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PLANIFICATION DES OPÉRATIONS D'ENTRETIEN HIVERNAL DES RÉSEAUX ROUTIERS

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Cette thèse intitulée :

PLANIFICATION DES OPÉRATIONS D'ENTRETIEN HIVERNAL
DES RÉSEAUX ROUTIERS

présentée par: PERRIER Nathalie
en vue de l'obtention du diplôme de: Philosophiae Doctor
a été dûment acceptée par le jury d'examen constitué de:

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RÉSUMÉ

Cette thèse traite de deux problèmes reliés à l'entretien hivernal des réseaux routiers: le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige, et le problème de tournées de véhicules pour les opérations de déblaiement des rues.

Nous présentons d'abord une revue complète de la littérature concernant l'utilisation de modèles d'optimisation et d'algorithmes de résolution dans le domaine de l'entretien hivernal des réseaux routiers. Cette revue porte sur les principaux problèmes rencontrés dans le domaine de l'entretien hivernal des réseaux routiers. Pour chaque classe de problèmes, nous proposons une classification des modèles et décrivons leurs principales caractéristiques en insistant sur leur structure et sur les algorithmes de résolution utilisés. Les modèles décrits sont regroupés en deux grandes catégories: les modèles de design des systèmes d'entretien hivernal et les modèles de routage des véhicules. La première catégorie inclut le problème du partitionnement d'un réseau routier en secteurs de déneigement et le problème d'affectation des secteurs aux sites de déversement.

Nous proposons ensuite deux méthodes de résolution approximatives pour résoudre le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. Étant donné un réseau routier et un ensemble de sites de déversement disponibles, le problème consiste à créer des groupes de segments de rues, appelés secteurs de déneigement, et à affecter chaque secteur à un site de déversement, tout en respectant certaines contraintes relatives à la forme et à la taille des secteurs ainsi qu'à l'utilisation des sites de déversement. L'objectif poursuivi consiste à minimiser les coûts variables de transport de la neige dans des camions vers les sites de déversement, les

coûts variables d'élimination de la neige aux sites, ainsi que les coûts fixes des camions. Les deux méthodes se fondent sur un modèle très complet qui traduit les difficultés fondamentales du problème découlant des contraintes relatives à la forme et à la taille des secteurs ainsi qu'à l'utilisation des sites de déversement. La première méthode permet de tenir compte de l'interdépendance entre le problème du partitionnement d'un réseau routier en secteurs et le problème d'affectation des secteurs aux sites tandis que la seconde méthode traite séparément ces deux composantes. Les comparaisons avec la seconde méthode indiquent que la première approche permet très souvent de réduire de façon considérable les coûts variables d'élimination de la neige aux sites.

La dernière partie de la thèse présente deux méthodes approximatives pour résoudre le problème de tournées de véhicules pour les opérations de déblaiement des rues. Étant donné un réseau routier, un garage et une flotte de véhicules, le problème consiste à déterminer un ensemble de tournées partant et revenant au garage tel que chaque segment de rue est desservi par un seul véhicule, tout en respectant plusieurs contraintes opérationnelles reliées aux caractéristiques du réseau routier, des segments de rues, des véhicules, ainsi qu'à la configuration des tournées. L'objectif poursuivi consiste soit à minimiser le temps d'achèvement des opérations, soit à minimiser successivement les temps pour desservir chaque classe de priorité. Les méthodes sont basées sur un modèle général et complet incorporant une grande variété de contraintes et de possibilités essentielles dans un cas réel. Ce modèle a été développé en fonction des besoins spécifiques d'une ville canadienne mais peut cependant être modifié pour traiter d'autres cas. En plus des contraintes générales de préséance, des vitesses de service et de passages à vide différentes, des passages répétés obligatoires pour les rues à voies multiples, de la possibilité d'augmenter l'ordre de préséance des rues non prioritaires, et des restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules, la formulation inclut des contraintes d'équilibre de durée des tournées, la possibilité de desservir certaines artères en tandem, et des pénalités pour limiter l'utilisation de certains types de virages. Les comparaisons avec les tournées produites manuellement par les employés de la ville indiquent que nos deux méthodes permettent très souvent de réduire à la fois le temps d'achèvement et les temps pour desservir chaque classe de priorité.

En somme, les principales contributions de cette thèse sont de proposer un modèle détaillé et flexible pour le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige ainsi que pour le problème des tournées de véhicules pour les opérations de déblaiement des rues, et de développer différentes méthodes approximatives pour résoudre chaque modèle. Par ailleurs, l'utilité pratique des méthodes présentées dans cette thèse est confirmée par leur application à des cas réels.

ABSTRACT

This dissertation addresses two problems related to winter road maintenance: the combined problem of partitioning a road network into sectors and allocating sectors to snow disposal sites for snow disposal operations and the problem of vehicle routing for snow plowing operations.

We first present a complete review of the literature concerning optimization models and solution algorithms for winter road maintenance planning. This survey describes the main problems that are treated in winter road maintenance planning. For each class of problems, we propose a classification of the models and describe their important characteristics by focusing on their structure and the solution algorithms proposed to solve them. The models are grouped in two main categories: system design models and vehicle routing models. The first category includes the problem of partitioning a road network into sectors and the problem of allocating sectors to snow disposal sites.

We then propose two heuristic methods for solving the combined problem of partitioning a road network into sectors and allocating sectors to snow disposal sites for snow disposal operations. Given a road network and a set of planned disposal sites, the problem consists in determining a set of non-overlapping subnetworks, called sectors, and to assign each sector to a single snow disposal site while satisfying supplementary constraints pertaining to the shape and size of the sectors as well as disposal site characteristics. The objective is to minimize the variable transportation costs for hauling snow from the street segments to the disposal sites, the variable costs to operate the disposal sites, and the fixed costs for the trucks. Both methods are based on a very complete model that addresses the fundamental difficulties of the problem resulting from the shape and size of the sectors as well as disposal site characteristics. The first method takes into account the interdependencies between the problem of partitioning a road

network into sectors and the problem of allocating sectors to disposal sites while the second method consists in separating these decisions. The computational experiments performed show that the first method can result in substantial disposal site cost savings compared to the second method.

The last part of the dissertation presents two heuristic methods for solving the problem of vehicle routing for snow plowing operations. Given a road network and a single depot where a number of vehicles are based, the problem is to determine a set of routes, each performed by a single vehicle that starts and ends at the depot, such that all road segments are serviced while satisfying a set of operational constraints related to the characteristics of the road network, street segments, vehicles, as well as to the configuration of the routes. The objective is to minimize either the plowing completion time or the completion time of each priority class. The methods are based on a general and complete model including a large variety of constraints and possibilities that are essential in a real case. This model was developed according to the specific needs of a Canadian city but can however be modified to deal with other cases. Besides general precedence relation constraints, different service and deadhead speeds, separate pass requirements for multi-lane road segments, class upgrading possibilities, and vehicle-road segment dependencies, the formulation incorporates load balancing constraints, tandem service possibilities, and turn restrictions. Our test results indicate that the two methods can produce sets of routes that dominate the existing set of routes of the city with respect to both plowing completion time and completion time of each priority class.

In short, the main contributions of this dissertation are to present a detailed and flexible model for the combined problem of partitioning a road network into sectors and allocating sectors to snow disposal sites for snow disposal operations as well as for the problem of vehicle routing for snow plowing operations, and to develop different heuristic methods to solve each model. In addition, the practical usefulness of the methods presented in this dissertation is confirmed by their application to real cases.

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INTRODUCTION

Les opérations reliées à l'entretien hivernal des réseaux routiers impliquent une variété de problèmes qui peuvent être étudiés à l'aide des méthodes de la recherche opérationnelle. Ces opérations comprennent l'épandage de fondants et d'abrasifs, le déblaiement des rues, le chargement de la neige dans des camions et le transport de la neige vers des sites de déversement. L'importance de ces problèmes se justifie par l'ampleur des dépenses directes nécessaires pour la conduite des opérations ainsi que par les coûts indirects résultant de la perte de productivité due à une mobilité réduite et des effets des fondants et des abrasifs sur les infrastructures, les véhicles et l'environnement. Aux États-Unis, les opérations reliées à l'entretien hivernal des réseaux routiers représentent une part importante des budgets annuels dans plusieurs juridictions, coûtant à l'état et aux gouvernements locaux approximativement deux milliards de dollars annuellement (MINSK, 1998). Au Japon et en Europe, les dépenses directes reliées à l'entretien hivernal des réseaux routiers sont près de deux à trois fois plus élevées que celles des États-Unis (TRANSPORTATION RESEARCH BOARD, 1995). En plus des dépenses directes, peuvent s'ajouter des coûts indirects associés à la détérioration des infrastructures et des véhicules, à la corrosion, à la dégradation de la qualité de l'eau, et à d'autres impacts environnmentaux résultant de l'utilisaiton des fondants pour l'entretien hivernal des réseaux routiers. Ces coûts indirects totalisent approximativement plus de cinq milliards de dollars par année aux États-Unis (TRANSPORTATION RESEARCH BOARD, 1995). Des coûts indirects additionnels encourus par les automobilistes et les entreprises comprennent les coûts associés aux accidents, aux retards, et à la perte de salaires et de productivité. Ces coûts sont difficles à quantifier monétairement mais peuvent devenir significatifs dans certaines situations. Par exemple, la tempête qui a eu lieu dans le nord-est des États-Unis en janvier 1996 a forcé la fermeture d'entreprises et de bureaux gouvernementaux fédéraux entraînant ainsi des pertes de production et de ventes d'environ 20 milliards de dollars (LORD, 1996).

Les opérations reliées à l'entretien hivernal des réseaux routiers impliquent une variété de problèmes stratégiques, tactiques, opérationnels et en temps réel. La planification stratégique consiste principalement en des décisions ayant des implications durant plusieurs années telles que le partitionnement du réseau routier en secteurs de déneigement, l'horaire de remplacement de la flotte de véhicules, et les décisions de localisation des sites de déversement et des garages. Le niveau tactique concerne la planification à moyen et à court terme qui doit être révisée à tous les ans ou à tous les trois ou quatre mois selon l'évolution des précipitations. L'affectation des secteurs de déneigement aux sites de déversement et la détermination de la taille de la flotte de véhicules en sont des exemples. La planification opérationnelle touche aux décisions de très court terme qui exigent un suivi sur une base mensuelle ou quotidienne telles que le routage des véhicules et la création d'horaires de travail. Le niveau en temps réel concerne finalement les situations nécessitant une réponse très rapide à des problèmes se posant en temps réel tels que les bris d'équipement ou les changements de température. La modification des tournées de véhicules fondée sur une information météo routière ponctuelle et détaillée est un exemple de contrôle en temps réel. Certains problèmes de décision peuvent être classés à différents niveaux selon l'horizon de planification considéré. Par exemple, les décisions reliées à l'affectation des secteurs de déneigement aux sites de déversement sont des décisions tactiques puisqu'elles sont généralement révisées à tous les ans. Cependant, ces décisions sont reliées au niveau opérationnel lorsque des ajustements mensuels doivent être faits pour pallier à une augmentation des précipitations.

Les problèmes reliés à l'entretien hivernal des réseaux routiers sont très difficiles et spécifiques à chaque réseau routier à cause de la diversité des conditions courantes influençant la conduite des opérations et de la grande variété des contraintes opérationnelles. Une approche inévitable en raison de la très grande taille des problèmes consiste donc à traiter ces problèmes de façon séquentielle. En premier lieu, les sites de déversement et les garages sont localisés dans le réseau routier de la ville. Le réseau est ensuite divisé en plusieurs secteurs de déneigement où s'effectuent simultanément les opérations afin d'offrir un certain niveau de service. Une fois les sites de déversement et les secteurs de déneigement définis, le problème est de déterminer vers quel site de déversement la neige de chaque secteur doit être transportée. Les tournées de véhicules et les horaires de travail sont déterminés en dernier lieu. Évidemment, la séparation de la planification stratégique et de la planification tactique en un problème de partitionnement d'un réseau routier en secteurs et un problème d'affectation des secteurs aux sites simplifie l'analyse, mais peut cependant conduire à une solution sous-optimale.

L'objet de cette thèse est le développement de modèles mathématiques et de méthodes d'optimisation pour le partitionnement d'un réseau routier en secteurs de déneigement, l'affectation des secteurs aux sites de déversement, et le routage des véhicules. Nous nous intéressons plus particulièrement au problème combiné du partitionnement et d'affectation pour la planification des opérations d'enlèvement de la neige ainsi qu'au problème de tournées de véhicules pour la planification des opérations de déblaiement des rues.

De nombreuses contraintes régissent le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. D'abord, chaque secteur doit être contigu, c'est-à-dire que le sous-graphe induit par les segments de rues qui le composent doit être connexe. Pour des raisons administratives et opérationnelles, des secteurs non contigus ne sont pas souhaitables étant donné que des passages à vide sont inévitables entre les groupes non adjacents de segments de rues. Ensuite, les secteurs doivent avoir à peu près la même charge de travail afin de garantir que les opérations se terminent en même temps dans tous les secteurs, tout en admettant des ressources équivalentes dans les secteurs. La charge de travail des secteurs est généralement déterminée par le niveau de service requis en termes de limite de temps imposée pour le déneigement des secteurs. Chaque secteur doit également être orienté en arc de cercle autour du site de déversement qui lui est associé de façon à réduire le nombre de camions servant au transport de la neige. Les souffleuses enlèvent la neige tout en remplissant les camions selon un procédé pratiquement continu pour minimiser le temps d'achèvement des opérations: dès qu'un camion est rempli, il se dirige vers un site de déversement tandis qu'un autre prend sa place à côté de la souffleuse. Le nombre de camions utilisé dans un secteur doit donc être suffisamment élevé pour que la souffleuse soit toujours en activité lors des opérations d'enlèvement de la neige. Ce nombre est petit si la souffleuse se trouve près du site de déversement mais il augmente à mesure que la souffleuse s'éloigne du site. Par conséquent, pour réduire le nombre de camions, des secteurs en forme d'arcs de cercle autour des sites sont préférables à des secteurs étroits et profonds. Plusieurs contraintes sont également associées aux sites de déversement. Chaque site possède une limite horaire de capacité (en m^3/heure) qui dépend de la logistique et de la configuration des quais de déversement du site. De plus, un site peut avoir une limite annuelle de capacité correspondant à un espace d'entreposage limité. Certains sites, tels les carrières, ont une capacité horaire élevée à cause de leurs nombreux quais de déchargement. D'autres sites, tels les chutes à l'égout, ont une capacité annuelle illimitée. Toutefois, leur capacité horaire est limitée par la logistique sur le site et par le fait que la température de l'eau du système d'égout ne doit pas être inférieure à 2°C. Finalement, pour des raisons opérationnelles, l'affectation de chaque secteur peut être limitée à un seul site de déversement. L'objectif poursuivi pour les décisions de partitionnement d'un réseau et d'affectation des secteurs consiste à minimiser les coûts variables de transport de la neige dans des camions vers les sites de déversement, les coûts variables d'élimination de la neige aux sites, ainsi que les coûts fixes des camions.

Plusieurs contraintes opérationnelles régissent également le routage des véhicules pour les opérations de déblaiement. D'abord, toutes les classes de rues doivent être desservies en respectant un certain ordre de préséance établi selon le taux de circulation. Les contraintes de préséance sont dites linéaires lorsque l'ordre pour desservir les classes de priorité est unique. Dans ce cas, les grandes artères doivent être desservies en premier lieu, ensuite les rues collectrices et les rues locales en dernier. Les contraintes de préséance sont dites générales lorsque la séquence pour desservir les classes de priorité est partielle. Ces contraintes apparaissent lorsque, par exemple, les grandes artères doivent être desservies avant les rues locales, mais les rues collectrices peuvent être desservies à n'importe quel moment dans la séquence de service. Ensuite, pour réduire le temps d'achèvement des opérations de déblaiement, certaines villes permettent d'augmenter l'ordre de préséance d'une rue non prioritaire. De plus, chaque véhicule peut avoir une restriction sur le type de rues qu'il peut desservir ou traverser. Cette restriction dépend de la taille du véhicule, de sa vitesse et de sa forme. Finalement, contrairement aux opérations d'épandage de fondants et d'abrasifs, puisque les véhicules utilisés pour les opérations de déblaiement ne peuvent pas desservir plus d'une seule voie à la fois, les rues à voies multiples nécessitent donc plusieurs passages de service avant d'être desservies. L'objectif poursuivi pour les décisions reliées au routage des véhicules pour les opérations de déblaiement consiste soit à minimiser le temps d'achèvement des opérations, soit à minimiser successivement les temps pour desservir chaque classe de priorité.

Puisque le problème combiné du partitionnement d'un réseau et d'affectation des secteurs ainsi que le problème de tournées de véhicules n'ont été l'objet que de très peu de recherches, le premier objectif de la thèse est de proposer un cadre de modélisation de chaque problème qui en capte les difficultés fondamentales tout en possédant la flexibilité nécessaire pour l'adaptation à divers contextes pratiques. Un second objectif, intimement lié au premier, est de présenter différentes méthodes heuristiques pour résoudre les modèles proposés et comparer leur performance. Le dernier objectif de la thèse est de démontrer l'utilité pratique des modèles et des méthodes proposées. À cet effet, des tests sont réalisés à partir de données réelles fournies par les villes de Montréal et de Dieppe.

Au premier chapitre, nous présentons une synthèse des contributions au regard des six articles composant cette thèse.

Aux chapitres 2, 3, 4, et 5, nous présentons une revue détaillée de la littérature récente concernant l'emploi de modèles d'optimisation et d'algorithmes de résolution dans le domaine de l'entretien hivernal des réseaux routiers. Cette revue déborde largement du cadre du partitionnement et de l'affectation pour la planification des opérations d'enlèvement de la neige ainsi que du routage des véhicules pour la planification des opérations de déblaiement, et traite de la plupart des problèmes de planification des opérations d'épandage de fondants et d'abrasifs, de déblaiement des rues, et d'enlèvement de la neige. Nous proposons une classification des différents modèles proposés dans la littérature et insistons plus particulièrement sur la structure de ces modèles ainsi que sur les algorithmes utilisés pour les résoudre. Au second chapitre, nous présentons les modèles de détermination du niveau de service et de partitionnement d'un réseau routier en secteurs de déneigement qui sont utilisés pour les opérations d'épandage de fondants et d'abrasifs et/ou de déblaiement des rues. Au troisième chapitre, nous décrivons les modèles de partitionnement d'un réseau routier en secteurs de déneigement, de localisation de sites de déversement, d'affectation des secteurs de déneigement à des sites de déversement, et d'affectation des secteurs de déneigement à des entrepreneurs utilisés pour les opérations d'enlèvement de la neige. Le quatrième chapitre décrit les modèles de routage des véhicules utilisés pour les opérations d'épandage de fondants et d'abrasifs. Une revue des modèles de localisation de garages, de localisation de dépôts de fondants et d'abrasifs, et d'affectation des équipes de travail aux garages est aussi présentée. Finalement, le cinquième chapitre présente une revue des modèles de routage des véhicules, de détermination de la taille de la flotte de véhicules, et d'horaire de remplacement de la flotte de véhicules utilisés pour les opérations de déblaiement et/ou d'enlèvement de la neige.

Le sixième chapitre présente un modèle et une méthode de résolution pour le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. Le modèle incorpore des contraintes sur la contiguïté, l'équilibre et la forme des secteurs de déneigement ainsi que sur les capacités horaire et annuelle des sites de déversement. La méthode de résolution consiste en une heuristique en deux phases qui tient compte de l'interdépendance entre le problème de partitionnement d'un réseau routier en secteurs de déneigement et le problème d'affectation des secteurs aux sites de déversement. Nous comparons également notre approche avec une autre méthode constructive en deux phases qui sépare ces deux composantes.

Au dernier chapitre, nous décrivons finalement un modèle et deux méthodes de résolution pour le routage des véhicules pour les opérations de déblaiement développés en fonction des besoins spécifiques de la ville de Dieppe, Nouveau-Brunswick. Ce modèle tient compte des très nombreuses caractéristiques du réseau, des rues, et des véhicules ainsi que des contraintes opérationnelles imposées sur la configuration des tournées. En particulier, il incorpore des contraintes générales de préséance, des vitesses différentes de service et de passages à vide, des passages répétés obligatoires pour les rues à voies multiples, la possibilité d'augmenter l'ordre de préséance d'une rue non prioritaire, et des restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules. Plusieurs extensions importantes du modèle sont également décrites, telles que l'ajout de pénalités pour limiter l'utilisation de certains types de virages, l'ajout de contraintes d'équilibre pour que les durées des tournées soient approximativement les mêmes, et la possibilité de desservir certaines artères en tandem. Le problème est résolu à l'aide de deux méthodes constructives qui permettent d'alléger le modèle, mais qui entraînent cependant une certaine détérioration de la qualité de la solution.

CHAPITRE 1

SYNTHÈSE DES CONTRIBUTIONS

La contribution des quatre premiers articles est de tracer un portrait récent et complet de l'utilisation de la recherche opérationnelle dans le domaine de l'entretien hivernal des réseaux routiers. En plus de proposer une classification des différents problèmes ayant été étudiés dans la littérature, ils fournissent une description détaillée des principaux modèles en insistant plus particulièrement sur leur structure et sur la méthode de résolution retenue. Les quatre premiers articles et les très nombreuses références qu'ils contiennent constituent donc un excellent point de départ pour quiconque s'intéresse à l'application de la recherche opérationnelle dans le domaine de l'entretien hivernal des réseaux routiers.

La principale contribution du cinquième article est de présenter le premier véritable modèle pour le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. Les résultats obtenus indiquent que ce modèle peut être résolu de manière approximative en des temps de calcul raisonnables. Les tests effectués montrent également qu'une approche tenant compte de l'interdépendance entre le problème du partitionnement d'un réseau routier et le problème d'affectation des secteurs aux sites peut être plus efficace qu'une approche traitant séparément ces deux composantes.

Le dernier article décrit un modèle général et complet pour le routage des véhicules pour les opérations de déblaiement des rues. Le modèle incorpore plusieurs facettes importantes du problème qui n'avaient jamais été prises en compte ensemble par aucun autre modèle. Le niveau de détail considéré dans ce modèle et les résultats obtenus confirment qu'il est possible de développer des modèles relativement complets pour ce type de problèmes, et que ces modèles peuvent être résolus de manière approximative en des temps de calcul raisonnables compte tenu du fait que la planification des tournées de véhicules n'est revue, en pratique, qu'à chaque saison hivernale.

CHAPITRE 2

A SURVEY OF MODELS AND ALGORITHMS FOR WINTER ROAD MAINTENANCE. PART I: SYSTEM DESIGN FOR SPREADING AND PLOWING

Nathalie Perrier, André Langevin et James F. Campbell, *Computers & Operations Research* 33, pages 209–238, 2006.

Les opérations reliées à l'entretien hivernal des réseaux routiers comportent de nombreux problèmes qui n'ont reçu que peu d'attention des chercheurs. Plusieurs raisons peuvent expliquer ce constat. Tout d'abord, les problèmes pratiques rencontrés dans le domaine de l'entretien hivernal des réseaux routiers sont spécifiques à chaque ville à cause des différences géographiques, météorologiques, démographiques, économiques, et technologiques. Les opérations reliées à l'entretien hivernal des réseaux routiers sont également très diversifiées étant donné qu'elles sont affectées par la densité de la population, la topographie des régions, le réseau de transport, les politiques de niveau de service, ainsi que plusieurs facteurs climatiques tels l'importance de la tempête, la température, le vent, et le nombre d'heures de lumière par jour. Ensuite, ces problèmes sont généralement de très grande taille. Enfin, les politiques des villes sont souvent difficiles à traduire en langage mathématique et donnent lieu à des modèles dont la résolution est difficile.

Cet article présente une revue de la littérature concernant l'utilisation de méthodes d'optimisation pour le design des systèmes d'entretien hivernal des réseaux routiers pour les opérations d'épandage de fondants et d'abrasifs et de déblaiement des rues. Nous décrivons d'abord brièvement les opérations reliées à l'entretien hivernal des réseaux routiers. Cette description permet d'introduire une taxonomie des problèmes étudiés. Les modèles décrits dans l'article sont regroupés en deux grandes catégories: les modèles de détermination du niveau de service et les modèles de partitionnement d'un réseau routier en secteurs.

Les problèmes de détermination du niveau de service pour les opérations d'épandage de fondants et d'abrasifs et de déblaiement des rues sont généralement définis en termes de conditions de la chaussée ou d'utilisation des ressources. Les rues sont classées en différentes catégories selon le taux de circulation et la détermination du niveau de service consiste en fait à identifier l'ensemble des politiques de service pour chaque classe de priorité. Certaines politiques visent l'état de la chaussée à la fin des opérations pour chaque classe de priorité dans le réseau. D'autres politiques visent plutôt l'utilisation de l'équipement, le taux d'application des fondants et des abrasifs, et les heures de couverture pour chaque classe de priorité dans le réseau. Le compromis fondamental dans l'établissement du niveau de service se situe entre la minimisation des coûts totaux des opérations, incluant les coûts directs et indirects, et la minimisation des inconvénients pour le public et le commerce.

Les problèmes de partitionnement d'un réseau routier en secteurs pour les opérations d'épandage de fondants et d'abrasifs et de déblaiement des rues comportent des similitudes avec les problèmes de création de circonscriptions électorales, de commissions scolaires, et de secteurs de vente. Toutefois, contrairement à ces problèmes qui ont été l'objet d'innombrables publications en recherche opérationnelle au cours des dernières décennies, les problèmes de partitionnement d'un réseau routier en secteurs pour l'entretien hivernal ne sont parvenus à attirer l'attention des chercheurs que plus récemment. De plus, les problèmes de partitionnement d'un réseau routier sont intimement liés au problème de localisation de garages et au problème de détermination de la taille de la flotte de véhicules. Pourtant, ces problèmes sont généralement traités indépendamment.

A Survey of Models and Algorithms for Winter Road Maintenance. Part I: System Design for Spreading and Plowing

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Abstract

Winter road maintenance operations involve a host of decision-making problems at the strategic, tactical, operational, and real-time levels. Those operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. As the first of a four-part survey, this paper reviews optimization models and solution algorithms for the design of winter road maintenance systems for spreading and plowing operations. System design problems for snow disposal operations are discussed in the second paper. The two last parts of the survey mainly address vehicle routing, depot location, and fleet sizing models for winter road maintenance. The present paper surveys research on determining the level of service policy and partitioning a region or road network into sectors for spreading and plowing operations. We also describe the applied setting in which these problems arise.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Operations research.

2.1 Introduction

Planning the operations of winter road maintenance involves a host of decision problems that can be addressed with operations research methodologies. The importance of these problems is obvious from the magnitude of the expenditures required to conduct winter road maintenance operations, as well as the indirect costs from both the lost productivity due to decreased mobility and the effects of chemicals (especially salt) and abrasives on infrastructure, vehicles and the environment. In the United States, winter road maintenance operations account for a substantial portion of annual operating budgets in many northern jurisdictions, costing state and local governments an estimated $2 billion each year (MINSK, 1998). Expenditures for winter road maintenance in Japan and Europe can be nearly two to three times higher than those in the United States (TRANSPORTATION RESEARCH BOARD, 1995). In addition to the direct expenditures are substantial indirect costs associated with deterioration of infrastructure and vehicles, corrosion, water quality degradation, and other environmental impacts resulting from the use of chemicals for winter road maintenance. These indirect costs are estimated to total more than $5 billion a year in the United States (TRANSPORTATION RESEARCH BOARD, 1995). Additional indirect costs incurred by motorists and businesses include accident costs, delay costs, and lost wages and productivity costs. These costs are difficult to quantify monetarily but can be significant in specific situations. For example, the "Blizzard of '96" (January 1996) in the northeastern United States shut down businesses (and federal government offices) and cost an estimated $20 billion in lost production and lost sales (LORD, 1996).

As highlighted by KUEMMEL (1994), in the past few decades there have been many important developments in winter road maintenance technologies to improve operations and minimize environmental impacts. These developments include use of alternative deicing materials, anti-icing methods, improved snow removal equipment, more accurate spreaders, better weather forecasting models and services, road weather information systems, etc. The microcomputer revolution has also had an immense impact on winter road maintenance practices by enhancing management systems, improving access to weather forecasting information, and aiding in the development of control systems in vehicles for spreading salt and other materials. These advances, and their growing use by state and local government agencies, have improved the effectiveness and efficiency of winter maintenance operations, benefiting government agencies, users, and the general public.

Unfortunately, advances in system design for winter road maintenance have not matched improvements in technology, as indicated by the relatively small number of contributions in the operations research literature. A study by NIXON and FOSTER (1996) to determine the current state of practice in winter road maintenance in the United States found that several needs are not being met, particularly in effectively utilizing new technologies to better plan and manage winter road maintenance operations. This suggests that mathematical optimization techniques, leading to even a small increase in efficiency or effectiveness, could result in substantial savings, improved mobility, and reduced environmental and societal impacts.

In fact, the limited progress of operations research in winter road maintenance highlights the considerable difficulty of the problems studied. Problems facing winter road maintenance planners are especially complex and site specific because of the tremendous differences in initial conditions such as geography, meteorology, demographics, economics, and technology. Winter road maintenance operations are incredibly diverse as they are affected by population density, the topography of a region, the road network, the level of service policies, and the climatic factors of snowfall rate, temperature profile, hours of daylight, and wind. Differences in these conditions necessitate differences in the planning and operation of winter road maintenance.

Winter road maintenance presents a variety of decision-making problems at the strategic, tactical, operational, and real-time levels. The *strategic level* involves the acquisition or construction of long-lasting resources intended to be utilized over a long time period. Decisions related to the partitioning of a region or road network into sectors, the scheduling of fleet replacement, and the location of facilities such as snow disposal sites and vehicle depots may be viewed as strategic. The *tactical level* includes medium and short term decisions that are usually updated every few months. For example, the assignment of sectors to snow disposal sites and the sizing of vehicle fleets could be termed tactical. The *operational level* is related to the winter tasks that require ongoing attention on a day-to-day basis. Various decisions concerning the routing and scheduling of vehicles and the staffing of such vehicles with crews belong to the operational level. Finally, the *real-time level* involves decision-making situations in which operations must be undertaken or altered in a very short time frame (e.g., minutes) in response to the sudden change of the system (equipment breakdowns, weather change, etc.). The modification of routes based on new weather and pavement information is an example of real-time control. Some decision problems may be viewed at different levels according to the planning horizon considered. For example, decisions related to the assignment of sectors to snow disposal sites could be termed tactical since they are usually updated every winter season. However, these decisions belong to the operational level when monthly adjustments must be made to account for snowfall variability.

This paper is the first part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. The aim of the two first parts is to provide a comprehensive survey of optimization models and solution methodologies for the design of winter road maintenance systems. These problems include determining the level of service policy, partitioning a region or road network into sectors, locating winter maintenance facilities (snow disposal sites, vehicle depots, and materials depots), allocating sectors to snow disposal sites, allocating sectors to private companies or governmental agencies, and sizing and replacing vehicle fleets. In this paper, the contributions dealing with the level of service policy and the design of sectors with regard to spreading and plowing operations are reviewed. System design models for snow disposal operations are discussed in the second part of the survey (PERRIER *et al.*, 2006b). The two last parts (PERRIER *et al.*, 2005a,b) mainly concentrate on vehicle routing, depot location, fleet sizing, and fleet replacement problems but several other important winter road maintenance problems are also considered.

The paper is organized as follows. Section 2.2 describes the techniques and operations of winter road maintenance, the system design problems related to spreading and plowing operations, particularly the level of service policy and the sector design problem, and the critical factors that have an impact on winter road maintenance operations. Models dealing with the level of service policy are reviewed in Section 2.3. Models that address the partitioning of a region or road network into sectors for spreading and plowing operations are described in Section 2.4. Conclusions and directions for future research are presented in the last section. The terms roadways, highways, roads, and streets are used interchangeably throughout the text with the intent that the models discussed in the survey are usually applicable to all levels of jurisdiction.

2.2 Winter road maintenance

The following section contains a brief description of winter road maintenance techniques and operations. A more detailed review of the available technology for winter road maintenance, and the scientific underpinnings of that technology, is presented in the book by MINSK (1998). For further details on winter road maintenance state-of-the-art technologies and processes in North America, Europe and Japan, see THE AMERICAN ASSOCIATION OF STATE HIGHWAY AND TRANSPORTATION OFFICIALS (1999), PÖYRY (2002), KUEMMEL (1994, 1999), and NIXON and FOSTER (1996). Following the description of winter road maintenance techniques and operations, this section describes the system design problems that have been addressed by operations researchers. This section concludes with a discussion of the operating conditions that have a strong impact on winter road maintenance operations.

2.2.1 Winter road maintenance operations

As was highlighted by MINSK (1998), winter road maintenance techniques to clear a roadway of snow can be classified into three categories: chemical, mechanical, or thermal. Chemical methods include the application of a freezing-point depressant on a surface and incorporation of the freezing-point depressant within the surface itself. Chemicals are applied to pavements to melt ice that has formed on pavement (deicing), to prevent formation of ice (anti-icing), and to prevent the building of pack, namely snow compacted by traffic action that becomes nearly as tightly bonded to pavement as ice, and that is frequently much thicker and more irregular. Several chemicals are available, but salt is most commonly used because of its low cost, ready availability, ease of application, high solubility in water, and effectiveness as a melting agent at temperatures near 0°C. The determination of the application rate depends on the level of service required, weather conditions, form (liquid or solid) and characteristics of the chemicals used, time of application, traffic density at the time of, and subsequent to, chemical application, as well as topography and the type of road surface.

The objective of mechanical removal is to pick up the snow that is loose or not bonded to the pavement surface from the road, shearing it from the road if necessary, and cast it to a storage area off the road. Mechanical methods include plowing and brooming. The role of snow plowing in either deicing or anti-icing operations is to remove as much snow and loose ice as possible before applying chemicals. Brooming can significantly reduce the need for chemical methods and is most effective on areas that receive little or no traffic between broomings.

Thermal methods involve applying heat to the roadway surface from either above or below to remove snow and ice or to prevent its formation. The purpose of heating a pavement is to reduce traffic delays, personal injury, and property damage from accidents caused by black ice, glaze ice, or packed snow. The cost of installing a fixed heating system and operating it, or of making and operating a mobile heating apparatus, is too high for general use. Critical locations where heating systems are usually installed include bridge decks, toll plazas, on and off ramps, and steep grades.

Chemicals assume a major role for winter road maintenance because of their effective performance and relatively low cost in comparison to alternatives. However, most of the adverse effects on infrastructure, vehicles and the environment from winter road maintenance operations stem from the use of salt and other materials for deicing. The main side effects (and indirect costs) of salting are motor vehicle corrosion, infrastructure damage, degradation of the roadside vegetation, and sodium infiltration of drinking water. Environmental problems arising from the use of ice control chemicals are reviewed by MINSK (1998).

Sand and other abrasives are also commonly used in winter road maintenance. Although they are often mixed with salt and other chemicals for deicing purposes, abrasives are used primarily to improve traction, particularly when pavement temperatures are too low for chemical treatments to be effective. However, abrasives are not chemicals and neither prevent or break the bond between pavement and ice. According to KETCHAM *et al.* (1995), application of abrasives provides no significant increase in friction or improvement in pavement condition on a road receiving properly timed anti-icing treatments. Abrasives are inexpensive, but they can be difficult to apply and they have several potential negative consequences (damage to cars, need for additional highway cleanup operations, and airborne dust problems).

The response to winter precipitation on a road depends on the type and magnitude of the precipitation. If the frozen precipitation is snow of depth sufficient to impede pedestrian and vehicular traffic, plowing is the appropriate response. If the precipitation is ice or a thin snow layer, spreading of chemicals and abrasives may be the best method. Effective spreading of chemicals and abrasives at the proper time, in the proper manner, and at proper locations is a critical operation. The decision must take into account such factors as type of snow (wet or dry), expected temperature conditions at the time of, and following, application, anticipated variations at the critical freeze-thaw point, methods of application, and types of material. Ice may be removed from pavement through a combination of plowing, chemical treatment, natural melting, and traffic action. However, because ice adherence to pavement is often quite strong, removal by mechanical means becomes very difficult and often impossible, even with repeated passes by plows. Weakening the ice-pavement bond by heat or by a deicing chemical becomes necessary, so that the resulting ice sheets can then be removed mechanically by plowing or traffic action.

In urban areas, the large volumes of snow cleared from roads and walkways may exceed the available space along roadways and walkways for snow storage, and therefore require disposal by some means. Loading snow into trucks for hauling to disposal sites is the most common solution. Loading and hauling of snow are usually post-storm operations, although they may be required during the precipitation to remove snow from alleys, narrow channelled sections, and other areas where there is no space for snow storage. Snowblowers or rotary plows are commonly used to pick up snow to load into trucks. The trucks may be adjacent to, or in some cases following, the vehicle loading it, though adjacent trucks will further restrict traffic during the operation (MINSK, 1998). Hauled snow is often dumped into rivers and other water bodies, or on large open fields. Where there is sufficient flow in the sewer system and the volume of snow is not too great, dumping the snow into sewers is a practicable snow disposal method (this method is in use in Montréal, Canada). Generally, however, the snow must be melted or reduced to slush before it is discharged into a sewer system. Fixed melting pits are installed in some cities, notably in eastern Canada (MINSK, 1998), or mobile snow melters can be transported to a problem area. Surface and underground flowing water systems are also used to carry snow away. In Sapporo, Japan, channels are constructed along streets, and river water or treated sewage diverted to them carries away snow dumped in them, rather than hauling the snow to disposal sites (CITY OF SAPPORO, 1991). These snow disposal methods all present different costs and benefits. Some of the issues are haul distances, traffic patterns into and out of disposal areas, the effect of snow melt on stream flows, and the impact of contaminants in the snow and ice plowed from roadways on the environment when the accumulations melt and flow into surface waters or seep into the groundwater.

Many agencies have parking regulations to facilitate winter road maintenance operations. Examples are regulations that prohibit street parking at all times on designated snow routes, allow alternate side street parking, prescribe alternate times for parking, and ban overnight parking. These restrictions are often limited to critical roads and streets that carry large traffic volume during the winter months or only during a snow emergency. The principal advantage of the winter-long set of parking restrictions is that road maintenance operations can quickly begin after a storm without encountering delays waiting for a snow emergency declaration and for people to move their vehicles.

Not all jurisdictions perform all winter road maintenance operations each winter. Spreading of chemicals and abrasives and snow plowing are operations common to almost all road networks that experience significant snowfall or frozen precipitation. Snow loading and hauling are generally performed on a regular basis only in urban areas with large snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow loading and hauling in response to infrequent but very heavy winter storms.

The fundamental operations in winter road maintenance involve a range of different benefits and cost. Safety effects and impacts on local economies that should be considered in valuing the benefits of winter road maintenance programs include savings in accident costs, in delay costs, and in lost wages and productivity costs. Adverse effects and indirect costs of winter road maintenance programs include the effects of deicing chemicals on the environment, infrastructure, and motor vehicles. The most direct cost of winter road maintenance is the expenditure required (generally of regional and local governments) to conduct winter road maintenance operations. Table 2.1 presents the major fixed and variable labor, materials, and equipment costs for each operation.

Table 2.1: Operating costs in the context of winter road maintenance

	Operations		
Costs	**Spreading**	**Plowing**	**Loading and hauling**
Variable costs	Fuel costs Crew costs Vehicle maintenance Material costs Variable costs of vehicle depots Variable costs of materials depots	Fuel costs Crew costs Vehicle maintenance Variable costs of vehicle depots	Fuel costs Crew costs Vehicle maintenance Variable costs of vehicle depots Variable costs of disposal sites
Fixed costs	Fixed costs of spreaders Fixed costs of vehicle depots Fixed costs of materials depots	Fixed costs of plowing equipment Fixed costs of vehicle depots	Fixed costs of loading and hauling equipment Fixed costs of vehicle depots Fixed costs of disposal sites

Since some vehicles, equipment and infrastructure may be shared among different winter road maintenance operations, and some may be used for non-winter operations, the allocation of costs for winter road maintenance is a challenging task. Over the years, many analytical models were proposed to help planners in properly allocating winter maintenance funds. For example, MILLER (1970) described a multiple regression model to predict labor, equipment and material costs for snow and ice control on various roadway types for each county of the Ohio state highway system for given snowfall amounts and traffic volumes. DUNLAY (1971) proposed a simultaneous equation stochastic model to explain or predict U.S. county expenditures for winter road maintenance based on selected measures of a county's need for spreading and plowing operations. DUAAS *et al.* (1994) described an analytical model to help planners at the directorate of public works in Norway secure a fair distribution of the national funds allocated to winter maintenance among the 19 counties. For each winter road maintenance operation and for each road section, the maintenance cost is calculated as a product of three elements. The first element is a quantity estimating the number of passes multiplied by the road length. The second element is the frequency of the maintenance operation during a year. The unit cost for providing personnel and equipment for the operation is the last element. The model also serves as a tool for analysing the impact of changes in factors that affect the unit cost. Finally, THORNES (1999) reviewed the literature on the benefits and costs of the use of salt on roads and proposed a benefit/cost model for the salting of roads in the United Kingdom.

2.2.2 System design problems for spreading and plowing

System design in winter road maintenance includes determining the level of service policy, partitioning the geographic region into sectors for efficient operations, locating the needed facilities (vehicle depots, materials storage facilities, and disposal sites), assigning the sectors obtained from the partitioning to various facilities, allocating contracts for various operations to private organizations, and replacing and sizing the vehicle fleet. This section describes two system design problems related to spreading and plowing operations that have been addressed by operations research techniques: the level of service policy and the sector design problem. Operations research models for those two types of problems are reviewed in Sections 2.3 and 2.4, respectively. System design problems related to snow disposal operations are described in the second part of the survey (PERRIER *et al.*, 2006b).

The level of service for spreading and plowing operations is generally defined in terms of snow conditions on the roadway, including evenness and wetness, and pavement skid characteristics. These conditions and characteristics are controlled by the type and frequency of winter road maintenance operations. A high level of service is characterized by little snow accumulation before plowing, absence of an ice-pavement bond during precipitation, and a rapid return to near normal road surface conditions after the precipitation ends. A low level of service may involve snow plowing only once after the precipitation is over. In this case, the road surface may be very slippery and very uneven with possible projecting bumps, making passage difficult in some places. In general, the higher the level of service, the greater the resource investment. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a *service hierarchy*, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. *Class continuity* requires that each winter maintenance vehicle route be homogeneous in class to allow for a clear hierarchy in the importance of a particular route. In urban areas, streets serving bus routes, hospitals, firehouses, schools, and similar important places are often given a high priority. In rural areas, school bus routes also rate a high priority. Note that priority treatment locations for spreading operations may not be the same as priority plowing locations.

Because of the difficulty and impracticability of organizing winter road maintenance operations in a wide geographical region, the spreading and plowing operations are generally carried out concurrently by separate crews and equipment in many small subareas. The *sector design problem* consists of partitioning a large geographic region or road network into non-overlapping subregions or subnetworks, called *sectors*, according to several criteria related to the operational effectiveness and the geographical layout. It is often more convenient to partition a transportation network in a region instead of the region itself. In that case, the sectors correspond to subnetworks in the larger network. The number of sectors to design may be given or may be part of the design. Moreover, one or several physical facilities such as disposal sites, vehicle depots, or chemical and abrasive depots are generally specified.

Common criteria for designing sectors for winter road maintenance include *compactness* or *shape*, *balance in workload*, and *contiguity*. The compactness or shape criterion depends on the number of sectors, the number and type of facilities, and the type of winter road maintenance operations. If the number of sectors to be designed corresponds to the number of facilities, then compact sectors with centrally located facilities lead to more efficient routing of vehicles for winter road maintenance operations. However, if the number of sectors exceeds the number of facilities, then the appropriate shape of a set of sectors depends on the type of winter road maintenance operations. The general guideline for forming sectors for efficient routing of vehicles for chemical spreading and snow plowing is that sectors should be elongated towards the vehicle depot or materials depot to reduce travel distance in each route. Conversely, for loading and hauling snow to disposal sites, sectors should be elongated in a direction perpendicular to the direction to the disposal site to reduce the number of trucks required (LABELLE *et al.*, 2002).

To balance the workload across sectors, they are often approximately the same size and are assigned equivalent resources (equipment and manpower). This helps ensure that operations will be completed at the same time in all sectors. The size of the sectors is usually determined by the level of service required and the operating capabilities of the equipment and manpower. Size and workload can be measured in terms of the number of basic entities, length of streets, or annual amount of snow. Basic entities or *basic units* are the units of analysis used to design sectors and can be defined either as single street segments or as small geographic zones. A micro approach to design sectors is to use a single street segment as the unit of analysis. However, for large road networks, this approach may lead to long computation times. One tractable approach is to define the basic units as geographic zones containing a collection of neighboring street segments. The contiguity criterion requires that sectors do not include distinct parts separated by other sectors. Non-contiguous sectors are undesirable from an administrative standpoint and from an operational standpoint given that deadheading trips would be necessary between the disjoint collections of street segments of each non-contiguous sector. *Deadheading* occurs when a vehicle must traverse a road segment without servicing it. Sector design may also need to conform to existing infrastructure (roadways, rail lines, bridges, etc.), geography (for example, rivers, hills, etc.), and jurisdictional boundaries. At a strategic planning level, the same sectors should usually be used for different types of winter road maintenance operations. Consequently, sectors should be robust and not influenced by minor changes in the characteristics of the operations performed within the sectors. Conversely, at the tactical and operational levels, different guidelines should be used to design sectors for spreading, snow plowing, loading trucks, and hauling snow to disposal sites.

2.2.3 Operating conditions

Many factors influence the conduct of winter road maintenance operations. Winter road maintenance methods vary widely according to geographical location and climatic conditions. Mechanical methods are the primary method used in North America whereas chemical treatment is the most effective in the British Isles, where the main problems arise from very light snowfalls or from thin layers of ice which form after water condenses on the roads and freezes. Also, in areas without prolonged sub-freezing temperatures, snow that is plowed from streets and sidewalks may quickly melt and need not be hauled away.

In addition to the climatic characteristics of the area, accurate and timely information on current weather conditions and forecasts of emerging conditions are critical to ensuring efficient and effective winter road maintenance operations. With access to good weather information, agencies can better time chemical treatments and more effectively deploy winter road maintenance equipment, which can lead to reducing expenditures and adverse environmental effects by eliminating unnecessary applications of salt and other chemicals and abrasives. For example, future warm weather may obviate the need for snow disposal, while a forecast for a second soon-to-arrive snowstorm may cause an agency to wait until after the subsequent storm to begin loading snow into trucks and hauling it to disposal sites.

Technological innovations can potentially improve the effectiveness and efficiency of winter road maintenance operations and, in some cases, reduce costs associated with that maintenance. NIXON and FOSTER (1996) present a review of new technologies related to winter road maintenance and methods for integrating these technologies into current practice. New technologies include materials application (deicing, anti-icing), mechanical methods, and weather information systems. Weather information capabilities such as road weather information systems, radiometers, and thermal mapping can lead to accurate lead-time for mobilization of resources to specific points of the road network. This results in less patrolling by maintenance crews, more timely spreading of chemicals and abrasives, and guidance on when winter road maintenance operations can begin to wind down. Geographical information systems and geographical positioning systems are also valuable tools for planning winter road maintenance operations. Geographical information systems can show what roads have been chemically treated or plowed, and geographical positioning systems can show the location of vehicles in real time. Advances in weather forecasting are also important and it is becoming more common to use "nowcasting" rather than forecasting to predict weather conditions. Nowcasting refers to the use of real-time information for short-term forecasting of the probable weather and pavement conditions or temperatures within one or two hours. Nowcasting is a valuable tool for deciding when to call-in and discharge personnel. A more detailed account of technology now available to provide real-time information about the microclimate of road segments to meteorologists can be found in the book by MINSK (1998).

Winter road maintenance operations are increasingly influenced by legislative requirements. Agencies in some states and local jurisdictions in North America are required by law to curtail use of deicing chemicals. Many reports have appeared during the last several years describing the effects of deicing chemicals on soils, vegetation, and structural materials. The legislation has led to experimentation with reduced salting programs and alternative chemicals and application methods, including anti-icing. Demands that highway agencies reduce salt use to control adverse environmental effects have also raised concerns about the potential effects on service levels, safety, and operations.

Many other factors influence the conduct of winter road maintenance operations, including demographics, economics, interagency cooperation, traffic information, and resource information. A region with a large population and a number of high-traffic roads has different priorities and requirements, than a very rural region with low population density. Information such as the variation of traffic rate throughout a twenty-four hour period is important for making operational decisions. Plowing and spreading of chemicals during storms should be made prior to peak traffic intervals. Information on the status of personnel, equipment and materials provides the basis for decisions relative to treatment and resource allocation options such as reallocation of personnel and equipment, interagency agreements to provide increased capability, and identification of materials depots and disposal sites that can be accessed from neighboring agencies.

This diversity of operating conditions dictates the wide variety of winter road maintenance methods and operations. Figure 2.1 summarizes the decisions and the operating conditions surrounding the winter road maintenance operations.

OPERATING CONDITIONS →	DECISIONS →	OPERATIONS
Geography	Level of service	Spreading of chemicals and abrasives
Climate	Snow disposal assignment	Snow plowing
Meteorology	Contract assignment	Loading snow into trucks
Demographics	Sector design	Hauling snow to disposal sites
Economics	Vehicle depot location	
Technology	Materials depot location	
Environment	Snow disposal site location	
Legislation	Fleet sizing and replacement	
Interagency cooperation	Crew assignment	
Traffic information	Crew scheduling	
Resource information	Vehicle routing	

Figure 2.1: Winter road maintenance operating conditions, decisions, and operations

2.3 Service level models

The level of service for spreading and plowing operations is generally defined in terms of desired results, resources to be used, or both. As explained in Section 2.2.2, roadways are generally classified into several categories based on traffic volume and the level of service is actually a set of policies for each class of roadways. *Results-oriented policies* specify how different classes of roadway should appear after winter road maintenance operations, namely bare pavement, bare wheel paths, slush-covered roadways, or snow-packed roadways. *Resource-oriented policies* specify the level of equipment usage, chemical and abrasive application rates, and hours of coverage that should be provided on each roadway class. Agencies may also have service policies that specify both desired results and resources to be used according to roadway class. For example, level of service guidelines may define several highway types based on traffic levels, each with different equipment coverage times and recommended pavement conditions. The fundamental tradeoff in establishing level of service policies is between minimizing the total costs of winter road maintenance, including the direct and indirect costs, and maximizing the benefits of winter road maintenance for safer travel and smaller delays.

Since level of service policies constitute an important strategic component of winter road maintenance operations (affecting sector design, depot location, and fleet sizing), models developed to assess and quantify level of service policies for spreading and plowing operations are reviewed in this section. Analytical and optimization models dealing primarily with resource-oriented policies are reviewed first, followed by optimization models that address results-oriented policies.

2.3.1 Resource-oriented models

Several models were proposed to assess and quantify resource-oriented level of service policies by road segment or highway class. Early work was contributed by ROSS and MILLER (1971) who proposed a multiple regression model to predict the maximum time to service a road for given values of storm duration, snowfall amount, traffic volume, temperature, percentage of daylight, wind velocity, route priority, and time interval between start of storm and initiation of maintenance operation on the road. LINDSEY and SEELY (1999) described a multicriteria approach to the problem of classifying the road segments of a network into various categories for snow plowing. Each road segment is evaluated on six criteria: average annual snowfall, estimated highest elevation of the road, annual average daily traffic, pavement type, functional classification (interstate, national highway system or state road), and number of lanes. A weighted additive multicriteria value is calculated for each road segment, and these values are grouped into separate categories. Each category covers a specific range of values, and all road segments falling into a single category receive the same level of service. A method to survey citizens to assess the appropriate level of service was described by HAYASHIYAMA *et al.* (2001). Results are based on how much respondents indicate they would pay to achieve an improved level of service, and how much compensation they would require if the level of service was reduced. Finally, ADAMS *et al.* (2003) described a comprehensive set of resource-oriented performance measures implemented in the state of Wisconsin, United States, for analyzing winter road maintenance level of service policies, for evaluating performance of materials, labor, and equipment, and for developing reliable evidence of compliance with standards and policies. The measures are computed from data collected by differential global positioning system receivers and sensors on winter maintenance vehicles.

SAGE (1979) proposed an optimization model to determine a cost minimizing service level. Let x be the snow removal rate expressed as centimeters per hour. Let b and m represent two delay cost parameters (detailed by RUSSELL (1971)), and let h and g represent two winter road maintenance cost parameters estimated by simulation. Define also L_{max} as the maximum allowable service level. The optimization model is a nonlinear programming model stated as follows.

$$\text{Minimize} \quad bx^m + h \ln x + g \tag{2.1}$$

subject to

$$0 < x \le L_{\max}. \tag{2.2}$$

The objective function (2.1) seeks to minimize the sum of the total cost defined by delay costs for travel and winter road maintenance costs. Constraint (2.2) requires that variable x assumes a positive value not larger than the maximum allowable service level. SAGE (1979) gave the second-order sufficient conditions for a strict relative minimum service level of the total cost function.

In order to balance the workload among various facilities or sectors, DECKER *et al.* (2000) developed a measure of a maintenance facility's efficiency in performing winter road maintenance operations. The approach is to normalize winter road maintenance costs for labor, equipment, and spreading materials for a specific maintenance facility by considering the number of lane-kilometers of given service levels in the service area and a storm severity index. The storm severity index is calculated from weighted daily snowfall totals and minimum and maximum temperatures.

Decision support systems have also been developed to help planners in determining the appropriate level of service. For example, MCBRIDE (1978) described a decision support system to aid planners in the selection of level of service for a specific storm, based on incremental cost-benefit ratio analysis. The benefits considered when choosing a high level of service over a lower one are defined by the cost savings of traffic delay and safety. The additional costs required to achieve the higher level of service are defined by the additional equipment costs, material costs and manpower costs. Also, KERANEN (2002) described an operations management system to help planners in the Minneapolis-St. Paul metropolitan area in evaluating times to regain bare pavement according to three highway types based on traffic levels (annual average daily traffic). For each class of roads, thresholds for acceptable "regain" times for bare pavement are defined based on previous data on regain times, decision rules dictated by experience, sample surveys of citizens, and weather conditions. The system, coupled with data from road weather information systems, is used for both planning and controlling winter maintenance operations by providing guidelines for service level performance.

Some agencies have level of service policies that emphasize both desired results and resources to be used. For example, ROUSSEL (1994) described service policies implemented in France that contain both performance and coverage guidelines. The country has defined thresholds for acceptable snow levels and pavement conditions during a snowstorm, and times to regain bare pavement. Also, BAROGA (2001) proposed two performance measures that emphasize both desired resources and results for rating the level of service provided on a road segment: the condition of the road segment and the elapsed time from the end of precipitation to attainment of bare pavement. Scores for different pavement conditions and different hour thresholds are aggregated to assess the service level delivered on each road segment.

2.3.2 Results-oriented models

UNGERER (1989) described a solution approach to select the sequence of winter road maintenance operation alternatives that gives the lowest maintenance cost over a storm duration planning horizon considering the given snowstorm type and roadway class, while ensuring that the road surface condition be dry by the end of the storm. The type of a snowstorm depends on many variables such as snowfall rate, storm duration, snow accumulation on the pavement, temperature, and visibility. The problem is modeled as a deterministic dynamic program. In this model, a stage is defined as the cumulative total number of hours completed and the decision variable at a given stage is the winter road maintenance operation to begin. The states associated with each stage are defined in terms of the road surface condition. Thus, the model tends to be results-oriented. Figure 2.2, taken from UNGERER (1989), provides an acyclic graph depicting the possible road surface condition (states) for each stage of a winter storm of three hour duration and one clean-up period. At each node, at most three decisions can be imposed. These decisions, which are "do nothing", "spread sand", and "plow snow", use arcs represented by full lines, short dotted lines, and long dotted lines, respectively. With every arc in the network is associated a cost which depends upon the cumulative number of hours completed and whether dry, hard-pack or slush-covered pavement states are associated with the two arc endpoints. The initial pavement condition and final desired state are both specified as dry.

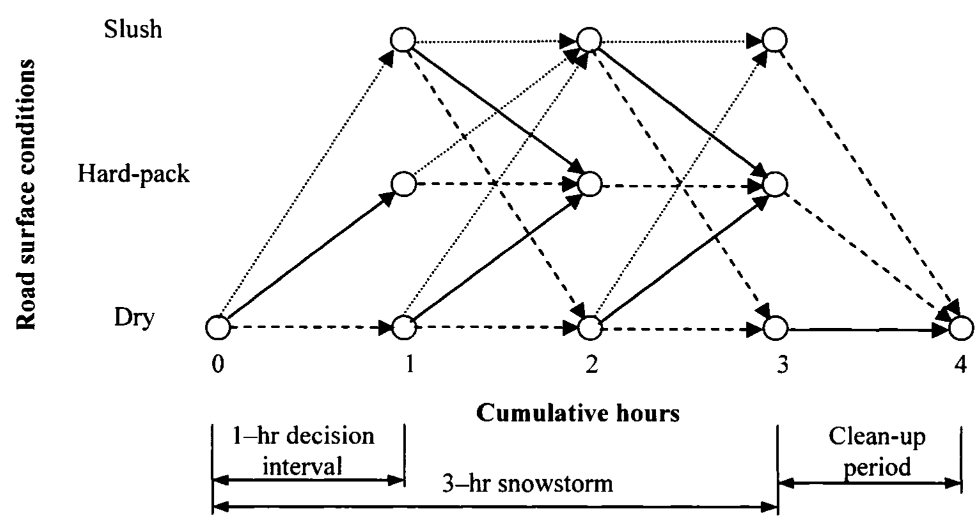

Figure 2.2: Possible road surface condition (states) for each stage

Every path in the network from the starting node (initial state) at stage 0 to the end node (final state) at stage 4 specifies a strategy indicating which winter road maintenance operation to begin in each time interval. Since the cost of a strategy sums the cost at each stage, the total cost corresponds to the length of a path from the starting node to the ending node. The minimum-cost strategy then is just the shortest path.

The author also discussed the use of the model in a multiobjective arena where many possibly conflicting or competing objectives need to be considered such as the minimization of maintenance cost, road user cost or congestion, and the maximization of vehicle volume-to-capacity ratio or safety index. The multiobjective version of the strategy selection problem is solved using compromise programming. Let I be the set of objectives. For each individual objective $i \in I$, the best shortest paths (or longest paths as appropriate) in the acyclic directed network associated with the strategy selection problem are identified first. Let P be the set of paths. The set $J \subseteq P$ of noninferior strategies is then identified. For each objective $i \in I$, define λ_i and z_i^* as the nonnegative weight for objective i, $\sum_{i \in I} \lambda_i = 1$, and the normalized length of a shortest path (or a longest path as appropriate) in the acyclic network in terms of the individual objective i, respectively. For each objective $i \in I$ and for each strategy $j \in J$, define z_{ij} as the normalized length of path j in terms of objective i in the acyclic network. For each strategy $j \in J$, let D_j be the square root of the total weighted sum of square deviations for strategy j calculated according to (2.3). The strategies with the smallest criteria D_j represent the best compromise strategies. The method was tested on a small hypothetical problem based on the network of Figure 2.2.

$$D_j = \sqrt{\sum_{i \in I} \lambda_i^2 \left(z_i^* - z_{ij}\right)^2} \,. \qquad (2.3)$$

In the same paper, UNGERER (1989) extended his solution methodology for simultaneously deciding the optimal strategy for various expected annual roadway classes and snowstorm types subject to a budgetary constraint as well as material, equipment and manpower availabilities. This strategic problem is formulated as a 0-1 linear integer program with an objective function that maximizes the total hourly vehicular traffic rate, which is defined as the total number of vehicles per hour circulating through a section of each roadway class during each snowstorm type. This objective function is weighted to provide superior maintenance to certain roadway-storm combinations, such as high-volume roads during severe storms. Let I be the set of roadway-storm combinations. For every roadway-storm combination $i \in I$, let G_i be the acyclic directed network associated with the strategy selection problem for roadway-storm combination i, and let S_i be the set of noninferior strategies identified by solving the multiobjective problem in the associated network G_i. The multiobjective version of the strategy selection problem is solved using the method of compromise programming described above. Also, for every roadway-storm combination $i \in I$, define W_i as the weight associated with roadway-storm combination i. For every roadway-storm combination $i \in I$ and for every strategy $j \in S_i$, let x_{ij} be a binary variable equal to 1 if and only if strategy j is assigned to roadway-storm combination i. Also, for every roadway-storm combination $i \in I$ and for every strategy $j \in S_i$, let M_{ij}, c_{ij}, g_{ij}, and n_{ij} represent the hourly vehicular traffic rate, the maintenance cost per lane-kilometer per storm, the material utilization per lane-kilometer per storm and the number of workers per snowplow and spreader of applying strategy j to roadway-storm combination i, respectively. Let K be the set of regions considered. For each roadway-storm combination $i \in I$ and for each region $k \in K$, define e_{ik} and l_{ik} as the expected yearly number of storms and the number of roadway lane-kilometers associated with roadway-storm combination i in region k, respectively. For every roadway-storm combination $i \in I$, for every strategy $j \in S_i$, and for every region $k \in K$, let t_{ijk} be the number of snowplows and spreaders required per lane-kilometer per storm of applying strategy j to roadway-storm combination i in region k. Finally, let C, G, T, and N be the annual budget level, the annual material availability, the number of snowplows and spreaders available per storm, and the number of workers available per storm, respectively. The extended model for the strategy selection problem can be stated as follows:

$$\text{Maximize} \quad \sum_{i \in I} \sum_{j \in S_i} W_i M_{ij} x_{ij} \qquad (2.4)$$

subject to

$$\sum_{j \in S_i} x_{ij} \leq 1 \qquad (i \in I) \qquad (2.5)$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} e_{ik} l_{ik} c_{ij} x_{ij} \leq C \qquad (2.6)$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} e_{ik} l_{ik} g_{ij} x_{ij} \leq G \qquad (2.7)$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} l_{ik} t_{ijk} x_{ij} \leq T \tag{2.8}$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} l_{ik} t_{ijk} n_{ij} x_{ij} \leq N \tag{2.9}$$

$$x_{ij} \in \{0,1\} \qquad (i \in I, j \in S_i). \tag{2.10}$$

The objective function (2.4) maximizes the total weighted hourly vehicular traffic rate. Constraints (2.5) require that at most one strategy be applied to each roadway-storm combination. Annual budget level and annual material availability are respected via constraints (2.6) and (2.7), respectively. Constraints (2.8) and (2.9) ensure that the number of snowplows, spreaders and workers available per storm be respected. Finally, all x_{ij} variables are restricted to be binary.

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1. *Strategy selection phase*
   a. For each roadway-storm combination $i \in I$, find the maximum-vehicular traffic rate strategy by solving a longest path problem in G_i. If the resource availabilities are satisfied, then STOP. The strategy assignment is optimal.
   b. For each roadway-storm combination $i \in I$, select the best compromise strategy $j \in S_i$ with the smallest criteria D_j.
   c. If the resource availabilities are satisfied, go to step 2. Otherwise, go to step d.
   d. Order the criteria from the largest to the smallest and select the roadway-storm combination i with assigned strategy j at the top of the criteria list.
   e. Select the best compromise strategy $s \in S_i$, $s \neq j$, $D_s \geq D_j$. If such a strategy exists, reassign strategy s to roadway-storm combination i and return to step c. Otherwise, move down the criteria list and repeat step e.
2. *Improvement phase*
   a. For each roadway-storm combination i with assigned strategy j, let $c(i) = \max_s \{W_i M_{is} - W_i M_{ij} \mid j, s \in S_i, s \neq j\}$ represent the best objective function improvement realized by reassigning strategy s to roadway-storm combination i while satisfying resource availabilities.
   b. Select the roadway-storm combination i such that $c(i) = \max_{l \in I} \{c(l)\}$. If $c(i) > 0$, reassign strategy s to roadway-storm combination i.
   c. Return to the beginning of step 2 until no improvement is obtained.

Figure 2.3: The two-phase heuristic for the strategy selection problem (UNGERER, 1989)

This model was applied to a very small instance and solved with the two-phase heuristic detailed in Figure 2.3. The first phase finds an initial, possibly infeasible, solution by assigning the best compromise strategy to each roadway-storm combination. A feasible solution is then obtained by iteratively considering the next best compromise strategy for the roadway-storm combination with the largest criteria that satisfies the resource availabilities. Note that if the maximum-vehicular traffic rate strategies found by solving a longest path problem in each of the $|I|$ acyclic directed networks associated with the roadway-storm combinations constitute a feasible solution to model (2.4)–(2.10), then this solution is optimal. The second phase is an exchange procedure that tries to improve the solution by iteratively identifying the combination-strategy pair that yields the best improvement. This is done as long as the total value of the objective function (2.4) increases.

2.4 Sector design models for spreading and plowing

The design of sectors consists in partitioning a region or transportation network into a mutually exhaustive and exclusive collection of small sectors according to several criteria related to the operational effectiveness and the geographical layout. As explained in Section 2.2.2, each sector must be a contiguous collection of basic units, balanced in workload, and appropriately shaped according to the operations. Several criteria may also be used to assess the quality of the sector design. One important criterion is that physical facilities such as materials and vehicle depots should be centrally located (relative to sectors they serve) for efficient routing of spreaders and plows. KANDULA (1996) studied the impact of the location of depots on deadheading. Computational experiments indicated that centrally located depots appear to result in routes with less deadheading than non-centrally located depots. Alternatively, a depot may merely serve as a home base, but not necessarily as a sector center. Other commonly used criteria for sector design include: the unicursality of the graph generated by the arcs and edges of each sector to enable routes with less deadheading, the minimization of the fleet size, the assignment of both directions of a two-way segment to the same sector, respect of natural boundaries, respect of some existing administrative or political subsectors, and similarity to the existing network partitioning plan. Typically, sector design problems involve medium and long-term planning decisions. Thus, sector design plans may have long-term consequences on the overall efficiency and effectiveness of the operations to be performed within the sectors.

The sector design problem in spreading and plowing operations is similar to the arc partitioning problem studied by BODIN and LEVY (1991) in the context of postal delivery. The sector design problem also shares several characteristics with districting problems for arc routing applications such as the design of service regions among one or more vehicle depots when each vehicle can visit several clients in a tour (WONG and BEASLEY, 1984; NOVAES and GRACIOLLI, 1999; GOLDEN and WASIL 1987) and the design of sectors for refuse collection (MALE and LIEBMAN, 1978; SILVA GOMES, 1983; HANAFI *et al.*, 1999). In comparison with other districting applications such as political districting, sales territory alignment, health care districting, school district design, and emergency services, the problem of districting in connection with vehicle routing for collection or distribution services has received very little attention. One of the main difficulties is to incorporate aspects related to the efficiency of vehicle routing within the solution method. This difficulty is also present in sector design problems for winter road maintenance operations.

Optimization models for sector design can be grouped into categories according to the winter road maintenance operations considered. Sector design models for spreading operations are reviewed first, followed by compound sector design models that address the sector design, vehicle depot location, and fleet sizing for both spreading and plowing operations. Compound models that integrate sector design, fleet sizing, and snow disposal assignment decisions for loading trucks and hauling snow to disposal sites are discussed in the second paper (PERRIER *et al.*, 2006b).

2.4.1 Sector design models for spreading

MUYLDERMANS *et al.* (2002) proposed a solution method to design sectors for spreading operations that produces contiguous, balanced and geographically compact subnetworks with centralized depots. The objective is to minimize the number of spreader trucks, while ensuring that the graph generated by the edges of each sector is Eulerian to enable tours with less deadheading. The locations of the depots are given and they coincide with nodes of an undirected road network G. Furthermore, every edge of G has a length and a weight corresponding to the demand for chemicals or abrasives. The authors proposed a four-phase composite heuristic that builds all sectors simultaneously by assigning basic entities represented by small cycles to the depots. The number of sectors to be designed equals the number of depots. The first phase corresponds to the cycle decomposition approach suggested by MALE and LIEBMAN (1978) in the context of districting and routing for solid waste collection. If G is not even, then edges of least total cost are added to G to create an even graph. As shown by EDMONDS and JOHNSON (1973), these edges can be determined by solving a perfect matching problem on an auxiliary graph whose vertices are the odd-degree vertices of G. The basic entities are then determined by partitioning the Eulerian graph G' into small cycles using a "checkerboard pattern" to obtain a set of faces with associated cycles as illustrated in Figure 2.4.

