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TARGET ALLOCATION UNDER UNCERTAINTY DURING THE VEHICLE DEVELOPMENT PROCESS

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TARGET ALLOCATION UNDER UNCERTAINTY DURING THE VEHICLE DEVELOPMENT PROCESS

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Dedication

To the memory of my beloved father Chokri Jilali

To my mother Lannani Salha

To my wife Ilham and my son Jalal Abderrahmane

To my brothers and sisters

To my friends

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Résumé

L'industrie automobile est l'une des plus concurrentielles où les besoins des consommateurs et la technologie sont en changement perpétuel. Pour être compétitif dans ce marché, les constructeurs automobiles ont adopté une stratégie orientée clients où ils enquêtent en permanence sur les besoins des consommateurs pour déceler le plus tôt possible les performances désirées des futurs véhicules, concevoir et commercialiser des voitures innovatrices qui permettent de combler les attentes des consommateurs.

Le développement d'un nouveau véhicule suppose la traduction des performances utopiques, qui peuvent être définies par le département de marketing, dans des cibles pour les caractéristiques ingénieurs de ses composantes. Cette approche nécessite lors de la conception du véhicule de prendre des décisions critiques qui peuvent influencer considérablement la compétitivité et la profitabilité de la compagnie.

Pendant les premières étapes du processus de développement des véhicules (PDV), les ingénieurs manquent souvent d'informations précises et complètes qui peuvent leurs permettre de prédire les possibilités de rencontrer les performances utopiques du véhicule en question, et ce, à cause de plusieurs facteurs (technologiques, régulation et règlementation, ressources, etc.). Pour cette raison, l'identification, la quantification et la gestion des incertitudes inhérentes aux différentes phases du PDV sont devenues un problème majeur dont dépend l'efficacité du PDV.

La présente étude propose une méthodologie pour l'allocation des cibles et pour la prise de décision sous incertitudes durant le processus de développement des véhicules. La méthode commence par la décomposition du nouveau véhicule en une structure hiérarchique à multi-niveaux. Cette structure représente l'élément de base pour la définition du modèle de véhicule à multi-niveaux (MVM). Nous avons considéré qu'une ou plusieurs caractéristiques ainsi que leurs cibles peuvent être associées à chaque composante du MVM, et ce, en concordance avec les performances utopiques du véhicule. Les opinions des experts sont exprimées avec des incertitudes inhérentes à la faisabilité de chaque cible. Les opinions des experts sont données sous forme de distributions de probabilité ou d'ensembles d'intervalles associés à des crédibilités subjectives pour les valeurs possibles des caractéristiques. Ces opinions sont ensuite agrégées et propagées depuis les feuilles vers le sommet du modèle. La théorie des Évidences a été utilisée pour exprimer les incertitudes sous forme de deux mesures: la crédibilité et la plausibilité. En

utilisant ces informations, deux mesures concernant la désirabilité et la réalisabilité des caractéristiques ont été définies. Une approche pour l'allocation des cibles sous incertitudes dans le modèle du véhicule à multi-niveaux basée sur la maximisation des mesures de réalisabilité et désirabilité des caractéristiques est proposée.

Une méthode pour traiter les systèmes grands et complexes a été développée. Cette méthode consiste à réduire le nombre d'intervalles traités par leur fusion en contrôlant la granularité d'information, et ce, sans altérer les courbes de crédibilité et de plausibilité sur un ensemble discret de points. Cette méthode permet d'alléger la quantité des calculs et ainsi diminuer le temps nécessaire.

Un système de prise de décision basé sur l'intégration des techniques proposées pour la caractérisation des incertitudes et l'allocation des cibles dans le MVM est proposé. Ce système consiste à modéliser le processus de conception sous la forme d'une série de processus (étape et porte) parallèles qui alternent la génération des connaissances et la prise de décision. Itérativement, les valeurs des caractéristiques sont raffinées de manière à orienter le processus de conception vers un produit final désirable et réalisable.

Le fait d'inclure les incertitudes lors des processus d'allocation des cibles et de prise de décision durant le PDV présente des avantages potentiels pour les constructeurs automobiles. En fait, dans le cas de la conception de véhicule sous certitude, les caractéristiques du véhicule et de ses composantes sont supposées être connues à l'avance. Dans ce cas, le processus de conception se limite au choix d'une solution parmi un certain nombre de solutions déjà connues. Un tel processus peut donner naissance à des conceptions peu fiables qui ont été poussées à leurs limites en matière de fabrication, modélisation et conception. Conséquemment, l'équipe de conception peut manquer l'opportunité d'améliorer le produit au niveau désiré suite à la non-exploration de toutes les possibilités offertes, de satisfaire les attentes des clients et de rencontrer les objectifs de la compagnie. En contrepartie, inclure les incertitudes dans les processus d'allocation des cibles et de prise de décision implique l'exploration de nouvelles possibilités, la collecte de plus d'information et le développement de nouvelles connaissances. Cela mènera à de nouveaux concepts, de nouvelles conceptions et de nouvelles technologies qui permettront de combler les attentes des clients et en même temps l'atteinte des objectifs de la compagnie.

En conclusion, le présent travail représente un pas vers la formulation d'une méthodologie intégrée qui tient compte des incertitudes durant le PDV. Les méthodologies pour la caractérisation des incertitudes, l'allocation des cibles et la prise de décision sous incertitudes ont été développées pour la conception des systèmes complexes. Puisque le PDV est seulement une application spécifique du système préposé, ce denier peut être directement appliqué à n'importe quel secteur d'ingénierie concerné par le développement des systèmes complexes comme les industries aéronautique, aérospatiale et navale.

Abstract

Under the increasing pressure of the evolving customers' expectations, the speed and competitiveness of the competitors, automakers have become customer-oriented. They continuously survey the customers' needs in order to early identify the desired or utopian vehicle performances and strive to fulfill these expectations by designing and marketing quickly new innovative products. The development of a new vehicle supposes the translation of the vehicle performances into its components' characteristics. Such approach requires making critical design decisions that can impact noticeably the competitiveness and profitability of the company.

In the early stages of the vehicle development process, the engineers lack precise and complete information about the possibility to meet the initial utopian vehicle performances due to many factors (technological, regulation, resources, etc.). For that reason, identifying, quantifying and handling the inherent uncertainty throughout the vehicle development process (VDP) became a serious issue, which affects the effectiveness of the design process.

This study proposes a methodology for target allocation and decision-making under uncertainty during the VDP. The method starts by the decomposition of the vehicle in hierarchical multilevel structure, which represents the basic framework required for the definition of the vehicle multilevel model (VMM). We have considered that each component in the VMM may have several characteristics, and that a target is defined for every component and characteristic in accordance with the utopian vehicle performances. Experts' opinions are expressed with uncertainty regarding the feasibility of achieving each target. Experts' opinions are given in the form of probability distributions or intervals associated with their subjective beliefs for the possible values of the characteristics and then are aggregated and propagated from the leaf nodes of the multilevel model up to the vehicle level. Evidence theory has been used to express uncertainty in the form of belief and plausibility measures. Using this information, two measures regarding the desirability and the achievability of the characteristics are defined. An approach for targets allocation under uncertainty based on the maximization of achievability and desirability measures of the characteristics is proposed and discussed.

A methodology to handle large-scale problem based on the merging of intervals by the control of the information granularity without affecting the precision of the belief and plausibility measures is presented.

A decision-making framework based on the integration of both exposed techniques for uncertainty characterization and target allocation under uncertainty in the vehicle multilevel model is proposed. This framework consists in modeling the design process in the form of a series of parallel stage-gate processes that alternate knowledge generation and decision-making. Iteratively the characteristics are set and refined in such a way to orient the design process towards an achievable and desirable final design.

In brief, including uncertainty in target allocation and decision-making processes presents many potential benefits to the automakers. In fact, in the case of vehicle design under certainty, the characteristics of the vehicle and its components are supposed to be known with certainty and the design process is restricted to only a choice among few existing alternatives. Such approach may yield unreliable designs that are pushed to the limits of design constraints boundaries. Consequently the design team may miss the opportunity: to improve the design to a desired level because of lack of exploration of all offered possibilities, to satisfy the customers' expectations and to meet the company's goals. In return considering uncertainty in targets allocation and decision-making processes implies exploring new possibilities, collecting more information and developing new knowledge that leads to new concepts, new designs and technologies allowing at best the fulfilment of the customers' needs and the attainment of the company's objectives.

In conclusion, the present work represents a step in the formulation of an integrated methodology to take into account uncertainty during the early stages of the vehicle development process. The proposed methodologies and the approaches for uncertainty management, target allocation under uncertainty and decision-making under uncertainty were developed for the design of complexes systems. Since, the VDP is only a specific application of the proposed systems. This later can be directly applied to any engineering field concerned by the development of complex system such as aeronautical, aerospace and naval industries.

Condensé en français

Introduction

Dans le monde de l'industrie automobile, le développement d'un nouveau véhicule qui satisfait les besoins des consommateurs passe par un processus itératif et complexe dit processus de développement des véhicules (PDV) visant à combler les attentes des clients. Ce processus alterne deux activités principales: la génération des connaissances et la prise de décision.

Au début du PDV, le nouveau véhicule est modélisé sous forme d'un modèle de véhicule à multi-niveaux (MVM) où il est décomposé en systèmes, sous-systèmes et pièces. La décomposition du véhicule peut être étendue à plusieurs niveaux dépendamment des besoins des concepteurs. Un ensemble de caractéristiques du véhicule désirées et leurs estimations définissent les objectifs utopiques du processus de conception. Ces caractéristiques sont passées à l'équipe de conception qui a pour première tâche de les traduire sous forme de cibles pour les caractéristiques ingénieurs aux niveaux des systèmes, sous-systèmes et pièces du véhicule. Les cibles des caractéristiques servent à guider l'équipe de conception et à orienter le processus de conception vers les niveaux les plus désirables qui peuvent être réalisés. L'allocation des cibles des caractéristiques de chaque composante du véhicule doit être consistante avec les caractéristiques du véhicule et doit respecter les couplages et les contraintes entre les systèmes, sous-systèmes et pièces.

À chaque itération du PDV, pour prendre des décisions cruciales qui influencent la valeur du nouveau véhicule, il est nécessaire de collecter des informations pertinentes au niveau des feuilles du MVM sous forme des opinions des experts. En fait, les experts ont le contrôle seulement sur les caractéristiques ingénieurs qui représentent les spécifications techniques des composantes du véhicule. Les caractéristiques véhicules sont obtenues par la combinaison des caractéristiques ingénieurs à travers des fonctionnelles.

Les opinions des experts sont générées en fonction des cibles, du niveau de progrès réalisé pendant les itérations précédentes du PDV et l'ensemble des connaissances disponibles. En fait, les experts basent leurs opinions sur une grande variété de sources d'information (analyse des résultats numériques de modèles mathématiques avec différents niveaux de fidélité, données historiques des véhicules actuels ou passés, étude comparative des véhicules des concurrents, objectifs d'amélioration des performances du véhicule).

Les opinions des experts sont aussi caractérisées par des incertitudes provenant de plusieurs sources à savoir: les variables aléatoires de conception, le manque d'information concernant l'évolution technologique et les processus de fabrication, les spécifications incomplètes des composantes et les interactions entre elles.

Les opinions des experts sont générées de façon relativement indépendante en considérant les cibles fournies à tous. Les cibles sont raffinées et les couplages entre les caractéristiques des composantes sont résolus itérativement en favorisant les compromis pour des conceptions désirables et réalisables.

Comme mentionné précédemment, l'incertitude est inévitable tout au long du PDV et affecte grandement la réalisabilité des projets de conception. De ce fait, l'efficacité du PDV dépend du niveau de son contrôle. Puisque les compagnies sont en recherche continuelle de nouvelles méthodes de développement pour faire face à la concurrence, l'incertitude est devenue un problème majeur durant le processus de conception. Tenir compte des incertitudes présente une opportunité de gérer les risques associés à la réalisabilité et la considération ou le rejet des solutions possibles en se basant sur cette mesure et la valeur ajoutée du produit.

Dans le présent projet, nous allons étudier les sujets suivants : les méthodologies de caractérisation des incertitudes, l'allocation des cibles et la prise de décision sous incertitudes durant le PDV. Nous allons développer une méthodologie pour l'allocation des cibles des spécifications du niveau de véhicule vers les niveaux les plus bas du MVM (systèmes, sous-systèmes et pièces) et nous allons développer une stratégie pour la prise de décision. Ceux-ci vont inclure une méthodologie spécifique pour la prise en compte des incertitudes inhérentes à l'avancement du processus de développement.

Axe de recherche

Cette recherche vise le développement d'une approche capable d'inclure les incertitudes durant le processus de conception des systèmes complexes. Cela est dans le but d'investiguer l'intérêt et l'impact de l'inclusion des incertitudes dans le processus de conception. L'évaluation des bienfaits et de l'impact de l'inclusion des incertitudes nécessite l'application de l'approche proposée à un produit réel et puisque seulement de simples exemples ont été utilisés, cette évaluation est considérée hors étendue du projet. Cependant, des observations préliminaires sur l'intérêt et l'impact de l'inclusion des incertitudes sur le processus de conception vont être présentés.

Objectifs

L'objectif principal de ce projet est le développement d'une méthodologie pour l'allocation des cibles sous incertitudes comme composante d'un processus de prise de décision. Pour cela, trois objectifs spécifiques ont été identifiés :

1. Définition, implémentation et validation d'une approche pour caractériser les incertitudes.
2. Définition, implémentation et validation d'une méthodologie pour l'allocation des cibles sous incertitudes.
3. Proposition d'une stratégie de prise de décision pour l'allocation des cibles durant les itérations du processus de conception.

Dans le contexte de ce projet, l'approche proposée sera supportée par le processus de développement des véhicules. Conséquemment une terminologie spécifique reliée au processus de développement des vehicules sera utilisée.

Méthodologie

Cette thèse comporte six chapitres et quatre annexes qui traitent des objectifs susmentionnés. Deux de ces chapitres sont des articles de journal soumis avec leur propre résumé, introduction, recherche de littérature, méthodologie, résultats, discussion et liste de références. Suite à ces chapitres, une discussion et une conclusion générale concernant la réussite des objectifs ainsi qu'un certain nombre de recommandations pour des études futures sont présentés.

La liste des articles soumis au cours de ce doctorat considéré pour la publication sont:

- A. Chokri, J-Y. Trépanier, C. Tribes, P. Fenyes, and S. Gu, "Managing uncertainty in a multi-characteristic vehicle multilevel model," Journal of Computing and Information Science in Engineering, (Soumis le 10 Décembre 2010).
- A. Chokri, J-Y. Trépanier, C. Tribes, P. Fenyes, and S. Gu, " Target allocation under uncertainty in a multi-characteristic vehicle multilevel model," Journal of Computing and Information Science in Engineering, (Soumis le 20 Décembre 2010).

Un autre article de conférence élaboré en collaboraiom avec Dr. Christophe Tribes est présenté sous forme d'annexe D.

- C. Tribes, A. Chokri, J-Y. Trépanier, P. Fenyes, and S. Gu, " Propagation and Merging of Uncertain Expert Opinions in a Hierarchical Multilevel System,"AIAA MDO Conference 2010, FortWork, Texas, USA.

Contributions et discussion

Comme il a été mentionné auparavant, l'objectif principal du projet est le développement d'une méthodologie pour l'allocation des cibles et pour la prise de décision sous incertitudes, et ce, dans le but de supporter le processus de développement des nouveaux véhicules. Nous présumons que la réalisation de ces objectifs va permettre d'améliorer l'efficacité du processus de conception. En fait, la méthodologie proposée va orienter le processus de conception en identifiant tôt les meilleurs compromis réalisables. De cette manière, les erreurs vont être évitées et le retravail épargné. De plus, la méthodologie est supposée aider à distinguer entre les composantes les plus difficiles à réaliser de celles qui ne le sont pas. Sous réserve d'ajouter une composante pour l'allocation des ressources à la méthodologie proposée, cette méthode pourrait aider à mieux allouer les ressources de manière équilibrée pour maximiser les possibilités de réalisation de toutes les composantes.

Pour répondre aux objectifs susmentionnés, nous avons été amenés à examiner les sujets suivants: la caractérisation des incertitudes durant le processus de conception, l'allocation des cibles sous incertitude, la prise de décision sous incertitude.

- **Caractérisation des incertitudes durant le processus de conception**

La méthode proposée pour la caractérisation des incertitudes est basée sur le principe de décomposition du véhicule en une structure hiérarchique qui constitue le squelette du modèle de véhicule à multi-nivaux (MVM). Au niveau supérieur du modèle, les performances utopiques du véhicule sont définies puis cascadées sous la forme de caractéristiques ingénieurs vers les niveaux inférieurs des systèmes, sous-systèmes et pièces. Au niveau des feuilles du modèle, les experts expriment leurs opinions sous la forme de groupes d'intervalles et des crédibilités associées. Cette manière d'expression est une représentation naturelle de l'expérience et des prévisions des experts leur permettant d'inclure différents types d'incertitude sans aucun besoin de les distinguer. La théorie des Évidences a été choisie et utilisée pour l'agrégation et la propagation des incertitudes dans le MVM. En fait, cette théorie permet de gérer des données sous forme d'intervalles ou de distributions de probabilités. De plus, elle permet aussi de tenir comptes des conflits entre les opinions des experts.

- **Allocation des cibles sous incertitudes**

Le processus d'allocation des cibles sert à propager des spécifications désirables au niveau du véhicule à tous les niveaux du MVM. Notre approche pour l'allocation des cibles consiste à représenter le véhicule sous forme d'un système de prise de décision qui sera dirigé par des objectifs mesurables et des contraintes vis-à-vis aux spécifications du véhicule.

La méthodologie proposée d'allocation des cibles consiste à maximiser la désirabilité et la réalisabilité des caractéristiques. La réalisabilité des caractéristiques est basée sur les mesures de plausibilité et de crédibilité qui sont déterminées à partir des opinions des experts propagés dans le MVM.

Plusieurs approches pour l'allocation des cibles basées sur la désirabilité et la réalisabilité des caractéristiques ont été présentées dans la référence (Chokri, A. et coauteurs, 2009). Cependant, seulement la formulation utilisant un objectif unique (Global Utility of Design) a été appliquée dans ce projet. Les autres approches peuvent présenter des opportunités pour supporter les processus de développement des nouveaux produits pour un éventail de compagnies qui ont des cultures d'entreprise très distinctes. Par conséquent, l'exploration des autres approches présente une bonne avenue pour des recherches futures.

- **Prise de décision sous incertitude durant le processus de conception**

La stratégie de prise de décision sous incertitude consiste à modéliser le processus de développement du véhicule sous la forme d'une série de processus (étape/porte), un processus pour chaque composante du produit. Ces processus alternent la génération de la connaissance et la prise de décision. Le système proposé pour la prise de décision intègre les méthodologies de caractérisation des incertitudes et d'allocation des cibles sous incertitude. D'une manière itérative, les cibles des caractéristiques sont raffinées de manière à orienter le processus de développement vers un produit final désirable et réalisable.

- **Stratégie de traitement des problèmes grands et complexes**

Lors de la conception des systèmes grands et complexes (taille de véhicule réel), la quantité d'information fournie par les experts et propagée dans le modèle à multi-niveaux peut devenir rapidement énorme au point qu'il serait difficile voir impossible de la traiter avec les moyens disponible. Par exemple dans un modèle à multi-niveaux constitué de 7 systèmes, 42 sous-systèmes et 252 pièces; le nombre d'intervalles propagés au niveau du véhicule est de 2.89e76 si seulement 2 intervalles sont collectés par pièce. Ce nombre passera à 1.71e120 si 3 intervalles sont collectés par pièce.

Dans ce cas, la stratégie proposée pour traiter de tel problème consiste dans la réduction du nombre des intervalles propagés en les fusionnant tout en contrôlant la granularité de l'information, et ce, sans altérer les mesures de plausibilités et de crédibilités sur un certain nombre de points.

Conclusion et recommandations

Cette étude a permis d'adresser le problème d'inclusion des incertitudes dans le processus de développement des nouveaux véhicules. Elle couvre le management des incertitudes, l'allocation des cibles sous incertitude, la prise de décision sous incertitude et l'intégration de l'ensemble de ces concepts dans un seul système qui va supporter le processus de développement des véhicules chez GM.

Le présent travail constitue un pas vers la formulation d'une méthodologie intégrée qui tient compte des incertitudes dans les premières étapes du processus de développement des véhicules. Il est constitué de trois contributions principales.

La première contribution concerne le développement d'une méthodologie pour caractériser les incertitudes. La méthode consiste à représenter le véhicule sous la forme d'un modèle à multi-niveaux ou les cibles des caractéristiques sont cascadées dans le modèle. Des experts aux niveaux des feuilles du modèle fournissent leurs opinions en fonction de la faisabilité des cibles sous forme d'intervalles associés à leur crédibilité subjective. Ces opinions sont agrégées et propagées du bas vers le haut du modèle. La théorie des Évidences a été utilisée pour propager les mesures de plausibilité et de crédibilité.

La deuxième contribution se rapporte au développement d'une méthodologie pour l'allocation des cibles sous incertitudes. Cette méthodologie utilise les mesures de plausibilité et crédibilité.

Deux nouvelles mesures de désirabilité et réalisabilité des caractéristiques sont introduites. La réalisabilité est définie à partir des mesures de plausibilité et crédibilité. L'approche pour l'allocation des cibles sous incertitudes « GUD » est basée sur la maximisation de la désirabilité et la réalisabilité du produit final. Elle consiste à cascader des problèmes d'optimisation du haut vers le bas permettant ainsi de définir les cibles des caractéristiques.

La troisième contribution concerne le développement d'un système de prise de décision qui intègre les méthodologies de management d'incertitude et d'allocation des cibles. Le processus de prise de décision a été modelé sous forme d'une série de processus parallèles (étape-porte) qui alterne itérativement la génération de connaissance et la prise de décision.

Une contribution additionnelle a été élaborée en collaboration avec Dr. Christophe Tribes visant le développement d'une méthodologie permettant de traiter les systèmes grands et complexes. La méthode consiste à la réduction du nombre des intervalles propagés en les fusionnant tout en contrôlant la granularité de l'information, et ce, sans altérer les mesures de plausibilités et de crédibilités sur un certain nombre de points.

Les méthodologies proposées pour le management des incertitudes, l'allocation des cibles sous incertitudes et de prise de décision sous incertitude ont été développées sous un angle général que l'on peut appliquer à la conception de toute sorte de système complexe provenant d'autres domaines d'ingénierie tel que l'aéronautique, l'aérospatiale et le naval, etc.

Notre méthodologie peut être étendue pour contenir d'autres concepts dont nous n'avons pas traité dans cette thèse. Dans le cadre des travaux futurs, nous recommandons d'adresser les sujets suivants:

- Exploration d'autres moyens pour orienter le processus de conception basé sur l'approche proposée via d'autres paramètres tel que le facteur de prise de décision Φ qui représente la tolérance au risque lors des décision d'allocation des cibles.
- Les opinions des experts sous la forme d'intervalles et des crédibilités associés est une manière simplifiée pour représenter les connaissances des experts. Cette représentation est purement quantitative supposant que les experts sont capable de quantifier n'importe qu'elle information même leurs visions ou sentiments. La quantification de ce type d'information implique sa distorsion et la perte de sa qualité

ce qui pourra mener à des conceptions non satisfaisantes. De ce fait, l'investigation de nouvelles manières de représentation des opinions des experts pourrait mener à de meilleures conceptions.

- L'allocation des ressources sous incertitude doit être investiguée dans le but de développer une méthodologie basée sur un modèle mathématique qui supporte la méthodologie de prise de décision proposée et remplace les méthodes d'allocation des ressources qui sont basées sur les estimations et l'expérience.
- Explorer la possibilité de supporter des opinions d'experts qui sont fournis par des modèles d'analyses ou l'incertitude est complètement probabiliste. Cela va permettre l'intégration de la méthodologie proposée et les outils d'analyse existants et de la comparer avec les méthodes existantes.

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List of abbreviations

ATC: Analytical Target Cascading

BBA: Basic Belief Assignment

Bel: Belief

BPA: Basic Probability Assignment

CO: Collaborative Optimization

CPT: Characteristic Propagation Tree

DBD: Decision-Based Design

DOM: Document Object Model

EO: Expert's Opinion

GA: Genetic Algorithm

GM: General Motors

GUD: Global Utility of Design

HP: Horse Power

Kg: Kilogram

MAUT: Multi-Attribute Utility Theory

Min_{DesVal} and Max_{DesVal}: The bounds of desirability interval

MCA: Multi-Characteristic Achievability

MCDA : the Multi-Criteria Decision Analysis

MCS: Monte Carlo simulation

MCPP: Multi-characteristic Probability Product

MDO: Multidisciplinary Design Optimization

MPP: Most Probable Point

MVM: Modèle de véhicule à multi-niveaux

NPD: New Product Development

NPDP: New Product Development Process

PDF: Probability Density Function

PDV: Processus de Développement des Véhicules

PEO: Propagated Expert's Opinion

PERT: Program Evaluation and Review Technique

Pl: Plausibility

QFD: Quality Function Deployment

TAU: Target Allocation under Uncertainty

VC: Vehicle Characteristic

VDP: Vehicle Development Process

VMM: Vehicle Multilevel Model

XML: Extensible Markup Language

List of symbols

A_i : Focal element given by the experts

$A_j^{Bel>}$, $A_j^{Pl>}$, $A_j^{Bel<}$, $A_j^{Pl<}$: Achievability measures

B : An event

$C_i^{Component}$: Characteristic 'i' of the component

C_i^{Sj}: Characteristic 'i' of the system 'j'

C_i^{SSj}: Characteristic 'i' of the subsystem 'j'

$\mathrm{C}_{\mathrm{power}}^{\mathrm{Eng}}$: Power characteristic of the engine

$C_{power}^{Eng^*}$: Selected point of reference for the power characteristic of the engine

D_j: Desirability measure

EO_{C_i}: Expert Opinion of the characteristic 'i'

f: A functional

GLI_0: Global length of the initial interval at the beginning of the VDP

GLI_i: Global length of the expert opinion interval at the iteration “i”

I_i: Interval 'i'

l, u: Lower and upper bounds of intervals

k_0: Initial value of the kurtosis excess factor

k_{it}: Kurtosis excess factor value at the iteration "it"

$m_j(A_i)$: Basic Belief Assignment for the focal element A_i

n_0: Initial maximum number of intervals that can constitute the expert's opinion.

PEO_{C_i}: Propagated Expert's opinion of the engineering characteristic 'i'

PEO_{VC_i}: Propagated Expert's opinion of the vehicle characteristic 'i'

$S^{Component}$: Shared parameter

sb_i: Subjective belief 'i'

T_{C_i} : Target of the characteristic 'i'

T_{VC_i} : Target of the vehicle characteristic 'i'

U_j: Utility function

V: Vehicle speed in (km/h)

$VC_{fueleconomy}$: Fuel economy characteristic of the vehicle

$\mathrm{VC}_{\mathrm{Fe}}^*$: Selected point of reference for the fuel economy characteristic of the vehicle

VC_{mass}: Mass characteristic of the vehicle

$\mathrm{VC}_{\mathrm{mass}}^{*}$: Selected point of reference for the vehicle mass

$\mathrm{VC}_{\mathrm{time}}$: Time needed to accelerate from rest to $\mathrm{V} = 100$ km/h

w_j : Scaled weight assigned according to the reliability of the sources

X : Frame of Discernment

$X^{\mathrm{Component}}$: Design parameter of the component

Z : Set of the various propositions that the experts can express

α, β: Weighting factors for the effect of variation of the engine power and the vehicle mass on the fuel economy

δ_k: Parameter equal to 1 or 0

Φ: Decision factor

List of appendices

Introduction

Context

Developing a new vehicle that satisfies consumer's needs is accomplished through a complex iterative vehicle development process (VDP) aiming the allocation of the characteristics targets and the needed resources. This process alternates knowledge generation and decision-making leading to a satisfying and feasible design.

At the beginning of the VDP, the overall vehicle is modeled as a vehicle multilevel model (VMM) where it is decomposed into a number of systems. The systems are further decomposed into subsystems, which in turn are decomposed into parts and so on. Hence, the VMM can be viewed as a hierarchical tree of vehicle components. A set of desirable vehicle characteristics and their estimated values, which may be identified by the marketing department, defines the utopian goals of the design process. These characteristics values/targets are passed on to the design team, whose first task is to translate them into engineering characteristics targets for the systems, subsystems and parts. The targets serve to guide the design teams and to orient the process towards the highest desirable design that can be achieved. The target allocation for each component of the vehicle must be consistent with the vehicle characteristics and must respect the couplings and constraints among systems, subsystems and parts.

At any iteration, to make crucial decisions impacting the value of the new vehicle, the VDP requires collecting relevant information at the leaf nodes of the VMM in the form of experts' opinions because the engineers have control only over the engineering characteristics. The latter represent the technical specifications of the vehicle components. The vehicle characteristics result from the combination of the engineering characteristics through functional relationships.

The experts' opinions are generated depending on the targets, the progress performed in the previous iteration of the VDP and the available knowledge. In fact, the experts base their opinions on a wide variety of sources (analysis results from computer models of various levels of fidelity, historical data for current and past vehicles, benchmark of competitive vehicles and selected goals for the performance improvements). However, the experts' opinions are tainted by uncertainty because several aspects are inherently uncertain during the design of complex systems owing to many factors such as aleatory design variables, lack of information about evolving technologies and manufacturing processes, incomplete specification of components and interactions amongst vehicle components.

The experts' opinions are generated independently, in the sense that each expert aims his/her own targets with limited interactions during the design progress of the other experts. Iteratively, the targets are refined and couplings between components' characteristics are resolved. Indeed, the iterativity of the VDP fosters communication and discussion among engineers at specific stages of the process leading to implicit consensus in the form of desirable and feasible trade-off expressed in target selection.

As mentioned previously, uncertainty is inevitable throughout the VDP. It greatly affects the realizability of a design and may conduct to unforeseen rework. Thus, the effectiveness of the VDP depends on the level of its control. Until recently, only aleatory uncertainty was explicitly considered whereas other forms of uncertainty were included implicitly without using a rigorous mathematical formalism. Since companies are continuously in search of better design methods to face the competition and the market pressure, the uncertainty became a major concern during the design process. Taking into account the uncertainty presents an opportunity to better manage risks associated to realizability and to consider/reject possible candidates' solutions based on this measure and the value of the product.

Research focus

Many commonalities can be discerned between the VDP and other engineering design processes. One of the key aspects is the inevitable presence of uncertainties. Hence, the present research focuses on the management of uncertainties and its impact on the design process of complexes engineering systems. In this case, managing uncertainty first implies a formal mathematical formalism characterization and then to control its reduction by selecting proper targets. A realistic evaluation of the impact and benefit of inclusion of uncertainties requires the application to a complex product. Since, for practical reasons only simplified examples are available, this evaluation is out of the scope of this project. However, preliminary observations can be provided.

Presuppositions

The context of this study being the vehicle development process, a terminology stemming from the automotive is used throughout of this document for the sake of practicality. Nevertheless, this research project can be generalized to other complex design processes verifying a series of not restrictive presuppositions. These presuppositions have been identified during consultation with our industrial partner and constitute the starting point for the project. Some of the reasons explaining the following presuppositions are briefly discussed. Please note that detailed discussions about some specific aspects are provided along this document.

- ***Presupposition 1:***

Complex system such as a vehicle can be modeled in the form of hieratical tree structure.

A common way to decompose a complex product in engineering design is in the form of hierarchical tree structure where the nodes of the structure represent the systems, subsystem and parts of the product. Each node of the structure is characterized by a set of characteristics.

- ***Presupposition 2:***

The experts are responsible of component design and are the only source of information available for target allocation.

Design variable for all components are under experts' responsibility. At any stage of the design process, the experts can select their own source of information to conduct component design. Also, it is considered as part of the experts work to design components with an acceptable probability of failure.

- ***Presupposition 3:***

The experts' opinions are in the form of set of intervals and their associated subjective beliefs (confidence measure).

Intervals and subjective beliefs are a common and natural way to express uncertain information.

- ***Presupposition 4:***

All types of uncertainty are included in the experts' opinions.

The experts' opinions are the result of a process of synthesis of the available information. Depending on the level of experts' knowledge and the sources of information used, the nature of uncertainty varies (epistemic, aleatory or interaction). We consider that all types of uncertainty are combined and no decomposition is available.

- ***Presupposition 5:***

The development process is iterative and oriented by the characteristics targets.

Characteristics targets are defined at the beginning of the development process. These targets are provided to the experts who iteratively make progress, evaluate the feasibility of each characteristic and make adjustments until the obtainment of the desired product.

- ***Presupposition 6:***

The development process starts with high level of uncertainty that decreases as the process progresses.

At the beginning of the development process, due to the lack of sufficient and precise information on the specifications of the vehicle, the material and manufacturing processes, the uncertainty is naturally high. As the development progresses, new knowledge is generated and experts' opinions are refined and consequently reducible uncertainty is diminished.

Research questions

The dissertation is organized in a manner to answer comprehensively the following questions considering the presuppositions presented in the previous section:

- ***Question 1:***

How to characterize explicitly uncertainties throughout a hierarchical tree of components?

- ***Question 2:***

How to allocate the product performance characteristics targets at top level and how to cascade them into the form of engineering characteristics targets in lowest levels of the multilevel model during the development process?

- ***Question 3:***

How to orient the development process in order to maximize the value of the product?

Objectives of the research

The main objective of this project is to develop a methodology for target allocation under uncertainty as part of a decision-making process. For that, three specific objectives have been identified:

1. Define, implement and validate approaches for the characterization of uncertainties.
2. Define and implement a methodology for target allocation under uncertainty.
3. Propose a decision-making strategy for target allocation during iterations of the design process.

Research contributions

The scientific contribution of this thesis is three fold where all the contributions relate to the management of uncertainty and its impact on the targets allocation and decision-making during the vehicle development process. Each contribution represents the achievement of one of the three aforementioned specific objective and the three together participate to achievement of the global objective of the project. These contributions are presented as follow:

The first contribution consists in the development of a methodology for the characterization of uncertainty in design process. This methodology consists in modeling the vehicle in the form of a multi-characteristics multilevel model and applying the principles of Evidence theory for the aggregation and the propagation of uncertain information provided by the experts at the leaf nodes of the model. The uncertainty is synthesized at each node of the model by two measures, the belief and the plausibility. Chapter 3 provides a comprehensive presentation of this methodology.

The second contribution consists in the development of a methodology for targets allocation under uncertainty in the multi-characteristics multilevel model. The method demonstrates how to rigorously allocate system, subsystem and parts targets while actively managing the uncertainties associated with a vehicle program. The targets allocation is performed by taking into account two conflicting dimensions: the customers’ expectations and the engineers concerns about achievement of the product. This methodology is presented in details in Chapter 4.

The third contribution consists in the proposition of a strategy for decision-making under uncertainty during the iterations of the design process. This strategy integrates both methodologies for uncertainty management and target allocation in the multi-characteristics multilevel model. It consists in an iterative process that alternates knowledge generation and decision-making. This strategy is presented in Section 5.4.

An additional contribution, developed in collaboration with Dr. Christophe Tribes, concerns the development of intervals merging technique that facilitates the handling of large amount of information in the case of large-scale system. It consists to reduce the number of intervals handled by controlling the information granularity while keeping the accuracy of the uncertainty measures on a given discrete set of characteristic values. This contribution is presented in Appendix D.

Dissertation overview

This dissertation is organized in six chapters and four appendices with an introduction and a conclusion. Chapters 3 and 4 are in the format of journal papers with their own introduction, literature review, methodology, results, discussion and list of references.

- A. Chokri, J-Y. Trépanier, C. Tribes, P. Fenyes, and S. Gu, “Managing uncertainty in a multi-characteristic vehicle multilevel model," Journal of Computing and Information Science in Engineering, (Submitted on december 10th, 2010).
- A. Chokri, J-Y. Trépanier, C. Tribes, P. Fenyes, and S. Gu, " Target allocation under uncertainty in a multi-characteristic vehicle multilevel model," Journal of Computing and Information Science in Engineering, (Submitted on december 20th, 2010).

Another conference paper elaborated in collaboration with Dr. Christophe Tribes is presented in Appendix D.

- C. Tribes, A. Chokri, J-Y. Trépanier, P. Fenyes, and S. Gu, " Propagation and Merging of Uncertain Expert Opinions in a Hierarchical Multilevel System," AIAA MDO Conference 2010, FortWork, Texas, USA.

In the Introduction, we present an overview of the problem, the research focus, the research questions and presuppositions, the objectives and the organization of the dissertation.

In Chapter 1, we provide a literature review on the vehicle development process, the foundational concepts for uncertainty management, the target allocation and the decision-making under uncertainty. We conclude the chapter by positioning our research project and scientific contributions compared to the state of the art.

In Chapter 2, we present the research methodology pursued in the project to achieve the different objectives.

In Chapter 3, we propose a methodology to decompose a vehicle in a multi-characteristics multilevel model and to manage uncertainty using the Evidence theory (Evidence theory itself is presented in Appendix B). In Section 5.2, we validate the proposed methodology for uncertainty management by the mean of a Monte Carlo simulation proposed in Appendix C.

In Chapter 4, we present a methodology for targets allocation in the multi-characteristics vehicle multilevel model. In Section 5.3, we verify the proposed methodology for targets allocation by comparison of its results to those of a methodology with a probabilistic approach and using the Monte Carlo simulation presented in Appendix C.

In Chapter 5, we present the validation and verification aspects of the proposed methodologies for uncertainty management and target allocation. We propose also a strategy for decision-making under uncertainty in Section 5.4. This strategy consists in the integration of the methodologies of uncertainty management and target allocation under uncertainty presented consecutively in Chapters 3 and 4. In this chapter, we point out also the problem of large-scale system and we propose a methodology that can handle such problem (see Appendix D).

In Chapter 6, we present a general discussion on different aspects of the project.

The dissertation is concluded with a summary of the research contributions and the recommendations for future works.

Chapter 1 Literature review

In this chapter a comprehensive literature review is presented on the vehicle development process (Section 1.1), the uncertainty management (Section 1.2), the target allocation and the decision-making under uncertainty in (Sections 1.3 and 1.4) respectively.

1.1 Vehicle development process

The VDP is a specific case of new product development process applied to the automotive industry [1, 2]. The VDP means different things to different people even to the persons responsible of the vehicle development and can be viewed from different perspectives because of the diversity of activities that it requires [3]. The VDP can be considered as a complex multi-phases process that consists in a set of activities (research, design, choices, decisions, etc.) that must be steered and coordinated in order to translate the customer requirements in a final product [4, 5]. The VDP starts with the identification of the customers' expectations which are then translated into the form of vehicle performances targets and cascaded down in the form of engineering characteristics to the systems, subsystems, components and parts of the vehicle.

Like any product development process, the VDP has evolved under the imperative and pressures of the automotive market, which is more and more competitive.

In the early decades of the automotive industry history, the phases of the VDP were sequential. This implies frequent reworks were required, low quality and high costs of development were occasioned. As a result, long time to market and less commercial advantage were guaranteed to the automakers [6, 7]. To overcome the drawbacks of the sequential VDP, the phases became concurrent. This helps the automakers to shorten the development cycle, minimize the reworks, improve the quality, reduce the development cost and consequently answer more effectively to the customers' expectations [6-8]. However, this new approach of conducting the VDP raised new technical and managerial problems and challenges that cannot be handled by a human without specialized methods and tools. Recently, with the apparition of the information technology, many software applications were developed and deployed through the automakers organizations. These new tools support all activities of the VDP and have proved their effectiveness in improving all its aspects. The VDP aims the following two objectives:

- The determination of targets for the vehicle performances specifications and the corresponding engineering characteristics of its components that will meet the customers' requirements.
- The determination and allocation of the needed resources to maximize the likelihood of fulfillment of the customers' expectations.

In practice, qualified experts participate in achieving these two objectives. They provide their opinions based on their previous experiences with similar situations and considering the customers' expectations.

Since only partial or imprecise information about the material, manufacturing process and candidate technologies for the new vehicle is available, the experts have incomplete knowledge of the possible technical specifications of the final product and consequently their opinions are tainted with different types of uncertainty. In a recent paper, Cafeo and coauthors [9] discussed the importance of taking into account uncertainty in the VDP. Their paper proposes to use the Decision Analysis Cycle together with uncertainty information managed using the probabilistic framework. They point out the need to later extend this process to include other theories for uncertainty management, such as Evidence theory or Possibility theory, to better account for all kinds of uncertainties. The Decision Analysis Cycle concept includes an informational phase where managers decide to reduce uncertainties in specific aspects of the design based on the value of information - an estimate of the value of eliminating uncertainty versus the cost of its elimination. However, the paper is not clear on how to estimate the value of eliminating uncertainties.

1.2 Uncertainty management

1.2.1 Sources of uncertainty

As mentioned previously, the VDP begins with the strategic vehicle performances that may be suggested by the marketing service. Then, the experts seek the specific systems, subsystems and parts characteristics that will satisfy these strategic requirements.

At the early stage of the VDP (conceptual phase), the vehicle is only an abstraction and the characteristics are approximated because of the uncertain design environment of innovative vehicle, which often requires new technologies, new materials, new manufacturing processes and even new design techniques. For that reason, the engineers face various uncertainties and are constrained to develop new knowledge in order to deal with this uncertain environment. Their greater challenge comes from specifications and attributes uncertainties inherent to the product and process. The sources of these uncertainties are related to the sources of information used by the engineers. Many authors [10-13] agree that there are mainly two sources of information that could potentially be accounted for during the design process: (1) engineering analysis, modeling and simulations and (2) “archived” experiences. The first source of information enables simulation-based (model-based) design optimization and is now routinely used due to the availability of numerical simulation analysis tools. The second source of information comes in many forms including empirical data, knowledge databases, interpolation or extrapolation, rules-of-thumb, design handbooks and guidelines. Furthermore, many decisions made in the process are based upon individual or corporate experience knowledge that is not formally archived in a database. This type of information can be approximated by experts in the form of set of intervals, which constitute a natural way for them to express their opinions when they lack precise or complete knowledge about a situation. In addition, these sources of information are potentially tainted with imprecision or uncertainty. As mentioned in Ref. [14], the benefit of distinguishing the nature of uncertainty includes an improved interpretation of the consequences of uncertainty on the system’s possible behaviors and an improved ability to allocate resources to decrease, if necessary, the system uncertainty (or risk). Consequently, taking into account the uncertainty during the VDP may represent an opportunity for engineers to improve the vehicle design through the exploration of more candidates solutions which were not considered before [11, 15, 16].

The identification of the sources of uncertainty is the key to develop a general methodology to quantify epistemic uncertainty, variability, interaction and ambiguity. Any proposed tool for target allocation under uncertainty must be able to incorporate these sources of information and the inherent uncertainties.

1.2.2 Uncertainty definitions, classifications and representations

For a long time, the uncertainty was used to encompass a multiplicity of concepts such as error, ambiguity, imprecision, etc. It is only during the last decades that a particular attention was given to this concept when specific problems were manifested to engineers such as: the need of optimization of consumed resources, the needs of control of the industrial processes and prediction of the systems' behaviors. Many research works have been driven to provide deep understanding of the subject. A comprehensive description and classification of uncertainty is presented in Refs. [10, 14, 17].

Several authors such as Der Kiureghian [18], Isukapalli and co-authors [19], Haukaas [16], Oberkampf and co-authors [20], Thunnissen [15], Agarwal and al. [21] and Nikolaidis [22] have offered many comprehensive taxonomies of uncertainty. These authors recognize many distinct kinds of uncertainty with considerable subtlety. Each one of them brings a classification that meets the specific needs of his activity field.

In this document we present only the classification proposed by Thunnissen [15]. We consider that this classification responds to our requirements for the VDP. Moreover, it was especially elaborated for the development of complex and multidisciplinary systems.

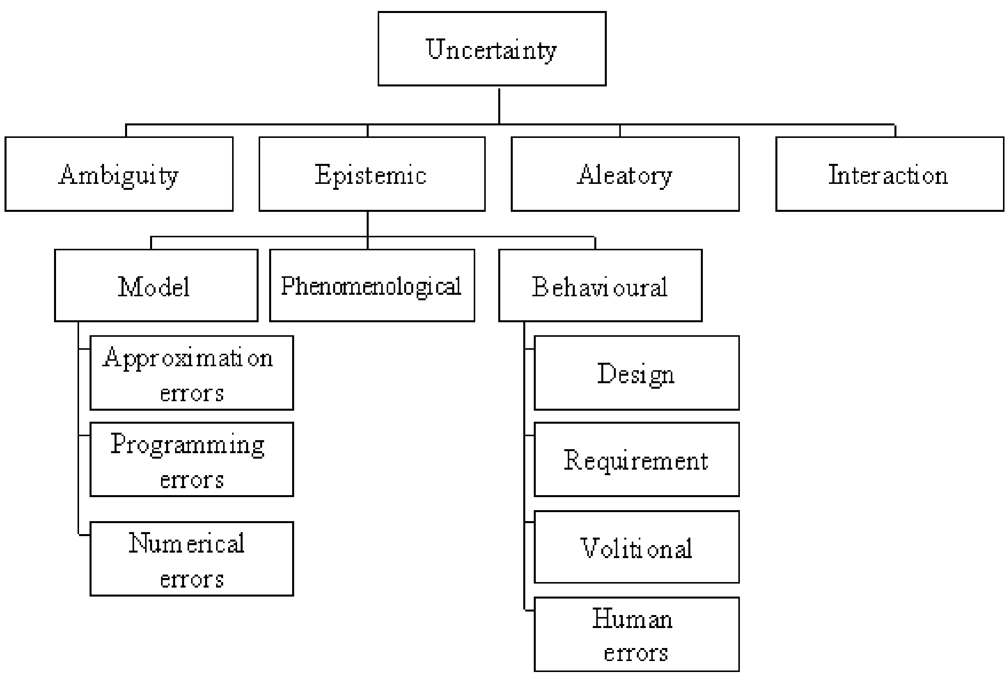

Figure 1-1: Uncertainty classification by Thunnissen [15]

Thunnissen identifies four categories of uncertainty: ambiguity, epistemic, aleatory and interaction uncertainty, which are presented in Figure 1-1 and discussed in the following subsections.

1.2.2.1 Ambiguity uncertainty

In real life, individuals often use imprecise terms and expressions to express their idea, feelings, needs, etc. When used, they cause misunderstanding or ambiguity if the interlocutor is not familiar with these expressions. Ambiguity is also called imprecision, linguistic imprecision or vagueness. Ambiguity remains an unavoidable aspect of human discourse. However, using linguistics conventions and careful definitions can reduce this kind of uncertainty. Theoretically, we can reduce any ambiguity to any desired level. Potential method to represent and to deal with the ambiguity is the Fuzzy set theory [23].

