Titre: Title:	On the numerical issues of the NorSand model in the simulation of undrained behavior of granular soils
Auteurs: Authors:	Seyyed Kazem Razavi, & Samuel Yniesta
Date:	2023
Type:	Communication de conférence / Conference or Workshop Item
Référence: Citation:	Razavi, S. K., & Yniesta, S. (octobre 2023). On the numerical issues of the NorSand model in the simulation of undrained behavior of granular soils [Communication écrite]. GeoSaskatoon 2023: 76th Canadian Geotechnical Conference, Saskatoon, Saskatchewan, Canada (8 pages). https://www.cgsconferences.ca/en/GEO2023/GEO2023_24.html

Document publié chez l'éditeur officiel
Document issued by the official publisher

Nom de la conférence: Conference Name:	GeoSaskatoon 2023: 76th Canadian Geotechnical Conference
Date et lieu: Date and Location:	2023-10-01 - 2023-10-04, Saskatoon, Saskatchewan, Canada
Maison d'édition: Publisher:	Canadian geotechnical society
URL officiel: Official URL:	https://www.cgsconferences.ca/en/GEO2023/GEO2023_24.html
Mention légale: Legal notice:	

On the numerical issues of the NorSand model in the simulation of undrained behavior of granular soils

Seyyed Kazem Razavi & Samuel Yniesta
Department of Civil, Geological, and Mining Engineering, Polytechnique Montreal, Montreal, QC, Canada

ABSTRACT
The NorSand model is one of the most used soil models to simulate the undrained and drained response of granular materials. The model requires the following parameters: two elastic material properties, three to four parameters to define the critical state in e-p and q-p space, two parameters to define the image stress ratio and limiting dilatancy, and two hardening parameters. These few material properties and a straightforward procedure to determine them are the main reasons for the considerable growth of the NorSand application in the simulation of the stress-strain behavior of granular materials. However, the material properties are obtained mainly based on drained test results conducted on dense samples. Therefore, the model uses some assumptions and mathematical techniques to modify its response to simulate the undrained behavior of soil states looser than the critical state. This paper explains how the NorSand model employs this mathematical technique and discusses the effectiveness of this approach.

RÉSUMÉ
La loi de comportement NorSand est un des modèles de sol les plus utilisés pour simuler le comportement non-drainé et drainé des matériaux granulaires. Le modèle demande la définition des paramètres d'entrée suivant : deux propriétés élastiques, trois ou quatre paramètres pour définir l'état critique dans l'espace e-p et q-p, deux paramètres pour définir le ratio des contraintes image et la dilatance maximum, et deux paramètres d'écrouissage. Le faible nombre de paramètres et une procédure de calibration simple sont les principales raisons pour l'augmentation de l'usage de NorSand dans la simulation du comportement contrainte-déformation des matériaux granulaires. Cependant, ces propriétés sont obtenues principalement sur la base de tests drainés, réalisés sur des échantillons denses. Par la suite, le modèle repose sur des hypothèses et des techniques mathématiques pour modifier sa réponse pour simuler le comportement non-drainé des sols lâches. Cet article présente comment NorSand effectue cette modification, et discute de l'efficacité de cette approche.

1 INTRODUCTION

It is essential to study in depth the performance of a constitutive soil model in order to understand its principles, underlying concepts, and limitations, and be able to develop novel ideas to improve said model. Jefferies (1993), for instance, conducted a thorough analysis of traditional Cambridge-type models (Roscoe et al. 1963) to reveal why they fail to simulate accurately the behavior of granular soils. His research showed that the attachment of the yield surface to the critical state was the root cause of the issue, resulting in a single normally consolidated line. This, however, is inconsistent with the behavior of granular soils, which exhibit infinite normally consolidated lines. To address this problem, Jefferies (1993) proposed a new model, the NorSand model, in which the yield surface is separated from the critical state line and the model only reaches it at large strains. This necessitated that the location and size of the NorSand yield surface be controlled by the state parameter (Been and Jefferies, 1985). This added feature enhanced the capability of Cambridge-type models to simulate the behavior of granular soils. The influence of the state parameter on the size and location of the yield surface and the use of an associated flow rule implies that the dilatancy is also a state parameter-dependent quantity. The concept of state-dependent dilatancy later has been recognized thoroughly and is used in models with non-associated flow rules (e.g. Li et al. 1999, Dafalias and Manzari, 2004, Lu and Huang, 2014), resulting in the development of models well capable of simulating sand's behavior.

Likewise, understanding the performance of the NorSand model in simulating the drained and undrained behavior of granular soils is necessary to identify its limitations. The principal drawback of the original NorSand model (Jefferies, 1993) is its inability to accurately simulate the undrained behavior of loose states, such as flow liquefaction (Chang and Jefferies, 2020, Jefferies and Been, 2015). To overcome this limitation, Jefferies and Been (2015) proposed modifying the hardening rule by adding a new term known as the cap-softening term, where the user is only capable to accept or reject its presence. This paper delves into the reasons behind the need for this modification and evaluates the effectiveness of the proposed approach.