In the second phase, the sector building process is initiated by assigning first the cycles that are adjacent to a depot and next, the cycles that are very close to a depot. Only eligible cycles are assigned to ensure the contiguity of the partially built sectors. A

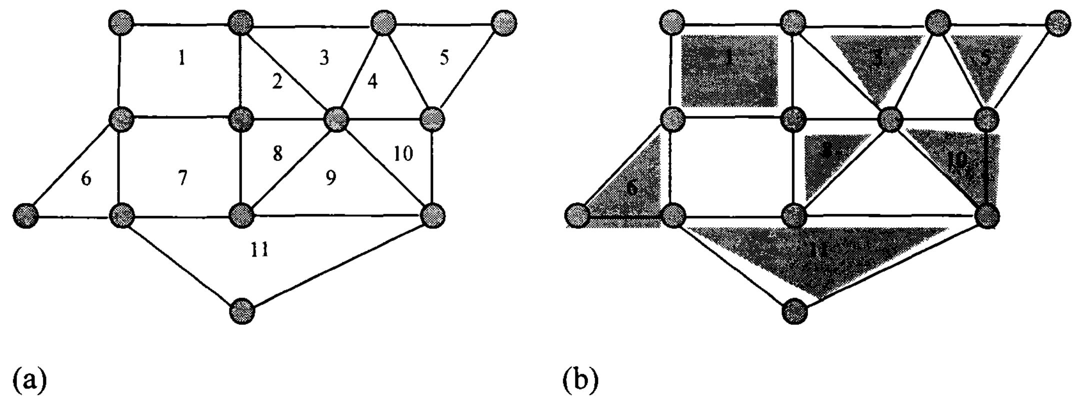

Figure 2.4: Example of seven cycles designated on an Eulerian graph:
(a) Eulerian graph *G'*, (b) Checkerboard pattern.

cycle is *eligible* for a certain depot if it is not yet assigned to a depot, and if it has at least one node in common with a cycle already assigned to that depot. Isolated cycles are then checked for and assigned to the nearest depot. In the third phase, all sectors are built simultaneously in order to balance the total workload. The first part of the third phase operates as a simple bin packing heuristic and is reminiscent of the partitioning algorithm of LEVY and BODIN (1989). The second part of the third phase uses a multicriteria approach for the assignment of the remaining cycles. Each candidate cycle-to-depot assignment is evaluated on three criteria: sector balance, compactness and fleet size. All criteria are treated through the calculation of a weighted additive multicriteria score for each cycle-depot pair. The candidate pair with the lowest score is then assigned. In the fourth phase, cycle shifts or interchanges between the depots are performed so as to decrease the number of required spreader trucks, while satisfying the contiguity constraints. No specific exchange procedure was developed. However, the authors suggested a user interactive procedure to allow sectors to be adjusted to incorporate topographic, climatic or other constraints that are difficult to quantify. Users can also modify parameter values, determine new basic entities and define new criteria and weights in the multicriteria approach. The four-phase heuristic procedure is presented in detail in Figure 2.5.

1. *Preprocessing*
   a. Partition the road network into elemental cycles. Let J be the set of cycles and let I be the set of depots or sectors.
   b. For each cycle $j \in J$, let CN_j represent the set of nodes on cycle j. For every depot $i \in I$ and for every node $k \in CN_j$, define D_{ik} as the length of a shortest path from depot i to node k. For each depot $i \in I$ and for each cycle $j \in J$, calculate the ratio R_{ij} as follows:

$$R_{ij} = \frac{\overline{D}_{ij}}{\min_{\substack{i' \in I \\ i' \neq i}} \{\overline{D}_{i'j}\}} \text{ where } \overline{D}_{ij} = \frac{\sum_{k \in CN_j} D_{ik}}{|CN_j|}.$$

2. *Initial partial assignment*
   a. For each cycle $j \in J$, if cycle j is adjacent to a depot, then assign cycle j to that depot.
   b. For each depot $i \in I$ and for each eligible cycle $j \in J$, if $R_{ij} \leq 0.5$, then assign cycle j to depot i.
   c. For each isolated cycle $j \in J$, assign cycle j to the nearest depot.

3. *Two-part iterative assignment*
   Part I
   a. Select the sector $i \in I$ with the smallest workload assigned to it.
   b. For each cycle $j \in J$, let the cycle weight CW_j represent the amount of chemicals or abrasives to service the roads in cycle j. For each depot $i \in I$, let BR_i be a ratio such that $0.5 < BR_i < 1$. Select the eligible cycle $j \in J$ with the largest weight such that $R_{ij} < BR_i$. If no such cycle is available, go to Part II. Otherwise, assign cycle j to sector i.
   c. For each isolated cycle $j \in J$, assign cycle j to the nearest depot. Return to the beginning of Part I.
   Part II
   d. For each depot $i \in I$, select the largest weight eligible cycle $j \in J$ with $R_{ij} < BR_i$. If no such cycle is available for a depot i, select the eligible cycle j with the smallest ratio R_{ij}.
   e. For every depot $i \in I$, define CD_i as the set of cycles in sector i. Define also Q as the vehicle capacity and N as an estimate for the number of vehicles required in the final partition calculated as follows:

$$N = \sum_{i \in I} \left\lceil \frac{\sum_{j \in CD_i} CW_j}{Q} \right\rceil.$$

   f. For each candidate cycle-depot pair, let N_1 and N_2 be two estimates for the number of vehicles when assuming that the cycle-to-depot assignment is carried out and dividing the remaining unassigned cycles over the sectors such that N is minimized and maximized, respectively.
   g. Let S_1, S_2, and S_3 be the score of the compactness measure, the sum of both estimates for the number of vehicles required and the score of the balance measure of the partially built sectors, respectively. Let also $W_1 = 1$, $W_2 = 3$, and $W_3 = 2$ be three weights. For each candidate cycle j depot i pair, calculate the score $S_{ij} = W_1S_1 + W_2S_2 + W_3S_3$ where $S_1 = 0$ if $\max\{BR_i, R_{ij}\} \leq BR_i$, $S_1 = 1$ if $BR_i < \max\{BR_i, R_{ij}\} \leq 1$, $S_1 = 2$ if $1 < \max\{BR_i, R_{ij}\} \leq 1.5$, $S_1 = 3$ if $1.5 < \max\{BR_i, R_{ij}\} < 2$, $S_2 = N_1 + N_2$, $S_3 = 0$ if the addition of cycle j to depot i decreases the imbalance, and $S_3 = 1$ if the assignment increases the imbalance.
   h. Select the assignment with the smallest total score and assign cycle j to sector i.
   i. For each isolated cycle $j \in J$, assign cycle j to the nearest depot. Return to the beginning of Part II.

4. *Improvement and user interaction*
   Shift or interchange cycles between the depots so as to decrease the number of required trucks, while ensuring the contiguity of the sectors.

Figure 2.5: The four-phase heuristic for the sector design problem

(MUYLDERMANS *et al.*, 2002)

The heuristic was tested on a real network from the province of Antwerp in Belgium with 244 edges, 154 nodes, and four depots. The new sectors found by the heuristic were not as well balanced as those actually used by the province. However, solving the capacitated arc routing problem for spreader trucks for these new sectors with the PEARN (PEARN, 1991) augment-insert algorithm allowed deadheading cost savings of about 14% over the solution produced with the same algorithm in the sectors actually used by the province.

In a subsequent paper, MUYLDERMANS *et al.* (2003) suggested and compared three heuristics for generating contiguous, balanced, and geographically compact sectors with centralized vehicle depots for spreading operations and road maintenance. The first two heuristics differ only in the definition of the basic units. Let $G = (V, E)$ be a connected, undirected, and planar graph with vertex set V and edge set E. With every edge $(v_r, v_s) \in E$ are associated a positive length c_{rs} and a positive demand q_{rs} for service. Define $X \subset V$ as the set of vehicle depots, each depot housing identical vehicles with a finite capacity Q. The first heuristic aggregates individual edges in E into sectors by allocating them to the nearest vehicle depot. The second heuristic aggregates groupings of edges into sectors in the same greedy manner as the first heuristic. Groupings of edges in G are defined by the cycle decomposition approach described previously by MUYLDERMANS *et al.* (2002) and an edge exchange heuristic (MUYLDERMANS, 2003) that we have not described in detail here. Let $H = (W, F)$ be an adjacency graph such that the vertex set W has a vertex w_j for each cycle and for each vehicle depot in the Eulerian graph G' and the edge set F contains an edge f_{ij} between w_i and w_j if the corresponding cycles c_i and c_j in G' have a vertex in common or if the vehicle depot associated with w_i is located on the cycle represented by w_j. Let $W_C \subset W$ and $W_D \subset W$ be two vertex subsets corresponding to cycles and vehicle depots, respectively. For every vertex $w_j \in W$, define the constant $q(w_j)$ equal to the total demand on cycle c_i if and only if $w_j \in W_C$ and zero otherwise. The second heuristic is described in Figure 2.6.

1. Define the basic entities in G by the cycle decomposition approach and the exchange heuristic. Construct the adjacency graph H.
2. Let sc_{ij} be the length of the shortest chain linking vertex v_i to vertex v_j in G. For every vehicle depot $w_i \in W_D$ and for every cycle $w_j \in W_C$, define D_{ij} as the sum of shortest deadheading distances in G for servicing each edge of cycle c_j separately from depot v_i. For every vehicle depot $w_i \in W_D$ and for every cycle $w_j \in W_C$, calculate the ratio R_{ij} as follows:

$$R_{ij} = \frac{D_{ij}}{\min_{\substack{v_k \in X \\ v_k \neq v_i}} \{D_{kj}\}} \text{ where } D_{ij} = \sum_{(v_r, v_s) \in c_j} (sc_{ir} + sc_{is}).$$

3. If all cycles $w_j \in W_C$ are assigned, go to step 5.
4. For each vehicle depot $w_i \in W_D$, select the unassigned cycle $w_j \in W_C$ with the lowest ratio value R_{ij}, adjacent to w_i or to a cycle w_k already assigned to vehicle depot w_i. Among these candidate depot-cycle assignments (w_i, w_j), allocate the cycle to the vehicle depot in the pair with the lowest R_{ij} value. Return to step 3.
5. Translate the cycle allocations in H into a sector design in G.

Figure 2.6: Greedy heuristic for the sector design problem (MUYLDERMANS *et al.*, 2003)

In the third heuristic, basic units obtained by the cycle decomposition are allocated to the vehicle depots through the solution of a linear mixed integer program. Prior to solving the model, the adjacency graph H is reduced in size. The idea is to allocate immediately the cycles that are considered as very near the depots, and further to merge some cycles by exploiting structural properties in H. The adjacency graph reduction is described in detail by MUYLDERMANS *et al.* (2003). To present the formulation, the following notations apply. For every cycle $w_j \in W_C$ and for every vehicle depot $w_i \in W_D$, let x_{ij} be a binary variable equal to 1 if and only if cycle w_j is assigned to depot w_i. For every vehicle depot $w_i \in W_D$, let y_i be a nonnegative integer variable corresponding to the continuous lower bound $\Sigma_i \lceil q(E_i)/Q \rceil$ on the number of vehicles scheduled from w_i, with $q(E_i)$ being the total demand in G serviced by vehicle depot w_i. MUYLDERMANS (2003) showed that by partitioning G into p sectors, in the worst case this lower bound can grow to $\lceil q(E)/Q \rceil + p - 1$ where $q(E)$ is the total demand in G. For each vehicle depot $w_i \in W_D$, let $R_{\max,i}$ be a nonnegative variable representing the maximum R_{ij} value among the cycles assigned to depot w_i, and let $H_i = (W_i \cup \{w_i\}, F_i)$ be the subgraph induced by w_i and the cycles $w_j \in W_i$ that can be assigned to vehicle depot w_i. A cycle w_j can be assigned to a vehicle depot w_i if $R_{ij} \leq R_{\text{lim},i}$ where $R_{\text{lim},i}$ is a limit value for the ratios R_{ij} associated to depot w_i. For each cycle $w_j \in W_\text{C}$, let W_j be the set of vehicle depots that can receive cycle w_j ($R_{ij} \leq R_{\text{lim},i}$). Finally, for every cycle $w_j \in W_\text{C}$ and for every vehicle depot $w_i \in W_\text{D}$, let $W_{ij} = \{w_k \in W_i : n_i(w_j) = n_i(w_k) + 1 \text{ and } (w_k, w_j) \in F_i\}$ be the set of vertices in H_i to express the sector connectivity when cycle $w_j \in W_i$ is assigned to depot w_i with $n_i(w_j)$ corresponding to the minimum number of edges needed for reaching w_j from w_i in H_i. The formulation is then:

$$\text{Minimize} \quad \sum_{w_i \in W_\text{D}} y_i + \alpha \sum_{w_i \in W_\text{D}} R_{\max i} \tag{2.11}$$

subject to

$$\sum_{w_i \in W_j} x_{ij} = 1 \qquad (w_j \in W_\text{C}) \tag{2.12}$$

$$\sum_{w_j \in W_i} q(w_j) x_{ij} + q(w_i) \leq Q y_i \qquad (w_i \in W_\text{D}) \tag{2.13}$$

$$x_{ij} \leq \sum_{w_k \in W_{ij}} x_{ik} \qquad (W_{ij} \neq \varnothing, w_i \in W_\text{D}, w_j \in W_i) \tag{2.14}$$

$$R_{ij} x_{ij} \leq R_{\max,i} \qquad (w_i \in W_\text{D}, w_j \in W_i) \tag{2.15}$$

$$x_{ij} \in \{0,1\} \qquad (w_i \in W_\text{D}, w_j \in W_i) \tag{2.16}$$

$$y_i \geq 0 \text{ and integer} \qquad (w_i \in W_\text{D}) \tag{2.17}$$

$$R_{\max,i} \geq 0 \qquad (w_i \in W_\text{D}). \tag{2.18}$$

The objective function (2.11) minimizes the sum of the lower bounds on the number of vehicles to be used from the vehicle depots and the ratio values of the most distant cycles that are assigned to the depots. Minimizing these ratio values penalizes the non-compactness of each sector. The scale factor α is chosen suitably small in order to make the contribution of the second term in (2.11) less than one so that it does not affect the minimum value of the first term. Constraints (2.12) require that each cycle be assigned to exactly one vehicle depot. The total capacity of the vehicles is respected at any vehicle depot via constraint set (2.13). Constraints (2.14) ensure network connectivity within a sector. For each vehicle depot $w_i \in W_D$, the cycles $w_j \in W_i$ are partially ordered according to the minimum number of edges $n_i(w_j)$ needed for reaching w_j from w_i in H_i. Constraints (2.14) require that at least one cycle of the set W_{ij} be allocated to w_i, before w_j can be assigned to w_i. Constraints (2.15) state that the maximum ratio value within each sector must be greater than the ratio value of any cycle in the sector and the vehicle depot to which it is assigned. Finally, all x_{ij} variables are restricted to be binary, while y_i and $R_{\max,i}$ variables must assume nonnegative integer values and nonnegative values, respectively. Two simplified versions of this model are also proposed. In the first version, the second term in the objective function (2.11) is removed as well as constraint sets (2.15) and (2.18). In the second version, the objective function (2.11) and constraint sets (2.15) and (2.18) are modified by replacing the variables $R_{\max,i}$ by a single variable $R_{\max}$. Some valid inequalities are added to the formulations and the resulting models are solved by the standard branch-and-bound algorithm of CPLEX 6.5. For details, see MUYLDERMANS (2003).

The three heuristics were tested on a large set of graphs constructed from the road network in Flanders (Belgium) with up to 27 vehicle depots and 1692 edges. The larger test problem required 598 vertices in the adjacency graph H defined by the cycle decomposition and 216 cycles after reducing H. The quality of the partitions generated by the three heuristics is evaluated by solving capacitated arc routing problems in the sectors by a local search heuristic (BEULLENS *et al.*, 2003) and by comparing the solution values with a cutting plane lower bound based on the supersparse formulation for the capacitated arc routing problem (BELENGUER and BENAVENT, 1998). The authors concluded that the first heuristic is most effective when Q is very small, whereas the second heuristic performs better for average Q values. The three versions of the linear mixed integer program based heuristic are particularly suitable when Q is large, but these require more time than the other heuristics.

2.4.2 Compound sector design, depot location, and fleet sizing models for spreading and plowing

As was highlighted by CATTRYSSE *et al.* (1997) and VAN OUDHEUSDEN *et al.* (1999), there are strong interactions between the design of sectors, the location of disposal sites and depots, and the routing of vehicles for plowing, spreading, loading, and hauling operations. However, these interdependent problems are most often solved separately. Typically, disposal sites are first located, sectors are then designed and assigned to disposal sites, and routes are determined last. Obviously, this sequential approach may lead to suboptimal decisions at all planning levels. Compound models reviewed in this section integrate the three components of sector design, depot location and fleet sizing for winter road maintenance.

KANDULA and WRIGHT (1995) proposed a linear mixed integer programming formulation for the combined sector design, depot location and fleet sizing problem. Their model addresses the combined problem for both spreading and plowing operations in a largely rural area with a sparse road network, but can also serve to model other districting problems in the context of arc routing. A solution to their model indicates the depot sites to open, the assignment of each road segment to an opened depot as well as the number of vehicles required to service simultaneously all road segments. Thus, each sector includes one opened depot at which a number of vehicles are based. The formulation incorporates contiguity and compactness constraints for each sector. Service hierarchy, class continuity for each vehicle, and constraints which place different limits on sector sizes are also enforced. The objective considered is a surrogate compactness measure.

Let $G = (V, E)$ be a connected undirected graph where $V = \{v_1, v_2, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. With every edge (v_i, v_j) are associated a nonnegative length c_{ij} and a positive number of circulation lanes l_{ij}. Let sc_{ij} be the length of the shortest chain linking vertex v_i to vertex v_j in G. Let $D \subset V$ be a set of potential depot sites. For every depot $v_d \in D$, define dhf_d as the deadhead factor used for road segments associated with depot v_d ($dhf_d \geq 1$ for all $v_d \in D$), cap_d as the maximum number of kilometers assigned to depot v_d, and $sumsc_d$ as the limit on the sum of the lengths of the shortest chains from depot v_d to both ends of road segments that are assigned to depot v_d. For every edge $(v_i, v_j) \in E$ and for every potential depot site $v_d \in D$, let x_{ijd} be a binary variable equal to 1 if and only if edge (v_i, v_j) is assigned to depot v_d. For every potential depot site $v_d \in D$, let y_d be a binary variable equal to 1 if and only if depot site v_d is opened.

Let $P_K = \{E_1, E_2, \ldots, E_K\}$ be a partition of E with $E_1 \cup E_2 \cup \ldots \cup E_K = E$ and $E_i \cap E_j = \varnothing$ for all $i, j \in \{1, 2, \ldots, K\}$, $i \neq j$. For every depot $v_d \in D$ and every class $E_k \subseteq P_K$, let n_{kd} be a nonnegative integer variable representing the number of vehicles based at depot v_d to service edges of class E_k assigned to depot v_d. For every depot $v_d \in D$ and every class $E_k \subseteq P_K$, define cl_{kd} as the maximum number of class k kilometers assigned to depot v_d. For every class $E_k \subseteq P_K$, define f_k as the frequency of service in hours that must be provided to road segments of class k. The vehicle speed, expressed as kilometers per hour, is denoted by s.

KANDULA and WRIGHT modeled the contiguity constraints as a circulation multi-commodity network flow problem with supplementary variables and constraints. Each commodity corresponds to a potential depot site and shares the same directed graph $G' = (V \cup \{v_0\}, A_1 \cup A_2 \cup A_3)$ constructed from G where v_0 is an artificial vertex and A_1, A_2 and A_3 are three sets of arcs defined as follows. The arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E, $A_2 = \{(v_0, v_i): v_i \in D\}$ and $A_3 = \{(v_i, v_0): v_i \in V \setminus D\}$.

For every depot $v_d \in D$ and every arc $(v_i, v_j) \in A_1 \cup A_2 \cup A_3$, let w_{ijd} be a nonnegative real variable representing the flow on arc (v_i, v_j) assigned to depot v_d. An example of network G' with three potential depots v_4, v_5 and v_{15} is illustrated in Figure 2.7. The three sets of arcs A_1, A_2 and A_3 are shown as dashed lines, solid lines, and dotted lines, respectively. For reasons of clarity, the arcs of A_3 are not shown as incoming arcs of vertex v_0.

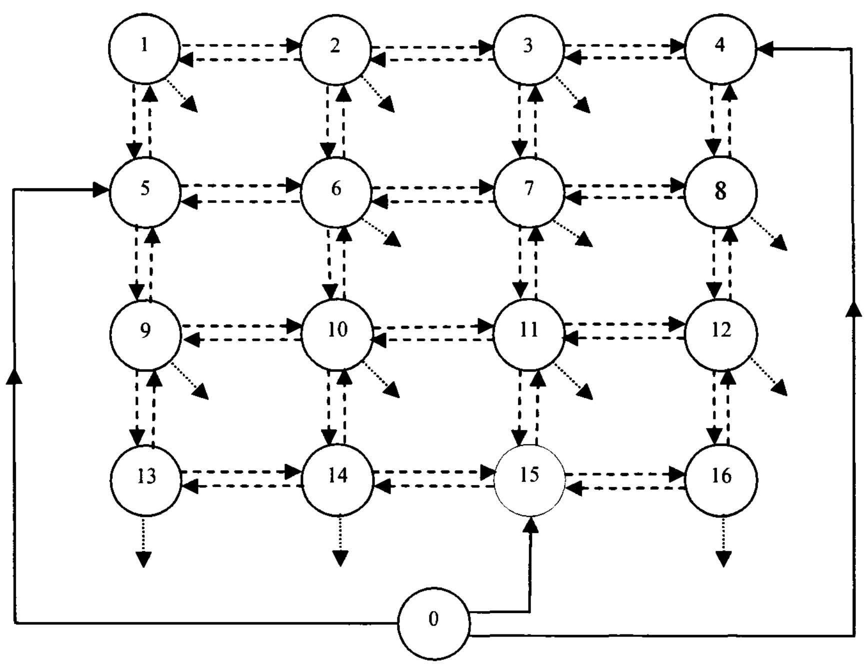

Figure 2.7: Example of network G' with three potential depots v_4, v_5 and v_{15}

Finally, define *numv* as the maximum number of vehicles to be used, *numd* as the maximum number of depots to be operative and *sumsc* as the limit on the sum of the lengths of the shortest chains from operative depots to both ends of road segments that are assigned to these depots. We present here a slightly simplified version of the Kandula and Wright formulation for the combined sector design, depot location and fleet sizing problem. (We eliminate some variables used by KANDULA and WRIGHT to clarify the interpretation of results).

$$\text{Minimize} \quad \sum_{v_d \in D} \sum_{(v_i, v_j) \in E} (sc_{id} + sc_{jd}) x_{ijd} \tag{2.19}$$

subject to

$$\sum_{(v_i, v_j) \in E_k} l_{ij} c_{ij} x_{ijd} \le cl_{kd} \qquad (v_d \in D, E_k \in P_K) \tag{2.20}$$

$$n_{kd} \ge \frac{dhf_d cl_{kd}}{f_k s} \qquad (v_d \in D, E_k \in P_K) \tag{2.21}$$

$$\sum_{v_d \in D} \sum_{E_k \in P_K} n_{kd} \le numv \tag{2.22}$$

$$\sum_{v_d \in D} y_d = numd \tag{2.23}$$

$$\sum_{(v_i, v_j) \in E} l_{ij} c_{ij} x_{ijd} \le cap_d y_d \qquad (v_d \in D) \tag{2.24}$$

$$\sum_{v_d \in D} x_{ijd} = 1 \qquad ((v_i, v_j) \in E) \tag{2.25}$$

$$w_{ijd} \le M x_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \tag{2.26}$$

$$w_{jid} \le M x_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \tag{2.27}$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1 \cup A_3\}} w_{ijd} - \sum_{\{v_j : (v_j, v_i) \in A_1\}} w_{jid} = 0 \qquad (v_i \in V \setminus D, v_d \in D) \tag{2.28}$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1\}} w_{ijd} - \sum_{\{v_j : (v_j, v_i) \in A_1 \cup A_2\}} w_{jid} = 0 \qquad (v_i \in D, v_d \in D) \tag{2.29}$$

$$\sum_{v_d \in D} w_{ijd} \ge 1 \qquad ((v_i, v_j) \in A_3) \tag{2.30}$$

$$\sum_{v_d \in D} \sum_{(v_i, v_j) \in A_2} w_{ijd} \ge |V \setminus D| \tag{2.31}$$

$$w_{ijd} = 0 \qquad ((v_i, v_j) \in A_2, v_d \in D, v_j \ne v_d) \tag{2.32}$$

$$x_{ijd} \le w_{i0d} \qquad ((v_i, v_j) \in E, v_d \in D) \tag{2.33}$$

$$x_{ijd} \le w_{j0d} \qquad ((v_i, v_j) \in E, v_d \in D) \tag{2.34}$$

$$(sc_{id} + sc_{jd}) x_{ijd} \le sumsc \qquad ((v_i, v_j) \in E, v_d \in D) \tag{2.35}$$

$$\sum_{(v_i,v_j)\in E}(sc_{id}+sc_{jd})x_{ijd} \le sumsc_d \qquad (v_d \in D) \qquad (2.36)$$

$$w_{ijd} \ge 0 \qquad ((v_i,v_j)\in A_1 \cup A_2 \cup A_3, v_d \in D) \qquad (2.37)$$

$$n_{kd} \ge 0 \text{ and integer} \qquad (v_d \in D, E_k \in P_K) \qquad (2.38)$$

$$x_{ijd} \text{ and } y_d \in \{0,1\} \qquad ((v_i,v_j)\in E, v_d \in D). \qquad (2.39)$$

The objective function (2.19) minimizes the sum of all lengths of the shortest chains between operative depots and both ends of road segments in G. Constraints (2.20) impose an upper bound on the number of kilometers of each roadway class assigned to each depot. Constraints (2.21) impose a lower bound on the number of vehicles based at every depot for each class. Equipment availability is respected at all depots for all roadway classes via constraints (2.22). Constraint sets (2.23)–(2.25) are identical to those of the P-median problem except that a capacity constraint is included. Constraint (2.23) imposes the number of depots to operate. The capacity constraints (2.24) link the location variables (y_d) and the allocation variables (x_{ijd}). They assure that each edge is assigned to a depot that is operative. Constraints (2.25) require each edge to be assigned to exactly one depot. Constraints (2.26)–(2.34) guarantee that each sector is contiguous. The linking constraints (2.26) and (2.27) ensure that the flow on arc (v_i, v_j) or arc (v_j, v_i) assigned to depot v_d is positive if the edge (v_i, v_j) is assigned to that depot. M is a sufficiently large positive number. Flow conservation at every node, except node v_0, for each potential depot is imposed by constraints (2.28) and (2.29). Constraints (2.30) assure that every non-depot node is part of at least one sector. Constraint (2.31) imposes a lower bound on the number of flow units out of vertex v_0 so that every vertex is part of at least one sector. Constraints (2.32) assure that every sector includes exactly one depot (operative or not). The linking constraints (2.33) and (2.34) ensure that an edge (v_i, v_j) is assigned to a depot v_d if both ends v_i and v_j of that edge are part of the sector including that depot. Constraints (2.35) and (2.36) assure that every sector is compact. Finally, all

w_{ijd} variables must assume nonnegative values and all n_{kd} variables must assume nonnegative integers values, while x_{ijd} and y_d variables are restricted to be binary.

KANDULA and WRIGHT used the CPLEX Mixed Integer Optimizer with barrier code (version 2.1) to solve model (2.19)–(2.39) with a set of data from the La Porte district of Indiana with three priority classes of roadways and four depots on a network of 63 vertices and 79 edges. This gave 1,554 variables and 2,295 constraints. The optimal solution provided sectors that were more compact than the sectors in use by the district. However, the number of vehicles used increased by 16%, probably on account of the class continuity constraints. Moreover, the computational effort to solve the model was significant with 178 branch-and-bound nodes and 5,780 iterations. To accelerate the computational time, the contiguity constraints (2.26)–(2.34) and the integrity constraints on the x_{ijd} variables were relaxed. Although the new solution was not integer, the resulting sectors were contiguous, with the exception of only one. Computational experiments performed by KANDULA (1996) on five other regions of Indiana with up to 62 nodes, 73 edges and three potential depots showed that the mixed integer model and its relaxation resulted in the same sectors in all cases.

In a subsequent paper, KANDULA and WRIGHT (1997) extended the original model to introduce cost characteristics related to route configuration such as service and deadhead costs for road segments. This is done by introducing new variables and new constraint sets, as well as by modifying the objective function and some constraints. The directed graph G' constructed from G to formulate the contiguity constraints as a circulation multi-commodity network flow problem is augmented by introducing an arc from each depot node $v_d \in D$ to the artificial vertex v_0. For every depot $v_d \in D$ and every arc $(v_i, v_j) \in A_1 \cup A_2 \cup A_3 \cup A_4$ where A_1, A_2 and A_3 are defined as above and $A_4 = \{(v_i, v_0): v_i \in D\}$, the flow variable w_{ijd} representing the flow on arc (v_i, v_j) assigned to depot v_d is now expressed as time units. With each edge $(v_i, v_j) \in E$ are now associated two traversal times: ts_{ij} is the time for servicing edge (v_i, v_j) and td_{ij} is the traversal time of (v_i, v_j) no matter if it has already been serviced or not. However, the service hierarchy of the network is no longer considered. Then, variable n_{kd} representing the number of vehicles based at depot v_d to service edges of class E_k assigned to depot v_d is now replaced by variable n_d representing the number of vehicles based at depot v_d. For every edge $(v_i, v_j) \in E$ and for every potential depot site $v_d \in D$, let q_{ijd} be a binary variable equal to 1 if and only if edge (v_i, v_j) assigned to depot v_d is traversed from v_i to v_j. For every depot $v_d \in D$ and every edge $(v_i, v_j) \in E$, let p_{ijd} be a nonnegative real variable representing the number of deadhead time units traversing edge (v_i, v_j) assigned to depot v_d from v_i to v_j. For every depot $v_d \in D$, let $SMAX_d$ be the maximum sum of the lengths of the shortest chains from operative depot d to both ends of a road segment that is assigned to this depot. Finally, define $numv_d$ as the maximum number of vehicles based at depot v_d and t as the average time to service edges on one vehicle route. The time t does not include the deadheading time. The formulation is given next.

$$\text{Minimize} \sum_{v_d \in D} SMAX_d + \sum_{v_d \in D} \sum_{(v_i,v_j) \in E} (p_{ijd} + p_{jid}) + \sum_{v_d \in D} \sum_{(v_i,v_j) \in A_1 \cup A_3 \cup A_4} w_{ijd} + \sum_{v_d \in D} n_d \qquad (2.40)$$

subject to

$$\sum_{(v_i,v_j) \in E} (sc_{id} + sc_{jd}) x_{ijd} \le SMAX_d \qquad (v_d \in D) \qquad (2.41)$$

$$\sum_{v_d \in D} x_{ijd} = 1 \qquad ((v_i, v_j) \in E) \qquad (2.42)$$

$$x_{ijd} \le y_d \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.43)$$

$$\sum_{v_d \in D} y_d = numd \qquad (2.44)$$

$$(sc_{id} + sc_{jd}) x_{ijd} \le sumsc \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.45)$$

$$n_d \ge \frac{\sum_{(v_i,v_j) \in E} l_{ij} c_{ij} x_{ijd}}{ts} \qquad (v_d \in D) \qquad (2.46)$$

$$n_d \le numv_d \qquad (v_d \in D) \qquad (2.47)$$

$$w_{ijd} = tn_d \qquad ((v_i, v_j) \in A_2, v_d \in D) \qquad (2.48)$$

$$w_{ijd} = 0 \qquad ((v_i, v_j) \in A_2, v_d \in D, v_j \neq v_d) \qquad (2.49)$$

$$\sum_{v_d \in D} w_{ijd} \geq 0.05 \qquad ((v_i, v_j) \in A_3 \cup A_4) \qquad (2.50)$$

$$x_{ijd} \leq 100\, w_{i0d} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.51)$$

$$x_{ijd} \leq 100\, w_{j0d} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.52)$$

$$w_{ijd} \leq Mx_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.53)$$

$$w_{jid} \leq Mx_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.54)$$

$$\sum_{v_d \in D} (w_{ijd} + w_{jid}) \geq \frac{ts_{ij}}{2} \qquad ((v_i, v_j) \in E) \qquad (2.55)$$

$$w_{ijd} \leq Mq_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.56)$$

$$w_{jid} \leq Mq_{jid} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.57)$$

$$\sum_{v_d \in D} (q_{ijd} + q_{jid}) = 1 \qquad ((v_i, v_j) \in E) \qquad (2.58)$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1 \cup A_3\}} w_{ijd} - \sum_{\{v_j : (v_j, v_i) \in A_1\}} w_{jid} = 0 \qquad (v_i \in V \setminus D, v_d \in D) \qquad (2.59)$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1 \cup A_4\}} w_{ijd} - \sum_{\{v_j : (v_j, v_i) \in A_1 \cup A_2\}} w_{jid} = 0 \qquad (v_i \in D, v_d \in D) \qquad (2.60)$$

$$w_{ijd} \leq t + p_{ijd} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.61)$$

$$w_{jid} \leq t + p_{jid} \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.62)$$

$$w_{ijd} \geq \sum_{\substack{\{v_k : (v_k, v_i) \in E\} \\ \{\text{or } (v_i, v_k) \in E\}}} \left(\frac{ts_{ki}}{2} x_{kid} \right) + \sum_{\{v_k : (v_k, v_i) \in E\}} \left(\frac{l_{ki} td_{ki}}{t} p_{kid} \right) \qquad ((v_i, v_j) \in A_3 \cup A_4, v_d \in D) \qquad (2.63)$$

$$w_{ijd} \geq 0 \qquad ((v_i, v_j) \in A_1 \cup A_2 \cup A_3 \cup A_4, v_d \in D) \qquad (2.64)$$

$$p_{ijd} \geq 0 \qquad ((v_i, v_j) \in E, v_d \in D) \qquad (2.65)$$

$$n_d \geq 0 \text{ and integer} \qquad (v_d \in D) \tag{2.66}$$

$$x_{ijd}, y_d \text{ and } q_{ijd} \in \{0,1\} \qquad ((v_i, v_j) \in E, v_d \in D). \tag{2.67}$$

In this formulation, the objective function (2.40) minimizes the total sum of the maximum sums of the lengths of the shortest chains from operative depots to both ends of road segments that are assigned to these depots, the total deadheading time, the total number of time units of all the commodities on each arc, except arcs of set A_2, and the total number of vehicles used. Minimizing the total flow of all the commodities on each arc ensures splitting of flows at nodes near the vehicle depots, so that most deadheading occurs on edges close to the vehicle depots. Constraints (2.41) state that the maximum sum of the lengths of the shortest chains between every operative depot and both ends of a road segment that is assigned to the depot must be greater than the sum of the lengths of the shortest chains from the depot to both ends of any road segment that is assigned to the depot. Constraint sets (2.42)–(2.44) are identical to those of the *P*-median problem. Constraint set (2.42) requires each edge to be assigned to exactly one depot. Constraints (2.43) state that an edge can only be assigned to a depot if it is operative. Constraint (2.44) states that exactly *numd* depots are to be located. Constraint set (2.45) is identical to its counterparts (2.35) of the model (2.19)–(2.39). Constraint sets (2.46) and (2.47) impose lower and upper bounds on the number of vehicles based at every depot. Constraints (2.48)–(2.63) ensure that each sector is contiguous. Constraint sets (2.48) and (2.49) assure that every sector includes exactly one depot (operative or not). Constraint sets (2.50)–(2.52) are identical to their respective counterparts (2.30), (2.33), (2.34) of the model (2.19)–(2.39), except that the constraint set (2.50) is now defined on the augmented graph G' and the right-hand sides of (2.50)–(2.52) are modified since flow units are now expressed as time units. Constraint sets (2.53) and (2.54) are identical to their respective counterparts (2.26) and (2.27) of the model (2.19)–(2.39). Constraint set (2.55) imposes lower flow bounds on all pairs of arcs of opposite direction associated with each edge. The time for servicing an edge is divided between the endpoints of that edge. Constraint sets (2.56)–(2.58) assure that the flow may be positive for exactly one depot on only one of the two arcs associated with each edge. Flow conservation at every node of the augmented graph G' for each disposal site is imposed by constraints (2.59)–(2.60). For each commodity, constraint sets (2.61)–(2.62) state that for every pair of arcs associated with each edge, any flow in excess of average time required to service edges on one vehicle route must be due to deadheading. For each commodity, constraint set (2.63) states that the total outflow of any node of the augmented graph G' must at least correspond to half of the time spent in servicing all edges assigned to the depot and incident to that node at the specified service speed, as well as deadhead travel into that node.

Again, the model was solved using CPLEX. Numerical experiments performed on five regions of Indiana with up to 62 nodes, 73 edges and three potential depots showed that the sectors created in every region were more compact than the existing sectors. However, the computation times were rather long with up to 1,927 branch-and-bound nodes and 55,673 iterations[1]. As mentioned by KANDULA and WRIGHT (1997), the computation time may be reduced by exploiting the spatial nature of the problem. Each edge is considered a candidate for assignment to all sectors. Nevertheless, in practice, there are likely to be very few reasonable alternatives for each edge assignment. Fixing such obvious variables at zero or one, or eliminating some variables might help to limit the size of the branch-and-bound search tree and therefore reduce the computation time. The quality of the configuration of the sectors was also evaluated on the basis of the quality of the vehicle routes produced in each sector with both a lower-bound based composite heuristic suggested by KANDULA and WRIGHT (1997) and a tabu search heuristic developed by WANG *et al.* (1995). Results indicated that the routes produced in the new sectors satisfied more constraints, such as service time intervals and class

[1]The authors did not, however, provide the timing results obtained with the branch-and-bound method of CPLEX.

continuity, were less numerous and had fewer deadhead miles than the routes determined in the existing sectors. KANDULA (1996) compared the two models solved using CPLEX and showed that the extended model (2.40)–(2.67) produced better results than the basic model (2.19)–(2.39) in half the real-life instances described above, but required excessive computing time. KANDULA (1996) also demonstrated how the basic and extended models may be used to address location issues such as opening, closing, or relocating depots, as well as repartitioning areas of a network where depots are located near the boundaries so as to improve compactness.

2.5 Conclusions

This paper is the first of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. (The second part of the survey (PERRIER *et al.*, 2006b) discusses system design models for snow disposal operations. The two last parts of the review (PERRIER *et al.*, 2005a,b) mainly address vehicle routing, depot location, and fleet sizing models for winter road maintenance problems.)

This paper addresses the level of service policy and the sector design problem for spreading and plowing operations. This represents an important part of the system design planning performed by regional and local government agencies. Table 2.2 summarizes the characteristics of the service level optimization models and the sector design models related to spreading and plowing operations.

The level of service policy related to spreading and plowing operations is usually given as an input in system design or vehicle routing models. Several analytical models (multiple regression and cost-benefit analyses) were proposed to assess and quantify the resource-oriented level of service policy for spreading and plowing operations. For the

Table 2.2: Characteristics of service level and sector design models for spreading and plowing

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
SAGE (1979)	Resource-oriented service level	Strategic	Snow removal rate and maximum service level	Min delay and maintenance costs	Nonlinear	Analytical
UNGERER (1989)	Results-oriented service level	Operational	Storm duration and surface road condition	Min maintenance costs	Shortest path	Dynamic programming
UNGERER (1989)	Results-oriented service level	Operational	Storm duration and surface road condition	Multi-objective	Shortest paths	Compromise programming
UNGERER (1989)	Results-oriented service level	Strategic	Service hierarchy, storm types, and resource availabilities	Max weighted vehicular traffic rate	Linear 0-1 IP	Composite heuristic
MUYLDERMANS *et al.* (2002)	Sector design for spreading	Strategic	Contiguity, compactness, balanced sectors, grouping of street segments, fixed depot location, and fixed number of sectors	Min spreader fleet size and deadheading	Assignment problems	Composite heuristic
MUYLDERMANS *et al.* (2003)	Sector design for spreading	Strategic	Contiguity, compactness, balanced sectors, grouping of street segments, vehicle capacity, fixed depot location, and fixed number of sectors	Min spreader fleet size and deadheading	Assignment problems and linear MIP	Construction heuristics
KANDULA and WRIGHT (1995)	Combined sector design, depot location, and fleet sizing for spreading and plowing	Strategic	Contiguity, compactness, service hierarchy, class continuity, maximum sector sizes, and fixed number of depots and sectors	Min total distance	Linear MIP	CPLEX Mixed Integer Optimizer
KANDULA and WRIGHT (1997)	Combined sector design, depot location, and fleet sizing for spreading and plowing	Strategic	Contiguity, compactness, and fixed number of depots and sectors	Min maximum distances, fleet size, and deadheading	Linear MIP	CPLEX Mixed Integer Optimizer

results-oriented policy, deterministic dynamic programming based algorithms, such as those proposed by UNGERER (1989), are promising optimization solution methods. Multiobjective analysis is also showing much promise to assist planners in making results-oriented policy decisions. However, the service level policy should ideally be determined endogenously in system design and vehicle routing models to identify the most efficient way of, for example, designing sectors while satisfying a set of technological constraints on contiguity, size or workload, and compactness or shape. Future research directions in service level planning for spreading and plowing operations should thus be oriented towards the development of new mathematical formulations that fully integrate the results of service level models with the design of systems or the routing of vehicles for spreading and plowing operations.

The design of sectors for spreading and plowing operations is closely linked to the location of vehicle and materials depots and the routing of spreaders and plows. However, the design of sectors is most commonly treated as a separate problem. Very frequently, sectors are designed by assuming that depot location decisions are given. The traditional approach for the sector design problem has thus consisted in partitioning the road network into sectors by assigning basic units to their closest depot. MUYLDERMANS *et al.* (2002) and MUYLDERMANS *et al.* (2003) used this approach for designing sectors for spreading and plowing operations. Since the quality of the vehicle routes produced in each sector is highly dependent on the size and shape of the sectors, this approach obviously leads to suboptimal routing decisions. A better approach could consist of designing sectors following the development of the vehicle routing plans similar to the "route first" approaches for the solution of multiple vehicle node routing problems (BEASLEY, 1983).

In addition to the development of new sequential approaches, the sector design problem can be incorporated in compound models that address the integration of sector design with other decisions related to spreading and plowing operations. Models that integrate multiple interdependent subcomponents of the planning process can significantly help to improve benefits and reduce costs of spreading and plowing operations. The compound sector design, depot location, and fleet sizing models proposed by Kandula and Wright (1995, 1997) are good examples of integrated models.

In short, as new mathematical formulations and solution strategies are developed for the service level policy and the sector design problem, the challenge of the future is to build broader models that address the integration of various subcomponents of the planning process related to spreading and plowing operations.

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CHAPITRE 3

A SURVEY OF MODELS AND ALGORITHMS FOR WINTER ROAD MAINTENANCE. PART II: SYSTEM DESIGN FOR SNOW DISPOSAL

Nathalie Perrier, André Langevin et James F. Campbell, *Computers & Operations Research* 33, pages 239–262, 2006.

Cet article présente une revue des modèles d'optimisation et des méthodes de résolution pour le design des systèmes d'entretien hivernal des réseaux routiers pour les opérations d'enlèvement de la neige. Nous décrivons d'abord brièvement les opérations d'enlèvement de la neige et les problèmes de design des systèmes pour ces opérations. Ces problèmes comprennent le partitionnement d'un réseau routier en secteurs de déneigement, la localisation des sites de déversement, l'affectation des secteurs de déneigement aux sites de déversement, et l'affectation des secteurs de déneigement aux entrepreneurs. Les modèles de localisation des sites de déversement et d'affectation des secteurs de déneigement aux sites décrits dans l'article sont regroupés en deux grandes catégories: les modèles où chaque site de déversement possède seulement une capacité annuelle et où chaque secteur peut être affecté à plus d'un site de déversement et les modèles où chaque site possède une capacité horaire et une capacité annuelle et où l'affectation de chaque secteur est limitée à un seul site. Les problèmes de partitionnement d'un réseau routier en secteurs de déneigement pour les opérations d'enlèvement de la neige sont intimement reliés au problème d'affectation des secteurs de déneigement aux sites de déversement et au problème de détermination de la taille de la flotte. Pourtant, ces problèmes sont généralement traités indépendamment en considérant comme donnés les différents secteurs de déneigement.

A Survey of Models and Algorithms for Winter Road Maintenance. Part II: System Design for Snow Disposal

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May 2004

Abstract

This is the second part of a four-part survey of optimization models and solution algorithms for winter road maintenance planning. The first part addresses system design problems for spreading and plowing operations. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the design of systems for snow disposal operations. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies. The two last parts of the survey mainly concentrate on vehicle routing for winter road maintenance.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Operations research.

3.1 Introduction

This is the second part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. Winter road maintenance planning involves a variety of decision-making problems relating to the system design and to the routing and scheduling of vehicles and crews. *System design* in winter road maintenance includes determining the level of service policy, partitioning the geographic region into sectors for efficient operations, locating vehicle depots, materials storage facilities, and disposal sites, assigning the sectors obtained from the partitioning to various facilities, allocating contracts for various operations to private organizations, and sizing and replacing vehicle fleets. Most commonly, decisions relating to the design of winter road maintenance systems belong to the *strategic* or *tactical planning levels*, while decisions concerning the routing and scheduling of vehicles and crews pertain to the *operational planning level* and *real-time control*. The distinction between the three levels of strategic, tactical, and operational planning, and the real-time control is explained in detail in the first part of the survey (PERRIER *et al.*, 2006a).

Winter road maintenance problems are very difficult and site specific because of the diversity of factors influencing the conduct of winter road maintenance operations, including geographical location, climatic and weather conditions, demographics, economics, technological innovations (for materials application, mechanical removal, and weather monitoring), legislative requirements, interagency agreements, variations of traffic rate, and information on the status of personnel, equipment and materials. Also, winter road maintenance planners have a multicriteria environment in which they have to address problems in terms of the three conflicting criteria of efficiency, effectiveness, and equity. For example, the fundamental tradeoff in determining the level of service policy is that a higher level of service requires greater costs for ensuring that roadways (and sidewalks) are safe for travel, but reduces costs for travelers, and for lost production and lost sales when travel is restricted. Furthermore, the benefits and costs of winter road maintenance range from some factors that are easy to quantify (for example, the expenditures for winter road maintenance and the effects of deicing chemicals on the environment, infrastructure, and motor vehicles) to others that are difficult to quantify but are likely to be important in individual situations (for example, the safety effects and impacts on local economies such as savings in accident costs, in delay costs, and in lost wages and productivity costs).

Spreading and plowing operations are usually performed on a regular basis in almost all rural and urban regions with frozen precipitation or significant snowfall. However, in urban areas with large snowfalls and prolonged subfreezing temperatures, the large volumes of snow plowed from roadways and walkways generally exceed the available space along roads for snow storage, and therefore require disposal by some means. The most common solution is to load snow into trucks for transport to disposal sites. Conversely, in rural regions, snow is often simply pushed to the sides of roadways without being removed and hauled.

The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the design of systems for snow disposal operations. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies. The level of service policy and the design of sectors for spreading and plowing operations were reviewed in the first part of the survey (PERRIER *et al.*, 2006a). Vehicle depot and materials depot location, and fleet sizing and replacement problems are discussed along with the routing of vehicles for winter road maintenance in the two last parts of the survey (PERRIER *et al.*, 2005a,b).

The paper is organized as follows. Section 3.2 describes the operations of snow disposal and the system design problems related to those operations. Models for the assignment of sectors to snow disposal sites are described in Section 3.3. Models that address the assignment of sectors to private companies or governmental agencies are reviewed in Section 3.4. Section 3.5 focuses on disposal site location problems for snow disposal. Models dealing with the partitioning of a region or road network into sectors for snow disposal are presented in Section 3.6. Conclusions and future research paths in winter road maintenance planning are presented in the last section.

3.2 Snow disposal for winter road maintenance

Winter road maintenance operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. State and local governments spend about $2 billion in the United States (MINSK, 1998) and approximately $4 to $6 billion in Japan and Europe (TRANSPORTATION RESEARCH BOARD, 1995) each year on these operations. Small percentage savings in these expenditures through optimization could result in substantial total savings over a number of years. The following section contains a brief description of snow loading and hauling operations. System design problems related to snow disposal that have been addressed with operations research methodologies are then discussed. A detailed review of the available technology for winter road maintenance is presented in the book by MINSK (1998).

3.2.1 Snow disposal operations

Snow disposal operations involve loading snow into trucks for hauling to disposal sites. These operations are generally post-storm operations, although they may be required during a snowfall to remove snow from areas, such as alleys or narrow channelled sections, with insufficient space for snow storage. Loading and hauling of snow are usually performed in urban areas with significant snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow disposal following uncommon but very large storms. During snow disposal, parking regulations are generally put into effect to facilitate loading snow into trucks for hauling to disposal sites.

Snow disposal sites are the destinations for snow hauling trucks originating in each sector, and must be visited many times during the snow disposal operations. There are several different types of disposal sites that may be considered, including surface sites, quarry sites, sewer chutes, snow melting machines and water sites. With every disposal site are associated a fixed location cost, an operating cost, and an annual capacity due to the limited space available to store snow. Each disposal site may also have an hourly capacity for unloading trucks depending on the configuration of the disposal site, and the available equipment and manpower. Surface sites typically require large plots of open land and may have very large capacities. They also may have other uses when snow is not present. Melters, in contrast, can be small mobile machines, but are typically quite expensive. Disposal in rivers or lakes represents the most economical disposal method, although disposal sites that allow melted snow to be processed in waste water treatment facilities provide environmental benefits.

Snow disposal operations require a fleet of trucks to haul snow to disposal sites and a fleet of snowblowers, rotary plows, or other types of snow loaders to transfer snow from the roadway into the trucks. In order to minimize the completion time for winter road maintenance, snowblowers (or other types of snow loaders) generally operate in a continuous process loading trucks. In practice, there may be several empty trucks moving slowly in a queue alongside each snowblower to ensure the snowblowers are never idle. As soon as a truck is filled with snow, it departs for the assigned disposal site while another truck takes its place to begin being filled. The truck that departed for the disposal site will travel to the disposal site, dumps its load of snow, possibly after waiting in line, and then return to the end of the queue alongside the assigned snowblower. This closed cyclic continuous system is illustrated in Figure 3.1. There may be more than one such cyclic closed system if a sector contains more than one snowblower or if a sector can be assigned to multiple disposal sites.

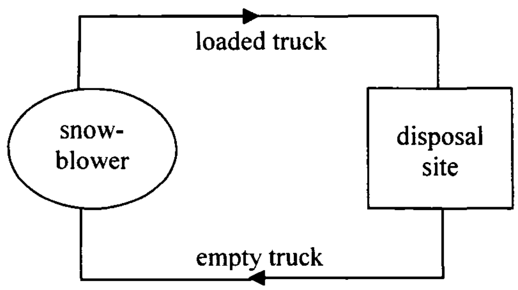

Figure 3.1: The snowblower–truck–disposal site cyclic closed system

Snow disposal operations involve a range of fixed and variable labor, materials, and equipment costs. Variable costs of snow disposal include fuel costs, crew costs, vehicle maintenance costs, and variable costs of operating vehicle depots and disposal sites. Fixed costs of snow disposal include fixed costs of snow removal equipment and fixed costs of acquiring vehicle depots and disposal sites.

3.2.2 System design problems for snow disposal

This section describes system design problems of snow disposal operations that have been addressed by operations research techniques. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies.

Section 3.3 of this survey covers models for assigning sectors to disposal sites. The primary costs for hauling snow to disposal sites include variable costs for transporting snow from sectors to disposal sites and elimination costs for operating disposal sites. Therefore, a good assignment plan of sectors to disposal sites is desirable to minimize these costs. Disposal sites have annual capacities based on their physical size. Disposal sites may also have hourly capacities for processing snow based on the operating and unloading practices at the site. Some disposal sites, such as large quarries and river disposal sites, have relatively high hourly capacities due to multiple unloading stations. Other disposal sites, such as sewer chutes, have effectively unlimited annual capacities but the unloading capabilities are restricted by the limited size of the openings into the sewer system, and the requirement that the temperature of the water not fall too low. Furthermore, for contractual reasons, all the snow of a given sector may be hauled to a single disposal site. This is called the *single assignment* requirement, as opposed to the *multiple assignment* case where snow from a sector can be hauled to several disposal sites. The *snow disposal assignment problem* consists of assigning a set of sectors to a set of disposal sites at minimum cost while satisfying some side constraints such as disposal site capacities and single or multiple assignment requirements.

Section 3.4 of this review addresses models for assigning contracts for snow removal to a set of contractors. Winter road maintenance operations are generally the responsibility of municipal or regional government public works agencies. In regions with low-snowfall or mild climate, agencies usually have sufficient in-house manpower and equipment to perform winter road maintenance operations for a light snowfall. When moderate to heavy snowfall occurs, contract maintenance forces are brought in to supplement the in-house maintenance capability. The contract maintenance forces may be integrated with in-house forces or assigned to particular sectors. Also, winter road maintenance can be fully contracted through private companies, other municipalities, in-house employees (contract with union), or a combination of these options. The *contract assignment problem* consists of assigning a set of sectors to a set of contractors, so as to minimize the total cost bid by the contractors.

Section 3.5 of this survey is devoted to disposal site location problems in the context of winter road maintenance. These problems are generally formulated as network location problems, in which disposal sites can be located only on the nodes or links of the network. In large cities, there are usually several snow disposal sites, possibly of different types, including surface sites, unused quarries, sewer chutes (openings into the storm sewer system), and water sites. Snow disposal sites may generate a number of environmental, social and economic impacts such as soil and water contamination, noise, increased traffic, and infringements on the aesthetics. The environmental impacts of "waste" snow, which is contaminated with deicing chemicals, as well as roadway pollutants (heavy metals, oil and fats, abrasives) is becoming a major concern, and is limiting options for snow disposal. (Quebec prevented dumping snow in rivers in 1996). This entails the redesign of the snow disposal system and gives an impetus to develop economical and environmentally sound snow disposal methods. Snow disposal sites can also present risks to the safety of the neighboring residents, so locating such sites in population centers can be undesirable. However, locating disposal sites far away from the sectors increases the costs due to the longer travel distances and times. Thus, in locating snow disposal sites, the key tradeoff is between minimizing transportation costs and minimizing the number of people adversely affected by the disposal sites. The *snow disposal site location problem* consists of locating disposal sites and assigning a set of sectors to the operative disposal sites at minimum cost while satisfying disposal site capacities and single or multiple assignment requirements.

Section 3.6 of this review is devoted to the sector design problem. Given the large geographic extent of most winter road maintenance operations, an agency generally partitions its service region (and transportation network) into subregions (subnetworks), called *sectors*. All sectors are treated simultaneously by separate crews to facilitate the organization of the operations. A sector is thus a bounded, organizational or administrative subarea in a larger geographical region. The *sector design problem* consists of partitioning a region or transportation network into a mutually exhaustive and exclusive collection of small sectors while satisfying side constraints such as contiguity, size or workload, and compactness or shape. A sector is contiguous if every pair of its basic units is connected. *Basic units* are the units of analysis used to partition the road network into sectors. A basic unit can be defined either as a single street segment or as a small geographic zone that contains a collection of neighboring street segments. Sectors are balanced in workload if they are approximately the same size and are assigned equivalent resources. Finally, sectors may either be compact or elongated in a direction perpendicular to the direction to the disposal site depending on the number of sectors and disposal sites. These criteria are explained in greater detail in the first paper (PERRIER *et al.*, 2006a). Common criteria to design sectors also include the need to conform to existing infrastructure, geography, and jurisdictional boundaries. In sector design models, the number of sectors to construct may be either determined endogenously or given as an input. Also, the sector design process is most commonly performed on a network rather than in the plane.

3.3 Snow disposal assignment models

Urban areas are generally partitioned into geographic sectors that are cleared of snow simultaneously by loading the snow into trucks which then haul the snow to assigned disposal sites. With every sector is usually associated an hourly removal rate and an annual volume of snow to remove. The hourly removal rate in a sector is the rate at which snow is sent out of the sector (in trucks) to a disposal site. This rate is usually expressed as cubic meters of snow per hour and depends on the snowblower and truck fleet sizes, as well as vehicle types. The annual volume of snow generated in a sector depends on the snowfall accumulation and the length of streets and sidewalks to be cleared of snow in the sector. This volume can be estimated based on historical data. Similarly, with every disposal site is associated an hourly capacity and an annual capacity for receiving snow. The hourly receiving rate capacity of a disposal site is usually expressed as cubic meters of snow per hour and depends on the logistics and configuration of unloading facilities at the disposal site. The annual capacity of a site depends on the finite space available for storing snow throughout the winter season.

The snow disposal assignment problem consists of assigning a set of sectors to the snow disposal sites so as to respect the hourly and annual capacities of the disposal sites. Since private contractors may be used for snow loading and hauling, for managerial and contractual reasons the assignment of each sector may be restricted to a single disposal site. The objective is usually to minimize the sum of transportation and operating costs associated with hauling snow and operating disposal sites. Snow disposal assignment plans are usually updated every winter season, but monthly adjustments can be made during the winter season to account for snowfall variability.

Models for the assignment of sectors to snow disposal sites are now reviewed. The case where each disposal site has only an annual capacity and each sector can be assigned to multiple disposal sites is discussed first, followed by models for addressing the more realistic case where disposal sites have annual and hourly capacities and each sector must be assigned to a single site.

3.3.1 Multiple assignment models with annual disposal site capacities

A transportation formulation for the snow disposal assignment problem with annual disposal site capacity constraints and possibly multiple disposal sites per sector was proposed by the BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975). Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$ and for every disposal site $j \in J$, let x_{ij} be a nonnegative variable representing the number of cubic meters of snow from sector i sent to disposal site j, and let c_{ij} represent the transportation cost per cubic meter for hauling snow from sector i to site j. For every site

$j \in J$, define b_j as the variable cost per cubic meter of snow to operate site j, and V_j as the annual capacity of site j in cubic meters. For every sector $i \in I$, let v_i represent the annual volume of snow in sector i in cubic meters. The formulation for the snow disposal assignment problem with multiple assignment and annual disposal site capacities can be stated as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} (b_j + c_{ij}) x_{ij} \tag{3.1}$$

subject to

$$\sum_{j \in J} x_{ij} = v_i \qquad (i \in I) \tag{3.2}$$

$$\sum_{i \in I} x_{ij} \le V_j \qquad (j \in J) \tag{3.3}$$

$$x_{ij} \ge 0 \qquad (i \in I, j \in J). \tag{3.4}$$

The objective function (3.1) minimizes the sum of the disposal site variable costs and the total transportation cost. Constraints (3.2) ensure that all the snow for each sector is sent to some disposal site. Constraints (3.3) ensure that the capacity of each disposal site is not exceeded. Finally, all x_{ij} variables must assume nonnegative values.

The model was solved using IBM's MPSX mathematical programming package. Results on a real-life instance from a large Canadian city with 80 sectors and seven disposal sites produced cost savings on the order of 5% over the current assignment plan for the city. The transportation formulation (3.1)–(3.4) for the snow disposal assignment problem was also proposed by LECLERC (1985) who suggested, in a subsequent paper (LECLERC, 1989), two procedures for post-optimal analysis of a degenerate optimal solution. In order to obtain a feasible solution to the single assignment case where each sector is restricted to be assigned to a single site, LECLERC (1981) also proposed an interactive heuristic procedure that modifies the optimal solution to the transportation problem by slightly adjusting the annual capacity of the disposal sites. The approach was tested on data from the city of Montreal. Finally, a decision support system for the single assignment case has been developed by LECLERC *et al.* (1981). The system incorporates the stepping stone solution method (DANTZIG, 1951), along with the heuristic capacity adjustment procedure.

3.3.2 Single assignment models with annual and hourly disposal site capacities

The most difficult version of the snow disposal assignment problem occurs when each disposal site has an annual capacity as well as an hourly capacity, and each sector must be assigned to a single disposal site. A model dealing with this version of the problem was proposed by CAMPBELL and LANGEVIN (1995b). Their formulation is a linear 0-1 integer program. Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$ and for every disposal site $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if sector i is assigned to site j, and let d_{ij} represent the distance from the centroid of sector i to site j. For every site $j \in J$, define R_j as the maximum hourly capacity for receiving snow at site j. For every sector $i \in I$, let r_i represent the hourly snow removal rate in sector i. The hourly capacity of a disposal site and the hourly removal rate in a sector are expressed as cubic meters of snow per hour. Finally, define V_j and v_i as the annual capacity of disposal site j and the annual volume of snow in sector i, respectively, as above. The formulation is given next.

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} d_{ij} v_i x_{ij} \tag{3.5}$$

subject to

$$\sum_{i \in I} v_i x_{ij} \leq V_j \qquad (j \in J) \qquad (3.6)$$

$$\sum_{i \in I} r_i x_{ij} \leq R_j \qquad (j \in J) \qquad (3.7)$$

$$\sum_{j \in J} x_{ij} = 1 \qquad (i \in I) \qquad (3.8)$$

$$x_{ij} \in \{0,1\} \qquad (i \in I, j \in J). \qquad (3.9)$$

The objective function (3.5) minimizes the total volume-weighted distance. Constraints (3.6) and (3.7) limit the assignments of sectors to disposal sites according to the annual and hourly receiving capacity of the disposal sites. The single assignment constraints (3.8) ensure that each sector is assigned to exactly one disposal site. Finally, all x_{ij} variables are restricted to be binary. This model can be viewed as a two-resource generalized assignment problem, a particular case of the multi-resource generalized assignment problem. As defined by GAVISH and PIRKUL (1991), the multi-resource generalized assignment problem involves the identification of a minimum-cost assignment of tasks to agents in a way that permits assignment of multiple tasks to an agent subject to the availability of a set of multiple resources consumed by that agent. Note that the coefficients v_i and r_i are the same for all sites in the formulation (3.5)–(3.9) whereas the amount of a resource used by an agent in performing a task can differ from one agent to another in a two-resource generalized assignment problem. Since the well-known generalized assignment problem is a special case of the two-resource generalized assignment problem, it follows that the two-resource generalized assignment problem is NP-hard. The multi-resource generalized assignment problem has important applications in database allocation in distributed computing systems (PIRKUL, 1986), in large distributed-computer-system design problems (GAVISH and PIRKUL, 1982; GAVISH and PIRKUL, 1986), in vehicle routing (MURPHY, 1986), and in flexible manufacturing systems in a material requirements planning environment (MAZZOLA *et al.*, 1989).

CAMPBELL and LANGEVIN (1995b) proposed a two-phase heuristic to solve the model (3.5)–(3.9). Their heuristic is a modification of the two-stage procedure proposed by MARTELLO and TOTH (1981) for the generalized assignment problem. In the first phase, a feasible assignment that satisfies the annual and hourly capacities of the disposal sites is obtained by iteratively considering all unassigned sectors and determining the sector with maximum difference in the objective function value between the closest and second closest disposal site. This sector with maximum difference is then assigned to its closest disposal site and the remaining hourly and annual capacities for this site are adjusted accordingly. This phase is repeated until all sectors are assigned. In the second phase, interchanges are performed to improve the solution by considering sectors two at a time and reassigning if a new assignment decreases the total value of the objective function (3.5). The heuristic algorithm is outlined in Figure 3.2.

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1. *Penalty-based assignment*
   a. For each unassigned sector $i \in I$, let $d_{ik} = \min_{j \in J}\{d_{ij} \mid v_i \leq V_k \text{ and } r_i \leq R_k\}$ denote the distance from the centroid of sector i to the closest site $k \in J$ and let $d_{il} = \min_{j \in J \setminus \{k\}}\{d_{ij} \mid v_i \leq V_l \text{ and } r_i \leq R_l\}$ represent the distance from the centroid of sector i to the second closest site $l \in J \setminus \{k\}$. Compute $penalty(i) = v_i\,(d_{il} - d_{ik})$.
   b. Select an unassigned sector $i \in I$ such that $penalty(i) = \max_{i \in I}\{v_i\,(d_{il} - d_{ik})\}$. Assign sector i to site k and set $V_k = V_k - v_i$ and $R_k = R_k - r_i$.
   c. If all sectors are assigned to a site, go to step 2. Otherwise, for any unassigned sector $i \in I$, if $v_i > V_k$ or $r_i > R_k$ or if $v_i > V_l$ or $r_i > R_l$, recalculate $penalty(i)$ as defined above. Return to b.

2. *Two-opt exchange*
   a. For each pair of sectors i and j with assigned sites m and n, respectively, let $c(i, j) = v_i d_{im} + v_j d_{jn} - v_i d_{ik} - v_j d_{jl}$, $k, l \in J$, $k \neq m$ or $l \neq n$, represent the objective function improvement realized by reassigning sectors i and j to sites k and l, respectively. If $c(i, j) > 0$ and the hourly and annual capacities of sites k, l, m, and n are satisfied, reassign sector i to site k, reassign sector j to site l, and adjust hourly and annual capacity utilizations of sites k, l, m, and n accordingly.
   b. Return to the beginning of step 2 until no improvement is obtained.

Figure 3.2: The two-phase heuristic for the snow disposal assignment problem (CAMPBELL and LANGEVIN, 1995b)

Tests performed with data from the city of Montreal involving 60 sectors and 20 disposal sites showed an improvement of 4.2% over the solution in use by the city. The two-phase heuristic achieved the optimal solution in a few seconds, while using CPLEX to optimally solve the model (3.5)–(3.9) required a few hours. Moreover, the heuristic can be used to calculate minimum cost solutions for sensitivity analyses performed to evaluate changes in the total snowfall amount, decreasing use of the river disposal sites, and closing of disposal sites. This model was extended in CAMPBELL and LANGEVIN (1995a) to include both operating and fixed costs for disposal sites. The extended model is presented in Section 3.5.

3.4 Contract assignment models

Because of the seasonal nature of winter road maintenance operations and the magnitude of the manpower and equipment to be deployed, agencies may rely fully or partly on contract service for winter road maintenance operations. There are a variety of contract options for obtaining resources to perform winter road maintenance operations. There may be single or multi-year contracts with private companies or other governmental agencies, and contractor forces may be mixed with in-house forces or given complete responsibility for particular sectors. These choices depend on financial analysis and assessment, political issues, climate, level of service requirements, in-house resources, etc. Maintenance management systems have been developed to assist planners in selecting the least-cost contract options for maintenance activities such as winter road maintenance operations. Such systems were described, for example, by BAUMAN and JORGENSON (1985) and BLAINE (1984). The contract assignment problem consists of assigning a given number of sectors to a set of contractors, so as to minimize the total cost bid by the contractors. Some contractors may require that they be awarded a minimum number of sectors, or none at all. Contractors may also quote a price for a specific collection of sectors. Finally, an agency may want to use its own forces to provide winter maintenance in some sectors.

The contract assignment problem can be represented as a simple network flow problem, even with constraints on the minimum number of sectors to be awarded to each contractor, or with bids allowed on collections of sectors. A transportation formulation of this problem for winter road maintenance was proposed by the BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975). Let I be the set of sectors and H be the set of contractors. For every sector $i \in I$ and for every contractor $h \in H$, let x_{ih} be a binary variable equal to 1 if and only if sector i is assigned to contractor h, and let c_{ih} represent the cost of the bid for sector i by contractor h. For each contractor $h \in H$, define N_h as the maximum number of sectors that can be awarded to contractor h. Define C as the number of sectors to assign. The basic model for the contract assignment problem can be stated as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{h \in H} c_{ih} x_{ih} \tag{3.10}$$

subject to

$$\sum_{h \in H} x_{ih} \leq 1 \qquad (i \in I) \tag{3.11}$$

$$\sum_{i \in I} x_{ih} \leq N_h \qquad (h \in H) \tag{3.12}$$

$$\sum_{i \in I} \sum_{h \in H} x_{ih} = C \tag{3.13}$$

$$x_{ih} \geq 0 \qquad (i \in I, h \in H). \tag{3.14}$$

The objective function (3.10) minimizes the sum of all bidding costs. Constraints (3.11) require that each sector be assigned to at most one contractor. Constraints (3.12) impose a limit on the maximum number of sectors assigned to each contractor. The total number of contracts to grant is satisfied via constraint (3.13). Finally, all x_{ih} variables must assume nonnegative values. This is an unbalanced transportation model. Computational tests using IBM's MPSX mathematical programming package were performed on data from a major Canadian city containing 51 sectors and 27 contractors with 40 contracts to grant. The model was also used to analyze a variety of scenarios for extensions to the basic model, including constraints on the minimum number of sectors to be awarded to each contractor, and bids on collections of sectors.

3.5 Snow disposal site location models

Given a set of planned sectors, the snow disposal site location problem consists of locating disposal sites and assigning sectors to the operating disposal sites so as to respect the hourly and annual capacities of the disposal sites. Like the snow disposal assignment problem, the assignment of each sector may also be restricted to a single disposal site. The objective is to minimize the sum of the fixed disposal site location costs, the variable costs to operate the disposal sites, and the transportation costs for hauling snow from sectors to the disposal sites. Models for the disposal site location problem can also be used to determine the most economical disposal method at the strategic level. Since snow disposal sites generate a large volume of truck traffic and around-the-clock activities during snow disposal operations, snow disposal sites may be considered as seasonally obnoxious facilities. Good disposal site locations will minimize the transportation costs for hauling snow from sectors to the disposal sites, and be in isolated or non-residential locations far from population centers. However, in urban areas, non-residential locations are rare, or require great travel distances, and cost. In an effort to estimate traffic impacts around disposal sites, BRAAKSMA *et al.* (1992) developed a three-phase procedure. The first phase solves the snow disposal assignment problem with annual capacities (not hourly capacities), where each sector may be assigned to more than one site. The second phase estimates the number of truck trips for each site by dividing the total amount of snow assigned to the site by the average capacity of the trucks. The truck traffic at a disposal site is then estimated by distributing the number of trips over the time period for the snow hauling operation and multiplying by a peak hour factor. Truck routing can also be accomplished at that stage and scenarios analysis can be examined. Analysis techniques to determine the traffic impacts for the time period are used last. The truck traffic is converted to an equivalent volume of passenger cars and the impacts on the roads and intersections are then determined. The procedure was tested with data from the regional municipality of Ottawa-Carleton, Canada.

