

Online Appendix for Branch-Price-and-Cut for the Electric Vehicle Routing Problem with Heterogeneous Recharging Technologies and Nonlinear Recharging Functions

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Appendix A: Proof of Theorem 1

As given in (8), function $f_k(y)$ writes:

$$f_k(y) = \min_{y_0 \in [0, y]} \{f_k^{\text{TW}}(y_0) + h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_0)\}. \quad (48)$$

Let us express the same function at $y + \Delta$ for some $\Delta > 0$:

$$f_k(y + \Delta) = \min_{y_0 \in [0, y + \Delta]} \{f_k^{\text{TW}}(y_0) + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y_0)\} \quad (49)$$

$$= \min \left\{ \min_{y_0 \in [0, y]} \{f_k^{\text{TW}}(y_0) + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y_0)\}, \right. \quad (50)$$

$$\left. \min_{x \in [0, \Delta]} \{f_k^{\text{TW}}(y + x) + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y + x)\} \right\}$$

$$= \min \left\{ \min_{y_0 \in [0, y]} \{f_k^{\text{TW}}(y_0) + h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_0)\} + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y), \right. \quad (51)$$

$$\left. \min_{x \in [0, \Delta]} \{f_k^{\text{TW}}(y + x) + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y + x)\} \right\}$$

$$= \min \left\{ f_k(y) + h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y), \right. \quad (52)$$

$$\left. h_{i_k}^{-1}(y + \Delta) + \min_{x \in [0, \Delta]} \{f_k^{\text{TW}}(y + x) - h_{i_k}^{-1}(y + x)\} \right\}.$$

Subtracting $f_k(y)$ on both sides and dividing by Δ , we obtain:

$$\frac{f_k(y + \Delta) - f_k(y)}{\Delta} = \min \left\{ \frac{h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y)}{\Delta}, \right. \quad (53)$$

$$\left. \frac{h_{i_k}^{-1}(y + \Delta)}{\Delta} - \frac{f_k(y)}{\Delta} + \min_{x \in [0, \Delta]} \left\{ \frac{f_k^{\text{TW}}(y + x)}{\Delta} - \frac{h_{i_k}^{-1}(y + x)}{\Delta} \right\} \right\}$$

$$= \min \left\{ \frac{h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y)}{\Delta}, \right. \quad (54)$$

$$\left. \frac{h_{i_k}^{-1}(y + \Delta) - h_{i_k}^{-1}(y)}{\Delta} + \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta} + \min_{x \in [0, \Delta]} \left\{ \frac{f_k^{\text{TW}}(y + x) - f_k^{\text{TW}}(y)}{\Delta} - \frac{h_{i_k}^{-1}(y + x) - h_{i_k}^{-1}(y)}{\Delta} \right\} \right\}$$

$$= \min \left\{ \frac{h_{i_k}^{-1}(y+\Delta) - h_{i_k}^{-1}(y)}{\Delta}, \frac{h_{i_k}^{-1}(y+\Delta) - h_{i_k}^{-1}(y)}{\Delta} + \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta} + \min_{x \in [0, \Delta]} \left\{ \frac{f_k^{\text{TW}}(y+x) - f_k^{\text{TW}}(y)}{x} \frac{x}{\Delta} - \frac{h_{i_k}^{-1}(y+x) - h_{i_k}^{-1}(y)}{x} \frac{x}{\Delta} \right\} \right\}. \quad (55)$$

Now, let $\Delta \rightarrow 0^+$ which also implies $x \rightarrow 0^+$ since $x \leq \Delta$. Then, by definition:

$$\lim_{\Delta \rightarrow 0^+} \frac{f_k(y+\Delta) - f_k(y)}{\Delta} = \frac{df_k(y)}{dy^+}, \quad (56)$$

$$\lim_{\Delta \rightarrow 0^+} \frac{h_{i_k}^{-1}(y+\Delta) - h_{i_k}^{-1}(y)}{\Delta} = \frac{dh_{i_k}^{-1}(y)}{dy^+}, \quad (57)$$

$$\lim_{x \rightarrow 0^+} \frac{h_{i_k}^{-1}(y+x) - h_{i_k}^{-1}(y)}{x} = \frac{dh_{i_k}^{-1}(y)}{dy^+}, \quad (58)$$

$$\lim_{x \rightarrow 0^+} \frac{f_k^{\text{TW}}(y+x) - f_k^{\text{TW}}(y)}{x} = \frac{df_k^{\text{TW}}(y)}{dy^+} \quad (59)$$

$$\lim_{\Delta \rightarrow 0^+} \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta} = \begin{cases} 0 & \text{if } f_k^{\text{TW}}(y) = f_k(y), \\ \infty & \text{otherwise,} \end{cases} \quad (60)$$

and we get:

$$\frac{df_k(y)}{dy^+} = \min \left\{ \frac{dh_{i_k}^{-1}(y)}{dy^+}, \frac{dh_{i_k}^{-1}(y)}{dy^+} + \lim_{\Delta \rightarrow 0^+} \left(\frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta} + \min_{x \in [0, \Delta]} \left\{ \frac{df_k^{\text{TW}}(y)}{dy^+} \frac{x}{\Delta} - \frac{dh_{i_k}^{-1}(y)}{dy^+} \frac{x}{\Delta} \right\} \right) \right\}. \quad (61)$$

