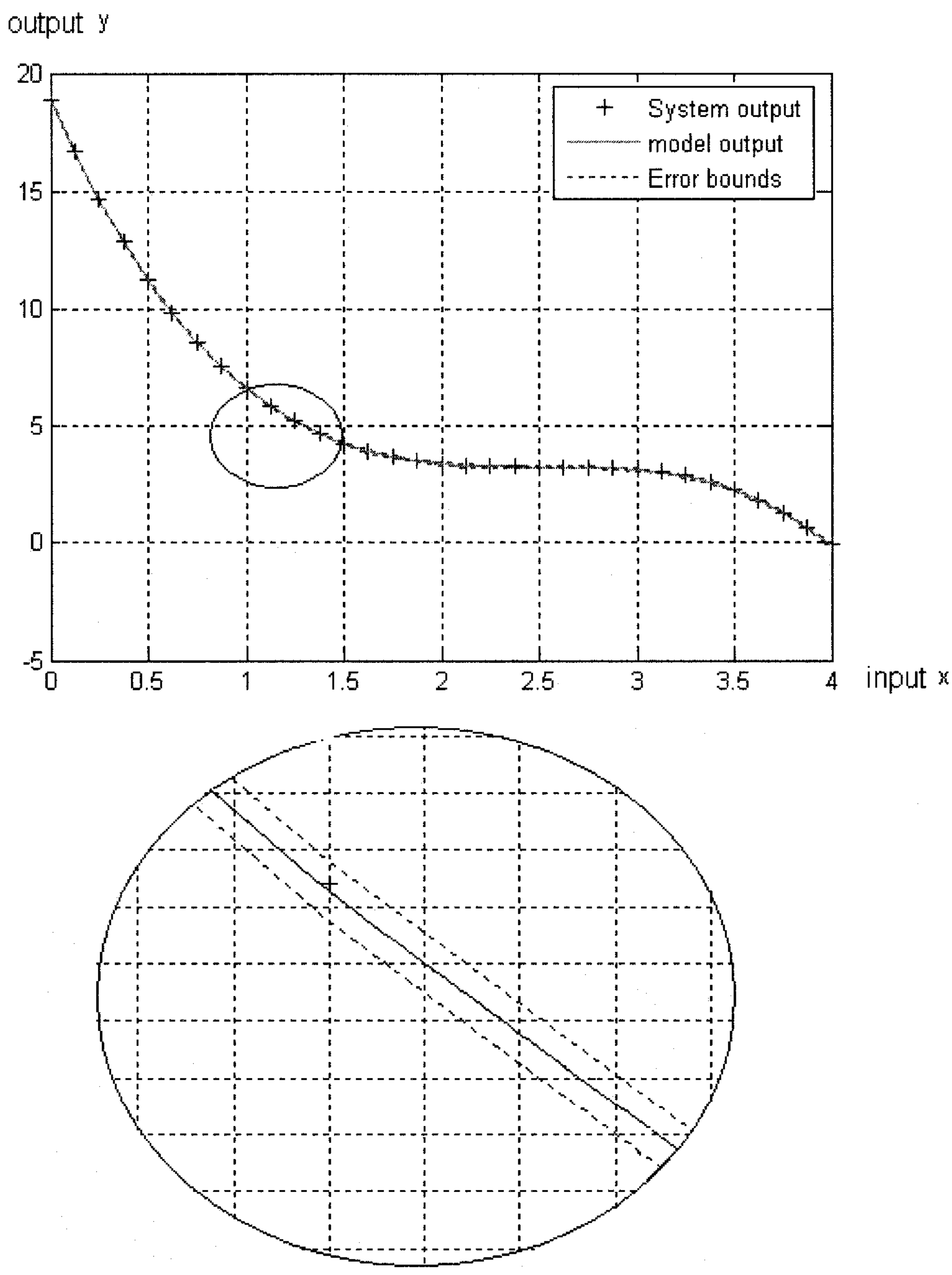


Figure 4.4 Type-1 TSK model error bounds

4.3 Uncertainties in a subtractive clustering based type-1 TSK FLS

From Mendel's book [4], there are at least four sources of uncertainties that can occur for type-1 FLSs -- uncertainty about the meanings of the words that are used in the rules; uncertainty about the consequent that is used in a rule; uncertainty about the measurements that activate the FLS; uncertainty about the data that is used to tune the parameters of an FLS. For TSK models, the uncertainties are about the meanings of words by using *precise* membership functions, the consequent that is used in a rule are extracted directly from data which is leading to a histogram of possibilities.

Based only on measured data without prior knowledge, there is no systematic way to obtain a TSK fuzzy model with a simple structure and sufficient accuracy. By using subtractive clustering, it is easy to obtain an efficient Type-1 TSK FLS. But there are still some uncertainties on the algorithm.

Subtractive clustering algorithm (Figure 4.5) has various parameters to be set. Not knowing the best parameters to be used for a given data, a parameter search is performed to identify the best model. As a result of parameter search, ranges of clustering parameters that provide the best models are also identified.

Recommended values for the four parameters form Chiu's subtractive clustering method [2, 3] and form Demirli's extended subtractive clustering method [44, 55] are listed in Table 4.6. These parameters have influence on the cluster estimation, the number of rules and error performance measures.

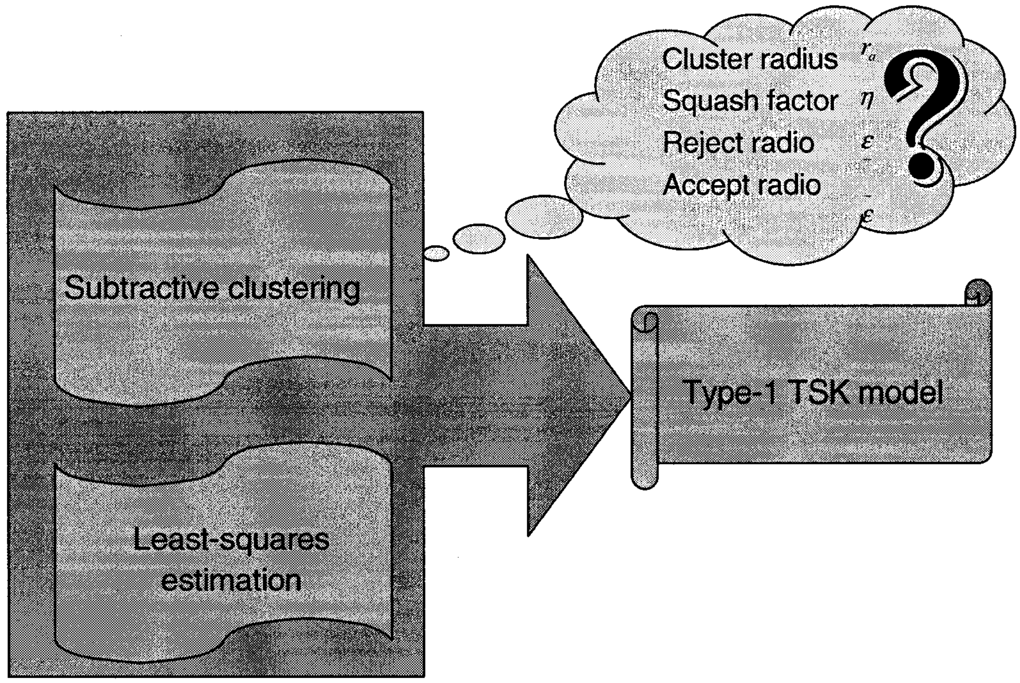


Figure 4.5 Subtractive clustering based type-1 TSK modeling

Table 4.6 Initialization of parameters for subtractive clustering

Symbol	Chiu	Demirli
Cluster radius r_a	[0.25, 0.50]	[0.15, 1]
Squash factor η	1.25	[0.05, 2]
Reject radio $\underline{\varepsilon}$	0.15	[0, 0.9]
Accept radio $\bar{\varepsilon}$	0.5	[0, 1]

In his paper [55], Demirli described in detail the influence of the value of those four parameters, especially cluster radius r_a and squash factor η. Example 4.2 explains the influence of parameter initialization to a two antecedents type-1 TSK FLS model.

Numerical example 4. 2: Two antecedents type-1 TSK FLS

Let us consider the following MISO system which is represented by 36 discrete data points in Table 4.7. The input data are in Column 1 and the system outputs are in Column 2

The identification algorithm used to obtain the TSK system in Table 4.8 is under these conditions:

1) total eight fuzzy rules ;
2) using Demirli's extended subtractive clustering method [55]; the range of the parameters r_a, η, $\underline{\varepsilon}$ and $\bar{\varepsilon}$ selected for enumerative search are given in Table 4.6 Column 3, the step size are 0.02, 0.05, 0.10 and 0.10.

Column 3 in Table 4.7 is the output compatible by using the best eight rules type-1 TSK model shown in the Table 4.8. The best eight rules type-1 TSK model in Table 4.8 is obtained when $r_a = 0.914$, $\underline{\varepsilon} = 0.2$, $\bar{\varepsilon} = 0.2$ and $\eta = 0.95$. The least LSE obtained is 0.021.

An important question is "**is this the absolute best model?**" In fact, there are around a total of 20763 TSK models with eight rules that can be obtained by using system data of Table 4.8. Among them, there are around 5546 models which have the same cluster center as the best eight rules TSK model in these two tables. Each of them has different consequent parameters.

Table 4.7 System data for type-1TSK FLS - Two antecedent case

Input System Data			Model Data
x_1	x_2	output	output
110	0.066	0.630	0.6240
91	0.066	0.730	0.7301
110	0.042	0.790	0.7546
123	0.054	0.320	0.3413
104	0.042	0.790	0.7547
97	0.048	0.760	0.7368
104	0.054	0.450	0.4140
117	0.066	0.590	0.5907
91	0.060	0.380	0.3755
110	0.036	0.900	0.8992
123	0.048	0.550	0.5278
97	0.042	0.580	0.6174
104	0.048	0.490	0.5580
123	0.066	0.540	0.5450
123	0.042	0.770	0.7972
97	0.036	0.690	0.6716
110	0.048	0.600	0.6190
117	0.054	0.430	0.4569
123	0.060	0.340	0.3253
91	0.048	0.650	0.6752
97	0.054	0.780	0.7561
104	0.060	0.340	0.3448
123	0.036	0.610	0.5892
104	0.036	1.000	1.0153
117	0.048	0.570	0.5255
91	0.042	0.620	0.5868
110	0.060	0.540	0.5572
97	0.066	1.030	1.0248
117	0.042	0.540	0.5665
91	0.036	0.830	0.8440
110	0.054	0.580	0.5668
117	0.060	0.430	0.4195
91	0.054	0.620	0.6210
97	0.060	0.690	0.7143
104	0.066	0.420	0.4203
117	0.036	0.340	0.3537

Table 4.8 Best eight rules type-1 TSK Model

Rule	Cluster Center (if)	Consequent $p_1x_1 + P_2x_2 + p_0$ (then)
1	110 0.054 0.580	$0.0823x_1 + 65.7967_2x_2 - 11.7197$
2	91 0.048 0.650	$0.0834_1x_1 + 34.5726x_2 - 8.3673$
3	110 0.036 0.900	$-0.1966x_1 + 11.1147x_2 + 22.6022$
4	123 0.060 0.340	$0.1138x_1 + 47.7914x_2 + 47.4714$
5	117 0.042 0.540	$0.0526x_1 + 37.6186x_2 - 6.8991$
6	91 0.066 0.073	$0.4206x_1 + 100.2997x_2 - 43.2717$
7	104 0.066 0.060	$0.1308x_1 - 77.6697x_2 + 15.4449$
8	97 0.036 0.690	$0.2085_1x_1 + 37.6833x_2 - 25.1954$

From example 4.2, it is not difficult to point out that parameter initialization causes the uncertainties in subtractive clustering based type-1 TSK model identification algorithm. In other word, there are uncertainties in model structure identification, like the number of rules and the variables involved in the rule premises as cluster centers and Gaussian MFs spread width. Moreover, there are uncertainties in model parameter identification, like the MF parameters and the consequent regression coefficients.