Models for the location of snow disposal sites are now reviewed. We first discuss models that include annual, but not hourly, disposal site capacities and allow multiple assignment (the snow from a sector can be hauled to several disposal sites). We then present models addressing the more realistic case where disposal sites have annual as well as hourly capacities and each sector must be assigned to a single site.

3.5.1 Multiple assignment models with annual disposal site capacities

One of the first models for snow disposal site location belongs to the BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975) which suggested a linear, mixed integer programming formulation of the problem. The model, which is a capacitated fixed charge facility location problem, simultaneously determines the optimal subset of the locations at which to place disposal sites and the optimal assignment of given sectors to these disposal sites. Let I be the set of sectors and J be the set of disposal sites. For every disposal site $j \in J$, let y_j be a binary variable equal to 1 if and only if site j is operative, define V_j as the annual capacity of site j, and let b_j and f_j represent the variable cost per unit volume of snow to operate site j and the fixed cost to locate at candidate site j, respectively. For every sector $i \in I$ and for every site $j \in J$, define x_{ij} as the volume of snow transported from sector i to site j, and let c_{ij} represent the transportation cost per unit volume of snow from sector i to site j. Finally, for every sector $i \in I$, v_i corresponds to the annual volume of snow in sector i. The capacitated fixed charge formulation for the snow disposal site location problem can be stated as follows:

$$\text{Minimize} \quad \sum_{i\in I}\sum_{j\in J}(b_j + c_{ij})x_{ij} + \sum_{j\in J} f_j y_j \tag{3.15}$$

subject to

$$\sum_{j\in J} x_{ij} = v_i \qquad (i\in I) \tag{3.16}$$

$$\sum_{i\in I} x_{ij} \le V_j y_j \qquad (j\in J) \tag{3.17}$$

$$x_{ij} \ge 0 \qquad (i\in I, j\in J) \tag{3.18}$$

$$y_j \in \{0,1\} \qquad (j\in J). \tag{3.19}$$

The objective function (3.15) minimizes the sum of three costs: the variable cost to operate the disposal sites, the transportation cost for hauling snow from sectors to the disposal sites, and the fixed disposal site cost. Constraints (3.16) stipulate that each sector be cleared of snow. Constraints (3.17) limit the assignments of sectors to operating disposal sites according to the annual capacity of the disposal sites. Finally, all x_{ij} variables must assume nonnegative values, while y_j variables are restricted to be binary. Note that this model allows a sector to send snow to more than one disposal site, which is often prohibited in practice. Also, we note that if we are given values y_j for the location variables, then the snow disposal site location problem reduces to the snow disposal assignment problem of the sort discussed in Section 3.3.1. This formulation for the snow disposal site location problem was also proposed by AUDETTE (1982) who developed a two-phase heuristic to solve the model. The first phase finds an initial solution by solving a relaxation of model (3.15)–(3.19) obtained by relaxing the integrality requirements on the y_j location variables and by replacing the cost function in (3.15) by the linear approximation (3.20).

$$\sum_{i \in I} \sum_{j \in J} (b_j + c_{ij} + \frac{f_j}{V_j}) x_{ij} \,. \tag{3.20}$$

A starting value for the cost function can be calculated by fixing the x_{ij} variables to their optimal values in this initial solution and by setting $y_j = 1$ if and only if any sectors are assigned to site j in this solution. The second phase then tries to reduce the value of the cost function by closing disposal sites selected by inspection. By iteratively fixing $V_j = 1$ for a given disposal site $j \in J$ in the model obtained from the relaxation and solving it at each iteration, an improved solution can be found. Tests performed on data from the city of Montreal involving 76 sectors and 28 sites showed that the second phase allowed an improvement of less than 1.5% over the initial solution, but the number of sites decreased by more than 18%.

3.5.2 Single assignment models with annual and hourly disposal site capacities

Depending on the logistics and configuration of unloading facilities at the disposal sites, the previous formulation may not be very realistic by disregarding the maximum snow receiving rate of the disposal sites. For example, sewer chutes have high annual capacities, but relatively small hourly capacities due to the small opening into the sewer system. Moreover, according to operating rules, the assignment of each sector may also be restricted to a single site. CAMPBELL and LANGEVIN (1995a) extended the basic model (3.15)–(3.19) to include both annual and hourly capacities as well as assignment of each sector to a single site. This model is formulated as a single-source capacitated facility location problem in which each facility has two capacities: an hourly capacity for receiving snow and an annual capacity for storing snow. In a single-source capacitated facility location problem, each customer has a demand which must be satisfied by a single facility. In the snow disposal site location problem considered here, each sector must be assigned to a single disposal site.

Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$, let r_i represent the snow removal rate in sector i, expressed as cubic meters per hour. For every disposal site $j \in J$, define R_j as the hourly snow receiving rate capacity of site j, also expressed as cubic meters per hour. Then, the snow disposal site location problem can be formulated as a single-source capacitated facility location problem with two capacities for each facility as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} (b_j + c_{ij}) v_i x_{ij} + \sum_{j \in J} f_j y_j \qquad (3.21)$$

subject to

$$\sum_{i \in I} v_i x_{ij} \leq V_j \qquad (j \in J) \qquad (3.22)$$

$$\sum_{i \in I} r_i x_{ij} \leq R_j \qquad (j \in J) \qquad (3.23)$$

$$\sum_{j \in J} x_{ij} = 1 \qquad (i \in I) \qquad (3.24)$$

$$x_{ij} \leq y_j \qquad (i \in I, j \in J) \qquad (3.25)$$

$$x_{ij}, y_j \in \{0,1\} \qquad (i \in I, j \in J). \qquad (3.26)$$

In this model, the binary variables x_{ij} indicate the assignment of sectors to disposal sites. The objective function (3.21) minimizes the sum of the volume-weighted variable cost to operate the disposal sites, the volume-weighted transportation cost for hauling snow from sectors to the disposal sites, and the fixed disposal site cost. Constraint sets (3.22) and (3.23) ensure that the annual and hourly capacities of each disposal site are not exceeded. Constraint sets (3.24) and (3.25) ensure that each sector is assigned to exactly one operative disposal site. As noted by CAMPBELL and LANGEVIN (1995a), when the location variables y_j are known, then the snow disposal site location problem with single assignment constraints and hourly and annual site capacities reduces to a two-resource generalized assignment problem of the sort discussed in Section 3.3.2. Thus, the authors suggested solving model (3.21)–(3.26) using a heuristic that incorporates the two-phase heuristic of CAMPBELL and LANGEVIN (1995b) for the snow disposal assignment problem presented in Section 3.3.2. However, they do not provide an algorithm.

MARÉCHAL (1997) proposed a model and a column generation algorithm for the snow disposal site location problem with single assignment constraints and two capacities for each site. The model is a linear, 0-1 integer, set-partitioning problem with additional side constraints that limit the assignments of sectors to disposal sites according to the annual and hourly receiving capacity of the disposal sites. To present the formulation, let I be the set of sectors and J be the set of disposal sites. Given the sets I and J, let $M = I \cup J$ and let M_k for $k \in K = \{1,\ldots, |J|(2^{|I|} - 1)\}$ be a subset of M consisting of one disposal site and at least one sector to be assigned to site j. For any disposal site $j \in J$, define also $K_j = \{k \in K | j \in M_k\}$ as the set of all subsets of M associated with site j. For every sector $i \in I$ and for every subset $k \in K$, define the binary constant a_{ik} equal to 1 if and only if sector $i \in M_k$. For every subset $k \in K$, let z_k be a binary variable equal to 1 if and only if subset M_k is selected. The operational parameters v_i, r_i, V_j and R_j are defined as above. The cost parameters $f_{j(k)}$ and $b_{j(k)}$ are now defined for each subset $M_k \in K$ and for each site $j \in M_k$ and the costs $c_{ij(k)}$ are defined for each subset $M_k \in K$, for each sector $i \in M_k$ and for each site $j \in M_k$. Then the snow disposal site location problem amounts to choosing a minimum-cost collection of subsets of M such that each sector is assigned to exactly one disposal site according to the annual and hourly receiving capacities of the disposal sites. The formulation is given next.

$$\text{Minimize} \quad \sum_{k\in K}\left(f_{j(k)} + \sum_{i\in I}(b_{j(k)} + c_{ij(k)})v_i a_{ik} \right) z_k \tag{3.27}$$

subject to

$$\sum_{k\in K} a_{ik} z_k = 1 \qquad (i \in I) \tag{3.28}$$

$$\sum_{k\in K_j} (\sum_{i\in I} v_i a_{ik}) z_k \le V_j \qquad (j \in J) \tag{3.29}$$

$$\sum_{k\in K_j} (\sum_{i\in I} r_i a_{ik}) z_k \le R_j \qquad (j \in J) \tag{3.30}$$

$$z_k \in \{0,1\} \qquad (k \in K). \tag{3.31}$$

The objective function (3.27) minimizes the sum of all subset costs. For the correctness of the objective function, MARÉCHAL showed that there is always at most one subset M_k selected for each disposal site $j \in J$ in an optimal solution to the model (3.27)–(3.31). Constraints (3.28) require that each sector be part of exactly one subset and constraints (3.29) and (3.30) are the disposal sites annual and hourly capacity constraints, respectively. MARÉCHAL showed that the model (3.27)–(3.31) is equivalent to CAMPBELL and LANGEVIN formulation (3.21)–(3.26). The MARÉCHAL formulation contains ($|I| \cdot |J|$) fewer constraints than CAMPBELL and LANGEVIN formulation, but the number of variables is exponential. MARÉCHAL proposed to decompose the model into a master problem and a set of $|J|$ different independent subproblems. The master problem is obtained by relaxing the capacity constraints (3.29) and (3.30) in the original formulation (3.27)–(3.31) and by imposing a limit of one subset M_k, $k \in K_j$, for each disposal site $j \in J$. The model for the master problem can be stated as follows:

$$\text{Minimize} \quad \sum_{k\in K}\left(f_{j(k)} + \sum_{i\in I}(b_{j(k)} + c_{ij(k)})v_i a_{ik} \right) z_k \tag{3.32}$$

subject to

$$\sum_{k \in K} a_{ik} z_k = 1 \qquad (i \in I) \tag{3.33}$$

$$\sum_{k \in K_j} z_k \leq 1 \qquad (j \in J) \tag{3.34}$$

$$z_k \in \{0,1\} \qquad (k \in K). \tag{3.35}$$

Moreover, for every site $j \in J$, the subproblem is of the following form:

$$\text{Minimize} \quad f_j + \sum_{i \in I} \left[(b_j + c_{ij}) v_i - u_i \right] a_i - u_{m+j} \tag{3.36}$$

subject to

$$\sum_{i \in I} v_i a_i \leq V_j \tag{3.37}$$

$$\sum_{i \in I} r_i a_i \leq R_j \tag{3.38}$$

$$a_i \in \{0,1\} \qquad (i \in I) \tag{3.39}$$

where a_i is equal to 1 if and only if sector i is assigned to site j. For each site $j \in J$, the objective function (3.36) corresponds to the reduced cost of variable z_k, $k \in K_j$. Model (3.32)–(3.35) is solved with a branch-and-bound procedure using linear programming relaxations that are solved by column generation. Columns of the master problem are generated by solving, for each site $j \in J$, subproblem (3.36)–(3.39) with an objective that is iteratively updated to reflect the new values of the dual variables u_i. Branching is performed on the binary variables indicating whether a subset is chosen or not. The subproblem (3.36)–(3.39) for each disposal site is a bidimensional knapsack problem that is solved using a simple branch-and-bound method. Computational experiments on instances with up to 80 sectors and 25 disposal sites produced optimal solutions within

30 minutes. However, to address the slow convergence of the column generation algorithm, MARÉCHAL suggested applying the stabilized column generation method proposed by DU MERLE *et al.* (1999) to stabilize and accelerate the procedure. Computational experiments were also performed to evaluate the impact of increasing the fixed costs and the problem size on performance measures such as computation times.

3.6 Sector design models for snow disposal

The traditional approach for the sector design problem consists in partitioning the road network into sectors by assigning basic units to their closest facility. MUYLDERMANS *et al.* (2002, 2003) and KANDULA and WRIGHT (1995, 1997) used this approach for designing sectors for spreading and plowing operations. Similar approaches for solving the sector design problem in the context of snow disposal operations are described in this section. Sector design models can be classified according to the winter road maintenance operations considered. Optimization models that address the sector design related to spreading and plowing operations were reviewed in the first part of the survey (PERRIER *et al.*, 2006a). In this section, sector design issues for snow disposal operations are discussed first, followed by compound models that integrate sector design, fleet sizing, and snow disposal assignment decisions for loading trucks and hauling snow to disposal sites.

3.6.1 Sector design issues for snow disposal

As highlighted by PERRIER *et al.* (2006a), the compactness or shape criterion for designing sectors for winter road maintenance depends on the number of sectors, the number and type of facilities, and the type of winter road maintenance operations (spreading, plowing, loading snow into trucks, hauling snow to disposal sites). When the number of sectors to be designed corresponds to the number of facilities, compact sectors with centrally located facilities (vehicle depots, materials depots, disposal sites) lead to more efficient routing of vehicles. However, when the number of sectors exceeds the number of facilities, the appropriate shape of a set of sectors for efficient routing thus depends on the type of operations. Typically, for efficient routing of spreaders and snow plows, sectors should be elongated towards the vehicle depot or materials depot to reduce travel distance in each route. This is the general guideline for forming sectors for a vehicle routing problem.

As explained in Section 3.2.1, snowblowers generally operate in a continuous process loading trucks to minimize the completion time for snow disposal operations. For hauling snow to disposal sites, the travel time depends on the location of the truck relative to the assigned disposal site when it departs from, and returns to the snowblower. If the snowblower is far from the disposal site, then a truck must travel a long distance to and from the disposal site. Therefore, the number of trucks assigned to the sector must be large enough to ensure that an empty truck will always be available to be filled by the snowblower. However, if the snowblower is near the disposal site, then only a small number of trucks are required to prevent the snowblower to become idle. For efficient routing of snowblowers and trucks, sectors should thus be elongated in a direction perpendicular to the direction to the disposal site to reduce the number of trucks required (LABELLE *et al.*, 2002).

3.6.2 Compound sector design, snow disposal assignment, and fleet sizing models for snow disposal

The combined sector design, snow disposal assignment, and fleet sizing problem addressed in this section involves partitioning an urban area into sectors, assigning the sectors to disposal sites, and determining the number of trucks assigned to each sector for snow loading and hauling operations. As was highlighted by CAMPBELL and LANGEVIN (1995a), the sector design and snow disposal assignment problems are interdependent. Indeed, the size and shape of a sector may influence its assignment and vice versa. In an effort to integrate both sector design and snow disposal assignment decisions into a single optimization model, LABELLE *et al.* (2002) proposed a formulation for the combined problem of sector design, snow disposal assignment, and truck fleet sizing. The model is based on a set of geographic zones each containing a collection of neighbouring street segments and incorporates a limit on sector size, hourly and annual disposal site capacities, as well as contiguity constraints.

To present the LABELLE *et al.* formulation, we first define the decision variables and the operational and cost parameters. Let I, J, and K be the sets of geographic zones, sectors, and disposal sites, respectively. For every zone $i \in I$ and for every sector $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if zone i is assigned to sector j. The number of snowblowers for loading snow into trucks is given and each sector must contain exactly one snowblower. Thus, the cardinality of J corresponds to the number of snowblowers available and there may be sectors to which no zones are assigned in a feasible solution. For every sector $j \in J$ and for every site $k \in K$, let y_{jk} be a binary variable equal to 1 if and only if sector j is assigned to site k. For each zone $i \in I$, define v_i as the annual volume of snow in zone i to be hauled to a disposal site, expressed as cubic meters of snow per year. The annual volume of snow in a zone is estimated based on the historical amount of snow per linear meter of street. Thus, the total length of streets in a zone determines the annual volume of snow generated by the sector to be sent to a disposal site. For every zone $i \in I$ and every site $k \in K$, define d_{ik} as the distance from the farthest part of zone i to site k, expressed as kilometers, and C_{ik} as the operational cost per cubic meter for hauling snow from zone i to site k. The distances d_{ik} are calculated with a shortest path algorithm specifically developed for this application that uses a reduced network of major roadways likely to be traveled by the heavy trucks hauling snow. For details, see LABELLE (1995) and CAMPBELL *et al.*, (2001).

For each disposal site $k \in K$, let V_k and R_k be the annual and hourly capacities of site k, respectively, and let CV_k represent the variable operating cost for disposal site k. For every sector $j \in J$, let r_j be the snow removal rate in sector j, expressed as cubic meters of snow per hour, and let N_j be the number of snow hauling trucks assigned to sector j. The number of snow hauling trucks assigned to a sector is defined so that there should always be a truck available to be filled by the snowblower, while other trucks are traveling to and from the disposal site. This allows snowblowers to operate continuously and will minimize the time required to clear the streets of snow. Thus,

$$N_j = \left\lceil \frac{2\max_{i,k}\{d_{ik}x_{ij}y_{jk}\}}{t_s} \times \frac{r_j}{t_v} \right\rceil + \tau \tag{3.40}$$

where t_v is the truck size, t_s is the truck speed, and τ is the number of "additional" trucks assigned to a sector. The hourly removal rate from sectors is estimated based on the capabilities of snowblowers for filling trucks (described below). The first ratio in N_j is the time taken by a truck to travel from the farthest zone in sector j to its assigned disposal site and back. The second ratio is the snow removal rate in trucks per hour. The product of these two ratios provides the number of trucks, possibly fractional, that would be filled by a continuously operating snowblower during the longest trip to and from the disposal site. The additional number of trucks τ helps to account for variability in the truck travel time. If $\tau = 0$, then the blower would be idle whenever the actual travel time is greater than the average travel time to the farthest zone. Values of τ greater than zero allow the blower to stay busy when travel times exceed the average value.

Finally, let CT be the fixed cost for trucks and M, the maximum number of zones in a sector. Then, LABELLE *et al.* (2002) formulated the combined sector design, snow disposal assignment, and truck fleet sizing problem as a nonlinear 0-1 integer program as follows:

$$\text{Minimize} \quad \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ik} v_i x_{ij} y_{jk} + \sum_{k\in K} CV_k \sum_{i\in I}\sum_{j\in J} v_i x_{ij} y_{jk} + CT \sum_{j\in J} N_j \qquad (3.41)$$

subject to

$$\sum_{j\in J} x_{ij} = 1 \qquad (i \in I) \qquad (3.42)$$

$$x_{ij} \le \sum_{k\in K} y_{jk} \qquad (i \in I, j \in J) \qquad (3.43)$$

$$\sum_{i\in I} x_{ij} \le M \qquad (j \in J) \qquad (3.44)$$

$$\sum_{i\in I}\sum_{j\in J} v_i x_{ij} y_{jk} \le V_k \qquad (k \in K) \qquad (3.45)$$

$$\sum_{i\in I}\sum_{j\in J} r_i x_{ij} y_{jk} \le R_k \qquad (k \in K) \qquad (3.46)$$

each sector is a contiguous collection of zones (3.47)

$$x_{ij}, y_{jk} \in \{0,1\} \qquad (i \in I, j \in J, k \in K). \qquad (3.48)$$

The nonlinear objective function (3.41) minimizes the sum of three costs: the transportation cost for hauling snow from the sectors to the disposal sites, the variable cost to operate the disposal sites, and the fixed cost for the trucks. Constraint set (3.42) assures that each zone is assigned to exactly one sector. Constraint set (3.43) links the zone and snow disposal assignments. It states that a zone can be assigned to a sector only if this sector is assigned to some site. Recall that there may be sectors to which no zones are assigned given that the number of snowblowers available is specified and that exactly one snowblower must be operative in each sector. Constraint set (3.44) limits the size of the sectors to at most M zones. Nonlinear constraint sets (3.45) and (3.46) limit the assignment of sectors to disposal sites according to the annual and hourly receiving capacity of each disposal site. The snow removal rate r_i in (3.46) is defined for each zone $i \in I$ rather than for each sector as in (3.40). Constraints (3.47) require that each sector is composed of a contiguous set of zones. LABELLE (1995) proposed a set of linear constraints requiring that each zone assigned to a disposal site must be contiguous to at least two other zones assigned to the same disposal site. For every pair of zones i, $h \in I$, $i \neq h$, define the binary constant a_{ih} equal to 1 if and only if zone i is adjacent to zone h. Then,

$$2x_{ij} - \sum_{h} a_{ih} x_{hj} \leq 0 \qquad (i \in I, j \in J) \qquad (3.49)$$

requires that every sector contains at least three zones. Note that this simple constraint set does allow non contiguous sectors, but each subsector will have at least three zones. Finally, all x_{ij} and y_{jk} variables are restricted to be binary. Note that the model (3.41)–(3.48) allows each sector to be assigned to several disposal sites. However, for operational reasons in many cities, the assignment of each sector is restricted to a single site.

LABELLE (1995) proposed a linear 0-1 integer program for the combined sector design and snow disposal assignment problem. The nonlinearities in the objective function (3.41) and in the constraint sets (3.45) and (3.46) of the LABELLE *et al.* formulation are removed by eliminating the fixed cost for the trucks and by replacing the two groups of assignment variables by a single composite variable. For every zone $i \in I$, for every sector $j \in J$ and for every site $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if zone i is assigned to sector j and to site k. Define C as the cost per cubic meter-weighted distance for hauling snow. All other operational and cost parameters as well as constants are defined as above. The formulation is given next.

$$\text{Minimize} \quad C \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} v_i d_{ik} x_{ijk} + \sum_{k \in K} CV_k \sum_{i \in I} \sum_{j \in J} v_i x_{ijk} \qquad (3.50)$$

subject to

$$\sum_{j \in J} \sum_{k \in K} x_{ijk} = 1 \qquad (i \in I) \qquad (3.51)$$

$$\sum_{i \in I} \sum_{k \in K} x_{ijk} \le M \qquad (j \in J) \qquad (3.52)$$

$$\sum_{i \in I} \sum_{j \in J} v_i x_{ijk} \le V_k \qquad (k \in K) \qquad (3.53)$$

$$\sum_{i \in I} \sum_{j \in J} r_i x_{ijk} \le R_k \qquad (k \in K) \qquad (3.54)$$

$$2\sum_{k \in K} x_{ijk} - \sum_{h \in I} a_{ih} \sum_{k \in K} x_{hjk} \le 0 \qquad (i \in I, j \in J) \qquad (3.55)$$

$$x_{ijk} \in \{0,1\} \qquad (i \in I, j \in J, k \in K). \qquad (3.56)$$

The objective function (3.50) minimizes the sum of the transportation annual cost for hauling snow from sectors to disposal sites and the variable cost to operate the disposal sites. In this model, constraint sets (3.42) and (3.44) of the LABELLE *et al.* (2002) formulation are modified by replacing each variable x_{ij} by the sum of x_{ijk} for $k \in K$. In addition, contiguity constraints (3.49) are modified by replacing each variable x_{hj} by the sum of x_{hjk} for $k \in K$ and constraint sets (3.45) and (3.46) are modified by replacing each product $x_{ij}y_{jk}$ by the variable x_{ijk}. Though the model (3.50)–(3.55) is linear and contains ($|I| \cdot |J|$) fewer constraints than the nonlinear model (3.41)–(3.46) with the contiguity constraints (3.49), the number of variables increases rapidly as the number of zones, sectors and sites increases.

In contrast to the sequential approach that consists of first partitioning a road network into sectors, and then assigning the sectors to disposal sites, LABELLE (1995) and LABELLE *et al.* (2002) developed a two-phase heuristic of "assign first, partition second" for the combined problem of sector design and snow disposal assignment. In the first phase (assign), the assignment of zones to disposal sites is determined to define each disposal site's "area of influence". In the second phase (partition), sectors are designed for each area of influence by agglomerating neighbouring zones into sectors. The objective for the "assign" phase is to minimize relevant operational costs while the "partition" phase seeks to minimize the number of trucks for the given zone assignments. The problem of assigning zones to disposal sites is solved using an adaptation of a composite heuristic proposed by CAMPBELL and LANGEVIN (1995b) for the snow disposal assignment problem. This composite heuristic is presented in detail in Section 3.3.2. In the constructive phase, zones are assigned to disposal sites based on a penalty calculation. Then, interchanges are performed to improve the solution by considering reassignment of every pair of zones to different sites.

The "partition" phase for aggregating zones into sectors considers each area of influence separately. Recall that the number of snow hauling trucks required in a sector depends on the maximum travel time between a sector and its assigned site. Under the assumption that minimizing distance minimizes travel time, the number of trucks can thus be minimized by designing sectors to minimize the sum of the maximum distances from sectors to disposal sites. The basic idea behind the sector aggregation algorithm is to combine two zones that satisfy the sector size constraint and whose union results in the greatest decrease in the sum of the maximum distances from the zones to the disposal site. This ideally produces sectors that are circular arcs centered on the disposal site. In practice, it tends to produce sectors elongated in the direction perpendicular to the direction to the disposal site. Combining two zones produces a "savings" corresponding to the trip to the closer of the two zones. (This is somewhat analogous to the CLARKE and WRIGHT (1964) savings procedure for the capacitated vehicle routing problem.) The LABELLE *et al.* sector aggregation algorithm is presented in Figure 3.3. The term "zone" is used to refer to the original set of geographic zones assigned to disposal sites and to the agglomeration of several of these original zones. The truck travel distances d_{ik} are estimated with a hybrid distance approximation developed by CAMPBELL *et al.* (2001).

The "assign first, partition second" heuristic was imbedded in a geographical information system to form a decision support system allowing manual adjustments to

1. Set $k = 1$.
2. Repeat the following steps until $k = |K|$:
   a. Let d_{ik} be the distance between the zone i centroid and site k. For each pair of adjacent zones i and j assigned to site k, compute *savings* $s_{ij} = \min\{d_{ik}, d_{jk}\}$.
   b. If a zone assigned to site k has only one adjacent zone assigned to site k and their union will not exceed the sector size limit, then join the two zones. Repeat step b while there are zones assigned to site k with only one adjacent zone assigned to site k.
   c. Order the savings from largest to smallest.
   d. Starting at the top of the savings list, join two adjacent zones i and j whose union will not exceed the sector size limit. If the sector size constraint is not satisfied, move to the next largest savings in the list.
   e. Repeat steps a, b, c, and d until all zones assigned to site k belong to a sector.
   f. Set $k = k + 1$.

Figure 3.3: The sector aggregation algorithm for the sector design problem (LABELLE, 1995; LABELLE *et al.*, 2002)

address selected geographic, political and economic concerns. The system was tested on a real-life instance from the city of Montreal involving 390 zones and 20 disposal sites. Results showed that the system produced sectors having the desired shape in less than 15 seconds and was useful in analyzing a variety of scenarios related to the modification of transportation and elimination costs as well as disposal site capacities. The solution produced by the heuristic had one disposal site with only one isolated zone assigned to it. Such a situation is addressed by manual adjustments, or by taking into account the fixed costs of the disposal sites in the "assign" phase.

3.7 Conclusions

This paper is the second part of a four-part survey of optimization models for winter road maintenance. (The first part of the survey (PERRIER *et al.*, 2006a) discusses system design models for spreading and plowing operations. The two last parts of the review (PERRIER *et al.*, 2005a,b) mainly address vehicle routing, depot location, and fleet sizing

Table 3.1: Characteristics of system design models for snow disposal

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
TRANSPORT CANADA (1975)	Snow disposal assignment	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Transportation problem	MPSX mathematical programming
LECLERC (1985)	Snow disposal assignment	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Transportation problem	Stepping stone
LECLERC (1981) LECLERC *et al.* (1981)	Snow disposal assignment	Tactical	Annual disposal site capacities and single assignment	Min transport costs and disposal site variable costs	Transportation problem	Constructive heuristic
CAMPBELL and LANGEVIN (1995b)	Snow disposal assignment	Tactical	Hourly and annual disposal site capacities, and single assignment	Min total snow volume-weighted distance	Two-resource generalized assignment problem	Composite heuristic
TRANSPORT CANADA (1975)	Contract assignment	Tactical	Annual disposal site capacities and multiple assignment	Min bidding costs	Transportation problem	MPSX mathematical programming
BRAAKSMA *et al.* (1992)	Disposal site traffic impacts	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Snow disposal assignment problem	Heuristic
TRANSPORT CANADA (1975)	Disposal site location	Strategic	One contractor per sector and maximum number of sectors per contractor	Min transport costs, and disposal site variable and fixed costs	Capacitated facility location problem	MPSX mathematical programming
AUDETTE (1982)	Disposal site location	Strategic	Annual disposal site capacities and multiple assignment	Min transport costs, and disposal site variable and fixed costs	Capacitated facility location problem	Heuristic
CAMPBELL and LANGEVIN (1995a)	Disposal site location	Strategic	Hourly and annual disposal site capacities, and single assignment	Min transport costs, and disposal site variable and fixed costs	Single-source capacitated facility location problem	Heuristic
MARÉCHAL (1997)	Disposal site location	Strategic	Hourly and annual disposal site capacities, and single assignment	Min transport costs, and disposal site variable and fixed costs	Linear 0-1 IP	Branch-and-bound
LABELLE (1995)	Combined sector design and snow disposal assignment	Strategic	Contiguity, elongated sectors, maximum sector size, grouping of street segments, hourly and annual disposal site capacities, and multiple assignment	Min transport costs and disposal site variable costs	Linear 0-1 IP	Heuristic assign first, partition second
LABELLE *et al.* (2002)	Combined sector design, snow disposal assignment, and truck fleet sizing	Strategic	Contiguity, elongated sectors, maximum sector size, grouping of street segments, hourly and annual disposal site capacities, and multiple assignment	Min transport costs, disposal site variable costs, and truck fixed costs	Nonlinear 0-1 IP	Heuristic assign first, partition second

models for winter road maintenance problems.) Table 3.1 summarizes the characteristics of the reviewed system design models related to snow disposal operations.

As mentioned in the introduction, these problems are often site specific and highly difficult because of the many significant differences in operating conditions surrounding the winter road maintenance operations. Hence, most research contributions have been case study oriented. Early works usually proposed simplified models of special structure (linear programming or network optimization) that often neglected to incorporate the characteristics of applications arising in practice. Later research generally focused on the design of heuristics to solve more realistic problems.

However, the use of operations research methodologies for winter road maintenance problems is still in its infancy. Even though most problems studied grew out of applications, and the proposed models were tested on real-life instances, few of them have been applied in practice. A survey carried out by GUPTA (1998) in 50 U.S. state departments of transportation and other agencies shows that most departments still rely in large part on decision rules dictated by experience when making vehicle and materials depot location and relocation decisions. Operations research holds great promise for improving winter road maintenance and efforts towards reducing the gap between theory and practice must be made. Also, to deal with almost any real-world applications, proposed models need to be extended in a variety of ways. However, advances in computing power now allow near-optimal solution of problems of realistic size. Thus, some promising directions for future research in winter road maintenance planning are the development of more realistic mathematical formulations, the use of multiobjective analysis, and the development of more comprehensive models that integrate multiple decisions.

The development of more realistic mathematical formulations is crucial not only to take into consideration the characteristics of applications arising from practice, but also to exhibit problem structures that may be readily utilized to design heuristic algorithms. Since winter road maintenance operations lead to large and complex problems, heuristics may continue to be the prevailing approach to such problems. These heuristics could then use the structure revealed by new mathematical models.

As mentioned in the introduction, a common characteristic of nearly all winter road maintenance problems is that multiple and often conflicting objectives need to be addressed. While a few researchers have started to consider multiple objective models for certain problems in winter road maintenance, most problems still await a multiobjective analysis. In particular, in locating snow disposal sites, the tradeoff between minimizing transportation costs and minimizing the number of people affected by the disposal sites remains largely unexplored. Also, in determining the truck fleet size for hauling snow to disposal sites, there may be a tradeoff between minimizing the fixed and variable costs for the trucks and minimizing the length of time for the snow loading and hauling operations. Therefore, another direction worth pursuing involves the use of multiobjective analysis to assist in quantifying these sorts of tradeoffs.

Another promising direction of research is the development of models that address the integration of various decisions in system design. Given the high difficulty of winter road maintenance problems and the large size of the problems encountered in practice, the traditional approach has been to deal with these problems sequentially. Commonly, disposal sites are located first, sectors are then designed and assigned to disposal sites, and vehicle routes and schedules are determined last. This approach simplifies the analysis, but is likely to produce a suboptimal system. The integration of system design models with vehicle routing decisions for winter road maintenance may prove to have a great impact on the future use of operations research in winter road maintenance.

Also, one interesting question might be the potential savings from a dynamic assignment plan of sectors to disposal sites. Generally, a static assignment plan is made at the beginning of the season based on anticipated snowfall totals. However, there might be savings from changing the sector assignments dynamically during the season. This would entail additional administrative costs at the operational planning level, but if savings were substantial, it might be worthwhile.

Finally, new technologies in the application of road weather information systems and weather forecasting services are being implemented in many highway agencies in North America, Europe, and Japan. These technologies are likely to have major repercussions on winter road maintenance operations. More accurate temporal and spatial knowledge of weather and pavement conditions both in real-time and for the near-future can lead to better use of scarce winter road maintenance resources. New problems are arising from the implementation of these new technologies in winter road maintenance. Hence, there will be plenty of challenging problems in the field of winter road maintenance for many years to come.

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CHAPITRE 4

A SURVEY OF MODELS AND ALGORITHMS FOR WINTER ROAD MAINTENANCE. PART III: VEHICLE ROUTING AND DEPOT LOCATION FOR SPREADING

Article écrit par Nathalie Perrier, André Langevin et James F. Campbell; accepté pour publication dans *Computers & Operations Research* en 2005.

Cet article présente une revue de la littérature concernant l'utilisation de modèles d'optimisation et d'algorithmes de résolution pour le routage des véhicules pour les opérations d'épandage de fondants et d'abrasifs. Nous décrivons d'abord brièvement les opérations d'épandage de fondants et d'abrasifs ainsi que les caractéristiques des problèmes de tournées de véhicules pour ces opérations. Les méthodes décrites dans l'article sont des approches de résolution approximatives qui peuvent être regroupées en trois familles distinctes. La première regroupe les approches de type constructif qui construisent leur solution un élément à la fois, sans jamais remettre en question les choix passés. La deuxième famille regroupe quant à elle les méthodes composites qui utilisent comme point de départ une solution réalisable du problème et cherchent à obtenir de meilleures solutions en effectuant une séquence de modifications locales qui améliorent chacune la solution en main. Ces deux familles de méthodes partagent la même carence fondamentale: elles sont incapables de progresser au-delà du premier optimum local rencontré. La dernière catégorie regroupe les adaptations de métaheuristiques, telles que le recuit simulé et les méthodes de recherche avec tabous, qui permettent de surmonter l'obstacle de l'optimalité locale. Ces méthodes sont donc des alternatives particulièrement attrayantes par rapport aux méthodes approximatives traditionnelles. Finalement, cet article présente une revue des modèles de localisation de garages et de dépôts intermédiaires de fondants et d'abrasifs et des modèles d'affectation des équipes de travail aux garages.

A Survey of Models and Algorithms for Winter Road Maintenance. Part III: Vehicle Routing and Depot Location for Spreading

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Abstract

Winter road maintenance planning involves a variety of decision-making problems related to the routing of vehicles for spreading chemicals and abrasives, for plowing roadways and sidewalks, for loading snow into trucks, and for transporting snow to disposal sites. These problems are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. As the third of a four-part survey, this paper reviews optimization models and solution algorithms for the routing of vehicles for spreading operations. We also review models for the location of vehicle and materials depots and for the assignment of crews to vehicle depots. The two first parts of the survey address system design problems for winter road maintenance. The fourth part of the survey covers vehicle routing problems for plowing and snow disposal operations.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Arc routing; Operations research.

4.1 Introduction

This paper is the third part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. This paper surveys optimization models and solution methodologies for vehicle routing and depot location problems for spreading chemicals and abrasives. It also addresses related problems for crew assignment, fleet sizing, and fleet replacement. The fourth part of the survey (PERRIER *et al.*, 2005b) addresses vehicle routing, fleet sizing, and fleet replacement models for plowing and snow disposal operations. The two first parts of the survey (PERRIER *et al.*, 2006a,b) address system design models for winter road maintenance.

Winter road maintenance operations involve a large number of interesting and expensive problems that can be modeled and solved using operations research techniques. These operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. In the United States, winter road maintenance operations consume over $2 billion in direct costs each year (TRANSPORTATION RESEARCH BOARD, 1991). Application of chemicals, including material, labor, and equipment costs, accounts for about one-third of the direct expenditures (TRANSPORTATION RESEARCH BOARD, 1991). Expenditures on sand and other abrasives account for more than 10 percent of winter maintenance budgets, excluding application costs (TRANSPORTATION RESEARCH BOARD, 1991). The use of chemicals and abrasives is also linked with many indirect costs, including damage to motor vehicles, infrastructure, and the environment, totaling more than $5 billion per year in the United States (TRANSPORTATION RESEARCH BOARD, 1991).

In recent years, new technologies in the application of road weather information systems, weather forecasting services, and thermal mapping have been implemented in many agencies in Europe, Japan, and the Unites States to help control the total costs associated with spreading materials, while enhancing their effectiveness. These developments are improving the timeliness and accuracy of weather information at the regional and road network levels, thus facilitating the use of more efficient and effective techniques for spreading operations, including anti-icing and prewetting techniques. New technologies can also make the use of chemicals alternatives more feasible by permitting use of smaller amounts of chemicals and targeted applications in environmentally sensitive areas.

However, progress in the development of optimization models for the routing of vehicles and the location of depots for winter road maintenance has experienced a slow growth compared to improvements in new technologies. Until recently, most contributions addressed simulation methods, simple constructive methods, and simplified models with little consideration of practical characteristics. Previous surveys by GUPTA (1998) and CAMPBELL and LANGEVIN (2000) suggest that many agencies still rely in large part on decision rules dictated by field experiences when making vehicle routing and depot location decisions. The limited progress in the use of optimization models is somewhat surprising given that even a small increase in efficiency or effectiveness through optimization could result in significant savings, improved mobility, and reduced environmental and societal impacts.

In fact, the slow progress in the development of optimization models for the routing of vehicles and the location of depots for winter road maintenance highlights the difficulty of the problems studied. Vehicle routing and depot location problems related to winter road maintenance are especially complex and site specific because of the tremendous diversity in operating conditions such as geography, meteorology, demographics, economics, and technology. (These operating conditions are described in detail in the first part of the survey (PERRIER *et al.*, 2006a)). In addition, winter road maintenance operations are incredibly diverse as they are affected by different operational constraints, which depend on the level of service policies and on the characteristics of the transportation network, road segments, sectors, depots, vehicles, and drivers.

In spite of the difficulty of winter road maintenance vehicle routing problems, recently proposed models tend to take into account a larger variety of characteristics of the problems arising in real-world applications, and the proposed solution methods are often based on local search techniques. Recent developments in modeling and algorithmic tools, the increased performance of computers, and the increased pressure facing state and local agencies to reduce expenditures on winter road maintenance operations, while maintaining or enhancing service levels and minimizing environmental impacts, all motivate the more widespread use of optimization models.

Vehicle routing and depot location problems facing state and local agencies can be classified into four categories according to the planning horizon considered: strategic level, tactical level, operational level, and real-time control. The *strategic* level involves the acquisition or construction of long-lasting resources intended to be utilized over a long time period. The *tactical* level includes medium and short term decisions that are usually updated every few months. The *operational* level is related to the winter tasks that require ongoing attention on a day-to-day basis. Finally, the *real-time* level involves decision-making situations in which operations must be undertaken or altered in a very short time frame (e.g., minutes) in response to the sudden change of the system (equipment breakdowns, weather change, etc.). Though each storm is unique in duration, intensity, and composition, vehicle routes are generally fixed at the beginning of the winter season. A 1995 survey in Minnesota reported that 62% of counties and 55% of cities re-evaluate or change their routes for plowing operations every year (OFFICE OF THE LEGISLATIVE AUDITOR, 1995). However, vehicle routes can be modified based on real-time weather and pavement information. Decisions relating to the location of depots for winter road maintenance may be viewed as strategic or tactical.

The paper is organized as follows. Section 4.2 describes the operations of spreading chemicals and abrasives, the vehicle routing and depot location problems related to those operations, and the crew assignment problem. Models dealing with the routing of vehicles for spreading operations are reviewed in Section 4.3. Models that address the location of vehicle and materials depots for spreading operations are described in Section 4.4. Models for the assignment of crews to vehicle depots are presented in Section 4.5. Conclusions and directions for future research are presented in the last section. The term roadway used in this paper refers to any highway, road street, or other pavement surface that carries motor vehicles.

4.2 Operations context and decision problems

The following section contains a brief description of spreading operations for winter maintenance. More detailed information on the state of the practice in managing winter road maintenance operations, including spreading operations, is presented in the synthesis report by KUEMMEL (1994). That report also includes additional information on estimating winter maintenance benefits and costs; personnel and management issues; weather information systems; and materials, equipment, and facilities for winter road maintenance. Also, a detailed review of the available technology for winter road maintenance, and the scientific underpinnings of that technology, is presented in the book by MINSK (1998). Finally, guidelines for selecting winter road maintenance strategies and tactics for a wide range of operating conditions found in the United States are provided in the report by BLACKBURN *et al.* (2004). These strategies and tactics refer to the combinations of materials, equipment, and techniques (both chemical and mechanical) used in winter road maintenance to achieve a defined level of service. They also include road weather information systems and weather forecasting.

Following the description of spreading operations, this section describes the characteristics of vehicle routing problems related to spreading operations. This section concludes with a brief discussion on vehicle and materials depot location problems, followed by crew assignment problems that have been addressed by operations researchers.

4.2.1 Spreading operations

Spreading operations are directed at achieving three specific goals in winter road maintenance: anti-icing, deicing, and traction enhancement. Anti-icing operations involve the application of chemicals in advance of precipitation to prevent the bonding of snow and ice to the pavement surface. The idea is to use ultimately less chemicals by preventing ice from forming, rather than melting it once it has formed, and to remove snow that has not frozen and bonded to the road surface by less plowing. Anti-icing is most effective when most or all of a lane is treated uniformly. Spraying a liquid chemical permits good control of application rate and coverage and results in much less material loss during application and from traffic compared to spreading a solid chemical. The two major pieces of information to consider when deciding on the start time of anti-icing operations are pavement surface condition and weather forecast. Pavement surface temperature and its trend indicate the probability of snow or ice freezing on the pavement, and the weather forecast indicates the likelihood of precipitation and its most likely form. The advent of road weather information systems and thermal maps of highway segments now provides the means for assessing the pavement surface condition in real time.

Deicing operations involve allowing the bonding of snow and ice to the pavement surface during the precipitation and periodically weakening it with chemicals until the ice-pavement bond is broken and the resulting ice sheets can be removed mechanically by plowing or traffic action. Deicing operations require greater quantities of chemicals than anti-icing operations as a result of dilution of the particles as they bore through the snow and ice layer to reach the ice-pavement bond. Prewetting of solid chemicals improves adhesion to the roadway and speeds the melting into the ice. This reduces bounce and scatter and accelerates deicing operations.

The most widely used chemical is salt because of its low price, ready availability, ease of application, and reliable ice-melting performance. However, over the years, evidence has grown that salt has many negative side effects on infrastructure, vehicles and the environment. The literature in this area was reviewed and evaluated by a special TRB study committee in 1991 (TRANSPORTATION RESEARCH BOARD, 1991). The committee reported that damage attributable to salt include accelerated corrosion of metals in bridges, parking structures, and motor vehicles, increased sodium levels in drinking water, and injury to roadside vegetation. Environmental problems arising from the use of deicing chemicals are also reviewed by MINSK (1998). Salt storage facilities are usually located at highway maintenance yards as well as at other intermediate points along highways.

The most common technique for enhancing traction on thick snow-packed and ice surfaces when temperatures are too low for chemicals to be effective is to spread abrasive materials such as sand, cinders, ash, tailings, and crushed stone and rock. These materials may be applied alone or with varying amounts of chemicals in a mixture. Abrasives are mixed with small quantities of chemicals to keep depots of abrasives from freezing or chunking. They are also mixed with sufficient quantities of chemicals to support both anti-icing and deicing operations. Abrasives are inexpensive, offer some immediate traction on slippery surfaces, and, where cinders and other dark materials are used, provide visible evidence of action by crews. However, the excessive use of abrasives can have several negative consequences, including problems of clogging of drainage channels and sewers, significant cleanup efforts following storms and the winter season, adverse effects on cars, and the pollution resulting from airborne fine particles. A further problem results from the reduced distance a truckload of abrasive can cover compared to a load of salt or other chemical, thus requiring frequent reloading.

The selection of the appropriate spreading operation is based on economics, environmental constraints, climate, level of service, material availability, and application equipment availability. The level of service policies determine the extent of the resource investment. The environmental constraints influence the choice of a chemical or nonchemical operation.

4.2.2 Vehicle routing problems for spreading

The operations of spreading chemicals and abrasives concern the service of a set of road segments by a fleet of vehicles, which are based at one or more depots located in one or more sectors, and travel over an appropriate transportation network. In particular, vehicle routing problems for spreading operations consist of determining a set of routes, each performed by a vehicle that starts and ends at its own depot, such that all road segments are serviced, all the operational constraints are satisfied, and the global cost is minimized. This section describes the typical characteristics of vehicle routing problems related to spreading operations by considering their main components (transportation network, road segments, sectors, vehicle and materials depots, vehicles, and drivers), the different operational constraints that can be imposed on the configuration of the routes, and the possible objectives to be achieved in the optimization process. These characteristics are summarized in Table 4.1. Models and algorithms proposed for the solution of vehicle routing problems related to spreading operations are reviewed in Section 4.3. Vehicle routing problems related to plowing and snow disposal operations are described in the fourth part of the survey (PERRIER *et al.*, 2005b).

Table 4.1: Characteristics of vehicle routing problems for spreading

Components	Characteristics
Transportation network	undirected, directed or mixed
	service hierarchy
	maximum time for spreading completion
Road segments	resource-oriented or results-oriented level of service policies
	length
	service and deadhead traversal times
	service time windows
	service frequencies
	required road segments
	one or multiple passes per road segment
	one or two lanes in a single pass
Sectors	sector design
	number of sectors
	compactness or shape
	balance in sector size or workload
	contiguity
	basic units
Vehicle and materials depots	vehicle and materials depot locations
	centrally located depots relative to sectors
	number of vehicle and materials depots
	materials depot capacities
	variable costs of vehicle and materials depots
	fixed costs of vehicle and materials depots
Vehicles	home depot
	spreader capacity
	spreader type and road segment dependencies
	one or multiple routes per spreader
	variable costs of spreaders
	fixed costs of spreaders
Drivers	maximum duration of driving and working periods
	number and duration of breaks
	overtime
	variable driver costs
Routes	start and end locations of routes
	start times of routes
	load balancing
	class continuity
	class upgrading
	both-sides service
	turn restrictions
	alternations between deadheading and servicing
	service connectivity
	sector boundaries
Objectives	minimize deadheading and fixed costs of spreaders and depots
	minimize fleet size
	minimize alternations between deadheading and servicing
	minimize operational constraint violations

The transportation network is generally described through a graph, whose arcs and edges represent the one-way streets and two-way streets to be serviced, respectively, and whose nodes correspond to the road junctions and to the vehicle and materials depot locations. The corresponding graph can be *directed*, *undirected*, or *mixed* depending on the topology of the transportation network and on the operating policies involved. If the two sides of some streets can be serviced at the same time, as is often the case in spreading operations, the mixed graph can be the appropriate representation. Conversely, if the two sides of the street must be serviced separately, as is the case in plowing and snow disposal operations, arcs may have to be duplicated and edges are replaced by two arcs of opposite direction. The resulting graph is then directed. Associated with the transportation network is a *maximum time* for completing spreading operations based on political and economic considerations. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a *service hierarchy*, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. Associated with each class of roads can also be a *maximum time* for spreading completion.

Most policies for winter road maintenance define levels of service for classes of highways based on their priority. *Level of service policies* tend to be *results-oriented* (e.g., bare pavement), *resource-oriented* (e.g., 24-hour equipment coverage), or a combination of both. Associated with each road segment is a cost, which generally represents its *length*, and three *traversal times*, which are possibly dependent on the vehicle type: the time required to service the road segment, the time of deadheading the road segment if it has not yet been serviced, and the time of deadheading the road segment if it has already been serviced. *Deadheading* occurs when a vehicle must traverse a road segment without servicing it. In general, the longest operation consists of spreading materials on a road segment, followed by deadheading an unserviced road segment, followed by deadheading a serviced road segment. However, the time of deadheading an unserviced road segment can exceed the time of servicing it if, for example, traversing an unserviced road segment is extremely difficult or impossible. Associated with each road segment is also a time interval, called *service time window*, during which the road segment can be spread with materials, which is possibly dependent on the hierarchy of the network, and a *service frequency* (e.g., the road segment should be covered at least once every two hours). Road segments that require spreading at least once according to the level of service policy are called *required* road segments. Lane configurations and road segment widths may necessitate *multiple passes* to service some road segments. Finally, chemicals and abrasives are often spread onto the road segment through a spinner which can be adjusted so that *two lanes* are treated *on a single pass*.

Given the large geographic extent of most winter road maintenance operations, an agency generally partitions its transportation network into subnetworks, called *sectors*. All sectors are treated simultaneously by separate crews to facilitate the management of the operations. A sector is thus a bounded, organizational or administrative subarea in a larger geographical region. Details on the design of sectors for winter road maintenance are given in the first part of the survey (PERRIER *et al.*, 2005b).

The routes performed for spreading operations start and end at one or more *vehicle depots*, located at the vertices of the graph. Associated with every vehicle depot is a given number of vehicles of each type. In spreading operations, vehicle routes often consist of materials spreading and intermediary trips to intermediate facilities, called *materials depots*. These facilities contain chemicals and abrasives to provide opportunities for vehicles to refill with materials without returning to the original starting point. Vehicle and materials depots should be *centrally located relative to sectors they serve* to reduce the distance covered by deadheading trips (KANDULA, 1996). *Costs* associated with vehicle and materials depots include variable costs of operating vehicle and materials depots and fixed costs of acquiring vehicle and materials depots.

Spreading operations are performed using a fleet of vehicles, called *spreaders*, whose size and composition can be fixed or can be defined according to the level of service policies, the configuration of the streets and sidewalks, land use (e.g., residential or commercial) and density of development, and times for spreading completion for each class. A spreader may end service at a depot other than its *home depot*. The *capacity* of the spreader is expressed as the maximum quantity of chemicals or abrasives the spreader can discharge. Application rates for chemicals and abrasives are usually specified in kilogram-per-lane-kilometer. The subset of road segments of the transportation network which can be traversed by the spreader is *dependent* on road segment widths and on the spreader type. Large road segments may require large spreaders. Narrow road segments may require small or medium-sized spreaders. In some applications, each spreader can cover *multiple routes* in the considered time period. Finally, each vehicle type is associated with a *fixed* leasing or acquiring *cost* and a *variable cost* that is proportional to the distance traveled. The variable cost component encompasses the costs of fuel, materials, and maintenance.

Drivers operating the spreaders must satisfy several constraints laid down by union contracts and agency regulations. Examples are *working periods* during the day, maximum duration of working and *driving periods*, number and duration of *breaks* during service, and *overtime*. *Costs* associated with drivers depend on the pay structure (e.g., regular or premium time, single or dual working periods).

The routes must satisfy several operational constraints, which depend on the level of service policies, and on the characteristics of the transportation network, road segments, sectors, spreaders, and drivers. The routes can *start* and *end* at one or more depot *locations* and each route can end service at a depot other than the original starting depot. In anti-icing operations, routes must *start* at the proper *time* for effective spreading of chemicals. The decision must take into account such factors as type of snow (wet or dry), expected temperature conditions at the time of, and following, application, anticipated variations at the critical freeze-thaw point, methods of application, and types of chemical. To *balance* the *workload* across routes, they are often approximately the same length or duration. This helps ensure that all spreading operations will be completed in a timely fashion. Since most arterial roads have multiple lanes that require separate passes, the total workload is usually measured in lane-kilometers. *Class continuity* requires that each route services road segments with the same priority classification. Thus, if a lower-class road is included in a route servicing higher-class roads, its service level may be *upgraded*. Sometimes, it is desirable that both sides of a two-lane, two-way road (one lane each way) be serviced by the same vehicle in a single route. However, *both-sides service* constraints usually arise in plowing operations. Also, the impact of undesirable *turns*, such as U-turns and turns across traffic lanes, is generally greater in routing snow plows as compared to spreading operations. Certain locations may be pre-specified as turn-around locations while turns may be simply prohibited in some regions. Good routes typically have long stretches of spreading and long stretches of deadheading. Too many *alternations between deadheading and servicing* must be avoided form an operational standpoint. *Service connectivity* requires that the subgraph induced by the set of road segments serviced by a spreader is connected. The configuration of routes may also need to conform to existing *sector boundaries*. Routes crossing these boundaries must be avoided from an administrative standpoint. Some operational constraints can be treated as hard constraints and others as soft requirements or as *terms* in an objective function. For example, it may be reasonable to exceed the spreading completion time deadlines for lower-class roads if this leads to reducing the fleet size.

Finally, several, and often conflicting, objectives can be considered for the routing of spreaders. Typical objectives are minimization of the global cost, dependent on the distance covered by *deadheading* trips (or on the deadhead travel time) and on the *fixed costs* associated with the used spreaders and depots; minimization of the *number of spreaders* required to service all the required road segments; minimization of the *alternations* between deadheading and servicing; minimization of the *terms* penalizing the violation of some operational constraints; or any weighted combination of these objectives. Note that minimizing deadhead distance and minimizing total distance are equivalent objectives since all required road segments must be serviced.

4.2.3 Vehicle and materials depot location problems for spreading

Section 4.4 of this survey is devoted to depot location problems in the context of winter road maintenance. A number of different depots are needed for spreading operations, including vehicle depots or equipment repair locations, and materials depots. Depot location problems in the context of winter road maintenance are generally formulated as network location problems, in which facilities can be located only on the nodes or links of the network. Since the same type of vehicles may be used for winter road maintenance as for other maintenance activities, the vehicle depot locations may need to be based on year-round operations. Some material depots may be associated with vehicle depots, while other material re-supply points may be placed strategically in a region to allow spreaders to re-supply without returning to the home depot. Many designs of buildings are used for storage of chemicals and abrasives, from simple sheds with fixed roofs, to sheds with slide-back roofs, to large dome-shaped structures, to silos. Ground-level storage facilities require some device for loading mechanized spreader trucks, such as a front-end loader. The silos enable faster filling of spreaders because they permit several trucks to be filled simultaneously and drivers can load without additional help or equipment.

4.2.4 Crew assignment problems

Section 4.5 of this survey discusses crew assignment models in the context of winter road maintenance. Agencies in areas that frequently experience severe weather year-round sometimes choose to have extra manpower capability to assure timely recovery. When winter road maintenance equipment and vehicles are idle, personnel are used for other tasks such as litter patrol, shop maintenance, equipment repair, or highway facility maintenance. However, in most agencies, the number of workers required for winter road maintenance operations typically exceeds the amount of permanent staff available. Options for obtaining staff to perform winter road maintenance operations include reassigning staff from other agencies and temporary or seasonal employees, and part time workers. Assigning specific highway sections to staff from these other sources usually involves disruption of existing permanent staff assignment plans. Given planned vehicle routes for winter road maintenance operations, the *crew assignment problem* consists of assigning a set of crews to vehicle depots from which emanate the planned routes, so as to satisfy the demand for crews for vehicle routing while minimizing travel costs.

4.3 Vehicle routing models for spreading

Vehicle routing problems related to spreading operations are generally formulated as arc routing problems. ASSAD and GOLDEN (1995) presented an extensive review of the literature in arc routing with special emphasis on applications. In a series of two papers, EISELT *et al.* (1995a,b) presented an integrated overview of the most relevant operations research literature on arc routing. More recently, a book on the subject was edited by DROR (2000).

Over the years, many decision support systems using optimization methods have been developed to assist planners in making vehicle routing decisions for spreading operations. Such systems were described, for example, by DURTH and HANKE (1983), JAQUET (1994), BLESIK (1994), MCDONALD (1998), and ANDERSON *et al.* (2000). These authors did not, however, provide a description of the optimization methods on which their systems rely. A survey of decision support systems for spreader and plow routing is presented in the report by GINI and ZHAO (1997). Also, TURCHI (2002) identified existing decision support systems for winter road maintenance and proposed a taxonomy scheme to categorize them.

Several rule-based decision support tools have also been developed to help planners in selecting chemical applications appropriate to winter weather conditions. For example, MALMBERG and AXELSON (1991) described a project initiated by the Swedish National Road Administration in the early 1990s to develop an expert system, called VVEXP, to recommend the starting time for salting, the salt quantity and the width of application. The project was a comprehensive plan for an integrated road weather information system network, including development of methods for interpolation of road weather information between stations, and automated selection of chemical applications based on rules and facts gathered from field interviews and meetings with experts. However, for financial reasons, further development of the system was stopped (LJUNGBERG, 2000). KETCHAM *et al.* (1995) described a table-based menu developed by the US Federal Highway Administration (FHWA) to suggest maintenance actions to field personnel in the selection and applications of chemicals for anti-icing operations according to current and forecast pavement temperature, snowfall and dew point conditions. Most of the maintenance actions involve the application of chemicals in either a dry solid, liquid, or prewetted solid form. Application rates are given for each chemical form where appropriate. The guidance is based upon the results of four years of anti-icing field testing conducted by 15 US state highway agencies and supported by the Strategic Highway Research Program (SHRP) and the FHWA. It has also been augmented with practices developed outside the US. Later, the De-icing Anti-icing Response Treatment (DART) program was developed for the Ministry of Transportation of Ontario, Canada, to adapt the FHWA menu to a computerized format and to evaluate the effectiveness of the recommended treatments (MINISTRY OF TRANSPORTATION OF ONTARIO, 1999). A more detailed review of published information on the VVEXP, FHWA, and DART systems is presented in the report by PERCHANOK *et al.* (2000).

Several heuristics procedures have been proposed for the routing of vehicles for spreading operations. These can be broadly classified into three categories: constructive methods, composite methods, and adaptation of metaheuristics. These three classes of methods are reviewed in the next three sections, respectively. The characteristics of the contributions are then summarized in Table 2.1 at the end of the section.

4.3.1 Constructive methods

Constructive methods for the routing of vehicles for spreading operations can be divided into four classes: sequential route construction methods, parallel route construction methods, cluster first, route second methods, and optimization methods based on capacitated arc routing or capacitated minimum spanning tree formulations. These four classes of methods are discussed in the next four sections, respectively.

Sequential constructive methods

EVANS (1990) and EVANS and WEANT (1990) described a decision support system, called SnowMaster, to assist planners in constructing feasible routing plans for salt spreading operations that satisfy maximum time constraints for spreading completion, maximum route duration, and vehicle capacities. The time for spreading completion does not include the return time to the depot after salting the last road segment on the route. Road segments that need to be serviced can be specified as requiring one single pass from one direction (as with narrow county roads) or two passes from both directions (as in the case of divided highways). Multiple depots can also easily be taken into account. The problem is solved using the path-scanning algorithm developed by GOLDEN *et al.* (1983) for the capacitated arc routing problem. The path-scanning algorithm uses several selection rules to extend a route under construction. GOLDEN *et al.* described five such rules. However, other selection rules can be preferred. In particular, EVANS (1990) proposed the "1-degree rule" that tries to insert into the emerging route the road segment with one endpoint with degree 1. With this rule, one tends to avoid the formation of isolated required road segments that are likely to increase deadheading if they are inserted into the emerging route at a later stage. The system can also be used to aid planners in the development of routing plans for plowing operations. Results on the road network of Butler County, Ohio involving 185 road segments and a total of 284 lane miles indicated that the system reduced the fleet size by 30% and cut the spreading completion time by 40% over the routing plan in use by the county. The system was useful at both the operational and strategic levels in analyzing a variety of scenarios regarding travel speeds, salt spreading rates, response/coverage times, general fleet requirements, fleet size and mix, and the scheduling of fleet replacement. GUPTA (1998) used SnowMaster at the strategic level to analyze various scenarios related to the opening or closing of vehicle or materials depots. The system has also been used successfully by several different US counties. Details on these applications are given by WADDELL (1994) who also mentioned the use of SnowMaster to help planners in making decisions about fleet size and mix, materials depot location, and materials type and usage.

LI and EGLESE (1996) proposed a three-stage heuristic for the salt spreader routing problem of Lancashire County Council, UK. This problem may be characterized as a multi-depot capacitated arc routing problem with a heterogeneous fleet, roadways of a rural mixed network $G = (V, E \cup A)$ partitioned into classes, a set of planned sectors with centrally located vehicle depots, fixed materials depot locations, and with both sides of a roadway spread in a single pass. The vehicle capacity constraints translate into maximum spreading distances. Associated with each roadway class is a completion time for salting treatment. A route can cover roadways of different classes but the time by which roadways in each class are treated must not be exceeded. The time constraint does not consider the deadheading trip to the depot. Each route must start and end at the same depot location. The objective is to minimize the sum of fixed costs of spreaders and transport costs. Considering a given sector and its centrally located depot v_0, the heuristic determines feasible routes one at a time in three stages. The first stage chooses the farthest non-serviced required link (v_i, v_j) from the depot v_0. Let v_i be the nearest vertex from the depot among the two endpoints v_i and v_j. The second stage finds a feasible path $P_1 = (v_0, ..., v_i)$ starting from v_i (in the reverse direction) using the following link-selection rule: if v_{start} is the current last vertex of the partial path P_1, extend P_1 with the next link (v_k, v_{start}) that fits within capacity and time constraints and minimizes the distance from the depot v_0 to v_k. The last stage determines another feasible path $P_2 = (v_j, ..., v_0)$ starting from v_j using the farthest link-selection rule with additional decision rules to determine if the spreader should head back towards the vehicle depot or the nearest materials depot to refill with salt. Given a partial path P_2 that ends at vertex v_{end}, the farthest link-selection rule chooses the link (v_{end}, v_k) that fits within capacity and time and maximizes the distance from v_k to the depot v_0. When either no non-serviced required link (v_{end}, v_k) of current class can be found or the time constraint would be exceeded by including (v_{end}, v_k), the third stage tries to insert a link of a lower roadway class following the farthest link-selection rule. The time left is then set to the time remaining until the deadline for the lower roadway class. Links of current or lower class with one endpoint with degree 1 are always chosen first in the two last stages to avoid the formation of isolated required roadways. A route ends when no more non-serviced required links of a suitable roadway class can be found. The heuristic, called *Time*

1. Determine the farthest non-serviced required link (v_s, v_t) from the depot v_0. Set $h = \alpha(v_s, v_t)$ where $\alpha(v_s, v_t)$ denotes the priority index of link (v_s, v_t) in G, $\alpha(v_s, v_t) \in \{1, ..., p\}$ with 1 being the highest priority. If the length of the shortest path SP_{0s} from v_0 to v_s is shorter than the length of the shortest path SP_{0t} from v_0 to v_t in G, set $v_{start} := v_s$ and $v_{end} := v_t$ (v_{start} and v_{end} denote the endpoints at the start and end of the route, respectively). Otherwise, set $v_{start} := v_t$ and $v_{end} := v_s$. Set $P_1 := \varnothing$ and $P_2 := \varnothing$.
2. *Phase I*
   a. Let $d(v_i)$ be the number of links incident to v_i. Choose a non-serviced required link (v_{start}, v_i) such that $\alpha(v_{start}, v_i) \geq h$ and $d(v_i) = 1$. If the capacity and time limit constraints permit, set $P_1 := P_1 + (v_{start}, v_i) + SP_{i,start}$ (the link (v_{start}, v_i) must be serviced only once).
   b. Choose a non-serviced required link (v_i, v_j) such that v_i is adjacent to v_{start} in G, $\alpha(v_{start}, v_i) \geq h$, $\alpha(v_i, v_j) \geq h$, and $d(v_j) = 1$. If the capacity and time limit constraints permit, set $P_1 := P_1 + (v_{start}, v_i, v_j) + SP_{j,start}$ (the links (v_{start}, v_i) and (v_i, v_j) are serviced only once).
   c. Choose two non-serviced required links (v_i, v_j) and (v_j, v_k) such that v_i is adjacent to v_{start} in G, $\alpha(v_{start}, v_i) \geq h$, $\alpha(v_i, v_j) \geq h$, $\alpha(v_j, v_k) \geq h$, and $d(v_k) = 1$. If the capacity and time limit constraints permit, set $P_1 := P_1 + (v_{start}, v_i, v_j, v_k) + SP_{k,start}$ (the links (v_{start}, v_i), (v_i, v_j),and (v_j, v_k) are serviced only once).
   d. Repeatedly apply Steps a, b, and c until no such links can be found.
   e. Choose the non-serviced required link (v_i, v_{start}) of priority h in G such that v_i is the nearest vertex to the depot. If the capacity and time limit constraints permit, set $P_1 := (v_i, v_{start}) + P_1$ and $v_{start} := v_i$. Otherwise, set $P_1 := SP_{0,start} + P_1$ (the links of the shortest path $SP_{0,start}$ are not serviced) and $v_{start} := v_0$.
   f. If $v_{start} := v_0$, go to Step 3. Otherwise, return to Step a.
3. *Phase II*
   a. Apply Steps a, b, c, and d of Phase I but by replacing v_{start} by v_{end} and P_1 by P_2.
   b. If the vehicle is less than half full and the remaining distance it can spread is less than the distance to the nearest materials depot location (possibly v_0), choose the non-serviced required link (v_{end}, v_j) of priority h in G such that v_j is the nearest vertex to the materials depot. Otherwise, choose a non-serviced required link (v_{end}, v_j) of priority h in G such that v_j is the farthest vertex from the depot v_0. If no non-serviced required link (v_{end}, v_j) of priority h can be found according to one of these two rules, go to Step e. If there is no non-serviced required link of any priority incident to v_{end}, go to Step f.
   c. If the capacity and time limit constraints permit, set $P_2 := P_2 + (v_{end}, v_j)$, $v_{end} := v_j$, and return to Step a. If the capacity constraint would be exceeded by including (v_{end}, v_j), go to Step d. If the time limit constraint would be exceeded by including (v_{end}, v_j), go to Step e.
   d. Choose the nearest materials depot location v_d (possibly v_0) to v_{end}. If the time limit permits, set $P_2 := P_2 + SP_{end,d}$ (the links of the shortest path $SP_{end,d}$ are not serviced), $v_{end} := v_d$ (the vehicle is refilled with salt at depot v_d), and return to Step a. Otherwise, go to Step e.
   e. If all required links of priority index higher than or equals to h are serviced, go to Step g. Otherwise, set $h = h + 1$ (the time left is the time remaining until the deadline for roadways of priority h) and return to Step a.
   f. Choose the nearest non-serviced required link (v_i, v_j) of priority h to v_{end}. If no such link exists, go to Step g. Otherwise, set $P_2 := P_2 + SP_{end,i} + (v_i, v_j)$ (the links of the shortest path $SP_{end,i}$ are not serviced), $v_{end} := v_j$, and return to Step a.
   g. Set $P_2 := P_2 + SP_{end,0}$ (the links of the shortest path $SP_{end,0}$ are not serviced) and $v_{end} := v_0$. $P_1 + (v_s, v_t) + P_2$ is a feasible vehicle route. If all required links are serviced, stop. Otherwise, return to Step 1.

Figure 4.1: Time constraint two phases algorithm (LI and EGLESE, 1996)

constraint two phases algorithm, is described in Figure 4.1. The following notation is used. Given two paths $P' = (v_i, ..., v_j)$ and $P'' = (v_j, ..., v_k)$ having a common endpoint v_j, the union of the links of these two paths is a longer path $P = (v_i, ..., v_j, ..., v_k)$ which is denoted $P' + P''$.

The authors compared the performance of the Time constraint two phases algorithm and the greedy algorithm proposed by EGLESE (1994) (see Section 4.3.3) on salt spreading problems for three service areas in the County with 77, 140 and 254 nodes and 111, 203 and 380 road segments, respectively. With some manual intervention, this algorithm produced better routes than the greedy algorithm, in terms of both the number of spreaders required and the total distance travelled. This algorithm has also been used successfully by two counties in the northwest of England.

Parallel constructive methods

SOYSTER (1974) described a computerized system for the routing of trucks for spreading of salt and abrasives. The system starts by generating feasible routing plans that satisfy the maximum route durations constraints using the heuristic described in Figure 4.2.

