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Auteur: Kaveh Rezvani Dehaghani
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Vulnerability Analysis of Public Transportation Systems

KAVEH REZVANI DEHAGHANI

Département des génies civil, géologique et des mines

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présentée par **Kaveh REZVANI DEHAGHANI**

en vue de l'obtention du diplôme de *Philosophiæ Doctor*

a été dûment acceptée par le jury d'examen constitué de :

Martin TRÉPANIÉ, président

Catherine MORENCY, membre et directrice de recherche

Francesco CIARI, membre

Oded CATS, membre externe

DEDICATION

To my beloved mother, Mansoureh, whose boundless love and sacrifice have been the cornerstone of my strength and perseverance.

To my father, Danial, whose wisdom and guidance have shaped the person I am today, always inspiring me to seek knowledge.

To my brothers, Roham and Sourena, for being my pillars of strength, making every step of this journey a little lighter.

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RÉSUMÉ

La rapide urbanisation et industrialisation des villes a entraîné une augmentation significative des volumes de déplacements urbains, intensifiant les demandes sur les systèmes de transport public, qui constituent une infrastructure urbaine essentielle. Les systèmes de transport en commun jouent un rôle crucial dans la mobilité quotidienne, en particulier pour les individus de divers milieux socio-économiques qui dépendent de ces services. Les perturbations dans les systèmes de transport en commun peuvent donc avoir des impacts sociaux étendus. Par ailleurs, des incidents tels que les accidents, les conditions météorologiques défavorables et les problèmes opérationnels provoquent fréquemment des interruptions temporaires ou prolongées des services de transport en commun. Par exemple, un examen des données historiques de perturbations dans le métro de Montréal des cinq dernières années montre que ce système subit plus de 7 400 interruptions annuellement de moins de 14 minutes, environ 100 interruptions de 15 à 29 minutes, et entre 70 et 75 interruptions par an de plus de 30 minutes. Ces statistiques soulignent l'importance d'étudier la vulnérabilité des systèmes de transport public.

Introduit en 2002 comme concept en ingénierie des transports, la vulnérabilité est définie comme une « susceptibilité aux incidents pouvant entraîner une réduction considérable de la serviceabilité du réseau routier » (Traduit de l'anglais). Depuis son introduction, des progrès substantiels ont été réalisés dans ce domaine, avec des chercheurs développant des méthodologies et indicateurs innovants. Les études ont évolué, passant de l'examen des seules caractéristiques topologiques des réseaux de transport à l'analyse des aspects fonctionnels, ce qui a permis d'améliorer la compréhension de la serviceabilité du réseau. L'introduction de concepts tels que la vulnérabilité dynamique et les défaillances en cascade a également perfectionné la précision des simulations de perturbations dans les réseaux de transport en commun. De plus, diverses mesures d'atténuation – notamment pour les interruptions dans le métro – ont été optimisées grâce à des techniques avancées d'optimisation pour réduire les effets des interruptions de moyen et long terme. Cependant, malgré ces avancées, il reste des lacunes importantes dans l'analyse de la vulnérabilité des systèmes de transport en commun. Cette thèse vise à combler certaines de ces lacunes à travers une série de quatre articles, chacun contribuant à une compréhension et évaluation plus approfondies de la vulnérabilité des systèmes de transport en commun.

La première étude examine la vulnérabilité topologique du réseau de métro de Montréal, en mettant particulièrement l'accent sur les limites des hypothèses trop simplificatrices souvent utilisées dans la simulation des perturbations. Elle remet en question la stratégie de défaillance fondée sur les nœuds (Node-Based Failure Strategy, NBFS), en soutenant que les modèles de défaillance qui négligent les caractéristiques infrastructurelles essentielles du réseau peuvent produire des résultats très éloignés de la réalité. Pour pallier cette lacune, l'étude introduit le concept de blocs, définis comme des séquences de stations consécutives délimitées par deux stations successives disposant d'installations permettant le changement de direction (Stations with Direction Change Facilities, SDCFs), éléments cruciaux pour garantir une flexibilité opérationnelle en cas de perturbation. S'appuyant sur ce cadre, l'étude propose une stratégie alternative plus réaliste : la stratégie de défaillance fondée sur les blocs (Block-Based Failure Strategy, BBFS). Les résultats montrent que, dans le cas du métro de Montréal, la BBFS reflète plus fidèlement la vulnérabilité réelle du réseau : selon cette approche, le système peut supporter jusqu'à 9 perturbations ciblées consécutives, contre 35 dans le cas de la NBFS. Ces résultats concordent étroitement avec les données empiriques fournies par l'exploitant du réseau de transport, renforçant ainsi la validité de l'approche proposée.

La deuxième étude vise à faire progresser les méthodes classiques d'évaluation de la vulnérabilité — à savoir les analyses basées sur la structure du réseau et la performance du service — en introduisant un cadre novateur fondé sur la redondance. En complément des approches traditionnelles, ce travail évalue l'impact des perturbations sur la qualité et la disponibilité des itinéraires alternatifs dans le réseau de métro, un aspect encore peu exploré dans la littérature. Une contribution majeure de l'étude réside dans le développement d'une approche fondée sur plusieurs indicateurs pour simuler les perturbations ciblées. Contrairement aux recherches antérieures qui s'appuyaient généralement sur un seul indicateur topologique (comme le degré ou l'intermédiarité), cette méthode combine simultanément sept indicateurs pour identifier les stations critiques, offrant ainsi une évaluation plus complète de la vulnérabilité. Dans le contexte des scénarios de défaillances aléatoires, l'étude révèle une grande variabilité des résultats, soulignant la nécessité d'adopter un processus itératif et de considérer plusieurs cas de perturbations aléatoires pour renforcer la robustesse des conclusions. Les résultats de cette analyse intégrée montrent que les approches conventionnelles comme celles fondées sur la redondance identifient de manière cohérente le réseau de métro de Montréal comme étant plus vulnérable aux perturbations ciblées. Cependant, certaines différences notables apparaissent : les méthodes classiques sont plus sensibles

au nombre de stations perturbées, tandis que l'approche fondée sur la redondance est davantage influencée par la localisation géographique des perturbations.

La troisième étude explore le rôle que peut jouer le réseau régulier de bus pour préserver la qualité du service de transport collectif lors de perturbations du métro. Alors que les recherches antérieures se sont principalement concentrées sur des mesures d'atténuation telles que la réaffectation du matériel roulant ou la mise en place de services de navettes dédiés — des solutions souvent coûteuses en temps et plus adaptées aux interruptions prolongées —, cette étude met en avant le rôle immédiat et spontané du réseau de bus existant, qui constitue généralement la première alternative empruntée par les usagers en cas de perturbation inattendue. Les résultats montrent que, même en présence d'une interruption majeure du métro, la majorité des déplacements restent réalisables au sein du système de transport collectif, bien que les temps de trajet soient allongés. Par ailleurs, l'augmentation du temps de déplacement liée au remplacement du métro par les bus réguliers est à la fois quantifiable et corrélée à l'ampleur de la perturbation. Fait intéressant, dans le contexte montréalais, le moment de la perturbation — qu'elle survienne pendant les heures de pointe du matin, du soir ou en dehors des périodes de pointe — n'a pas d'impact significatif sur la capacité du réseau de bus à absorber le choc et à maintenir la continuité du service.

La quatrième étude propose un cadre novateur pour évaluer et comparer la vulnérabilité des usagers face aux perturbations du métro en intégrant des caractéristiques sociodémographiques. Cette approche segmente la population en dix groupes distincts selon l'âge, le genre, l'occupation principale et le motif du déplacement. En analysant l'augmentation du temps de trajet causée par le recours au réseau régulier de bus lors des interruptions du métro, l'étude mesure la vulnérabilité relative de chaque segment. Les résultats, issus du cas de Montréal, indiquent que les enfants âgés de 5 à 15 ans constituent le groupe le moins vulnérable, tandis que les hommes actifs de 25 à 34 ans sont les plus affectés. Il est important de noter que l'analyse adopte une perspective relative, comparant les niveaux de vulnérabilité entre groupes sans établir de classement absolu. Cette approche permet une compréhension plus fine et différenciée des impacts des perturbations selon les profils d'usagers.

ABSTRACT

The rapid urbanization and industrialization of cities have led to a significant increase in urban travel volumes, intensifying demands on public transportation systems, which serve as essential urban infrastructure. Public transit systems play a crucial role in daily mobility, particularly for individuals from diverse socio-economic backgrounds who rely on these services. Disruptions to transit systems, therefore, can have widespread societal impacts. Meanwhile, incidents such as accidents, adverse weather conditions, and operational issues frequently cause temporary or prolonged disruptions to transit services. An examination of a five-year disruption history in Montreal's metro system, for instance, shows that this system experiences over 7,400 annual disruptions of up to 14 minutes, around 100 disruptions lasting between 15 and 29 minutes, and between 70 to 75 disruptions each year that extend beyond 30 minutes. These statistics underscore the importance of studying the vulnerability of public transportation systems.

Vulnerability, introduced as a concept in transportation engineering in 2002, is defined as “a susceptibility to incidents that can result in considerable reductions in road network serviceability.” Since its introduction, substantial progress has been made in this field, with researchers developing innovative methodologies and indicators. Studies have evolved from focusing solely on the topological features of transportation networks to examining functional aspects, enhancing the understanding of network serviceability. The introduction of concepts like dynamic vulnerability and cascading failures has further advanced the precision of disruption simulations in transit networks. Additionally, various mitigation measures—particularly for metro disruptions—have been optimized through advanced optimization techniques to alleviate the effects of medium- and long-term disruptions. However, despite these advancements, there remain significant research gaps in vulnerability analysis of public transportation systems. This dissertation seeks to address some of these gaps through a series of four articles, each contributing to a deeper understanding and assessment of transit system vulnerability.

The first article investigates the topological vulnerability of the Montreal subway system, with a particular focus on addressing the limitations of overly simplified disruption simulation assumptions. Specifically, it critiques the commonly used Node-Based Failure Strategy (NBFS), arguing that failure models which overlook key infrastructural characteristics of the network may yield results that significantly diverge from real-world outcomes. To address this issue, the study

introduces the concept of blocks, defined as sequences of consecutive stations situated between two consecutive Stations with Direction Change Facilities (SDCFs), which are critical for operational flexibility during service disruptions. Building upon this framework the Block-Based Failure Strategy (BBFS) as a more realistic alternative to NBFS is introduced. The findings demonstrate that, in the case of the Montreal subway system, the BBFS more accurately reflects the network's actual vulnerability: under BBFS, the system can withstand up to 9 consecutive targeted disruptions, compared to 35 disruptions under the NBFS. These results align closely with empirical data provided by the transit service operator, thereby reinforcing the validity of the proposed approach.

The second article aims to advance conventional vulnerability assessment methods—namely, network-based and serviceability-based analyses—by introducing a novel redundancy-based framework. In addition to applying traditional approaches, this work evaluates the impact of disruptions on the quality and availability of alternative routes within the metro network—an aspect that has received limited attention in prior research. A key contribution of the study is the development of a multi-indicator approach for simulating targeted disruptions. Unlike earlier studies that typically rely on a single topological indicator (such as degree or betweenness centrality), this method integrates seven indicators simultaneously to identify critical stations, thereby providing a more comprehensive assessment of vulnerability. In the context of random failure scenarios, the study reveals a wide variability in outcomes, highlighting the necessity of adopting an iterative process and considering multiple random failure cases to improve result robustness. The findings from this integrated vulnerability analysis suggest that both conventional and redundancy-based approaches consistently identify the Montreal metro system as being more susceptible to targeted disruptions. However, notable distinctions emerge: while conventional methods tend to exhibit greater sensitivity to the number of disrupted stations, the redundancy-based approach is more responsive to the spatial location of these disruptions.

The third article explores the role of the regular bus system in maintaining transit service quality during metro disruptions. While previous research has primarily focused on mitigation strategies such as rolling stock reallocation or dedicated shuttle services, these measures often require significant planning and are better suited for long-duration disruptions. In contrast, this study emphasizes the immediate and spontaneous role of the existing bus network, which is typically the first alternative accessed by commuters during unexpected disruptions. The findings indicate that,

even in the event of major metro service interruptions, the majority of trips remain feasible within the transit system, albeit with increased travel times. Furthermore, the additional travel time incurred by substituting metro segments with regular bus services is shown to be both quantifiable and positively correlated with the extent of the disruption. Interestingly, in the case of Montreal, the occurrence time of the disruption—whether it occurs during the morning peak, evening peak, or off-peak hours—does not significantly affect the bus system’s ability to absorb the shock and maintain service continuity.

The fourth article proposes a novel framework for assessing and comparing user vulnerability to metro disruptions by incorporating socio-demographic characteristics. The approach segments the population into ten distinct user groups based on age, gender, occupation, and trip purpose. By analyzing the increase in travel time resulting from reliance on the regular bus network during metro disruptions, the study evaluates the relative vulnerability of these segments. The findings, based on the Montreal case study, indicate that children aged 5 to 15 constitute the least vulnerable group, while male workers aged 25 to 34 are the most affected. Importantly, the analysis adopts a relative perspective, comparing vulnerability levels across segments without assigning absolute vulnerability classifications. This approach facilitates a more nuanced understanding of how disruption impacts vary among different user profiles.

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LIST OF SYMBOLS AND ABBREVIATIONS

ATL	Average Travel Length
ATS	Average Travel Speed
ATT	Average Travel Time
BBFS	Block-Based Failure Strategy
CDNAN	Corresponding Disruption Number to the Annihilation of the Network
CSA	Connection Scan Algorithm
EoD	Extent of Disruption
GTFS	General Transit Feed Specification
MADM	Multi-Attribute Decision-Making
MSE	Mean Squared Error
MTS	Multimodal Transportation system
NBFS	Node-Based Failure Strategy
ND	Network Diameter
NE	Network Efficiency
NRI	Network Robustness Index
OD	Origin-Destination
ORFS	Optimistic Random Failure Scenarios
PC	Pure Connectivity
PRFS	Pessimistic Random Failure Scenarios
PT	Public Transportation
PTN	Public Transportation Network
PTS	Public Transportation System
RC	Relative Connectivity
RH	Ride-Hailing
RRFS	Realistic Random Failure Scenarios
RS GC	Relative Size of the Giant Component
RTL	Réseau de transport de Longueuil
SDCF	Station with Direction Change Facilities
SST	Substandard Trips
STL	Société de transport de Laval
STM	Société de transport de Montréal
TFS	Targeted Failure Scenario
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
TRI	Transit Robustness Index
TUSRBD	Trips Utilizing the Same Route as Before the Disruption
UMN	Urban Metro Network

CHAPTER 1 INTRODUCTION

This chapter provides an overview of the research, outlining its key components and focus areas. It begins by setting the stage for the study, discussing the significance of public transportation systems and the impact of disruptions. Following this, the evolution of vulnerability concepts in transportation research is reviewed, providing insights into existing methodologies and their relevance. The discussion then highlights current challenges and research gaps in vulnerability analysis, underscoring the need for innovative approaches. The chapter also outlines the objectives and novel contributions of this research, which include the development of advanced methods for analyzing system vulnerability and user resilience. Lastly, an overview of the structure of the thesis is presented, offering a concise summary of the topics covered in the subsequent chapters.

1.1 Context

The rapid expansion of urbanization and industrialization has fueled the transformation of cities in recent years, prominently impacting the rise in urban travel. Statistical analysis underscores that the growth rate of urban trips within cities notably outpaces the growth rate of their populations. For example, in Montreal, data from 1982 to 2016 demonstrates a substantial divergence between population growth and the escalation of urban travel (Ville de Montréal). Consequently, urban transportation systems represent vital infrastructure in contemporary society.

Public transportation plays a pivotal role in urban mobility, often hailed as a solution to mitigate the impacts of population growth and increased urban trips, particularly in large cities. Despite the setback caused by the COVID pandemic on public transportation usage in recent years, statistics reveal that in September 2023, approximately 135 million trips were made using public transportation networks in Canada (Statistic Canada), underscoring its enduring importance. Furthermore, historical analysis and comparison of public transportation ridership suggest that this figure is poised to increase in the coming years. Consequently, a significant segment of society relies on this transportation system for their daily activities, highlighting its critical role in enhancing people's quality of life. Nonetheless, this dependency also entails inherent risks: any disruption to the system, whether due to incidents, accidents, or events, can have profound

repercussions affecting various societal classes, placing a substantial burden on both operators and users alike.

The analysis of transportation system vulnerability, focusing on the causes of disruptions within these systems and their impact on network performance and users' travel patterns, represents a relatively new concept in transportation engineering. There is no universally accepted definition for this concept; however, its emergence as a significant research area can be attributed to the work of (Berdica, 2002). Berdica defined transportation network vulnerability as *"a susceptibility to incidents that can lead to significant reductions in road network serviceability."* In subsequent years, this definition has been elaborated upon and broadened by numerous scholars, leading to the introduction of various approaches for evaluating the vulnerability of different transportation systems, including public transportation networks.

Urban transportation networks, particularly public transportation systems, are more vulnerable to damage and threats compared to road networks. This heightened susceptibility is due to the lower flexibility and redundancy inherent in public transit networks when compared to other urban transportation modes. Consequently, disruptions in transit networks are more likely to occur than in other urban transportation systems. Given the fundamental role that public transportation plays in people's daily lives, disruptions within these networks can have significant social and economic repercussions. These impacts may be more pronounced than disruptions occurring in other urban transportation modes, highlighting the critical importance of ensuring the resilience and reliability of public transit systems.

Unplanned rail disruptions are a prevalent and impactful issue in major cities, particularly affecting subway systems. According to data from the City of Montreal, the Montreal metro system operated by Société de transport de Montréal (STM) experienced a total of 769 disruptions lasting between 10 to 30 minutes between 2019 and 2022 (with 208 disruptions in 2019, 176 in 2020, 187 in 2021, and 210 in 2022) (ouvertes). Similarly, in 2015, the Toronto Transit Commission (TTC) recorded 144 disruptions within its subway system, as documented in subway disruption data (Itani & Shalaby, 2021). Moreover, the New York City Comptroller (2017) conducted a comprehensive analysis that projected significant economic impacts resulting from subway delays. Based on ridership, delay durations, and an average hourly wage of \$34, the analysis estimated economic losses for the city in 2016 to range from \$170 million to \$389 million (Stringer, 2017). Notably,

these substantial costs were solely attributed to subway delays. However, transit systems encompass various modes beyond just subways, including bus rapid transit (BRT), regular buses, light rail transit (LRT), and trains. If the economic costs of delays across the entire transit system had been considered by the New York City Comptroller, the projected figures could have been significantly higher. This underscores the broader economic implications associated with transit disruptions beyond subway delays alone.

Given the increasing use of public transportation systems and their high susceptibility to disruptions, as well as the severe economic and social consequences resulting from disruptions in public transportation systems, extensive research has been conducted in recent decades on the vulnerability of public transportation worldwide which has transformed this subject into a nearly independent domain within transportation engineering studies. The conducted studies have presented diverse methods and indicators for evaluating the vulnerability of public transportation networks, with some demonstrating notable effectiveness. However, due to the extensive characteristics and varied modes of public transportation systems, as well as the complexities inherent in network modeling and understanding supply-demand dynamics within these systems, there remains considerable scope for further research and discussion within the field of transportation engineering.

1.2 Background

The concept of "vulnerability" originated in disaster literature in the early 1970s and has since been adopted and expanded across various sectors. Following the 1995 Kobe earthquake, vulnerability assessments began to include road networks in the late 1990s (Lu & Peng, 2011). Minor incidents, as noted by (Abrahamsson, 1997), can amplify the vulnerability of road networks, leading to significant disruptions. For instance, if a minor incident occurs at a critical time or location, the resulting impact can escalate, affecting the surrounding area and potentially causing a network-wide failure (Lu & Peng, 2011).

As previously mentioned, the concept of transportation vulnerability, as currently understood, was introduced by (Berdica, 2002). Shortly thereafter, (Crucitti, Latora, Marchiori, & Rapisarda, 2003) applied this concept to assess the vulnerability of urban rail transit systems, marking one of the earliest studies to investigate the vulnerability of a public transportation system in research. Early studies primarily focused on static network topology, aiming to identify key network nodes or links

where failures would have the most significant and far-reaching impacts. This line of research, which remains widely popular, often utilizes indicators such as node degree, betweenness centrality, and global efficiency. Subsequently, serviceability-based methods were introduced, which consider the transport-related characteristics of the network, including link and node capacities, link lengths, and travel time attributes. These methods combine network topology with operational aspects to assess the potential impacts of network failures on traffic operations and the satisfaction of travel demand. In addition to serviceability-based methods, accessibility-based methods emerged as another approach to transportation vulnerability analysis, although they are less compatible with public transportation systems. These methods focus on broader travel demand considerations and examine the ability of a region (and specific segments within that region) to maintain routine economic and social activities amid transportation network disruptions. A detailed technical review of the literature reveals that the first category of vulnerability analysis, namely topological analysis, has been extensively utilized in research on public transportation network vulnerability (J. Lin & Ban, 2013). Mattsson & Jenelius, (2015) suggest that the primary reason for this preference is the reliance of serviceability and demand-based methodologies on sophisticated simulations which are primarily designed for road infrastructure but are not readily available or applicable to public transportation systems.

Another significant concept in this field, pioneered by (Scott, Novak, Aultman-Hall, & Guo, 2006), is the robustness analysis of transportation networks, which has conceptual similarities with vulnerability but offers a reverse approach to addressing vulnerability problems. Originally utilized in computer science and subsequently integrated into complex network science, robustness analysis has emerged within transportation engineering to complement the concept of vulnerability. These two concepts exhibit strong similarities, with robustness analysis essentially representing a form of vulnerability analysis applied in reverse. Nevertheless, concepts like network resilience, which examine the recovery of a network to its normal state following a disruption, differ conceptually from vulnerability and robustness, despite their apparent relevance. It is important not to conflate these concepts, as they are distinct and fall outside the scope of this research.

The process of conducting vulnerability and robustness analyses in public transportation networks involves simulating disruptions within the network (Massobrio & Cats, 2024). However, in this procedure, two critical considerations must be addressed: 1) determining how the location and/or timing of the disruption is selected, and 2) applying an appropriate failure strategy showing how

disruptions affect the network. Disruption locations in transportation networks can be deliberately chosen to simulate worst-case scenarios or specific situations (see (Candelieri, Galuzzi, Giordani, & Archetti, 2019); (Lin Zhang, Cheng, Wen, Lei, & Li, 2022)), or selected randomly to model unpredictability. Nonetheless, a literature review exposes shortcomings in both methods of location selection, often overlooking key details and highlighting areas for improvement. When selecting a failure strategy within the context of L-space transit modeling, which represents the transit network as a graph where each station is a node and links between nodes indicate consecutive service routes between stations, two options are available: node-based and link-based strategies. In the node-based strategy, disruptions are assumed to occur at nodes (stations), leading to the removal of the node along with all connected links from the network. Conversely, in the link-based strategy, disruptions are assumed to occur on specific links (connections between stations), resulting in the removal of only the affected link from the network. However, these failure strategies may not be well-suited for simulating disruptions in metro systems, integral components of urban transit networks in major cities. Therefore, emphasizing the critical role of disruption simulation in vulnerability analysis for transit systems highlights the need for further refinement. Enhancing the accuracy of the simulation is essential to better align vulnerability assessments with real-world scenarios, underscoring the importance of ongoing research in this area.

Although significant advancements have been made in both topological-based and serviceability-based methodologies over the years, especially with the adoption of dynamic vulnerability analysis that incorporates time-related considerations (see (Zhong et al., 2020); (Pan, Ling, Jia, Liu, & He, 2024); (Yichi, Hanping, Haoyue, & Jinfan, 2024)), these methodologies still exhibit certain limitations. Specifically, they do not adequately address network redundancy and the availability of alternate routes for users during disruptions, crucial factors impacting travel quality. Hence, in addition to ongoing efforts to enhance existing methods, there is a recognized need to introduce a new method that can effectively address the current limitations and bridge existing gaps in vulnerability analysis of transit systems.

While network resilience analysis is considered distinct from vulnerability analysis, a crucial aspect within vulnerability analysis involves studying mitigation measures during disruptions and evaluating their effectiveness in minimizing disruption impacts. This area represents one of the most critical components of vulnerability analysis and has been the subject of extensive research. Based on the reviewed literature, mitigation measures are typically categorized into internal and

external measures (S. Zhang, Lo, Ng, & Chen, 2021). Internal measures pertain to actions that can be implemented within the disrupted transportation mode to alleviate the negative impacts of disruptions, such as timetable adjustments and modifications to rolling stock. External measures, on the other hand, involve actions that rely on assistance from other modes of transportation or utilizing reserve systems to mitigate disruption effects. Some research focused on minimizing the rolling stock rescheduling costs (Veelenturf, Kroon, & Maróti, 2017), while others aimed to assess the inconvenience experienced by passengers during disruptions (Durand, Van Oort, & Hoogendoorn, 2018). Additionally, following the introduction of the term "bus bridging" by (Kepaptsoglou & Karlaftis, 2009), many studies have concentrated on optimizing bus substitute services, like (Kang et al., 2019) aimed to maximize connections for passengers transferring from rail to bus while minimizing waiting times. However, tasks such as mobilizing buses from different lines for shuttle services, providing drivers with route information, and notifying them of schedule changes and shift adjustments can be quite time-consuming. These activities may not be practically feasible for addressing short-term disruptions, which statistical studies suggest occur more frequently than other types of disruptions. Therefore, there remains an opportunity to further explore this topic, particularly regarding short-term disruptions, which have not yet been precisely defined in the existing literature within the field of transportation engineering.

1.3 Motivation

Analyzing the vulnerability of public transportation systems poses one of the most complex and challenging problems in transportation vulnerability analysis. Challenges include the intricate process of collecting supply data from various operators, the complexity involved in modeling multi-layered public transportation networks, and, notably, the difficulty in defining disruption scenarios and accurately simulating them.

The initial phase of this study focused on enhancing disruption simulation within transit networks, comprising three distinct steps. The first step involved the development of a novel failure scenario specifically tailored for urban rail transit systems. Given that not all stations in metro networks have facilities for changing direction, conventional failure strategies (node-based or link-based) represent a significant oversimplification. Consequently, an advanced failure strategy termed Block-Based Failure Strategy (BBFS) was introduced, building upon previous approaches. In BBFS, stations situated between two consecutive stations with direction change facilities are

defined as blocks. Under this strategy, disruption of a regular station results in the removal of the entire block to which it belongs from the network. Conversely, disruption of a station with direction facilities necessitates the removal of all blocks connected to this station from the network.

As previously discussed, disruptions can be simulated randomly or purposefully. When purposefully simulating a disruption in a specific location—referred to by different names such as targeted disruption, deliberate attack, malicious attack, etc., in various research—the disruption location(s) are typically selected based on their importance. In node-based or station-based (in L-space transit network modeling), while factors such as boarding/alighting volumes of stations have been used to identify the most critical station(s), strategic importance has generally been prioritized in most research studies. In research that prioritizes station importance based on location, commonly one of the criteria considered is node degree, betweenness, or closeness to identify the most critical station. However, it has been observed that changing the criterion from node degree to betweenness can lead to a different selection of important nodes. We advocate for a comprehensive approach that considers not only these traditional indicators (degree, betweenness, and closeness) simultaneously but also explores additional indicators and algorithms from other disciplines, such as computer or electronic sciences, to identify the most critical nodes in transit graphs. Therefore, one of the primary aims in the initial phase of this research was to introduce new indicators for identifying the most important stations in transit networks and to develop a general framework for utilizing all these indicators in simulations.

In the last step of the first phase of this research, we aimed to address another significant oversimplification assumption related to simulating random disruptions. Our literature review revealed that a common method for simulating random failure scenarios involves using random numbers to select stations for removal. However, this approach can yield outcomes that diverge significantly from realistic scenarios, such as selecting consecutive nodes, terminals, transfer stations, or other specific types. To mitigate this effect, we propose iteratively simulating scenarios multiple times and comparing the results to assess their consistency. The initial simulation may yield a rare scenario that is unlikely to occur in practice, potentially leading to unrealistic vulnerability assessments. Therefore, conducting multiple iterations of random disruption simulations or defining optimistic and pessimistic scenarios can provide a more accurate evaluation. We emphasize the importance of dedicating adequate attention to simulating random

disruptions alongside targeted disruptions, as neglecting this aspect may skew the overall assessment of network vulnerability.

A review of the existing literature highlighted that while current methodologies for assessing vulnerability in public transportation systems have been well-developed, certain critical issues remain unaddressed by these approaches. Topological analysis, the most commonly employed method for transit system vulnerability assessment, focuses on the network's structural characteristics to identify vulnerable segments. Dynamic vulnerability analysis has advanced this approach by considering variations throughout the day. Additionally, advanced methodologies in serviceability analysis evaluate disruption impacts on travel time, network capacity, and demand satisfaction. However, these methods fall short in assessing the impact of disruptions on users' route alternatives. Route diversity and reliable alternatives play a crucial role in reassuring public transportation users and encouraging their continued use of the system. However, frequent disruptions, particularly in metro networks, can erode this confidence if users perceive a lack of dependable route alternatives (a "plan B"). The availability of route alternatives is itself a measure of travel quality. In the second phase of this research, we introduce a methodology that not only assesses the impact of disruptions on trip feasibility but also investigates their effects on the quantity and quality of available trip alternatives.

Subway systems, as pivotal components of metropolitan transit networks, frequently experience short-term disruptions. Despite extensive research on mitigation measures for these disruptions, many of these measures are time-consuming and may not be implemented or evaluated before disruptions are resolved. Consequently, existing measures are often ineffective for managing short-term disruptions in metro systems. During such disruptions, the burden of mitigating impacts falls primarily on regular public transport services, particularly bus systems. Therefore, the third final phase of this research focuses on investigating the role of bus systems in mitigating disruption impacts within metro systems. This phase aims to assess how effectively and with what quality bus systems can manage the different types of disruptions within metro networks.

To date, numerous researchers have concentrated on developing new indicators and refining existing methodologies to assess network robustness and performance under various disruption scenarios. However, these studies have largely overlooked the comparative robustness of different user groups within public transportation systems. Conducting such an analysis could provide

valuable insights for transportation planners, allowing them to identify which population segments are more resilient to transit disruptions and to design targeted strategies for less robust groups. Therefore, the final phase of this research is focused on ranking transit system users based on their robustness to disruptions.

1.4 Objective and contribution

The primary aim of this thesis is to introduce a comprehensive, innovative, and adaptable framework for assessing the vulnerability of public transportation networks. Given that this topic represents a relatively new concept in transportation engineering, there are still research gaps in this area. Therefore, this thesis also seeks to identify current scientific gaps and propose corresponding solutions. To achieve this primary objective, specific goals and contributions have been defined for this thesis, which are summarized as follows.

1.4.1 To improve disruption simulation process

Objective: The vulnerability assessment of transportation systems relies heavily on disturbance simulation, where simulation assumptions critically influence the outcomes. Certain research in this field necessitates simplifying assumptions, which can compromise the integrity of the study and impede accurate judgments and conclusions. This research aims to identify and address specific simplifying assumptions to enhance the reliability and validity of public transportation vulnerability assessments.

Contribution: The primary contribution in this context involves the introduction of the Block-Based Failure Strategy (BBFS). Implementing BBFS enhances the accuracy of disruption simulations in metro systems and prompts a critical reevaluation of prior research, particularly studies focusing on cascading disruptions. Furthermore, the emergence of BBFS during metro network design provides valuable insights to network planners for strategically locating stations with directional facilities. Another significant contribution of this thesis is the proposal of a novel approach for simulating random disruptions, aimed at identifying rare scenarios without prioritizing them as primary random failure scenarios. Additionally, this research presents new indicators and utilizes Multi-Attribute Decision Making (MADM) methods to identify critical stations, enhancing the precision of targeted disruption simulations.

1.4.2 To introduce a novel vulnerability analysis method

Objective: Previous methodologies used in vulnerability assessment of public transportation systems did not adequately evaluate the impact of disruptions on users' route diversity. This research aims to enhance existing methodologies and introduce a novel approach to assess how disruptions affect the availability of route alternatives for users, thereby indicating the impact of disruptions on trip quality from this particular perspective.

Contribution: The main contribution of this research was the introduction of the "redundancy-based vulnerability method," which utilized novel indicators to evaluate transportation network vulnerability from a new perspective. This method demonstrated that the geometric configuration (shape) of the network significantly influences transit network vulnerability, underscoring the importance of considering this factor during the design phase. Additionally, the research introduced novel connectivity-related indicators aimed at enhancing topological vulnerability assessment methodologies.

1.4.3 To assess the quality of alternative transit service during metro disruptions.

Objective: Metro networks are prone to more frequent disruptions compared to other transit modes, leading to extensive research on measures to mitigate their negative impacts. However, there is no specific definition or threshold to distinguish short-duration from medium- or long-duration disruptions. As a result, past researchers have often neglected to mention the duration of disruptions. Instead, they focused on actions that may not be suitable for short-duration disruptions. Statistics indicate that most disruptions in metro networks are relatively short-lived. Therefore, this research aims to assess the ability and quality of regular transit services to moderate the effects of short-term disruptions (or pre-mitigation measures for medium-duration disruptions) in metro networks, particularly during the period before implementing mitigation measures for disruptions of medium duration.

Contribution: The main contribution of this research is the presentation of a five-step framework designed to assess the effectiveness of one or multiple transit modes in mitigating disruptions within metro systems. Another key contribution is the development of an approach to investigate how the timing of disruptions affects the effectiveness and quality of mitigation measures. Additionally, this research introduces a system to evaluate the quality of alternative routes provided by transit systems to users.

1.4.4 To rank the vulnerability of users to disruptions in transit systems

Objective: Disruptions in transit systems compel some users to alter their routes or modes of transportation, often resulting in increased travel time. However, the question of which user groups exhibit greater vulnerability to such disruptions remains a critical yet underexplored area of study. This research aims to classify users based on their socio-demographic and travel characteristics and subsequently rank them according to their robustness to transit system disruptions.

Contribution: The primary contribution of this research is the development of a four-step adaptable framework for ranking transit users based on their vulnerability to subway disruptions. This research introduces a novel metric, the Transit Robustness Index (TRI), which quantifies the proportion of robust trips within each population segment by applying different thresholds. Robust trips are defined as those for which disruptions in the public transportation system have minimal impact on their feasibility and travel time.

1.5 Thesis Structure

This thesis is structured into several chapters. Chapter 1 introduces the research context, emphasizing its significance and the necessity for this investigation. Additionally, four research objectives are formulated to address the identified problems and related challenges. Chapter 2 reviews the relevant literature, organized according to the four specified objectives, and identifies the key research gaps. Chapter 3 details the case study, focusing on the Montreal transit system, presents the data utilized, and provides supplementary information on the methodologies employed in this research. Chapters 4 through 7 constitute the core contributions of this dissertation, presented as four distinct articles, as outlined below:

- **Article 1**, titled "*A New Failure Strategy to Improve the Vulnerability Assessment of Urban Metro Networks*," was initially presented in July 2023 at the WCTR Conference. A revised version was subsequently published in *Transportation Research Procedia* in January 2025.
- **Article 2**, titled "*Beyond Traditional Metrics: Redefining Urban Metro Network Vulnerability with Redundancy Assessment*," was first presented in January 2024 at the TRB Conference. The initial version was submitted in July 2024 to *Physica A: Statistical Mechanics and Its Applications* and is currently undergoing the second round of revision.

- **Article 3**, titled "*Are Conventional Bus Systems Reducing the Impact of Metro Disruptions? The Case of Montreal*," was initially presented in May 2024 at the CTRF Conference. A revised version was submitted to the *Public Transport* journal in January 2025 and is currently under review.
- **Article 4**, titled "*Assessing Population Segments' Vulnerability to Metro Disruptions: Introducing the Transit Robustness Index (TRI)*," was presented at the Polytechnique Chair Mobility Conference in June 2024. A revised version was submitted to the *Transport Policy Journal* in February 2025.

Chapter 8 synthesizes the key findings and discusses their broader implications. Lastly, Chapter 9 concludes the thesis, outlining the contributions and limitations of each article and offering recommendations for future research directions.

CHAPTER 2 LITERATURE REVIEW

In this chapter, the relevant research on the vulnerability and robustness analysis of public transportation systems is examined. The discussion begins by tracing the history and conceptual development of vulnerability and robustness within the context of transportation engineering, particularly focusing on public transportation. Subsequently, the methodologies employed for evaluating the vulnerability and robustness of transportation systems are reviewed. The chapter then shifts to an in-depth examination of studies focused on the vulnerability of various transportation networks, with an emphasis on public transportation systems, highlighting relevant network analysis approaches and the indicators used in these studies. Following this, research on mitigation strategies during metro disruptions is explored. Attention is then directed towards studies that investigate public transportation systems at a micro-level, particularly those that categorize users based on their socio-demographic characteristics. Finally, research gaps in the field are identified and discussed.

2.1 History and definition

2.1.1 Vulnerability

The conceptualization of transportation network vulnerability is credited to Berdica, (2002) and D'este & Taylor, (2003). Berdica defined vulnerability as *"a susceptibility to incidents that can cause considerable reductions in road network serviceability,"* with serviceability referring to the ability to utilize a link, route, or road network within a specific time frame (Berdica, 2002). This comprehensive vulnerability concept remains widely applied, as evidenced by subsequent studies (instance Berdica & Mattsson, 2007; Jenelius & Mattsson, 2012; and Jenelius, Petersen, & Mattsson, 2006).

Jenelius & Mattsson, (2015) succinctly described transportation system vulnerability as "the risk of disruption and degradation of the transportation system." D'este & Taylor, (2003) associated vulnerability with accessibility, defining it as the ease with which individuals from specific locations can engage in activities occurring in other physical locations within and around the region, facilitated by the transportation system.

In the field of transportation engineering, M. Taylor, (2017) introduced the concepts of "vulnerable nodes" and "critical links". A network node is considered vulnerable if the loss or significant

degradation of a small number of links considerably reduces the node's accessibility, as measured by a standard accessibility index. Similarly, a network link is deemed critical if its loss or substantial degradation significantly diminishes the accessibility of the network or specific nodes, as measured by the same accessibility index.

These definitions necessitate the use of accessibility metrics sensitive to network topology and operational conditions. Consequently, two distinct yet complementary definitions of transport network vulnerability are commonly utilized, each with its unique application. Berdica, (2002) serviceability definition is apt for addressing short-term (hours, days, weeks) network disruptions, degradations, and failures, offering insights into network operations and their implications for network deterioration. It primarily reflects a supply-side perspective on vulnerability. Conversely, the D'este & Taylor, (2003) definition may be more suitable for long-term (weeks, months, years) disruptions, degradations, and network failures, emphasizing broader socioeconomic consequences and representing a demand-side perspective on vulnerability (M. Taylor, 2017).

2.1.1.1 Short definitions of vulnerability from the point of view of different scholars

Khademi et al., (2015) assert that vulnerability within the road transport system manifests as susceptibility to incidents capable of significantly diminishing road network serviceability. Conversely, Baroud, Barker, & Ramirez-Marquez, 2014 and Baroud, Ramirez-Marquez, Barker, & Rocco, (2014) define vulnerability as the potential for a disruptive event to detrimentally affect system performance. Colicchia, Dallari, & Melacini, 2010 posit that supply chain vulnerability encompasses the occurrence of stochastic disturbances leading to deviations from normal, anticipated, or planned activities within the supply chain, resulting in adverse effects or consequences. Nicholson, 2007 delineates vulnerability as the ease with which a system can be perturbed from its state of stability or equilibrium. Lastly, Knoop, Snelder, van Zuylen, & Hoogendoorn, (2012) characterize vulnerability as indicative of the fragility of a network.

2.1.2 Robustness

Various disciplines offer definitions of the robustness of networked systems. The IEEE standard computer vocabulary (Geraci, 1991) defines robustness as *"the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions."* In the realm of electrical networks, Holmgren, (2007) describes robustness as the ability to maintain its structure under disruptions. Immers, Yperman, Stada, & Bleukx, (2004) view

robustness as "the degree to which a system is capable of functioning according to its design specifications in the case of serious disruptions," while Boccaletti, Latora, Moreno, Chavez, & Hwang, (2006) define it as the network's ability to avoid malfunctioning due to damage to a fraction of its constituents. Robustness is consistently linked with system performance (Schillo, Bürckert, Fischer, & Klusch, 2001).

2.1.2.1 Robustness of transportation networks

The transportation system exemplifies a complex network, and research on transportation network robustness has seen significant advancements in recent years. Two main categories of studies exist regarding the robustness of transportation networks. The first category focuses on network topology and involves static robustness evaluation, which examines the impact of removing nodes or edges on the network's topological properties.

For instance, X. Li et al., (2018) examined the robustness of Chongqing's combined bus and metro transportation network, highlighting the enhanced robustness achieved through collaboration between bus and metro systems. Yang, Liu, Zhou, Li, & Sun, (2015) employed complex network theory to assess the resilience of the Beijing metro network against random and deliberate attacks, revealing its resilience against random attacks but susceptibility to deliberate targeting of critical nodes. Zhu, Liu, Wang, & Yan, (2018) utilized a defense-attack model to analyze subway network robustness, identifying loop lines and smaller transfer stations as contributors to increased resilience in case of accidents or attacks. J. Wang, Jiang, & Qian, (2014) evaluated the robustness of 33 metro networks globally against random and targeted attacks, identifying Tokyo and Rome as the most robust, with factors such as shorter transfer distances and ample alternative lines contributing to their resilience.

(Rong, 2010) proposed a method to investigate the robustness of bus networks, finding that nodes with low degrees but high loads are critical for network robustness, as failure of associated nodes or edges due to accidents or congestion impacts the network's overall performance.

The second category of studies focuses on cascade failures, where node or edge failures can propagate to adjacent nodes or edges due to passenger flow redistribution. This phenomenon, akin to cascading failures observed in other complex networks like power grids (Dobson, 2015);(Kinney, Crucitti, Albert, & Latora, 2005), the Internet (X. Wang et al., 2017), and cyber-

physical networks (Yang et al., 2015) underscores the interconnected nature of transportation networks and the potential amplification of disruptions across the system.

2.1.2.2 Short definitions of robustness from the point of view of different scholars

“Network Robustness Index (NRI) is The change in travel-time cost associated with rerouting all traffic in the system should that segment become unusable.” (Scott et al., 2006).

“Network robustness is defined as the degree to which the transportation network can function in the presence of various capacity disruptions on component links” (Sullivan, Novak, Aultman-Hall, & Scott, 2010).

“Robustness is the extent to which, under prespecified circumstances, a network is able to maintain the function for which it was originally designed” (Snelder, Van Zuylen, & Immers, 2012).

“Robustness is the network’s capacity to maintain its original service level in the presence of incidents” (de Oliveira, da Silva Portugal, & Junior, 2016).

2.2 Employed methodologies

Different classifications have been proposed to categorize the methodologies of network vulnerability analysis in the realm of transportation engineering over the past decade. One of the first categorizations was presented by Mattsson & Jenelius, (2015); they split the vulnerability analysis methodologies into topological methodologies and system-based methodologies. Similarly, Pan, Yan, He, & He, (2021) categorized the methodologies of the previous research into two categories entitled “topological methodologies” and “system performance” methodologies. They considered mathematical models, simulations, and data-driven methods as the sub-categories of system performance methods. Jing Zhang & Zhang, (2019) classified the methods of previous research into three basic categories called “methods based on network topology”, “methods based on accessibility”, and “methods based on mathematical models”. However, one of the most comprehensive categorizations presented by M. Taylor, (2017), classified the methodologies of vulnerability analysis of transportation networks as follow: **Topological based methods** focus on analyzing a network's structure and connections to identify the nodes or links whose failure would cause the most significant or widespread impact across the network. **Serviceability-based methods** incorporate transport-related network characteristics—such as link capacity, node capacity, link length, and travel time attributes—alongside the network’s topological structure to assess the

potential consequences of network failures on traffic performance and the ability to accommodate travel demand. **Accessibility-based methods** take a broader perspective on travel demand, evaluating the extent to which the population, including specific demographic segments within a region, can sustain routine economic and social activities despite disruptions in the transportation network.

2.2.1 Topological-based methods

Graph theory, complex network, network science, or topological methods are the names that evoke the same concept, considering the networks as a finite set of nodes and links. When these methods are used to model the network, the most frequently employed *topological indicators* (L. Gao et al., 2019);(Ma, Liang, et al., 2020) -which are also called *structural indicators* (Ma, Liang, et al., 2020)- are as follow :

2.2.1.1 Degree

This indicator quantifies the total edges linked to Node i . In a directed graph, it is categorized into indegree and outdegree (Lu Zhang, Zhao, Chen, & Zhang, 2021); (J. Zhang et al., 2021).

$$D=K_i \quad \text{Equation 2-1}$$

Where:

K_i represents the number of links connected to node i

2.2.1.2 Average degree

This indicator shows the average of the degrees of all nodes i in the network (Ma, Shi, et al., 2020); (Tran, Cheong, & Bui, 2019).

$$k = \frac{1}{N} \sum_{i=1}^N k_i \quad \text{Equation 2-2}$$

Where:

K_i represents the number of links connected to node i , and N represents the number of nodes in the graph.

2.2.1.3 Betweenness

This indicator measures the proportion of shortest paths that pass through a specific link or node relative to the overall total number of shortest paths in the entire network (L. Zhang et al., 2021);

(H. Chen, Zhang, Liu, Wang, & Dai, 2021); (H. Zhang & Yao, 2019); (Chaoqi, Ying, Xiaoyang, & Yangjun, 2018).

$$B_i = \sum_{p \neq q \neq i} \frac{\tau_{pq}(i)}{\tau_{pq}} \quad \text{Equation 2-3}$$

Where:

$\tau_{pq}(i)$ represents the number of shortest paths passing through node i between node p and node q , and τ_{pq} represents the total number of shortest paths between node p and node q .

2.2.1.4 Relative Size of the Giant Component (RSGC)

This indicator shows the relative size of the largest connected subgraph (after some nodes breakdown) in comparison with the size of the original graph, denoted as S and characterizes the robustness of a network from the aspect of network scale, which is calculated by:

$$S = \frac{N'}{N} \quad \text{Equation 2-4}$$

Where:

N' denotes the giant component's size following a node failure, while N represents the initial network size. A larger S value signifies greater network stability when subjected to disruptions..

2.2.1.5 Clustering Coefficient (CC)

This indicator shows the proportion of existing edges linking node i to its adjacent nodes relative to the total possible edges, calculated as $k_i(k_i - 1)/2$ (Ma, Shi, et al., 2020); (Tran et al., 2019).

$$CC_i = \frac{2n_i}{k_i(k_i - 1)} \quad \text{Equation 2-5}$$

Where:

n_i represents the number of the neighbor of node i connected to each other, k_i represents the degree of node i .

2.2.1.6 Clustering coefficient of the entire network

This indicator shows the average of all node clustering in the network (Ma, Shi, et al., 2020). Typically, all the graph's nodes are involved in the clustering system. For this reason, the sum of CC_i is divided by N (number of nodes).

$$CC = \frac{1}{N} \sum_{i=1}^N CC_i \quad \text{Equation 2-6}$$

Where:

CC_i for node i is calculated by using the formula presented in section 2.2.1.5, and N represents the number of nodes in the graph.

2.2.1.7 Average of the shortest distance

This indicator reflects the inherent efficiency—or, conversely, the cost—of the network, measured as the average shortest distance between all pairs of nodes (X. Li et al., 2018).

$$D(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in g}^N d_{ij} \quad \text{Equation 2-7}$$

Where:

d_{ij} represents the shortest distance between node i and j , and N represents the number of nodes in the graph.

2.2.1.8 Reciprocal of the average of the shortest distances

This indicator measures the efficiency index, representing the network's overall efficiency. It is computed as the average of the reciprocals of the shortest path distances between all node pairs (Tran et al., 2019); (J. Zhang et al., 2021).

$$D(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in g}^N \frac{1}{d_{ij}} \quad \text{Equation 2-8}$$

This index is particularly useful for evaluating the efficiency of a disconnected network. It applies when Node i and Node j in the network are not directly connected, i.e., $d_{ij} = \infty$, then $1/d_{ij} = 0$.

2.2.1.9 Global efficiency of network

This indicator reflects the directness of the connections between the specified node pairs.

$$E_{\text{glob}}(G) = E(G) / E(G_{id}) \quad \text{Equation 2-9}$$

G_{id} proposed an abstract network sharing the same nodes as the network G , where each pair of nodes is directly connected. The distance between any two nodes is defined by their Euclidean distance.