In the next chapter, we proposed a type-2 TSK FLS identification algorithm that can be used to handle uncertainties in subtractive clustering based type-1 TSK FLS.

CHAPTER 5

TYPE-2 TSK FLS IDENTIFICATION ALGORITHM

Subtractive clustering based type-2 TSK fuzzy system identification algorithm [1] and fuzzy model evaluation is described in this chapter. Experimental results are given to show the effectiveness of our method.

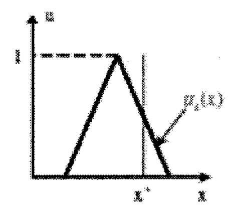
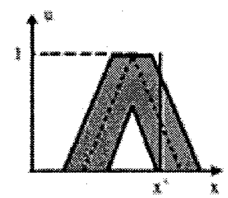
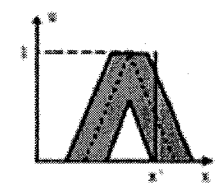
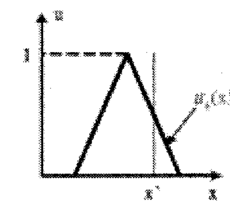
5.1 Comparison between type-1 and type-2 TSK FLS

While using subtractive clustering, a type-1 TSK model has uncertainties because of its needs to pre-initialized parameters. Conversely, a type-2 FLS is able to handle uncertainties because it can model them and minimize their effects.

Type-1 and type-2 TSK FLSs are characterized by IF-THEN rules and no defuzzification is needed in the inference engine, but they have different antecedent and consequent structures. Assuming FLSs with m rules and n antecedents in each rule, a type-1 TSK FLS is compared with a type-2 TSK FLS in Table 5.1.

From Table 5.1, a type-2 TSK FLS has more design degrees of freedom than does a type-1 TSK FLS because its type-2 fuzzy sets are described by more parameters than type-1 fuzzy sets [4]. This suggests that a type-2 TSK FLS has the potential to outperform a type-1 TSK FLS because of its larger number of design degrees of freedom.

Table 5.1 Comparison between type-1 and type-2 TSK FLS

		Type-1 TSK FLS (A1-C0)	**Type-2 TSK FLS** (A2-C1)	(A2-C0)	(A1-C1)
structure	Antecedents	Type-1 fuzzy set 	type-2 fuzzy set 	type-2 fuzzy set 	Type-1 fuzzy set 
	consequent Parameters	crisp number	Fuzzy number	crisp number	Fuzzy number
Output		A crisp point	An interval set of output A point output		
Number of design parameters		(3n +1)m*	(5n +2)m*	(4n+1)m*	(4n+2)m*

In summary, we need type-2 TSK FLSs to directly model uncertainties and minimize their effects, all within the framework of rule-based FLSs.

5.2 Proposed type-2 TSK FLS identification algorithm using subtractive clustering

Subtractive clustering based type-2 TSK fuzzy system identification algorithm that we proposed in [1] is an extension of the type-1 TSK modeling algorithm proposed by Chiu in [2, 3]. In our method, subtractive clustering method is combined with least-square estimation algorithm to pre-identify a type-1 fuzzy model form input/output data. Then with type-2 TSK fuzzy logic theory [4], considering the type-1 membership functions as the principal MFs of type-2 FLS, the antecedent MFs are extended as interval type-2 fuzzy memberships by assigning uncertainty to cluster centers. The consequent parameters are extended as fuzzy numbers (type-1 fuzzy subsets) by assigning uncertainty to consequent parameter values in type-1 TSK model. Minimum error models are obtained through enumerative search of optimum values for spreading percentage of cluster centers and consequent parameters.

The proposed type-2 TSK FLS identification algorithm using subtractive clustering [1] is illustrated in Figure 5.1 following step 1 to 4 as below.

Step 1: Use Chiu's subtractive clustering method [2, 3] combined with least-square estimation algorithm to pre-identify a type-1 fuzzy model form input/output data;

Step 2: Calculate LSE, if LSE is bigger than error limitation expected, go to Step 3. If not, end program, which means the model is acceptable, no need to use type-2 TSK model. The best type-1 TSK model is obtained.

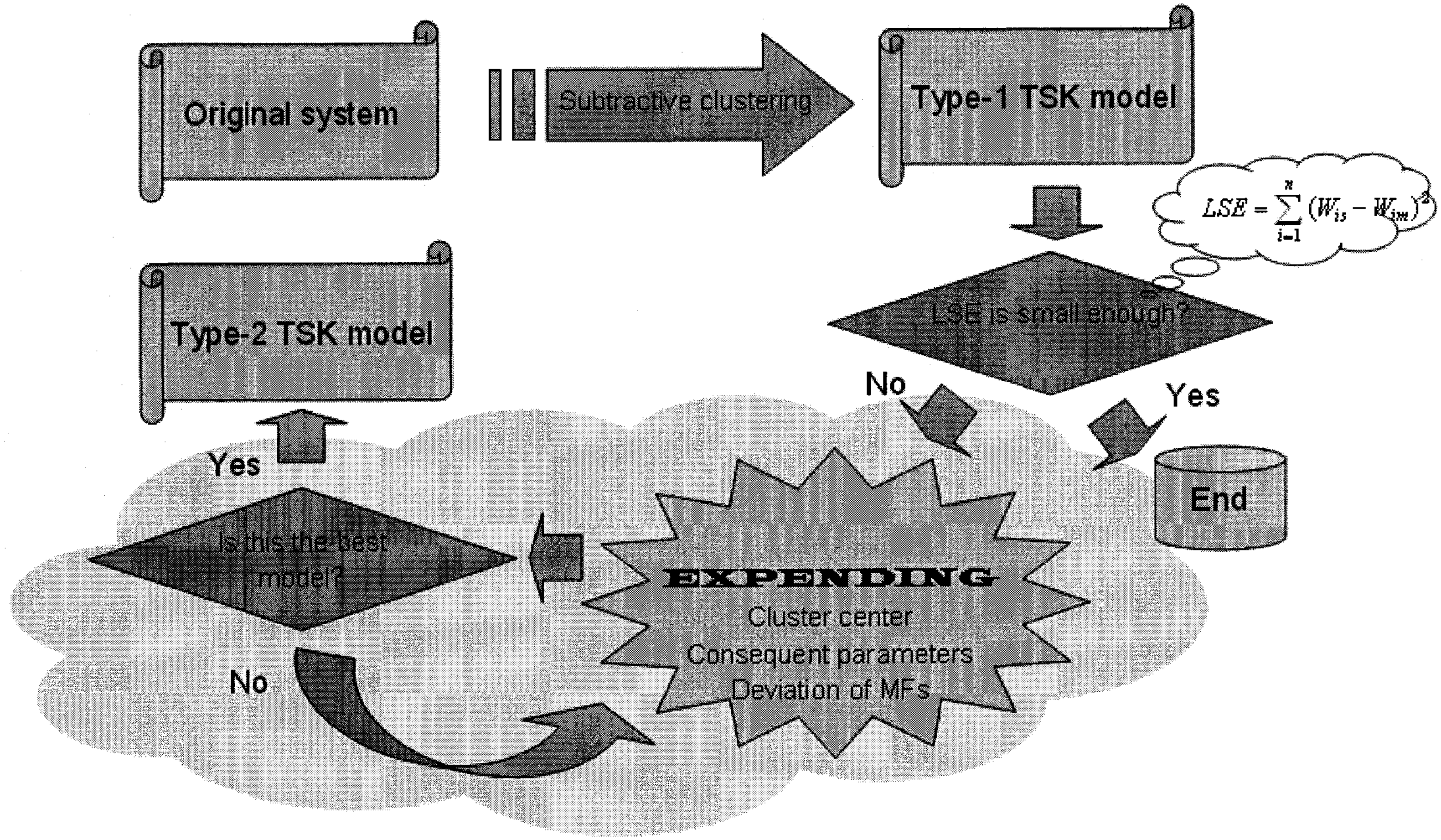


Figure 5.1 Type-2 TSK FLS identification algorithm using subtractive clustering

Step 3: Use type-1 Gaussian membership function as principal membership function to expand type-1 TSK model to type-2 TSK model.

Step 4: Calculate the interval set of output and crisp output by using Mendel's type-2 TSK FLS inference engine [4]:

- Determine the explicit dependence of the total firing interval for each rule by using eq.(3.4) .
- Obtain the interval value of the consequent for each rule by using eq.(3.5) and eq.(3.6) .
- Obtaining the total output interval set and two end-points of output interval set following eq.(3.7) and eq.(3.8) .
- Crisp output is the average value of the two end-points output following eq.(3.9)

In Step 3 of the type-2 TSK FLS identification algorithm, there are two important FL problems that have to be solved to make sure that a best type-2 TSK model is obtained:

1) How to spread uncertainties to a type-1 TSK FLS to change it to a type-2 TSK FLS?
2) How to obtain the best type-2 TSK FLS?

5.2.1 Expending a type-1 TSK FLS to a type-2 TSK

Comparing a type-1 TSK FLS to a type-2 TSK (the most general model A2-C1) in Table 5.1, they have different antecedent and consequent structures. To extend a type-1 TSK FLS to its type-2 counterpart with emphasis on interval set, antecedent MFs have to be changed from type-1 fuzzy sets to type-2 fuzzy sets. Consequent parameters have also to be changed from a certain number to a fuzzy number. The method is described in detail below:

- Expend a width of $x^*_{jk}\ a^k_j$ to both two directions of cluster center x^*_{jk} as shown in Figure 5.2. By so, cluster center is expanded from a certain point to a fuzzy number

$$\tilde{x}^*_{jk} = [x^*_{jk}(1-a^k_j), x^*_{jk}(1+a^k_j)] \tag{5.1}$$

where a^k_j is the spread percentage of the cluster center. The cluster center x^*_{jk} becomes a constant width interval valued fuzzy set $\tilde{x}^*_{jk}$ because it is extended to equal width to both directions.