Note: All the mean confining pressures (*p*) in this research are effective stresses.

2 NORSAND YIELD SURFACE

The formulation of the NorSand model presented by Jefferies and Been (2015) is considered here. NorSand uses an associated flow rule to define its yield surface by integrating the stress-dilatancy rule (Equation 1). Depending on this rule, the NorSand yield surface (Equation 2) takes a bullet-type shape, similar in shape to the yield surface of the original Cam-Clay model (Roscoe et al. 1963).

$$D = M_i - \eta \quad [1]$$

$$\eta = M_i\left(1 - ln\left(\frac{p}{p_i}\right)\right) \quad [2]$$

The point on the yield surface that exhibits zero dilatancy is referred to as the image state, which is characterized by three parameters: image stress ratio (M_i), image confining pressure (p_i), and image state parameter (ψ_i). In order to determine the image stress ratio, the NorSand model combines various concepts and assumptions, including the State-Dilatancy law, Nova's rule, and the Stress-Dilatancy rule. The State-Dilatancy law describes the relationship between limiting dilatancy (D_L) and its corresponding state parameter (ψ_L).

During a drained triaxial compression (DTC) test, there is a specific point on the stress-strain path where the soil experiences minimal dilatancy (D_{tc-min}) and maximum stress ratio (η_{tc-max}). At this point, the state parameter can be defined as ψ_{tc-min}. NorSand considers D_{tc-min} and ψ_{tc-min} as limiting dilatancy (D_L) and its corresponding state parameter (ψ_L), respectively. Moreover, the stress ratio at the limiting dilatancy (η_L) is equal to η_{tc-max}. In most cases, the following linear equation fits well with experimental data:

$$D_{tc-min} = \chi_{tc} \cdot \psi_{tc-min} \quad [3]$$

Where, χ_{tc} is a material constant called the state-dilatancy parameter. For a general drained path conducted on dense samples, state-dilatancy law has been modified as follows:

$$D_L = \frac{M}{M_{tc}}\chi_{tc}\psi_L \quad [4]$$

Where M_{tc} is the critical stress ratio for triaxial compression test, and M is the critical stress ratio depends on the Lode angle (θ).

$$M = M_{tc} - \frac{{M_{tc}}^2}{3+M_{tc}}cos\left(\frac{3}{2}\theta + \frac{\pi}{4}\right) \quad [5]$$

Lode angle represents the shear mode from triaxial compression to triaxial extension where the shearing takes place. D_L is used to control the NorSand yield surface expansion. It must be noted that for the loose states under drained condition, there is no measurable quantity as minimum dilatancy. Therefore, NorSand assumes that Equation (4) can be extended in its linear format for states looser than critical (see Figure 1).

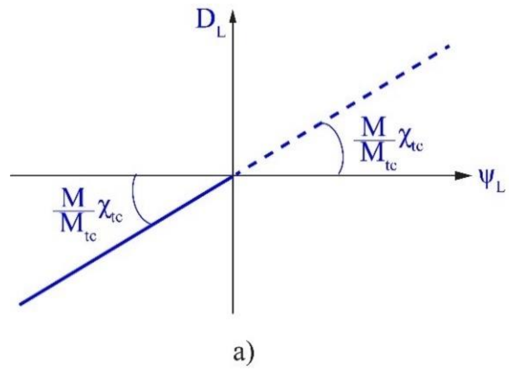
According to Equation (4), ψ_L is required to calculate D_L. Therefore, NorSand presents the following relationship to calculate ψ_L from the image state parameter ψ_i.

$$D_L = \frac{M}{M_{tc}}\chi_{tc}\psi_L = \frac{M}{M_{tc}}\chi_i\psi_i \quad [6]$$

Where, χ_i is called the NorSand state-dilatancy parameter and is a positive value. For linear critical state line shown in Equation (7), χ_i can be easily obtained as Equation (8).

$$e_c = \Gamma - \lambda_l \ln(p) \quad [7]$$

$$\chi_i = \chi_{tc}/\left(1 - \lambda_l \frac{\chi_{tc}}{M_{tc}}\right) \quad [8]$$



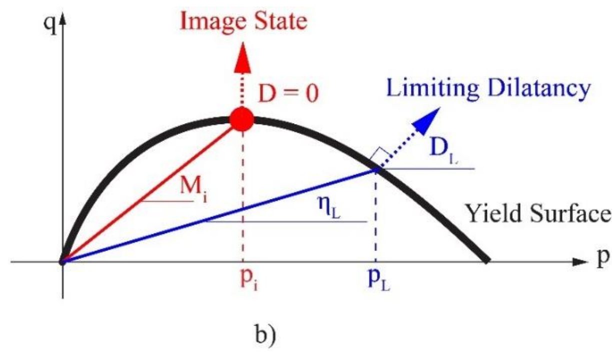


Figure 1 a) Relationship between limiting Dilatancy (D_L) and its corresponding state parameter, b) the concept of limiting Dilatancy on the NorSand yield surface