1. For each spreader truck $k \in K$, start with any edge incident to the depot in G as the beginning of route k.
2. For each spreader truck $k \in K$, choose any edge incident to the last edge added to route k that has not been serviced. If such an edge exists and if the maximum route duration t_k permits, add this edge for servicing to route k. Otherwise, choose any edge incident to the last edge added to route k and add this deadheading edge to route k if the maximum route duration t_k permits.
3. If all edges in G have been serviced, for each spreader truck $k \in K$, add the shortest chain between the endpoint of route k and the depot in G to route k in order to get a set of tours and stop.
4. For each spreader truck $k \in K$, if the maximum route duration t_k has elapsed, add the shortest chain between the endpoint of route k and the depot in G to route k and return to step 1.
5. Return to step 2.

Figure 4.2: The heuristic procedure for the spreader routing problem (SOYSTER, 1974)

This heuristic is embedded into a discrete event simulation approach to model spreader movements and interactions. Each feasible routing plan is then evaluated on the number of deadheading trips and on the respect of the service hierarchy. These two criteria are treated through the calculation of a weighted additive score S_p for each routing plan R_p defined as

$$S_p = \sum_{(v_i, v_j) \in R_p} \left(n_{ij} \cdot f_{ij}(t) \right)$$

where n_{ij} is the priority of roadway $(v_i, v_j) \in E$, $n_{ij} \in \{0, 1, 2, 3, 4, 5\}$ with 5 being the highest priority, and $f_{ij}(t)$ is the value of a benefit function corresponding to the time t at which service begins on roadway (v_i, v_j). If roadway (v_i, v_j) is deadheaded, then $n_{ij} = 0$ for the corresponding units of time. The benefit to the public from servicing a particular roadway decreases as a function of the time span between the onset of the storm and the beginning of the service. The routing plans with the highest scores then represent the best routing plans. The author reports that the system performed well on theoretical networks with about a dozen or so intersections and 30 to 40 road segments. The author did not, however, provide a model for the determination of a valid benefit-time function.

COOK and ALPRIN (1976) considered the salt spreader routing problem in the city of Tulsa, Oklahoma. The authors proposed a parallel route construction heuristic to balance the total workloads assigned to the different spreader trucks while satisfying the vehicle capacity and the both-sides service constraint. The heuristic starts by determining, for each spreader truck, the nearest street segment (v_i, v_j) from the materials depot such that the amount of salt to service both sides of the street segment in two separate passes does not exceed the capacity of the vehicle. For each spreader truck, a feasible vehicle route servicing both sides of the street segment (v_i, v_j) in two separate passes is then created (i.e. a route made of a shortest path between the materials depot and v_i, the street segment serviced from v_i to v_j, the street segment serviced from v_j to v_i, and a shortest path between v_i and the materials depot). This process is repeated until all street segments are contained in a route. The heuristic is embedded into a discrete event simulation model of the salt spreading operations for Tulsa. The simulation model incorporates waiting times incurred by trucks when queueing for the operational safety check of spreaders or for reloading at the materials depot. Tests performed with data from the city of Tulsa showed a 36% reduction in total spreading time over the solution in use by the city. The simulation model was also useful to evaluate the benefits of increasing the number of materials depots, the fleet size, and the vehicle capacity.

UNGERER (1989) studied a salt spreading problem where bridge decks and some critical highway locations require immediate salting so as to minimize total distance traveled, while satisfying vehicle capacities and maximum route lengths. This real-time problem is formulated as a capacitated vehicle routing problem with customers representing bridge decks and critical highway locations. It is solved with a parallel version of the CLARKE and WRIGHT (1964) savings algorithm in which the merge step is repeated until no further improvement is possible.

Cluster first, route second methods

LIEBLING (1973) developed a traditional cluster-first, route-second approach for the salt spreader routing problem in the city of Zurich. The approach takes into account vehicle capacities, working periods, and service frequencies. Basic units are first aggregated into a minimum number of feasible sectors, and a spreader route is constructed for each sector by solving a directed Chinese postman problem. The two-phase heuristic can also serve to solve routing problems encountered in other road maintenance operations such as snow plowing or street sweeping. The approach has been successfully used by the city of Zurich. Indeed, the author reports considerable savings in the amount of salt required over the solution used by the city. Finally, the author suggested formulating the spreader routing problem on sidewalks as an undirected Chinese postman problem.

ENGLAND (1982b) described a decision support system to assist planners at the South Yorkshire County Council's department of engineering in developing feasible routes for road maintenance operations such as gully emptying, road sign cleaning, grass cutting, road sweeping, street lighting, and salt spreading. The system first organizes road segments into balanced and compact sectors by using cluster analysis and an interchange method that tries to produce the desired workload ENGLAND (1982a). Several vehicle routes are generated for each sector by using an insertion procedure that takes into account service frequencies and turn restrictions while minimizing deadhead travel time and the number of alternations between deadheading and servicing. The insertion procedure is embedded into a discrete event simulation approach to model vehicle movements for selecting the best identified vehicle route for each sector.

Optimization-based methods

SOYSTER (1974) treated a spreader truck routing problem in which all two-lane, two-way road segments (one-lane each way) must be serviced in one single pass from either direction so as to minimize the total distance traveled, while satisfying vehicle capacities. Let $G = (V, E)$ be an undirected graph where V is the vertex set and E is the edge set and let K be the set of heterogeneous spreader trucks. For each node $v_i \in V$, let $E(v_i) = \{v_j \in V: (v_i, v_j) \in E\}$ be the set of nodes adjacent to node v_i. With every edge $(v_i, v_j) \in E$ is associated a nonnegative length c_{ij}. For each spreader truck $k \in K$, define b_k as the maximum distance truck k can cover depending on its capacity. For each edge $(v_i, v_j) \in E$ and for each spreader truck $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if edge (v_i, v_j) is serviced from v_i to v_j by truck k. Then the problem can be formulated as a linear 0-1 integer program as follows.

$$\text{Minimize} \sum_{k \in K} \sum_{(v_i, v_j) \in E} c_{ij} (x_{ijk} + x_{jik}) \tag{4.1}$$

subject to

$$\sum_{k \in K} (x_{ijk} + x_{jik}) \geq 1 \qquad ((v_i, v_j) \in E) \tag{4.2}$$

$$\sum_{(v_i, v_j) \in E} c_{ij} (x_{ijk} + x_{jik}) \leq b_k \qquad (k \in K) \tag{4.3}$$

$$\sum_{\{v_j : (v_j, v_i) \in E\}} x_{jik} - \sum_{\{v_j : (v_i, v_j) \in E\}} x_{ijk} = 0 \qquad (v_i \in V, k \in K) \tag{4.4}$$

$$\sum_{\{v_j : (v_0, v_j) \in E\}} x_{0jk} \geq 1 \qquad (k \in K) \tag{4.5}$$

$$\sum_{\{v_j : (v_j, v_0) \in E\}} x_{j0k} \geq 1 \qquad (k \in K) \tag{4.6}$$

$$x_{ijk} \in \{0,1\} \qquad ((v_i, v_j) \in E, k \in K). \tag{4.7}$$

The objective function (4.1) minimizes the total distance traveled. Constraints (4.2) require that each road segment be serviced at least once from either direction by a spreader truck. Constraints (4.3) impose a limit on the distance each truck can travel. Flow conservation at every node for each spreader truck type is imposed by constraints (4.4). Constraints (4.5) and (4.6) ensure that each truck starts and ends its route at the depot v_0, respectively. Finally, all x_{ijk} variables are restricted to be binary. Note that this model does not prevent the formation of disconnected subtours, which is often undesirable from an administrative standpoint given that deadheading trips would be necessary between the disconnected subtours associated to each truck. The model (4.1)–(4.7) was applied to a very small hypothetical problem. The LP relaxation of the model was solved using IBM's MPS system. The constraints

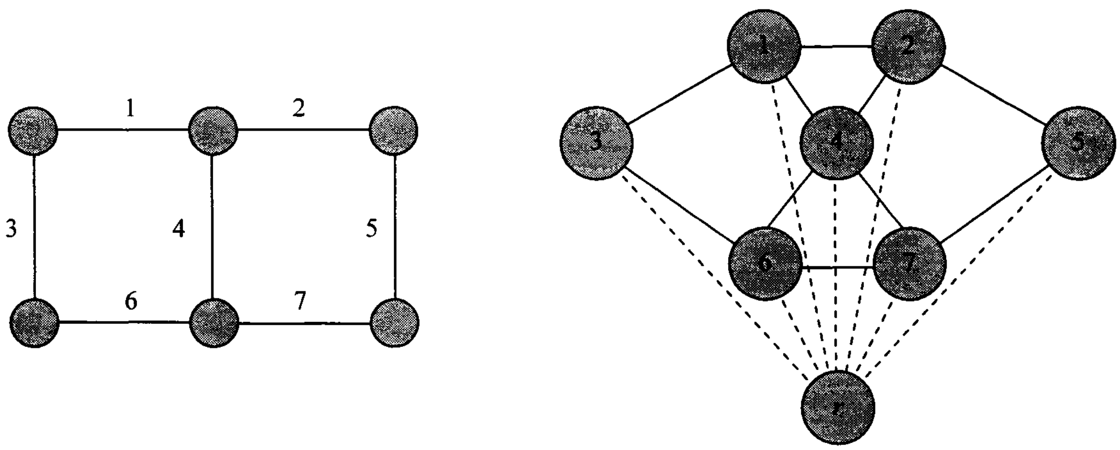
$$\sum_{k \in K} \sum_{\{v_j : (v_j, v_i) \in E\}} x_{jik} \geq \begin{cases} (|E(v_i)| + 1)/2 & \text{if } |E(v_i)| \text{ is odd} \\ |E(v_i)|/2 & \text{if } |E(v_i)| \text{ is even} \end{cases} \qquad (v_i \in V) \tag{4.8}$$

$$\sum_{k\in K}\sum_{\{v_j:(v_i,v_j)\in E\}} x_{ijk} \geq \begin{cases} (|E(v_i)|+1)/2 & \text{if } |E(v_i)| \text{ is odd} \\ |E(v_i)|/2 & \text{if } |E(v_i)| \text{ is even} \end{cases} \qquad (v_i \in V) \qquad (4.9)$$

$$x_{ijk} \geq p \qquad ((v_i, v_j) \in E, k \in K) \qquad (4.10)$$

were added to the LP relaxation in an effort to obtain integrality. Constraints (4.8) and (4.9) ensure that the number of trucks entering and leaving each node, respectively, satisfies the minimum requirement depending on the degree of the node. Integer solutions are also searched for by varying the value of the parameter p in constraints (4.10).

HAGHANI and QIAO (2002) proposed a heuristic decomposition approach for routing salt spreader trucks in Calvert County, Maryland. The approach takes into account service connectivity and vehicle capacity. Moreover, all road segments are treated as two-lane, two-way segments and both lanes can be serviced by the same spreader truck. The approach uses a decomposition of the problem into two subproblems: allocation of road segments to salt spreader trucks and routing. This approach is similar to cluster first, route second methods for vehicle routing problems. In a first step, the problem of assigning road segments to salt spreader trucks is formulated as a capacitated minimum spanning tree problem. Consider the undirected graph $G = (V, E)$ where $V = \{v_0, v_1, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. Let $H = (W \cup \{w_r\}, F_1 \cup F_2)$ be an auxiliary graph such that the vertex set W has a vertex w_j for each edge in the undirected graph G, w_r is the root node, the edge set F_1 contains an edge between w_i and w_j if the corresponding edges in G have a vertex in common, and the edge set F_2 contains an edge between the root w_r and each vertex $w_i \in W$. An example of such a network transformation is illustrated in Figure 4.3. The numbers in G correspond to edge numbers. The two sets of edges F_1 and F_2 are shown as solid lines and dashed lines, respectively.



(a) (b)

Figure 4.3: Example of a network transformation: (a) a graph G;
(b) the corresponding graph H.

Every tree rooted at vertex w_r in graph H specifies a set of road segments to service in the same route that satisfies the service connectivity requirement. With every vertex $w_i \in W$ is associated a nonnegative amount of required salt q_i equal to the total amount of salt to spread on both sides of the corresponding edge in G. Then the subproblem of allocating road segments to salt spreader trucks amounts to choosing a spanning tree rooted at vertex w_r in H such that the total amount of required salt in each subtree does not exceed the truck salting capacity Q. The objective is to minimize the required truck fleet size. For every edge $(w_i, w_j) \in F_1 \cup F_2$, let z_{ij} be a binary variable equal to 1 if and only if edge (w_i, w_j) is included in the optimal spanning tree, and let also f_{ij} be a nonnegative real variable representing the flow on edge (w_i, w_j) from w_i to w_j, expressed as salt units. The formulation is then:

$$\text{Minimize} \quad \sum_{w_i \in W} z_{ir} \tag{4.11}$$

subject to

$$\sum_{\{w_j:(w_i,w_j)\in F_1\cup F_2\}} z_{ij} = 1 \qquad (w_i \in W) \qquad (4.12)$$

$$\sum_{\{w_j:(w_i,w_j)\in F_1\cup F_2\}} f_{ij} - \sum_{\{w_j:(w_j,w_i)\in F_1\}} f_{ji} = q_i \qquad (w_i \in W) \qquad (4.13)$$

$$f_{ir} \le Qz_{ir} \qquad (w_i \in W) \qquad (4.14)$$

$$z_{ij} \in \{0,1\} \qquad ((w_i, w_j) \in F_1 \cup F_2) \qquad (4.15)$$

$$f_{ij} \ge 0 \qquad ((w_i, w_j) \in F_1 \cup F_2). \qquad (4.16)$$

The objective function (4.11) seeks to minimize the total number of edges incident to the root node w_r, which corresponds to the number of subtrees and translates into the number of trucks. (Note that the objective followed in a capacitated minimum spanning tree problem is usually to minimize the sum of costs of the edges in the spanning tree.) Constraints (4.12) assure that each node in H (except the root node) is connected to some other node. Summing up these constraints implies that a spanning tree of H contains exactly $|W|$ edges. Flow conservation at every node (except the root node) is guaranteed by constraints (4.13). Constraints (4.14) impose a limit on the flow on each edge incident to the root node. Finally, all z_{ij} variables are restricted to be binary, while f_{ij} variables must assume nonnegative values. The authors also considered the following objectives for the allocation of road segments to spreader trucks: the minimization of the distance covered by deadheading trips (for a fixed truck fleet size), and a weighted combination of the number of trucks needed and the distance covered by deadheading trips. A distance approximation requiring that each truck traverses a road segment (with or without servicing it) twice from both directions is used to estimate the distance covered by deadheading trips.

Once the set R of required road segments belonging to a vehicle route is determined, routes are then constructed by solving a series of directed rural postman problems. Let

$G' = (V, A_1)$ be a directed graph constructed from G where the arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E. With every arc $(v_i, v_j) \in A_1$ is associated a nonnegative length c_{ij}. Let $A_2 = \{(v_i, v_j), (v_j, v_i) \in A_1: (v_i, v_j) \in R \text{ or } (v_j, v_i) \in R\}$ be a set of two-lane, two-way roads (one lane each way) that can be serviced only once from one direction, and let also $A_3 = \{(v_i, v_j), (v_j, v_i) \in A_1: (v_i, v_j) \in R \text{ and } (v_j, v_i) \in R\}$ be a set of two-lane, two-way roads that need to be serviced twice from both directions, $A_2 \cup A_3 = R$, $A_2 \cap A_3 = \varnothing$. For every arc $(v_i, v_j) \in A_1$, let x_{ij} be a binary variable equal to 1 if and only if arc (v_i, v_j) is traversed (with or without servicing it), and let f_{ij} be a nonnegative real variable representing the flow on arc (v_i, v_j). HAGHANI and QIAO (2002) proposed the following linear, mixed integer program for the problem of determining a minimum cost covering tour for the set R of required road segments assigned to a given salt spreader truck.

$$\text{Minimize} \quad \sum_{(v_i, v_j) \in A_1} c_{ij} x_{ij} \tag{4.17}$$

subject to

$$\sum_{\{v_j : (v_j, v_i) \in A_1\}} x_{ji} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ij} = 0 \qquad (v_i \in V) \tag{4.18}$$

$$x_{ij} + x_{ji} = 1 \qquad ((v_i, v_j), (v_j, v_i) \in A_2) \tag{4.19}$$

$$x_{ij} = 1 \qquad ((v_i, v_j) \in A_3) \tag{4.20}$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1\}} f_{ij} - \sum_{\{v_j : (v_j, v_i) \in A_1\}} f_{ji} = \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ij} \qquad (v_i \in V \setminus \{v_0\}) \tag{4.21}$$

$$f_{ij} \leq |V|^2 x_{ij} \qquad ((v_i, v_j) \in A_1) \tag{4.22}$$

$$f_{ij} \geq 0 \qquad ((v_i, v_j) \in A_1) \tag{4.23}$$

$$x_{ij} \in \{0,1\} \qquad ((v_i, v_j) \in A_1). \tag{4.24}$$

The objective function (4.17) minimizes the total distance traveled. Constraints (4.18) ensure that the indegree of every vertex is equal to its outdegree. Constraints (4.19) and (4.20) assure that each arc is traversed as required. Constraints (4.21)–(4.23) are the subtour elimination constraints (GOLDEN and WONG, 1981). Constraints (4.21) state that the outflow minus inflow of a node must equal the number of outgoing arcs of the node that are traversed. Constraints (4.22) specify that the flow on an arc can be positive only if the arc is in the route. Finally, all f_{ij} variables must assume nonnegative values, while x_{ij} variables are restricted to be binary. The model (4.11)–(4.16) for the problem of assigning road segments to spreader trucks was solved using CPLEX. The authors did not, however, solve the model (4.17)–(4.24). Computational tests on three subnetworks of the existing road network of salting operations in Calvert County showed that the model with the objective function of minimizing the distance covered by deadheading trips obtained the best solutions within fifteen minutes.

4.3.2 Composite methods

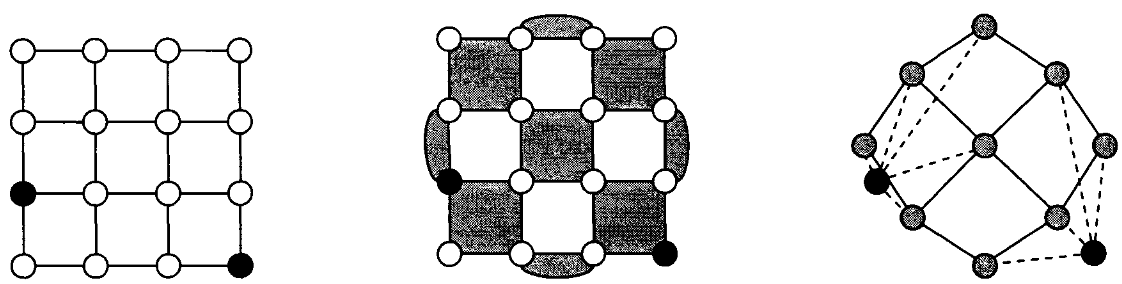
When the vehicle capacity constraints are dominated by the time limit constraint for spreading completion, each route based at a depot location should be covered by a different spreader to complete the salting treatment within the time limit at minimum cost. Conversely, when the time limit constraint is dominated by the capacity constraints, each spreader should cover more than one route within the time limit to minimize the fleet size. A heuristic dealing with this version of the problem was proposed by XIN and EGLESE (1989). The problem considered is to design a set of spreader routes so as to minimize the number of spreaders and the distance covered by deadheading trips, while satisfying the capacities of the spreaders and the time limit for spreading completion. The capacities of the spreaders are again given as maximum distances which can be spread in one route. Each required road segment must be spread exactly once. The heuristic also considers the presence of multiple depots but each route and each spreader must start and end at the same depot. This problem is solved with a heuristic decomposition strategy that iterates between two subproblems until a given number of iterations have been executed or until no further improvement is possible. The first subproblem determines a set of spreader routes based at each depot location such that the total distance of all required road segments to service in a route emanating from a depot does not exceed the maximum distance that can be covered by a spreader based at the same depot, while minimizing the distance covered by deadheading trips. Given the planned spreader routes, the second subproblem partitions the set of routes based at a given depot location into a mutually exhaustive and exclusive collection of subsets of routes so that each subset of routes can be covered within the time limit and the routes included in a subset are associated with the same spreader type, while minimizing the number of spreaders. The first subproblem for each depot location is solved with a heuristic, called *cycle-node scanning algorithm*, analogous to the path-scanning algorithm proposed by GOLDEN *et al.* (1983) for the capacitated arc routing problem. The authors formulated the second subproblem as a bin packing problem with items representing spreader routes and bins representing spreaders or subsets of routes. The second subproblem for each depot location is solved using a heuristic similar to the first-fit decreasing heuristic for the bin packing problem, called the *merging-combining algorithm*. The heuristic for the salt spreading problem studied by XIN and EGLESE is summarized in Figure 4.4.

This heuristic is based on the construction of an auxiliary graph, called *cyclenode graph*, suggested by MALE and LIEBMAN (1978) to solve the routing problem of waste collection vehicles. Let G be an undirected graph. In a first step, required road segments in G are specified as the ones of highest priority and the time window during which these road segments must be spread is replaced by a time limit for service completion (not counting the time to return to the depot after finishing spreading). If G is not Eulerian, then copies of some edges must be added to G so that the augmented graph G'

1. *Cycle-node scanning algorithm*
   a. Let G be an undirected graph. If G is not Eulerian, transform G into an Eulerian graph by solving a minimum cost matching problem. Construct the cyclenode network G'' from G.
   b. For each depot vertex $v_d \in V_2$ in G'', execute the following.
      i) Choose a cycle vertex v_i adjacent to the depot vertex v_d in G'' that is not yet added to a subtree. If no such cycle exists, go to Step iv). Add (v_d, v_i) to T and set $v_k := v_i$ (T denotes a subtree rooted at depot vertex v_d in G'').
      ii) Choose a cycle vertex v_j adjacent to v_k in G'' that is not yet added to a subtree, according to one of the above mentioned cycle-selection rules such that the vehicle capacity is satisfied. If no such cycle vertex exists, go to Step iii). Add (v_k, v_j) to T, set $v_k := v_j$, and repeat Step ii).
      iii) Choose a cycle vertex v_j of T according to one of the above mentioned searching modes. If no such cycle vertex exists, go to Step i). Set $v_k := v_j$, and go to Step ii).
      iv) Choose another depot vertex and go to Step i).
2. *Merging-combining algorithm*
   a. Select two route trees T and T' rooted at the same depot or not.
   b. Combine T and T' if the capacity and time limit constraints permit (If these constraints are broken slightly, move the service of a leaf node serviced in T or T' to another route tree, while satisfying the capacity and time limit constraints. Repeat this "pruning" process until T and T' can feasibly be combined or no leaf node serviced in T or T' can be moved to another route).
   c. Repeat Steps a and b until no more route trees can be combined.
   d. For each depot vertex $v_d \in V_2$ in G'', execute the following.
      i) Let R be the set of routes based at depot v_d. Let R_H be the route of highest duration in R. Set $i = 1$ and $S_i := \{R_H\}$ (S_i denotes the i^{th} subset of routes based at depot v_d). Remove R_H from R.
      ii) Choose the route R_L of lowest duration in R. If $R := \emptyset$, stop. If the time limit permits and if R_L is based at the same depot as the routes of S_i, add R_L to S_i and go to Step iii). Otherwise, set $i = i + 1$ and go to Step iv).
      iii) Remove R_L from R and return to Step ii).
      iv) Choose the route R_k of highest duration in R. If $R := \emptyset$, stop. Otherwise, set $R_H := R_k$, $S_i := \{R_H\}$, remove R_H from R, and return to Step ii).

Figure 4.4: Heuristic for the salt spreading problem (XIN and EGLESE, 1989)

becomes Eulerian. The Eulerian graph G' can be obtained from G by solving a perfect matching problem on a graph whose vertices are the odd-degree vertices of G (EDMONDS and JOHNSON, 1973). A lower class road segment is treated as a required road segment if it is included in the set of edges added to G to create the Eulerian graph G'. Road segments are then grouped into cycles by partitioning the Eulerian graph G' into small cycles using a "checkerboard pattern" to obtain a set of faces with associated cycles. An algorithm for decomposing G into such cycles is given by EGLESE (1994). The cyclenode graph $G'' = (V_1 \cup V_2, E_1 \cup E_2)$ is constructed from the Eulerian graph G' where the vertex sets V_1 and V_2 have a vertex for each cycle and for each vehicle depot in the Eulerian graph G', respectively, and E_1 and E_2 are two edge sets defined as follows. The edge set E_1 contains an edge between two vertices in V_1 if the corresponding cycles in G' have a vertex in common and the edge set E_2 contains an edge between two vertices in $V_1 \cup V_2$ if the depot associated with the vertex in V_2 is the closest depot to any node on the cycle associated with the vertex in V_1. All edges in E_1 have zero cost. With every edge $(v_d, v_i) \in E_2$ is associated a cost c_{di} equal to twice the length of a shortest chain in the original graph G between the depot v_d and the closest node in the cycle to the depot. The construction of the cyclenode graph is illustrated in Figure 4.5. Vehicle depots correspond to black vertices and the two sets of edges E_1 and E_2 are shown as solid lines and dashed lines, respectively.



(a) (b) (c)

Figure 4.5: An example of a cyclenode construction: (a) Original graph G, (b) Eulerian graph G', (c) Cyclenode graph G''

XIN and EGLESE use two cycle-selection rules in Step ii) for attempting to construct a feasible route. Given a subtree T and a leaf node v_i of T, one can choose the cycle vertex v_j adjacent to v_i in the cyclenode network G'' so that the remaining vehicle capacity in T is either minimized or maximized. The authors also use two searching modes in Step iii) to identify a cycle vertex in T from which the route can be extended. They select the cycle vertices in either a first-in, first-out order or in a last-in, first-out order.

The authors proposed a modified version of the cycle-node scanning algorithm where the cycle-selection rule is chosen randomly at each step and the types of spreaders based at each depot location are chosen randomly at each run of the algorithm, following given probability distributions. The authors also developed a somewhat more sophisticated merging-combining algorithm that treats the time limit constraint as a soft constraint by adding to the objective function a term penalizing time limit constraint violations. A *soft time limit constraint* allows the spreaders to complete the salting treatment before or after the time limit. The soft time limit is chosen at random from a uniform distribution which is within a given percentage of the time limit. The heuristic was tested on actual data from the County of Lancashire in England. Computational tests showed that the modified versions of the two algorithms obtained the best solutions but required more time than the original heuristic. The largest instance solved contained 380 road segments, 141 cycle vertices, and three depots.

HAGHANI and QIAO (2001) proposed a model and a heuristic for routing salt spreader trucks in Calvert County, Maryland. The model is a linear, mixed integer, capacitated arc routing problem with time windows and additional side constraints. Besides flow conservation equations and vehicle capacities, the model incorporates priority classes of roadways, maximum route durations, and level of service policies. In particular, each two-lane, two-way segment (one lane each way) is associated with two time intervals, one for each direction, called *time windows*, within which salting must be completed. High-class road segments are given strict service time windows that must be before the time intervals associated with low-class road segments. The number of times required for servicing each two-lane, two-way segment from each direction is also taken into consideration and guarantees that the appropriate level of service is achieved. Finally, most multi-lane, two-way road segments with more than two lanes in each direction must be serviced more than once, but spreader trucks can usually make one single pass over a two-lane, two-way road segment in either direction to spread salt covering both lanes.

The model is based on the formulation proposed by GOLDEN and WONG (1981) for the undirected capacitated arc routing problem. Let $G = (V, E)$ be an undirected graph where $V = \{v_0, v_1, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. The depot is represented by the node v_0. Let K be the set of vehicles of multiple types. For every vehicle $k \in K$, let W_k and t_k be the salting capacity and the maximum route duration of vehicle k, respectively. Let $G' = (V, A_1)$ be a directed graph constructed from G where the arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E. With every arc $(v_i, v_j) \in A_1$ are associated a nonnegative length c_{ij}, a nonnegative number of times n_{ij} arc (v_i, v_j) should be spread, a nonnegative amount of required salt q_{ij} for each pass, and a time window $[0, b_{ij}]$ within which spreading must be completed. Note that since $n_{ij} \geq 0$, the arc routing problem considered is a rural postman problem. For every arc $(v_i, v_j) \in A_1$ and for every vehicle $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is *traversed* by vehicle k while deadheading, let y_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is *serviced* by vehicle k, and let t^s_{ijk} be a nonnegative variable representing the starting time of service or traversal of arc (v_i, v_j) by vehicle k. Note that an arc can not be serviced or deadheaded more than once by the same vehicle. In addition, with every arc $(v_i, v_j) \in A_1$ and with every vehicle $k \in K$ are associated two positive durations t_{ijk} and t'_{ijk} for the service and traversal of arc (v_i, v_j) by vehicle k, respectively. The formulation is then as follows.

$$\text{Minimize } \sum_{k \in K} \sum_{(v_i, v_j) \in A_1} c_{ij} x_{ijk} \tag{4.25}$$

subject to

$$\sum_{\{v_j : (v_j, v_i) \in A_1\}} x_{jik} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ijk} + \sum_{\{v_j : (v_j, v_i) \in A_1\}} y_{jik} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} y_{ijk} = 0 \qquad (v_i \in V, k \in K) \tag{4.26}$$

$$\sum_{k\in K}(y_{ijk}+y_{jik})=1 \qquad ((v_i,v_j),(v_j,v_i)\in A_2) \qquad (4.27)$$

$$\sum_{k\in K} y_{ijk}=n_{ij} \qquad ((v_i,v_j)\in A_1\setminus A_2) \qquad (4.28)$$

$$\sum_{(v_i,v_j)\in A_1} q_{ij}y_{ijk}\le W_k \qquad (k\in K) \qquad (4.29)$$

$$t^s_{ijk}+t_{ijk}y_{ijk}+t'_{ijk}x_{ijk}\le t^s_{jlk}+M(1-x_{ijk}-y_{ijk}) \qquad ((v_i,v_j),(v_j,v_l)\in A_1,k\in K) \qquad (4.30)$$

$$t^s_{ijk}\le b_{ij}+M(1-y_{ijk}) \qquad ((v_i,v_j)\in A_1,k\in K) \qquad (4.31)$$

$$\sum_{(v_i,v_j)\in A_1}(t_{ijk}y_{ijk}+t'_{ijk}x_{ijk})\le t_k \qquad (k\in K) \qquad (4.32)$$

$$\sum_{\{v_j:(v_0,v_j)\in A_1\}}(x_{0jk}+y_{0jk})=1 \qquad (k\in K) \qquad (4.33)$$

$$\sum_{\{v_i:(v_i,v_0)\in A_1\}}(x_{i0k}+y_{i0k})=1 \qquad (k\in K) \qquad (4.34)$$

$$\sum_{v_i,v_j\in S}(x_{ijk}+y_{ijk})\le|S|-1+|V|^2u^S_k \qquad (S\subseteq V\setminus\{v_0\},S\ne\emptyset,k\in K) \qquad (4.35)$$

$$\sum_{v_i\in S}\sum_{v_j\notin S}(x_{ijk}+y_{ijk})\ge 1-w^S_k \qquad (S\subseteq V\setminus\{v_0\},S\ne\emptyset,k\in K) \qquad (4.36)$$

$$u^S_k+w^S_k\le 1 \qquad (S\subseteq V\setminus\{v_0\},S\ne\emptyset,k\in K) \qquad (4.37)$$

$$u^S_k,w^S_k\in\{0,1\} \qquad (S\subseteq V\setminus\{v_0\},S\ne\emptyset,k\in K) \qquad (4.38)$$

$$t^s_{ijk}\ge 0 \qquad ((v_i,v_j)\in A_1,k\in K) \qquad (4.39)$$

$$x_{ijk},y_{ijk}\in\{0,1\} \qquad ((v_i,v_j)\in A_1,k\in K). \qquad (4.40)$$

The objective function (4.25) minimizes the total distance covered by deadheading trips. Constraints (4.26) are flow conservation equations for each vehicle. Define $A_2 = \{(v_i, v_j), (v_j, v_i) \in A_1: n_{ij} = ½ \text{ and } n_{ji} = ½\}$ as the set of two-lane, two-way roads (one lane each way) that can be serviced only once from one direction. Constraints (4.27) and (4.28) state that each arc is serviced as required. Constraints (4.29) guarantee that the capacity of each vehicle is never exceeded. Constraints (4.30) ensure that the time when each vehicle completes the service or traversal of an arc is ahead of the starting time of service or traversal of the next arc (M is a sufficiently large positive number). Constraints (4.31) are the time window restrictions. Maximum route duration for each vehicle is imposed by constraints (4.32). Constraints (4.33) and (4.34) force all vehicles to start and end at the depot, respectively. Constraints (4.35)–(4.38) prohibit the formation of disconnected subtours but allow tours that include two or more closed cycles. These constraints are explained in detail by GOLDEN and WONG (1981). Finally, all t^s_{ijk} variables must assume nonnegative values, while x_{ijk} and y_{ijk} variables are restricted to be binary.

HAGHANI and QIAO (2001) developed a four-stage solution procedure for a less difficult version of the problem where time windows are not considered and all vehicles have the same capacity. The four-stage heuristic procedure is presented in detail in Figure 4.6.

In the first stage, feasible vehicle routes are constructed one at a time as follows. Initial service directions are randomly selected for two-lane, two-way road segments (one lane each way) that can be serviced from one direction. An initial route is created by first determining the furthest required arc from the depot. Then, the nearest required arcs from the route are sequentially inserted into the route as long as vehicle capacity and maximum route duration permit. The arc insertion procedure is analogous to the ADD algorithm for the undirected rural postman problem (HERTZ *et al.*, 1999). The route generation process is repeated until all required arcs are part of one route or more according to the level of service policy. The other stages are improvement procedures that attempt to reduce the total distance traveled while satisfying the vehicle capacity and maximum route duration constraints. The service directions of the two-lane, two-way road segments can be changed in the improvement stages.

1. *Initial solution*
   a. For each pair of arcs (v_i, v_j), $(v_j, v_i) \in A_2$, select a service direction arbitrarily (say from v_i to v_j), set $n_{ij} = 1$, and set $n_{ji} = 0$.
   b. Let sc_{ij} be the length of the shortest path between vertex v_i and vertex v_j in G'. Determine the farthest required arc $(v_i, v_j) \in A_1$ from the depot v_0 with $n_{ij} \geq 1$. The arc (v_i, v_j) is the farthest arc from the depot if it yields the maximum value $sc_{0i} + sc_{j0}$.
   c. Create a feasible vehicle route servicing (v_i, v_j) (i.e. a route made of a shortest path between the depot and v_i, the required arc (v_i, v_j), and a shortest path between v_j and the depot). Set $n_{ij} = n_{ij} - 1$.
   d. Let SP_{ij} be the shortest path between vertex v_i and vertex v_j in G'. If every arc on the route is serviced, then identify a required arc (v_k, v_l) that does not appear on the route and a vertex v_i on the route yielding the minimum value of $sc_{ik} + sc_{li}$, add the circuit $SP_{ik} \cup \{(v_k, v_l)\} \cup SP_{li}$ to the route, and set $n_{kl} = n_{kl} - 1$. Otherwise, if some arcs on the route are not serviced, identify a path $P = (v_s, \ldots, v_t)$ of non-required arcs on the route and a required arc (v_k, v_l) that does not appear on the route yielding the best objective function improvement by replacing P by the path $SP_{sk} \cup \{(v_k, v_l)\} \cup SP_{lt}$, replace P by this path, and set $n_{kl} = n_{kl} - 1$.
   e. Repeat step d as long as vehicle capacity and maximum route duration constraints permit.
   f. If $n_{ij} = 0$ for each required arc $(v_i, v_j) \in A_1$, go to step 2. Otherwise, return to step b.
2. *First improvement*
   a. Augment algorithm
      i) Select two routes R_1 and R_2.
      ii) Let R_1 be the longest route. As long as vehicle capacity and maximum route duration permit, change the status of traversed arcs on R_1, from non-serviced to serviced, if these arcs are serviced by the shorter route R_2. Remove R_2 if all serviced arcs on R_2 are now serviced by the longer route R_1.
      iii) Return to the beginning of step a until no improvement can be obtained.
   b. Merge algorithm
      i) Select two routes R_1 and R_2.
      ii) Identify a common vertex v_c of R_1 and R_2 yielding the best objective function improvement by merging R_1 and R_2 at v_c while satisfying vehicle capacity and maximum route duration constraints. Combine the routes R_1 and R_2.
      iii) Return to the beginning of step b until no improvement can be obtained.
   c. Delete and Insert algorithm
      i) Select two routes R_1 and R_2.
      ii) Identify a serviced arc (v_i, v_j) in R_1 yielding the best objective function improvement by deleting (v_i, v_j) from R_1 (by means of the "Delete" phase described in step iii) and by inserting (v_i, v_j) into R_2 (by means of the "Insert" phase described in step iv) while satisfying vehicle capacity and maximum route duration constraints.
      iii) Delete (v_i, v_j) from R_1 and then try to get a shorter route R_1 by means of the two following strategies:
         - a path P of non-required arcs in R_1 can be reduced by replacing P by a shortest path between the endpoints of P;
         - if a serviced arc is traversed twice in route R_1 in the same direction, interchange the service and deadhead of that arc and try to reduce the length of R_1 by applying the former strategy.
      iv) Insert (v_i, v_j) into R_2 by means of the following procedure. If every arc on R_2 is serviced, then identify a vertex v_k on R_2 yielding the minimum value of $sc_{ki} + s_{jk}$ and add the circuit $SP_{ki} \cup \{(v_i, v_j)\} \cup SP_{jk}$ to R_2. Otherwise, if some arcs on R_2 are not serviced, identify a path $P = (v_s, \ldots, v_t)$ of non-required arcs on R_2 yielding the best objective function improvement by replacing P by the path $SP_{si} \cup \{(v_i, v_j)\} \cup SP_{jt}$ on R_2 and replace P by this path.
      v) Return to the beginning of step c until no improvement can be obtained.
3. *Second improvement*
   a. Try to improve the solution by means of the Merge algorithm.
   b. Try to improve the solution by means of the Delete and Insert algorithm.
   c. Link Exchange algorithm
      i) Select two routes R_1 and R_2.
      ii) Identify a serviced arc (v_i, v_j) in R_1 and a serviced arc (v_k, v_l) in R_2 yielding the best objective function improvement by deleting (v_i, v_j) from R_1 and (v_k, v_l) from R_2 (by means of the "Delete" phase of the Delete and Insert algorithm) and by inserting (v_i, v_j) into R_2 and (v_k, v_l) into R_1 (by means of the "Insert" phase of the Delete and Insert algorithm) while satisfying vehicle capacity and maximum route duration constraints.
      iii) Delete (v_i, v_j) from R_1 and (v_k, v_l) from R_2 and insert (v_i, v_j) into R_2 and (v_k, v_l) into R_1.
      iv) Return to the beginning of step c until no improvement can be obtained.
4. *Third improvement*
   a. Try to improve the solution by means of the Delete and Insert algorithm.
   b. Try to improve the solution by means of the Link Exchange algorithm.

Figure 4.6: The four-stage heuristic (HAGHANI and QIAO, 2001)

The second stage tries to improve the solution obtained at the first stage by applying three post-optimization procedures successively. The first procedure, called Augment, selects two routes and discards the shorter route if its required arcs can be serviced by the longer route. The second procedure, called Merge, combines two routes at the common node that provides the best objective function improvement. The Augment and Merge procedures are clearly analogous to the approaches used in the "Augment" and "Merge" phases of the augment-merge algorithm developed by GOLDEN *et al.*, (1983) for the capacitated Chinese postman problem. The last post-optimization procedure, called Delete and Insert, deletes a required arc from a route, reduces the length of the route by covering the same set of required arcs, but not necessarily in the same order, and inserts the arc into another route. This procedure is similar to the DROP-ADD algorithm used for the undirected rural postman problem (HERTZ *et al.*, 1999). However, the DROP-ADD algorithm removes a required edge from a route, shortens the solution, and reinserts it into the same route.

The third stage then tries to improve the solution obtained at the second stage by applying the Merge, Delete and Insert, and Link Exchange algorithms consecutively. The Link Exchange algorithm is similar to the Delete and Insert algorithm except that a required arc is removed from each route and inserted in the other route. A detailed analysis of the performance of the Augment, Merge, Insert, and Link Exchange algorithms and combinations of these was performed by QIAO (1998). As a complement of the Link Exchange algorithm, QIAO (2002) proposed two improvement procedures in which the number of arcs to remove or to insert into a route is randomly generated (Random Link Exchange) or the arcs to delete or to insert are randomly chosen and the number of arcs removed or inserted into a route can be more than two (Multiple Link Exchange).

Finally, the fourth stage tries to improve the solution obtained at the third stage by applying the Delete and Insert and Link Exchange algorithms successively. The heuristicwas tested on three subnetworks of the existing road network of salting operations in Calvert County, Maryland, with up to 42 nodes and 104 edges. The system reduced the distance covered by deadheading trips by 15-54% over the solution in use by the County with fewer vehicles used (up to three) and with computing times less than two minutes.

4.3.3 Metaheuristics

We are aware of three types of metaheuristic that have been applied to vehicle routing problems related to spreading operations: simulated annealing, tabu search, and elite route pool.

A simulated annealing approach was used by EGLESE (1994) to address the salt spreader routing problem of Lancashire County Council, UK. The approach takes into account the service hierarchy of the network, given in terms of time windows, as well as the capacities of the spreaders and the salt application rates, given in terms of maximum distances which can be spread in one route. For each road segment, both sides must be spread in one single pass. The approach can also deal with multiple depot locations but each route must start and end at the same depot. The simulated annealing approach is again based on the concept of cyclenode network (see Figure 4.5). Each subtree (tree) rooted at a depot node in the cyclenode graph G'' corresponds to a route (set of routes) in the original graph G. A solution of the salt spreader routing problem is thus defined as a spanning collection of trees of the graph G'' rooted at the depot nodes. An initial solution is obtained by means of a greedy algorithm similar to the CLARKE and WRIGHT (CLARKE and WRIGHT, 1964) savings procedure for the capacitated vehicle routing problem. When two subtrees or trees, one containing cycle vertex v_i incident to depot v_d and the other containing cycle vertex v_j adjacent to v_i in the graph G'', can be combined so that the maximum spreading distance and time limit constraints are satisfied, a distance saving $s_i = c_{di}$ is generated. The algorithm is described in Figure 4.7.

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1. *Savings computation*
   a. For each cycle vertex $v_i \in V_1$, compute the *saving* $s_i = c_{di}$.
   b. For each cycle vertex $v_i \in V_1$, create a route made of the edge (v_d, v_i). The resulting spanning collection of trees corresponds to the subgraph induced by the set of edges in E_2 (At this stage, if the maximum spreading distance and time limit constraints are not satisfied, partition the graph G into two or three smaller subgraphs, construct a cyclenode network for each subgraph, compute the savings, and create a route for each cycle vertex).
   c. Order the savings from largest to smallest.
2. *Best feasible combination*
   a. Starting at the top of the savings list and moving downwards, execute the following. Given a saving s_i, determine whether there exists two subtrees or trees, one containing cycle vertex v_i incident to the depot vertex v_d and the other containing cycle vertex v_j adjacent to v_i in the graph G'' that can be combined so that the maximum spreading distance and time limit constraints are satisfied. If so, combine these two subtrees or trees by deleting (v_d, v_i) and introducing (v_i, v_j). If the maximum spreading distance and time limit constraints are not satisfied, move to the next largest saving in the list.

Figure 4.7: Greedy algorithm for the salt spreading problem (EGLESE, 1994)

Solutions violating the maximum spreading distance and time limit constraints are allowed during the simulated annealing search. These intermediate infeasible solutions are however penalized through the minimization of an artificial objective function $f(S) = R(S) + \alpha_D E_D(S) + \alpha_T E_T(S)$, where $R(S)$ is the total number of routes in solution S, $E_D(S)$ is the sum over all routes of S of the excess distance with respect to maximum spreading distance, $E_T(S)$ is the sum over all routes of S of the time exceeding time limit for service completion, and α_D and α_T are the corresponding penalty parameters. A neighbor solution S' is obtained from a solution S by moving the service of a leaf node v_i from a route tree T to another route tree T', by replacing the edge incident to v_i in T by the edge joining v_i to the closest depot, or by combining two route trees. The service of v_i is removed from T by deleting the edge incident to v_i while it is introduced into T' by adding to T' the edge joining v_i to a cycle vertex serviced in T' that is adjacent to v_i in the graph G''. Two route trees T and T' are combined by adding to S the edge joining v_i

to a cycle vertex serviced in T' that is adjacent to v_i in G'' and by removing from S the highest-cost edge of the two edges incident to the root nodes. The cooling schedule employed by EGLESE is typical of what is commonly done in simulated annealing. The temperature is decreased as a step function: initially, the temperature is set equal to a value chosen experimentally and is multiplied by a constant factor α ($0 < \alpha < 1$) after every K iterations, where K is a fixed multiple of the size of the neighborhood. The search ends when the number of routes has not decreased for at least five consecutive cycles of K iterations and the number of accepted moves for the more recent cycle of K iterations has been less than a given percentage of K. To account for the service hierarchy of the network, the problem for each roadway class is solved using the cyclenode construction as well as the simulated annealing approach. Routes are then assigned to spreaders so that each covers a route of highest priority first, followed by a route of lower priority, and so on.

The heuristic decreased the number of depots by more than 50% over the solution in use by the County without increasing the fleet size or the number of routes. The heuristic was also used to evaluate the impacts of fleet size and depot location changes on service levels and costs. EGLESE and LI (1992) evaluated the efficiency of the heuristic by computing a ratio of salting distance to the total distance travelled. This ratio varied from 56% to 80% in 31 County divisions. An upper bound on this ratio can be obtained by evaluating it for the unconstrained Chinese postman solution. Computational tests on four subsets of roads in Lancashire showed that even the upper bound rarely exceeded 70%, a result that the authors explained by noting the large incidence of "T-junctions" (nodes at which three roads meet) in the rural areas considered. Such nodes of degree 3 generate important deadheading trips reflected in the low efficiency ratios. In contrast, urban areas forming a square grid pattern have odd nodes only on the borders of the area and, as they are close to each other, the artificial links added to join them in pairs are relatively short, thus yielding higher ratios. In practice, in addition to the topology of the network, the sector design can also affect the routing efficiency measure.

BENSON *et al.* (1998) proposed the SIRMM system (Snow and Ice Removal Monitoring and Management) to assist planners in developing feasible routes for salt spreading maintenance vehicles. The system can deal with different vehicle capacities, service and deadhead speeds, different times for service completion, and one or multiple passes per road segment depending on lane configurations and road widths. The problem is modeled as a nonlinear mixed integer program in which vehicle capacities and time limit constraints are treated as soft constraints. Soft time limit constraints allow a vehicle to complete spreading road segments before or after its time limit. Similarly, soft vehicle capacity constraints allow a spreader vehicle to be assigned to a route requiring more salt than the vehicle can contain. As a result, the vehicle incurs a lower level of service.

Let $G = (V, A)$ be a directed graph where $V = \{v_1, \ldots, v_n\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the arc set and let $R \subseteq A$ be the subset of required arcs. It is convenient to also denote an arc by a. With every arc $(v_i, v_j) \in R$ are associated a traversal time t_{ij} of servicing arc (v_i, v_j) and a positive number of passes l_{ij} required to service arc (v_i, v_j). For every pair of required arcs (v_i, v_j), $(v_p, v_q) \in R$, let sp_{ijpq} be the traversal time of the shortest path between arc (v_i, v_j) and arc (v_p, v_q) in G. Let H be the set of spreader routes. The number of spreader vehicles is given and each route must be covered by exactly one spreader. Thus, the cardinality of H corresponds to the number of spreader vehicles available and there may be routes to which no required arcs are assigned in a feasible solution. For every route $h \in H$, define a_h, z_h, d_h, and r_h as the home depot of the vehicle assigned to route h, the destination depot at which the vehicle assigned to route h ends service, the time required by deadheading trips on route h, and the rate at which salt is applied by the vehicle assigned to route h, expressed as kilograms per unit of time, respectively. Let $K = \{1, 2, \ldots, |R|\}$ be the set of order indices in which a required arc serviced in a route can be visited with 1 being the first order position in the sequence of required arcs serviced in the route, 2 being the second order position, and so on. For every arc $(v_i, v_j) \in R$, for every route $h \in H$, and for every order index $k \in K$, let x_{ijk}^h be a binary variable equal to 1 if and only if required arc

$(v_i,\ v_j)$ appears on route h in order position k. For every route $h \in H$, let t_h be a nonnegative variable representing the time for service completion of route h and let T_h denote the time limit of route h. The time limits T_h, $h \in H$, can be violated at a cost and by an unlimited length of time. Unlimited time limits are defined together with the following penalty functions:

$$f_1^h(t_h) = \begin{cases} 0 & \text{if } t_h \le T_h \\ w_1(t_h - T_h) & \text{if } t_h - T_h \le \alpha_1 T_h \\ \exp\left(w_1^+(t_h - T_h)\right) & \text{if } t_h - T_h > \alpha_1 T_h \end{cases}$$

where w_1 and w_1^+ are two positive constants and $0 \le \alpha_1 \le 1$. For every route $h \in H$, let s_h be a nonnegative variable representing the amount of salt applied to the roads assigned to route h and let C_h denote the salt capacity of the vehicle servicing route h. Similarly, the vehicle capacities C_h, $h \in H$, can be violated at a cost and by an unlimited amount of salt. Unlimited vehicle capacities are defined together with the following penalty functions:

$$f_2^h(s_h) = \begin{cases} 0 & \text{if } s_h - C_h < \alpha_2 C_h \\ w_2(s_h - C_h)^2 & \text{if } s_h - C_h \ge \alpha_2 C_h \end{cases}$$

where w_2 is a positive constant and $0 \le \alpha_2 \le 1$. The problem is then formulated as follows.

$$\text{Minimize} \quad \sum_{h \in H} \left(f_1^h(t_h) + f_2^h(s_h) + w_3 d_h \right) \tag{4.41}$$

subject to

$$\sum_{\left\{ \substack{v_j:(v_i,v_j) \in A, \\ v_i = a_h} \right\}} x_{ij1}^h = 1 \qquad (h \in H) \tag{4.42}$$

$$\sum_{\{v_i:(v_i,v_j)\in A,\ v_j=z_h\}} x_{ij1}^h = 1 \qquad (h\in H) \tag{4.43}$$

$$\sum_{h\in H}\sum_{k=1}^{|R|} x_{ijk}^h = l_{ij} \qquad ((v_i,v_j)\in R) \tag{4.44}$$

$$\sum_{(v_i,v_j)\in R} x_{ijk}^h \le 1 \qquad (h\in H, k\in K) \tag{4.45}$$

$$d_h = \sum_{k=1}^{|R|-1} \sum_{\substack{(v_i,v_j)\in R,\\(v_p,v_q)\in R}} sp_{ijpq} x_{ijk}^h x_{pq(k+1)}^h \qquad (h\in H) \tag{4.46}$$

$$s_h = r_h \sum_{k=1}^{|R|} \sum_{(v_i,v_j)\in R} t_{ij} x_{ijk}^h \qquad (h\in H) \tag{4.47}$$

$$t_h = \sum_{k=1}^{|R|} \left(\sum_{(v_i,v_j)\in R} t_{ij} x_{ijk}^h \right) + d_h \qquad (h\in H) \tag{4.48}$$

$$x_{ijk}^h \in \{0,1\} \qquad ((v_i,v_j)\in R, h\in H, k\in K) \tag{4.49}$$

$$d_h, s_h, t_h \ge 0 \qquad (h\in H) \tag{4.50}$$

where w_3 is a positive constant. The objective function (4.41) minimizes a weighted combination of time limits constraints violations, vehicle capacities constraints violations, and the deadhead traversal time. Constraints (4.42) and (4.43) ensure that each vehicle starts service at its home depot and ends service at its destination depot, respectively. Constraints (4.44) guarantee that the appropriate number of passes is achieved on each required arc. Constraints (4.45) impose that at most one required arc appears in each order position of a route. Constraints (4.46), (4.47), and (4.48) define, for each route, the time required by deadheading trips, the amount of salt consumed, and the time for service completion, respectively. Finally, all x_{ijk}^h variables are restricted to be binary while d_h, s_h, and t_h variables must assume nonnegative values. The model is solved using a tabu search heuristic. The authors report an implementation in Wayne County, Michigan. They did not, however, provide an algorithm or numerical results.

QIAO (2002) extended the original model (4.25)–(4.40) for routing salt spreader trucks in Calvert County, Maryland, to incorporate service connectivity and suggested two metaheuristic methods to solve the model. The extension is also an attempt at integrating both allocation of road segments to salt spreader trucks and routing decisions into a single optimization model. However, the service hierarchy of the transportation network and the maximum route duration constraints are no longer imposed. For each edge $(v_i, v_j) \in E$ and for every vehicle $k \in K$, let y_{ijk} be a binary variable equal to 1 if and only if edge (v_i, v_j) is serviced from v_i to v_j by vehicle k. Let $H = (W \cup \{w_r\}, F_1 \cup F_2)$ be an auxiliary graph defined as in Section 4.3.1 (see Figure 4.3). For each edge $(w_m, w_n) \in F_1 \cup F_2$ and for each vehicle $k \in K$, let z_{mnk} be a binary variable equal to 1 if and only if edge (w_m, w_n) is in the subtree associated with vehicle k, and let also f_{mnk} be a nonnegative real variable representing the flow on edge (w_m, w_n) from w_m to w_n, expressed as salt units, in the subtree associated with vehicle k. For each node $w_m \in W$ and for each vehicle $k \in K$, let s_{mk} (also represented by $s_{m(ij),k}$) be a binary variable equal to 1 if and only if vertex w_m (corresponding to edge (v_i, v_j) in E) is serviced by vehicle k. Finally, for each edge $(w_m, w_n) \in F_1$ and for each vehicle $k \in K$, let r_{mnk} be a binary variable equal to 1 if and only if edge (w_m, w_n) is in the subgraph induced by the nodes in the adjacency graph H that are serviced by vehicle k. Then, the constraints

$$y_{ijk} + y_{jik} \geq s_{m(ij),k} \qquad ((v_i, v_j) \in A_1, w_m \in W, k \in K) \qquad (4.51)$$

$$y_{ijk} + y_{jik} \leq Ms_{m(ij),k} \qquad ((v_i, v_j) \in A_1, w_m \in W, k \in K) \qquad (4.52)$$

$$M(s_{mk} + s_{nk} - 2) + (1 - r_{mnk}) \leq 0 \qquad ((w_m, w_n) \in F_1, k \in K) \qquad (4.53)$$

$$(s_{mk} + s_{nk} - 2) + M(1 - r_{mnk}) \geq 0 \qquad ((w_m, w_n) \in F_1, k \in K) \qquad (4.54)$$

$$z_{mnk} \leq r_{mnk} \qquad ((w_m, w_n) \in F_1, k \in K) \qquad (4.55)$$

$$\sum_{\{w_n : (w_m, w_n) \in F_1\}} z_{mnk} = s_{mk} \qquad (w_m \in W, k \in K) \qquad (4.56)$$

$$\sum_{w_m \in W} z_{mrk} \leq 1 \qquad (k \in K) \qquad (4.57)$$

$$\sum_{\{w_n:(w_m,w_n)\in F_1\cup F_2\}} f_{mnk} - \sum_{\{w_n:(w_n,w_m)\in F_1\}} f_{nmk} = s_{mk} \qquad (w_m \in W, k \in K) \qquad (4.58)$$

$$f_{mnk} \le Qz_{mnk} \qquad ((w_m, w_n) \in F_1 \cup F_2, k \in K) \qquad (4.59)$$

$$r_{mnk}, z_{mnk} \in \{0,1\} \qquad ((w_m, w_n) \in F_1 \cup F_2, k \in K) \qquad (4.60)$$

$$s_{mk} \in \{0,1\} \qquad (w_m \in W, k \in K) \qquad (4.61)$$

$$f_{mnk} \ge 0 \qquad ((w_m, w_n) \in F_1 \cup F_2, k \in K) \qquad (4.62)$$

are added to model (4.25)–(4.40) to impose service connectivity. Constraints (4.51) and (4.52) ensure that if either or both directions of a road segment in the original graph *G* are serviced by a vehicle, then the corresponding node in the adjacency graph *H* should also be serviced by the vehicle (selected in the associated subtree). Constraints (4.53) and (4.54) define the set of edges in the adjacency graph *H*. These constraints ensure that an edge is included in the subgraph induced by the nodes in *H* that are serviced by a vehicle only if the two endpoints of the edge are serviced by the vehicle. Constraints (4.55) link the spanning tree variables z_{mnk} and the adjacency graph variables r_{mnk}. They state that an edge can be part of the subtree associated with a vehicle only if this edge is in the subgraph induced by the nodes that are serviced by the vehicle. Constraints (4.56)–(4.59) are similar to their counterparts (4.12)–(4.14) of the capacitated minimum spanning tree model. Constraints (4.56) assure that each node in *H* (except the root node) is connected to some node (except the root node) in the optimal spanning tree only if both nodes are serviced by the same vehicle. Constraints (4.57) guarantee that at most one node in each subtree associated with a vehicle should be connected to the root node. Constraints (4.58) ensure that flow conservation is satisfied for each vehicle at all nodes of the adjacency graph *H* (except the root node). Finally, the vehicle salting capacity *Q* is respected at every edge in each subtree associated with a vehicle via constraints (4.59). Qiao also showed how load balancing constraints can be introduced in the formulation and how multiple salt depots can be taken into account. The model is solved with a classical tabu search algorithm and an elite route pool procedure. The *elite route pool* procedure is similar to the technique of genetic algorithms. The population is formed by a pool of good routes found in the best solutions, called the elite route pool. Associated with every route in the elite route pool is a weight corresponding to the frequency with which the route appears in the best solutions. New offspring routes are produced by selecting the routes with the highest weights in the elite route pool while avoiding duplications of serviced required arcs. Mutations are then obtained by applying the improvement methods described in Figure 4.6 (see Section 4.3.2). Tests showed that the various heuristics were useful in analyzing a variety of scenarios related to the modification of load balancing parameters and depot or salt dome locations as well as vehicle capacities. The thesis by QIAO (2002) provided an interesting comparison of the various heuristics (Merge, Delete and Insert, Link Exchange, Random Link Exchange, Multiple Link Exchange, tabu search, and elite route pool) and four popular constructive methods for the capacitated arc routing problem: PEARN's algorithm (PEARN, 1984), augment-merge (GOLDEN *et al.*, 1983), path-scanning (GOLDEN *et al.*, 1983), and construct-strike (CHRISTOFIDES, 1973). Computational tests on 23 networks derived from the test problems used by PEARN (PEARN, 1984) showed that the elite route pool procedure obtained the largest number of best solutions on sparse networks with $7 \leq |V| \leq 27$ and arc densities between 13% and 40%. On dense networks, PEARN's algorithm produced the best solutions in most cases.

4.4 Vehicle and materials depot location models

A number of different depot location problems arise in the context of winter road maintenance. These include determining the locations of vehicle depots and materials depots. Vehicle depots serve as starting and ending points for spreader trucks, as well as plows and snow loaders. Because vehicle depots are usually used year-round for the various road maintenance activities, the vehicle depot location problem is not exclusive to winter road maintenance. However, as indicated by GUPTA (1998), winter road

Table 4.2: Characteristics of vehicle routing models for spreading

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
EVANS (1990), EVANS and WEANT (1990)	Salt spreader truck routing	Operational	Vehicle capacities, maximum time for spreading completion, maximum route duration, and one or two lanes in a single pass	Min total distance	Capacitated arc routing problem	Sequential constructive method
LI and EGLESE (1996)	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, multiple vehicle and materials depots, and two lanes in a single pass	Min fixed spreader costs and transport costs	Capacitated arc routing problem	Sequential constructive method
SOYSTER (1974)	Salt and abrasives spreader truck routing	Operational	Service hierarchy and maximum route durations	Min service hierarchy violations and deadheading	Capacitated arc routing problem	Parallel constructive method
COOK and ALPRIN (1976)	Salt spreader truck routing	Operational	Vehicle capacity, load balancing, and both-sides service	Min total spreading time	Capacitated arc routing problem	Parallel constructive method
UNGERER (1989)	Salt spreader truck routing	Real-time	Vehicle capacities and maximum route lengths	Min total distance	Capacitated vehicle routing problem	Parallel constructive method
LIEBLING (1973)	Combined sector design and spreader routing	Strategic	Vehicle capacities, working periods, and service frequencies	Min deadheading	Directed Chinese postman problem	Cluster first, route second
ENGLAND (1982a,b)	Combined sector design and vehicle routing	Strategic	Compactness, workload balance, service frequencies, and turn restrictions	Min deadheading and alternations between servicing and deadheading	Arc routing problem	Cluster first, route second
SOYSTER (1974)	Salt spreader truck routing	Operational	Vehicle capacity and two lanes in a single pass	Min total distance	Linear 0–1 IP	Optimization -based method
HAGHANI and QIAO (2002)	Salt spreader truck routing	Operational	Service connectivity, vehicle capacity, and both lanes of two-lane, two-way roads in a single route	Min fleet size and/or deadheading	Capacitated minimum spanning tree	Optimization -based method
XIN and EGLESE (1989)	Salt spreader truck routing	Operational	Vehicle capacities, time limit for spreading completion, multiple routes per vehicle, and multiple depots	Min fleet size and deadheading	Capacitated arc routing problem	Composite method
HAGHANI and QIAO (2001)	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, maximum route durations, and level of service policies	Min deadheading	Linear MIP	Composite method
EGLESE (1994)	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, multiple depots, grouping of street segments, class upgrading, and two lanes in a single pass	Min fleet size	Spanning trees	Simulated annealing
BENSON *et al.* (1998)	Salt spreader routing	Operational	Vehicle capacities, service/deadhead speeds, times for service completion, and one or multiple passes per road	Min vehicle capacities and time limits violations and deadheading	Nonlinear MIP	Tabu search
QIAO (2002)	Salt spreader truck routing	Operational	Service connectivity, vehicle capacity, and level of service policies	Min deadheading	Linear MIP	Tabu search and elite route pool

maintenance is the most resource intensive activity that impacts the decision to establish a new vehicle depot or close an existing one.

Materials depots are intermediate facilities containing chemicals and abrasives to provide opportunities for spreader vehicles to refill with materials without returning to the original starting point. Materials depots should be located so as to minimize non-productive travel time, maximize use by multiple crews, minimize possible environmental damage, and not create a nuisance to adjoining properties. The number and locations of materials depots depend on many considerations such as capacity of the spreaders, maximum time allowed for a spreading operation, level of service of the road segments to be treated, and special treatment features such as bridges, tunnels, and intersections. The number and locations of materials depots are usually reviewed periodically to incorporate new technology. For example, the number of materials depots may be reduced through the use of anti-icing operations.

We now review optimization models aimed specifically at the location of vehicle and materials depots in the context of winter road maintenance. Models that are mainly concerned with the efficient location of vehicle and materials depots are discussed first, followed by compound models which integrate depot location and routing or fleet sizing decisions. A summary of these models is presented in Table 4.3 at the end of the section.

4.4.1 Depot location models

Since the location of vehicle and materials depots influences the cost for spreading and plowing operations, models for vehicle and materials depot location include costs for different aspects of the operations, such as vehicle routes, materials or the vehicles themselves.

KORHONEN *et al.* (1992) described a decision support system developed to assist planners in Finland in locating vehicle depots for winter road maintenance. The objective is to minimize transport costs and fixed vehicle depot costs. For economical and administrative reasons, all vehicle depots located for ten years or less are forced into their location in the solution and transport costs are calculated for high-class roadways only. The system incorporates a construction heuristic similar to the "add heuristic" devised by KUEHN and HAMBURGER (1963) for the solution of the uncapacitated facility location problem. The construction heuristic opens vehicle depots sequentially until it fails to find a vehicle depot whose addition will result in a decrease in the total cost. However, the author did not provide a rule for selecting the vehicle depot to add to the solution at each step. The effects of vehicle depot location on accidents and travel delays are also considered. The system yielded an annual cost reduction of 11% over the solution in use by the Finnish National Road Administration.

RAHJA and KORHONEN (1994) described a computerized tool to assist planners in Finland in locating sand and salt storage facilities for anti-icing. The system is based on the national road network divided into balanced geographic zones defined as groups of neighboring road segments. Each zone has a demand given in terms of sand and salt consumption. The system relies on the exchange heuristic proposed by TEITZ and BART (1968) for the *P*-median problem. This heuristic tries to find improved locations based on an objective of minimizing the demand-weighted distance between each demand zone and the nearest storage facility. The system was also used to select Finnish seaports and inland ports for importing salt by ship, while minimizing the sum of the cost of transporting salt from the ports to the storage facilities, the port charges, and the cost for unloading and shipping. The authors also proposed a transshipment model to determine transportation and storage decisions for imported salt. In this model, salt from a port may progress through several storage facilities before reaching a road segment. The objective considered is the minimization of salt purchase costs, port charges, unloading and shipping costs, transportation costs, and storageing costs, as well as the cost of salt spreading defined by equipment cost, salt solution production cost, loading cost, and crew cost.

For spreading operations, GUPTA (1998) described a GIS-based computerized tool to assist planners in analyzing the opening or closing of vehicle or materials depots. The routing of spreader trucks is dealt with by the Snowmaster decision support system (EVANS, 1990) described in Section 4.3.1, which helps planners in generating feasible spreader truck routes using the constructive path-scanning heuristic proposed by GOLDEN *et al.* (1983) for the capacitated arc routing problem. The objectives guiding the opening of a new depot, or the closing of an existing one, include operating costs (vehicle costs, crew wages and road materials costs), fixed costs associated with the depot (depreciation of permanent structures, maintenance cost of permanent infrastructure, and depreciation and maintenance costs of trucks), and costs for opening a new depot or closing an existing depot (salvage value of closed depot, plant and machinery reassignment cost, personnel reassignment cost, environmental treatment cost for closing an existing depot and construction cost of a new depot). The usefulness of the GIS-based computerized system is demonstrated using data from Hamilton County, Ohio involving 360 nodes and 855 arcs. Finally, GUPTA suggested formulating the materials depot location problem as a P-median problem.

4.4.2 Compound depot location and routing or fleet sizing models

A combined model for vehicle depot location, materials storage facility location and spreader truck fleet sizing was described by HAYMAN and HOWARD (1972). The problem considered is to determine the spreader truck fleet size based at each depot and assigned to each intermediate facility containing chemical and abrasive materials that gives the lowest operational and depreciation costs for long-term planning, while ensuring that the total roadway system be covered within a specific time period following the beginning of a storm. Maximum service times are provided within each class of roadways to reflect priorities. In the optimal solution of the model, if no truck at all is required at a given depot, then no depot is located at this candidate site. Similarly, no intermediate facility is located at a candidate site if no spreader truck is assigned to this intermediate facility in the optimal solution.

To present the formulation, let I, J, and K be the sets of vehicle depots, materials depots, and roadway sections, respectively. For every vehicle depot $i \in I$ and for every materials depot $j \in J$, let x_{ij} be a nonnegative integer variable representing the number of trucks based at vehicle depot i and having to refill with materials at intermediate facility j, and let b_{ij} and d_{ij} represent the unit time cost in traveling from vehicle depot i to materials depot j and the travel distance from vehicle depot i to materials depot j, respectively. For each materials depot $j \in J$ and for every roadway section $k \in K$, let y_{jk} be a nonnegative integer variable representing the number of trucks assigned to materials depot j to effect the servicing of roadway k, and let also z_{jk} be a nonnegative integer variable representing the number of loads of chemical and abrasive product to be hauled from materials depot j and spread on roadway k. A single load of materials corresponds to one ton. For each materials depot $j \in J$ and for every roadway $k \in K$, let also c_{jk} and d_{jk} represent the unit time cost for trucks plus loaders involved in the spreading operation from materials depot j to roadway k and the travel distance from materials depot j to roadway k, respectively. For every depot $i \in I$, define a_i as the depot depreciation cost per storm or spreading event applied to each truck based at vehicle depot i. For every materials depot $j \in J$, let f_j and t_j represent the cost per ton of the spreading material delivered to materials depot j and the average time in traveling from a vehicle depot to materials depot j, respectively. For every roadway $k \in K$, define l_k, t_k, and n_k as the length of roadway k, the time available to complete the spreading of roadway k measured from the beginning of the storm, and the number of loads of material required to service roadway k, respectively. Finally, if we let s represent the average vehicle speed, then the formulation is as follows.

$$\text{Minimize} \quad \sum_{i\in I}\sum_{j\in J}\left(a_i + b_{ij}\frac{2d_{ij}}{s}\right)x_{ij} + \sum_{j\in J}\sum_{k\in K}c_{jk}\left(\frac{2z_{jk}(d_{jk}+0.5l_k)+6z_{jk}}{s}\right) + \sum_{j\in J}\sum_{k\in K}f_j z_{jk} \qquad (4.63)$$

subject to

$$\frac{2z_{jk}(d_{jk}+0.5l_k)+6z_{jk}}{s} \le (t_k - t_j)y_{jk} \qquad (j\in J, k\in K) \qquad (4.64)$$

$$\sum_{j\in J} z_{jk} \ge n_k \qquad (k\in K) \qquad (4.65)$$

$$\sum_{i\in I} x_{ij} \ge \sum_{k\in K} y_{jk} \qquad (j\in J) \qquad (4.66)$$

$$x_{ij}, y_{jk}, z_{jk} \ge 0 \text{ and integer} \qquad (i\in I, j\in J, k\in K). \qquad (4.67)$$

In this formulation, the objective function (4.63) minimizes the total cost per storm or spreading event. This objective includes three costs: the time cost in deployment of the trucks from the vehicle depots to the materials depots, including depot depreciation, the time cost of delivering the material from the materials depot to the roadways, and the cost of the material. The quantity $2d_{ij} / s$ in (4.63) is the time required to travel between vehicle depot i and materials depot j and back to the vehicle depot. The quotient $2z_{jk} (d_{jk} + 0.5l_k) + 6z_{jk} / s$ represents the time required to service roadway k when refilling the spreader vehicles z_{jk} times with materials at materials depot j, regardless of the actual number of vehicles involved. This quotient accounts for z_{jk} trips over the distance between materials depot j and the beginning of roadway section k to be serviced and is determined based on studies of the relationship between the total distance driven and the distance spread. If we assume that the total time required to service roadway k when reloading with materials at materials depot j may be equally divided among y_{jk} spreader vehicles, each traveling at the same speed s, then constraint set (4.64) ensures that the total time available measured from the storm beginning to service each roadway is respected. Constraint set (4.65) requires that each roadway be properly spread. Constraints (4.66) link the total number of trucks required at each materials depot to effect the spreading operation and the number of trucks dispatched from the various vehicle depots to each materials depot. Finally, all x_{ij}, y_{jk} and z_{jk} variables must assume nonnegative integer values.