Note that both $\frac{df_k^{\text{TW}}(y)}{dy^+}$ and $\frac{dh_{i_k}^{-1}(y)}{dy^+}$ are non-negative constants in the minimization over x , so that the optimal solution value of x is either 0 or Δ . Then we can split the second case in two, depending on the value of x , giving the minimum over three cases:

$$\frac{df_k(y)}{dy^+} = \min \left\{ \begin{aligned} &\frac{dh_{i_k}^{-1}(y)}{dy^+}, \\ &\frac{dh_{i_k}^{-1}(y)}{dy^+} + \lim_{\Delta \rightarrow 0^+} \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta}, \\ &\frac{dh_{i_k}^{-1}(y)}{dy^+} + \lim_{\Delta \rightarrow 0^+} \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta} + \frac{df_k^{\text{TW}}(y)}{dy^+} - \frac{dh_{i_k}^{-1}(y)}{dy^+} \end{aligned} \right\} \quad (62)$$

$$= \min \left\{ \begin{aligned} &\frac{dh_{i_k}^{-1}(y)}{dy^+}, \\ &\frac{dh_{i_k}^{-1}(y)}{dy^+} + \lim_{\Delta \rightarrow 0^+} \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta}, \\ &\frac{df_k^{\text{TW}}(y)}{dy^+} + \lim_{\Delta \rightarrow 0^+} \frac{f_k^{\text{TW}}(y) - f_k(y)}{\Delta}. \end{aligned} \right\} \quad (63)$$

Note that $f_k^{\text{TW}}(y) \geq f_k(y)$, so the only situation where $\frac{df_k(y)}{dy^+} = \frac{df_k^{\text{TW}}(y)}{dy^+}$ is when $f_k^{\text{TW}}(y) = f_k(y)$ and $\frac{df_k^{\text{TW}}(y)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y)}{dy^+}$. Thus we can re-write the last expression as:

$$\frac{df_k(y)}{dy^+} = \begin{cases} \frac{df_k^{\text{TW}}(y)}{dy^+} & \text{if } f_k^{\text{TW}}(y) = f_k(y), \frac{df_k^{\text{TW}}(y)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y)}{dy^+} \\ \frac{dh_{i_k}^{-1}(y)}{dy^+} & \text{otherwise.} \end{cases} \quad (64)$$

Let y_0 be a point where $f_k^{\text{TW}}(y_0) = f_k(y_0)$ and $\frac{df_k^{\text{TW}}(y_0)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y_0)}{dy^+}$. Further, let y_1 be the minimum between the maximal SoC B and the largest value larger than y_0 so that $\frac{df_k^{\text{TW}}(y)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y)}{dy^+}, \forall y \in [y_0, y_1]$, i.e., $y_1 = \min \{B, \inf \{y' | y' > y_0, \frac{df_k^{\text{TW}}(y')}{dy^+} > \frac{dh_{i_k}^{-1}(y')}{dy^+}\}\}$. Then, from (64), we can define $f_k(y)$ as follows:

$$\int_{y_0}^y \frac{df_k(v)}{dv^+} dv = \int_{y_0}^y \frac{df_k^{\text{TW}}(v)}{dv^+} dv \quad (65)$$

$$\Rightarrow f_k(y) - f_k(y_0) = f_k^{\text{TW}}(y) - f_k^{\text{TW}}(y_0) \quad (66)$$

$$\Rightarrow f_k(y) = f_k^{\text{TW}}(y), \quad \forall y \in [y_0, y_1]. \quad (67)$$