1.2.2.2 Epistemic uncertainty

Epistemic uncertainty is caused by subjectivity and/or a lack of knowledge (ignorance), and can in general be reduced by collecting more data and by increasing knowledge. It is often present in advanced or conceptual design situations where it reflects the expert's perceptive opinion and subjectivity.

Epistemic uncertainty is also called reducible uncertainty, subjective uncertainty, model form uncertainty and state of knowledge. It can be classified into model, phenomenological or behavioral uncertainty. Model uncertainty can be classified into approximation errors, programming errors, and numerical errors. Behavioral uncertainty can be classified into design uncertainty, requirement uncertainty, volitional uncertainty, and human errors. It can be handled by Evidence theory, Possibility theory, and upper and lower previsions theory (see Refs. [10, 14, 17] for further discussion on this subject).

1.2.2.3 Aleatory uncertainty

Aleatory uncertainty describes the inherent variability affecting a system, for example the random variability in operating conditions or in manufacturing processes. It is also called, variability, irreducible uncertainty, inherent uncertainty, stochastic uncertainty, intrinsic uncertainty, underling uncertainty, physical uncertainty, probabilistic uncertainty, noise and risk. In most situations, this variability cannot be reduced by additional study but can be relatively well described within the classical probability framework using probability density functions (PDF). The probability distribution can be quantified by statistic estimation. In reality, engineers have little control over aleatory uncertainty in the design of complex systems. However, they must make sure that it is correctly modeled with sufficient statistical data [10, 14, 17].

1.2.3 Interaction uncertainty

In the case of complex multidisciplinary systems design, an interaction uncertainty can appear. It is due to unanticipated interaction of many events and/or disciplines. It can also arise from the disagreement between informed experts when only subjective estimates are available or when new sources of data are discovered. Many methods are used to handle this type of uncertainty, including simulation, multidisciplinary design optimization (MDO), complexity science, Evidence theory, weighted averages and Bayesian techniques [23].

1.2.4 Theories to represent and propagate uncertainty

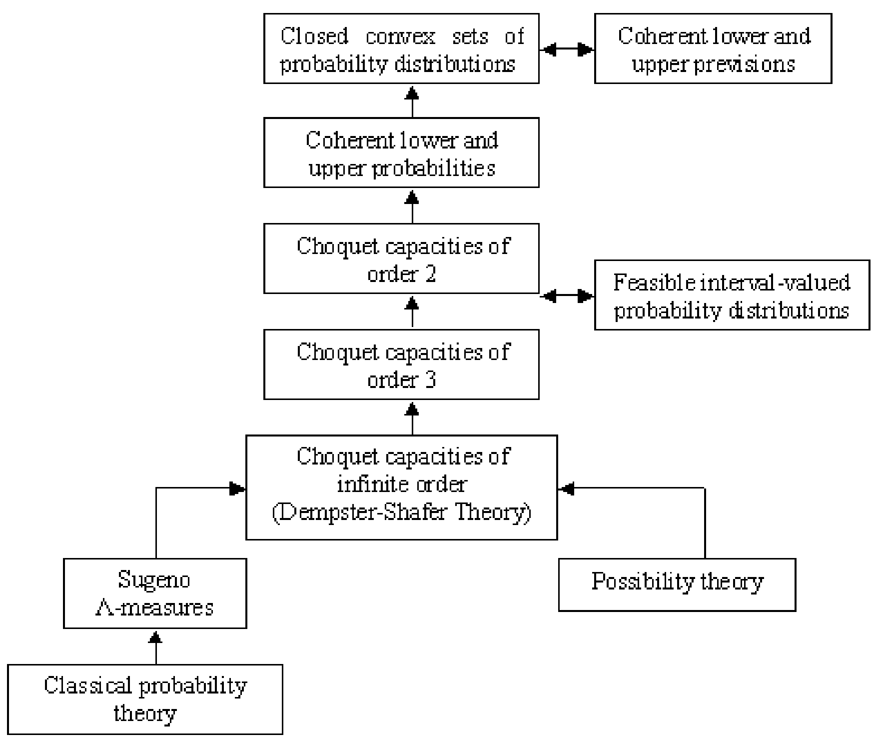

Figure 1-2: Hierarchical relationship among theories of uncertainty [21]

Numerous methods to include and to propagate uncertainty measures into engineering design processes have been proposed in recent years. A hierarchical relationship representation of the various theories for uncertainty has been given by Klir and Smith in a recent article cited in Ref. [21]. The Figure 1-2 shows that the theory of upper and lower previsions is the most general of the various theories. Classical Probability and Possibility theory are subsets of Evidence theory. There is no overlap between the probability and possibility measures, although both are special classes of the plausibility measures. In the upcoming subsections, we present only the Probability theory, the Possibility theory, the Evidence theory and the Fuzzy set theory.

1.2.4.1 Probability theory

By far, the largest and most used uncertainty theory is the Probability theory. The basics of this theory were well established by the early 20th century. It provides the mathematical structures traditionally used to represent uncertainty. However, the scientific community criticizes this theory in its capacity to represent epistemic uncertainty, for two reasons mentioned in Refs.[24-26]: (1) The identification of a probability distribution requires more information than what an expert is able to supply (2) Experts prefer to supply intervals rather than point-values because their knowledge is not only of limited reliability but also tainted with imprecision and possibly vagueness.

The authors of the References [10, 14, 17] demonstrate that an epistemic uncertainty on input variables modeled by uniform probabilistic distribution and propagated through functional will lead to incorrect results for output variables. In fact, even a uniform probabilistic distribution within a given interval is a possibly incorrect inference to express uncertainty about probability itself (comprehensive examples of this behavior are given in Ref. [14]). In other words, the classical probability framework cannot be used directly to quantify epistemic uncertainty without introducing additional information that can lead to unjustified results.

However, the classical probability theory is certainly recommended when the uncertainty on input variables is aleatory by nature and its probability distribution function is known with a sufficient accuracy (that is, useful and sufficient statistical data are available). In order to assess the probability of system failure (i.e., reliability analysis), the effect of uncertainty on system response due to a randomly distributed input variable has been studied by many authors (e.g., Refs. [27-31]). In a recent paper the effect of uncertainty has been explored in the probabilistic design of multi-level system with target allocation (see Ref. [32]).

Whether for single or multi-level systems, the propagation of aleatory uncertainty into the system response is essential to assess the product's ability to perform a prescribed function (i.e., product reliability).

Monte Carlo simulation (MCS) is the traditional approach to estimate the probabilistic properties of uncertain system responses resulting from aleatory uncertain inputs. Using the stochastic properties of random variables, the Monte Carlo technique consists in the simulation of a population of designs (see Ref. [33] for an application of the method). This explains why MCS usually requires a large number of evaluations for accurate estimates of response statistics. Some methods for reliability analysis use sensitivity derivatives to construct approximations of analyses outputs and to estimate mean and variance of the output value for small deviations of the inputs. Another family of algorithm requires the solution of an optimization problem to identify the most likely failure point, or most probable point (MPP).

Instead of using the classical probability theory, a different approach explored by several authors consists in representing uncertainties in terms of the upper and lower bounds on variables, i.e., interval-valued quantities (see Refs. [34-36]). However, it is important to outline that propagating uncertainties by using interval arithmetic in all circumstances (that is even when uncertainties can be expressed not just by intervals) can result in unnecessarily conservative answers.

1.2.4.2 Evidence theory

Shafer developed Dempster's work and presented a new theory, called Evidence or Dempster-Shafer theory. This theory provides an alternative manner to the traditional Probability theory to represent uncertainty in a formal way. The Evidence theory represents directly epistemic uncertainties without assumptions about a probability distribution function. Ref. [14] gives a description of Evidence theory for engineering applications.

Compared with probability theory, Evidence theory allows one to make less restrictive statements about likelihood (see Refs. [14, 24, 37-40] for a general presentation of Evidence theory). However, when a probability distribution is specified, Evidence theory yields the same measures of likelihood. Two types of fuzzy measures, called belief and plausibility measures are the basis of the Evidence theory. The belief and plausibility measures can be interpreted as lower and upper probability estimates. Several types of expert statements can be used to determine belief and plausibility measures. Also, the Evidence theory allows combining into the system several conflicting opinions on the same subject. Oberkampf and coauthors [14] describe in details a methodology for uncertainty analysis in order to construct belief and plausibility measures for the system response (Section B.1.3.1 of Appendix B presents an example illustrating the calculus of the belief and plausibility measures). Several combination methods of evidences exist such as Dempster's rule, Yager's rule, Dubois and Prade's rule, averaging rule, convolutive x-averaging rule, etc. These methods will directly impact the uncertainty analysis.

Dempster's rule of combination ignores conflicting evidence and generates counter-intuitive results when the information is not consistent. Yager's rule tries to remedy to weakness of Dempster's rule by regarding conflicting evidence as a contribution to the overall uncertainty. Dubois and Prade's rule is a disjunctive version of Dempster's rule. The result is uninformative in the sense that it tends to generate wide bounds on the quantity of interest. The averaging rule is the most known and easy to use. Indeed, it weights equally all the sources of information [12-14, 37, 38].

For more details on the Evidence theory, Appendix B presents a comprehensive review of its fundamentals.

1.2.4.3 Possibility theory

Possibility theory provides another alternative to Probability theory for the representation of uncertainty. The Possibility theory is a subclass of the general Evidence theory. This theory can be used to characterize epistemic uncertainty when incomplete data is available. It can be applied only when there is no conflict in the provided body of evidence. In such case, the focal elements of the body of evidence are nested and the associated belief and plausibility measures are called consonant. In contrary, when there is conflicting evidence, the belief and plausibility measures are dissonant [24, 41, 42].

The possibility theory is based on a fuzzy set approach at various confidence levels; it can be used to bracket the "true" probability. In fact, possibility measures are equivalent to fuzzy sets (see Ref. [23]). A fuzzy set is an imprecise set defined by a membership function that provides a gradual transition from 'belonging' to not 'belonging' to the set. For each imprecise input variable the expert must provide a possibility distribution or membership function (this is sometime called the "fuzzification" process). After that, the uncertainty can be propagated into the system response using fuzzy calculus and the extension principle (see Ref. [43] about fuzzy sets theory application). An example of fuzzification and several propagation methods are provided in Ref. [42]. Also, the authors of Ref. [42] propose an efficient hybrid (global-local) optimization method to obtain the possibility distribution of the output (investigators in Ref. [44] also successfully applied this method).

1.2.4.4 Fuzzy set theory

The concept of fuzzy sets was proposed by L. A. Zadeh in his paper published in 1965 [45]. Based on that, a Fuzzy set theory emerged as powerful tool and as another alternative to the classical Probability theory to represent and to handle uncertainty. This theory allowed uncertainty analysis where the source of uncertainty come from vagueness rather than from randomness [46]. That is in the sense that the Fuzzy set theory is capable to represent quantitatively and manipulate the imprecision due to the natural language thanks to the extension of the conventional (Boolean) logic to handle the concept of partial truth [45]. The basis of the Fuzzy set theory is a membership function $m_A(x)$ describing the degree to which a statement is true [47]. This membership function map the members of a set A into the entire unit interval [0,1] where the value of the $m_A(x)$ is called the grade of memebership of x in A. So compared to crisp set, fuzzy sets corresponds to continuous logic [43, 48].

In the classical set theory, the truth value of a statement can be given by the membership function $m_A(x)$ defined as:

$$m_A\,(x) = \begin{cases} 0 & if \;\; x \notin A \\ 1 & if \;\; x \in A \end{cases}$$

However, the Fuzzy set theory allows a continuous value of $m_A(x)$ between 0 and 1, and the membership function can be defined as:

$$m_A\,(x) = \begin{cases} 0 & if \;\; x \notin A \\ 1 & if \;\; x \in A \\ p;\; 0 < p < 1 & if \;\; x\; parially\; belongs\; to\; A \end{cases}$$

We note that the membership function can be represented by any function F: U → [0, 1] independently of its shape. The choice of a specific function can be done depending on the application and the properties of the chosen function. In practice, the most commonly used functions are triangular, trapezoidal, Gaussian and sigmoid [49-51].

The fuzzy sets or fuzzy numbers can be manipulated through the fuzzy arithmetic by the means of different operations [47, 50]. The most common operators are presented herein for two fuzzy sets A and B:

The complement operation is defined as:

$$m_{\overline{A}}(x) = 1 - m_A(x)$$

The intersection operation is:

$$m_{A\cap B}(x) = \min\,(m_A(x), m_B(x))$$

The union operation is:

$$m_{A\cup B}(x) = \max\,(m_A(x), m_B(x))$$

Fuzzy set theory has been studied extensively during the last three decades and is now applied in different domains. For example, in engineering design, it has been applied to characterize uncertainty [52]. It was also used to study uncertainty associated with incomplete and subjective information in engineering process [53]. In project management, it was applied for decisions making for imprecise project [54]. Also, it was applied for uncertainty calculation in business, finance and management problems [55].

We note that there are many published literature reviews on the Fuzzy set theory and its applications. As examples, we cite the following studies: a review of the Fuzzy logic and its applications [56], a topical classification on the Fuzzy set theory and its application in industrial engineering [57], a survey on the Fuzzy set theory applications in production management [58], a literature review and opportunities on the use of Fuzzy logic in product family development [59].

The analysis of the available literature on the Fuzzy set theory shows that even though the application of this theory is largely widespread, it was noticed that it present a major limitation.

Indeed, this theory is more suitable for qualitative reasoning than for quantitative estimation of uncertainty [60].

1.2.4.5 Conclusion

The Probability theory is intended mainly for aleatory uncertainty and it is generally inappropriate for the epistemic uncertainty. Possibility theory and Evidence theory can deal with both aleatory and epistemic uncertainties. In other word, the Probability theory is ideal to represent uncertainty when sufficient statistical information is available. When there is insufficient information, Possibility theory or Evidence theory can be used. Evidence theory is applicable even if there are conflicting evidences, otherwise, the Possibility theory can be applied only when there is no conflicting evidences [11, 21]. Concerning the Fuzzy set theory, it presents a major drawback. Indeed, it can deal with uncertainty coming from vagueness rather than from randomness; thereafter it is more suitable for qualitative reasoning than for quantitative estimation of uncertainty.

Since we are concerned by the early stages of the VDP, in addition to the inevitable aleatory uncertainty, the experts will face the epistemic uncertainty due to the lack of knowledge about the majority of the vehicle aspects. Moreover, the experts' opinions may be conflicting due to the insufficiency and the imprecision of information. For these reasons, handling uncertainties during the VDP by the Evidence theory became an obvious choice.

1.3 Target allocation under uncertainty

The development process of complex systems, such as cars, is often a downward process in a sense that the marketing service determines the strategic specifications or the initial product's attributes from which the experts determine the systems, subsystems and parts specifications as well as the manufacturing methods. Because of the interactions among components and the relative lack of knowledge of the experts, these specifications may be conflicting or even unachievable. To limit conflicts and interactions between disciplines, experts are provided with targets for a few key characteristics to drive the design process. The difficulty is now shifted onto the target allocation process. Moreover, at each stage of the VDP, a variety of uncertainties inherent to the product attributes or to manufacturing process is present. The engineers have to deal with these uncertainties in order to ensure that the final product meet the initial requirements.

Several techniques have been developed to allocate targets under uncertainty at all phases of the VDP. These techniques lay between two extreme approaches depending on the companies' cultures. The first extreme approach privileges discussion and consensus among engineers. As example for this approach, The Quality Function Deployment (QFD) was developed by a Japanese company and was largely adopted all around the world. This technique consists in bringing experts together from all the company departments (marketing, finance, design, planning, manufacturing, etc.). During a series of meetings, the specialists discuss all the aspects related to the product development and manufacturing techniques. They raise all conflicts and interactions among disciplines. By consensus, they reach the best compromises to carry out the product that will meet the needs of the market [61-63].

The second extreme approach for targets allocation under uncertainty consists in the use of mathematical modeling, simulation and prototyping. This approach represents the early VDP (including target allocation) as a formal decision-making framework driven by measurable objectives and constraints with respect to the vehicle specifications at each level. The Multidisciplinary Design Optimization (MDO) methodologies offer a systematic approach for optimization of complex engineering systems. The MDO formalism uses mathematical programming models to express design aims. Based upon design evaluations, optimization algorithms may be used to automate the decision process instead of engineers making decisions interactively. Multi-level MDO methodologies have been applied successfully on complex systems (e.g., aerospace vehicle design and automotive design in Refs. [64-66]). Multi-level approaches offer the possibility to partition a complex system design among specialists and permit tasks to be executed concurrently, each specialist concentrating on his/her task. As interactions among the specialized tasks are present in such systems, a consistency must be maintained at the different levels. To solve this difficulty, one approach consists of propagating targets in the framework and minimizing the discrepancy between the target value and the actual value (see Collaborative Optimization (CO) framework in Ref. [64] and the Target Cascading methodology in Ref. [66]).

The analytical target cascading (ATC) is a specific application of the MDO approach that was presented as a methodology for hierarchical multilevel system design (see Refs. [66-69]). A multilevel hierarchical structure similar to the VMM is used in ATC to model a complex system. Also, the components characteristics are related through functional dependencies in a bottom-up direction. The ATC framework handles the selection of local and shared design variables of components and the cascading of targets from the top level to the bottom levels. The optimal design variables and targets are obtained by ATC solving a series of minimum deviation optimization problem for each component of the hierarchy. The consistency between targets and components characteristics and the overall system design optimality are achieved by ATC after several iterations. The ATC optimization tasks require integration of the analyses modules or simulation models to produce component responses dependent upon local, shared and coupling variables. The analysis modules produce consistent results throughout the iterative process.

An extension of ATC with a probabilistic formulation to encompass uncertainty has been developed (see Refs. [32, 70, 71] for details). The solution of ATC probabilistic optimization problems requires the propagation of uncertainty to obtain variance information for all component responses dependent upon the random variables. The generation of variance information is handled by ATC from random variables through analysis modules. Then, the uncertainty information is used in reliability constraints during optimization from which the design variables and the targets are obtained. The reliability constraints serve to ensure that the probability of failure of the component will be below a prescribed value.

Other approaches for target allocation based on the Multi-Criteria Decision Analysis (MCDA) consist in the choice of the best alternative of possible design [72]. These approaches are applicable in the case of discretionary design and are largely used in the automotive industry where the automakers have the majority of components already designed and have only to choose the best combination in order to fulfill the specific customers' expectations. The Evidential reasoning is an example of these approaches [73, 74]. It consists in the evaluation of alternatives considering quantitative and qualitative criteria under different uncertainties including randomness and ignorance (epistemic and aleatory uncertainties) by the mean of utility function that combine the different criteria.

From the previous discussion, we can detect the opportunity of development of new targets allocation technique under uncertainty, which is more in line with current conceptual design practices for complex hierarchical systems. Such technique would orient the development process to fulfill the customers' needs using the experts' knowledge and confidence which varies during the process unlike the analysis modules used in ATC. In addition, experts generate the uncertainty information while the source of uncertainty is unknown and may contain subjective aspects. More importantly, local and shared variables of design are under experts' responsibility and are not handled by the target allocation process. The resulting experts' opinions account for design reliability because it is considered as part of the experts work to design components with an acceptable probability of failure.

1.4 Decision-making strategy in the vehicle development process

It is only after the proposition of the concept of Decision-based Design (DBD) that the product development process started to be considered as a series of decisions instead of a series of tasks. The DBD is an approach aiming the maximization of the value of designed product even under uncertainty [75]. However, this approach presents an issue due to the lack of consensus on how the design utility function can be constructed to take into account the objectives of all stakeholders [76].

As mentioned previously, the VDP is a specific case of new product development process that can be viewed from different perspectives [3]. Hence, based on the concept of DBD, the VDP can be viewed from a decision-making perspective. Consequently, the development of a new vehicle will require making critical design decisions, throughout the VDP, that can impact noticeably the competitiveness and profitability of the company. These decisions are required to ensure that the subsequent actions or absence of action fit with the company's objectives [77-79].

Our main objective is to identify a decision-making strategy that could be applicable to targets allocation during the vehicle development process (VDP). This decision-making strategy must be adapted to dynamic targets allocation to account for varying uncertainties on different characteristics of the vehicle and on available resources.

In this context, decision-making processes have been explored and only two of them were the most relevant for the engineering design [80]. The first one advocate an Incremental Decision

Process Model, in sense that the design process is divided in a series of small decisions leading to a fulfillment of a big decision [81]. The second one, called Design Optimization, consists in solving optimization problem to define the set of parameters or characteristics values that maximize the value of a designed product [82]. However, the design optimization process can present issues because of multidisciplinary design of complex and large scale product. This problem has been addressed in many researches that proposed various decompositions of a large scale problem in a set of smaller problems [66, 68, 83].

So, developing a strategy of decision-making throughout the VDP requires the integration of both concepts presented previously in order to address two major problems: the first one concern the identification of a methodology based on the optimisation approach in order to determine the characteristics targets and the progress performed at any moment of the VDP using relevant utility functions (objective function and constraints of mathematical problem); the second one consists in the identification of a monitoring strategy that will allow managing consecutive decisions throughout the VDP, archiving the historical data for the design process progress and coordinating the decision processes for the whole components of the vehicle in order to meet the utopian requirements of the customers.

1.4.1 Multi-objective optimization

The automotive design is an area characterized by multiple stakeholders, which are involved within the design process. These stakeholders have different objectives that may be conflicting or even contradictory. Technically, a large number of parameters must be considered to comply with all kinds of constraints and to provide the best product [77-79]. For this reason, a multi-attribute decision-making approach is needed to perform tradeoff analysis among conflicting criteria. The parameters have to be combined in one or many objectives and have to be optimized in order to identify the set of characteristics targets leading to desirable and feasible vehicle.

References [75, 84] provide a comprehensive literature review on the multi-objective optimization method for engineering. In these references, the pros and cons in addition to the validity of each method are discussed. From this discussion, it can be deduced that the Genetic Algorithm is a good candidate method for the formulated decision-making optimization problem. In fact, the GAs are appropriate for single objective as well as multiple objectives optimization problems and they are relatively robust. The GAs are global optimization techniques, in the sense that they converge to a global solution rather than to a local solution. They combine the use of random numbers and information of previous iterations to evaluate and improve a population of points (a group of potentials solutions) rather than a single point at a time. GAs do not require gradient information consequently they can be effective regardless of the nature of the objective functions and constraints.

1.4.2 Monitoring strategy of decision-making process

The concepts of management, archiving and coordination required for the monitoring strategy of decision-making process are included in almost all the new product development processes [1, 2]. Consequently, developing such strategy goes through the choice of a relevant new product development process (NPDP). This choice must take into account the nature of the performed activities and their organisation during the VDP.

Since, the activities of the VDP are concurrent and alternating knowledge generation and decision-making, a stage-gate process [85, 86] will present a good candidate model to monitor the decision-making process. Indeed, modeling the VDP in the form of a series of parallel stage-gate processes for the whole components of the vehicle will ensure that the requirements of the VDP and monitoring strategy fit together.

1.5 Synthesis

In this chapter, we have reviewed the vehicle development process, the uncertainty management, the target allocation and the decision-making under uncertainty. This analysis leads to the following conclusions:

The Vehicle Development Process (VDP) can be considered as an iterative and complex multi-phases process alternating multiple activities such as research, design, choices and decisions. In this process, the vehicle is represented in the form of a multilevel model composed of systems, subsystems and parts and their associated engineering characteristics, which combine into vehicle performance characteristics through functional.

At the early stages of the VDP, the design is only an abstraction and the available information is imprecise and/or incomplete. For that reason, the experts' opinions are tainted with uncertainty.

Set of intervals associated with subjective beliefs constitute a natural way for experts to express uncertain opinions during the VDP.

Four types of uncertainty inherent to design process of complex systems that can be included in the experts' opinions were identified: 1) the ambiguity is related to the manner in which engineers express their opinions. This uncertainty can be reduced by the adoption of linguistics conventions and careful definitions 2) the interaction uncertainty is due to unanticipated interaction of many events or disagreement between informed experts when only subjective estimates are available 3) the epistemic uncertainty is caused by subjectivity and/or a lack of knowledge. It can be reduced by collecting more data and by increasing knowledge 4) the aleatory uncertainty describes the inherent variability affecting a system. This type of uncertainty cannot be reduced by the generation of more knowledge. However it can be relatively well described by classical probability distributions.

In this research, we take into account the four types of uncertainty. Indeed, we adopt a uniform manner to express the experts' opinions (set of intervals and associated the subjective beliefs), which reduce the ambiguity and we develop a methodology to handle interaction and another to handle both the epistemic and aleatory uncertainties (see Chapter 3).

Numerous methods to include and to propagate uncertainty measures into engineering design processes have been proposed such as the classical Probability, Possibility theory, Fuzzy set theory and Evidence theory. In this study, the Evidence theory is selected to manage uncertainty for the following reasons: the first one is that it is capable to handle both aleatory and epistemic uncertainties without any need to distinguish between them, the second one is that it allows handling conflicting experts' opinions and finally it is capable to manage experts' opinions in the form of intervals with the associated subjective beliefs without assuming any kind of probability distribution.

For the target allocation under uncertainty, we discussed the difference between the extreme possible approaches depending on the company culture. A methodology using mathematical modeling will be considered because it is more in line with current conceptual design practices for complex hierarchical systems. This methodology consists in cascading optimization problems allowing setting vehicle performance targets at the top level and the corresponding engineering characteristic targets at the lowest levels. Two conflicting measures have been identified for the optimization objectives. One is related to the customers' satisfaction and the other one to the experts concerns about design feasibility.

Concerning the decision-making under uncertainty, we pointed out its requirements during the VDP and we proposed an approach to ensure that. This approach consists in modeling the design process in the form of a set of stage-gate processes that alternate knowledge generation and decision-making. At the stage, the generated information is handled by the proposed methodology for uncertainty management. At the gate, decisions are performed based on this information by the mean of the proposed methodology for target allocation under uncertainty.

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Chapter 2 Research methodology

2.1 Introduction

The development of methods capable of target allocation during the design process is an endeavour with a long history. Many approaches were proposed to handle this problem. In this thesis, our goal is the proposition of a formal framework for target allocation under uncertainty during the vehicle design process. Our premise is that the target allocation process could be improved by taking into account the various uncertainties into an overall decision-making framework under uncertainty. Indeed as mentioned in Ref. [1], the benefit of distinguishing the nature of uncertainty includes an improved interpretation of the consequences of uncertainty on the system's possible behaviours and an improved ability to allocate resources to decrease, if necessary, the system uncertainty. Consequently, taking into account the uncertainty during the VDP may represent an opportunity for engineers to improve the vehicle design through the exploration of more candidates solutions which were not considered before [2-4].

To achieve our goal, three specific objectives were identified: 1) the development of a methodology for uncertainty characterisation 2) the development of a methodology for target allocation under uncertainty 3) and the proposition of a framework for decision-making under uncertainty that accompanies the design process.

The pursued research methodology combined the analysis of both the practical and theoretical aspects of the project, then the development of approaches for solving the identified problems, then the validation and/or verification of these approaches and finally application.

From a practical point of view, the project is motivated by industrial concerns that need effective solutions. For that reason, General Motors offered us its support and bring its expertise during all the phases of the project. GM participated to the definition of the problem, the determination of the scope of the project, the evaluation and the validation of proposed methods and approaches. Along this PhD, their support was threefold: intern training in GM Research and Development Technical Center, visits to GM facilities, discussion with experts and periodic distant meeting with two dedicated experts who have extensive experience in vehicle design.

From a theoretical point of view, the project intersects three engineering and scientific fields: the mechanical engineering, the industrial engineering and applied mathematics. Hence, knowledge in the three domains was required to achieve the previous objectives. In this context, we conducted an extensive literature review on the following topics: new product development processes, uncertainty characterisation, target allocation under uncertainty, decision-making under uncertainty. In addition, familiarization with optimization algorithms, Matlab programming and XML was required.

2.2 Research Methodology

In this section, we present the details of our research methodology as depicted in the organization chart in Figure 2-1. Mainly, we treat the uncertainty characterisation in the VMM, target allocation under uncertainty, decision-making under uncertainty, simulation of the decision-making process and approach for handling large-scale problems. For each topic, we position the problem, present a relevant literature review and propose a solution. We present also possible avenues for validation or verification of the proposed methodologies. The support of GM and its participation aspects to the achievement of the objectives are presented in subsection 2.2.1.

- **Vehicle Multilevel model**

The concept of decomposition of system in the form of hierarchical structure is commonly used in design process of complex system. In mechanical design, the decomposition can be performed considering different criteria such as geometric disposition, functional relation or a combination of both [5-9]. In the automotive industry, since most functionalities of the vehicle are already well defined and the technological solutions have high levels of maturity a hierarchical decomposition of the vehicle is used in both conceptual and detailed design.

In the context of this project, we have adopted a decomposition that combines the geometric disposition and the functional relations among the components. The nodes of the hierarchical tree structure represent the systems, subsystems and parts of the vehicle. The numbers of levels and elements are defined according to the complexity of the vehicle and the required level of detail in the design process. The motivation of this choice is dictated by General Motors actual practices expressed in the first presupposition «*Complex system such as a vehicle can be modeled in the form of hierarchical tree structure*».

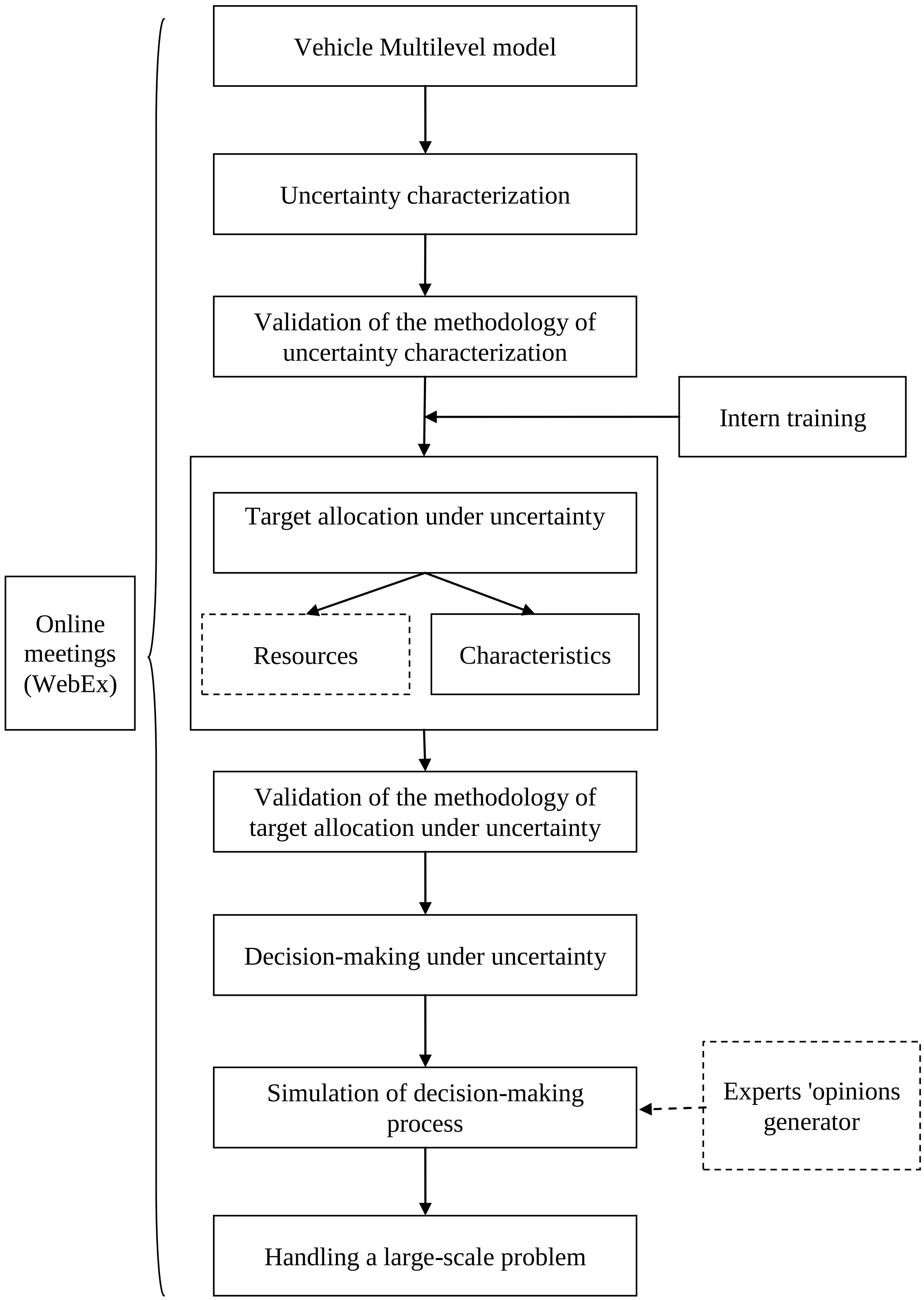

Figure 2-1: Research methodology organization chart

We associated characteristics and their targets to each node of the model. Two types of characteristics were identified: the vehicle characteristics and engineering characteristics. The engineering characteristics are controlled by engineer at the leaf nodes of the model and represent physical specifications or technical measures that quantify the responses of the components. The vehicle characteristics are calculated based on the engineering characteristics through functionals and are defined only at the top level of the model. These characteristics represent the vehicle technical specifications. The targets of the vehicle characteristics are the initial available information for a new vehicle. At the beginning of the design process, these targets are utopian and will be refined as the design process progresses.

Also as stipulated in presupposition 2, General Motors invited us to consider experts as the only source of information in the design process. For that reason, experts are affected to the leaf nodes of the model. These experts are responsible of the design variables of the component. They choose their own source of information to produce their opinions.

Associating characteristics, targets, functionals between engineering and vehicles characteristics and experts to the hierarchical tree structure constitutes the concept of the vehicle multilevel model (VMM). This model will provides a framework for handling and propagating information top-down and reallocating targets bottom-up in the model.

The structure of the multilevel model and the uncertain experts 'opinions will be organized in an XML file. An ad hoc structure of the XML file was developed and expressed in an XML schema. A software for reading and writing the XML files was developed in MATLAB using standard DOM methods.

- **Uncertainty characterization**

Once, the vehicle is decomposed in the form of a multilevel model, the utopian vehicle characteristics are cascaded top down in the form of targets for the engineering characteristics. At the leaf nodes of the model, experts evaluate the feasibility of each characteristic and return their opinions in a specific form which results from a process of synthesis of the available information. Since the design is only an abstraction at the early stages of the development process, the available information is imprecise and incomplete because the experts may lack knowledge about the achievability of the characteristics. For that reason, the experts’ opinions are tainted with different types of uncertainty. Our industrial partner proposed that the uncertain information will be provided in the form of set of intervals and the associated subjective belief because this is a natural and common way to express uncertainty (see the presupposition 3). This manner of expressing uncertainty allows including different types of uncertainty without the need of distinction between them (presupposition 4 stipulates that all types of uncertainty are included in the experts' opinions). We note that the identification of the different types of uncertainties is a key to develop any methodology to handle uncertainty during the decision making process.

In this context, the problem is the proposition of a formal methodology able to capture the uncertainty included in the experts' opinions. This methodology must be able to aggregate multiple experts' opinions, propagate them bottom up in the VMM and measure the uncertainty at any nodes of the model.

In Section 1.2, we presented an extensive literature review on the uncertainty management in design process including the sources of uncertainty, its classification and the candidates' theories that can handle it in the context of engineering design. Four types of uncertainty were identified aleatory, epistemic, ambiguity and interaction uncertainties. In this research, we will take into account the four aforementioned types of uncertainty. Indeed, we adopt a uniform manner to express the experts' opinions in the form of set of intervals and associated subjective beliefs which will allow the control of the ambiguity uncertainty. We develop also a methodology to handle interaction and another one to handle both the epistemic and aleatory uncertainties.

The proposed methodologies to handle the uncertainties will be based on the Evidence theory because this theory can deal with both aleatory and epistemic uncertainties without distinction, can treat conflicting evidences and can handle uncertain information in the form of set of intervals. In Chapter 3, we present in details the proposed methodologies to handle the different types of uncertainty in the VMM.

- **Validation of the proposed methodology for uncertainty management**

The proposed methodology for uncertainty management in the multilevel model is based on Evidence theory. To validate this methodology, we consider few simple multilevel models and we analyze them using both Evidence and Probability theories. Uncertain information, in the form of experts' opinions at the leaf nodes is aggregated and propagated bottom-up in the models using two approaches. The first approach uses the Evidence theory and the second one uses the Monte Carlo simulation. In order to apply the MCS, we suppose uniform probability distribution over the intervals that constitute the experts' opinions.

Based on the aggregated and propagated information, belief and plausibility measures are calculated using the Evidence theory and compared with cumulative probability obtained by the Monte Carlo simulation that in order to understand the meaning of the belief and plausibility measures and to illustrate the validity of the approach. In addition, an application of the proposed approach to an example from the literature and comparison of the results were performed to confirm the validity of the proposed approach for uncertainty management in the VMM.

- **Target allocation under uncertainty**

The target allocation or target distribution among components during the early stages of design process of any new product is an everlasting concern for engineers. The characteristics of a new product are usually conflicting and evolving in opposite directions. In the case of complex system such a vehicle, the design process starts by the definition of the utopian characteristics targets of system as a whole. Then, cascading these utopian targets to lowest levels of the model in the form of engineering characteristics targets through the functionals that link both types of characteristics. The reallocated targets are obtained through tradeoffs among the characteristics at any node of the VMM.

The problem to address herein is the development of a formal methodology able to reallocate the characteristics targets under uncertainty during the early stages of the design process at both top and components levels of the model.

A literature review on the methodologies for target allocation was conducted (see Section 1.3). The first type of these methodologies uses discussion and consensus among experts to reallocate targets. The Quality Function Deployment (QFD) is an example which is applicable to both conceptual and detailed design. The second type uses a formal mathematical modeling, simulation and prototyping to allocate target under uncertainty. Hence, target allocation can be driven by measurable objectives and constraints with respect to the vehicle specifications at each level. This type is mainly used during the detailed or advanced design where more information is available and the relations among the components are well defined. The third type uses the Multi-

Criteria Decision Analysis and consists in the choice of the best alternative among all the possible designs by the evaluation of a utility function that combines different criteria. This type is applicable in the case of discretionary design and is largely used in the automotive industry where the majority of components already designed. The automakers have only to choose the best combination in order to fulfill the specific customers' expectations. This approach is not applicable in our case because vehicle characteristics vary continuously.

Based on this discussion and the requirements of our industrial partner, the proposed methodology for target allocation under uncertainty must be applicable to early stages of the design process where the available information are fuzzy, imprecise and incomplete (information containing both aleatory and epistemic uncertainties), the relation among the components are not clearly defined and the only source of information to take decision are provided by experts in the form of sets of intervals. In order to make the decision making process automatic after collecting experts' opinions we have selected a target allocation based on objectives and constraints (see second type above). We have identified two conflicting objectives: the achievability representing the engineers concerns and the desirability representing the customers wants. The desirability can easily be measured by the means of an appropriate utility function while the achievability can be calculated based on the belief and plausibility measures. So, once the uncertainty is characterized through the model, the process of target reallocation can be performed. The main idea for the targets reallocation is to define an original measure which combines the achievability and desirability of characteristics in a single measure (a multi-objectives utility function) to guide the reallocation process. This measure is a major contribution of the present work.

We recall that initially the project aim was the development of a methodology for allocation of characteristics targets and resources. Unfortunately, because of lack of time the scope of the project was restricted to treat only the characteristic target allocation.

In Chapter 4, we present in details the proposed methodology for targets allocation under uncertainty in the VMM.

- **Verification of the proposed methodology for target allocation under uncertainty**

The proposed methodology for target allocation under uncertainty in the multilevel model, proposed in Chapter 4, is based upon cascading optimization problems where the objective functions use the belief and the plausibility measures obtained by the Evidence theory. Since this approach is new, scarce literature on analogous methods with examples is available. Moreover, the proposed approach is complex to the point where no analytical solution is possible. For that reason, we resort to the Monte Carlo simulation for the propagation of the uncertain information in the VMM and we adapt the proposed approach in order to provide an equivalent one. The equivalent approach will use probability density function from aleatory variables obtained by the MCS instead of the belief and plausibility measures. The target allocation will be performed based on the basis of highest likelihood.

Comparison of the results of both approaches will allows the observation of the characteristics of the proposed methodology, the discussion of its various advantages and validity.

- **Decision-making under uncertainty**

Decision-making in the product development process is a vast domain that concern all aspects such as concept development, supply chain design, product design, performance testing and validation, production ramp-up and launch, etc [5, 10]. In the present project, we are concerned mainly by the product design and we are interested especially in the decision-making related to the target and resource allocation. For that reason, we consider the vehicle development process from a decision-making perspective in the sense that the process will be viewed as a series of decisions rather than a series of tasks.

In this context, the problem to address is the choice of a decision-making model appropriate for the vehicle design process. This model will constitute an application by integration of the proposed methodologies for uncertainty management and targets allocation in the VMM.

In Section 1.4, we presented a literature review on the decision-making during the design process. We have identified two relevant models for the engineering design. The first model is an Incremental Decision Process that consists in a series of small decision leading to the achievement of a big decision. The second model is a Design Optimization Process that consists in solving optimization problems to define the set of characteristic values that maximize the value of the designed product.

Two presuppositions (5 and 6) concerning the vehicle design process were formulated by our industrial partner. These presuppositions stipulate that the development process is iterative, oriented by the characteristics targets and that the uncertainty diminish as the process progress. Indeed, utopian characteristic targets are defined at the beginning of the process and are iteratively evaluated and adjusted until the obtainment of the desired product thanks to the measure and control of the uncertainty.

Considering the previous elements, the proposed decision-making strategy must be adapted to a dynamic targets allocation to account for varying uncertainties on characteristics and on available resources. This strategy must monitor iterative targets reallocation. From that, the strategy will integrate the targets allocation methodology and the monitoring strategy.

The targets allocation will be performed by the maximization of an appropriate utility function that combine multiple criteria into single or multiple objectives in order to measure the progress performed and to adjust the targets at any moment of the development process. An approach for targets allocation under uncertainty was discussed in the previous subsection.

The monitoring strategy will be concerned by managing the information and ensuring the coordination of decisions during the design process. These aspects are parts of any new product development process and consequently a stage-gate process will present a good candidate model to monitor the alternating activities of knowledge generation and decision-making.

In Section 5.4, we present in details the proposed framework for decision-making during the design process.

- **Validation of decision-making under uncertainty**

The proposed approach for decision-making under uncertainty during the vehicle design process, presented in Section 5.4, consists in the integration of both uncertainty characterisation and target allocation methodologies in an iterative process. This approach is based on an assumption that this method has the potential to lead to a better design process and a better final product at the end of the VDP. Of course, a validation of this assumption on a real VDP is out of the scope of the current thesis, since a real VDP can be extended over many years and involves hundreds of participants. As a surrogate, we will rely on a simulation of the VDP using an experts' opinions generator. This simulation of the design process will allow obtaining information about the validity of the approach by assessment of the variation of the components characteristics during the iterations of a simple VMM.

- **Handling large scale problem**

Application of the proposed methodology for target allocation to large scale problems was an objective and a concern from the first day of the formulation of this research project. As mentioned previously, the target allocation methodology presented in this thesis is based on the collection of experts' opinions in the form of sets of intervals at the leaf nodes and their propagation towards the top level through functionals to determine the belief and plausibility curves. The propagation of intervals can result in a possibly overwhelming number of intervals to be handled. To overcome this problem, instead of resorting to classical approach to handle large scale design problem such as problem decomposition or modularization, we proposed a new approach to handle the large amount of information. This approach is a procedure for the propagation and merging of intervals leading to reduction of the number of intervals by controlling the information granularity while keeping the accuracy of the belief and plausibility on a given set of discrete values. This procedure will allow the reduction of the computational burden of uncertainty aggregation and propagation through the VMM. This makes the proposed approach competitive compared to Monte Carlo simulation, which is known to be numerically expensive. Furthermore, at a given node, the proposed merging procedure results in the same discrete belief and plausibility values as obtained from unmerged intervals.

In Appendix D, we present in details this methodology, its applications and its advantages and limitations.

- **Experts' opinions generator (not presented in the thesis)**

As stated previously, the target allocation and decision-making processes use mainly the uncertain information collected at the leaf nodes and propagated to the top of the model. This information is provided by experts that evaluate the achievability of the components and return their opinions in the form of sets of intervals with the associated subjective beliefs.

In practice, it is impossible to proceed to the implementation, comparison and the validation of proposed decision-making strategy within a real vehicle design team because of time, cost and resources considerations. Therefore, a computerized simulation of the VDP and the decision-making process is required. In order to perform such simulation, a behavioural model for the experts has been developed and implemented in the form of an experts' opinions generator. This generator is capable of simulating different behaviours of fictitious experts and artificially generates their opinions in accordance to predefined rules. Since, these rules are arbitrary; the experts' opinions generator was not applied to validate the proposed strategy for decision-making and consequently was not presented in this thesis.

2.2.1 GM support and participation

In this section, we describe the participation of General Motors to the achievement of the project in terms of coaching, mentoring, training and support. We present principally my intern training into the GM Research and Development Technical Center, the visits to GM facilities, the discussion with GM experts and the periodic online meeting with two GM's experts.

- **Resources assigned to the project**

Two GM researchers (Drs. Peter Fenyes and Stacey Gu) were assigned to the project. These researchers have extensive knowledge and understanding of the complexities of vehicle development through observation and participation in GM's early vehicle development process. Their mandate consists in leading the GM efforts to properly define the research problem, executing the research, and carrying out testing and validation of the developed methods.

During the project, they provide information on the VDP and the inherent uncertainty. They collaborate to definition of the objectives and the scope of research. Finally, they test and validate the resulting methods within GM environment.

- **Intern training**

The first year of my PhD, GM offered me an intern training for five months at General Motors Research and Development Technical Center under the supervision of Drs. Peter Fenyes and Stacey Gu. The objectives of this training were:

- Learn about the design process in GM facilities.
- Meet professionals in design of vehicles and learn about their methods, approaches, constraints and problems.
- Understand terminology and current trends in vehicle design process.

- Ask questions to obtain new information.
- Read about relevant topics inherent to the research project.
- Gather and analyze data in order to identify opportunities to improve the design process.
- Recognize organizational constraints.
- Specify the objectives, steps, approaches of the research project.

During this training, I realized a project entitled "***Simulation of mass evolution during the Vehicle Development process***". In this project, I developed an approach for target allocation of the vehicles technical specifications. The proposed approach consists in the application of Monte Carlo simulation and specific rules to guide the process of target allocation. The approach and its results are GM propriety that we cannot present herein.