2.2.1.10 Local efficiency of network

G_i represents a subgraph of network G , encompassing all nodes adjacent to Node i and the links that connect them. This indicator measures the degree of directness in the connections among the nodes within this subgraph.

$$E_{loc} = \frac{1}{N} \sum_{i \in g}^N E_{glob}(G_i) \quad \text{Equation 2-10}$$

In a nutshell, vulnerability research based on network topology attributes entails identifying key network locations, comparing the vulnerability of a specific transportation network to various disruption methods comparing the vulnerabilities of transportation networks in different cities or regions, and determining the vulnerability of a transportation network (Jing Zhang & Zhang, 2019). Network topology characteristics encompass elements such as connectivity, transfer capacity, average path length, and diameter, each offering a unique perspective on the network's structure. Among these, network connectivity is regarded as particularly crucial. Prior studies have classified topological indicators into static and dynamic categories. Static indicators, including node degree, degree distribution, and connectivity, describe the fundamental properties of the network, while dynamic indicators, such as global network efficiency and the ratio of the largest connected subgraph, reflect the broader trends in network behavior during disruptions (J. Hu, Yang, & Zhen, 2024). While parameter selection based on network topology effectively describes the structural properties of the network, this approach overlooks the critical role of passenger travel in urban transit operations and management, failing to capture the relationship between network topology and passenger service quality.

2.2.2 Serviceability-based methods

Serviceability refers to a transportation network's ability to fulfill its functional requirements—such as facilitating traffic flow across an area and ensuring access to properties and facilities—under different conditions, including the degradation or failure of certain network components (M. Taylor, 2017).

2.2.2.1 Total delay time

The travel time $t_\ell(x_\ell)$ for a vehicle on link ℓ is determined as a function of the flow x_ℓ (He, Navneet, van Dam, & Van Mieghem, 2021):

$$t_{\ell}(x_{\ell}) = t_{0\ell} \left(1 + \alpha \left(\frac{x_{\ell}}{c_{\ell}}\right)^{\gamma}\right) \quad \text{Equation 2-11}$$

Where:

c_{ℓ} represents the capacity of link ℓ , while $t_{0\ell}$ denotes the free flow travel time. The parameter α and γ are the shape coefficients.

Then, total delay time is estimated using the following equation:

$$C_G = \sum t_{\ell}(x_{\ell}) \quad \text{Equation 2-12}$$

C_G typically increases to $C_G + \Delta C_G$ as a result of node disruptions and the degradation of link attributes. The robustness of the network can be quantified by the normalized increase in total travel time caused by these disruptions, which is represented by the robustness indicator η_G .

$$\eta(G) = \frac{C_{G'} - C_G}{C_G} \quad \text{Equation 2-13}$$

Where:

G' is the network after the disruption.

2.2.2.2 T^{post}/T^{pre}

This indicator shows the portion of the total number of completed trips before and after disruption (Kermanshah & Derrible, 2016) :

$$T^{post} = T_A^{post} + T_B^{post} + T_C^{post} + T_D^{post} \quad \text{Equation 2-14}$$

T_A^{post} : The completed trips that used the exact same route before and after the disruption.

T_B^{post} : The completed trips that were forced to use longer routes after disruption.

T_C^{post} : The trips that could not be completed after the disruption, because the trip's origins/destinations are isolated and become unreachable

T_D^{post} : The trips that could not be completed because a trip origin/destination is directly located in an affected region

$$\tau_A = \frac{T_A^{post}}{T^{pre}}, \quad \tau_B = \frac{T_B^{post}}{T^{pre}}, \quad \tau_C = \frac{T_C^{post}}{T^{pre}}, \quad \tau_D = \frac{T_D^{post}}{T^{pre}}, \quad \tau_{AB} = \frac{T_B^{post}}{T_A^{post} + T_B^{post}}$$

Where:

T^{pre} represents the total number of completed trips before the disruption, τ_A is the proportion of intact trips, τ_B is the proportion of trips that used longer routes, τ_C is the proportion of trips that were affected and not completed after the disruption, τ_D is the proportion of trips that could not be completed due to the removal of the origins/destinations, τ_{AB} is the proportion of completed trips that were forced to use longer routes.

2.2.2.3 Passenger flows

This indicator shows the fraction of available operating trains (Ouyang, Zhao, Hong, & Pan, 2014).

$$F_\sigma = \frac{1}{N_T} \sum_i X_i^\sigma \quad \text{Equation 2-15}$$

Where:

N_T is the number of trains that can run on a normal day, and $X_i^\sigma = 1, 2, \dots, N_T$, is the status of the i^{th} train after service disruption. If i^{th} train can still operate $X_i^\sigma = 1$, otherwise, $X_i^\sigma = 0$.

2.2.2.4 Total volume of passengers

This indicator shows the sum of passengers on all links in the largest connected component (Ma et al., 2019).

$$F_{total} = \sum_{i \in G}^{N|G} \sum_{j \in G, j > i}^{N|G} F_{ij} \quad \text{Equation 2-16}$$

Where:

$|N|_G$ represents the total number of nodes within the largest connected component of the network, and F_{ij} is the passenger flow on edge ℓ_{ij} .

The changes in the total volume of passenger flow ($\eta(F)$) were measured by the formulation below:

$$\eta(F) = \frac{F_0 - F_T}{F_T} \quad \text{Equation 2-17}$$

Where:

F_0 denotes the total volume of passengers in the network, and F_T represents the total volume of passengers in the largest connected component of the network after disruption, $\eta(F) \in (0,1)$.

2.2.3 Accessibility-based methods

The accessibility-based metric primarily evaluates how easily service facilities and critical nodes can be reached within a transportation network. To assess system vulnerability, accessibility is

compared before and after a disruption. This method is particularly effective for identifying optimal sites for emergency and service facilities by considering their exposure to network failures (Jing Zhang & Zhang, 2019).

2.2.3.1 Relative and integral accessibility

Accessibility indicators can be formally defined by categorizing the indices into two distinct classifications: relative accessibility and integral accessibility (M. Taylor, 2017): Relative accessibility refers to the extent of connectivity between two specific locations, such as a point of origin and an activity site, based on factors like distance, travel time, or cost. For instance, it is often applied in determining the optimal locations for emergency services, such as fire departments, ambulance stations, or police posts, where the closest facility is considered first. Once separation C_{ij} (distance, time, or cost) is known, relative accessibility A_{ij} between two points i and j can be expressed as:

$$A_{ij} = \frac{1}{C_{ij}} \quad \text{Equation 2-18}$$

Where:

A_{ij} represents relative accessibility from nodes i to j ; C_{ij} denotes the level of separation between two nodes, which can be measured in terms of distance, time, or cost.

Network vulnerability, as conceptualized under the D'este & Taylor, (2003) framework, evaluates the loss of travel utility between two points caused by network degradation. This type of vulnerability, referred to as connective vulnerability, is addressed within the scope of serviceability-based vulnerability analysis. It emphasizes the diminished capacity of the network to facilitate travel between specific locations when disruptions occur.

Conversely, integral accessibility assesses the overall connectivity of a given location to all other destinations or activities across a region. This measure provides an overarching assessment of accessibility from a designated location to all defined destinations. Denoted as A_i , the integral accessibility measure of an entity i accounts for its spatial location within the study area. The entity i may represent various objects, such as a region, a city, a subarea, a household, or an individual, depending on the context of the analysis. At a strategic scale, i often refers to cities or towns, whereas at an urban scale, it may correspond to zones, households, or individuals. At the regional

or rural level, it typically pertains to zones or settlements. Drawing on the definition by Morris, Dumble, and Wigan (1979), the integral accessibility index quantifies the overall accessibility of i to all relevant activity locations in the study area. Additionally, the summation $\sum_i A_i$ offers an aggregate measure of accessibility across the entire study area.

2.2.3.2 Hansen integral accessibility

Originally introduced by Hansen, (1959) this indicator is particularly useful for evaluating national or state transportation networks, as it allows for the comparison of a city's accessibility to all other cities within the study network.

$$AH_i = \sum_j B_j f(C_{ij}) \quad \text{Equation 2-19}$$

Where:

$f(C_{ij})$ indicates the separation degree between two cities and acts as the impedance function for the travel cost C_{ij} between i to node j . Typically $f(C_{ij})$ is defined as $= 1/d_{ij}$, where d_{ij} represents the travel distance within the network, At times, this may also be expressed as a negative exponent function, $f(C_{ij}) = \exp(-\beta C_{ij})$, and B_j is the attractiveness of the location j , which in national and regional studies, can be the population of town/city.

This formulation also is seen in a normalized form as follows:

$$AH_i = \frac{\sum_j B_j f(C_{ij})}{\sum_j B_j} \quad \text{Equation 2-20}$$

2.2.3.3 ARIA

The Accessibility/Remoteness Index of Australia (ARIA), introduced by M. A. Taylor, (2012), serves as an accessibility-based indicator. The ARIA index is calculated based on the road distance d_{iL} from location i to the nearest service center within category L , where L represents the categories A, B, C, D, E as specified in the table below.

$$ARIA_i = \sum_L \min\{3, \frac{d_{iL}}{\bar{d}_L}\} \quad \text{Equation 2-21}$$

Where:

$\bar{d}_L$ represents the average distance from all locations to the closest service center in category

L . To mitigate the influence of extreme values, an upper limit of 3 is applied to the ratio of d_{iL} to d_L . The ARIA index has an upper limit of 15, representing an extremely remote location.

Table 2-1 ARIA service center categories A–E (GISCA, 2009)

Service center category	Population	Mean distance to service center (km)
A	250,000	413
B	48,000–249,999	239
C	18,000–47,999	139
D	5000–17,999	88
E	1000–4999	43
F	(200–999)	(na)

Generally speaking, the accessibility-based vulnerability study is an effort to go beyond some of the topological research's shortcomings, though it comes at the cost of requiring additional data, such as travel demand and capacity to respond to changes (Jing Zhang & Zhang, 2019).

Upon completing the review of existing methods and indicators, a significant gap becomes evident. To the best of the author's knowledge, no existing indicator or method in the field of transit network vulnerability analysis has been designed to evaluate the impact of disruptions on both the quantity and quality of available route alternatives.

2.3 Network and services

Choosing the method of transport vulnerability analysis depends largely on the transport network or service to examine. While this research focuses on the vulnerability analysis of public transportation systems, reviewing studies on the vulnerability of other transportation networks can offer valuable insights. Transportation networks, regardless of their type—such as road, rail, or aviation systems—often share foundational principles in network modeling, which can inform vulnerability analysis. This section provides a technical review of recent studies on network vulnerability, with a particular emphasis on urban public transportation networks and multimodal systems. Additionally, it briefly examines research on other transportation networks or services that utilize indicators or methodologies relevant to or capable of inspiring this study.

2.3.1 Public Transportation Networks (PTNs)

Even though the network topologies of public transportation systems may appear simpler, they can be more intricate than road networks. Factors such as transferring between lines, using of

transportation hubs, and walking connections tied to transfers are examples of multimodal operations and passenger movement. In general, public transit networks have fewer connections than urban road networks and are less dense, offering fewer possible routes between nodes. Additionally, vehicle flow in public transit systems tends to be irregular, operating in discrete blocks that align with daily, weekly, and seasonal schedules, as well as the limited number of vehicles available. This pattern of flow suggests that public transportation systems might be more vulnerable than road networks. (M. Taylor, 2017).

2.3.1.1 PTN modeling

The application of network topology in evaluating the vulnerability and resilience of transit networks has been extensively explored in research. This is largely due to the fact that serviceability and demand-based approaches typically rely on advanced modeling tools, which, while standard for road networks, are not easily accessible or adaptable to public transit systems, as noted by Mattsson & Jenelius, (2015). Nevertheless, there are instances where these serviceability and demand-based methods have demonstrated their effectiveness, proving useful in specific contexts.

A transit network can be thought of as an undirected graph $G=\{V, A\}$, consisting of a set of nodes (vertices) and a set of links (edges), based on graph theory and network science.

Two indicators, in this regard, were proposed by (Derrible & Kennedy, 2010), defining the complexity and connectivity of networks.

The first one is β showing the ratio between number of links (N) and number of nodes (S):

$$\beta = \frac{N}{S} \quad \text{Equation 2-22}$$

The higher the value of β , the more complex the network is.

The second one is γ which is a topological indicator demonstrating the ratio between the current connectivity of a network and the maximum connectivity a network could have. In this indicator, the number of links represents the connectivity. In general, the maximum links an undirected graph can have (number of nodes of a Complete graph) is $\frac{N(N-1)}{2}$. Because transit networks fall under planar networks, such a node count is unrealistic, as intersecting links would naturally create additional nodes. Derrible & Kennedy, (2010) stated that the maximum possible number of links in this kind of networks is $3(N-2)|N>3$, and they defined the γ as below:

$$\gamma = \frac{S}{3N-6} \quad \text{Equation 2-23}$$

Von Ferber, et al. (2009) presented a comprehensive view of transit systems by modelling the system with alternative network topologies in multiple spaces (L-Space, B-Space, P-Space, and C-Space). The analyst can use some unique features of the network and consider the different indications of the network and its connectedness by modeling the network within each of these spaces.

L-space: This graph represents each station by a node, a link between nodes indicates that there is at least one route that services the corresponding stations consecutively.

P-space: The network is represented as a graph in this model, with stations represented as nodes that are linked if they are served by at least one common route. The neighborhood of a particular node in a P-space representation corresponds to all stations that can be reached without changing modes of transportation.

B-space: Both routes and stations are represented by nodes in this graph. Each route node is connected to all the station nodes it serves. There is no direct link between nodes of the same kind.

C-space: All nodes in this space are routes (or lines), and links show a common station between two lines, meaning that if there is a common station between two lines, a link connects their corresponding nodes to each other.

M. Taylor, (2017) provided descriptions of the various space networks as well as their attributes. In any network, the degree of a node and the characteristic path length are of interest. These attributes show how much movement is required to travel between stations, as well as whether or not transfers are required. The characteristics of S-space, L-space, B-space, and C-space are listed in Table 2-2.

Table 2-2 Characteristics of different transit networks representations (M. Taylor, 2017)

Space	Description	Properties
L-space	- A simplified physical or geographic representation of the network. - Stations are represented as nodes. - A link connects two nodes if they are consecutive stations on at least one transit line.	- Characteristic path length ($\bar{L}$) represents the average shortest number of stops between any two stations on a line. - The degree of a node (station) corresponds to the number of directly connected stations accessible with a single stop.
B-space	- Both transit lines and stations are represented as nodes. - A line node connects to all station nodes it serves. - No direct connections exist between nodes of the same type.	- A station node's degree corresponds to the number of transit lines serving it. - A line node's degree represents the total number of stations it.
P-space	- Stations are represented as nodes. - A link exists between two stations if they share at least one transit line..	- $\bar{L}$ represents the average minimum number of transfers required to travel between any two stations. - A node's (station's) degree corresponds to the number of stations that can be reached without changing lines.
C-space	- Lines are represented as nodes. - A link exists between two nodes if the corresponding lines share a station.	- $\bar{L}$ represents the average minimum number of transfers required to move between any two lines. - The degree of a node (line) indicates the number of other lines that can be directly transferred to from it.

2.3.1.2 Double-layer PTN

For transit networks, which are complex transportation systems, focusing on just one transportation mode often results in an incomplete analysis. Adjetey-Bahun, et al. (2016) explored the

interconnection between multiple subsystems, including transportation, power, telecommunications, and organizational subsystems. Additionally, some studies have broken down the network into multiple layers to examine how faults propagate across different systems.

Adjetei-Bahun et al., (2016) proposed a methodology based on a multi-layer network, where each layer represents a specific mode of public transit. X. Li et al., (2018) proposed a two-layer multiplex network, consisting of two single-layers, modeled based on graph theory and both P-space and L-space modeling to demonstrate the network. The methodology was presented to analyze the robustness of a bus-subway network in a mountain city by using topological indicators including “relative size of the giant component, average path length, diameter, and global efficiency”. Ma et al., (2019) defined a bilayer network to evaluate the vulnerability of bus-metro networks to rainstorm weather conditions. Both P-space and L-space modeling were employed in this research to model the transit network. Using the L-space, Ma et al., (2019) proposed a double layer network to assess the robustness of Beijing’s bus-subway network. In this research, average degree, clustering coefficient, average path length, relative size of giant component, and global network efficiency were the indicators used to quantify the robustness of the transit network.

Q. Du et al., (2024) propose an integrated resilience assessment framework for bus and metro systems, modeled as a Bus-Metro Double-Layer Network (B-M DLN). Their framework evaluates resilience by considering both network structure and system function. Using a Genetic Algorithm (GA), they develop a resilience optimization model to prioritize the recovery of damaged stations based on factors like repair time, node strength, and degree. A case study in Xi’an City demonstrates that applying the optimal recovery strategy significantly enhances the resilience of the B-M DLN, particularly in large-scale failure scenarios. Guo et al., (2023) propose a cascading failure and recovery model for metro-bus networks, addressing the often-overlooked recovery propagation after disasters. Their study investigates how failure and recovery spread, considering factors such as initial loads, node capacities, and repair speeds. Applied to Xi’an City, the model reveals that node capacity and repair speed exhibit a marginal decreasing effect, while the coexistence of failure and recovery propagation improves network performance. The findings offer strategic insights for developing more sustainable and resilient transportation systems. M. Zhang, Huang, Guo, & He, (2022) provide a comprehensive review of applying complex network theory to traffic networks, treating them as geometric graphs with nodes representing infrastructures and

edges as links. The paper introduces key measures and properties of complex traffic networks and examines their traffic flow characteristics and congestion dynamics. They also discuss the robustness of these networks and how complex network theory can enhance urban transportation systems. Finally, they highlight unresolved challenges and potential research opportunities in the field.

2.3.1.3 Multimodal PTN

B. Wang, Su, & Chin, (2021) presented a new method to topologically model the multimodal transportation networks. This method, which is called ML-space, is a multi-layer L-space model, providing prospective researchers in the field of PTN and Multimodal Transport Network (MTN) modeling to consider transshipment characteristics. This research considering a road-rail MTN as a case study, examined both random and intentional disruptions as well as the cascading effects of disruptions on the network.

Lin Zhang, Xu, & Wang, (2024) address the vulnerability of multimodal public transit systems (MPTS) to interdependent cascading failures across rail, bus, and road networks, which hinder resilient megacity development. They propose a real-time operational control method based on System Emergency Capability (SEC) to prevent the spread of severe failures by considering network topology and dynamic evolution. Unlike prior qualitative approaches, their method enables immediate emergency response. Through a three-stage SEC design process, they optimize loading strategies, and a case simulation demonstrates the method's adaptability for emergency resource allocation and future intelligent control algorithms.

N. Wang, et al. (2023) present an innovative approach for evaluating the resilience of urban multimodal transportation systems by incorporating both passenger demand and infrastructure supply. Their approach evaluates how well the system can adapt and recover from disruptions by considering both the demand side (passenger needs) and the supply side (infrastructure capacity). This method offers a more comprehensive view of resilience, supporting improved planning and optimization of urban multimodal transportation system (UMTS).

J. Li, Lu, Xu, Wang, & Xie, (2024) address the gap in understanding cascading failures between multimodal public transportation systems. They proposed a multimodal coupled map lattice model to assess station coupling strength during failures. Station coupling strength measures the interdependence of transit stations across modes and their impact on intermodal connections and

cascading failures. The model explores failure spread across metro-bus networks and examines the effects of station coupling asymmetry. Applied to Shenzhen's metro-bus network, the study reveals that coupling with the bus network alleviates metro failures but magnifies failures on the bus network. Failures of stations with higher station coupling strength trigger more severe cascading effects than key stations on single networks, offering insights for robust metro-bus system planning and emergency response strategies.

2.3.2 Multimodal networks

Jing, Yong, & Lei, (2021) by using the complex network and graph theory, topologically analyzed the vulnerability of CIM Kaitong's multimodal transport business in China. Path length and the percentage of the largest subgraph (giant component) were employed to measure the vulnerability of multimodal transportation system after random and intentional disruptions. The results showed that the studied multimodal transportation network is more vulnerable to intentional failures than to random failures. He et al., (2021) presented a paper assessing the robustness of multimodal freight transportation network. Graph theory and multi-layer networks were the concepts employed in this research to model the multimodal transportation network consisting of railway, road, and inland waterway. Their study models nodes as junctions, terminals, and crossings, while links represent travel paths. The network structure captures both interconnection and interdependency features. At interconnection terminals, freight can transfer between different transport modes, whereas the failure of a single interdependent node can disrupt multiple modalities. Interdependent nodes, such as crossings, impact the operation of several transport modes (e.g., bridges, tunnels). Network robustness is evaluated based on the increase in total travel time resulting from infrastructure disruptions and capacity reductions along travel paths, both classified as disruptions.

2.3.3 Urban highway networks

Vivek & Conner, (2022) assess the vulnerability of the urban road network of Boston to large-scale disruptions. Using the concept of graph theory (intersections are considered as nodes and roads as edges), they concentrated their research on the consequences of targeted disruptions as well as determining the vulnerable nodes. For this, they used two methods; betweenness centrality of nodes was the first indicator analyzed to determine the nodes playing the most important roles in shortest time paths. The second method was greedy node removal algorithms. This is an algorithm finding the node that appears in the greatest number of routes, as compared to other nodes. This node

represents the most important node, as its failure affects the highest number of routes. The algorithm then removes all routes that contain this top node, finds the next most important node with the highest number of remaining routes, and removes routes that contain this node. All shortest-time routes are disrupted as the algorithm iterates until the minimal number of nodes to target is found.

Mhatre, Richmond, Qu, & Davis, (2018), by concentrating on natural disasters, assessed the vulnerability of the Southeastern North Carolina Highway Transportation System to a Hurricane. They employed the FHWA's vulnerability assessment framework as a guide to estimate the impact of Hurricane Matthew on the highway transportation system in southeastern North Carolina. There are three steps in this structure: (1) identifying a project's scope, (2) determining vulnerability, and (3) incorporating vulnerability into decision-making.

C. Chen, Wang, Zhang, Gu, & Huang, (2022) evaluated the vulnerability of the Haikou transportation network from both functional and topological points of view. Giant Component and Travel Efficiency were employed as the topological indicators which were calculated by graph theory. However, for functional vulnerability analysis, the lost flow and travel time were employed as indicators which are related to percentage of unfeasible trips after disruption and changes in travel time after the disruption respectively which are calculated by equilibrium models. They considered both random and intentional disruptions. For intentional disruptions, the node importance is calculated by using a Multi-Criteria Decision-Making Methods (MCDM), called TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution). For this, four indicators were employed: strength, flow betweenness, closeness centrality, and PageRank. The closeness centrality measures how close a node is to the outer geometric center of a road network. PageRank is an indicator assessing the importance of each node in a graph by considering the recursive relationship between the significance of each node's neighbors.

2.3.4 Rail networks

Khademi, et al. (2021) analyzed the vulnerability of inter-urban sparse rail networks using a network scan methodology. The study examined two scenarios following a disruption: (i) demand rerouted to an alternative path, leading to increased costs due to longer travel distances, and (ii) cases where demand remains unmet because no available routes exist between the connected origin-destination (OD) pairs. To capture the effects of the disruption, five criteria were

established: Passenger Unsatisfied Demand (PUD), Additional Passenger Transport Cost (APC), Freight Unsatisfied Demand (FUD), Additional Freight Transport Cost (AFC), and Reconstruction Cost (RC).

The study introduced a comprehensive approach to vulnerability assessment in sparse transportation networks with the following key elements:

- The analysis of link failure impact incorporated the distance elasticity of demand, reflecting changes in demand as travel distances increase.
- Both passenger and freight demands were considered in the evaluation.
- The duration of network element closures was accounted for, as it affects travelers' decisions to continue their journey amid network disruptions.
- The study addressed both unsatisfied demand (for passengers and freight unable to complete their trips) and the additional costs incurred from detours.

In their methodology, distance elasticity of demand for a specific OD pair is defined as the percentage of passengers or freight unable to finish their journey when travel distance increases by one percent due to the failure of a particular link.

2.3.5 Rural road networks

Morelli & Cunha, (2021) investigated the vulnerability of various transportation networks in São Carlos, Brazil, to extreme events. Their research modeled the road network as a directed graph, but due to the differing access to infrastructure across transportation modes, separate graphs were created for each mode. By comparing pedestrian and vehicular infrastructures, the study revealed notable structural differences, such as the fact that cars typically follow one-way routes, while pedestrians have more freedom of movement in both directions. This disparity can result in situations where vehicle movement is obstructed by disruptions, but walking remains possible. To account for these distinctions, the researchers utilized the OSMnx library to extract networks for walking, driving, and cycling from OpenStreetMap, converting them into nonplanar graphs. The findings of the study highlighted that walking and cycling demonstrated higher resilience to disruptions, while private motorized transport proved more vulnerable. Furthermore, the analysis indicated that longer trips are more susceptible to flooding, suggesting that urban designs focused on car use may exacerbate network weaknesses. Conversely, a compact urban layout that promotes short walking and cycling trips could improve resilience to flooding.

2.3.6 Aviation Transportation Networks (ATNs)

Over the past years, several research has been conducted in the field of robustness analysis ATNs. X. Sun & Wandelt, (2021) presented a systematic literature review about research that has been published in the last 15 years related to robustness analysis of air transportation. They identified around 70 studies that were classified according to topics such as centrality identification, failure versus attack effectiveness, network recovery, and networks of networks approaches.

In this section we concisely review the presented definitions for robustness of ATNs from the point of view of recent published papers and have a glance on methodologies and indicators used in this research.

Sano & Berton, (2021) defined robustness as the network's ability to keep connected after a failure or attack from some vertex. They used the concepts of temporal network and event graph in evaluating robustness of ATNs Brazilian an US domestic flights. They considered airports and flight routes as nodes and links respectively and used giant component indicator in quantifying the robustness. X. Sun & Wandelt, (2017) presented a topological method based on complex network and graph theory to evaluate the robustness of ATN in China. In this research, airports are considered nodes and direct flights from one airport to others as links. Giant component was used as an indicator to quantify the robustness for both random and intentional failure.

2.4 Disruption mitigation strategies by duration

As metro systems in numerous cities continue to age, service disruptions have become more frequent. For instance, an analysis of the Montreal metro system over the past five years indicates that it experiences over 7,400 annual disruptions lasting 14 minutes or less, approximately 100 disruptions lasting between 15 and 29 minutes, and between 70 to 75 disruptions each year lasting 30 minutes or longer (DonnéesOuvertes, 2024). Consequently, the management of metro disruptions has become a crucial component of urban emergency management, and significant research has been conducted in this area in recent years.

Disruptions in metro systems can be handled internally within the network or by deploying external substitute bus (SB) services (S. Zhang et al., 2021). Internal disruption management typically involves adjusting timetables, rescheduling rolling stock, and reallocating crew members, depending on the rail system design and the availability of crossovers for train turnovers. In

contrast, SB services, also referred to as bus bridging, are a prominent approach to mitigating the negative impacts of metro disruptions.

However, metro operators decide whether to implement mitigation measures based on the (anticipated) duration of the disruption. For instance, tasks such as redirecting buses from different lines for shuttle services, informing drivers of selected routes, and adjusting schedules and shifts are time-consuming processes. These activities may not be practical for addressing disruptions of short duration. In some cases, the metro system may return to normal operations before these measures are fully implemented or their effects can be evaluated. For short-term disruptions, the responsibility for mitigating their impact often falls solely on conventional public transport services, particularly bus systems.

2.4.1 Internal and external mitigation strategies for metro disruptions

Several studies have explored disruption management strategies in public transportation systems. For example, Malucelli & Tresoldi, (2019). formulated an optimization problem aimed at minimizing operational costs resulting from changes in driver scheduling. Y. Huang et al. (2020) developed a mixed-integer formulation to simultaneously address supply and demand aspects of metro disruptions, integrating train scheduling and passenger flow modeling. Veelenturf et al., (2017) presented a model that reschedules rolling stock and adjusts timetables while accounting for changes in demand during disruptions.

The concept of substitute bus services gained traction with the introduction of "bus bridging" by Kepaptsoglou & Karlaftis, (2009), and it has been the subject of extensive research since then. In practice, bus bridging is widely employed by many transit agencies. Pender, Currie, Delbosc, & Shiwakoti, (2013) observed that most transit agencies utilize some form of bus bridging as part of their disruption management strategies. H. Hu, et al. (2016) proposed a nonlinear integer programming model for planning bus bridging services for a disrupted rail transit line, which was implemented using a genetic algorithm in the context of the Shanghai metro system. (Van der Hurk, et al. (2016) proposed a model to optimize the selection of shuttle lines and frequencies while adhering to budgetary constraints, which was implemented in Boston.

2.4.2 Short-term metro disruptions

A review of the literature on public transportation vulnerability reveals a lack of studies focusing on short-duration disruptions, with no clear threshold defined to identify them. One of the few

studies on short-term disruptions was conducted by Arslan Asim, Weiss, Kattan, & Wirasinghe, (2021), which examined users' mode choice behavior during short-term planned light rail transit (LRT) disruptions in Calgary, Canada. In this context, short-term disruptions were defined as planned weekend disruptions lasting up to three days, which differs conceptually from the scope of the present research. Another relevant study by Srikukenthiran & Shalaby, (2017) simulated various disruptions in the Toronto transit network. While this study did not explicitly categorize disruptions by duration, the findings highlighted differences in network resilience for disruptions lasting 10 minutes, 20-30 minutes, and more than 30 minutes.

The absence of a precise definition for short-duration disruptions has led previous research to focus primarily on medium- and long-duration disruptions, despite the fact that short-duration disruptions are statistically more frequent. In practice, some metro operators have empirically established a critical threshold below which no mitigation measures, either internal or external, are implemented. Instead, they rely on conventional transit services to mitigate the effects of these disruptions. To date, no studies have evaluated the efficiency and quality of conventional systems in mitigating the impact of metro disruptions. Furthermore, there has been no attempt to establish a relationship between the extent of the disruption and the additional travel time imposed on users. While previous research has considered disruption duration and recovery time, few studies have examined the specific timing of disruptions. The effect of disruption timing on travel time increases and the effectiveness of mitigation measures throughout the day remains largely unexplored.

2.5 Vulnerability of users

In recent years, the study of vulnerability in transit systems, particularly metro systems, has gained considerable scholarly attention, with researchers examining a wide array of failure types. These disruptions encompass random failures, targeted failures, and more complex cascading failures, each affecting the performance of metro systems in distinct ways. In addition to the methodologies explored, various approaches, such as node-based and link-based failure strategies, have been developed to simulate these disruptions and assess their impacts on users and the network as a whole. Nevertheless, relatively few studies have specifically focused on users, with the majority concentrating on macroscopic analyses that examine broader transportation metrics, such as total travel time and system efficiency. In contrast, research at the microscopic level, addressing individual users, remains limited. Even within existing microscopic analyses, the focus has

typically been on issues of equity and accessibility or the effects of disruptions on users' mode choices, rather than on their vulnerability or robustness. To the best of our knowledge, no research to date has undertaken the task of categorizing users of public transportation systems and evaluating their vulnerability to disruptions within these systems.

2.5.1 Equity and accessibility in public transportation

Research in public transportation examines users in detail, particularly through comparative analysis among different passenger groups, and primarily addresses concepts such as safety, equity, and accessibility. For instance, Cardoso et al., (2023) explored the resilience of transit trips in the context of urban violence from a gender perspective, revealing that women are more vulnerable to violence in public transport than men.

Mohri, Mortazavi, & Nassir, (2021) developed a method to identify and address inequities in job accessibility through public transportation services by introducing a Modified Cumulative Accessibility (MCA) measure. This measure facilitates the identification of inequities based on users' travel purposes. In a similar vein, Deboosere & El-Geneidy, (2018) devised a metric to assess accessibility to low-income employment for vulnerable residents, with an emphasis on the actual travel times these individuals experience. This modified place-based accessibility measure, which incorporates aspects of person-based metrics, was applied in 11 key cities in Canada.

2.5.2 Users' mode choice after disruption in metro system

Empirical data-based methods employ data from sources such as smart cards and vehicle location systems to assess the real-world impacts of incidents, producing metrics related to vehicle load, travel delays, and the spatial distribution of disruptions. These studies can be performed at both macro and micro levels, with micro-level analyses centering on individual user behavior. For instance, Mo, Von Franque, et al. (2022) examined the impact of metro disruptions at both aggregate and individual levels, calculating changes in demand across various rail lines, stations, bus routes, and stops to elucidate the redistribution of passenger flows during incidents. Passenger behavior was also examined using a binary logit model to determine mode choice preferences during disruptions.

Similarly, Tian & Zheng, (2018) explored commuter behavior during unexpected train delays, utilizing six months of real-world data collected from Singapore's automated fare collection system

to develop a classification model. This model aimed to predict whether commuters would switch from mass rapid transit to alternative transportation modes in response to delays.

Survey-based approaches often focus on passenger behavior and perceptions during disruptions, employing econometric models to analyze individual-level responses. For example, T. Lin, Sriukenthiran, Miller, & Shalaby, (2018) investigated transit users' mode choices in response to rapid transit service disruptions in Toronto, considering various factors, including the type of disruption, the stage of the trip (pre-trip or en-route), weather conditions, and the uncertainty surrounding delay duration. Another study by Currie and Muir (2017) aimed to understand passenger behavior, perceptions, and priority interventions in response to unplanned disruptions in urban passenger rail services. This research reported on a comprehensive survey of rider responses among users of the Metropolitan heavy rail system in Melbourne, Australia.

Moreover, some studies have evaluated alternative transportation modes, such as ride-hailing (RH) trips, which are commonly employed during metro disruptions. For instance, Young & Farber, (2019) concentrated on ride-hailing as a potential alternative during metro disruptions, finding that RH services are predominantly utilized by younger individuals, particularly those aged 20 to 39 years. Notably, RH trips made by individuals aged 20 to 29 account for 2% of all trips taken by this age group, which is more than double the average for RH trips overall. Conversely, RH usage among older adults is notably low, with less than 2% of RH users being 60 years or older. This trend is likely attributed to the digital divide between younger and older generations, as well as the fact that only 30% of Canadians aged 71 and older own smartphones capable of ordering RH trips.

R. Liu, et al. (2020) examined 78 significant metro disruptions in Toronto and discovered that bus routes serving disadvantaged areas experienced greater impacts from these disruptions than those in wealthier neighborhoods. Their study also showed that while ride-hailing demand rose during metro disruptions, this trend was not observed in disadvantaged areas. Their study highlights the disproportionate impact of subway disruptions on lower-income communities and illustrates how ride-hailing services may exacerbate rather than alleviate these inequalities.

Despite notable progress in understanding the effects of metro system disruptions, research dedicated to user vulnerability and robustness remains relatively underexplored. A stronger focus on individual-level analyses that examine user resilience in response to disruptions is crucial for

devising more comprehensive strategies aimed at improving the overall robustness of metro systems.

2.6 Research gaps

As previously discussed, the concept of vulnerability analysis of transportation systems to disruptions is relatively new within the domain of transportation engineering, with numerous aspects yet to be thoroughly explored. This section will highlight the key research gaps in the field of vulnerability analysis of public transportation that this study aims to address.

2.6.1 Research gap 1: inadequate disruption simulation

Current methodologies for simulating disruptions in transit networks are limited by oversimplified node-based and link-based strategies, failing to reflect the complexity of metro systems, especially at stations without direction-change facilities. Additionally, the selection of critical stations is often based on single indicators like node degree or betweenness, leading to inaccurate results. Furthermore, random disruption scenarios are simulated using simple random selection methods, which may lead to unrealistic outcomes. This study addresses these limitations by introducing the Block-Based Failure Strategy (BBFS) to simulate disruptions more accurately, developing a comprehensive framework for identifying critical stations with multiple indicators, and proposing an iterative approach to simulate random disruptions to improve accuracy and consistency in network vulnerability assessments.

2.6.2 Research gap 2: limited assessment of route alternatives in vulnerability analysis

Traditional vulnerability assessments in public transportation systems primarily focus on structural characteristics, serviceability, and dynamic variations. However, these methods neglect the impact of disruptions on the availability and reliability of users' route alternatives, a key factor influencing user mode choice. The study aims to develop a new methodology to assess not only trip feasibility during disruptions but also the diversity and reliability of alternative routes available to users, addressing a critical aspect of travel quality.

2.6.3 Research gap 3: ineffective short-term disruption mitigation measures

Despite extensive research on disruption mitigation, current measures are often time-consuming and ineffective for short-term disruptions in metro systems. During such disruptions, mitigation

efforts rely heavily on regular bus services, whose role in handling different types of disruptions remain understudied. The study intends to examine the effectiveness of bus systems in managing short-term metro disruptions and ensuring service continuity.

2.6.4 Research gap 4: lack of comparative vulnerability analysis of user groups

Most studies on transit network vulnerability focus on system-level robustness but overlook the differences in resilience among various user groups. Understanding the comparative vulnerability of different population segments when faced with metro disruptions can provide valuable insights for designing targeted mitigation interventions. This study aims to fill this gap by ranking transit system users based on their vulnerability to disruptions, allowing planners to prioritize improvements for less resilient groups.

CHAPTER 3 METHODOLOGY

In this chapter, we begin by introducing the case study, which focuses on the public transit systems of Montreal. This is followed by a detailed presentation of the datasets utilized in this research, encompassing various aspects such as transit supply, demand, and historical records of service disruptions. The classification and impact of disruptions are then examined, establishing a structured approach to characterizing disruptions and quantifying their effects on transit operations and users. The chapter proceeds with a transit system vulnerability assessment, employing network-, serviceability-, and redundancy-based analyses to capture different dimensions of system resilience. This is followed by an intermodality-focused evaluation, exploring how alternative transit modes, particularly buses, mitigate the impact of metro disruptions. Finally, a user vulnerability assessment is conducted, identifying population segments most affected by service interruptions based on travel patterns and resilience metrics.

3.1 Case study

This research examines the public transit networks of the greater Montreal metropolitan area, focusing primarily on the metro and bus systems operated by the Société de transport de Montréal (STM). Montreal, a city with a population of 1.76 million, lies within a larger metropolitan region home to 4.29 million residents. Each day, the city attracts significant commuter traffic from neighboring areas such as Laval and Longueuil, making it essential to consider the transit systems of these regions as well. Therefore, the study includes the transit services provided by the Société de transport de Laval (STL) and the Réseau de transport de Longueuil (RTL), alongside the STM.

The Montreal transit network also encompasses an automated light rail system operated by the Réseau express métropolitain (REM), along with a regional train network and bus services managed by Exo. However, these components fall outside the direct scope of the present study and have therefore not been explicitly analyzed.

In total, the case study covers one metro system and three bus systems. The Montreal metro, one of the largest in Canada, consists of four lines (Orange, Green, Blue, and Yellow), encompassing 68 stations and covering a total distance of 70 kilometers. This network facilitates approximately 820,000 trips daily. The bus networks, operated by STM, STL, and RTL, collectively manage

around 395 bus routes and over 15,000 stops. Specifically, STM operates 216 bus routes with 8,747 stops, STL manages 99 routes and 2,827 stops, and RTL operates 80 routes with 3,510 stops.

3.2 Utilized data

The analysis relies on multiple data sources to assess transit system vulnerability. These include the Montréal Origin-Destination Survey, which provides detailed travel behavior insights, open data such as GTFS for transit schedules and real-time information, and historical disruption records. Additionally, proprietary STM data offers network-specific operational details, enhancing the robustness of the study.

3.2.1 Montréal Origin-Destination (OD) survey

The Montreal Origin-Destination (OD) Survey is a key transportation study conducted in the Montreal metropolitan area every five years since 1970, utilizing telephone-based data collection methods, and since 2018, web-based methods as well. It serves as one of the most comprehensive and reliable sources of data on travel patterns in Quebec, capturing a wide range of modes of transportation including walking, cycling, bus, metro, train, and car usage within the greater Montreal area. The survey samples approximately 4 to 5% of resident households during an average autumn day through a stratified random sampling process without replacement. This sampling is conducted over a four-month period, from September to December, and respondents are required to provide detailed information about their trips on the previous day, along with sociodemographic data for all household members.

For this study, the 2018 edition of the Montreal OD Survey was used, which involved the participation of 74,000 households during the fall of 2018. This iteration aimed to capture a comprehensive picture of the travel behaviors of residents across the Montreal metropolitan area on a typical weekday, regardless of the mode of transportation.

In this research, we utilized both sociodemographic data—such as gender, age, and occupation—and trip attributes, including the purpose of the trip, geographic coordinates (latitude and longitude) of origin and destination, departure times, and the sequence of transportation modes used. Additionally, each recorded trip was assigned a weighting factor, denoted as f_{per} , which represents the number of real-world trips that each surveyed trip corresponds to. This factor was applied to weight the dataset, ensuring that it accurately represents the travel behavior of the overall

population. By incorporating f_{per} , the analysis accounts for potential sampling biases, thus providing a more representative picture of actual trip frequencies and distributions.

3.2.2 Open data

3.2.2.1 General Transit Feed Specification (GTFS) data

The General Transit Feed Specification (GTFS) is a standardized data format used to represent public transportation schedules and geographic information. It provides a consistent way to describe the details of transit services, allowing transit agencies to share their data with application developers, researchers, and other stakeholders. A typical GTFS feed contains a collection of text files (usually in CSV format) that provide information such as transit routes, stops, schedules, service frequencies, and fares. These files also include spatial data like the geographical coordinates of stops, and information on service calendars, indicating on which days and times certain services operate.

GTFS data is widely used for a variety of purposes, including powering transit apps (like Google Maps or Transit), helping riders plan their journeys, and facilitating analysis by transportation planners and researchers. By analyzing GTFS feeds, professionals can assess transit service coverage, optimize schedules, evaluate network accessibility, and develop new tools for improving public transportation systems. The standardized format ensures that transit data from different agencies can be easily integrated and used across different platforms and geographic regions.

In this research, General Transit Feed Specification (GTFS) data was utilized for the topological modeling of the transit network, spatial analysis, and the simulation of transit trips. We employed GTFS datasets from three major transit agencies : Société de transport de Montréal (STM), Société de transport de Laval (STL), and Réseau de transport de Longueuil (RTL). These datasets were obtained directly from the respective agencies' websites.

3.2.2.2 Disruption history data

Given that a significant portion of this research involves simulating metro disruptions, it is essential to examine the history of such events within the Montreal metro system. The Société de transport de Montréal (STM), which operates the Montreal metro, records detailed information on metro disruptions and makes this data publicly available on the donnees.montreal.ca website. The dataset includes comprehensive details such as the name of the affected station, the cause of the disruption, the date, time of occurrence, the duration of the disruption, etc. covering the last five years. In this

research, this data was utilized to define disruption scenarios, enabling a realistic and data-driven approach to modeling the impact of metro interruptions.

3.2.3 STM data

A significant portion of the transit network analyzed in this research, particularly the metro system, is operated by the Société de transport de Montréal (STM). Throughout the course of this research, we maintained regular communication with STM to obtain necessary data, validate our results, and stay informed about their policies regarding various types of disruptions. For instance, we requested data on the network topology and the locations of Stations with Direction Change Facilities (SDCFs), which were instrumental in defining the Block-Based Failure Strategy (BBFS). Additionally, we sought other data to validate the BBFS algorithm.

Through regular meetings and ongoing correspondence with STM, we received valuable information that helped us define disruption scenarios. This included details on mitigation measures in the event of metro disruptions, as well as specific thresholds for disruption durations that trigger the implementation of these measures. Such collaboration was crucial in ensuring the accuracy and applicability of the disruption models developed in this research.

3.3 Disruption classification and impact

Given that this research focuses on assessing the vulnerability of transit systems to disruptions, a comprehensive understanding of transit network disruptions is crucial for accurately evaluating system vulnerability. Disruptions can arise from various sources, including accidents, infrastructure failures, adverse weather conditions, planned maintenance, or large-scale events, leading to partial or complete service interruptions. Their duration can range from minutes to hours, with varying consequences depending on the affected stations and network configuration. By establishing a structured framework for disruption classification and evaluation, this section lays the foundation for a comprehensive vulnerability assessment of the transit system.

3.3.1 Disruption types

This study focuses on node-based disruptions, where service interruptions occur at metro stations rather than along transit links. The research categorizes disruptions into two primary types: targeted disruptions and random disruptions. These classifications enable a structured approach to evaluating the resilience of the transit network under various failure conditions.

3.3.1.1 Targeted disruptions

Targeted disruptions are predetermined and based on systematic criteria to ensure a realistic assessment of network vulnerabilities. The station selection process follows two principal methodologies: historical frequency of disruption and topological importance.

Historical Frequency-Based Selection:

The study analyzes metro system disruption records spanning a five-year period (2018–2023) to identify stations that have experienced recurrent operational failures. These disruptions may result from infrastructure failures, maintenance issues, or external factors. Stations are ranked based on the frequency of recorded incidents, ensuring that selected scenarios reflect empirically observed vulnerabilities and common failure points in the network.

Importance-Based Selection:

Instead of prioritizing stations with high passenger flow, this research employs graph-theoretic centrality measures to determine the structural significance of stations within the network. The evaluation considers seven key indicators that each capture a unique aspect of station influence within the network:

- Degree centrality (station connectivity)
- Betweenness centrality (control over transfer flows)
- Closeness centrality (proximity to all other stations)
- Hub score (influence in transit operations)
- Authority score (importance in receiving traffic)
- PageRank (global significance within the network)
- Eigenvector centrality (relative influence based on connections)

To calculate these indicators, the network is modeled using L-space representation in Python, and the NetworkX library is employed to calculate these metrics. To ensure an objective assessment, weights for each indicator are determined using the Shannon Entropy method, which evaluates the relative importance of each measure. The final station ranking is established through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which prioritizes stations with the highest cumulative centrality scores. For multi-disruption scenarios, a dynamic analysis is

performed, recalculating station importance after each simulated failure to account for the evolving network configuration.

3.3.1.2 Random disruptions

Random disruptions serve to model failure scenarios without pre-selection biases, providing insights into network robustness under unpredictable conditions. Traditional random selection methods may yield unrepresentative results, such as clusters of failures in a single corridor or disruptions concentrated at terminal stations. To mitigate these issues, a structured probabilistic approach is implemented:

The random station selection process is conducted 100 times to generate diverse failure scenarios.

- The scenario occurring most frequently across iterations is designated as the *Realistic Random Failure Scenario (RRFS)*, representing typical system failures.

To account for extreme variations, two additional cases are analyzed:

- *Pessimistic Random Failure Scenario (PRFS)*: The scenario with the lowest connectivity post-disruption, representing the worst-case scenario.
- *Optimistic Random Failure Scenario (ORFS)*: The scenario with the highest connectivity, indicating minimal network degradation.

By employing an iterative simulation process, the study ensures that random failures reflect realistic operational vulnerabilities rather than statistical anomalies. This approach enhances the reliability of resilience assessments and allows for a comprehensive understanding of potential disruptions in the transit network.

3.3.2 Failure strategy

A disruption at a single metro station can extend beyond its immediate location, potentially affecting an entire segment of the network. To accurately model such impacts, a structured failure strategy is required. This study employs the *Block-Based Failure Strategy (BBFS)* to simulate cascading disruptions and evaluate the resilience of the transit network.

BBFS builds upon conventional turn-back operation principles but introduces a specialized framework for urban metro systems. The core concept of BBFS revolves around the definition of

a block, which refers to a segment of consecutive regular stations bounded by two Stations with Direction-Change Facilities (SDCFs). These SDCFs are categorized into two types:

- Fully equipped SDCFs: Facilitate direction changes in both directions.
- Semi-equipped SDCFs: Allow direction changes only in one direction.

Under BBFS, the failure of a regular station leads to the failure of its entire block, while the disruption of an SDCF affects all adjacent blocks. The degree of impact varies depending on whether the affected SDCF is fully or semi-equipped. A semi-equipped SDCF results in asymmetric service interruptions, where the failure of a station on one side of the SDCF may impact only a single block, while a failure on the opposite side may disrupt multiple blocks.

By systematically incorporating these principles, BBFS provides a structured approach to modeling metro disruptions, ensuring that both localized and widespread failures are captured effectively. This methodology enables a more realistic assessment of transit network vulnerability and enhances the ability to develop mitigation strategies for service continuity.

3.3.3 Disruption scenario

In this research, a disruption (or failure) scenario represents a state of the system following one or more disruptions, wherein the network's structural integrity and capacity to accommodate travel demand are assessed. A key characteristic of such scenarios is Extent of Disruption (EoD), which quantifies the number or proportion of affected transit nodes (e.g., stations) that have become inoperative. This measure provides a standardized way to evaluate the impact of disruptions, enabling comparative assessments across different failure scenarios.

3.3.4 Simulation tool: Transition Platform

The Transition Platform (www.transition.city) is an open-source application developed by Polytechnique Montréal, designed primarily for simulating transit trips and calculating travel times within public transportation networks. It employs the Connection Scan Algorithm (CSA) to efficiently generate multiple route alternatives for each transit trip. Although the software was originally intended for trip simulation and travel time estimation under normal operating conditions, this study extends its application by utilizing it as a tool for simulating disruption scenarios and assigning trips under such conditions.