Conversely, let y_0 be a point such that $f_k^{\text{TW}}(y_0) > f_k(y_0)$ or $\frac{df_k^{\text{TW}}(y_0)}{dy^+} > \frac{dh_{i_k}^{-1}(y_0)}{dy^+}$, and let y_1 be the minimum between the maximal SoC B and the least value y' larger than y_0 for which $f_k^{\text{TW}}(y') = f_k(y')$ and $\frac{df_k^{\text{TW}}(y_0)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y_0)}{dy^+}$, i.e., $y_1 = \min \{B, \min\{y' | y' > y_0, f_k^{\text{TW}}(y') = f_k(y'), \frac{df_k^{\text{TW}}(y_0)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y_0)}{dy^+}\}\}$. Note that $f_k^{\text{TW}}(y) \geq f_k(y)$, $\forall y \in [y_0, y_1]$, by the definition of y_1 and the assumption that $f_k^{\text{TW}}(y_0) > f_k(y_0)$ or $\frac{df_k^{\text{TW}}(y_0)}{dy^+} > \frac{dh_{i_k}^{-1}(y_0)}{dy^+}$. Then, from (64), $f_k(y)$ can be derived as follows:

$$\int_{y_0}^y \frac{df_k(v)}{dv^+} dv = \int_{y_0}^y \frac{dh_{i_k}^{-1}(v)}{dv^+} dv \quad (68)$$

$$\Rightarrow f_k(y) - f_k(y_0) = h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_0) \quad (69)$$

$$\Rightarrow f_k(y) = f_k(y_0) + h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_0), \quad \forall y \in [y_0, y_1]. \quad (70)$$

Since we now have an expression for $f_k(y)$, we can rewrite the definition of y_1 as

$$y_1 = \min \left\{ B, \min\{y' | y' > y_0, f_k^{\text{TW}}(y') = f_k(y_0) + h_{i_k}^{-1}(y') - h_{i_k}^{-1}(y_0), \frac{df_k^{\text{TW}}(y_0)}{dy^+} \leq \frac{dh_{i_k}^{-1}(y_0)}{dy^+}\} \right\}.$$

□

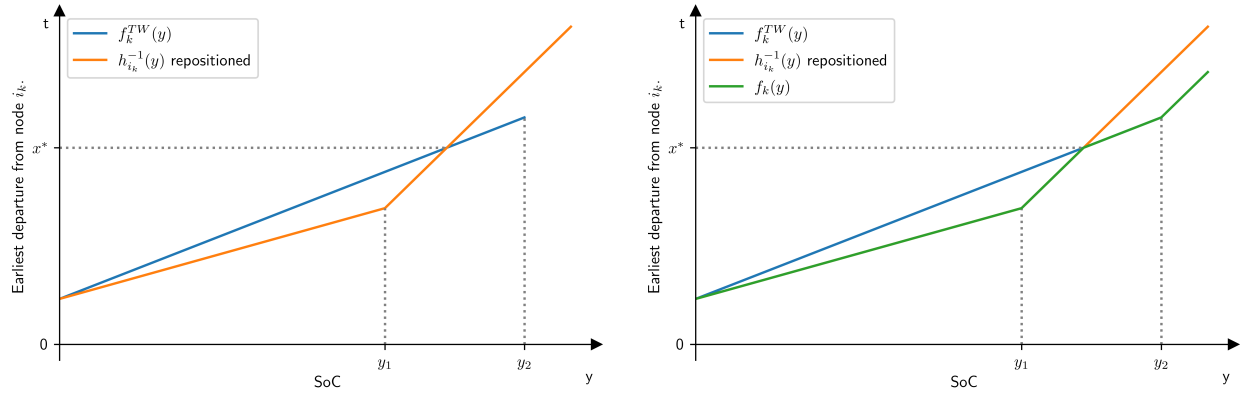


Figure 11 Illustration of case 3 with and without $f_k(y)$.

Appendix B: Charging procedure for piecewise linear recharging functions in forward recursion

This section presents a more detailed version of Algorithm 1 for the special case where all recharging functions are piecewise linear. Let $K(h_{i_k}, f_k^{TW})$ be the set of y -values where h_{i_k} or f_k^{TW} has a break or end point. Assume that y_1 and y_2 are two break points such that $y_1 < y_2$ and $(y_1, y_2) \cap K(h_{i_k}, f_k^{TW}) = \emptyset$, i.e., there are no break points between them. We use Theorem 1 to express the following cases for $f_k(y)$:

$$\begin{aligned} \text{Case 1: } \frac{dh_{i_k}^{-1}(y_1)}{dy^+} &\leq \frac{df_k^{TW}(y_1)}{dy^+} \\ \Rightarrow f_k(x) &= f_k(y_1) + \frac{dh_{i_k}^{-1}(y_1)}{dy^+}(x - y_1), \forall x \in [y_1, y_2], \end{aligned} \quad (71)$$

$$\begin{aligned} \text{Case 2: } \left(\frac{dh_{i_k}^{-1}(y_1)}{dy^+} > \frac{df_k^{TW}(y_1)}{dy^+} \right), (f_k(y_1) &= f_k^{TW}(y_1)) \\ \Rightarrow f_k(x) &= f_k(y_1) + \frac{df_k^{TW}(y_1)}{dy^+}(x - y_1), \forall x \in [y_1, y_2], \end{aligned} \quad (72)$$