In Equation (7), Γ is the critical void ratio at p=1 kPa, and λ_l is a material constant used to define the linear CSL. As mentioned above, for a drained triaxial compression (DTC) test conducted on dense states, there is a point where the soil experiences a minimum dilatancy (D_{tc-min}) with maximum stress ratio (η_{tc-max}). Nova's rule relates D_{tc-min} to η_{tc-max} through the following formulation:

$$D_{tc-min} = (M_{tc} - \eta_{tc-max})/(1 - N) \quad [9]$$

Where, N is a material constant called shear-volume coupling factor. The original NorSand formulation assumes that it can be applied for the whole stress path as follows:

$$D_L = (M - \eta)/(1 - N) \quad [10]$$

By combining Equations (10), (6) and (1), the image stress ratio can be obtained:

$$M_i = M\left(1 + N\frac{\chi_i\psi_i}{M_{tc}}\right) \quad for \quad \psi_i < 0 \quad [11]$$

For loose states (ψ_i), however, Equation (11) is not valid, because in that case, M_i becomes greater than M. The developers of the NorSand model assume that a symmetric version of Equation (11) can be used for loose states. Subsequently, the image stress ratio can be written in its general format as follows:

$$M_i = M\left(1 - N\frac{\chi_i|\psi_i|}{M_{tc}}\right) \quad [12]$$

For a nonlinear CSL defined with Equation (13), Equation (8) is no longer valid.

$$e_c = \Gamma - \lambda\left(\frac{p}{p_{ref}}\right)^{\xi} \quad [13]$$

In Equation (13), Γ is the critical void ratio at p=1 kPa, and λ and ξ are material constants used to define a nonlinear CSL. For nonlinear CSL, it can be proved that in Equation (8), λ_l should be calculated from the following equation to use a nonlinear CSL in the NorSand formulation.

$$\lambda_l = \lambda\xi\left(\frac{p}{p_{ref}}\right)^{\xi} \quad [14]$$

The formulation of the NorSand model considering the CSL, stress-dilatancy rule, yield surface, and image stress ratio is summarized in Table 1.

Table 1 general formulation used to obtain the yield surface of NorSand model

Characteristic	Formulation
CSL	$e_c = \Gamma - \lambda_l \ln(p)$
Stress – dilatancy rule	$D = M_i - \eta$
Yield surface	$\eta = M_i\left(1 - ln\left(\frac{p}{p_i}\right)\right)$
Image Stress ratio	$M_i = M\left(1 - \frac{\chi_i N|\psi_i|}{M_{tc}}\right)$ $\chi_i = \frac{\chi_{tc}}{1 - \lambda_l \frac{\chi_{tc}}{M_{i,tc}}}$
Nova's rule	$D_L = \frac{M - \eta_L}{1 - N}$
State-Dilatancy law	$D_L = \frac{M}{M_{tc}}\chi_{tc}\psi_L$

3 NORSAND HARDENING RULE IN THE FRAMEWORK OF CRITICAL STATE THEORY (CST)

The classic CST framework posits that critical state can be achieved at large strains only when two conditions are met: (i) plastic volumetric strain rate ($d\varepsilon_v{}^p$) or dilatancy (D) should be zero, and (ii) there should be continuous zero dilatancy under continuous shearing at large strains ($d(d\varepsilon_v{}^p) = 0$). In the case of the NorSand model with an associated flow rule, zero dilatancy occurs when the current confining pressure (p) reaches the image confining pressure (p_i). Therefore, to be at critical state at large strains, both p and p_i should reach p_c, the confining pressure at the critical state. Additionally, q, the current deviatoric stress, should also reach q_c, the deviatoric stress at critical state.

$$\varepsilon_q{}^p \to \infty \quad \Rightarrow \begin{cases} p \to p_i \to p_c & or \quad \psi \to \psi_i \to 0 \\ q \to q_c & or \quad \eta \to M \end{cases} \quad [15]$$

The second requirement of CST forces that at large strain $d\psi_i = 0$. This means that at large strains the state parameter after reaching critical state should not change.

$$\psi_i = e - (\Gamma - \lambda_l \ln(p_i)) \quad [16]$$

$$\frac{d\psi_i}{d\varepsilon_q{}^p} = \frac{de}{d\varepsilon_q{}^p} + \lambda_l \frac{\frac{dp_i}{p_i}}{d\varepsilon_q{}^p} = 0 \quad [17]$$

de at critical state is zero, therefore the second term in the above equation must reach zero at critical state. The second term can be used as a hardening rule to control the size of yield surface as follows:

$$\frac{dp_i}{d\varepsilon_q{}^p} = H.p.\left(\frac{p_i}{p_L} - \frac{p_i}{p}\right) \quad [18]$$

Therefore, whenever $p_L < p$, the size of the yield surface will increase, otherwise, the yield surface will shrink. For $\psi_i \geq 0$, D_L is positive and $p_L \geq p_i$, and for $\psi_i \leq 0$ the opposite condition will apply. These conditions along with the NorSand hardening concept are shown schematically in Figure 2, for ($\psi_i < 0$), $\eta_L = \eta_{max}$ (maximum stress ratio that can be developed) and $D_L = D_{min}$ (minimum dilatancy) observed under drained tests.