3.3.4.1 Disruption simulation

The transit network is modeled as an undirected graph $G = (V, E)$, following complex network theory and L-space representation. Here, V represents the set of nodes (stations), and E represents the set of edges (connections between stations). The network is implemented within the Transition Platform, where disruptions are simulated based on the Block-Based Failure Strategy (BBFS). This allows for a realistic assessment of how station failures propagate through the network.

3.3.4.2 Trip assignment

As mentioned, the Transition Platform is employed to simulate transit trips and determine optimal travel alternatives for specific origin-destination pairs. The total travel time for each trip is computed by considering the following components:

- Vehicle travel time (time spent onboard transit vehicles)
- Waiting time (time spent waiting for a vehicle)
- Access and egress time (time required to reach the transit system and exit upon arrival)
- Transfer walking and waiting time (where applicable)

To ensure a realistic assessment, the study applies predefined constraints, including:

- A maximum access/egress time of 15 minutes
- A maximum waiting time at the first station of 15 minutes
- A maximum transfer access time of 10 minutes

By integrating disruption modeling with transit trip simulation, the Transition Platform provides a robust analytical framework for evaluating the resilience of metro networks and identifying critical vulnerabilities.

3.4 Transit system vulnerability analysis

In this research, transit system vulnerability is assessed through three perspectives: network-based, serviceability-based, and redundancy-based approaches. The network-based assessment examines how disruptions alter the structural properties of the transit network. The serviceability-based assessment focuses on the impact of disruptions on trip-related characteristics such as travel time and accessibility. Lastly, the redundancy-based assessment evaluates the availability and quality

of alternative routes, determining the network's ability to maintain service continuity. Together, these approaches provide a comprehensive framework for understanding transit system resilience.

3.4.1 Network-based vulnerability analysis

Topological indicators are commonly employed to evaluate metro network connectivity and assess its resilience to disruptions. Traditional approaches use well-established metrics such as the Relative Size of Giant Components (SGC), Network Efficiency, and Network Diameter to quantify structural robustness. These indicators help measure the overall connectivity and efficiency of a transit network under different failure scenarios. The network is modeled using L-space representation in Python, and the NetworkX library is utilized to compute these metrics.

3.4.1.1 Novel connectivity indicators

In addition to conventional metrics, this research introduces two novel indicators: Pure Connectivity (P.C) and Relative Connectivity (R.C). These indicators provide enhanced insights into network cohesion and fragmentation by integrating both SGC and Number of Connected Components (NCC).

Pure Connectivity (P.C) measures the average number of directly connected stations in the network, reflecting the level of interconnectivity among accessible nodes. It is defined as:

$$P.C = \frac{\sum_{i=1}^{N_A} |N_G(i)|}{N_A - 1} \quad \text{Equation 3-1}$$

Where $|N_G(i)|$ represents the number of adjacent stations connected to station, derived from the sum of entries in the corresponding row of the adjacency matrix. N_A denotes the total number of accessible stations. This metric efficiently captures how well stations remain linked to post-disruption.

The computation of P.C involves summing adjacency matrix rows, which has a complexity of $O(N^2)$ for dense networks but reduces to $O(E)$ for sparse metro networks, where E is the number of edges. This makes it computationally feasible, particularly when utilizing sparse matrix representations.

Relative Connectivity (R.C) evaluates the proportion of accessible stations in the disrupted network compared to its original state, incorporating the degree of fragmentation. It is expressed as:

$$R.C = \frac{N_A}{N.C.C \times N} \quad \text{Equation 3-2}$$

where N.C.C is the number of connected components and represents the total number of stations in the network. A higher R.C value indicates better preservation of accessibility, whereas lower value signals increased fragmentation.

Computing R.C requires identifying connected components through graph traversal algorithms such as Depth-First Search (DFS) or Breadth-First Search (BFS), with a complexity of $O(N + E)$. While analyzing extensive networks may demand computational optimizations, this indicator remains practical for metro systems, effectively capturing the impact of disruptions on overall accessibility.

3.4.2 Serviceability-based analysis

The serviceability-based analysis evaluates the feasibility of passenger trips under different disruption scenarios. This assessment focuses on quantifying the extent to which users can complete their trips despite network failures, capturing the degree of service degradation and disruption-induced travel adjustments.

To achieve this, three key indicators are introduced to measure the proportion of trips that remain feasible before and after disruptions:

$$\tau_A = \frac{T_i^A}{T_O} \quad \text{Equation 3-3}$$

$$\tau_B = \frac{T_i^B}{T_O} \quad \text{Equation 3-4}$$

$$\tau_C = \frac{T_i^C}{T_O} \quad \text{Equation 3-5}$$

where:

τ_A represents the number of trips that continue to follow their original shortest path after the i th disruption,

τ_B denotes the number of trips that must reroute onto longer paths due to the i th disruption,

τ_C indicates the number of trips that become infeasible and cannot be completed after the i th disruption, and

T_O is the total number of trips under normal (pre-disruption) conditions.

To compute these metrics, an origin-destination (OD) dataset and transit network data are required. Travel demand data, obtained from OD surveys, is combined with transit network specifications, derived from General Transit Feed Specification (GTFS) datasets. Using the Transition software, trips are initially routed under normal operating conditions and their travel times are calculated. Subsequently, the same procedure is applied under various failure scenarios, allowing for a comparative assessment of trip feasibility, rerouting patterns, and unfulfilled travel demand.

This approach provides valuable insights into the resilience of the transit system by quantifying how disruptions impact passenger mobility.

3.4.3 Redundancy-based analysis

Redundancy is assessed by evaluating the number of available routes connecting each OD pair. While transit systems often provide multiple route alternatives, many of these options may be impractical due to excessive travel times. To ensure that only meaningful alternatives are considered, a quality threshold is applied: a route is classified as an *acceptable alternative* if its travel time does not exceed twice the duration of the shortest available route under normal operating conditions.

This threshold ensures that passengers are not merely presented with a theoretical set of alternative routes but rather with practical alternatives that remain reasonable in terms of travel time. The analysis, therefore, focuses on understanding how disruptions affect both the *quantity* and *quality* of available routes.

3.4.3.1 Redundancy indicators

The redundancy-based analysis examines the impact of disruptions on the availability and quality of alternative routes between origin-destination (OD) pairs in a transit network. A higher degree of redundancy enhances network robustness, providing passengers with multiple viable travel options and improving the system's resilience to disruptions. To quantify the impact of disruptions on network redundancy, the following indicators are defined for each OD pair:

Theoretical Redundancy (R_t): The total number of available simple paths connecting an OD pair.

Acceptable Redundancy (R_p): The subset of routes that meet the predefined travel time threshold.

Unacceptable Redundancy (R_u): The number of routes that fail to meet the quality threshold.

By definition, the total redundancy can be expressed as:

$$\mathbb{R}_t = \mathbb{R}_p + \mathbb{R}_u \quad \text{Equation 3-6}$$

A specific subset of $\mathbb{R}_u$ corresponds to cases where, despite the OD pair remaining physically connected, no acceptable routes exist after a disruption. In such cases, passengers experience unsatisfied demand due to insufficient route quality.

The redundancy indicators are formulated as follows:

$$\overline{\mathbb{R}}_t = \frac{\sum^{D_i} \mathbb{R}_t}{T_i^A + T_i^B} \quad \text{Equation 3-7}$$

$$\overline{\mathbb{R}}_p = \frac{\sum^{D_i} \mathbb{R}_p}{T_i^A + T_i^B} \quad \text{Equation 3-8}$$

$$\overline{\mathbb{R}}_u = \frac{\sum^{D_i} \mathbb{R}_u}{T_i^A + T_i^B} \quad \text{Equation 3-9}$$

$$\tau_D = \frac{T_i^D}{T_i^A + T_i^B} \times 100 \quad \text{Equation 3-10}$$

Where:

T_i^D represents the number of trips that could not be completed after the i th disruption due to a lack of acceptable alternative routes.

$T_i^A + T_i^B$ corresponds to the total satisfied demand after the disruption.

The indicator τ_D captures the proportion of trips rendered infeasible due to degraded route quality, providing insights into how disruptions compromise passenger accessibility within the transit network.

By applying these redundancy indicators, this study evaluates how transit network disruptions influence the availability and usability of alternative routes. The analysis highlights the extent to which redundancy mitigates the negative impacts of disruptions and identifies critical OD pairs that are highly vulnerable due to a lack of alternative routes. These insights contribute to the development of more resilient transit planning strategies, ensuring that redundancy considerations are incorporated into future network designs.

3.5 Intermodality in vulnerability analysis

This section presents a comprehensive framework designed to systematically assess the effectiveness of conventional bus services, as well as other transit and paratransit modes, in mitigating the impacts of metro disruptions. The proposed methodology consists of five sequential steps: (1) data preparation, (2) route assignment and travel time computation under normal operating conditions, (3) disruption scenario definition and simulation, (4) quality assessment of alternative routes, and (5) result modeling. While certain aspects of steps 1 to 3 have been introduced in previous sections, this section provides a concise overview of these elements and focuses primarily on the novel components of the methodology.

The first step involves data preparation, which requires OD and GTFS data. Additionally, disruption data may be necessary depending on the type of disruption being analyzed. Given that this study considers targeted disruptions and scenarios are defined based on both historical disruptions and station importance, disruption data is incorporated into the analysis.

The second step concerns route assignment and travel time computation under normal conditions. This process follows the methodology previously described and is implemented in the Transition Platform software, maintaining consistent settings for traffic assignment procedure.

The third step involves defining disruption scenarios. In this study, all disruptions are station-based and are simulated using BBFS within the Transition software. A total of 12 disruption scenarios are defined, incorporating both single and multiple simultaneous disruptions. Specifically, six scenarios are developed based on station importance, while the remaining six are derived from historical disruption frequencies within the system. Defined disruption scenarios should encompass a broad spectrum of Extent of Disruption (EoD) levels to ensure a comprehensive analysis of network vulnerability. By incorporating scenarios ranging from minor, localized failures to severe, system-wide disruptions, the assessment can capture varying degrees of network stress and resilience. This approach enables a more accurate evaluation of how different levels of disruption impact system performance and user accessibility. A higher number of disruption scenarios enhances the robustness and accuracy of the results, allowing for a more comprehensive evaluation of transit system resilience.

Upon reassignment of the trips in disrupted networks within Transition software, trips are categorized into three distinct groups:

- Trips Utilizing the Same Route as Before Disruption (TUSRBD): Trips that remain unchanged despite the disruption.
- Trips Utilizing Alternative Routes (TUA): Trips that remain feasible but require either route or mode adjustments.
- Substandard Trips (SST): Trips that are no longer feasible within the transit network due to excessive travel time increases or loss of connectivity.

3.5.1 Quality assessment of route alternatives

In metropolitan areas, when disruptions occur in the subway system, most trips still have alternative routes available within the transit network. However, these alternatives are not always of comparable quality to the original routes used before the disruption. The additional travel time imposed by rerouting may lead users to reconsider their reliance on the transit system. Some users may tolerate moderate increases in travel time, while others may shift to alternative modes such as walking, cycling, or private vehicles if the travel time increase exceeds their acceptable threshold. To quantify this effect, it is essential to define quality thresholds for route alternatives, providing a systematic way to evaluate the usability of available transit options during disruptions.

To assess the impact of disruptions on travel times, we introduce a quality indicator, K_i , which measures the ratio of travel time under disrupted conditions to travel time under normal operations for each individual trip i :

$$K_i^S = \frac{ADTT_i^S}{BDTT_i} \quad \text{Equation 3-11}$$

Where:

$ADTT_i^S$ is the travel time of the fastest available alternative for trip i in disruption scenario S ,

$BDTT_i$ is the travel time of trip i under normal (non-disrupted) conditions,

S denotes the disruption scenario, and

i represents an individual trip, ranging from 1 to D_{Total} (the total number of trips).

If a disruption does not impact the chosen route for a trip, the travel time remains unchanged, resulting in $K_i = 1.1$. However, in most cases, disruptions necessitate changes to the route or mode, leading to increased travel times ($K_i > 1$). To evaluate the quality of alternative routes, we define multiple thresholds based on the percentage increase in travel time that users may find acceptable:

- $K_i \leq 1.1$: Alternative routes where travel time increases by 10% or less.
- $K_i \leq 1.25$: Alternative routes where travel time increases by 25% or less.
- $K_i \leq 1.5$: Alternative routes where travel time increases by 50% or less.
- $K_i \leq 1.75$: Alternative routes where travel time increases by 75% or less.
- $K_i > 1.1$: Alternative routes where travel time increases by more than 75%.

For each threshold, we calculate the proportion of trips that remain feasible within the transit system. A feasible trip is defined as one for which an alternative route exists within the transit network, even if the travel time is increased. The percentage of feasible trips for each threshold in scenario S is computed using:

$$FT_K^S (\%) = \frac{|FT_K^S|}{D_{Total}} \times 100 \quad \text{Equation 3-12}$$

where $|FT_K^S|$ represents the number of trips that remain feasible under threshold K in scenario S . By comparing these values across different scenarios, we gain insight into how disruptions of varying magnitudes affect transit accessibility and the extent to which the conventional bus system can serve as a viable substitute.

Beyond measuring the proportion of feasible trips, we also assess the additional travel time burden imposed on users. The Surplus of Travel Time (τ) is an indicator that quantifies the average percentage increase in travel time across all feasible trips in a given scenario. This is expressed as:

$$\tau = \frac{ATT_{N,S} - ATT}{ATT_{N,S}} \times 100 \quad \text{Equation 3-13}$$

Where:

$ATT_{N,S}$ represents the average travel time under normal conditions,

ATT represents the average travel time of feasible trips under disrupted conditions.

This measure provides a system-wide perspective on the overall increase in travel time due to the disruption and helps evaluate the effectiveness of the bus system in mitigating these impacts.

Modeling the Relationship Between Extent of Disruption (EoD) and Travel Impacts

To quantitatively analyze the relationship between disruption magnitude and its consequences on the transit system, we employ regression models. Regression analysis provides a structured way to

examine the extent to which disruptions influence both (1) the percentage of feasible trips after applying each quality threshold and (2) the increase in travel time for those trips.

3.5.2 Regression models for predicting travel outcomes

The explanatory variable in these models is the EoD, which represents the severity of the disruption in the metro network. This can be quantified based on the number of stations affected, the spatial distribution of disruptions, or the loss of connectivity within the network.

The dependent variables in the models include:

Feasible Trips After Applying Each Quality Threshold (FT_K): This measures the percentage of trips that remain possible within the transit system at different quality levels.

Surplus of Travel Time (τ): This quantifies the additional burden imposed on users due to the increased travel times caused by rerouting.

The basic form of linear regression can be expressed as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \epsilon \quad \text{Equation 3-14}$$

Where:

- y is the dependent variable,
- $x_1, x_2, \dots, x_n$ are the independent variables,
- β_0 is the intercept,
- $\beta_1, \beta_2, \dots, \beta_n$ are the coefficients,
- ϵ is the error term (residual). In this study, a comprehensive set of diagnostic evaluations was performed to assess the assumptions of linear regression before selecting it as the modeling approach:

- *Linearity*: Scatter and residual plots.
- *Homoscedasticity*: Breusch-Pagan test.
- *Independence of Errors*: Durbin-Watson test.
- *Normality of Residuals*: Shapiro-Wilk test & Q-Q plots

3.5.3 The influence of the occurrence time of the disruption

The impact of a disruption is not only a function of its spatial extent but also of its timing. Travel demand fluctuates significantly throughout the day, with peak periods experiencing higher passenger volumes and more severe consequences in the event of disruption. To capture this temporal effect, we segment the dataset into three distinct time periods:

- Morning Peak (MP): 6:00 AM – 8:59 AM
- Evening Peak (EP): 3:00 PM – 5:59 PM
- Off-Peak (OP): All other times

For each disruption scenario, we create three separate datasets corresponding to these time periods. We then apply the same regression framework to examine whether the effect of disruption extent on travel outcomes differs across time periods.

Additionally, we introduce an indicator $FT_K^j(\%)$ to quantify the percentage of feasible trips within each time period:

$$FT_K^j(\%) = \frac{|FT_K^j|}{|FT^j|} \times 100 \quad \text{Equation 3-15}$$

where:

$|FT_K^j|$ represents the number of feasible trips under threshold K during time segment j (MP, EP, or OP),

$|FT^j|$ represents the total number of trips in time segment j .

To determine whether the effect of disruption extent varies by time of day, a two-sample hypothesis tests can be conducted. The null hypothesis (H_0) assumes no significant difference in the means of feasible trips across different time periods. Rejecting H_0 would indicate that the impact of disruptions is time-dependent, which has important implications for transit planning and mitigation strategies. In our research, due to a small sample size and non-normality of the data, we utilized the Mann-Whitney U test.

3.5.3.1 Mann-Whitney U test

The Mann-Whitney U test assesses whether the distributions of two independent groups differ significantly. Rather than comparing means, as the t-test does, it compares the ranks of the data

from both groups. The null hypothesis (H_0) assumes that the distributions of the two groups are identical, while the alternative hypothesis (H_a) posits that there is a difference between the two distributions.

The Mann-Whitney U statistic is calculated as follows:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \quad \text{Equation 3-16}$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 \quad \text{Equation 3-17}$$

Where:

n_1 and n_2 are the sample sizes of the two groups,

R_1 and R_2 are the sums of the ranks for each group.

The test statistic U is the smaller of U_1 and U_2 . The sum of the ranks R_1 and R_2 is obtained by ranking all observations from both groups together, assigning the lowest rank to the smallest observation, and so on. The Mann-Whitney U statistic is then compared to critical values from the Mann-Whitney distribution, or alternatively, a p-value can be obtained.

For large sample sizes, the U statistic can be approximated by a normal distribution, and the z-value is calculated as:

$$z = \frac{U - \mu_U}{\sigma_U} \quad \text{Equation 3-18}$$

$$\text{Where: } \mu_U = \frac{n_1 n_2}{2}, \sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

In our study, we interpreted the results by comparing the p-value obtained from the test with a predetermined significance level (usually 0.05). If the p-value was less than 0.05, we rejected the null hypothesis, concluding that the distributions of the two groups were significantly different.

The Mann-Whitney U test offers a powerful alternative to parametric methods, especially when working with small or non-normally distributed datasets. It allowed us to make robust inferences about our data, providing reliable results despite the limited sample size.

3.6 User vulnerability assessment

User vulnerability analysis can be assessed using two primary approaches: relative vulnerability and absolute vulnerability. Relative vulnerability analysis ranks population segments based on their susceptibility to disruptions, offering a comparative perspective rather than an absolute determination of whether a segment is vulnerable. In contrast, absolute vulnerability analysis relies on predefined thresholds to classify specific population groups as vulnerable or not.

This study primarily adopts a relative vulnerability approach, proposing a systematic framework to rank population segments based on their exposure to subway disruptions. While this framework is designed for relative assessment, it shares methodological similarities with absolute vulnerability analysis and can serve as a foundation for future research exploring absolute vulnerability classifications.

The focus of this framework is on short-duration subway disruptions, where no mitigation measures are implemented by transit authorities. Given the complementary role of bus networks in urban transit systems, particularly in cities where subway and bus services are integrated under a common fare structure, it is, in this study, assumed that affected passengers primarily rely on conventional bus routes during disruptions. However, the proposed methodology is adaptable and can be extended to include other alternative modes such as bikesharing, carsharing, or on-demand mobility services.

3.6.1 User segmentation

A critical step in assessing transit user vulnerability is the segmentation of passengers into distinct population groups. This segmentation enables a more precise evaluation of how different user categories experience subway disruptions, facilitating targeted analyses and policy interventions. By differentiating user groups based on key demographic and trip characteristics, we can systematically assess their exposure to disruptions and identify the most vulnerable segments. This structured approach ensures that resilience-enhancing strategies are designed to address the specific needs of affected populations, promoting both robustness and equity in transit planning.

To establish meaningful segmentation, we utilized OD data to categorize users based on age, gender, occupation, and primary trip purpose. The segmentation process focused on frequent and simple trip chains, ensuring consistency in the analysis of travel behaviors. For instance, for

individuals whose primary trip purpose is commuting to work, we identified Home-Work-Home trips, which typically occur during morning and evening peak hours. More complex trip chains, such as Home-Work-Leisure-Home, were excluded to maintain analytical clarity and focus on essential commuting patterns. A similar approach was applied to students, where Home-School-Home trips were prioritized while disregarding additional discretionary trips such as Home-School-Leisure-Home. This filtering ensures that the assessment reflects typical, recurrent travel patterns, allowing for a more controlled evaluation of disruption impacts.

Following this structured methodology, ten population segments were defined based on distinct demographic and travel attributes (Table 1). These segments capture key variations in subway usage, ensuring a comprehensive representation of transit-dependent groups:

- S1 (Children): Students aged 5–15 making school-related trips.
- S2 (Men Students) & S3 (Women Students): Male and female students aged 16–24, whose transit usage includes both education-related trips and part-time employment commutes.
- S4 (Young Men Workers) & S5 (Young Women Workers): Employed individuals aged 25–34, representing a workforce segment highly reliant on transit for daily commuting.
- S6 (Men Workers) & S7 (Women Workers): Workers aged 35–64, accounting for a broad range of job types and travel patterns.
- S8 (Adults, Men & Women): Individuals aged 35–64 who undertake both work and non-work-related trips, reflecting diverse transit usage beyond employment needs.
- S9 (Elderly Men) & S10 (Elderly Women): Individuals aged 65 and above, a group with unique mobility challenges and distinct travel behaviors.

The segmentation framework employed in this study is designed to capture the heterogeneity in travel behavior among different transit user groups while maintaining analytical clarity and policy relevance. By categorizing users based on age, gender, occupation, and trip purpose, the segmentation accounts for variations in trip regularity, travel flexibility, and reliance on transit services. For example, children (S1) and students (S2, S3) typically exhibit highly structured travel patterns, with predictable school schedules and limited alternative travel options, making them particularly susceptible to disruptions. Young workers (S4, S5) and adult workers (S6, S7) represent the backbone of peak-hour transit demand, and their vulnerability is tied to work-related

time constraints and modal dependence. The inclusion of a distinct adult segment (S8) acknowledges that not all individuals in this age range travel solely for work, capturing a wider range of transit dependency. Finally, elderly users (S9, S10) are classified separately due to potential physical mobility limitations and different time-of-day travel patterns, ensuring that their specific needs are considered.

This segmentation strategy balances granularity and interpretability, providing a structured basis for subsequent analyses of transit vulnerability. By defining user groups based on demographic attributes and travel behavior, it enables a targeted assessment of vulnerability while remaining practical for policymakers aiming to enhance transit resilience and equity. This approach ensures that disruptions are assessed with greater precision, ultimately supporting data-driven decision-making in transit system resilience planning.

3.6.2 Transit Robustness Index (TRI) calculation

To systematically evaluate the impact of subway disruptions on different population segments, we introduce the Transit Robustness Index (TRI) as a measure of users' resilience to disruptions. The TRI quantifies the proportion of trips that remain viable under a disrupted network, considering predefined thresholds that define acceptable travel time increases. This metric provides a standardized approach to assessing transit system robustness across different disruption scenarios and user groups.

Given that travel time increases may have different implications for different types of trips, we employ a dual-threshold approach:

1. **Relative thresholds** (percentage-based): These capture proportional increases in travel time. A 10% or 25% increase ensures that disruptions are assessed based on their proportional impact, rather than absolute delays.
2. **Absolute thresholds** (fixed time-based): These account for delays that might be more perceptible for short trips. A 5-minute or 10-minute additional delay may be acceptable for most users, making this threshold crucial for evaluating disruption tolerance.

Thus, a trip is considered **resilient** if it falls within one of the following categories:

- No increase in travel time (baseline resilience).
- Travel time increase of $\leq 10\%$.

- Travel time increase of $\leq 25\%$.
- Travel time increase of ≤ 5 minutes.
- Travel time increase of ≤ 10 minutes.

By applying these thresholds, the TRI is computed for each population segment across all disruption scenarios, allowing a structured comparison of vulnerability levels. A lower TRI value indicates that a segment is more severely affected by disruptions, while a higher TRI suggests a greater ability to withstand transit interruptions. The robustness index serves as a foundational metric for identifying the most and least resilient population groups, guiding transportation planners toward targeted mitigation strategies.

3.6.3 Cross-scenario vulnerability comparison

While the TRI effectively ranks user segments within a single disruption scenario, a direct comparison across multiple scenarios presents challenges. The severity of disruptions varies significantly, with some scenarios involving minor interruptions and others leading to extensive network failures. A naive comparison of TRI values without accounting for the EoD can lead to misleading conclusions.

To address this, we employ a regression-based approach that models the relationship between EoD (independent variable) and TRI (dependent variable) for each population segment. This method ensures that vulnerability assessments remain robust across varying disruption scales.

3.6.3.1 Regression models for robustness analysis

1. *Linear regression:*

A straightforward yet interpretable model, linear regression estimates the direct effect of EoD on TRI, expressed as:

$$TRI = \beta_0 + \beta_1 \cdot EoD + \epsilon \quad \text{Equation 3-19}$$

Where β_1 represents the sensitivity of a population segment's resilient to disruption severity. Given the limited sample size (12 disruption scenarios), linear regression provides a transparent analytical framework but requires validation due to potential linearity assumptions.

2. *Decision tree regression:*

Recognizing the possibility of non-linear patterns in the data, we apply a decision tree regression model, which segments EoD into discrete intervals and models TRI

accordingly. This method identifies thresholds at which disruptions begin to significantly impact different user groups. However, decision trees can be sensitive to overfitting, necessitating careful tuning of depth and pruning strategies.

3. ***Random forest regression:***

To mitigate overfitting and enhance model robustness, we further employ a random forest approach. By averaging predictions across multiple decision trees trained on different data subsets, random forest regression captures complex relationships between EoD and TRI while improving generalizability.

CHAPTER 4 ARTICLE 1: A NEW FAILURE STRATEGY TO ENHANCE THE VULNERABILITY ASSESSMENT OF URBAN METRO NETWORKS

Authors: Kaveh Rezvani Dehaghani¹, Catherine Morency¹

¹Department of Civil, Geological and Mining Engineering, Polytechnique Montréal.

Montreal, QC, H3C 3A7, Canada.

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ABSTRACT

Urban Metro Networks (UMNs), as an integral component of transit systems, play a crucial role in the overall efficiency of urban transportation. Disruptions in their operations can significantly impact daily travel. Consequently, numerous studies have focused on assessing the vulnerability of UMNs. However, many of these studies have not adequately considered the infrastructural characteristics of UMNs. This study seeks to emphasize the importance of incorporating infrastructural features—particularly Stations with Direction-Change Facilities (SDCFs)—to enhance the accuracy of research findings. To address this, we introduce the Block-Based Failure Strategy (BBFS), a novel approach that accounts for SDCFs. The Montreal UMN is selected as the case study, and its vulnerability to targeted disruptions is assessed using both BBFS and a conventional strategy known as the Node-Based Failure Strategy (NBFS). The results reveal a significant disparity between simulations conducted using BBFS and NBFS. When SDCFs are considered and BBFS is applied, the network is deemed fully degraded after 9 targeted disruptions. In contrast, the NBFS simulation suggests that the network can withstand up to 35 targeted disruptions before becoming completely degraded—a figure much higher than what would realistically occur.

4.1 Introduction

Public Transportation Systems (PTS) play an important role in people's daily lives, with Urban Metro Networks (UMNs) serving as a cornerstone of PTS in large metropolises, significantly contributing to the efficient mobility of passengers. As such, ensuring the reliable operation of

these systems is a constant concern for authorities. Various events, such as incidents, technical problems, and human behavior, can impact the performance of UMN, leading to delays that result in substantial time losses and significant costs for both operators and users. For example, the New York City Comptroller (2017) provided a basic calculation predicting the large-scale economic consequences of subway delays, estimating that the city's losses in 2016 ranged between \$170 million and \$389 million based on factors such as ridership, delay duration, and an hourly average wage of \$34 (Stringer, 2017). Unplanned rail disruptions are common in large cities and are among the primary causes of delays in UMN. For instance, data on subway disruptions show that the Toronto subway system, managed by the TTC, experienced 144 disruptions in 2015 (Itani & Shalaby, 2021).

The frequency of disruptions in UMN and their significant socio-economic impacts have been addressed in several studies (X. Wu, Dong, Tse, Ho, & Lau, 2018) (H. Xu & Li, 2019) (Y. Wang & Tian, 2020), (Y. Wu et al., 2021). These studies have proposed various approaches for analyzing the vulnerability and robustness of UMN. Although these studies have employed diverse methodologies and produced valuable results, certain key features have been overlooked.

Previous research has seldom considered the infrastructural and operational characteristics of the network, which can significantly affect the validity of topological vulnerability analyses. This paper aims to bridge this gap by demonstrating how incorporating the infrastructural features of UMN can enhance the accuracy of vulnerability assessments

The paper begins with a comprehensive literature review in Section 2, where the research gap is detailed. Section 3 presents the proposed methodology, followed by a discussion of the case study on Montreal's Urban Rail Transit Network (URTN) in Section 4. The findings are presented and analyzed in Section 5, and the paper concludes with a summary in Section 6.

Nomenclature

PTS	Public Transportation System
UMN	Urban Metro Network
SDCF	Station with Direction Change Facilities
DCF	Direction Change Facilities
BBFS	Block-Based failure Strategy
NBFS	Node-Based failure Strategy

4.2 Background

Vulnerability is a relatively new concept within the field of transportation engineering, and there is no universally accepted definition for it. However, its origins as a research topic can be traced back to (Berdica, 2002) and (D'este & Taylor, 2003). Berdica (2002) defined the vulnerability of transportation networks as *"a susceptibility to incidents that can result in considerable reductions in road network serviceability"*. This definition has been generalized by various scholars over the past decade, including (Jenelius et al., 2006), (Berdica & Mattsson, 2007), and (Jenelius & Mattsson, 2012). Another related term that has been gaining considerable attention in this field of transportation engineering in recent years is robustness that versus the term vulnerability is used. In other words, the vulnerability of a network decreases with increasing robustness. This term owes its emergence to (Scott et al., 2006) when Network Robustness Index (NRI) was defined. Then, Sullivan et al., (2010) defined network robustness as "the degree to which the transportation network can function in the presence of various capacity disruptions on component links". Next definitions were presented by Snelder et al., (2012) and de Oliveira et al., (2016).

Using system science and complex network theory, Jianhua Zhang & Wang, (2019) abstracted the topological structure of the Nanjing metro network to analyze its vulnerability. This research focused on targeted failures based on two strategies: degree-based removal and boarding volume-based removal which were called LDNBA and HLNBA respectively. Moreover, in this research, a dynamic failure analysis was presented in which it was assumed that when a station is disrupted, the passengers of this station will be evacuated to its neighbor stations, that was conceptually similar to cascading failure in (Lu, Zhang, Xu, Cui, & Li, 2022). Zhu et al., (2018) topologically studied the robustness of 5 metro networks around the world. The relative size of the first giant connected component (GCC) was considered as the robustness indicator, and seven targeted failure strategies including degree centrality, betweenness centrality, fracture coefficient, collective influence, K-shell decomposition method, levels of tree-based attack coefficient, and Breadth-tree coefficient were used. The random failure strategy was also employed in this research. The results of this research illustrated that the most effective failure strategy is the classical highest betweenness centrality strategy. Jianhua Zhang, Wang, & Wang, (2018) by using the complex network, analyzed and compared the vulnerability of the metro network in three cities (Shanghai, Beijing, and Guangzhou) in China. This research concentrated only on targeted failures; both degree-based and betweenness-based removal were employed in this research. X. Wang et al.,

(2017) quantified and compared the robustness of 33 metro networks around the world. They employed both random and targeted (degree-based removal). In this research, ten robustness indicators were calculated for each network: robustness indicator, effective graph conductance, reliability, average efficiency, clustering coefficient, algebraic connectivity, average degree, natural connectivity, degree diversity, and meshedness coefficient.

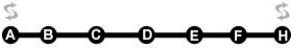
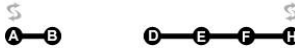
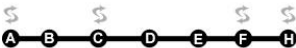
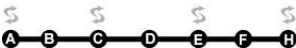
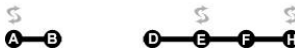
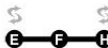
The primary gap in the vulnerability assessment of UMN lies in an erroneous assumption frequently observed in topological assessment studies: the disregard of infrastructural and operational features of UMN that distinguish transit network analysis from pure graph analysis. Specifically, it is important to recognize that in most metro systems, trains cannot change direction at every station; Direction-Change Facilities (DCFs) are installed only at specific stations. Adding such facilities requires extensive construction efforts, such as tunnel digging, which entails significant time and financial investment. Moreover, if these facilities are to be added after a line has been launched, construction must be limited to non-operational hours (e.g., from 1 am to 5 am), making it practically infeasible to equip every station with such facilities.

The basis of the node-failure mechanism is that when a node is disrupted, the node and its connected links are removed from the network (X. Li et al., 2018), and it is generally assumed that the remaining network will continue to function as before. However, this assumption holds true only if every station is equipped with a facility to change direction. If a station is disrupted and lacks such a facility, trains in motion will eventually reach the last operational station in their direction within a few minutes, as the next station will be occupied by the preceding train. Table 4-1 provides further details on this concept.

Table 4-1 demonstrates a hypothetical metro line consisting of 7 stations. To well clarify the role that Station with Direction Change Facilities (SDCF) play in UMN, 4 scenarios are defined. For each scenario, the metro line is demonstrated in 3 situations: Normal state (before the disruption), a while after the disruption, and a few minutes after the disruption.

- Scenario #1: Only terminals are SDCF. By assuming a disruption in station C, firstly this station and the connected links are removed (situation 2). Then, since the train cannot return from B to A, or from D to H, after a few minutes all the trains have to stop at the stations, and the network inevitably will completely be disrupted.

Table 4-1 Role of SDCFs in a metro line

	Normal state Before the disruption	Disrupted station	After disruption	
			Situation (2): A while after the disruption	Situation (3): A few minutes after the disruption
Scenario #1		C		
Scenario #2		C		
Scenario #3		D		
Scenario #4		C		

- Scenario #2: All of the stations are SDCF, meaning that after occurrence a disruption in station C, trains easily can return from B to A, or from D to H. Hence, the rest of the network can function as before.
- Scenario #3: Station C, F and terminals are SDCF. After the occurrence of a disruption in station D, firstly this station and the connected links are removed (situation 2), then since trains cannot return to H from E, station E will be disrupted very soon. Hence, two segments: A to C, and F to H can continue as before.
- Scenario #4: Terminals and stations C and E are SDCF. If a disruption occurs in station C, firstly this station and links connected to the station are removed (situation 2). Then, since trains cannot return from B to A, this section of the network will be disrupted very soon, and section E to H is the only segment that can continue after occurrence a disruption in station C.

In previous research, typically the second situation is considered as a disrupted network which is an unstable state. Networks can only continue to work for a few minutes under such circumstances, and after a few minutes, it inevitably turns into situation 3. Hence, it is not logical to study the vulnerability of the networks under such circumstances. However, since previous research did not consider the CDFs, they do not have a conception of situation 3, meaning that in such research,

only a graph is analyzed instead of a real rail network. This matter gains more significance when the authors try to compare the vulnerability of some UMN, because their results are significantly different from what in the real world will happen. Furthermore, the mentioned concept also calls the main logic behind the cascading failures into question. The pillar of cascading failure is that after the occurrence of a disruption in one station, the passengers of the station will try to take themselves to neighbor stations (H. Chen et al., 2021). While according to the above-mentioned concept, this assumption only can be correct once both the previous and next stations have CDFs. Otherwise, by removing one station, all of its previous and subsequent stations before a SDCF will be disrupted, and it is not possible for passengers to use them.

4.3 Methodology

4.3.1 UMN modeling

(Von Ferber et al., 2009) presented a comprehensive view of transit systems by modelling the system with alternative network topologies in multiple spaces. Currently, one of the most often used techniques for representing the topological structures of transit networks is L-space considering the transit network as a graph. This graph represents each station by a node, and a link between nodes indicates that there is at least one route that services the corresponding stations consecutively (M. Taylor, 2017). The metro network will be formulated in L-space as follow:

$$G_M = (V_M, E_M) \quad \text{Equation 4-1}$$

$V_M = \{m_i\}_{i=1}^{N_M}$ refers to a set of nodes (i.e., metro stations)

$E_M = \{(m_i, m_j) | 1 \leq i, j \leq N_m, i \neq j\}$ is a set of edges, showing the direct connectivity between stations.

$N_M = |V_m|$ is the total number of metro stations

How may operational features of the system affect the UMN model?

As mentioned above, the graph's order is related to the stations of the UMN. On the other hand, this matter should be known that the service offered by operators may vary over time. For example, some stations serve the users only in specific hours of the day (like what happened in line #3 of the Tehran metro during the first months of launching the line), or some lines only work during rush

hours. Therefore, the number of nodes and links, topology, and all network configurations may change over time. Hence, vulnerability should be analyzed as a function of time, even in topological assessments. For this, GTFS data for one year should be studied. If a change in the number of active lines or stations is found, this matter should be considered in the vulnerability assessment; otherwise, the topology of the network is not a function of time, and only one topological vulnerability analysis would be sufficient.

How may infrastructural features of the system affect the UMN model?

As it was broadly discussed in section 2, Stations with Change-Direction Facilities (SCDFs) as one of the most important infrastructural features of the network, could considerably impact the results of the vulnerability assessment of UMN. To consider the role that these facilities play in a network, we defined rail blocks. Each block consists of consecutive stations and rail trunks that are located between two SCDFs. For example, in the third scenario of table 1, the line is made up of 3 blocks: block #1 consists of stations A, B, and C; block #2 consists of stations C, D, E, and F; block #3 consists of stations F and G. As is visible, SCDFs belong to all blocks connected to each other by the SCDF.

4.3.2 Fundamental topological attributes of UMN

Each of the following attributes demonstrates the significance of the stations using a separate reasoning.

The *node degree* of V_i is shown by k_i which is the total number of edges connected to V_i . The average node degree of an UMN is calculated by formulation below:

$$\bar{k} = \frac{1}{N} \sum_{i=1}^N k_i \quad \text{Equation 4-2}$$

Where N is the total number of nodes(stations) in a UMN.

The *betweenness centrality* of V_i is shown by B_i which is the proportion of shortest paths passing through node i to all the paths connecting nodes.

$$B_i = \sum_{p \neq q \neq i} \frac{\tau_{pq}(i)}{\tau_{pq}} \quad \text{Equation 4-3}$$

Where: $\tau_{pq}^{(i)}$ represents the number of shortest paths passing through node i between node p and node q , and τ_{pq} represents the number of all the shortest paths between node p and node q which can be calculated in an undirected network by the following formulation:

$$\tau_{pq} = \frac{(N-1) \times (N-2)}{2} \quad \text{Equation 4-4}$$

4.3.3 Vulnerability assessment indicators

This section describes the stability, accessibility, and node proximity of the network using two key topological features. In network-based vulnerability analysis, these topological indicators are frequently used as vulnerability indicators.

4.3.3.1 Relative size of the giant component

Due to disruption, the network may be split into several connected subgraphs. The term giant connected component refers to the subgraph with the biggest order. This indicator, which is represented by RSGC, evaluates the UMN's vulnerability from the perspective of network distance. The higher the RSGC, the better the stability of the network.

$$RSGC = \frac{N'}{N_0} \quad \text{Equation 4-5}$$

Where N' represents the size of the giant component after some nodes break down, and N is the original size of the network.

4.3.3.2 Network diameter

This indicator, which is denoted as D , is related to maximum distance between two nodes in the giant component. This indicator is formulated as follows:

$$D = \max \{d_{ij}\} \quad \text{Equation 4-6}$$

Where d_{ij} is the distance between V_i and V_j , and g is giant component, meaning that:

$$\begin{cases} i \text{ and } j \in g & d_{ij} = \text{finite value} \\ i \text{ or } j \notin g & d_{ij} = 0 \end{cases}$$

4.3.3.3 Degradation

This indicator is related to the percentage of the stations that are no longer in service, which can be calculated by using the formulation below:

$$Deg = (1 - \frac{N''}{N}) \times 100 \quad \text{Equation 4-7}$$

Where N'' represents the number of the station which are out of service due to the disruption(s).

Another concept that is employed to illustrate the level of degradation in a network, is the degradation threshold that can be defined as the number of disruptions that should occur in the network, such that the number of active stations is equal to a predetermined fractions of the stations of the original network. For instance, $Deg_{25\%}$ is related to the number of the disruption(s) at which more than a quarter of the network's stations are degraded; in other words, the number of the disruption(s) at which the network degradation exceeds 25%. Similarly, $Deg_{50\%}$ represents the number of the disruptions at which more than 50% of the stations are out of service, and $Deg_{100\%}$, in a similar way, corresponds to the number of the disruptions at which network is completely degraded.

4.3.4 Failure mechanism

In this section, two questions should be addressed: 1) Disruptions occur in which component(s) of the network? 2) How are the components undergoing a disruption chosen?

Regarding the first question, there exist two general cases: the disruption may occur at the links (rail trunks) or in the nodes (stations). In the link-based failure strategy, by occurrence a disruption in a link, only that link is removed from the network. While the core of the node-based failure strategy is that when a node is disrupted, that node and all the links connected to the node are removed. Concentrating on station disruption, we, in this research, by the combination of the two mentioned strategies introduced a new failure strategy entitled *Block-Based Failure Strategy (BBFS)*. In BBFS, if a disruption occurs in a node not only the node and the connected links are removed, but also the block(s) that the node belongs to, should be removed. In simple terms, if a disruption occurs in a station other than the first or last station of a block, only that block is removed. However, if one of the first or last stations -which is SCDF- is disrupted, all the blocks connected to that SCDF will be deleted. Indeed, to implement the BBFS the network needs to be considered as an abstract graph in which the nodes correspond to SCDFs, and the edges are blocks representing regular stations. To clarify the concept, exemplifications of how this strategy works are included in section 5.

To address the second question, there are two general cases as well: nodes can be removed randomly and selectively. In this research, we employ the latter pattern called targeted failure by which -once research concentrates on network topology, nodes could be removed by descending order of degrees or betweenness centrality. Since there exists quite little diversity in the node degree of the Montreal metro network ($k=2$ for more than 85% of stations), this research applied the betweenness-based pattern to assess the vulnerability of MN, meaning that at each step, the network betweenness centrality distribution is computed, and in the simulation scenario, the node with the highest betweenness centrality is chosen to be removed.

4.4 Case study

The Montreal metro consisting of 4 lines and 68 stations is considered as the case study of the research. As Figure 4-1 indicates, the highest node degrees are related to the 4 transfer stations in the network including station #18: BERRI-UQAM ($k_{18}=5$) connecting the yellow line, green line, and orange line to each other, station #11: LIONEL-GROULX ($k_{11}=4$) connecting the green line to the orange line, stations #24: JEAN-TALON ($k_{24}=4$), and station 7: SNOWDON ($k_7=3$) connecting the blue line to the orange line. More details concerning the network are presented in Table 4-2.

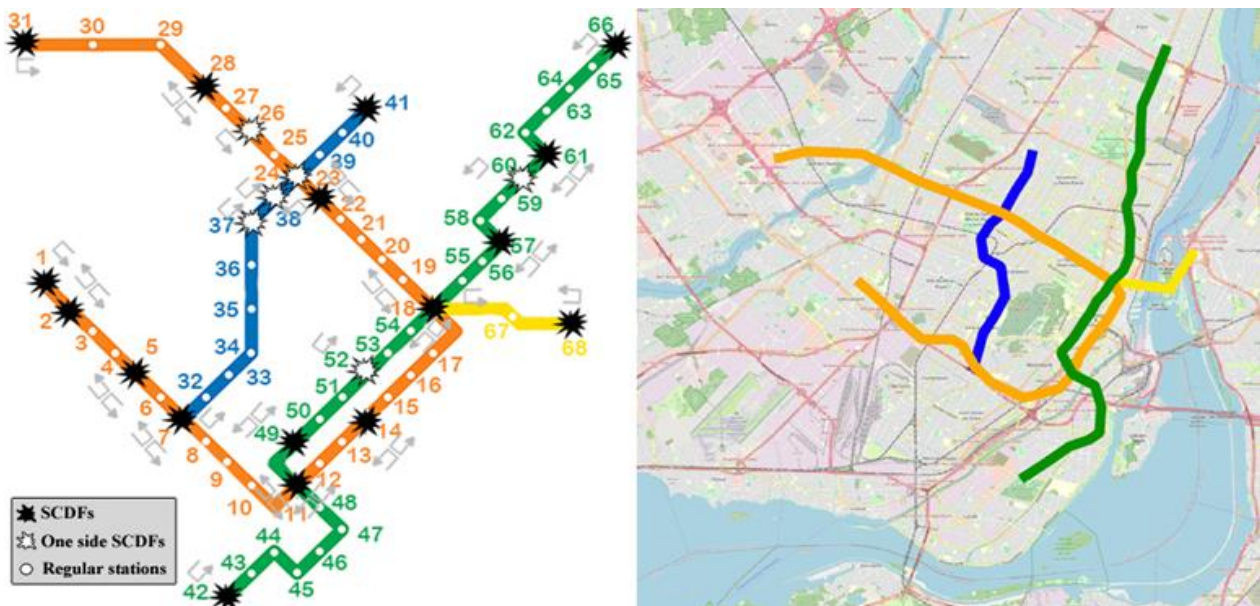


Figure 4-1 Montreal urban metro network

Table 4-2 Montreal UMN's characteristics (Normal state)

General information		Indicators	
No. of accessible hubs (transfer stations)	4	Diameter	29
No. of accessible edges (rail trunks)	69	Degradation	0%
Network attributes		Average Degree	2.029
		No. of the connected component	1
Maximum Degree	Value	5	
	Correspond station	Berri-UQAM	
Maximum Betweenness	Value	0.48	
	Correspond station	Berri-UQAM	

4.4.1 Defining the blocks

According to data received from the system operator (STM), there exist 24 SCDFs in the network demonstrated by stars in Figure 4-1. Six of them are the first/last stations of the lines needing only to provide the DCF to one side (stations #1 and 31 in the orange line, stations #42 and 66 in the green line, station #41 in the blue line, and station 68 in the yellow line). In six of them (stations #23, 26, 37, 38, 52, and 60) which are marked by white stars, it is possible for trains to change their direction only to the side indicated by the arrow. In 12 SCDFs (stations # 2, 4, 7, 11, 14, 18, 28, 43, 49, 57, and 61) trains can change direction to both sides if necessary. Hence, 24 blocks are defined on Montreal's URTN represented in Table 4-3.

Table 4-3 Details of Montreal UMN blocks

Block#	Stations	Block#	Stations	Block#	Stations
1	1,2	9	24,25,26	17	11,49
2	2,3,4,5	10	26,27,28	18	49,50,51,52
3	5,6,7	11	28,29,30,31	19	52,53,54,18
4	7,8,9,10,11	12	7,32,33,34,35,36,37	20	18,55,56,57
5	11,12,13,14	13	37,38	21	57,58,59,60
6	14,15,16,17,18	14	38,24	22	60,61
7	18,19,20,21,22,23	15	24,39,40,41	23	61,62,63,64,65,66
8	23,24	16	42,43,44,45,46,47,48,11	24	18,67,68

4.4.2 Implementation of BBFS

Based on the betweenness-based removal pattern, the station with the highest betweenness value should be selected in the targeted failure strategy. According to Table 4-2, the highest betweenness

is related to station #18 (Berri-UQAM). Hence, this station should be selected to be removed from the network. As Berri-UQAM is an SCDF, based on BBFS rules, all the blocks connected to this SCDF will be out of service. Therefore, after the first targeted disruption, six blocks (#6, 7, 18, 19, 20, and) including 17 stations would be out of service. Figure 4-2 is intended to illustrate the network configuration of after the first targeted failure.