Hence, the premise MF is changed from type-1 fuzzy sets of eq.(4.4) into type-2 fuzzy set, *i.e.*,

$$\tilde{Q}_{jk} = \exp\left[-\frac{1}{2}\left(\frac{x_j - x^*_{jk}(1 \pm a^k_j)}{\sigma}\right)^2\right] \tag{5.2}$$

The membership grades of type-1 TSK model is changed from certain number to fuzzy number as

$$\tilde{\mu}_{jk} = \left[\underline{\mu}_{jk}, \bar{\mu}_{jk}\right]$$

(5.3)

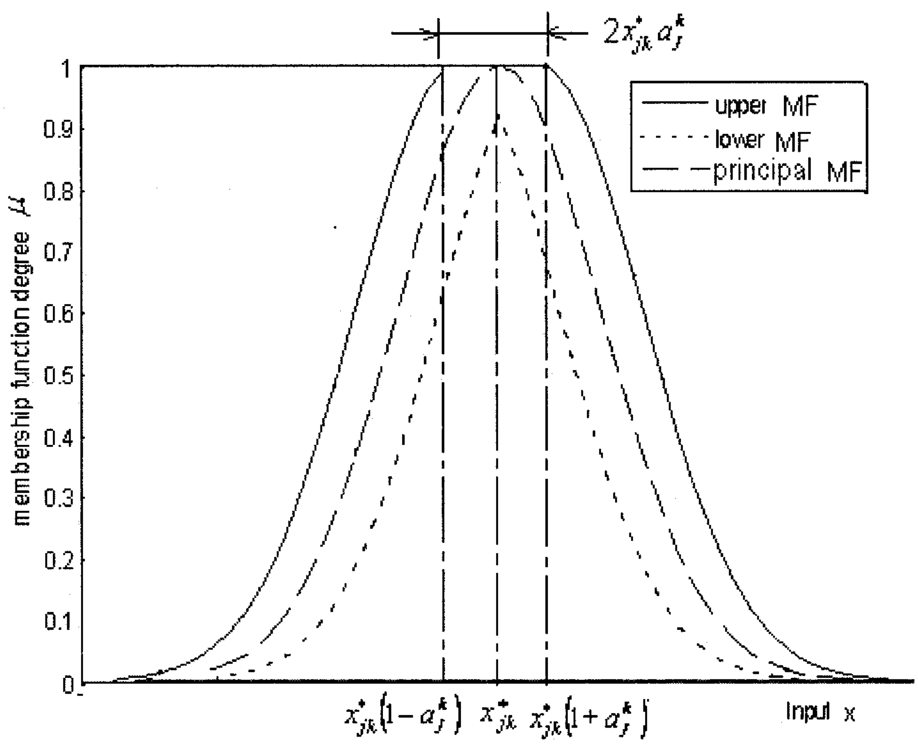


Figure 5.2 Spread of the cluster center

- Spread the consequent parameters to expanding consequent parameters from a certain value to fuzzy numbers, *i.e.*, as in eq.(2.6).

If consequent parameters of a type-1 TSK FLS defined as in eq.(5.4), the consequent parameters of a type-2 TSK FLS defined as

$$p_j^k = c_j^k \tag{5.4}$$

$$\tilde{p}_j^k = \left[c_j^k - c_j^k * b_j^k, c_j^k + c_j^k * b_j^k \right]$$

(5.5)

where b_j^k is the spread percentage of fuzzy numbers $\tilde{p}_j^k$, which is constant width interval valued fuzzy set.

The s_j^k of eq.(3.2) is changed to

$$s_j^k = c_j^k * b_j^k \tag{5.6}$$

- The deviations for rules varie from each other to get the best model. Eq.(5.2) is changed to eq.(5.7) as illustrated in Figure 5.3 where constant σ is replaced by σ_j^k.

$$\tilde{Q}_{jk} = \exp\left[-\frac{1}{2}\left(\frac{x_j - \tilde{x}_{jk}\left(1 \pm a_j^k\right)}{\sigma_j^k}\right)^2\right] \tag{5.7}$$

5.2.2 Choosing the best type-2 TSK model

To get the best type-2 TSK model, there are two important questions to answer:

1) How the uncertainties influence the type-2 TSK FLS?
2) What can be used to evaluate a type-2 TSK FLS?

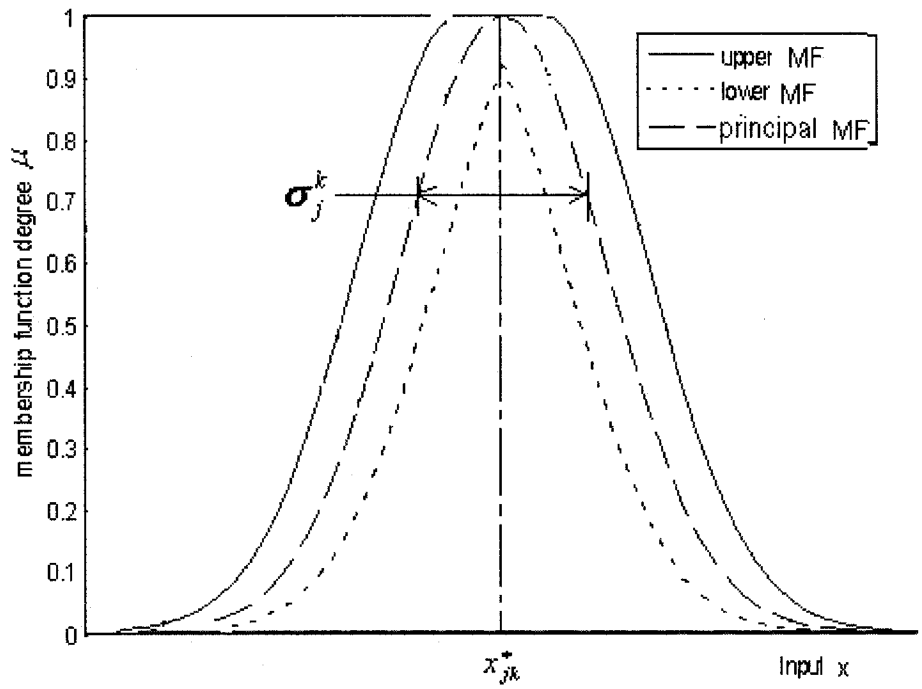


Figure 5.3 Standard deviation of Gaussian MF

5.2.2.1 Influence of uncertainties on a type-2 TSK FLS

Uncertainties on cluster center x_{jk}^*, consequent parameters p_j^k and deviation of MF σ_j^k have influence over different factors on the type-2 TSK FLS such as RMSE, model output, Gaussian MFs and model error. The example 5.1 analyzes them in detail from a specified system.

<u>Numerical Example 5.1</u> : Influence of uncertainties

Consider the following SISO system

$$y = (x-2.5)^3 + x + 1 \qquad where \quad x \in [0,4] \tag{5.8}$$

which is represented by 65 discrete data points in Figure 5.4.

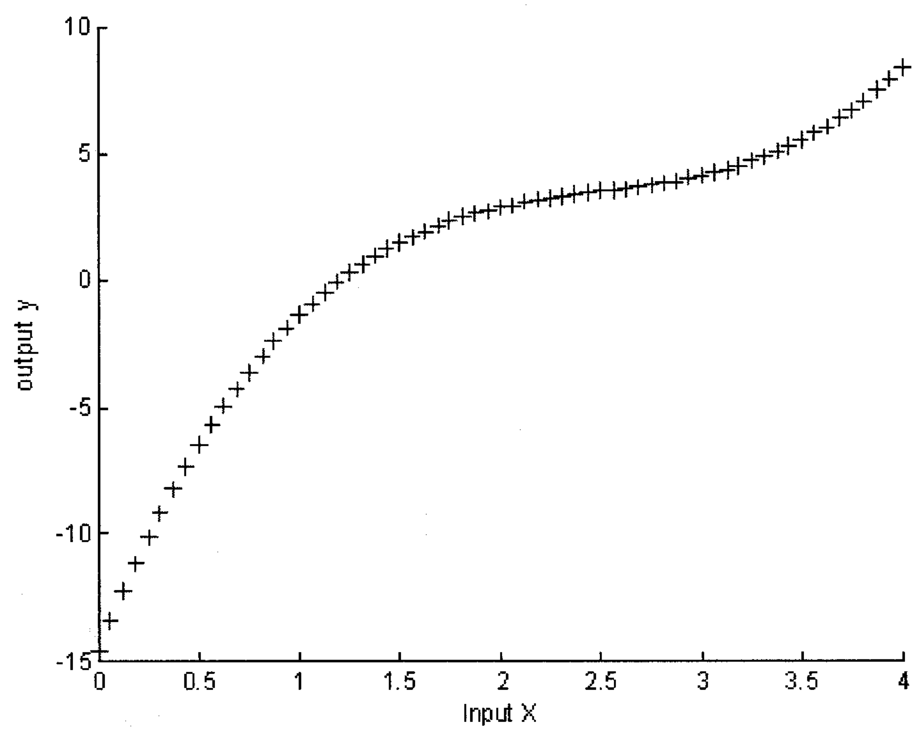


Figure 5.4 System $y = (x-2.5)^3 + x + 1$

Using recommended values form Chiu's subtractive clustering method [2, 3], $r_a = 0.25$, $\underline{\varepsilon} = 0.15$, $\bar{\varepsilon} = 0.5$, $\eta = 1.25$, cluster centers and consequent parameters are obtained. By using Gaussian membership function, the type-1 TSK model can be identified in Table 5.2.

Table 5.2 Six rules type-1 TSK model of $y = (x-2.5)^3 + x + 1$

Rule	If x, then $y = p_1 x + p_0$
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.5}{0.35355}\right)^2)$, then $y = 2.1948x - 1.9308$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.5}{0.35355}\right)^2)$, then $y = 3.8481x - 4.3995$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x-3.5}{0.35355}\right)^2)$, then $y = 4.6866x - 10.734$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.8125}{0.35355}\right)^2)$, then $y = 8.9872x - 11.736$
5	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.3125}{0.35355}\right)^2)$, then $y = 31.698x - 32.089$
6	If $x = \exp(-\frac{1}{2}\left(\frac{x}{0.35355}\right)^2)$, then $y = 41.433x - 3.0343$

Based on the type-1 TSK model rules in Table 5.2, a six rules type-2 TSK model can be expanded by assigning uncertainties spread percentage a_j^k , b_j^k to cluster centers, consequent parameters and choose separate standard deviation of Gaussian MF σ_j^k to different rules . To discuss the influences of a_j^k , b_j^k and σ_j^k to a type-2 TSK FLS model,

a_j^k , b_j^k and σ_j^k are assigned to the six rules in Table 5.2 separately, and each time the same value is assigned to different rules.

As shown in Figure 5.5, when a_j^k is chosen from 0 to 0.1, it doesn't influence significantly the RMSE. However, the RMSE grows significantly when a_j^k grows from 0.1 to 0.4 is not significant, but RMSE is up obviously when it grows bigger.