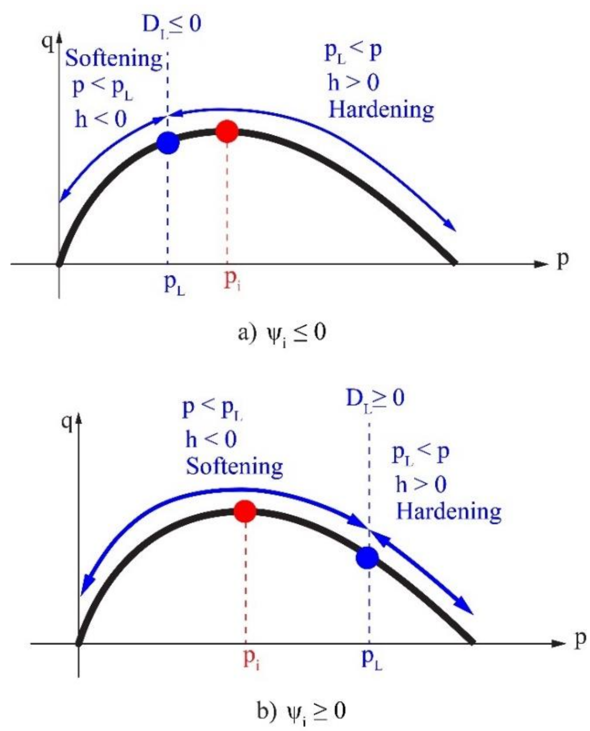


Figure 2 Hardening rule used in the NorSand model.

4 RANGE OF APPLICABILITY OF THE NORSAND MODEL

From Equation (12), the image stress ratio is a linear function of the image state parameter. It is obvious that for $|\psi_i| \geq \frac{M}{N.\chi_i}$, $M_i \leq 0$, and the model loses its applicability. It must be noted that near to zero image stress ratio also affects the volumetric response of the NorSand model, and it is recommended that M_i should not be lower than a certain value:

$$\frac{M}{1+N} \leq M_i \qquad [19]$$

By using this condition in Equation (12), the range of state parameter where the NorSand model is applicable can be obtained:

$$|\psi_i| \leq \frac{M}{(1+N)\chi_i} \qquad [20]$$

Beyond this range, the NorSand model reaches critical state at higher mean confining pressure because of lower image stress ratio.

5 CLASSIC NORSAND MODEL AND UNDRAINED LOADING

In this remainder of this article the formulation of the NorSand model presented in the previous sections is referred to as the classic NorSand model. This version has two main limitations in the simulation of granular soils under undrained loading:

- Inability of simulating flow liquefaction properly (In order to enhance the simulation results, adjustments were required either in the hardening rule or the bulk modulus, as noted by Jefferies and Been (2015) and Ng et al. (2023))

- Inability of simulating limited flow feature (Jefferies, 1993)

5.1 Classic NorSand Model and Flow Liquefaction

Flow liquefaction is a type of cohesionless soil behaviour that occurs at states initially looser than critical state ($\psi_i > 0$). According to the explanations presented in Section 3, and visually depicted in Figure 2b, a positive image state parameter yields $p_i < p_L$ and $M_i < \eta_L$. At the test's inception, $p_L < p$, hence h > 0, and the yield surface expands. Given that $p_i < p$, normality rules mandate a reduction in mean confining pressure during the yield surface expansion, this indicates that p and p_i converge (with p_i increasing due to hardening and p declining due to normality). Zone 1 in Figure 3 shows this part of the stress path. Once $p_L = p$, the yield surface expansion ceases. Subsequently, when $p < p_L$, normality, and hardening rules work in tandem, leading to a reduction in p and p_i towards the critical state. This section of the stress path is represented by zone 2 in Figure 3. Flow liquefaction emerges at a particular stress ratio (η_{FL}) in zone 2, ranging from η_L to M_i.