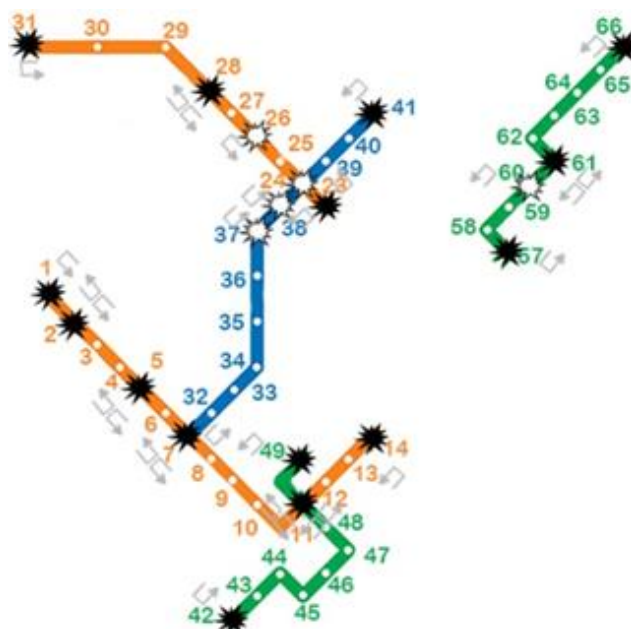


Figure 4-2 Configuration of Montreal UMN after the first targeted disruption

Table 4-4 Characteristics of Montreal UMN after the first disruption

General information		Indicators	
No. of accessible nodes (stations)	51	Relative Size of the Giant Component	80%
No. of accessible hubs (transfer stations)	3	Diameter	26
No. of accessible edges (rail trunks)	49	Degradation	25%
Network attributes		Average Degree	1.922
		No. of the connected component	2
Maximum Degree	Value	4	
	Correspond station	Lionel-Groulex (#11) & Jean-Talon (#24)	
Maximum Betweenness	Value	0.399	
	Correspond station	Snowdon (#7)	

After each disruption, the network should be analyzed, and the characteristics of the network should be recalculated to identify the next element that should be removed. More details about the network after the first targeted disruption is presented in the Table 4-4. The maximum betweenness after the first disruption, according to Table 4-4, is related to station #7 (Snowdon); so, Snowdon is the next station that should be removed in the second step. Similar to step 1, Snowdon is a SCDF, and by removing this station, three blocks (#3, 4, and 12) will be out of service.

4.5 Results and Discussion

In this section, we study the changes in the topological vulnerability indicators of Montreal UMN. Both BBFS and NBFS are separately employed to simulate the targeted disruptions and results are compared to illustrate the importance of possession of a correct assumption and employing an appropriate failure strategy.

4.5.1 Simulation based on BBFS

The mechanism of simulations based on BBFS was described in detail in the previous sections and the first disruption was correspondingly simulated. By recommencing the same failure strategy, after the ninth targeted disruption, no active stations remain in the network. Table 4-5 provides more information about the selection order of stations and characteristics of the network after each disruption, and Figure 4-3 shows the network configuration after each disruption.

Table 4-5 Characteristics of Montreal UMN during the degradation

Disruption Number	Betweenness centrality		Topological Vulnerability Indicators					
	Highest amount	correspond station	RSGC (%)	ND	Degradation	No. of Connected components	Average Degree	No. of Accessible Stations
1 st D	0.399	Snowdon (#7)	80	26	25.00%	2	1.922	51
2 nd D	0.068	Jean-Talon (#24)	34	10	39.71%	4	1.805	41
3 rd D	0.071	Lionel-Groulex (#11)	39	10	54.41%	4	1.742	31
4 th D	0.131	Viau (#60)	52	9	72.06%	3	1.684	19
5 th D	0.061	De La Savane (#3)	38	4	80.88%	3	1.538	13
6 th D	0.055	Cartier (#29)	40	3	85.29%	3	1.4	10
7 th D	0.2	Prefontaine (#58)	67	3	91.18%	2	1.333	6
8 th D	0	-	100	1	97.06%	1	1	2
9 th D	-	-	0	0	100.00%	0	0	0

As the above table indicates, after each disruption, the vulnerability indicators are recalculated to demonstrate the network's gradual decline and how the network is impacted during a complete degradation process. Diagrams in Figure 4-4 confirm that in the targeted failure pattern, the significant changes in the indicators are related to the first disruptions. After the first and the second disruption, RSGC decreased to 60% and 35% respectively, showing that only after two disruptions, the network considerably loses its stability. The diagram also shows that the last six disruptions do not make significant changes in RSGC. Considering the Network Diameter (ND) diagram, after the first disruption (when Deg=25%), ND decreased from 29 to 26 (around 10%), and after the second disruption (when Deg=40%), ND decreased from 26 to 10 (more than 60%). This matter shows that after two targeted disruptions, travel length significantly decreased, and the network loses its proximity and accessibility. Similar to RSGC, the ND diagram also demonstrates the last disruptions do not create important changes.

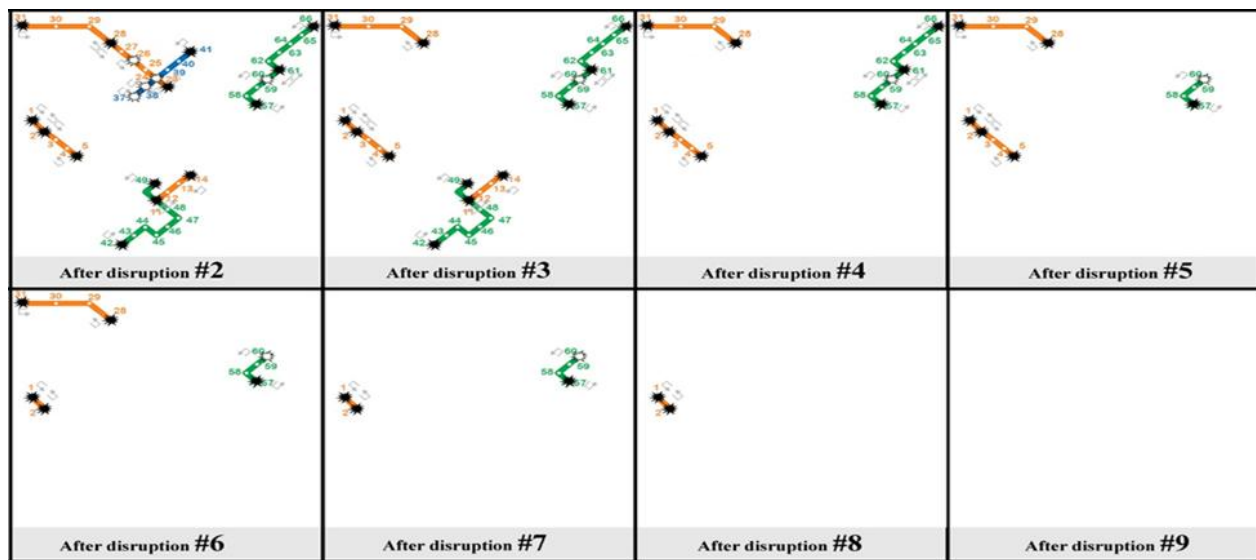


Figure 4-3 Degradation process of Montreal UMN

4.5.2 Simulation based on NBFS

Based on NBFS, when a node is disrupted, the node and the links connected to the node are removed. Hence, by adopting the NBFS, after 35 targeted disruptions based on descending order of betweenness centrality, all the stations would be out of service. According to Figure 4-4, the first disruption (when Deg=1.5%), decreased the RSGC to 78%, showing that it was less impactful than the first disruption simulated based on BBF. However, after the second disruption, RSGC reached 34% which was more impactful than the second disruption simulated based on BBF.

Results also show that after the fifth disruption (when $Deg=7.5\%$), there are no significant changes in RSGC. This matter shows that in NBFS-based simulation, the network loses its stability sooner than simulation based BBFS.

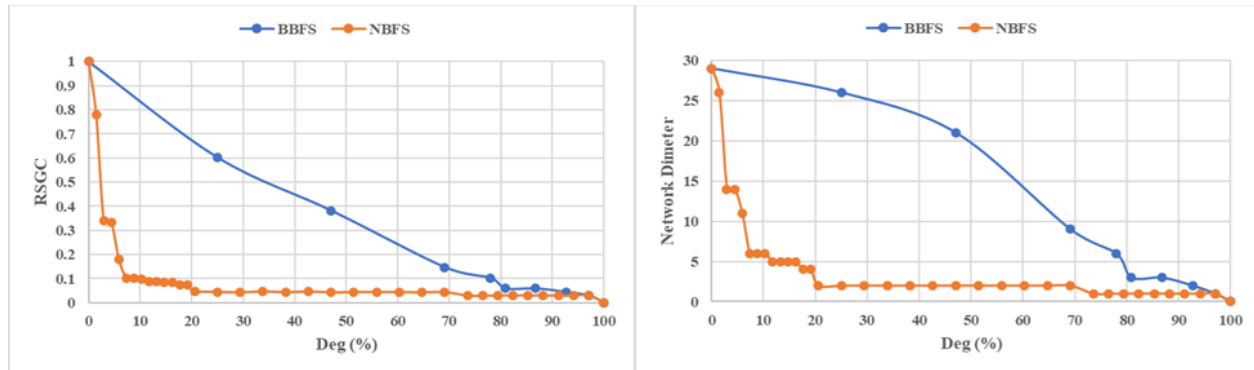


Figure 4-4 Comparing the vulnerability assessment indicators of BBFS and NBFS implementations.

Considering the ND diagram (Figure 4-4), the first disruption simulated based NBFS had exactly the same impact as the first BBFS-based disruption, decreasing ND from 29 to 26. However, the second NBFS-based disruption creates a more significant change in ND than the second BBFS, showing that the network loses its proximity and accessibility sooner when NBFS is considered. Moreover, after the 5 disruptions (when $Deg=7.5\%$), ND decreased by around 80%, strengthening the assumption that in the targeted disruptions, the significant changes correspond to the first disruptions.

4.5.3 Comparing the simulations based on BBFS and NBFS

With regard to comparison of these two simulation strategies, the most important distinction is related to number of the stations that in each disruption will be out of service, or in the other word, the number of disruptions leading to a complete degradation in the network. As Figure 5 indicates, when BBFS is implemented, $Deg_{25\%}=1$, $Deg_{50\%}=3$, and $Deg_{100\%}=9$, meaning that in first disruption at least 25% of the stations, in the third disruption at least 50% of stations, and at the ninth disruption all the stations are degraded. While by implementation of NBFS, $Deg_{25\%}=15$, $Deg_{50\%}=21$, and $Deg_{100\%}=35$, meaning that by adopting this strategy after the fifteenth disruption, 25% of stations are out service, and the network can tolerate 35 disruptions to be completely degraded.

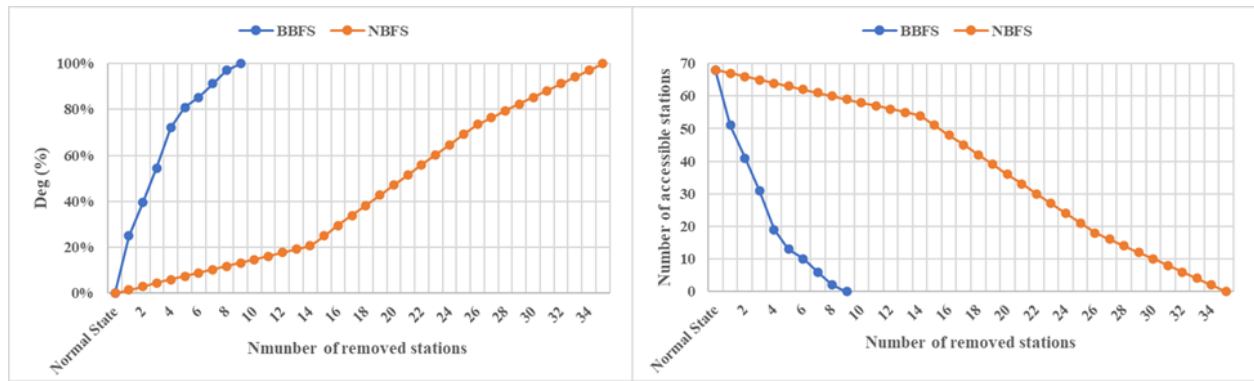


Figure 4-5 Degradation process based on BBFS and NBFS

Since the number of removed stations in the network's complete degradation simulation of these two strategies is significantly different, the fraction of removed stations (degradation percentage) is used to differentiate between BBFS and NBFS. As Figure 5 indicates, in BBFS implementation, RSGC decreased to 80% when 25% of stations are degraded, while approximately the same percentage decreased in NBFS when only 1.5% of stations are degraded. Similarly, RSGC decreased by 65% in BBFS implementation when 40% of stations are out of service, whereas in NBFS implementation RSGC reached the same percentage once only 3% of the stations are out of service. There exists similar trend for other indicators where average path length decreased by 72% in BBFS implementation when 71% of stations are degraded, while in NBSF implementation, the average path length touches this percentage once only 6% of stations are degraded. Changes in network diameter follow a similar pattern, decreasing by 10% in BBFS simulation when 25% of stations are out of service, while in NBFS reach this percentage after the first disruption (1.5% degradation). However, the most considerable difference is related to the number of connected components after each disruption, where in adopting BBFS the network is maximum split into 4 connected components, while the results of NBFS simulation show that after the sixteenth disruption, the network is split into 21 connected components.

4.6 Conclusion and future works

This paper proposed a new failure strategy in the realm of vulnerability assessment of metro networks. The necessity of introducing this failure strategy called Block Based Failure Strategy (BBFS) is the inefficiency of previous strategies -such as Node-Based Failure Strategy (NBFS)- in considering the disruption duration and infrastructural features of the metro network. Old Failure strategies cannot consider the situation of the network if the disruptions are prolonged,

concentrating on network configurations only exactly at the moment of the disruption. Hence, employing such methods in simulating a disruption in the real world is inexplicable. It is tried, in this research, to show how not paying attention to infrastructural features of a metro network such as Stations with Direction Change Facilities (SDCFs) and using the old failure strategies (such as NBFS) instead of BBFS, makes the results of the research bias. For this, Montreal metro network was taken as the case study and its topological vulnerability to targeted disruptions was investigated. In this research, both NBFS and BBFS were implemented separately, and the results were compared. Based on the analyses, there are significant differences in the outputs of the simulations. For instance, when BBFS is implemented, the $Deg_{100\%} = 9$, while in the simulation conducted based on NBFS, the $Deg_{100\%} = 35$, meaning that if the SDCFs are identified and this matter is considered in research, after 9 targeted disruptions, entire the network would be degraded. While if these stations are not identified and based on an erroneous simplifying assumption all the stations are considered as SDCFs, the network can tolerate 35 targeted disruptions that is significantly far from the real robustness of the network.

Considering the important role played by SDCFs, one of the contributions of this paper is to give an idea to the metro network designers to locate SDCFs in optimal locations in order to increase the robustness of the network. Although the process of network design is a sophisticated process in which a wide range of technical and financial constraints should be considered, our simplest suggestion is to locate the SDCFs as close as possible to the network's hubs. Moreover, to determine the precise location of SDCFs in a metro network, this matter can be proposed and solved as an optimization problem.

Further studies on vulnerability investigation may be considered with more indicators, concentrating on functional (transportation-related) features of the network. Moreover, by adding regional rail lines to the current network, the vulnerability of Montreal Urban Rail Transit Network could comprehensively be assessed. Taking the random failures into account as well as presenting a weighted random removal strategy are other interesting matters that can consummate this research.

CHAPTER 5 ARTICLE 2: BEYOND TRADITIONAL METRICS: REDEFINING URBAN METRO NETWORK VULNERABILITY WITH REDUNDANCY ASSESSMENT

Authors: Kaveh Rezvani Dehaghani¹, Catherine Morency¹

¹Department of Civil, Geological and Mining Engineering, Polytechnique Montréal.

Montreal, QC, H3C 3A7, Canada.

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ABSTRACT

Previous studies have predominantly analyzed Urban Metro Network (UMN) vulnerability from topological and functional perspectives, often neglecting the impact of disruptions on alternative route availability. This research introduces a novel redundancy-based vulnerability analysis, assessing the reduction in travel alternatives following disruptions. The Montreal UMN is used as a case study, utilizing General Transit Feed Specification (GTFS) data from the Montreal Transit Authority and trip data from the 2018 Montreal Origin-Destination survey. Using the open-source platform Transition, we simulate shortest transit routes for each trip, generate alternative routes, and compute travel times. We define one targeted and three random failure scenarios, selected from 100 simulations, to evaluate network vulnerability to various disruption types. Indicators are formulated, calculated, and compared across all scenarios. Each failure scenario involves a sequence of consecutive metro station disruptions, leading to complete network shutdown. Findings reveal that the metro network is significantly more vulnerable to targeted disruptions than random ones. Among all indicators, functional ones related to users' travel time show greater sensitivity to disruption type, be it targeted or random. Vulnerability indicators exhibit the most substantial changes during initial disruptions, highlighting their critical impact. Although traditional approaches (topological and functional) show a direct relationship between the number of disruptions and changes in vulnerability indicators, this is not true for the redundancy-based vulnerability indicator. In this case, the primary determinants are the locations of disrupted stations and the network's geometry, rather than the number of disruptions.

5.1 Introduction

Metro systems are the mainstay of transit systems and a crucial component of urban transportation infrastructures. Because of its higher speed and reliability, the metro is also considered as an efficient transportation mode (Jianhua Zhang et al., 2018). Moreover, due to its beneficial effects on noise reduction, energy conservation, and emission reduction (Jianhua Zhang & Wang, 2019), the urban metro system is extremely popular in metropolises. For instance, in Montreal with a population of 1.78 million -embedded in a 4.2 million people metropolitan area, more than 820,000 metro trips are made every day (STM), demonstrating the key role that this transit mode plays in this city. The increasing importance of metro systems globally has heightened scholarly interest in examining the vulnerability of Urban Metro Networks (UMNs) over the past decade.

The increasing importance of subway systems worldwide has intensified the need to assess the vulnerability of UMNs. Vulnerability, in the context of subway systems, refers to their capacity to maintain service in the face of disruptions. Two principal approaches have traditionally been employed to analyze this vulnerability: topological and functional methods, though they may be referred to under different terminologies. The former, often termed "topological" (Hong, Ouyang, Peeta, He, & Yan, 2015), (L. Sun, Huang, Chen, & Yao, 2018), (Jiang, Lu, & Peng, 2018), (Y. Zhang, Ayyub, Saadat, Zhang, & Huang, 2020), (Ma, Shi, et al., 2020), (Meng et al., 2020), "network-based" or "complex network" (Z. Xu & Chopra, 2022), (X. Li et al., 2018), (Jianhua Zhang et al., 2018), or "structural" (Ma, Liang, et al., 2020), (H. Zhang & Yao, 2019), focuses on identifying critical locations within the network—either nodes or links—where disruptions would result in the greatest impact. In contrast, the latter, known as functional analysis (Jianhua Zhang & Wang, 2019), (Hong, Zhong, Ouyang, Tian, & He, 2019), (Khademi et al., 2021), serviceability-based analysis, system-based analysis, or transportation performance analysis (J. Zhang et al., 2021), evaluates transportation characteristics such as node and link capacity, flow, length, and travel time. Serviceability-based methods are complex to apply to public transportation systems. As a result, topological analysis is more commonly used for studying transit network vulnerabilities, while serviceability-based methods are typically applied to road networks. (Mattsson & Jenelius, 2015).

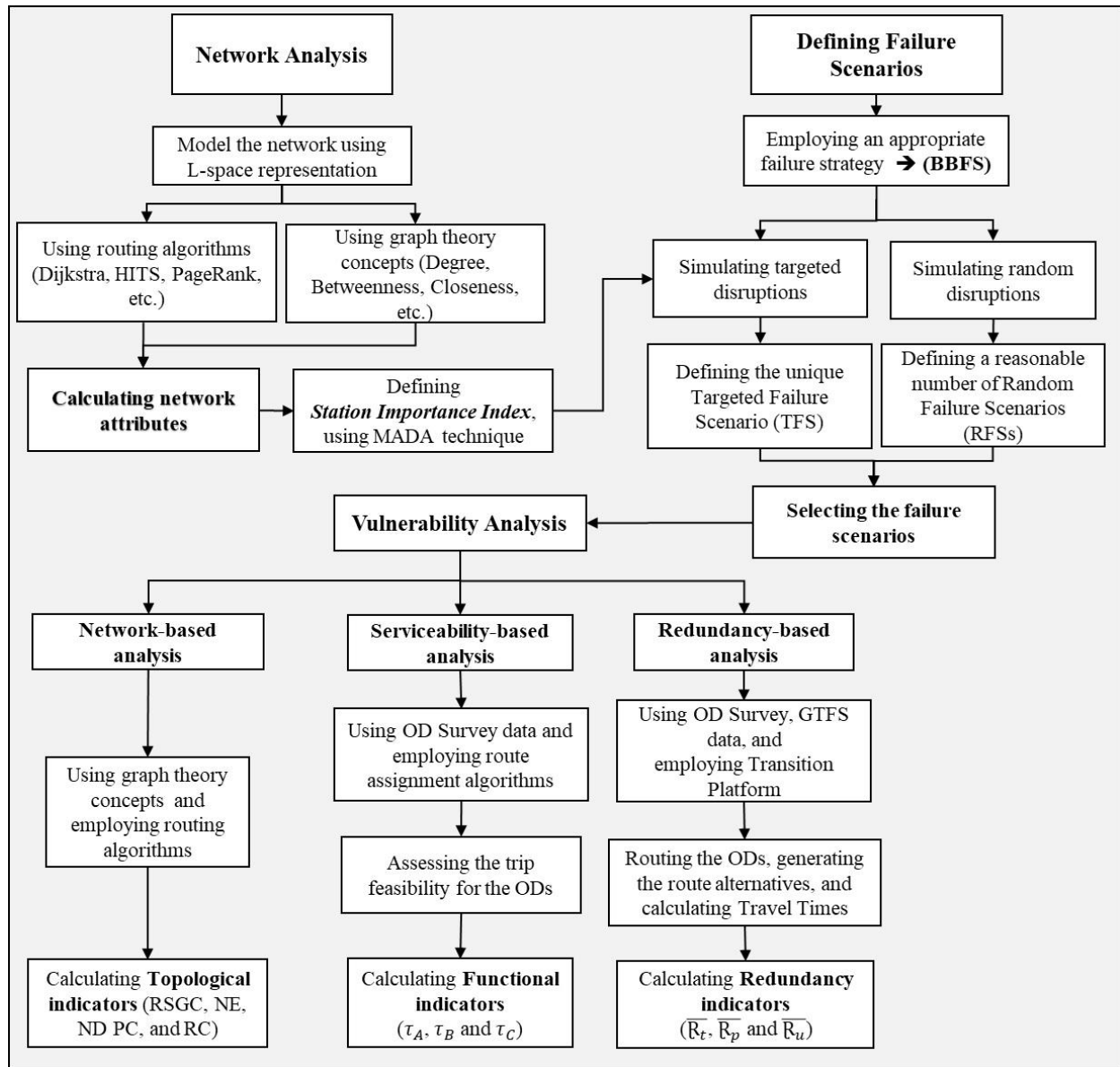


Figure 5-1 Methodology framework

Previous research has made efforts to combine both topological and functional indicators for conducting systematic vulnerability assessments of transit networks. However, a critical consideration has been overlooked: the role of redundancy. Neither topological nor functional approaches sufficiently address the quantity and quality of alternative routes within the network during disruptions. This gap is particularly important as redundancy plays a key role in maintaining network functionality under failure conditions. To address this issue, the current study proposes a redundancy-based vulnerability assessment framework, using the Montreal UMN as a case study. By gathering relevant data, utilizing appropriate analytical tools, and detailing the methodology,

this approach can be extended to other urban metro systems. Figure 5-1 presents an overview of the research structure.

This study employs a Block-Based Failure Strategy (BBFS), which accounts for the role of Stations with Direction Change Facilities (SDCFs), enhancing the accuracy of the simulation. Additionally, the research simulates both targeted and random disruptions within the subway system, allowing for a comparative analysis of their respective impacts. In addition to introducing a redundancy-based vulnerability assessment, this study makes several other notable contributions:

- Two new indicators namely pure-connectivity and relative-connectivity, are proposed to assess network connectivity. These indicators represent a departure from prior studies that primarily relied on measures such as network efficiency, average path length, or average degree.
- A Multi-Attribute Decision-Making (MADM) technique is applied to develop a Station Importance Index by integrating multiple indicators. This approach contrasts with previous studies that focused on individual indicators to identify critical stations for simulating the targeted disruption.
- This study demonstrates that significant variations can occur among simulated random failure scenarios. Therefore, it is important to conduct iterative simulations of random failures multiple times. By selecting diverse scenarios with varying characteristics, one can ensure a comprehensive understanding of potential outcomes. This approach helps avoid focusing solely on uncommon or rare conditions.

5.2 Background

The initial definitions of vulnerability in the realm of transportation engineering are traced back to the research by Berdica, (2002) and D'Este and D'este & Taylor, (2003). Berdica defined the vulnerability of transportation networks as *"a susceptibility to incidents that can result in considerable reductions in road network serviceability"*. During the first years of the emergence of this concept in transportation engineering, scholars tried to generalize the initial definition by examining the importance of investigating the vulnerability of transportation networks from different perspectives (see (Berdica & Mattsson, 2007), and (Jenelius & Mattsson, 2012)).

Upon reviewing the related literature, it is obvious that vulnerability assessment studies of transit networks have been classified in various ways. Mattsson & Jenelius, (2015) introduced an initial categorization into "topological" and "system-based" analyses. M. Taylor, (2017) further delineated these studies into "topologically-based analysis" and "serviceability-based analysis". Jing Zhang & Zhang, (2019) proposed a classification based on "analysis using network topology," "mathematical models," and "accessibility-based analysis," noting that the first two categories are relevant to transit systems. Similarly, Pan, Pan et al., (2021)) categorized previous research into "topological" and "system performance" analyses. Despite differing terminology used by scholars, these categorizations share fundamental concepts.

Several studies have focused on network topology in vulnerability analysis without considering Origin–Destination (OD) passenger flows. For instance, Meng et al., (2020) analyzed the topological complexity of Urban Rail Transit Networks (URTNs) with multi-line transfer stations, using the Shenzhen Metro as a case study. They developed and compared Space L and Space P models based on multiple evaluation parameters and key node rankings. Their findings indicated that both models exhibited scale-free network characteristics, displaying robustness against random faults but vulnerability to targeted attacks. Similarly, Mussone, Salgado, & Notari, (2024) examined network robustness through graph theory and introduced a new centrality measure, the Sum of Weighted Distances Index (SWDI). This index quantified the percentage change in travel distances when a node and its edges were disrupted. By comparing SWDI values in both standard and degraded network states, the study identified the edges and nodes most susceptible to performance reductions. In contrast, some studies have exclusively focused on functional indicators, such as demand and travel time, without addressing topological aspects. For example, Jianhua Zhang, Shao, Yang, Zhao, & Liu, (2023) investigated the robustness of URTNs under disruptions using a user equilibrium model with a time compensation mechanism, using the Shanghai subway network as a case study. Their analysis assessed changes in average travel time, link flow, and the demand-supply ratio under three distinct attack modes, identifying critical nodes and links that significantly impacted the network's robustness.

More recent studies, however, have incorporated both topological and functional features, including passenger flows, to explore network vulnerability more comprehensively. For example, Du et al. (2020) proposed a novel method for evaluating node significance in metro networks, using the Improved Topological Potential with Entropy (ITPE) model. The ITPE model aggregated

multiple centrality measures, with topological entropy determining their weights, to assess node importance. This method's effectiveness was validated through invulnerability measurements, and its application to the Shenzhen Metro system successfully identified critical nodes in terms of both network structure and passenger flow, outperforming traditional undirected and unweighted models (Z. Du et al., 2020).

In another study, J. Zhang et al., (2021) introduced the Integrated Coupled Map Lattices (ICML) model to assess node states and investigate the vulnerability of weighted URTNs under external disturbances. By refining passenger flow into in-flow and out-flow at stations, they introduced a new redistribution rule to analyze cascading failures. Their results revealed a linear relationship between node and edge betweenness, demonstrating that URTNs are highly vulnerable to malicious attacks, with increasing susceptibility as external disturbances grow. Lastly, Song, Ding, Gui, & Zhu, (2024) evaluated the vulnerabilities of rail transit networks (RTNs) through the Chongqing Rail Transit (CRT) system. They developed a network topology model using the spatial L method and analyzed key nodes based on degree, betweenness centrality, network efficiency, and passenger flow volume (PFV). The study identified six node clusters with varying levels of vulnerability and transportation capacity, proposing management strategies to improve planning and safety for RTNs.

A review of traditional vulnerability analysis classifications and recent literature reveals ongoing advancements in integrating topological and functional approaches. However, despite these developments, current methods do not consider the impact of disruptions on the frequency and quality of alternative routes within transit systems. This study aims to address this gap by introducing a redundancy-based vulnerability assessment method.

5.3 Network analysis

The Montreal Urban Metro Network (UMN), comprising four lines—Orange, Blue, Green, and Yellow—along with 68 stations, is selected as the case study for this research. Figure 5-2 provides additional details about the Montreal UMN. Following the principles of complex network theory and L-space representation (Von Ferber et al., 2009), this network is modeled as an undirected graph $G = (V, E)$, where $V = \{v_i | i = 1, 2, 3 \dots N\}$ represents the set of nodes, and $E = \{e_{ij} | v_i, v_j \in V\}$ represents the set of edges. In this model, stations are considered as nodes, while the lines connecting them are represented as edges.

The importance of stations within the metro system can be determined through two main approaches: functional features, such as boarding and alighting volumes, and topological features, such as network centrality. While these two methods are conceptually distinct, the functional importance of stations can fluctuate depending on factors like the time of day, whereas topologically important stations remain consistent. Although both approaches provide valuable insights, this research, like the majority of studies in this field, focuses on the network's topological features. However, unlike prior research that typically relied on a single indicator (often degree or betweenness centrality), this study introduces multiple independent indicators to assess the topological importance of stations. Each indicator serves as a criterion for determining the Station Importance Index.

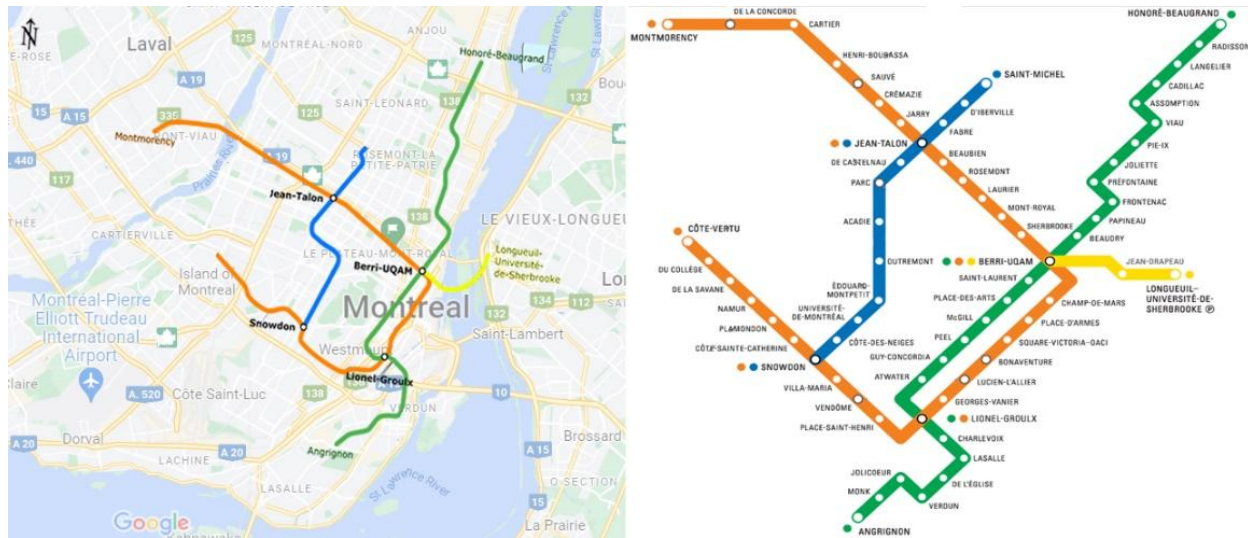


Figure 5-2 Montreal metro network (STM)

5.3.1 Degree centrality

The *degree centrality* of v_i is shown by $C_D(v_i)$ which is the total number of edges connected to v_i .

$$C_D(v_i) = \deg(v_i) = |\{v_j \in V: (v_i, v_j) \in E\}| \quad \text{Equation 5-1}$$

Where v_j represents any other node in the graph that is connected to v_i ; V is the entire set of nodes in the network; E represents the set of edges in the network.

5.3.2 Betweenness centrality

The *betweenness centrality* of V_i is shown by B_i which is the proportion of shortest paths passing through node i among all the shortest paths of the graph.

$$B_i = \sum_{p \neq q \neq 1} \frac{\tau_{pq}(i)}{\tau_{pq}} \quad \text{Equation 5-2}$$

Where: $\tau_{pq}(i)$ represents the number of shortest paths passing through node i between node p and node q , and τ_{pq} represents the number of shortest paths between node p and node q . These can be calculated in an undirected network using equation below:

$$\tau_{pq} = \frac{(N-1) \times (N-2)}{2} \quad \text{Equation 5-3}$$

Where N is the total number of nodes in an UMN

5.3.3 Closeness centrality

In an UMN, the station's *closeness centrality* is determined by taking the reciprocal of the lengths of its shortest paths to all other stations in the network. In a rail network, nodes that have a higher closeness centrality are located closer to one another. This indicates that a trip is convenient because the total cost of traveling between nodes is the shortest.

$$CC_i = \frac{N-1}{\sum_{j \in V, i \neq j} d_{ij}} \quad \text{Equation 5-4}$$

Where d_{ij} is the average length of the shortest path between node i and j .

5.3.4 PageRank

This indicator evaluates the importance of nodes in a graph by considering the recursive relationship between significant nodes and the significance of the nodes around them. In a metro network, PageRank identifies the most important stations by considering both the number of direct connections a station has (i.e., the degree of the station) and the importance of the stations it connects to. Stations with many well-connected neighboring stations (such as major transfer hubs) will have higher PageRank values, reflecting their critical role in network connectivity and passenger flow.

$$PR_{v_i} = \frac{1-\lambda}{N} + \lambda \sum_{u \in \Gamma(i)} \frac{PR(v_j)}{\deg(v_j)} \quad \text{Equation 5-5}$$

Where V_i is the node for which the PageRank is being calculated, V_j represents the neighboring nodes of V_i , $\Gamma(i)$ is a set of neighboring nodes of V_i , and λ is the damping coefficient which is generally 0.85 (Jain & Purohit, 2011), representing the probability of continuing to follow edges.

5.3.5 Hub/Authority

Hubs and Authorities were inspired by a specific observation about web page development during the early stages of the internet. Some websites, known as hubs, served as extensive directories that aggregated a diverse range of resources, directing users to other authoritative pages rather than providing authoritative information themselves. In the context of the UMN, a good authority is a station linked by many different hubs, while a good hub is a station that links to many other stations. The Hyperlink-Induced Topic Search (HITS) algorithm is employed to calculate hub and authority scores (Prajapati, 2012). In an undirected graph, the Hub and Authority scores in the HITS algorithm are iteratively calculated, with each node's score depending on the scores of its neighbors. In each iteration, the Hub score of a node is updated based on the current Authority scores of its connected nodes, and vice versa, the Authority score is updated using the Hub scores of the neighboring nodes. This iterative process continues until the scores converge, reflecting the mutual reinforcement between Hub and Authority values.

$$h_i = \sum_{j \in N(v_i)} a_j \quad \text{Equation 5-6}$$

$$a_i = \sum_{j \in N(v_i)} h_j \quad \text{Equation 5-7}$$

Where h_i is Hub score for node v_i , a_i is Authority score for node v_i , and $N(v_i)$ represents the set of neighbors of node v_i , i.e. all nodes v_j that are connected to v_i by an edge

5.3.6 Eigenvector centrality

This indicator illustrates how nodes influence each other within a network, with a node's score being more significantly increased by connections from other high-scoring nodes. A node that is connected to many nodes with high scores will have a higher eigenvector centrality. The algorithm computes the eigenvector corresponding to the largest absolute eigenvalue, typically using the power iteration method. In each iteration, the centrality score of each node is updated based on the scores of its neighbors, and this process continues until convergence.

The Eigenvector centrality for node is defined as the solution to the following equation:

$$Ax = \lambda x \quad \text{Equation 5-8}$$

Where A is the adjacency matrix of the graph, $x = (x_1, x_2, \dots, x_n)$ is the vector of eigenvector centralities for all nodes, x_i represents the centrality of node v_i , and λ is the principal eigenvalue associated with the adjacency matrix.

5.4 Failure scenarios

In this section, two key concepts must be defined: disruption and failure scenarios. In this research, a disruption is characterized as an accident, incident, or event occurring within a metro station. Depending on the network topology, a single disruption may impact either the affected station alone or a group of stations, necessitating the simulation of such disruptions. To simulate disruptions, a failure strategy must be employed. Furthermore, a failure scenario involves a sequence of consecutive disruptions at metro stations, ultimately resulting in the complete collapse of the Urban Metro Network (UMN).

5.4.1 Failure strategy

The Block-Based Failure Strategy (BBFS) employed in this study builds upon the previously established concept of turn-back operations [32] but introduces specific enhancements tailored to urban rail networks. While both approaches share a common foundation in modeling disruptions and their cascading impacts, BBFS distinguishes itself by defining the concept of a "block." In BBFS, a block refers to a segment of consecutive regular stations situated between two consecutive Stations with Direction-Change Facilities (SDCFs). Additionally, BBFS incorporates a nuanced categorization of SDCFs, distinguishing between fully equipped SDCFs, which facilitate direction changes for both paths (both directions), and semi-equipped SDCFs, which provide direction-changing capabilities for only one path.

In BBFS, the general rule is that the failure of a regular station results in the failure of the entire block to which it belongs, and the disruption of an SDCF affects all connected blocks. However, as illustrated in Figure 3, semi-equipped SDCFs can lead to more localized impacts. For example, station 52 is a semi-equipped SDCF that provides direction-changing capabilities only to the right path. Thus, if a station on the left side of station 52 (e.g., station 51 or 50) is disrupted, only block 18 will be affected, and stations 50 and 51 will be disrupted. Conversely, if a station on the right side of station 52 is disrupted, both blocks 18 and 19 will be out of service. These adaptations render BBFS a specialized extension of the turn-back operation concept, offering greater

applicability to the structural and operational complexities of urban rail systems. Further details about the Montreal UMN, which consists of 24 blocks, are provided in Figure 5-3.

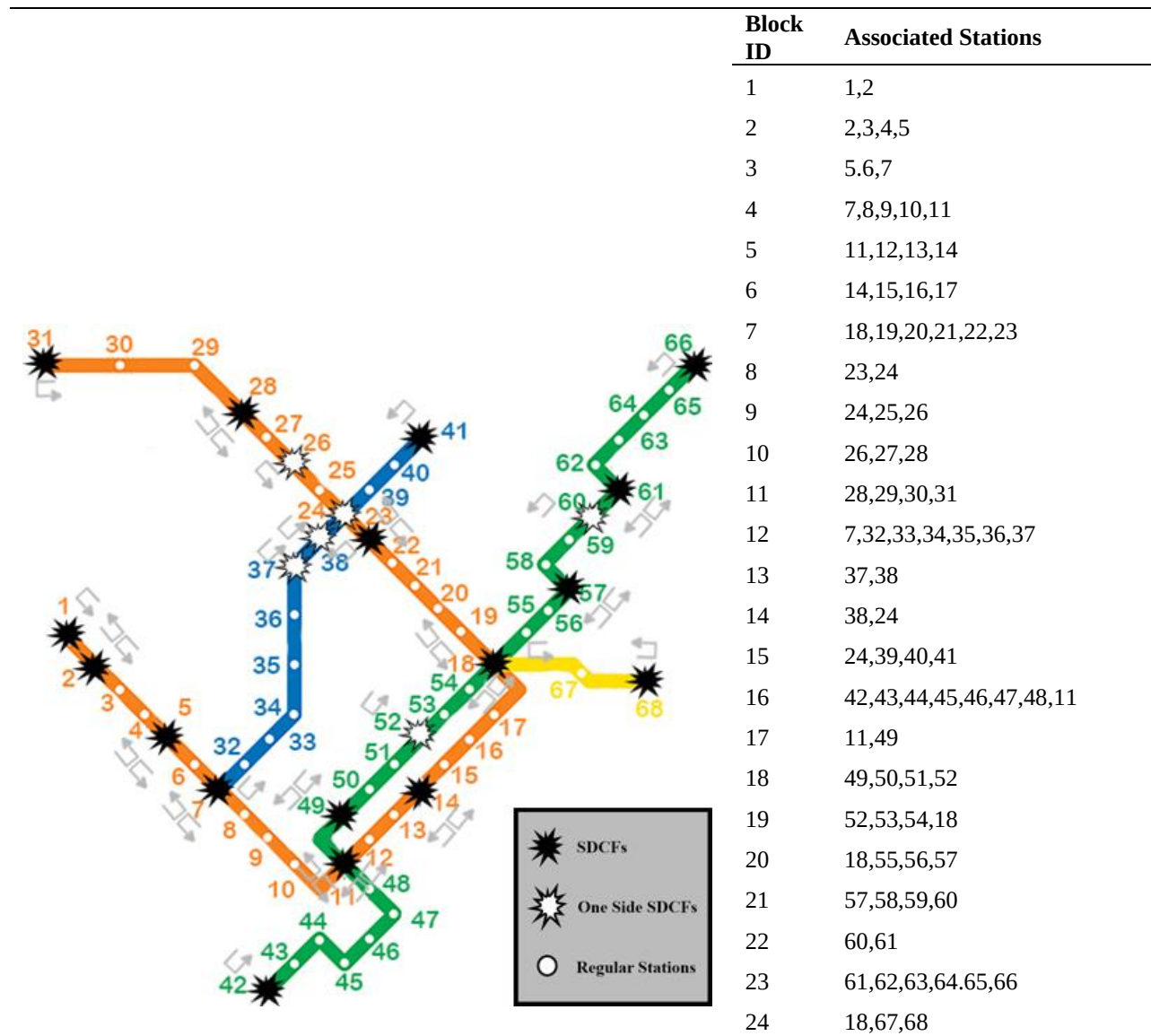


Figure 5-3 Montreal UMN blocks

5.4.2 Targeted disruptions

In earlier studies, targeted failure scenarios were often simulated by selecting nodes for removal based on descending order of degree or betweenness centrality. However, in this research, we introduced a Station Importance Index as the primary criterion for selecting the most crucial station. This index incorporates various centrality-related attributes of the network simultaneously, providing a ranking of station importance based on centrality metrics. The station identified with

the highest station importance according to this index is chosen for targeted disruptions in our approach.

5.4.2.1 Calculating the station importance index

In this study, we applied the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Tzeng & Huang, 2011), a method of Multi-Attribute Decision Making, to identify the most significant station within a specified network context. Our analysis considered 68 alternatives (station) and 7 network-centrality attributes under normal conditions. The importance of stations was assessed using various logics and algorithms associated with these attributes. To facilitate the TOPSIS methodology, attribute weighting was implemented using Shannon's entropy method (Shannon, 1948), as elaborated in the following discussion.

A decision matrix is needed in the first step, in which the elements must match the values of the aforementioned attributes of each node (stations). Hence, in the decision matrix $[X]_{m \times n}$, m represents the number of choices (stations) and n is the value of attributes.

$$X = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1N} \\ X_{21} & X_{22} & \dots & X_{2N} \\ \dots & \dots & \dots & \dots \\ X_{M1} & X_{M2} & \dots & X_{MN} \end{bmatrix}$$

The second step deals with normalising the decision matrix. Because of the potential for a large difference between the value ranges of different attributes and the variations in the measurement units, the values of the attributes should be normalized.

$$p_{ij} = \frac{X_{ij}}{\sum_{i=1}^n X_{ij}} \quad \text{Equation 5-9}$$

The next step is related to the computation of the entropy of attributes using the equation below:

$$E_j = -k \sum_{i=1}^m p_{ij} \ln p_{ij} \quad \text{Equation 5-10}$$

$$\text{Where } k = \frac{1}{\ln(m)}$$

The third step entails computing the attributes' weights using the entropy concept:

$$w_j = \frac{1-E_j}{\sum_{j=1}^n (1-E_j)} \quad \text{Equation 5-11}$$

The ranking results of the node (station) importance should be established by the TOPSIS approach after determining the weight of the attributes. Five steps must be taken in this regard.

First, determine the normalized matrix using:

$$\bar{X}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^n x_{ij}^2}} \quad \text{Equation 5-12}$$

Second, determine the weighted normalized matrix:

$$V_{ij} = \bar{X}_{ij} \times w_j \quad \text{Equation 5-13}$$

Third, calculate the ideal best and ideal worst value:

$$\begin{cases} v^+ = \max (V_1^+, V_2^+ \dots, V_p^+) \\ v^- = \min (V_1^-, V_2^- \dots, V_p^-) \end{cases}$$

Fourth, calculate the Euclidean distance from the ideal best and ideal worst:

$$S_i^+ = \sqrt{\sum_{j=1}^m (V_{ij} - V_j^+)^2} \quad \text{Equation 5-14}$$

$$S_i^- = \sqrt{\sum_{j=1}^m (V_{ij} - V_j^-)^2} \quad \text{Equation 5-15}$$

Fifth, determine the performance score:

$$P_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad \text{Equation 5-16}$$

A summary of the required steps to calculated station importance is given in Figure 5-4.

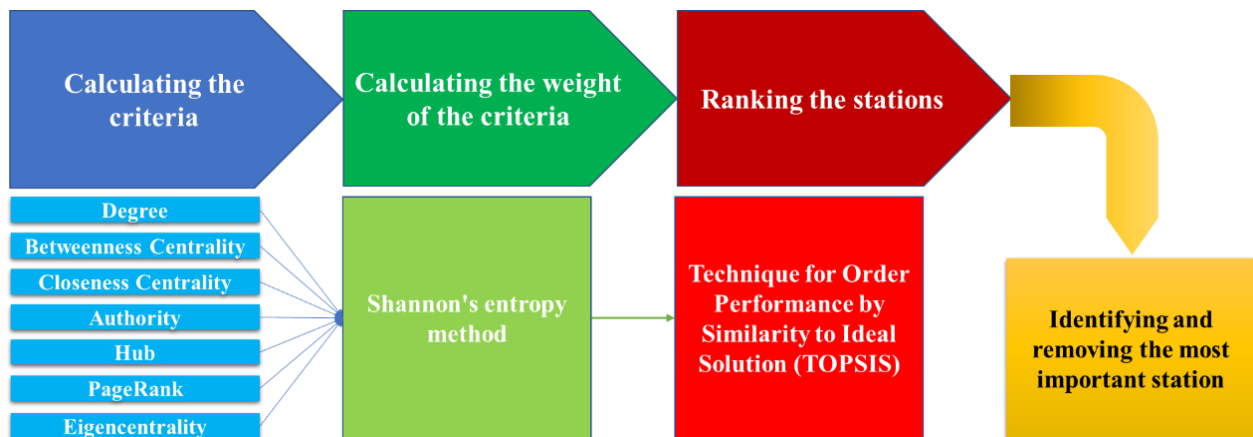


Figure 5-4 Schema of steps to identify the most important station.

Table 5-1 provides detailed insights into the station selection process during the targeted failure scenario. For each network scenario, the weights of seven criteria used to identify the most important station are computed. The total weight of criteria in each table row sums up to one.

Therefore, while considering the impact of each criterion (based on its weight) in identifying the most important station, Table 1 highlights the most influential criterion in each network scenario (i.e., the criterion with the highest weight).

For instance, in the normal state (no disruption), closeness centrality emerges as the most influential indicator, with "Berri-UQAM" (station #18) identified as the highest in importance. Following the first simulated disruption (removal of the most important station), according to BBFS (Block-Based Failures Strategy), seventeen stations within the same blocks as Berri-UQAM would be out of service. After each disruption simulation, the network undergoes analysis, recalculating and reweighting all indicators.

Based on the details presented in Table 5-1 regarding successive targeted disruptions, the network is predicted to collapse after nine targeted disruptions. Thus, the Corresponding Disruption Number to the Annihilation of the Network (CDNAN) in the targeted failure scenario is 9.

Table 5-1 Order of selected stations in the targeted failure scenario

Network situation	Criteria weight							Code and Name of the Purposefully Selected Station (Station with highest importance index)
	Degree	Closeness centrality	Betweenness centrality	Authority	Hub	Page Ranks	Eigen centrality	
N.S	0.09	0.29	0.16	0.11	0.16	0.09	0.11	18 (Berri-UQAM)
D#1	0.10	0.19	0.24	0.09	0.14	0.10	0.13	11 (Lionel-Groulx)
D#2	0.08	0.20	0.25	0.12	0.18	0.07	0.11	24 (Jean-Talon)
D#3	0.23	0.08	0.14	0.18	0.18	0.13	0.06	61 (Viau)
D#4	0.24	0.12	0.13	0.14	0.17	0.14	0.07	4 (Namur)
D#5	0.20	0.11	0.17	0.15	0.15	0.12	0.10	58 (Prefontaine)
D#6	0.19	0.13	0.15	0.16	0.16	0.11	0.10	29 (Cartier)
D#7	0.13	0.20	0.13	0.11	0.20	0.11	0.13	6 (Cote-Sainte-Catherine)
D#8	0.14	0.14	0.14	0.14	0.14	0.14	0.14	1 (Cote-Vertu)

5.4.3 Random disruptions

Using random numbers to select stations for removal is a common method for simulating random failure scenarios. However, this approach can sometimes produce outputs that create unusual graph

states that differ significantly from actual conditions. For instance, all randomly selected nodes might be consecutive, located at the beginning or end of lines, or at transfer stations.