- **Online meeting (Web Ex)**

One distant meeting was held every two or three weeks with GM researchers. In this meeting the progress of the project is evaluated and discussed. Moreover decisions concerning the next steps are taken to ensure the achievement of the project's objectives.

- **Visits and meetings**

Meeting were organised with the GM researchers in Montréal and in GM Research and Development Technical Center. Meeting at GM facilities allowed direct contact with other researchers in order to extend our knowledge and understanding of GM procedures while those in Montréal allowed for easier demonstration of the developed software and discussion of the project progress, directions and planning. We also meet at an AIAA MDO conference.

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Chapter 3 Managing Uncertainty in a Multi-Characteristic Vehicle Multilevel Model

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This paper presents a methodology to propagate uncertainty in a hierarchical multilevel model, as used during the conceptual design phase of a vehicle. We have considered that each component in the multilevel model may have several characteristics, and that a target is defined for every component and characteristic. Experts' opinions are expressed with uncertainty regarding the feasibility of achieving each target. Experts' opinions are given in the form of probability distributions or intervals associated with their subjective beliefs for the possible values of the characteristics. The paper describes how the uncertainty from multiple experts' opinions are aggregated and propagated from the nodes of the multilevel model up to the vehicle level. Evidence theory has been used to express uncertainty in the form of belief and plausibility measures, which are compared with the probability measures obtained by a Monte Carlo simulation.

3.1 Introduction

The automotive industry is one of the largest in the world, and the automotive market is

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dominated by relatively few large corporations. In order to remain competitive and profitable, these companies continuously survey customer needs and desires, and develop new products and innovative technologies that deliver the desired characteristics. At the outset of the vehicle development process (VDP), a set of desirable vehicle characteristics identified by the marketing department defines the utopian goal of the design process. These vehicle characteristics are passed on to the design team, whose first task is to translate them into targets for the vehicle systems, subsystems, and parts. The target allocation for each component of the vehicle must be consistent with the vehicle characteristics and must respect the couplings and constraints among systems, subsystems, and parts. In practice, these targets are refined iteratively as the VDP progresses based on experts' opinions for the component characteristics. At each phase of the VDP, the experts must provide their opinions with respect to the components characteristics targets under limited resources and time.

Recently, stakeholders have been concerned about the influence of uncertainty on this target allocation process. The main question is: can we build a better target allocation process by taking uncertainty into account?

Even without considering uncertainty, automating design decisions in a hierarchically decomposed vehicle is a complex task. Recently, analytical target cascading (ATC) was presented as a methodology for hierarchical multilevel system design (see Refs. [1-4]). The vehicle is modeled as a multilevel hierarchical structure (see Figure 3-1) where the components characteristics are related through functional dependencies in bottom-up fashion. In ATC, system design consists of selecting local and shared design variables of components, cascading targets from the top level and propagating components responses bottom-up. The design variables and targets are obtained by the formulation and solution of a minimum deviation optimization problem for each component of the hierarchy, in order to achieve consistency among components and overall system design optimality after several iterations. An extension of ATC with a probabilistic formulation to encompass uncertainty has recently been developed (see Refs. [5-7] for details), where the uncertain quantities are random design variables represented by probability distributions. The solution of ATC probabilistic optimization problems requires the propagation of uncertainty to obtain variance information for all design variables dependent upon the random variables.

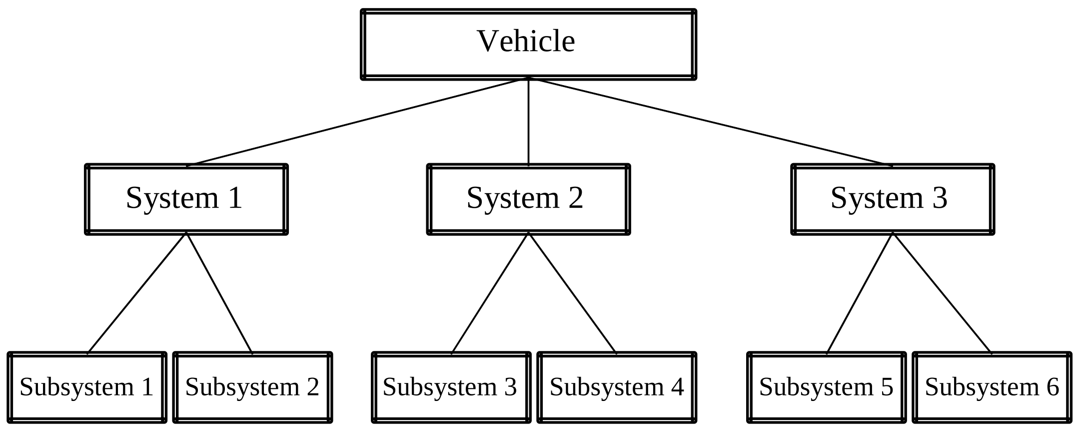

Figure 3-1: Example of a simplified vehicle hierarchical structure

In ATC, the component responses are provided directly by analysis or simulation models without human intervention. In contrast, current conceptual design practices for complex hierarchical systems often involve distinct experts for targets allocation and for components design with respect to those targets. The current practice for target reallocation is a human-driven decision making process in search of compromises and design improvements based on experts' opinions. In this paper we consider that upon receiving targets, each expert conducts component design and returns values of characteristics leading to a feasible design in the vicinity of the targets if possible. Experts' opinions can encompass several sources of uncertainty that can be expressed under different forms without distinction.

During the VDP, the target allocation starts at the vehicle level; hence, global uncertainty measures at the vehicle level are required to make informed decisions about target allocation. However, uncertainties can be best evaluated by experts, located at the lower component levels. As a result, a method is needed to propagate uncertainty from the lower levels to the higher level. Several theories for managing uncertain data can be considered such as Imprecise probability theory [8], P-boxes [9], Evidence theory [10-12], Possibility theory [13] and Probability theory [14, 15]. A hierarchical relationship exists among the various theories of uncertainty that has been given in Ref. [16].

The Probability theory is intended mainly for aleatory uncertainty and it is generally inappropriate for the epistemic uncertainty. Possibility theory and Evidence theory can deal with both aleatory and epistemic uncertainties. Probability theory is ideal to represent uncertainty when sufficient statistical information is available. However, when there is insufficient information, possibility theory or Evidence theory can be used. Evidence theory is applicable even if there are conflicting evidences, otherwise, the possibility theory can be applied only when there is no conflicting evidences [16, 17]. The Evidence theory has been selected to handle uncertainties of different natures without distinction.

Our contribution in this paper is to develop a methodology that manages uncertainties in a VMM in order to evaluate globally vehicle characteristics uncertainties propagated from experts' opinions with respect to component characteristics. For example, we propose a method to evaluate interaction uncertainty between different components that are not decoupled by the introduction of targets.

The present work constitutes a first step towards answering the fundamental question about the benefit of including uncertainty in a formal decision making methodology during the conceptual design phase of a vehicle. The decision-making aspects for target allocation will be the subject of subsequent studies and are not presented in this paper.

In the next section, the vehicle multilevel model concepts and definitions are introduced. The uncertainty management theoretical framework is described in Section 3.3 and illustrated by several examples.

3.2 Vehicle multilevel model concepts and definitions

3.2.1 Vehicle multilevel model

The vehicle multilevel model (VMM) represents a decomposition of a vehicle in a hierarchical tree structure where the components of the tree represent the systems, subsystems and parts of the vehicle. The numbers of levels and elements are defined according to the complexity of the vehicle and the required level of detail during the VDP. The VMM provides a framework for handling and propagating information. An example of a simple VMM, including characteristics, targets, and experts' opinions is presented in Figure 3-2.

In the present work, we distinguish two types of characteristics: vehicle characteristics and engineering characteristics. Engineering characteristics (C) are returned by experts during the design process as measures to represent physical specifications, characterize behaviors, and quantify the responses of components (systems, subsystems and parts) of the VMM. Vehicle characteristics (VC) appear only at the top of the VMM; they express measures that are palpable and understandable to the customer and represent technical criteria often used to compare similar vehicles.

3.2.2 Functional relations among characteristics

Vehicle characteristics have a functional dependency with respect to various engineering characteristics of the VMM components at different levels. For example, the handling performance is influenced by many factors such as mass distribution, type of suspension, quality and dimension of the tires. The VMM represents a hierarchical decomposition similar to what is used in ATC with a functional dependency between component characteristics oriented from lower levels to higher levels (see Refs. [1-7]). Figure 3-2 illustrates how, in the VMM, components characteristics are linked by functional relations in the bottom up direction. A functional relation may vary from a simple inheritance to a complex relationship involving several components characteristics. An example of simple inheritance is provided in Figure 3-2: the characteristics C_1^{SS1} and C_2^{SS2} are transferred directly from subsystems 1 and 2 to system 1 ($C_1^{S1} = C_1^{SS1}, C_2^{S1} = C_2^{SS2}$). Also illustrated in Figure 3-2 is the characteristic C_6^{S2} obtained with a more complex functional: $C_6^{S2} = f(C_0^{S2}, C_4^{SS3}, C_5^{SS4})$. In vehicle design, mass is a characteristic of primary importance because many other characteristics depend on it. The mass of all components sums along the branches of the VMM into the vehicle mass. For example, the mass of system 2 is obtained by adding the mass of subsystems 3 and 4: $C_0^{S2} = C_0^{SS3} + C_0^{SS4}$.

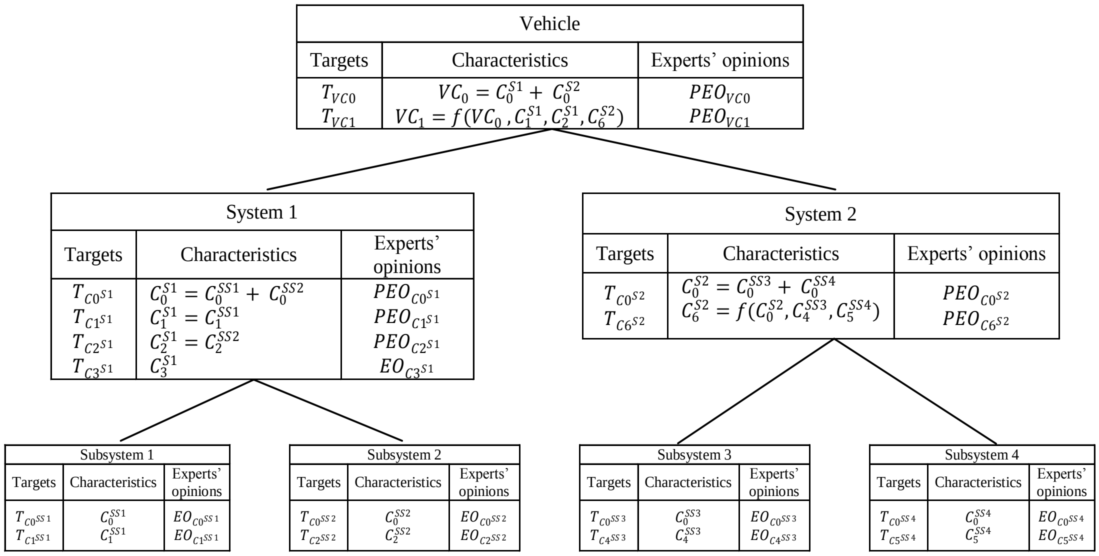

Figure 3-2: Example of a multi-characteristic vehicle multilevel model

3.2.3 Targets and experts' opinions

A leaf component for a given characteristic C_i possesses at least one expert's opinion EO_{C_i}. Moreover, each component of the VMM encompasses experts' opinions either expressed by experts at leaf nodes (EO_{C_i}) or propagated from them (PEO_{C_i} and PEO_{VC_i}), and characteristic targets (T_{VC_i} for vehicle characteristics and T_{C_i} for other components). The targets and experts' opinions for all characteristics in the VMM are described in an XML definition file and handled by MATLAB (see Appendix A).

In the context of the vehicle development process (VDP), experts are guided by a given set of targets but have limited resources and time. Their opinions for a component characteristic synthesize uncertain knowledge and several design results. Hence, experts possess enough information to express uncertain opinions (see Sections 3.3.1 and 3.3.2).

Once the EOs are obtained in an appropriate format, they have to be aggregated (see Section 3.3.4) and propagated bottom-up. Based on propagated EOs, the Evidence theory is used to evaluate two measures of uncertainty for events that are usable during the decision making process (see Section 3.3.3). The propagation of experts' opinions is achieved through the functional relations, leading to PEO_{C_i} or PEO_{VC_i} (see Section 3.3.6).

For the present work, we assume that the targets have been determined by a suitable process, that is, all T_{C_i} and T_{VC_i} are consistent with respect to the functional relations that link the engineering and vehicle characteristics. The target allocation process is not the subject of the present work but will be addressed in subsequent publications.

3.3 Uncertainty management

Creating innovative products with challenging specifications most likely requires the development of new design methods, new technologies, new materials and/or new manufacturing methods. So, until the product is completed, the product characteristics and performances are approximate and subject to uncertainty.

3.3.1 Uncertainty definition and classification

A number of authors such as Der Kiureghian [18], Isukapalli et al. [19], Haukaas [20], Oberkampf et al. [21], Thunnissen [22], Agarwal et al. [16] and Nikolaidis [23] have offered many comprehensive taxonomies of uncertainty. They distinguish many distinct types of uncertainty with considerable subtlety, and each proposes a classification that meets the needs of a specific field of research. We have adopted the classification of Thunnissen [22], which is appropriate for the design of complex systems. Thunnissen considers four categories of uncertainty: ambiguity, epistemic, aleatory and interaction uncertainties. For our purpose, the types that will be formalized, aggregated, propagated and interpreted are aleatory, epistemic, and interaction uncertainty. Ambiguity uncertainty is not considered because, in essence, experts' opinions express possible values of measurable quantities using precise terms and expressions based on consensual definitions (for example, an expression such as "the mass is around 30 kg" is prohibited because the term "around" does not have the same range for all parties).

Aleatory uncertainty, also called probabilistic uncertainty, is the inherent and irreducible variation or randomness associated with a physical system. Epistemic uncertainty arises from a lack of knowledge or information. Increasing knowledge and collecting more data can reduce epistemic uncertainty. Finally, interaction uncertainty arises in the case of complex system design as a result of interaction of many systems and/or disciplines. When using a target allocation approach in vehicle design, interaction uncertainty arises because the targets are indicative characteristic values that may not be attained (see Section 3.3.7).

3.3.2 Uncertainty representation

In this work, we have supposed that, during the VDP, experts' opinions are expressed under limited resources and time based on a given set of targets. For this reason, the experts' opinions synthesize uncertain knowledge and design work.

The design could be subjected to parameters having inherent and irreducible randomness. The design maybe incomplete because all parameters affecting the component may not be selected accurately: some of them could be approximately bracketed or subjectively ignored depending on the level of details required for the component characteristic evaluation. Moreover, several possible designs may be explored by experts.

But, knowledge is required to conduct design work and to evaluate the component characteristic value. The uncertainty depends on the sources of information used by experts to form their knowledge. When sufficient data are available about stochastic variables, experts can create, verify, and use strong statistical models. In this case, the uncertainty is purely aleatory, and is commonly represented by a probability distribution (e.g., Refs.[17, 24-27]). More generally, experts may rely on sparse statistical data, experience, empirical methods, approximation functions, and computation-based analyses as valuable sources of information during the design process. So, experts' opinions may contain epistemic and aleatory uncertainty because of partially characterized randomness, subjectivity and incomplete physical models in the analyses.

Based on discussions with expert engineers from GM, it appears that experts possess enough information to provide a range of possible values (further described as bodies of information) around the targeted characteristic value. For real-value engineering or vehicle characteristics, the bodies of information can naturally be provided as real intervals.[6] Moreover, because experts are recipients and developers of knowledge, the probability given to a body of information can reflect their confidence/subjective belief in the knowledge used or that the component value of the final vehicle will ultimately lie within the interval. Both interpretations of the probability reflect an uncertain opinion.

[6] Discretized probability density functions can also be handled as bodies of information in the proposed approach.

Hence, we describe an uncertain expert's opinion about a characteristic as a set of real intervals associated with their subjective beliefs. For example, the value of characteristic C lies in the interval $[a, b]$ with $x\ \%$ subjective belief, or in the interval $[c, d]$ with $y\ \%$ subjective belief, etc. Note that the information is imprecise since C can take any value between the interval bounds, but not fuzzy, because the bounds are clearly identified; however, the presence of C in the interval is uncertain because of a subjective belief (even if the subjective belief is $100\ \%$).

Due to the various forms of uncertainty present in experts' opinions, we take an approach similar to Ref. [11], and select the Evidence theory to handle both epistemic and aleatory uncertainty.

3.3.3 Evidence theory

Evidence theory, or Dempster-Shafer theory [12], allows less restrictive statements about uncertainty than in the case of probabilistic specification. The main concept of the Evidence theory is that our knowledge of a given problem can be inherently imprecise. So, Evidence theory uses two specifications of likelihood - belief and plausibility - for each subset of the universal set under consideration. In this section we present an overview of Evidence theory based on Refs. [11, 17, 28-30] and applied to uncertainty given as real intervals with the associated subjective beliefs.

3.3.3.1 Frame of discernment

A frame of discernment is defined as a set of mutually exclusive elementary propositions that can be viewed as a finite space in probability theory. The subsets of this set might be nested in one another or might partially overlap. In our case, for a single characteristic, the frame of discernment noted X is the set of all real numbers: $X = \mathbb{R}$. Let Z be a set of the various propositions that the experts can express by union of elementary propositions (i.e., subsets of X). In our case, $Z = \mathbb{R}$, that is the set of all real intervals:

$$[\mathbb{R}] := \{[a, b] \mid a < b \text{ and } (a, b) \in \mathbb{R}^2 \} \quad (1)$$

3.3.3.2 Basic belief assignment

The basic measure in Evidence theory is called the basic belief assignment (BBA) or basic probability assignment (BPA). This measure expresses the degree of belief in a proposition. It is a function that maps Z to the interval $[0, 1]$.

$$m: Z \rightarrow [0, 1] \tag{2}$$

This function allows subjective belief to be expressed with numbers included in the interval $[0, 1]$. For a subset A_i of Z, called a focal element, $m(A_i)$ represents the portion of total belief assigned to the proposition A. The basic belief assignments function must satisfy the three axioms below:

$$m\,(A_i) \geq 0 \text{ for any } A_i \in Z \tag{3}$$

$$m\,(\emptyset) = 0 \tag{4}$$

$$\sum_{A_i \in Z} m(A_i) = 1 \tag{5}$$

Basic belief assignment axioms look similar to those of probability theory, except that they are less stringent [11, 16, 29]. In our case, when considering the uncertainty of a characteristic, A_i is a specific interval provided by an expert. Hence, with Evidence theory, the uncertainty provided by intervals is handled naturally.

The previous definitions and axioms can be generalized to a combination of n independent characteristics by considering $X = \mathbb{R}^n$ and Z=[$\mathbb{R}^n$].

3.3.3.3 Belief and plausibility functions

Unlike probability theory which uses only one measure: "the probability of an event", Evidence theory uses two measures, belief (Bel) and plausibility (Pl), to describe the inherent uncertainty of an event B:

$$Bel(B) = \sum\nolimits_{A_i \subset B} m(A_i) \tag{6}$$

$$Pl\,(B) = \sum\nolimits_{A_i \cap B \neq \emptyset} m(A_i) \tag{7}$$

The belief in the event B is the summation of the BBA of all propositions $A_i \in Z$ included in B. The plausibility of the event B is the summation of the BBA of all propositions $A_i \in Z$ that intersect with B, and the intersection is not empty. The belief measure can be viewed as the minimum likelihood associated with an event B of the frame of discernment. Similarly, the plausibility measure can be viewed as the maximum likelihood associated with the same event B.

To illustrate the concepts and the calculation of belief and plausibility, we consider an experts' opinion EO_{C_0} given as a set of three focal elements:

$$A = ([150, 180]\ , [180,190], [190,200])$$

for which the basic belief assignment $m(A_i)$ is equal to the subjective belief of the expert associated with the interval A_i:

$$m(A_1) = 0.33, m(A_2) = 0.33 \text{ and } m(A_3) = 0.34$$

Let us consider the event B of Z that $C_0 > x$. With $x = 184$, we calculate the belief and the plausibility using Eqs. (6) and (7) above for the characteristic C_0. In this case:

$$Bel(C_0 > 184) = m([190, 200]) = 0.34$$

$$Pl(C_0 > 184) = m([180, 190]) + m([190, 200]) = 0.67$$

By varying x, we obtain the belief and plausibility curves plotted in Figure 3-3. The belief can be interpreted as the minimum likelihood that the proposition $C_0 > x$ is true, while the plausibility is the maximum likelihood for the same proposition to be true. For a given x, the difference between the belief and the plausibility represents the uncertainty associated with the proposition $C_0 > x$.

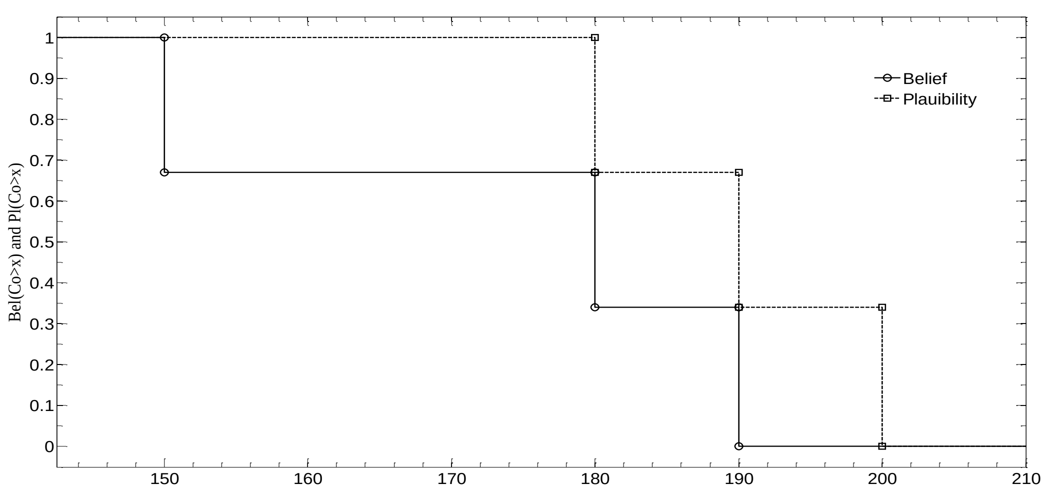

Figure 3-3: Belief and plausibility curves for the characteristic C_0>x

3.3.4 Aggregation of experts' opinions

In the case where there is more than one expert to provide opinions on a given characteristic, the experts' opinions must be aggregated to create an equivalent expert before the resulting belief and plausibility can be evaluated. For this purpose, many authors have proposed rules of combination. We can cite Dempster's rule, Yager's rule, Dubois and Prade's rule, the averaging rule, and the convolutive x-averaging rule [11, 26, 30, 31]. In this paper, and without loss of generality for the rest of the methodology, we will apply the averaging rule.

The averaging rule is the simplest and most common way to combine evidence. It is a generalization of averaging for probability distributions. The expression for the averaging rule is the following:

$$m(A_i) = \sum_{j=1}^{k} w_j * m_j(A_i) \tag{8}$$

where the $m_j(A_i)$ are the BBAs for a focal element A_i given by the experts to be aggregated and the w_j are the scaled weights assigned according to the reliability of the sources $(\sum_{j=1}^{k} w_j = 1)$. A discussion of the pros and cons of the various aggregation rules can be found in reference [30].

In Table 3-1 two experts' opinions for a characteristic C_1 are given with the resulting equivalent experts' opinion obtained using the averaging rule with unit weights.

Table 3-1: Example of experts' opinions aggregation

Characteristic C_1^{SS1}		
Expert A	Expert B	Equivalent expert
[100, 120], 0.14		[100,120], 0.07
[120, 140], 0.21	[120, 140], 0.25	[120,140], 0.23
[140, 160], 0.3	[140, 160], 0.50	[140,160], 0.40
[160, 180], 0.21	[160, 180], 0.25	[160,180], 0.23
[180, 200], 0.14		[180,200], 0.07

3.3.5 Incomplete or conflicting experts' opinions

The combination rules usually suppose that the sum of the subjective belief associated with the intervals equals 1. No incompleteness in the experts' opinions is tolerated. In the case where there is some "ignorance" or "incompleteness" in an expert's opinion, a special operation must be performed to distribute this ignorance in some manner. Also, another problem may arise due to conflicts among experts' opinions. There is a conflict between two experts' opinions when the intersection of the focal elements they provided is empty. Approaches to deal with ignorance and conflicts can be found in Refs. [32-34]. In this paper, we will assume experts' opinions are provided without conflicts or ignorance.

3.3.6 Uncertainty propagation

In the present work, we recall that experts' opinions are given only at the leaf nodes of the characteristic propagation tree (CPT) of the multilevel model. The target allocation process requires that the uncertainties be available at all nodes where decisions are taken. So, the experts' opinions for each characteristic must be propagated from the leaf nodes of the CPT to the top level of the VMM using the functional relations among characteristics. Based on these functional relationships, a mapping between the uncertainties of the children nodes (input space) and the uncertainty of the parent node (output space) must be established at each level in the tree.

To present the uncertainty propagation method, let us suppose the simple VMM example given in Figure 3-4.

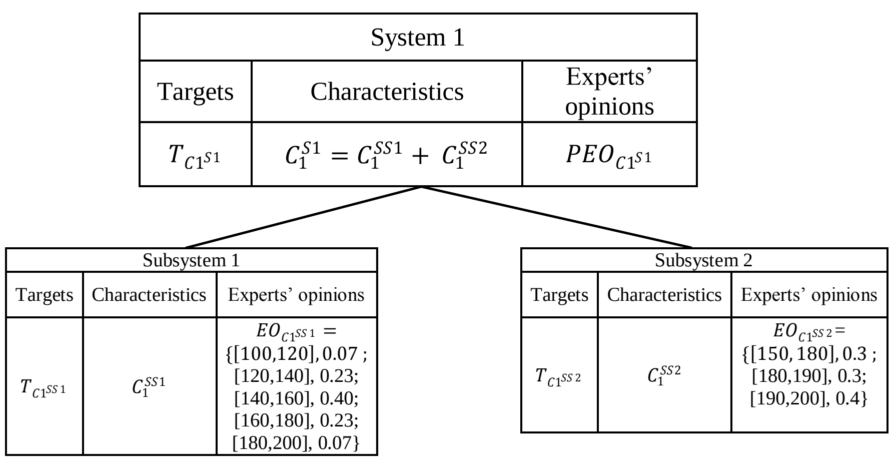

Figure 3-4: Simple VMM to illustrate uncertainty propagation

A functional relation between characteristic C_1 of subsystems 1 & 2 and system 1 is given by:

$$C_1^{S1} = f(C_1^{SS1}, C_1^{SS2}) = C_1^{SS1} + C_1^{SS2} \tag{9}$$

That is, $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is a function from a real two-dimensional vector to a real number. Because it is considered that the experts' opinions are independent, all combinations of the input intervals from $EO_{C_1^{SS1}}$ and from $EO_{C_1^{SS2}}$ must be propagated through the functional to obtain the output intervals for $PEO_{C_1^{S1}}$. So, the function f must be extended to take intervals in $\mathbb{R}^2$ as inputs and produce intervals in $\mathbb{R}$ as outputs, that is $[f]: [\mathbb{R}^2] \rightarrow [\mathbb{R}]$. For input intervals $[a, b]$ of $EO_{C_1^{SS1}}$ and $[c, d]$ of $EO_{C_1^{SS2}}$ we have considered an interval extension of the functional with the following output interval, lower bound l and upper bound u.

$$l = \underset{C_1^{SS1} \times C_1^{SS2} \in [a,b] \times [c,d]}{\text{minimum}} f(C_1^{SS1}, C_1^{SS2}) \tag{10}$$

$$u = \underset{C_1^{SS1} \times C_1^{SS2} \in [a,b] \times [c,d]}{\text{maximum}} f(C_1^{SS1}, C_1^{SS2}) \tag{11}$$

For any combination of input intervals at the leaf nodes, there is only one corresponding output interval defined at the parent node level. The subjective belief associated with the interval $[\mathrm{l}, \mathrm{u}]$ is equal to the product of the subjective beliefs associated with intervals $[\mathrm{a}, \mathrm{b}]$ and $[\mathrm{c}, \mathrm{d}]$. For the case presented in

Figure 3-4 the functional relation is linear and the intervals of system 1 are obtained by the summation of the interval bounds of Subsystems 1 and 2: $[l, u] = [a + c, b + d]$. The results of the propagation of the experts' opinions for this example are presented in Table 3-2 ($PEO_{C_1^{S1}}$).

Table 3-2: Propagation through summation of intervals and multiplication of subjective beliefs

$PEO_{C_1^{S1}}$		$EO_{C_1^{SS1}}$				
		[100,120], 0.07	[120,140], 0.23	[140,160], 0.4	[160,180], 0.23	[180,200], 0.07
$EO_{C_1^{SS2}}$	[150,180], 0.3	[250,300], 0.021	[270,320], 0.069	[290,340], 0.12	[310,360], 0.069	[330,380], 0.021
	[180,190], 0.3	[280,310], 0.021	[300,330], 0.069	[320,350], 0.12	[340,370], 0.069	[360,390], 0.021
	[190,200], 0.4	[290,320], 0.028	[310,340], 0.092	[330,360], 0.16	[350,380], 0.092	[370,400], 0.028

Using Eqs. (6) and (7) on $PEO_{C_1^{S1}}$ makes it possible to calculate the belief and plausibility for the characteristic C_1^{S1} of system 1. To automate this task, a MATLAB module has been developed to calculate and plot the belief and plausibility curves (see Figure 3-5).

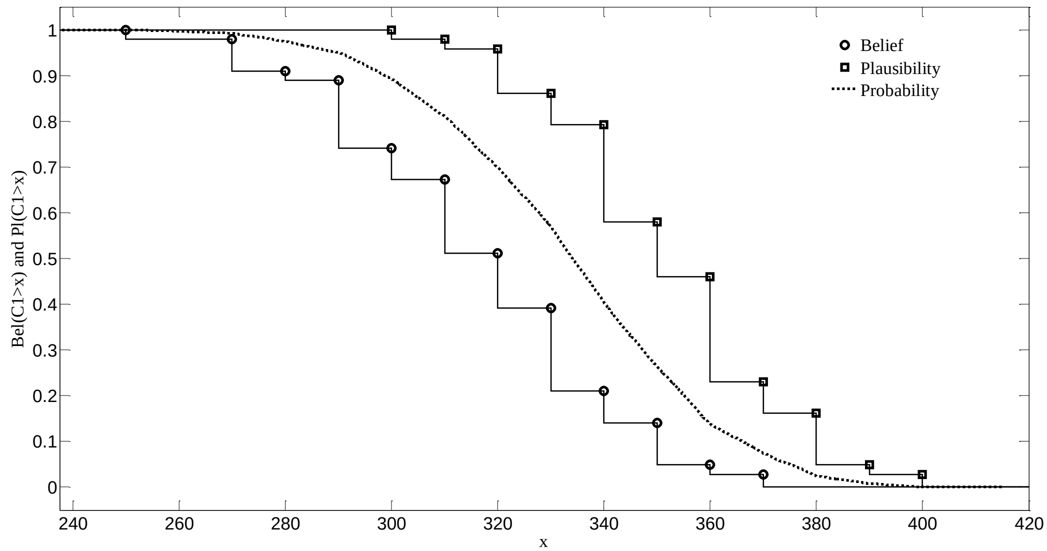

Figure 3-5: Belief, plausibility and probability curves for $PEO_{C_1}^{S1}$ without interaction

A probability distribution obtained by a Monte Carlo simulation is also plotted in Figure 3-5. This curve was obtained by assuming a uniform PDF in each interval with a cumulative probability equal to the subjective belief of the interval (see Ref. [35] for a complete description of the Monte Carlo simulation). As predicted by the Evidence and probability theories, the belief and plausibility distributions bracket the probability distribution. We note that the belief and plausibility curves provide more information than the probability curve: for a given value of x, the larger the difference between belief and plausibility, the more uncertain the experts are about this information.

In the example presented above, the experts' opinions are given in the form of intervals and subjective beliefs. However, the framework and the MATLAB module we have developed (see Ref. [35] for implementation details) can also accept experts' opinions given in the form of probability density functions. In such a case, these functions are first discretized into intervals and the corresponding probability that the characteristic lies within each interval is used as the subjective belief.

3.3.7 Managing interaction uncertainty

Thunnissen [22] points out that the interaction uncertainty may occur in the design of complex systems. This is because of the interactions of many systems and/or disciplines and a disagreement among informed experts. In order to illustrate the source of the interaction uncertainty let us consider again the simple VMM presented in

Figure 3-4 involving two experts opinions. Let us consider that the values of characteristics C_1^{SS1} and C_1^{SS2} depend on local variables (X^{SS1} and X^{SS2}), shared variables (S^{SS1} and S^{SS2} are copies of shared design parameters used by experts) and coupling variables (C_1^{SS1} and C_1^{SS2}), and that, for the sake of simplicity, this relation can be expressed by functional relations:

$$C_1^{SS1} = f_1{}^{SS1}(X^{SS1}, S^{SS1}, C_1^{SS2}) \tag{12}$$

$$C_1^{SS2} = f_1{}^{SS2}(X^{SS2}, S^{SS2}, C_1^{SS1}) \tag{13}$$

3.3.7.1 Effect of shared design variables

Through these exact functional relations, the range of values taken by the design parameters X^{SS1}, X^{SS2} can produce parametric or aleatory uncertainty on the characteristics C_1^{SS1} and C_1^{SS2}. To reach the overall design realizability experts must interact to obtain a fixed value for the shared parameters such that $S^{SS1} = S^{SS2}$. Even if X^{SS1} and X^{SS2} are fixed without uncertainty, a disagreement or a lack of interaction on the value taken by S^{SS1} and S^{SS2} induces uncertainty because the compromise value for a realizable design is not selected with certainty. In the context of our study, the reduction of this type of interaction uncertainty is supposed to be resolved implicitly by experts reaching agreements as the VDP progresses. However, if the effect of this interaction uncertainty on the characteristic uncertainty can be estimated subjectively and expressed by each expert then it can be accounted for as a generic uncertainty to better allocate the characteristics targets.

3.3.7.2 Effect of coupled characteristics

The treatment of the interaction uncertainty for the coupled characteristics is done differently because characteristics are deployed explicitly on all nodes of the VMM.

By considering characteristics targets dispatched in the VMM instead of uncertain characteristic values, experts can work more independently and concurrently by reducing the effect of coupled characteristics. The equations (12) and (13) in the example above can be decoupled by using characteristics targets:

$$C_1^{SS1} = f_1{}^{SS1}(X^{SS1}, S^{SS1}, T_{C_1^{SS2}}) \tag{14}$$

$$C_1^{SS2} = f_1{}^{SS2}(X^{SS2}, S^{SS2}, T_{C_1^{SS1}}) \tag{15}$$

Uncertain characteristic values from C_1^{SS1} do not affect the uncertainty of C_1^{SS2} and vice versa. Therefore, the interaction uncertainty due to the coupling effect is not considered. This strategy to decouple characteristics is very commonly used in multidisciplinary design optimization (MDO) problems where consistency constraints are added for coupled characteristics and resolved while optimizing [36-38]. In our case, it prevents the propagation of coupled uncertainty between experts. Otherwise, characteristic uncertainties would have a larger extent because independent sources of uncertainties combine and accumulate as illustrated on Section 3.3.6. In the context of the VDP, it is supposed that, in most situations, expert's opinion about a characteristic explores the vicinity of the provided target. Also, during the iteration of the VDP, targets may be reallocated such that experts can converge more effectively around the targets and reduce independently their uncertainty. By doing so, the unaccounted interaction uncertainties are reduced as the VDP progresses.

However, neglecting interaction may result in misevaluating the characteristic uncertainties; especially in situations were targets are not achievable by experts. To illustrate this effect on the example above, a unidirectional interaction between C_1^{SS1} and C_1^{SS2} is considered:

$$C_1^{SS1} = f_1{}^{SS1}(X^{SS1}, C_1^{SS2}) \tag{16}$$

$$C_1^{SS2} = f_1{}^{SS2}(X^{SS2}) \tag{17}$$

Let us consider that a target values $T_{C_1^{SS2}} = 200$. With the target $T_{C_1^{SS2}}$ replacing C_1^{SS2} in Eq. (16) independent experts' opinions are provided. The propagation of the two independent experts' opinions to the System 1 level is done by applying the methodology described in Section 3.3.6 to obtain belief and plausibility curves (see Figure 3-5). However, in the case considered here, the target value for C_1^{SS2} ($T_{C_1^{SS2}}$ = 200) does not match the provided expert's opinion $EO_{C_1^{SS2}}$ and interaction uncertainty is not considered. Let us suppose that an approximation of $f_1{}^{SS1}$ built on historical data of existing vehicles is available under the following form:

$$C_1^{SS1} = 0.5 * C_1^{SS2} + [20{,}60] \tag{18}$$

This approximation can be viewed as a valuable source of information to be combined with experts' opinions. Using the target value in Eq. (18) instead of C_1^{SS2}, we obtain $C_1^{S1} \in [120{,}160]$, that is an interval contained in expert's opinion $EO_{C_1^{SS1}}$. In addition, Eq. (18) can be fed with expert's opinion for C_1^{SS2} to evaluate an additional expert's opinion for C_1^{SS1}:

$$EO_{C_1^{SS1/SS2}} = 0.5 * EO_{C_1^{SS2}} + [20{,}60] \tag{19}$$

$$EO_{C_1^{SS1/SS2}} = \{[95{,}150], 0.3; [110{,}155], 0.3; [115{,}160], 0.4; \} \tag{20}$$

This additional expert's opinion disagrees with the original expert's opinion $EO_{C_1^{SS1}}$. Nevertheless, it includes the interaction effect between C_1^{SS1} and $C_1^{SS2}$and the resulting $EO_{C_1^{SS1/SS2}}$ has globally lower values than $EO_{C_1^{SS1}}$. To illustrate how this additional expert's opinion accounts realistically for the interaction uncertainty we compare the belief and plausibility curves at System 1 level with and without interaction.

First, the propagation of $EO_{C_1^{SS1/SS2}}$ to System 1 level through Eq. (9) must account for the dependence between $EO_{C_1^{SS1/SS2}}$ and $EO_{C_1^{SS2}}$. The procedure of Section 3.3.6 for propagating independent experts' opinions cannot be used in this case. However, the functional relations given in Eq. (9) and Eq. (18) can be added as follows:

$$C_1^{S1} = 1.5 * C_1^{SS2} + [20{,}60] \tag{21}$$

and fed with expert's opinion for C_1^{SS2} to obtain:

$$PEO_{C_1^{S1/SS2}} = 1.5 * EO_{C_1^{SS2}} + [20{,}60] \tag{22}$$

$$PEO_{C_1^{S1/SS2}} = \{[245{,}330], 0.3; [290{,}345], 0.3; [305{,}360], 0.4\} \tag{23}$$

After that, the two sources of information available at System 1 ($PEO_{C_1^{S1/SS2}}$ and $PEO_{C_1^{S1}}$) can be aggregated by using the procedure described in Section 3.3.4 and the corresponding belief and plausibility curves are obtained.

Figure **3-6** illustrates the effect of accounting for or not interaction uncertainty on belief and plausibility curves. Because the additional expert's opinion $\mathrm{EO}_{C_1^{SS1/SS2}}$ gives more subjective belief to lower values than $EO_{C_1^{SS1}}$, the resulting belief and plausibility curves are shifted to lower values. Also, the addition of interaction uncertainty, in this case, broadens the gap between belief and plausibility curves.

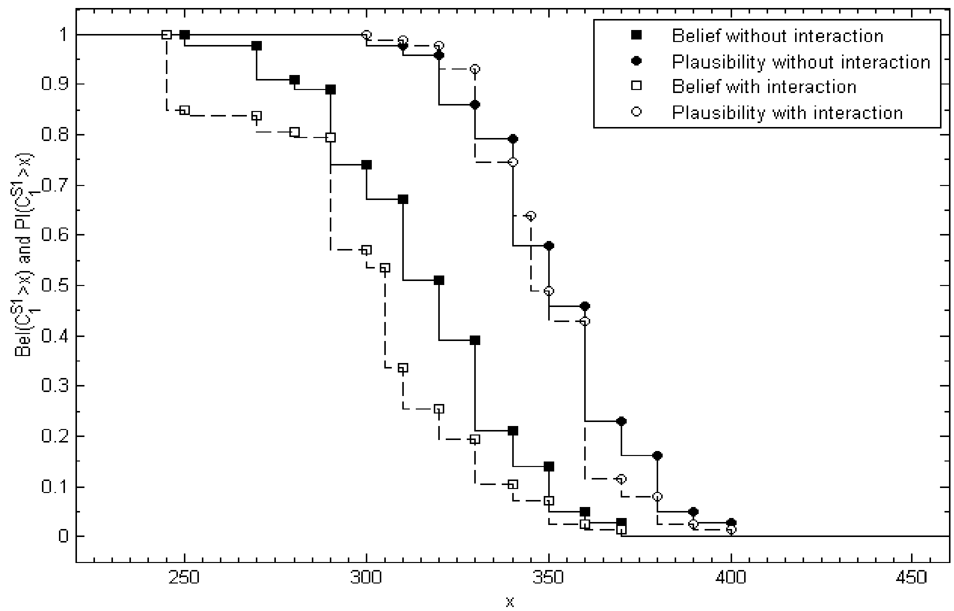

Figure 3-6: Belief and plausibility curves for System 1 with and without evaluation of interaction uncertainty

As illustrated on the above example, in the situation where experts' opinions cannot approach sufficiently the provided targets, the combination of additional sources of information with existing experts' opinions will help capturing the interaction uncertainty.

3.3.8 Application example

In this section, we apply the strategy of uncertainty propagation to a simplified hierarchical vehicle description (see Figure 3-7) to demonstrate its extensibility to multiple characteristics.

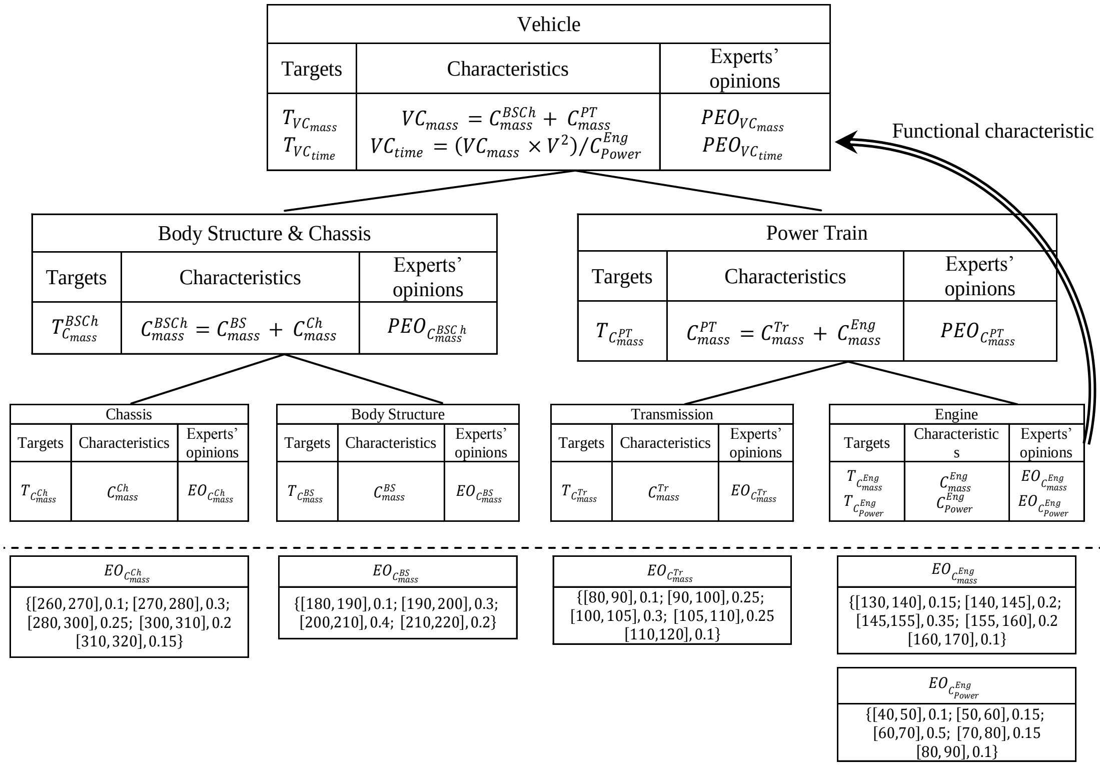

Figure 3-7: Simplified vehicle with functional characteristic and interaction among elements

3.3.8.1 Propagation of a mass characteristic

The mass characteristic is present in every component of a vehicle, and it has an additive property from the leaf nodes to the vehicle node. The independent experts' opinions are provided at the leaf elements of the model in the form of intervals and their subjective beliefs (see Figure 3-7). The information collected in the VMM is propagated by the process proposed in Section 3.3.6.

Figure 3-8 presents the results of mass characteristic uncertainty propagation in the VMM using the MATLAB module. The results take the form of plausibility (Pl) and belief (Bel) curves calculated by the Evidence theory for the vehicle level. The same curves can be plotted for any element or node of the model.

Using simple interval arithmetic, the lowest possible value for VC_{mass} is 650 kg and the highest possible value is 830 kg, according to the experts' opinions, hence the belief and plausibility that $VC_{mass} > 650\ kg$ is 100%, and the belief and plausibility that $VC_{mass} > 1140\ kg$ is 0%. For a specific proposition $VC_{mass} > x\ kg$, the belief and the plausibility can be graphically determined. For example, based on the curves presented on Figure 3-8, the belief that $VC_{mass} > 730\ kg$ is 0.42 and the plausibility is 0.93.