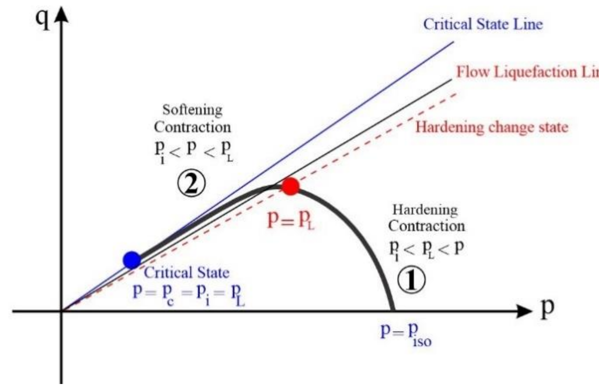


Figure 3 Schematic view of the location where static liquefaction is initiated with the NorSand model

Based on the above discussion, the classic NorSand model can simulate flow liquefaction. However, the major limitation is the accuracy of the simulation results in comparison with measured data. By using the stress dilatancy rule and applying the limiting dilatancy, the following expression can be derived:

$$\eta_L = M_i + D_L = M_i + \frac{M}{M_{tc}}\chi_i\psi_i \qquad [21]$$

χ_i is the material property that is obtained from tests conducted on dense states ($\psi_i < 0$); and then used to simulate the mechanical behaviour of loose states. This assumption produces η_L that is slightly lower than $M_i \cong M$ for loose states. As explained earlier, flow liquefaction happens somewhere between η_L and M (see Figure 3). meaning that η_{FL} takes place at high stress ratio close to M. This is not in agreement with the observed loose soil behaviour, where measured η_{FL} is much lower. Because of these reasons, researchers such as Shuttle and Jefferies (2016), Reid and Smith (2021), and Ng et al. (2023) have attempted to manipulate the bulk modulus to enable NorSand to achieve a better η_{FL} that aligns with experimental data.

5.2 Classic NorSand Model and Limited Flow Behavior

The classic NorSand model cannot simulate the limited flow behaviour due to its hardening rule. Equation (18) is only capable to simulate flow liquefaction as shown in Figure 3. To simulate the limited flow behaviour, the hardening rule should be modified in such a way that produces the desired stress-path drawn schematically in Figure 4. As it can be seen the path contains three zones. Zone 1, presenting an increase in q and a decrease in p, Zone 2, having both p and q decreasing, and Zone 3, where increases in q and p are observed. The new modified hardening rule must be able to simulate strain hardening (zone 1), strain softening (zone 2), and strain hardening (zone 3). However, the hardening rule in the classic NorSand model only allows for the simulation of zone 1 and zone 2 (strain hardening followed by strain softening before reaching critical state) for loose states.

6 NORSAND MODEL WITH CAP-SOFTENING FEATURE

To solve the classic NorSand limitations under undrained conditions, developers of the NorSand model modified the hardening rule by adding a new term called the cap-softening term (Jefferies and Been, 2015). This additional term is only used for the undrained response and is not considered for the drained response. Although, this kind of modification improves the model's response, it implies that the model is unable to reach a unique formulation for both drained and undrained loading conditions. The modified hardening rule for undrained loading condition is written as follows:

$$h_{undrained} = \frac{dp_i}{d\varepsilon_q{}^p} = H\left(p\frac{p_i}{p_L} - p_i\right) + S\frac{\eta}{\eta_L}\frac{d\left(p\frac{p_i}{p_L}\right)}{d\varepsilon_q{}^p} \qquad [22]$$

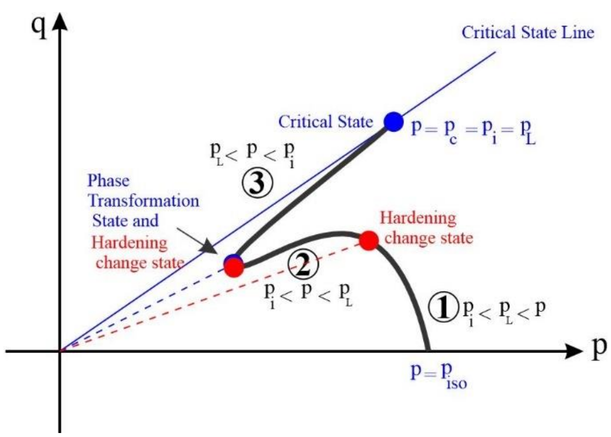


Figure 4 Ideal simulation of limited flow feature through the concept of hardening rule and normality rule in Cambridge type models

Where S is a switch, 0 for drained loading and 1 for undrained loading. By expanding the cap-softening term in Equation (22), the undrained hardening rule becomes:

$$h_{undrained} = \frac{dpi}{d\varepsilon_q{}^p} = \frac{H\left(p\frac{p_i}{p_L} - p_i\right) - S\frac{\eta}{\eta_L}\left(\frac{p_i}{p_L}\right)KD}{1 + p\frac{\eta}{\eta_L}\frac{d\left(\frac{p_i}{p_L}\right)}{d\psi_i}} \qquad [23]$$

Where K is the elastic bulk modulus. In the new hardening rule, p_L and everything associated with it, including D_L and η_L, lose their meaning as a limiting quantity. This fact can be proven by Equation (23), where at $p = p_L$, it is not necessary for $h_{undrained}$ to be zero. In other words, the cap-softening term shifts the point where the hardening regime reverses the expansion of the yield surface to shrinkage. The new point ($p = p_{L-cap}$) can be obtained by finding the root of Equation (23).

$$p_{L-cap} = p_L + S\frac{\eta}{\eta_L}\frac{K}{H}D \qquad [24]$$

Therefore, in the new hardening rule, the location of p in comparison with p_{L-cap}, controls the hardening-softening behaviour of the NorSand yield surface. Figure 5 shows schematically the effect of cap-softening term on the location of the limiting hardening quantity. It must be mentioned that, at the beginning of any simulation with initial isotropic condition, $p_{L-cap} = p_L$ due to the term $\frac{\eta}{\eta_L} = 0$. At critical state, $p_{L-cap} \to p_L$, since $D \to 0$.