To mitigate the impact of these outcomes, we conduct the simulation 100 times and adopt the most frequently occurring scenario as the realistic random scenario. This iterative process helps to reduce the influence of anomalous outcomes and ensures that the simulated random failures better represent typical or expected scenarios within the network graph.

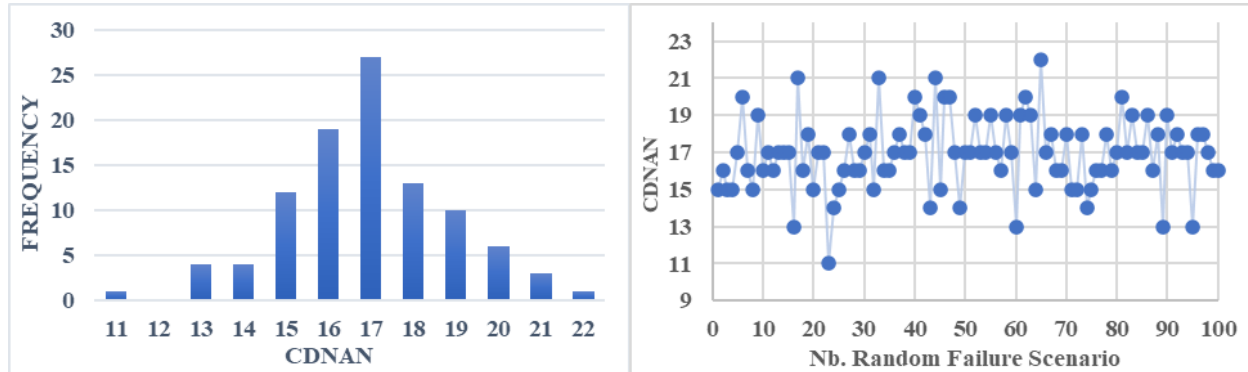


Figure 5-5 Details of simulated random failure scenarios

Figure 5-5 illustrates that among 100 simulated random scenarios, the most frequently repeated CDNAN is 17, occurring 27 times. This scenario is referred to as Realistic Random Failure Scenarios (RRFS). In one scenario, the CDNAN is 11, characterized as the Pessimistic Random Failure Scenario (PRFS). Conversely, in the most optimistic case, the CDNAN is 22, representing the Optimistic Random Failure Scenario (ORFS). These different scenarios provide insights into the range of potential outcomes resulting from random failure simulations.

5.4.4 Selected scenarios

In this study, we identify one Targeted Failure Scenario (TFS) with the lowest possible Corresponding Disruption Number to the Annihilation of the Network (CDNAN) as the worst-case scenario. Additionally, we select three random failure scenarios corresponding to the lowest (PRFS), the most frequent (RRFS), and the highest (ORFS) CDNAN among 100 simulated scenarios.

It is important to note that the CDNAN values corresponding to TFS, PRFS, and ORFS are unique. However, since CDNAN equals 17 in 27 simulated scenarios, the RRFS used in our analysis was randomly selected from these 27 scenarios. Table 5-2 provides detailed information about the

sequence of selecting stations in each scenario, namely how stations are chosen during the simulation of targeted and random failure scenarios to assess network resilience and vulnerability.

Table 5-2 Additional information on failure scenarios

Scenario	Description	Nb. Disruption																					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
TFS	CSS	18	11	24	61	4	58	29	6	1													
	NRS	51	36	21	15	13	9	5	2	0													
PRFS	CSS	7	26	19	40	11	63	1	18	60	31	4											
	NRS	51	53	48	44	34	29	27	12	7	3	0											
RRFS	CSS	39	31	2	49	56	28	20	7	13	45	52	18	63	38	24	61	59					
	NRS	65	62	58	55	53	51	47	36	34	26	23	16	11	9	5	4	0					
ORFS	CSS	33	8	51	7	29	61	19	45	52	59	49	16	39	57	28	12	4	68	37	1	38	26
	NRS	63	60	58	56	53	47	43	36	33	30	29	26	23	20	18	14	11	8	7	5	4	0

CCS: Code of the Selected Station

NRS: Number of Remaining Stations

5.5 Vulnerability analyses

5.5.1 Network-based analysis using topological indicators

Topological indicators are primarily used to evaluate network connectivity. Previous studies have employed various metrics, such as the Relative Size of Giant Components, Network Efficiency, and Network Diameter, to assess connectivity. In addition to these conventional indicators, this research introduces two novel metrics: Pure Connectivity and Relative Connectivity. These new indicators are based on two factors: the Size of the Giant Component (SGC) and the Number of Connected Components (NCC). The SGC directly reflects network connectivity, with a larger SGC indicating that more stations remain connected, signifying greater overall cohesion. In contrast, NCC has an inverse relationship with connectivity. As NCC increases, the network fragments into more isolated components, reducing the relative connectivity of the remaining network compared to the original. Thus, while SGC measures the strength of connectivity, NCC reflects the degree of network fragmentation.

Relative Size of the Giant Component (RSGC)

This indicator is defined as the order ratio of the largest connected subgraph of the network to the original graph. The *RSGC* is calculated using equation below:

$$RSGC = \frac{N'}{N_o} \quad \text{Equation 5-17}$$

Where N' represents the order of the giant component (the largest connected subgraph after the disruption), and N_o is the node number of the original graph.

Network Efficiency (NE)

This indicator, which is also called global efficiency, is used to assess the efficiency of the entire network. It is obvious that lower values for shortest paths suggest a greater efficiency, showing the shortest paths have an adverse relationship to network efficiency (Rubinov & Sporns, 2010):

$$E = \frac{\sum_{i \neq j} \frac{1}{d_{ij}}}{N(N-1)} \quad \text{Equation 5-18}$$

Where d_{ij} is the shortest distance between v_i and v_j ,

Network Diameter (ND)

This indicator is defined as the maximum distance between two nodes in the giant component, calculated using follow equation:

$$ND = \max \{d_{ij}\} \quad \text{Equation 5-19}$$

Pure Connectivity (P.C)

This indicator quantifies the average number of directly connected stations in the network and is formulated as:

$$P.C = \frac{\sum_{i=1}^{N_A} |N_G(i)|}{N_A - 1} \quad \text{Equation 5-20}$$

Where $|N_G(i)|$ denotes the number of adjacent nodes to station i calculated as the sum of the entries in the i^{th} row of the adjacency matrix. N_A is the total number of accessible stations in the network.

Computing $P.C$ requires summing the adjacency matrix rows, which has a complexity of $O(N^2)$ for dense networks but reduces to $O(E)$ for sparse networks, where E is the number of edges. Thus, this indicator is computationally feasible for metro networks, especially when leveraging sparse matrix representations.

While analyzing very large networks may demand computational optimizations, $P.C$ is straightforward to implement for metro systems with manageable sizes. Its simplicity and reliance on fundamental network properties make it a practical and reliable indicator for understanding network connectivity under disruptions. These advantages outweigh the potential challenges in its application to more complex or large-scale systems

Relative Connectivity (R.C)

This indicator evaluates the proportion of accessible stations in the disrupted network relative to its pre-disruption state and is defined as:

$$R.C = \frac{N_A}{N.C.C \times N} \quad \text{Equation 5-21}$$

Where $N.C.C$ represents the number of the connect components.

The computation of $R.C$ requires identifying the connected components of the disrupted network. This task is performed using graph traversal algorithms like Depth-First Search (DFS) or Breadth-First Search (BFS), with a complexity of $O(N + E)$, where N and E are the number of nodes and edges, respectively. Although connected component analysis can become more resource-intensive for larger networks, it remains feasible for most metro systems. The indicator provides significant insights into network fragmentation, enabling clear comparisons between pre- and post-disruption scenarios. Its ability to capture the level of disconnection makes $R.C$ a valuable indicator for urban transit systems, where ensuring accessibility is critical.

Generally speaking, $P.C$ indicates the present situation of the network after each disruption in terms of connectivity, while $R.C$ mainly indicates the extent of fragmentation of the network after each disruption compared to the normal state.

5.5.1.1 Vulnerability analysis

The network-based vulnerability assessment in this research incorporates topological and connectivity vulnerability indicators, along with a novel foolproof indicator known as Cumulative Degradation percentage ($C.Deg\%$). This indicator quantifies the ratio of out-of-service stations to the total number of accessible stations in the normal state. $C.Deg\%$ provides a straightforward measure of network degradation, reflecting the proportion of stations that are unavailable relative to the total number of stations accessible under normal conditions.

Normal state

The initial network analysis reveals that the network comprises a single connected graph, signifying that all stations are part of the same component. Consequently, $RSGC$ equals 1. The original network efficiency (NE) is calculated as 0.157, indicating a network that is not well-connected. The longest shortest path in the graph, with a length of 29 links, corresponds to the route connecting station #1 (Côte-Vertu) to station #66 (Honoré-Beaugrand), denoted as ND , which equals 29. The shortest path lengths used in computing NE and ND are determined using the Dijkstra algorithm (Dijkstra, 1959) implemented in Python. Moreover, based on the above defined indicators, $P.C = 67$, meaning that in normal state, each station is connected to 67 stations, and relative connectivity ($R.C$) equals 1.

After the disruptions

As depicted in Figure 5-6-a, the rate of Relative Size of Giant Component ($RSGC$) decline in the Targeted Failure Scenario (TFS) is notably more pronounced compared to random failures, indicating rapid network destabilization under targeted disruptions. Additionally, a comparison among the three random failure scenarios reveals that the Optimistic Random Failure Scenario (ORFP) and the Realistic Random Failure Scenario (RRFP) exhibit similar decreasing trends in $RSGC$, suggesting that changes in $RSGC$ are slower in scenarios with larger Corresponding Disruption Number to the Annihilation of the Network (CDNAN) than those with smaller CDNAN.

Initial disruptions, particularly targeted ones, induce the most significant and swift reduction in Network Efficiency (NE) (Figure 5-6-b). Comparisons between scenarios demonstrate a 50% decrease in NE after the first disruption in TFS (from 0.157 to 0.080), whereas NE in RRFS shows less than 5% reduction after the first disruption, highlighting NE 's sensitivity to targeted disruptions. Furthermore, it is noteworthy that across all scenarios, initial disruptions exert a more pronounced effect on NE reduction compared to subsequent disruptions.

The trend of Network Diameter (ND) changes in TFS notably differs from random failure scenarios, indicating a more rapid decline in node closeness and network accessibility under targeted disruptions (Figure 5-6-c). Moreover, the steepness of the tangent gradient on the TFS diagram suggests that ND changes due to initial disruptions are more substantial than subsequent disruptions, whereas random failure scenarios, particularly RRFS and ORFS, show no significant variance in ND changes.

Figure 5-6-d illustrates a distinct decrease in Pure-Connectivity ($P.C$) during the initial disruptions within TFS. This reduction amounts to approximately 90% after the third disruption. In contrast, random disruptions (especially RRFS and ORFS) exhibit different trends, indicating a higher sensitivity of network connectivity to targeted disruptions. As expected, the initial disruptions in TFS have a greater impact on reducing $P.C$ compared to subsequent disruptions across all scenarios. There is no significant difference observed in $P.C$ changes during the final disruptions in these scenarios.

In Figure 5-6-e, both targeted and random failure scenarios exhibit a similar trend of Robustness Coefficient ($R.C$) decrease, notably declining during initial disruptions. However, this trend is more pronounced in targeted disruptions than random ones (especially in RRFS and ORFS), underscoring targeted disruptions' greater impact on reducing network connectivity.

Cumulative Degradation percentage ($C.Deg(\%)$), representing network deterioration, shows a steep tangent gradient in the TFS diagram, indicating the sharpest increase in $C.Deg(\%)$ under targeted disruptions. After the first disruption, 25% of stations are out-of-service in TFS, whereas in RRFS, $C.Deg(\%)$ remains under 5% after the first disruption (Figure 5-6-f).

Key observations from this network-based analysis include:

1. The network is significantly more vulnerable to targeted disruptions.
2. Initial disruptions exert a greater impact on network vulnerability across all scenarios.
3. Networks with lower CDNAN exhibit higher vulnerability.
4. Meaningful differences in connectivity reduction rates are observed between newly introduced indicators ($R.C$ and $P.C$) and the previously used (ND), particularly when comparing TFS to PRFS, suggesting that ND alone does not offer a precise interpretation of connectivity.

5.5.2 Serviceability-based analysis using functional indicators

In this section, we analyze the feasibility of users' travel under different failure scenarios. To assess this, we employ three indicators inspired by (Kermanshah & Derrible, 2016) that show the number of feasible trips before and after the disruptions.

$$\tau_A = \frac{T_i^A}{T_O}, \quad \text{Equation 5-22}$$

$$\tau_B = \frac{T_i^B}{T_O}, \quad \text{Equation 5-23}$$

$$\tau_C = \frac{T_i^C}{T_O} \quad \text{Equation 5-24}$$

Where: T_i^A represents the number of trips that still utilize the initial route (the shortest possible path) after the i^{th} disruption, T_i^B denotes the number of trips that had to use longer routes after the i^{th} disruption occurred, T_i^C denotes the number of trips that could not be completed after the i^{th} disruption., and T_O is the total number of trips done before the disruption occurred (in the normal state).

It should be noted that $(T_i^A + T_i^B)$ represents the possible satisfied demand after the i^{th} disruption, and T_i^C represents the unsatisfied demand after the i^{th} disruption. This is also clear that $T_O = T_i^A + T_i^B + T_i^C$

To compute the indicators mentioned, considering travel demand is necessary. To achieve this, we utilized data from the 2018 Montreal Origin-Destination survey. After data processing and weighting the data, we identified a total of 350,879 trips (after data expansion) exclusively using the metro system, which serves as the dataset for this research. Subsequently, to evaluate trip feasibility, we implemented a network graph in Python to validate the existence of routes between nodes (stations).

5.5.2.1 Vulnerability analysis

Normal State

Since in the normal state all trips are feasible using the shortest path, it is obvious that $T_O = T_0^A = 1$, and $T_0^B = T_0^C = 0$.

After the disruptions

Based on Figure 5-6-g and Figure 5-6-h, values of τ_A and τ_C reveal that the most significant reduction in satisfied demand is associated with the Targeted Failure Scenario (TFS). After the first, second, and third disruptions in TFS, τ_C reaches 62%, 82%, and 95% respectively, indicating pronounced vulnerability of network serviceability to targeted disruptions. This vulnerability is further supported by comparing the tangent gradient of the TFS diagram with random scenarios in

τ_A and τ_C . Among the three random failure scenarios, the Pessimistic Random Failure Scenario (PRFS) exhibits a higher decrease rate in satisfied demand, suggesting that each disruption has a more substantial impact on satisfied demand in this scenario compared to the other two random failure scenarios.

Furthermore, the τ_A and τ_C diagrams reveal that in failure scenarios with smaller Corresponding Disruption Number to the Annihilation of the Network (CDNAN), the initial disruptions have a more significant impact on satisfied demand (number of feasible trips on the subway network). As CDNAN increases, the curvature of the τ_A and τ_C diagrams decrease. For instance, in the Optimistic Random Failure Scenario (ORFS), the relationships between τ_A and τ_C with the number of disruptions are approximately linear, indicating that all disruptions have a nearly equal impact on reducing satisfied demand.

According to Figure 5-6-i, in the TFS, after the first disruption, only one route remains between each possible origin and destination in the network. Consequently, no user can continue his trip by accepting a longer route after the disruption. In such scenarios, the following relationships hold after each disruption: $\tau_B=0$, $\tau_A=1-\tau_C$. In contrast, among random failure scenarios, after some initial disruptions, it is observed that some users can still continue their metro travels by accepting longer routes after the disruption. The frequency of such trips in each scenario is directly related to the CDNAN, meaning that higher CDNAN values in a scenario correspond to a higher number of passengers who can continue traveling by metro after disruptions (provided they accept longer routes).

The key findings of the serviceability-based analysis are summarized as follows:

1. Network serviceability is significantly more vulnerable to targeted disruptions compared to random disruptions.
2. Initial disruptions have a greater impact on network serviceability across all scenarios.
3. Scenarios with a lower Critical Degree of Network Affiliation Number (CDNAN) exhibit higher vulnerability in network serviceability.
4. Comparing the tangent gradient of the diagrams reveals a more significant difference between targeted and random failure scenarios in serviceability-based analysis compared

to network-based analysis. This suggests that functional indicators are more sensitive to the type of disruption than topological and connectivity indicators.

The sensitivity of indicators to disruption type reflects their susceptibility to decline during different failure scenarios, with greater susceptibility indicating higher sensitivity.

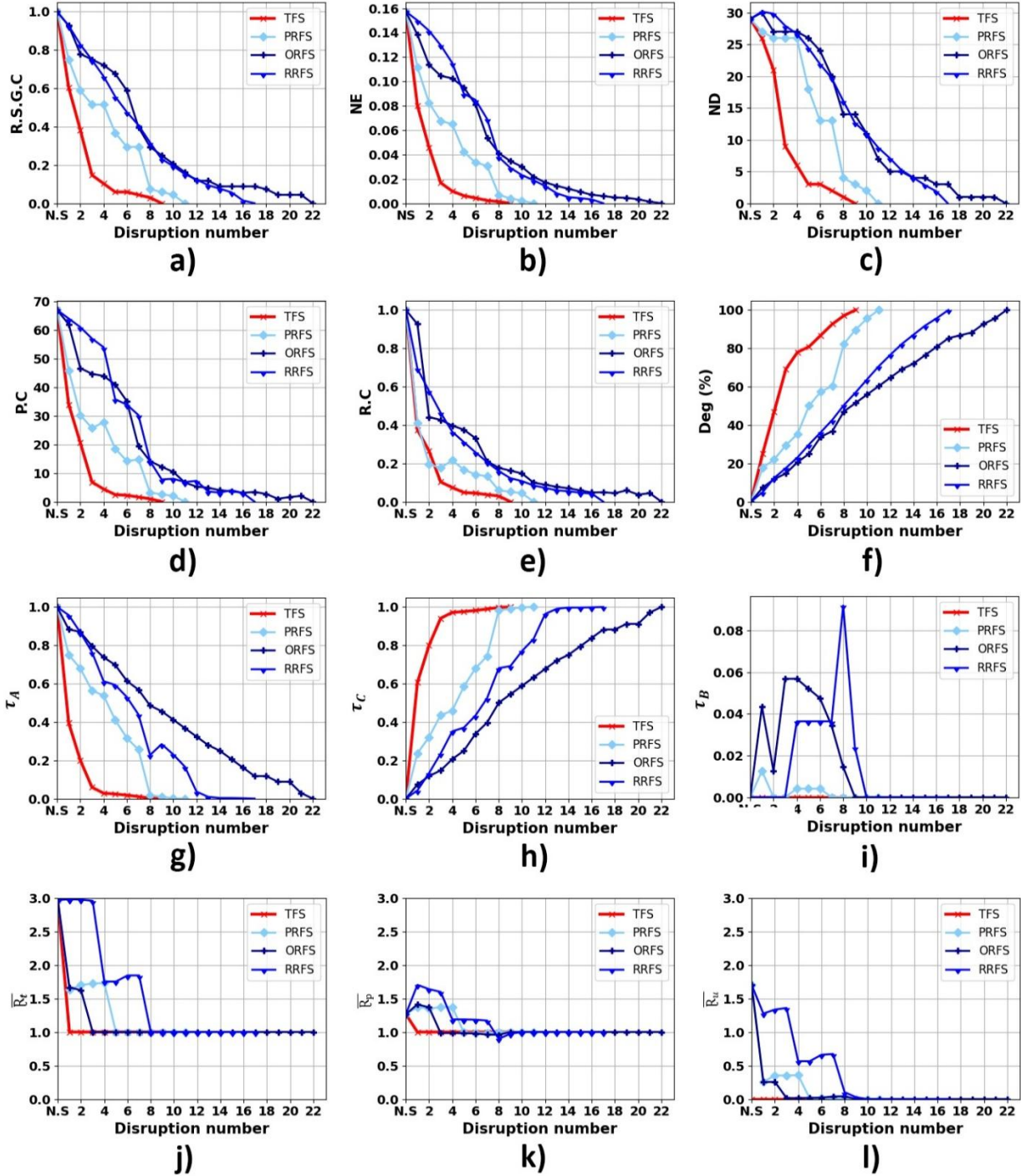


Figure 5-6 Results of network-based vulnerability analysis

5.5.3 Redundancy-based analysis

The redundancy of a transportation network is evaluated based on the number of available routes between stations, with each route considered a simple path where no node is traversed more than once. In this study, data from the 2018 Montreal Origin-Destination survey and General Transit Feed Specification (GTFS) from the STM (STM) website were utilized. To compute trip routes, generate alternatives, and calculate associated travel times, the Transition Platform—a Polytechnique Montreal-developed open-source software (www.transition.city)—was employed. This platform leverages the Connection Scan Algorithm (CSA) for transit route generation.

To determine total travel time for each trip, several components were considered, including total vehicle time, waiting time, access and egress time, and transfer walking time where applicable. The study applied a maximum access travel time of 10 minutes and a maximum egress travel time of 15 minutes, with a 10-minute limit on access time during transfers. These parameters allowed the calculation of travel times for 308,721 trips, representing 88% of the total trips captured in the dataset.

5.5.3.1 Quality of alternatives

This study evaluates route alternatives within the Montreal metro network by considering both the availability of multiple routes between origin-destination (OD) pairs and the quality of these routes in terms of travel time. A key focus is on defining a quality threshold based on trip duration. This threshold distinguishes acceptable from unacceptable route alternatives. Based on observations of Montreal metro users, it is hypothesized that passengers are unlikely to consider alternate routes if the travel time is more than twice that of their shortest available option.

For instance, Figure 5-7 depicting theoretical routes between station #1 (Côte-Vertu) and station #13 (Lucien-L'Allier). Four routes are identified with durations of 24, 45, 52, and 61 minutes (during workdays at 8:00 AM). The shortest path between these stations takes 24 minutes. According to the defined quality threshold, an acceptable alternative route should have a duration less than twice the time of the shortest path in normal conditions (i.e., before disruptions). In this specific example, the second-best route option has a duration of 45 minutes, which is less than twice the duration of the shortest path (48 minutes). Therefore, it qualifies as an acceptable alternative. Conversely, the third and fourth route options exceed this threshold, with durations of

52 and 61 minutes respectively, and are therefore deemed unacceptable alternatives based on the defined quality criteria.

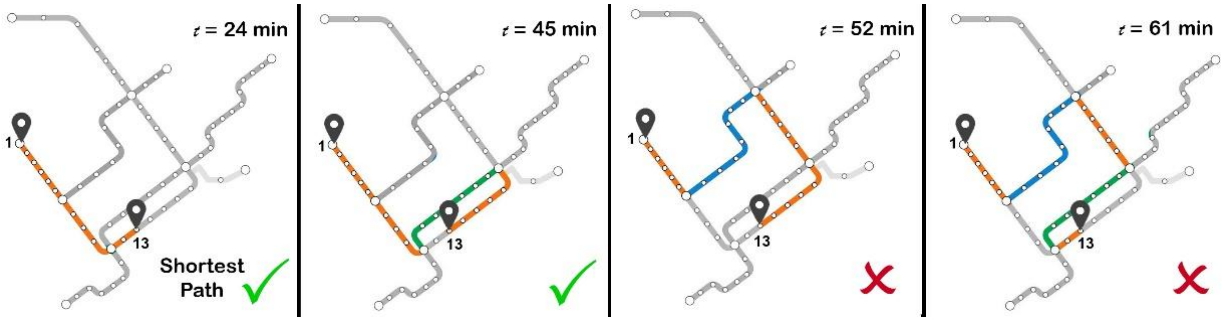


Figure 5-7 Quality threshold of route alternatives

Redundancy indicators

To introduce the redundancy indicators, the following concepts for each OD pair must be defined:

Theoretical Redundancy (R_t): The total number of available routes (simple paths).

Acceptable Redundancy (R_p): The number of routes meeting the predefined quality threshold.

Unacceptable Redundancy (R_u): The number of routes failing to meet the quality threshold.

Consequently, for each OD pair, a numerical value is assigned to each of the aforementioned indicators. Generally, it holds that $R_t = R_p + R_u$.

A specific subset of R_u represents unsatisfied demand during disruptions (when $R_t > 0$ and $R_p = 0$). This occurs when both the origin and destination remain physically accessible and theoretically connected, but the available route(s) fail to meet the quality threshold. In this research, two types of unsatisfied demand are defined:

1- Unsatisfied demand caused by physical degradation (denoted by τ_c).

2- Unsatisfied demand caused by decreased route quality (denoted by τ_d), stemming from degradation in travel route quality.

The employed indicators to assess redundancy are as follow:

$$\overline{R}_t = \frac{\sum^D R_t}{T_i^A + T_i^B} \quad \text{Equation 5-25}$$

$$\overline{R}_p = \frac{\sum^D R_p}{T_i^A + T_i^B} \quad \text{Equation 5-26}$$

$$\overline{R}_u = \frac{\sum_i^{D_i} R_u}{T_i^A + T_i^B} \quad \text{Equation 5-27}$$

$$\tau_D = \frac{T_i^D}{T_i^A + T_i^B} \times 100 \quad \text{Equation 5-28}$$

Where T_i^D is the number of trips that could not be completed after the i^{th} disruption because of the insufficient quality of alternative routes.

5.5.3.2 Vulnerability analysis

Normal state

Using the OD data and developed routing algorithms, we found that for 14% of the trips, theoretical redundancy equals 1 (no alternative is available), and that for around 86% of trips the theoretical redundancy is greater than 1. However, when considering the quality of the alternatives, it is seen that for 59% of trips, the acceptable redundancy equals 1 (there is no alternative). The corresponding percentages of trips with one, two, and three acceptable alternatives in the network are 15%, 21%, and 4% respectively. Figure 5-8 provides more details.

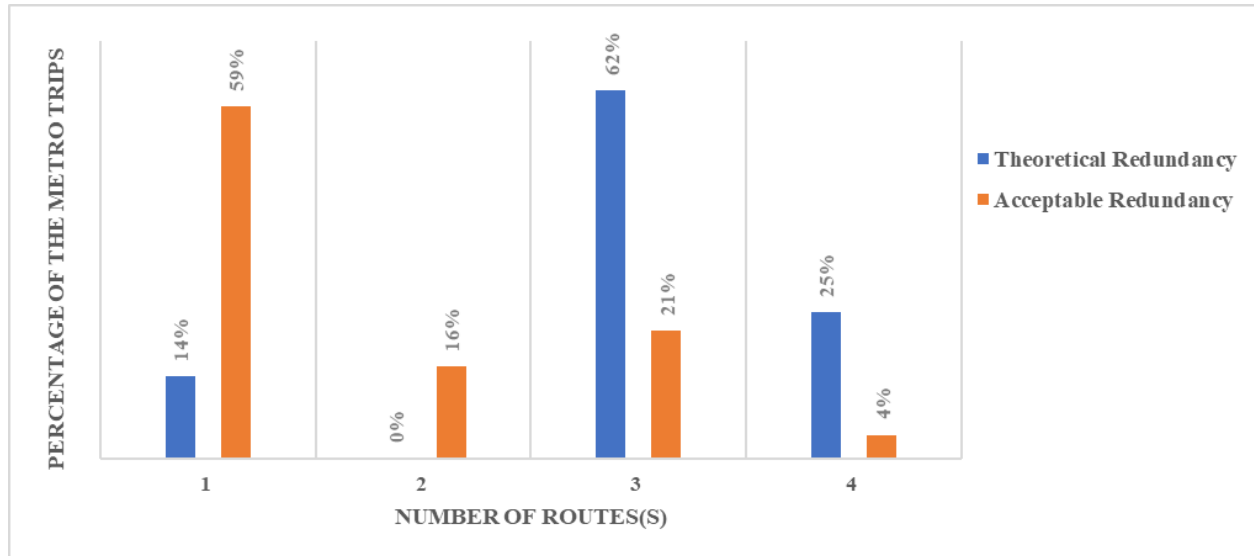


Figure 5-8 Distribution of theoretical and acceptable routes in normal state

The results show that the theoretical redundancy ($\overline{R}_t$) in the normal state is 2.97, the acceptable redundancy ($\overline{R}_p$) is 1.70, and the unacceptable redundancy ($\overline{R}_u$) is 1.27. It is also obvious that since all the demand is satisfied in the normal state, both τ_C and τ_D equal 0.

After the disruption

Figure 5-6-j illustrates that $\overline{R}_t$ is highly sensitive to targeted disruptions. In Targeted Failure Scenario (TFS), following the initial disruption, the theoretical redundancy decreased to 1, indicating that no route alternatives could be identified for OD pairs after subsequent disruptions ($\overline{R}_t = \overline{R}_p = 1$, and $\overline{R}_u = 0$). In Pessimistic Random Failure Scenario (PRFS) and Optimistic Random Failure Scenario (ORFS), a significant reduction in theoretical redundancy was observed after the first disruption, indicating that many trips lost their alternative routes. Specifically, in PRFS, after the third random disruption, and in ORFS, after the fifth random disruption, the network ceased to provide any alternative routes to users.

Conversely, in Realistic Random Failure Scenario (RRFS), although Corresponding Disruption Number to the Annihilation of the Network (CDNAN) value in this scenario was lower than in ORFS, the first substantial decrease in theoretical redundancy occurred after the fourth disruption, with the network able to provide alternative routes up until the eighth disruption.

Notably, contrary to previous analyses, the network exhibited greater vulnerability in ORFS compared to RRFS, suggesting that there is no significant relationship between CDNAN and network vulnerability from a redundancy perspective. Therefore, additional investigation into factors such as the location of disrupted stations and the geometric configuration of the network post-disruption is warranted.

As Figure 5-9 indicates, the Montreal UMN contains three cycles: C#1 (7→11→15→18→24→35→7), C#2 (7→11→52→18→24→35→7), and C#3 (11→15→18→52→11). In TFS, the first selected station (Berri-UQAM: station#18) is strategically located at the connection of 3 out of 4 lines. Based on BBFS, by removing this station, 25% of stations are out of service, and the residual graph is a non-cyclic graph. Therefore, based on graph theory, the network has become a tree that cannot provide any route alternative between two nodes.

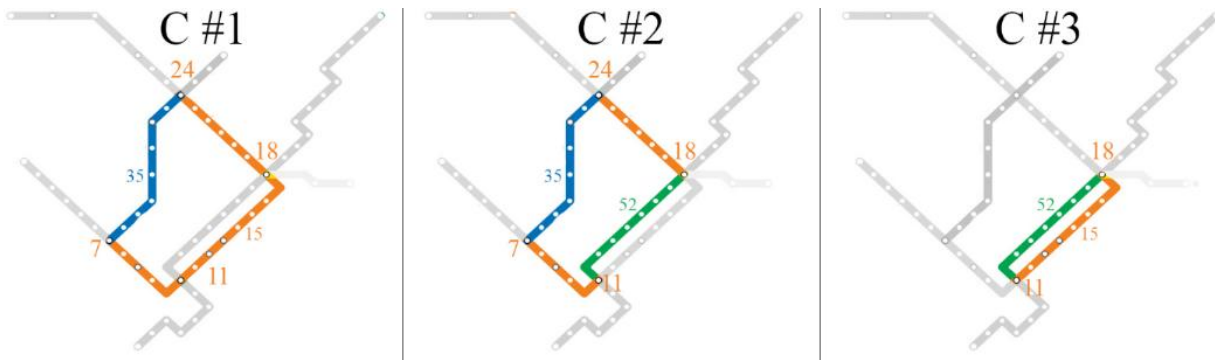


Figure 5-9 Cycles in Montreal UMN

In PRFS, the first disrupted station is Snowdon (station #7), impacting 18% of stations and causing the elimination of cycles C1 and C2, significantly reducing theoretical redundancy. Conversely, in ORFS, station #33 (Université-de-Montréal) is the initial disrupted station, resulting in a mere 5% physical degradation but leading to the loss of cycles C1 and C2 in the network. While in RRFS, we see that after the 3rd disruption (when stations #39 (Fabre), #31 (Montmorency), and #2 (Du College) are removed and physical degradation is around 15%), still retains three cycles without a substantial decrease in theoretical redundancy. However, after the fourth disruption (when station #49 (Atwater) is disrupted), two cycles (C2 and C3) are lost, and the theoretical redundancy considerably decreased.

Figure 5-6-k demonstrating acceptable redundancy, reveals that in TFS, $\overline{R}_p$ decreased to 1 after the first disruption and remained unchanged in subsequent disruptions due to the presence of only one route between each OD pair. Similar patterns are observed in PRFS and ORFS during the initial random disruptions. Surprisingly, despite ORFS having a considerably higher CDNAN than PRFS, $\overline{R}_p$ decreases to 1 sooner in ORFS, indicating that acceptable redundancy is not directly related to CDNAN. Notably, RRFS exhibits the highest acceptable redundancy among all scenarios, highlighting that a high CDNAN value does not necessarily ensure greater redundancy. Moreover, focusing on RRFS, it is observed that after the 8th disruption, the average acceptable redundancy drops below 1, suggesting that for certain OD pairs, single theoretical routes fail to meet the defined quality threshold. Table 5-3 is intended to show more details concerning each scenario. The key findings of the redundancy-based vulnerability analysis are as follows:

Table 5-3 Changes of some vulnerability indicators in different failure scenarios

Disruption Type	Nb. disruptions	$\bar{R}_t$	$\bar{R}_p$	$\bar{R}_u$	Deg (%)	τ_C (%)	τ_D (%)	Disruption Type	Nb. disruptions	$\bar{R}_t$	$\bar{R}_p$	$\bar{R}_u$	Deg (%)	τ_C (%)	τ_D (%)
TFS	Normal State	2.97	1.27	1.70	0.00	0.00	0.00	PRFS	Normal State	2.97	1.27	1.70	0.00	0.00	0.00
	1	1.00	1.00	0.00	25.00	60.58	0.00		1	1.63	1.37	0.26	14.71	23.68	0.00
	2	1.00	1.00	0.00	47.06	79.98	0.00		2	1.70	1.35	0.35	19.12	32.08	0.00
	3	1.00	1.00	0.00	69.12	94.09	0.00		3	1.72	1.37	0.35	26.47	43.60	0.00
	4	1.00	1.00	0.00	77.94	97.17	0.00		4	1.73	1.37	0.36	30.88	45.84	0.00
	5	1.00	1.00	0.00	80.88	97.59	0.00		5	1.00	0.99	0.01	45.59	58.57	0.79
	6	1.00	1.00	0.00	86.76	98.19	0.00		6	1.00	0.99	0.01	52.94	67.97	1.18
	7	1.00	1.00	0.00	92.65	98.93	0.00		7	1.00	0.99	0.01	60.29	74.40	1.23
	8	1.00	1.00	0.00	97.06	99.76	0.00		8	1.00	1.00	0.00	82.35	98.31	0.00
	9	1.00	1.00	0.00	100.00	100.00	0.00		9	1.00	1.00	0.00	89.71	98.95	0.00
ORFS	1	1.67	1.41	0.25	7.35	12.27	0.48	RRFS	10	1.00	1.00	0.00	95.59	99.90	0.00
	2	1.62	1.37	0.26	11.76	20.56	0.00		11	1.00	1.00	0.00	100.00	100.00	0.00
	3	1.00	0.98	0.02	14.71	29.38	1.53		1	2.98	1.70	1.27	4.41	4.27	0.00
	4	1.00	0.98	0.02	20.59	33.99	1.54		2	2.97	1.64	1.33	8.82	13.29	0.00
	5	1.00	0.98	0.02	25.00	41.34	1.75		3	2.96	1.60	1.36	14.71	23.51	0.00
	6	1.00	0.98	0.02	33.82	52.09	2.18		4	1.75	1.19	0.56	19.12	35.08	1.20
	7	1.00	0.96	0.04	39.71	61.22	3.82		5	1.84	1.18	0.66	22.06	37.05	1.56
	8	1.00	0.96	0.04	50.00	69.36	4.18		6	1.84	1.17	0.67	25.00	43.37	1.77
	9	1.00	1.00	0.00	54.41	76.09	0.00		7	1.00	0.90	0.10	30.88	52.44	10.21
	10	1.00	1.00	0.00	58.82	79.11	0.00		8	1.00	0.97	0.03	50.00	68.07	2.64
	11	1.00	1.00	0.00	63.24	84.92	0.00		9	1.00	1.00	0.00	52.94	69.14	0.00
	12	1.00	1.00	0.00	67.65	86.14	0.00		10	1.00	1.00	0.00	64.71	76.98	0.00
	13	1.00	1.00	0.00	72.06	87.27	0.00		11	1.00	1.00	0.00	69.12	83.34	0.00
	14	1.00	1.00	0.00	75.00	90.06	0.00		12	1.00	1.00	0.00	79.41	96.37	0.00
	15	1.00	1.00	0.00	79.41	93.70	0.00		13	1.00	1.00	0.00	86.76	99.06	0.00
	16	1.00	1.00	0.00	83.82	94.90	0.00		14	1.00	1.00	0.00	92.65	99.68	0.00
	17	1.00	1.00	0.00	88.24	98.61	0.00		15	1.00	1.00	0.00	92.65	99.68	0.00
	18	1.00	1.00	0.00	88.24	98.61	0.00		16	1.00	1.00	0.00	94.12	99.80	0.00
	19	1.00	1.00	0.00	91.18	99.31	0.00		17	1.00	1.00	0.00	100.00	100.00	0.00
	20	1.00	1.00	0.00	91.18	99.31	0.00								
	21	1.00	1.00	0.00	97.06	99.88	0.00								
	22	1.00	1.00	0.00	100.00	100.00	0.00								

- 1-The network is significantly more vulnerable to targeted disruptions.
- 2-Unlike prior vulnerability analyses emphasizing $C.Deg(\%)$ and CDNAN are the most impactful factors, in redundancy-based analysis, there is no explicit link between the mentioned factors and vulnerability.
- 3-Using the theorems of graph theory and studying the geometry of the network show that the most impactful factors in network redundancy are the geometric shape of the network (in the normal state) and the location of disrupted stations (if there are disruptions).

5.6 Conclusion

This research has presented a comprehensive vulnerability analysis of the Montreal Urban Metro Network (UMN). The study not only developed widely used methods such as network-based and serviceability-based approaches but also introduced a new method called the redundancy-based vulnerability method. In the network-based method, two novel connectivity-related indicators, namely P.C and R.C, were introduced to assess station connectivity and network fragmentation compared to the normal state. Additionally, three innovative indicators—theoretical, acceptable, and unacceptable redundancy—were introduced to evaluate the redundancy of the network, taking into account the quality of route alternatives.

Failure scenarios were considered, including one targeted and three random scenarios—pessimistic, realistic, and optimistic—which were simulated using Block-Based Failure Strategy (BBFS). The results of all analyses, ranging from network-based to serviceability-based and redundancy-based, indicated that the Montreal UMN is significantly more vulnerable to targeted disruptions than random ones. Moreover, it was observed that the difference between targeted and random failure scenarios is more significant in serviceability-based analysis than in other analyses, indicating that functional indicators are more sensitive to the type of disruption than topological, connectivity, and redundancy indicators. When comparing random failure scenarios, it was evident that the smaller the value of the Corresponding Disruption Number to the Annihilation of the Network (CDNAN), the greater the network's vulnerability. Thus, the network was more vulnerable to the Pessimistic Random Failure Scenario (PRFS) than the Realistic Random Failure Scenario (RRFS) and was more vulnerable to RRFS than the Optimistic Random Failure Scenario (ORFS). Changes in vulnerability indicators were most significant during the initial disruptions of

the failure scenarios, highlighting that the first disruptions have the most impact. This effect was more noticeable in the Targeted Failure Scenario (TFS) than in random scenarios and in functional indicators than in other categories of indicators. Generally, there was a correlation between changes in vulnerability indicators and CDNAN: the larger the CDNAN in a scenario, the lower the rate of change for indicators during the initial disruptions. However, there was a different trend in redundancy-based analysis. The results demonstrated that changes in network redundancy were not sensitive to CDNAN and instead were more sensitive to the location of disrupted stations and the geometric shape of the network.

In this research, our analysis focused exclusively on trips made solely by the metro, omitting users who combined metro travel with other modes such as buses, bike sharing, or carsharing. Future research will incorporate a broader range of travel demand scenarios, including trips where the metro is used either independently or in combination with other modes. This expanded scope will enable us to assess the potential role of different urban transport services, such as buses and bike sharing systems, in mitigating disruptions within the metro system. Additionally, we plan to conduct a detailed socio-demographic study from a microscopic perspective to identify population segments that may be disproportionately affected by disruptions, providing insights into how disruptions impact various user demographics.

The findings of this research offer network operators a fresh perspective for assessing station importance, particularly valuable in crisis situations to prioritize stations for reopening operations. Traditionally, station evaluation focused solely on topological (e.g., centrality) or functional (e.g., passenger volumes) aspects. However, this study underscores the significance of station location and its impact on network geometry, notably in maintaining travel quality. Therefore, in crisis management scenarios, stations that enhance network redundancy (sustaining network cycles) should take precedence in reopening efforts, even over stations with higher traffic but lesser impact on network redundancy, as they contribute to improved travel quality for all passengers. Furthermore, with the introduction of SDCFs and BBFS in this study, network designers are encouraged to integrate the BBFS algorithm into their design processes and strategically plan for SDCFs, considering their limitations in terms of cost, complexity, and construction time, to optimize their location.

CHAPTER 6 ARTICLE 3: ARE CONVENTIONAL BUS SYSTEMS MITIGATING THE IMPACTS OF METRO DISRUPTIONS? THE CASE OF MONTREAL

Authors: Kaveh Rezvani Dehaghani¹, Catherine Morency¹

¹Department of Civil, Geological and Mining Engineering, Polytechnique Montréal.

Montreal, QC, H3C 3A7, Canada.

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ABSTRACT

In this research, we present a comprehensive framework designed to analyze the potential role of conventional bus systems in mitigating the repercussions of metro disruptions. This framework is applicable to short-duration disruptions and can be extended to medium-duration ones. It is also adaptable to include additional modes, supporting multimodal transportation solutions. We tested this approach using the Montreal transit network as a case study. Initial modeling of the transit network was performed using GTFS data, followed by route assignment for all transit trips under normal conditions using Montreal OD 2018 data and the Transition platform. Twelve station-based metro disruption scenarios were defined based on station importance and historical disruption frequency. After simulating these scenarios, trips were rerouted in the disrupted network. By introducing various trip quality thresholds, we compared feasible metro-bus trips post-disruption with normal conditions. Results show a consistent linear relationship between disruption extent and feasible trips across all trip quality levels. Additionally, the travel time surplus at each quality level exhibited predictability and a linear correlation with disruption extent. Notably, the occurrence time of disruptions within the metro system did not significantly impact feasible trips or surplus travel time following disruptions.

6.1 Introduction

In contemporary urban transportation, the metro has evolved into an inseparable component, significantly influencing urban mobility within metropolises. Hence, disruptions within the metro system can potentially bring chaos to the transit network. Accidents, incidents, events, and adverse weather conditions are common occurrences that frequently disrupt metro systems worldwide. For

instance, the Beijing metro system experienced 408 disruptions from January 2014 to December 2018. These encompassed 73 disruptions in 2014, 173 in 2015, 95 in 2016, 40 in 2017, and 27 in 2018 (K. Liu, Zhu, & Wang, 2021). In 2015, 144 disruptions were recorded in the metro system in Toronto (Itani & Shalaby, 2021), and based on data received from Société de transport de Montréal, between 2019 and 2023 (5 years), the Montreal metro experienced, 1,327 disruptions lasting more than 10 minutes (ouvertes).

Certain disruptions are swiftly addressed, often within minutes, while others persist for extended periods, spanning hours or even days. Metro operators adapt their mitigation measures based on the duration of disruptions, implementing diverse strategies to alleviate their impact. They may address metro disruptions through internal measures like timetable adjustments and rolling stock modifications, or externally via substitute bus services, commonly referred to as bus bridging (S. Zhang et al., 2021). However, tasks such as calling buses from various lines for shuttle services, providing drivers with information about selected routes, and notifying them of the new schedule and shift changes are relatively time-consuming. These activities may not be practically feasible for addressing short-duration disruptions. In some instances, it has been observed that disruptions are resolved, and the metro network returns to normalcy even before these measures can be implemented, or their results assessed. In the event of such disruptions, the responsibility for mitigating the impact of disruptions in metro systems rests solely on conventional public transport services, with bus systems taking the forefront. This is due to the widely recognized status of bus systems as complements to metro systems (Ma, Shi, et al., 2020), offering comparable attributes in terms of cost and service coverage area.

Previous research has predominantly concentrated on presenting innovative mitigation measures and optimizing those introduced. However, they have frequently overlooked the assessment of the functionality of the unaffected sections of the transit network in mitigating the impact of metro disruptions. Not only are these mitigation measures often inadequate for addressing short-duration disruptions, but also there is currently no suitable method to compare the overall performance of the transit system before and after the implementation of these measures during metro disruptions. For instance, the widely adopted bus bridging system imposes significant costs and supply reductions. It is crucial to evaluate how the remaining operational parts of the transit system can manage metro disruptions in the absence of such measures. Therefore, detailed studies on the quality of conventional transit systems during metro disruptions are essential to provide insights

into system performance during short-duration disruptions and to evaluate the necessity of external mitigation measures, thereby determining their actual requirement.

To address the mentioned research gaps, this research aims to present an adaptable approach to evaluate the quality of conventional bus service during instances of metro disruptions, specifically those aligned with short-duration disruptions. This adaptable approach is applied to evaluate the quality of route alternatives during disruption(s) in the Montreal metro network, and it can be implemented for various networks and different alternative transit modes. The main objectives of this research are as follows:

- To estimate the proportion of trips within the metro network that remain unaffected, considering both route and mode choices, under various disruption scenarios.
- To assess the effectiveness of conventional bus services in maintaining travel quality amidst disruptions within the metro system.
- To predict the surplus of travel time incurred when utilizing conventional bus services during metro disruptions.
- To investigate the influence of the timing of disruptions within the metro system on the quality of alternative transportation options and the resulting surplus of travel time.

6.2 Background

Due to the increasing importance of subway systems in modern urban transportation, invaluable studies have been conducted to investigate the impact of disruptions, assess vulnerability and robustness, and analyze the resilience of metro networks. System vulnerability denotes the extent to which a system and its components are susceptible to attacks or external influences, characterized by their ease of being compromised and their lack of ability to resist or recover from disruptions (Asteris et al., 2019) (Salas & Yepes, 2018). However, in transportation engineering, vulnerability is defined as *"a susceptibility to incidents that can cause considerable reductions in road network serviceability"* (Berdica, 2002). Additionally, other definitions have been proposed by several authors (see (D'este & Taylor, 2003), (Nicholson, 2007), (Colicchia et al., 2010), (Knoop et al., 2012), (Baroud, Ramirez-Marquez, et al., 2014), (Baroud, Barker, et al., 2014), (Khademi et al., 2015)). Conversely, robustness, a concept utilized across various disciplines related to networked systems, has been introduced in transportation engineering through the

Network Robustness Index (Scott et al., 2006) for evaluating the critical importance of highway segments. Sullivan, Novak et al. (2010) defined network robustness as "the degree to which the transportation network can function in the presence of various capacity disruptions on component links" (Sullivan et al., 2010). Subsequent definitions have also been presented for robustness (see (Snelder et al., 2012) and (de Oliveira et al., 2016)).

Early studies on metro system assessments have primarily concentrated on examining the robustness and vulnerability of metro networks from the perspective of complex network theory. For instance, Z. Du et al., (2020). introduced an innovative method for determining node significance in metro networks using the Improved Topological Potential model considering Entropy (ITPE). This method was subsequently applied to the transit system in Shenzhen Utilizing Complex Network Theory (CNT), Kopsidas & Kepaptsoglou, (2022) introduced a method for identifying critical stations within the metro system. This methodology was then applied to the Athens metro system. Some researchers took the concept further and considered bus and metro networks simultaneously. Ma, Shi, et al., (2020) created a double layer transit network consisting of bus and metro networks and explored the robustness of public transportation systems. X. Li et al., (2018) presented a two-layer multiplex network to assess the public transportation system. These kinds of research are considered as the first step towards developing mitigation measures, providing public transportation planners with insights to mitigate the impacts of disruptions in metro networks.