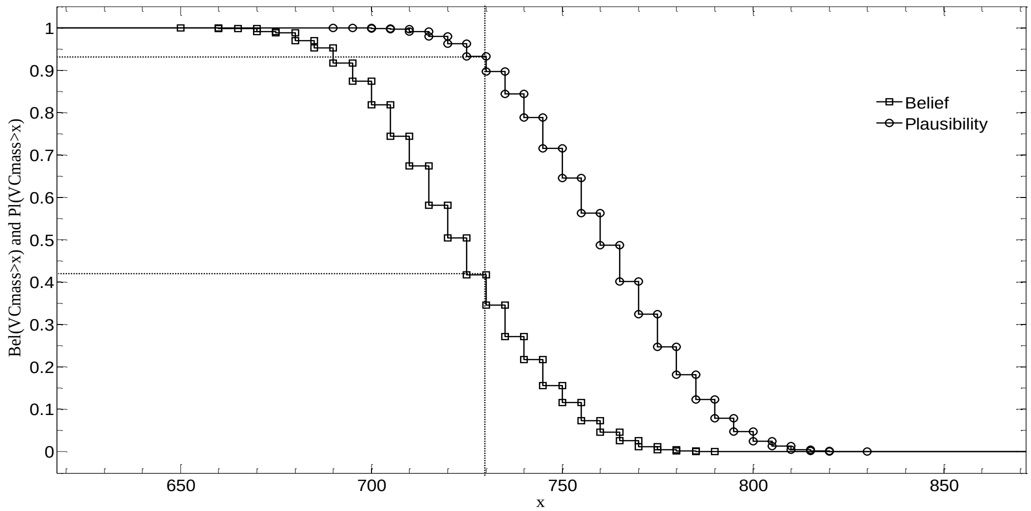

Figure 3-8: Belief and plausibility curves for the vehicle mass

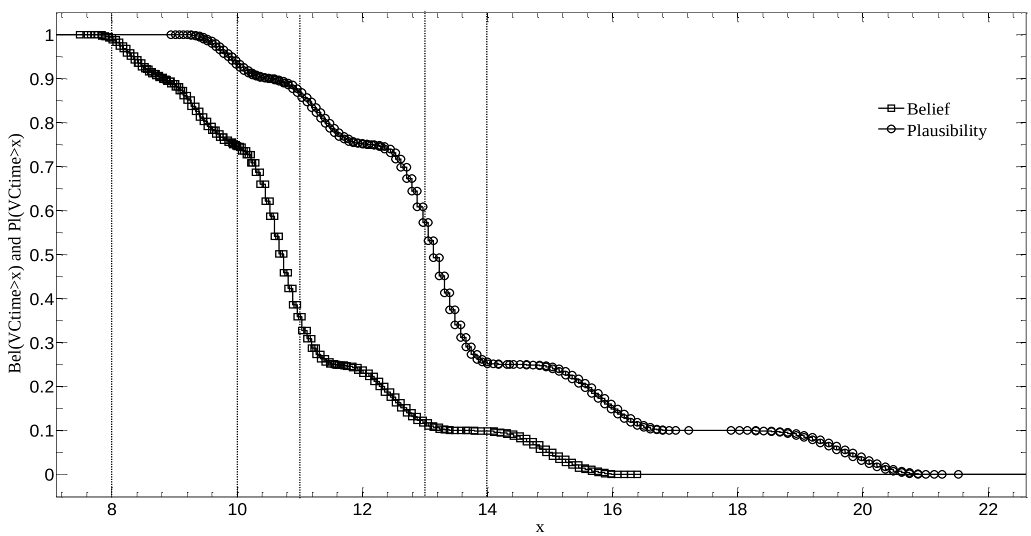

Figure 3-9: Belief and plausibility curves for the vehicle acceleration characteristic

3.3.8.2 Propagation of characteristics with functional relations

For the VMM in Figure 3-7, the time needed to accelerate from 0 to $V = 100$ km/h is presented in the form of a non-linear functional characteristic (VC_{time}). This characteristic depends on the vehicle mass (VC_{mass}) and the engine power (C_{power}^{Eng}):

$$VC_{time} = f\left(VC_{mass}, C_{power}^{Eng}, V\right) = \frac{VC_{mass} * V^2}{C_{Power}^{Eng}} \tag{24}$$

In this example, because of the dependence of VC_{time} on VC_{mass}, we first propagate the mass VC_{mass} (see Section 3.3.8.1) and then evaluate the acceleration time characteristic VC_{time}. An expert's opinion concerning the engine power characteristic is provided in Figure 3-7.

Figure 3-9 presents the uncertainties for the characteristic VC_{time} in the form of belief and plausibility curves. We can see that the acceleration time VC_{time} varies between 8 and 21. The belief and plausibility that $VC_{time} > 10$ are 0.75 and 0.94 respectively, indicating a high probability for this event. The belief and plausibility that $VC_{time} > 14$ are 0.1 and 0.25 respectively, indicating a low probability for this event. The region $11 > VC_{time} > 13$ shows the highest uncertainty for the present data, as represented by the large difference between the belief and plausibility curves in that region.

3.4 Conclusion

This paper has presented a methodology for performing uncertainty estimations during the conceptual design of a vehicle. The vehicle is modeled as a multilevel hierarchical structure where the targets associated with various characteristics are cascaded top-down and given to the experts of systems, subsystems and parts. Experts' opinions with respect to the feasibility of matching these targets are collected in the form of intervals for the possible values of characteristics associated with their subjective beliefs. This paper describes how, from multiple experts' opinions, uncertainties are propagated from the leaf nodes of the multilevel model up to the vehicle level. Evidence theory has been used to propagate belief and plausibility measures. Monte Carlo simulation based on probability distribution has been applied for some cases as a verification method.

Several simple examples were developed and presented to illustrate the process of uncertainty aggregation and propagation in the VMM. These examples involve different types of characteristics combined in several typical situations to provide insight into the meaning of the belief and plausibility measures. A final example illustrating application to a simplified vehicle design is given.

Work is ongoing to use these propagated uncertainties to drive the target reallocation process in the VMM during the VDP.

The validation of this methodology for uncertainty management in the MVM is proposed in Section 5.2.

Acknowledgements

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Chapter 4 Target Allocation under Uncertainty in a Multi-Characteristic Vehicle Multilevel Model

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In this paper, we consider setting performance targets for a vehicle design. The vehicle is modeled by a multilevel hierarchical tree structure. We have considered that each leaf of the structure may have several characteristics, and that for each characteristic a target is defined. Experts' opinions are expressed with uncertainty regarding the feasibility of achieving these targets. Experts' opinions are given in the form of intervals associated with their subjective beliefs for the possible values of characteristics. The collected information is propagated in the model to determine the plausibility and the belief for characteristics. Using this information, two measures regarding the desirability and the achievability of the characteristics are defined. An approach for target allocation under uncertainty based on the maximization of achievability and desirability measures of the characteristics is proposed and discussed.

4.1 Introduction

In the automotive industry, developing a new vehicle that satisfies consumer needs and desires is accomplished through a complex vehicle development process (VDP). This process requires

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collecting relevant information from a wide variety of sources to make crucial decisions impacting the value of the new vehicle.

At the early stages of the VDP, a set of vehicle characteristics desirable to the consumer are identified by a planning team and passed to a design team to guide the development of the vehicle. The design team translates the vehicle characteristics into targets for the systems, subsystems and parts. The target allocation for each component of the vehicle must be consistent with the vehicle characteristics and must respect the couplings and constraints among systems, subsystems and parts. Subject matter experts are in charge of designing components with respect to the component targets based on a range of analyses, historical and competitive data, etc. However, the initial utopian vehicle characteristics may be unattainable with the existing technologies, facilities and resources. So, in practice, during the conceptual design phase of complex systems, the design targets are refined iteratively as the VDP progresses and the components are designed in greater detail. The current practice for target allocation is a human-driven decision making process in search of compromises and design improvements based on experts' opinions. This task will most likely always remain under human supervision because objectives and constraints in complex systems are the result of subjective compromises evolving during the process [1-5].

It is important to understand the influence of uncertainty on the target allocation process and how it can impact the performance of the vehicle at the end of the development process. Several aspects are inherently uncertain during the design of complex systems owing to many factors such as random design variables, a lack of information about evolving technologies and manufacturing processes, the incomplete specification of components and the interactions among vehicle components. Inclusion of uncertainty during the VDP allows considering new compromises while controlling associated risks [1, 2, 5-7]. However, the amount of information to comprehend for human decision making may increase drastically. Several techniques for decision-making and target allocation were developed depending on the companies' cultures. There is a full range of techniques from those based purely on consensus and discussion such as the Quality Function Deployment (QFD) (see Refs. [8, 9]) to those based on mathematical programming and aiming the process automation.

In support to this trend, this work aims to demonstrate the capacity to perform automatic target allocation under uncertainty in complex systems and, most importantly, to provide synthesized information in the form of desirability and achievability measures in order to facilitate human decision.

The present work shares some concepts and terminology with the Analytical Target Cascading (ATC) methodology. The ATC was presented as a methodology for multilevel system design (see Refs. [10-13] for the original ATC and Refs. [14-16] for ATC under uncertainties), where all design variables and targets throughout the system are obtained by solving system level and component level minimum deviation problems. The ATC optimization tasks require integration of the analysis modules or simulation models to evaluate component characteristics. In contrast, the proposed target allocation process uses uncertain experts' opinions possibly obtained from multiple sources of information to reassign targets, which is more in line with the current practice for the VDP.

Section 4.2 recalls briefly the concepts and definitions of uncertainty management in a vehicle multilevel model that were introduced during previous work and presented in Ref. [17]. In Section 4.3, the target allocation process under uncertainty in the VDP is presented, followed by the application of this process to a simplified vehicle multilevel model in Section 4.4.

4.2 Uncertainty management in a vehicle multilevel model

Uncertainty management involves the aggregation and the propagation of uncertainty from multiple experts, in the form of probability distributions or intervals associated with their subjective beliefs. In this paper, it is considered for the sake of simplicity that the information is produced synchronously at the leaf nodes and then propagates up to the higher level of the multilevel model.

4.2.1 Vehicle multilevel model (VMM)

The vehicle multilevel model (VMM) is a hierarchical decomposition of a vehicle into systems, subsystems and parts. The VMM serves as the framework for handling and propagating component characteristics and uncertain experts' opinions.

In the VMM, we distinguish two types of characteristics: vehicle characteristics and engineering characteristics. At a given node in the VMM, an engineering characteristic C_{name}^{node} is a measure to provide physical specifications, characterize behaviours, and quantify the responses of components. A vehicle characteristic (VC_{name}) is a measure of a technical criterion often used to compare similar vehicles. Vehicle and engineering characteristics are related through functional relationships.

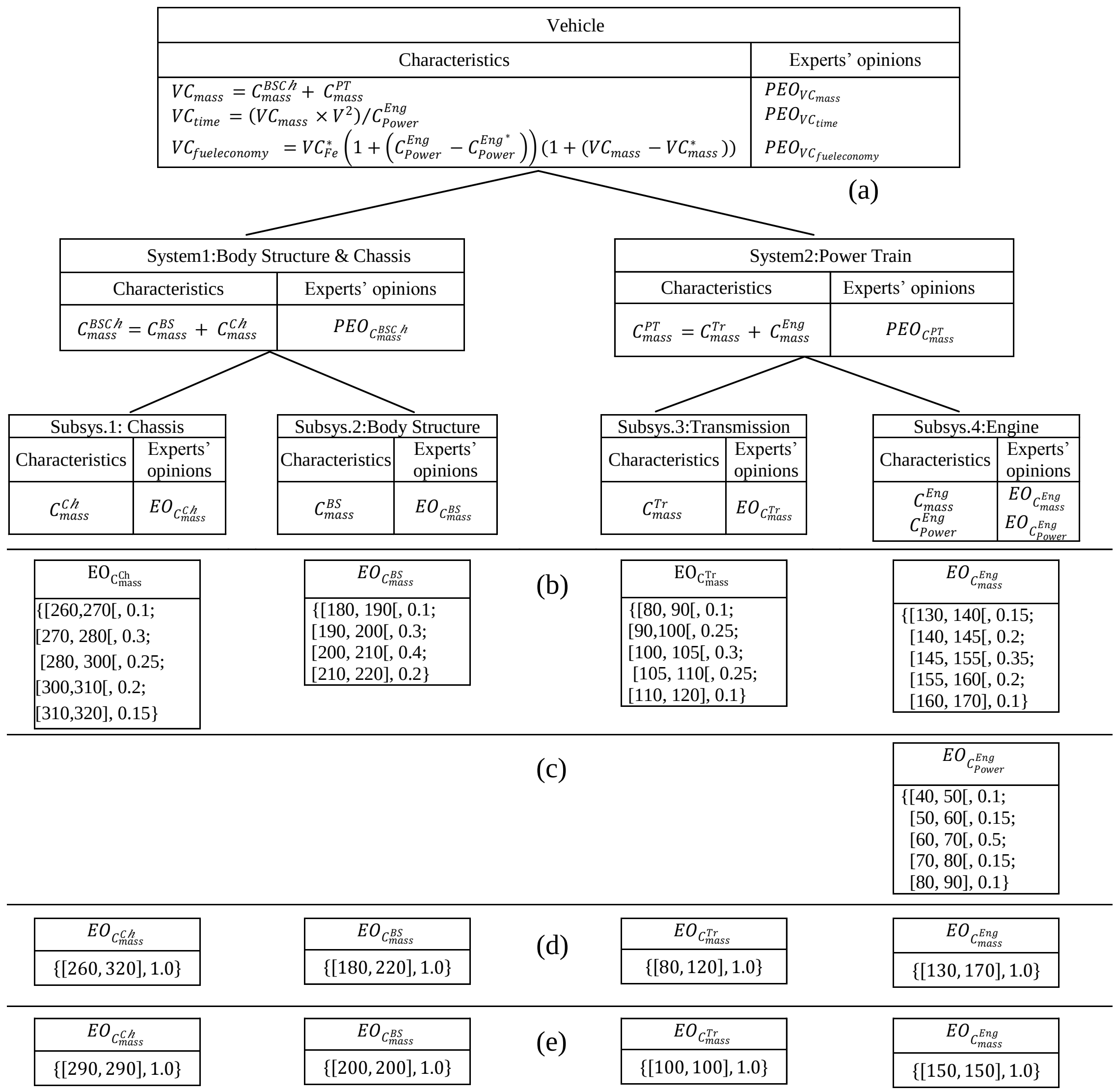

Figure 4-1: Example of vehicle multilevel model

Figure 4-1(a) presents a simple VMM, which contains two engineering characteristics (the mass of the components and the engine power) and three vehicle characteristics (vehicle mass, time for acceleration and fuel economy). Experts' opinions on characteristics are available in the VMM: $EO_{C^{node}_{name}}$ when provided at leaf nodes and $PEO_{C^{node}_{name}}$ when propagated from leaf nodes. For the mass characteristic, experts' opinions are provided in three different formats (multi-intervals, unique interval and discrete point) presented consecutively in Figure 4-1(b, d and e). The experts' opinions related to the engine power are given in Figure 4-1(c).

4.2.2 Uncertain experts' opinions

During any iteration of the VDP, the experts must evaluate the feasibility of targets. At any stage of the design process, the experts' opinions are subject to epistemic and aleatory uncertainty because the design may not be fully detailed and some design variables may be stochastic. It is also considered that uncertain experts' opinions encompass the effect of the interactions between components characteristics. Reference [17] provides details about the nature of uncertainty in experts' opinions and a discussion on the impact of interactions on uncertain experts' opinions.

Continuous probability density functions or probability given on bodies of information are classical approaches to represent uncertain information (e.g., Refs. [18-22]). We consider only real-valued engineering characteristics, and the bodies of uncertain information are provided by experts as real intervals associated with subjective beliefs. The probability given by an expert to a body of information reflects his confidence or subjective belief. For example, the value of characteristic C lies in the interval $[a, b[$ with x % subjective belief or in the interval $[c, d[$ with y % subjective belief, etc. Such expression provides no information about the probability distribution within the bodies of information, that is, within the intervals in our case. The granularity of uncertain information (i.e., interval span) must be selected by the expert to express at best his subjective opinion. Finer granularity of information tends toward continuous probability distribution.

Due to the various forms of uncertainty present in experts' opinions, we take an approach similar to Ref. [23] and select Evidence Theory to represent both epistemic and aleatory uncertainties.

4.2.3 Uncertainty representation

The uncertainty of an event B expressed as a subset of all the possible values of a characteristic at a node of the VMM is quantified by two measures: the belief (Bel) and the plausibility (Pl) in the interval $[0, 1]$. By considering the bodies of information A_i about the characteristic values and associated subjective beliefs sb_i provided by experts, the Evidence theory allows the quantification of these two measures [18, 23-27]. The belief in the event B is the summation of the subjective beliefs of all bodies of information A_i included in B; the plausibility of the event B is the summation of the subjective beliefs sb_i of all propositions A_i that intersect with B, and the intersection is not empty, as given by:

$$Bel(B) = \sum_{A_i \subset B} sb_i \tag{1}$$

$$Pl\,(B) = \sum_{A_i \cap B \neq \emptyset} sb_i \tag{2}$$

The belief and plausibility measures can be viewed as the minimum and maximum likelihood of occurrence of an event.

In the scope of this work, we have considered events such as the value of characteristic C being larger (or smaller) than a threshold value $x \in \mathbb{R}$, i.e., $B = \{C \in \mathbb{R} | C \geq x\}$. The uncertainty of events such as “the vehicle mass is larger than 730 kg” will serve as a measure of the confidence in minimally achieving a given value for a characteristic such as the vehicle mass.

When the uncertainty is given as real intervals with associated subjective beliefs, the belief and plausibility measures can be evaluated as follows. Let us consider $A = \{(I_1, sb_1), \ldots, (I_m, sb_m)\}$ the set of intervals $I_i = [l_i, u_i[$ and subjective beliefs sb_i of cardinality m representing an expert’s opinion. Based on Eqs. (1) and (2), the belief and plausibility that the characteristic be larger than a given value x are determined using the following expressions:

$$Bel(C \geq x) = \sum_{i=1}^{m} sb_i \cdot \delta_k(I_i) \text{ with } \begin{cases} \delta_k = 1 \text{ if } x \leq l_i \\ \delta_k = 0 \text{ if } x > l_i \end{cases} \tag{3}$$

$$Pl(C \geq x) = \sum_{i=1}^{m} sb_i \cdot \sigma_k(I_i) \text{ with } \begin{cases} \sigma_k = 1 \text{ if } x < u_i \\ \sigma_k = 0 \text{ if } x \geq u_i \end{cases} \tag{4}$$

Figure 4-2(a, b and c) provides three examples of how belief and plausibility change when the threshold value changes. The belief and plausibility for the vehicle mass are determined using experts’ opinions at leaf nodes in the form of a single interval given in Figure 4-1(d), sets of intervals given in Figure 4-1(b) or discrete values given in Figure 4-1(e).

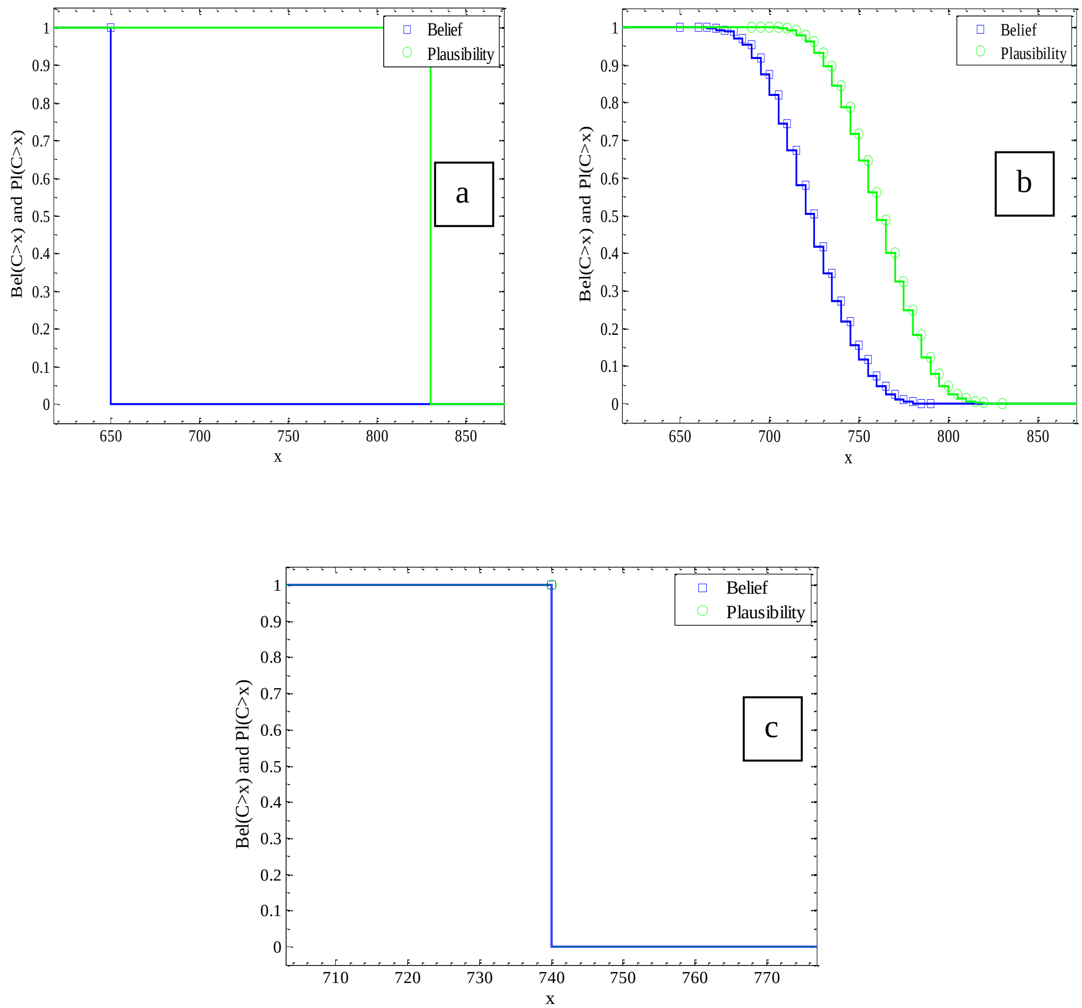

Figure 4-2: Belief and plausibility for the mass characteristic (experts' opinions in the form of single intervals (a), sets of intervals (b) and discrete values (c)).

4.2.4 Propagation of uncertain experts' opinions

In the proposed vehicle model, uncertain experts' opinions and characteristic targets are associated with every component. Experts' opinions ($EO_{C_{name}^{leaf}}$) are collected at the leaf elements of the VMM (Figure 4-1 (a)) and propagated into the VMM through functional relations in the form of propagated experts' opinions ($\mathrm{PEO}_{\mathrm{C}_{\mathrm{name}}^{\mathrm{node}}}$ at intermediate nodes and $\mathrm{PEO}_{\mathrm{VC}_{\mathrm{name}}}$ at the vehicle level).

Based on the functional relationships between the vehicle and engineering characteristics, a mapping between the uncertainties of the children nodes (input space) and the uncertainty of the parent node (output space) must be established at each level in the tree. To explain the uncertainty propagation method, let us consider a functional relation between real-valued characteristic C_{mass} of Subsytsem 1 ($C_{\text{mass}}^{Subsys1}$), Subsystem 2 ($C_{\text{mass}}^{Subsys2}$), and System 1 ($C_{\text{mass}}^{Sys1}$) given by:

$$C_{\text{mass}}^{Sys1} = f(C_{\text{mass}}^{Subsys1}, C_{\text{mass}}^{Subsys2}) = C_{\text{mass}}^{Subsys1} + C_{\text{mass}}^{Subsys2} \tag{5}$$

That is, $f: \mathbb{R}^2 \to \mathbb{R}$ is a function from a real bi-dimensional vector to a real number. All combinations of the input intervals from $C_{\text{mass}}^{Subsys1}$ and $C_{\text{mass}}^{Subsys2}$ must be propagated through the functional to obtain the output intervals for C_{mass}^{Sys1}. Hence, the function f must be extended to take intervals in $\mathbb{R}^2$ as inputs and produce intervals in $\mathbb{R}$ as outputs, that is noted $[f[: [\mathbb{R}^2[\to [\mathbb{R}[$. For input intervals $[a, b[$ of $C_{\text{mass}}^{Subsys1}$ and $[c, d[$ of $C_{\text{mass}}^{Subsys2}$, we consider an interval extension of the functional with the output interval $[l, u[$:

$$l = \underset{C_{\text{mass}}^{Subsys1} \times C_{\text{mass}}^{Subsys2} \in [a,b[\times[c,d[}{\text{minimum}} f(C_{\text{mass}}^{Subsys1}, C_{\text{mass}}^{Subsys2}) \tag{6}$$

$$u = \underset{C_{\text{mass}}^{Subsys1} \times C_{\text{mass}}^{Subsys2} \in [a,b[\times[c,d[}{\text{maximum}} f(C_{\text{mass}}^{Subsys1}, C_{\text{mass}}^{Subsys2}) \tag{7}$$

In the present case, the minimum and maximum operators are readily evaluated because f is a monotonic function of $C_{mass}^{Subsys1}$ and $C_{mass}^{Subsys2}$: $l = a + c$ and $u = b + d$. The subjective belief associated to the interval $[l, u[$ is equal to the product of the subjective beliefs associated with intervals $[a, b[$ and $[c, d[$.

In the general case where f is a nonlinear function, Eqs. (6) and (7) are solved using a general function minimizer.

4.3 Target allocation under uncertainty

The vehicle development process alternates knowledge accumulation and decision making through a series of development stages with reviews (or gates) as described in Refs. [4, 28]. One element is shown in Figure 4-3.

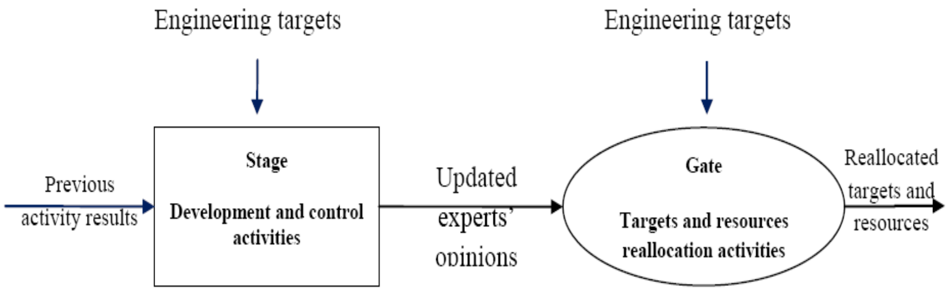

Figure 4-3: Stage/gate development process

The vehicle development process starts with subjective and uncertain estimates of the characteristics. As the design matures, experts' opinions are refined based on data generated from development activities such as analyses, simulations, prototypes, and demonstrations. Collected information provided as experts' opinions about engineering characteristics is transformed into information for the vehicle characteristics. Obviously, highly accurate assessments of the vehicle characteristics are possible only when the product is designed in detail and uncertainty has greatly diminished. However, design decisions in the VDP must be made even with uncertain information. Decision makers must choose among alternative technical solutions, and then select the characteristic targets and the engineering resources needed to achieve them. Ultimately and ideally, all decisions guide the design toward a vehicle meeting all the customer expectations.

The overall design of a new vehicle can be conducted following many strategies. Wheelwright and Clark [4, 28] consider several development processes which focus on different degrees of resource utilization, technical advancement, risk management, system solution, speed, and technical performance. In the scope of the current work, we focus on technical performance by proposing an automatic target allocation method applicable during a single stage/gate pass. The resource allocation problem will be the subject of subsequent work.

The target allocation at each stage of the vehicle development process can be guided by evaluating the ability to achieve the vehicle characteristics targets and how desirable they are. Accordingly, two measures are introduced in the following subsections: achievability and desirability. A single measure called the global utility of design (GUD) combining achievability and desirability is also introduced.

4.3.1 Desirability measure

We use the term desirability to describe a non-dimensional utility of a vehicle characteristic value with respect to customer expectations and desires [29]:

$$D_j : vc_j \rightarrow U_j\left(vc_j\right) \in [0,1]$$

The definition of the utility function should reflect consumer input and may be based on marketing studies. For the purpose of the present paper and without loss of generality, we have developed our own function defined by a fixed and somewhat arbitrary desirability interval bounds Min_{DesVal} and Max_{DesVal} for each vehicle characteristic. A sigmoid function having an "S" shape is mapped into the desirability interval, supposing that $U_j(Min_{DesVal}) = a$ and $U_j(Max_{DesVal}) = 1 - a$. We have selected $a = 0.05$ for evaluating all the desirability measures. Outside the Min_{DesVal} and Max_{DesVal} interval bounds, the utility tends to 0 or 1. This type of utility function is adapted when vehicle characteristic has clear preference articulation such as “higher is better” or “lower is better” (see hypothetical examples in Figure 4-4 and Figure 4-5). In what follows and without loss of generality, we only consider these two types of preference articulation.

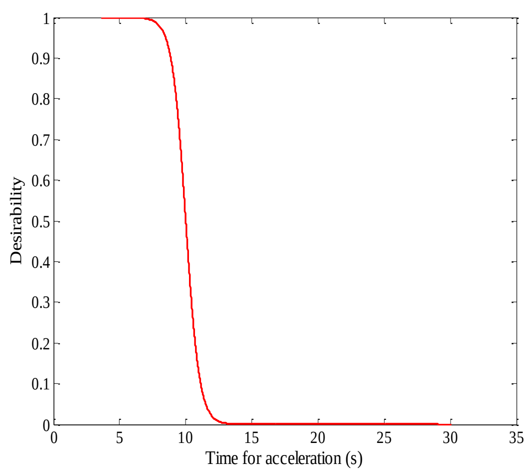

Figure 4-4: Lower is better desirability function for time for acceleration characteristic

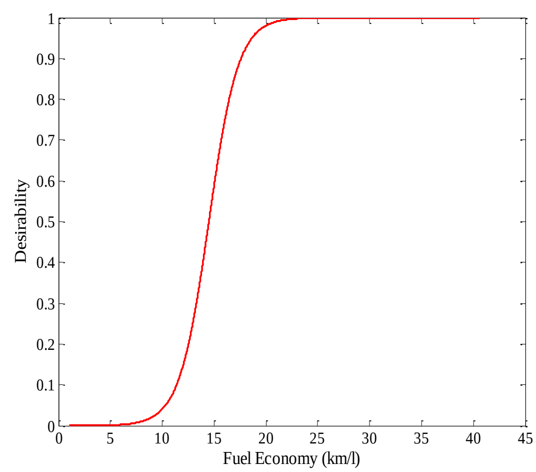

Figure 4-5: Higher is better desirability function for fuel economy characteristic

The desirability is a measure of the customer’s perception of the vehicle characteristics. While the customer does not directly perceive the engineering characteristics, we recall that vehicle characteristics values vc_j are obtained from the functional relations that exist between vehicle and engineering characteristics. Hence, by providing a set of engineering characteristic values, a vehicle characteristic desirability can be evaluated.

4.3.2 Achievability measure

We define the achievability as a measure of the confidence in achieving a characteristic value (engineering or vehicle) based on propagated experts' opinions.

Experts' opinions express the confidence that a characteristic value will be effectively within the provided intervals. Hence, the confidence in achieving a given value or a narrow range of values is meaningless because the probability distribution in the intervals is unknown. Instead, we have considered the belief and plausibility for the events that a variable will be larger or lower than a given value. This can be used to define four measures of achievability in the case of a vehicle characteristic VC_j:

$$A_j^{Bel>}: \ vc_j \rightarrow Bel\left(VC_j \geq vc_j\right) \tag{8}$$

$$A_j^{Bel<}: \ vc_j \rightarrow Bel\left(VC_j < vc_j\right) \tag{9}$$

$$A_j^{Pl>}: \ vc_j \rightarrow Pl\left(VC_j \geq vc_j\right) \tag{10}$$

$$A_j^{Pl<}: \ vc_j \rightarrow Pl\left(VC_j < vc_j\right) \tag{11}$$

Similarly to the desirability, each achievability measure is between 0 and 1, independent of the unit of the characteristic being considered.

In addition, the belief and plausibility of an event $\mathcal{A}$ and its contrary $\overline{\mathcal{A}}$ are linked with the following relations (see Refs. [18, 25, 27, 30]):

$$Bel(\mathcal{A}) = 1 - Pl(\overline{\mathcal{A}}) \tag{12}$$

$$Pl(\mathcal{A}) = 1 - Bel(\overline{\mathcal{A}}) \tag{13}$$

Hence, we have the following relations:

$$A_j^{Bel<} = 1 - A_j^{Pl>} \tag{14}$$

$$A_j^{Pl<} = 1 - \ A_j^{Bel>} \tag{15}$$

and only $Bel(VC_j \geq vc_j)$ and $Pl(VC_j \geq vc_j)$ need to be evaluated from experts' opinions to determine the four achievability measures. Similar achievability measures can be defined for the engineering characteristics.

4.3.3 Global utility of design (GUD)

The proposed global utility of design (GUD) combines the achievability and the desirability of a set of p characteristics $\{vc_1, \dots, vc_p\}$ in a single measure of design merit. We define the GUD for vehicle characteristics as:

$$GUD(vc_1, \dots, vc_p) = \sum_{j=1}^{p} w_j * D_j(vc_j) * A_j(vc_j) \quad (16)$$

The weights w_j serve to balance the importance of the different characteristics. In the absence of knowledge about the relative importance of characteristics, we have considered $w_j = 1$.

The term $A_j(vc_j)$ represents an overall achievability for a single characteristic defined as:

$$A_j(vc_j) = \left((1-\Phi)A_j^{Pl}(vc_j) + \Phi A_j^{Bel}(vc_j)\right) \quad (17)$$

If a vehicle characteristic VC_j is identified as "higher is better" then the measures of achievability used in the GUD are the following:

$$A_j^{Pl}(vc_j) = A_j^{Pl>}(vc_j) = Pl(VC_j \geq vc_j) \quad (18)$$

$$A_j^{Bel}(vc_j) = A_j^{Bel>}(vc_j) = Bel(VC_j \geq vc_j) \quad (19)$$

For these characteristics, the higher the vc_j value, the larger the corresponding desirability of the vehicle. However, higher vc_j values are most likely harder to achieve, which corresponds to lower belief/plausibility and lower achievability value (see Figure 4-6).

If a characteristic VC_j is identified as "lower is better" then the achievability measures are taken as follows:

$$A_j^{Pl}(vc_j) = A_j^{Pl<}(vc_j) = Pl(VC_j < vc_j) \quad (20)$$

$$A_j^{Bel}(vc_j) = A_j^{Bel<}(vc_j) = Bel(VC_j < vc_j) \quad (21)$$

In general, for any given characteristic vc_j, $A_j(vc_j)$ and $D_j(vc_j)$ are thus conflicting measures. Hence, when varying vc_j there is a trade-off between achievability and desirability. Figure 4-6 and Figure 4-7 illustrate the trade-off between achievability and desirability for fuel economy and time for acceleration in the example presented earlier.

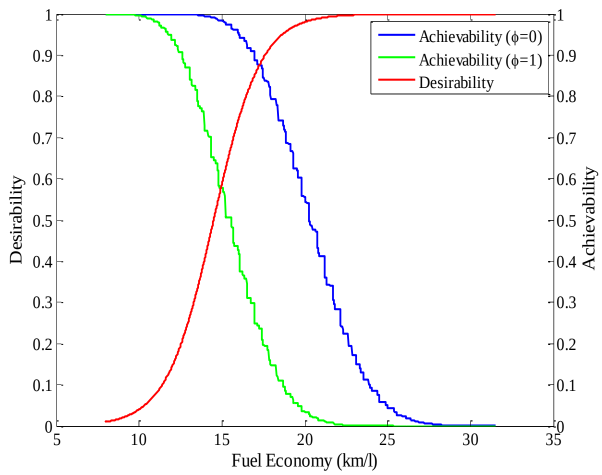

Figure 4-6:Achievability/desirability compromise for "higher is better" characteristic

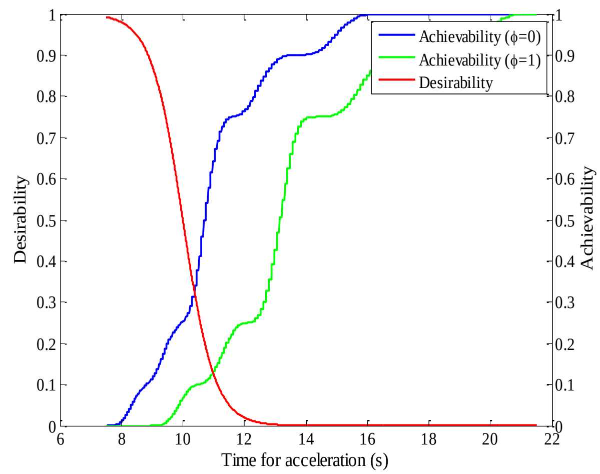

Figure 4-7: Achievability/desirability compromise for "lower is better" characteristic

The overall achievability measure is determined by weighing $A_j^{Pl}(vc_j)$ and $A_j^{Bel}(vc_j)$ with a decision coefficient $\Phi \in [0,1]$. The level of risk tolerance is defined by the value of Φ. For $\Phi = 1$, we consider only the belief, which represents the lowest probability for the event occurrence. For $\Phi = 0$, only the plausibility is considered which represents the highest probability for the event occurrence.

With the above definition, the GUD provides a compromise between two competing measures for a set of characteristics. Low GUD values may correspond to a low desirability or a low achievability for any characteristic. But GUD values close to one represent designs with high achievability and desirability for all characteristics.

4.3.4 Multi-characteristic achievability (MCA)

The MCA is a global measure for the achievability of a component with a set of n characteristics $\{c_1, \ldots, c_n\}$. It is defined by the following equation:

$$MCA(c_1, \dots, c_n) = \prod_{i=1}^{n} A_i(c_i) \tag{22}$$

where $A_i(c_i)$ represents an overall achievability for a single characteristic (c_i), as given by Eq. (17). The product of the achievability of single characteristic in Eq. (22) guarantees that feasible trade-offs among characteristics will have a MCA strictly greater than zero and lower than or equal to 1.

4.3.5 Target allocation by cascading optimization

The current VDP practice for target reallocation at a gate is a human-driven decision making process in search of design improvements based on experts' opinions. Because the definition of competing and evolving objectives and constraints in complex systems is subjective, this task will most likely always remain under human supervision. In the scope of this work, a fully automated process is desired to demonstrate the capacity to perform acceptable target allocation based on the aforementioned definition and constraints. However, the proposed approach was also developed to give access to synthesized information such as desirability and achievability measures in order to facilitate human decision when the amount of information increases beyond what a person can comprehend.

At the vehicle level, an utopian objective after many iterations is to converge to a competitive design, that is, to achieve a narrow range of highly desirable vehicle characteristics. This is obtained throughout iterations by selecting targets corresponding to high desirability, because, as previously stated, targets guide the range of possible characteristics values as expressed in experts' opinions. However, experts' opinions cannot always meet the targets. In this case, to achieve convergence, targets must be attracted towards highly achievable characteristic values as identified in experts' opinions. Also, concentration of experts' opinions within a narrow range of value requires smooth target variation between iterations. Moreover, smooth variations will also help stabilizing the design process in the situation of experts' opinions sudden shift when new unanticipated information becomes available. In summary, the proposed approach must allocate targets in accordance with mutual improvements of achievability and desirability to ensure convergence to a competitive design.

Several target allocation approaches applicable to the design of a vehicle under uncertainty were presented in Ref. [31]. These approaches use both single-objective and multi-objective optimization formulations, where desirability and achievability of characteristics are components of the objective functions or constraints. In this paper, we retain the approach where targets corresponding to high achievability and desirability are sought by maximizing the global utility of design (GUD):

$$\underset{vc_1,\dots,vc_m}{\text{maximize}}\, GUD(vc_1, \dots, vc_m) \tag{23}$$

The vehicle targets need to be propagated to obtain the engineering characteristic targets. First, let us consider the propagation to the system level. To simplify the notation, let us suppose that the vehicle characteristics values are linked to the system characteristics with the functional relations:

$$vc_j = f(c_1^{sys}, \dots, c_n^{sys})$$

Potentially, several combinations of system targets $t_{c_1^{sys}}, \dots, t_{c_n^{sys}}$ can produce the desired vc_j. The new targets are intended to concentrate experts' opinions around a narrower range of engineering characteristics without consideration of the desirability of the vehicle which is already fixed. To do that, targets corresponding to the highest global achievability are sought such that the system targets are consistent with the vehicle targets:

$$\begin{aligned} &\underset{t_{c_1^{sys}},\dots,t_{c_n^{sys}}}{\text{maximize}}\, MCA\left(t_{c_1^{sys}}, \dots, t_{c_n^{sys}}\right) \\ &\text{subject to: } vc_j = f(t_{c_1^{sys}}, \dots, t_{c_n^{sys}}) \text{ for all } j = 1, \dots, m \end{aligned} \tag{24}$$

The equality consistency constraint given as $vc_j = f(t_{c_1^{sys}}, \dots, t_{c_n^{sys}})$ can be relaxed to an inequality constraint if the vehicle characteristic is articulated as "lower is better" or "higher is better".

$$\begin{aligned} &\underset{t_{c_1^{sys}},\dots,t_{c_n^{sys}}}{\text{maximize}}\, MCA\left(t_{c_1^{sys}}, \dots, t_{c_n^{sys}}\right) \\ &\text{subject to: } vc_j \le f(t_{c_1^{sys}}, \dots, t_{c_n^{sys}}) \text{ for all } j = 1, \dots, m \text{ with "higher is better" } VC_j \end{aligned} \tag{25}$$

$$vc_j \geq f\left(t_{c_1^{sys}}, \dots, t_{c_n^{sys}}\right) \text{ for all } j = 1, \dots, m \text{ with "lower is better" } VC_j$$

Having identified all system targets, the subsystem targets must be allocated. For each system Sk, again for notation simplicity, let us suppose that there is a functional relation between the characteristics c_i^{Sk} and the characteristics from all its subsystems:

$$c_i^{Sk} = f(c_1^{subsys}, \dots, c_n^{subsys})$$

The targets for the subsystems of system Sk are obtained by solving a problem similar to the formulation given in Eq. (25).

The GUD and the MCA curves are not smooth because of the belief and plausibility are not continuous functions as given in Eqs. (3) and (4) (see Figure 4-6 and Figure 4-7). The maximisation of GUD and the MCA functions requires a global search optimization algorithm. We have used the genetic algorithm provided in Matlab (Genetic Algorithm Optimization Toolbox). At the system, subsystem and part levels multiple solution of the optimization problems are possible because of the stairs form of the MCA function.

4.3.6 Independence of the characteristics

The independence of the objectives is an axiom of the utility analysis to be verified when formulating an objective function in the form of additive utility [1, 32, 33]. The proposed GUD was formulated in the form of an additive utility function of the considered vehicle characteristics. The objective or the marginal utility of each characteristic was defined as the product of the related achievability and desirability. This formulation of the GUD was based on an assumption of independency of the objectives to meet the utility theory requirement.

In the engineering design, characteristics interdependency is almost unavoidable because of the shared design parameters. In the example presented in Figure 4-1, it is obvious that the vehicle characteristics (fuel economy and time for acceleration) are interdependent since they share two engineering characteristic (the mass and the engine power). In this context, the relevance of the GUD as an additive utility function can be contested since the vehicle characteristics are interdependent. Consequently the validity of the GUD for decision-making and target allocation can also be contested.

In Ref. [34], D. L. Thurston stated that in the engineering design there is a mistaken belief that the independence conditions of utility analysis are not valid when the characteristics are interdependent. This misconception is due to confusion about distinction between independence of characteristics and independence of preferences on the objectives. The independence conditions used in utility analysis concern the preferences over each objective. When achieved, it leads simply to the facilitation of the assessment of the multi attributes utility function. In contrast, The independence of characteristics imply that the objectives are not conflicting and can be improved independently without influencing each other. However, when the characteristics are interdependent, unavoidable trade-off between the objectives must be reached.

Considering this statement, the proposed GUD remains relevant for target allocation and decision-making despite the interdependency of the vehicle characteristics. The problem will be shifted from the maximization of each objective alone to the search of a feasible compromise between the characteristics that maximize the multi-attribute utility function.

In the case of the problem presented in Figure 4-1, the target allocation will performed by the search of a feasible combination of engineering characteristics (mass and engine power) corresponding to the compromise between the vehicle characteristics (fuel economy and time for acceleration) that maximize the GUD.

4.4 Application

As previously mentioned, a target allocation process for the whole VDP is beyond the scope of this work. Instead, we have considered a possible situation of an early iteration during the VDP where experts' opinions correspond to a wide range and relatively low value of desirability as presented in Figure 4-1.

To illustrate the target allocation, first, we consider the case of a single characteristic (the mass), followed by the case of multiple characteristics (time for acceleration and fuel economy).

4.4.1 Target allocation of the mass characteristic

Mass is one of the most important engineering characteristic in the VDP since it is present in every component and it impacts many of the vehicle characteristics. The vehicle mass has, in general, a “lower is better” articulation. The corresponding experts' opinions at the leaf nodes of the VMM are presented in Figure 4-1(b). The corresponding achievability and desirability are plotted as functions of the vehicle mass in Figure 4-8.

The problem consists in determining the proper balance between the achievability and the desirability of the vehicle mass target (see Figure 4-8). The mass target is obtained by applying the optimization problem of Eq. (23), to the vehicle level of the VMM. The vehicle mass target is located on the maximum of the GUD function curve for any value of the decision factor Φ, as illustrated in Figure 4-9. This vehicle mass target is cascaded down by the application of the optimization problem, given in Eq. (25), to the systems, subsystems and parts levels.

In the example, two designs are considered corresponding to the limit values that can be taken by the decision factor Φ. With the decision factor $\Phi = 0$, the decisions are optimistic and are based on an achievability calculated from the plausibility measure (see Eq. (17)). In this case, the vehicle mass target obtained is 730 kg with a desirability of 72.57% and an achievability of 65.39%.

With the decision factor $\Phi = 1$, only the belief measure is considered in the formulation of the achievability. In this case the decisions are conservative. The vehicle mass target is 745 kg with a desirability of 38.07% and an achievability of 28.43%.

As shown in Table 4-1, the achievability of the mass targets of the systems and the subsystems can vary significantly whether $\Phi = 1$ or $\Phi = 0$ even if it corresponds to relatively small variations of the characteristic targets.

In practice, the overall target allocation strategy depends on the selection of the decision factor Φ with more achievable but less desirable designs being obtained for larger values of Φ. We can also anticipate that during a smooth course of design, experts' opinions will more likely concentrate (i.e., lower number of intervals and small interval span) with new targets that they have previously identified as being highly achievable. To demonstrate the effectiveness of this approach, we have to consider many iterations of the VDP which is out of the scope of this paper. In future work, this approach will be deeply investigated and verified.