6.1 NorSand Model with Cap-Softening and Flow Liquefaction

As shown schematically in Figure 5b, for loose states, the presence of cap-softening results in shifting of the limiting hardness quantity from p_L to p_{L-cap}. Due to the shape of the yield surface, $\eta_{L-cap} < \eta_L$. Based on the new hardening rule, flow liquefaction (η_{FL}) must occur between η_{L-cap} and M instead of η_L and M. As a result, the new hardening rule reduces the simulated η_{FL}, and improves the agreement of the simulations with real data.

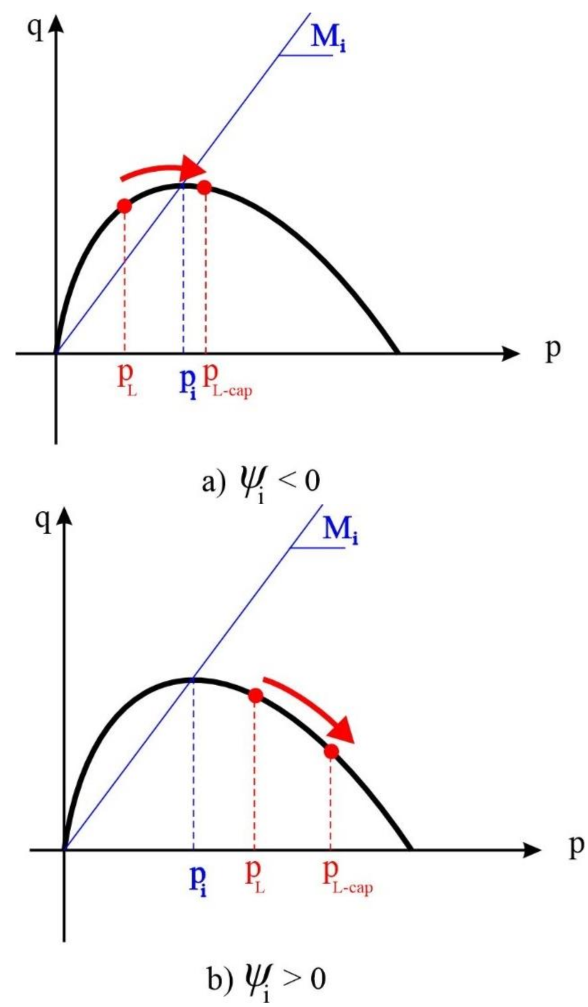


Figure 5 Effect of cap-softening term on the location of limiting dilatancy by shifting it to a new location

6.2 NorSand Model with Cap-Softening and Limited Flow Feature

As explained in Section 5.2, the limited flow behavior requires a strain hardening-softening-hardening feature. This fact means that for NorSand to model limited flow behavior, it should experience at least two changes of hardening regime before reaching critical state. The undrained hardening regime change takes place when $p = p_{L-cap}$ and as a result $dp_i = 0$. This condition is reached several times at $p = p_i = p_{L-cap} \neq p_c$ or at $\psi = \psi_i = \psi_{L-cap} \neq 0$. However, at large strains, the NorSand reaches the critical state when dp_i and ψ_i both reach zero.

The developers of the NorSand model restricted the value of S in Equation (24) to be 0 or 1, which limits the number of times where undrained hardening regime change would occur. Figure 6, presents examples of simulations using the NorSand model, where the value of S varies.