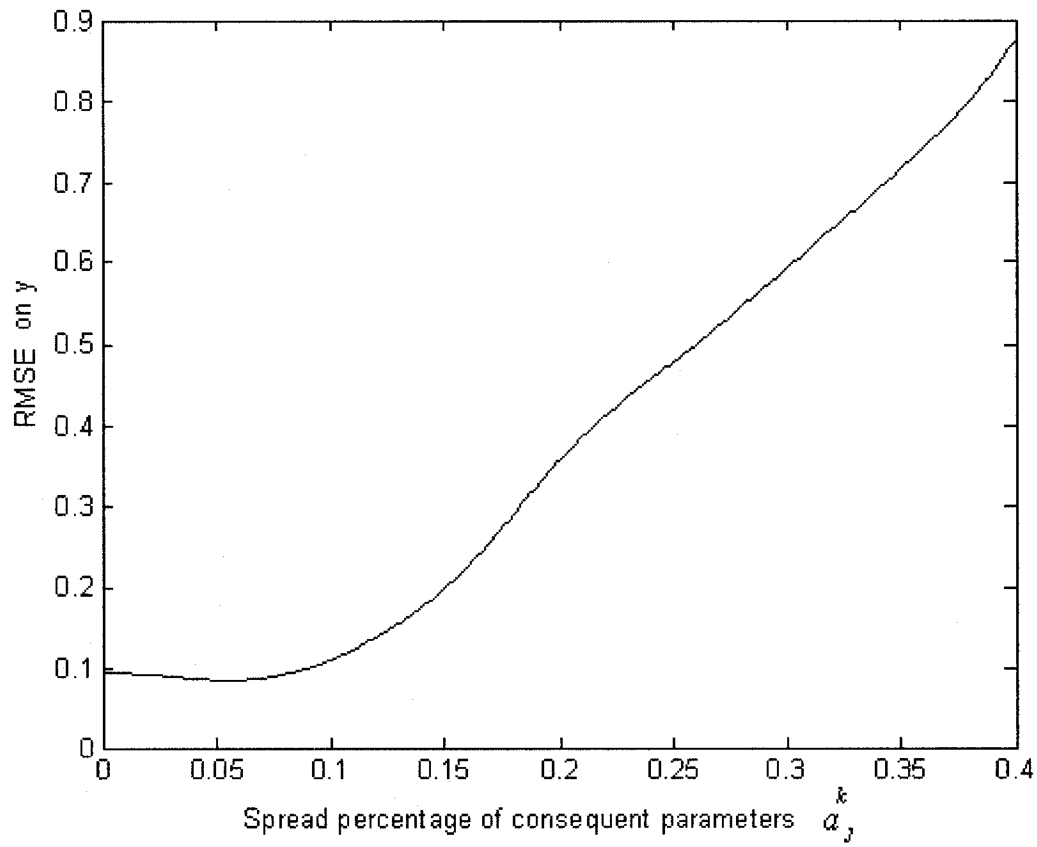


Figure 5.5 Influence of a_j^k to RMSE

Figure 5.6 to 5.8 shows influences of a_j^k to model output, Gaussian MFs and model error of the fuzzy model in Table 5.2.

In Figure 5.6, a_j^k has influence on the shape of type-2 Gaussian MFs. Bigger is the value of cluster center, wider is MF expansion.

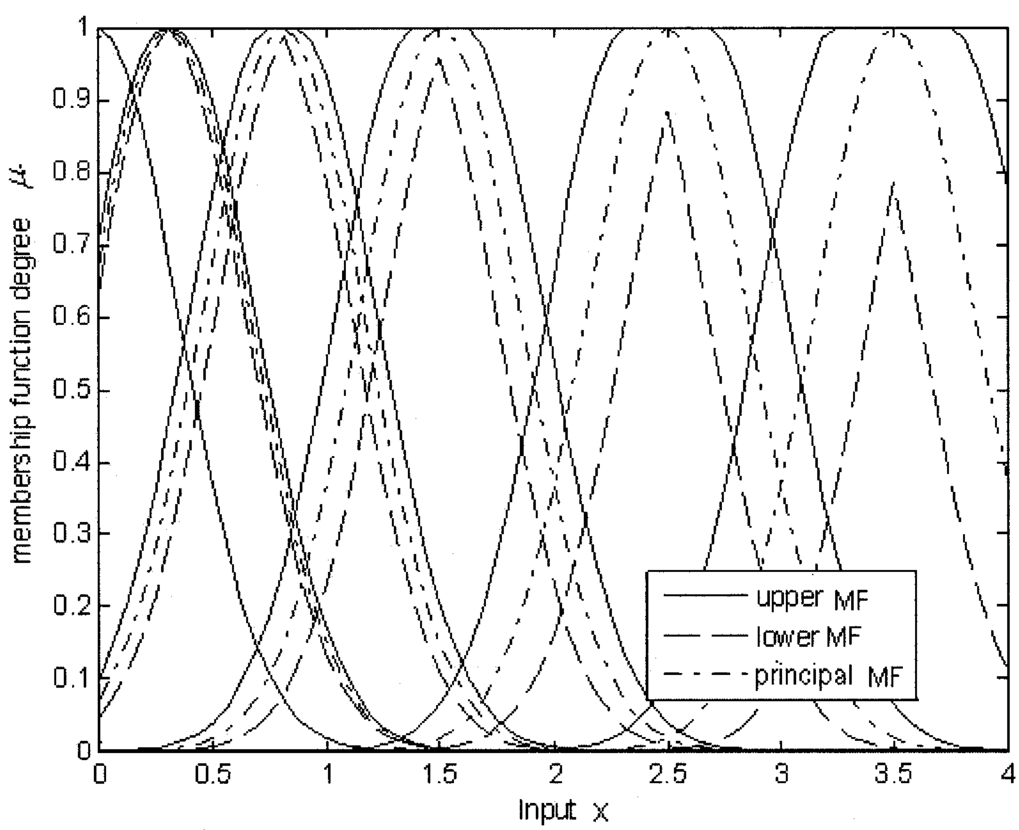


Figure 5.6 Influence of a_j^k to Gaussian MFs

Figure 5.7 depicts that influence of a_j^k to the interval value of type-2 model output is small.

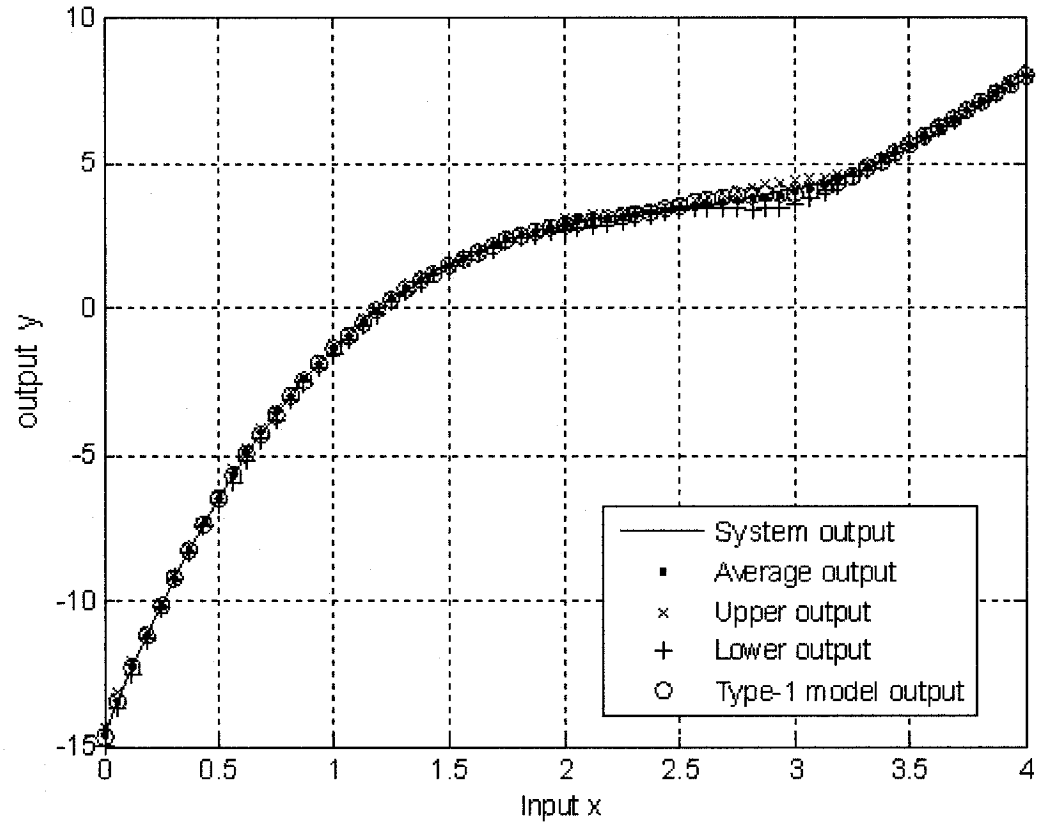


Figure 5.7 Influence of a_j^k to type-2 fuzzy model output

Figure 5.8 shows that when the clusters are expanded, better model can be gotten.

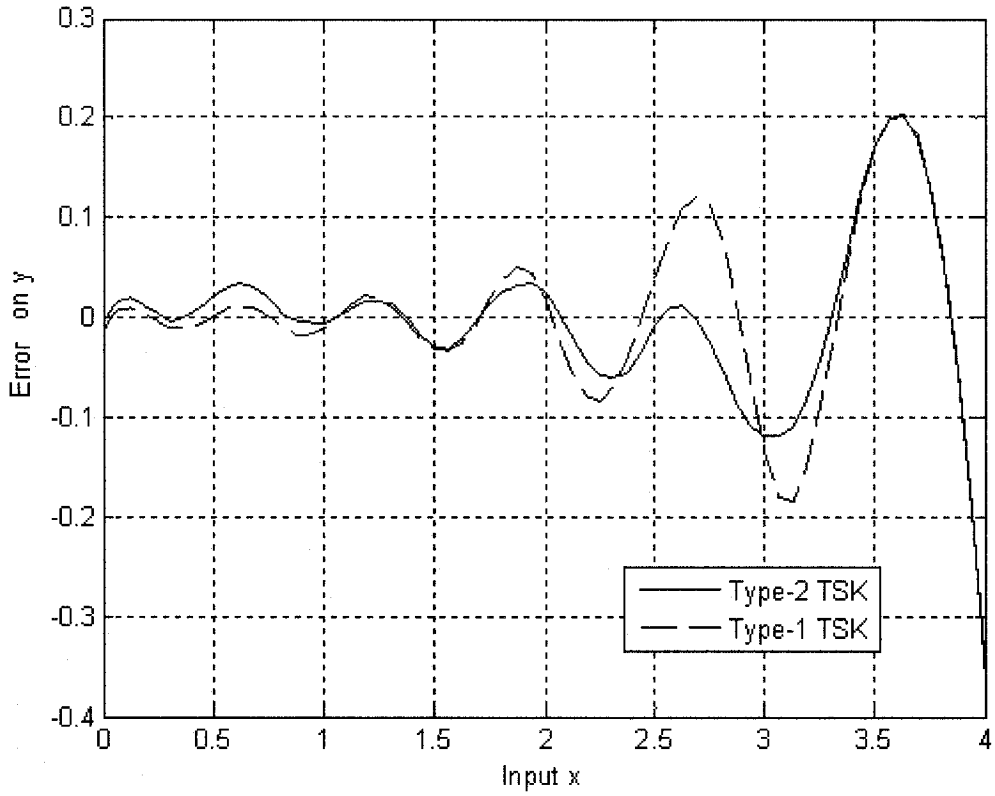


Figure 5.8 Influence of a_j^k to type-2 TSK model error

From Figure 5.9 to 5.12, the influences of b_j^k is discussed. This time only b_j^k is assigned separately to the six rules of Table 5.2. Its value is also chosen within [0, 0.4].

Figure 5.9 shows that b_j^k has nothing to do with RMSE of the system output.