Table 4-1: Allocated mass targets for the VMM nodes

	$\boldsymbol{\Phi = 0}$			$\boldsymbol{\Phi = 1}$		
	Target (kg)	Achievability	Desirability	Target (kg)	Achievability	Desirability
Vehicle	730	65.39%	72.57%	745	28.43%	38.07%
Power Train	250	77%	X	256.87	45.75%	X
Engine	145.38	70%	X	147.6	35%	X
Transmission	104.61	65%	X	109.27	65%	X
Body struct & Chassis	480	62 %	X	488.12	20%	X
Chassis	278.43	40 %	X	282.38	40%	X
Body structure	201.56	80 %	X	205.73	40%	X

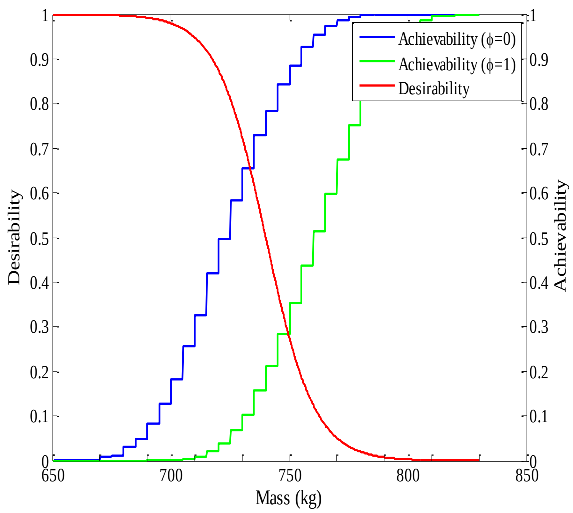

Figure 4-8: Desirability and achievability ($\Phi = 0$ and $\Phi = 1$) curves for the vehicle mass characteristic

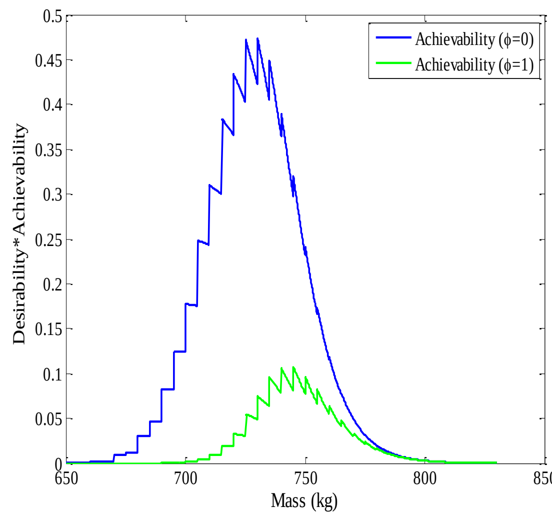

Figure 4-9: GUD ($\Phi = 0$ and $\Phi = 1$) curves for the vehicle mass characteristic

4.4.2 Target allocation for multiple vehicle characteristics

To illustrate the target allocation of multiple characteristics, we consider the example of a VMM with two vehicle characteristics (time for acceleration and fuel economy).

The time for acceleration (noted characteristic VC_{time}) or the time needed to accelerate from rest to V = 100 km/h is approximated in the form of a non-linear functional characteristic. This characteristic depends on the vehicle mass (VC_{mass}) and the engine power (C_{power}^{Eng}), as approximated by the equation below:

$$VC_{time} = f\left(VC_{mass}, C_{power}^{Eng}, V\right) = \frac{VC_{mass} * V^2}{C_{Power}^{Eng}} \tag{26}$$

The desirability of the VC_{time} characteristic is articulated as “lower is better”. The achievability and desirability curves are presented in Figure 4-7.

The fuel economy characteristic (noted $VC_{fueleconomy}$) represents the distance traversed by the vehicle for each litre of fuel. This characteristic depends on the vehicle mass and the engine power. We propose a simple approximation formula:

$$VC_{fueleconomy} = VC_{Fe}^{*}(1 + \alpha(C_{power}^{Eng^{*}} - C_{power}^{Eng}))(1 + \beta(VC_{mass}^{*} - VC_{mass})) \qquad (27)$$

where VC_{Fe}^{*}, VC_{mass}^{*} and $C_{power}^{Eng^{*}}$ represent selected points of references of the fuel economy, the vehicle mass and the engine power. In our case, they are equal to the middle of the intervals of the characteristics defined from the experts’ opinions. The parameters ($\alpha = 0.01$) and ($\beta = 0.005$) are two weighting factors allowing us to account for the effect of variation of the engine power and the vehicle mass on the fuel economy around the reference points. The fuel economy has a “higher is better” articulation (see desirability and achievability curves for the time for acceleration characteristic in Figure 4-6).

The allocation of targets for multiple vehicle characteristics is performed by applying the optimization problem, given in Eq. (23), to the vehicle level of the VMM. The solution consists in the identification of the vehicle characteristic targets that maximize the product of achievability and desirability for each characteristic. This solution is located at the maximum of the GUD curve of Figure 4-10.

Once the vehicle characteristics targets are defined, they are cascaded down in the form of engineering characteristics targets by applying the optimization problem, given in Eq. (25), to the systems, subsystems and parts levels of the VMM.

Table 4-2 illustrates the results of the process of target allocation for multiple characteristics applied to the VMM with the decision factor Φ equal to 0 and 1 consecutively. These results, compared to those of Table 4-1, show that when adding vehicle characteristics in the GUD function, the target values of the masses and their corresponding achievability have smaller values. That implies that the target allocation process is influenced by the vehicle characteristics considered. Our formulation can then propose a balance of the characteristics which comprehends all the customer requirements.

Table 4-2: Allocated targets for multiple characteristics

	$\Phi = 0$			$\Phi = 1$		
Characteristic Name	**Target**	**Desirability**	**Achievability**	**Target**	**Desirability**	**Achievability**
	Vehicle characteristics					
Time for acceleration	9.59 (s)	70.24%	68.71%	10.11 (s)	44.46%	8.18%
Fuel economy	17.4 (km/l)	88.6%	87.6%	15.22 (km/l)	62.55%	55.7%
	Engineering characteristics					
Vehicle mass	724.99 (kg)	X	49.58%.	753.66 (kg)	X	35.39%.
Power Train mass	251.75 (kg)	X	77%	261.95 (kg)	X	63.75%
Engine mass	146.72 (kg)	X	70%	156.52 (kg)	X	70%
Transmission mass	105.02 (kg)	X	90%	105.41 (kg)	X	65%
Body structure & Chassis mass	473.23 (kg)	X	44%	491.43 (kg)	X	36.5%
Chassis mass	272.56 (kg)	X	40%	280.74 (kg)	X	40%
Body structure mass	200.66 (kg)	X	80%	210.96 (kg)	X	80%
Engine power	78.37 (hp)	X	25%.	77.29 (hp)	X	10%.

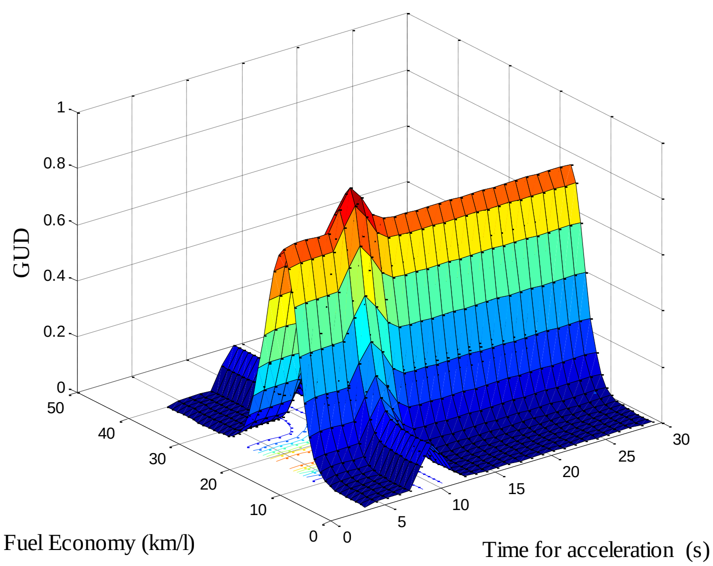

Figure 4-10: GUD measure of fuel economy and time for acceleration

4.5 Conclusion

In this paper a methodology for target allocation during the vehicle development process (VDP) is proposed. The VDP has been modeled as a multilevel decision making framework based upon a component decomposition of the vehicle in the form of a hierarchical tree structure. The vehicle characteristics are determined by top-down functional relations with the engineering characteristics of the components in the tree structure.

Targets are provided to guide experts design work and estimates. For the model of VDP considered, experts determine the possible values of the characteristics around the targets in the form of intervals associated with their subjective beliefs.

The proposed methodology handles the experts' uncertain opinions provided at the leaf node components of the vehicle model. This information is propagated bottom-up in the vehicle multilevel model and the uncertainty of the vehicle characteristics is obtained based upon the Evidence theory. Two measures of uncertain knowledge about characteristics values are calculated: the plausibility and the belief; from which a measure of achievability of a characteristic value is determined.. A desirability measure of a vehicle characteristic value is also defined to account for the customer preferences. The utopian objective of a target allocation procedure is to favour concentration of design towards a competitive design, that is, to achieve a narrow range of highly desirable vehicle characteristics. To achieve this, an approach for target allocation under uncertainty based on the maximization of achievability and desirability measures of the characteristics targets in a mono-objective problem is proposed, discussed and applied to a simplified vehicle multilevel model. The proposed target allocation approach consists in cascading multilevel optimizations top-down in the model allowing the setting of the characteristics' targets. Two examples have been provided and the results show that automatic target allocation under uncertainty has been achieved for different cases. The influence of a selection factor to control the concentration of experts' opinions is also discussed.

The validation of this methodology for target allocation under uncertainty in the multi-characteristic vehicle multilevel model is presented in the Section 5.3.

Acknowledgements

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Chapter 5 Validation and application

5.1 Introduction

In this chapter, we present the validation and verification of the proposed methodologies for uncertainty management and target allocation under uncertainty in the multilevel model presented consecutively in chapters 3 and 4. We note that the validation was a big issue during the project. Indeed, the proposed approaches are new and only scarce literature on analogous methods with example is available. Moreover, the proposed approaches are complex to the point where no analytical solutions are possible. For that reason, we resort to the Monte Carlo simulation in order to provide approaches equivalent to the proposed ones. This allows a comparison of results providing insight into the validity and the advantages of each proposed approach.

The chapter is organized in three sections. In Section 5.2, we present the validation of the uncertainty management methodology proposed in Chapter 3. In Section 5.3, we present the validation of the proposed methodology for target allocation under uncertainty presented in Chapter 4. In Section 5.4, we propose a decision-making framework that will includes both the uncertainty management and the target allocation methodologies. This framework will help illustrating how the proposed methodologies behave during a real iterative process.

5.2 Validation of the methodology for uncertainty management

To validate the methodology for uncertainty management proposed in Chapter 3, two test cases are considered. The first one comes from the literature and the second one is a simplified multilevel model. Both test cases are analyzed using the Probability theory and Evidence theory approaches. Uncertain information, in the form of experts' opinions collected at the leaf nodes of the multilevel model, is aggregated and propagated bottom-up. Based on this information, belief and plausibility measures are calculated and compared with cumulative probability obtained by a Monte Carlo simulation. This comparison will provide an insight into the meaning of the belief and plausibility measures and also the validity of the proposed approach for uncertainty management in the VMM.

5.2.1 Problem from Sandia laboratory

In the context of an uncertainty workshop held on 2002, Sandia Laboratory presented a challenge problem [1, 2] to the proponents and practitioners of all available candidates methods that can represent, aggregate and propagate the different types of uncertainty through a computational model. The main objective was the identification of promising approaches to handle uncertainty.

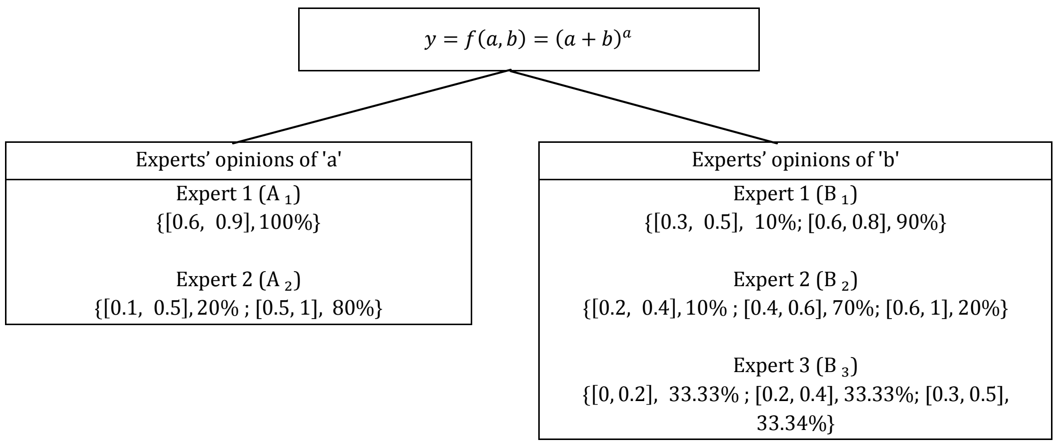


Figure 5-1: Sandia challenge problem

The challenge problem was presented in the form of a simple multilevel model (see Figure 5-1) with two uncertain input parameters (a & b) and one system response variable y given as an algebraic equation:

$$y = f(a,b) = (a+b)^a \tag{1}$$

The input variables are considered independent in the sense that knowledge about the value of one parameter implies nothing about the value of the other. Multiple experts provide their opinions for the input variables. The emphasis of this study is on comparing uncertainty estimation results obtained by the application of Probability and Evidence theories.

5.2.1.1 Evidence theory approach

a) Aggregation of uncertain information

The combination of n experts' opinions provided at a leaf node is performed by the mixing or averaging rule (see Section B.2.1.2 in Appendix B for details). The result of the aggregation is an equivalent expert. The aggregation is performed in two steps. First, the uncertain information is organized in the form of lower triangle matrix with the same dimension for all the experts' opinions. Second, these triangular matrices are averaged.

To represent the lower triangular matrix, let $l_1, l_2, \ldots, l_m$ be the lower values for m intervals bounds, where $l_1 < l_2 < \cdots < l_m$. Let $u_1, u_2, \ldots, u_m$ be the upper values of the intervals' bounds, where $u_1 < u_2 < \cdots < u_n$. The intervals can be expressed as $[l_i, u_j]$ with $l_i \leq u_j$. Let $m([l_i, u_j])$ be the BPA for the interval $[l_i, u_j]$. In this case, the $m \times m$ triangular matrix representing an expert's opinion can then be written as:

$$\begin{array}{c c} & \begin{array}{ccccc} l_1 & \ldots & l_i & \ldots & l_n \end{array} \\ \begin{array}{c} u_1 \\ \ldots \\ u_j \\ \ldots \\ u_n \end{array} & \begin{pmatrix} m([l_1, u_1]) & \ldots & & & \\ \ldots & \ldots & & & \\ m([1, u_j]) & \ldots & m([l_i, u_j]) & \ldots & \\ \ldots & \ldots & \ldots & \ldots & \\ m([l_1, u_n]) & \ldots & m([l_i, u_n]) & \ldots & m([l_n, u_n]) \end{pmatrix} \end{array} \tag{2}$$

Once all experts' opinions are provided for a given node then the mixing rule is applied using the following equation:

$$A = \frac{1}{n} \sum_{i=1}^{n} w_i . A_i \tag{3}$$

where the w_i are the weights assigned according to the reliability of the sources of information. In this case, the sources are equally credible and thereafter the w_i are equal to 1.

Input variable a:

The set $\{0.1, 0.5, 0.6, 0.9, 1\}$ represents the list of the intervals' bounds of the experts' opinions (1 and 2) provided for the variable a (see Figure 5-1). From this list we deduce the sets of lower values $\{0.1, 0.5, 0.6, 0.9\}$ and upper values $\{0.5, 0.6, 0.9, 1\}$ needed for the construction of the triangular matrices A_1 and A_2. The matrix A representing the aggregated information provided by the experts (1 and 2) is obtained by the averaging of A_1 and A_2.

$$A_1 = \begin{array}{c c} & \begin{array}{cccc} 0.1 & 0.5 & 0.6 & 0.9 \end{array} \\ \begin{array}{c} 0.5 \\ 0.6 \\ 0.9 \\ 1 \end{array} & \left[\begin{array}{cccc} 0 & & & \\ 0 & 0 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}, \quad A_2 = \begin{array}{c c} & \begin{array}{cccc} 0.1 & 0.5 & 0.6 & 0.9 \end{array} \\ \begin{array}{c} 0.5 \\ 0.6 \\ 0.9 \\ 1 \end{array} & \left[\begin{array}{cccc} 0.2 & & & \\ 0 & 0 & & \\ 0 & 0 & 0 & \\ 0 & 0.8 & 0 & 0 \end{array}\right] \end{array}$$

$$A = \frac{1}{2}\sum_{i=1}^{2} A_i = \frac{1}{2}(A_1 + A_2) = \begin{array}{c|cccc|} & 0.1 & 0.5 & 0.6 & 0.9 \\ \hline 0.5 & 0.1 & & & \\ 0.6 & 0 & 0 & & \\ 0.9 & 0 & 0 & 0.5 & \\ 1 & 0 & 0.4 & 0 & 0 \\ \hline \end{array}$$

Input variable b:

The same process is performed for the representation and aggregation of the three experts' opinions related to the variable *b*.

$$B_1 = \begin{bmatrix} 0 & & & & & & \\ 0 & 0 & & & & & \\ 0 & 0 & 0 & & & & \\ 0 & 0 & 0.1 & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0.9 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & & & & & & \\ 0 & 0 & & & & & \\ 0 & 0.1 & 0 & & & & \\ 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0.7 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.333 & & & & & & \\ 0 & 0 & & & & & \\ 0 & 0.333 & 0 & & & & \\ 0 & 0 & 0.333 & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \frac{1}{3}\sum_{i=1}^{3} B_i = \frac{1}{3}(B_1 + B_2 + B_3) = \begin{bmatrix} 0.111 & & & & & & \\ 0 & 0 & & & & & \\ 0 & 0.144 & 0 & & & & \\ 0 & 0 & 0.144 & 0 & & & \\ 0 & 0 & 0 & 0.233 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0.3 & \\ 0 & 0 & 0 & 0 & 0 & 0.067 & 0 \end{bmatrix}$$

b) Propagation of uncertain information

The propagation of the uncertain information to the top level of the model consists in the consideration of the combinations of all intervals that constitute the equivalent expert's opinion at the leaf nodes of the VMM. The bounds (l, u) of a resulting interval from the propagation process of one interval of *A* and one interval of *B* are determined by the following formulas:

$$l = \underset{a\times\, b\in\, Interval\ of A \times Interval\ of\ B}{\text{minimum}} (a+b)^{a} \tag{4}$$

$$u = \underset{a\times\, b\in\, Interval\ of A\times Interval\ of\ B}{\text{maximum}} (a+b)^{a} \tag{5}$$

The associated subjective belief (Sb) to the interval $[\,l, u]$ is equal to the product of the subjective beliefs associated to the input intervals.

Since the evidences are in the form of set of intervals, the belief and plausibility that $y \geq y_0$ for a given y_0 are obtained by the application of the equations 6 and 7. These equations are equivalent to the belief and plausibility functions given in Section 3.3.3.3 (see also Section B.1.3 of the Appendix B for details).

$$Bel(y \geq y_0) = \sum_{i=1}^{m} sb_i \cdot \delta_k(I_i) \text{ with } \begin{cases} \delta_k = 1 \text{ if } y_0 \leq l_i \\ \delta_k = 0 \text{ if } y_0 > l_i \end{cases} \tag{6}$$

$$Pl(y \geq y_0) = \sum_{i=1}^{m} sb_i \cdot \sigma_k(I_i) \text{ with } \begin{cases} \sigma_k = 1 \text{ if } y_0 < u_i \\ \sigma_k = 0 \text{ if } y_0 \geq u_i \end{cases} \tag{7}$$

where I_i is an interval from the set of propagated intervals and (l_i, u_i) the bounds of this interval. The complete methodology for the propagation of uncertainty in the form of intervals and the associated subjective belief is described in details in Section 3.3.6 of Chapter 3.

5.2.1.2 Probability approach

Applying the Probability theory to the challenge problem requires also the aggregation of evidences. Since the emphasis of the problem is the comparison with the results obtained with the Evidence theory, the same aggregation method is applied (see Section 5.2.1.1). The propagation of uncertain information is performed by the mean of Monte Carlo simulation presented in detail in Appendix C. The sampling for each uncertain variable is done by considering a uniform distribution over each interval with a probability equals to the subjective belief.

5.2.1.3 Comparison and interpretation

The aggregation of the experts' opinions provided at the leaf nodes of the model for the input data leads to the following combined evidence $a \in [0.1, 1]$ and $b \in [0, 1]$. Applying rules of intervals arithmetic, we deduce that the response variable y is inevitability included in the interval $[0.7, 2]$. Hence, $Bel\,(y > 2) = Pl(y > 2) = \Pr(y > 2) = 0$ because there is no evidence that y should be larger than 2 and $Bel\ (y < 0.7\ ) = Pl(y < 0.7\ ) = \Pr(y < 0.7\ ) = 1$ because the combined evidence suggests that y is greater than 0.7. This result is also obtained when applying equations 6 and 7 as depicted on Figure 5-2.

Now let us look in the detailed evidences provided by experts. As predicted in Appendix B, the curves of the belief and the plausibility bracket the probability curve. The gap between the belief and the plausibility curves characterize the uncertainty embedded in the experts' opinions for any value of the output variable y. This gap can be seen as an uncertainty on the probability measure, that is the belief and the plausibility measures can be viewed respectively as a minimum and maximum likelihood for a specific event.

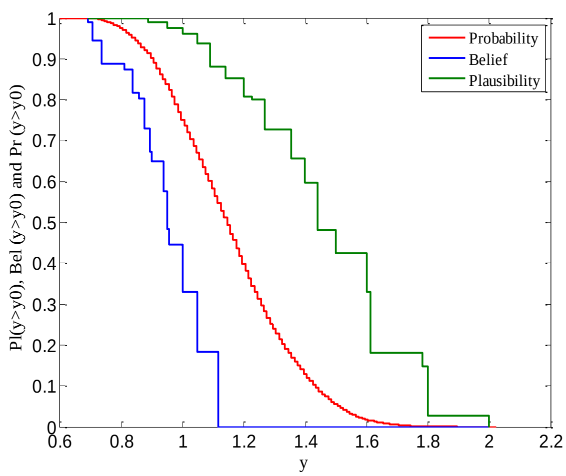


Figure 5-2: Belief, plausibility and probability representations for Sandia challenge problem

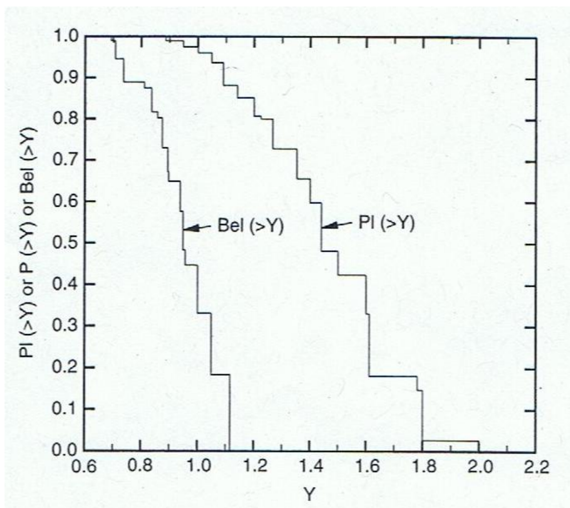


Figure 5-3: Belief, plausibility and representations for Sandia challenge problem form Ref. [4].

Finally, the comparison of the belief, plausibility and probability curves depicted on Figure 5-2 generated by our multilevel model program applied to Sandia challenge problem and those of Figure 5-3 taken form Ref. [4] shows that they are exactly the same. That validates our methodology for uncertainty management in the vehicle multilevel model.

5.2.2 Simple Multilevel problem

The simple example of Figure 5-4 was provided firstly in Section 3.3.6 of Chapter 3. This example is presented in order to complete the illustration of the meaning of the uncertainty included in the experts' opinions and its effect on the belief, plausibility and probability measures. The functional connecting the characteristic in this multilevel model is linear and additive:

$$C_1^{S1} = C_1^{SS1} + C_1^{SS2} \tag{8}$$

At the leaf nodes of the model, we consider two beta distributions that will be discretized with different steps to generate the associated experts' opinions. The parameters of the Beta distributions are $(\alpha = 2.00085\,, \beta = 2.0008)$ and $(\alpha = 2.5427\,, \beta = 1.0912)$ for the experts of subsystem 1 and subsystem 2 respectively. Since, the beta distributions are supported only on the interval [0, 1]; it is required to scale them on the real intervals of the characteristics. The value of the subjective belief $Sb_{[l_i,u_j]}$ associated to a generated interval $[l_i, u_j]$ is equal to the difference of the cumulative destiny function of the beta distribution on the interval bounds as given by Equation (9).

$$Sb_{[l_i,u_j]} = BetaCdf(u_j) - BetaCdf(l_i) \tag{9}$$

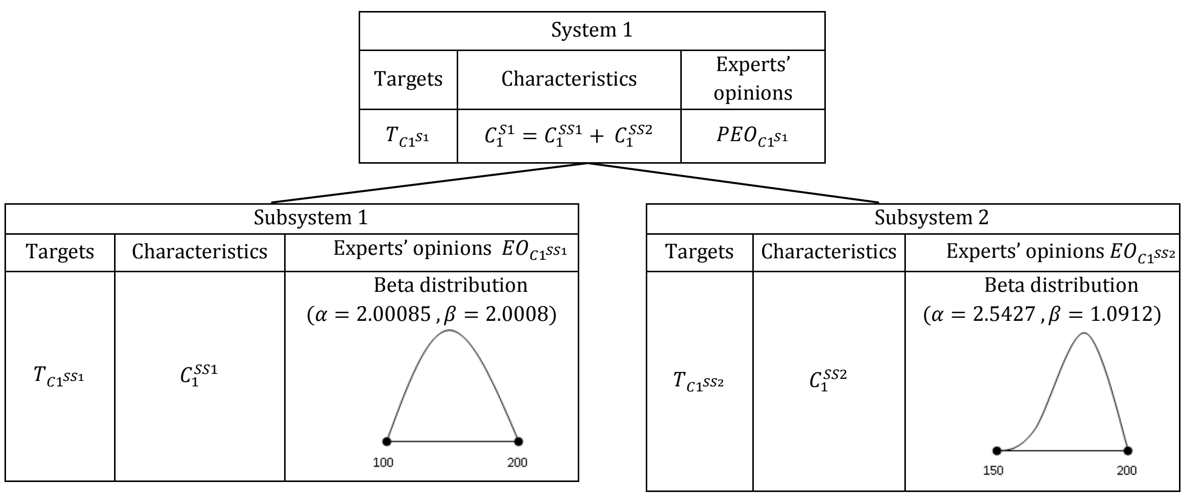


Figure 5-4: Simple VMM to illustrate uncertainty propagation

Since, only one expert will be considered at each leaf node, there is no need for evidences aggregation. The propagation of the uncertain information was presented in details in Chapter 3.

Three simulations will be performed using an increasing level of discretization of both Beta probability distributions. The first simulation is done with a coarse granularity. The second one uses a medium granularity (Ndiscr=20) and the third one is performed with a fine granularity (Ndiscr=100). The increase of discretization number is a manner to simulate the increase of expert's knowledge and consequently the amount of information provided. In the case of coarse granularity, the experts' opinions are expressed in the following form:

$$EO_{C1^{ss_1}} = \{[100,120], 0.07;\ [120,140], 0.23;\ [140,160], 0.40;\ [160,180], 0.23;\ [180,200], 0.07\}$$

$$EO_{C1^{ss_2}} = \{[150,180], 0.3;\ [180,190], 0.3;\ [190,200], 0.4\}$$

A Monte Carlo simulation is performed with a sample size of *Ns = 10000*. Calculation with such sample size ensures a good match with the theoretical results and the results are quickly obtained.

The uncertainty is mainly epistemic and aleatory. Since the aleatory uncertainty is irreducible, only the epistemic uncertainty can be reduced by the development of more knowledge. The reduction of the epistemic uncertainty can be translated in the form of the gap reduction and consequently convergence of the belief and plausibility curves towards the probability curve, which represents the irreducible aleatory aspects of uncertainty.

Figure 5-5 to Figure 5-7 show the results of the three simulations. These simulations confirm that as the discretization number of the beta distributions increases, the gap between the belief and plausibility measures diminishes and the corresponding curves converge to the probability curve obtained by the Monte Carlo simulation. This implies that as far as the experts provides more information; the epistemic uncertainty diminishes until only the aleatory uncertainty remains in the form of a probability distribution. From that, we deduce that the Evidence theory provides a consistent framework that can handle the type of available information during the VDP. Indeed, at the early stages of the development process, the engineers have only partial and imprecise information about the engineering characteristics of the vehicle that can be expressed in the form of few large intervals with their associated subjective beliefs. Once, the process progresses sufficiently, more information is available in the form of many short intervals. The Evidence theory presents also the advantage of not hiding any assumption and consequently the information is not distorted. Moreover, compared to the probability theory, instead of getting one probability measure, the Evidence theory provides two measures: the belief and the plausibility representing the maximum and minimum likelihood for any specific event. The difference between these two measures represents the uncertainty included in the experts' opinions. Unless the belief and plausibility are the same, this uncertainty contains epistemic and aleatory parts.

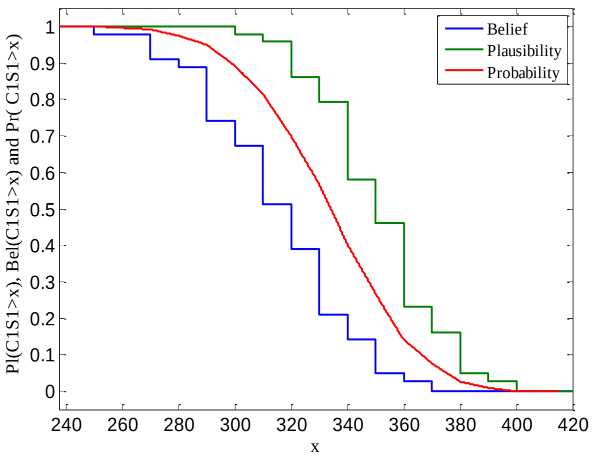


Figure 5-5: Coarse granularity

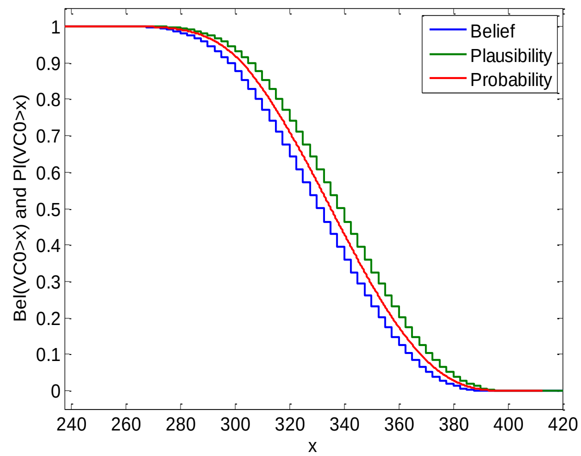


Figure 5-6: Medium granularity (Ndiscr=20)

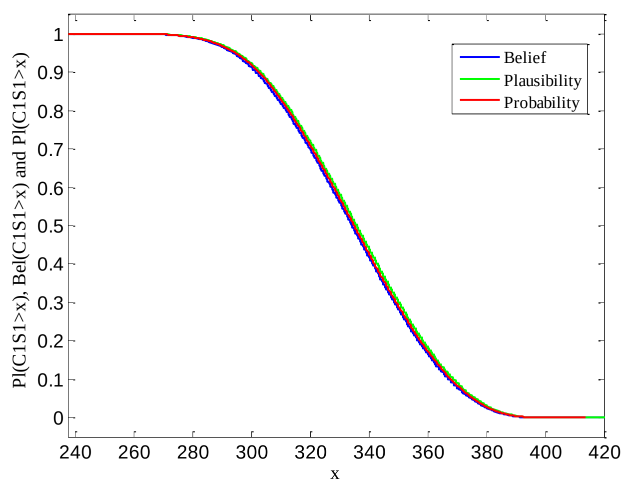


Figure 5-7: fine granularity (Ndiscr =100)

5.3 Verification of the target allocation under uncertainty methodology

As presented in Chapter 4, the proposed methodology for target allocation under uncertainty in the multilevel model is based upon cascading optimization problems. At the vehicle level the performance targets are selected by the maximization of the global utility of design (GUD) involving the desirability and the achievability of each characteristic while at the lowest levels the selection is performed by the maximization of multiple characteristic achievability (MCA) involving only the achievability of each engineering characteristic. The proposed achievability measure relies on the belief and plausibility, which can account for both aleatory and epistemic uncertainties. Nevertheless, the proposed approach can be adapted to use probability density function from aleatory variables.

To better illustrate our approach, we will compare the result obtained with the methodology proposed in Chapter 4 to those obtained with a methodology based on a probabilistic approach where the selection of the characteristics' targets is performed on the basis of highest likelihood. This method is presented in the next section.

5.3.1 Probabilistic formulation of target allocation

We consider the case where the experts provide their opinions with a set of intervals associated with subjective beliefs. A simple and natural way to allocate the targets would be to maximize the probability of the realization of the whole system. For that reason, the Multi-characteristic Probability Product (MCPP) is thus defined in Eq. (10):

$$MCPP(c_1, \dots, c_n) = \prod_{i=1}^{n} pdf_i(c_i) \tag{10}$$

The $pdfs$ are calculated at every node of the multilevel model thanks to Monte Carlo simulation (see Appendix C), which propagate the experts' opinions from the leaf nodes to the vehicle level.

The cascading optimization problems allowing the targets selection in the multilevel model can be formulated as follows: at the top level, the vehicle characteristics vc_j are set by the resolution of the optimization problem of Equation (11).

$$\underset{vc_1,\dots,vc_m}{\text{Maximize}}\, MCPP(vc_1, \dots, vc_m) \tag{11}$$

Since, the vehicle characteristics are linked to the system characteristics with the functional relations:

$$vc_j = f(c_1^{sys}, \dots, c_n^{sys})$$

The allocation of the system characteristics targets can be performed by the resolution of the optimization problem presented in Equation (12).

$$\underset{t_{c_1^{sys}},\ldots,t_{c_n^{sys}}}{\text{Maximize}}\, MCPP\left(t_{c_1^{sys}},\ldots,t_{c_n^{sys}}\right)$$
$$\text{Subject to: } vc_j = f\left(t_{c_1^{sys}},\ldots,t_{c_n^{sys}}\right) \text{ for all } j = 1,\ldots,m \qquad (12)$$

5.3.2 Application to a simple multilevel model

In this section, we will apply the above described approach and the approach proposed in Chapter 4 to the simple example of Figure 5-8. The results of both approaches will be compared and a discussion will be given on their merits.

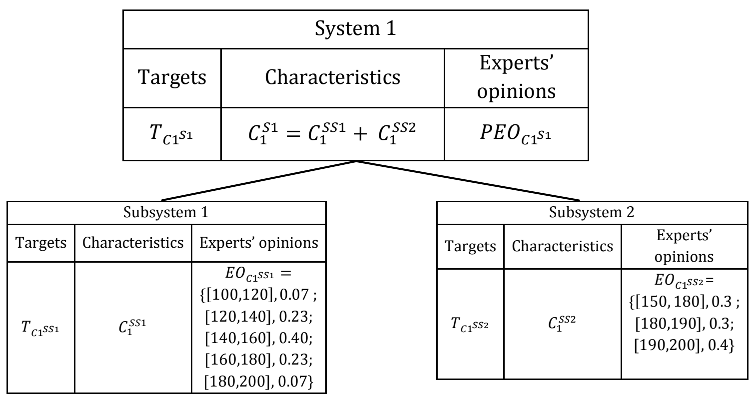


Figure 5-8: Simple multilevel model with experts 'opinions in the form of set of intervals

In this example the bounds of the desirability interval for the characteristic C_1^{S1} are Min-Des-Bnd=200 (kg), Max-Des-Bnd= 400 (kg).

5.3.2.1 Target allocation using probability theory

Applying the probabilistic approach consists in this case to first select the C_1^{S1} by the solution of the optimization problem given by the Equation (13).

$$t_{C_1^{S1}} = \text{maximize}\, pdf(c_1^{S\,1}) \qquad (13)$$

Figure 5-9 shows the plot of the probability distribution function of the response variable C_1^{S1} obtained from the propagated experts' opinions at the leaf nodes of the multilevel model by a Monte Carlo simulation (see Appendix C). This function has a maximum at $C_1^{S1} = 337.01$ kg with a probability of achievement equal to 54.24% (see the cumulative probability function on Figure 5-10). This maximum is selected as target.

Once C_1^{S1} is selected, the targets for the characteristics C_1^{SS1} and C_1^{SS2} of the subsystems nodes 1 and 2 can also be determined by the solution of the optimization problem given by Equation (14).

$$\underset{\left(t_{c_1^{SS1}},t_{c_1^{SS2}}\right)}{\text{maximize}}\ pdf(c_1^{SS1}) * pdf(c_1^{SS2}) \qquad (14)$$

$$\text{Subject to: } t_{c_1^{S1}} = t_{c_1^{SS1}} + t_{c_2^{SS2}}$$

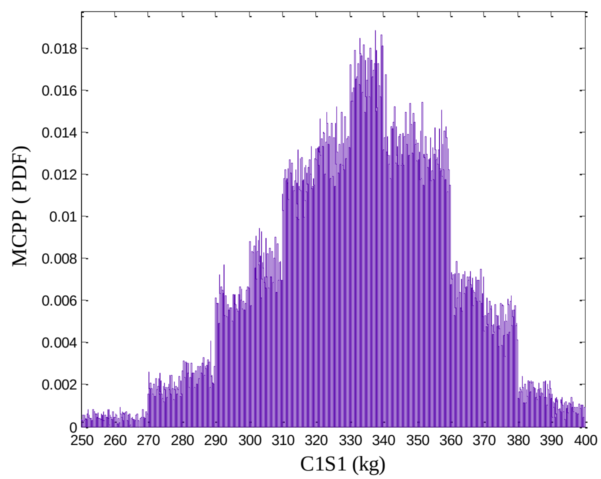


Figure 5-9: Probability density function of C_1^{S1}

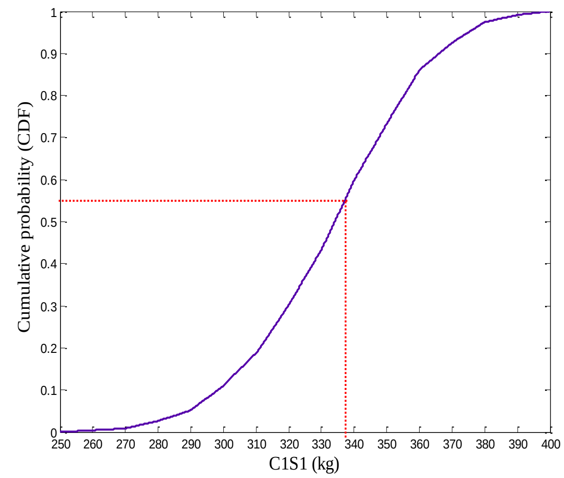


Figure 5-10: Cumulative probability function of C_1^{S1}

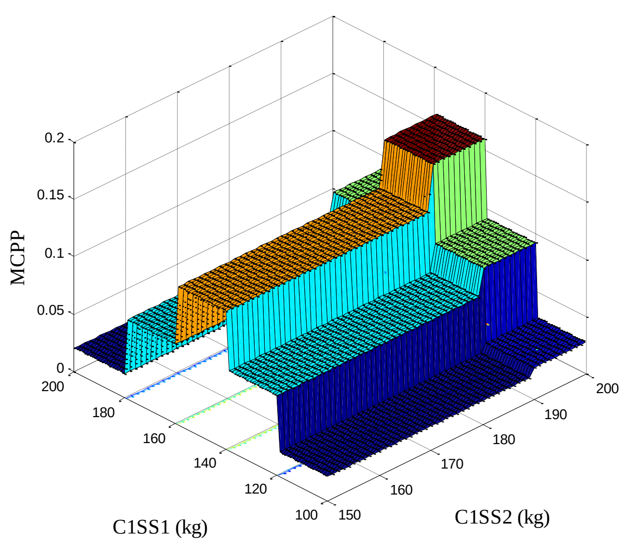


Figure 5-11:MCPP for C_1^{SS1} and C_1^{SS2}

Figure 5-11 shows the plot of the evolution of the MCPP $\left(t_{c_1^{SS_1}}, t_{c_1^{SS_2}}\right)$ in terms of the variation of both input variables $t_{c_1^{SS_1}}$ and $t_{c_1^{SS_2}}$. This graph has a maximum at $t_{c_1^{SS_1}} = 145.4$ (kg) with a probability of 70% and $t_{c_1^{SS_2}} = 191.6$ (kg) with a probability of 100%.

5.3.2.2 Target allocation using Evidence theory

Now, we apply to the problem of Figure 5-8 our approach for target allocation based on Evidence theory and presented in Chapter 4.

Figure 5-13 shows the plots of the GUD at the top level of the multilevel model of Figure 5-8 for two extreme values of the decision-making factor $\Phi = 0$ and $\Phi = 1$. From an achievement perspective, these two values of the decision factor correspond to either a challenging (optimistic) or a conservator (pessimistic) target for the engineers. Both curves have a maximum constituting the target to be chosen.

The maximum of the GUD curve for $\Phi = 0$ is located at $t_{c_1^{S_1}} = 320$ (kg) as seen on Figure 5-13 with a desirability of 31.47% and an optimistic achievability of 49.9% as can be observed on Figure 5-12. The targets of the characteristics C_1^{SS1} and C_1^{SS2} can be deduced from Figure 5-14. This figure shows the plot of the MCA in terms of the characteristics C_1^{SS1} and C_1^{SS2} for the same value of the Φ factor. The plot has a maximum at $t_{c_1^{SS_1}} = 130$ (kg) with an optimistic achievability of 19.31% and $t_{c_1^{SS_2}} = 190$(kg) with an optimistic achievability of 100%.

The maximum of the GUD curve for $\Phi = 1$ is located at $t_{c_1^{S_1}} = 350$ (kg) with a desirability of 12.5% and a pessimistic achievability of 41.24%. The corresponding targets for C_1^{SS1} and C_1^{SS2} can be determined from an analogous curve of the MCA in terms of C_1^{SS1} and C_1^{SS2} with $\Phi = 1$ (see Figure 5-15). In this case, the maximum of the curve gives $t_{c_1^{SS_1}} = 151.6$ (kg) with a pessimistic achievability of 3.46% and $t_{c_1^{SS_2}} = 198.4$ (kg) with a pessimistic achievability of 29.7%.

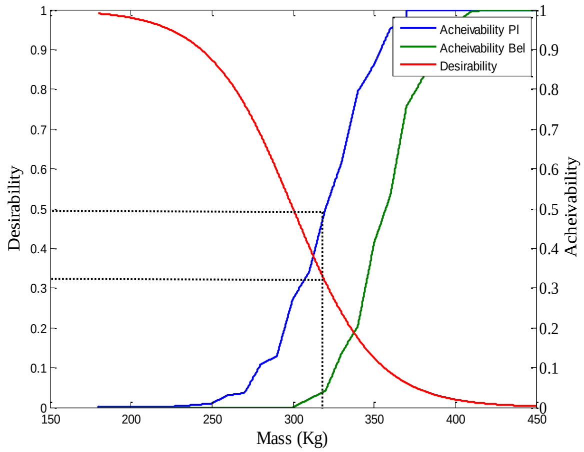


Figure 5-12: Belief, plausibility and desirability curves

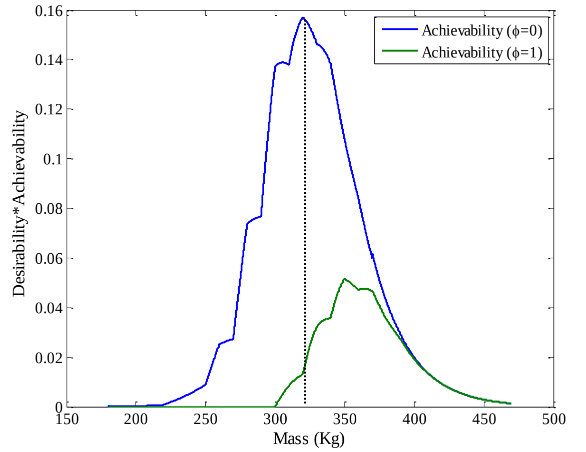


Figure 5-13: GUD ($\Phi = 0$ and $\Phi = 1$) curves for the vehicle mass characteristic C_1^{S1}

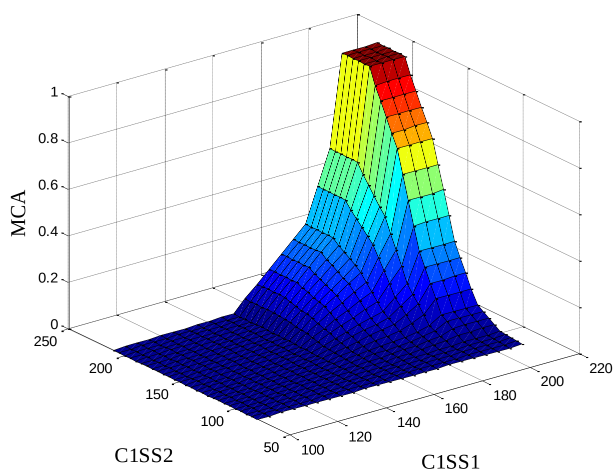


Figure 5-14: MCA ($\Phi = 0$) for subsystems 1& 2 mass characteristics C_1^{SS1} and C_1^{SS2}

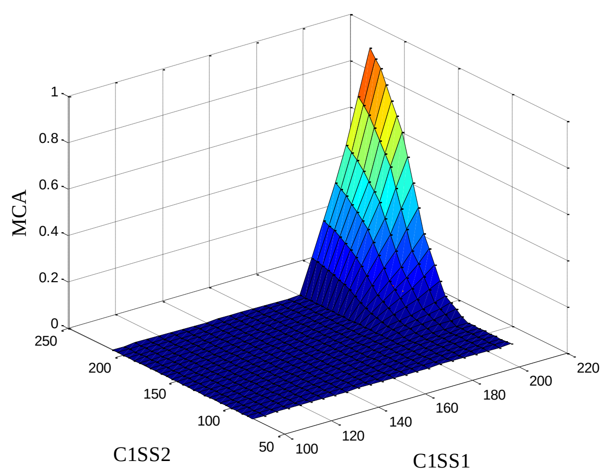


Figure 5-15: MCA ($\Phi = 1$) for subsystems 1& 2 mass characteristics C_1^{SS1} and C_1^{SS2}

The comparison between the challenging (optimistic) and conservator (pessimistic) sets of targets shows that the allocated targets are more desirable in the case of the challenging design, which is consistent and intuitive. However, the higher achievability at the top level of the model in the case of challenging design compared to the conservator one seems inconsistent and counterintuitive. The point here is that the achievability for both approaches is calculated on different basis: the optimistic achievability for the challenging case is based on plausibility measure while the pessimistic achievability for the conservator case is based on a belief measure.