In order to reduce the size of the problem, some x_{ij} variables are set to zero if t_j, the travel time from vehicle depot i to materials depot j, is excessively larger than t_k, the time allotted to service any roadway from material depot j. Similarly, some y_{jk} and z_{jk} variables are discarded if too much deadhead time is consumed by travelling from materials depot j to roadway k. Computational results were reported on a real-life instance with 15 potential vehicle depot sites, 21 materials depots and 41 roadway sections[1]. The LP relaxation of the model (4.63)–(4.67) was solved using the simplex algorithm and the total number of trucks required at any materials depot was rounded up to the nearest integer value, as was the total number of trucks required from any vehicle depot.

REINERT et *al.* (1985) proposed a model for the combined problem of locating materials storage depots and assigning predetermined spreader truck routes to these depots. The problem is formulated as a P-median problem with depot capacities. Let I be the set of vehicle routes and let J be the set of materials depots. For every vehicle route $i \in I$ and for every materials depot $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if route i is assigned to materials depot j and let d_{ij} represent the distance between the median point of route i and the materials depot j. For every materials depot $j \in J$, let y_j be a binary variable equal to 1 if and only if a materials depot is located at potential site j and let k_j represent the capacity of a materials depot at candidate site j. Finally, for every route $i \in I$, define a_i and s_i as the length and the materials requirement associated with

[1]The real-life instance was obtained from the state of Wyoming, US.

route i, respectively. The formulation is then as follows.

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} a_i d_{ij} x_{ij} \tag{4.68}$$

subject to

$$\sum_{j \in J} x_{ij} = 1 \qquad (i \in I) \tag{4.69}$$

$$\sum_{j \in J} y_j \leq P \tag{4.70}$$

$$x_{ij} \leq y_j \qquad (i \in I, j \in J) \tag{4.71}$$

$$\sum_{i \in I} s_i x_{ij} \leq k_j \qquad (j \in J) \tag{4.72}$$

$$x_{ij}, y_j \in \{0,1\} \qquad (i \in I, j \in J)\,. \tag{4.73}$$

The objective function (4.68) minimizes the total length-weighted distance between each route median point and the nearest materials depot. Constraints (4.69) require each spreader truck route to be assigned to exactly one materials depot. Constraints (4.70) state that at most P materials depots are to be located. Constraints (4.71) link the location variables y_j and the assignment variables x_{ij}. They ensure that each route is assigned to a materials depot that is selected. Storage capacity is respected for every materials depot via constraints (4.72). Finally, all variables x_{ij} and y_j are restricted to be binary. The problem was solved using IBM's MPSX mathematical programming package. Tests performed with data from the District of Columbia involving 14 potential materials depot sites showed a 27% reduction in deadheading over the solution in use by the district.

LOTAN *et al.* (1996) proposed a three-stage procedure for the combined depot location and spreader routing problem in the province of Antwerp, Belgium. The procedure takes into account the service hierarchy, the spreader capacities (all the same), and the times for service completion for each class of roads. For high-priority roads, two lanes must be spread in one pass in each direction, whereas low priority roads can be spread either in two directions (servicing one lane at a time), or in one pass (servicing the two lanes together). The authors define a priority class network for each class of roads. The first stage simultaneously locates vehicle depots and constructs feasible routes in the network induced by the set of high-priority roads. The authors observed that the network induced by the set of high-priority roads is characterized by having a tree structure rooted around the ring of Antwerp. Given a fixed number of vehicle depots, feasible routes are thus defined by traversing the tree from its leaves towards the root and vehicle depots are located on high-priority roads so as to minimize the distance covered by deadheading trips. By varying the number of vehicle depots to locate and iteratively solving the location-routing problem on the tree structure, the proper tradeoff between minimizing the number of vehicle depots and minimizing the distance covered by deadheading trips can be identified. The second stage then allocates low priority roads to vehicle depots by assigning roads to their closest vehicle depot, while ensuring that the graph generated by the links assigned to each depot is Eulerian. This facilitates the creation of routes with less deadheading during route construction. The last stage constructs feasible routes and locates materials storage silos for each depot independently. The AUGMENT-INSERT algorithm of PEARN (1991) is used to produce a solution to the capacitated arc routing problem depot by depot. Silos are then located to improve the resulting solution. The three-stage procedure can be seen as a location-allocation-routing heuristic scheme typified by LAPORTE (1998) for node location-routing problems. The procedure was tested on the network of Brecht, Belgium with 33 nodes and 43 road segments. Results indicated that the procedure reduced the total distance travelled by 28% and the fleet size by 17% over the solution in use by the district when allowing two lanes to be spread in one pass. These improvements increase to 34% and 33%, respectively, when including a silo, but the duration of the route that utilises the silo then increases.

Table 4.3: Characteristics of vehicle depot and materials depot location models

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
KORHONEN *et al.* (1992)	Vehicle depot location	Strategic	Service hierarchy and fixed location of new vehicle depots	Min transport and fixed vehicle depot costs	Uncapacitated facility location problem	Constructive method
RAHJA and KORHONEN (1994)	Materials depot location	Tactical	Grouping of road segments into fixed and balanced zones	Min demand-weighted distance	*P*-median problem	Improvement method
GUPTA (1998)	Opening or closing vehicle and materials depots	Strategic	Vehicle capacities and maximum route times	Min vehicle and materials depot variable and fixed costs	Capacitated arc routing problem	Composite method
HAYMAN and HOWARD (1972)	Combined vehicle and materials depot location, and fleet sizing	Strategic	Service hierarchy, homogeneous fleet, and maximum spreading route times	Min transport, depot depreciation, and material costs	Linear IP	Simplex algorithm
REINERT *et al.* (1985)	Combined materials depot location and route assignment	Tactical	Maximum number of materials depots, materials depot capacities, and fixed spreader routes	Min total length-weighted distance	*P*-median problem	MPSX mathematical programming
LOTAN *et al.* (1996)	Combined depot location and spreader routing	Strategic	Service hierarchy, vehicle capacity, and times for service completion	Min fleet size, deadheading, and number of vehicle depots	Location-arc routing problem	Constructive method

4.5 Crew assignment models

For most northern countries, the extent of winter road maintenance operations involves seasonal reassignment of workers from summer maintenance activities to winter maintenance operations. This reassignment avoids seasonal hiring and firing, but may disrupt existing worker assignment plans by reassigning crews to work out of different depots for the winter season. This section contains a review of optimization models for the assignment of crews to vehicle depots. A summary of these models is presented in Table 4.4 at the end of the section.

The information related to the first optimization model for the crew assignment problem is derived from WRIGHT *et al.* (1987), EGLY and WRIGHT (1987), and WRIGHT and EGLY (1986) who developed a decision support system to assist planners at the Indiana Department of Highways in making worker assignment decisions at the operational planning level. Given a planned vehicle routing for plowing and spreading, the worker assignment problem considered consists of assigning a set of workers to the depots from which emanate the planned routes, so as to satisfy the demand for workers at each depot while respecting the availability of vehicles that can be issued to certain workers to travel from their residence to their respective depot. According to the Indiana Department of Highways' policy, a state-owned vehicle must be provided to a worker during the winter season if the distance between the residence of the worker and his/her assigned depot is higher than some maximum allowable distance, and if the assigned depot is not the nearest depot to the residence of the worker. Planners may also take into consideration worker seniority requirements.

The problem is formulated as a 0-1 integer program with two objective functions. The first objective function seeks to minimize the total distance between each residence and the nearest depot located within some maximum allowable distance. The second objective function is the minimization of the maximum distance between a worker and the closest depot located within some maximum allowable distance. Let I be the set of workers and let J be the set of vehicle depots. For every worker $i \in I$ and for every depot $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if worker i is assigned to depot j and let d_{ij} represent the distance between the residence of worker i and depot j. For every worker $i \in I$, define $N_i = \{j \in J \mid d_{ij} \leq D\}$ as the set of candidate depots to which worker i

may be assigned if the shortest path distance d_{ij} between the residence of worker i and depot j is less than or equal to the maximum travel distance D. For every worker $i \in I$ and for every depot $j \in J$, define the binary constant a_{ij} equal to 1 if and only if the assignment of worker i to depot j requires a vehicle. For every depot $j \in J$, define T_j as the total number of workers required at depot j. Finally, let $DMAX$ and C be the maximum distance between a worker and the nearest depot located within the maximum distance D and the number of vehicles available, respectively. The formulation is as follows.

Minimize

$$\sum_{i \in I} \sum_{j \in N_i} d_{ij} x_{ij} \qquad (4.74)$$

$$DMAX \qquad (4.75)$$

subject to

$$\sum_{j \in N_i} x_{ij} \leq 1 \qquad (i \in I) \qquad (4.76)$$

$$\sum_{i \in I} x_{ij} \geq T_j \qquad (j \in J) \qquad (4.77)$$

$$\sum_{i \in I} \sum_{j \in J \setminus N_i} a_{ij} x_{ij} \leq C \qquad (4.78)$$

$$DMAX \geq \sum_{j \in N_i} d_{ij} x_{ij} \qquad (i \in I) \qquad (4.79)$$

$$x_{ij} \in \{0,1\} \qquad (i \in I, j \in J). \qquad (4.80)$$

Constraints (4.76) require that each worker be assigned to at most one depot located within the maximum distance D. Constraints (4.77) ensure that demand for workers is satisfied at each depot. Vehicle availability is respected via constraint (4.78). Constraints (4.79) state that the maximum distance between a worker and the nearest depot located within the maximum distance D must be greater than the distance between any worker and the depot located within the maximum distance D to which he or she is assigned.

The two-objective model is solved using a modified constraint method of multiobjective optimization. First, the method finds the endpoints of the noninferior solution set in objective space. The model with the objective function (4.74) for minimizing the total distance is solved as a single objective linear program using a branch-and-bound algorithm. In the same way, the formulation with the single objective function (4.75) for minimizing the maximum distance is solved. The two optimal solutions correspond to points A and B, respectively, in Figure 4.8. Then, the method identifies the set of noninferior solutions by systematically varying the value of D between the smallest and the largest maximum distance measure (as provided by the initial endpoint solutions), and iteratively solving the model with the single objective function (4.74). A solution s is inferior if there exists some other solution s' that is as good as s in terms of the two objectives and s' is strictly better than s in terms of at least one objective. This method allows identification of convex-dominated solutions such as solution C in Figure 4.8[1].

Numerical experiments showed that the dual simplex algorithm produced an integer solution to the linear programming relaxation of the model with the single objective function (4.74) in excess of 97% of tested instances. In fact, non-integer solutions occurred only when the constraint (4.78) on available vehicles was binding at optimality. The authors observed that when the availability of state-owned vehicles is relaxed, the linear programming relaxation of the model with the single objective function (4.74) reduces to that of a transportation problem. Consequently, the simplex algorithm is used to obtain the optimal solution to the linear programming relaxation of the model with the

[1]A solution is convex-dominated if it is above the line connecting two other noninferior solutions.

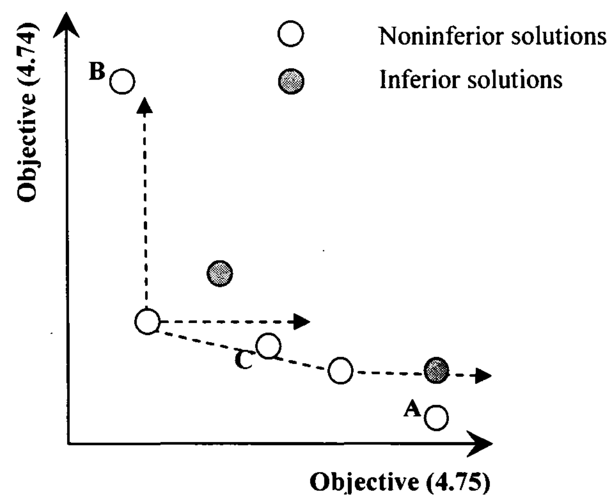

Figure 4.8: Tradeoff curve for distance objectives

single objective function (4.74) followed by a rounding heuristic that consists of decreasing the number of available vehicles by one until an integer solution is obtained.

The system was tested on a problem involving 118 workers and 20 depots. Computational results indicated significant cost savings over the solution used by the Indiana Department of Highways. The system may also be used at the real-time level to aid planners in making decisions about modifications of existing routes and related demand for workers. Planners at the Indiana Department of Highways can solve the worker assignment problem via the Internet. Upon submission of a problem, the server activates CPLEX to solve the problem, and then reports the solution.

BOGARDI *et al.* (1998) studied the problem of assigning crews to vehicle depots for plowing and spreading operations in Lincoln, Nebraska. The time horizon considered is a single winter season. All possible scenarios of allocating the total number of crews among the vehicle depots are first generated[1]. For each scenario, the optimal assignment

[1]The data used contained three facilities and three crews, thus leading to ten different crew scenarios.

of crew-days from vehicle depots to vehicle routes is then found by solving a transportation problem with supply nodes representing vehicle depots and demand nodes representing either plowing routes or spreading routes. Let I be the set of vehicle depots and let J be the set of plowing or spreading routes. For every vehicle depot $i \in I$ and for every route $j \in J$, let x_{ij} be a nonnegative variable representing the number of crew-days for servicing route j from vehicle depot i, and let c_{ij} represent the sum of the distance between vehicle depot i and midpoint of route j and back to the depot per crew-day, and the distance to service route j per crew-day. For spreading routes, the lengths of the round trips for reloading of materials per crew-day are also taken into account. For every vehicle depot $i \in I$, define s_i as the total number of crew-days provided by vehicle depot i. For every route $j \in J$, define d_j as the total number of crew-days required for servicing route j. For every scenario, the following transportation problem is solved.

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{4.81}$$

subject to

$$\sum_{j \in J} x_{ij} = s_i \qquad (i \in I) \tag{4.82}$$

$$\sum_{i \in I} x_{ij} = d_j \qquad (j \in J) \tag{4.83}$$

$$x_{ij} \geq 0 \qquad (i \in I, j \in J)\,. \tag{4.84}$$

The objective function (4.81) minimizes the sum of all distances. Constraints (4.82) stipulate that the total number of crew-days provided by each vehicle depot must be equal to the capacity of the depot. Similarly, constraints (4.83) ensure that each route be served by the required number of crew-days. The scenario with the lowest total distance gives the optimal crew assignment plan. The authors also applied this approach to other maintenance activities provided by the city such as concrete, asphalt, drainage, and traffic sign engineering. Travel distance reductions on the order of 3-5% were obtained over the solution in use by the city for snow plowing, asphalt service, and drainage. (An improvement on the order of 30% was obtained for traffic sign engineering.) We note that the crew assignment problem studied by BOGARDI *et al.* (1998) can simply be formulated as a network flow problem with one supply node, transhipment nodes representing vehicle depots, and demand nodes representing either plowing routes or spreading routes.

In the same paper, BOGARDI *et al.* (1998) extended the original transportation model to consider a multi-season planning horizon. For each planning season, maintenance service demands are predicted on the basis of city development trends. Since the single-period problems do not interact in any way, crew distribution scenarios are defined, and each single-period problem is solved separately using the transportation model for each scenario. The scenario with the lowest total distance in time corresponds to the optimal crew assignment plan.

As mentioned before, part of the winter road maintenance can be performed by workers from other maintenance services such as concrete, asphalt, or drainage. However, the allocation of crews to vehicle depots is often treated separately for each of these maintenance activities. This can lead to unbalanced vehicle depots having a shortage or excess of crews. BOGARDI *et al.* (1998) proposed to integrate crew assignment decisions for concrete, asphalt, drainage, curb-cut, plowing, and spreading operations into a single optimization model. Let I be the set of vehicle depots and let J be the set of road maintenance activities. For every vehicle depot $i \in I$ and for every maintenance service $j \in J$, let x_{ij} be a nonnegative integer variable representing the number of crews based at vehicle depot i for service j, and let $w_{ij} = c_{ij}/\max_{k \in I}\{c_{kj}\}$ represent the nonnegative weight associated with vehicle depot i for service j with c_{ij} corresponding to the total travel distance incurred by assigning all crews to vehicle depot i for service j. For each maintenance service, this entails placing a weight of 1 on the vehicle depots incurring the maximum travel distance and smaller weights on the others vehicle depots. For every maintenance service $j \in J$, define s_j, d_j, and D_j as the total number of worker-days associated with each crew for service j, the number of workers in each crew for service j, and the total number of workers required for service j, respectively. The optimization model is an integer linear program stated as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} w_{ij} x_{ij} \tag{4.85}$$

subject to

$$\sum_{j \in J} s_j x_{ij} \leq S \qquad (i \in I) \tag{4.86}$$

$$\sum_{i \in I} d_j x_{ij} = D_j \qquad (j \in J) \tag{4.87}$$

$$x_{ij} \geq 0 \text{ and integer} \qquad (i \in I, j \in J)\,. \tag{4.88}$$

The objective function (4.85) minimizes the total weighted number of crews. Constraints (4.86) limit the total number of worker-days associated with each vehicle depot to a maximum of S. Constraints (4.87) ensure that the appropriate number of workers is assigned to each service. Finally, all x_{ij} variables must assume nonnegative integer values. The authors did not propose a solution methodology to solve the model.

Finally, BOGARDI *et al.* (1998) studied the combined problem of assigning concrete, asphalt, curb-cut, drainage, spreading, and plowing crews to vehicle depots from a multiobjective perspective. In particular, the problem is concerned with the selection of a preferred crew assignment scenario for these maintenance activities among twelve possible such scenarios, while considering economic, social, environmental, and political criteria. The problem is solved using composite programming (BOGARDI and BARDOSSY, 1983). The method of composite programming identifies solutions which are closest to the ideal solution as determined by some measure of distance. The method starts with the selection of the decision criteria, called *basic indicators*, for constructing a hierarchy of criteria. The decision criteria are identified based on experience and constitute the first level of the structure. Based on their characteristics, these basic indicators are then grouped into successively broader clusters of higher level indicators, called *composite indicators*, until one final composite indicator is obtained at the highest level of the hierarchy. A composite index associated with this final composite indicator is calculated for each scenario. The crew assignment scenario with the highest composite index corresponds to the best scenario. Figure 4.9 presents the hierarchy of decision criteria developed by BOGARDI *et al.* (1998) for the crew assignment problem.

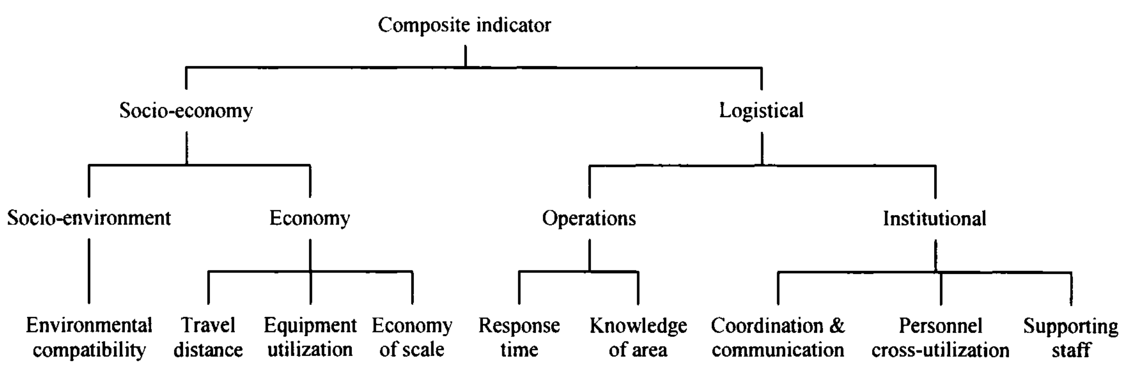

Figure 4.9: Hierarchy of decision criteria

For each scenario, the travel distance criterion is measured in kilometers while subjective and numerical values are assigned to the other criteria. Since the basic indicators are not expressed in commensurate terms, a scaling function is defined to ensure the same range for each basic indicator. This range corresponds to the interval [0, 1]. Let I be the set of basic indicators. For each basic indicator $i \in I$, define z_i as the numerical value of basic indicator i for a given scenario, and let z_i^* and z_i^{**} be the best and the worst numerical values of basic indicator i among all scenarios, respectively. For each basic indicator $i \in I$, the scaling function S_i is then calculated as follows:

$$S_i = \frac{z_i - z_i^{**}}{z_i^{*} - z_i^{**}}.$$

Let $P_J = \{I_1, I_2, \ldots, I_J\}$ be a partition of I with $I_1 \cup I_2 \cup \ldots \cup I_J = I$ and $I_k \cap I_l = \varnothing$ for all $k, l \in \{1, 2, \ldots, J\}$, $k \neq l$. For each group $I_j \subseteq P_J$ and for each basic indicator i in group I_j, let α_{ij} and S_{ij} be the weight value expressing the relative importance of basic indicator i in group I_j ($\Sigma_i \alpha_{ij} = 1$, $i \in I_j$, for each group $I_j \subseteq P_J$) and the normalized value of basic indicator i in group I_j, respectively. For each group $I_j \subseteq P_J$, define p_j as the balancing factor among indicators in group I_j. The parameter p_j reflects the importance of the maximal deviation. The measure of closeness used by BOGARDI *et al.* (1998) to identify scenarios which are closest to the ideal solution is a family of L_j composite indices, defined as follows:

$$L_j = 1 - \left[\sum_{i \in I_j} \alpha_{ij} (1 - S_{ij})^{p_j} \right]^{1/p_j}.$$

For each scenario, the composite indices L_j are calculated iteratively for each level of the hierarchy of Figure 4.9 until the final composite index is obtained. The composite programming method leads to (1, 1) as the best possible point, and (0, 0) as the worst possible point of the solution space depicted in Figure 4.10.

The composite programming method was embedded in a decision support system. Computational tests on data from the city of Lincoln, Nebraska, showed that the scenario actually used by the city ranked eleventh best among the twelve scenarios. BOGARDI *et al.* (1998) also extended the composite programming method to consider a multi-period planning horizon. Results on the same data indicated that the scenario actually used by the city ranked thirteenth best among the seventeen scenarios considered for future

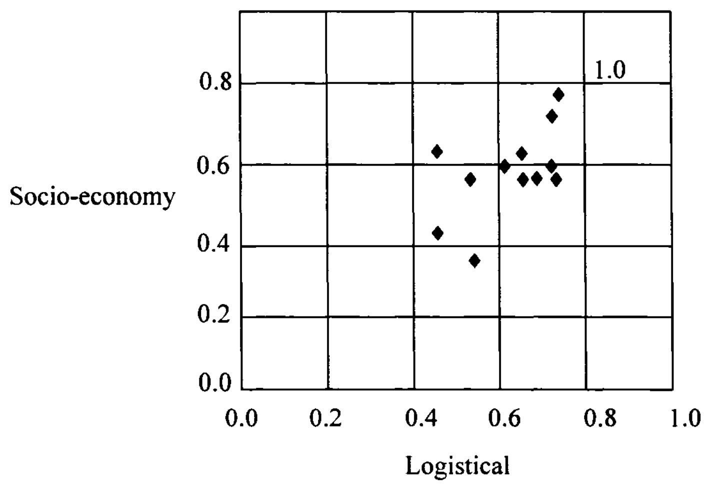

Figure 4.10: Composite indices for the twelve scenarios

maintenance services. Moreover, the multiple criteria approach appeared robust, namely, neither the best or worst scenarios changed under different weights α_{ij}. The authors concluded that the system can be used for other applications such as the selection of landfill sites, water resources planning, facility location studies, or transportation planning.

Computerized tools have also been developed to help planners in scheduling crews for winter road maintenance operations. Such tools were described, for example, by RISSEL and SCOTT (1985), GAGNON (1985), and SAPLING CORPORATION (1998).

4.6 Conclusions

This paper is the third part of a four-part survey of optimization models and solution algorithms for winter road maintenance. (The two first parts of the survey (PERRIER *et al.*, 2006a,b) discuss system design models for winter road maintenance operations. The

Table 4.4: Characteristics of crew assignment models

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
WRIGHT *et al.* (1987) EGLY and WRIGHT (1987) WRIGHT and EGLY (1986)	Assignment of workers to vehicle depots	Operational	Fixed vehicle depots location and fixed routes	Min total distance Min maximum distance	Linear 0-1 IP	Constraint method
BOGARDI *et al.* (1998)	Crew assignment for plowing or spreading	Operational and strategic	Fixed number of crews, fixed vehicle depots location, and fixed routes	Min total distance	Transportation problems	Scenario analysis
BOGARDI *et al.* (1998)	Multi-service crew assignment	Operational	Fixed number of crews, fixed vehicle depots location, and balanced depots	Min relative distance	Linear IP	–
BOGARDI *et al.* (1998)	Multi-service crew assignment	Operational and strategic	Fixed number of crews and fixed vehicle depots location	Multi-objective	Multiple criteria	Composite programming

last part of the review (PERRIER *et al.*, 2005b) addresses vehicle routing, fleet sizing, and fleet replacement models for plowing and snow disposal operations.) This paper addresses vehicle routing, depot location, and crew assignment models for spreading operations. Vehicle routing problems in winter road maintenance are the most studied of any winter road maintenance problems. Because of the inherent difficulties of these problems, most solution methods that have been proposed are heuristics. Much early work for the routing of vehicles for spreading operations adapted or extended simple capacitated arc routing algorithms with little consideration of practical characteristics. Early attempts to apply simple heuristics, such as parallel route construction methods or cluster first, route second methods, produced nice results from simulation studies, but these heuristics were rarely implemented and used in spreading operations. Recent models are solved with more sophisticated local search techniques (e.g., composite methods and metaheuristics), and are showing much promise to assist planners in making routing decisions for spreading operations in practice. One interesting line of research would be the further development of compound models that address the integration of spreader routing with other decisions related to spreading operations.

In summary, despite the increasing realism of recent spreading operations applications and the important progress in solution methods, considerable work remains to be accomplished on the design of fast heuristic algorithms that produce good approximate solutions, and on the development of more comprehensive models that address the integration of depot location models with spreader routing decisions.

Acknowledgements

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CHAPITRE 5

A SURVEY OF MODELS AND ALGORITHMS FOR WINTER ROAD MAINTENANCE. PART IV: VEHICLE ROUTING AND FLEET SIZING FOR PLOWING AND SNOW DISPOSAL

Article écrit par Nathalie Perrier, André Langevin et James F. Campbell; accepté pour publication dans *Computers & Operations Research* en 2005.

Cet article présente une revue des modèles d'optimisation et des méthodes de résolution pour le routage des véhicules pour les opérations de déblaiement et d'enlèvement de la neige. Nous décrivons d'abord brièvement les opérations de déblaiement et d'enlèvement de la neige ainsi que les caractéristiques des problèmes de tournées de véhicules pour ces opérations. Les méthodes décrites dans l'article sont regroupées en trois grandes catégories: les méthodes constructives, les méthodes composites et les métaheuristiques. Nous présentons également une revue des modèles pour les problèmes de détermination de la taille de la flotte de véhicules et d'horaire de remplacement de la flotte de véhicules.

A Survey of Models and Algorithms for Winter Road Maintenance. Part IV: Vehicle Routing and Fleet Sizing for Plowing and Snow Disposal

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Abstract

This is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance planning. The two first parts of the survey address system design problems for winter road maintenance. The third part concentrates mainly on vehicle routing problems for spreading operations. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the routing of vehicles for plowing and snow disposal operations. We also review models for the fleet sizing and fleet replacement problems.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Arc routing; Operations research.

5.1 Introduction

This is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the routing, the sizing, and the replacement of vehicle fleets for plowing and snow disposal operations. The third part of the survey (PERRIER *et al.*, 2005a) primarily addresses vehicle routing and depot location problems for spreading chemicals and abrasives. The two first parts of the survey (PERRIER *et al.*, 2006a,b) address system design models for winter road maintenance.

Winter road maintenance planning involves a variety of decision-making problems relating to the routing of vehicles for spreading chemicals and abrasives, for plowing roadways and sidewalks, for loading snow into trucks, and for transporting snow to disposal sites. These problems are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. Spreading and plowing operations are usually performed on a regular basis in almost all rural and urban regions with frozen precipitation or significant snowfall. However, in urban areas with large snowfalls and prolonged subfreezing temperatures, not all the snow that is plowed to the roadside can remain there. The most common solution is to load snow into trucks for transport to disposal sites. Conversely, in rural regions, snow is often simply pushed to the sides of roadways without being removed and hauled. These operations consume over $2 billion in direct costs each year in the United States (TRANSPORTATION RESEARCH BOARD, 1991). Indirect costs associated with corrosion and environmental impacts add at least $5 billion (TRANSPORTATION RESEARCH BOARD, 1991).

The paper is organized as follows. Section 5.2 describes the operations of plowing and snow disposal, and the vehicle routing, fleet sizing, and fleet replacement problems related to those operations. Models for the routing of vehicles for plowing and snow disposal operations are described in Sections 5.3 and 5.4, respectively. Models that address the sizing and replacement of vehicle fleets are reviewed in Section 5.5. Conclusions and future research paths in winter road maintenance planning are presented in the last section.

5.2 Operations context and decision problems

This section contains a brief description of plowing, snow loading, and hauling operations and a discussion of associated problems of vehicle routing and fleet sizing and replacement. More detailed information on the state of the practice in managing winter road maintenance operations, including plowing and snow disposal operations, is presented in the synthesis report by KUEMMEL (1994). For further details on winter road maintenance technologies, strategies, and tactics, see MINSK (1998) and BLACKBURN *et al.* (2004). Decisions relating to the routing of vehicles for winter road maintenance usually belong to the *operational* planning level or *real-time* control, while decisions concerning the sizing and replacement of fleets for winter road maintenance vehicles pertain to the *strategic* or *tactical* planning levels.

5.2.1 Plowing and snow disposal operations

Snow falling on a paved surface may be removed by chemical, thermal, or mechanical techniques. Chemical methods include the external application of a freezing-point depressant and incorporation of the freezing-point depressant within the surface itself. Thermal methods involve applying heat to the surface from either above or below to remove snow and ice or to prevent its formation. Mechanical removal is the physical process of attempting to pick up the snow from the road, shearing it from the road if necessary, and casting it to a storage area off the road. Plowing and snow disposal operations fall into this category.

Plowing operations can be used alone or in conjunction with anti-icing, deicing, or abrasives spreading operations. Plowing operations alone are suitable for use during and/or after frozen precipitation has occurred at very low pavement temperatures (lower than about 12°F), when it is too cold for chemicals to work effectively, and on low-volume and unpaved roads. Anti-icing is the practice of attempting to prevent the formation of bonded snow and ice to a pavement surface by timely applications of a chemical freezing-point depressant. Chemical applications can be coordinated with timely plowing of snow and ice during anti-icing operations to produce the highest level of service during and after the precipitation. Deicing is necessary when ice or compacted ice is strongly bonded to the pavement and the bond has to be destroyed in order to remove the frozen layer. The practice of plowing operations in conjunction with deicing primarily results in controlling the depth of loose snow and ice on the roadway. Plowing operations together with abrasives spreading is a combination of winter road maintenance operations in which sand or other abrasives (or a mixture of abrasives and a chemical) are applied to the plowed or scraped roadway surface that may have a layer of compacted snow or ice already bonded to the pavement surface. This combination of operations is used to provide increased friction for vehicular traffic. However, abrasives are not chemicals and do not support the fundamental objective of either anti-icing or deicing operations. Plowing plus spreading with abrasives can be used in most snow and ice situations, particularly in very low pavement temperature situations where chemical spreading operations are not likely to be effective and on roads having a low traffic volume. This combination is also a viable option for unpaved roads if there is no, or very little, chemical in the mixture.

Plowing is most commonly accomplished by displacement plows mounted on the front, side, or beneath their truck carriers, or by rotary plows which pull the snow into a rotating element and cast it to the side. Blade plows are also used for scraping and cutting compacted snow and ice in the attempt to remove them. Frequently, the same equipment is used to remove both snow and ice. However, because of the high-strength adhesive bonds which may form between ice or compacted snow and pavement, specialized equipment is frequently required. Plowing operations are limited to one lane at a time. This contrasts with materials spreading operations where materials are spread onto the road through a spinner which can be adjusted so that more than one lane of a road segment can be treated in a single pass.

Agencies employ different plow patterns for two-lane roads and multi-lane highways. For example, some agencies extend plows over the centerline of two-lane roads on the first pass or on the second pass, while other agencies plow the centerline on all passes or as often as possible without conflicting with traffic. Also, many agencies have developed tandem plow patterns in echelon formations for multi-lane highways given that plowing operations can treat only one lane at a time.

The large volumes of snow plowed from roads and walkways may exceed the available space along roadways and walkways for snow storage, and therefore require disposal by some means. Loading snow into trucks using snowblowers, rotary plows, or other types of snow loaders for transport to disposal sites is the most common solution. The trucks may be adjacent to, or in some cases following, the vehicle loading it, though adjacent trucks will further restrict traffic during the operation (MINSK, 1998). Loading and hauling of snow are generally post-storm operations, although they may be required during a snowfall to remove snow from areas, such as alleys or narrow channelled sections, with insufficient space for snow storage. These operations are usually performed in urban areas with heavy snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow disposal following infrequent but very large storms. Parking regulations are generally put into effect during snow disposal to facilitate loading snow into trucks for hauling to disposal sites. Examples are regulations that prohibit street parking at all times on designated snow routes, allow alternate side street parking, prescribe alternate times for parking, and ban overnight parking.

Disposal sites are the destinations for snow hauling trucks originating in each snow removal sector, and must be visited many times during the snow disposal operations. There are several different types of disposal sites that may be considered, including surface sites, quarry sites, sewer chutes, snow melting machines and water sites. Associated with every disposal site are a fixed location cost, an operating cost, and an annual capacity due to the limited space available to store snow. Each disposal site may also have an hourly capacity for unloading trucks depending on the configuration of the disposal site, and the available equipment and manpower. Surface sites typically require large plots of open land and may have very large capacities. They also may have other uses when snow is not present. Melters, in contrast, can be small mobile machines, but are typically quite expensive. Disposal in rivers or lakes represent the most economical disposal method, although disposal sites that allow melted snow to be processed in waste water treatment facilities provide environmental benefits.

In order to minimize the completion time for winter road maintenance, snowblowers (or other types of snow loaders) generally operate in a continuous process loading trucks. In practice, there may be several empty trucks moving slowly in a queue alongside each snowblower to ensure the snowblowers are never idle. As soon as a truck is filled with snow, it departs for the assigned disposal site while another truck takes its place to begin being filled. The truck that departed for the disposal site will travel to the disposal site, dumps its load of snow, possibly after waiting in line, and then return to the end of the queue alongside the assigned snowblower. This closed cyclic continuous system is illustrated in Figure 5.1. There may be more than one such a cyclic closed system if a sector contains more than one snowblower or if a sector can be assigned to multiple disposal sites.

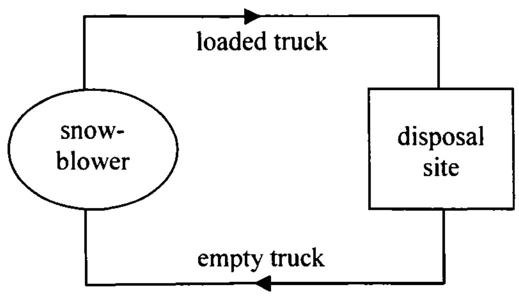

Figure 5.1: The snowblower–truck–disposal site cyclic closed system

5.2.2 Vehicle routing problems for plowing and snow disposal

Plowing, snow loading, and hauling operations involve a number of vehicle routing problems where streets or roads have to be traversed by plows, snowblowers, and trucks. The *plow routing* and *snowblower routing* problems consist of determining a set of routes, each performed by a vehicle that starts and ends at its own depot, such that all road segments are serviced, all the operational constraints are satisfied, and the global cost is minimized. Another type of routing problem for snow disposal, called the *truck routing problem*, calls for the determination of a set of itineraries for the trucks filled with snow that travel from the assigned snowblower to disposal sites and back to the snowblower. This is a difficult shortest path problem, because of the movement of the snowblower while a truck travels to and from the disposal site. Truck routing also involves political and equity issues due to the disruptive nature of large numbers of trucks converging on disposal sites.

This section describes the typical characteristics of vehicle routing problems related to plowing and snow disposal operations by considering their main components (transportation network, road segments, sectors, disposal sites and vehicle depots, vehicles, and crews), the different operational constraints that can be imposed on the configuration of the routes, and the possible objectives to be achieved in the optimization process. These characteristics are summarized in Table 5.1. Models and algorithms proposed for the solution of vehicle routing problems related to plowing and

Table 5.1: Characteristics of vehicle routing problems for plowing and snow disposal

Components	Characteristics
Transportation network	undirected, directed or mixed
	urban or rural
	required road segments
	service hierarchy
	maximum time for service completion
Road segments	resource-oriented or results-oriented level of service policies
	length
	service and deadhead traversal times
	parking restrictions
	service time windows
	service frequencies
	service in tandem for multi-lane roads or separate passes
Sectors	sector design
	number of sectors
	compactness or shape
	centrally located sites and depots relative to sectors
	balance in sector size or workload
	contiguity
	basic units
Snow disposal sites and vehicle depots	disposal site and vehicle depot locations
	number of disposal sites and vehicle depots
	variable costs of disposal sites and vehicle depots
	fixed costs of disposal sites and vehicle depots
Plows, snowblowers and trucks	home depot
	truck capacity
	vehicle type and road segment dependencies
	one or multiple routes per vehicle
	variable costs of plows, snowblowers, and trucks
	fixed costs of plows, snowblowers, and trucks
Crews	number of crews per sector
	maximum duration of driving and working periods
	number and duration of breaks
	overtime
	variable crew costs
Routes	start and end locations of routes
	start times of routes
	load balancing
	class continuity
	class upgrading
	both-sides service
	sector boundaries
	one or multiple vehicles per route
	turn restrictions
	block design or length design
Objectives	minimize deadheading
	minimize service completion time
	minimize turn penalties
	minimize operational constraint violations

snow disposal operations are reviewed in Sections 5.3 and 5.4, respectively. Vehicle routing problems related to spreading operations are described in the third part of the survey (PERRIER *et al.*, 2005a).

Vehicle routing problems related to winter road maintenance can be defined on *directed*, *undirected*, or *mixed* graphs depending on the topology of the transportation network and on the operating policies involved. As a rule, one-way streets are represented by arcs and two-lane, two-way streets (one lane each way) are represented by edges. If the two sides of the street can be serviced at the same time, as is often the case in spreading operations, the mixed graph can be the appropriate representation. Conversely, if the two sides of the street must be serviced separately, as is the case in plowing and snow disposal operations, arcs may have to be duplicated and edges are replaced by two arcs of opposite direction. The resulting graph is then directed. The transportation network can be *urban* or *rural*, depending on whether it is required to service all road segments or only a subset of road segments, respectively. The road segments that require to be serviced are called *required* road segments. Rural problems are often simpler due to the sparser road networks and the service requirement, in many cases, to remove snow only from the roadways and leave it beside the road to accumulate over the winter without hauling the snow to disposal sites. The transportation network is usually associated with a *maximum time* for completing spreading operations based on political and economic considerations. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a *service hierarchy*, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. Associated with each class of roads may be a *maximum time* for service completion.

Most policies define level of service for classes of highway based on their priority. *Level of service policies* tend to be *results-oriented* (e.g., bare pavement), *resource-oriented* (e.g., 24-hour equipment coverage), or a combination of both. Models dealing with level of service policies are presented in the first part of the survey (PERRIER *et al.*, 2006a). Associated with each road segment is a cost, which generally represents its *length*, and three *traversal times*, which are possibly dependent on the vehicle type: the time required to service the road segment, the time of deadheading the road segment if it has not yet been serviced, and the time of deadheading the road segment if it has already been serviced. *Deadheading* occurs when a vehicle must traverse a road segment without servicing it. In general, the longest operation consists of removing snow on a road segment, followed by deadheading an unserviced road segment, followed by deadheading a serviced road segment. With each road segment are generally associated *parking regulations* to facilitate loading snow into trucks for hauling to disposal sites. These restrictions are often limited to critical road segments that carry large traffic volume. Also associated with each road segment is a time interval, called *service time window*, during which the road segment can be plowed, which is possibly dependent on the parking restrictions or on the hierarchy of the network, and a *service frequency* (e.g., the road segment should be covered at least once every two hours). Service time windows can also be associated with classes of roads or routes. Finally, since plowing operations are limited to one lane at a time, multi-lane road segments necessitate either multiple *separate passes* or *tandem* plow patterns in echelon formations.

Because of the difficulty and impracticability of organizing winter road maintenance operations in a wide transportation network, these operations are generally carried out concurrently by separate crews and equipment in many small subnetworks, called *sectors*. See PERRIER *et al.* (2006a,b) for details on the design of sectors for plowing and snow disposal.

The routes performed for plowing and snow disposal operations start and end at one or more *vehicle depots*, located at the vertices of the graph. With every vehicle depot is associated a given number of vehicles of each type. In large cities, there are usually several snow *disposal sites*, possibly of different types, including surface sites, unused quarries, sewer chutes (openings into the storm sewer system), and water sites. *Costs* associated with disposal sites and vehicle depots include variable costs of operating disposal sites and vehicle depots, and fixed costs of acquiring disposal sites and vehicle depots.

Plowing and snow disposal operations are performed using a fleet of plows, snowblowers, and trucks whose size and composition can be fixed or can be defined according to the level of service policies, the configuration of the streets and sidewalks, land use (e.g., residential or commercial) and density of development, and times for service completion for each class. A vehicle may end service at a depot other than its *home depot*. The *capacity* of the truck is expressed as the maximum volume of snow the truck can load for hauling to disposal sites. The subset of road segments of the transportation network which can be traversed by the vehicle is *dependent* on road segment widths and on the vehicle type. Large road segments may require large vehicles. Narrow road segments may require small or medium-sized vehicles. In some applications, each vehicle can cover *multiple routes* in the considered time period. Finally, associated with each vehicle type is a *fixed* leasing or acquisition *cost* and a *variable cost* that is proportional to the distance traveled. The variable cost component encompasses the costs of fuel and maintenance.

Crews of personnel assigned to sectors must satisfy several constraints from union contracts and agency or company regulations. Examples are *working periods* during the day, maximum duration of working and *driving periods*, number and duration of *breaks* during service, and *overtime*. A separate crew of personnel is usually assigned to each sector. *Costs* associated with crews depend on the pay structure (e.g., regular or premium time, single or dual working periods).

The routes must satisfy several operational constraints imposed by the level of service policies and the characteristics of the transportation network, road segments, sectors, vehicles, and crews. The routes can *start* and *end* at one or more depot *locations* and each route can end service at a depot other than the original starting depot. In plowing operations, the route *start times* are dependent on the accumulation of snow and ice on the roadway surface. For example, in Montreal, snow plowing operations begin as soon as the accumulation of snow and ice reaches two centimeters and a half (LABELLE *et al*., 2002). In Japan, snow plowing begins when accumulation reaches five centimeters (TRANSPORTATION RESEARCH BOARD, 1995). To *balance* the *workload* across routes, they are often approximately the same length or duration. This helps ensure that plowing and snow disposal operations will be completed in a timely fashion. Since most arterial roads have multiple lanes that require separate passes, the total workload is usually measured in lane-kilometers. *Class continuity* requires that each route services road segments with the same priority classification. Thus, if a lower-class road is included in a route servicing higher-class roads, its service level may be *upgraded*. In plowing and snow disposal operations, it is often desirable that *both sides* of a two-lane, two-way road (one lane each way) be serviced by the same vehicle in a single route. The configuration of routes may also need to conform to existing *sector boundaries*. Routes crossing these boundaries must be avoided from an administrative standpoint. In some applications, each route can be operated by *multiple vehicles*. For example, if tandem service is required, the vehicles need to be assigned to a single route and operate in parallel. In practice, some operational constraints may often be treated as soft constraints or as *terms* in an objective function rather than hard constraints.

In urban areas, the impact of undesirable *turns*, such as U-turns and turns across traffic lanes, is generally greater in routing snow plows and snowblowers as compared to spreading operations. Since most plows are designed always to cast the snow to the right side of the roadways, a left turn or a street crossing at an intersection results in a trail of snow in the middle of the intersection. Thus, the general guideline for constructing routes for snow plowing is that each plow should remain on the right side of a roadway using a *block pattern* by accomplishing a series of right turns to avoid compromising safety. Conversely, for loading snow into trucks, each snowblower should cast the snow to a truck alongside or following directly behind using a *length pattern* where servicing one street at a time is preferred to frequent right turns. To deal with these situations, a *penalty* can be assigned to each type of turn (e.g., left, right, U-turn, and go straight).

Finally, several objectives can be considered for the routing of vehicles for plowing and snow disposal operations. Typical objectives are minimization of the distance covered by *deadheading* trips (or on the deadhead travel time); minimization of the time for service completion; minimization of the *penalties* associated with turns; minimization of the *terms* penalizing the violation of some operational constraints; or any weighted combination of these objectives.

5.2.3 Fleet sizing and replacement problems

Section 5.5 of this survey discusses models for fleet sizing and fleet replacement in the context of winter road maintenance. The most common types of mobile winter road maintenance equipment used on a routine basis, in approximate order of their total numbers, are trucks, plows, material spreaders, wheel loaders, motor graders, snowblowers or rotary plows, sweepers, and melters. Several factors determine the type, size, and design of appropriate equipment, including the frequency and severity of frozen precipitation, the nature and range of tasks, the environment, the level of service required, the type of road surface, and the extent of roads to be maintained, as well as geographic factors. In general, because winter road maintenance is seldom a constant, year-round, everyday activity, the vehicles and equipment used for winter road maintenance operations are typically powered equipment or vehicles primarily designed for other activities such as concrete, asphalt, drainage, curb-cut, or traffic sign engineering. There are some conflicting objectives to consider in determining the sizes and replacement schedules of fleets for winter road maintenance vehicles. The important tradeoff in fleet sizing is that larger fleets are desired to more quickly clear the roadways, but larger fleets require greater expenditures. In fleet replacement, one usually wants to balance maintenance costs for keeping old vehicles and costs for acquiring new vehicles.

5.3 Vehicle routing models for plowing

Vehicle routing problems related to plowing operations are generally formulated as arc routing problems. An integrated overview of the most relevant operations research literature on arc routing was presented by EISELT *et al.* (1995a,b). A more extensive review of the literature in arc routing with special emphasis on applications was proposed by ASSAD and GOLDEN (1995). More recently, a book on the subject was edited by DROR (2000).

Several heuristics procedures have been proposed for the routing of vehicles for plowing operations. These can be broadly classified into three categories: constructive methods, composite methods, and adaptation of metaheuristics. These three classes of methods are covered in Sections 5.3.3, 5.3.4, and 5.3.5, respectively. The characteristics of the contributions are then summarized in Table 5.2 at the end of the section. Some contributions emphasize both plowing and spreading operations, while others cover snow disposal in addition to plowing and spreading. Before, a brief review of simulation methods and rule-based decision support systems developed to help planners in making vehicle routing decisions for winter road maintenance operations is presented in Sections 5.3.1 and 5.3.2, respectively.

5.3.1 Simulation methods

Most early systems for the routing of vehicles for winter road maintenance operations relied in large part on simulation techniques either as a tool to assist planners in constructing feasible routing plans or as a tool to evaluate the quality of the configuration of a given set of routes and to help guide manual modifications.

BROWN (1972) proposed an interactive computer simulation tool, called AID, to model vehicle movements and interactions for plowing and snow disposal operations in small urban areas. Inputs of the system are road segment characteristics (length, width, snow depth, etc.), vehicle characteristics (weight, hourly cost, speed, etc.), vehicle routes, and climatic and weather conditions. A vehicle can be assigned to more than one route. The system, which can handle multiple winter seasons, allows the user to specify maintenance actions (plow snow or do nothing) and starting times for plowing, select vehicle types, and assign vehicles to routes. The system can also run without user interaction by using decision rules resident in the simulation model. Routes can be added or modified during the simulation run. The output of the model is the total cost defined by accident costs, delay costs, vehicle operating costs, lost wages, and productivity costs. The model may also be used for defining the level of service, partitioning the road network into priority classes and sizing and replacing vehicle fleets by evaluating the consequences of modifying road segment and vehicle characteristics. The accuracy of the model was validated using data from Hanover, New Hampshire. The simulation model was also useful in analyzing a variety of scenarios related to the modification of road segment and vehicle parameters such as snow depth, snow density, operator wage rate, and vehicle hourly cost.

TUCKER and CLOHAN (1979) developed an interactive program to assist planners in constructing feasible routing plans for plowing operations. The number of routes to build must correspond to the number of vehicles available for plowing. Also, each route must start and end at the same location and routes must have nearly equal service traversal times with minimum route overlap. Analytical formulas are derived to estimate the service traversal time per route when plows have the same or different blade widths. In addition, multiple passes may be required to clear a road segment, depending on the segment width and the effective width of the plow blade. Finally, penalties are used to restrict the total number of U-turns and left turns. Computer graphics are used to display the road network, and feasible routes are constructed one at a time on the screen using a sequential insertion heuristic (TUCKER, 1977). The interactive program is embedded into a simulation program to model plow movements and interactions. The simulation program incorporates meteorological conditions (storm length, rate of snowfall, snow density, storm starting time), plow characteristics (fleet size, plow weight, plow width, plowing speed) and route configuration characteristics (snow accumulation before starting plowing, route starting time). The simulation model was validated using data from the town of Newington, Connecticut. The model can be used to assess the relative efficiency of various routing plans by comparing route duration, number of U-turns and left turns, as well as distance covered by deadheading trips. The authors also demonstrated how their simulation model may be used to analyze the sensitivity of plowing time to storm length, accumulation rate and snow accumulation before starting plowing.

Simulation models have also been developed to help planners for the strategic planning of winter road maintenance operations. For example, PRUETT and KUONG LAU (1981) proposed a computer-aided simulation model to assist planners at the Louisiana Department of Transportation and Development in making decisions about manpower, equipment, and materials utilization for the strategic planning of many types of highway maintenance activities including snow removal. Also, WELLS (1984) proposed a discrete event simulation approach to aid planners in metropolitan and larger urban areas for the strategic planning of various maintenance activities including salt and sand spreading, snow plowing, and snow disposal. The main goal of her approach is to help in evaluating the consequences of variations in the level of service, availability of resources, budget, and weather conditions. The approach is also proposed as a way to control maintenance activities by setting standards for crew performance. Inputs of the system are data regarding resource levels and costs, workload standards, performance characteristics, desired level of service, and weather conditions. Outputs of the system include activity delays caused by adverse weather conditions, resource shortages, times dedicated to each activity, expenditures on manpower, equipment and materials, as well as frequency of activity, total working hours and total workload for each activity. The accuracy of the simulation model was validated using historic data from a Midwestern city in the US. The system also permitted useful scenarios analysis related to outputs versus resources expended.

5.3.2 Rule-based decision support systems

More recently, several rule-based decision support systems were proposed to assist planners in making vehicle routing decisions for plowing and snow disposal. For example, FUKUCHI *et al.* (1996) described a rule-based decision support system for planning of snow removal operations in Japan. The system provides instructions regarding operation details, vehicle types, fleet size, and personnel to be put on alert up to 24 hours in advance of need by following the rules of a knowledge database based on input weather forecasts. The knowledge database includes data on mobilization conditions of vehicles distinguished by type, snow removal capabilities, operation patterns for different weather conditions, standard operation hours, mobilization priority, conditions for altering operations for situations requiring temporary allocation of a large number of vehicles, and conditions for changes in commuting time zones. The system may also be used at the real-time level to provide new instructions directly to vehicle operators by employing satellite circuits in response to sudden changes in weather and road surface conditions. Also, KANEMURA (1998) described a decision support system that has been implemented in the city of Sapporo, Japan. The system, which relies on the city's meteorological observation facilities, collects, analyzes, and provides information on snowfall conditions, meteorological conditions, short-range forecasts, and status of snow removal operations (e.g. being conducted or completed) in various parts of the city. These data are used to estimate the starting and completion time for road maintenance as well as to determine if a road surface improvement service should be performed, if personnel and equipment should be kept waiting for mobilization on the same day, and if the current road maintenance service should stop or continue.

Some agencies have developed rule-based decision support systems that emphasize both plowing and spreading operations. GRENNEY and MARSHALL (1991) described a prototype rule-based decision support system developed to help planners in the western US decide when to dispatch crews and equipment for plowing and sand spreading operations. The Minnesota Department of Transportation developed a winter road maintenance management system to help in planning, organizing, directing and controlling winter road maintenance operations by determining resource requirements, allocating resources, scheduling crews, and evaluating performance (MINNESOTA DEPARTMENT OF TRANSPORTATION, 1993). The system is based on the application of management principles dictated by experience to winter road maintenance operations. Inputs of the system are employee hourly and overtime rates, equipment and materials unit costs and inventories, as well as service levels, road segment characteristics, and routes for spreading and plowing. For each sector, for each road class and for each route, outputs of the system are quantities of sand and salt to spread, application rates and material inventory balances, as well as labor, materials and equipment costs. ALFELOR *et al.* (1999) described a prototype decision support system developed by the Minnesota Department of Transportation for defining, collecting, analyzing, and applying performance and evaluation data to manage maintenance activities such as sanding and plowing, as well as to help efficiently allocate labor, equipment, and material resources. The categories of maintenance activities were initially developed by using internal department knowledge of customer needs and were verified through direct customer research. The system incorporates analytical models that calculate the value added to customers in terms of road-user costs (i.e. travel time and accident costs) to help operations managers respond to customer needs.

The Swedish Road and Transport Research Institute (VTI) developed a rule-based expert system to assist planners in making real-time winter maintenance operation and scheduling decisions (LJUNGBERG, 2000). The system focuses on choice of operation (plowing, sanding, or salting), starting time, and material type (dry, prewetted or brine) and quantity for different road classes. Besides weather conditions and road weather forecasts, data for the proposed system consist of information about the network, including available vehicles and equipment, and a knowledge database of maintenance practices gathered from literature studies and interviews with experts. For further details on the VTI system, see the review by PERCHANOK *et al.* (2000). Recently, MAHONEY and MYERS (2003) described a project initiated by the FHWA to develop a prototype winter Maintenance Decision Support System (MDSS) to provide winter maintenance decision makers with real-time road maintenance guidelines (e.g., chemical use, plowing, timing of operation, and location) regarding vehicle routes. The system merges weather forecasting with road condition information, chemical concentration algorithms, and anti-icing and deicing rules of practice. The system allows users to select treatment constraints (chemical used, route times, application rates) for each route. In addition, users can tailor the rules of practice for each route.

Computer assisted route design systems have also been developed to assist planners in constructing feasible plow and/or spreader routes. For example, LAPPALAINEN (1988) described a computerized route design system to minimize the number of plows and the distance covered by deadheading trips in Finland. PREDIERI (1988) described a computerized road map used for routing vehicles for various municipal services, including plowing, salt spreading, and anti-icing in the city of Bologna. The system allows the user to construct routes by selecting road segments on the screen. A similar computer-aided system for the design of snow plow routes in Finland was described by KORHONEN *et al.* (1992). The roads of the Finnish network are partitioned into several classes based on traffic volume, each with different service levels and time limits for service completion. The system is based on the use of a digital map that allows the user to construct routes by pointing and clicking with a mouse on roads of the map while providing information such as route length, route duration, distance covered by deadheading trips, and service level data. MARTIKAINEN and KERANEN (1997) described a project supported by the Minnesota Department of Transportation to develop an automated route planning system for summer and winter maintenance operations. FARKAS and CORBLEY (1998) mentioned the use of a GIS-based snow tracking system for plowing operations in the city of Newark, New Jersey. Besides interactive on-screen route design, the system permits display of road segment conditions (e.g., plowed to pavement, in progress, plowed but snow-packed, or unplowed) at any given time on the screen, thereby ensuring real-time control. Finally, CORTINA and LOW (2001) described an interactive route design system, called Snowman, to build spreader and plow routes in the town of Brighton, New York. Each route must start and end at the same depot and each road segment must be covered by the required number of passes. The system, which relies in large part on the individual expertise of the planner, helps the user construct one route at a time interactively by pointing and clicking with a mouse on road segments of the schematic transportation network, thereby adding each road segment to the current route. As a route is created, on-screen statistical information regarding distance traveled, elapsed time, and materials utilization can be viewed. Once all road segments are covered by the appropriate number of passes, the planner can attempt to interactively improve any route. Several criteria are used to evaluate the quality of the routes generated by the planner: the minimization of total deadhead travel time, balance in route times, the number of passes for each road segment, and violations of acceptable times for service completion for each class.

5.3.3 Constructive methods

One of the first contributions dealing with the plow routing problem within the context of arc routing is due to MARKS and STRICKER (1971). Given a fleet of m homogeneous plows, the problem considered is to design a set of m plow routes such that each road segment is cleared within either two or four passes, depending on its width, while minimizing the distance covered by deadheading trips. Multiple pass requirements are taken into account by duplicating each road segment as many times as the required number of passes on the road segment. The problem is modeled as a m-vehicle undirected Chinese postman problem. Two approaches are presented for solving the problem. In the first approach, the transportation network is partitioned into m subnetworks by solving a districting problem, and a Chinese postman problem is solved for each of them using a decomposition heuristic. In the second approach, a unicursal graph is first derived from the original network, and arbitrarily partitioned into m mutually exclusive, collectively exhaustive subgraphs of approximately the same size so that an Eulerian cycle can be defined for each of them without additional duplication of edges. For details, see STRICKER (1970). The first approach was tested on real data from the city of Cambridge, Massachusetts, involving one vehicle for urban waste collection but not for snow plowing. The authors also suggested three strategies to handle the hierarchy of the network when class connectivity is satisfied. The first strategy tries to allow the highest level of equipment usage on road segments of highest priority by multiplying the length of each road segment by its priority (with 1 being the highest priority) and solving a Chinese postman problem using these weighted lengths so as to favour the duplication of edges associated with road segments of highest priority. The second strategy solves a Chinese postman problem on each connected subgraph induced by the set of edges of a specific priority class and assigns exactly one vehicle to each postman tour. Finally, the last strategy generates several Eulerian cycles while disregarding road priorities, and chooses the cycle which best adheres to the hierarchy of the network.

MOSS (1970) proposed a cluster-first, route-second approach to solve the vehicle routing problem for plowing and spreading operations in Centre County, Pennsylvania. Road segments are first organized into balanced sectors, and a vehicle route is obtained for each of them by solving a directed Chinese postman problem. Each direction of a two-lane, two-way road (one lane each way) can be serviced by a different vehicle. The cluster phase tries to ensure that the graph generated by the edges of each sector is Eulerian to reduce deadheading in the routing phase. Service hierarchy, class continuity for each sector, and maximum route times are enforced.

The BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975), modeled a plow routing problem, with a homogeneous fleet of plows and multiple pass requirements for large road segments, as a multi-vehicle undirected Chinese postman problem. Multiple pass requirements are taken into account by duplicating each road segment the required number of times. The problem is solved using a cluster first, route second heuristic, based on earlier work by STRICKER (1970). The cluster phase breaks the original graph into small subgraphs according to several rules so as to enable routes with less deadheading. In particular, each subgraph should contain an even number of odd degree nodes forming as compact and centralized a location as possible. The route phase then solves an undirected Chinese postman problem in each subgraph and Fleury's algorithm (KAUFMANN, 1967, p.309) is used for determining an Eulerian cycle in the resulting Eulerian subgraph. The heuristic was tested on real data from a major Canadian city. The BUREAU OF MANAGEMENT CONSULTING also proposed to handle the hierarchy of the network and the direction of the traffic flow directly within Fleury's algorithm by selecting, at each iteration, the next edge of highest priority whose removal does not disconnect the Eulerian subgraph, while trying to respect the direction of the traffic flow.

LEMIEUX and CAMPAGNA (1984) studied the problem of determining a plow route that starts at a depot, traverses every road segment only once in both directions, and ends at the depot, while respecting precedence relation constraints and U-turn restrictions. The problem is modeled as a directed hierarchical postman problem. Let $G = (V, A)$ be a directed network with counterpart arcs in opposite directions between intersection nodes. Since G is symmetric, an Eulerian circuit in G can be determined by means of Fleury's algorithm (KAUFMANN, 1967, p.309). The authors proposed to handle precedence relation constraints directly within Fleury's algorithm by selecting the next arc (whose removal does not disconnect graph G) so as to cover high-class road segments as quickly as possible. However, since there is no guarantee that there exists an Eulerian circuit in G that strictly satisfies precedence relation constraints without introducing any deadheading, these constraints are treated as soft constraints. Soft precedence relation constraints allow a low-class road segment to be plowed before a high-class road segment. Let $G' = (V', A')$ be the subgraph induced by the subset of arcs not yet included in the circuit. Given a partial circuit that ends at arc (v_i, v_j), the authors always choose the arc (v_j, v_k) with highest priority so that $G' = (V', A' \setminus \{(v_j, v_k)\})$ is connected. However, if $v_k = v_i$ (implying a U-turn), the two following rules are used for attempting to restrict the number of U-turns:

Rule 1: Choose the arc (v_j, v_l), $v_l \neq v_i$, with the same priority class as that of arc (v_j, v_k) so that $G' = (V', A' \setminus \{(v_j, v_l)\})$ is connected. If such an arc does not exist, choose the arc (v_j, v_k).

Rule 2: Choose the arc (v_j, v_l), $v_l \neq v_i$, with highest priority so that $G' = (V', A' \setminus \{(v_j, v_l)\})$ is connected.

With the first rule, the heuristic tries to cover the arcs with the highest priorities as quickly as possible, even if it implies U-turns. In contrast, with the second rule, the heuristic can compromise the importance of observing priorities so as to avoid U-turns. The authors applied the heuristic to a very small hypothetical problem.

In a mathematical modeling competition in 1990, students from various colleges and universities in the US studied the problem of designing routes for two plows to clear the county roads in a district of Wicomico County, Maryland. The county roads are two-way with one lane in each direction and form a strongly connected, directed graph G with 139 vertices and 374 arcs. Each plow can service exactly one lane at a time and each route must start and end at the same location. The objective considered is to minimize the plowing completion time. Since G is unicursal, the plow routing problem considered consists of determining two Eulerian circuits of approximately the same length in G to minimize the plowing completion time.

A first solution method was that of ATKINS *et al.* (1990) who developed a traditional cluster-first, route-second heuristic. The graph G is first partitioned into two subgraphs of approximately the same size (in terms of total road lengths). This is done by iteratively choosing a two-way road and adding the two associated arcs of opposite direction to the subgraph that has the smaller current total length. An Eulerian circuit is then constructed for each subgraph by tracing a closed walk on a spanning tree of the associated undirected graph, into which the nontree edges are inserted in a simple fashion.

CHERNAK *et al.* (1990) studied a more realistic problem in which the service hierarchy of the transportation network is also considered, with an objective of minimizing the distance covered by deadheading trips, in addition to minimizing the plowing completion time. This problem is solved using a heuristic approach that constructs, for each plow, a primary route servicing roads of highest priority and a second route servicing the other roads. The heuristic was also used to evaluate the impacts of changing the width of the plow blade on service completion times.

ROBINSON *et al.* (1990) suggested a cluster-first, route-second method and a route-first, cluster-second method for determining two Eulerian circuits of approximately the same length in G. In the cluster-first, route-second method, roads are first organized into two balanced subgraphs by means of the procedure described by ATKINS *et al.* (1990), and an Eulerian circuit is constructed in each subgraph in a depth-first search fashion. In the route-first, cluster-second method, an Eulerian circuit $C = (x, ..., y, ..., x)$ is first built on G in a depth-first search fashion and is then segmented into two feasible routes $R_1 = (x, ..., y)$ and $R_2 = (y, ..., x)$ where x and y are the starting points of routes R_1 and R_2, respectively. Comparisons showed that the cluster-first, route-second method produced better routes than the route-first, cluster-second method, in terms of both the number of U-turns implied and the times for plowing completion.

Finally, HARTMAN *et al.* (1990) extended the end-pairing algorithm (EDMONDS and JOHNSON, 1973) to simultaneously build two Eulerian circuits of approximately the same length in G.

In a series of two papers, SALIM *et al.* (2002a,b) proposed the SRAM (Snow Removal Asset Management) system to solve the vehicle routing problem for plowing and spreading operations in Black Hawk County, Iowa. The SRAM system can deal with service hierarchy and maximum route service times. Although the system relies in large part on decision rules drawn from interviews with experts, it also uses a simple constructive method that builds feasible routes one at a time for each class of roads using the following greedy criterion. Given a partial route that ends at a road of a given priority class, choose the nearest road of the same class that fits within the maximum route service time limit. For each class of roads, the optimal assignment of vehicles to routes is then found by solving a transportation problem with supply nodes representing vehicles and demand nodes representing either plowing routes or spreading routes. The assignment variables correspond to the time spent by a vehicle on a route. A vehicle can be assigned to more than one route, and a route can have more than one vehicle assigned to it. The quantity of materials required for each spreading route is determined last. Related field testing showed that the system was useful in analyzing a variety of scenarios concerning the number of required road segments, vehicles and drivers and the status of the materials inventory, and reduced the total traversal time (service and deadheading) by 1.9-9.7% (depending on snowfall conditions) over the solution in use by the county.

5.3.4 Composite methods

HASLAM and WRIGHT (1991) developed an interactive route generation procedure for the plow routing problem at the Indiana Department of Transportation (INDOT), U.S. In this problem, routes of total minimal length that start and end at a given depot are sought and class continuity as well as maximum route length constraints must be satisfied. INDOT is only responsible for servicing state roads, highways, and Interstates, but county roads may be traversed (while deadheading) to provide service to state roads. A fleet of homogeneous plows are based at the depot and the underlying network is assumed to be directed (an arc for each lane). When ignoring class continuity constraints, the problem can be formulated as a directed capacitated arc routing problem with vehicle capacity representing maximum route length and arc demands representing arc lengths.

The route generation procedure starts by calculating a lower bound L_r on the number of routes to construct by dividing the total workload in each class (in terms of total road lengths) by the maximum distance a plow may travel when servicing that class. The user then provides s seed nodes, $s \geq L_r$, with associated classes out of which feasible routes are constructed one at a time using the three-stage algorithm described in Figure 5.2. Given a seed node and its class, the first stage of the algorithm constructs a feasible route made of a path from the seed node to the depot and another path in the reverse direction, without violating class continuity and maximum route length constraints. In the second stage, pairs of non-covered required arcs of opposite direction are sequentially inserted into the route as long as class continuity and maximum route length permit. Finally, in the last stage, if required arcs have not been covered, then the class continuity constraint is relaxed and the second stage is repeated by permitting class upgrading.

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1. Let $G = (V, A)$ be a directed graph where $V = \{v_0, v_1, \ldots, v_n\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the arc set. The depot is represented by the node v_0. Let P be a path in G. Given two paths $P = (v_i, \ldots, v_j)$ and $P' = (v_j, \ldots, v_k)$ having a common endpoint v_j in G, let $P + P' = (v_i, \ldots, v_j, \ldots, v_k)$ denote the union of the arcs of these two paths. Let SP_{ij} be the shortest path from v_i to v_j in G and let sp_{ij} be its length. Let $v_s \in V$ be a seed node of class k, $k \geq 1$. Set $P := \varnothing$ and $v_{end} := v_s$ (v_{end} denotes the endpoint of P different from the seed node).
   a. Choose a non-serviced required arc (v_{end}, v_i) of class k in G such that $sp_{i0} < sp_{end,0}$. If such an arc does not exist, set $P := P + SP_{end,0}$, $v_{end} := v_0$ and go to Step b. If the maximum route length constraint permits, set $P := P + (v_{end}, v_i)$ and $v_{end} := v_i$. Otherwise, set $P := P + SP_{end,0}$, $v_{end} := v_0$ and go to Step b. If $v_{end} \neq v_0$, then repeat Step a. Otherwise, go to Step b.
   b. Choose a non-serviced required arc (v_{end}, v_j) of class k in G such that $sp_{js} < sp_{end,s}$. If such an arc does not exist, set $P := P + SP_{end,s}$ and go to Step 2. If the maximum route length constraint permits, set $P := P + (v_{end}, v_j)$ and $v_{end} := v_j$. Otherwise, set $P := P + SP_{end,s}$ and stop. If $v_{end} \neq v_s$, then repeat Step b. Otherwise, go to Step 2.
2. *First insertion strategy*
   If all required arcs are serviced, stop. Otherwise, choose any vertex v_i on P incident to a pair of non-serviced required arcs (v_i, v_j) and (v_j, v_i) of class k not on P. If such a pair of arcs does not exist, go to Step 3. If the maximum route length constraint permits, add the circuit (v_i, v_j, v_i) to P and repeat Step 2. Otherwise, stop.
3. *Second insertion strategy*
   If all required arcs are serviced, stop. Otherwise, repeat Step 2 by relaxing the class continuity constraint and permitting class upgrading.