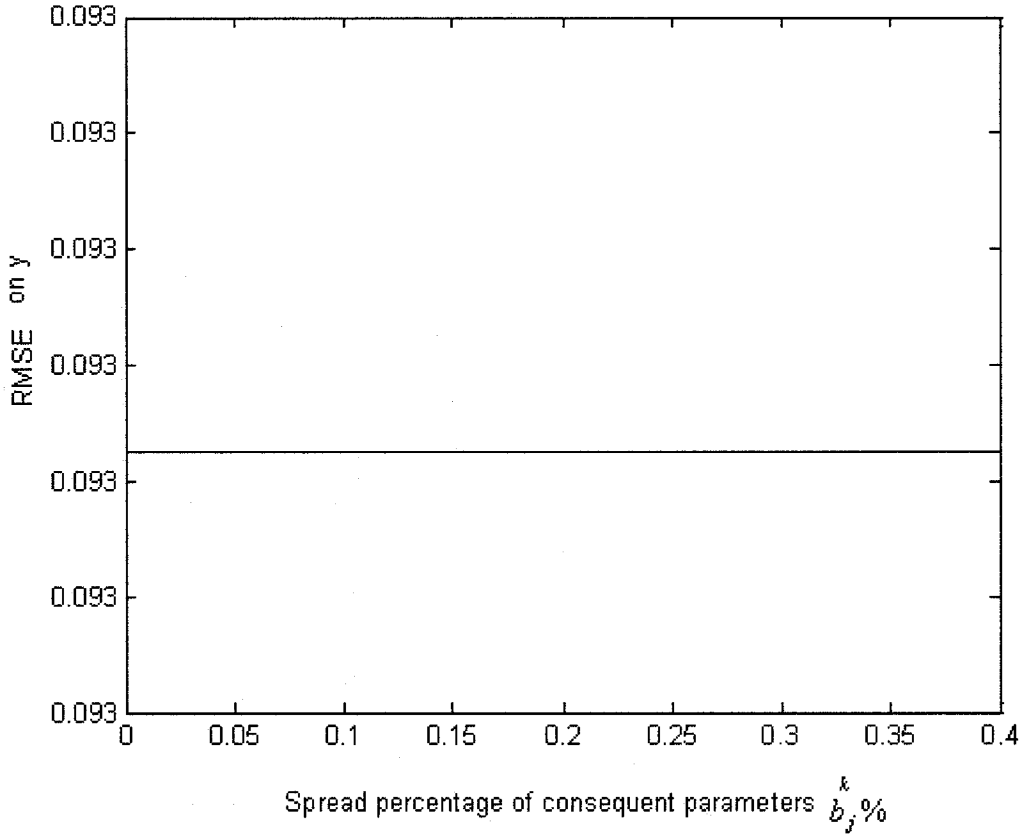


Figure 5.9 Influence of b_j^k to RMSE

Also Figures 5.10 and 5.11 depicts that spread of consequent parameter does not influence the shape of the Gaussian MFs and model error. Gaussian MFs and model error for both models are same curves.

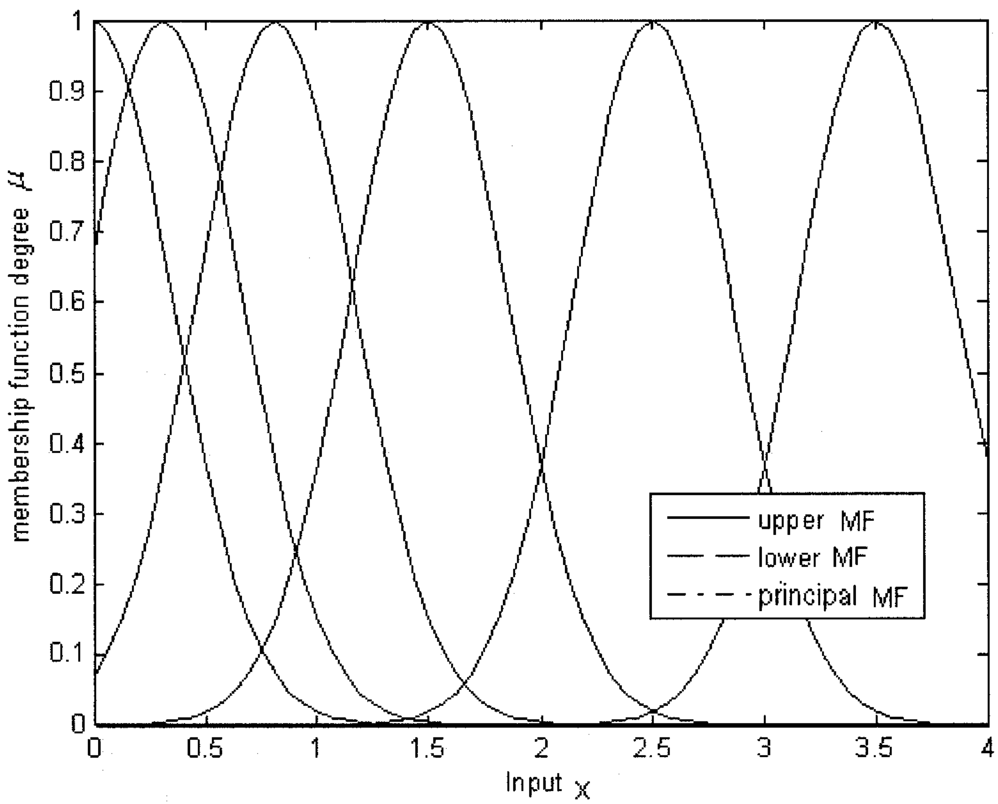


Figure 5.10 Influence of b_j^k to Gaussian MFs

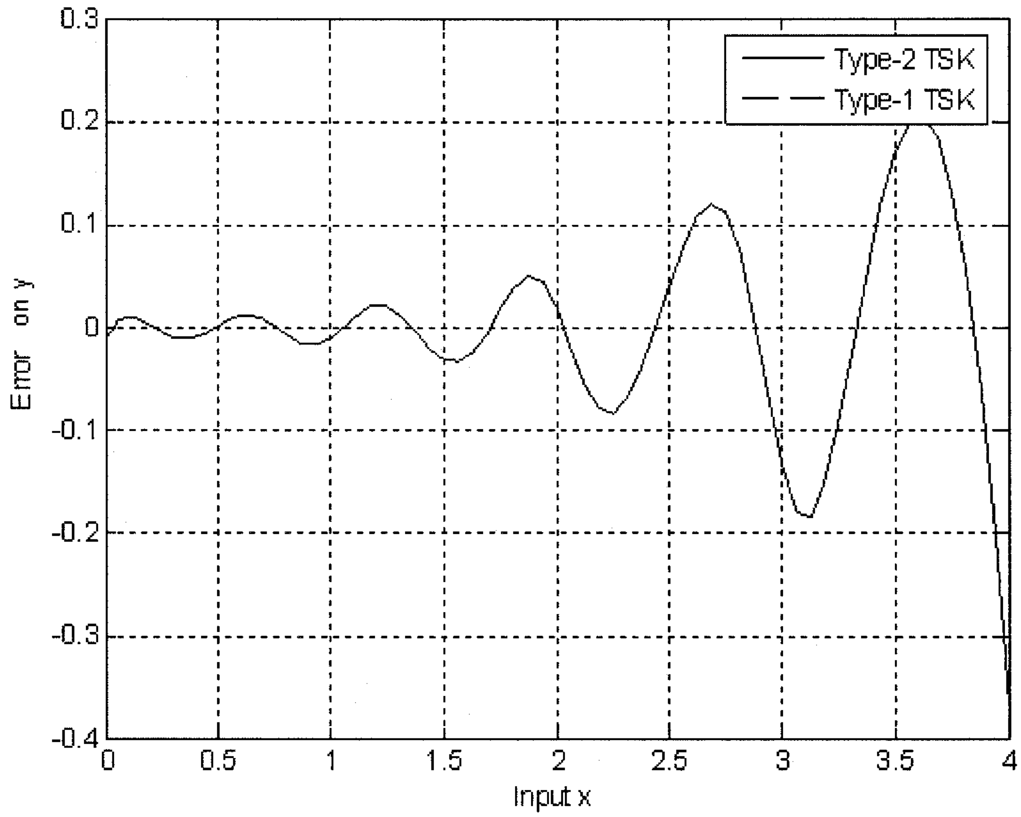


Figure 5.11 Influence of b_j^k to type-2 TSK model error

However Figure 5.12 illustrates that the interval set of output, in some sense, depends on the value of b_j^k.

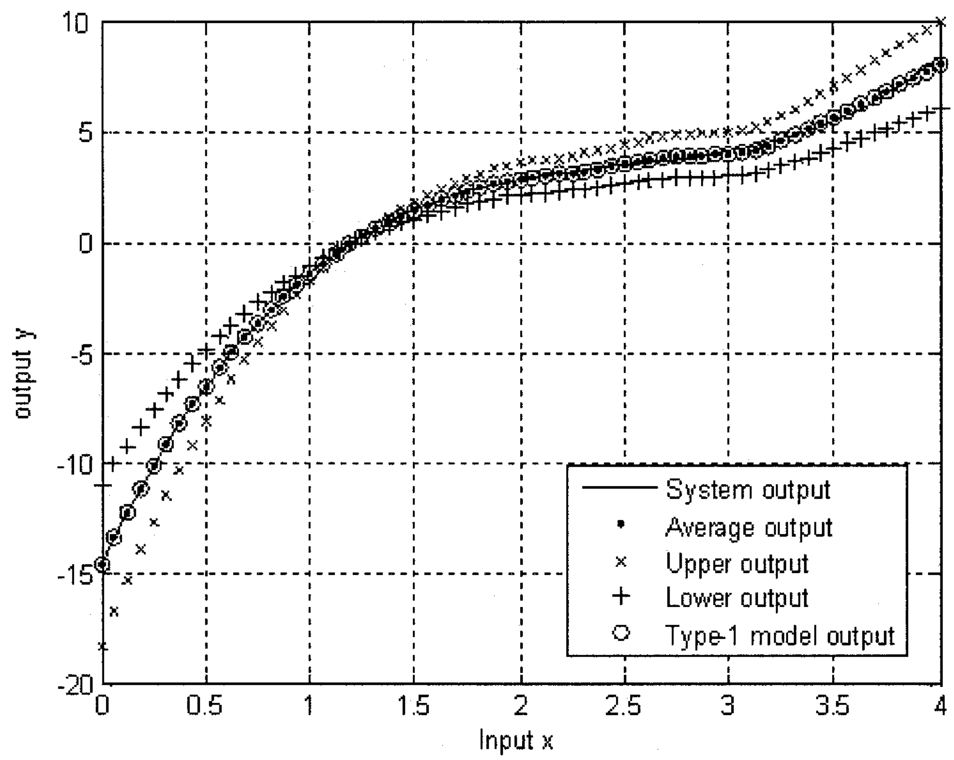


Figure 5.12 Influence of b_j^k to type-2 fuzzy model output

The deviation of MFs σ_j^k is chosen from [0.2, 0.6] instead of that constant value of the type-1 model (0.35355) in Table 5.2. Figure 5.13 shows that a type-2 TSK FLS using different σ_j^k has different RMSE.