The difference between these two values can be seen as an uncertainty on the probability of achievability.

5.3.2.3 Comparison between the two methods for target allocation under uncertainty

Table 5-1 summarizes the results of target allocation by both methodologies. The comparison of the allocated targets, their achievability and desirability shows that the probabilistic approach provides target more desirable than the conservator design and less desirable than the challenging design. These results are consistent with the prediction made in Chapter 4. Indeed, since the plausibility and belief can be considered as a minimum and a maximum likelihood for an event, it is expected that the targets provided by the approach using these two measures bracket the target provided by the probabilistic approach.

Table 5-1: Allocated targets by both methodologies using evidence and probability theories

Characteristic name	Evidential approach						Probabilistic approach	
	$\Phi = 0$			$\Phi = 1$				
	Target	Desirability	Optimistic Achievability	Target	Desirability	Pessimistic Achievability	Target	Probability
C_1^{S1}	320 (kg)	31.47%	49.9%	350 (kg)	12.5%	41.24%	337.01(kg)	54.24 %
C_1^{SS1}	130 (kg)	X	19.3%.	151.6 (kg)	X	3.47%.	145.4 (kg)	70%
C_1^{SS2}	190 (kg)	X	100%	198.4 (kg)	X	29.7%	191.6(kg)	100 %

The target allocation method under uncertainty proposed in Chapter 4 is more general than the probabilistic approach. That is in the sense that two limit designs given by $\Phi = 0$ or $\Phi = 1$ bracket the values obtained by the probabilistic approach. Moreover, the proposed methodology presents many advantages compared to probabilistic approach. Indeed, with the probability theory, it is necessary to assume a *pdf* on the interval. This information is not always available and assuming a *pdf* can lead to a distortion of the information. With the Evidence theory, there is no need for assumption of the type of probability distribution on the intervals provided by the experts; which is consistent with the type of information and the level of knowledge at the early stages of the design process. In this context, the proposed approach allows taking decision even with insufficient information pending the generation of more knowledge. It can also handle the risk associated to decision thanks to the decision-making factor $\Phi \in [0, 1]$. This factor allows choosing characteristics' targets from conservator to challenging levels.

In conclusion, the verification approach for the proposed target allocation methodology confirms our predictions made in Chapter 4.

5.4 Decision-making under uncertainty during the design process

One of the objectives of this thesis is the exploration of a decision-making approach that can handle the uncertainty during a design process. The previous chapters (3 & 4) have presented respectively approaches to propagate uncertainties from the experts to the vehicle level and to reassign targets for systems, subsystems and parts such as to maximize achievability and desirability of characteristics. In this section, we propose a methodology for decision-making based on the integration of these methodologies.

Our assumption is that this method has the potential to lead to a better design process and a better final product at the end of the VDP. Of course, a validation of this assumption on a real VDP is out of the scope of the current thesis, since a real VDP can be extended over many years and involves hundreds of participants. As a surrogate, we will rely on a model of the VDP to obtain information about the validity of our approach.

Based on the analysis of the literature review on the decision-making during the design process (see Section 1.4 in Chapter 1), we propose a generic stage-gate process to model the decision-making framework as presented in Figure 5-16. The stage represents a period of time when development activities such as analyses, measurements, reviews and prototyping are performed. The principal inputs to the stage are the engineering characteristic targets for the components considered and all the other components of the vehicle. The latter is necessary for the components to be designed concurrently but with limited coupling. In addition, the results of the previous stage are also required to continue the engineering work. The gate represents a barrier between two consecutive stages where a standpoint is marked. The experts' opinions are collected and analyzed in order to reallocate the targets for the next stage. In our case, the modeled VDP is extremely simplified. It is considered that experts' opinions are generated instantaneously and the target reallocation is performed synchronously for all components.

The repetition of target reallocation as described in Chapter 4 is the decision-making process considered in this work. The validity of our approach is assessed on the basis of component characteristics variation during the iterations on a simple VMM.

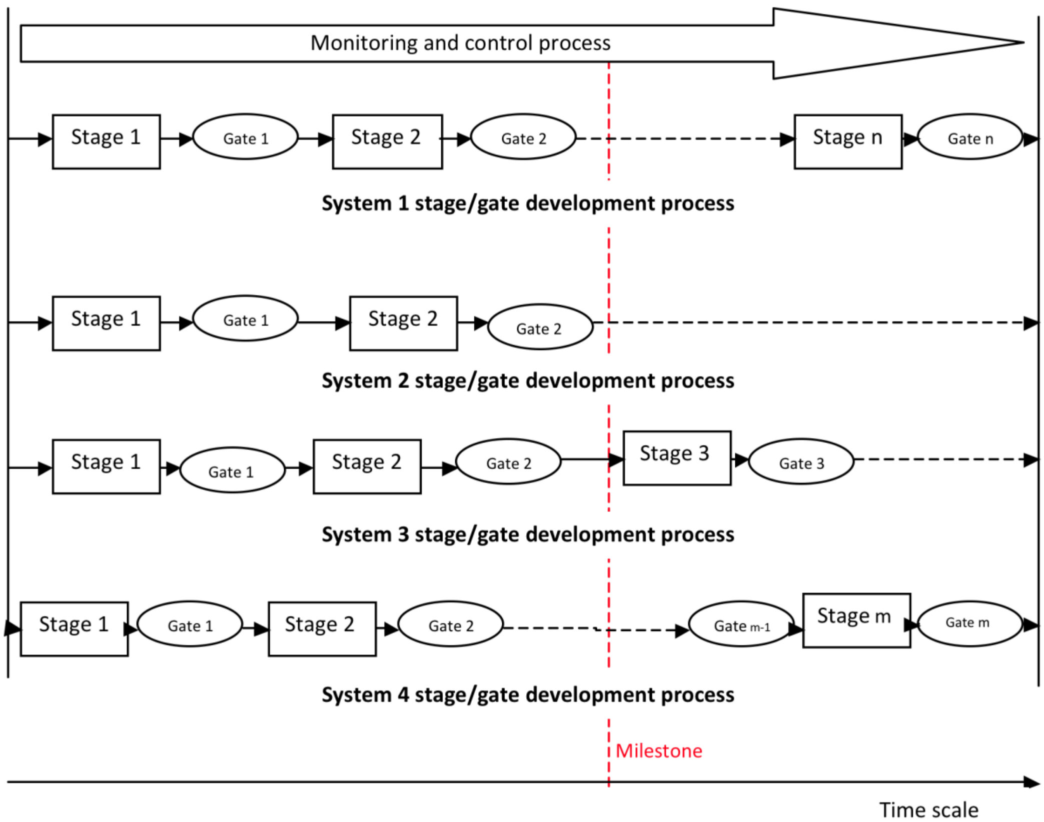


Figure 5-16: Stage-gate development process

5.4.1 Simulation of decision-making in the VDP

We consider the VMM presented in Chapter 4 and reproduced in Figure 5-17. For the sake of clarity, this VMM contains two engineering characteristics (the mass of the components "$C_{mass}^{component}$" and the engine power "C_{powers}^{engine}") and two vehicle characteristics (the time for acceleration "VC_{time}" and the fuel economy "$VC_{fuel\ economy}$").

The time for acceleration or the time needed to accelerate from rest to $V = 100$ km/h is approximated in the form of a non-linear functional characteristic (VC_{time}). This characteristic depends on the vehicle mass (VC_{mass}) and the engine power (C_{power}^{Eng}), as given by the equation (15).

$$VC_{time} = f\left(VC_{mass}, C_{power}^{Eng}, V\right) = \frac{VC_{mass} * V^2}{C_{Power}^{Eng}} \quad (15)$$

The desirability of this characteristic is articulated as “lower is better” and the achievability is articulated as “lower is difficult”. These articulations influence the way the desirability and achievability are evaluated (see Sections 4.3.1 and 4.3.2 in Chapter 4).

The fuel economy characteristic represents the distance traversed by the vehicle for each liter of fuel. This characteristic depends on the vehicle mass and the engine power. We will use a simple evaluation formula:

$$\mathrm{VC}_{\mathrm{fueleconomy}} = \mathrm{VC}^{*}_{\mathrm{Fe}}(1+\alpha(\mathrm{C}^{\mathrm{Eng}^{*}}_{\mathrm{power}} - \mathrm{C}^{\mathrm{Eng}}_{\mathrm{power}}))(1+\beta(\mathrm{VC}^{*}_{\mathrm{mass}} - \mathrm{VC}_{\mathrm{mass}})) \qquad (16)$$

where $\mathrm{VC}^{*}_{\mathrm{Fe}}$, $\mathrm{VC}^{*}_{\mathrm{mass}}$ and $\mathrm{C}^{\mathrm{Eng}^{*}}_{\mathrm{power}}$ represent selected points of references of the fuel economy, the vehicle mass and the engine power. We have selected $\mathrm{VC}^{*}_{\mathrm{Fe}} = 14.5\ \mathrm{km/l}$, $\mathrm{VC}^{*}_{\mathrm{mass}} = 740$ kg and $\mathrm{C}^{\mathrm{Eng}^{*}}_{\mathrm{power}} = 90$ hp. The parameters α and β are two weighting factors allowing accounting for the effect of variation of the engine power and the vehicle mass on the fuel economy around the reference points. We have selected ($\alpha = 0.01$) and ($\beta = 0.005$). The fuel economy has a “higher is better” desirability articulation and “higher is difficult” achievability articulation.

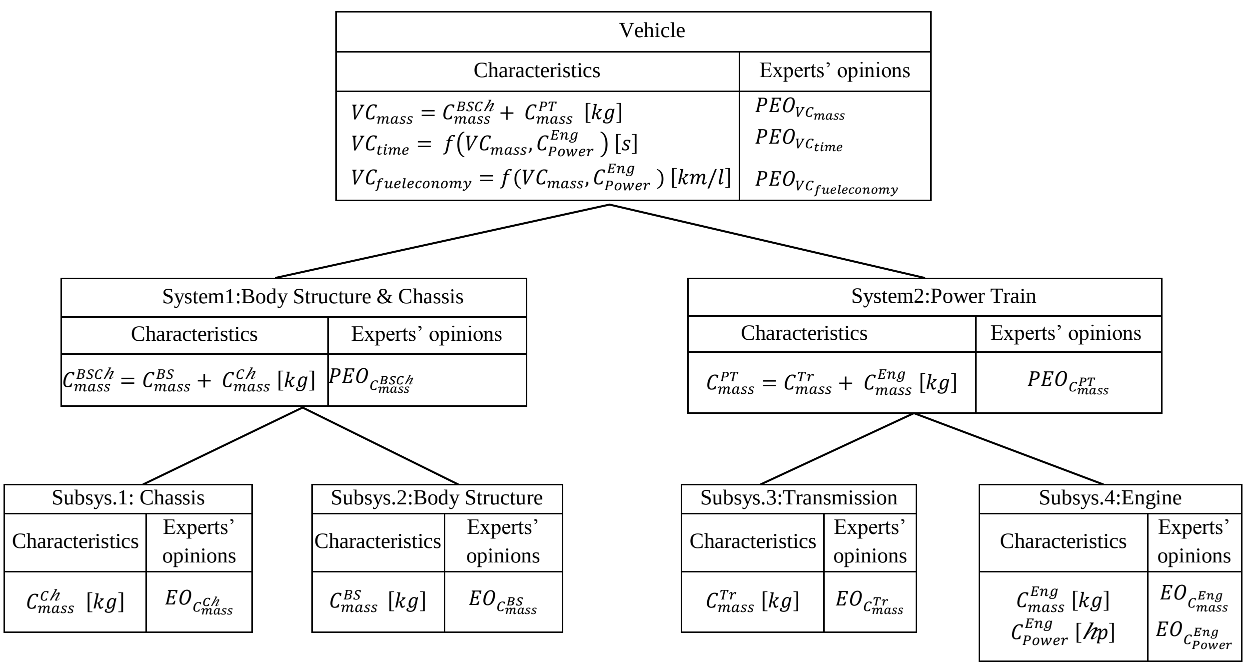


Figure 5-17: Example of vehicle multilevel model

	Chassis's mass [kg]	Body structure's mass [kg]	Transmission's mass [kg]	Engine's mass [kg]	Engine's power [hp]
Iteration 1	$T^{ch}_{mass} = 300$ $EO_{C^{Ch}_{mass}}$ {[260,270[, 0.1; [270, 280[, 0.3; [280, 300[, 0.25; [300,310[, 0.2; [310,320], 0.15}	$T^{BS}_{mass} = 200$ $EO_{C^{BS}_{mass}}$ {[180, 190[, 0.1; [190, 200[, 0.3; [200, 210[, 0.4; [210, 220], 0.2}	$T^{Tr}_{mass} = 100$ $EO_{C^{Tr}_{mass}}$ {[80, 90[, 0.1; [90,100[, 0.25; [100, 105[, 0.3; [105, 110[, 0.25; [110, 120], 0.1}	$T^{Eng}_{mass} = 150$ $EO_{C^{Eng}_{mass}}$ {[130, 140[, 0.15; [140, 145[, 0.2; [145, 155[, 0.35; [155, 160[, 0.2; [160, 170], 0.1}	$T^{Eng}_{Power} = 100$ $EO_{C^{Eng}_{Power}}$ {[40, 50[, 0.1; [50, 60[, 0.15; [60, 70[, 0.5; [70, 80[, 0.15; [80, 90], 0.1}
Iteration 2	$T^{ch}_{mass} = 296.3$ $EO_{C^{Ch}_{mass}}$ {[292.2, 293.2 [, 0.01; [293.2, 295.9 [, 0.23; [295.9, 300.7 [, 0.32; [300.7, 304.3 [, 0.23; [304.3,308.4], 0.21}	$T^{BS}_{mass} = 198.4$ $EO_{C^{BS}_{mass}}$ {[194, 194.7 [, 0.04; [194.7, 199.5 [, 0.36; [199.5, 205.7 [, 0.48; [205.7, 206.6], 0.04; [206.6, 207.8], 0.08}	$T^{Tr}_{mass} = 102.9$ $EO_{C^{Tr}_{mass}}$ {[95.2, 99.0 [, 0.23; [99.0, 109.1 [, 0.58; [109.1, 111.2 [, 0.09; [111.2, 112.5 [, 0.05; [112.5, 113.5], 0.05}	$T^{Eng}_{mass} = 158.9$ $EO_{C^{Eng}_{mass}}$ {[153.5, 156.9 [, 0.41; [156.9, 157.2 [, 0.02; [157.2, 158.8 [, 0.16; [158.8, 158.9 [, 0.02; [158.9, 163.4], 0.39}	$T^{Eng}_{Power} = 94.3$ $EO_{C^{Eng}_{Power}}$ {[76.1, 81.8 [, 0.15; [81.8, 85.1 [, 0.57; [85.1, 87.4 [, 0.03; [87.4, 96.7 [, 0.23; [96.7, 96.8], 0.02}
Iteration 3	$T^{ch}_{mass} = 292.2$ $EO_{C^{Ch}_{mass}}$ {[283.3, 288.1 [, 0.13; [288.1, 294.7 [, 0.31; [294.7, 299.0 [, 0.23; [299.0, 305.5 [, 0.28; [305.5,306.6], 0.05}	$T^{BS}_{mass} = 201.0$ $EO_{C^{BS}_{mass}}$ {[194.2, 194.4[, 0.28; [197.5, 197.5 [, 0.08; [198.2, 201.5 [, 0.28; [201.5, 202.7], 0.13; [202.7, 207.9], 0.23}	$T^{Tr}_{mass} = 99.9$ $EO_{C^{Tr}_{mass}}$ {[96.4, 97.5[, 0.01; [97.5, 97.8 [, 0.02; [97.8, 98[, 0.03; [98, 99.9 [, 0.14; [99.9, 111.5], 0.8}	$T^{Eng}_{mass} = 147.9$ $EO_{C^{Eng}_{mass}}$ {[143.6, 161.0 [, 0.63; [161.0, 162.2 [, 0.05; [162.2, 163.1 [, 0.01; [163.1, 165.7 [, 0.14 [165.7, 171.2 [, 0.17}	$T^{Eng}_{Power} = 98.2$ $EO_{C^{Eng}_{Power}}$ {[67.1, 78.4 [, 0.09; [78.4, 81.9 [, 0.05; [81.9, 107.8 [, 0.6; [107.8, 117.4 [, 0.18; [117.4, 123.8], 0.08}
Iteration 4	$T^{ch}_{mass} = 286.3$ $EO_{C^{Ch}_{mass}}$ {[281.8, 288.5 [, 0.27;]288.5, 291.1 [, 0.16; [291.1, 292.6 [, 0.05; [292.6,301.5[, 0.52}	$T^{BS}_{mass} = 200.5$ $EO_{C^{BS}_{mass}}$ {[194.7, 198.3 [, 0.3; [198.3, 200.7 [, 0.35; [200.7, 202.5 [, 0.29; [202.5, 202.9], 0.06}	$T^{Tr}_{mass} = 98.3$ $EO_{C^{Tr}_{mass}}$ {[83.2, 95.1 [, 0.26; [95.1, 105.8 [, 0.61; [105.8, 107.0 [, 0.05; [107.0, 108.5[, 0.08}	$T^{Eng}_{mass} = 149.4$ $EO_{C^{Eng}_{mass}}$ {[142.3, 144.5 [, 0.11; [144.5, 148.2 [, 0.24; [148.2, 149.5 [, 0.08; [149.5, 163.0[, 0.57}	$T^{Eng}_{Power} = 99.7$ $EO_{C^{Eng}_{Power}}$ {[74.5, 98.7 [, 0.6; [98.7, 104.7 [, 0.24; [104.7, 105 [, 0.01; [105, 108.9[, 0.15}
Iteration 5	$T^{ch}_{mass} = 284.7$ $EO_{C^{Ch}_{mass}}$ {[283.2, 283.5 [, 0.06; [283.5, 286.6 [, 0.78; [286.6, 287.2[, 0.16}	$T^{BS}_{mass} = 198.8$ $EO_{C^{BS}_{mass}}$ {[196.9, 198.3 [, 0.17; [198.3, 199.4 [, 0.22; [199.4,201.9[, 0.61 }	$T^{Tr}_{mass} = 101.0$ $EO_{C^{Tr}_{mass}}$ {[98.9, 99.2 [, 0.07; [99.2, 101.0 [, 0.41; [101.0, 104.1[, 0.52}	$T^{Eng}_{mass} = 148.4$ $EO_{C^{Eng}_{mass}}$ {[143.8, 147.0 [, 0.28; [147.0, 148.4 [, 0.25; [148.4, 153.1[, 0.57}	$T^{Eng}_{Power} = 100.1$ $EO_{C^{Eng}_{Power}}$ {[84.5, 86.2 [, 0.3; [86.2, 102.3 [, 0.72; [102.3, 107.8[, 0.25}
Iteration 6	$T^{ch}_{mass} = 284.8$ $EO_{C^{Ch}_{mass}}$ {[284.5, 284.6 [, 0.23; [284.6, 284.8 [, 0.51; [284.8, 284.9[, 0.26}	$T^{BS}_{mass} = 198.9$ $EO_{C^{BS}_{mass}}$ {[198.6, 198.9 [, 0.58; [198.9, 199.1 [, 0.12; [199.1, 199.8[, 0.30}	$T^{Tr}_{mass} = 101.5$ $EO_{C^{Tr}_{mass}}$ {[101.3, 101.4 [, 0.24; [101.4, 101.5 [, 0.61; [101.5, 101.7[, 0.15}	$T^{Eng}_{mass} = 148.5$ $EO_{C^{Eng}_{mass}}$ {[148.3, 148.4 [, 0.08; [148.4, 148.5 [, 0.39; [148.5, 148.7[, 0.53}	$T^{Eng}_{Power} = 99.8$ $EO_{C^{Eng}_{Power}}$ {[99.1, 99.7 [, 0.58; [99.7, 99.8[, 0.09; [99.8, 100.2[, 0.33}
Iteration 7	$T^{ch}_{mass} = 284.8$ $EO_{C^{Ch}_{mass}}$ {[284.5, 284.8 [, 0.38; [284.8, 285.0 [, 0.62}	$T^{BS}_{mass} = 198.8$ $EO_{C^{BS}_{mass}}$ {[198.7, 198.8[, 0.01; [198.8, 199.2[, 0.99}	$T^{Tr}_{mass} = 101.4$ $EO_{C^{Tr}_{mass}}$ {[101.3, 101.4[, 0.13; [101.4, 101.6[, 0.87}	$T^{Eng}_{mass} = 148.6$ $EO_{C^{Eng}_{mass}}$ {[148.4, 148.6 [, 0.56; [148.6, 148.7[, 0.44}	$T^{Eng}_{Power} = 99.8$ $EO_{C^{Eng}_{Power}}$ {[99.8, 100[, 0.94; [100.0, 100.1[, 0.06}

Figure 5-18: Experts' opinions for seven iterations of the VDP

Two applications of the proposed decision-making process are considered, the first one is with a single characteristic and the second is with multiple characteristics.

We consider that the decision factor is Φ=0 and we limit the presented results only to the vehicle level and a single leaf node of the multilevel model. The experts' opinions for all iterations of the simulations are provided in Figure 5-18. We have generated the experts' opinions iteratively by applying the following principles that respect the presuppositions given in the introduction. At the beginning of the process, targets for characteristics are provided. In response, the experts evaluate the feasibility of each characteristic and provide their opinions in the form of set of intervals with the associated subjective beliefs. At this stage, the experts explore many solutions, generate a lot of information and produce many intervals around the targets. Once progress is made, the experts concentrate their opinions and consequently the number and the width of intervals decrease. At the end of the process, ideally the experts' opinions converge to one interval with a null length around or near the target.

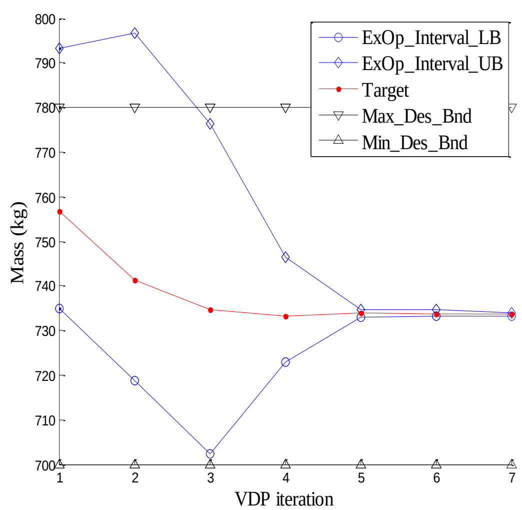
5.4.1.1 Multilevel model with a single characteristic (Mass)

Figure 5-19 illustrates the evolution of mass target, the desirability and achievability at both vehicle level and a single leaf node of the multilevel model. The principles for experts' opinions production accounts for target value and process iteration number.

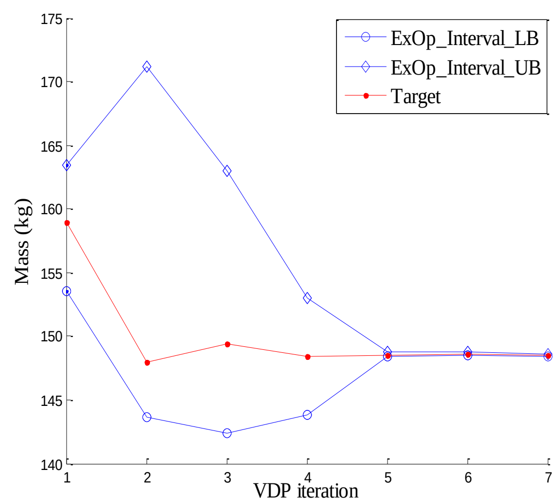
Figure 5-19(a) shows the results at the vehicle level. The width of the intervals of the combined experts' opinions diminishes continuously until convergence to a constant value into the desirability interval (the bounds Min-Des-Bnd and Max-Des-Bnd of the desirability interval are presented on the figure; Mass> Max-Des-Bnd→ Desirability (Mass) = 0 and Mass< Min-Des-Bnd→ Desirability (Mass) = 1). The target converges to the same value as the experts' opinions. The target selection is guided by experts' opinions in order to improve the desirability and achievability of the vehicle (see Figure 5-19 (c)). The targets search proposed by the decision-making strategy to guide experts' opinions towards maximum desirability (iterations1-4). Once this is achieved, the target is kept almost constant (i.e., constant desirability). This allows experts' opinions concentration and make sure high achievability can be obtained (iterations 5-7).

Figure 5-19(b) shows how the propagated intervals span diminishes and converges to a specific value through iterations. It shows also how the target oscillates before converging. The variation of the target results from a series of tradeoffs among the engine characteristics.

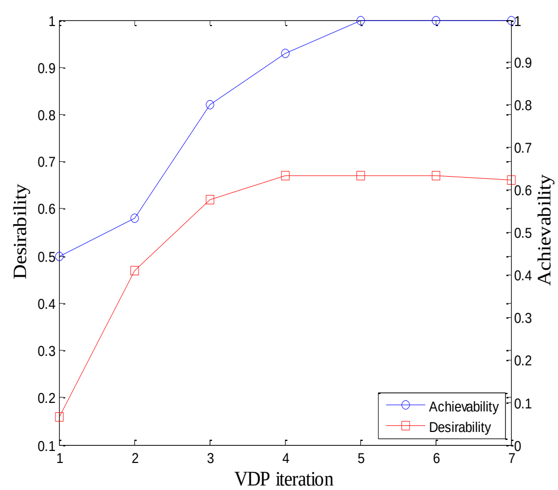
Figure 5-19(d) presents the evolution of the achievability of the engine mass and shows that it can vary drastically between two consecutive iterations. A decrease is obtained when more desirable targets for the vehicle (less achievable engine target) are selected. An increase of achievability is obtained when less desirable targets are selected and/or when new knowledge to achieve these targets is available.



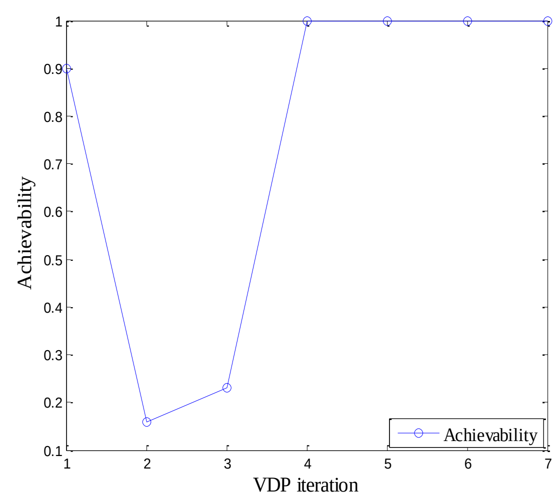
(a) Evolution of the vehicle mass



(b) Evolution of engine mass



(c) Evolution of the desirability and achievability of the vehicle mass



(d) Evolution of the achievability of the engine mass

Figure 5-19: Results of the VDP with a single characteristic vehicle multilevel model

The principles used to generate experts' opinions rely on the common sense and suppose that experts provide consistent opinions varying smoothly in the limit of what can possibly be done in terms of technological constraints. The results obtained show that with such experts' opinions the decisions for target selection converge smoothly towards a high desirability and fully achievable solution. This is a prerequisite property of the proposed decision-making strategy to be applied in real life situation.

5.4.1.2 Effect of the decision-making factor Φ

The decision-making factor Φ was introduced firstly in the definition of the achievability and the GUD (equations 16 and 17 in Chapter 4). It refers to the risk tolerance of the target allocation decision. This factor is a dimensionless measure lying between 0 and 1.

The decision-making factor represents a control parameter for the design team that allows steering the design during the VDP. A factor close to 0 means maximum challenge is given to the design team which implies highest desirability is sought. A factor close to 1 implies low challenge to the engineers and a more conservative final product is sought. Figure 5-20 illustrates this effect on the evolution of the mass target at the vehicle level of VMM of Figure 5-17. Knowing that the mass has a lower is better desirability articulation; the graph confirms the assumption of the effect of the factor Φ on the orientation of the design activities.

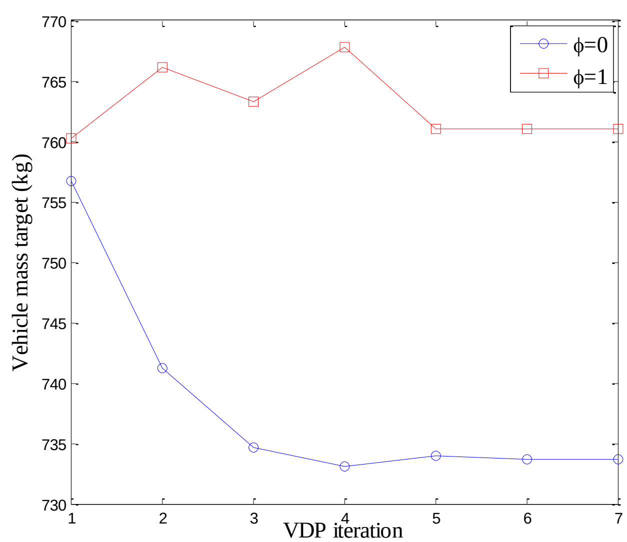


Figure 5-20: Evolution of the vehicle mass target for Φ=0 and Φ=1

5.4.1.3 Handling large-scale problem

In order to be applicable in real life situation, the proposed framework has been developed to handle large amount of information, comparable to that of a real vehicle. We have applied the proposed framework to a VMM constituted by:

- 1 characteristic (mass).
- 4 levels, 10 systems, 57 subsystems and 245 parts.
- 9 iterations of the VDP.

Here again the experts' opinions are generated following the same principles as previously stated. Similar outputs of the process were noticed as those of Sections 5.4.1.1 and 5.4.1.2 in terms of convergence and desirability of the final design. However, the time for simulation could be unacceptable as the scale of the problem and/or the number of intervals of the experts' opinions at the leaf nodes of VMM increase. Indeed, as mentioned in Chapters 3 and 4, the uncertainty information is provided as intervals and subjective beliefs at the leaf nodes of the VMM. This information must be propagated through functional relations linking the nodes characteristics of the vehicle multilevel model. Propagating intervals from children nodes to parent node implies to combine all intervals through functional relations that results in a possibly large number of intervals to be handled. For a VMM of the same proportion as a real vehicle, the number of intervals to be processed becomes overwhelming. That constituted a serious issue for an effective implementation of the proposed system in a real context.

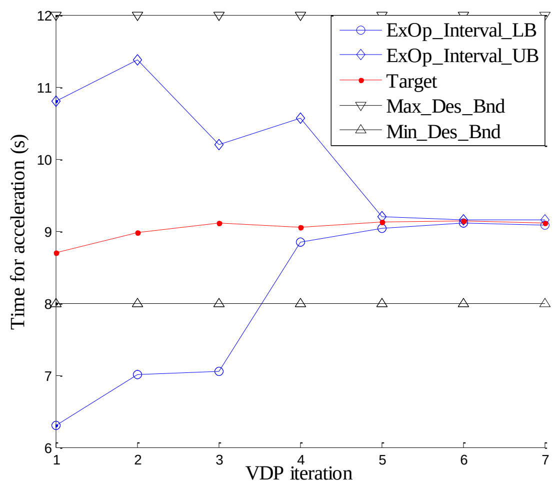
In Appendix D, a propagation and merging procedure is proposed to reduce the number of intervals handled while keeping the accuracy of the belief and plausibility for a given discrete set of characteristic values. This strategy has been applied to obtain results for large-scale problems.

5.4.2 Application of decision-making process under uncertainty to a multi-characteristics multilevel model

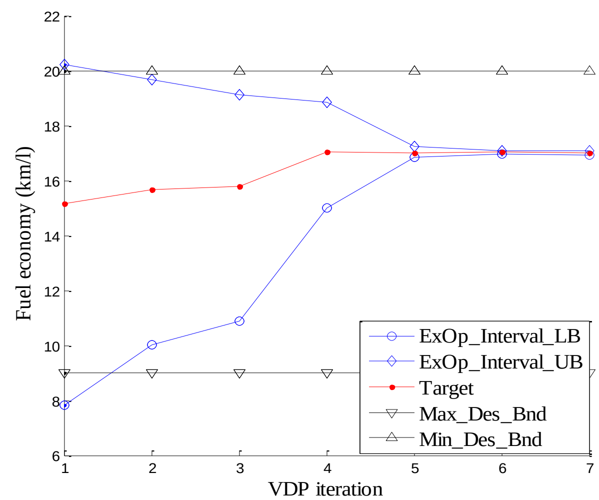
The following application is provided in order to demonstrate the capacity of the proposed process to handle multiple characteristics during the VDP. The example presented in Figure 5-17 is taken.

The results obtained are similar to those presented in Section 5.4.1.1 in terms of convergence and effectiveness of the process and also in term of desirability and achievability of the final design.

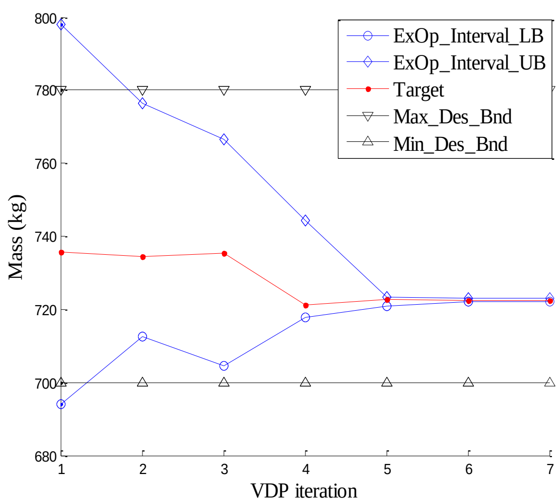
The evolution of the four characteristics targets is presented in Figure 5-21. We can notice that the desirability does not improve for all characteristics because of the series of tradeoffs that must be reached iteratively.



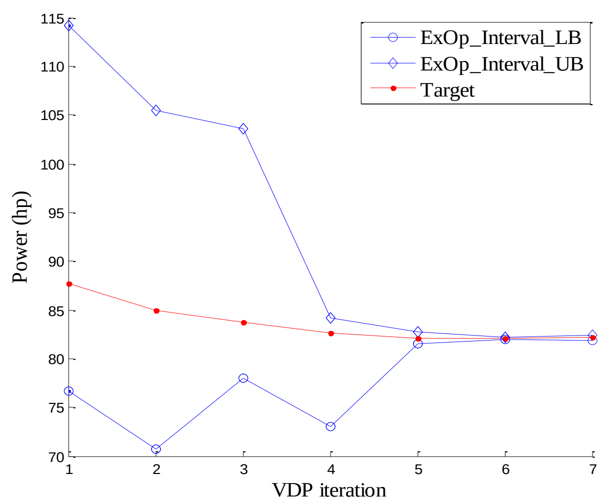
(a) Evolution of the vehicle's time for acceleration



(b) Evolution of the vehicle's fuel economy



(c) Evolution of the vehicle's mass



(d) Evolution of the engine's power

Figure 5-21: Results of the VDP of multi-characteristics vehicle multilevel model

The convergence and effectiveness of the targets allocation and decision-making processes were confirmed. Figure 5-21 (a, b, c and d) show that the VDP converges after few iterations and the final targets of the characteristics remain stable somewhere in the desirability interval. The bounds Min-Des-Bnd and Max-Des-Bnd of the desirability interval for each characteristic are presented on the figures.

5.5 Conclusion

In this chapter, we addressed three topics: the validation of the methodology for uncertainty management in the multilevel model, the verification of methodology for target allocation under uncertainty and the proposition of a decision-making framework.

In Section 5.2, we presented the validation of the proposed methodology for uncertainty aggregation and propagation in the multilevel model. Monte Carlo simulation was used for the propagation of uncertainty taken as probabilistic. Application of the proposed approach to an example coming from the literature and comparison of the results confirm the validity of our approach. Moreover, analysis of the meaning of the belief and plausibility curves showed that the difference between these two measures characterize the uncertainty included in the experts' opinions. This difference can be viewed as a risk measure.

In Section 5.3, we addressed the issues of verification of our approach for target allocation that use the Evidence theory. We presented a methodology for target allocation based on a probabilistic approach, compared the results of both approaches and discussed their merits. The conclusions were that our approach presents the advantages of allowing the managers to produce an infinity of possible design ranging between a challenging design (when an optimistic achievability is used) and a conservator design (when an pessimistic achievability is used). Moreover, a design obtained by the probabilistic approach is included in the previous range, which confirms the predictions made in Chapter 4.

In Section 5.4, we proposed a framework for decision-making under uncertainty during the vehicle development process. This framework is based on the integration of the vehicle multilevel model, the methodology to handle uncertainty in the VMM and the target allocation under uncertainty. The decision-making strategy is based on a series of parallel stage-gate processes that alternate knowledge generation and decision-making. This strategy was applied to a simple example. The convergence was studied and two cases of VMM were used. The first case is a mono-characteristic and the second one is a multi-characteristics.

An analysis of the effect of the decision-making factor Φ demonstrates that it is possible to influence the process by selecting specific value of this factor (level of risk that can be tolerated by the experts). So, for different values of Φ, different outputs can be obtained for the same inputs. Concerning the scalability, the process can handle small VMM as well as complex VMM with the size of a real vehicle.

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Chapter 6 General Discussion

As stated earlier, the research project focuses on the management of uncertainties and its impact on the design process of complexes engineering systems.

Based on an analysis of all project's aspects and following a consensus with GM, the scope and the objectives of the project were defined. The main objective of the project was the development of a methodology for target allocation under uncertainty during the design process. To accomplish this objective, the proposed approach consists in the decomposition of the vehicle in a hierarchical tree structure, cascading the utopian targets of vehicle performances top-down in the model in the form of engineering characteristics, collecting information from the experts about the achievability of the characteristics of the components, measuring the uncertainty included in the experts' opinions, allocating the targets based on these measures of uncertainty and additional measures of customers’ satisfaction (desirability of characteristic). Form that, three major issues were identified. The first one concerns the development of an approach for the uncertainty management in the design process. The second one concerns the development of an approach for target allocation and finally the third issue concerns the proposition of decision-making framework that integrate both previous approaches to see how the proposed approach for target allocation behave during the design process. Addressing these issues constitutes the achievement of the objectives and the same time the scientific contributions of this research project.

To achieve these objectives, the pursued research methodology combined both the practical and theoretical aspects of the subject. The practical aspects represent GM's needs, objectives and constraints. However the theoretical aspects come from the extensive literature reviews on the available approaches and scientific fields related to different facets of the project. The proposed solution for each issue is validated or verified using an appropriate methodology.

In the following sections, we discuss specifics issues related to the scientific contributions of the thesis and also general issues related to project as a whole.

❖ Contributions

In this section, we present the proposed solutions to the different problems presented previously, their pros and cons and eventually their validations or verifications:

➢ **Uncertainty management approach**

The methodology for uncertainty management or uncertainty characterization starts by the decomposition of the vehicle in the form of a vehicle multilevel model. This representation is adopted because it is commonly used for both conceptual and detailed design when the architecture of the product is known, its most functionalities are well defined and the technological solutions have a certain level of maturity.

As required by GM, the only source for information available through the process is experts who provide their opinions concerning the achievability of vehicle components in the form of sets of intervals and their associated subjective beliefs. This representation is commonly used by engineers to express their level of knowledge (and/or lack of information), their uncertainty and their forecast. This representation allows including in addition to aleatory uncertainty which is irreducible and inevitable, the epistemic uncertainty due to the lack of knowledge and which is a specific characteristic of early stages of the design process. Moreover, this representation is a uniform manner to express experts' opinions, which allows including the ambiguity uncertainty.

To handle these uncertainties in the multilevel model, the Evidence theory was chosen because compared to the available theories; it can deal with both aleatory and epistemic uncertainties without distinction, it can treat conflicting evidences and can handle uncertain information in the form of sets of intervals. With this theory, uncertainty can be aggregated, propagated and measured at any node of the VMM by the means of the belief and the plausibility measures which can be considered as the minimum and maximum probabilities for any values of the characteristic. The difference between these two measures can be considered as a risk indicator that can be used in the decision-making process.

An approach based on the Evidence theory was developed to aggregate and propagate the uncertain information using the structure and the functional defined in the vehicle multilevel model. Another approach was also proposed to handle specifically the interaction uncertainty (see Chapter 03 for details).

The validation of the proposed approach for uncertainty management was performed by the comparison to an example published in the literature. Monte Caro Simulation was also applied to simplified examples as a tool for the propagation of uncertainty in the VMM and the results were compared to those obtained by the Evidence theory in order to bring insight into the meaning of uncertainty measures, the belief and the plausibility (see Section 5.2 of Chapter 5 for more details).

- **Target allocation under uncertainty**

Developing a methodology for target allocation under uncertainty through the vehicle multilevel model during the design process constitutes the second scientific contribution of this project. The proposed methodology consists in searching compromise between two contradictory dimensions concerning two major stakeholders of design process. The customers' expectations (desirability measure) and the engineers' concerns about the achievement of these expectations (achievability measure).

The desirability of vehicle performance is calculated by the means of a utility function in the form of sigmoid function defined on a certain range of characteristic values, called desirability interval. However, the achievability is defined by the means of uncertainty measures (belief and plausibility measures) calculated from the uncertain information provided by the experts at the leaf nodes, aggregated and propagated bottom-up in the multilevel model.

The process of target allocation consists in cascading multilevel optimization problems top-down in the model allowing the setting of the characteristics’ targets. The optimization consists in the maximization of the desirability and achievability of the characteristics taking into account the constraints of the allocated targets at the highest levels of the multilevel model. Two types of objective function are used in this process. At the top level of the model, the optimization function is a utility function that involves the desirability and the achievability of each the vehicle performance. This function called the Global Utility of Design consists in the sum of the product of the desirability and achievability of each vehicle characteristic. This formulation allows that for the chosen tradeoff of the desirability and achievability measures of the vehicle performances are not null. Also, with a GUD tending to 1, both the desirability and achievability of the vehicle characteristics tend to 1.

At the lowest levels of the multilevel model, since there is no desirability for the characteristics, the objective function is in the form of achievability product (Multiple characteristics achievability: MCA). This formulation ensures that the chosen compromise corresponds necessarily to a no null achievability of all characteristics. Also a higher MCA imply a higher achievability of the characteristics. (Details on the methods are provided in Chapter 4)

The validation of the methodology was a big issue because of the scarcity of equivalent methods and examples in the literature in addition to the complexity of process to be verified analytically. For that reason, we adapted our approach (called probabilistic approach) to use probability density function of aleatory variables for the verification of the methodology. In this case, the uncertain information is propagated by the means of Monte Carlo Simulation and the characteristics targets allocation is performed on the basis of the maximization of the probability of the whole system by the means of the Multi-characteristic Probability Product (MCPP). It was demonstrated that the proposed approach using Evidence theory is more general than the probabilistic approach for target allocation under uncertainty. Indeed, the results obtained by the probabilistic approach always lay between two extreme values obtained by the optimistic and pessimistic approaches of our methodology corresponding to the values of 0 or 1 of the decision-making or tolerance to risk factor Φ (Details on this verification are provided in Section 5.3 of Chapter 5).

- **Decision-making framework**

The proposition of a framework for decision-making under uncertainty during the vehicle development process constitutes the third contribution of this project. The proposed framework is based on the integration of both methodologies for uncertainty characterization and target allocation under uncertainty in the vehicle multilevel model presented consecutively in Chapters 3 and 4. The decision-making process was modeled in the form of a series of parallel stage-gate process that alternate knowledge generation and decision-making because it is an approach which is the most in line with GM practices and constraints. An assumption stipulates that this decision-making strategy will lead in the end of the process to a better design. Moreover, the proposed framework will help to simulate the VDP and to illustrate the behavior of the proposed methodology for target allocation during the design process.

Validation of this decision-making process and the assumption is out of the scope of the project because a real VDP can be extended over many years and involves hundreds of participants and resources. However, as a surrogate, we relied on a simulation of the VDP to obtain information about the validity of our approach. The validity of our approach is assessed on the basis of component characteristics variation during the iterations on a simple VMM.

The simulation of the design process and the analysis of the results brought some elements of responses that confirm our assumption in terms of convergences and effectiveness of the process after some iteration. It was also demonstrated that is possible to influence the process by selecting specific value of decision-making factor Φ (factor representing the level of risk that can be tolerated by the experts). (Details of the proposed decision-making process are presented in Section 5.4 of Chapter 5).

- **Handling large scale problem with intervals propagation and merging technique**

As presented previously the process of target allocation starts by the collection of experts' opinions at the leaf nodes. These experts' opinions must be aggregated and propagated towards the top level of the multilevel model to determine belief and plausibility curves in order to capture the overall uncertainties in experts' opinions using the Evidence theory. The propagation of intervals can result in possibly overwhelming number of intervals to be handled when we perform our process in the case of large-scale systems. In this context the need for the development of propagation and merging procedure is justified.

The proposed methodology to solve this problem is based on intervals propagation and merging. It consists in reducing the number of intervals handled by controlling the information granularity while keeping the accuracy of the belief and plausibility on a given discrete set of characteristic values. This method helped reducing the computational burden of uncertainty aggregation and propagation through the VMM. (Details on this methodology is proposed in Appendix D)

Application of the proposed procedure for intervals propagation and merging diminished considerably the needed processing time from several hours to few minutes even in the case where our approach is applied to a real VMM.

❖ General issues related to the proposed approaches

The objective of the project was mainly dictated by the operational aspects of the vehicle design process. However, the considerations to adopt the proposed strategy must be more general and must touch to different issues inherent to other activities such as resources allocation, research and development, innovation, knowledge management, costs minimization and control of design process. Some issues related to these considerations with respect to the proposed decision-making strategy are presented below:

- The automakers accumulate large quantities of information and knowledge about design and manufacturing of vehicle. Using these information and knowledge to develop new winning design, may constitute a real advantage against the competitor in term of the reduction of time and cost of development. The proposed approach allows the experts to explore and to use the accumulated knowledge of the company as well as their own.
- The weakness of cooperation and conflicts among the multidisciplinary development teams is an everlasting problem that influences negatively the performance of the teams. A multitude of approaches that are supposed to foster cooperation among teams exist but some limitations occur. For example, the Japanese methods which are very effective in Japan were adopted here in North America. Unfortunately, they failed because of the cultural aspects. The proposed strategy is based on a virtual collaboration relying on targets that help avoiding conflicts among teams due to direct contact.
- The automakers try unceasingly to maintain a climate propitious for innovation where engineers can investigate and achieve their ideas. Again the proposed method provides the opportunity to the engineers to consider all the possible solutions not only the most achievable ones.
- The allocation of resource is one of the keys elements for the effectiveness of the design process. In fact, the optimization of the resource allocation (by the determination of what, how, when and where resources will be allocated) will ensure the achievement of the objectives even those that are the most challenging. The proposed approach will ensure answering to all these questions by the inclusion of resources distribution in the proposed process (see conclusion and recommendation of this thesis).
- The need for the minimization of development and production costs is a serious issue. That because it influences the company profit, the performances and the quality level of the developed vehicle. The proposed approach consists in reaching compromise between the customers' requirements and the automakers' constraints. That implies less rework, less time to market and consequently lower development and production costs.
- Orienting and measuring the VDP progress is a major issue for the effectiveness of design process. The proposed strategy will ensure that through the definition of the characteristics' targets and the use of two measures: the desirability and achievability of

all components of the vehicle. That constitutes a dashboard for the decision-makers to steer the VDP.