Certain researchers extended the conceptual framework, concentrating on exploring measures that can be implemented to mitigate the impact of disruptions in metro systems. As an illustration, Malucelli & Tresoldi, (2019) conducted research on disruption management in local public transportation systems. This involved formulating an optimization problem aimed at minimizing operational costs arising from changes in the originally planned driver scheduling. Huang, (Y. Huang et al., (2020) simultaneously addressing supply and demand aspects, introduced a mixed-type formulation for disruption management during metro disruptions. This approach involved integrating a big-M block with a time-indexed block to effectively handle both train scheduling and passenger flow modeling. Despite the presence of such research emphasizing internal mitigation measures in metro systems, the predominant focus among scholars has leaned towards external measures. This emphasis, particularly on bus substitute services, has gained prominence since the introduction of the term "bus bridging" by Kepaptsoglou & Karlaftis, (2009). H. Hu et

al., (2016) proposed a nonlinear integer programming model designed for planning bus bridging services along a rail transit line during disruptions. This model was solved using a genetic algorithm and implemented in the context of the Shanghai metro system. Van der Hurk et al., (2016) put forth a model for the optimal selection of shuttle lines and frequencies within budget constraints. This model found practical application in addressing a shuttle design challenge within the Massachusetts Bay Transportation Authority network in Boston.

Reviewing the literature regarding the vulnerability assessment of public transportation reveals a scarcity of studies on short-duration disruptions, with no specific threshold identified to differentiate short-duration disruptions. One of the limited research endeavors examining short-term disruptions was conducted by Arslan Asim et al., (2021), which investigated users' mode choice behavior in Calgary, Canada, during short-term planned service disruptions in light rail transit. Given the nature of the referenced research, short-term planned service disruptions for Light Rail Transit (LRT) were defined as scheduled weekend disruptions lasting up to three days which is conceptually different from the focus of our current research. In another related study conducted by Srikukenthiran & Shalaby, (2017), various disruptions were simulated in the Toronto transit network. While explicit categorization into short-, medium-, and long- duration disruptions was not performed in this research, the findings highlighted a significant difference in network resilience among disruptions lasting 10 minutes, disruptions lasting 20-30 minutes, and disruptions exceeding 30 minutes.

The research gap in offering a precise definition for short-duration disruptions has led prior researchers to concentrate on medium- and long-duration disruptions. While statistical studies indicate that short-duration disruptions occur more frequently than other types. In practical terms, however, it is observed that certain metro operators empirically set a critical threshold. For disruptions shorter than this established threshold, no specific mitigation measures -neither internal nor external- are implemented. Instead, they rely on the efficiency of conventional transit services to mitigate the impact of metro disruptions. Up to this point, no research has been conducted to assess the efficiency and quality of conventional systems in mitigating the impact of metro disruptions. Additionally, there has been no attempt to establish a logical relationship between the disruption extent in the metro network and the surplus travel time imposed on the network's users. Moreover, while some previous studies have considered disruption duration and network recovery time, few have examined the precise timing of disruptions in transit systems. The impact of

disruption timing on travel time surplus and the effectiveness of mitigation measures throughout the day remains largely unexplored. This gap in the literature underscores the need for a thorough investigation into how the occurrence time of disruptions influences transit system performance and the quality of mitigation efforts.

6.3 Methodological approach

In this study, we introduce an adaptable five-step framework designed to assess the quality of conventional bus-service in mitigating the impact of disruption in metro network. During the initial step, referred to as data collection, we introduce and prepare our primary sources, namely GTFS and OD data. These datasets serve as representations of travel supply and demand, respectively. The second step pertains to route assignment and the travel time calculation under normal network operations. In this step, we utilize Transition, an open-source platform developed at Polytechnique Montreal (www.transition.city), which operates through the Connection Scan Algorithm (CSA). In the third step, we utilize the Block Based Failure Strategy as the failure mechanism. Through assessing the importance of stations and reviewing their disruption history, we define hypothetical disruptions. Subsequently, various disruption scenarios are defined and simulated in Transition. Then OD trips are rerouted for each disruption scenario within the disrupted networks, and related travel times are calculated. In the fourth step, various thresholds are introduced to rank the quality of route alternatives based on their travel times. The last step involves modeling the relationship between the disruption extent and the imposed surplus travel time on the network as well as the relationship between disruption extent and percentage of feasible trips for each quality threshold. Figure 6-1 presents more details about the introduced framework. Complementary information about each step and the transferability of the framework is presented below.

6.3.1 Step1: Data collection

To execute this framework, it is essential to acquire supply-related data including routes, stations, and operational details such as headways, operational speed, etc. for both the metro and the alternative mode(s) (specifically, the conventional bus system in this research), along with travel demand information. If accessible, the best choices for these requirements are GTFS files - published by service operators- for the former, and OD matrices for the latter. It is worth stating that to have accurate results only trips exclusively made by the metro and trips made using a combination of the metro and the alternative mode(s) are considered.

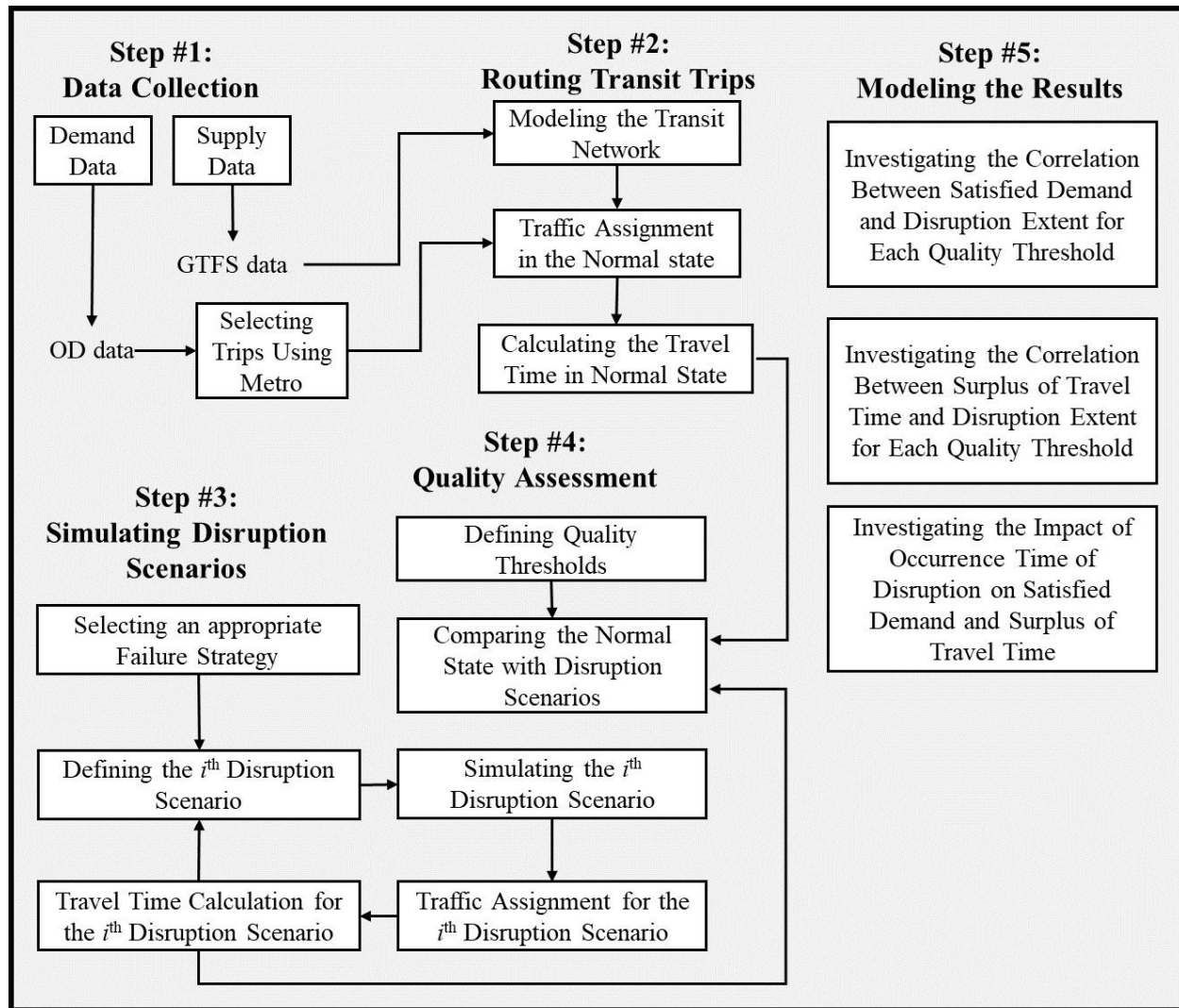


Figure 6-1 Methodological framework schema

6.3.2 Step 2: Route assignment and travel time calculation in normal state

After collecting the data and creating models for transit networks, the next step involves routing transit trips and calculating their travel times. In this process, we utilized the Transition Platform which is an open-source software developed at Polytechnique Montreal. This platform employs the Connection Scan Algorithm (CSA) to generate transit alternatives. To determine the total travel time for each trip, we consider the summation of total vehicle time, total waiting time, access and egress time, and transfer walking time (if applicable). Moreover, before the trip assignment, based on travel behavior in the studied network, three variables, namely Maximum Access and Egress Travel Time, Maximum Access Time During Transfers, and Maximum First Waiting Time are initialized. The total number of trips is denoted by D_{Total} .

6.3.3 Step 3: Disruption simulation and disruption scenarios definition

This step constitutes the core of the research and consists of three major phases:

Initially, the adoption of a failure mechanism is necessary, choosing from available strategies such as node-based, link-based, or block-based failure strategies. Given the compatibility of the Block-Based Failure Strategy (BBFS) with metro networks, it is employed in this study. To implement BBFS, it is crucial to comprehend that a block comprises several consecutive regular stations positioned between two Stations with Direction-Changed Facilities (SDCF). Therefore, following BBFS, if one regular station is disrupted, the entire block to which it belongs is removed. Similarly, if one of the SDCF is disrupted, all the blocks connected to it are also removed. The accompanying Table 6-1 provides further details in this context. It is noteworthy that each metro line consists of two paths (e.g., North and South or East and West). Consequently, in coding the BBFS algorithm, consideration should be given to the existence of two types of SDCF in Urban Metro Networks (UMNs): Fully equipped SDCFs with embedded direction-changing facilities for both paths, and semi-equipped SDCFs providing direction-changing facilities only for one path.

After adopting the appropriate failure strategy, the next step is to define various disruption scenarios. This involves assuming disruptions at different metro stations and simulating these disruptions. Hypothetical disrupted stations can be selected either randomly or purposefully. In the scope of this research, our emphasis lies on targeted disruptions, with a specific focus on two approaches: station selection grounded in station importance and station selection predicated on the historical frequency of disruptions encountered at the station. All the defined scenarios, wherein different extents of disruption in the metro network are observed, need to be simulated. In this research, simulations are conducted using the Transition Platform.

Ultimately, within the simulated scenarios, all trips must undergo routing, and their respective travel times need to be calculated. It is crucial to ensure that all configurations of route assignment mirror exactly what was executed in the second step for trips under normal operating conditions. Following route assignment in the disrupted network, the trips undergo classification into three distinct groups: Trips Utilizing the Same Route as Before the Disruption (TUSRBD), denoting trips that adhere to their original routes; Substandard Trips (SST), representing trips that are no longer executable within the transit system; and TUA that denotes trips that remain feasible within the transit system, albeit requiring alterations to the route or both route and mode.

6.3.4 Step 4: Quality assessment of route alternatives

In metropolitan areas, an anticipation exists that, in the event of partial or even complete disruptions to the metro system, most trips will still have alternative routes within the transit network. Nonetheless, it is noteworthy that not all trips possess alternatives characterized by quality equivalent or nearly equivalent to that of the fastest route utilized before the disruption. At times, the increase in travel time reaches a point where users are no longer willing to sustain their reliance on the transit system; this point may vary among different users and situations. Consequently, it is imperative to establish distinct thresholds that can be applied to discern whether users would accept or reject the alternative in each scenario.

We, in this research, introduce an indicator called K_i that can be employed to define the quality threshold for the alternatives. This indicator demonstrates the ratio of the travel time of trip i in each scenario and in the normal state:

$$K_i^S = \frac{ADTT_i^S}{BDTT_i} \quad \text{Equation 6-1}$$

Where $ADTT_i^S$ represents the travel time of the fastest alternative for trip i in scenario S , i represents the trip number, with values spanning from 1 to D_{Total} . S represents the scenario number, and $BDTT_i$ represents the travel time of trip i in the normal state.

It is evident that if the disruption does not alter the route of trip i , the travel time remains constant both before and after the disruption. Consequently, for all TUSRBD the value of K is equal to 1. However, if the disruption impacts trip route/mode choice, there is a possibility that the alternative route has an equivalent or more plausibly a longer travel time. In the given circumstances, employing the threshold $K_i \leq 1.1$ indicates that users express willingness to persist in utilizing the transit system only if their travel time increases by 10% or less. Correspondingly, the feasible trips denoted as $FT_{1.1}$, is associated with $K_i \leq 1.1$. Analogously, the condition $K_i \leq 1.25$ signifies that users accept to continue using transit only if their travel time increases by 25% or less, and $FT_{1.25}$ denotes the corresponding feasible trips when $K_i \leq 1.25$. Extending this rationale, three other thresholds ($K_i \leq 1.5$, $K_i \leq 1.75$, $K_i > 1.75$) are delineated, leading to the assignment of three distinct values of feasible trips ($FT_{1.5}$, $FT_{1.75}$, and FT_{all}) to these thresholds. It is evident that every member (trip) belonging to set $FT_{1.1}$ is also a member of sets $FT_{1.25}$ through FT_{all} ; however, the converse is not necessarily true. In other words, $FT_{1.1} \subset FT_{1.25} \subset FT_{1.5} \subset FT_{1.75} \subset FT_{all}$. The

percentage of feasible trips following the application of each threshold in each scenario, represented by SD_n (%), is computed using the formulation below:

$$FT (\%) = \frac{|FT|}{D_{Total}} \times 100 \quad \text{Equation 6-2}$$

Where $|FT|$ is the cardinality of the set SD_n .

For each scenario, an indicator is estimated for feasible trips after applying each threshold, referred to as the Surplus of Travel Time (τ). This indicator corresponds to the percentage increase in average travel time in each scenario after applying a specific quality threshold, compared to the normal state. This indicator is estimated using the following equation:

$$\tau = \frac{ATT_{N.S} - ATT}{ATT_{N.S}} \times 100 \quad \text{Equation 6-3}$$

Where: $ATT_{N.S}$ shows the average travel time in the normal state, ATT represents the average travel time of feasible trips (in each scenario).

6.3.5 Step 5: Modeling the results

Since regression models represent an important class of mathematical models that commonly serve as the foundational framework for numerous other model types (Lee, Im, & Lee, 2023), we use regression models for our prediction processes. Therefore, by considering the "disruption extent in the metro network" as the explanatory factor, we utilize regression models to predict both the "feasible trips after applying each quality threshold" and the "surplus of travel time caused by using alternative route/mode after applying each quality threshold" during the metro disruption. In these models, the observation number corresponds to the number of simulated scenarios. Hence, more disruption scenarios should lead to a more accurate model.

However, to understand how the occurrence time of a disruption might impact the percentage of feasible trips after applying each quality of alternative, we need a more detailed approach. This requires dividing the day into specific time segments for analysis. In this research, we define three time segments : AM rush hours (6:00 to 8:59), PM rush hours (15:00 to 17:59), and off-peak hours. Three distinct datasets are created for each scenario based on trips departure times. These datasets illustrate Morning Peak hour trips ($j=MP$), Evening Peak hour trips ($j=EP$), and off-peak trips ($j=OP$). In each dataset, as an initial step, the total number of trips is divided by duration of time period to determine the trip rate per hour for each (FT^j). Subsequently, within each scenario, every

dataset is examined separately. We utilize an indicator denoted by FT_K^j , which indicates the percentage of feasible trips for a specific quality threshold within each scenario and specific time segment. For instance, $FS_{1.1}^{MP}$ within each scenario represents the percentage of feasible trips during morning peak hours when $K \leq 1.1$. To calculate this indicator, the following formulation is used:

$$FT_K^j(\%) = \frac{|FT_K^j|}{|FT^j|} \times 100 \quad \text{Equation 6-4}$$

Where $|FT_K^j|$ represents the number of SD_K trips in time segment j , and $|FT^j|$ indicates the number of all trips made during time segment j .

After applying each quality threshold, three sets are delineated based on disruption timing (corresponding with $j=MP$, $j=EP$, and $j=OP$), and the cardinality of these sets aligns with the number of simulated scenarios (12 in this study). Then, two-sample hypothesis tests are applied. The null hypothesis (H_0) in this test is a statement that there is no significant difference between the means of the two groups being compared.

6.4 Framework implementation

6.4.1 Case study

The study is centered on the Montreal metropolitan area, recognized as the second-largest population center in Canada, boasting a population of 4.3 million. The metro system under investigation features 4 lines, 68 stations, and has a length of 70 km; it is one of the largest metro systems in Canada. Each day, 820,000 metro trips are recorded within this extensive network (STM). Based on BBFS, the Montreal metro network is delineated into 24 blocks, as illustrated in Table 6-1. Furthermore, the region is served by three bus operators; STM, with 216 bus routes and 8,747 stops; STL, encompassing 99 bus routes and 2,827 bus stops; and RTL, operating 80 bus routes with 3,510 bus stops.

Table 6-1 Montreal UMN blocks

	Block ID	Associated Stations
	1	1,2
	2	2,3,4,5
	3	5,6,7
	4	7,8,9,10,11
	5	11,12,13,14
	6	14,15,16,17
	7	18,19,20,21,22,23
	8	23,24
	9	24,25,26
	10	26,27,28
	11	28,29,30,31
	12	7,32,33,34,35,36,37
	13	37,38
	14	38,24
	15	24,39,40,41
	16	42,43,44,45,46,47,48,11
	17	11,49
	18	49,50,51,52
	19	52,53,54,18
	20	18,55,56,57
	21	57,58,59,60
	22	60,61
	23	61,62,63,64,65,66
	24	18,67,68

6.4.2 Step 1

In this research, we use data from the 2018 Montreal Origin-Destination survey. Our focus is specifically on trips undertaken exclusively via metro and a combination of metro and bus, yielding a total of 29,477 recorded trips (representing 813,503 trips after expansion). Furthermore, we incorporate General Transit Feed Specification (GTFS) data from the three previously mentioned operators to model the transit network.

6.4.3 Step 2

As previously indicated, the Transition platform is used for the routing of transit trips; it provides features of the best transit route along with travel times components (access, in-vehicle, waiting, transfer). Transition parameters are set to: 15 minutes for maximum waiting time at the first station, 15 minutes for maximum access and egress travel time as well as for transfers. Consequently, the trip assignment resulted in a total of 717,807 trips, encompassing over 88% of the initial trips. The findings reveal that transit trips have an average travel length of 7.9 kilometers and average duration of 43 minutes. Distributions of both trip length and duration are presented in Figure 6-2.

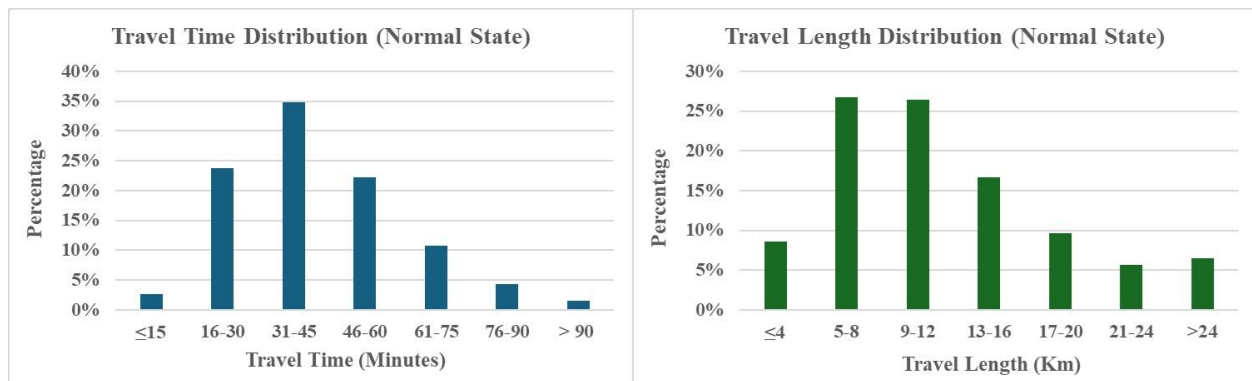


Figure 6-2 Trip attributes in the normal state

6.4.4 Step 3

The research design includes the simulation of 12 distinct disruption scenarios: 3 scenarios with a single disruption at pivotal stations (Figure 6-3 a, b, and c), 3 scenarios with two simultaneous disruptions at pivotal stations (Figure 6-3 d, e, and f), 3 scenarios with a single disruption at the most frequently disrupted stations (Figure 6-3 g, h, and i), and 3 scenarios with two simultaneous disruptions at the most frequently disrupted stations (Figure 6-3 j, k, and l). Historical records of metro disruptions globally indicate that occurrences of more than two simultaneous disruptions are exceedingly rare. Consequently, our research confines itself to the simulation of two simultaneous disruptions.

Our scenarios comprehensively span all potential network degradation ranges resulting from one or two disruptions, encompassing degradation levels from 3% to 47%. Station 18 is notable for its specificity; in tandem with its designation as the top-ranked among the pivotal stations in the network, this station also has the highest frequency of disruptions. Consequently, in scenario #1,

we simulate a single disruption at station #18 and explore three combinations involving simultaneous disruptions at station #18 and other stations in scenarios #4 to #6. Thus, in delineating disruption scenarios based on the frequency of disruptions, considering the four simulations of disruptions in Station 18 in scenarios #1 to #6, we designate Station #50 as the station experiencing the highest frequency of disruptions (ranked 2 based on historical disruptions). To facilitate further identification, scenarios are named using the following logic:

- The first character refers to the number of simultaneous disruptions: “S” for single and “D” for double.
- The second term relates to the way stations facing disruptions are selected: “SI” stands for station importance while “DF” refers to disruption frequency (based on historical data);
- The third term indicates the disruption extent, showing the percentage of the metro network affected (e.g., "25" means 25% of the network is disrupted).
- The final element, enclosed in parentheses, is used to differentiate scenarios that share identical characteristics in the preceding terms but represent different instances.

Table 6-2 reveals that across all scenarios, at least 99% of trips have alternative routes. The maximum value observed for Substandard Trips (SST) is 0.9, aligning with scenario #5, characterized by a disruption extent of 40%. Moreover, another noteworthy observation pertains to TUSRBD, with its maximum value reaching 60%. This signifies that in all scenarios, even when only 3% of the network is disrupted, at least 40% of trips must change route.

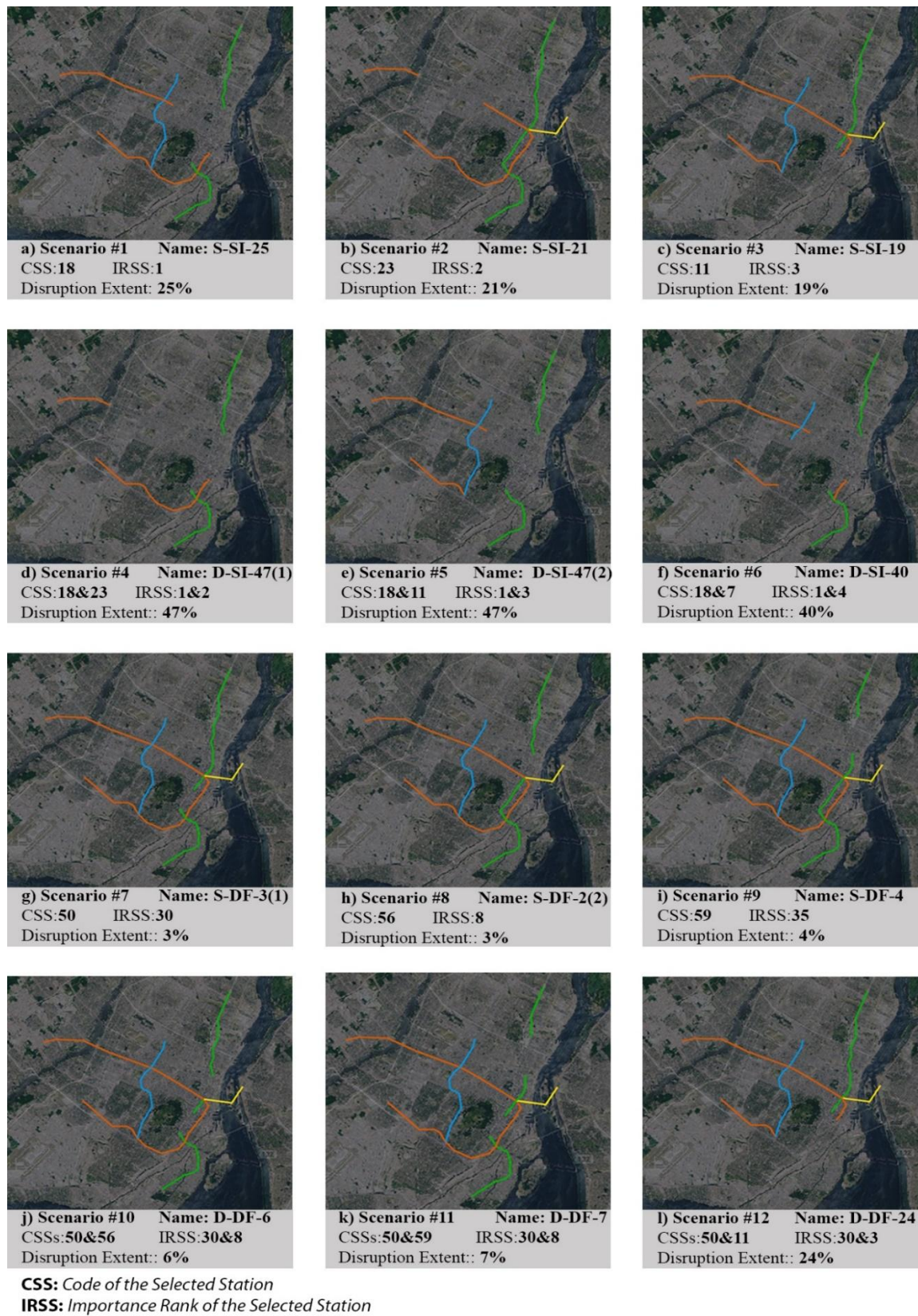


Figure 6-3 Simulation of disruption scenarios

Table 6-2 presents various trip attributes for the various scenarios. In almost all scenarios, the most significant decrease in travel time distribution share pertains to trips lasting 30 to 45 minutes. This category experiences a reduction of approximately 40% when the network undergoes major disruptions. Conversely, across all scenarios, the most notable relative surge in trip proportion is observed in the longest trips, lasting more than 90 minutes. While constituting a mere 1% in the Normal State, in certain scenarios, this category increases to 10% or more. Furthermore, as expected, in all scenarios, there is a reduction in the frequency percentage of short trips (those lasting less than 45 minutes) compared to the Normal State. At the same time, there is a corresponding rise in the percentage of long trips, causing an overall lengthening of trips. This becomes evident when considering the Disruption extent, Average Travel Time (ATT), and surplus of travel time (τ_A). Surplus of travel time indicates the percentage by which travel time has increased compared to the Normal State. Raising the disruption extent in each scenario results in an increase in the ATT and τ_A .

Table 6-2 Trip attributes in disruption scenarios

Scenario Name	Disruption Extent	All Feasible Trips (combination of TUSRBD + TAU)										TAU	SST
		Trips Attributes		Travel Time distribution (min)									
		ATT (min)	τ_A	USRBD (%)	<15	(15-30]	(30-45]	(45-60]	(60-75]	(75-90]	>90	τ_B	SST (%)
		43	-	-	3%	24%	35	22%	11%	4%	1%	-	-
S-SI-25	25%	54	25%	26%	1%	12%	26%	26%	17%	10%	7%	35%	0.2%
S-SI-21	21%	48	12%	48%	2%	18%	30%	24%	15%	7%	4%	23%	0.1%
S-SI-19	19%	48	12%	49%	2%	17%	31%	25%	14%	7%	4%	24%	0.1%
D-SI-47 (1)	47%	58	35%	15%	1%	10%	22%	25%	19%	12%	11%	41%	0.3%
D-SI-47 (2)	47%	58	34%	18%	1%	10%	23%	26%	18%	12%	10%	41%	0.3%
D-SI-40	40%	60	39%	11%	1%	9%	20%	25%	19%	13%	12%	45%	0.9%
S-DF-3 (1)	3%	45	4%	59%	2%	20%	35%	24%	12%	5%	2%	11%	0.2%
S-DF-3 (2)	3%	47	8%	56%	2%	19%	32%	25%	13%	6%	3%	19%	0.3%
S-DF-4	4%	46	7%	58%	2%	20%	33%	23%	13%	6%	2%	16%	0.3%
D-DF-6	6%	47	10%	49%	2%	18%	32%	25%	14%	6%	3%	20%	0.3%
D-DF-7	7%	47	9%	49%	2%	18%	33%	24%	13%	6%	3%	18%	0.3%
D-DF-24	24%	49	14%	44%	2%	16%	30%	25%	15%	8%	4%	26%	0.2%

ATT represents the average travel time for all feasible trips, encompassing both TUSRBDs and TAU. Considering the significant share of TUSRBDs in each scenario, particularly in those with a lower disruption extent, a detailed analysis would be insightful. Specifically, comparing the travel time of TUAs to their travel time in the Normal State could provide valuable insights.. Hence, τ_B indicates the surplus of travel of TUAs. Analyzing τ_A and τ_B in each scenario reveals a notable trend where τ_B consistently surpasses τ_A in all scenarios.

6.4.5 Step 4

In all scenarios, a noteworthy portion of trips must change route or even mode for their realization. Consequently, we use the defined threshold in each scenario to determine feasible trips according to each quality level. Table 6-3 shows that in the first scenario, characterized by the disruption of the network's pivotal station, merely 26% of trips are not affected and still use the same route while for 14% of trips, travel time increases by 10% or less, resulting in a value of $FT_{1.1}(\%)$ equals to 40%. Similarly, the analysis reveals that travel time increases between 10% to 25% for 16% of the trips, leading to a corresponding value of $FT_{1.25}(\%)$ amounting to 56%. Similarly, $FT_{1.5}(\%) = 78\%$, $FT_{1.75}(\%) = 91\%$, and $FT_{all}(\%) = 99.8$, and 0.2% of trips in this scenario can no longer be made using transit.

Table 6-3 also confirms a trend between Disruption extent in various scenarios and reduction in the percentage of Trips Using the Same Route as Before the Disruption (TUSRBD). Scenario#7, characterized by a minimal disruption extent (3%), exhibits the highest values for both TUSRBD and $FT_{1.1}(\%)$. In this scenario, 59% of trips remain identical to the normal state, and the travel time for 87% of the trips increases by 10% or less. However, in contrast to our initial expectations, scenarios #4 and #5, featuring the highest disruption extent at 47%, do not yield the minimum shares for these indicators. Instead, scenario #6, with a disruption extent of 40%, emerges as the scenario associated with the minimum values for TSRBD and $FT_{1.1}(\%)$.

Table 6-3 indicates a noticeable correlation between the increasing Disruption extent and the corresponding rise in the surplus of travel time (τ). This correlation becomes more pronounced as parameter K increases. Across nearly all scenarios, the augmentation of K corresponds to a concurrent rise in the value of τ .

Table 6-3 Satisfied demand and surplus of travel time after applying quality thresholds

Scenario Name	K=1	K≤1.1		K≤1.25		K≤1.5		K≤1.75		K>1.75		SST
	USRBD	$FT_{1.1}$	$\tau_{FT_{1.1}}$	$FT_{1.25}$	$\tau_{FT_{1.25}}$	$FT_{1.5}$	$\tau_{FT_{1.5}}$	$FT_{1.75}$	$\tau_{FT_{1.75}}$	FT_{all}	$\tau_{FT_{all}}$	
S-SI-25	26%	40%	9%	56%	16%	78%	24%	91%	26%	100%	28%	0.2%
S-SI-21	48%	68%	5%	81%	8%	91%	12%	96%	14%	100%	15%	0.1%
S-SI-19	49%	69%	5%	80%	9%	91%	12%	96%	14%	100%	14%	0.1%
D-SI-47 (1)	15%	26%	11%	42%	19%	68%	29%	85%	35%	100%	37%	0.3%
D-SI-47 (2)	18%	28%	9%	43%	18%	69%	29%	87%	34%	100%	36%	0.3%
D-SI-40	11%	20%	8%	34%	19%	62%	34%	83%	40%	99%	41%	0.9%
S-DF-3 (1)	59%	84%	6%	95%	7%	98%	6%	99%	6%	100%	6%	0.2%
S-DF-3 (2)	56%	78%	4%	87%	6%	94%	9%	97%	10%	100%	10%	0.3%
S-DF-4	58%	80%	3%	90%	6%	96%	8%	99%	9%	100%	9%	0.3%
D-DF-6	49%	71%	6%	84%	9%	93%	11%	97%	12%	100%	12%	0.3%
D-DF-7	49%	72%	6%	85%	8%	95%	10%	98%	11%	100%	11%	0.3%
D-DF-24	44%	62%	5%	75%	10%	88%	15%	95%	16%	100%	16%	0.4%

6.4.6 Step 5

In the present analysis, two regression models are estimated. There is a linear relationship between feasible trips or satisfied demand (Y for model 1) at various quality levels and the disruption extent (X for model 1) as well as between surplus of travel time (Y for model 2) at various quality levels and disruption extent (X for model 2). Two linear regression models are estimated using the following general equation.

$$Y = c + \beta X \quad \text{Equation 6-5}$$

Where Y is the independent variable, c is constant, and β is the regression coefficient.

After fitting the models, the suitability of the regression model for the datasets is assessed through various diagnostic checks. Residual plots against both predicted values and independent variables reveal no evidence of a violation of the independence of errors. Furthermore, a Q-Q plot (theoretical quantiles vs. sample quantiles) and the Shapiro-Wilk test are used to verify the normal distribution of residuals, with the associated p-value provided in Table 6-4. The examination of residual scatterplots ensured homoscedasticity. These results collectively indicate that the regression model meets the necessary assumptions and is well-suited for the analyzed dataset.

When K exceeds 1.75, the percentage of feasible trips in nearly all disruption scenarios consistently fluctuates very close to 100%. This condition results in non-normality, contravening the assumptions inherent in the linear regression model. Consequently, the last row of Table 6-4 in the model pertaining to the prediction of feasible trips in various quality levels remains unassigned. Furthermore, given that there is no surplus of travel time for TUSRBD, where $K = 1$, it is excluded from consideration in the model related to the prediction of the surplus of travel time. Consequently, the first row of the corresponding table has no value.

In the prediction of feasible trips at various quality levels, we observe that the R-squared values consistently hover around 0.87 across all scenarios, indicating the acceptable accuracy of our models. Additionally, the negative coefficients across all models suggest that an increase in the disruption extent leads to a decrease in feasible trips. When examining the absolute values of the coefficients, it becomes evident that the largest coefficient corresponds to $K \leq 1.1$. This implies that at this specific quality level, feasible trips are more sensitive to the disruption extent compared to other quality levels. In simpler terms, an escalation in the disruption extent causes a more substantial reduction in feasible trips at this particular quality level than at others, a trend shown by the slopes of the diagrams in Figure 6-4.

Table 6-4 Linear regression results

Quality Level	Satisfied demand at various quality levels				Surplus of travel time at various quality levels			
	Shapiro-Wilk p-value	Intercept	Coefficient	R-squared	Shapiro-Wilk p-value	Intercept	Coefficient	R-squared
K=1	0.470	0.602	-0.983	0.874	-	-	-	-
K≤1.1	0.496	0.852	-1.312	0.888	0.066	0.043	0.104	0.575
K≤1.25	0.749	0.953	-1.192	0.884	0.999	0.054	0.284	0.859
K≤1.5	0.717	0.996	-0.697	0.874	0.915	0.058	0.532	0.871
K≤1.75	0.428	1.001	-0.319	0.864	0.644	0.055	0.654	0.880
K>1.75	-	-	-	-	0.787	0.051	0.700	0.893

In the context of predicting surplus travel time at various quality levels, upon analyzing the initial data, the Q-Q plot for the first model ($K \leq 1.1$) reveals a greater deviation from the straight line when contrasted with the other models. Moreover, although a $p\text{-value} > 0.05$ in the Shapiro-Wilk test typically supports the application of linear regression models, the significant difference in the p-

value of this model and the remaining models shows that residuals in this model are not as normally distributed as in the other models. Consequently, this model exhibits a relatively lower R-squared value ($R^2=0.57$) compared to the other models. Generally, as depicted in Figure 6-4, there is a discernible trend where an increase in the disruption extent correlates with a rise in the surplus of travel time, and this effect becomes more pronounced in models with higher values of K, as expected.

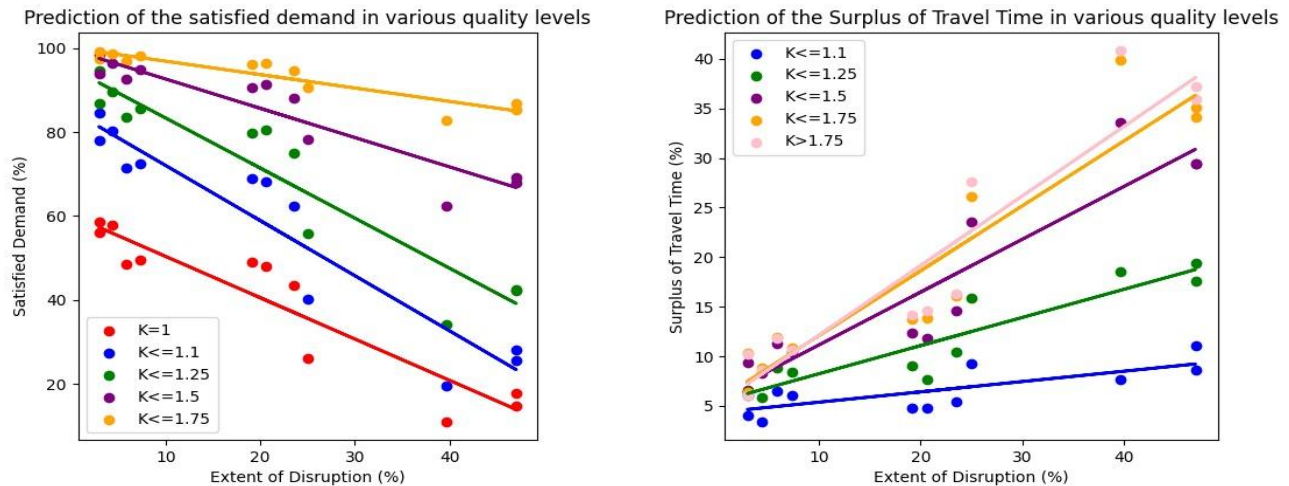


Figure 6-4 Linear regression diagrams

Table 6-5 comprises twelve sections, each corresponding to a distinct disruption scenario. This table provides insights into the percentage distribution of trips categorized by specific quality attributes across three distinct time segments throughout the day. For instance, for scenario #1, we see that if the disruption occurs during the Morning peak hours, 20% of trips maintain their original route, while 11% experience a 10% increase in travel time, and 19% a travel time increase ranging from 10% to 25%, etc. Conversely, if disruption occurs during the evening rush hours, 26% of trips retain their original route, with 14% experiencing at most 10% increase in travel time, and 16% facing a travel time increase ranging from 10% to 15%, etc.

Table 6-5 Satisfied demand distribution in different segments throughout the day after applying quality thresholds

	Scenario #1: S-SI-25			Scenario #2: S-SI-21			Scenario #3: S-SI-19		
	MP	EP	OP	MP	EP	OP	MP	EP	OP
K=1 (TUSBD)	20%	26%	26%	40%	48%	48%	44%	51%	48%
(1<K≤1.1)	11%	14%	13%	18%	20%	19%	17%	20%	20%
(1.1<K≤1.25)	19%	16%	15%	17%	12%	13%	12%	10%	12%
(1.25<K≤1.5)	28%	22%	22%	15%	11%	11%	15%	10%	11%
(1.5<K≤1.75)	14%	13%	12%	6%	5%	5%	7%	5%	6%
(1.75>K)	8%	10%	11%	4%	4%	4%	5%	4%	4%
	Scenario #4: D-SI-47 (1)			Scenario #5: D-SI-47 (2)			Scenario #6: D-SI-40		
	MP	EP	OP	MP	EP	OP	MP	EP	OP
K=1 (TUSBD)	10%	14%	15%	16%	18%	20%	7%	11%	13%
(1<K≤1.1)	8%	10%	11%	8%	10%	10%	6%	8%	8%
(1.1<K≤1.25)	19%	16%	16%	16%	14%	15%	16%	14%	15%
(1.25<K≤1.5)	30%	26%	26%	32%	26%	25%	33%	27%	27%
(1.5<K≤1.75)	20%	17%	17%	20%	18%	16%	22%	20%	19%
(1.75>K)	13%	16%	16%	12%	14%	14%	15%	19%	18%
	Scenario #7: S-DF-3 (1)			Scenario #8: S-DF-3 (2)			Scenario #9: S-DF-4		
	MP	EP	OP	MP	EP	OP	MP	EP	OP
K=1 (TUSBD)	56%	61%	57%	50%	58%	55%	52%	59%	58%
(1<K≤1.1)	25%	26%	23%	20%	23%	20%	21%	23%	21%
(1.1<K≤1.25)	14%	9%	11%	12%	8%	10%	13%	9%	10%
(1.25<K≤1.5)	4%	3%	5%	10%	6%	7%	10%	6%	7%
(1.5<K≤1.75)	1%	1%	2%	5%	3%	4%	3%	2%	3%
(1.75>K)	0%	0%	1%	3%	2%	3%	1%	1%	2%
	Scenario #10: D-DF-6			Scenario #11: D-DF-7			Scenario #12: D-DF-24		
	MP	EP	OP	MP	EP	OP	MP	EP	OP
K=1 (TUSBD)	43%	50%	48%	44%	51%	49%	38%	46%	44%
(1<K≤1.1)	20%	23%	21%	21%	23%	21%	16%	19%	17%
(1.1<K≤1.25)	16%	12%	13%	18%	13%	13%	15%	12%	14%
(1.25<K≤1.5)	12%	8%	10%	12%	9%	10%	16%	12%	14%
(1.5<K≤1.75)	6%	4%	5%	4%	3%	4%	8%	6%	7%
(1.75>K)	3%	3%	4%	1%	1%	3%	6%	5%	5%

In all the scenarios, a consistent pattern emerges: when the value of the parameter K falls within the range of 1 to 1.1, and particularly when K equals 1 (meaning that the route taken after the

disruption has the same duration as the one taken in normal state), the metrics EP and OP exhibit proximity, while MP demonstrates a lower value. This observation suggests that disruptions during Morning Peak hours has more impacts on travel time than those occurring at other times. However, to rigorously assess the statistical significance of the observed disparity between MP and the two other metrics (EP and OP), we employ the two-sample hypothesis test. In this study, given the modest sample size of 12 observations, we opted for the Mann-Whitney U test, a non-parametric method known for its suitability with small samples. The use of the exact significance level (p-value) derived from precise permutation methods was necessary due to the small sample size (less than 20). This approach ensured robust and accurate statistical inference.

Although differences were observed between MP and the other groups (EP and OP), the p-values from the Mann-Whitney U test suggest that these differences are not statistically significant. This lack of significance applies to most of the measured feasible trips across different quality levels. As Table 6-6 shows, the sole statistically significant disparity is observed between the Morning Peak hours and Evening Peak hours when the K value is between 1.1 and 1.25. Consequently, the available evidence does not support the assertion that disruptions during Morning Peak hours exert a significantly greater impact on travel time compared to disruptions occurring at other times throughout the day.

Table 6-6 Mann-Whitney U test results

	K=1	K=1.1	K=1.25	K=1.5	K=1.75	K>1.75
MP & EP	0.198	0.319	0.004	0.114	0.347	1.000
MP & OP	0.319	0.410	0.140	0.198	0.410	0.671
EP & OP	0.603	0.590	0.378	0.799	0.932	0.630

6.5 Conclusions

This study addresses a notable gap in existing research by focusing on short-duration disruptions in metro systems, an area that has often been overlooked in previous studies. While past research has primarily concentrated on evaluating substitute bus service as a mitigation measure, which may not always be suitable for short-duration disruptions, our research shifts attention to the role of conventional bus services in alleviating the impact of disruptions on metro systems.

The study focuses on the Montreal transit system, which comprises a single metro system with four lines and 68 stations, alongside three distinct bus systems totaling 395 lines and 15,084 stations.

Through a comprehensive 5-step framework, we presented a systematic approach to assess the effectiveness of conventional bus services in mitigating disruption impacts on the metro system. Importantly, this framework was designed to be adaptable and applicable to other transit networks and various mitigation strategies beyond conventional bus systems.

In this study, we proposed a modeling of the Montreal's transit network using GTFS data from all transit service operators, followed by trip routing and travel time estimation using the Transition platform, an open-source application developed by Polytechnique Montreal. Then, simulations of metro system disruptions lead to trip rerouting within the disrupted network, enabling the calculation of travel times during the disruption period. By comparing pre- and post-disruption travel times, we establish different quality thresholds, elucidating the efficacy of the conventional bus and metro systems' combination in mitigating disruption impacts across varying trip's quality levels. Moving forward, regression models were employed to investigate the correlation between disruption extent and feasible trips at each quality level, alongside the correlation between disruption extent and surplus of travel time. Additionally, hypothesis testing was utilized to assess whether disruptions occurring at different times of the day yield significant differences in the quality of the conventional bus and metro systems' combination in mitigating metro disruption impacts.

The findings of this study underscore a clear linear correlation between disruption extent and feasible trips, with an anticipated decrease in feasible trips as disruption extent increases, particularly evident at higher quality thresholds. Another noteworthy observation is that in nearly all disruption scenarios when no quality threshold is applied, the feasible trips reach 100%. This suggests that even in the face of substantial disruptions in the metro system, all transit trips could still be accommodated, albeit at the expense of longer travel times. Furthermore, our analysis reveals the predictability of surplus of travel time by accounting for disruption extent, especially at lower quality levels, where disruptions exert a more pronounced effect. Interestingly, the occurrence time of disruptions within the metro system does not significantly alter the quality of alternatives or the surplus of travel time, as demonstrated through the simulation of various disruption scenarios.

The most popular method for connecting the disrupted sections of the network is to deploy a substitute bus service or bus bridging (Zhang, Lo et al. 2021). However, because of the high cost

of buying and maintaining buses, the difficulty of locating depots in different parts of the city, as well as the expense of standby drivers, operators typically prefer to mobilize buses from other lines to serve as shuttles rather than establishing a dedicated reserved service. This phenomenon will inevitably lead to a reduction in supply, an increase in travel time, and a decrease in the quality of travel in other areas of the network, particularly during peak rush hours. Hence, conducting such research not only fills a significant research gap by evaluating the influence of short-duration metro disruptions on users' travel quality but also provides operators with valuable insights regarding the necessity of implementing bus substitute services for all medium-term disruptions.

As a limitation of the research, it is important to note that we only examined weekdays. It would be valuable to extend this analysis to weekends as well, allowing for a comprehensive comparison of results across different time periods. Additionally, in the routing of trips, we assumed that all passengers would actively seek out and select the fastest available alternative routes following a disruption. However, it is worth acknowledging that in reality, some passengers may opt to remain within the metro system and wait for the disruption to conclude, which could impact the dynamics of travel behavior and route selection. Furthermore, this study does not account for the capacity of buses. It is assumed that the conventional bus system possesses sufficient capacity to accommodate all passengers.

In the subsequent phase of this study, besides considering the capacity of the alternative modes, we propose expanding our analysis to incorporate regional train services, shared modes such as carsharing and bikesharing, as well as active modes of transportation. This broader scope aims to evaluate the effectiveness of multimodal options in alleviating the impact of disruptions within metro systems. Additionally, we intend to investigate the deployment of bus substitute services, both with and without reserved services, for disrupted segments. Through comparative analysis, we aim to assess the performance of the transit system under scenarios where bus substitutes are utilized versus scenarios where multimodal options are fully operational. This comprehensive approach will provide valuable insights into the comparative effectiveness of various mitigation strategies for disruptions in urban transit systems.