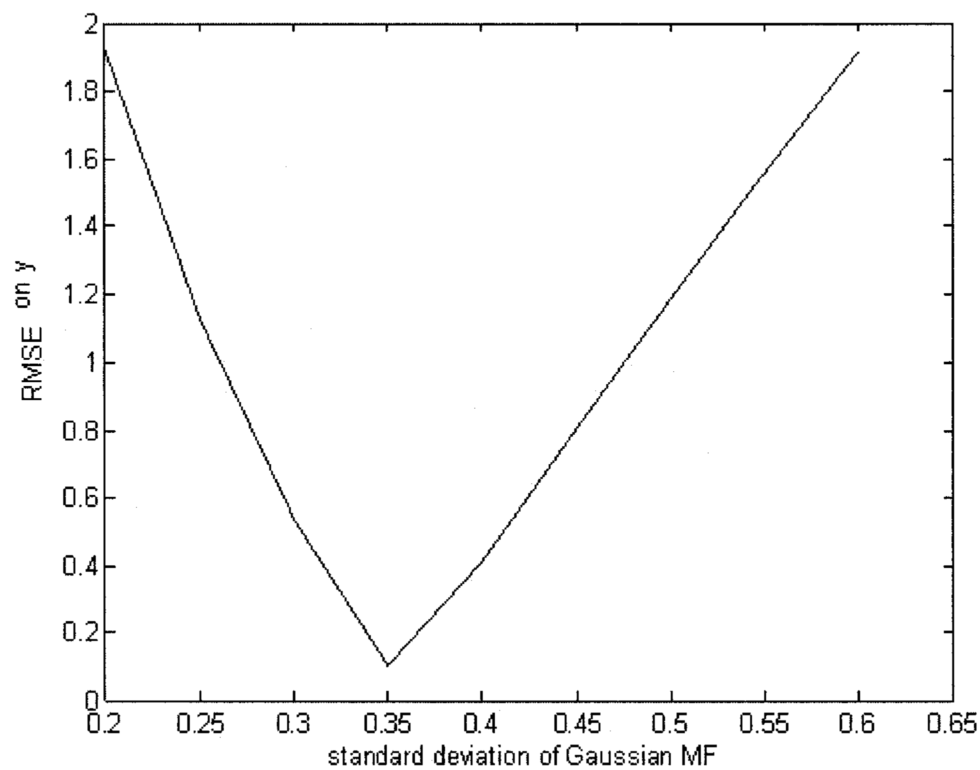


Figure 5.13 Influence of σ_j^k to RMSE

Figures 5.14 to 5.16 show that model output, Gaussian MFs and model error of a type-2 TSK fuzzy model are under the influence of the value of σ_j^k.

Table 5.3 summarizes the influences of a_j^k, b_j^k and σ_j^k to RMSE, type-2 fuzzy model output, Gaussian MFs and type-2 TSK model error which are depicted in Figure 5.5 to Figure 5.16.

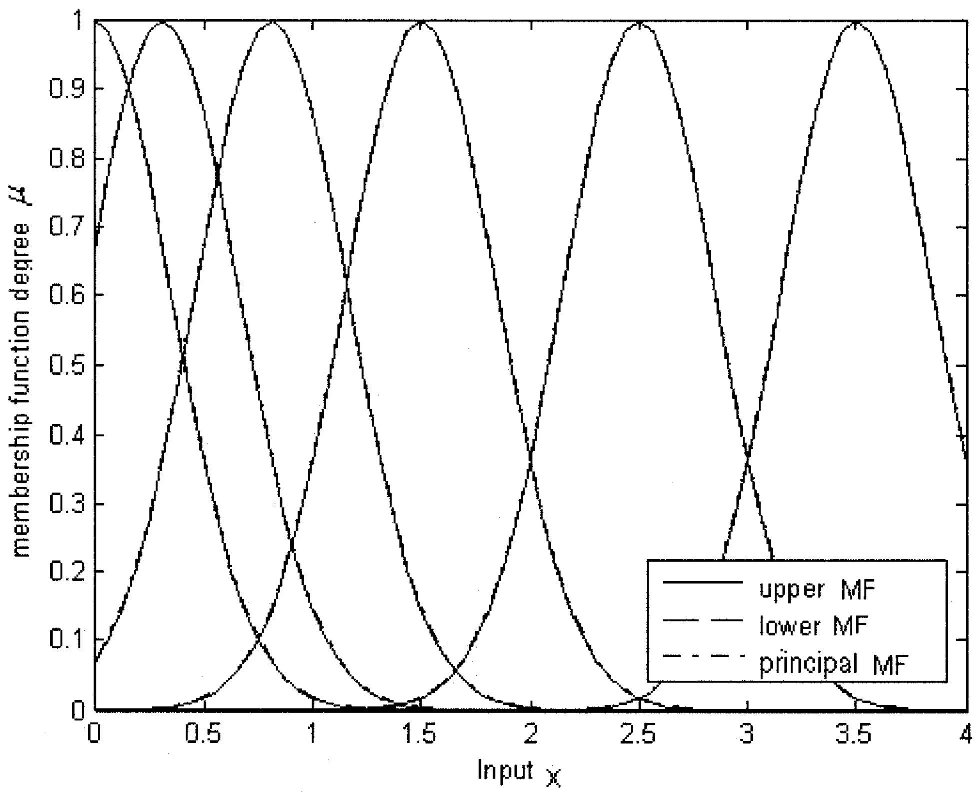


Figure 5.14 Influence of σ_j^k to Gaussian MFs

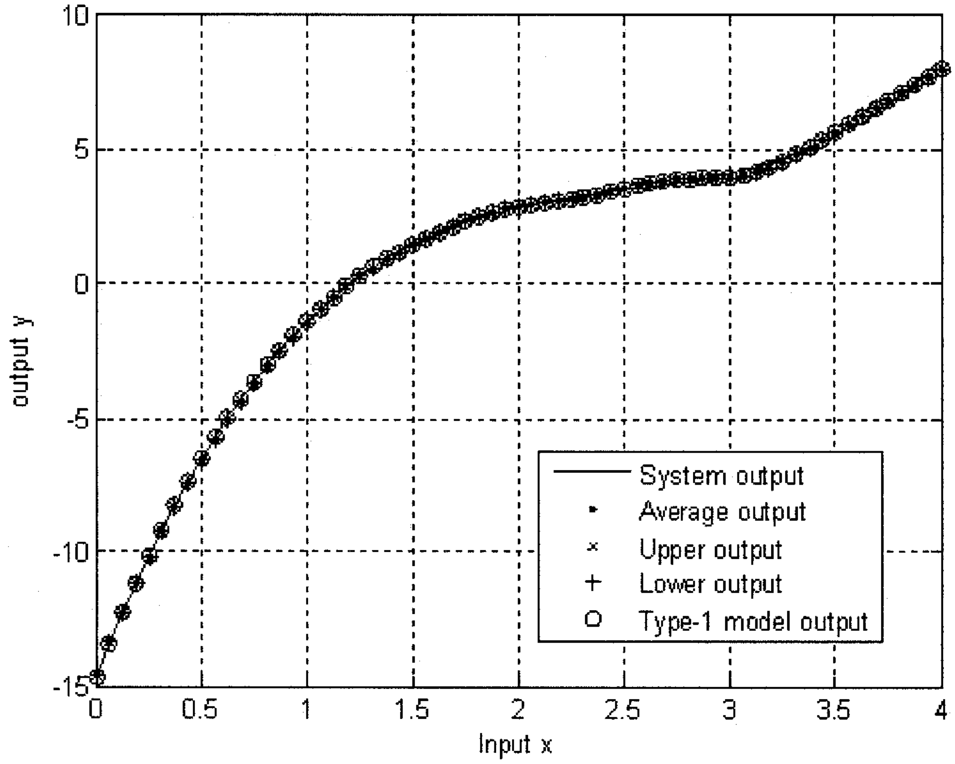


Figure 5.15 Influence of σ_j^k to type-2 fuzzy model output

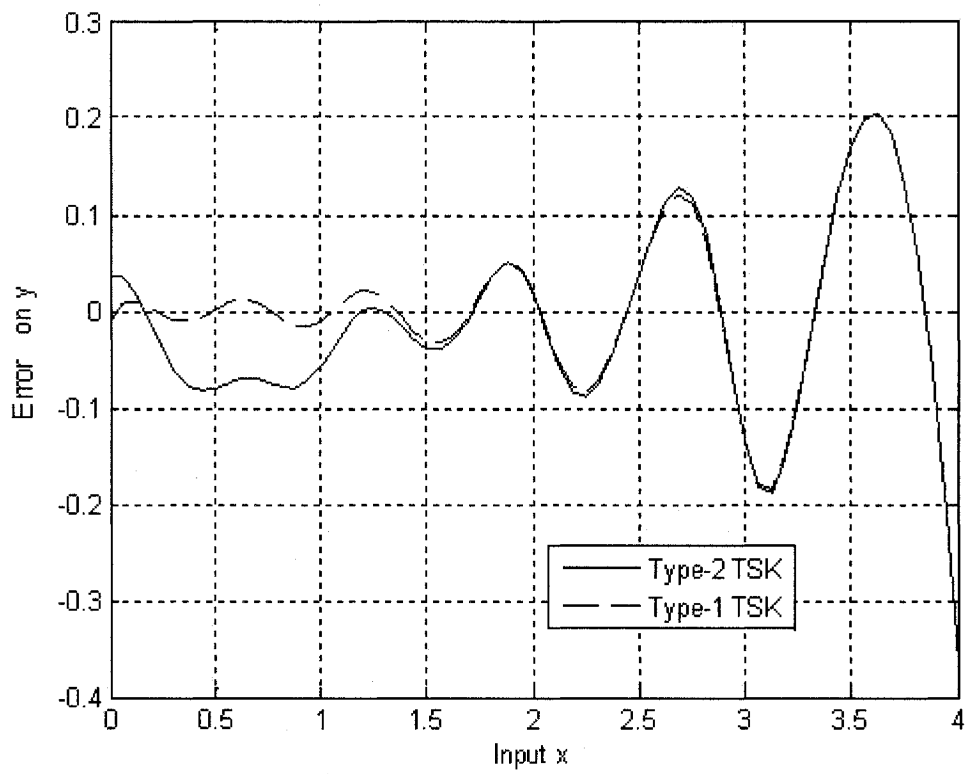


Figure 5.16 Influence of $\overset{k}{\sigma}_j$ to type-2 TSK model error

Table 5.3 Summary of influence of $\overset{k}{a}_j$, $\overset{k}{b}_j$ and $\overset{k}{\sigma}_j$

Influence	$\overset{k}{a}_j$	$\overset{k}{b}_j$	$\overset{k}{\sigma}_j$
RMSE	Yes	No	Yes
Model output	Yes	Yes, Significant	Yes
Gaussian MFs	Yes, Significant	No	Yes
Model error	Yes	No	Yes

5.2.2.2 Performance evaluation of a type-2 TSK FLS

From the preliminary experiments of example 5.1, it has been found that $\overset{k}{a_j}$, $\overset{k}{b_j}$ and $\overset{k}{\sigma_j}$ have strong influences on different factors of performance of a type-2 TSK FLS. It is recommended an search in certain rang to get the optimum value of $\overset{k}{a_j}$, $\overset{k}{b_j}$ and $\overset{k}{\sigma_j}$ to get the best type-2 TSK fuzzy model.

In subtractive clustering based type-1 TSK FLS identification algorithm, the model with LSE is chosen as the best model. Because that the staring point for the least-squares method to design a type-1 TSK FLS is a type-1 fuzzy basis function (FBF) expansion [56], the performance of a type-2 TSK FLS is evaluated using the following RMSE:

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(W_{js} - W_{jm})^2} \tag{5.9}$$

where the initial system has a group of data with n vector, W_{js} and W_{jm} are the system output and model output for jth vector, $j \in [1, n]$.

The best model has the least RMSE.

5.3 Application

Experimental results from example 5.2 are given to show the effectiveness of our type-2 TSK identification method. Furthermore, a comparison of these two TSK FLSs is also given.

<u>Numerical example 5.2</u>: Type-2 TSK FLS obtained by using proposed type-2 TSK FLS identification algorithm.