In resume, the investigation and integration of the previous issues to the proposed strategy for target allocation and decision-making under uncertainty constitute enrichment and a motivation lever to its promotion.

Conclusion and recommendations

In this thesis, we studied the effect of uncertainty on the development process in order to improve our understanding of the early phases of the process and to demonstrate the importance of including the uncertainties. The developed framework during this project helped us to define strategies to improve target allocation and decision-making under uncertainty during the development process.

The proposed research constitutes an original contribution by tackling, in a single project, a comprehensive study of advanced conceptual design for complex engineering systems. This contribution is intended to be used to increase the productivity and competitiveness of industrial companies in the global market.

Three specifics contributions are achieved and presented during the project. The first one consists in the development of a methodology for characterization, aggregation and propagation of uncertainty in the multilevel model based on the Evidence theory. The second contribution consists in the development of a methodology for target allocation under uncertainty in the multilevel model that take into account two contradictory objectives; the customers' expectations and the engineers concern about the achievement of these expectations. In this methodology, the characterized uncertainty in the multilevel model is used as a measure for achievability of the characteristics. The third contribution consists in the proposition of a decision-making framework that integrates both previous methodologies. This framework can be directly implemented to support the development process. An additional achievement consists in the development of a methodology to handle large-scale problem. This methodology is a procedure for intervals propagation and merging that reduces the computational burden of uncertainty aggregation and propagation through the multilevel model.

The structure of the multilevel model and the uncertain information are given in an XML file while the subroutines for uncertainty management, target allocation and decision-making are coded in MATLAB.

In conclusion, the present work represents a step in the formulation of an integrated methodology to take into account uncertainty during the early stages of the vehicle development process. This methodology can be enriched by the inclusion of the resources allocation aspect and can be extended to other fields where it can contribute to their advancement.

Concerning our recommendations for the possible future works, we think it will be very interesting to address the following topics:

- Investigation of different approaches to orient the deign process through different parameters such as the decision–making factor Φ that represents the risk tolerance of the target allocation decision.
- The presentation of the experts' opinions in the form of intervals and their associated subjective beliefs is a simplified manner to represent the expert knowledge and confidence. This representation is purely quantitative supposing that the engineers are capable of quantifying any information, feeling, vision and even supposition in the form of number (supposition not usually verified). The quantification of any information implies its distortion and loss of its quality, which can leads to no satisfactory design. Furthermore, investigating new representation of the experts' opinions and new ways to handle this information may lead to better design.
- The resource allocation under uncertainty during the VDP must be investigated in order to develop a new methodology based on a mathematical model that supports the proposed methodology for decision-making and replaces the existing methods of resources allocation based on the estimate and experience of the experts.
- Application of the proposed approaches for target allocation and decision making under uncertainty to other engineering fields such as aerospace and aeronautical engineering.
- Investigation of the proposed methodology for targets allocation in the case where the experts' opinions are provided by analysis modules and the uncertainties are completely probabilistic. That will allow the integration of the proposed methodology with the existing analysis modules and its comparison to the existing methods for targets allocation and decision-making.

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Appendix A MATLAB and XML implementation

The structure of the multilevel model is given in an XML file. XML stands for Extensible Markup Language. As a markup language XML is similar to HTML except that the tags are not predefined. Thus, XML is designed to describe any type of data by providing specific tags and structures. An example of the XML file for the vehicle multilevel model is provided in what follows.

```xml
<?xml version="1.0" encoding="ISO-8859-1"?>
<MultiLevelModel Version="2.0" xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance">
   <Vehicle Name="TestVehicle" Type="unknown">
     <Characteristic Name="Mass" TargetValue="1730" Unit="kg"/>
       <System Name="Chassis">
         <Characteristic Name="Mass" TargetValue="1350" Unit="kg"/>
           <Subsystem Name="Engine">
             <Characteristic Name="FuelEconomy" TargetValue="36" Unit="MPG">
                <ExpertOpinion Name="E1">
                    <FocalElement LowerBound="37.5" UpperBound="38.5" SubjectiveBelief="0.2"/>
                    <FocalElement LowerBound="37.0" UpperBound="37.5" SubjectiveBelief="0.3"/>
                    <FocalElement LowerBound="36" UpperBound="37" SubjectiveBelief="0.3"/>
                    <FocalElement LowerBound="35.6" UpperBound="36" SubjectiveBelief="0.2"/>
                </ExpertOpinion>
             </Characteristic>
                <Characteristic Name="Mass" TargetValue="380" Unit="kg">
                   <ExpertOpinion Name="E1">
                     <ProbabilityDensityFunction  Mean="380" StdDev="10.0" Ndiscr="25" />
                   </ExpertOpinion>
                </Characteristic>
          </Subsystem>
          <Subsystem Name="Drivetrain">
           …….

         </Subsystem>

              ……

      </System>
   </Vehicle>
</MultiLevelModel>
```

A MATLAB module has been developed to access data in the XML file and propagate uncertainty in the multilevel model. The MATLAB module uses the Document Object Model (DOM) platform and language-neutral interface (see http://www.w3schools.com/dom/ for details). The XML DOM defines a standard way for accessing and manipulating XML documents. The DOM represents an XML document as a tree-structure (a node tree), with the elements, attributes, and text defined as nodes. The nodes have a hierarchical relationship to each other. The tree starts at the document node and continues to branch out until it has reached all next nodes at the lowest level of the tree. The terms “parent” and “child” are used to describe the relationships between nodes. Some nodes may have child nodes, while other nodes called leaf nodes do not have children.

Because the XML data is structured in a tree form, it can be traversed without knowing the exact structure of the tree and without knowing the type of data contained within. The DOM interface provides a series of methods accessible from MATLAB to traverse the tree structure and collect pertinent data. Once available as MATLAB variables, the data can be used to determine uncertainties at all levels of the model.

Appendix B Fundamentals of the Evidence theory

In this document, an overview of the Evidence theory including the concepts of the frame of discernment, the basic belief assignment and the belief and plausibility functions is presented. The document includes also a presentation of methods for uncertainty aggregation and propagation.

B.1 Evidence theory

B.1.1 Frame of discernment

The frame of discernment is a set of mutually exclusive “elementary” propositions and it can be viewed as a finite space in probability theory [1, 2]. The subsets of this set might be nested in one another or might partially overlap.

For example, suppose that our frame of discernment is noted X and defined as X={x1, x2, x3}. Let Z be a set of the various propositions (subsets of X called also power set 2^X) that can be expressed by the combinations of the elementary propositions. The number of all combinations is given by the formula 2^n, with n the number of elementary propositions of X: Z={Ø, {x1}, {x2}, {x3}, {x1, x2}, {x1, x3}, {x2, x3}, {x1, x2, x3}}.

B.1.2 Basic belief assignment (BBA)

The basic measure in Evidence theory is known as the basic probability assignment (BPA) or basic belief assignment (BBA). It expresses a degree of belief in a proposition. It is a function (m) that maps Z to the interval $[0,1]$. This function allows expressing belief with numbers included in the interval $[0,1]$.

$$m: Z = 2^X \rightarrow [0,1] \tag{1}$$

For a subset A of Z, m(A) represents the portion of total belief assigned exactly to proposition A. The basic belief assignments function must satisfy the three axioms quoted below:

$$m(A) \geq 0 \;\; for\ any\ \ A \in Z \tag{2}$$

$$m(\emptyset) = 0 \tag{3}$$

$$\sum_{A \in Z} m(A) = 1 \tag{4}$$

Basic belief assignment axioms look similar to those of probability theory except that they are less stringent [2, 3].

B.1.3 Belief and plausibility functions

Contrary to probability theory which uses only one measure (the probability of an event), Evidence theory uses two measures: the belief (Bel) and plausibility (Pl) to describe the inherent uncertainty of an event. The belief measure can be viewed as the minimum amount of likelihood associated with an event (A) of the frame of discernment. Similarly, the plausibility measure can be viewed as the maximum amount of likelihood associated with the same event (A) (see Figure B-1). Bel(A) and Pl(A) are given by the following expressions:

$$Bel(A) = \sum_{C \subset A} m(C) \tag{5}$$

$$Pl(A) = \sum_{A \cap C \neq \emptyset} m(C) \tag{6}$$

Bel(A) can be calculated by the summation of the basic belief assignments of all propositions which are included in A. Pl(A) can be obtained by adding basic belief assignments of all propositions that intersect with A and the intersection is not empty [1-4].

The belief and plausibility measures verify the following equations:

$$Bel(A) + Pl(\overline{A}) = 1 \tag{7}$$

$$Bel(A) + Bel(\overline{A}) \leq 1 \tag{8}$$

$$Pl(A) + Pl(\overline{A}) \geq 1 \tag{9}$$

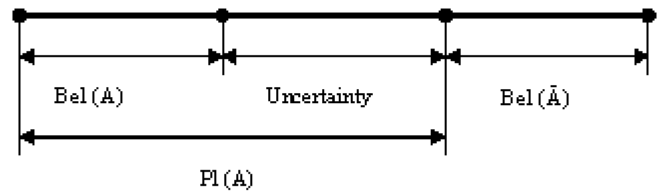

Figure B-1: Representation of belief (Bel), plausibility (Pl) and uncertainty [1]

B.1.3.1 Belief and plausibility measures calculus

The equations 5 and 6 present the general formula for the calculation of the belief and the plausibility measures. In the case where the information is in the form of set of interval and the associated subjective belief, the determination of the belief and plausibility measures can be calculated by the specific equations. The belief and plausibility that $y \geq y_0$ for a given y_0 are obtained by the application of the following equations:

$$Bel(y \geq y_0) = \sum_{i=1}^{m} sb_i \cdot \delta_k(I_i) \text{ with } \begin{cases} \delta_k = 1 \text{ if } y_0 \leq l_i \\ \delta_k = 0 \text{ if } y_0 > l_i \end{cases} \tag{10}$$

$$Pl(y \geq y_0) = \sum_{i=1}^{m} sb_i \cdot \sigma_k(I_i) \text{ with } \begin{cases} \sigma_k = 1 \text{ if } y_0 < u_i \\ \sigma_k = 0 \text{ if } y_0 \geq u_i \end{cases} \tag{11}$$

where I_i an interval from the set intervals, (l_i, u_i) are the bounds of this interval and Sb_i is the associated subjective belief.

The complete methodology for the aggregation of uncertainty in the form of intervals and the associated subjective belief is described in details in Section 3.3.4 and Section 5.2.1.1.

Example:

Figure B-2 presents a simple example of an expert's opinion constituted by a set of four intervals and their associated subjective belief {[790,840[0.1, [840,890[0.3, [890,960[0.2, [960,1000[0.4} and the values of the belief and plausibility measures for the set of values {1000, 900, 800, 700} To calculate the plausibility and belief measures at these specific values, the conditions in the equations 10 and 11 must be verified for any interval in order to participate in the belief, the plausibility or both.

- For $y > 1000$, $Pl = Bel = 0$ because no interval verify the conditions in the equations 10 and 11.
- For $y > 900$, $Pl = 0.4 + 0.2 = 0.6$ $and\ Bel = 0.4$ because the intervals {[890,960[0.2, [960,1000[0.4} participate to the plausibility measure while only the [890,960[0.2 participate to the belief.
- For $y > 800$, $Pl = 0.4 + 0.2 + 0.3 + 0.1 = 1$ $and\ Bel = 0.4 + 0.2 = 0.6$ because the intervals {[840,890[0.3, [890,960[0.2, [960,1000[0.4} participate to the plausibility measure while only the {[890,960[0.2, [960,1000[0.4} participate to the belief.
- For $y > 700$, $Pl = Bel = 0.1 + 0.3 + 0.2 + 0.4 = 1$ because all the intervals verify the conditions in the equations 10 and 11.

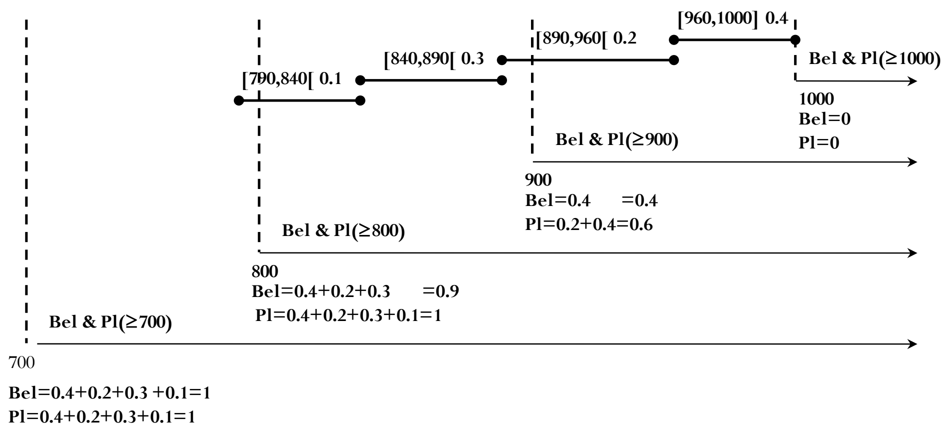

Figure B-2: Example of belief and plausibility calculus

B.2 Uncertainty aggregation

B.2.1 Evidence combination rules

During the development process, engineers and managers need sufficient information to take pertinent decisions concerning uncertain parameters. In Evidence theory, specific rules can be used to combine information provided from different sources. Many authors have proposed different rules of combination such as Dempster's rule, Yager's rule, Dubois and Prade's rule, averaging rule, convolutive x-averaging rule, etc.

Dempster's rule of combination ignores conflicting evidence and generates counter-intuitive results when the information is not consistent. Yager's rule tries to remedy to weakness of

Dempster's rule by regarding conflicting evidence as a contribution to the overall uncertainty. Dubois and Prade's rule is a disjunctive version of Dempster's rule. The result is uninformative in the sense that it tends to generate wide bounds on the quantity of interest. The averaging rule is the most known and easy to use. Indeed, it weights equally all the sources of information [2, 3, 5-7].

B.2.1.1 The Dempster's rule of combination

Evidence obtained from independent sources of experts must be combined. If the BPAs m_1 and m_2 express evidences from two experts, the combined evidence m can be calculated by the following Dempster's rule of combining:

$$m(A) = \frac{\sum_{B\cap C=A} m_1(B).m_2(C)}{(1-K)} \; for \;\; all \; A \neq \varnothing \tag{12}$$

where

$$K = \sum_{B\cap C=\varnothing} m_1(B).m_2(C) \tag{13}$$

This formula expresses the combination for two experts but can be generalized to include any number of experts.

The factor K in Eq. (13) represents the conflict between the two independent experts. Dempster's rule filters out any conflict, or contradiction among the provided evidence, by normalizing with complementary degree of conflict. This method is not applicable in the situation where the evidence are completely in conflict, that is, $K = 1$. It is usually appropriate for relatively small amounts of conflicts where there is some consistency or sufficient agreement among the opinions of the experts. Many authors have proposed different alternative rules of combination to remedy to the drawbacks of Dempster's rule [1, 2, 6, 8].

B.2.1.2 Mixing or averaging

Mixing or averaging is the simplest and most common way to combine evidences. It is a generalization of averaging for probability distributions. The probability distribution describes the frequency of different values within an interval of possible values in continuous case or in the discrete case, the possible simple events. The formula for "mixing" combination rule is the following:

$$m_{1\dots n}(A) = \frac{1}{n}\sum_{i=1}^{n} w_i . m_i(A) \tag{14}$$

Where m_i are the BPAs for the belief structures being aggregated and the w_i are weights assigned according to the reliability of the sources [2, 6].[1]

The mixing, or averaging, method is the most popular method used to combine evidences. Its biggest drawback comes from the fact that it ignores the conflict between the evidences.

B.2.1.3 Convolutive X-averaging

Convolutive X-averaging or C-averaging is a generalization of the average for scalar number. This is given by the formula:

$$m_{12}(A) = \sum_{\frac{(B+C)}{2}=A} m_1(B). m_2(C) \tag{15}$$

Like the mixing average, this can be formulated to include any number of BPAs, in the following equation [6]:

$$m_{1\dots n}(A) = \sum_{\frac{(A_1+A_2+\dots A_n)}{n}=A} \prod_{i=1}^{n} m_i(A_i). \tag{16}$$

The interval A in Eq. (16) is defined as the average of the upper and the lower bounds of the intervals A_i provided by the experts.

B.2.1.4 Dubois and Prade's Disjunctive rule

Suppose there are three items of evidence expressed as mass assignments m_1, m_2 and m_3. Then the disjunctive rule proposed by Dubois and Prade is:

---

[1] When the w_i are equal to one the evidence sources are equivalent or no information about these sources is available in last case we put forward the hypothesis that the sources are equivalent.

$$m_{1,2,3}(A) = \sum_{B \cup C \cup D = A} m_1(B).m_2(C).m_3(D) \quad (17)$$

This formula can be directly generalized to include any number of BPA, n, in the following equation:

$$m_{\cup}(A) = \sum_{B_1 \cup B_2 \cup \ldots B_n} \prod_{i=1}^{n} m_i(B_i) \quad (18)$$

The union does not generate any conflict and does not reject any of the information asserted by the sources. As such, no normalization procedure is required. The drawback of this method is that it may yield a more imprecise result than desirable [1, 2, 6].

The union can be more easily performed via belief measure: let $Bel_1 \cup Bel_2$ is the belief measure associated with $m_1 \cup m_2$. Then for every subset A of the universal set X:

$$Bel_1(A) \cup Bel_2(A) = Bel_1(A).Bel_2(A) \quad (19)$$

Due to the union operation between the intervals in the Dubois and Prade’s rule, this method is the most imprecise among the cited methods. The bounds of the intervals are equal or greater then those provided by the other methods. The BPAs are calculated in the same way as with the convolutive x-averaging method. We can note that this method creates uncertainty by taking the largest intervals of the possible values.

B.3 Uncertainty propagation

As discussed earlier, the present project requires propagating uncertainty for different variables from the parts level to the subsystems level ("vertically") and among subsystems, components and parts ("horizontally"). For example, the mass of all parts of a subsystem and their uncertainty must be combined, and also the mass of a given part may influence directly or indirectly the mass of other parts.

In addition, for most levels, engineering design practices make use of analyses (ranging from nonlinear partial differential equations to empirical approximations and historical data). When the input variables are subjected to uncertainty, the value and uncertainty of the output variables must be determined by analyses.

For both situations, uncertainty propagates through functional relations between input and output variables: $Y = F(X_1, X_2, \dots, X_n)$. Thus, a mapping between input and output uncertainties must be established. If a relational dependency exists between several of the input variables, the variables and their uncertainties are said to be dependent or interactive [9]. This situation is the most complex to manage in particular when coupling is present in multidisciplinary analyses systems. In some cases it is possible to establish a new functional relationship between the independent variables (e.g., if $X_2 = A(X_1)$, than $Y = F(X_1, A(X_1), \dots, X_n)$ otherwise some simplifications must be adopted to propagate the uncertainty. A simplification proposed by Dubois and Prade aims to calculate "pessimistic" fuzzy quantities with interactive variables. We limit ourselves to independent (non-interactive) input variables, that is, input variables that values can be selected independently and which uncertainties are not influenced by one another.

Depending on how the uncertainty is modeled, the mapping between the input and output uncertainty can be expressed in different ways. First we introduce how the mapping is done when the Evidence theory is used (see Refs. [10, 11] for details). For a subset Z of the output space we have:

$$Bel_Y(Z) = Bel_X\left(F^{-1}(Z)\right) \tag{20}$$

and

$$Pl_Y(Z) = Pl_X\left(F^{-1}(Z)\right) \tag{21}$$

Determining $F^{-1}(Z)$ could be complicated and expansive to evaluate, especially when F is a complex non-linear function.

When modeling uncertainty with the Possibility theory, the mapping is written as:

$$\prod_Y(Z) = \prod_X\left(F^{-1}(Z)\right) = Sup(\prod(w) | F(w)\ in\ Z) \tag{22}$$

The supremum operator (Sup) is used because among all the w such that F(w) in Z, only the one that has the higher possibility is considered. This is called the extension principle [9, 12]. A practical application of the extension principle is presented in Chapter 4.

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Appendix C Probability representation of uncertainty and Monte Carlo simulation

In this document, we explain the basic principles of probability representation and propagation of uncertainty in a multilevel model using Monte Carlo Simulation. A specific validation is also provided to illustrate how things work.

C.1 Probability representation of uncertainty

The goal of propagating uncertainties in a multilevel model is to determine the uncertainty in the model outputs y that results from uncertainty in the input elements $\boldsymbol{x}=[x_1,x_2,\ldots,x_n]$. We consider here that y is a scalar variable and that each x_i is an independent variable affected by uncertainty. The propagation of uncertainties can be viewed as the study of functions of the form $y=f(\boldsymbol{x})$. The uncertainty in the elements of $\boldsymbol{x}$ is characterized in our case by experts' opinions $\boldsymbol{EO}x_1,\ldots,\boldsymbol{EO}x_n$ providing a quantitative representation of subjective uncertainties. The resulting uncertainty in the model outputs will be presented as a cumulative probability distribution function $g(Y)=P(y>Y)$, where $P(y>Y)$ is the probability that y is larger than Y.

For an input element x_i, several opinions may be available, but for simplicity we consider the case where only one opinion is available for each input element. The generalization to multiple experts' opinions relies on aggregation of opinions resulting in one equivalent expert opinion $\boldsymbol{EO}x_i$. Please note that the aggregation should be conducted before applying Probability theory. For now, we consider two types of opinions: intervals with subjective beliefs (i.e., imprecise uncertain opinion) or normal distribution probability density functions (i.e., precise uncertain opinion).

With the first approach an expert's opinion $\boldsymbol{EO}x_i$ is expressed as a series of intervals, each associated with a subjective belief that the variable x_i will effectively be in the interval:

$$EO_{x_i} = \{[\,l_1, u_1]sb_1, \ldots, [\,l_m, u_m]sb_m\} \qquad (1)$$

The only constraint on $\boldsymbol{EO}x_i$ is that $l_j<u_j$ (but the intervals can overlap) and that the sum of all the subjective belief is equal to one. With this type of uncertainty the expert does not provide any information on how the probability is distributed within each interval. In particular, it is not exact

to consider a priori that this expert "means" that the probability is uniform in the intervals. Using Probability theory one must consider that each interval is an elementary event called a focal element that cannot be subdivided. In this case, the probability density function for any value in the intervals is not accessible. However, as will be presented later a probability density function is required to conduct a Monte Carlo Simulation. In order to remove this limitation, we make an approximation on the probability distribution within the interval. We consider here a uniform probability distribution in the intervals because it is the most natural assumption. However, one can note that this probability distribution will not be continuous for experts' opinions given as intervals. Nevertheless, a MCS can be done in this case.

In the second approach considered, an expert opinion providing uncertainty as a normal distribution is expressed with a mean and standard deviation. This representation is naturally amenable to Probability theory. To enable the aggregation of opinions of different types (intervals with subjective belief and normal distributions) we translate probability distribution functions into focal elements by asking the expert to provide a discretization step in addition to the mean and standard deviation. Once this is done, both types of opinions are handled as intervals on which the probability is considered uniform.

As mentioned before, we are seeking a probability representation of the uncertainty of *y*. A density function could be used to characterize the uncertainty of *y*. The cumulative distribution function provides a convenient and informative representation:

$$g(\mathrm{Y}) = \mathrm{Prob}(\mathrm{y} > Y) = \int_{\Omega_\mathrm{Y}} \mathrm{d_Y(y)dV} \qquad (2)$$

where, $d_Y(y)$ denotes the density function associated with the distribution of *y*. As mentioned by Oberkampf and Helton [1], a closed-form representation for the density function d_Y can be derived from *f* and $d_X(\boldsymbol{x})$ (see § C.2.1.1 for an example on a validation test case). However, in real problems, this is rarely done due to the complexity of the distributions. An alternative form is the following:

$$\mathrm{Prob}(\mathrm{y} > Y) = \int_{\Omega_X} \delta_\mathrm{Y}\, [f(\mathrm{x})] d_x(\mathrm{x}) dV \qquad (3)$$

where $d_X(\boldsymbol{x})$ represents the density function corresponding to the experts' opinions for each input variable x_i, and,

$$\delta_Y[f(x)] = \begin{cases} 1 \text{ if } f(x) \geq Y \\ 0 \text{ if } f(x) < Y \end{cases} \tag{4}$$

C.2 Monte Carlo simulation

Conducting a Monte Carlo simulation of the multilevel model requires a sampling procedure and a way to map model inputs and outputs. With this mapping it is possible to evaluate an approximation of the integral in Eq. (3). A sample k for all design variables,

$$\mathbf{x_k} = [x_{1k}, x_{2k}, \ldots, x_{nk}], k = 1, \ldots, Ns \tag{5}$$

must be generated. Then, the evaluations of f $y_k = f(x_k)$ on all samples create a mapping:

$$[\boldsymbol{x_k}, y_k] \; k = 1, \ldots, Ns \tag{6}$$

with Ns the number of samples.

The integral in Eq. (3) can be approximated by

$$\text{Prob}(y > Y) \cong \sum_{k=1}^{Ns} \delta_Y(y_k)\omega_k \tag{7}$$

The ω_k must be selected in conjunction with the sampling techniques used (Helton and Davis [2]). For random sampling, $\omega_k=1/Ns$, and Eq. (7) consists in counting the number of points such that $y_k>Y$ and divide by the sample size. The accuracy of the approximation depends on the number of points; this aspect has been investigated on test cases and is discussed in Section C.2.1.2.

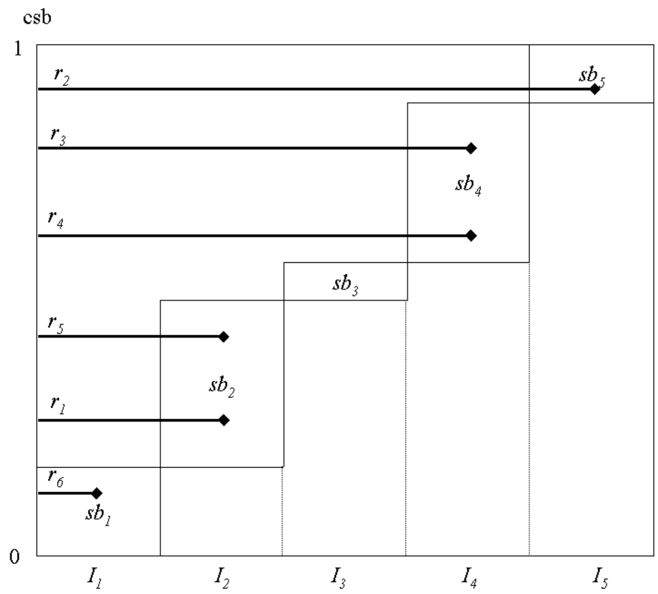

Figure C-1: Random sampling to select intervals according to their subjective belief

The methodology implemented to conduct the Monte Carlo Simulation on a multilevel model relies on Eq. (7). However, the random sampling must be adapted to account for the uncertainty representation. On each interval, a subjective belief is given; therefore we have strata of uniform probability distributions, one for each interval. The number of sample points dedicated to each stratum must account for the associated probability (subjective belief). The higher the probability of a stratum the more points should be taken in it. The total number of sample points must be equal to the number of points per stratum: $Ns=Ns_1+\ldots+Nsj\ldots+Ns_m$. Ideally, we would have $Nsj=sbj \bullet Ns$ to dispatch the points without bias between intervals. Another constraint to avoid bias is that the points are shuffled among the intervals. This is achieved with the proposed sampling strategy. The idea is to put more points in the intervals having higher subjective beliefs. First, we consider a cumulative subjective belief function (*csb*) that varies from 0 to 1 (see Figure C-1). After that, we make a mapping between a random sample Ω drawn between 0 and 1, and the number of the corresponding interval:

$$[r_k, I_j]\ k=1,\ldots,Ns \qquad (8)$$

Then, for each I_j, a random value between $[l_j, u_j]$ is drawn and participates as a sample point. Different draws will produce different $Ns_1,\ldots,Ns_m$, but with a sufficiently large sample size, we would approach $Ns_j=sb_j\ Ns$ for $j=1,\ldots,m$. The influence of the sample size is studied in the next section.

C.2.1 Validation of the Monte Carlo simulation

In this document, for the seek of simplicity, we are mainly interested in mass input elements with additive properties while aggregating upward in the multilevel model where we consider parts, components, subsystems, and systems. The model output of interest is also a mass; for generality we note: $y=f(x)=x_1+x_2+\ldots+x_n$. With this type of function theoretical results are available that we use for comparison. In the next two subsections the theoretical results are presented, followed by the validation of the Monte Carlo simulation on a test case.

C.2.1.1 Opinions given as intervals with subjective belief – Test Case #1

To illustrate the theoretical development we use the test case # 1 given in Figure C-2. For this test case we have:

$$\text{Prob}(900 \le m_B \le 1000) = \int_{900}^{1000} f_B(m)dm = 1 \tag{9}$$

and

$$\text{Prob}(800 \le m_C \le 950) = \int_{800}^{950} f_C(m)dm = 1 \tag{10}$$

We suppose that on each interval the probability density function is constant, thus we have:

$$\begin{cases} f_B = \frac{1}{100} \text{ if } 900 \le \mathrm{m_B} \le 1000 \\ f_B = 0 \ \ otherwise \end{cases} \tag{11}$$

and

$$\begin{cases} f_C = \frac{1}{100} \text{ if } 800 \le \mathrm{m_C} \le 950 \\ f_C = 0 \ \ otherwise \end{cases} \tag{12}$$

The objective is to propagate the uncertainty to the element A, i.e., to determine

$$\text{Prob}\ (m_A \ge Y) = \int_{Y}^{+\infty} f_A(m)dm \tag{13}$$

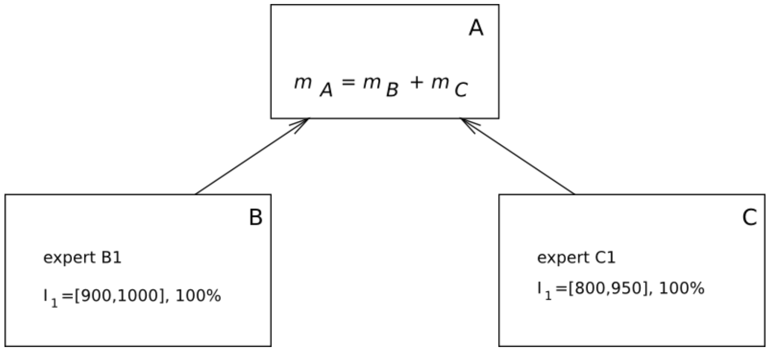

Figure C-2: Test case #1

M_B and M_C being two continuous random variables, f_B and f_C their respective density probability functions, if M_B and M_C are independent, then the density probability f_A of the random variable M_A=M_B+M_C is given by the convolution product:

$$\begin{aligned} f_A(m_A) &= f_B \otimes f_C(m_A) \\ &= \int_{-\infty}^{+\infty} f_B(m)\, f_C(m_A - m)\, dm \\ &= \int_{-\infty}^{+\infty} f_B(m_A - m)\, f_C(m)\, dm \end{aligned} \quad (14)$$

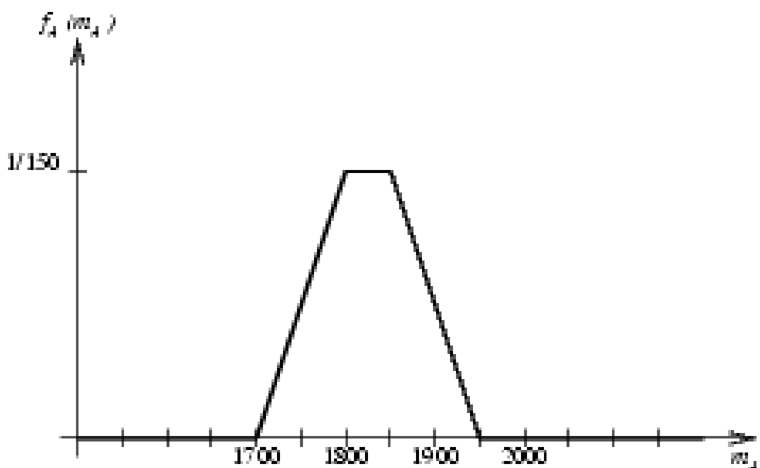

Figure C-3: Resulting function fA (test case #1)

When considering f_B and f_C for the test case we have:

$$f_A(m_A) = \frac{1}{100} \int_{900}^{1000} f_C(m_A - m) dm \quad (15)$$

Evaluating the integral consists in determining the area of a moving rectangular function whose position depends on m_A inside fixed boundaries. The resulting function is presented on Figure

C-3. Using Eq. (13), we can calculate the probability that $m_A \geq Y$ when Y is varying. This function is presented on Figure C-4 and is called exact probability in what follows. It is worthwhile noting that the cumulative probability function approaches unity when $m_A \rightarrow 1700$ and approaches zero as $m_A \rightarrow 1950$.

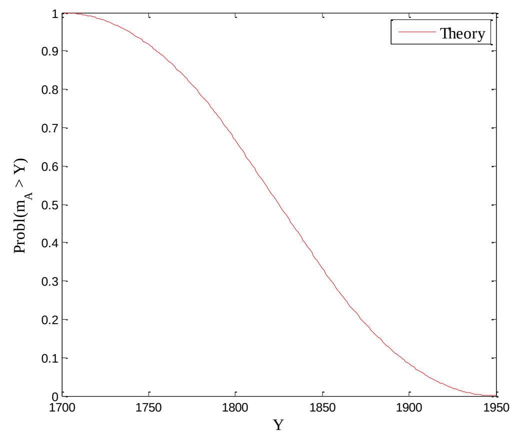

Figure C-4: Curve of the cumulative probability distribution for test case #1

In order to select the adequate sample size Ns, simulation results are compared with exact solutions. Based on a series of results (e.g., see Figure C-5), Ns = 10000 has been selected. Calculation with such sample size ensures a good match with the theoretical results and the results are obtained quickly.

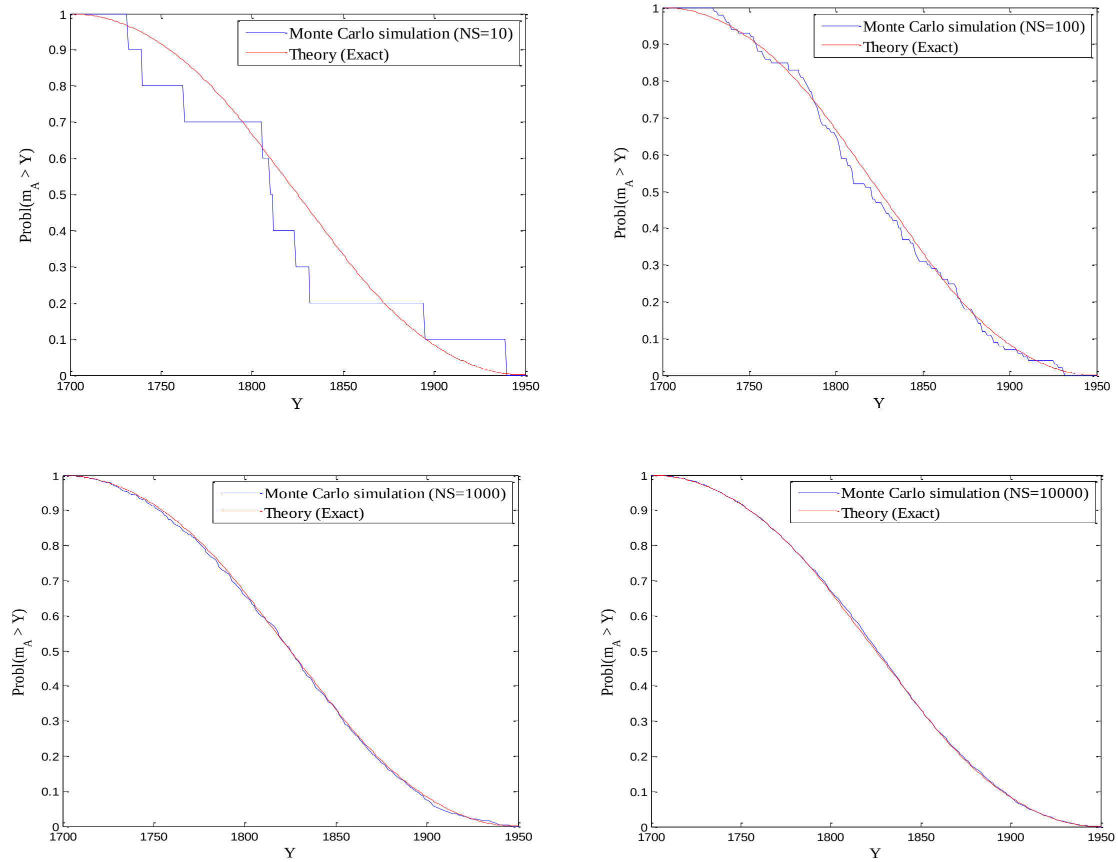

Figure C-5: Influence of Ns on the results for test case #1

C.2.1.2 Opinions given as normal distributions – Test Case #2

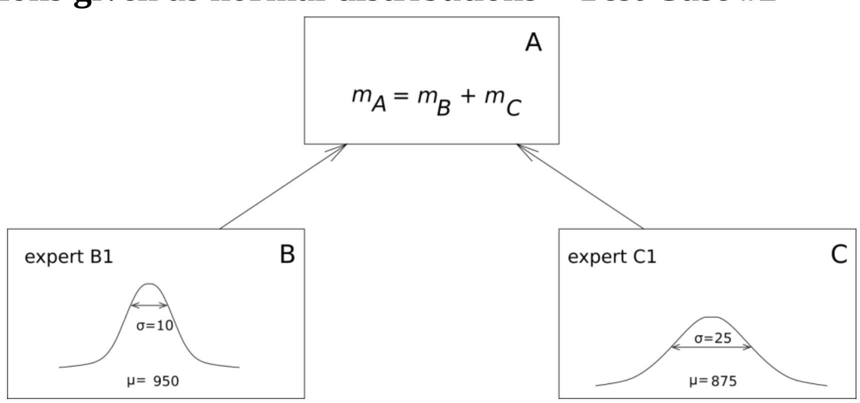

Figure C-6: Test case #2 (normal distributions)

We use test case #2 presented in Figure C-6 where opinions are provided as normal distribution functions. The cumulative probability function $Prob(m_A \geq Y)$ is evaluated using Eq. (13) and Eq.

(14). The integral is evaluated numerically with MATLAB. Thus the integral in Eq. (13) is evaluated for discrete values of Y. We have chosen a granularity of one to present the results for the probability function (theory and MCS). Please also note that the normal distribution functions f_B and f_C are truncated to allow numerical calculation. We have considered that f_B and f_C are null outside of the interval [μ-4σ, μ+4σ].

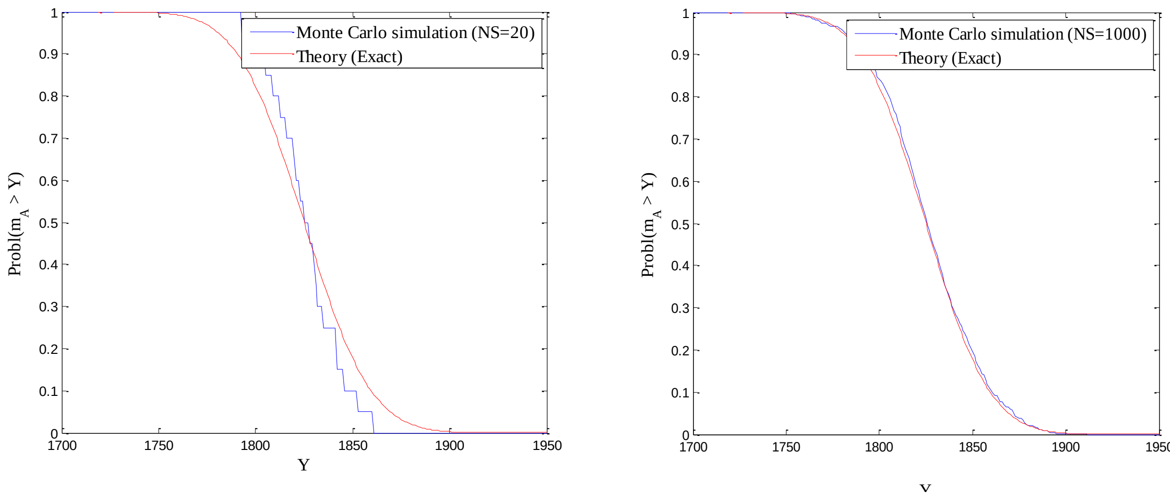

Figure C-7: Curve of the cumulative probability distribution for test case #2

The cumulative probability outside of this interval is almost equal to zero with respect to the accuracy expected during the project: $Prob(m_B \leq \mu_B\text{-}4\sigma_B) = Prob(m_C \leq \mu_C\text{-}4\sigma_C)=3E\text{-}5$. Figure C-7 shows a plot of the results for the probability obtained with the theoretical results given previously using the normal distribution functions and those obtained by Monte Carlo Simulation with different steps of discretization of these normal distributions. We can note also that the probability function obtained by MCS converges to the one obtained theoretically as far as the number of discetization increase.

C.3 Conclusion

In this appendix, we presented the basic principles of probability representation and propagation of uncertainty in a multilevel model using Monte Carlo Simulation and we validated this approach.

References

[1] W. L. Oberkampf and J. C. Helton, "Evidence Theory for engineering applications," in *Engineering Design Reliability Handbook*, ed: CRC Press LLC, 2005, pp. 10.1 --10.30.

[2] J. C. Helton and F. J. Davies, "Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems," *Reliability Engineering & System Safety*, vol. 81, pp. 23-69, 2003.

Appendix D Propagation and Merging of Uncertain Experts' Opinions in Hierarchical Multilevel System

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A procedure for propagating and merging uncertain experts' opinions provided as intervals and subjective beliefs is presented in this paper. The proposed approach can be applied to hierarchical multilevel models with functional relations linking the characteristics of the multilevel model nodes. When experts' opinions are provided at the leaf nodes, it is necessary to propagate intervals towards the top level to determine belief and plausibility curves that capture the overall uncertainties in experts' opinions using the Evidence theory. The propagation of intervals can result in a possibly overwhelming number of intervals to be handled. Hence, a propagation and merging procedure is proposed to reduce the number of intervals. A test case example is used to illustrate the efficiency of the procedure.

D.1 Introduction

Creating innovative products with challenging specifications requires iterative design approaches based on new technologies, new materials and/or new manufacturing methods. Hence, until the product is completed, the product characteristics and performance are approximate and subject to uncertainties.

Complex systems such as a vehicle can be modeled as a multilevel hierarchical structure (see

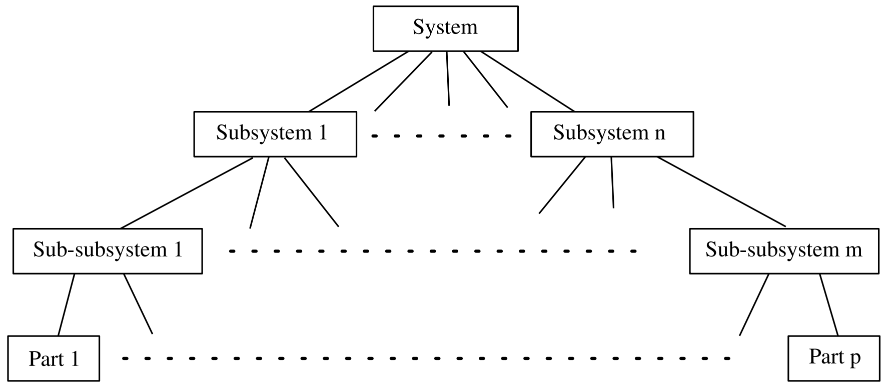


Figure D.1 Multilevel hierarchical structure

Figure D.1) where the component characteristics are related through functional dependencies in the bottom-up direction. Conceptual design practices for complex hierarchical systems often involve system experts for targets allocation and subject matter experts for designing components with respect to those targets. The current practice for target reallocation is a human driven decision making process in search of compromises and design improvements for the overall system based on experts' opinions. In this paper we consider that the component design process is conducted by experts and applied only at the leaf components of the multilevel hierarchical system. Upon receiving targets, each expert conducts component design and returns values of characteristics feasible with respect to his/her targets, if possible. Moreover, in order to allocate targets that will guide system design improvements, the information provided by experts at the leaf components must be propagated through all the multilevel structure to reach the top level.

The design of complex systems is inherently uncertain due to many factors such as random design variables, the lack of information about evolving technologies and manufacturing process, the incomplete specifications of components and the interactions among components. Thus, the experts' opinions about component characteristics may include multiple sources of uncertainty having objective or subjective nature. Here again, the uncertain information must be propagated through the multilevel structure to appraise its impact on the system. As will be illustrated in this paper, due to the multilevel hierarchical structure, the amount of information to be propagated could become overwhelming. However, depending on the design phase, the granularity of uncertainty information may be finer than what is required for target allocation decisions. Therefore, the amount of uncertainty information may be reduced by carefully merging some bodies of information.

The present work represents a preliminary step in the development of a formal decision making methodology including uncertainties during the conceptual design phase of complex systems. This paper presents a merging procedure to control propagated information granularity and to reduce the computational burden of uncertainty propagation. The decision making aspects for target allocation will be the subject of subsequent papers.

In Section D.2 uncertainty representation using Evidence theory is introduced. Sections D.3 and D.4 describe respectively the propagation and merging of uncertain experts' opinions.