Figure 5.2: The three-stage algorithm (HASLAM and WRIGHT, 1991)

The three-stage algorithm was tested on a subnetwork of the Fowler subdistrict with 21 nodes, 54 arcs (all required) and three classes of roads. The routes produced by the algorithm failed to cover all required arcs of the network. However, with some manual intervention, the authors found a set of routes covering all required arcs and satisfying more constraints than the route configuration in use by the subdistrict, but having a higher deadheading cost. The authors also described two improvement methods that operate on several routes at a time. The first improvement method is an exchange heuristic that tries to reduce the distance covered by deadheading trips and better enforce class continuity and route compactness by swapping arcs between routes. The second improvement method tries to reduce the number of routes and the distance covered by deadheading trips by eliminating a route of a given class and inserting its required arcs into other routes of the same class. The two improvement methods were applied to the existing set of routes in use by the Fowler subdistrict. The instance contained 99 nodes, 362 arcs (all required) and three classes of roads. Computational tests showed that the swap heuristic and the route elimination method reduced the distance covered by deadheading trips by 23% and 9%, respectively, over the route configuration in use by the subdistrict. Moreover, the second heuristic decreased the number of routes by more than 3%. The swap heuristic was embedded in a prototype decision support system (WRIGHT *et al.*, 1988).

CAMPBELL and LANGEVIN (2000) described the commercially available vehicle routing software GeoRoute developed by the firm GIRO, based in Montreal, Canada, for postal delivery, winter maintenance, meter reading, street cleaning and waste collection applications. The GeoRoute software allows three types of winter road maintenance operations: plowing, spreading and snowblowing (for loading snow into trucks). The software can accommodate service time windows, service frequency, vehicle capacities, spreading rates, turn restrictions, street segment dependencies, and both-sides service restrictions (servicing both sides of a road segment in a single route). In addition, for spreading operations, the software can determine the number of passes required to service each road segment given the street width, the vehicle type and whether both lanes of a two-lane, two-way street (one lane each way) should be spread in a single pass or not. GeoRoute uses a two-phase method similar to the cluster first, route second method, but constructs instead one route at a time. The first phase selects a seed basic unit to initialize a cluster and allocate the basic units closest to the seed to a cluster. The second phase determines a vehicle route on the cluster with an arc routing adaptation of the GENIUS composite procedure proposed by GENDREAU *et al.* (1992) for the traveling salesman problem. The objective function is a weighted additive multicriteria function defined by the user. The user may also specify the basic units that must be serviced on the same route. GeoRoute has been implemented in Ottawa, Canada (MINER and BRETHERTON, 1996; MINER, 1997) for snow plowing and in Suffolk County, United Kingdom (GUTTRIDGE, 2004) for salt spreading. CAMPBELL and LANGEVIN (2000) also report three implementations in the cities of Laval, Charlesbourg, and Nepean in Canada.

The design of block and length pattern routes for winter road maintenance operations was studied by GENDREAU *et al.* (1997) who modelled the problem as a mixed rural postman problem with turn penalties (MRPPTP). Given a mixed graph $G = (V, A \cup E)$ with a subset of required links $R \subseteq A \cup E$, nonnegative costs c_{ij} associated to its edges or arcs $(v_i, v_j) \in A \cup E$, and nonnegative penalties associated to its turns, the MRPPTP consists of finding a minimum-cost closed chain in G containing each link of R at least once. The objective function to be minimized is a weighted combination of the deadheading travel time, the number of left turns, U-turns, straight crossings, and street changes. The authors conducted a study to properly calibrate the various weights of the objective function to obtain particular types of routes (e.g. block or length pattern). To this end, several combinations of parameter values are first tested on 30 street networks in three Montreal suburbs for each type of pattern. The MRPPTP corresponding to each experiment is solved using GeoRoute, an arc routing package developed by the Montreal-based firm GIRO. For each pattern, the combinations of parameter values are then identified based on decision rules dictated by experience. These sets of parameter values were embedded in the GeoRoute system. Computational experiments were also performed to properly calibrate the various penalty costs so as to generate length patterns for waste collection.

Finally, the authors showed that it is possible to solve the MRPPTP by transforming it into an asymmetric traveling salesman problem (ATSP) based on the transformation procedure proposed by LAPORTE (1997). The first step of the transformation consists of replacing each edge $(v_i, v_j) \in E$ by a pair of arcs (v_i, v_j) and (v_j, v_i) of opposite direction and cost c_{ij}. Let $G' = (V, A \cup A')$ be the resulting directed graph and denote $R' = \{(v_i, v_j), (v_j, v_i): (v_i, v_j) \in E \cap R\} \cup (A \cap R)$ as the subset of required arcs in G'. The second step consists of transforming the MRPPTP on G' into an equivalent generalized traveling salesman problem (GTSP) on a directed graph $H = (W, B)$. In this graph, the vertex set W has a vertex w_{ij} for each required arc $(v_i, v_j) \in R'$ and the arc set B contains an arc (w_{ij}, w_{kl}) between w_{ij} and w_{kl} if the two required arcs (v_i, v_j) and (v_k, v_l) are not associated with the same required edge in G. Let $G'' = (V'', A_1 \cup A_2)$ be a directed graph such that V'' contains two vertices for each arc $(v_i, v_j) \in A \cup A'$, A_1 contains an arc with cost c_{ij} for each arc $(v_i, v_j) \in A \cup A'$, and A_2 contains an arc with the appropriate turn penalty for each turn in G. The cost c_{jk} of an arc $(w_{ij}, w_{kl}) \in B$ is equal to the sum of the cost of arc (v_i, v_j) and the length of a shortest path from v_j to v_k in G''. This cost includes all turn penalties starting from the arc terminating at node v_j and finishing with the arc emanating from node v_k. Solving a MRPPTP on G then amounts to solving an asymmetric GTSP on H which consists of determining a least cost Hamiltonian circuit visiting each of several clusters at least once. Any vertex $w_{ij} \in W$ corresponding to a required arc (v_i, v_j) of $A \cap R$ or any pair of vertices $\{w_{ij}, w_{ji}\} \subseteq W$ corresponding to the same required edge (v_i, v_j) of $E \cap R$ defines such a cluster. The third and last step consists of transforming the asymmetric GTSP on H into an ASTP on an associated complete directed graph $H' = (W, B \cup B')$ by means of a procedure proposed by NOON and BEAN (1993). For each cluster $\{w_{ij}, w_{ji}\} \subseteq W$ corresponding to an edge (v_i, v_j) of $E \cap R$, the arc set B' contains two arcs (w_{ij}, w_{ji}) and (w_{ji}, w_{ij}) of opposite direction, both with cost $-M$, where M is a large positive constant. In addition, the cost c_{jk} of every arc (w_{ij}, w_{kl}) ∈ B linking two clusters is replaced by the cost c_{ik} of arc (w_{ji}, w_{kl}) ∈ B. A MRPPTP defined in G can then be solved by the resolution of an ATSP defined in H'.

To illustrate, consider the mixed graph G shown in Figure 5.3(a), where the links of R are shown in bold lines and the numbers correspond to link costs. The construction of G'' is represented in Figure 5.3(b) for the following turn penalties: 0 for a straight crossing, 1 for a right turn, 3 for a left turn, and 9 for a U-turn. The two sets of arcs A_1

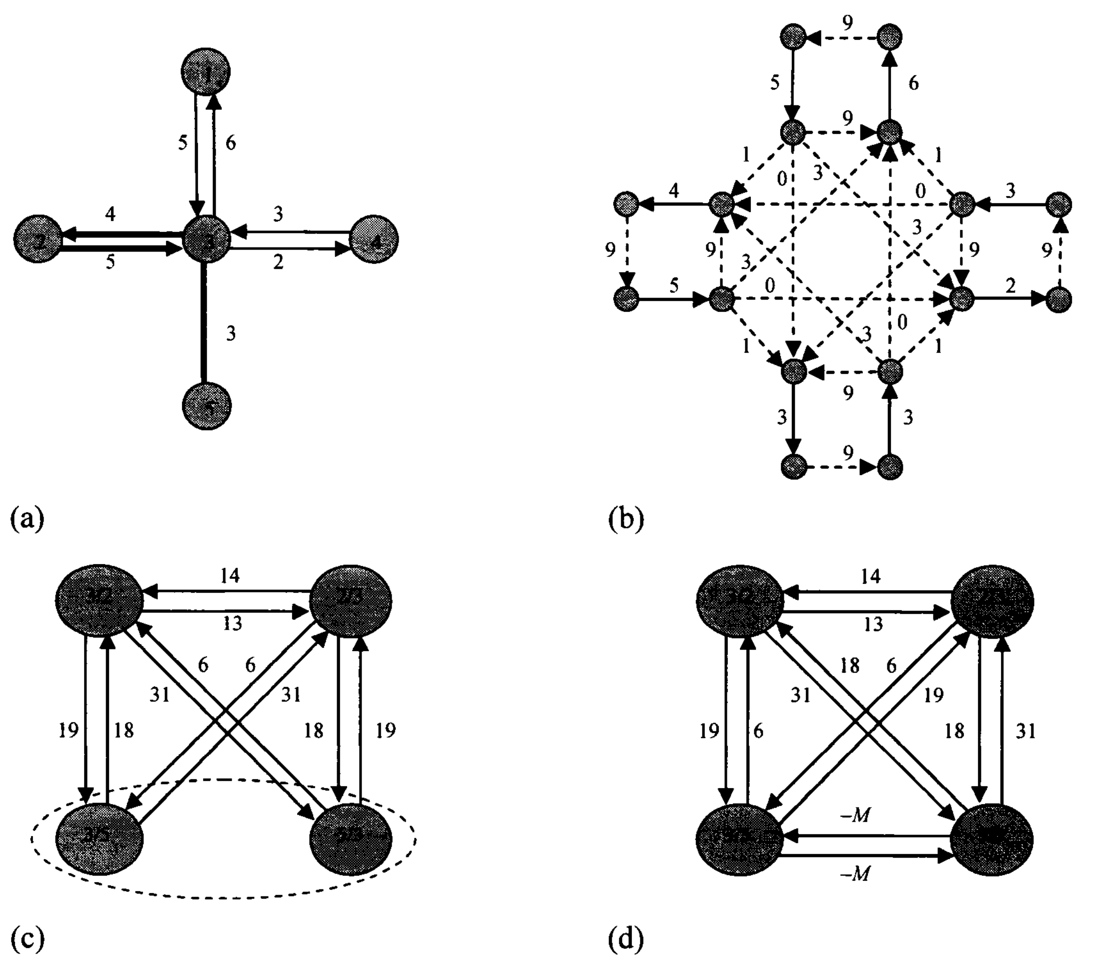

Figure 5.3: Example of a graph transformation: (a) graph G, (b) corresponding graph G'', (c) graph H and (d) complete graph H'.

and A_2 are shown as solid lines and dashed lines, respectively, with the corresponding arc cost or turn penalty. A first transformation leads to the graph H represented in Figure 5.3(c). The graph H has four vertices, $w_{3/2}$, $w_{2/3}$, $w_{3/5}$, and $w_{5/3}$, and three clusters $\{w_{3/2}\}$, $\{w_{2/3}\}$, and $\{w_{3/5}, w_{5/3}\}$. The numbers correspond to arc costs computed in G''. The graph H is further transformed by first including two arcs $(w_{3/5}, w_{5/3})$ and $(w_{5/3}, w_{3/5})$ in B', both with cost $-M$, where M represents a very large positive number, and by replacing the cost of arcs $(w_{3/5}, w_{3/2})$, $(w_{3/5}, w_{2/3})$, $(w_{5/3}, w_{3/2})$, and $(w_{5/3}, w_{2/3})$ by the cost of arcs $(w_{5/3}, w_{3/2})$, $(w_{5/3}, w_{2/3})$, $(w_{3/5}, w_{3/2})$, and $(w_{3/5}, w_{2/3})$, respectively. The resulting graph H' is shown in Figure 5.3(d).

As highlighted by LAPORTE (1997), this type of transformation induces a fair amount of degeneracy in the cost structure of the ATSP, which may limit its computational interest.

Finally, a three-stage composite heuristic was proposed by KANDULA and WRIGHT (1997) for routing plows and spreaders in the state of Indiana. The heuristic takes into account class continuity and a maximum route duration for each class. In addition, both sides of a road segment must be serviced by the same vehicle. For spreading operations, the vehicle capacity constraints are given in terms of maximum route durations. Given an undirected graph, the first phase identifies a set of seed nodes in sufficient number to respect the time limits, and then determines the maximum number of routes that can be constructed out of each seed node by means of an adaptation of the node scanning lower bound procedure introduced by ASSAD *et al.* (1997) for the capacitated Chinese postman problem. A good set of seed nodes close to the depot helps to reduce the distance covered by deadheading trips. The second phase then constructs routes one at a time out of each seed node using the following greedy optimality criterion: given a partial route that ends at vertex v_i, choose the non-serviced edge (v_i, v_j) that fits within the time limits and maximizes the distance between v_j and the depot. If no such edge can be found, then a route containing the partial route that ends at vertex v_i is created (i.e., a route made of a shortest chain of deadheaded edges between the depot and the seed node, the partial route that ends at vertex v_i, the partial route in the reverse direction to satisfy the both-sides service constraint, and a shortest chain of deadheaded edges between the seed node and the depot). An improvement procedure that tries to reduce the distance covered by deadheading trips and the number of kilometers violating the class continuity constraints without exceeding the time limits is used last. This is done by swapping edges among the routes or by transferring edges from one route to another. Comparisons with the tabu search algorithm proposed by WANG and WRIGHT (1994) for a vehicle routing problem for plowing and spreading operations (see Section 5.3.5) on five networks of Indiana showed that the heuristic obtained the best solutions. However, it should be emphasized that the tabu search algorithm was stopped after a given number of iterations.

2.3.5 Metaheuristics

WANG and WRIGHT (1994) described an interactive decision support system, called CASPER (Computer Aided System for Planning Efficient Routes), to assist planners at the Indiana Department of Transportation (INDOT) in the design of vehicle routes for plowing and spreading operations. The sectors are given and each of them contains exactly one depot. The system, which can accommodate service time windows, class continuity, and class upgrading, starts by calculating the number of routes to construct in a given sector for each class of roadways. Let $G = (V, A)$ be the directed graph associated with the sector. The depot is represented by the vertex v_0. Let $P_K = \{A_1, A_2,\ldots, A_K\}$ be a partition of A with $A_1 \cup A_2 \cup \ldots \cup A_K = A$ and $A_i \cap A_j = \varnothing$ for all $i,j \in \{1, 2, \ldots, K\}$, $i \neq j$. For every class $A_k \subseteq P_K$, define N_k, w_k and t_k as the number of class k routes to construct, the total workload of class k (in terms of total number of class k kilometers requiring service) and the time limit of a class k route, respectively. Thus,

$$N_k = \left\lceil \frac{w_k}{t_k \cdot s} + dhf \right\rceil$$

where s is the average vehicle service speed and dhf is a nonnegative number to account for deadheading trips. For every class $A_k \subseteq P_K$, the system builds N_k vehicle routes starting and ending at the depot using a tabu search algorithm. An initial solution is obtained by means of a route growth heuristic described in WANG (1992), which is a refinement of the three-stage algorithm proposed by HASLAM and WRIGHT (1991) and described in Section 5.3.4. In this heuristic, N_k feasible vehicle routes are first generated for every class $A_k \subseteq P_K$ following a look-ahead procedure. Given two paths $P = (v_i, ..., v_j)$ and $P' = (v_j, ..., v_k)$ having a common endpoint v_j in G, let $P + P' = (v_i, ..., v_j, ..., v_k)$ denote the union of the arcs of these two paths. Let SP_{ij} be the shortest path from v_i to v_j in G. Given a partial class k route P that ends at vertex v_i, the look-ahead procedure selects the non-serviced required arc (v_i, v_j) of class k that maximizes the number of non-serviced required arcs (v_j, v_l) of class k adjacent to v_j such that the duration of the route $P + (v_i, v_j) + SP_{j,0}$ does not violate the time limit. The remaining non-serviced required arcs are then sequentially inserted into vehicle routes using four complementary insertion strategies that allow class upgrading and overduration. The possible resulting infeasible routes are however penalized by the objective function $f(r) = \alpha_1 D(r) + \alpha_2[M(r)]^3 + \alpha_3 C(r)$, where $D(r)$ is the total distance covered by deadheading trips in route r, $M(r)$ the number of minutes route r is below or above the lower or upper bound of the service time window, $C(r)$ the total distance of all class upgraded road segments in route r, and α_1, α_2, and α_3 are the corresponding penalty parameters. The four insertion strategies are defined as follows:

strategy 1: given a class k route and a vertex v_i on the route incident to a pair of non-serviced required arcs (v_i, v_j) and (v_j, v_i) of class k, determine if the circuit (v_i, v_j, v_i) can be added on the route without violating the time limit;

strategy 2: use strategy 1 by relaxing the class continuity constraint and permitting class upgrading;

strategy 3: given a non-serviced required arc of class k, determine the existing class k route into which this arc should be inserted in order to minimize the objective function f without violating the time limit;

strategy 4: use strategy 3 by relaxing both class continuity and time limit constraints.

The route growth heuristic proposed by WANG (1992) is summarized in Figure 5.4. The insertion of an arc into a given vehicle route at Step 4 is performed by means of a

---

1. Set $k = 1$.
   a) Set $P := \varnothing$. Determine the closest non-serviced required arc (v_i, v_j) of class k from the depot, set $P = SP_{0i} + (v_i, v_j)$ and $v_{end} := v_j$ (v_{end} denotes the endpoint of P different from the depot).
   b) Choose a non-serviced required arc (v_{end}, v_i) of class k that maximizes the number of non-serviced required arcs (v_i, v_j) of class k adjacent to v_i. If such an arc does not exist, set $P = P + SP_{end,0}$ and go to Step c. If the time limit constraint permits, set $P := P + (v_{end}, v_i)$, $v_i = v_{end}$ and repeat Step b. Otherwise, set $P := P + SP_{end,0}$ and go to Step c.
   c) If the current number of routes of class k is smaller than N_k, return to Step a. If $k < K$ and the current number of routes of class k is equal to N_k, set $k = k + 1$ and return to Step a. Otherwise, go to Step 2.
2. *First insertion strategy*
   If all required arcs are serviced, stop. Otherwise, for each class $A_k \subseteq P_K$, identify a vertex v_i on any route of class k incident to a pair of non-serviced required arcs (v_i, v_j) and (v_j, v_i) of class k not on the route. If the time limit constraint permits, add the circuit (v_i, v_j, v_i) on the route. Repeat Step 2 until no additional vertex v_i incident to a pair of non-serviced required arcs (v_i, v_j) and (v_j, v_i) can be found.
3. *Second insertion strategy*
   If all required arcs are serviced, stop. Otherwise, repeat Step 2 by relaxing the class continuity constraint and permitting class upgrading.
4. *Third insertion strategy*
   If all required arcs are serviced, stop. Otherwise, select a non-serviced required arc (v_i, v_j) and determine the route for which the insertion of (v_i, v_j) minimizes the objective function f while satisfying time limit and class continuity constraints. If such a route has been determined, then insert (v_i, v_j) into the route. Repeat Step 4 until no additional non-serviced required arc can be inserted into any route.
5. *Fourth insertion strategy*
   If all required arcs are serviced, stop. Otherwise, repeat Step 4 by relaxing both time limit and class continuity constraints. Repeat Step 5 until all required arcs are serviced.

Figure 5.4: The route growth heuristic (WANG, 1992)

procedure similar to the ADD algorithm developed by HERTZ *et al.* (1999) for the undirected rural postman problem.

Solutions violating the class continuity and time limit constraints are also allowed during the tabu search process. These infeasible solutions are penalized by the objective function $F(S) = \sum_r f_r(S)$, where $f_r(S)$ is the value of the objective function $f(r)$ assigning a value for route r to any given solution S. A neighbor solution S' is obtained from a solution S by moving the service of an arc (v_i, v_j) or a pair of arcs (v_i, v_j) and (v_j, v_i) from a route T to another route T' of S. The service of an arc or an arc pair is removed from T by means of an algorithm similar to the ADD algorithm (HERTZ *et al.*, 1999) while it is introduced into T' using an algorithm similar to the DROP algorithm (HERTZ *et al.*, 1999). Two tabu lists L_1 and L_2 of limited length are used to register the most recent moves that have been performed during the search process. If the service of an arc a (arc pair p) has been moved from a route T to another route T', then either the arc a (arc pair p) enters the list L_1, and the service of a (p) cannot be removed from T' until the maximum length of L_1 has been reached, or the trio (a, T, T') (trio (p, T, T')) enters the list L_2, and the service of a (p) cannot be removed from T' and reintroduced into T until the maximum length of L_2 has been reached. Clearly, the tabu list L_1 is more restrictive (i.e., it forbids a larger collection of moves) than the tabu list L_2. The structure of the tabu search algorithm is described in Figure 5.5.

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1. Construct an initial set S_0 of vehicle routes by means of the route growth heuristic described in Figure 5.4. Set $S := S_0$, $S^* := S_0$, and $F^* := F(S_0)$. Set one of L_1 and L_2 (say L_1) equal to the empty set.
2. Determine the neighborhood $N(S)$ of S as follows. For each required arc (v_i, v_j) and for each pair of required arcs (v_i, v_j) and (v_j, v_i), consider the route T servicing it, as well all other routes T' in S. Remove the service from T, and introduce it into T'.
3. Choose the non-tabu neighbor S' in $N(S)$ which minimizes the objective function F (i.e., $F(S') \leq F(S'')$ for all S'' in $N(S) \setminus L_1$). If $F(S') < F^*$, then set $S^* := S'$ and $F^* := F(S')$. Set $S := S'$, and update the tabu list L_1.
4. If the maximum number of iterations has been reached, stop (S^* is the best solution encountered during the search process). Otherwise, return to Step 2.

Figure 5.5: The tabu search algorithm (WANG and WRIGHT, 1994)

The authors proposed to reduce the size of the neighborhood of a solution by performing service exchanges only between two routes with the worst values of the objective function $f(r)$ or by introducing the service of an arc (v_i, v_j) or a pair of arcs (v_i, v_j) and (v_j, v_i) only into a route T' whose shortest path from any vertex on T' to v_i or v_j does not exceed a given threshold. The system allows the user to change the setting of the tabu search parameters, to modify the routes manually, to modify road segment or node attributes, and to add route configuration constraints, such as restrictions on mixing road classes and tandem servicing restrictions. A permanent tabu list can also be used to retain user specified moves that cannot be reversed. The system was tested on data from four northern districts of Indiana (WANG *et al.*, 1995). On average, the system reduced the value of the objective function F, the distance covered by deadheading trips, and the number of routes by more than 96%, 4%, and 7%, respectively, over the routing plan in use by INDOT. The large decreases in objective function values result mainly from the CASPER routes better satisfying the service time windows. WANG *et al.* (1995) discussed the importance of the central location of the depot (with respect to the sector to be serviced from that depot) and the compactness of the sector in achieving routes with less deadheading. The problem of partitioning a road network into compact sectors with centrally located depots for plowing and/or spreading operations was studied by KANDULA and WRIGHT (1995, 1997) and MUYLDERMANS *et al.* (2002, 2003).

5.4 Vehicle routing models for snow loading

Very little work has addressed the routing of snowblowers for loading snow into trucks. A decision support system, called GeoRoute, was developed by the firm GIRO, based in Montreal, Canada, for arc routing applications including the routing of snowblowers for loading snow into trucks. The system uses a cluster first, route second method in which the route phase is an adaptation of the GENIUS algorithm proposed by GENDREAU *et al.*

Table 5.2: Characteristics of vehicle routing models for operational planning of plowing

Authors	Problem type	Problem characteristics	Objective function	Model structure	Solution method
MARKS and STRICKER (1971)	Plow routing	Two or four passes per road and service hierarchy	Min deadheading	*m*-vehicle undirected Chinese postman problem	Cluster first, route second
MOSS (1970)	Plow and spreader routing	Service hierarchy, class continuity, and maximum route durations	Min deadheading	Directed Chinese postman problem	Cluster first, route second
TRANSPORT CANADA (1975)	Plow routing	Multiple passes per road and service hierarchy	Min deadheading	*m*-vehicle undirected Chinese postman problem	Cluster first, route second
LEMIEUX and CAMPAGNA (1984)	Plow routing	Service hierarchy and turn restrictions	Min number of U-turns	Directed hierarchical postman problem	Constructive method
ATKINS *et al.* (1990)	Plow routing	Two passes per road and balance in route length	Min plowing completion time	2-vehicle directed Chinese postman problem	Cluster first, route second
CHERNAK *et al.* (1990)	Plow routing	Two or four passes per road, balance in route length and service hierarchy	Min plowing completion time and deadheading	2-vehicle directed Chinese postman problem	Constructive method
ROBINSON *et al.* (1990)	Plow routing	Two passes per road and balance in route length	Min plowing completion time and number of U-turns	2-vehicle directed Chinese postman problem	Cluster first, route second and route first, cluster second
HARTMAN *et al.* (1990)	Plow routing	Two passes per road and balance in route length	Min plowing completion time	2-vehicle directed Chinese postman problem	Constructive method
SALIM *et al.* (2002a,b)	Plow and spreader routing	Service hierarchy, maximum route service times, and one or multiple vehicles (routes) per route (vehicle)	Min deadheading	Arc routing problem	Constructive method
HASLAM and WRIGHT (1991)	Plow routing	Multiple vehicles, class continuity and maximum route length	Min deadheading	Directed capacitated arc routing problem	Composite method
CAMPBELL and LANGEVIN (2000)	Plow routing, spreader routing, and snowblower routing	Time windows, service frequency, vehicle capacities, spreading rates, turn restrictions, street segment dependencies, and both-sides service	Min multicriteria additive function	Arc routing problem	Composite method
GENDREAU *et al.* (1997)	Plow routing	Turn restrictions	Min deadheading and turn penalties	Mixed rural postman problem with turn penalties	GeoRoute
KANDULA and WRIGHT (1997)	Plow and spreader routing	Class continuity, maximum route length, and both-sides service	Min deadheading	Capacitated Chinese postman problem	Composite method
WANG and WRIGHT (1994)	Plow and spreader routing	Time windows, class continuity and class upgrading	Min deadheading, time windows violations, and class continuity violations	Directed capacitated arc routing problem	Tabu search

(1992) for the traveling salesman problem. Further details on the GeoRoute system are given in Section 5.3.4.

GILBERT (1990) proposed a model and a heuristic method for the snowblower routing problem for loading snow into trucks in the city of Montreal, Canada. Considering a given sector with its fixed depot location and the number of possible workdays to complete snow loading operations measured from the end of the storm, the problem consists of designing a single snowblower route that starts and ends at the depot for each workday, so that the large volumes of snow pushed to the sides of the roadways and sidewalks are loaded into trucks by the snowblower for hauling to disposal sites, while satisfying some side constraints such as service hierarchy and special restrictions put into effect during snow loading. For example, the city bans snow loading on both sides of a roadway during the same time interval and on high-class roadways during the AM and PM peak periods. Also, the right side of a high-class one-way street must be serviced before its left side. Finally, the side of a high-class two-way street carrying the heaviest traffic during the AM peak or during the PM peak must be serviced before the other side. The objective is to minimize the total deadhead travel time over the given set of workdays.

The arc routing problem is formulated as a node routing problem with nodes representing sides of roadways to be serviced by the snowblower. Consider a mixed graph $G = (V, E \cup A)$ in which the two link sets E and A are used to represent two-lane, two-way streets (one lane each way) and one-way streets, respectively. The depot is represented by the vertex v_0. Associated with every edge or arc (v_i, v_j) are two lengths or traversal times: s_{ij} is the time of servicing edge or arc (v_i, v_j) and d_{ij} is the traversal time of deadheading edge or arc (v_i, v_j). The graph G is augmented by adding a copy $(v_{i'}, v_{j'})$ of each arc (v_i, v_j) in A with lengths s_{ij} and d_{ij} to model one-way streets requiring service on each side. Similarly, let $G' = (V, A \cup A')$ be a directed graph constructed from the multigraph G by replacing each edge (v_i, v_j) in E with a pair of arcs (v_i, v_j) and (v_j, v_i) of opposite direction and lengths s_{ij} and d_{ij} to model two-lane, two-way streets requiring separate service on each side. The arc routing problem on G' is transformed into an equivalent node routing problem on a complete directed graph $H = (W, B)$ where the vertex set W has a vertex w_{ij} for each arc (v_i, v_j) in $A \cup A'$. The cost t_{jk} of an arc $(w_{ij}, w_{kl}) \in B$ is equal to the traversal time of a shortest path from v_j to v_k in G'. Solving a directed Chinese postman problem on G' then amounts to solving an asymmetric traveling salesman problem on H.

In Montreal, the number of possible workdays to complete snow loading operations measured from the end of the storm depends on total snow accumulation. For example, a sector must be cleared within four days for a snow accumulation of less than 20 cm, within four and a half days for an accumulation of 20 cm to 25 cm, and within five days for 25 cm of snow or more. Let D be the set of possible workdays to complete snow loading operations. Every workday is divided into a given number of periods of work, which is dependent on parking restrictions. For every workday $d \in D$, define $P(d)$ as the set of periods of work associated with workday d. The sets $P'(d)$ and $P''(d)$ contain all periods of work associated with workday d except the first and last period, respectively. Let $P_K = \{P_1(d), P_2(d),\ldots, P_K(d)\}$ be a partition of $P(d)$ with $P_1(d) \cup P_2(d) \cup \ldots \cup P_K(d) = P(d)$ and $P_i(d) \cap P_j(d) = \varnothing$ for all $i, j \in \{1, 2, \ldots, K\}$, $i \neq j$. For every vertex $w_{ij} \in W$ corresponding to an arc (v_i, v_j) of $A \cup A'$, define $D_{ij} \subseteq D$ as the subset of workdays for which the arc (v_i, v_j) can be serviced. For every workday $d \in D$, for every period of work $p \in P(d)$, and for every vertex $w_{ij} \in W$ corresponding to an arc (v_i, v_j) of $A \cup A'$, define the binary constant a_{ij}^{pd} equal to 1 if and only if arc (v_i, v_j) can be serviced during period p of workday d. The subsets of workdays and periods of work for which an arc can be serviced are defined based on the hierarchy of the network and the snow loading regulations mentioned above. For every workday $d \in D$ and for every period of work $p \in P(d)$, define T_p as the time limit of period p. For every workday $d \in D$ and for every period of work $p \in P'(d)$, let $p^{-}(d)$ be the period of work preceding period p of workday

d. For every workday $d \in D$, for every period of work $p \in P(d)$, and for every vertex $w_{ij} \in W$ corresponding to an arc (v_i, v_j) of $A \cup A'$, let y_{ij}^{pd} be a binary variable equal to 1 if and only if vertex w_{ij} is visited during period p of workday d. For every workday $d \in D$, for every period of work $p \in P(d)$, and for every arc $(w_{ij}, w_{kl}) \in B$, let x_{ijkl}^{pd} be a binary variable equal to 1 if and only if arc (w_{ij}, w_{kl}) is traversed during period p of workday d in the optimal solution. If the end location of one period of work of a given workday (except the last period) and the start location of the next period of work of the same workday do not coincide, then the snowblower must use the transportation network to go from one location to another. In order to model the additional deadheading time incurred by the snowblower, GILBERT suggested extending H by adding an artificial vertex w_f representing the transfer point between two consecutives periods of the same workday. Two arcs of opposite direction. are included between a vertex $w_{ij} \in W$ and the transfer vertex w_f. A vertex w_0 representing the depot is also added to H and linked to each other vertex in W with a pair of arcs of opposite direction. Let $H' = (W \cup \{w_0, w_f\}, B \cup B_1 \cup B_2)$ be the extended complete directed graph where $B_1 = \{(w_0, w_{ij}), (w_{ij}, w_0): w_{ij} \in W\}$ and $B_2 = \{(w_f, w_{ij}), (w_{ij}, w_f): w_{ij} \in W\}$. For each workday $d \in D$, the transfer vertex w_f must be visited $|P(d)| - 1$ times along the route, and the depot vertex w_0 must be visited at the beginning and at the end of the first and last period of the workday, respectively. For every workday $d \in D$, for every period of work $p \in P(d) \setminus P'(d)$ ($p \in P(d) \setminus P''(d)$), and for every arc $(w_0, w_{ij}) \in B_1$ ($(w_{ij}, w_0) \in B_1$), let $x_{0,ij}^{pd}$ ($x_{ij,0}^{pd}$) be a binary variable equal to 1 if and only if arc (w_0, w_{ij}) ((w_{ij}, w_0)) is traversed during period p of workday d in the optimal solution. Finally, for every workday $d \in D$, for every period of work $p \in P'(d)$ ($p \in P''(d)$), and for every arc $(w_f, w_{ij}) \in B_2$ ($(w_{ij}, w_f) \in B_2$), let $x_{f,ij}^{pd}$ ($x_{ij,f}^{pd}$) be a binary variable equal to 1 if and only if arc (w_f, w_{ij}) ((w_{ij}, w_f)) is traversed during period p of workday d in the optimal solution. We present here a slightly modified version of the GILBERT nonlinear 0–1 integer programming model for the snowblower routing problem. (We eliminate some variables used by GILBERT to clarify the interpretation of results).

$$\text{Minimize} \quad \sum_{d\in D}\sum_{p\in P(d)}\sum_{w_{ij}\in W}\sum_{w_{kl}\in W} t_{jk}x_{ijkl}^{pd} + \sum_{d\in D}\sum_{p\in P'(d)}\sum_{w_{ij}\in W}\sum_{w_{kl}\in W} t_{jk}x_{ij,f}^{p^{-}d}x_{f,kl}^{pd} \tag{5.1}$$

subject to

$$\sum_{w_{kl}\in W\cup\{w_0\}} x_{ijkl}^{pd} = y_{ij}^{pd} \qquad (d\in D, p\in P(d)\setminus P''(d), w_{ij}\in W) \tag{5.2}$$

$$\sum_{w_{kl}\in W\cup\{w_f\}} x_{ijkl}^{pd} = y_{ij}^{pd} \qquad (d\in D, p\in P''(d), w_{ij}\in W) \tag{5.3}$$

$$\sum_{w_{kl}\in W\cup\{w_0\}} x_{klij}^{pd} = y_{ij}^{pd} \qquad (d\in D, p\in P(d)\setminus P'(d), w_{ij}\in W) \tag{5.4}$$

$$\sum_{w_{kl}\in W\cup\{w_f\}} x_{klij}^{pd} = y_{ij}^{pd} \qquad (d\in D, p\in P'(d), w_{ij}\in W) \tag{5.5}$$

$$\sum_{w_{ij}\in W} x_{0,ij}^{pd} = 1 \qquad (d\in D, p\in P(d)\setminus P'(d)) \tag{5.6}$$

$$\sum_{w_{ij}\in W} x_{ij,0}^{pd} = 1 \qquad (d\in D, p\in P(d)\setminus P''(d)) \tag{5.7}$$

$$\sum_{w_{ij}\in W} x_{f,ij}^{pd} = 1 \qquad (d\in D, p\in P'(d)) \tag{5.8}$$

$$\sum_{w_{ij}\in W} x_{ij,f}^{pd} = 1 \qquad (d\in D, p\in P''(d)) \tag{5.9}$$

$$\sum_{d\in D_{ij}}\sum_{p\in P(d)} y_{ij}^{pd} = 1 \qquad (w_{ij}\in W) \tag{5.10}$$

$$y_{ij}^{pd} \le a_{ij}^{pd} \qquad (d\in D, p\in P(d), w_{ij}\in W) \tag{5.11}$$

$$\sum_{p\in P_k(d)} y_{ij}^{pd} + y_{i'j'}^{pd} \le 1 \qquad (d\in D, P_k(d)\in P_K, (v_i,v_j),(v_{i'},v_{j'})\in A) \tag{5.12}$$

$$\sum_{p\in P_k(d)} y_{ij}^{pd} + y_{ji}^{pd} \le 1 \qquad (d\in D, P_k(d)\in P_K, (v_i,v_j),(v_j,v_i)\in A') \tag{5.13}$$

$$\sum_{w_{ij}\in W} s_{ij}y_{ij}^{pd} + \sum_{w_{ij}\in W}\sum_{w_{kl}\in W} t_{jk}x_{ijkl}^{pd} \le T_p \qquad (d\in D, p\in P(d)\setminus P'(d)) \tag{5.14}$$

$$\sum_{w_{ij}\in W} s_{ij}y_{ij}^{pd} +$$

$$\sum_{w_{ij} \in W} \sum_{w_{kl} \in W} (t_{jk} x_{ijkl}^{pd} + t_{jk} x_{ij,f}^{p^- d} x_{f,kl}^{pd}) \leq T_p \qquad (d \in D, p \in P'(d)) \tag{5.15}$$

$$\text{subtour elimination} \qquad (d \in D, p \in P(d)) \tag{5.16}$$

$$x_{ijkl}^{pd} \in \{0,1\} \qquad (d \in D, p \in P(d), (w_{ij}, w_{kl}) \in B) \tag{5.17}$$

$$x_{0,ij}^{pd} \in \{0,1\} \qquad (d \in D, p \in P(d) \setminus P'(d), w_{ij} \in W) \tag{5.18}$$

$$x_{ij,0}^{pd} \in \{0,1\} \qquad (d \in D, p \in P(d) \setminus P''(d), w_{ij} \in W) \tag{5.19}$$

$$x_{f,ij}^{pd} \in \{0,1\} \qquad (d \in D, p \in P'(d), w_{ij} \in W) \tag{5.20}$$

$$x_{ij,f}^{pd} \in \{0,1\} \qquad (d \in D, p \in P''(d), w_{ij} \in W) \tag{5.21}$$

$$y_{ij}^{pd} \in \{0,1\} \qquad (d \in D, p \in P(d), w_{ij} \in W). \tag{5.22}$$

The nonlinear objective function (5.1) minimizes the total deadheading traversal time. Constraints (5.2)–(5.5) require that the snowblower enters and leaves each vertex associated with a side of a roadway during the same workday and the same period of work. Constraints (5.6) and (5.7) ensure that the snowblower leaves and enters the depot during the first and last period of each workday, respectively. Analogously, constraints (5.8) and (5.9) ensure that exactly one arc leaves and enters the transfer vertex during each period of a given workday except the first and last period, respectively. Constraints (5.10) require that each vertex associated with a side of a roadway be visited during exactly one of the appropriate workdays. Constraints (5.11) state that a vertex associated with a side of a roadway can be visited during a period of work only if it is allowed. Constraints (5.12) and (5.13) ensure that only one side of a one-way and two-way street can be serviced during each subset of periods of work associated with a given workday, respectively. Constraints (5.14) and (5.15) are the time limit restriction for the first period of work and for each period of work except the first period associated with a given workday, respectively, whereas constraints (5.16) are the subtour elimination constraints. Finally, all variables are restricted to be binary.

GILBERT proposed to solve the model using a heuristic approach that first initializes feasible partial routes one at a time by inserting higher priority vertices into the first periods of work, and then balances out the workload of the partial routes by filling the emptiest initial partial routes with lower priority vertices, mainly in the last workdays. The heuristic is described in Figure 5.6.

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1. For every workday $d \in D$ and for every period of work $p \in P(d)$, let C_{pd} be the partial route associated with period p of workday d. The first insertion strategy starts with the first period p of the first workday d.
2. *First insertion strategy*
   Let $W_{pd} \subseteq W$ be the set of higher priority vertices not in a partial route that can be visited during period p of workday d without exceeding the time limit of period p. Select the vertex w_{ij} in W_{pd} for which the insertion into C_{pd} creates the smallest detour. If such a vertex has been selected, insert w_{ij} into C_{pd} and repeat Step 2. If no such vertex exists, then move to the next period and repeat Step 2. If no such vertex exists and if p is the last period of workday d, then move to the first period of the next workday and repeat Step 2. Declare a partial route C_{pd} "closed" if no additional higher priority vertex not in a partial route can be visited during period p of workday d without violating the time limit of period p. All other existing partial routes are "open".
3. *Second insertion strategy*
   Select the open partial route C_{pd} with the lowest load. Let $W_{pd} \subseteq W$ be the set of lower priority vertices not in a partial route that can be visited during period p of workday d without exceeding the time limit of period p. Select the vertex w_{ij} in W_{pd} that induces the smallest detour. If no such vertex exists, then declare route C_{pd} "closed". Otherwise, insert w_{ij} into C_{pd}. Repeat Step 3 until all vertices are serviced or all partial routes are closed.

Figure 5.6: The constructive heuristic for the snowblower routing problem (GILBERT, 1990)

The second insertion strategy operates as a parallel insertion method and is reminiscent of the parallel-insert algorithm proposed by CHAPLEAU *et al.* (1984) for the capacitated arc routing problem. Other orderings of the two insertion strategies can be implemented to generate other procedures. The insertion of a vertex into a given partial route is performed by means of the cheapest insertion procedure (BODIN *et al.*, 1983) for the traveling salesman problem. Given a partial route C_{pd}, the arc (w_{ij}, w_{kl}) in C_{pd} and the vertex w_{mn} not in C_{pd} are first chosen such that $t_{jm} + t_{nk} - t_{jk}$ is minimal, and then w_{mn} is inserted between w_{ij} and w_{kl}. However, this rule can not be used to choose the first vertex to insert into C_{pd} if no vertex is visited during the two periods preceding and succeeding period p. Then, the vertex w_{ij} that maximizes the length s_{ij} is inserted into C_{pd}. In addition to the two complementary insertion strategies, GILBERT also suggested reoptimizing the partial route C_{pd} for which the set of vertices W_{pd} has just become empty. This is achieved by using an adaptation of the CARPANETO and TOTH (1980) algorithm for the asymmetric traveling salesman problem (ATSP). For small instances (about 15 vertices or less), the ATSP was solved exactly. Otherwise, the algorithm was stopped. The heuristic was tested on actual data from one district in the city of Montreal, Canada, involving 470 vertices that represent sides of roadways to be serviced by the snowblower. In order to reduce the size of the problem, these vertices are aggregated into 122 geographic zones that contain a collection of neighboring sides of roadways. The snowblower routing plan was obtained in two minutes and had a smaller distance covered by deadheading trips than the plan used by the city.

5.5 Fleet sizing and replacement models

Relatively little work has been accomplished for determining the sizes and replacement schedules of fleets for winter road maintenance vehicles. Large fleets are required to clear the roadways and sidewalks promptly to allow safer travel, but expenditures increase as fleet size increases. Thus, the tradeoff between minimizing the completion time for winter road maintenance and minimizing the expenditures is key. In vehicle fleet replacement, the important tradeoff is between minimizing maintenance costs for keeping old vehicles and minimizing acquisition costs for new, perhaps more efficient, vehicles. At the strategic and tactical planning levels, replacement schedules specify the sequence of vehicles to replace in the future and the length of time each vehicle in the sequence is to be kept in service. At the operational and real-time levels, replacement schedules specify whether to keep an existing vehicle or to replace it immediately with a new vehicle. Optimization and analytical models for vehicle fleet sizing and replacement in the context of winter road maintenance are reviewed in this section. Models for fleet sizing are discussed first, followed by an optimization model to determine replacement schedules of old vehicles by new vehicles. The characteristics of these models are then summarized in Table 5.3 at the end of the section.

5.5.1 Fleet sizing models

The combined optimization models of KANDULA and WRIGHT (1995, 1997), LABELLE *et al.* (2002), and HAYMAN and HOWARD (1972) reviewed in the first (PERRIER *et al.*, 2006a), second (PERRIER *et al.*, 2006b), and third parts of the survey (PERRIER *et al.*, 2005a), respectively, are an attempt at integrating fleet sizing with other components of winter road maintenance operations such as sector design, depot location, and snow disposal assignment. However, these closely interdependent problems are most often treated separately: strategic and tactical plans are developed first, followed by fleet sizing to determine the number of plows, spreader vehicles, and trucks for the planned sectors and depots. Fleet sizing problems can be grouped into two classes according to the winter road maintenance operation involved: fleet sizing for snow plowing and truck fleet sizing for hauling snow to disposal sites. An optimization model for the snowplow fleet sizing problem is first reviewed, followed by two analytical models for the truck fleet sizing problem.

Fleet sizing models for snow plowing

HAYMAN and HOWARD (1972) treated a snowplow fleet sizing problem in which the number of plows dispatched from depots to clear the roadways is determined so as to minimize operational and depot depreciation costs, while satisfying a specified level of service for each road class. The number of snowplows for high-class roadways must be large enough to limit the average snowfall accumulation to some critical depth, whereas maximum service times are imposed on every other road class. The problem is formulated as a linear integer programming problem. Let I be the set of vehicle depots and let J be the set of roadways to be serviced. For every depot $i \in I$ and for every roadway $j \in J$, let x_{ij} be a nonnegative integer variable representing the number of snowplows based at depot i to clear roadway j, and let b_{ij} and d_{ij} be the unit time cost in traveling from depot i to roadway j, and the distance from depot i to the centroid of roadway j, respectively. For each depot $i \in I$, define a_i as the cost of deployment of the snowplows to depot i including the depot depreciation cost applied to each snowplow based at depot i. For every roadway $j \in J$, let d_j, l_j, p_j, and t_j be the average distance from a depot to roadway j, the length of roadway j, the number of plow passes required to clear roadway j, and the allowable time for plowing roadway j calculated from the beginning of the snowfall, respectively. Let also s, r and d be the average vehicle speed, the rate of snowfall and the critical snow depth, respectively. Finally, if we let $A \subseteq J$ be a set of high-class roadways, then the formulation is as follows.

$$\text{Minimize} \quad \sum_{i\in I}\sum_{j\in J} a_i x_{ij} + \sum_{i\in I}\sum_{j\in J}\left(b_{ij}\frac{d_{ij}}{s}\right)x_{ij} \tag{5.23}$$

subject to

$$\sum_{i\in I} x_{ij} \geq \frac{p_j l_j r}{s \cdot d} \qquad (j \in A) \tag{5.24}$$

$$\frac{\left(\dfrac{p_j l_j}{\sum_{i\in I} x_{ij}}\right)}{s} \leq t_j - \left(\frac{d_j}{s}\right) \qquad (j \in J \setminus A) \tag{5.25}$$

$$x_{ij} \geq 0 \text{ and integer} \qquad (i \in I, j \in J). \tag{5.26}$$

The objective function (5.23) minimizes the sum of all depreciation and operational costs. Constraint set (5.24) imposes a minimum number of plows to clear each high-class roadway. The specification for high-class roadways requires that the snowfall should not be allowed to accumulate beyond the critical depth d. The snow will accumulate to d at time d / r. The distance a plow will travel during this time is sd / r. If a group of plows are to follow one another down a roadway and are spaced according to sd / r, then the maximum snow accumulation between them would be d. This corresponds to the service level desired. Therefore, the required number of plows to clear each high-class roadway is given by the right-hand side of (5.24). BROHM and COHEN (1973) gave a similar expression to calculate the number of vehicles required to plow a roadway segment with a specified level of service and to estimate the number of spreader vehicles required. If we assume that the plowing of a lower-class roadway may be equally divided among the plows dispatched from all depots, then constraint set (5.25) ensures that the total time available to plow each lower-class road is respected. Finally, all variables x_{ij} naturally assume nonnegative integer values. In order to reduce the size of the problem, some x_{ij} variables are discarded if the total time available to plow lower-class road j from vehicle depot i is higher than a specific threshold. Tests were performed on a real problem involving 15 potential depot sites and 41 roadway sections. The LP relaxation of the model (5.23)–(5.26) was solved using the simplex algorithm and the total number of plows required was rounded up to the nearest integer value.

UNGERER (1989) described a simulation model to help planners in determining the number of snowplows to suit regional winter maintenance demands, operating costs and the desired service level. Inputs of the model are the numbers defining the cumulative probability function of the number of snowstorms per year of each type, the number of annual simulation runs, the level of service, the target storm type for which the number of snowplows available is sufficient to satisfy performance demands, the highway lane-kilometers, and the highway classification, as well as various costs associated with purchasing and maintaining snowplows and rentals if not enough equipment is available to meet the desired level of service. Monte Carlo simulation is used to generate distributions of snowplow requirements and associated costs per year for each storm type. Insights for assigning the required number of snowplows annually are provided by varying the input values of the model and analyzing the resulting snowplow number and operational cost distributions and the various tradeoffs.

Fleet sizing models for hauling snow to disposal sites

In determining the fleet size for snow hauling trucks in a sector, one usually wants to balance the fixed and variable costs for the trucks, and the length of time for the snow loading and hauling operations. A detailed analysis of snow loading and hauling operations was performed by the BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975) who proposed two analytical models to determine the size of a homogeneous fleet of trucks assigned to a snowblower. The first model estimates the number of trucks required by a continuously operating snowblower. This number corresponds to one plus the quotient of the round trip travel time between the snowblower and the disposal site, and the time for filling a truck with snow. The round trip travel time for a truck includes the average time required to travel from the snowblower to the disposal site and back to the blower, and the average time to unload a truck at a disposal site. This model tends to underestimate the real truck fleet size required to allow continuous operation of a snowblower since it does not include time for queuing alongside a snowblower or at a disposal site, nor does it include variability in travel time or unloading time.

The second model is based on a two stage cyclic closed queuing system with transit times developed by POSNER and BERNHOLTZ (1967). The model is a finite closed system with two stations in which the time taken to move from one station to the next is assumed to be a random variable with a general distribution. At each station, the service times are independent, exponentially distributed random variables with expected service times that may be arbitrary functions of the number of units at that station. In the snowblower – truck – disposal site cyclic closed system shown in Figure 5.1, the two stations represent the snowblower and the disposal site, and trucks circulate cyclically between them. The two stage cyclic closed queuing model of Posner and Bernholtz (1967) was used to compute the marginal probability P that a snowblower never becomes idle for various numbers of trucks in the system given known loading and unloading times as well as truck speed. This marginal probability then serves to evaluate alternative fleet sizes. Let C_b and C_t be the hourly cost for a snowblower and the hourly cost for a truck, respectively. Let m be the total number of trucks in the snowblower – truck – disposal site system. The total operating cost C of the system is given by (5.27).

$$C = \frac{C_b + C_t m}{P}. \tag{5.27}$$

Setting m in (5.27) to a very small number minimizes the total cost whereas setting m equal to a very large number minimizes the completion time. As m varies between these extremes, candidate compromise fleet sizes can be identified. The accuracy of the model was validated using historic data from one Canadian city. Tests showed that the model was useful in analyzing a variety of scenarios related to the modification of loading and unloading times as well as truck speed and truck capacity.

However, the two-stage closed queuing model with transit times involves certain restrictive assumptions. For a first-in-first-out loading and unloading discipline, loading and unloading times are required to be exponentially distributed. It is also assumed that trucks transit instantaneously from the snowblower to the disposal site or inversely. To relax these assumptions, CHUGHA and POSNER (1980) proposed a two-stage cyclic queue model with general service time distributions of the Erlang class and with time lags between stations. The authors developed a good estimate of the utilization of a station defined as the proportion of time that a station is busy over a long period of time. This estimate could serve in the context of winter road maintenance to study disposal site utilizations by developing operating cost characteristics which could then be used for solving the snow disposal site location problem.

5.5.2 Vehicle fleet replacement models

Very little work has been published concerning fleet replacement for winter road maintenance. The BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975) proposed a basic replacement model to determine a cost minimizing replacement schedule for snow and ice control vehicles, where cost is measured by estimates of the operating, maintenance, and net replacement costs. Equations are derived for these costs. The problem is formulated as a nonlinear programming problem. Let x be a nonnegative variable representing the number of cumulative hours utilized by a vehicle of a homogeneous fleet. Define Q, r and p as the initial purchase cost of the vehicle, the average annual rate of replacement of a vehicle in the fleet, and the average annual number of hours utilized by a vehicle in the fleet, respectively. Define also A, B, C and D as four constants whose values can be determined by regression with a suitable set of data. The formulation is as follows.

$$\text{Minimize} \quad \frac{Ax^B + Q(1+r)^{\frac{x}{p}} - Ce^{\frac{-Dx}{p}}}{x} \tag{5.28}$$

subject to

$$x > 0. \tag{5.29}$$

The objective function (5.28) minimizes the hourly cost for the utilization of the vehicle during x cumulative hours. This is the sum of the operating and maintenance costs for the utilization of a vehicle in the fleet during x cumulative hours and the net replacement cost divided by the number of cumulative hours utilized by the vehicle. The net replacement cost is defined as the difference between the cost of acquiring a new vehicle after x cumulative hours of utilization of the vehicle and the resale value of a vehicle in the fleet after x cumulative hours operated. Computational experiments on historic data from a Canadian city showed that the model was useful in determining replacement schedules for snowplows, sidewalk snowplows and snowblowers. In addition to deciding when to replace individual vehicles, the model can also be used to make replacement scheduling decisions for a homogeneous fleet of vehicles having the same acquisition and operating cost structures. The authors discussed the case where newer vehicles have different acquisition or operating cost structures than older less sophisticated vehicles. Computational results indicated that the model can still be used to find the optimal buy and sell policy for a newer or an older vehicle by calibrating the operating, maintenance, and net replacement costs through regression analysis.

Computerized systems have also been developed to help planners in determining the sizes or replacement schedules of fleets for winter road maintenance vehicles. Such systems were described, for example, by HAMMOND (1978) and NIELSON (1987).

5.6 Conclusions

Winter road maintenance operations involve a host of system design and vehicle routing problems that can be addressed with operations research techniques. This paper is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance. It addresses vehicle routing, fleet sizing, and fleet replacement

Table 5.3: Characteristics of fleet size and replacement models

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
HAYMAN and HOWARD (1972)	Fleet sizing for snow plowing	Tactical	Service hierarchy, maximum snow depth, maximum service times, and fixed vehicle depot location	Min transport and depot amortization costs	Linear IP	Simplex algorithm
UNGERER (1989)	Fleet sizing for snow plowing	Tactical	Service level, service hierarchy, and storm types	Min snowplow fleet size and operating costs	Simulation model	Monte Carlo simulation
TRANSPORT CANADA (1975)	Truck fleet sizing for hauling	Tactical	Homogeneous fleet, fixed snowblower routes, and fixed snow disposal assignments	Min completion time and operating costs	Two stage cyclic closed queuing system	Scenario analysis
TRANSPORT CANADA (1975)	Fleet replacement scheduling	Strategic	Homogeneous fleet	Min operating, maintenance, and net replacement costs	Nonlinear P	Analytical

models for plowing and snow disposal operations. (The two first parts of the survey (PERRIER *et al.*, 2006a,b) discuss system design models for winter road maintenance operations. The third part of the review (PERRIER *et al.*, 2005a) addresses vehicle routing, depot location, and crew assignment models for spreading operations.)

Vehicle routing problems for winter road maintenance are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. Hence, all algorithms developed for the routing of vehicles for winter road maintenance are heuristics. Early models were generally solved with simple constructive methods for undirected and directed versions of the Chinese postman problem, and used simulation models to evaluate benefits. Implementation details and operational constraints were rarely considered. One then witnessed a gradual consideration of more realistic vehicle routing problems and a gradual introduction of local search techniques. While some recent models are solved with composite heuristic methods, which blend route construction and improvement algorithms, others are solved using metaheuristics, which have proven to be very effective for several classes of discrete optimization problems.

However, even though recently proposed models tend to incorporate many of the characteristics of applications arising in practice, they have not yet been widely implemented. A 1995 survey in Minnesota reported that only a single agency out of 414 jurisdictions (counties, cities, townships) used a computerized routing software for snow and ice control (OFFICE OF THE LEGISLATIVE AUDITOR, 1995). This gap between theory and practice may be reduced with the documentation of recent successful routing software packages for winter road maintenance. The CASPER system and the GeoRoute Municipal package are two illustrative examples of this progress. The factors explaining the success of these routing software packages are discussed by CAMPBELL and LANGEVIN (2000).

Although there has been some work on vehicle routing problems for spreading and plowing operations, there is almost no research on the routing of snowblowers and trucks for snow loading and hauling operations. Most models for the routing of trucks for hauling snow to disposal sites are based on a simple cyclic closed system based on a static allocation of trucks. A more sophisticated approach that dynamically redeploys trucks between different snowblowers according to changing needs would be worth exploring. For the snowblower routing problem, composite methods, such as the one embedded in the GeoRoute package, are promising optimization approaches.

There are strong interactions between the various winter road maintenance problems of routing of spreaders, plows, snowblowers, and trucks, locating disposal sites, designing sectors, assigning sectors to disposal sites, assigning crews, and managing vehicle fleets. However, models that take all these aspects of winter road maintenance into consideration get extremely complex if not simply intractable. The traditional approach has thus been to deal separately and sequentially with each problem. Very frequently, disposal sites are first located, sectors are then designed and assigned to disposal sites, and routes are determined last. Since the quality of the routes produced in each sector is highly dependent on the quality of the configuration of the sectors, this approach obviously leads to suboptimal routing decisions. As highlighted by GHIANI and LAPORTE (2001), a better sequential approach could consist of designing the routes first and locating facilities last. Several researchers have employed this second approach for the solution of location-arc routing problems. Another direction worth pursuing involves the use of multiobjective analysis to assist planners in making fleet sizing decisions. In particular, in determining the truck fleet size for hauling snow to disposal sites, the tradeoff between minimizing the fixed and variable costs for the trucks and minimizing the length of time for the snow loading and hauling operations remains largely unexplored.

Finally, technology now available provides new opportunities for the planning of vehicle routing for winter road maintenance. For example, road weather information systems could lead to real-time vehicle routing to treat only those areas in need at a particular time. Truck mounted pavement sensors could help better determine when and how to treat a road and electronic spreader controls on trucks can adjust the amount of materials being spread based on vehicle speed. Automatic vehicle location using positioning system technology could permit real-time reallocation of vehicles and crews, real-time status reports to inform road users of current operations and road conditions (e.g., spread or not, plowed or not, etc.), and to identify unauthorized travel. Many winter maintenance management systems that include one or more of these technologies have been developed. However, to fully achieve the benefits promised by these new technologies, further developments are required to integrate them with optimization techniques for the planning of vehicle routing for winter road maintenance. The greater power and sophistication of road weather information systems, weather forecasting services, winter road maintenance equipment, geographic information systems, global positioning systems, communication systems, computer systems, and optimization techniques can now be merged to address the full scope of vehicle routing for winter road maintenance.

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CHAPITRE 6

THE SECTOR SESIGN AND ASSIGNMENT PROBLEM FOR SNOW DISPOSAL OPERATIONS

Article écrit par Nathalie Perrier, André Langevin et James F. Campbell; soumis pour publication à *European Journal of Operational Research.*

Comme en témoigne la revue de littérature du troisième chapitre, un seul modèle, développé par LABELLE *et al.* (2002), avait précédemment été proposé pour résoudre le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. Le modèle est non linéaire et les unités de base à agglomérer en secteurs sont représentées par de petites zones contenant chacune un ensemble de segments de rues adjacents. De plus, le modèle permet d'affecter un secteur de déneigement à plus d'un site de déversement. Enfin, aucune contrainte ne garantit la contiguïté des secteurs. Dans cet article, nous proposons un premier modèle linéaire mixte pour le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige. Dans ce modèle, les unités de base à agglomérer en secteurs sont représentées par les segments de rues. L'avantage de cette représentation est qu'elle peut permettre de générer des tournées de souffleuses de meilleure qualité que celle utilisant de petits ensembles de segments de rues adjacents comme unités de base. En effet, l'inconvénient majeur de représenter les unités de base comme de petites zones contenant chacune un ensemble de segments de rues adjacents provient du fait que tous les segments de rues appartenant à une zone doivent inévitablement être affectés au même secteur et donc desservis dans la même tournée de souffleuse. Puisque la construction des zones est généralement traitée indépendamment de celle des tournées, l'agglomération des zones en secteurs peut ainsi entraîner un nombre excessif de passages à vide.

Après avoir défini le problème étudié, nous donnons une formulation mathématique du problème qui est basée sur un graphe représentant le réseau de transport. La définition de ce graphe sert en outre à imposer certaines contraintes telles que la contiguïté des secteurs et le fait que chaque secteur de déneigement ne puisse être affecté à plus d'un site de déversement. Le modèle comprend également des contraintes sur la forme des secteurs et sur la charge de travail maximale des secteurs, ainsi que des contraintes de capacités horaire et annuelle des sites de déversement. La formulation suppose que les deux côtés des artères à voies multiples sont affectés au même secteur de déneigement. Nous expliquons par la suite comment généraliser le modèle afin de relâcher cette hypothèse.

Deux méthodes constructives en deux phases sont présentées pour résoudre ce modèle. La première méthode consiste tout d'abord à construire une zone d'influence contiguë pour chaque site de déversement en affectant les segments de rues aux sites de déversement tout en respectant les capacités des sites, de façon à minimiser les coûts variables de transport de la neige et d'élimination de la neige aux sites. Pour cela, un modèle de programmation linéaire mixte est utilisé. Chaque zone d'influence associée à un site de déversement est ensuite divisée en secteurs de déneigement contigus et ayant approximativement la même charge de travail, tout en minimisant les coûts fixes des camions. Le partitionnement d'une zone d'influence en secteurs de déneigement se fait en résolvant un modèle de programmation linéaire mixte. La seconde méthode consiste tout d'abord à diviser le réseau routier en secteurs contigus, compacts, et ayant approximativement la même charge de travail en résolvant un modèle de programmation linéaire mixte. Les secteurs de déneigement sont ensuite affectés aux sites de déversement tout en respectant les capacités des sites ainsi que le fait que chaque secteur ne puisse être affecté à un plus d'un site, de façon à minimiser les coûts variables de transport de la neige et d'élimination de la neige aux sites. Pour cette deuxième phase, un modèle linéaire en variables binaires est utilisé. La première méthode permet de tenir compte de l'interdépendance entre le problème du partitionnement d'un réseau routier et le problème d'affectation des secteurs aux sites. En effet, la grandeur et la forme d'un secteur peuvent dépendre de l'affectation. Toutefois, la seconde méthode traite séparément ces deux composantes.

Les résultats numériques montrent que les deux méthodes peuvent résoudre des problèmes inspirés de données réelles avec tout l'éventail des contraintes en quelques minutes de calcul sur un ordinateur personnel. De plus, les comparaisons avec la seconde méthode indiquent que la première approche permet très souvent de réduire de façon considérable les coûts variables d'élimination de la neige aux sites.

The Sector Design and Assignment Problem for Snow Disposal Operations

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Abstract

Winter road maintenance operations involve a host of decision-making problems at the strategic, tactical, operational, and real-time levels. Those operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. In this paper, we present a model and two heuristic solution approaches based on mathematical optimization for the strategic problem of partitioning a road network into sectors and allocating sectors to snow disposal sites for snow disposal operations. Given a road network and a set of planned disposal sites, the combined problem is to determine a set of non-overlapping subnetworks, called sectors, according to several criteria related to the operational effectiveness and the geographical layout, and to assign each sector to a single snow disposal site so as to respect the capacities of the disposal sites, while minimizing relevant variable and fixed costs. Traditionally, these two closely intertwined problems are treated separately: sector design plans are developed first, followed by snow disposal assignments that specify the disposal site to which the snow in each planned sector is transported to by truck. In this paper, it is shown that a more integrated approach may be more efficient.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Operations research.

6.1 Introduction

In urban areas with substantial snowfalls and prolonged subfreezing temperatures, the large volumes of snow plowed from roads and walkways generally exceed the available space along roads for snow storage, and therefore require disposal by some means. The most common solution is to load snow into trucks for transport to disposal sites. Given the large geographic extent of most snow disposal operations, an agency generally partitions its transportation network into a mutually exhaustive and exclusive collection of small subnetworks, called sectors, according to several criteria related to the operational effectiveness and the geographical layout. All sectors are treated simultaneously by separate crews to facilitate the management of the snow disposal operations. A sector is thus a collection of streets, usually serviced in a single snowblower route. KANDULA and WRIGHT (1995, 1997) and MUYLDERMANS *et al.* (2002, 2003) addressed the problem of partitioning a road network into non-overlapping subnetworks, which they denote "districts", for spreading and plowing operations. However, a district is defined as a bounded, organizational or administrative subnetwork that includes one local depot where a number of vehicles are based. Their definition of districts as independent, geographical areas, each containing several routes, does not correspond to our notion of sectors as small clusters of streets, each serviced in a single route. TOOBAIE and HAGHANI (2004) used our notion of sectors, but their research focuses on spreading operations.

Typically, sector design is performed after the location of the disposal sites has been determined and before the snowblower routes are fixed. The traditional planning approach consists in separating the design of sectors from their assignment to disposal sites: Service level policies first determine the design of sectors, and a snow disposal assignment problem is next solved to assign the planned sectors to the operating disposal sites. In this paper, we propose a model and two heuristic decomposition approaches for designing sectors and assigning them to disposal sites. The first approach solves the combined problem of sector design and snow disposal assignment, while the second approach is a traditional sequential approach. Because of the high degree of interdependence between these decisions (CAMPBELL and LANGEVIN, 1995a), the first approach can result in substantial savings for most agencies.

Sector design and assignment plans define the set of sectors and indicate which disposal site receives the snow from each sector. In a medium-term planning horizon (i.e., a few months), the locations of the disposal sites are fixed and the objective followed in making these decisions is usually to minimize the sum of variable costs for transporting snow from sectors to disposal sites, elimination costs for operating disposal sites, and fixed costs for the trucks. The problem is generally defined over a planning horizon that corresponds to the length of the winter season. However, monthly snow disposal assignment adjustments can be made during the winter season to account for snowfall variability. Hence, the sector design and assignment problem may be viewed as tactical or operational. The model and solution methods introduced next could also be applied to the strategic problem in which disposal site location decisions are taken into account.

Many operational constraints must be considered for the problem of simultaneous design of sectors and assignment of sectors to disposal sites. First, each sector must be contiguous, balanced in workload, and appropriately shaped according to the snow disposal operations. A sector is contiguous if the subgraph induced by its basic units is connected, but not necessarily strongly connected. *Basic units* are the indivisible units of analysis used to build sectors. A basic unit can be defined either as a single street segment or as a small zone that contains a collection of connected street segments. Non-contiguous sectors are undesirable from an administrative standpoint and from an operational standpoint given that deadheading trips would be necessary between the disjoint collections of street segments of each non-contiguous sector. *Deadheading* occurs when a vehicle must traverse a street segment without servicing it. However, if the subgraph induced by the set of basic units assigned to a sector is not Eulerian not even strongly connected, then deadheading trips are necessary within the sector and/or between the sector and its neighbor sectors. Sectors are balanced in workload if they are approximately the same size and are assigned equivalent resources. This helps ensure that operations will be completed at the same time in all sectors assuming equal resources in all sectors. The size of the sectors is usually determined by the level of service required and the operating capabilities of the equipment and manpower. As was highlighted by LABELLE *et al.* (2002), sectors should be elongated in a direction perpendicular to the direction to the disposal site (i.e., sectors that are circular arcs centered on the disposal site) to reduce the number of trucks required. Snowblowers generally operate in a continuous process for loading trucks to minimize the completion time for snow disposal operations. In practice, there may be several empty trucks moving slowly in a queue alongside each snowblower to ensure the snowblowers are never idle. As soon as a truck is filled with snow, it departs for the assigned disposal site while another truck takes its place to begin being filled. The truck that departed for the disposal site will travel to the disposal site, dumps its load of snow, possibly after waiting in line, and then return to the end of the queue alongside the assigned snowblower. This closed cyclic continuous system is illustrated in Figure 6.1.

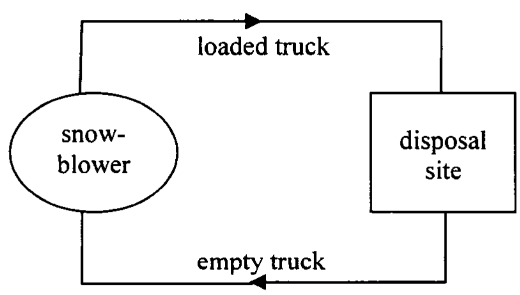


Figure 6.1: The snowblower–truck–disposal site cyclic closed system

For hauling snow to disposal sites, the travel time depends on the location of the truck relative to the assigned disposal site when it departs from, and returns to the snowblower. If the snowblower is far from the disposal site, then a truck must travel a long distance and time to and from the disposal site. Therefore, the number of trucks assigned to the sector must be large enough to ensure that an empty truck will always be available to be filled by the snowblower. However, if the snowblower is near the disposal site, then only a small number of trucks are required to prevent the snowblower to become idle.