Using the same data as that in example 5.1, based on the type-1 TSK model rules of Table 5.2, a six rules type-2 TSK model is obtained in Table 5.4 and the premise MFs are given in Figure 5.17 by using the proposed type-2 TSK FLS identification algorithm in Figure 5.1. In this case, the selected range of a_j^k , b_j^k and σ_j^k selected for enumerative search are [0, 0.3], [0, 0.3] and [0.2, 0.4] The step sizes are selected as 0.001, 0.001 and 0.00001. RMSE of the type-2 model output is 0.0815.

Figure 5.18 illustrates the comparison of the system output to outputs of type-1 and type-2 TSK FLSs. The type-2 TSK FLS provides more information, not only crisp output as that of type-1 TSK FLS, but also the interval set of the output. This interval set of the output has the information about the uncertainties that are associated with the crisp output, and this information can only be obtained by working with type-2 TSK FLS.

Table 5.4 Six rules type-2 TSK model of $y = (x-2.5)^3 + x + 1$

Rule	If x, then $z = p_1 \times x + p_0$
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.5*(1\pm 22.057\%)}{0.26133}\right)^2)$, then $y = 2.1948x(1\pm 26.361\%) - 1.9308(1\pm 10.524\%)$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.5*(1\pm 10.67\%)}{0.26539}\right)^2)$, then $y = 3.8481x(1\pm 26.361\%) - 4.3995(1\pm 10.524\%)$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x-3.5*(1\pm 2.8392\%)}{0.39298}\right)^2)$, then $y = 4.6866x(1\pm 26.361\%) - 10.734(1\pm 10.524\%)$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.8125*(1\pm 5.087\%)}{0.26071}\right)^2)$, then $y = 8.9872x(1\pm 26.361\%) - 11.736(1\pm 10.524\%)$
5	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.3125*(1\pm 5.8798\%)}{0.28858}\right)^2)$, then $y = 31.698x(1\pm 26.361\%) - 32.089(1\pm 10.524\%)$
6	If $x = \exp(-\frac{1}{2}\left(\frac{x}{0.39063}\right)^2)$, then $y = 41.433x(1\pm 26.361\%) - 3.0343(1\pm 10.524\%)$

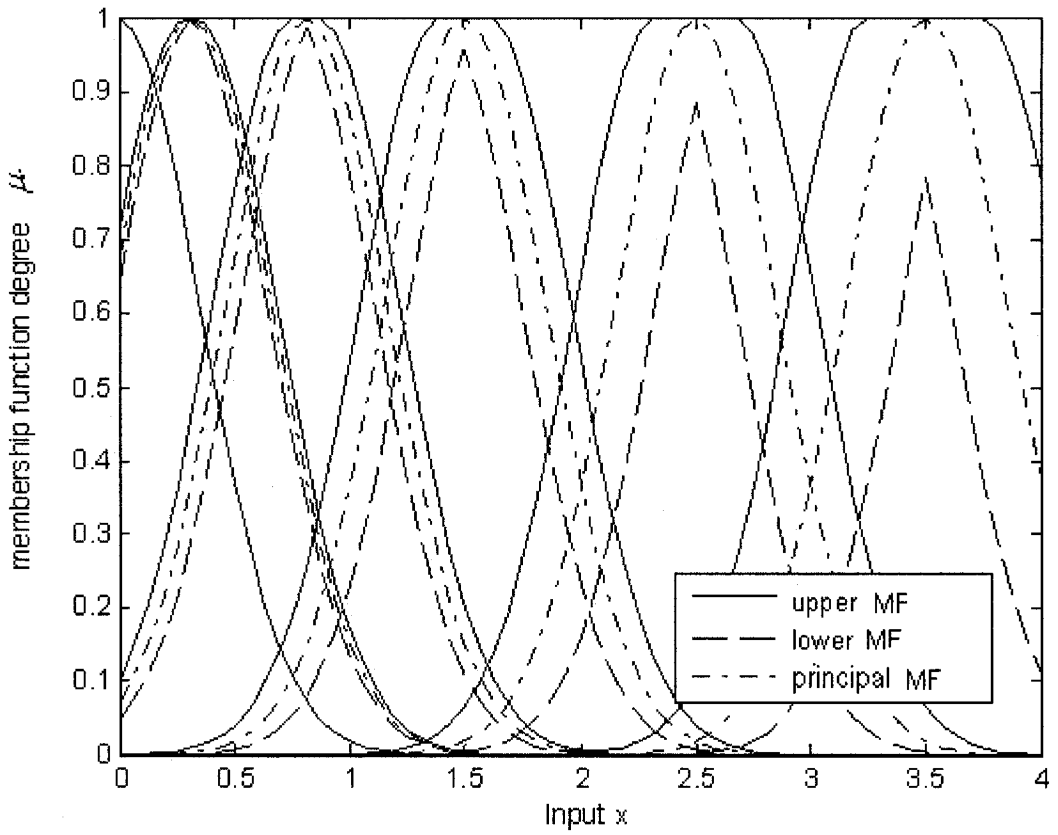


Figure 5.17 Premises of type-2 TSK FLS

The comparison of model error of type-1 TSK model and type-2 TSK model are shown in Figure 5.19. It is observed that better performance is obtained using a type-2 TSK FLS than those obtained using a type-1 FLS. Moreover Type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well [4].

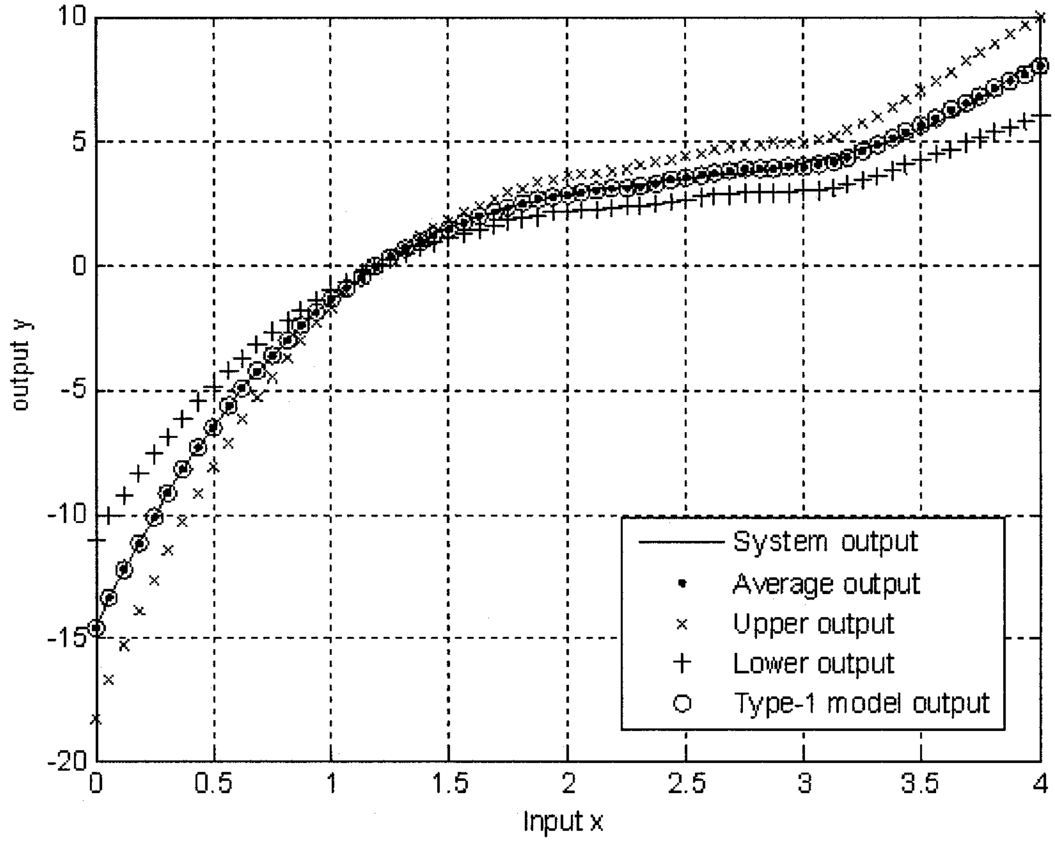


Figure 5.18 Type-2 model output of $y=(x-2.5)^3+x+1$

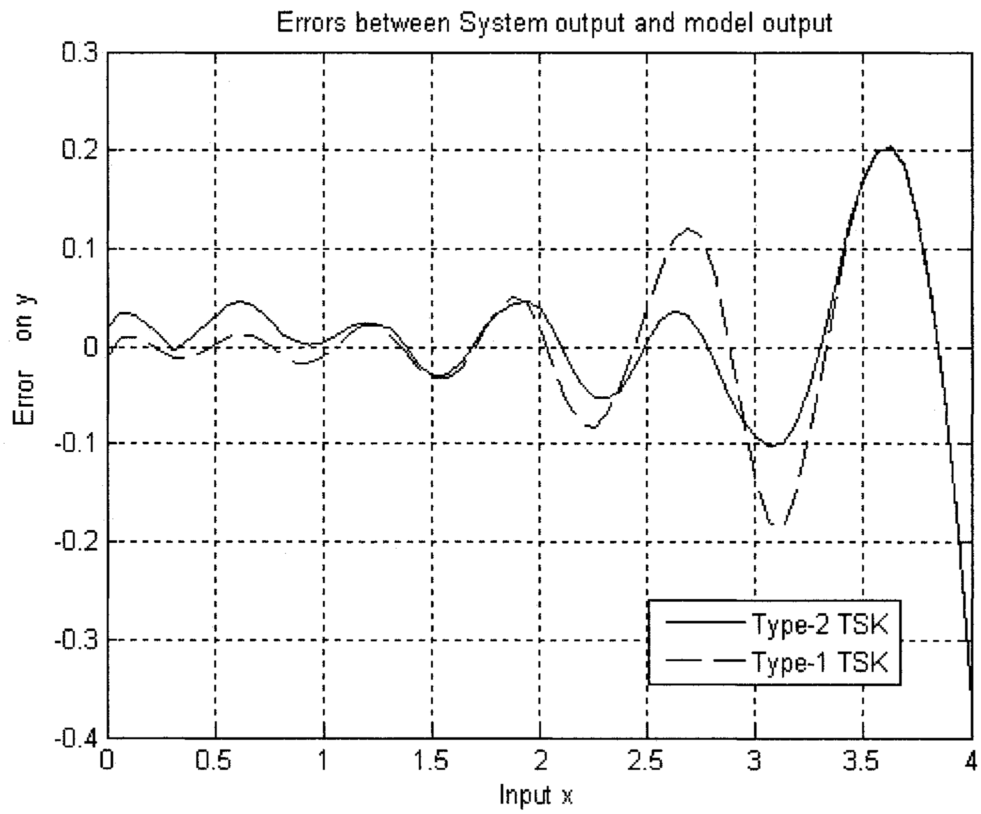


Figure 5.19 Comparison of type-1 and -2 model of $y=(x-2.5)^3+x+1$

Let's consider the same system $y=(x-2.5)^3+x+1$. Using the Demirli's extended subtractive clustering [55], a type-1 TSK model is obtained in Table 5.5. Through enumerative search for optimal values of the four parameters in the range shown in Table 4.6, cluster radius r_a, reject radio $\bar{\varepsilon}$, accept radio $\underline{\varepsilon}$, squash factor η are chosen as 0.75, 0.9, 0.6, and 0.15, respectively, LSE of this type-1 TSK model is 2.1236e-011. If considering the type-1 TSK FLS is acceptable, no need to continue expending to type-2 FLS.