D.2 Uncertainty representation

D.2.1 Sources of uncertainty

The sources of uncertainty depend on the sources of information used by experts. When sufficient data is available, experts can create and verify strong statistical models for stochastic variables. In this case, the uncertainty is purely aleatory and is commonly represented by a probability distribution. Moreover, epistemic uncertainty can be present in experts' opinions because of the lack of knowledge or information. More generally, experts may rely on sparse statistical data with partially characterized randomness, some subjective experience and empirical methods, approximation functions and computational-based analyses as valuable sources of information during the design (see Thunnissen (2003) for a complete description of the nature of uncertainty). Therefore, experts' opinions may contain epistemic and aleatory uncertainties.

In this work, we suppose that both type of uncertainties are present in experts' opinions without distinction. An additional hypothesis is that experts' opinions are provided independently of one another. Therefore, coupling between component characteristics is considered resolved by experts and uncertain experts' opinions encompass this effect in some way.

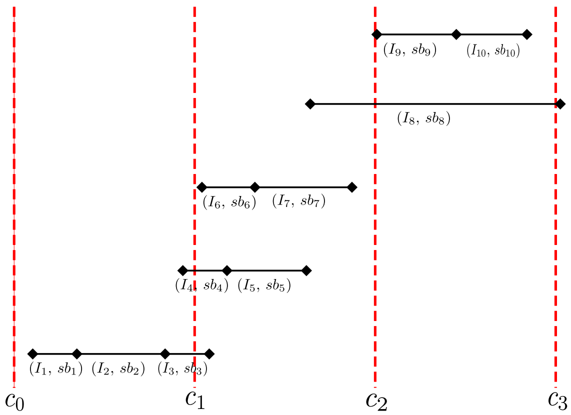
Continuous probability density functions or probability given on bodies of information are classical approaches to represent uncertain information (e.g., Batill et al. (2000), Oberkampf & Helton (2004)). In the scope of the present work, we have considered only real-valued engineering characteristics, and the bodies of uncertain information are provided by experts as real intervals associated with subjective beliefs. The probability given by an expert to a body of information reflects his/her confidence or subjective belief. For example, the value of characteristic C lies in the interval $[a, b]$ with $x\%$ subjective belief or in the interval $[c, d]$ with $y\%$ subjective belief, etc. One can note that the information is imprecise since C can take any value between the intervals bounds, but not fuzzy because the bounds are clearly identified; however, the presence of C in the interval is uncertain even if the belief is 100 % because of the experts' subjectivity. Due to the various forms of uncertainty present in experts' opinions, we take an approach similar to Oberkampf & Helton (2004), Helton et al. (2005) and select the Evidence theory that can handle both epistemic and aleatory uncertainties.

D.2.2 Belief and plausibility representation of uncertainty

The "Evidence theory", or "Dempster-Shafer theory" (Shafer (1976)), allows less restrictive statements about uncertainty than in the case of probabilistic specification. The main concept of the Evidence theory is that our knowledge of a given problem can be inherently imprecise. Hence, the Evidence theory uses two specifications of likelihood: the belief and the plausibility. The belief measure can be viewed as the minimum amount of likelihood associated with an event. Similarly, the plausibility measure can be viewed as the maximum amount of likelihood associated with the same event.

According to the Evidence theory, the belief and plausibility can be evaluated as follows when uncertainty is given as real intervals with associated subjective beliefs.

Let us consider $\mathcal{A} = \{(I_1, sb_1), \ldots, (I_m, sb_m)\}$ the set of intervals $I_i = [l_i, u_i[$ ($l_i \in I_i$ and $u_i \notin I_i$) and subjective beliefs sb_i of cardinality m such that $\sum sb_i = 1$ representing an expert's opinion. The belief Bel_k and plausibility Pl_k that the characteristic C be larger than a given value c_k is



c_3 : $\sigma_3 = 1$ only for $j = 8 \Rightarrow Pl_3 = sb_8$
$\delta_3 = 0$ for all $j \Rightarrow Bel_3 = 0$
c_2 : $\sigma_2 = 1$ only for $j = 8, 9, 10 \Rightarrow Pl_2 = sb_8 + sb_9 + sb_{10}$
$\delta_2 = 1$ only for $j = 9, 10 \Rightarrow Bel_2 = sb_9 + sb_{10}$
c_1 : $\sigma_1 = 1$ only for $j = 3, \ldots, 10 \Rightarrow Pl_1 = sb_3 + \ldots + sb_{10}$
$\delta_1 = 1$ only for $j = 5, \ldots, 10 \Rightarrow Bel_1 = sb_5 + \ldots + sb_{10}$
c_0 : $\sigma_0 = 1$ for all $j \Rightarrow Pl_0 = sb_1 + \ldots + sb_{10}$
$\delta_0 = 1$ for all $j \Rightarrow Bel_0 = sb_1 + \ldots + sb_{10}$

Figure D.2 Example of belief and plausibility calculation

determined using the following expressions:

$$Bel_k = Bel(C \geq c_k) = \sum_{j=1}^{m} sb_j \, \delta_k(I_j) \quad \text{with} \quad \begin{cases} \delta_k(I_j) = 1 \text{ if } c_k \leq l_j \\ \delta_k(I_j) = 0 \text{ if } c_k > l_j \end{cases} \tag{D.1}$$

$$Pl_k = Pl(C \geq c_k) = \sum_{j=1}^{m} sb_j \, \sigma_k(I_j) \quad \text{with} \quad \begin{cases} \sigma_k(I_j) = 1 \text{ if } c_k < u_j \\ \sigma_k(I_j) = 0 \text{ if } c_k \geq u_j \end{cases} \tag{D.2}$$

An illustration of belief and plausibility calculation is presented in Figure D.2.

D.3 Uncertainty propagation

D.3.1 Propagation through functional relations

Experts' opinions for each characteristic must be propagated from the leaf components to the top level of the multilevel system model using the functional relations among characteristics. Based on these functional relationships, a mapping between the uncertainties of the children nodes (input space) and the uncertainty of the parent node (output space) must be established at each level in the hierarchical tree.

To explain the uncertainty propagation method, let us consider a functional relation between real-valued characteristic C of Subsystems 1 & 2 and System 1 given by:

$$C^{S1} = P(C^{SS1}, C^{SS2}) = C^{SS1} + C^{SS2} \tag{D.3}$$

That is, $P : \mathbb{R}^2 \mapsto \mathbb{R}$ is a function from a real bi-dimensional vector to a real number. All combinations of the input intervals from C^{SS1} and C^{SS2} must be propagated through the functional to obtain the output intervals for C^{S1}. Hence, the function P must be extended to take intervals in $\mathbb{R}^2$ as inputs and produce intervals in $\mathbb{R}$ as outputs, that is noted $[P[\colon [\mathbb{R}^2[\mapsto [\mathbb{R}[$. For the input intervals $[a, b[$ of C^{SS1} and $[c, d[$ of C^{SS2}, we consider an interval extension of the functional with the output interval $[l, u[$:

$$\begin{cases} l = \underset{C^{SS1} \times C^{SS2} \in [a,b[\times [c,d[}{\text{minimize}} P(C^{SS1}, C^{SS2}) = a + c \\ u = \underset{C^{SS1} \times C^{SS2} \in [a,b[\times [c,d[}{\text{maximize}} P(C^{SS1}, C^{SS2}) = b + d \end{cases} \tag{D.4}$$

For any combination of input intervals, there is only one corresponding output interval. In the case of Eq. (D.3), the minimize and maximize operators are readily evaluated because P is a monotonic function of C^{SS1} and C^{SS2}: $l = a + c$ and $u = b + d$.[1] Similar to the probability associated with the occurrence of independent events, the subjective belief associated with independent bodies of information is equal to the product of the subjective beliefs associated with each body of information. Hence, in the previous example, the subjective belief associated with the event $C^{S1} \in [l, u[$ is equal to the product of the subjective beliefs associated with the events $C^{SS1} \in [a, b[$ and $C^{SS2} \in [c, d[$.

D.3.2 Uncertainty propagation in a large scale system

To illustrate the computational burden related to uncertainty propagation in a large scale hierarchical system, let us define a multilevel model with a simple structure repeated at each level. Four levels of components are considered. The system has n subsystems, each subsystem has n sub-subsystems, each sub-subsystem has n parts; thus, we have a total of $n*n$ sub-subsystems, $n*n*n$

[1]Please note that the results of this work is based on monotonic n dimensional functional relations but the maximize and minimize operators in Eq. (D.4) can be applied to any functional relations to obtain the propagated upper and lower bounds.

Table D.1 Number of intervals in multilevel model ($n = 4$, $m = 1, \ldots, 8$)

		Parts		Sub-subsystems		Subsystems		System	
# int. by EO	# children by level	# int. by component	# int. by level	# int. by component	# int. by level	# int. by component	# int. by level	# int. by component	# int. by level
1	4	1	64	1	16	1	4	1	1
2	4	2	128	16	256	65536	262144	1.8E19	1.8E19
3	4	3	192	81	1296	4.3E7	1.7E8	3.4E30	3.4E30
4	4	4	256	256	4096	4.3E10	1.7E10	3.4E38	3.4E38
5	4	5	320	625	10000	1.5E11	6.1E11	5.4E44	5.4E44
6	4	6	384	1296	20736	2.8E12	1.1E13	6.3E49	6.3E49
7	4	7	448	2401	38416	3.3E13	1.3E14	1.2E54	1.2E54
8	4	8	512	4096	65536	2.8E14	1.1E15	6.2E57	6.2E57

parts and $n*n*n+n*n+n+1$ components in the multilevel model. Let us consider a single characteristic and assume all experts' opinions are given with the same number of intervals m. The propagation from n children subcomponents to a parent component consists in combining intervals and subjective beliefs along a sub-tree of the hierarchy. The number of intervals propagated to the parent component is m^n. Hence, the number of intervals in a multilevel model may become huge as the intervals are propagated up to the top level. An example of the number of intervals to be managed during the propagation is given in Table D.1.

The results presented in Table D.1 illustrate the computational difficulty of handling and processing intervals in a large scale system with detailed experts' opinions.

D.3.3 Impact of information granularity on belief and plausibility

At a given node, an expert can provide characteristic information with a certain level of precision and granularity. The precision is the number of significant digits and the granularity is the typical interval size. Let us consider two experts whose independent opinions are expressed at the subsystem level and propagated to the system level. Expert's opinion $EO_{C^{SS1}}$ comprises intervals and subjective beliefs about the characteristic C of component $SS1$ with a granularity of the order 1 unit. Similarly, $EO_{C^{SS2}}$ comprises intervals and subjective beliefs about the characteristic C of component $SS2$ with a granularity of the order 0.1 unit. All combinations of intervals in $EO_{C^{SS1}}$ and $EO_{C^{SS2}}$ must be propagated because experts' opinions are assumed to be independent. Let us

Table D.2 Combination of intervals with different granularities

$EO_{C^{SS1}}$	$EO_{C^{SS2}}$	Propagated EO for $C^{S1} = C^{SS1} + C^{SS2}$		
$\{[120, 125[, 0.3$;	$\{[10.1, 10.3[, 0.4$;	$\{[130.1, 135.3[, 0.12$;	$[135.1, 137.3[, 0.16$;	$[137.1, 140.3[, 0.12$;
$[125, 127[, 0.4$;	$[10.3, 10.5[, 0.3$;	$[130.3, 135.5[, 0.09$;	$[135.3, 137.5[, 0.12$;	$[137.3, 140.5[, 0.09$;
$[127, 130[, 0.3\}$	$[10.5, 10.6[, 0.3\}$	$[130.5, 135.6[, 0.09$;	$[135.5, 137.6[, 0.12;$	$[137.5, 140.6[, 0.09\}$

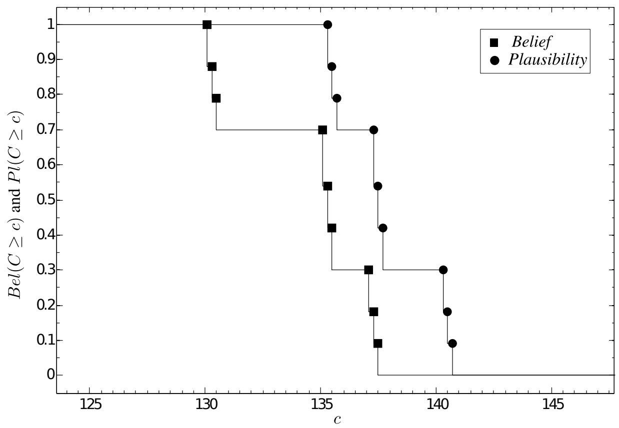


Figure D.3 Influence of granularity on belief and plausibility curves

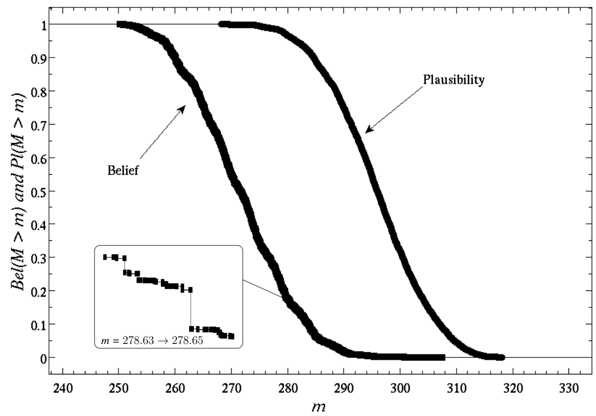


Figure D.4 Belief and plausibility curves for a characteristic M of a multilevel system

consider that characteristic C of the system equals the sum of C^{SS1} and C^{SS2} (see data provided in Table D.2). The resulting belief and plausibility curves (see Figure D.3) exhibit large steps made of several smaller steps. The larger steps are associated with the larger information granularity.

In a large scale multilevel model, the smallest granularity of information among experts' opinions given at leaf nodes is propagated to the system level and appears in the belief and plausibility curves as small characteristic steps (see Figure D.4). Such detailed curves do not possess high level of accuracy at the early stages of the design because characteristic targets given to experts are approximate and experts respond with coarse granularity uncertain opinions. Hence, coarse approximation of the belief and plausibility curves could be sufficient for target selection in this situation. However, belief and plausibility curves with smaller information granularity allow more flexibility to fine tune the characteristic targets as the design progresses and the experts' opinions become more certain, that is, opinions composed of fewer and smaller intervals. Depending on the design phase, a meaningful increment δc of system characteristic C can be identified and an approximation of the belief and plausibility curves can be sufficient for allocating the targets to guide the next design phase.

D.4 Merging of uncertain experts' opinions

Having identified a meaningful increment δc of characteristic C at a node of the multilevel model it is possible to calculate belief and plausibility curves on a series of discrete values $c_{k+1} = c_k + \delta c$ where c_0 is the lowest possible value taken by the characteristic C. At a given node, based on all propagated intervals, discrete belief and plausibility curves are given as a series of (c_k, Bel_k) and (c_k, Pl_k) pairs with $Bel_k = Bel(C \geq c_k)$ and $Pl_k = Pl(C \geq c_k)$.

In what follows, a procedure for merging intervals is presented that allows a reduction in the number of intervals to be processed at each node before it becomes intractable during the propagation toward the system level.

D.4.1 Conditions for merging intervals

Let us consider $\mathcal{A} = \{(I_1, sb_1), \ldots, (I_m, sb_m)\}$ the set of pairs of intervals $I_i = [l_i, u_i[$ ($l_i \in I_i$ and $u_i \notin I_i$) and subjective beliefs sb_i of cardinality m representing an expert's opinion for a characteristic C. For the sake of clarity, we recall that the belief Bel_k and plausibility Pl_k that the characteristic C be larger than a given value c_k is determined as follows:

$$Bel_k = Bel(C \geq c_k) = \sum_{j=1}^{m} sb_j \, \delta_k(I_j) \quad \text{with} \quad \begin{cases} \delta_k(I_j) = 1 \text{ if } c_k \leq l_j \\ \delta_k(I_j) = 0 \text{ if } c_k > l_j \end{cases} \tag{D.5}$$

$$Pl_k = Pl(C \geq c_k) = \sum_{j=1}^{m} sb_j \, \sigma_j(I_j) \quad \text{with} \quad \begin{cases} \sigma_k(I_j) = 1 \text{ if } c_k < u_j \\ \sigma_k(I_j) = 0 \text{ if } c_k \geq u_j \end{cases} \tag{D.6}$$

For an element $(I_j, sb_j) \in \mathcal{A}$, the contribution to the belief and plausibility can be expressed as the function f_k :

$$f_k : ([\mathbb{R}[, \mathbb{R}) \mapsto \mathbb{R}^2 \tag{D.7}$$

where $[\mathbb{R}[$ represents the set of all finite intervals in $\mathbb{R}$ and

$$f_k\left(I_j, sb_j\right) = \begin{pmatrix} \delta_k(I_j) \\ \sigma_k(I_j) \end{pmatrix} sb_j \tag{D.8}$$

Hence,

$$\begin{pmatrix} Bel_k \\ Pl_k \end{pmatrix} = \sum_{\mathcal{A}} f_k\left(I_j, sb_j\right) \tag{D.9}$$

Let us consider $\mathcal{B}$, a subset of $\mathcal{A}$ of cardinality $n \leq m$. The contribution of the elements of $\mathcal{B}$ to the belief and plausibility can be expressed by the function g_k:

$$g_k : ([\mathbb{R}[, \mathbb{R})^n \mapsto \mathbb{R}^2 \tag{D.10}$$

and

$$g_k(\mathcal{B}) = \sum_{\mathcal{B}} f_k\left(I_j, sb_j\right) \tag{D.11}$$

Let us define a merged interval and subjective belief from $\mathcal{B}$:

$$M_{\mathcal{B}} = (\tilde{I}_{\mathcal{B}}, \widetilde{sb}_{\mathcal{B}})$$

with

$$\tilde{I}_{\mathcal{B}} = \left[\min_{\mathcal{B}}(l_j), \max_{\mathcal{B}}(u_j)\right[$$

an interval containing all the elements of $\mathcal{B}$, and $\widetilde{sb}_{\mathcal{B}} = \sum_{\mathcal{B}} sb_j$ the contribution of the elements of $\mathcal{B}$ to an overall subjective belief. Hence, the bodies of information that $\mathcal{B}$ contains are merged into a less detailed single body of information $M_{\mathcal{B}}$.

Having defined a merged interval and subjective belief, which conditions make the contribution of the intervals and subjective beliefs in $\mathcal{B}$ to Bel_k and Pl_k equal the contribution of $M_{\mathcal{B}}$, i.e., the contribution of a merged interval and subjective belief? We have identified three conditions where $g_k(\mathcal{B}) = f_k(M_{\mathcal{B}})$:

- If $c_k \leq \min_{\mathcal{B}}(l_j)$ then

$$\forall I_j \in \mathcal{B},\ \delta_k(I_j) = \sigma_k(I_j) = 1 \ \Rightarrow\ g_k(\mathcal{B}) = \sum_{\mathcal{B}} \binom{1}{1} sb_j = \binom{1}{1} \sum_{\mathcal{B}} sb_j$$

and for $M_{\mathcal{B}}$

$$\delta_k(\tilde{I}_{\mathcal{B}}) = \sigma_k(\tilde{I}_{\mathcal{B}}) = 1 \ \Rightarrow\ f_k(M_{\mathcal{B}}) = \binom{1}{1} \tilde{sb} = \binom{1}{1} \sum_{\mathcal{B}} sb_j$$

- If $c_k \geq \max_{\mathcal{B}}(u_j)$ then

$$\forall I_j \in \mathcal{B},\ \delta_k(I_j) = \sigma_k(I_j) = 0 \ \Rightarrow\ g_k(\mathcal{B}) = \sum_{\mathcal{B}} \binom{0}{0} sb_j = \binom{0}{0}$$

and

$$\delta_k(\tilde{I}_{\mathcal{B}}) = \sigma_k(\tilde{I}_{\mathcal{B}}) = 0 \Rightarrow f_k(M_{\mathcal{B}}) = \binom{0}{0} \tilde{sb} = \binom{0}{0}$$

- If $c_k < \min_{\mathcal{B}}(u_j)$ and $c_k > \max_{\mathcal{B}}(l_j)$ then

$$\forall I_j \in \mathcal{B},\ \delta_k(I_j) = 0 \text{ and } \sigma_k(I_j) = 1 \ \Rightarrow\ g_k(\mathcal{B}) = \sum_{\mathcal{B}} \binom{0}{1} sb_j = \binom{0}{1} \sum_{\mathcal{B}} sb_j$$

and

$$\delta_k(\tilde{I}_{\mathcal{B}}) = 0 \text{ and } \sigma_k(\tilde{I}_{\mathcal{B}}) = 1 \Rightarrow f_k(M_{\mathcal{B}}) = \binom{0}{1} \tilde{sb} = \binom{0}{1} \sum_{\mathcal{B}} sb_j$$

Let us now consider a decomposition of $\mathcal{A}$ into a finite series of subsets $\mathcal{B}_j$: $\mathcal{A} = \{\mathcal{B}_1, \ldots, \mathcal{B}_p\}$ and $\mathcal{B}_i \cap \mathcal{B}_j = \varnothing\ \forall i \neq j, i = 1, \ldots, p$ and $j = 1, \ldots p$ which implies that

$$\sum_{j=1,\ldots,p} |\mathcal{B}_j| = m \tag{D.12}$$

Also, for each $\mathcal{B}_j \in \mathcal{A}$ a merged interval and subjective belief $M_{\mathcal{B}_j} = (\tilde{I}_{\mathcal{B}_j}, \widetilde{sb}_{\mathcal{B}_j})$ can be obtained. The belief Bel_k and plausibility Pl_k that the characteristic C be larger than a given value c_k can be expressed as:

$$\begin{pmatrix} Bel_k \\ Pl_k \end{pmatrix} = \sum_{\mathcal{A}} f_k\left(I_j, sb_j\right) = \sum_{j=1,\ldots,p} g_k\left(\mathcal{B}_j\right) \tag{D.13}$$

Also, based on the conditions on intervals bounds previously stated, we have:

$$\forall j = 1, \ldots, p,\ g_k(\mathcal{B}_j) = f_k(M_{\mathcal{B}_j}) \text{ if } \quad \begin{cases} c_k \leq \min_{\mathcal{B}_j}(l_i) \\ c_k \geq \max_{\mathcal{B}_j}(u_i) \\ c_k < \min_{\mathcal{B}_j}(u_j) \text{ and } c_k > \max_{\mathcal{B}_j}(l_j) \end{cases} \tag{D.14}$$

Hence, the belief and plausibility can be evaluated with the following expression:

$$\begin{pmatrix} Bel_k \\ Pl_k \end{pmatrix} = \sum_{j=1,\ldots,p} f_k(M_{\mathcal{B}_j}) \tag{D.15}$$

When the conditions on intervals bounds for a given $M_{\mathcal{B}_j}$ are verified for all c_k (see conditions stated in (D.14)) then the contribution of $M_{\mathcal{B}_j}$ on the belief and plausibility values for all c_k is the same with the merged interval as with the unmerged intervals.

The conditions on intervals bounds state whether a given $\mathcal{B}_j$ can be merged into a $M_{\mathcal{B}_j}$. But, practical application requires a merging rule to construct $\mathcal{B}_j$ from all the intervals in $\mathcal{A}$. Let us consider a base interval $I_1 = [l_1, u_1[$ and an interval $I_2 = [l_1, u_2[$ candidate for merging with I_1. For merging I_2 to I_1, one of the three intervals bounds conditions must be verified for all c_k which is equivalent to:

$$\forall k, c_k \in\,]-\infty, \min(l_1, l_2)] \cup\,]\max(l_1, l_2), \min(u_1, u_2)[\cup [\max(u_1, u_2), +\infty[\tag{D.16}$$

or

$$\forall k, c_k \notin\,]\min(l_1, l_2)], \max(l_1, l_2)]\ \cup\ [\min(u_1, u_2), \max(u_1, u_2)[\tag{D.17}$$

In other words, for merging I_1 and I_2, the two lower bounds must be between the same consecutive c_k values and the two upper bounds must be between the same consecutive c_k values. Any other candidate interval must verify the same merging rule. If an interval cannot be merged with any

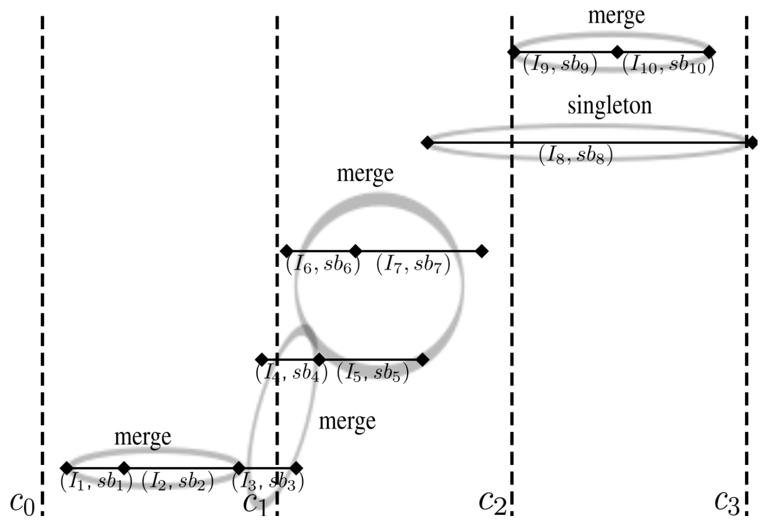


Figure D.5 Example of interval merging

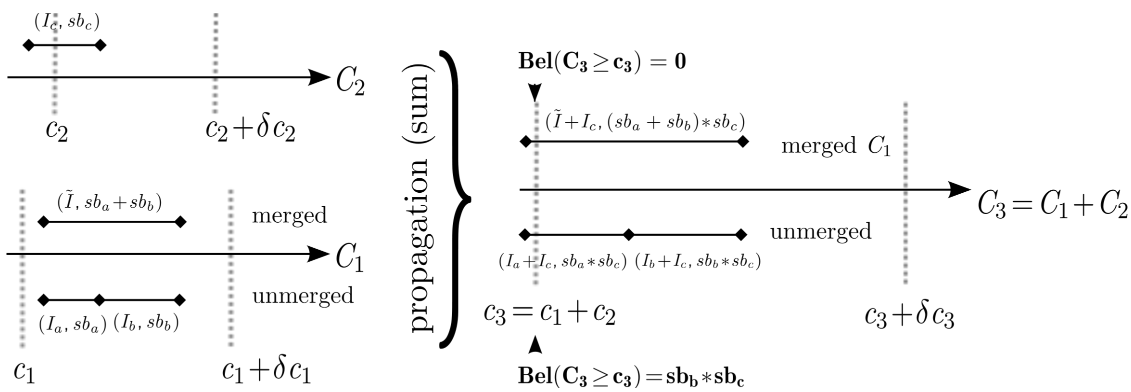


Figure D.6 Interval merging and propagation

existing $M_{\mathcal{B}_j}$ then it constitutes a new base interval for merging. The result of the merging process is illustrated by an example in Figure D.5.

D.4.2 Loss of uncertainty details during propagation with interval merging

In the multilevel framework the merging at a node can be systematically applied based on a specific characteristic increment δc after propagation from a lower level. A sequence of propagation and merging is repeated up to the top level which would keep the number of intervals manageable.

However, the loss of uncertainty information details inherent to the merging process can affect the accuracy of belief and plausibility obtained after propagation to the next level. It is important to

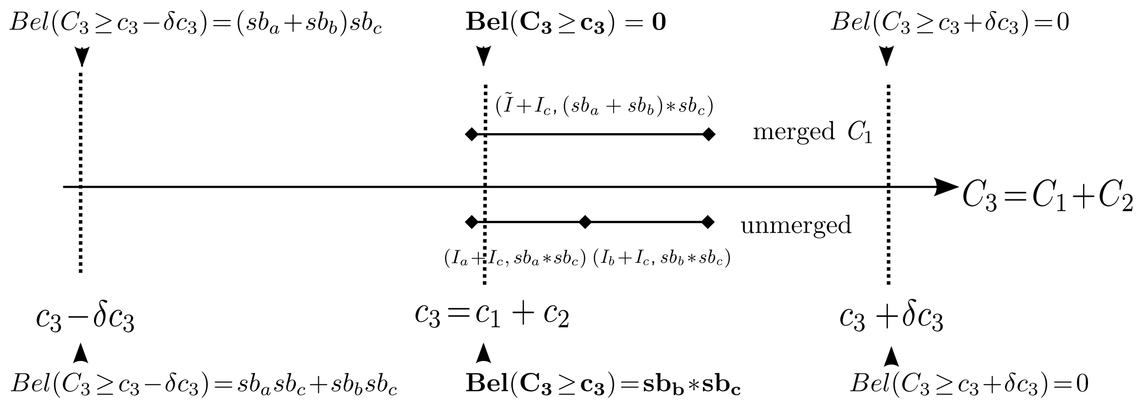


Figure D.7 Local modifications of belief values after merging and propagation

note that this can happen although we have shown that the belief and plausibility curves for all c_k are the same with or without merging before propagation (see a simple example on Figure D.6). The error introduced by the propagation after merging a series of intervals impacts the belief and plausibility curves only at certain characteristic values (see Figure D.7). In the simple example presented in Figures D.6 and D.7 it can be seen that it is the merging performed for the C_1 characteristic that alters the uncertainty information for the $C_3 = C_1 + C_2$ characteristic. Also, it can be seen that, based on the conditions previously set forth in subsection D.4.1 of this section, the propagated unmerged intervals for C_3 should not be merged at this level.

This observation could be generalized into a rule for flawless merging during propagation: at a given node, intervals should be merged only if the merging rule is verified for the current node and all subsequent propagations. Unfortunately, verifying this condition implies to propagate all intervals up to the vehicle level before deciding to merge which is against the objective of reducing the number of intervals during propagation. For this reason, keeping in mind the objective of reducing the number of intervals, we study in what follows the effect of propagation and merging on belief and plausibility curve.

D.4.3 Propagation procedure with interval merging

The propagation of uncertainties in a multilevel hierarchical model is dependent upon the functional relations between the characteristics at all levels. At the present stage of the project, we are

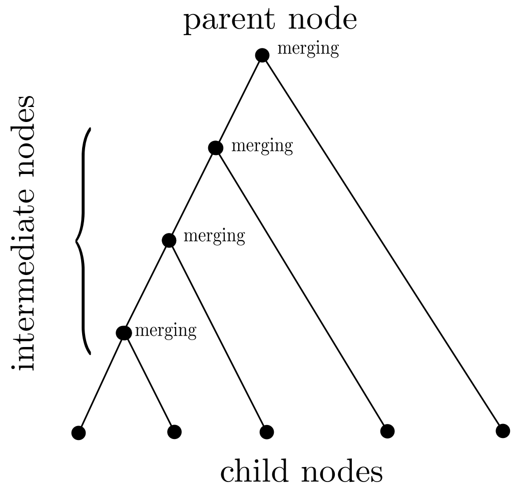


Figure D.8 Illustration of propagation with intermediate merging

investigating the propagation of uncertainties for a single characteristic with an additive property (more specifically the mass characteristic M) linking the child nodes with their parent node. Future work will investigate the case of characteristics linked with general functional relations.

We have considered two approaches for propagating uncertainties in a multilevel model with merging of intervals. The first one consists in propagating characteristic intervals from all the children nodes to their parent node, and then applying the merging once. The operation is repeated from leaf nodes to the top level. This level-by-level approach performs well when the number of intervals propagated from several child nodes to a parent node is not too large. However, in practical situations the number of child nodes and the number of intervals per node may be such that the propagated intervals become intractable for the merging process. For example, let us consider 12 children nodes with 10 intervals each. At the children node, 10 intervals may be reasonable to represent accurately the belief and plausibility curves, however, at the parent node this results in 10^{12} interval combinations to be handled for merging.

Hence, we have developed a 2-by-2 approach to prevent such problems. The idea is to propagate intervals only from two children nodes at a time before applying the merging. This results in an intermediate set of intervals to be propagated and merged with another child node. The procedure continues until all child nodes are propagated and merged to the parent node (see Figure D.8).

Because there are more merging and propagation steps in the second approach, it can be anticipated that the errors introduced by merging will be larger with the 2-by-2 approach than for the level-by-level approach. This aspect is investigated in the subsection D.4.5.

D.4.4 Meaningful characteristic increment

The merging is based upon a meaningful increment δc_n^* of the node characteristic, which can be determined based on the meaningful increment δc_v^* of the top level selected by a decision maker. The propagation from the leaf nodes to the top level node must be performed along the branches of the multilevel model through the functional relations between nodes characteristics.

This paper focuses on the mass characteristic which is of primary importance during the vehicle development process. Due to the additive property of the mass, the meaningful increments of mass at children nodes 1 to k are propagated to the parent node as $\delta m_1 + \ldots + \delta m_k$. The granularity of information obtained at the system level equals the sum of the δm of all leaf nodes. Knowing the desired granularity of information at top level δm_v^*, a correction factor for merging $\gamma = \delta m_v^* / \sum_{j=1,\ldots,k} \delta m_j$ is applied before propagation to all leaf nodes and subsequent parent nodes: $\delta m_n^* = \delta m_n \gamma$. A series of system δm_v^* have been considered to evaluate the impact on the error caused by merging and propagation: $\delta m_v^* = \{0.1, 1.0, 10\}$.

D.4.5 Application example

The simplified hypothetical vehicle multilevel model used to illustrate the merging process is presented in Figure D.9 along with the experts' opinions for the mass characteristic. The number of intervals and subjective beliefs provided by experts is small enough that we can propagate intervals to the vehicle level without merging, which makes it possible for us to compare the effect of the merging procedure on belief and plausibility curves. Without merging, the total is 50625 intervals propagated to the vehicle level.

The results obtained with $\delta m_v^* = 1.0$ for the 2-by-2 merging approach are presented in Figures D.10(a) and D.10(b). When interval merging was conducted, belief and plausibility were

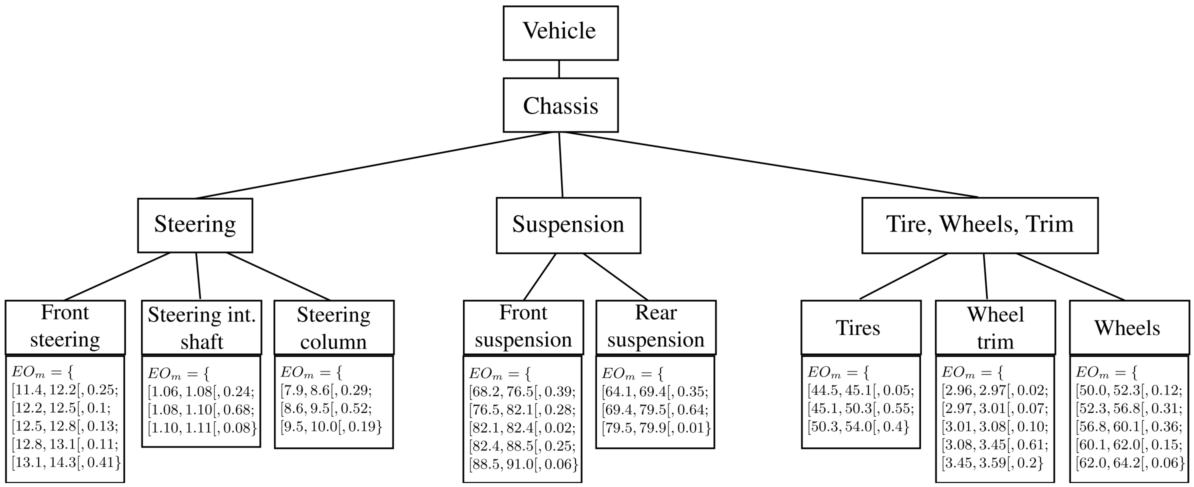


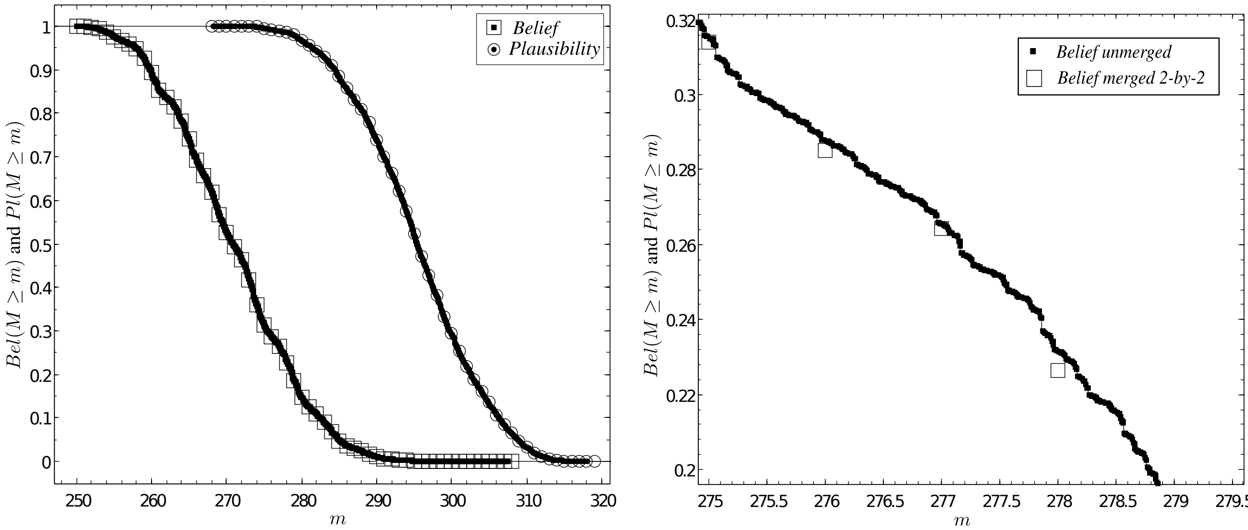
Figure D.9 Simplified vehicle multilevel model with mass characteristic experts' opinions

calculated at the discrete characteristic values based on δm_v^*. The comparison between belief and plausibility curves obtained with or without merging confirms that the 2-by-2 merging strategy preserves the shape of the curves. The number of intervals propagated to the vehicle level for the 2-by-2 merging procedure is 815, which is significantly reduced compared to the 50625 intervals without merging. However, small errors at the discrete characteristic values are introduced in the belief and plausibility curves due to the propagation and merging procedure (see Figure D.10(b)).

The results obtained with $\delta m_v^* = 1.0$ for the level-by-level merging approach are presented in Figures D.11(a) and D.11(b). The number of intervals propagated to the vehicle level is 841 compared with 50625 intervals without merging. When comparing Figures D.10(b) and D.11(b) it is clear that the errors on belief and plausibility values are smaller for the level-by-level method than for the 2-by-2 method. However, the errors are still present although not visible in Figure D.11(b).

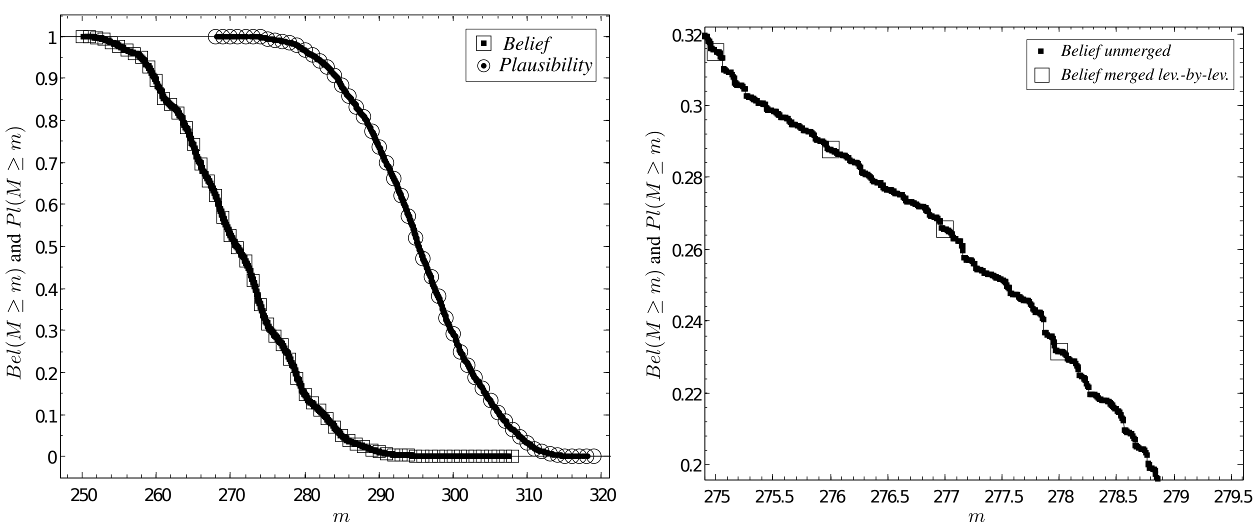
It is important to note that a sampling size of $\delta m_v^* = 1.0$ is able to capture very well the shape of the belief and plausibility while significantly reducing the number of intervals to be managed. To further explore the influence of δm_v on the belief and plausibility values two other δm_v^* have been considered.

The belief and plausibility values obtained by the two propagation and merging procedures with $\delta m_v^* = 0.1$ are very close to the unmerged values (see Figure D.12 for a detailed view of the belief curve with the 2-by-2 merging procedure). The errors introduced by the procedures are very small



(a) Overview (b) Details

Figure D.10 Belief and plausibility curves for $\delta \mathbf{m}_{\mathbf{v}}^{*} = \mathbf{1.0}$ with the 2-by-2 merging approach



(a) Overview (b) Details

Figure D.11 Belief and plausibility curves for $\delta \mathbf{m}_{\mathbf{v}}^{*} = \mathbf{1.0}$ with the level-by-level merging approach

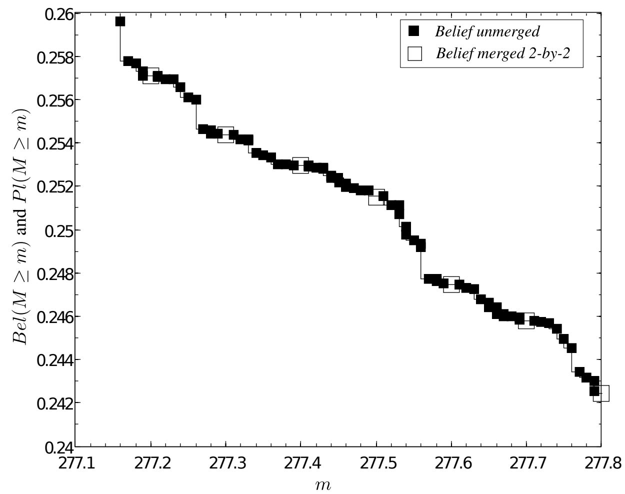


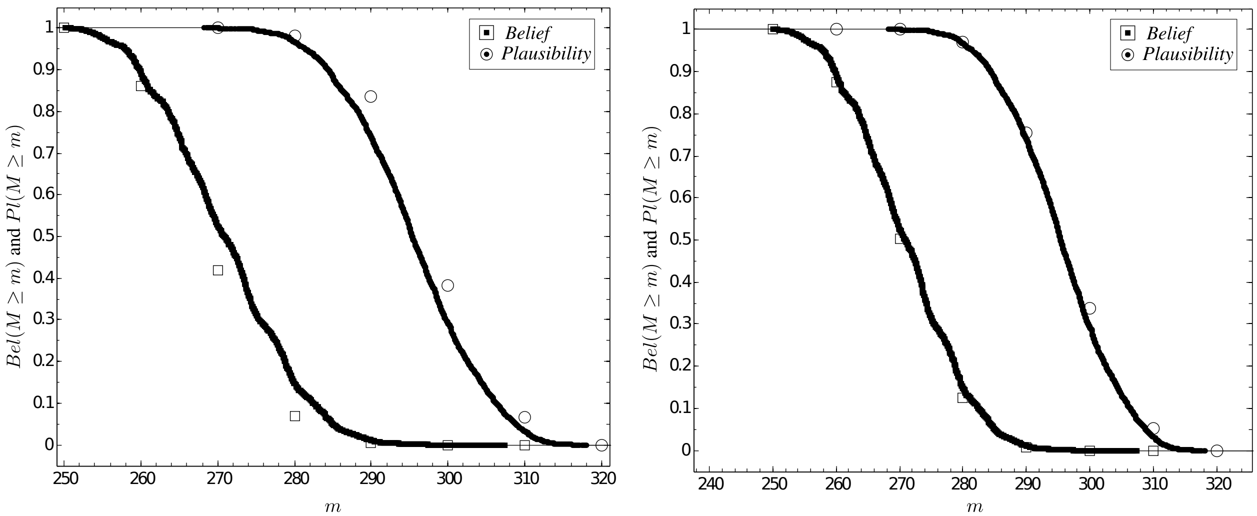
Figure D.12 Belief curve for $\delta \mathbf{m_v^*} = \mathbf{0.1}$ with the 2-by-2 merging procedure

but the number of intervals propagated is much higher: 14959 and 16558 respectively for the 2-by-2 and level-by-level procedures compared to 50525 without merging. On the contrary, with $\delta m_v^* = 10$, the number of intervals to be handled at the vehicle level is much smaller: 17 and 19 respectively for the 2-by-2 and level-by-level procedures. However, Figures D.13(a) and D.13(b) show that the errors introduced by the propagation and merging procedures are noticeable but the belief and plausibility values may still be usefull for decision making because the general shape is correctly captured.

D.5 Conclusion

This paper presents a merging procedure for uncertain experts' opinions given in a hierarchical multilevel model. The uncertainty information is provided as intervals and subjective beliefs which must be propagated through functional relations linking the nodes characteristics of the multilevel model. Propagating intervals towards the top level serves to determine belief and plausibility curves to characterize globally the uncertainties in experts' opinions. The uncertainty information can be used to reassign characteristic targets to guide the design toward a more desirable and achievable product.

Propagating intervals from children nodes characteristics to parent node characteristics requires the combination of all intervals through functionals which results in a possibly large number of



(a) 2-by-2 (b) level-by-level

Figure D.13 Belief and plausibility curves for $\delta \mathbf{m}_{\mathbf{v}}^{*} = \mathbf{10}$

intervals to be handled. A rough estimate presented herein, show that, when experts have sufficiently advanced into their work to explore different possible values of a characteristic, the number of intervals to be propagated becomes overwhelming.

In this context a propagation and merging procedure is proposed to reduce the number of intervals handled while keeping the accuracy of the belief and plausibility for a given discrete set of characteristic values. This paper demonstrates that, at a given node, the proposed merging procedure results in the same discrete belief and plausibility values as obtained from unmerged intervals. However, when the merging is followed by a propagation of the uncertainty information, we have demonstrated that errors are introduced into the subsequent belief and plausibility values.

A test case example has been used to illustrate that, even with errors introduced in the belief and plausibility values, the proposed merging procedure to reduce the number of propagated intervals is very efficient. Application to large examples in a decision making framework is ongoing.

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