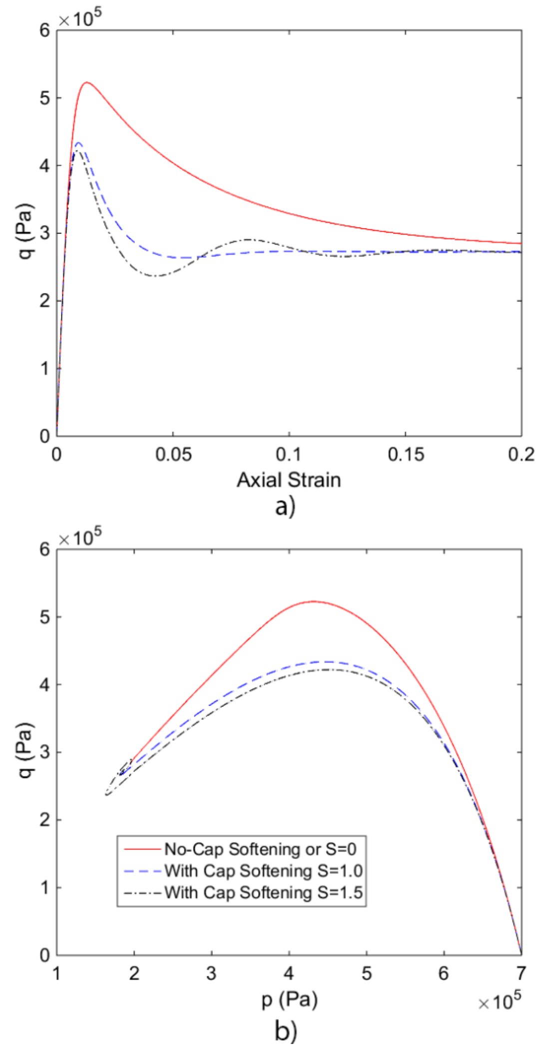


Figure 6 Effect of the cap-softening term on the undrained behaviour of loose states a) deviatoric stress-axial strain, b) deviatoric stress-confining pressure

The material properties and initial state condition (p_0, e_0) used in the simulation for Figure 6 are shown in Table 2. In the figure, $S = 0$ corresponds to the classic NorSand model where the cap-softening has not been applied, $S = 1$ is with the cap-softening term suggested by the developers of the NorSand model. An example of simulation with $S = 1.5$ is presented to illustrate the rigidity of the formulation, although the value is outside of the recommended range. As it can be seen, by increasing S outside of the recommended value, the model starts bouncing around the critical state, gets instable and unable to simulate the soil behaviour properly, not giving the necessary flexibility to adjust the behavior of the model.

Table 2 Material property of Fraser River Sand used to simulate the undrained behaviour of a loose sample with the NorSand model.

parameters	Value
M	1.47
Γ	1.01
λ	0.087
ξ	0.38
χ_{tc}	5
N	0.4
H_0	120
H_y	400
p_0	700e3 Pa
e_0	0.9

6.3 Outcomes of the Cap-Softening Feature

It appears that the NorSand model, in its current formulation may lack the flexibility to effectively reproduce a wide range of undrained response in loose states, instead, it replicates an undrained behavior pattern. To study the capability of their model to match the behavior of different soils, the developers of the NorSand model have compared its response to soil data using a wide range of material properties. For instance, in Figure 7, the normalized undrained peak shear strength of loose samples (s_u/p_0) is illustrated alongside their initial state parameters (ψ_0) for various soils. The results obtained from the NorSand model for different values of (H/I_r) are also displayed in the figure. Here, I_r denotes the shear rigidity and can be derived using the following equation:

$$I_r = \frac{G}{p} \quad [25]$$

Where G is the shear modulus of the soil. The graph reveals that for initially loose samples having high initial state parameters, ψ_0, the model's efficacy in accurately predicting undrained peak responses weakens. Additionally, the model may require different (H/I_r) values to match the behavior of a specific soil under different loading conditions and initial state parameter. This means that manipulation on the elastic material properties may be required to get more accurate results. The H value selected must be applicable not only for the undrained condition but also for the drained response.

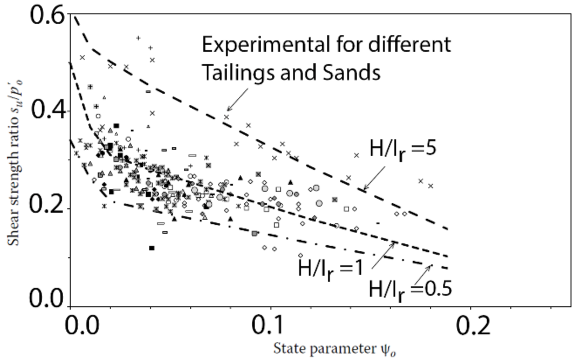


Figure 7 Effect of (H/I_r) on the normalized undrained peak shear strength simulated through the NorSand model in comparison with measured data (Modified after Jefferies and Been, 2015)

7 IMPROVING THE NORSAND MODEL

By going back to Sections 2 and 5, it is evident that the underlying issue comes from the extrapolation of the state-dilatancy law from dense states to loose states. To address this problem and enhance the model's performance, one possible solution is to introduce a second state-dilatancy law with a distinct formulation for loose states. The simplest form can be linear like Equation (4) with a new material property ($\chi_{tc-loose}$).

$$D_L = \frac{M}{M_{tc}} \chi_{tc-loose} \psi_L \qquad [26]$$

$\chi_{tc-loose}$ must be equal or greater than χ_{tc}. However, according to Section 4, higher $\chi_{tc-loose}$ further limits the range of state parameters where the NorSand model is applicable. Choosing another form of state-dilatancy law for loose states adds complexity to the model and may require further changes to the model's formulation. Nevertheless, by incorporating an appropriate state-dilatancy law and its associated material parameters and formulations, the transition point from hardening to softening under undrained loading can be more accurately controlled. The authors have tried to introduce new formulations of state-dilatancy law with a new parameter to simulate static liquefaction more precisely. Figure 8 illustrates the impact of this new parameter (called undrained dilatancy-state parameter) on controlling static liquefaction without modifying any other material properties in the model. A comprehensive presentation of the modified model, named PolySand, will be provided in a future publication (Razavi, 2023).