CHAPTER 7 ARTICLE 4: ASSESSING POPULATION SEGMENTS' VULNERABILITY TO METRO DISRUPTIONS USING A TRANSIT ROBUSTNESS INDEX (TRI)

Authors: Kaveh Rezvani Dehaghani¹, Catherine Morency¹

¹Department of Civil, Geological and Mining Engineering, Polytechnique Montréal.

Montreal, QC, H3C 3A7, Canada.

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ABSTRACT

Research on transit system vulnerability has traditionally focused on the impact of disruptions on network topology and macroscopic attributes such as flow, delay, and capacity. However, the impact of disruptions on individuals and specific population segments has been largely overlooked. This study introduces a four-step framework to assess the vulnerability of various population segments to metro disruptions. Using the Montreal transit system as a case study and data from the 2018 Montreal Origin-Destination survey, transit users are divided into ten segments. Twelve metro disruption scenarios, based on station importance and historical disruption frequencies, are developed with varying Extent of Disruption (EoD). Transit trips are then simulated under normal and disrupted conditions using the Transition platform. A novel threshold-based Transit Robustness Index (TRI) is introduced to measure travel time deviations for each population segment during disruptions. The impact of EoD on TRI for each segment is evaluated through Decision Tree, Random Forest, and Linear Regression models, which confirm a high level of consistency. The results show that, while the most vulnerable population segment changes depending on TRI thresholds, young men workers are more frequently identified as the most vulnerable group. Conversely, children are consistently the least vulnerable segment across all TRI thresholds.

7.1 Introduction

Metro systems are essential infrastructures, offering an efficient solution for daily urban transportation. As cities expand and travel demand rises, these systems are continuously growing

and have become an indispensable mode of transport in major cities (Liu, Zhu et al. 2021). The use of metro systems as a primary or supplementary mode of transportation for daily trips is not confined to a specific segment of society. A significant portion of the population, including individuals across various income levels, educational backgrounds, and occupation relies on this public transportation mode for their urban commutes. Therefore, disruptions in the metro system can have impacts on a diverse set of population segments.

Metro systems are highly susceptible to disruptions, including accidents, incidents, events, and adverse weather conditions. Such disruptions are common in metro systems worldwide. Open data from the Société de transport de Montréal reveals that the Montreal metro system experienced an average of 73 disruptions per year over the past five years, each lasting 30 minutes or more (Data 2024). Similarly, according to the Toronto Transit Commission (TTC) open data portal, the Toronto metro system recorded 78 disruptions lasting 30 minutes or longer between October 2017 and September 2018 (Liu, Palm et al. 2020). In other regions, the Beijing metro system reported 408 disruptions from January 2014 to December 2018 (Liu, Zhu et al. 2021). Even in the relatively new Hong Kong metro system, Mass Transit Railway (MTR), approximately 250 disruptions occur annually, with about 10% causing delays exceeding 30 minutes, some with significantly longer delays (Zhang 2018).

When discussing disruptions in public transportation systems, the focus typically centers on concepts such as vulnerability, robustness, and resilience analysis of the system. Researchers often assess the impact of disruptions on network topology or changes in macroscopic-level transportation indicators such as travel time, cost, and distance. However, microscopic-level research specifically examining the susceptibility of different population groups is quite rare. Most studies in public transportation that consider users often examine issues such as equity and accessibility during normal operations when the system is free from disruption. This research aims to bridge this gap by segmenting passengers based on sociodemographic features—such as age, gender, occupation, and travel purpose—and comparing their vulnerability to various metro disruption scenarios.

Transit user vulnerability can be categorized into relative and absolute vulnerability. Relative vulnerability analysis focuses on comparing and ranking different population segments based on their susceptibility to disruptions in the transit system. This approach does not directly determine

whether a specific population segment is vulnerable but provides a comparative assessment among them. In contrast, absolute vulnerability analysis establishes predefined thresholds and assumptions to determine whether a given population segment is vulnerable to transit disruptions.

This study primarily focuses on relative vulnerability analysis and proposes a four-step methodological framework for ranking population segments based on their vulnerability to subway disruptions. While the proposed framework is designed for relative vulnerability assessment, it shares significant methodological similarities with absolute vulnerability analysis. Therefore, it can serve as a foundation or inspiration for future research investigating absolute vulnerability.

7.2 Background

Over the past decade, several types of disruptions within metro systems have been extensively studied, ranging from random to targeted failures, single to multiple failures, and cascading failures. A diverse array of failure mechanisms and methodologies has been employed to simulate these disruptions and assess their impacts. Researchers have focused on identifying critical segments of the network. For instance, Ma, Liu et al. (2023) investigated methods for identifying critical stations within metro networks by considering the influence of passenger flow. Similarly, (Kopsidas and Kepaptsoglou (2022) proposed a metric for evaluating metro station criticality based on Complex Network Theory (CNT), which takes into account substitute services during disruptions.

Several studies have examined the impact of service disruptions, employing a range of methodologies to analyze their effects on system performance and service levels. Common approaches include graph theory, surveys, simulations, and the analysis of empirical data. Methods based on graph theory (complex network) typically derive resilience, vulnerability, or robustness indicators from the network topology, offering insights into high-level network properties related to incidents (e.g., (Qi, Meng et al. 2022); (Zhang, Ayyub et al. 2020); (Zhang, Wang et al. 2018); (Wu, Dong et al. 2018); (Berdica 2002)). Simulation-based methods model passenger flows across transit networks under various incident scenarios, providing insights into potential system-wide impacts (e.g., (Chen, Zhang et al. 2021); (Zhang, Graham et al. 2021); (Khademi, Bababeik et al. 2021); (Sun and Guan 2016)). Methods that study cascading failures typically account for both network topology and flow distribution. The central concept of cascading failure is that, following a disruption at one station, passengers from the affected station will attempt to reroute themselves

to nearby stations, potentially causing additional strain and subsequent failures within the network. (e.g., (Wang and Tian 2020); (Shen, Ren et al. 2021); (Yang, Dong et al. 2023); (Shen, Yang et al. 2024)).

Empirical data-based methods utilize data from sources such as smart cards and vehicle location systems to evaluate the real-world impacts of incidents, producing metrics related to vehicle load, travel delays, and the spatial distribution of disruptions. These studies can be performed at both macro and micro levels, with micro-level analyses focusing on individual user behavior. For instance, Mo, Von Franque et al. (2022) examined the impact of metro disruptions at both aggregate and individual levels, calculating changes in demand across different rail lines, stations, bus routes, and stops to better understand the redistribution of passenger flows during incidents. Additionally, individual behavior was analyzed using a binary logit model to infer passengers' mode choices. Similarly, (Tian and Zheng 2018) analyzed commuter behavior during unexpected train delays, using six months of real-world data collected from Singapore's automated fare collection system to build a classification model predicting whether commuters would switch from mass rapid transit to alternative transportation modes in response to delays. Survey-based approaches, on the other hand, often focus on passenger behavior and perceptions during disruptions and typically employ econometric models to analyze individual-level responses. For example, Lin, Srikukenthiran et al. (2018) investigated transit users' mode choices in response to rapid transit service disruptions in Toronto, considering factors such as the type of disruption, the stage of the trip (pre-trip or en-route), weather conditions, and the uncertainty surrounding delay duration. In another study (Currie and Muir (2017) aimed to understand passenger behavior, perceptions, and priority interventions in response to unplanned disruptions in urban passenger rail services. This study reported on a comprehensive survey of rider responses conducted among users of the Metropolitan heavy rail system in Melbourne, Australia. As observed, none of the aforementioned methods are capable of adequately assessing user vulnerability to disruptions. The two most recent microscopic approaches focus on individuals rather than on the network and aggregate transportation indicators. However, these approaches primarily emphasize the impact of disruptions on users' mode choice rather than on their vulnerability level.

On the other hand, research in the field of public transportation that examines users in detail and compares different passenger groups primarily focuses on concepts such as safety, equity, and accessibility. For instance, Cardoso, Santos et al. (2023) explored the resilience of transit trips in

the context of urban violence from a gender perspective, revealing that women are more vulnerable to violence in public transport than men (Cardoso, Santos et al. 2023). Mohri, Mortazavi et al. (2021) developed a method to identify and address inequities in job accessibility through public transportation services by introducing a Modified Cumulative Accessibility (MCA) measure, which facilitates the identification of such inequities based on users' travel purposes. Similarly, Deboosere and El-Geneidy (2018) devised a measure of accessibility to low-income employment for vulnerable residents using public transportation, with a particular focus on the actual travel times experienced by these individuals. This measure, calculated across 11 major Canadian cities, represents a modified place-based accessibility metric that integrates elements of person-based metrics.

Additionally, some research has focused on evaluating the alternative transportation modes, such as bus-bridging services and ride-hailing (RH) trips, which are commonly used during metro disruptions. For example, Young and Farber (2019) concentrated on RH that is a potential alternative in case of metro disruption. They found that RH services are predominantly used by younger individuals, particularly those aged between 20 and 39 years. Although there are slightly more RH users in the 30–39 age group, RH trips made by individuals aged 20–29 account for 2% of all trips taken by this age group. This proportion is more than double RH trips share of other age groups, indicating that individuals in their 20s are the most likely to use these services. RH usage is notably low among older adults, with less than 2% of RH users being 60 years or older. This trend is likely due to the digital divide between younger and older generations, as well as the fact that only 30% of Canadians aged 71 and older own a smartphone capable of ordering a RH trip. Liu, Palm et al. (2020) examined 78 major subway disruptions in Toronto and discovered that bus routes serving disadvantaged areas experienced greater service reductions due to bus bridging, compared to routes in wealthier neighborhoods. They also found that, although ride-hailing demand increased during subway disruptions, it was not the case in disadvantaged areas. Their research highlights how subway disruptions disproportionately impact lower-income communities and shows that ride-hailing services often exacerbate rather than alleviate these inequalities. One possible explanation is that fewer commuters in Neighbourhood Improvement Areas (NIAs) choose to use ride-hailing services such as Uber or Lyft during subway disruptions due to income constraints, as observed in previous studies. A more concerning possibility from an equity

perspective is that ride-hailing services may be less accessible for trips originating in NIAs, potentially due to longer wait times, which discourages commuters from switching to this mode.

7.3 Methodology

In this research, we focus on short-duration disruptions where no mitigation measures, such as bus-bridging services, timetable adjustments, or rolling stock modifications, are implemented by service operators. Given the widely recognized role of bus systems as complements to metro systems, offering comparable attributes in terms of cost and service quality (Ma, Shi et al. 2020), it is assumed that users primarily rely on regular bus services during these disruptions. In Montreal, where bus and metro services share a common subscription card, we hypothesize that, in the event of a metro disruption, passengers will select the fastest available bus-metro route alternatives. However, this methodology is adaptable and can incorporate other alternative transportation modes, such as bikesharing or carsharing, etc.

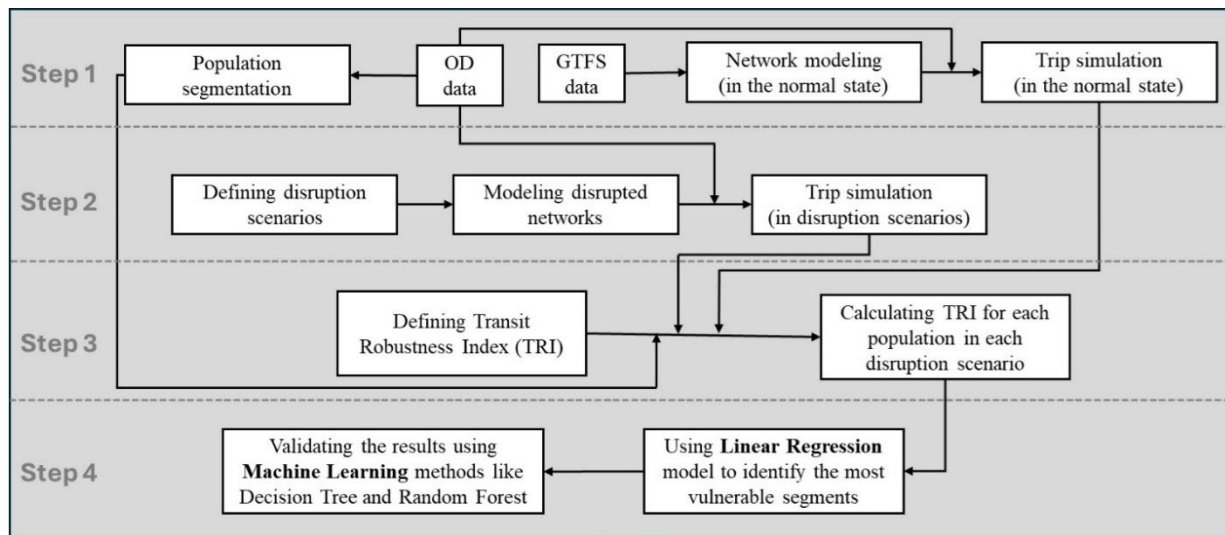


Figure 7-1 Schematic representation of the Methodology

This research introduces a four-step framework to identify the most vulnerable population segments to metro disruptions. In the first step, users are segmented using Origin-Destination (OD) data, and the transit network is modeled based on open-access General Transit Feed Specification (GTFS) data provided by service operators. Transit trips are then simulated on the network under normal conditions, and their travel times are calculated. The second step involves defining various disruption scenarios and modeling the disrupted network for each scenario. Transit trips are subsequently simulated on these disrupted networks, and their travel times are recalculated. In the

third step, we introduce the Transit Robustness Index (TRI), which quantifies the proportion of each population segment with resilient trips. Finally, in the fourth step, the least and most vulnerable population segments are identified using regression models and machine learning methods, specifically decision tree and random forest algorithms. Figure 7-1 presents more details about each step.

7.3.1 Case study

The study focuses on the Montreal metropolitan area, which is the second-largest population center in Canada with a population of 4.3 million. As Figure 7-2 below indicates, the metro system under analysis includes 4 lines, 68 stations. It spans over 70 kilometers, making it one of the largest in Canada. Data from 2023 indicates that this extensive network records an average of 820,000 metro trips per day (Société de transport de Montréal (STM)). Additionally, the region is served by three bus operators: STM, with 216 bus routes and 8,747 stops; STL, with 99 bus routes and 2,827 stops; and RTL, operating 80 bus routes with 3,510 stops.

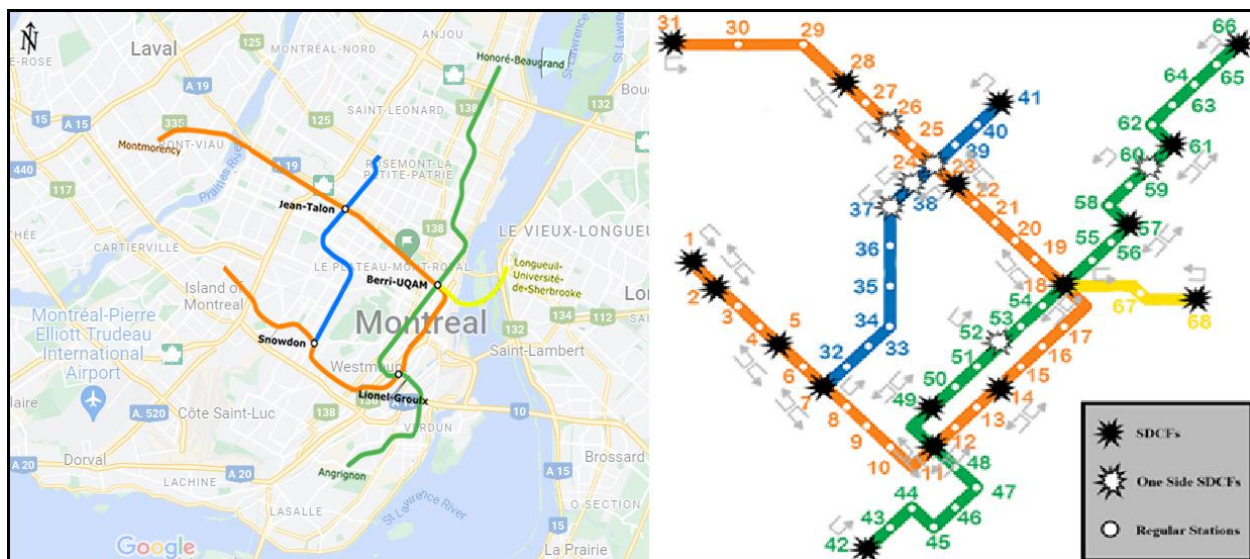


Figure 7-2 Montreal metro system

7.3.2 Step 1: Data analysis and baseline transit simulation

7.3.2.1 Network modeling data analysis

In this study, the Montreal transit network is represented using the L-space method (Von Ferber, Holovatch et al. 2009), which depicts the network as a graph where each station is a node. In this graph, a link between nodes signifies that at least one route consecutively services the corresponding stations. Utilizing the L-space method and GTFS data -published by previously

mentioned service operators-, the transit network is implemented in Transition, an open-source platform developed at Polytechnique Montreal (www.transition.city). We also utilize data from the 2018 Montreal Origin-Destination survey. Since our focus is on metro disruptions, with regular bus service as the only alternative mode, we restricted our analysis to trips undertaken exclusively by metro or a combination of metro and bus. We also focused on simple chain home-based trips to concentrate on the most common types of daily trips and prioritize those occurring during peak hours.

7.3.2.2 User segmentation

Segmentation of users in public transportation vulnerability analysis provides a nuanced understanding of how disruptions affect distinct groups. By identifying the vulnerabilities of segments such as the elderly and students, transportation planners can design targeted interventions that address the specific needs of each group, ensuring that resources are put where they are most required. This approach enhances the robustness of the transit system by implementing tailored solutions that mitigate the impacts of disruptions on the most vulnerable users. Moreover, segmentation promotes equity by ensuring transportation policies consider the diverse needs of all users, preventing solutions that might inadvertently favor certain groups.

Specifically, we analyzed the most frequent types of simple chain trips for each segment. For example, for groups whose primary trip purpose is work, we focused on Home-Work-Home trips, which typically begin during the morning peak hours and conclude during the evening peak. We excluded more complex trip chains, such as Home-Work-Leisure-Home, which may extend beyond the evening peak hours. Similarly, for groups whose primary purpose is study, we concentrated on Home-School-Home trips, excluding additional stops made by students, such as Home-School-Leisure-Home. This ensured a clear focus on simple trip chains during peak periods.

As Table 7-1 shows, considering the age, gender, occupation, and trip purpose of users, we create 10 population segments. Segment S1, renamed "Children", includes students aged 5 to 15, focusing on school-related trips, reflecting the uniform daily routines of this group. Segments S2 and S3, now called "Men Students" and "Women Students" respectively, differentiate male and female students aged 16 to 24, acknowledging that their metro usage diversifies as they balance education and part-time work. For young workers aged 25 to 34, segments S4 ("Young Men Workers") and S5 ("Young Women Workers") emphasize their work commutes, recognizing the significance of

employment in their daily metro usage. The segmentation of individuals aged 35 to 64 into five groups—S6 ("Men Workers"), S7 ("Women Workers"), S8 ("Adults, Men & Women")—accounts for both work-related and non-work-related trips, highlighting the diverse purposes of metro usage in this age group. Finally, segments S9 ("Elderly Men") and S10 ("Elderly Women") focus on individuals aged 65 and older, ensuring that the specific needs and vulnerabilities of elderly users are carefully considered in transportation planning. This segmentation approach allows for a more precise identification of the most and least vulnerable groups, guiding efforts to improve robustness and equity in metro services.

Table 7-1 Population Segmentation

	Description (Abbreviation)	Age	Gender	Occupation	Main Purpose
S1	Children (YS-MF-15-S)	5 - 15	M & F	Student	Study
S2	Men Students (S-M-16-24-S.W)	16 - 24	M	Part-time Worker, Student	Study, Work
S3	Women Students (S-F-16-24-S.W)	16 - 24	F	Part-time Worker, Student	Study, Work
S4	Young Men Workers (YW-M-25-34-W)	25 - 34	M	Full-time worker	Work
S5	Young Women Workers (YW-F-25-34-W)	25 – 34	F	Full-time worker	Work
S6	Men Workers (W-M-35-64-W)	35 - 64	M		Work
S7	Women Workers (W-F-35-64-W)	35 - 64	F		Work
S8	Adults, Men & Women (A-MF-35-64-L.SH)	35 - 64	M & F		Leisure, Shopping
S9	Elderly Men (E-M-65+)	65+	M		
S10	Elderly Women (E-F-65+)	65+	F		

7.3.2.3 Simulation transit trips

To simulate transit trips, the Transition Platform is employed. This platform utilizes the Connection Scan Algorithm (CSA) to generate transit alternatives. The total travel time for each trip is calculated by summing the total vehicle time, total waiting time, access and egress time, and, when

applicable, transfer walking and waiting time. In this application, the maximum waiting time at the first station, as well as the maximum access, egress, and transfer travel times, were all initialized at 15 minutes.

7.3.3 Step 2: Transit simulation under disruption scenarios

7.3.3.1 Disruption simulation

To simulate disruptions within transportation networks, the adoption of a failure mechanism is essential. In this research, we focus specifically on metro disruptions, and the Block-Based Failure Strategy (BBFS) has been selected due to its compatibility with metro systems. The implementation of BBFS requires an understanding that a block consists of several consecutive regular stations situated between two Stations with Direction-Change Facilities (SDCF). Consequently, according to the BBFS, if a regular station within a block experiences a disruption, the entire block to which it belongs becomes unavailable. Likewise, if one of the SDCFs is disrupted, all the blocks connected to it also become unavailable. Further details on this process are provided in Table 7-2. It is important to note that each metro line comprises two directional paths (e.g., North and South or East and West). As a result, when coding the BBFS algorithm, it is necessary to consider the existence of two types of SDCFs within metro networks: fully equipped SDCFs, which have direction-changing facilities for both paths, and semi-equipped SDCFs, which provide direction-changing facilities for only one path.

7.3.3.2 Disruption scenarios

After selecting the appropriate failure strategy, the next step involves defining various disruption scenarios. This process entails assuming disruptions at different metro stations and simulating their impacts. The disrupted stations can be selected either randomly or with specific intent. In this research, the focus is on targeted disruptions, employing two primary approaches: station selection based on the station's importance and station selection informed by the historical frequency of disruptions at the station.

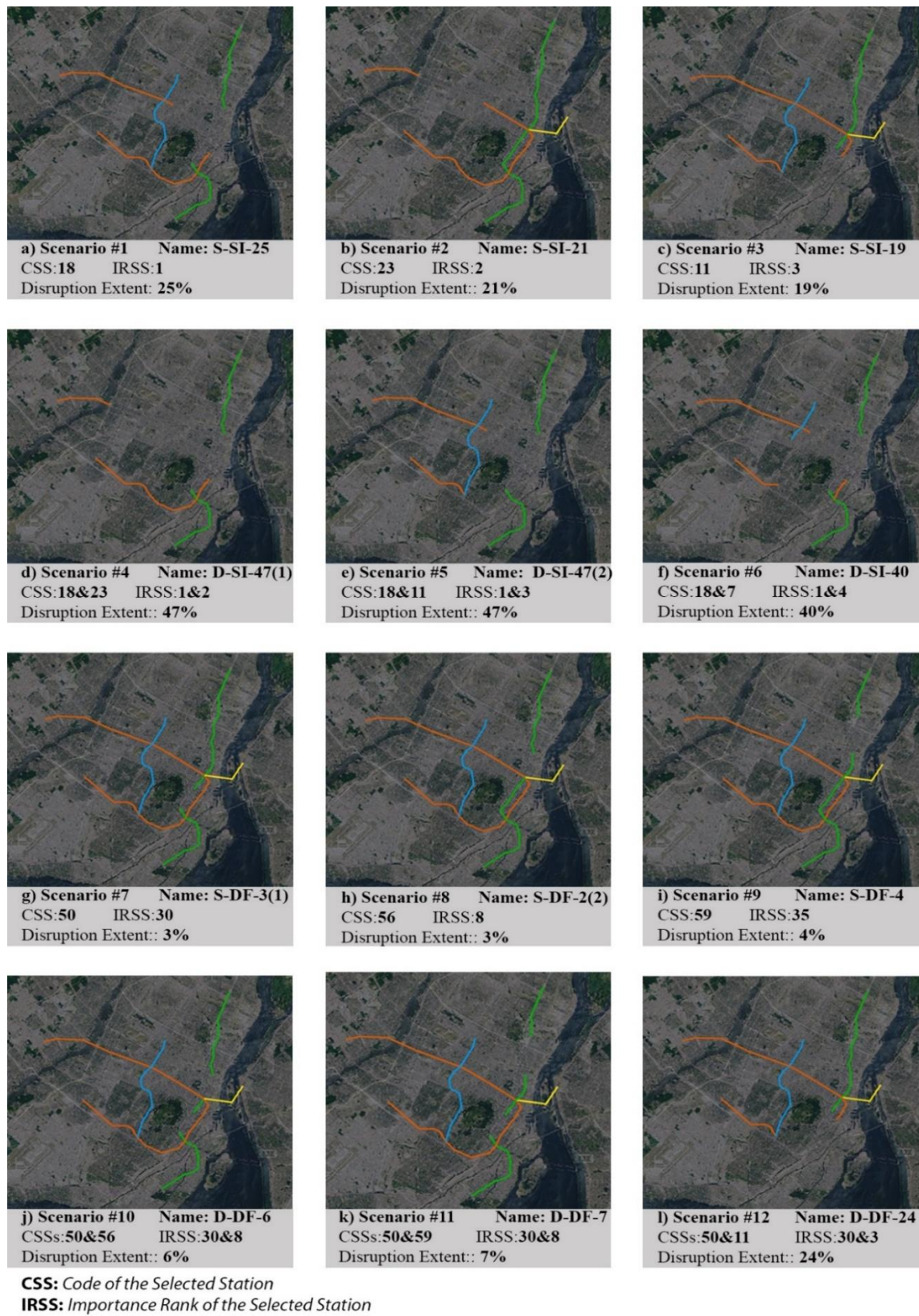


Figure 7-3 Disruption scenarios

The research design includes the simulation, in Transition, of 12 distinct disruption scenarios : three scenarios with a single disruption at key stations (Figure 7-3 a, b, and c), three scenarios with two simultaneous disruptions at key stations (Figure 7-3 d, e, and f), three scenarios with a single disruption at the most frequently disrupted stations (Figure 7-3 g, h, and i), and three scenarios with two simultaneous disruptions at the most frequently disrupted stations (Figure 7-3 j, k, and l). Historical data on global metro disruptions indicate that occurrences of more than two simultaneous disruptions are extremely rare. Consequently, this research confines itself to the simulation of two simultaneous disruptions. Our scenarios comprehensively cover all potential ranges of network degradation resulting from one or two disruptions, with degradation levels varying from 3% to 47%. To facilitate further identification, scenarios are named according to the following logic (see Figure 7-3):

- The first character refers to the number of simultaneous disruptions: "S" for single and "D" for double.
- The second term relates to the station selection method: "SI" denotes station importance, while "DF" refers to disruption frequency (based on historical data).
- The third term indicates the Extent of Disruption (EoD), representing the percentage of the metro network affected (e.g., "25" signifies that 25% of the network is disrupted).
- The final element, enclosed in parentheses, differentiates scenarios that share identical characteristics in the preceding terms but represent different instances.

Table 7-2 Montreal metro blocks

Block ID	Associated Stations	Block ID	Associated Stations	Block ID	Associated Stations
1	1,2	9	24,25,26	17	11,49
2	2,3,4,5	10	26,27,28	18	49,50,51,52
3	5,6,7	11	28,29,30,31	19	52,53,54,18
4	7,8,9,10,11	12	7,32,33,34,35,36,37	20	18,55,56,57
5	11,12,13,14	13	37,38	21	57,58,59,60
6	14,15,16,17	14	38,24	22	60,61
7	18,19,20,21,22,23	15	24,39,40,41	23	61,62,63,64,65,66
8	23,24	16	42,43,44,45,46,47,48,11	24	18,67,68

7.3.3.3 Simulation transit trips

In each disruption scenario, it is essential to simulate transit trips and calculate the corresponding travel times. It is imperative that the route assignment configurations replicate those employed in the first step for trips under normal operating conditions.

7.3.4 Step 3: Transit Robustness Index (TRI) calculation

The TRI measures the proportion of trips that remain feasible in the disrupted transit network, based on predefined thresholds for acceptable increases in travel time. Specifically, it represents the share of resilient trips, which are those where travel time in the disrupted network (TT_{DS}) is equal to or only slightly longer than in normal conditions (TT_{NS}). In this research, five different thresholds are applied to classify trips as resilient:

- Trips with no increase in travel time $TT_{DS} = TT_{NS}$
- Trips with a travel time increase of 10% or less ($TT_{DS}/TT_{NS} \leq 1.1$).
- Trips with a travel time increase of 25% or less ($TT_{DS}/TT_{NS} \leq 1.25$).
- Trips with an additional 5 minutes or less ($TT_{DS} - TT_{NS} \leq 5 \text{ minutes}$).
- Trips with an additional 10 minutes or less ($TT_{DS} - TT_{NS} \leq 10 \text{ minutes}$).

These thresholds are based on two approaches: percentage-based increases and absolute time increases. Percentage-based thresholds account for relative changes in travel time, while absolute thresholds recognize that a 5–10 minute delay may be acceptable for most passengers, regardless of their initial travel time. This is particularly relevant for short trips, where a percentage increase of 50% to 100% may still be reasonable.

The TRI is calculated for each population segment across all disruption scenarios and serves as the key metric for identifying the most and least vulnerable groups. A lower TRI indicates greater vulnerability to metro disruptions.

7.3.5 Step 4: Cross-scenario vulnerability comparison

When comparing different population segments within a single disruption scenario, the Trip Robustness Index (TRI) effectively ranks segments based on their vulnerability to metro disruptions. However, while this indicator provides valuable insights, it may not be sufficient for comparing the vulnerability of different population segments across multiple disruption scenarios. For instance, in a major disruption scenario (e.g., Scenario #6, where the EoD is 47%), a TRI of

40% might indicate a relatively low vulnerability segment. Conversely, in a minor disruption scenario (e.g., Scenario #7, with a EoD of just 3%), a TRI of 70% could suggest that the segment is among the most vulnerable.

Given that this research aims to provide a general comparison of population segments across various disruption scenarios, rather than focusing on a single scenario, we propose a more nuanced approach. Specifically, it is necessary to employ a method that considers the EoD when comparing the vulnerability of population segments across different scenarios. Hence, in this research, we employ the linear regression model to explore the relationship between the EoD (independent variable) and the percentage of what was defined as resilient trips (dependent variable) for each population segment. The simplicity of linear regression allows for clear interpretation, making it an ideal choice for examining the direct impact of disruptions on robustness. However, given the limited sample size of 12 observations (number of disruption scenarios) in this study, it is crucial to validate the results of the linear regression analysis using alternative modeling approaches, such as decision tree regression and random forest regression. The primary assumption of linear regression—the linearity of the relationship between the dependent and independent variables—may not be reliably assessed with such a small dataset. Therefore, employing machine learning methods that do not rely on this assumption provides a complementary perspective. Decision tree regression and random forest regression offer distinct, yet related, approaches to modeling the data. While decision tree regression partitions the data into discrete regions based on feature values, random forest regression aggregates multiple decision trees to enhance predictive accuracy and reduce variance. The consistency of results across these methods reinforces the validity of the linear regression findings, ensuring that the observed relationships are not artifacts of the modeling process but are reflective of the underlying data structure. This multi-method approach serves as an accurate validation strategy, enhancing the reliability of the conclusions drawn from the analysis.

7.3.5.1 Linear regression

Linear regression is a fundamental statistical method used to model the relationship between a dependent variable and one or more independent variables. In its simplest form, the model assumes a linear relationship between the dependent variable Y and a single independent variable X , which can be expressed mathematically as:

$$Y = \beta_0 + \beta_1 X + \varepsilon \quad \text{Equation 7-1}$$

Where:

- Y is the dependent variable, representing the outcome or the variable of interest.
- X is the independent variable, which is hypothesized to influence Y .
- β_0 is the intercept, indicating the value of Y when $X = 0$.
- β_1 is the slope, representing the change in Y for a one-unit change in X .
- ε is the error term, capturing the variability in Y not explained by the linear relationship with X .

In the context of our study, the dependent variable Y is the Trip Robustness Index (TRI), which quantifies the vulnerability of different population segments in response to disruptions in the metro system. The independent variable X represents the Extent of Disruption (EoD) in the metro system. The model coefficients, β_0 and β_1 , are estimated for each population segment to understand how vulnerability varies across different groups.

To ensure the reliability of linear regression models, several key assumptions must be validated. Linearity, which requires a linear relationship between the independent and dependent variables, is essential for accurate interpretation of regression coefficients and is visually assessed through scatter and residual plots in this research. The independence of errors, tested using the Durbin-Watson statistic, ensures that the residuals are not autocorrelated. Homoscedasticity, indicating constant variance of residuals across all levels of the independent variable, is evaluated using the Breusch-Pagan test. Lastly, the data normality and the normality of residuals are assessed through the Shapiro-Wilk test and visual inspection of Q-Q plots respectively. Validating these assumptions is crucial for the stability and interpretability of the regression model's results. Finally, the model's accuracy is evaluated using the coefficient of determination (R^2), which measures the proportion of variance in the dependent variable explained by the independent variables, providing an overall assessment of the model's fit.

7.3.5.2 Decision tree

Decision Tree is a non-parametric supervised machine-learning method that splits data into branches to make predictions based on key variables. In this study, a Decision Tree regression model is employed to analyze the vulnerability of different population segments to metro disruptions, specifically modeling the relationship between the EoD as the independent variable

and the TRI as the dependent variable. The Decision Tree is chosen for its simplicity and interpretability, enabling clear insights into how varying levels of disruption impact vulnerability across segments. The model construction involved recursively splitting the data based on EoD to minimize variance in TRI, with careful attention to avoiding overfitting through depth limitation and pruning. The accuracy of the model is assessed through the R^2 and Mean Squared Error (MSE), which evaluate the model's effectiveness in capturing the relationship between the EoD and the TRI. The Decision Tree's ability to visualize the decision-making process and highlight the vulnerability of specific segments based on the predicted slopes of TRI against EoD makes it a reliable tool for this analysis, offering profound insights into the more and less vulnerable population segments.

7.3.5.3 Random forest

Random Forest is an ensemble learning method that builds multiple decision trees and combines their predictions to improve accuracy and reduce the risk of overfitting. In this study, a Random Forest regression model is used to analyze the vulnerability of different population segments to metro disruptions, with the EoD as the independent variable and the TRI as the dependent variable. Unlike a single decision tree, Random Forest aggregates the predictions from multiple trees, which helps in capturing complex, non-linear relationships between EoD and TRI. The model is trained by constructing numerous decision trees using different subsets of the data, and the final predictions are averaged to ensure reliability and accuracy. Random Forest is chosen for its ability to handle these complexities, offering a more nuanced understanding of how different levels of disruption affect vulnerability across segments. Model validation is performed using the R^2 and MSE, to ensure Random Forest can effectively capture the intricate dynamics between EoD and TRI.

7.4 Results

As outlined earlier, this study concentrates specifically on trips that involve either the metro or a combination of metro and bus services. After filtering for trips embedded in simple chains and applying constraints such as a maximum waiting time of 15 minutes, along with limits for access, egress, and transfer times, the initial dataset comprised 25,385 recorded trips. Following data expansion, this corresponds to a total of 548,911 trips. These routed trips constitute the dataset analyzed in this study. The analysis reveals that in the normal state, transit trips have an Average Travel Length (ATL) of 12 kilometers, an Average Travel Time (ATT) of 45 minutes, and an

Average Transit Speed (ATS) of 16 km/h. Detailed information about ATL, ATT, ATS for each population segment is available in Table 7-3.

Following the segmentation process, 462,372 trips are assigned to one of the 10 defined population segments, representing approximately 84% of the total dataset. However, around 16% of the trips are not classified into any of these predefined segments. The proportion of each segment within the overall dataset ranges from 2% to 18%, illustrating the diversity and distribution of the users among the defined groups.

Table 7-3 trips attributes for each population segment in the normal state

	Segments	Segment Proportion	ATL (Km)	ATT (Min)	ATS (Km/h)
S1	Children	3%	9.6	41	14.0
S2	Men Students	10%	13.7	50	16.6
S3	Women Students	12%	13.8	49	16.8
S4	Young Men Workers	8%	11.0	41	15.9
S5	Young Women Workers	8%	11.0	42	15.8
S6	Men Workers	18%	12.8	46	16.6
S7	Women Workers	18%	12.7	46	16.6
S8	Adults, Men & Women	2%	9.2	36	15.3
S9	Elderly Men	3%	10.9	41	15.8
S10	Elderly Women	2%	10.2	39	15.7
	Unsegmented Users	16%	11.6	43	16.3

As anticipated, the segments with the smallest proportions are Children (S1), Adults, Men and Women whose travel purposes are leisure and shopping (S8), and Elderly (S9 and S10). These groups generally exhibit lower use of public transportation, particularly the metro, in their daily commutes. Children often attend schools situated near their homes or are driven by their parents, resulting in infrequent use of the metro for school-related travel. Adults typically engage in shopping at stores within a reasonable walking distance from their residences and prefer private vehicles for leisure activities. Furthermore, most elderly people in Montreal primarily rely on private vehicles, either as drivers or passengers, or walking for their trips, with only a small proportion who are using public transportation. As Table 7-3 indicates, the shortest trips are generally associated with segments that have the smallest proportions, while the longest trips correspond to Students (S2 and S3). However, a direct relationship between travel distance and travel time is not consistently observed. For instance, although the ATL in S1 is shorter than in

S10, the ATT in S1 exceeds that in S2. Overall, the slowest transit trips are typically linked to Children (S1), whereas the fastest trips are associated with Students (S2 and S3) as well as Workers (S6 and S7).

In the subsequent step, trips are assigned under various disruption scenarios. As shown in Table 7-4, disruption consistently leads to an increase in both ATL and ATT across all scenarios compared to the normal state. This trend is also obvious across all population segments. A comparison of ATL in the disruption scenarios with the normal state reveals that the most significant increases occur in scenarios where stations were disrupted based on their importance (scenarios 1 to 6). However, it is important to note that an increase in ATL is not always directly correlated with the EoD. For instance, while the largest EoD is 47% in scenarios 4 and 5, the ATL increase in scenario 1 (with a 25% EoD) exceeds that of scenario 4. Similarly, the ATL increase in scenario 6 is higher than in scenario 5. Furthermore, an examination of the population segments reveals that the greatest absolute difference in ATL, calculated as the post-disruption ATL minus the pre-disruption ATL, is observed in segments S6 and S7. However, when considering the relative increase in ATL, expressed as the ratio of post-disruption to pre-disruption ATL, the highest proportional increase is found in segment S5. Moreover, segment S1 exhibits the smallest increase in ATL, both in terms of the absolute difference and the ratio of post-disruption to pre-disruption values.

Similarly, the increase in ATT is more pronounced in scenarios 1 to 6, where the disrupted stations are selected based on their importance, compared to scenarios 7 to 12, where stations are chosen based on their historical frequency of disruption. In terms of population segments, the largest absolute difference in ATT (pre- and post-disruption) is observed in segments S7, S6, and S5. However, when considering the ratio of post-disruption to pre-disruption ATT, the highest values are found in segments S5, S10, and S8. On the other hand, the smallest absolute differences in ATT are associated with segments S1, S2, and S8, while the lowest ratios are found in segments S1, S2, and S3.

Table 7-4 Trip attributes for each population segment across all disruption scenarios

Scenarios	Trip Attributes	Total	Population Segments										Trips from other segments
			S1: (Y-S-MF-15-S)	S2: (S-M-16-24-S,W)	S3: (S-F-16-24-S,W)	S4: (Y-W-M-25-34-W)	S5: (Y-W-F-25-34-W)	S6: (W-M-35-64-W)	S7: (W-F-35-64-W)	S8: (A-MF-35-64-L,SH)	S9: (E-M-65+)	S10: (E-F-65+)	
Scenario #1: S-SI-25	ATL (Km)	13.7	10.0	15.1	15.4	12.6	12.6	14.5	14.5	10.5	12.2	11.6	12.9
	ATT (Min)	56	45	58	60	53	54	58	58	47	53	50	53
Scenario #2: S-SI-21	ATL (Km)	12.8	10.1	14.3	14.5	11.4	11.5	13.4	13.3	9.6	11.4	10.7	12.2
	ATT (Min)	50	46	55	56	46	46	52	51	40	46	44	49
Scenario #3: S-SI-19	ATL (Km)	12.7	9.9	14.2	14.2	11.5	11.5	13.4	13.3	9.6	11.4	10.6	12.0
	ATT (Min)	50	46	54	54	47	47	52	52	40	46	44	48
Scenario #4: D-SI-47 (1)	ATL (Km)	13.6	10.1	15.1	15.4	12.2	12.3	14.3	14.2	10.1	12.0	11.5	12.9
	ATT (Min)	60	49	63	65	57	58	62	62	50	55	53	58
Scenario #5: D-SI-47 (2)	ATL (Km)	13.2	9.9	14.7	14.8	12.1	12.1	13.9	13.9	9.9	11.9	11.3	12.5
	ATT (Min)	59	47	62	63	57	58	62	63	49	55	53	56
Scenario #6: D-SI-40	ATL (Km)	13.1	9.9	14.5	14.5	12.0	12.0	13.7	13.7	9.8	11.6	11.1	12.5
	ATT (Min)	62	51	65	66	58	59	64	64	50	56	54	60
Scenario #7: S-DF-3 (1)	ATL (Km)	12.5	9.8	14.0	14.2	11.1	11.2	13.0	12.9	9.4	11.2	10.4	11.8
	ATT (Min)	47	42	51	52	43	43	48	48	38	43	41	44
Scenario #8: S-DF-3 (2)	ATL (Km)	12.5	9.8	14.0	14.1	11.2	11.3	13.2	13.0	9.5	11.2	10.3	11.8
	ATT (Min)	48	43	52	53	45	46	50	50	40	45	42	46
Scenario #9: S-DF-4	ATL (Km)	12.5	9.8	14.0	14.1	11.2	11.3	13.1	13.0	9.4	11.1	10.3	11.8
	ATT (Min)	48	43	52	52	44	45	49	49	39	44	41	46
Scenario #10: D-DF-6	ATL (Km)	12.6	9.8	14.1	14.3	11.2	11.4	13.2	13.1	9.5	11.2	10.4	11.8
	ATT (Min)	49	44	53	54	46	47	51	51	41	46	43	47
Scenario #11: D-DF-7	ATL (Km)	12.6	9.8	14.1	14.3	11.2	11.3	13.2	13.1	9.5	11.2	10.4	11.9
	ATT (Min)	49	44	53	53	45	46	50	50	40	45	42	46
Scenario #12: D-DF-24	ATL (Km)	12.7	9.9	14.2	14.2	11.5	11.5	13.4	13.3	9.6	11.4	10.6	12.0
	ATT (Min)	51	46	55	55	47	48	53	53	41	47	45	49

An interesting observation about segment S8 is that when the vulnerability of the population segments is assessed based on the absolute difference in ATT from the normal state, this segment

ranks among the three most vulnerable ones. However, when the ratio of post- to pre-disruption ATT is used as the criterion, segment S8 ranks among the three least vulnerable ones.

Nevertheless, it is premature to draw conclusions about the vulnerability of different population segments at this stage. There are two main reasons for this. First, the ATT of each population segment in each scenario is an aggregate indicator that reflects the Average Travel Time but does not account for individual trip behavior. It is likely that the increase in ATT for some segments is driven by trips whose travel times have risen to a level that discourages users from continuing to use the transit system. Therefore, focusing on ATT alone may not provide a complete picture, and it would be more logical to concentrate on the proportion of resilient trips within each segment. Second, when comparing population segments across different disruption scenarios, relying solely on ATT is insufficient. Some scenarios, characterized by a more extensive disruption, may cause significant increases in ATT, which could skew the overall results. We aim to avoid drawing biased conclusions based on one or two extreme scenarios. Consequently, although this descriptive evaluation provides valuable insights into the vulnerability of population segments, it is essential to employ the TRI to more accurately identify the most and least vulnerable population segments.

As previously stated, the TRI serves as the primary criterion for evaluating the vulnerability of population segments. A higher TRI value indicates a greater percentage of resilient trips within the segment. In this study, TRI is determined by applying five distinct thresholds for each population segment, as shown in Table 7-5. For instance, when the threshold $TT_{DS} - TT_{NS} = 0$ is applied, the TRI for segment S1 in Scenario 1 is 53%, indicating that 53% of trips in this segment experience no change in travel time during Scenario 1 compared to normal conditions. The dark green color representing this segment in Scenario 1 reflects its lower vulnerability relative to other segments. Conversely, the lowest TRI value in this scenario is 21%, associated with segments S4 and S5, which are depicted in dark red, meaning that these population segments are the most vulnerable ones to disruption. When this threshold is applied, the greatest variation in the TRI across population segments is 32%, observed in Scenario 1. In contrast, the smallest variation of 8%, with a maximum TRI of 63% and a minimum of 54%, occurs in Scenario 7. When applying additional thresholds, a general trend emerges: the largest TRI range is associated with Scenario 1 or Scenario 5, while the smallest range is observed in Scenario 2 or Scenario 7.