Table 5.5 Five rules type-1 TSK model of $y = (x-2.5)^3 + x + 1$

Rule	If x, then $y = p_1 \times x + p_0$
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.3125}{1.0607}\right)^2)$, then $y = 2628.4x - 6233.2$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.8125}{1.0607}\right)^2)$, then $y = 1528.5x - 8063.9$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.8125}{1.0607}\right)^2)$, then $y = 1722x - 9.1$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.8125}{1.0607}\right)^2)$, then $y = 324 \times x - 1906$
5	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.375}{1.0607}\right)^2)$, then $y = 239x - 2340.8$

Even the five rules type-1 TSK model has very little error (RMSE is 5.7160e-007). Using the type-2 FLS identification algorithm, it is still possible to obtain a five rules type-2 TSK model better performance as shown in Table 5.6 (RMSE is 5.7159e-007). The values of a_j^k, b_j^k and σ_j^k are very small as shown in Table 5.6. This means that the uncertainties of the type-1 TSK model are very little.

Table 5.6 Five rules type-2 TSK model of $y = (x-2.5)^3 + x + 1$

Rule	If x, then $z = p_1 \times x + p_0$
1	If $x = \exp(-\frac{1}{2}\left(\frac{x - 2.3125(1 \pm 2.8235e-4)}{1.0607}\right)^2)$, then $y = 2628.4(1 \pm 3.9318e-6)x - 6233.2(1 \pm 4.2867e-6)$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x - 2.8125(1 \pm 7.5864e-7)}{1.0607}\right)^2)$, then $y = 1528.5(1 \pm 3.9318e-6)x - 8063.9(1 \pm 4.2867e-6)$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x - 1.8125(1 \pm 6.215e-5)}{1.0607}\right)^2)$, then $y = 1722(1 \pm 3.9318e-6)x - 9.1(1 \pm 4.2867e-6)$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x - 1.8125(1 \pm 1.3606e-4)}{1.0607}\right)^2)$, then $y = 324(1 \pm 3.9318e-6)x - 1906(1 \pm 4.2867e-6)$
5	If $x = \exp(-\frac{1}{2}\left(\frac{x - 1.375(1 \pm 2.5491e-5)}{1.0607}\right)^2)$, then $y = 239(1 \pm 3.9318e-6)x - 2340.8(1 \pm 4.2867e-6)$

The modeling of the output data for the two types FLS shown in Figure 5.20 is on same curve. Figure 5.22 depicts premise MFs of type-1 and type-2 which are a same curve. Model errors are also very similar (see Figure 5.21). This proved that if all uncertainties disappear, type-2 FL reduces to type-1 FL, in the similar way that, if randomness disappears, probability reduces to determinism. Type-1 FLS is thus a special case of type-2 FLS. If uncertainties disappear, all type-2 FL reduces to type-1 FL [4].

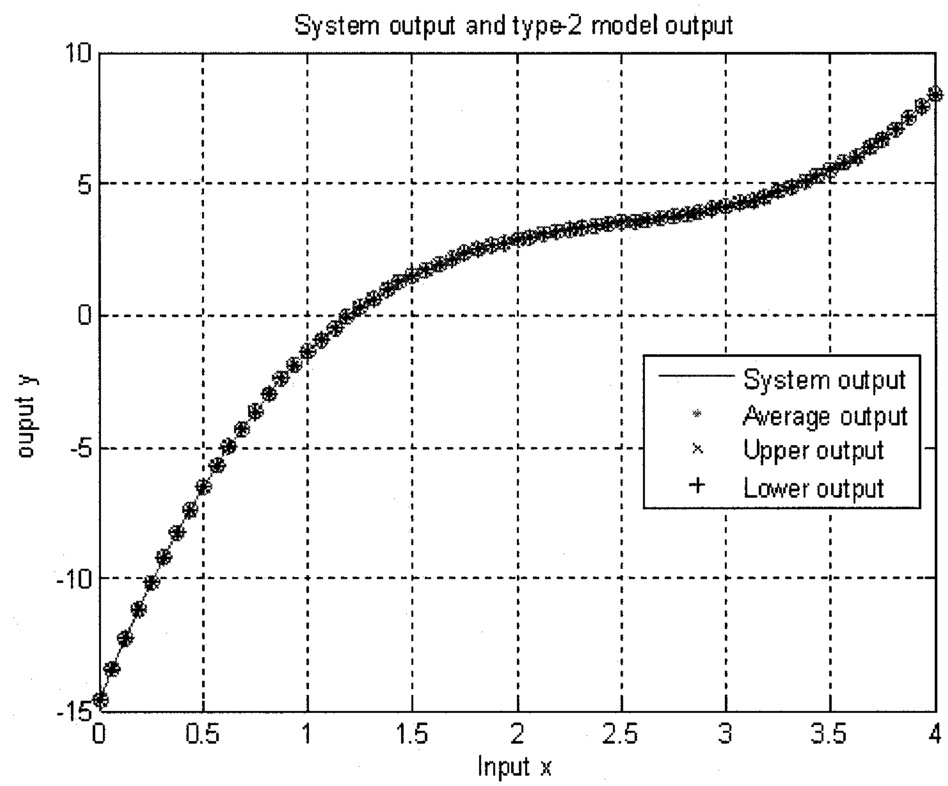


Figure 5.20 Modeling data from five rules TSK FLSs

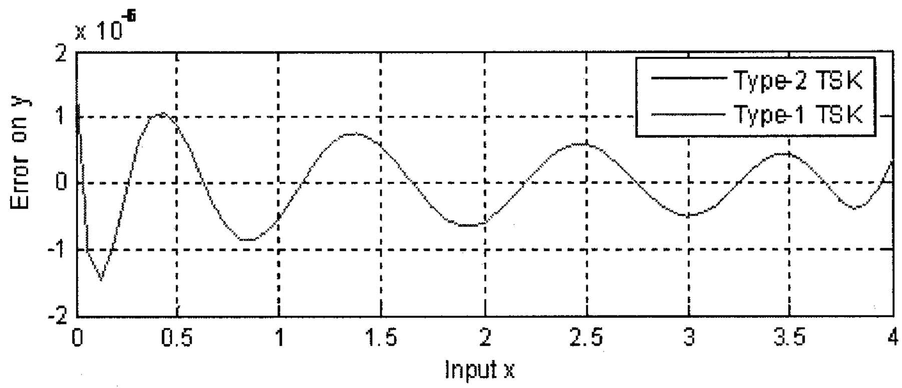


Figure 5.21 Model errors for five rules TSK FLSs

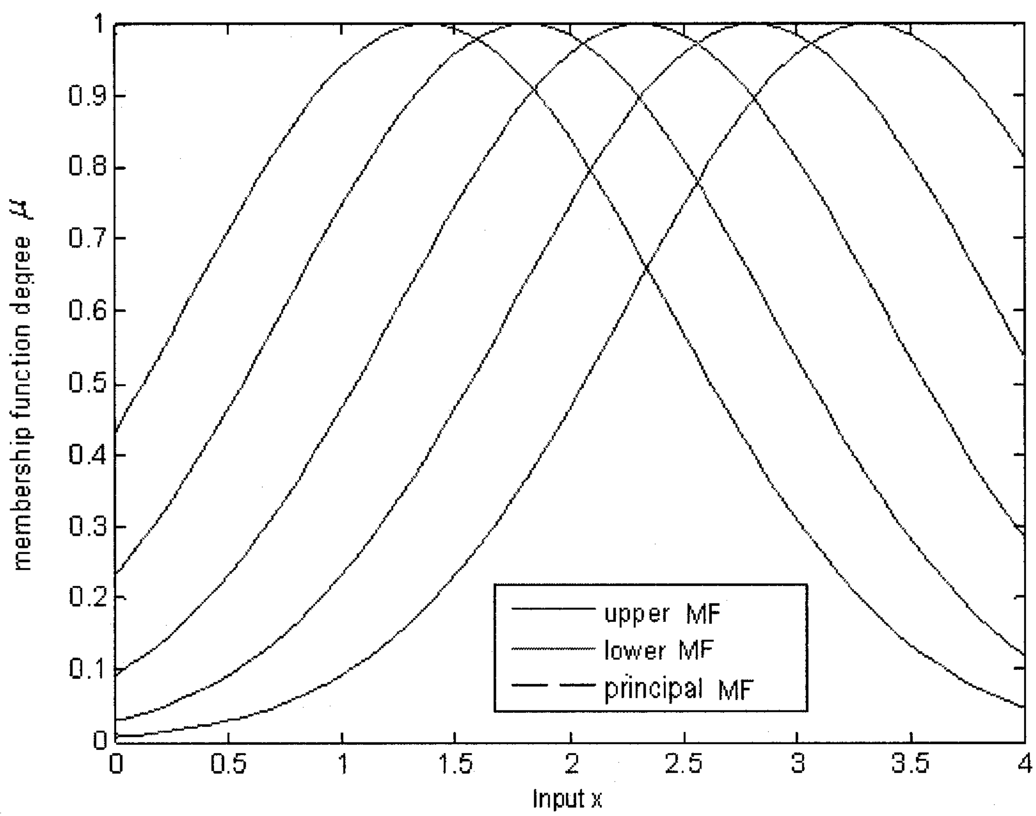


Figure 5.22 Premises for five rules TSK FLSs

Through by means of example 5.2, we have demonstrated that type-2 TSK modeling results are always better or equal to a that of type-1, although no mathematical prove is yet available. From Figure 5.18, type-1 crisp output is only one point within the type-2 interval output. Comparing systems from Table 5.4 to 5.6, in some sense, precision of type-2 TSK model is based on the precision of type-1 TSK model from which the expansion is made. Better precision of type-1 TSK model means also better precision for type-2 model. Percentage of expansion of cluster center a_j^k and consequent parameters b_j^k also depend on the precision of type-1 TSK model. Higher precision of type-1 TSK model means less value for a_j^k and b_j^k.

CHAPTER 6

CONCLUSION

Type-2 TSK FLSs has larger number of design parameters for each rule. This suggests that a type-2 TSK FLS has the potential to outperform a type-1 TSK FLS. When a system data has noise, type-2 FLS is better to model it. Also because of larger number of design parameters, a type-2 TSK FLS is more difficult to identify than a type-1 TSK FLS.

Proposed type-2 TSK FLS identification algorithm [1] can be compared relatively to Mendel's back–propagation method [4]. Its advantage and Disadvantage are summarized as below:

- Advantage:

    1) Easier to understand for type-1 TSK model expert.
    2) Easier to handle the precision of type-2 TSK modeling.
    3) Avoid enumerative parameter search for type-2 model.
    4) Easy to control number of rules and minimize RMSE.

- Disadvantage:

    1) In some sense, precision of type-2 TSK modeling is based on the precision of type-1 TSK model from which it is expanded.
    2) Percentage of expansion of cluster center and consequent parameters depend

on the precision of type-1 TSK model.

In the proposed subtractive clustering based type-2 TSK fuzzy system identification algorithm, cluster center and consequent parameter are expanded to constant width interval valued fuzzy set. No-constant width interval valued fuzzy set will be interesting subject for future research. Until right now, higher than-first-order type-1 TSK model has not been described in the literature. It also could be very good future research direction.

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