Disposal sites have annual capacities based on their physical size and hourly capacities for processing snow based on the operating and unloading practices at the site. Some disposal sites, such as large quarries and river disposal sites, have relatively high hourly capacities due to multiple unloading stations. Other disposal sites, such as sewer chutes, have effectively unlimited annual capacities but may have limited hourly capacities because the unloading capabilities are restricted by the limited size of the openings into the sewer system, and the requirement that the temperature of the water not fall too low.

Finally, since private contractors may be hired for snow loading and hauling, for managerial and contractual reasons all the snow of a given sector may be required to be hauled to a single disposal site. This is called the *single assignment* requirement, as opposed to the *multiple assignment* case where snow from a sector can be hauled to several disposal sites.

The simultaneous design of sectors and assignment of sectors to disposal sites have received very little attention in the operations research literature. LABELLE *et al.* (2002) proposed a solution approach for a nonlinear, integer programming formulation of the problem. The model, which allows multiple assignment, deals with the interactions between sector design, snow disposal assignment, and truck fleet sizing decisions. Basic units are represented by small geographic zones, each containing a collection of neighbouring street segments. The number of snowblowers for loading snow into trucks is given and each sector must contain exactly one snowblower. Thus, the number of sectors corresponds to the number of snowblowers available. The mathematical formulation includes a binary variable associated with each possible assignment of a zone to a sector. These variables provide a solution to the sector design problem. A second set of binary variables is used to represent the assignment of sectors to disposal sites. Additional variables are also introduced to represent the number of snow hauling trucks assigned to a sector. This number is defined so that there should always be a truck available to be filled by the snowblower, while other trucks are traveling to and from the disposal site. This allows snowblowers to operate continuously in order to minimize the time required to clear the streets of snow. The model incorporates constraints on the maximum number of zones per sector to ensure that the specified level of service can be achieved in terms of the maximum time to clear the snow from a sector. There are also nonlinear constraints to enforce hourly and annual disposal site capacities. However, there is no guarantee that the model will produce contiguous sectors. The objective function minimizes the sum of the transportation cost for hauling snow from the sectors to the disposal sites, the variable cost to operate the disposal sites, and the fixed cost for the trucks. This last term, which uses a simple approximation of the number of trucks assigned to a sector, makes the objective function nonlinear. The nonlinearities in the objective function and in the disposal site capacity constraints can be removed by eliminating the fixed cost for the trucks and by replacing the two groups of assignment variables by a single composite variable with three subscripts (LABELLE, 1995). The resulting model is linear and contains fewer constraints than the nonlinear model, but the number of variables increases rapidly as the number of zones, sectors and sites increases.

LABELLE *et al.* proposed to split the global sector design and assignment problem into two components to be solved sequentially. The assignment of zones to disposal sites is first determined to define the "area of influence" for each site, and sectors are then designed for each area of influence by agglomerating neighbouring zones into sectors. The objective for the first component is to minimize relevant operational costs while the second component seeks to minimize the number of trucks for the given zone assignments. The problem of assigning zones to disposal sites is solved using an adaptation of a composite heuristic proposed by CAMPBELL and LANGEVIN (1995b) for the snow disposal assignment problem. In the constructive phase, zones are assigned to disposal sites based on a penalty calculation. Then, interchanges are performed to improve the solution by considering reassignment of every pair of zones to different sites. The second component for aggregating zones into sectors for each area of influence separately is somewhat analogous to the CLARKE and WRIGHT (1964) savings procedure for the capacitated vehicle routing problem. The basic idea is to combine two adjacent zones that satisfy the sector size constraint and whose union results in the greatest decrease in the sum of the maximum distances from the zones to the disposal site. This ideally produces sectors that are circular arcs centered on the disposal site. In practice, it tends to produce sectors elongated in the direction perpendicular to the direction to the disposal site. The solution approach was tested on a real-life instance from the city of Montreal involving 390 zones and 20 disposal sites. Results showed that the approach produced sectors having the desired shape in less than 15 seconds and was useful in analyzing a variety of scenarios related to the modification of transportation and elimination costs as well as disposal site capacities. The solution produced by the heuristic had one disposal site with a single isolated zone assigned to it. Such a situation is addressed by manual adjustments, or by taking into account the fixed costs of the disposal sites in the first component.

The problem treated in the present paper is more complex because we use a single street segment as the unit of analysis to design sectors. This may lead to better snowblower routes than from previous models that use small geographic zones as the unit of analysis. Indeed, the main drawback of representing basic units by small geographic zones emanates from the fact that all neighboring street segments contained in a zone have to be assigned to the same sector and serviced in the same snowblower route. Since the construction of zones is independent of snowblower routing considerations, the aggregation of zones may thus involve excessive deadheading.

As highlighted by LABELLE *et al.* (2002), contiguity constraints are very difficult to write in an efficient or linear form (MACMILLAN and PIERCE, 1994). LABELLE (1995) proposed a set of linear constraints requiring that each zone assigned to a disposal site must be contiguous to at least two other zones assigned to the same disposal site. Thus, the constraint set does allow non contiguous sectors, but each subsector will have at least three zones. In this paper, we proposed to model the contiguity constraints as a circulation multi-commodity network flow problem with supplementary variables and constraints to avoid overlapping sectors and multiple assignment.

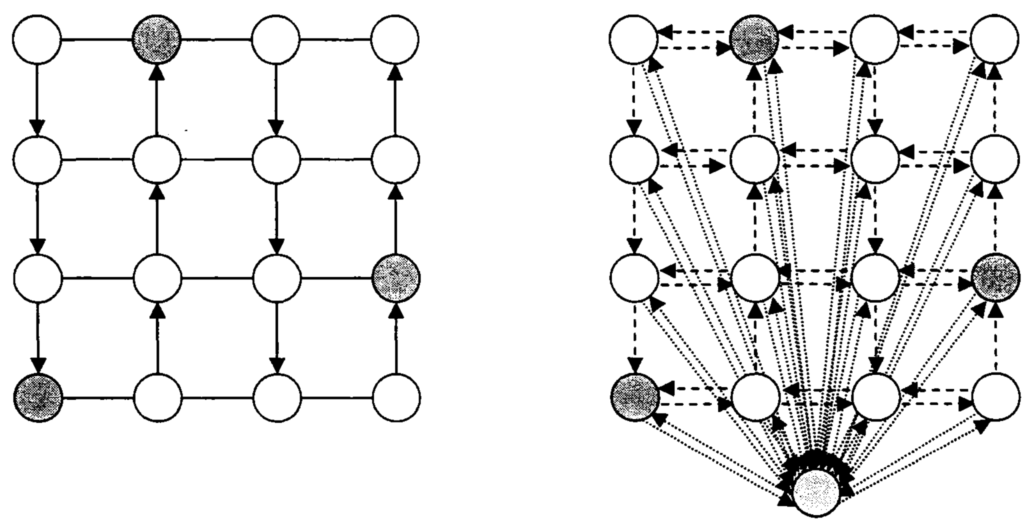
The problem of snow disposal assignment has, however, been the subject of more research compared to the problem of simultaneous design of sectors and assignment of sectors to disposal sites. The BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975) and LECLERC (1985) have studied the case where each disposal site has only an annual capacity and each sector can be assigned to multiple disposal sites. This version of the problem can be modeled as a transportation problem with supply nodes representing disposal sites and demand nodes representing sectors. The single assignment case where each sector is restricted to be assigned to a single disposal site was first studied by LECLERC (1981). The author proposed an interactive heuristic procedure that modifies the optimal solution to the transportation problem by slightly adjusting the annual capacity of the disposal sites. Also, a decision support system for the single assignment case has been developed by LECLERC *et al.* (1981). The system incorporates the stepping stone solution method (DANTZIG, 1951), along with the heuristic capacity adjustment procedure. Later, CAMPBELL and LANGEVIN (1995b) described a heuristic for a model that incorporates single assignment requirements and annual and hourly disposal site capacities. The model is a two-resource generalized assignment problem with tasks representing sectors and agents representing disposal sites. The heuristic is a modification of the two-stage procedure proposed by MARTELLO and TOTH (1981) for the generalized assignment problem. Computational experiments performed on data from the city of Montreal generated significant savings over the solution used by the city. For a recent survey of models and algorithms for winter road maintenance, the reader is referred to the work of PERRIER *et al.* (2005a,b, 2006a,b).

The rest of the paper is organized as follows. In Section 6.2, the notation that is used throughout the text and a mathematical formulation of the problem is presented. Two heuristic decomposition approaches are then described in Section 6.3. To investigate the efficiency of the methods, computational experiments were performed using data from the city of Montreal. The results of these experiments are reported in Section 6.4. Conclusions and paths for future research are presented in the last section.

6.2 Mathematical model

Let $G = (V, A \cup E)$ be a strongly connected mixed graph where $V = \{v_1, v_2, \ldots, v_n\}$ is the vertex set, $A = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the arc set, and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i < j\}$ is the edge set. It is convenient to also denote an arc (edge) by a (e). Vertices v_1, v_2, …, v_n correspond to the street intersections and to the disposal site locations. Each arc and each edge represents a one-way street segment (one-lane or multi-lane) and a two-way street segment (one-lane or multi-lane each way), respectively. For operational reasons, all the snow of a given multi-lane street segment (one-way or two-way) must be loaded in trucks by the same snowblower. Thus, all lanes of a multi-lane road segment must then be assigned to the same sector. Each arc or edge is then used to represent both one-lane and multi-lane street segments. With each arc and edge (v_i, v_j) in $A \cup E$ is associated a length l_{ij}, expressed as kilometers and an annual volume of snow v_{ij} to remove, expressed as cubic meters of snow per year. The annual volume of snow generated on a street segment depends on the snowfall accumulation and on the length and width of streets. This volume can be estimated based on historical data. For example, in Montreal, the annual volume of snow generated on a street segment is estimated based on the historical amount of snow per linear meter of street. Thus, the length and the number of lanes of a street segment determine the annual volume of snow generated by the street segment to be sent to a disposal site.

Let $D \subset V$ be the set of disposal sites. With every disposal site $v_d \in D$ is associated an hourly capacity R_d for receiving snow, an annual capacity V_d for receiving snow, and a variable operating cost CV_d. The hourly receiving rate capacity of a disposal site is usually expressed as cubic meters of snow per hour and depends on the logistics and configuration of unloading facilities at the disposal site. The annual capacity of a site depends on the finite space available for storing snow throughout the winter season. The mathematical formulation of the problem is based on a multi-commodity network flow problem to model the contiguity constraints with supplementary variables and constraints to avoid overlapping sectors and multiple assignment. Each commodity corresponds to a possible sector-site combination. The formulation requires the extended directed graph $G' = (V \cup \{v_0\}, A \cup A_1 \cup A_2 \cup A_3)$ constructed from G where v_0 is an artificial vertex and A_1, A_2 and A_3 are three sets of arcs defined as follows. The arc set A_1 contains arcs of opposite direction and length l_{ij}, hourly removal rate r_{ij}, and annual volume of snow v_{ij} for each edge (v_i, v_j) in E, $A_2 = \{(v_0, v_i): v_i \in V\}$ and $A_3 = \{(v_i, v_0): v_i \in V\}$. For every arc $(v_i, v_j) \in A \cup A_1$ and every disposal site $v_d \in D$, define $d_{ij}^{\,d}$ as the length of a shortest path from vertex v_j to site v_d in G, expressed as kilometers, and $C_{ij}^{\,d}$ as the operational cost per cubic meter for hauling snow from arc (v_i, v_j) to site v_d. Let S be the set of sectors. For every sector $s \in S$ and for every disposal site $v_d \in D$, let $z_s^{\,d}$ be a binary variable equal to 1 if and only if sector s is assigned to disposal site v_d. With every sector $s \in S$ is usually associated an hourly removal rate r_s, expressed as cubic meters of snow per hour. The hourly removal rate in a sector is the rate at which snow is sent out of the sector (in trucks) to a disposal site. This rate depends on the truck fleet size, as well as snowblower types. The number of sectors to construct is given as an input, based on the service level policies (e.g., completion time of snow disposal operations) and the size of the street network. Each sector must contain exactly one snowblower. For every arc $(v_i, v_j) \in A \cup A_1 \cup A_2 \cup A_3$, for every sector $s \in S$, and for every disposal site $v_d \in D$, let y_{ij}^{sd} be a nonnegative real variable representing the flow on arc (v_i, v_j) associated with sector s and disposal site v_d. An example of the construction of graph G' from G is illustrated in Figure 6.2. The arcs of $A \cup A_1$ and $A_2 \cup A_3$ are represented by dashed lines and dotted lines, respectively. The three disposal sites and the artificial vertex v_0 are shown as dark and pale circles, respectively.



(a) (b)

Figure 6.2: Construction of G' from G: (a) graph G, (b) graph G'

For every arc $(v_i, v_j) \in A \cup A_1 \cup A_2$, for every sector $s \in S$, and for every disposal site $v_d \in D$, let x_{ij}^{sd} be a binary variable equal to 1 if and only if arc (v_i, v_j) is assigned to sector s and to disposal site v_d. For every sector $s \in S$, let $DMAX_s$ be a nonnegative real variable representing the maximum distance from a street segment in sector s to its assigned disposal site, and let $N_s = \left\lceil \frac{2DMAX_s}{t_1} \times \frac{r_s}{t_2} \right\rceil$ be the number of snow hauling trucks assigned to sector s, where t_1 and t_2 are the truck speed and the truck size, respectively. The first ratio in N_s is the time taken by a truck to travel from the farthest street segment in sector s to its assigned disposal site and back. The second ratio is the snow removal rate in trucks per hour. The product of these two ratios provides the number of trucks, possibly fractional, that would be filled by a continuously operating snowblower during the longest trip to and from the disposal site. Finally, let CT be the fixed cost per year for trucks. The sector design and assignment problem can now be stated as

$$\text{Minimize} \sum_{v_d \in D}\sum_{s \in S}\sum_{(v_i,v_j) \in A \cup A_1} \min\{C_{ij}^d, C_{ji}^d\} v_{ij} x_{ij}^{sd} + \sum_{v_d \in D} CV_d \sum_{s \in S}\sum_{(v_i,v_j) \in A \cup A_1} v_{ij} x_{ij}^{sd} + CT\sum_{s \in S} N_s \quad (6.1)$$

subject to

$$\sum_{v_d \in D}\sum_{s \in S} x_{ij}^{sd} = 1 \qquad ((v_i,v_j) \in A) \quad (6.2)$$

$$\sum_{v_d \in D}\sum_{s \in S} (x_{ij}^{sd} + x_{ji}^{sd}) = 1 \qquad ((v_i,v_j),(v_j,v_i) \in A_1 : i < j) \quad (6.3)$$

$$\sum_{v_d \in D}\sum_{v_j \in V} x_{0j}^{sd} = 1 \qquad (s \in S) \quad (6.4)$$

$$\sum_{(v_i,v_j) \in A \cup A_1} l_{ij} x_{ij}^{sd} \le U \qquad (v_d \in D, s \in S) \quad (6.5)$$

$$\sum_{s \in S} r_s z_s^d \le R_d \qquad (v_d \in D) \quad (6.6)$$

$$\sum_{s \in S}\sum_{(v_i,v_j) \in A \cup A_1} v_{ij} x_{ij}^{sd} \le V_d \qquad (v_d \in D) \quad (6.7)$$

$$z_s^d \ge x_{ij}^{sd} \qquad ((v_i,v_j) \in A \cup A_1, v_d \in D, s \in S) \quad (6.8)$$

$$\sum_{v_d \in D} z_s^d = 1 \qquad (s \in S) \quad (6.9)$$

$$N_s \ge \frac{2r_s}{t_1 t_2} DMAX_s \qquad (s \in S) \quad (6.10)$$

$$DMAX_s \ge \min\{d_{ij}^d, d_{ji}^d\} x_{ij}^{sd} \qquad ((v_i,v_j) \in A \cup A_1, v_d \in D, s \in S) \quad (6.11)$$

$$\sum_{v_j \in V \cup \{v_0\}} y_{ij}^{sd} = \sum_{v_j \in V \cup \{v_0\}} y_{ji}^{sd} \qquad (v_i \in V \cup \{v_0\}, v_d \in D, s \in S) \qquad (6.12)$$

$$x_{ij}^{sd} \leq y_{ij}^{sd} \leq M x_{ij}^{sd} \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2, v_d \in D, s \in S) \qquad (6.13)$$

$$x_{ij}^{sd} \leq y_{i0}^{sd} \qquad ((v_i, v_j) \in A \cup A_1, v_d \in D, s \in S) \qquad (6.14)$$

$$y_{ij}^{sd} \geq 0 \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2 \cup A_3, v_d \in D, s \in S) \qquad (6.15)$$

$$DMAX_s \geq 0 \qquad (s \in S) \qquad (6.16)$$

$$N_s \geq 0 \text{ and integer} \qquad (s \in S) \qquad (6.17)$$

$$x_{ij}^{sd} \in \{0,1\} \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2, v_d \in D, s \in S) \qquad (6.18)$$

$$z_s^d \in \{0,1\} \qquad (v_d \in D, s \in S). \qquad (6.19)$$

The objective function (6.1) minimizes the sum of three costs: the annual transportation cost for hauling snow from the street segments to the disposal sites, the annual variable cost to operate the disposal sites, and the annual fixed cost for the trucks. Constraints (6.2) and (6.3) assure that each one-way and two-way street segment (one-lane or multi-lane each way) is assigned to exactly one sector and one disposal site, respectively. Constraints (6.4) require that each sector be associated with exactly one disposal site and one vertex of G, called *seed vertex*. For each sector, the seed vertex corresponds to the head of an arc of A_2. Constraints (6.5) limit the size of the sectors to a maximum number of U kilometres, based on the service level policies. Constraints (6.6) and (6.7) limit the assignment of street segments to disposal sites according to the hourly and annual capacities of the disposal sites. Constraints (6.8) link the street segment and sector assignments. They assure that if any street segments are assigned to a sector and a disposal site, then the sector must be assigned to the site. The single assignment constraints (6.9) ensure that each sector is assigned to exactly one disposal site. Constraints (6.10) define the number of snow hauling trucks assigned to each sector. Constraints (6.11) state that the maximum distance from a street segment in a given sector to its assigned disposal site must be greater than or equal to the distance from any street segment in the sector to the disposal site. The nonnegative $DMAX_s$ variables are used to create sectors that are circular arcs centered on the disposal site. Flow conservation at every node for each sector and for each disposal site is imposed by Constraints (6.12). Constraints (6.13) assure that the flow on every arc associated with a sector and a disposal site is positive if and only if the street segment represented by this arc is assigned to that sector and to that disposal site. $M = \min\{|A \cup A_1|, U / sl\}$ where sl is the length of the shortest arc in $A \cup A_1$. The ratio U / sl corresponds to the maximum number of arcs in any given sector. Constraints (6.14) ensure that each sector is composed of a contiguous set of street segments. To see how these constraints operate, observe that for given arc $(v_i, v_j) \in A \cup A_1 \cup A_2$, sector s, and disposal site v_d, y_{ij}^{sd} must take on positive values if and only if $x_{ij}^{sd} = 1$. Thus, any arc $(v_i, v_j) \in A \cup A_1$ assigned to sector s and disposal site v_d must be connected to the seed vertex v_{sd} associated with sector s and disposal site v_d ($v_{sd} \neq v_i$, $v_{sd} \neq v_j$) since

$$x_{ij}^{sd} = 1 \Rightarrow y_{i0}^{sd} \geq 1$$

and

$$\sum_{v_k \in V} x_{0k}^{sd} = 1 \Rightarrow y_{0k}^{sd} \geq 1, v_k = v_{sd} \text{ and } x_{0k}^{sd} = 0, v_k \neq v_{sd} \Rightarrow y_{0i}^{sd} = 0$$

imply that the flow variables associated with sector s and disposal site v_d must define a directed path P from the seed vertex v_{sd} to v_i to satisfy flow conservation at vertex v_i. This in turn implies that all arcs on P must be assigned to sector s and disposal site v_d. Finally, all y_{ij}^{sd} and $DMAX_s$ variables must assume nonnegative values, all N_s variables must assume nonnegative integer values, while x_{ij}^{sd} and z_s^d and variables are restricted to be binary.

In snow disposal operations, it is often desirable that both sides of a two-lane, two-way street segment (one lane each way) be assigned to the same sector. Some agencies allow, however, each side of a multi-lane, two-way street segment (multi-lane each way) to be assigned to different sectors. The formulation can easily be customized to deal with this additional situation by adding to A all pairs of arcs (v_i, v_j), $(v_j, v_i) \in A_1$ representing multi-lane, two-way street segments (multi-lane each way) such that each side can be assigned to a different sector.

The formulation contains a large number of variables and constraints, even for moderate-size instances. The large size of the model is a direct result of the need to consider each street segment as a candidate for assignment to all sectors and disposal sites. When there are very few reasonable alternatives for each street segment assignment based on the spatial nature of the problem, such obvious variables can be fixed at zero or one, or some variables can be eliminated. However, even in that case, the resulting model usually has a large number of alternatives for each street segment assignment. Solving it through a branch-and-bound method with bounds computed using the simplex algorithm may thus require long computation times. We thus consider the use of two heuristic decomposition approaches to solve this problem.

6.3 Two heuristic decomposition approaches

6.3.1 Assign first partition second

The "assign first partition second" heuristic decomposition first constructs the area of influence for each disposal site by allocating street segments to disposal sites while satisfying disposal site capacities and single assignment requirements. Then, in a second phase, the area of influence for each disposal site is partitioned into contiguous and balanced sectors. Assigning street segments to disposal sites in the assign phase allows sectors to be defined in the partition phase based on the known assignments. The objective for the assign phase is to minimize the sum of the transportation cost for hauling snow from the sectors to the disposal sites and the variable cost to operate the disposal sites; the objective for the partition phase is to minimize the number of trucks for the given street segment assignment. In the assign phase, street segments are assigned to disposal sites by solving the following linear, mixed integer programming model.

$$\text{Minimize} \sum_{v_d \in D} \sum_{(v_i,v_j) \in A \cup A_1} \min\{C_{ij}^d, C_{ji}^d\} v_{ij} x_{ij}^d + \sum_{v_d \in D} CV_d \sum_{(v_i,v_j) \in A \cup A_1} v_{ij} x_{ij}^d \qquad (6.20)$$

subject to

$$\sum_{d \in D} x_{ij}^d = 1 \qquad ((v_i,v_j) \in A) \qquad (6.21)$$

$$\sum_{d \in D} (x_{ij}^d + x_{ji}^d) = 1 \qquad ((v_i,v_j),(v_j,v_i) \in A_1 : i < j) \qquad (6.22)$$

$$x_{0j}^d = 1 \qquad (v_d \in D, v_j = v_d) \qquad (6.23)$$

$$\sum_{v_j \in V} x_{0j}^d = 1 \qquad (v_d \in D) \qquad (6.24)$$

$$r_s N_d \le R_d \qquad (v_d \in D) \qquad (6.25)$$

$$\sum_{(v_i,v_j) \in A \cup A_1} v_{ij} x_{ij}^d \le V_d \qquad (v_d \in D) \qquad (6.26)$$

$$\sum_{(v_i,v_j) \in A \cup A_1} l_{ij} x_{ij}^d \le N_d U \qquad (v_d \in D) \qquad (6.27)$$

$$\sum_{v_d \in D} N_d = |S| \qquad (6.28)$$

$$\sum_{v_j \in V \cup \{v_0\}} y_{ij}^d = \sum_{v_j \in V \cup \{v_0\}} y_{ji}^d \qquad (v_i \in V \cup \{v_0\}, v_d \in D) \qquad (6.29)$$

$$x_{ij}^d \le y_{ij}^d \le M x_{ij}^d \qquad ((v_i,v_j) \in A \cup A_1 \cup A_2, v_d \in D) \qquad (6.30)$$

$$x_{ij}^d \le y_{i0}^d \qquad ((v_i,v_j) \in A \cup A_1, v_d \in D) \qquad (6.31)$$

$$x_{ij}^d \in \{0,1\} \qquad ((v_i,v_j) \in A \cup A_1 \cup A_2, v_d \in D) \qquad (6.32)$$

$$y_{ij}^d \geq 0 \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2 \cup A_3, v_d \in D) \qquad (6.33)$$

$$N_d \geq 0 \text{ and integer} \qquad (v_d \in D). \qquad (6.34)$$

In this model, each commodity corresponds to a disposal site. The binary x_{ij}^d variables indicate the assignment of street segments to disposal sites whereas the nonnegative real y_{ij}^d variables and the nonnegative integer N_d variables represent the flow variables and the number of sectors associated with the disposal sites, respectively. The objective function (6.20) minimizes the sum of the annual transportation cost for hauling snow from the street segments to the disposal sites and the annual variable cost to operate the disposal sites. Constraints (6.21) and (6.22) assure that each one-way and two-way street segment is assigned to exactly one disposal site, respectively. Constraints (6.23) and (6.24) require that each disposal site's area of influence be associated with exactly one vertex of G corresponding to the disposal site's location to serve as the seed vertex for the area. Note that the extended directed graph G' illustrated in Figure 6.2 can here be reduced by removing all arcs from the artificial vertex v_0 to the vertices corresponding to street intersections, but not to disposal site locations. Then, $A_2 = \{(v_0, v_i): v_i \in D\}$. Constraints (6.25) and (6.26) limit the assignment of street segments to disposal sites according to the hourly and annual capacities of the disposal sites. Constraints (6.27) impose upper bounds on the size of all areas depending on the number of sectors associated with each disposal site. The total number of sectors to design in the "partition" phase is imposed by Constraint (6.28). Flow conservation at every node for each disposal site is imposed by Constraints (6.29). Constraints (6.30) assure that the flow on every arc associated with a disposal site is positive if and only if the street segment represented by this arc is assigned to that disposal site. The constant M is defined as in Constraints (6.13). Constraints (6.31) guarantee that each area of influence is contiguous. These constraints operate as Constraints (6.14).

Once the area of influence for each disposal site is determined, contiguous and balanced sectors are then designed for each area of influence separately by solving a series of linear, mixed integer programming models. For every disposal site $v_d \in D$, let $G_d = (V_d, A_d \cup E_d)$ be the connected subgraph induced by the set of street segments assigned to disposal site v_d where V_d is the vertex set, A_d is the arc set, and E_d is the edge set. Also, for each disposal site $v_d \in D$, let S_d be the set of sectors associated with disposal site v_d. For each disposal site $v_d \in D$, the cardinality of S_d corresponds to the number N_d of sectors associated with disposal site v_d. For every disposal site $v_d \in D$, the formulation is based on multi-commodity network flows to model the contiguity constraints with supplementary variables and constraints to avoid overlapping sectors. For every disposal site $v_d \in D$, each commodity corresponds to a sector in the set S_d and shares the same directed graph $G_d' = (V_d \cup \{v_0\}, A_d \cup A_1 \cup A_2 \cup A_3)$ constructed from G_d where v_0 is an artificial vertex, $A_1 = \{(v_i, v_j), (v_j, v_i): (v_i, v_j) \in E_d\}$, $A_2 = \{(v_0, v_i): v_i \in V_d\}$, and $A_3 = \{(v_i, v_0): v_i \in V_d\}$. For every disposal site $v_d \in D$, the problem is of the following form:

$$\text{Minimize} \quad CT\sum_{s\in S_d} N_s \tag{6.35}$$

subject to

$$\sum_{s\in S_d} x_{ij}^s = 1 \qquad ((v_i, v_j) \in A_d) \tag{6.36}$$

$$\sum_{s\in S_d} (x_{ij}^s + x_{ji}^s) = 1 \qquad ((v_i, v_j), (v_j, v_i) \in A_1 : i < j) \tag{6.37}$$

$$\sum_{v_j\in V} x_{0j}^s = 1 \qquad (s \in S_d) \tag{6.38}$$

$$\sum_{(v_i, v_j)\in A_d \cup A_1} l_{ij} x_{ij}^s \le U \qquad (s \in S_d) \tag{6.39}$$

$$N_s \ge \frac{2r_s}{t_1 t_2} DMAX_s \qquad (s \in S_d) \tag{6.40}$$

$$DMAX_s \geq \min\{d_{ij}^d, d_{ji}^d\} x_{ij}^s \qquad ((v_i, v_j) \in A \cup A_1, v_d \in D, s \in S_d) \qquad (6.41)$$

$$\sum_{v_j \in V \cup \{v_0\}} y_{ij}^s = \sum_{v_j \in V \cup \{v_0\}} y_{ji}^s \qquad (v_i \in V \cup \{v_0\}, s \in S_d) \qquad (6.42)$$

$$x_{ij}^s \leq y_{ij}^s \leq M x_{ij}^s \qquad ((v_i, v_j) \in A_d \cup A_1 \cup A_2, s \in S_d) \qquad (6.43)$$

$$x_{ij}^s \leq y_{i0}^s \qquad ((v_i, v_j) \in A_d \cup A_1, s \in S_d) \qquad (6.44)$$

$$x_{ij}^s \in \{0,1\} \qquad ((v_i, v_j) \in A_d \cup A_1 \cup A_2, s \in S_d) \qquad (6.45)$$

$$y_{ij}^s \geq 0 \qquad ((v_i, v_j) \in A_d \cup A_1 \cup A_2 \cup A_3, s \in S_d) \qquad (6.46)$$

$$DMAX_s \geq 0 \qquad (s \in S_d) \qquad (6.47)$$

$$N_s \geq 0 \text{ and integer} \qquad (s \in S_d) \qquad (6.48)$$

where the binary x_{ij}^s variables indicate the assignment of street segments to sectors and the nonnegative real y_{ij}^s variables represent the flow variables associated with the sectors. For every area of influence, the objective function (6.35) minimizes the annual fixed cost for the trucks. Constraints (6.36) and (6.37) assure that each one-way and two-way street segment of a given area is assigned to exactly one sector, respectively. For every disposal site $v_d \in D$, Constraints (6.38) require that exactly one vertex of G_d serves as a seed vertex for every sector associated with disposal site v_d. Constraints (6.39) limit the size of the sectors associated with a given area of influence to a maximum number of U kilometres. Constraints (6.40) define the number of snow hauling trucks assigned to each sector. For every disposal site $v_d \in D$, Constraints (6.41) state that the maximum distance from a street segment in a given sector $s \in S_d$ to disposal site v_d must be greater than or equal to the distance from any street segment in s to v_d. For every area of influence, Constraints (6.42) ensure that flow conservation is satisfied for each commodity at all vertices. For every area of influence, Constraints (6.43) assure that the flow on every arc associated with a sector is positive if and only if the street segment represented by this arc is assigned to that sector. $M = \min\{|A_d \cup A_1|, U / sl\}$ where sl is the length of the shortest arc in the area of influence. The ratio U / sl corresponds to the maximum number of arcs in any given sector. Finally, Constraints (6.44) ensure that all sectors of a given area of influence are contiguous. These constraints operate as Constraints (6.14). The assign first partition second heuristic decomposition is summarized next.

1. ("assign" phase) Solve model (6.20)–(6.34) to determine for each disposal site its area of influence.
2. ("partition" phase) For each disposal site, solve model (6.35)–(6.48) to partition its area of influence into sectors.

6.3.2 Partition first assign second

In contrast to the assign first partition second approach, the "partition first assign second" heuristic decomposition defines sectors independently of their assignment to disposal sites. This heuristic first finds a partition of the street segments into contiguous, balanced, and geographically compact sectors with centralized seed vertices, and then assigns the sectors to the disposal sites. We first describe an integer linear programming model for partitioning the street segments into contiguous, balanced, and compact sectors. For every arc $(v_i, v_j) \in A \cup A_1 \cup A_2$ and every sector $s \in S$, let x_{ij}^s be a binary variable equal to 1 if and only if arc (v_i, v_j) is assigned to sector s. For every arc $(v_i, v_j) \in A \cup A_1 \cup A_2 \cup A_3$ and every sector $s \in S$, let y_{ij}^s be a nonnegative real variable representing the flow on arc (v_i, v_j) associated with sector s. The formulation is given next.

$$\text{Minimize} \sum_{s \in S} \sum_{(v_i, v_j) \in A \cup A_1} y_{ij}^s \tag{6.49}$$

subject to

$$\sum_{s \in S} x_{ij}^s = 1 \qquad ((v_i, v_j) \in A) \tag{6.50}$$

$$\sum_{s \in S} (x_{ij}^s + x_{ji}^s) = 1 \qquad ((v_i, v_j), (v_j, v_i) \in A_1 : i < j) \tag{6.51}$$

$$\sum_{v_j \in V} x_{0j}^s = 1 \qquad (s \in S) \tag{6.52}$$

$$\sum_{(v_i, v_j) \in A \cup A_1} l_{ij} x_{ij}^s \leq U \qquad (s \in S) \tag{6.53}$$

$$\sum_{v_j \in V \cup \{v_0\}} y_{ij}^s = \sum_{v_j \in V \cup \{v_0\}} y_{ji}^s \qquad (v_i \in V \cup \{v_0\}, s \in S) \tag{6.54}$$

$$x_{ij}^s \leq y_{ij}^s \leq M x_{ij}^s \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2, s \in S) \tag{6.55}$$

$$x_{ij}^s \leq y_{i0}^s \qquad ((v_i, v_j) \in A \cup A_1, s \in S) \tag{6.56}$$

$$x_{ij}^s \in \{0,1\} \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2, s \in S) \tag{6.57}$$

$$y_{ij}^s \geq 0 \qquad ((v_i, v_j) \in A \cup A_1 \cup A_2 \cup A_3, s \in S). \tag{6.58}$$

The objective function (6.49) minimizes the total flow of all the commodities on each street segment. This ensures that the flow is distributed near the seed vertices, so as to assess the compactness of every sector. Constraints (6.50) and (6.51) assure that each one-way and two-way street segment is assigned to exactly one sector, respectively. Constraints (6.52) require that exactly one vertex of G serves as a seed vertex for every sector. Constraints (6.53) limit the size of the sectors to a maximum number of U kilometres. Flow conservation at every vertex for each sector is imposed by Constraints (6.54). Constraints (6.55) assure that the flow on every arc associated with a sector is positive if and only if the street segment represented by this arc is assigned to that sector. $M = \min\{|A \cup A_1|, U / sl\}$ where U / sl is the length of the shortest arc in the area of influence. The ratio U / sl corresponds to the maximum number of arcs in any given sector. Finally, Constraints (6.56) ensure that all sectors are contiguous. These constraints operate as Constraints (6.14).

The second phase assigns the sectors to the disposal sites through the solution of the snow disposal assignment problem (6.59)–(6.63). For every sector $s \in S$ and every disposal site $v_d \in D$, let x_{sd} be a binary variable equal to 1 if and only if sector s is assigned to disposal site v_d, and let C_s^d represent the operational cost per cubic meter for hauling snow from sector s to site v_d. For every sector $s \in S$, define r_s and v_s as the hourly removal rate in sector s expressed as cubic meters of snow per hour and the annual volume of snow generated in sector s in cubic meters, respectively. The formulation for the snow disposal assignment problem with single assignment and hourly and annual disposal site capacities can be stated as

$$\text{Minimize} \quad \sum_{s \in S} \sum_{v_d \in D} C_s^d v_s x_{sd} + \sum_{v_d \in D} CV_d \sum_{s \in S} v_s x_{sd} \tag{6.59}$$

subject to

$$\sum_{s \in S} v_s x_{sd} \le V_d \qquad (v_d \in D) \tag{6.60}$$

$$\sum_{s \in S} r_s x_{sd} \le R_d \qquad (v_d \in D) \tag{6.61}$$

$$\sum_{v_d \in D} x_{sd} = 1 \qquad (s \in S) \tag{6.62}$$

$$x_{sd} \in \{0,1\} \qquad (s \in S, d \in D)\,. \tag{6.63}$$

The objective function (6.59) minimizes the sum of the transportation cost for hauling snow from the sectors to the disposal sites and the variable cost to operate the disposal sites. Constraints (6.60) and (6.61) limit the assignments of sectors to disposal sites according to the annual and hourly receiving capacity of the disposal sites, respectively. The single assignment constraints (6.62) ensure that each sector is assigned to exactly one disposal site. As highlighted by CAMPBELL and LANGEVIN (1995b), model (6.59)–(6.63) can be viewed as a two-resource generalized assignment problem, a particular case of the multi-resource generalized assignment problem. Whereas the coefficients v_s and r_s are the same for all disposal sites in the formulation (6.59)–(6.63), the amount of a resource used by an agent in performing a task can differ from one agent to another in a two-resource generalized assignment problem. Since the well-known generalized assignment problem is a special case of the two-resource generalized assignment problem, it follows that the two-resource generalized assignment problem is NP-hard. The partition first assign second heuristic decomposition may be summarized as

1. ("partition" phase) Solve model (6.49)–(6.58) to partition the street network into contiguous, balanced, and compact sectors. Let S be the resulting set of sectors.
2. ("assign" phase) Solve model (6.59)–(6.63), taking S as an input, to assign sectors to disposal sites.

6.4 Computational experiments

In order to test the heuristic algorithms described above, computational experiments were performed on undirected graphs corresponding to subnetworks of the simplified street network of the City of Montreal. Simplifying the street network involved that we considered only the arterial, collecting, and local two-way streets. Define $|S|$ and TL as the number of sectors to construct in a subgraph and the total load of the subgraph measured in terms of length of streets, respectively. Let $G_M = (V_M, E_M)$ be the undirected graph corresponding to the simplified street network of the City of Montreal. For every edge (v_i, v_j) to E_M, let c_{ij} be the length associated with edge (v_i, v_j). Every undirected subgraph was generated as follows:

Step 1. Declare all vertices of V_M inactive, except only one, say v_i. Let L be the vertex list used to register the most recent vertices that have been declared active during the generation process. Set $E = \varnothing$ and $L = \{v_i\}$.

Step 2. Select an active vertex v_i in the list L according to the "first in first out" criterion.

Step 3. For every edge $(v_i, v_j) \in E_M$, if v_j is inactive, add (v_i, v_j) to E with cost c_{ij}, declare v_j active, update the vertex list L, and set $TL := TL + c_{ij}$.

Step 4. If $TL \geq (|S| \cdot U) - 1$, stop. $G = (V, E)$ is the resulting subgraph of G_M induced by the set of edges in E. Otherwise, return to Step 2.

This graph generation procedure always produces undirected connected graphs since every newly added edge in E originates from an active vertex. The procedure was repeatedly applied for every sector size $U = 2$, 3, and 4 (expressed in kilometres) and for every $|S| = 2, 3, 4, 5, 6, 7, 8$, giving 21 subgraphs in total. For every subgraph, the number of disposal sites is equal to $\lceil |S| \; / \; 2 \rceil$ and the locations of the disposal sites coincide with randomly selected nodes of the subgraph. The context of this real-life application of the sector design and assignment problem is described in detail by LABELLE *et al.* (2002), and we only briefly recall it here. Table 6.1 presents the hourly and annual rate capacities and the elimination costs for the 17 disposal sites in Montreal. For every subgraph, the disposal sites were randomly selected out of the 17 disposal sites. If one generated problem was infeasible due to the capacities of the disposal sites, we simply rejected it.

The cost of transporting snow in a truck has been approximated by the city of Montreal as

$$C_{ij} = \alpha d_{ij} + \beta \tag{6.64}$$

where C_{ij} is the transport cost per cubic meter of snow from street segment i to site j ($\$/m^3$), d_{ij} is the length of a shortest path from street segment i to site j (km), $\alpha = 0{,}1395$, and $\beta = 0{,}513$. The city of Montreal estimates the hourly removal rate from sectors as

Table 6.1: Disposal site capacities and elimination costs

Site	Type	Elimination cost ($\$/m^3$)	Hourly capacity (m^3/h)	Annual capacity (m^3/yr)
1. Anbar	Sewer chute	0,16	700	∞
2. Beauharnois	Sewer chute	0,38	400	∞
3. Brousseau	Sewer chute	0,25	1000	∞
4. De L'Épée	Sewer chute	0,48	400	∞
5. Dickson Nord	Sewer chute	0,25	600	∞
6. Iberville	Sewer chute	0,29	700	∞
7. Millen	Sewer chute	0,17	2000	∞
8. Poincarré	Sewer chute	0,38	700	∞
9. Sauvé	Sewer chute	0,32	600	∞
10. St-Pierre	Sewer chute	0,22	2800	∞
11. Armand-Chaput	Surface	0,68	6000	1,050,000
12. Contrecoeur	Surface	0,57	4000	700,000
13. M.A. Fortin	Surface	0,62	2000	400,000
14. Montée St-Léonard	Surface	0,34	2000	600,000
15. Parc Newman	Surface	0,43	1000	154,000
16. Royalmount	Surface	0,85	2000	250,000
17. Francon	Quarry	0,34	10,000	3 000,000

either 400 m^3/h or 700 m^3/h and the annual volume of snow generated on a street segment as four times the length of the street segment in metres. In all tests, the hourly removal rate, the truck speed, and the truck size are equal to 400 m^3/h, 15 km/h, and 20 m^3, respectively.

The characteristics of the test problems are summarized in Table 6.2. The column headings show the problem Gri-j-k, where $i = U$, $j = |S|$, and $k = |D|$, the maximum number of kilometres U in a sector, the number of vertices $|V|$ in the subgraph induced by the set of edges in E, the number of edges $|E|$ in the subgraph, the number of sectors $|S|$ to construct in the subgraph, and the set of randomly located disposal sites.

Table 6.2: Characteristics of the test problems

Problem	U	$\|V\|$	$\|E\|$	$\|S\|$	D
Gr2-2-1	2	20	20	2	{3}
Gr2-3-2	2	29	32	3	{8,15}
Gr2-4-2	2	36	42	4	{3, 14}
Gr2-5-3	2	45	52	5	{7, 14, 16}
Gr2-6-3	2	61	68	6	{4, 10, 13}
Gr2-7-4	2	77	87	7	{10, 11, 13, 16}
Gr2-8-4	2	91	104	8	{11, 12, 16, 17}
Gr3-2-1	3	29	32	2	{15}
Gr3-3-2	3	40	46	3	{1, 7}
Gr3-4-2	3	61	68	4	{12, 14}
Gr3-5-3	3	81	92	5	{7, 10, 11}
Gr3-6-3	3	100	116	6	{10, 12, 13}
Gr3-7-4	3	117	142	7	{3, 10, 11, 12}
Gr3-8-4	3	131	159	8	{11,12, 16,17}
Gr4-2-1	4	36	42	2	{3}
Gr4-3-2	4	61	68	3	{16, 17}
Gr4-4-2	4	91	104	4	{2, 7}
Gr4-5-3	4	113	134	5	{1, 5, 7}
Gr4-6-3	4	131	159	6	{2, 4, 10}
Gr4-7-4	4	154	194	7	{2, 4, 8, 10}
Gr4-8-4	4	186	229	8	{11, 12, 13, 17}

For the assign phase of the partition first assign second algorithm, the transportation costs are determined by equation (3.1) but C_{ij} is the transport cost per cubic meter of snow from sector i (rather than street segment i) to site j (\$/m^3). The distance d_{ij} from sector i to site j (km) corresponds to the average length of a shortest path from a street segment in sector i to site j. The annual volume of snow generated in a sector is equal to the sum of the annual volumes of snow generated on the street segments in the sector.

All linear integer models were programmed using OPL studio 3.7 running with CPLEX 9.0 (after tuning the parameters). All computational tests were executed on a Pentium 4 personal computer. A maximum running time of 1800 seconds was imposed for each linear integer programming problem.

For all but the four instances Gr2-2-1, Gr2-3-2, Gr2-4-2, and Gr3-2-1, we were unable to obtain a feasible solution to the subproblem (6.35)–(6.48) within the prescribed time limit. For these instances, we adapted a technique proposed by BENAVENT *et al.* (1990) for partitioning arcs into clusters. The procedure works as follows.

1. Randomly select a seed edge e_1.
2. Determine a set of $|S_d| - 1$ seed edges such that the product of the distances from among the seed edges is maximum. In particular, if seeds $e_1, \ldots, e_k$ have already been selected, seed e_{k+1} is chosen to maximize $\prod_{h=1,\ldots,k} (sp_{eh} + sp_{he})$ over all edges e where sp_{ij} represents the length of a shortest path between edge e_i and edge e_j.
3. For each sector $s \in S_d$, construct the graph G_s containing only edge e_s. Declare all sectors "open".
4. Select the open sector s with largest residual number of kilometres, and determine in G_d the edge (v_i, v_j) incident to any vertex of G_s that maximizes the distance $\min\{d_{ij}^d, d_{ji}^d\}$ between (v_i, v_j) and the disposal site v_d. If no such edge exists, or if its load exceeds the available residual number of kilometres in sector s, then declare sector s "closed". Otherwise, add this edge to G_s, and remove it from G_d. Repeat Step 4 until all sectors are closed.

For each required instance, this procedure was repeated ten times and the solution with the smallest number of trucks was selected. Also, for the partition phase of the partition first assign second algorithm, only the two instances Gr2-2-1 and Gr3-2-1 could be solved (optimally) in 1800 seconds. For all other instances, the partition phase was solved with the following adaptation of the above procedure: Given a sector s with largest residual number of kilometres, determine in G the edge (v_i, v_j) incident to any vertex of G_s that minimizes the sum of the distances between (v_i, v_j) and the seed edge e_s. The time used by the two procedures is negligible. The sectors are created almost immediately or within a few seconds.

For each heuristic, Table 6.3 presents the transportation and elimination cost (TEC) and the total number of snow hauling trucks (NT), with the best values shown in bold characters. The last column compares the transportation and elimination cost of both solution approaches and contains the percentage difference gap $((TEC_{PFAS} - TEC_{AFPS}) / TEC_{AFPS})$ between the cost produced by the partition first assign second and the cost produced by the assign first partition second algorithm. Some gaps are negative because the cost produced by the assign first partition second algorithm is sometimes larger than the cost produced by the partition first assign second algorithm. Only instances Gr2-2-1 and Gr3-2-1 of model (6.1)–(6.19) could be solved exactly by means of the simplex-based branch-and-bound method of CPLEX in eight hours. Optimal values are shown with an asterisk in Table 6.3. A value of "1800.00" in the Seconds column of the assign first partition second algorithm indicates that the assign phase of the corresponding instance could not be optimally solved in 1800 seconds.

These results reveal that the best heuristic is the assign first partition second algorithm in terms of transportation and elimination costs. Up to 20.21% can be saved in transportation and elimination cost by applying the assign first partition second algorithm. However, in terms of number of trucks, both algorithms compete with each other. With respect to the computing times, the partition first assign second algorithm clearly outperforms the other algorithm. The computing times of the assign first partition second algorithm seem however reasonable given that decisions relating to the partitioning of a road network into sectors belong to the strategic level. Thus, the same sectors are usually intended to be utilized over a long time period.

Table 6.3: Computational comparison of the two solution approaches

	Assign first partition second			Partition first assign second			
Problem	TEC_{AFPS}	NT_{AFPS}	Seconds	TEC_{PFAS}	NT_{PFAS}	Seconds	Gap (%)
Gr2-2-1	***10 015.00**	***5**	25.86	***10 015.00**	***5**	30.53	0.00
Gr2-3-2	**21 145.99**	**6**	30.44	21 198.45	7	2.41	0.25
Gr2-4-2	25 363.18	**9**	25.04	**25 326.69**	**9**	2.20	–0.14
Gr2-5-3	**27 371.64**	**14**	6.82	31 032.40	**14**	3.16	13.37
Gr2-6-3	**35 240.58**	14	7.73	42 361.62	**12**	2.41	20.21
Gr2-7-4	**41 896.09**	20	13.76	48 947.78	**18**	2.53	16.83
Gr2-8-4	**55 770.34**	25	14.71	60 460.21	**21**	2.76	8.41
Gr3-2-1	**21 703.00**	***5**	78.85	**21 703.00**	6	2.35	0.00
Gr3-3-2	**24 319.84**	**9**	13.31	**24 319.84**	**9**	2.48	0.00
Gr3-4-2	**41 550.43**	**12**	6.76	**41 550.43**	**12**	2.66	0.00
Gr3-5-3	**43 574.02**	17	334.38	43 869.07	**15**	2.92	0.68
Gr3-6-3	**55 336.34**	**19**	243.41	60 183.07	**19**	2.84	8.76
Gr3-7-4	65 751.28	**22**	278.96	**64 944.49**	23	2.92	–1.23
Gr3-8-4	**88 391.67**	**31**	152.38	91 832.14	**31**	3.36	3.89
Gr4-2-1	**24 222.06**	**6**	4.3	**24 222.06**	**6**	2.31	0.00
Gr4-3-2	**40 540.89**	**9**	6.47	47 923.83	**9**	2.22	18.21
Gr4-4-2	**47 703.95**	**15**	9.48	**47 703.95**	**15**	2.48	0.00
Gr4-5-3	**62 270.22**	23	69.83	**62 270.22**	**22**	2.41	0.00
Gr4-6-3	**76 994.90**	**24**	318.45	**76 994.90**	26	3.59	0.00
Gr4-7-4	**91 609.45**	**29**	1 800.00	95 692.82	**29**	2.73	4.46
Gr4-8-4	**122 214.37**	**39**	1 800.00	128 061.96	40	2.55	4.78

Table 6.4 presents the variable cost for transporting snow from sectors to disposal sites (TC) and the elimination cost for operating disposal sites (EC) for each algorithm, with the best values shown in bold characters.

Table 6.4: Transportation and elimination costs

	Assign first partition second		Partition first assign second		Percentage gap (%)	
Problem	TC_{AFPS}	EC_{AFPS}	TC_{PFAS}	EC_{PFAS}	Gap^1	Gap^2
Gr2-2-1	***6 933.00**	***3 082.00**	***6 933.00**	***3 082.00**	0.00	0.00
Gr2-3-2	**12 130.95**	**9 015.04**	12 154.41	9 044.04	–0.19	0.32
Gr2-4-2	**16 629.50**	8 733.68	16 630.09	**8 696.60**	0.00	–0.42
Gr2-5-3	21 243.48	**6 128.16**	**20 459.76**	10 572.64	3.83	72.53
Gr2-6-3	25 419.78	**9 820.80**	**24 844.98**	17 516.64	2.31	78.36
Gr2-7-4	30 330.25	**11 565.84**	**29 661.30**	19 286.48	2.26	66.75
Gr2-8-4	35 262.90	**20 507.44**	**34 638.69**	25 821.52	1.80	25.91
Gr3-2-1	**12 308.36**	***9 394.64**	**12 308.36**	***9 394.64**	0.00	0.00
Gr3-3-2	**18 872.36**	**5 447.48**	**18 872.36**	**5 447.48**	0.00	0.00
Gr3-4-2	**26 372.83**	**15 177.60**	**26 372.83**	**15 177.60**	0.00	0.00
Gr3-5-3	33 009.30	**10 564.72**	**32 036.35**	11 832.72	3.04	12.00
Gr3-6-3	40 342.02	**14 994.32**	**40 183.75**	19 999.32	0.39	33.38
Gr3-7-4	48 146.00	**17 605.28**	**41 970.93**	22 973.56	14.71	30.49
Gr3-8-4	56 930.79	**31 460.88**	**56 676.14**	35 156.00	0.45	11.75
Gr4-2-1	**16 744.06**	**7 478.00**	**16 744.06**	**7 478.00**	0.00	0.00
Gr4-3-2	25 363.29	**15 177.60**	**25 130.91**	22 792.92	0.92	50.17
Gr4-4-2	**37 450.23**	**10 253.72**	**37 450.23**	**10 253.72**	0.00	0.00
Gr4-5-3	**49 182.94**	**13 087.28**	**49 182.94**	**13 087.28**	0.00	0.00
Gr4-6-3	**56 637.86**	**20 357.04**	**56 637.86**	**20 357.04**	0.00	0.00
Gr4-7-4	67 764.09	**23 845.36**	**67 479.46**	28 213.36	0.42	18.32
Gr4-8-4	**79 978.21**	**42 236.16**	80 291.88	47 770.08	–0.39	13.10
				Average	1.41	19.65

The two last columns indicate the percentage difference $gap^1 = (TC_{AFPS} - TC_{PFAS}) / TC_{PFAS}$ in transportation cost between the assign first partition second and the partition first assign second algorithms and the percentage difference $gap^2 = (EC_{PFAS} - EC_{AFPS}) / EC_{AFPS}$ in elimination cost between the partition first assign second and the assign first partition second algorithms, respectively. Some gaps are negative because the transportation cost (elimination cost) produced by the partition first assign second (assign first partition second) algorithm is sometimes larger than the cost produced by the assign first partition second (partition first assign second) algorithm. For instance Gr3-2-1, the relative difference between the value of the transportation cost produced by the two heuristics and the optimal value of the transportation cost produced by solving model (6.1)–(6.19) is 0.01%.

Results presented in Table 6.4 indicate that, with respect to the transportation costs, there is on average no significant difference between the sectors generated by the two algorithms. However, considering the elimination costs, significant savings are obtained compared with the partition first assign second algorithm. The larger the number of disposal sites $|D|$, the more the assign first partition second algorithm outperforms the partition first assign second algorithm in terms of elimination costs. For $|D| = 4$, the assign first partition second algorithm allows elimination cost savings of up to 66.75% over the sectors produced by the partition first assign second algorithm.

6.5 Conclusions

The aim of this paper was to present a modeling and solution approach for the problem of simultaneous design of sectors and assignment of sectors to disposal sites. The proposed model captures the basic operational constraints of the problem and the solution approach is a two-phase constructive method that defines sectors based on the assignment of street segments to disposal sites. The computational experiments performed show that the heuristic can result in substantial savings compared to the traditional sequential approach that consists in separating the design of sectors from their assignment to disposal sites.

There are strong interactions between the various winter road maintenance problems of locating disposal sites, designing sectors, assigning sectors to disposal sites, and routing of snowblowers and trucks. However, models that take all these aspects of snow disposal operations into consideration get extremely complex if not simply intractable. The traditional approach has thus been to deal separately and sequentially with each problem. Very frequently, disposal sites are first located, sectors are then designed and assigned to disposal sites, and routes are determined last. Since the quality of the routes produced in each sector is highly dependent on the quality of the configuration of the sectors, this approach obviously leads to suboptimal routing decisions. As highlighted by GHIANI and LAPORTE (2001), a better sequential approach could consist of designing the routes first and locating facilities last. Several researchers have employed this second approach for the solution of location-arc routing problems.

Another direction worth pursuing involves the use of Benders decomposition to design exact or heuristic algorithms for the partition phase of the two proposed algorithms. Indeed, the structure of the models makes them well suited for a variable decomposition. In particular, for the partition phase of the partition first assign second algorithm, for any feasible solution to constraints (6.50)–(6.53) and (6.57) that involve only the x_{ij}^s variables, problem (6.49)–(6.58) decomposes into $|S|$ network flow subproblems. Hence, for given values of the x_{ij}^s variables that indicate the assignment of street segments to sectors, the resulting subproblems are relatively easy to solve and involve only the flow variables y_{ij}^s. This observation points out a method, such as Benders decomposition, that would iteratively adjust the values of the x_{ij}^s variables until optimality is reached or a good solution is found. Other issues for future research include the influence of sector shape and basic unit (street segment or small zone) on the routing of snowblowers, alternate aggregation schemes to create sectors, the impacts of changing parameters (e.g., increasing hourly capacities at disposal sites), opening new disposal sites, or closing existing ones.

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CHAPITRE 7

VEHICLE ROUTING FOR URBAN SNOW PLOWING OPERATIONS

Article écrit par Nathalie Perrier, André Langevin et Alberto Amaya; soumis pour publication à *Transportation Science*.

Comme en témoigne la revue du cinquième chapitre, plusieurs modèles avaient précédemment été proposés pour résoudre le problème de routage des véhicules pour les opérations de déblaiement des rues. Dans un article récent (PERRIER *et al.*, 2006), nous avons proposé une méthode constructive en deux phases pour le routage des véhicules pour les opérations de déblaiement des rues dans le contexte d'une application pratique à la ville de Dieppe, Nouveau-Brunswick. La méthode tient compte de nombreuses contraintes opérationnelles telles que les contraintes linéaires de préséance, la nécessité de passages répétés obligatoires ou de service en tandem (deux ou plusieurs véhicules desservant, côte à côte, chacun une voie de circulation) pour les rues à voies multiples, les restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules, les restrictions sur l'utilisation de certains types de virages, les contraintes d'équilibre pour que les durées des tournées soient égales, et la possibilité d'augmenter l'ordre de préséance d'une rue non prioritaire. La méthode suppose que la vitesse de service et de passage à vide est la même pour chaque type de véhicules. L'objectif poursuivi est de minimiser le temps d'achèvement des opérations. Toutefois, pour des raisons de niveau de service, l'objectif principal dans la gestion des opérations reliées à l'entretien hivernal des réseaux routiers consiste, pour plusieurs villes, à minimiser successivement les temps pour desservir chaque classe de priorité au lieu de minimiser le temps d'achèvement des opérations. De plus, pour plusieurs villes, il est important de pouvoir considérer un ordre partiel quant à la séquence pour desservir les classes de priorité ainsi qu'une vitesse de service différente de la vitesse de passage à vide pour chaque type de véhicules.

Cet article présente un modèle plus général pour le routage des véhicules pour les opérations de déblaiement des rues. Le modèle impose les contraintes générales de préséance, des vitesses différentes de service et de passages à vide pour chaque type de véhicules, des passages répétés obligatoires pour les rues à voies multiples, la possibilité d'augmenter l'ordre de préséance des rues non prioritaires, et des restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules. Le modèle est basé sur un graphe représentant le réseau de transport. La définition de ce graphe est similaire à celle utilisée pour imposer en outre les contraintes de contiguïté des secteurs dans le modèle du chapitre 6, mais sert ici à imposer les contraintes de connectivité de chaque tournée de véhicule. La formulation suppose que tous les types de virages peuvent être effectués sans difficulté et que les durées des tournées peuvent varier. Elle suppose également que seuls les passages répétés peuvent être utilisés pour desservir les rues à voies multiples. Nous proposons cependant une approche permettant d'imposer des pénalités pour limiter l'utilisation de certains types de virages. Nous expliquons également comment les contraintes d'équilibre de durée des tournées et la possibilité de desservir en tandem les rues à voies multiples peuvent être traitées en ajoutant des contraintes supplémentaires au modèle.

Afin de résoudre le problème, deux méthodes constructives sont développées. La première méthode construit plusieurs tournées en parallèle en résolvant, successivement pour chaque classe de priorité, un problème du postier rural avec plusieurs véhicules et des contraintes additionnelles, tout en considérant les segments de rues traversés comme étant déjà desservis. Les contraintes additionnelles du problème du postier rural sont les restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules, les restrictions sur l'utilisation de certains types de virages, et les contraintes d'équilibre de durée des tournées. La seconde méthode crée d'abord des groupes de segments de rues, chacun ayant approximativement la même charge de travail, tout en tenant compte des restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules. Un problème du postier chinois hiérarchique avec des contraintes additionnelles est ensuite résolu pour chaque groupe. Les contraintes additionnelles du problème du postier chinois hiérarchique comprennent la possibilité d'augmenter l'ordre de préséance des rues non prioritaires et les les restrictions sur l'utilisation de certains types de virages.

Les tests effectués montrent que les deux méthodes peuvent résoudre un problème réel avec les nombreuses contraintes opérationnelles en quelques heures de calcul sur un ordinateur personnel. De plus, les comparaisons avec les tournées produites manuellement par les employés de la ville de Dieppe indiquent que nos approches permettent très souvent de réduire à la fois le temps d'achèvement et les temps pour desservir chaque classe de priorité.

Vehicle Routing for Urban Snow Plowing Operations

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Abstract

Winter road maintenance planning involves a variety of decision-making problems related to the routing of vehicles for spreading chemicals and abrasives, for plowing roadways and sidewalks, for loading snow into trucks, and for transporting snow to disposal sites. In this paper, we present a model and two heuristic solution approaches based on mathematical optimization for the routing of vehicles for snow plowing operations in urban areas. Given a district and a single depot where a number of vehicles are based, the problem is to determine a set of routes, each performed by a single vehicle that starts and ends at the district's depot, such that all road segments are serviced while satisfying a set of operational constraints and minimizing a time objective. The formulation models general precedence relation constraints with no assumption on class connectivity, different service and deadhead speed possibilities, separate pass requirements for multi-lane road segments, class upgrading possibilities, and vehicle-road segment dependencies. Several extensions, such as turn restrictions, load balancing constraints, and tandem service requirements, which are required in a real-life application, are also discussed. Two objectives are considered: a hierarchical objective and a makespan objective. The resulting model is based on a multi-commodity network flow structure to impose the connectivity of the route performed by each vehicle. The two solution strategies were tested on data from the City of Dieppe in Canada.

Keywords: Winter road maintenance; Snow removal; Arc routing; Chinese postman problem; Operations research.

7.1 Introduction

Snow plowing operations are usually performed in almost all urban regions with frozen precipitation or significant snowfall. Though each storm is unique in duration, intensity, and composition, vehicle routes for plowing operations are generally fixed at the beginning of the winter season. To facilitate the management of the plowing operations, the geographical region (or network) is usually partitioned into non-overlapping subareas (subnetworks), called districts, each including one depot at which a number of vehicles are based. The traditional approach for the design of districts in the context of winter road maintenance consists in partitioning the road network into districts by assigning road segments to their closest depot. KANDULA and WRIGHT (1995, 1997) and MUYLDERMANS *et al.* (2002, 2003) used this approach for designing districts for plowing and spreading operations. A similar approach for designing small clusters of streets in the context of snow disposal operations was developed by LABELLE *et al.* (2002). For a recent survey of optimization models and algorithms for the design of districts for winter road maintenance, the reader is referred to the work of PERRIER *et al.* (2006a,b).

In this paper, we address the problem of vehicle routing within each district borders for snow plowing operations. For each district, the vehicle routing problem consists of determining a set of routes, each served by a single vehicle that starts and ends at the district's depot location, such that all road segments are serviced, all the operational constraints are satisfied, and a time objective is minimized. In addition, the configuration of routes needs to conform to existing district boundaries. Routes crossing these boundaries must be avoided from an administrative standpoint. In rural regions, only a subset of all road segments requires service, whereas most urban areas assume that all road segments of the district network must be serviced. Most naturally, each road segment is usually associated with two traversal times, which are possibly dependent on the vehicle type: the time required to plow the road segment and the time of deadheading the road segment. *Deadheading* occurs when a plow must traverse a road segment without servicing it. In general, a shorter time is associated with deadheading. Traversal times for servicing and deadheading a road segment have already been considered by HAGHANI and QIAO (2001) and BENSON *et al.* (1998).

Different operational constraints can be imposed on the snow plow routes. For example, since agencies have finite resources that generally do not allow the highest level of service on all road segments, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the road segments of a district network are partitioned into classes based on traffic volume and must be serviced while respecting a *hierarchy*, or precedence relation, between classes. Each subgraph induced by a class can be connected or not depending on the topology of the district network and on the level of service policies involved. One type of hierarchy constraint, called *linear precedence relation*, requires a unique ordering relation between classes in a route. This is the case where all roads carrying the heaviest traffic must be serviced first, followed by those that carry medium traffic volume, and so on. Another type of hierarchy constraint, called *general precedence relation*, imposes a weak partial ordering relation between classes in a route. This is the situation where all roads having a large traffic volume must be serviced before those having a low traffic volume in a route, but medium-volume roads can be serviced either before or after some high-volume and low-volume roads. However, some agencies allow *class upgrading*, the possibility of servicing road segments of a class in any of the classes of higher priority, in order to reduce the service completion time of this class and/or the total completion time. Class upgrading is also allowed if traversing unserviced road segments is extremely difficult or impossible. If so, plows must service each road segment the first time they traverse it while disregarding the hierarchy constraint.

Also, each vehicle type can have a restriction on the road segments that it can service and road segments that it can traverse. This constraint for each vehicle type is called *vehicle-road segment dependency*. In plowing operations, the vehicle fleet may consist of a collection of vehicles with varying size, service speed, and shape. Vehicles from the larger vehicle type cannot traverse small alleys. Vehicles from the slower vehicle type cannot service roads having a large traffic volume (for example, rotary plows). Some road segments allow vehicles from a vehicle type to traverse but not service the road segment because the road segment is too narrow to conduct service (for example, displacement plows mounted on the front, side, or beneath their truck carriers).

Finally, since plowing operations are usually limited to one lane at a time, multi-lane road segments necessitate multiple separate passes. This contrasts with materials spreading operations where materials are spread onto the road through a spinner which can be adjusted so that more than one lane of a road segment can be treated in a single pass.

The time objective considered for the routing of vehicles for plowing operations is to minimize the completion time of the first priority class, then the time of the second class, etc. This objective is called the *hierarchical* or *lexicographic* objective, as opposed to the *makespan* objective which minimizes the time at which all vehicles return to the depot, i.e., the shortest time required to service all road segments plus the shortest travel time from the last serviced road segment to the depot. The hierarchical criterion is well suited for snow plowing operations where road segments of higher priority classes must be serviced as soon as possible even if this requires a longer overall time. Moreover, the hierarchical objective is particularly appropriate when class upgrading possibilities are allowed since the vehicle routing problem with makespan objective and class upgrading possibilities is equivalent to the vehicle routing problem with makespan objective and no hierarchy constraint. The hierarchical objective has previously been considered by CABRAL *et al.* (2004) and KORTEWEG and VOLGENANT (2006). PERRIER *et al.* (2006)

studied a vehicle routing problem with makespan objective and class upgrading possibilities, but they impose a tolerance level on the total distance of lower-class road segments that can be serviced prior to higher-class road segments.

In a previous paper (PERRIER *et al.*, 2006), we proposed a two-phase constructive method for the problem of vehicle routing for urban snow plowing operations. The method was developed by focusing on the specific needs of a particular city and incorporates a wide range of constraints and possibilities such as linear precedence relations with no assumption on class connectivity, separate passes or tandem plow patterns for multi-lane road segments (two vehicles plowing at the same time almost side by side), vehicle-road segment dependencies, left turn restrictions, load balancing across routes, and class upgrading possibilities. However, the method supposes that every arc and every vehicle type is associated with a single traversal time no matter if the arc is traversed by the vehicle while servicing or deadheading. The first phase determines a partition of the arcs into clusters, each having approximately the same workload, with an adaptation of the technique proposed by BENAVENT *et al.* (1990) for the capacitated arc routing problem. A directed hierarchical rural postman problem with makespan objective and class upgrading possibilities is then solved heuristically on each cluster using an extension of a procedure introduced by DROR *et al.* (1987) for the HCPP. Test results indicated that the method produced sets of routes that dominate the existing set of routes of the city with respect to either makespan objective, total duration of the routes, total distance travelled, or total duration unbalance occurring between routes. However, to maintain or enhance service levels in many cities, the emphasis should be placed on service completion time (hierarchical objective) as opposed to the time at which the vehicles return to the depot (makespan objective). Moreover, several cities choose to have a general precedence relation between classes in a route and each vehicle type usually has different service and deadhead speeds.

In this paper, we propose a formulation and two solution approaches based on a more general framework that can be adapted to the characteristics of several different cities. The model incorporates the hierarchical objective, general precedence relation constraints with no assumption on class connectivity, different service and deadhead speed possibilities, separate pass requirements for multi-lane road segments, class upgrading possibilities, and vehicle-road segment dependencies. Turn restrictions, load balancing constraints, and tandem service requirements are also enforced. The model is based on a multi-commodity network flow structure to impose the connectivity of the route performed by each vehicle with supplementary variables and constraints to model the hierarchical objective and is optimized with two constructive methods.

The rest of the paper is organized as follows. A brief review of literature is presented in the next section. In Section 7.3, a mathematical formulation of the problem is presented. Section 7.4 describes the two constructive methods. Computational experiments performed using data from the City of Dieppe, New Brunswick, Canada, are reported in Section 7.5 and conclusions are given in the last section.

7.2 Literature review

The vehicle routing problem treated in the present paper can be viewed as a multiple hierarchical Chinese postman problem (m-HCPP) with class upgrading possibilities and vehicle-road segment dependencies. The m-HCPP generalizes the hierarchical Chinese postman problem (HCPP), calling for the determination of a *single* route starting and ending at a depot and servicing *all* road segments of a network in such a way that the service hierarchy is satisfied and a time objective (makespan or hierarchical) is minimized. The HCPP is NP-hard (DROR *et al.*, 1987), but can be solved in polynomial time if the precedence relation is linear and all subgraphs induced by the classes are connected. DROR *et al.* (1987), GHIANI and IMPROTA (2000), and KORTEWEG and

VOLGENANT (2006) have described exact algorithms for this case. The more realistic case, where the subgraph induced by a class is not connected, was first studied by ALFA and LIU (1988). The authors proposed a heuristic that first solves a rural postman problem on each subgraph induced by a class and then forms a giant tour satisfying the linear precedence relations. Later, CABRAL *et al.* (2004) showed that it is possible to solve the HCPP with linear precedence relations and no assumption on class connectivity by transforming it into a rural postman problem. GÉLINAS (1992) described a dynamic programming algorithm for the HCPP with general precedence relations and class connectivity. Since the HCPP with no assumption on class connectivity is a special case of the m-HCPP, it follows that the m-HCPP is NP-hard. Hence, all algorithms developed for the solutions of m-HCPPs are heuristics.