Table 7-5 TRIs of each population segment in different disruption scenarios

TRI Thresholds	Population Segments	Transit Robustness Indicator (TRI)											
		Disruption Scenarios:											
		Sc1: S-SI-25	Sc2: S-SI-21	Sc3: S-SI-19	Sc4: D-SI-47 (1)	Sc5: D-SI-47 (2)	Sc6: D-SI-40	Sc7: S-DF-3 (1)	Sc8: S-DF-3 (2)	Sc9: S-DF-4	Sc10: D-DF-6	Sc11: D-DF-7	Sc12: D-DF-24
$TT_{DS} - TT_{NS} = 0$	S1: (YS-MF-15-S)	53%	46%	51%	32%	43%	27%	61%	61%	59%	57%	54%	46%
	S2: (S-M-16-24-S.W)	33%	41%	47%	19%	25%	13%	55%	55%	54%	48%	46%	41%
	S3: (S-F-16-24-S.W)	31%	42%	49%	15%	24%	12%	54%	56%	56%	47%	46%	40%
	S4: (YW-M-25-34-W)	21%	51%	50%	14%	13%	11%	60%	56%	60%	49%	51%	44%
	S5: (YW-F-25-34-W)	21%	50%	49%	12%	14%	10%	58%	51%	53%	44%	45%	45%
	S6: (W-M-35-64-W)	23%	49%	47%	14%	14%	11%	61%	55%	57%	49%	50%	43%
	S7: (W-F-35-64-W)	23%	51%	49%	14%	14%	9%	60%	54%	57%	47%	49%	45%
	S8: (A-MF-35-64-L.SH)	29%	54%	55%	18%	23%	19%	63%	57%	61%	47%	49%	51%
	S9: (E-M-65+)	25%	48%	48%	17%	18%	13%	56%	54%	55%	47%	47%	40%
	S10: (E-F-65+)	21%	53%	47%	15%	14%	11%	58%	58%	60%	48%	50%	41%
$TT_{DS} / TT_{NS} = 1.1$	S1: (YS-MF-15-S)	74%	69%	71%	52%	60%	41%	89%	88%	86%	85%	81%	66%
	S2: (S-M-16-24-S.W)	54%	68%	73%	36%	40%	25%	85%	82%	80%	76%	74%	63%
	S3: (S-F-16-24-S.W)	48%	65%	74%	29%	36%	23%	81%	81%	81%	73%	72%	63%
	S4: (YW-M-25-34-W)	34%	71%	68%	23%	22%	20%	85%	78%	82%	71%	73%	63%
	S5: (YW-F-25-34-W)	34%	70%	68%	21%	23%	18%	83%	71%	75%	65%	67%	61%
	S6: (W-M-35-64-W)	36%	70%	67%	24%	23%	18%	87%	76%	79%	70%	72%	63%
	S7: (W-F-35-64-W)	35%	70%	67%	24%	22%	16%	86%	75%	79%	70%	72%	61%
	S8: (A-MF-35-64-L.SH)	40%	70%	70%	28%	32%	29%	86%	76%	78%	71%	70%	65%
	S9: (E-M-65+)	39%	67%	71%	27%	30%	24%	85%	79%	79%	71%	72%	63%
	S10: (E-F-65+)	33%	69%	63%	22%	23%	18%	81%	79%	82%	69%	71%	56%
$TT_{DS} / TT_{NS} = 1.25$	S1: (YS-MF-15-S)	87%	83%	82%	69%	76%	63%	97%	95%	95%	93%	91%	79%
	S2: (S-M-16-24-S.W)	68%	81%	85%	53%	55%	40%	96%	91%	91%	88%	88%	80%
	S3: (S-F-16-24-S.W)	64%	80%	85%	46%	53%	38%	94%	89%	90%	86%	86%	79%
	S4: (YW-M-25-34-W)	49%	82%	78%	39%	36%	33%	94%	85%	90%	82%	86%	74%
	S5: (YW-F-25-34-W)	48%	82%	78%	37%	36%	30%	93%	83%	87%	80%	82%	73%
	S6: (W-M-35-64-W)	53%	82%	78%	41%	38%	33%	96%	85%	89%	83%	85%	74%
	S7: (W-F-35-64-W)	54%	81%	78%	40%	37%	32%	96%	85%	89%	83%	86%	74%
	S8: (A-MF-35-64-L.SH)	52%	81%	81%	40%	45%	39%	94%	86%	88%	82%	82%	77%
	S9: (E-M-65+)	53%	83%	82%	43%	44%	38%	94%	87%	88%	83%	83%	78%
	S10: (E-F-65+)	49%	82%	76%	40%	38%	34%	93%	88%	90%	83%	84%	71%

Table 7-5 TRIs of each population segment in different disruption scenarios (continued)

TRI Thresholds		Transit Robustness Indicator (TRI)											
		Disruption Scenarios:											
		Population Segments	Sc1: S-SI-25	Sc2: S-SI-21	Sc3: S-SI-19	Sc4: D-SI-47 (1)	Sc5: D-SI-47 (2)	Sc6: D-SI-40	Sc7: S-DF-3 (1)	Sc8: S-DF-3 (2)	Sc9: S-DF-4	Sc10: D-DF-6	Sc11: D-DF-7
$TT_{DS} - TT_{NS} = 5 \text{ min}$	S1: (YS-MF-15-S)	86%	73%	74%	57%	64%	46%	92%	90%	89%	87%	85%	69%
	S2: (S-M-16-24-S.W)	66%	70%	74%	37%	40%	26%	88%	85%	82%	79%	77%	64%
	S3: (S-F-16-24-S.W)	61%	67%	75%	31%	39%	25%	84%	82%	83%	75%	74%	65%
	S4: (YW-M-25-34-W)	52%	73%	70%	28%	27%	24%	90%	80%	84%	76%	79%	66%
	S5: (YW-F-25-34-W)	52%	73%	71%	25%	26%	21%	88%	76%	80%	70%	73%	64%
	S6: (W-M-35-64-W)	51%	72%	69%	27%	25%	20%	89%	78%	81%	73%	75%	64%
	S7: (W-F-35-64-W)	51%	72%	69%	26%	25%	19%	90%	77%	81%	73%	75%	64%
	S8: (A-MF-35-64-L.SH)	55%	74%	75%	31%	33%	30%	90%	80%	82%	75%	75%	70%
	S9: (E-M-65+)	55%	73%	73%	32%	33%	27%	89%	82%	83%	75%	75%	68%
	S10: (E-F-65+)	53%	74%	67%	29%	28%	23%	88%	85%	86%	77%	79%	62%
$TT_{DS} - TT_{NS} = 10 \text{ min}$	S1: (YS-MF-15-S)	86%	83%	84%	69%	76%	63%	96%	94%	94%	92%	91%	80%
	S2: (S-M-16-24-S.W)	66%	79%	82%	51%	53%	38%	96%	88%	89%	87%	87%	77%
	S3: (S-F-16-24-S.W)	61%	77%	82%	44%	50%	36%	94%	87%	89%	86%	86%	75%
	S4: (YW-M-25-34-W)	52%	82%	77%	42%	39%	37%	96%	85%	90%	83%	87%	75%
	S5: (YW-F-25-34-W)	52%	82%	79%	41%	38%	33%	95%	82%	87%	80%	84%	73%
	S6: (W-M-35-64-W)	51%	80%	76%	40%	37%	31%	96%	84%	88%	82%	85%	73%
	S7: (W-F-35-64-W)	51%	80%	75%	39%	35%	30%	97%	83%	88%	82%	85%	73%
	S8: (A-MF-35-64-L.SH)	55%	84%	82%	48%	47%	43%	97%	85%	89%	84%	85%	78%
	S9: (E-M-65+)	55%	84%	82%	46%	46%	40%	96%	86%	88%	84%	85%	78%
	S10: (E-F-65+)	53%	83%	76%	45%	43%	39%	96%	89%	91%	86%	88%	73%

It is also noted that adjusting the threshold to allow for a slight increase in travel time for resilient trips—whether it's a 10% increase or just 5 minutes—causes a noticeable rise in the TRI. Furthermore, across both types of thresholds (i.e., absolute difference and the ratio of post- to pre-disruption travel times), an increase in the threshold value (e.g., raising the ratio from 1.10 to 1.25 or increasing the absolute time difference from 5 to 10 minutes) results in a corresponding increase in the TRI.

Focusing on individual population segments, S7 shows the highest sensitivity to the EoD, with TRI ranging from 61% to 9% (a variation of 52%). Meanwhile, Segment S1 exhibits the smallest variation (34%), indicating lower sensitivity to EoD compared to other segments. When applying

additional thresholds, a general trend emerges: the largest TRI range is associated with Scenario 1 or Scenario 5, while the smallest range is observed in Scenario 2 or Scenario 7. Moreover, across all thresholds, Segment S7 consistently exhibits the highest sensitivity to EoD, while Segment S1 remains the least sensitive.

When examining individual population segments with the application of the first threshold, Segment S7 demonstrates the most significant variation in the TRI across various disruption scenarios. Its TRI values fluctuate between 61% and 9%, resulting in a total variation of 52%. In contrast, Segment S1 shows the least variation at 34%, indicating a lower degree of change in response to disruption scenarios compared to other segments. This trend persists with the application of additional thresholds. Segment S7 continues to exhibit the highest rate of change across various disruption scenarios, while Segment S1 remains the least affected. Hence, even under a specific threshold, the TRI alone is insufficient as a criterion for determining the vulnerability of a population segment. The assessment is significantly influenced by the EoD in each scenario. For instance, it cannot be definitively stated whether a TRI of 50% (even after applying a particular threshold) indicates a favorable or unfavorable level of vulnerability. As illustrated in Table 7-5, when the threshold $TT_{DS} - TT_{NS} = 0$ is applied, a TRI of 32% in Scenario 4 suggests that segment S1 is the least vulnerable. In contrast, in Scenario 7, a TRI of 55% for segment S2 denotes greater vulnerability, rendering it the most vulnerable population group in that scenario.

Regression models can be utilized to identify the most vulnerable population segments by treating the EoD as the independent variable and the TRI as the dependent variable. In this approach, the slope of the regression model represents the influence of increasing EoD on the reduction of TRI for each population segment. As demonstrated in Table 5, the TRI values for all population segments decline as the EoD increases across various scenarios. Consequently, the slope of the regression model (for all segments and under each threshold) is expected to be negative. A steeper negative slope (indicating a larger absolute value) corresponds to a sharper decline in TRI, signifying greater vulnerability for that segment.

Table 7-6 Vulnerability ranking of population segments (From 1: most vulnerable to 10: least vulnerable)

Thresholds	Population Segments	Linear Regression						Decision Tree				Random Forest			
		Durbin-Watson	Shapiro-Wilk p-value	Breusch-Pagan p-value	R-squared	Slope	Rank	R-squared	MSE	Slope	Rank	R-squared	MSE	Slope	Rank
$TT_{DS} - TT_{NS} = 0$	S1: (YS-MF-15-S)	2.93	0.42	0.07	0.79	-0.58	10	0.92	8.31	-0.58	10	0.92	8.59	-0.57	10
	S2: (S-M-16-24-S.W)	2.50	0.31	0.10	0.88	-0.80	9	0.99	1.67	-0.80	9	0.98	4.46	-0.79	9
	S3: (S-F-16-24-S.W)	2.44	0.88	0.11	0.87	-0.86	8	0.98	4.10	-0.86	8	0.97	7.14	-0.85	8
	S4: (YW-M-25-34-W)	1.89	0.96	0.64	0.85	-1.06	1	1.00	0.94	-1.07	1	0.98	8.02	-1.05	1
	S5: (YW-F-25-34-W)	1.94	0.75	0.74	0.79	-0.95	5	0.99	3.42	-0.95	5	0.96	12.35	-0.95	5
	S6: (W-M-35-64-W)	2.05	0.99	0.54	0.88	-1.03	3	0.99	1.90	-1.02	4	0.97	8.12	-1.02	3
	S7: (W-F-35-64-W)	1.97	0.82	0.52	0.83	-1.02	4	0.99	2.17	-1.03	3	0.97	11.18	-1.00	4
	S8: (A-MF-35-64-L.SH)	2.06	0.26	0.95	0.78	-0.88	7	0.99	3.43	-0.88	7	0.95	12.57	-0.87	7
	S9: (E-M-65+)	1.93	0.81	0.44	0.86	-0.89	6	1.00	0.29	-0.90	6	0.98	4.45	-0.88	6
	S10: (E-F-65+)	2.05	0.94	0.65	0.85	-1.04	2	0.99	1.79	-1.04	2	0.96	11.79	-1.02	2
$TT_{DS} / TT_{NS} = 1.1$	S1: (YS-MF-15-S)	2.75	0.15	0.06	0.85	-0.83	10	0.98	4.56	-0.83	10	0.97	6.81	-0.81	10
	S2: (S-M-16-24-S.W)	2.28	0.17	0.13	0.89	-1.12	9	0.99	2.79	-1.11	9	0.99	4.34	-1.09	9
	S3: (S-F-16-24-S.W)	2.14	0.94	0.15	0.89	-1.19	7	0.99	2.21	-1.19	7	0.98	8.12	-1.17	7
	S4: (YW-M-25-34-W)	1.96	0.99	0.59	0.87	-1.40	1	0.99	3.15	-1.39	1	0.96	20.98	-1.33	4
	S5: (YW-F-25-34-W)	2.03	0.93	0.73	0.83	-1.28	5	0.99	7.25	-1.29	5	0.96	21.32	-1.28	5
	S6: (W-M-35-64-W)	2.08	0.96	0.49	0.87	-1.37	4	0.99	6.15	-1.36	4	0.97	18.39	-1.35	2
	S7: (W-F-35-64-W)	2.07	0.92	0.40	0.87	-1.38	3	0.99	6.27	-1.37	2	0.97	17.46	-1.35	3
	S8: (A-MF-35-64-L.SH)	2.10	0.74	0.76	0.86	-1.16	8	0.99	4.90	-1.16	8	0.97	12.69	-1.15	8
	S9: (E-M-65+)	1.94	0.74	0.42	0.88	-1.25	6	0.99	2.60	-1.25	6	0.97	13.18	-1.23	6
	S10: (E-F-65+)	2.16	0.69	0.49	0.89	-1.38	2	0.99	3.21	-1.37	3	0.98	10.62	-1.37	1
$TT_{DS} / TT_{NS} = 1.25$	S1: (YS-MF-15-S)	2.67	0.59	0.03	0.84	-0.60	10	0.97	3.15	-0.59	10	0.95	4.98	-0.58	10
	S2: (S-M-16-24-S.W)	2.15	0.04	0.13	0.86	-1.00	9	0.99	1.60	-1.00	9	0.98	7.40	-0.96	9
	S3: (S-F-16-24-S.W)	2.23	0.34	0.12	0.86	-1.05	8	0.99	3.26	-1.05	8	0.97	10.15	-1.00	8
	S4: (YW-M-25-34-W)	1.95	0.98	0.47	0.87	-1.27	1	0.99	5.10	-1.27	1	0.97	16.46	-1.24	1
	S5: (YW-F-25-34-W)	1.99	0.94	0.45	0.85	-1.25	3	0.99	5.13	-1.25	4	0.96	18.97	-1.23	2
	S6: (W-M-35-64-W)	2.08	0.97	0.36	0.88	-1.24	5	0.98	7.58	-1.22	5	0.97	13.77	-1.20	5
	S7: (W-F-35-64-W)	2.02	0.99	0.34	0.89	-1.26	2	0.99	6.29	-1.26	2	0.97	14.27	-1.20	4
	S8: (A-MF-35-64-L.SH)	2.07	0.71	0.45	0.85	-1.12	7	0.99	3.79	-1.11	7	0.96	14.08	-1.10	6
	S9: (E-M-65+)	1.92	0.74	0.40	0.84	-1.12	6	0.99	2.15	-1.12	6	0.96	16.26	-1.08	7
	S10: (E-F-65+)	2.10	0.92	0.39	0.88	-1.25	4	0.99	2.76	-1.25	3	0.97	12.97	-1.23	3

Table 7-6 Vulnerability ranking of population segments (From 1: most vulnerable to 10: least vulnerable) (continued)

Thresholds	Population Segments	Linear Regression						Decision Tree				Random Forest			
		Durbin-Watson	Shapiro-Wilk p-value	Breusch-Pagan p-value	R-squared	Slope	Rank	R-squared	MSE	Slope	Rand	R-squared	MSE	Slope	Rand
$TT_{DS} - TT_{NS} = 5 \text{ min}$	S1: (YS-MF-15-S)	2.28	0.31	0.09	0.78	-0.78	10	0.98	3.76	-0.77	10	0.95	9.59	-0.76	10
	S2: (S-M-16-24-S.W)	2.03	0.05	0.17	0.90	-1.15	9	0.99	2.64	-1.15	9	0.99	3.62	-1.13	8
	S3: (S-F-16-24-S.W)	2.25	0.52	0.17	0.89	-1.16	8	0.99	3.06	-1.16	8	0.98	6.72	-1.11	9
	S4: (YW-M-25-34-W)	2.01	1.00	0.52	0.92	-1.36	3	0.99	5.01	-1.36	2	0.98	9.51	-1.33	2
	S5: (YW-F-25-34-W)	1.88	0.98	0.76	0.88	-1.31	5	0.99	6.83	-1.31	5	0.98	11.22	-1.29	5
	S6: (W-M-35-64-W)	1.98	1.00	0.48	0.90	-1.35	4	0.99	7.12	-1.33	4	0.98	9.08	-1.32	3
	S7: (W-F-35-64-W)	2.07	0.99	0.54	0.90	-1.36	2	0.98	8.99	-1.35	3	0.98	12.56	-1.32	4
	S8: (A-MF-35-64-L.SH)	1.92	0.32	0.78	0.89	-1.20	7	0.99	4.40	-1.20	7	0.98	8.05	-1.18	7
	S9: (E-M-65+)	1.98	0.93	0.42	0.90	-1.23	6	0.99	2.76	-1.22	6	0.99	5.27	-1.18	6
	S10: (E-F-65+)	2.15	0.68	0.32	0.94	-1.38	1	0.99	2.64	-1.38	1	0.99	4.39	-1.36	1
$TT_{DS} - TT_{NS} = 10 \text{ min}$	S1: (YS-MF-15-S)	2.86	0.08	0.06	0.85	-0.58	10	0.97	3.03	-0.57	10	0.96	4.25	-0.57	10
	S2: (S-M-16-24-S.W)	2.35	0.14	0.03	0.87	-1.02	9	0.99	3.22	-1.01	9	0.98	7.33	-1.01	9
	S3: (S-F-16-24-S.W)	2.35	0.11	0.41	0.89	-1.10	6	0.99	3.83	-1.10	6	0.97	9.39	-1.05	6
	S4: (YW-M-25-34-W)	2.10	0.55	0.96	0.88	-1.21	3	0.98	7.14	-1.21	3	0.96	15.77	-1.21	3
	S5: (YW-F-25-34-W)	2.02	0.52	0.89	0.85	-1.19	4	0.98	8.65	-1.18	5	0.96	17.89	-1.17	5
	S6: (W-M-35-64-W)	2.15	0.31	0.80	0.88	-1.25	2	0.98	7.64	-1.25	2	0.97	15.96	-1.23	2
	S7: (W-F-35-64-W)	2.20	0.39	0.89	0.88	-1.29	1	0.98	10.47	-1.29	1	0.96	17.80	-1.26	1
	S8: (A-MF-35-64-L.SH)	2.07	0.61	0.98	0.84	-1.04	8	0.98	6.44	-1.04	8	0.95	15.25	-1.03	8
	S9: (E-M-65+)	2.01	0.43	0.87	0.84	-1.08	7	0.99	4.56	-1.07	7	0.96	14.59	-1.03	7
	S10: (E-F-65+)	2.28	0.36	0.72	0.89	-1.18	5	0.99	4.54	-1.18	4	0.97	11.56	-1.17	4

Table 7-6 shows that the Durbin-Watson values for most population segments fluctuate around 2 (ranging from 1.5 to 2.5), while the p-values for the Breusch-Pagan and Shapiro-Wilk tests are predominantly greater than or equal to 0.05. These results confirm the suitability of linear regression for most population segments after applying each threshold. However, the limited number of observations corresponding to the disruption scenarios necessitates the consideration of alternative methods. Therefore, decision tree regression and random forest regression models, which are not sensitive to linearity assumptions, were also employed. As shown in Table 7-6, after applying nearly every threshold, the machine learning models generally confirm the results of the

linear regression, with no significant differences in ranking population segments across the three models. The only slight discrepancy occurs when the threshold $TT_{DS}/TT_{NS} = 1.1$ is applied, where the random forest model deviates somewhat from the results of the linear regression and decision tree models, compared to its consistency under other thresholds. Moreover, R-squared values, used to evaluate model accuracy, demonstrate that, in general, machine learning models offer greater accuracy than linear regression models after the application of each threshold. Additionally, a comparison of R-squared and Mean Square Error (MSE) values between the two machine learning techniques indicates that the Decision Tree model exhibits slightly higher accuracy compared to the Random Forest model.

As previously stated, this research focuses on relative vulnerability, meaning that the analysis primarily compares the associated slopes of different population segments. A steeper negative slope for one group relative to another indicates higher vulnerability. For instance, as shown in Table 7-6, when the threshold $TT_{DS} - TT_{NS} = 0$ is applied, the slope for segment S1 is approximately -0.58, representing the least steep decline and indicating that this segment is the least vulnerable. Conversely, the slope for segment S4 is approximately -1.06, the steepest among all segments, suggesting that this group is the most vulnerable in the studied system.

However, given that the study examines relative vulnerability, two key points must be considered:

1. A ranking of 10 (indicating the least vulnerability) does not imply that the corresponding population segment is entirely resilient to disruptions. Similarly, a ranking of 1 (indicating the highest vulnerability) does not necessarily mean that the population segment is inherently vulnerable. Rather, these rankings indicate that within the analyzed system, S1 exhibits the lowest relative vulnerability, whereas S4 demonstrates the highest.
2. The slope values alone do not provide an absolute measure of vulnerability. For example, a slope of -0.58 does not necessarily indicate that a population segment is not vulnerable, nor does a slope of -1.06 confirm that the segment is vulnerable in an absolute sense. Instead, these values should be interpreted in a comparative framework, where a segment associated with a slope of -1.06 is more vulnerable than one with a slope of -0.58.

While slope can serve as a reliable indicator for assessing absolute vulnerability, its interpretation must consider the impact of applied thresholds. As illustrated in Table 6, the thresholds used for

the Trip Resilience Index (TRI) influence the calculated slopes of population segments. Therefore, slope alone is not sufficient to comprehensively assess vulnerability.

Table 7-6 presents the vulnerability ranking of population segments after applying various TRI thresholds. However, to establish a general ranking for the vulnerability of all population segments, it is necessary to focus on a consistent criterion, given that the rankings across different TRI thresholds are not uniform. The primary criterion in this context is the slope. Table 7-7 illustrates the average slope of each population segment across different models, following the application of each TRI threshold. Furthermore, the table introduces the "Global Slope Average," which represents the arithmetic mean of the slopes for each population segment across the various models. This indicator serves as the principal criterion for ranking population segments according to their vulnerability level.

Table 7-7 Global vulnerability analysis of population segments (From 1: most vulnerable to 10: least vulnerable)

Population Segments	Slope Average in Different Models			Global Slope Average	Rank
	Linear Regression	Decision Tree	Random Forest		
S1: (YS-MF-15-S)	-0.67	-0.67	-0.66	-0.67	10
S2: (S-M-16-24-S.W)	-1.02	-1.01	-1.00	-1.01	9
S3: (S-F-16-24-S.W)	-1.07	-1.07	-1.04	-1.06	8
S4: (YW-M-25-34-W)	-1.26	-1.26	-1.23	-1.25	1
S5: (YW-F-25-34-W)	-1.19	-1.20	-1.18	-1.19	5
S6: (W-M-35-64-W)	-1.25	-1.24	-1.22	-1.24	4
S7: (W-F-35-64-W)	-1.26	-1.26	-1.23	-1.25	2
S8: (A-MF-35-64-L.SH)	-1.08	-1.08	-1.06	-1.07	7
S9: (E-M-65+)	-1.11	-1.11	-1.08	-1.10	6
S10: (E-F-65+)	-1.25	-1.25	-1.23	-1.24	3

Across nearly all models and thresholds (Table 7-6), segments S1 and S2 consistently occupy the least vulnerable positions, ranked tenth and ninth, respectively. The Global Slope Average further substantiates that these segments are the least vulnerable to disruptions. Segment S3 frequently appears as the eighth most vulnerable group in most models, while segment S8 commonly ranks eighth or seventh. Thus, the assignment of S3 and S8 to the eighth and seventh ranks in vulnerability, respectively, aligns well with prior model results. Segment S9 maintains a stable position across all models and thresholds, consistently ranked as the sixth most vulnerable group,

making this assignment in Table 7-7 unsurprising. Segment S5, however, demonstrates more variability in Table 7-6, with its ranking fluctuating among fifth, fourth, third, or even second in some cases. As such, its precise ranking—absent the Global Slope Average—is more challenging to ascertain compared to the previous segments. Segment S6, ranked variably as fifth, fourth, third, or second across models, is ultimately designated as fourth in vulnerability based on the Global Slope Average. Segment S10, frequently appearing within the top three most vulnerable segments, now ranks as the third most vulnerable. The Global Slope Average assigns segment S7 as the second most vulnerable, while segment S4, consistently ranked first across models and thresholds, is confirmed as the most vulnerable segment.

7.5 Discussion

This research aimed to bridge the existing gap in assessing the impact of metro disruptions on individuals and various population segments. We proposed a four-step framework, which culminates in ranking population segments based on their vulnerability to such disruptions. Although we focused on metro disruptions and considered the regular bus system as a complementary mode of transport, the framework is adaptable and can be generalized to disruptions in other modes. Additionally, supplementary modes such as carsharing, bikesharing, etc. can be integrated to assess the effects of multimodality on users' vulnerability in transit systems.

The framework was tested on the Montreal transit system. In the first step, using GTFS data, we modeled the transit network in the Transition Platform. This network included 4 metro lines and 395 bus lines, operated by three distinct service providers. Subsequently, based on data from the 2018 Montreal Origin-Destination survey, users were grouped into 10 population segments according to their socioeconomic and travel attributes. We focused on simple home-based trips made either solely by metro or by a combination of metro and bus, and these trips were simulated using the Transition Platform.

In the second step, we defined and simulated 12 station-based metro disruption scenarios, using station importance and historical disruption frequencies. These scenarios encompassed a range of Extent of Disruption (EoD) levels. For each disruption scenario, transit trips were re-routed, and their new travel times were calculated. In the third step, we introduced a novel Transit Robustness

Index (TRI), which measured the proportion of resilient trips relative to all trips made by each population segment. Five distinct thresholds were tested to define resilient trips.

In the final step, we used the number of disruption scenarios as the sample size and employed regression models, using EoD as the independent variable, to predict TRI for each population segment at different thresholds. We conducted statistical tests to assess linearity, normality, and other assumptions necessary for linear regression, and the results substantially confirmed that linear regression was valid for our data. However, given the limited number of observations (12 disruption scenarios), we also used two non-parametric machine-learning methods, which are not sensitive to linearity and normality. These additional methods allowed us to validate the linear regression results and make our findings more reliable by comparing the outcomes of the three models. The slope in these regression models—indicating the rate at which TRI decreases as EoD increases—was the key metric for evaluating the vulnerability of each population segment. A greater absolute value of the slope corresponded to more vulnerability in a population segment.

The results showed that while the mainly linear regression models achieved an acceptable level of accuracy ($R\text{-squared} > 0.85$), machine-learning models provided higher accuracy, with Decision Trees slightly outperforming other methods. Moreover, across all TRI thresholds, the machine-learning models—particularly Decision Trees—strongly confirmed the results obtained from the linear regression models. Overall, the models consistently identified Children (S1) as the least vulnerable population segment. However, the identification of the most vulnerable population segment was less straightforward. At a specific TRI threshold, the models consistently agreed on the most vulnerable group. However, applying different TRI thresholds led to variations, suggesting different segments such as Young Men Workers (S4), Women Workers (S7), and Elderly Women (S10) as the most vulnerable. Ultimately, by averaging the slopes across the different TRI thresholds, we concluded that S4 (Young Men Workers) had the highest absolute average slope value, making them the most vulnerable segment.

This research had some limitations. We assumed that all passengers, upon being informed of metro disruptions, would continue their trips using the fastest bus-based alternative. Additionally, we assumed that the bus system could accommodate all passengers switching from the metro without causing further delays. In future research, we will account for the capacity limitations of the bus system and other complementary modes in handling passengers affected by metro disruptions.

Furthermore, absolute vulnerability analysis will be conducted as a complementary step to assess the overall severity of disruptions on different population segments, providing a more comprehensive understanding beyond relative comparisons. Additionally, we plan to define TRI thresholds based on trip duration to enhance the precision of our analysis. Furthermore, the integration of other transit modes, such as regional trains, light rapid rail, and bus rapid transit, will be considered. Shared mobility options, including carsharing and bikesharing, will also be incorporated to evaluate the impact of multimodal networks on users' vulnerability.

CHAPTER 8 GENERAL DISCUSSION

The primary contributions of this thesis lie in the development of methodologies for a comprehensive assessment of public transportation vulnerability, encompassing all networks, services, and users. Through an extensive review of the literature, significant research gaps were identified regarding the vulnerability of public transportation systems, leading to the formulation of research objectives designed to address these gaps effectively.

8.1 Improving the disruption simulation process

The first research objective was to improve the disruption simulation process. Disruption simulation serves as the cornerstone in studies of public transportation system vulnerability, as all vulnerability indicators are derived from the network configuration post-disruption. As one of the foundational steps in vulnerability analysis, disruption simulation must be conducted with rigor and accuracy; if approached imprecisely or grounded in incorrect assumptions, the simulation outcomes can diverge significantly from real-world scenarios, rendering subsequent research efforts ineffective. Thus, employing a precise failure strategy and avoiding erroneous simplifications are essential to enhancing the accuracy of simulations. The first objective of this research is to advance disruption simulation techniques within transit systems, with a particular focus on metro networks.

In Chapter 4, a significant research gap in the simulation of metro system disruptions is examined in detail, specifically addressing the limited consideration of infrastructural characteristics within transit networks. This gap particularly pertains to the role of Stations with Direction Change Facilities (SDCFs). To address this, the chapter introduces the Block-Based Failure Strategy (BBFS), which defines regular stations positioned between two SDCFs as a single block. The adoption of BBFS shifts the focus from a simplistic graph-based analysis to a comprehensive examination of the transportation network, enhancing the simulation's alignment with real-world conditions. Additionally, knowledge of the BBFS algorithm may provide valuable insights for network planners by informing strategic decisions on the placement of stations with directional facilities in metro system designs.

Chapter 5 further explores existing gaps in the formulation of disruption scenarios, both targeted and random. In conventional approaches to targeted disruption scenarios—often representing worst-case conditions—the most critical station in the network is typically identified through a single indicator, such as degree or betweenness, with occasional use of other indicators like closeness and PageRank. Chapter 5 demonstrates the advantages of a Multi-Attribute Decision-Making (MADM) approach, specifically using the Technique for Order of Preference by Similarity (TOPSIS), to account for multiple indicators simultaneously, thereby achieving a more accurate identification of critical stations.

This chapter also highlights the oversight commonly found in defining random disruption scenarios, particularly in the context of BBFS. In cases of random station selection, it is essential to recognize the potential occurrence of unusual, though rare, scenarios that could lead to misleading interpretations of network degradation. Chapter 5 illustrates that, across 100 random disruption scenarios, there was one instance where network degradation closely resembled that of a targeted disruption scenario and another instance where the impact appeared overly optimistic. Relying on a single random scenario may therefore introduce biases if it happens to be an outlier. Consequently, in this chapter it is suggested to iterate random scenarios multiple times is recommended to ensure that selected scenarios are representative and not anomalous.

8.2 Introducing a novel vulnerability analysis method

In studies examining the vulnerability of public transportation systems, the chosen methodology defines the perspective from which transit systems are analyzed. Traditionally, analyses have adopted either a topological or a functional approach. Topology-based analyses, which are more prevalent in transit system studies, focus on the structural or topological characteristics of the network. These analyses were foundational to vulnerability studies in transportation, identifying network segments that are particularly susceptible to disruptions, especially those whose failures could lead to severe network-wide impacts. Conversely, functional methods—now increasingly applied—concentrate on transportation-specific attributes, evaluating the effects of disruptions on factors such as travel time and passenger flow. Despite their insights, however, neither topological nor functional methodologies adequately account for the impact of disruptions on the quality and

quantity of alternative routes available to users. This limitation underscores the need for approaches that can assess how disruptions influence route diversity within transit systems.

In Chapter 5, a novel approach termed the *redundancy-based vulnerability method* is introduced to address a critical gap in transit system vulnerability analysis. This method specifically examines the availability of route alternatives for each metro journey. However, not all theoretically identified routes serve as viable alternatives to the primary (fastest) route. To refine the categorization of alternatives, a threshold is introduced based on the travel time ratio between the fastest route and other theoretical routes. This threshold allows for the differentiation of alternative routes into *acceptable* and *unacceptable*, effectively classifying them by quality. Subsequently, the method evaluates the effect of disruptions on both the quantity and quality of alternative routes.

The results derived from this approach reveal trends that diverge from those observed in conventional topological and functional vulnerability analyses within our case study. Redundancy indicators demonstrate a heightened sensitivity to the location of disrupted stations, rather than merely the number of stations affected, contrasting with traditional topological and functional metrics. This underscores the critical role of network configuration in sustaining redundancy. In some cases, a station may not exhibit high boarding or alighting volumes; however, its role in preserving network cycles enhances overall redundancy. Therefore, a station's significance is not always tied to functional metrics (such as boarding/alighting volume) or topological metrics (such as betweenness). This insight is valuable for transit planners, especially in network design or in prioritizing station reopening during major disruptions, where preserving redundancy or providing alternative options is essential for users.

8.3 Assessing the quality of alternative transit service during metro disruptions

Metro networks are subject to frequent disruptions compared to other transit modes, prompting substantial research focused on mitigating their adverse effects. However, for service operators to implement mitigation strategies, disruptions generally need to exceed a predefined duration. A review of the literature indicates that no standardized threshold or scientifically validated calculation method exists for this; instead, service operators empirically establish this threshold

based on network characteristics and trip patterns within their service areas. Consequently, prior research has often assumed that mitigation measures are enacted without specifying the disruption duration (e.g., short-, medium-, or long-term), analyzing or optimizing the effectiveness of these measures under this assumption.

A historical analysis of metro disruptions reveals that short-duration disruptions occur at a frequency several times greater than medium- or long-duration events, potentially influencing operators' decisions on mitigation strategy implementation. In cases where disruptions occur without the activation of mitigation measures, the conventional bus system becomes the primary alternative available to users to alleviate the impacts of metro outages. It is generally recognized that, even during major metro disruptions, nearly all transit trips can still be completed via buses. However, a critical consideration lies in assessing how much travel time increases, or how transit trip quality diminishes, when relying on buses instead of the metro.

Additionally, an important question remains regarding the relationship between the extent of metro disruption and the increase in travel time when using the bus system. Another open question is whether the timing of the disruption significantly affects the bus system's capacity to maintain transit quality during a metro disruption. These unresolved questions highlight research gaps that are addressed in this study.

Chapter 6 introduces a five-step framework that examines the impact of metro disruptions in Montreal's transit network. The process begins with modeling the transit network using GTFS data from all regional transit providers. Following this, trip routing and travel times are estimated for normal operating conditions. Metro disruption scenarios are then simulated, prompting alternative trip routing and enabling the estimation of travel times during disruptions. By comparing these travel times with those under regular conditions, quality thresholds are established. This approach highlights the effectiveness of the combined metro and bus systems in reducing disruption effects across different levels of travel quality.

The framework further applies regression models to analyze the relationship between disruption extent and the proportion of feasible trips at varying quality levels, as well as increases in travel time. Hypothesis testing is also conducted to evaluate whether disruption

timing significantly influences the combined metro and bus systems' performance in mitigating disruption impacts.

The results show a linear correlation between disruption extent and feasible trips. This indicates that feasible trips decline as disruption severity increases, particularly at higher quality thresholds. The analysis also confirms that the increase in travel time can be predicted based on the disruption extent. Interestingly, the timing of metro disruptions does not substantially impact the quality of transit alternatives or the increase in travel time, as demonstrated through simulations across different scenarios.

8.4 Ranking the vulnerability of users to disruptions in transit systems

Research on vulnerability in public transportation predominantly focuses on network-level and macro-scale transportation metrics, often neglecting the individual-level impacts of system disruptions. These disruptions frequently require users to adjust their routes or modes of travel, leading to increases in travel time. Yet, understanding which groups of users demonstrate greater resilience in the face of such disruptions remains a crucial but largely unexamined topic. Furthermore, a review of existing literature reveals the absence of established frameworks or indicators specifically designed to assess user vulnerability or to systematically rank population segments based on their susceptibility to transit system disruptions. These gaps represent significant research opportunities in the field.

Chapter 7 presents a structured, four-step framework for ranking population segments according to their vulnerability to metro disruptions. Although it focuses on metro systems, with the bus network as a complementary mode of transport, this framework can be adapted to assess disruptions in other modes of transportation. Additionally, it is flexible enough to incorporate supplementary modes, such as car-sharing or bike-sharing, to examine how multimodality affects user vulnerability within transit systems.

This framework is applied to the Montreal transit system. In the first step, GTFS data models the transit network, and users are grouped into 10 population segments based on socioeconomic and travel characteristics from the 2018 Montreal Origin-Destination survey. The second step defines 12 metro disruption scenarios, using station importance and historical disruption frequencies to capture varying levels of Extent of Disruption (EoD). For each scenario, transit trips are rerouted, and new travel times are recalculated.

In the third step, the framework introduces the Transit Robustness Index (TRI), which measures the proportion of robust trips for each population segment. Five distinct thresholds are tested to define robust trips. The final step uses the 12 disruption scenarios as the sample size in regression models, with EoD as the predictor variable for estimating TRI across different thresholds for each segment. Given the limited sample size, two non-parametric machine learning methods, which do not require linearity or normality assumptions, are employed alongside linear regression. This multi-model approach provided a cross-validation mechanism, ensuring more reliable findings. Within these regression models, the slope—indicating the rate at which TRI decreases with increasing EoD—become the key indicator of segment vulnerability, where greater absolute slope values signify higher vulnerability.

In this case study, the models consistently identify Pupils as the segment with the least vulnerability. Determining the most vulnerable segment, however, requires a more detailed approach. At certain TRI thresholds, the models align on the most vulnerable group, but different thresholds reveal variation, with segments such as Young Male Workers, Female Workers, and Elderly Females sometimes exhibiting the greatest vulnerability. By averaging the slopes across TRI thresholds, Young Male Workers emerge with the highest absolute average slope, marking them as the segment most impacted by disruptions.

8.5 Discussion

The analysis of metro system vulnerability in this thesis provides a comprehensive examination of the factors that influence the robustness of public transportation networks during disruptions. By adopting a multi-faceted approach, this research challenges conventional views of vulnerability and offers new perspectives on how transit systems can be assessed, planned, and improved to ensure operational continuity in the face of unforeseen events. The findings of this study underscore the complexity of vulnerability, demonstrating that a variety of network characteristics, operational practices, and user behaviors contribute to the overall robustness of a metro system.

One key observation from the research is that traditional models of vulnerability, which often focus on topological characteristics like station connectivity and network structure, may not fully capture the real-world impacts of service disruptions. The limitations of these methods become apparent when considering how actual service disruptions affect passengers. For instance, while many

studies focus on the number of stations affected by a disruption, this research highlights the importance of understanding where those disruptions occur within the network. The spatial location of disruptions has a more profound effect on the system's ability to maintain operational integrity than previously acknowledged, particularly when critical transfer points are affected. These findings suggest that vulnerability assessments must be refined to account for the nuanced nature of disruptions, moving beyond simple topological measures to include a more detailed analysis of station importance, operational flexibility, and redundancy.

Additionally, the role of alternative transit services, such as buses, in mitigating the effects of metro disruptions has been a central theme in this study. While the bus network serves as a crucial backup during metro service interruptions, its ability to effectively absorb displaced demand is not without limitations. The findings indicate that, while buses can often provide an alternative means of travel, the increased travel time and reduced service quality are significant drawbacks for users, especially as disruptions become more severe. These results emphasize the importance of considering not only the feasibility of alternative routes but also the quality of service they provide. The relationship between disruption extent and the increase in travel time underscores the need for transit agencies to carefully evaluate the trade-offs between service restoration times and the capability of alternative modes to sustain transit quality during disruptions.

The findings also challenge the conventional wisdom that metro disruptions follow predictable patterns. The lack of significant time-of-day effects in the case of Montreal, for example, suggests that disruption impacts are less contingent on peak versus off-peak hours than previously thought. However, it is important to note that this analysis does not account for actual bus network capacity. In practice, especially during peak periods, the bus system is often saturated, and passengers may face significant additional delays due to the inability to board overcrowded buses. This limitation suggests that while the model indicates similar impacts across time periods, the real-world experience during peak-hour disruptions may be more severe..

Finally, this research sheds light on the variation in vulnerability across different user groups. By incorporating socio-demographic characteristics into the analysis, it becomes clear that certain segments of the population are disproportionately affected by metro disruptions. For instance, younger men workers, who may rely heavily on metro services, face greater challenges when disruptions occur. This finding highlights the need for transit systems to adopt a more inclusive

approach when designing mitigation strategies, considering the specific vulnerabilities of various population groups. Tailored strategies that prioritize the needs of the most vulnerable users could significantly improve the overall resilience of the system and ensure that all passengers are adequately served during service disruptions.

In conclusion, this research offers valuable contributions to the field of transit vulnerability assessment, moving beyond traditional methods to provide a more nuanced understanding of how metro systems respond to disruptions. The findings emphasize the importance of a holistic approach that considers not only the technical aspects of network design but also the operational and social dimensions of service delivery. As cities continue to face growing challenges related to congestion, aging infrastructure, and climate change, the insights from this study can inform more resilient, equitable, and effective transit planning practices.

CHAPTER 9 CONCLUSION

9.1 Summary

This research presents a comprehensive and multi-dimensional investigation into the vulnerability of public transportation systems, with a particular emphasis on metro networks. It underscores the crucial role of accurate simulation methodologies, robust network assessment techniques, and user-focused analyses in understanding and mitigating the impacts of transit disruptions. Recognizing that public transportation systems are increasingly exposed to operational challenges and unexpected disruptions; this research introduces a suite of methodological innovations designed to enhance both analytical accuracy and policy relevance in the field of transportation vulnerability assessment.

At the core of this research is the recognition that disruption simulation serves as the foundation for any effective vulnerability analysis. Through the introduction of a Block-Based Failure Strategy (BBFS), the study addresses the limitations of simplistic graph-based simulations and demonstrates how incorporating infrastructural features such as direction change facilities can lead to more realistic and reliable modeling of disruptions. By advancing simulation precision, the study mitigates the risk of skewed conclusions that can arise from erroneous assumptions and strengthens the overall credibility of vulnerability assessments.

Beyond simulation improvements, the research challenges the adequacy of traditional topological and functional methodologies in capturing the complexity of real-world transit disruptions. While these conventional approaches offer structural and operational insights, they often neglect the availability and quality of alternative routes available to users. To bridge this gap, the study proposes a redundancy-based vulnerability framework that redefines how route alternatives are identified and evaluated. By introducing a quality-based threshold for alternative routes, this approach offers a more meaningful measure of system robustness, highlighting how critical the spatial location of disruptions is—often more so than the quantity of disrupted nodes.

In addition to evaluating infrastructure performance, this study explores the resilience of complementary transit services during metro outages. The role of the conventional bus system is critically examined within a five-step framework that models both normal and disrupted scenarios in the Montreal transit network. Through travel time comparisons and regression analyses, the

findings reveal a clear linear relationship between the extent of disruption and the proportion of feasible trips, confirming that increased disruption leads to reduced service quality. Moreover, under the assumption of sufficient bus capacity across all scenarios, the timing of disruptions was found to have minimal influence on the quality of alternative services during metro outages. It is important to acknowledge, however, that in real-world conditions—particularly during peak periods—the bus system is often operating near or at full capacity. This limitation implies that actual travel times and service reliability may be considerably worse than the model predicts.

Significantly, the study extends vulnerability analysis beyond infrastructure and operations to consider equity and user-specific impacts. It presents a pioneering framework for identifying and ranking population segments based on their susceptibility to transit system disruptions. By integrating socioeconomic data, travel behavior characteristics, and machine learning regression models, the study reveals substantial variability in how different groups experience disruptions. The research identifies segments such as pupils as generally more resilient, while others—particularly younger workers—are more vulnerable, depending on the travel time thresholds used. These findings emphasize the need for transit agencies to adopt more inclusive and data-driven strategies when allocating resources and planning interventions, ensuring that the needs of the most affected populations are addressed.

Overall, this research contributes a set of original tools and perspectives that advance the field of public transportation vulnerability analysis. It advocates for a more holistic understanding of resilience—one that integrates simulation fidelity, network redundancy, service quality, and socio-demographic vulnerability. The methodologies and findings presented here have direct implications for both academic research and practical transit planning, offering a roadmap for building more equitable, robust, and adaptive transportation systems in the face of growing urban and environmental uncertainties.

9.2 Limitations

Despite the valuable contributions of this study to the field of public transportation vulnerability analysis, several limitations must be acknowledged, particularly with regard to data availability, modeling assumptions, and simulation tools.

9.2.1 Data availability

A key limitation of this research lies in the restricted availability of comprehensive data. In particular, the absence of alighting data constrained the ability to fully analyze station-level passenger flows. In Montreal, the OPUS card system only records entries into the network, meaning that only boarding information is available. This limitation reduces the granularity and accuracy of demand estimation at individual stations, especially at key transfer points.

Additionally, the study relied on the most recent available Origin-Destination (OD) survey from 2018, which reflects pre-pandemic travel behavior. Given the substantial shifts in mobility patterns during and after the COVID-19 pandemic, this dataset may not accurately represent current trends. Moreover, the OD data pertains exclusively to weekday travel, thereby excluding potentially significant differences in weekend or holiday travel patterns. The lack of seasonal differentiation is another constraint, as weather conditions can significantly affect travel behavior and the viability of alternative modes such as walking and cycling.

9.2.2 Modal scope and network coverage

This study focused solely on metro disruptions and did not consider failures in other parts of the transit network, such as bus or regional rail systems. The exclusion of these components limits the comprehensiveness of the vulnerability assessment and may underestimate the interconnected effects of multimodal disruptions.

Furthermore, while the proposed framework is adaptable to various transit services, only conventional metro and regular bus modes were included in the analysis. Other services such as bus rapid transit (BRT), regional trains (EXO), and automated light rail systems (REM) were omitted, potentially overlooking important elements of the broader network.

9.2.3 Disruption modeling assumptions

All disruption scenarios in this research were modeled as full station closures, affecting both passenger access and train movement. However, this assumption does not reflect all real-world disruptions. In practice, many incidents only affect boarding and alighting while still permitting train passage. This simplification may lead to an overestimation of network degradation and passenger impact.

Additionally, disruptions were treated uniformly across all lines intersecting at a station. In reality, disruptions are often localized and may affect only the infrastructure of a specific line while others remain operational. The inability to model line-specific or link-based failures is a further limitation in terms of representing realistic failure scenarios..

9.2.4 Limitations of simulation platform

The Transition platform used in this study, while effective for simulating normal transit trips, is not specifically designed for modeling disruption scenarios. As a result, implementing and managing simulation runs—particularly under disrupted conditions—was time-consuming and limited the number of scenarios that could be practically analyzed. This technical constraint may affect the representativeness and robustness of the findings.

Furthermore, a key limitation of this study lies in the constraints of the trip simulation tool, which was unable to incorporate the capacity of the bus system. Consequently, the analysis was conducted under the simplifying assumption that the bus network always has sufficient capacity to accommodate all passengers, including those displaced from the metro system during disruptions, regardless of the time of day. While this assumption facilitated the modeling process, it does not fully reflect real-world conditions, particularly during peak hours when bus services frequently operate at or near full capacity. In practice, this can result in significantly longer waiting times and additional delays for passengers attempting to board overcrowded buses. This limitation should be taken into account when interpreting the findings, as the actual impact of metro disruptions may be more severe than indicated by the simulation results.

9.2.5 Behavioral assumptions in trip simulation

Several assumptions were made regarding passenger behavior that may not reflect actual decision-making. It was assumed that all passengers select the fastest route under both normal and disrupted conditions. In reality, factors such as walking distance, weather conditions, personal preferences, and familiarity with the network play important roles. Furthermore, it was assumed that displaced passengers would use regular bus services, and that these services had unlimited capacity to absorb the increased demand—an assumption that is unlikely to hold during peak hours. These simplifications may lead to an underestimation of delays and passenger dissatisfaction during disruption events.

9.3 Future research directions

Building upon the limitations identified in this study, several avenues for future research are proposed to enhance the depth, accuracy, and applicability of transit vulnerability analyses. These directions aim to refine methodological assumptions, integrate more diverse data sources and network components, and improve the behavioral realism and technical capacity of simulation frameworks. Advancing these areas will contribute to the development of more resilient and user-centered public transportation systems.

9.3.1 Improved data sources and granularity

Future studies should incorporate more detailed datasets, including both boarding and alighting volumes, which would improve the accuracy of passenger flow modeling. If such data become available through enhanced smart card technologies or sensor-based systems, more precise station-level and segment-level analysis can be conducted. In the absence of complete alighting data, destination imputation algorithms can be employed to estimate origin-destination (OD) pairs, allowing for the reconstruction of passenger travel patterns from partial information. Integrating such algorithms with existing datasets would enhance the granularity of the analysis and enable more robust modeling of passenger distribution and network resilience.

Additionally, future work should utilize post-pandemic OD data to reflect recent changes in travel behavior and public transport usage. A comparative analysis between pre- and post-pandemic datasets would offer valuable insights into the evolving resilience of transit systems. Expanding the temporal scope to include weekends and separating summer and winter trips would further enrich the understanding of temporal and seasonal variability in system performance, particularly regarding alternative mode viability.

9.3.2 Broader network and modal integration

Subsequent research could expand the network model to include other forms of public transportation such as BRT, light rail, regional rail (e.g., EXO), and newly operational systems like REM. This extension would enable more comprehensive resilience assessments of multimodal transit systems.

Incorporating emerging mobility services—including bike-sharing, car-sharing, and on-demand transport—would also allow researchers to evaluate how users adapt to disruptions with flexible

and decentralized alternatives. These additions would provide a more realistic picture of urban mobility under stress conditions and support more integrated planning solutions.

9.3.3 Differentiated disruption modeling

Future studies should distinguish between different types and severities of disruptions. Modeling partial station closures (e.g., boarding/alighting restrictions without train movement interruption) would reflect more realistic conditions. Similarly, line-specific or link-based disruptions should be considered, particularly in multi-line hubs where failures may affect only part of the infrastructure. This level of detail would allow for more targeted and accurate vulnerability assessments.

9.3.4 Expansion of disruption scenario coverage

Future research would benefit from simulating a larger and more diverse set of disruption scenarios, encompassing variations in location, severity, timing, and network topology. Increasing the number of analyzed scenarios would strengthen the statistical robustness of the findings, support more comprehensive sensitivity analyses, and enable the identification of broader system-level patterns in vulnerability and resilience. While this study demonstrated the adaptability of the Transition platform in modeling disruptions, future efforts could focus on automating or streamlining the scenario generation process—within or alongside existing tools—to efficiently explore a wider range of failure conditions.

9.3.5 Empirical study of user behavior

Future research should explore real-world traveler responses to disruptions through empirical methods such as surveys, interviews, or GPS-based behavior tracking. Investigating how passengers choose modes and routes during disruptions—especially under different weather and time-of-day conditions—would help validate simulation assumptions. Such research is particularly needed in the Montreal context, where user behavior during service interruptions remains underexplored.

9.3.6 Capacity analysis of alternative modes

It is also recommended that future studies examine the capacity constraints of bus and other alternative services in absorbing displaced passengers. Modeling how increased demand during peak hours affects waiting times, service reliability, and user satisfaction would allow transit agencies to better anticipate bottlenecks and improve mitigation strategies.

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