8 CONCLUSION

According to the above discussion, the NorSand model, while widely used for simulating the drained and undrained behavior of granular materials, has several limitations to capture some of the specific features of the behavior of granular materials. First, it can only simulate soil response within a particular range of initial state parameters. Secondly, to capture the flow behavior of loose material, the model employs a cap-softening term to modify its undrained hardening rule that does not provide flexibility in its response. This term is chosen to satisfy the NorSand approach to the critical state and avoid adding additional parameters to the model. By altering the limiting hardening point, the cap-softening term enhances NorSand's ability to simulate flow liquefaction and limited flow behavior in the undrained response. However, the value of the cap-softening term is either 1 or 0, which limits the capacity of a user to calibrate the model to match a target behavior.

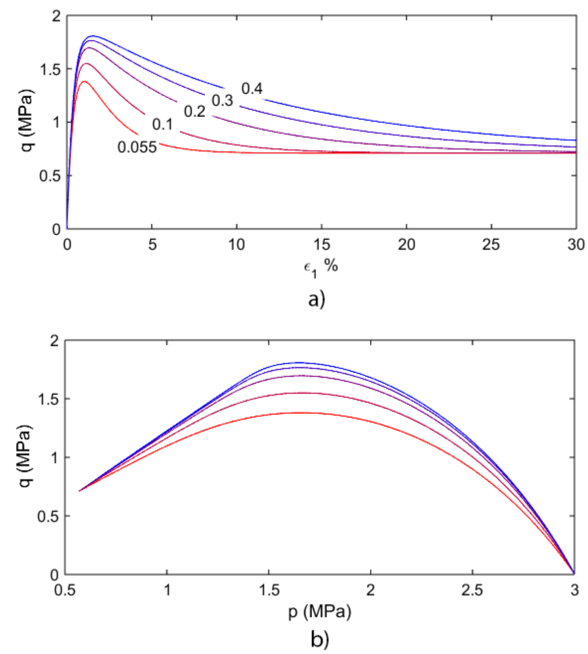


Figure 8 Influence of undrained dilatancy-state parameter used in a new state-dilatancy law for loose state, on the flow behaviour of loose states, a) stress-strain behaviour, b) stress path

9 REFERENCES

Been, K. and Jefferies, M.G., 1985. A state parameter for sands. Géotechnique, 35(2), pp.99-112.

Cheng, Z. and Jefferies, M., 2020, February. Implementation and Verification of NorSand Model in General 3D Framework. In Geo-Congress 2020: Geotechnical Earthquake Engineering and Special Topics (pp. 10-19). Reston, VA: American Society of Civil Engineers.

Dafalias, Y.F. and Manzari, M.T., 2004. Simple plasticity sand model accounting for fabric change effects. Journal of Engineering mechanics, 130(6), pp.622-634.

Jefferies, M. and Been, K., 2015. Soil liquefaction: a critical state approach. CRC press.

Jefferies, M.G., 1993. Nor-Sand: a simple critical state model for sand. Géotechnique, 43(1), pp.91-103.

Li, X.S., Dafalias, Y.F. and Wang, Z.L., 1999. State-dependant dilatancy in critical-state constitutive modelling of sand. Canadian Geotechnical Journal, 36(4), pp.599-611.

Lu, X. and Huang, M., 2015. Static liquefaction of sands under isotropically and K0-consolidated undrained triaxial conditions. Journal of Geotechnical and Geoenvironmental Engineering, 141(1), p.04014087.

Ng, C.W., Crous, P.A. and Jacobsz, S.W., 2023. Centrifuge and Numerical Modeling of Liquefied Flow and Nonliquefied Slide Failures of Tailings Dams. Journal of Geotechnical and Geoenvironmental Engineering, 149(9), p.04023075.

Razavi, S.K., 2023. Constitutive modeling of static liquefaction of tailings (Doctoral dissertation, in Preparation), Polytechnique Montreal, Montreal, QC.

Reid, D. and Smith, K., 2021. Interpretation of state parameter in partially drained tailings: a case history examination. Géotechnique Letters, 11(4), pp.276-280.

Roscoe, K.H., Schofield, A. and Thurairajah, A., 1963. Yielding of clays in states wetter than critical. Geotechnique, 13(3), pp.211-240.

Shuttle, D. and Jefferies, M., 2016. Determining silt state from CPTu. Geotechnical Research, 3(3), pp.90-118.