One of the first heuristic algorithms developed for the solution of the vehicle routing problem for snow plowing operations is due to MOSS (1970) who proposed a cluster-first, route-second approach to solve the vehicle routing problem for plowing and spreading operations in Centre County, Pennsylvania. Road segments are first organized into balanced sectors, and a vehicle route is obtained for each of them by solving a directed Chinese postman problem. The cluster phase tries to ensure that the graph generated by the edges of each sector is Eulerian to reduce deadheading in the routing phase.

MARKS and STRICKER (1971) presented two approaches for solving the problem of designing a set of m plow routes such that each road segment is cleared within either two or four passes, depending on its width, while minimizing the distance covered by deadheading trips. All plows are identical and multiple pass requirements are taken into account by duplicating each road segment as many times as the required number of passes on the road segment. The problem is modeled as a m-vehicle undirected Chinese postman problem. In the first approach, the transportation network is partitioned into m subnetworks by solving a districting problem, and a Chinese postman problem is solved for each of them using a decomposition heuristic. In the second approach, a unicursal graph is first derived from the original network, and arbitrarily partitioned into *m* mutually exclusive, collectively exhaustive subgraphs of approximately the same size so that an Eulerian cycle can be defined for each of them without additional duplication of edges. For details, see STRICKER (1970). The authors also suggested three strategies to handle the hierarchy of the network when class connectivity is satisfied. The first strategy tries to allow the highest level of equipment usage on road segments of highest priority by multiplying the length of each road segment by its priority (with 1 being the highest priority) and solving a Chinese postman problem using these weighted lengths so as to favour the duplication of edges associated with road segments of highest priority. The second strategy solves a Chinese postman problem on each connected subgraph induced by the set of edges of a specific priority class and assigns exactly one vehicle to each postman tour. Finally, the last strategy generates several Eulerian cycles while disregarding road priorities, and chooses the cycle which best adheres to the hierarchy of the network.

The BUREAU OF MANAGEMENT CONSULTING, Transport Canada (1975), modeled a similar plow routing problem, with a homogeneous fleet of plows and multiple pass requirements for large road segments, as a *m*-vehicle undirected Chinese postman problem. Again, multiple pass requirements are taken into account by duplicating each road segment the required number of times. The problem is solved using a cluster first, route second heuristic, based on earlier work by STRICKER (1970). The cluster phase breaks the original graph into small subgraphs according to several rules so as to enable routes with less deadheading. The route phase then solves an undirected Chinese postman problem in each subgraph and Fleury's algorithm (KAUFMANN, 1967, p.309) is used for determining an Eulerian cycle in the resulting Eulerian subgraph. The BUREAU OF MANAGEMENT CONSULTINg also proposed to handle the hierarchy of the network and the direction of the traffic flow directly within Fleury's algorithm by selecting, at each iteration, the next edge of highest priority whose removal does not disconnect the Eulerian subgraph, while trying to respect the direction of the traffic flow.

CHERNAK *et al.* (1990) studied the problem of designing routes for two plows to clear the county roads in a district of Wicomico County, Maryland. The objective considered is to minimize the distance covered by deadheading trips, in addition to minimizing the plowing completion time. This problem is solved using a heuristic approach that constructs, for each plow, a primary route servicing roads of highest priority and a second route servicing the other roads.

A three-stage composite heuristic was proposed by KANDULA and WRIGHT (1997) for routing plows and spreaders in the state of Indiana. The heuristic takes into account class continuity and a maximum route duration for each class. *Class continuity* requires that each route services road segments with the same priority classification. In addition, both sides of a road segment must be serviced by the same vehicle. Given an undirected graph, the first phase identifies a set of seed nodes in sufficient number to respect the time limits, and then determines the maximum number of routes that can be constructed out of each seed node by means of an adaptation of the node scanning lower bound procedure introduced by ASSAD *et al.* (1997) for the capacitated Chinese postman problem. The second phase then constructs routes one at a time out of each seed node using a greedy optimality criterion. An improvement procedure that tries to reduce the distance covered by deadheading trips and the number of kilometers violating the class continuity constraints without exceeding the time limits is used last. Comparisons with the tabu search algorithm proposed by WANG and WRIGHT (1994) for a vehicle routing problem for plowing and spreading operations on five networks of Indiana showed that the heuristic obtained the best solutions. However, it should be emphasized that the tabu search algorithm was stopped after a given number of iterations.

Finally, in a series of two papers, SALIM *et al.* (2002a,b) proposed the SRAM (Snow Removal Asset Management) system to solve the vehicle routing problem for plowing and spreading operations in Black Hawk County, Iowa. The SRAM system can deal with service hierarchy and maximum route service times. Although the system relies in large part on decision rules drawn from interviews with experts, it also uses a simple constructive method that builds feasible routes one at a time for each class of roads using a greedy optimality criterion. Related field testing showed that the system reduced the total traversal time (service and deadheading) by 1.9-9.7% (depending on snowfall conditions) over the solution in use by the county.

While several models have been proposed for the *m*-HCPP in the context of snow plowing operations, a recent survey of models and algorithms for vehicle routing and fleet sizing for plowing and snow disposal (PERRIER *et al.*, 2005b) indicates that very few have taken into account class upgrading possibilities and/or vehicle-road segment dependencies. One of the first efforts in this direction belongs to HASLAM and WRIGHT (1991) who developed an interactive route generation procedure for the plow routing problem at the Indiana Department of Transportation (INDOT), U.S. In this problem, routes of minimal total length that start and end at a given depot are sought and class continuity as well as maximum route length constraints must be satisfied. The route generation procedure starts by calculating a lower bound L_r on the number of routes to construct. The user then provides s seed nodes, $s \geq L_r$, with associated classes out of which feasible routes are constructed one at a time using a three-stage algorithm. Given a seed node and its class, the first stage of the algorithm constructs a feasible route made of a path from the seed node to the depot and another path in the reverse direction, without violating class continuity and maximum route length constraints. In the second stage, pairs of non-covered arcs of opposite direction are sequentially inserted into the route as long as class continuity and maximum route length permit. Finally, in the last stage, if arcs have not been covered, then the class continuity constraint is relaxed and the second stage is repeated by permitting class upgrading.

WANG and WRIGHT (1994) described an interactive decision support system, called CASPER (Computer Aided System for Planning Efficient Routes), to assist planners at the Indiana Department of Transportation (INDOT) in the design of vehicle routes for plowing and spreading operations. The sectors are given and each of them contains exactly one depot. The system, which can accommodate service time windows, class continuity, and class upgrading, starts by calculating the number of routes to construct in a given sector for each class of roadways. For every class, the system builds the required number of vehicle routes starting and ending at the depot using a tabu search algorithm. An initial solution is obtained by means of a route growth heuristic described in WANG (1992), which is a refinement of the three-stage algorithm proposed by HASLAM and WRIGHT (1991). The system was tested on data from four northern districts of Indiana (WANG *et al.*, 1995). On average, the system reduced the distance covered by deadheading trips and the number of routes by more than 4% and 7%, respectively, over the routing plan in use by INDOT.

Later, CAMPBELL and LANGEVIN (2000) described the commercially available vehicle routing software GeoRoute developed by the firm GIRO, based in Montreal, Canada, for postal delivery, winter maintenance, meter reading, street cleaning and waste collection applications. The GeoRoute software allows three types of winter road maintenance operations: plowing, spreading and snowblowing (for loading snow into trucks). The software can accommodate service time windows, service frequency, vehicle capacities, spreading rates, turn restrictions, vehicle-road segment dependencies, and both-sides service restrictions (servicing both sides of a road segment in a single route). GeoRoute uses a two-phase method similar to a cluster first, route second method, but constructs instead one route at a time. GeoRoute has been implemented in Ottawa, Canada (MINER, 1996, 1997) for snow plowing and in Suffolk County, United Kingdom (GUTTRIDGE, 2004) for salt spreading. CAMPBELL and LANGEVIN (2000) also report three implementations in the cities of Laval, Charlesbourg, and Nepean in Canada.

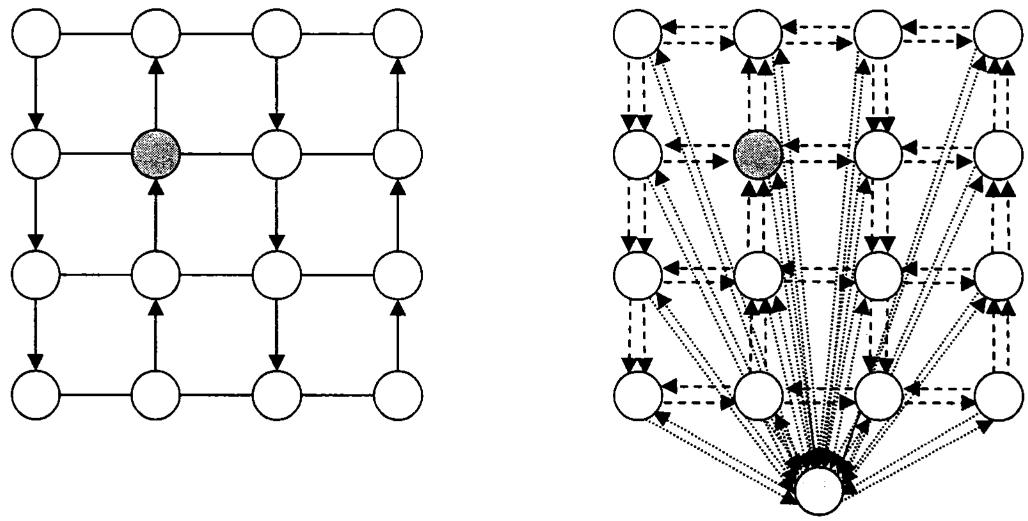
Very recently, CABRAL *et al.* (2004) proposed a decomposition heuristic for the undirected HCPP with linear precedence relations and no assumption on class connectivity and hierarchical objective. The heuristic consists of sequentially solving the HCPP for each class, starting with the highest class, considering all traversed edges in any of the classes of lower priority as already serviced. As highlighted by KORTEWEG and VOLGENANT (2006), declaring a deadheading edge as already serviced can not increase total completion time, but may generate routes with a shorter time. KORTEWEG and VOLGENANT (2006) did not, however, provide a model or an algorithm to handle class upgrading possibilities.

7.3 Mathematical model

Formally, the problem of vehicle routing for urban snow plowing operations is defined on a strongly connected mixed graph $G = (V, A \cup E)$, where $V = \{v_0, v_1,\ldots, v_n\}$ is the vertex set, $A = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the arc set, and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i < j\}$ is the edge set. Vertices $v_1,\ldots, v_n$ correspond to the road intersections, whereas vertex v_0 correspond to the depot at which are based m vehicles. Let $M = \{1,\ldots, m\}$ be the set of vehicles. Arcs and edges are used to represent one-way streets and multi-lane, two-way streets (one lane or more each way), respectively. For every arc and edge $(v_i, v_j) \in A \cup E$, let a_{ij}, e_{ij}, and e_{ji} be the number of circulation lanes associated with arc (v_i, v_j), with edge (v_i, v_j) from v_i to v_j, and with edge (v_i, v_j) from v_j to v_i, respectively. In plowing operations, since each lane must be serviced separately, each arc $(v_i, v_j) \in A$ is replaced by a_{ij} copies and each edge $(v_i, v_j) \in E$ is replaced by e_{ij} arcs from v_i to v_j and by e_{ji} arcs from v_j to v_i. The resulting multigraph $G' = (V, A')$ is then directed. The arc set A' is partitioned into $\{A^1, A^2,\ldots, A^K\}$ with $A^1 \cup A^2 \cup\ldots\cup A^K = A'$ and $A^i \cap A^j = \emptyset$ for $i \neq j$, which induce the service hierarchy, i.e., all arcs of class A^i must be serviced before those of class A^{i+1}. Classes $1,\ldots, K-1$ represent road segments having a given priority whereas class K represents road segments that can be serviced anywhere in the sequence. For every class $p = 1,\dots, K + 1$, let $TMAX_p$ be a nonnegative real variable representing the service completion time of class p. Class $K + 1$ is a fictitious class that allows to include the shortest travel path to the depot from the last serviced arc in class K for each vehicle. The graph G' is a multigraph, i.e., some arcs (v_i, v_j) may be replicated to model multi-lane road sections requiring separate servicing on each lane and road section widths requiring multiple servicing passes in addition to one-way streets requiring separate servicing on each side. Some arcs can be serviced by all types of vehicles, while others are restricted to certain types of vehicles only depending on the vehicle-road segment dependency requirements. For every vehicle $h \in M$, let $A_h \subseteq A'$ be the subset of arcs in G' that can be serviced by vehicle h. With every vehicle $h \in M$ and every arc $(v_i, v_j) \in A_h$ are associated two positive durations s_{ij}^h and d_{ij}^h for the service and deadheading of arc (v_i, v_j) by vehicles h, respectively. For every vehicle $h \in M$, for every arc $(v_i, v_j) \in A_h$, and for every class $p = 1,\dots, K$, let x_{ij}^{ph} be a binary variable equal to 1 if and only if arc (v_i, v_j) is serviced in class p by vehicle h.

The mathematical formulation of the problem is based on a multi-commodity network flow problem to impose the connectivity of the route performed by each vehicle with supplementary variables and constraints. In this model, each commodity corresponds to a possible class-vehicle combination and shares the same directed graph $G'' = (V \cup \{v_a\}, A' \cup A_1 \cup A_2)$ constructed from G' where v_a is an artificial vertex, $A_1 = \{(v_a, v_i): v_i \in V\}$ and $A_2 = \{(v_i, v_a): v_i \in V\}$. An example of the construction of graph G'' from G is illustrated in Figure 7.1. The arcs of A' and $A_1 \cup A_2$ are represented by dashed lines and dotted lines, respectively. The depot v_0 and the artificial vertex v_a are shown as dark and pale circles, respectively.

For every vehicle $h \in M$, for every arc $(v_i, v_j) \in A_h \cup A_1 \cup A_2$, and for every class $p = 1,\dots, K + 1$, let y_{ij}^{ph} be a nonnegative integer variable representing the number of times arc (v_i, v_j) is traversed (while servicing or deadheading) in class p by vehicle h. For every



(a) (b)

Figure 7.1: Construction of *G'* from *G*: (a) graph *G*, (b) graph *G'*

vehicle $h \in M$, for every arc $(v_i, v_j) \in A_h \cup A_1 \cup A_2$, and for every class $p = 1,\ldots, K + 1$, let w_{ij}^{ph} be a nonnegative real variable representing the flow on arc (v_i, v_j) associated with class p and vehicle h. Finally, for every class $p = 1,\ldots, K + 1$ and for every vehicle $h \in M$, let t_p^h be a nonnegative real variable representing the service completion time of class p on route h. We include the t_p^h variables to clarify the formulation and the interpretation of results. The basic model for the problem of vehicle routing for urban snow plowing operations can be stated as follows:

$$\text{Minimize} \quad \sum_{p=1}^{K+1} M_p TMAX_p \tag{7.1}$$

subject to

$$TMAX_p \geq t_p^h \qquad (p = 1,\ldots, K+1, h \in M) \tag{7.2}$$

$$t_p^h = t_{p-1}^h +$$

$$\sum_{(v_i,v_j)\in A_h} (s_{ij}^h x_{ij}^{ph} + d_{ij}^h (y_{ij}^{ph} - 1)) \qquad (p = 1,...K+1, h \in M) \tag{7.3}$$

$$t_0^h = 0 \qquad (h \in M) \tag{7.4}$$

$$\sum_{\substack{h\in M \\ (v_i,v_j)\in A_h}} \sum_{p=1}^{k} x_{ij}^{ph} = 1 \qquad ((v_i,v_j)\in A^k, k = 1,..., K-1) \tag{7.5}$$

$$\sum_{\substack{h\in M \\ (v_i,v_j)\in A_h}} \sum_{p=1}^{K+1} x_{ij}^{ph} = 1 \qquad ((v_i,v_j)\in A^K) \tag{7.6}$$

$$\sum_{(v_i,v_j)\in A_h\cup A_1\cup A_2} y_{ij}^{ph} = \sum_{(v_i,v_j)\in A_h\cup A_1\cup A_2} y_{ji}^{ph} \qquad (v_i \in V \cup \{v_a\}, p = 1,..., K+1, h \in M) \tag{7.7}$$

$$y_{ij}^{ph} \geq x_{ij}^{ph} \qquad ((v_i,v_j)\in A_h, p = 1,..., K, h \in M) \tag{7.8}$$

$$\sum_{(v_i,v_j)\in A'\cup A_1\cup A_2} w_{ij}^{ph} = \sum_{(v_j,v_i)\in A'\cup A_1\cup A_2} w_{ji}^{ph} \qquad (v_i \in V \cup \{v_a\}, p = 1,..., K+1, h \in M) \tag{7.9}$$

$$y_{ij}^{ph} \leq w_{ij}^{ph} \leq |A'| y_{ij}^{ph} \qquad ((v_i,v_j)\in A_h \cup A_1, p = 1,..., K+1, h \in M) \tag{7.10}$$

$$y_{ij}^{ph} \leq w_{ia}^{ph} \qquad ((v_i,v_j)\in A_h, p = 1,..., K+1, h \in M) \tag{7.11}$$

$$\sum_{v_i\in V} y_{ai}^{ph} = 1 \qquad (p = 1,..., K+1, h \in M) \tag{7.12}$$

$$\sum_{v_i\in V} y_{ia}^{ph} = 1 \qquad (p = 1,..., K+1, h \in M) \tag{7.13}$$

$$y_{ia}^{ph} = y_{ai}^{p+1,h} \qquad (v_i \in V, p = 1,..., K, h \in M) \tag{7.14}$$

$$y_{a0}^{1h} = 1 \qquad (h \in M) \tag{7.15}$$

$$y_{0a}^{K+1,h} = 1 \qquad (h \in M) \tag{7.16}$$

$$x_{ij}^{ph} \in \{0,1\} \qquad ((v_i,v_j)\in A_h, p = 1,..., K, h \in M) \tag{7.17}$$

$$y_{ij}^{ph} \geq 0 \text{ and integer} \qquad ((v_i,v_j)\in A_h \cup A_1 \cup A_2, p = 1,..., K+1, h \in M) \tag{7.18}$$

$$w_{ij}^{ph} \geq 0 \qquad ((v_i,v_j)\in A_h \cup A_1 \cup A_2, p = 1,..., K+1, h \in M) \tag{7.19}$$

$$TMAX_p \geq 0 \qquad (p = 1,..., K+1) \tag{7.20}$$

$$t_p^h \geq 0 \qquad (p = 0,\dots,K+1, h \in M) \tag{7.21}$$

where $M_1 >> M_2 >> \dots >> M_{K+1} = 1$. The objective function (7.1) minimizes the service completion time of the first priority class, then the completion time of the second class, and so on. As highlighted by CABRAL *et al.* (2004), the notation ">>" means that in any feasible solution, the relation

$$M_p TMAX_p > \sum_{k=p+1}^{K+1} M_k TMAX_k$$

must be satisfied for $p = 1,\dots, K$. Constraints (7.2) state that the maximum service completion time of a given class must be greater than or equal to the service completion time of this class on any route. Constraints (7.3) and (7.4) define the service completion time of each class on each route and the start time of each route, respectively. Constraints (7.5) and (7.6) assure that each arc of a given priority class is serviced either in this class or in any of the classes of higher priority by exactly one eligible vehicle and that all other arcs are serviced by exactly one eligible vehicle, respectively. A vehicle is *eligible* for a certain arc if it can service or deadhead this arc. Constraints (7.7) ensure route continuity for each possible class-vehicle combination. Constraints (7.8) state that an arc is serviced by an eligible vehicle in a given class only if it is traversed by the same vehicle in the same class. Flow conservation at every node for each class and for each vehicle is imposed by Constraints (7.9). Constraints (7.10) assure that the flow on every arc associated with a class and an eligible vehicle is positive if and only if this arc is traversed (while servicing or deadheading) in that class by that vehicle. Constraints (7.11) ensure that each partial route associated with a class and a vehicle does not contain any disconnected subtours. Constraints (7.12) and (7.13) require that each class-vehicle combination be associated with exactly two vertices of G: one start location at which the route must start service, called *start vertex*, and one end location at which the route must end service, called *end vertex*, respectively. Constraints (7.14) assure that the end vertex associated with a class and a vehicle corresponds to the start vertex associated with the next class and the same vehicle. Constraints (7.15) and (7.16) require that each route starts and ends at the depot location, respectively. A schematic representation of a feasible route h in G'' is illustrated in Figure 7.2. Recall that the partial route associated with class $K + 1$ corresponds to the shortest path from the last serviced arc by vehicle h in class K to the depot v_0.

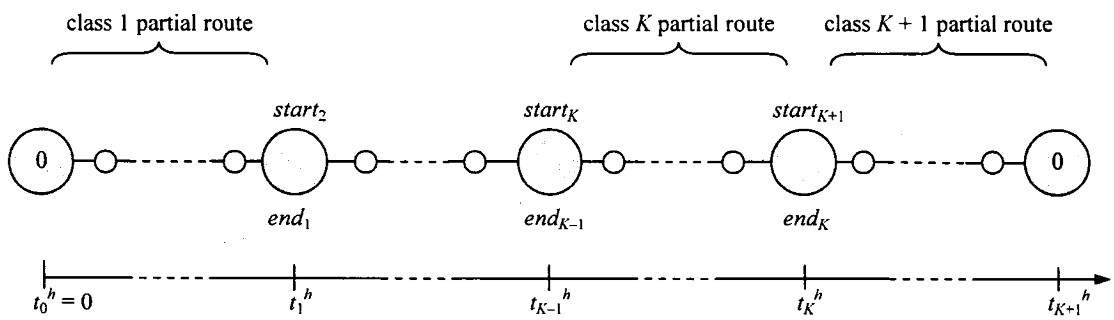


Figure 7.2: Schematic representation of a feasible route h in G''

Finally, all y_{ij}^{ph} variables must assume nonnegative integer values and all w_{ij}^{ph}, t_p^h, and $TMAX_p$ variables must assume nonnegative values, while x_{ij}^{ph} variables are restricted to be binary.

Proposition 1. *A feasible solution to the model (7.1)–(7.21) does not contain any disconnected subtours.*

Proof. We first show that the partial route associated with given vehicle h and class p does not contain any disconnected subtours. First, observe that for a given arc $(v_i, v_j) \in A_h \cup A_1$ serviced by vehicle h in class p, w_{ij}^{ph} must take on positive values if and only if $y_{ij}^{ph} = 1$. Thus, any arc $(v_i, v_j) \in A_h$ serviced by vehicle h in class p must be connected to the start vertex v_{start}^{ph} associated with vehicle h and class p ($v_{start}^{ph} \neq v_i$, $v_{start}^{ph} \neq v_j$) since

$$y_{ij}^{ph} = 1 \Rightarrow w_{i0}^{ph} \geq 1$$

and

$$\sum_{v_k \in V} y_{ak}^{ph} = 1 \Rightarrow w_{ak}^{ph} \geq 1, v_k = v_{start}^{ph} \text{ and } y_{ak}^{ph} = 0, v_k \neq v_{start}^{ph} \Rightarrow w_{ai}^{ph} = 0$$

imply that the flow variables associated with class p and vehicle h must define a directed path P from the start vertex v_{start}^{ph} to v_i to satisfy flow conservation at vertex v_i. This in turn implies that all arcs on P must be serviced or deadheaded by vehicle h in class p. Moreover, the partial route associated with vehicle h and class p must be connected to the depot v_0 since the partial route associated with vehicle h and every class $p = 1,\ldots, K+1$ does not contain any disconnected subtours and $v_{start}^{1h} = v_0$, $v_{end}^{1h} = v_{start}^{2h},\ldots, v_{end}^{p-1,h} = v_{start}^{ph}$, $v_{end}^{ph} = v_{start}^{p+1,h},\ldots, v_{end}^{K+1,h} = v_0$. Thus, the route h does not contain any disconnected subtours. □

The multi-commodity network flow structure can also be used to model the contiguity constraints in a linear form for the design of sectors for snow disposal operations. Contiguity constraints require that sectors do not include distinct parts separated by other sectors. Non-contiguous sectors are undesirable from both administrative and operational standpoints given that deadheading trips would be necessary between the disjoint collections of road segments of each non-contiguous sector. For details, the reader is referred to the recent work of PERRIER *et al.* (2006c).

The model (7.1)–(7.21) can be customized to deal with many additional situations. First, the model assumes that all types of turns made at intersections are allowed. However, in urban areas, vehicle routes must observe traffic rules such as the prohibition of making certain turns, mostly left turns and U-turns. More generally, even if they are not forbidden, the impact of undesirable turns, such as U-turns and turns across traffic lanes, is usually greater in routing snow plows as compared to spreading operations. Since most plows are designed to always cast the snow to the right side of the roadways, a left turn or a street crossing at an intersection results in a trail of snow in the middle of the intersection. Thus, the general guideline for constructing routes for snow plowing is that each plow should remain on the right side of a roadway using a block pattern by accomplishing a series of right turns to avoid compromising safety. To deal with these situations, a penalty can be imposed to each turn depending on its type (e.g., left, right, U-turn, and go straight). In snow plowing operations, right turns would be given the lowest penalty to provide safe roads. For each pair of arcs (v_i, v_j), $(v_j, v_k) \in A'$, denote $[(v_i, v_j), (v_j, v_k)]$ as the turn made going from arc (v_i, v_j) to arc (v_j, v_k) in G'. Let T denote the set of allowed turns in G'. For each turn $[(v_i, v_j), (v_j, v_k)] \in T$, for each class $p = 1,\ldots, K + 1$, and for each vehicle $h \in M$, let $n_{[ijk]}^{ph}$ be a nonnegative integer variable representing the number of times turn $[(v_i, v_j), (v_j, v_k)]$ is executed in class p by vehicle h. Then, the constraints

$$\sum_{(v_j,v_k)\in A'} n_{[ijk]}^{ph} = y_{ij}^{ph} \qquad ((v_i,v_j)\in A', p=1,\ldots,K, h\in M) \qquad (7.22)$$

$$\sum_{(v_k,v_i)\in A'} n_{[kij]}^{ph} = y_{ij}^{ph} \qquad ((v_i,v_j)\in A', p=1,\ldots,K, h\in M) \qquad (7.23)$$

must be added to Model (7.1)–(7.21) to impose turn penalties. Constraints (7.22) ensure that the number of times a turn beginning with arc (v_i, v_j) is executed by a given vehicle in a given class corresponds to the number of times this arc is traversed by the same vehicle in that class. Constraints (7.23) serve the same purpose for turns that terminate with arc (v_i, v_j). Turn penalties are imposed by adding the term $\sum_{h\in M}\sum_{p=1}^{K}\sum_{[(v_i,v_j),(v_j,v_k)]\in T} p_{[ijk]} n_{[ijk]}^{ph}$ to the objective function (7.1), where $p_{[ijk]}$ is the penalty associated with turn $[(v_i, v_j), (v_j, v_k)]$. Examples of plow routing applications where turn penalties are explicitly taken into account are provided in LEMIEUX and CAMPAGNA (1984), ROBINSON *et al.* (1990), GENDREAU *et al.* (1997), and CAMPBELL and LANGEVIN (2000).

Next, load balancing constraints can be introduced easily. Balancing the workload across routes means creating routes of approximately the same duration. If the minimum and maximum route durations are denoted by l and u, respectively, then the following constraints can be added to the original formulation:

$$l \leq \sum_{(v_i,v_j)\in A_h} \sum_{p=1}^{K+1} (s_{ij}^h x_{ij}^{ph} + d_{ij}^h (y_{ij}^{ph} - 1)) \leq u \qquad (h \in M). \tag{7.24}$$

Finally, the last situation concerns the inclusion of tandem service constraints. Since plowing operations are limited to one lane at a time, many agencies have developed tandem plow patterns in echelon formations for multi-lane road segments. Generally, these road segments must be serviced in any of the classes of higher priority. Let A^1 be the set of arcs that require tandem service. Given a set of pairs of vehicles to operate in parallel, the most popular approach is thus to assign each pair of vehicles to a single class 1 partial route starting at the depot and covering multi-lane road segments requiring tandem service. Let $A_{\text{tandem}} = \{(i_r, i_s), (i_t, i_u) \in A^1: (i_r, i_s)$ and (i_t, i_u) necessitate tandem plow patterns$\}$ and let M_{tandem} be the set of pairwise compatible vehicles to operate in parallel such that each vehicle belongs to at most one pair. Then, adding the constraints

$$x_{rs}^{1h_1} = x_{tu}^{1h_2} \qquad ((i_r, j_s) \in A_{\text{tandem}} \cap A_{h_1}, (i_t, j_u) \in A_{\text{tandem}} \cap A_{h_2}, (h_1, h_2) \in M_{\text{tandem}}) \tag{7.25}$$

ensures that pairs of arcs that necessitate tandem plow patterns are assigned to pairwise compatible vehicles in M_{tandem}.

7.4 Solution approaches

Even for small instances of the problem, model (7.1)–(7.25) contains a very large number of variables and constraints. We propose two constructive methods to solve this model. The first method, called *parallel algorithm*, constructs several routes in parallel by sequentially solving a multiple vehicle rural postman problem (m-RPP) with vehicle-road segment dependencies, turn restrictions, and load balancing constraints for each class $p = 1, ..., K$, considering all traversed arcs as already serviced. Recall that in the RPP, the arc set is partitioned into *required* and *non-required* arcs. The m-RPP consists of designing a set of m vehicle routes of least total cost, such that each route starts and ends at the depot and each required arc appears in at least one route and is serviced by exactly one vehicle. The second approach, called *cluster first, route second algorithm*, first determines a partition of the arcs into clusters, each having approximately the same workload, while taking into account vehicle-road segment dependencies. A hierarchical rural postman problem (HRPP) with class upgrading possibilities and turn restrictions is then solved on each cluster. We first present the parallel route constructive approach and then describe the cluster first, route second method.

7.4.1 Parallel algorithm

The parallel algorithm is based on a decomposition of the model into a set of K different subproblems. For every class $p = 1,..., K$, the subproblem determines the $|M|$ best partial class p routes for the given start times when the partial class p routes must start service and for the given start locations at which the partial class p routes must start service through the solution of a m-RPP with vehicle-road segment dependencies, turn restrictions, and load balancing constraints, considering all arcs of class p traversed in any of the classes of higher priority as non-required arcs to allow class upgrading. For every vehicle $h \in M$, the time required to return to the depot, i.e., t_K^h plus the shortest travel time to the depot from the last serviced arc in class K by vehicle h, is determined last. For every class $p = 1, ..., K$, the m-RPP is of the following form:

$$\text{Minimize} \quad TMAX_p + \sum_{h\in M} \sum_{[(v_i,v_j),(v_j,v_k)]\in T} p_{[ijk]} n_{[ijk]}^{ph} \tag{7.26}$$

subject to

$$TMAX_p \geq t_p^h \qquad (h \in M) \tag{7.27}$$

$$t_p^h = s_p^h + \sum_{(v_i,v_j)\in A_h} (s_{ij}^h x_{ij}^{ph} + d_{ij}^h (y_{ij}^{ph} - 1)) \qquad (h \in M) \tag{7.28}$$

$$\sum_{\substack{h\in M \\ (v_i,v_j)\in A_h}} x_{ij}^{ph} = 1 \qquad ((v_i,v_j) \in A^p) \tag{7.29}$$

$$\sum_{(v_i,v_j)\in A_h \cup A_1 \cup A_2} y_{ij}^{ph} = \sum_{(v_j,v_i)\in A_h \cup A_1 \cup A_2} y_{ji}^{ph} \qquad (v_i \in V \cup \{v_a\}, h \in M) \tag{7.30}$$

$$y_{ij}^{ph} \geq x_{ij}^{ph} \qquad ((v_i,v_j) \in A_h, h \in M) \tag{7.31}$$

$$\sum_{(v_i,v_j)\in A' \cup A_1 \cup A_2} w_{ij}^{ph} = \sum_{(v_j,v_i)\in A' \cup A_1 \cup A_2} w_{ji}^{ph} \qquad (v_i \in V \cup \{v_a\}, h \in M) \tag{7.32}$$

$$y_{ij}^{ph} \leq w_{ij}^{ph} \leq |A'| y_{ij}^{ph} \qquad ((v_i,v_j) \in A_h \cup A_1, h \in M) \tag{7.33}$$

$$y_{ij}^{ph} \leq w_{ia}^{ph} \qquad ((v_i,v_j) \in A_h, h \in M) \tag{7.34}$$

$$\sum_{v_i \in V} y_{ai}^{ph} = 1 \qquad (h \in M) \tag{7.35}$$

$$\sum_{v_i \in V} y_{ia}^{ph} = 1 \qquad (h \in M) \tag{7.36}$$

$$y_{a,start_p^h}^{ph} = 1 \qquad (h \in M) \tag{7.37}$$

$$\sum_{(v_j,v_k)\in A'} n_{[ijk]}^{ph} = y_{ij}^{ph} \qquad ((v_i,v_j) \in A', h \in M) \tag{7.38}$$

$$\sum_{(v_k,v_i)\in A'} n_{[kij]}^{ph} = y_{ij}^{ph} \qquad ((v_i,v_j) \in A', h \in M) \tag{7.39}$$

$$l \leq \sum_{(v_i,v_j)\in A_h} (s_{ij}^h x_{ij}^{ph} + d_{ij}^h (y_{ij}^{ph} - 1)) \leq u \qquad (h \in M) \qquad (7.40)$$

$$x_{ij}^{ph} \in \{0,1\} \qquad ((v_i,v_j) \in A_h, h \in M) \qquad (7.41)$$

$$y_{ij}^{ph} \geq 0 \text{ and integer} \qquad ((v_i,v_j) \in A_h \cup A_1 \cup A_2, h \in M) \qquad (7.42)$$

$$w_{ij}^{ph} \geq 0 \qquad ((v_i,v_j) \in A_h \cup A_1 \cup A_2, h \in M) \qquad (7.43)$$

$$TMAX_p \geq 0 \qquad (7.44)$$

$$t_p^h \geq 0 \qquad (h \in M). \qquad (7.45)$$

For every class p = 1, ..., K, the objective function (7.26) minimizes the sum of the service completion time of class p and the penalties associated with turns. For any given class, constraint set (7.27) is identical to its counterpart (7.2) of the model (7.1)–(7.25). For every class p = 1, ..., K, constraints (7.28) define the service completion time of class p on each route given the start time s_p^h of class p on route h. For every class p = 1, ..., K constraints (7.29) assure that each arc of class p is serviced by exactly one eligible vehicle. For any given class, constraint sets (7.30)–(7.36) are identical to their respective counterparts (7.7)–(7.13) of the model (7.1)–(7.26). For every class p = 1, ..., K and for every vehicle $h \in M$, constraints (7.37) require that each partial class p, route h starts service at its start location $start_p^h$. For any given class, constraint sets (7.38)–(7.40) are identical to their respective counterparts (7.22), (7.23), and (7.24) of the model (7.1)–(7.25). Finally, if p = 1, then the constraints (7.25) must be added to the model (7.26)–(7.45) to impose tandem plow patterns.

For every class p = 1, ..., K and for every vehicle $h \in M$, let $start_p^h$ and end_p^h represent the start and end locations at which the partial class p, route h starts and ends service, respectively, and let s_p^h represent the start time when the partial class p, route h must start service. Given two routes R_p^h = ($start_p^h$, ..., end_p^h) and R_{p+1}^h = ($start_{p+1}^h$ = end_p^h, ..., end_{p+1}^h) representing the partial class p, route h and the partial class p + 1, route h in G, respectively, and having a common endpoint end_p^h in G, let R_p^h + R_{p+1}^h =

($start_p^h$, ..., $end_p^h = start_{p+1}^h$, ..., end_{p+1}^h) denote the union of the arcs of these two partial routes. For every vehicle $h \in M$, let R_h be the (possibly partial) route h in G, let SP_K^h be the shortest duration path from the last serviced arc in class K by vehicle h to the depot, and let sp_K^h be its travel time. The parallel algorithm can be described more precisely as follows.

1. Set $p = 1$. For every vehicle $h \in M$, set $s_p^h := 0$, and $start_p^h := v_0$. For every vehicle $h \in M$, set $R^h := \varnothing$.
2. If $p = 1$, add constraints (7.25) to model (7.36)–(7.45) and solve the resulting model. Otherwise, solve model (7.36)–(7.45). Let R_p^h be the resulting partial class p, route h. For every vehicle $h \in M$, declare all traversed arcs on R_p^h as already serviced and set $R_h := R_h + R_p^h$. Set $TMAX_p = \max_{h \in M} \{t_p^h\}$.
3. If $p = K$, go to Step 4. Otherwise, set $p = p + 1$. For every vehicle $h \in M$, set $s_p^h := t_{p-1}^h$, $start_p^h := end_{p-1}^h$, and return to Step 2.
4. For every vehicle $h \in M$, set $R_h := R_h + SP_K^h$ and $t_{K+1}^h = t_K^h + sp_K^h$. **Stop.**

7.4.2 Cluster first route second algorithm

The cluster first, route second algorithm first determines a partition of the arcs to be serviced into compact clusters, each having approximately the same workload, while taking into account vehicle-road segment dependencies. Since vehicles plow at different speeds, the total workload is measured in time units (e.g., minutes). A vehicle route is then constructed in each cluster through the solution of a HRPP with class upgrading possibilities and turn restrictions. We adapted a technique proposed by BENAVENT *et al.* (1990) for partitioning the arcs into clusters. This technique determines the assignment of the arcs by solving a generalized assignment problem (GAP). It is inspired by the FISHER and JAIKUMAR (1981) generalized-assignment-based algorithm for the capacitated vehicle routing problem.

The algorithm starts by locating $|M|$ geographically dispersed arcs of A' to serve as seed arcs $s_1 \in A_1, \ldots, s_h \in A_h$ for the $|M|$ vehicles. The criterion for widely dispersing seed arcs over the graph G'' is to maximize the product of the shortest paths among the seed arcs and the depot v_0. In particular, if seeds $s_1, \ldots, s_h$ have already been selected, seed arc s_{h+1} is chosen to maximize $\prod_{k=0,\ldots,h} (sp_{ak} + sp_{ka})$ over all arcs a. For every vehicle $h \in M$ and for every arc $(v_i, v_j) \in A_h$, let x_{ij}^h be a binary variable equal to 1 if and only if arc (v_i, v_j) is assigned to vehicle h, let d_{ijh} and d_{hij} represent the lengths of the shortest paths from arc (v_i, v_j) to arc s_h and from s_h to (v_i, v_j), respectively. Then the problem of assigning each arc of A' to exactly one of the $|M|$ vehicles can be formulated as a linear 0–1 integer program as follows.

$$\text{Minimize} \sum_{h \in M} \sum_{(v_i, v_j) \in A_h} (d_{ijh} + d_{hij}) x_{ij}^h \tag{7.46}$$

subject to

$$\sum_{\substack{h \in M \\ (v_i, v_j) \in A_h}} x_{ij}^h = 1 \qquad ((v_i, v_j) \in A') \tag{7.47}$$

$$L \leq \sum_{(v_i, v_j) \in A_h} s_{ij}^h x_{ij}^h \leq U \qquad (h \in M) \tag{7.48}$$

$$x_{rs}^{h_1} = x_{tu}^{h_2} \qquad ((i_r, j_s) \in A_{\text{tandem}} \cap A_{h_1}, (i_t, j_u) \in A_{\text{tandem}} \cap A_{h_2}, (h_1, h_2) \in M_{\text{tandem}}) \tag{7.49}$$

$$x_{ij}^h \in \{0,1\} \qquad ((v_i, v_j) \in A', h \in M) \tag{7.50}$$

where

$L = (1 - \alpha)z$, $U = (1 + \alpha)z$, $z = \dfrac{\displaystyle\sum_{(v_i v_j) \in A'} \left(\frac{\sum_{k \in M} s_{ij}^h}{|M|} \right)}{|M|}$, and $0 \leq \alpha \leq 1$.

The objective function (7.46) minimizes the sum of all lengths of the shortest paths from the arcs in G'' to the seed arcs and from the seed arcs to the arcs, so as to assess the compactness of every cluster. Constraints (7.47) assure that each arc is assigned to exactly one eligible vehicle. Constraints (7.48) impose a specified lower bound L and an upper bound U on the total workload of each vehicle. The value of α is chosen suitably small in order to make the bounds on the total workload as tight as possible so that it does not affect the realisabilty of the problem. Pairs of arcs that necessitate tandem plow patterns by pairwise compatible vehicles are imposed by constraints (7.49).

Once the $|M|$ clusters have been determined, a HRPP with class upgrading possibilities, vehicle-road segment dependencies, and turn restrictions is then solved on each cluster. Model (7.1)–(7.23) can be used to this end. However, the assignment constraint sets (7.5) and (7.6) must be replaced with the constraints

$$\sum_{\substack{h\in M \\ (v_i,v_j)\in S_h}} \sum_{p=1}^{k} x_{ij}^{ph} = 1 \qquad ((v_i,v_j)\in A^k, k=1,\dots,K-1) \qquad (7.51)$$

$$\sum_{\substack{h\in M \\ (v_i,v_j)\in S_h}} \sum_{p=1}^{K+1} x_{ij}^{ph} = 1 \qquad ((v_i,v_j)\in A^K) \qquad (7.52)$$

where S_h is the subset of required arcs assigned to vehicle h in the solution to the linear 0–1 integer model. Hence, each vehicle is now required to service only a subset of arcs. The route phase works by iteratively solving the resulting model for every vehicle $h \in M$, considering all traversed arcs on routes $1,\dots, h-1$ as already serviced in order to reduce the total completion time. The cluster first, route second algorithm can be summarized as follows.

1. *Cluster phase*
   1. Determine a set $\{s_1,..., s_{|M|}\}$ of $|M|$ seed arcs, selecting for each vehicle $h \in M$ an arc s_h such that the product of the sum of the shortest paths between s_h and the seed arcs $s_1,..., s_{h-1}$ and between s_h and the depot v_0 is maximum.
   2. Solve model (7.46)–(7.50) to determine the assignment of each arc of A' to exactly one of the $|M|$ vehicles. For each vehicle $h \in M$, let S_h be the set of arcs assigned to vehicle h in the solution to the linear 0–1 integer model. Set $h = 1$.
2. *Route phase*
   a) Solve model (7.1)–(7.24) with constraint sets (7.5) and (7.6) replaced by constraint sets (7.51) and (7.52), taking S_h as an input, to construct the h-th vehicle route. Let R_h be the resulting route operated by vehicle h. Declare all traversed arcs on route h as non-required.
   b) If $h = |M|$, set $TMAX_p = \max_{h \in M} \{t_p^h\}$ for each class $p = 1,..., K + 1$ and **stop**. Otherwise, set $h := h + 1$ and return to step 2a.

7.5 Computational experiments

To measure the performance of the proposed solution approaches, computational experiments were performed using data from the City of Dieppe, New Brunswick, Canada. All linear integer models were programmed using OPL studio 3.7 running with CPLEX 9.0 (after tuning the parameters) and the procedure to determine the set of seed arcs in the cluster phase of the cluster first, route second algorithm was coded in VBA using Microsoft Visual Basic 6.3. All experiments were performed on a Pentium 4 personal computer. A maximum running time of 3600 seconds was imposed for each linear integer programming problem. We first describe the data requirements and then give a summary of the results obtained.

7.5.1 Data requirements

The Dieppe data were extracted from a digitalized map stored in an image format and the current vehicle routes were obtained from Public Services of the City of Dieppe. We used Forestry GIS (fGIS) to extract the topology of the transportation network and road segment lengths from the graphical file. The road network of Dieppe involves 462 vertices and 1234 arcs partitioned into three classes A^1, A^2, and A^3 representing arterial streets, collecting streets, and local streets, respectively, with $|A^1| = 244$, $|A^2| = 229$, and $|A^3| = 761$. The subgraph induced by the set of arcs of class A^1 is Eulerian while the subgraphs induced by the set of arcs of classes A^2 and A^3 are not strongly connected. The City of Dieppe currently uses eight vehicles of three different types: one grader, two plows, and five loaders. The grader and the plows can clear 1,5 lanes in each pass, whereas the loaders can only clear one lane at a time. Moreover, the latter are restricted to class 2 and 3 streets, while the grader and the plows do not have any such restrictions. Each vehicle type has the same service and deadhead speeds. Thus, $s_{ij}^h = d_{ij}^h$ for every vehicle $h \in M$ and every arc $(v_i, v_j) \in A_h$. Plows travel at a speed of 25 km/h on class 1 and 2 streets and at 10 km/h on class 3 streets. The grader can travel at 20 km/h on class 1 and 2 streets and at 10 km/h on class 3 streets. The loaders travel at a speed of 10 km/h. The precedence relation between classes in a route is linear, i.e. all arterial streets must be serviced before collecting streets and all collecting streets must be serviced before local streets, and the makespan objective is minimized. Linear precedence relations can be incorporated easily to the formulation by setting $A^K = \varnothing$.

The routes must also take into account forbidden left turns. These turns can be penalized by assigning them a high positive penalty and accounting for it in the model. However, as highlighted by BENAVENT and SOLER (1999) and CORBERÁN *et al.* (2002), a method that takes into account turn penalties, but not turn prohibitions, and tries to avoid the forbidden turns by assigning them a high penalty cannot guarantee to produce routes that avoid forbidden turns. A direct way of modelling forbidden turns is to transform the m-HCPP with forbidden left turns as a m-HRPP by adding artificial arcs to graph G'. For this, vertices have to be replicated. To illustrate, consider the street intersection v_k shown in Figure 7.3a, where the two left turns $[(v_i, v_k), (v_k, v_l)]$ and $[(v_j, v_k), (v_k, v_i)]$ are forbidden and all other turns are allowed. Figure 7.3b illustrates the replication of vertex v_k and the introduction of nine non-required arcs shown as dashed lines to represent straight crossings, right turns, left turns, and U-turns. The two forbidden left turns $[(v_i, v_k), (v_k, v_l)]$ and $[(v_j, v_k), (v_k, v_i)]$ can thus be avoided by eliminating their corresponding non-required arcs (k_2, k_6) and (k_1, k_3), respectively, in the augmented graph G', and the remaining non-required arcs have zero cost.

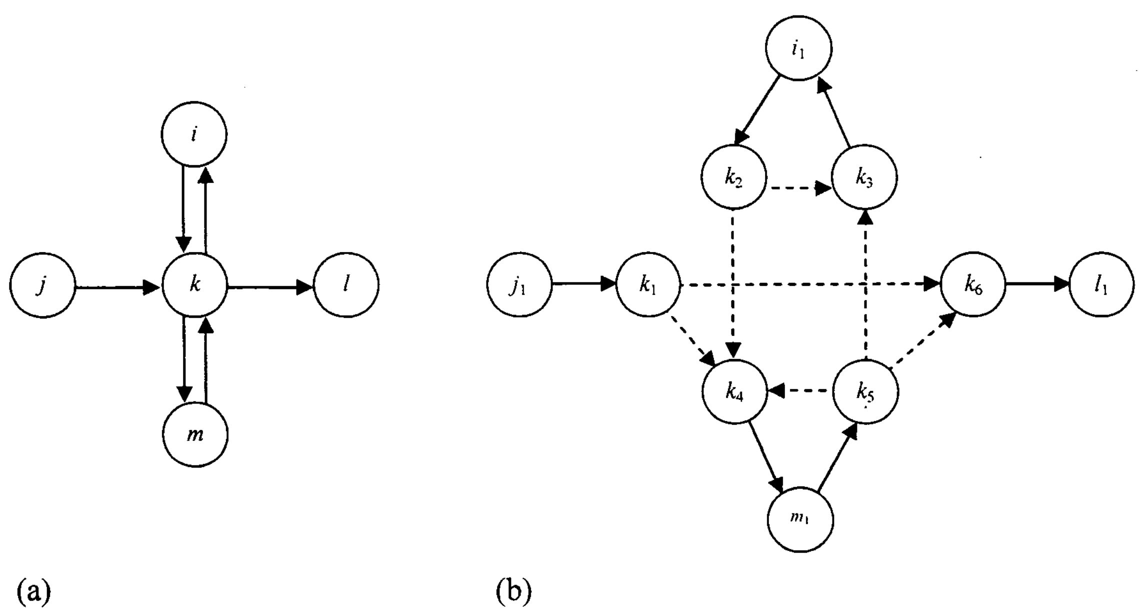


Figure 7.3: Replicating vertices and introducing artificial arcs: (a) street intersection (k), (b) same street intersection with replicated vertices and artificial arcs.

Finally, some class 1 multi-lane road segments necessitate tandem plow patterns in echelon formations. The city requires that service in tandem should be accomplished by the two plows.

7.5.2 Summary of results

Four scenarios were used in the experiments. In the first scenario, class upgrading possibilities are forbidden and the hierarchical objective is minimized. In the second scenario, the hierarchical objective is still used, but the possibility to service road segments of a class in any of the classes of higher priority is now allowed. The third scenario considers the makespan objective instead of the hierarchical objective and prohibits class upgrading possibilities. Finally, the fourth scenario disregards the linear precedence relations between classes in each route. Since the parallel algorithm constructs feasible routes for each class independently, it cannot be employed for solving the hierarchy relaxation.

In Scenarios 1 and 3, class upgrading prohibitions are treated by replacing constraints (7.5) and (7.6) with the constraints

$$\sum_{\substack{h \in M \\ (v_i, v_j) \in A_h}} x_{ij}^{kh} = 1 \qquad ((v_i, v_j) \in A^k, k = 1, \ldots, K-1) \qquad (7.53)$$

$$\sum_{\substack{h \in M \\ (v_i, v_j) \in A_h}} x_{ij}^{Kh} = 1 \qquad ((v_i, v_j) \in A^K) \qquad (7.54)$$

respectively, and by not considering all traversed arcs as already serviced in steps 2 and 2a of the parallel and cluster first route second algorithms. If the makespan objective is considered instead of the hierarchical objective, then *TMAX* may be a nonnegative real variable representing the time required to service all arcs of $A^1 \cup A^2 \cup \ldots \cup A^{K-1}$ plus the shortest travel time to the depot from the last serviced arc. One would then replace the objective function (7.1) by (7.55) and constraints (7.2) and (7.20) by (7.56) and (7.57), respectively.

$$TMAX \qquad (7.55)$$

$$TMAX \geq t_{K+1}^{h} \qquad (h \in M) \tag{7.56}$$

$$TMAX \geq 0 \; . \tag{7.57}$$

We were unable to obtain a feasible solution to the subproblem (3.1)–(3.20) for class 3 within the prescribed time limit. For this class, the m-RPP was solved with the following adaptation of the cluster first route second algorithm.

1. For every vehicle $h \in M$, let end_2^h be the seed vertex for vehicle h. Let R^3 be the set of non-serviced arcs of class 3. For every vehicle $h \in M$ and for every arc $(v_i, v_j) \in A_h \cap R^3$, let x_{ij}^h be a binary variable equal to 1 if and only if arc (v_i, v_j) is assigned to vehicle h, and let d_{hij} represent the length of the shortest path from seed vertex end_2^h to arc (v_i, v_j). Solve the following linear 0–1 integer model to determine the assignment of each arc of R^3 to exactly one of the $|M|$ vehicles.

$$\text{Minimize} \sum_{h \in M} \sum_{(v_i, v_j) \in A_h \cap R^3} d_{hij} x_{ij}^h \tag{7.58}$$

subject to

$$\sum_{\substack{h \in M \\ (v_i, v_j) \in A_h}} x_{ij}^h = 1 \qquad ((v_i, v_j) \in R^3) \tag{7.59}$$

$$L \leq t_2^h + \sum_{(v_i, v_j) \in A_h \cap R^3} s_{ij}^h x_{ij}^h \leq U \qquad (h \in M) \tag{7.60}$$

$$x_{ij}^h \in \{0,1\} \qquad ((v_i, v_j) \in R^3, h \in M) \tag{7.61}$$

where $L = t_2^1 + r^1 - \alpha(t_2^1 + r^1)$ and $U = t_2^1 + r^1 + \alpha(t_2^1 + r^1)$, $0 \leq \alpha \leq 1$. For each vehicle $h \in M$, the time r^h for servicing all arcs of R^3 by vehicle h is determined such that the equations

$$t_2^{h_1} + r^{h_1} = t_2^{h_2} + r^{h_2} \qquad (h_1, h_2 \in M) \qquad (7.62)$$

$$\sum_{h \in M} r^h = \sum_{(v_i v_j) \in R^3} \left(\frac{\sum_{h \in M} s_{ij}^h}{|M|} \right) \qquad (7.63)$$

are satisfied. For each vehicle $h \in M$, let S_h be the set of arcs of class 3 assigned to vehicle h in the solution to the linear 0–1 integer model. Set $h = 1$.

2. Set $A^3 = R^3$ and $A^k = \varnothing$, $k = 1, 2$, and solve model (7.1)–(7.24) with constraints (7.4) and (7.15) replaced with the constraints

$$t_0^h = t_2^h \qquad (h \in M) \qquad (7.64)$$

$$y_{a,end_2^h}^{1h} = 1 \qquad (h \in M) \qquad (7.65)$$

and constraints (2.5) and (2.6) replaced with the constraints (7.51) and (7.52), taking S_h as an input. Let R_3^h be the resulting partial class 3 route operated by vehicle h. Declare all traversed arcs on route R_3^h as already serviced and set $R_h := R_h + R_3^h$.

3. If $h = |M|$, set $TMAX_3 = \max_{h \in M} \{t_3^h\}$ and go to step 4 of the parallel algorithm. Otherwise, set $h := h + 1$ and return to step 2.

Table 7.1 presents the service completion times obtained when solving the instance under the first three scenarios with the parallel algorithm and the four scenarios with the cluster first route second algorithm. The last four rows compare the completion times of both solution approaches and contain the percentage difference in completion time between the parallel and the cluster first route second algorithms, based, respectively, on

Table 7.1: Completion times and percentage gaps

	Scenario			
	1	2	3	4
Completion time (h)				
Parallel algorithm				
$TMAX_{1P}$	1.2	1.2	1.2	–
$TMAX_{2P}$	2.0	1.9	2.0	–
$TMAX_{3P}$	5.0	5.3	5.0	–
$TMAX_{4P}$	5.7	5.7	5.2	–
Cluster first route second algorithm				
$TMAX_{1C}$	1.4	1.4	1.4	4.7
$TMAX_{2C}$	1.9	1.9	1.9	5.1
$TMAX_{3C}$	4.8	4.5	5.3	5.1
$TMAX_{4C}$	5.3	4.9	5.5	5.2
Percentage gap (%)				
$(TMAX_{1P} - TMAX_{1C}) / TMAX_{1C}$	−17.9	−15.3	−18.3	–
$(TMAX_{2P} - TMAX_{2C}) / TMAX_{2C}$	5.2	1.2	1.4	–
$(TMAX_{3P} - TMAX_{3C}) / TMAX_{3C}$	4.9	17.3	−5.7	–
$(TMAX_{4P} - TMAX_{4C}) / TMAX_{4C}$	8.9	15.6	−6.4	–

the completion times of classes 1, 2, and 3, and on the total completion times (class 4). Some gaps are negative because the service completion time produced by the cluster first route second algorithm is sometimes longer than the service completion time produced by the parallel algorithm.

When the hierarchical objective is minimized (Scenarios 1 and 2), we observe that for classes 2, 3, and 4, the cluster first route second algorithm improve upon the parallel algorithm. For classes 3 and 4, up to 17.3% and 15.6% can be saved in completion time, respectively, by applying the cluster first route second algorithm. For class 2, the partial routing can still be carried out with slightly less time with the cluster first route second algorithm. However, for class 1, the partial routes constructed by the cluster first route second algorithm incur at least 15.3% more time than those found by the parallel algorithm. This performance is easily explained from the following observations: since the subgraph induced by the set of arcs of class A^1 is Eulerian, in the parallel algorithm, only the deadheading travel time to the first serviced arc from the depot will make each partial class 1 route more expensive. However, in the cluster first route second algorithm, if the subgraph induced by the set of arcs of class 1 assigned to a vehicle is neither Eulerian nor strongly connected, then a partial class 1 route with more deadheading will be created during the route phase. Thus, the cluster phase should ensure that the graph generated by the arcs of class 1 of each cluster is Eulerian to reduce deadheading in the routing phase. When the makespan objective is minimized (Scenario 3), for all but one of the priority classes the completion time produced by the parallel algorithm is lower than that produced by the cluster first route second algorithm.

With respect to the computation times, we note that the cluster first route second algorithm solves faster than the parallel algorithm. For each scenario, the parallel and the cluster first route second algorithms require to solve 12 and 9 linear integer programming problems, respectively. In most of the 60 linear integer programming problems (24 problems for the parallel algorithm and 36 problems for the cluster first route second algorithm), the maximum running time of 3600 seconds was not sufficient to reach and prove optimality. Thus, the time it takes the cluster first route second algorithm to complete a single scenario is about 9 hours. This computing time seems reasonable given that decisions related to the routing of vehicles for plowing operations are generally updated every winter season.

Table 7.2 evaluates the quality of the solution produced by the City of Dieppe for winter 2004-2005 by comparison with the two solution approaches. We report the percentage difference in the completion time for the four classes. The City of Dieppe

Table 7.2: Percentage gaps: Dieppe's method vs. parallel and cluster first route second algorithms

	Scenario			
	1	2	3	4
Percentage gap (%)				
$(TMAX_{1D} - TMAX_{1P}) / TMAX_{1P}$	137.5	131.7	137.5	–
$(TMAX_{2D} - TMAX_{2P}) / TMAX_{2P}$	172.2	183.0	172.2	–
$(TMAX_{3D} - TMAX_{3P}) / TMAX_{3P}$	2.2	–2.8	3.0	–
$(TMAX_{4D} - TMAX_{4P}) / TMAX_{4P}$	–2.8	–2.1	8.1	–
$(TMAX_{1D} - TMAX_{1C}) / TMAX_{1C}$	95.0	96.2	93.9	–40.6
$(TMAX_{2D} - TMAX_{2C}) / TMAX_{2C}$	186.3	186.3	176.1	6.3
$(TMAX_{3D} - TMAX_{3C}) / TMAX_{3C}$	7.2	14.0	–2.9	1.1
$(TMAX_{4D} - TMAX_{4C}) / TMAX_{4C}$	5.8	13.1	1.1	6.7

produced the following solution for winter 2004-2005: $TMAX_{1D} = 2.8$, $TMAX_{2D} = 5.4$, $TMAX_{3D} = 5.2$, and $TMAX_{4D} = 5.6$. The city's completion time of class 3 is shorter than that of class 2 because the city allows the service of all arcs of class 3 in the higher class 2.

These results indicate that the two solution approaches can produce better routes than the city's method, in terms of service completion times. In fact, the parallel algorithm with the makespan objective (Scenario 3) reduces the time to service all arcs of A^1, A^2, and A^3 and the time required to service all arcs and return to the depot by more than 137%, 172%, 3%, and 8%, respectively, over the routing plan in use by the city. Moreover, the cluster first route second algorithm with the hierarchical objective and class upgrading possibilities (Scenario 2) cuts the service completion time of classes 1, 2, and 3 and the total completion time by more than 96%, 186%, 14%, and 13%, respectively, over the existing plan. The large decreases in completion times of higher priority classes result mainly from the parallel and cluster first route second routes better satisfying the hierarchy constraint.

We also analyzed the effects on service completion times of minimizing the hierarchical objective, allowing class upgrading possibilities, and satisfying the linear precedence relations between classes in each route. Table 7.3 presents, for every class $p = 1,..., 4$, the relative difference gap_p between the value of $TMAX_p$ produced by the parallel (cluster first route second) algorithm with makespan objective (Scenario 3) and the value of $TMAX_p$ produced by the parallel (cluster first route second) algorithm with hierarchical objective (Scenario 1).

Table 7.3: Percentage gaps: Makespan objective vs. hierarchical objective

Percentage gap (%)	Parallel algorithm	Cluster first route second algorithm
gap_1	0.0	0.6
gap_2	0.0	3.7
gap_3	−0.8	10.4
gap_4	−10.0	4.6

These results indicate that minimizing the hierarchical objective can reduce the service completion time of higher priority classes or increase the total completion time. With the parallel algorithm, the completion times of classes 1 and 2 remain the same because the makespan objective cannot be considered for solving subproblem (7.26)–(7.45) for these classes. Table 7.4 indicates, for every class $p = 1,..., 4$, the relative difference gap_p between the value of $TMAX_p$ produced by the parallel (cluster first route second) algorithm with hierarchical objective and no class upgrading possibilities (Scenario 1) and the value of $TMAX_p$ produced by the parallel (cluster first route second) algorithm with hierarchical objective and class upgrading possibilities (Scenario 2).

Table 7.4: Percentage gaps: No class upgrading vs. class upgrading

Percentage gap (%)	Parallel algorithm	Cluster first route second algorithm
gap_1	0.0	0.6
gap_2	4.0	0.0
gap_3	–4.9	6.3
gap_4	0.7	6.9

Clearly, permitting class upgrading possibilities can reduce both the service completion time of higher priority classes and the total completion time. Table 7.5 illustrates the benefits of imposing linear precedence relations between classes in each route. The gap corresponds to the relative difference between the service completion time produced by the cluster first route second algorithm when the hierarchy constraint is relaxed (Scenario 4) and the service completion time produced by the cluster first route second algorithm with service hierarchy (Scenario 1, 2, or 3).

Table 7.5: Percentage gaps: No service hierarchy vs. service hierarchy

Percentage gap (%)	Scenario 1	Scenario 2	Scenario 3
gap_1	228.1	230.0	226.3
gap_2	169.3	169.3	159.7
gap_3	6.1	12.8	–3.9
gap_4	–0.8	6.0	–5.2

These results show, not surprisingly, that the linear precedence relations have a very positive influence on the service completion time of higher priority classes. In particular, completion times are reduced considerably for the first two classes.

In our previous paper (PERRIER *et al.*, 2006), we proposed a cluster first route second method for the snow plow routing problem in the City of Dieppe. Nine variants were considered by changing the lower and upper bounds L and U on the total workload of each vehicle and the tolerance level on the total distance of class 3 road segments that can be serviced prior to higher-class road segments. According to these results, the cluster first route second algorithm with hierarchical objective and class upgrading possibilities (Scenario 2) presented here can produce a set of routes that mostly dominate the sets of routes produced by the other approach. Further comparisons with this method would thus be pointless. Table 7.6 presents, for every class p = 1,..., 4, the relative difference gap_p between the value of $TMAX_p$ produced by a given variant and the value of $TMAX_p$ produced by the cluster first route second algorithm with the hierarchical objective and class upgrading possibilities (Scenario 2).

Table 7.6: Percentage gaps: Nine variants from PERRIER *et al.* (2006) vs. cluster first route second algorithm (Scenario 2)

	Variant								
Percentage gap (%)	V1	V2	V3	V4	V5	V6	V7	V8	V9
gap_1	30.0	–0.1	141.0	30.0	182.3	266.5	243.6	–0.1	–0.1
gap_2	35.5	162.8	70.6	139.9	237.0	137.1	131.4	105.7	30.6
gap_3	24.2	5.4	18.1	2.8	32.1	5.2	26.9	7.4	19.2
gap_4	18.1	8.5	9.5	–1.5	42.9	5.3	20.6	1.5	10.6

7.6 Conclusions

We have proposed a basic model and two solution approaches for the problem of vehicle routing in the context of snow plowing operations. This problem can be viewed as a multiple hierarchical Chinese postman problem with class upgrading possibilities and vehicle-road segment dependencies. The proposed model incorporates a wide variety of operational constraints and can also be customized to deal with many additional situations. The two constructive methods can produce sets of routes that dominate the existing routing plan of City of Dieppe with respect to service completion times in a few hours of computing time. This performance is satisfactory given the fact that the model need only be solved once every winter season. A faster solution approach would however be required so that the model can be used in real-time to determine the changes to be made following an equipment breakdown or weather change. Also, several extensions are still needed to make the model more useful in practice. For example, each road segment should be associated with three traversal times, which are possibly dependent on the vehicle type: the time required to plow the road segment, the time of deadheading the road segment if it has not yet been plowed, and the time of deadheading the road segment if it has already been plowed. Other extensions concern the requirement to service only a subset of arcs, thus leading to a multiple hierarchical rural postman problem, and the possibility of servicing multi-lane road segments requiring tandem service anywhere in the sequence, in order to reduce service completion times.

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DISCUSSION GÉNÉRALE ET CONCLUSION

La première contribution de cette thèse est de présenter une revue détaillée de la littérature récente concernant l'emploi de modèles d'optimisation et d'algorithmes de résolution dans le domaine de l'entretien hivernal des réseaux routiers. Aux chapitres 2 et 3, nous avons d'abord présenté les modèles de design des systèmes d'entretien hivernal des réseaux routiers. Ces problèmes concernent le partitionnement d'un réseau routier en secteurs, la localisation des sites de déversement, l'affectation des secteurs de déneigement à des sites de déversement, et l'affectation des secteurs de déneigement à des entrepreneurs. Aux chapitres 4 et 5, nous avons ensuite décrit les modèles de routage des véhicules utilisés pour l'entretien hivernal des réseaux routiers. Ces deux chapitres présentent également une revue des modèles de localisation de garages et de dépôts intermédiaires de fondants et d'abrasifs, d'affectation des équipes de travail aux garages, de détermination de la taille de la flotte de véhicules et d'horaire de remplacement de la flotte de véhicules.

Une autre contribution importante de cette thèse est de présenter un cadre de modélisation de deux problèmes liés à l'entretien hivernal des réseaux routiers: le problème combiné du partitionnement d'un réseau routier en secteurs de déneigement et d'affectation des secteurs aux sites de déversement pour les opérations d'enlèvement de la neige et le problème de tournées de véhicules pour les opérations de déblaiement des rues. Dans le cas du problème combiné du partitionnement d'un réseau en secteurs et d'affectation des secteurs aux sites, ce cadre original s'inspire en partie des approches utilisées pour traiter les problèmes de partitionnement d'un réseau routier en secteurs pour les opérations d'épandage de fondants et d'abrasifs et de déblaiement des rues, mais introduit également de nombreux éléments de modélisation qui sont propres aux opérations d'enlèvement de la neige tels que la forme des secteurs en arcs de cercle autour des sites de déversement. Dans le cas du problème de tournées de véhicules pour les opérations de déblaiement des rues, le cadre de modélisation permet de traduire un très large éventail de contraintes et de possibilités qui peuvent être communes ou spécifiques à différentes applications pratiques telles que les contraintes générales de préséance, des vitesses différentes de service et de passages à vide, les passages répétés obligatoires pour les rues à voies multiples, la possibilité d'augmenter l'ordre de préséance des rues non prioritaires, et les restrictions sur les rues qui peuvent être desservies ou traversées par chaque type de véhicules. De plus, les contraintes d'équilibre de durée des tournées, la possibilité de desservir certaines artères en tandem, et les restrictions sur l'utilisation de certains types de virages peuvent être prises en compte sans trop affecter la structure du modèle.

Une dernière contribution importante de cette thèse est le développement et la comparaison de diverses approches de résolution pour les modèles proposés. Dans le cas du problème combiné du partitionnement d'un réseau en secteurs et d'affectation des secteurs aux sites pour les opérations d'enlèvement de la neige, deux méthodes constructives en deux phases ont été développées. La première méthode permet de tenir compte de l'interdépendance entre le problème du partitionnement d'un réseau routier et le problème d'affectation des secteurs aux sites tandis que la seconde méthode traite séparément ces deux composantes. Les deux méthodes peuvent résoudre, en quelques minutes de calcul sur un ordinateur personnel, des instances comportant quatre sites de déversement, huit secteurs, et plus de 200 arêtes. Les comparaisons avec la seconde méthode montrent que des économies considérables peuvent être réalisées au niveau des coûts variables d'élimination de la neige aux sites en utilisant la première approche. Dans le cas du problème de tournées de véhicules pour les opérations de déblaiement des rues, nous avons également utilisé deux méthodes constructives. Ces méthodes permettent de résoudre, en quelques heures de calcul sur un ordinateur personnel, un problème réel comportant trois classes de priorité, 462 sommets et 1234 arcs. Les comparaisons avec les tournées produites manuellement par les employés de la ville de

Dieppe montrent que les deux méthodes permettent habituellement de réduire à la fois le temps d'achèvement et les temps pour desservir chaque classe de priorité.

Dans leur forme actuelle, les approches développées pour le problème de tournées de véhicules pour les opérations de déblaiement des rues ne permettent pas de résoudre le problème de planification en temps réel des tournées. Rappelons que le niveau en temps réel concerne les situations nécessitant une réponse très rapide à des problèmes se posant en temps réel tels que les bris d'équipement ou les changements de température. À ce niveau, des méthodes de résolution très rapides fondées sur une information météo routière pontuelle et détaillée sont donc nécessaires afin d'obtenir une bonne solution.

Finalement, un développement important concernant le problème combiné du partitionnement d'un réseau routier en secteurs et d'affectation des secteurs aux sites pour les opérations d'enlèvement de la neige consisterait à évaluer l'influence de la forme des secteurs et du type d'unités de base utilisées (segments de rues ou petites zones contenant chacune un ensemble de segments de rues adjacents) sur la qualité des tournées de souffleuses qui peuvent être générées dans les secteurs.

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