$$\begin{aligned} \text{Case 3: } \left(\frac{dh_{i_k}^{-1}(y_1)}{dy^+} > \frac{df_k^{TW}(y_1)}{dy^+} \right), (f_k(y_1) < f_k^{TW}(y_1)) \\ \Rightarrow \begin{cases} f_k(x) = f_k(y_1) + \frac{dh_{i_k}^{-1}(y_1)}{dy^+}(x - y_1), \forall x \in [y_1, x^*], \\ f_k(x) = f_k^{TW}(y_1) + \frac{df_k^{TW}(y_1)}{dy^+}(x - y_1), \forall x \in [x^*, y_2], \end{cases} \\ \text{where } x^* = y_1 + \min \left\{ y_2 - y_1, \frac{f_k^{TW}(y_1) - f_k(y_1)}{\frac{dh_{i_k}^{-1}(y_1)}{dy^+} - \frac{df_k^{TW}(y_1)}{dy^+}} \right\}. \end{aligned} \quad (73)$$

An illustration of the third case is given in Figure 11, where $h_{i_k}^{-1}$ has been positioned to coincide with f_k^{TW} at $y = 0$. In this example, $f_k(y) = f_k(0) + h_{i_k}^{-1}(y), \forall y \in [0, y_1]$. The break point at y_1 leads to case 3, as $\frac{dh_{i_k}^{-1}(y_1)}{dy^+} > \frac{df_k^{TW}(y_1)}{dy^+}$ and $f_k(y_1) < f_k^{TW}(y_1)$.

In Algorithm 3, we exploit the three cases stated above to find $f_k(y), \forall y \in [0, B]$, by evaluating $f_k(y)$ only at the break points $K(h_{i_k}, f_k^{TW})$. The algorithm starts at the break point with the least SoC value, then uses the conditions in the cases above, to calculate the SoC value where the next break point will be and how to calculate $f_k(y)$ for that value. The algorithm moves from break point to break point until reaching the largest

value in $K(h_{i_k}, f_k^{\text{TW}})$, which is always equal to B . If we assume that the points in $K(h_{i_k}, f_k^{\text{TW}})$ are already sorted in increasing order of their SoC values, the time complexity of this algorithm is $O(|K(h_{i_k}, f_k^{\text{TW}})|)$, i.e. linear in the number of total break points in the two functions.

Algorithm 3 Compute f_k

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1: function CHARGE( $f_k^{\text{TW}}, h_{i_k}$ )
2:    $Y_{\text{values}} = K(h_{i_k}, f_k^{\text{TW}})$   $\triangleright$  ordered vector of increasing  $y$ -values where  $h_{i_k}$  or  $f_k^{\text{TW}}$  has a
   break or end point
3:    $y_1 = 0$ 
4:    $f_{\text{current}} = f_k^{\text{TW}}(y_1)$ 
5:    $\text{Result} = [(f_{\text{current}}, y_1)]$   $\triangleright$  vector of (time, SoC) points
6:   for  $i = 1, \dots, |Y_{\text{values}}| - 1$  do
7:      $y_2 = Y_{\text{values}}[i]$ 
8:     if  $\frac{df_k^{\text{TW}}(y_1)}{dy^+} \geq \frac{dh_{i_k}^{-1}(y_1)}{dy^+}$  then  $\triangleright$  (Case 1)
9:        $f_{\text{current}} = f_{\text{current}} + \frac{dh_{i_k}^{-1}(y_1)}{dy^+}(y_2 - y_1)$ 
10:    else if  $\frac{df_k^{\text{TW}}(y_1)}{dy^+} < \frac{dh_{i_k}^{-1}(y_1)}{dy^+}$ ,  $f_{\text{current}} = f_k^{\text{TW}}(y_1)$  then  $\triangleright$  (Case 2)
11:       $f_{\text{current}} = f_{\text{current}} + \frac{df_k^{\text{TW}}(y_1)}{dy^+}(y_2 - y_1)$ 
12:    else  $\triangleright$  (Case 3)
13:       $x^* = y_1 + (f_k^{\text{TW}}(y_1) - f_{\text{current}}) / (\frac{dh_{i_k}^{-1}(y_1)}{dy^+} - \frac{df_k^{\text{TW}}(y_1)}{dy^+})$ 
14:      if  $y_2 > x^*$  then
15:         $f_{\text{current}} = f_{\text{current}} + \frac{dh_{i_k}^{-1}(y_1)}{dy^+}(x^* - y_1)$ 
16:         $\text{Result.append}((f_{\text{current}}, x^*))$ 
17:         $f_{\text{current}} = f_{\text{current}} + \frac{df_k^{\text{TW}}(y_1)}{dy^+}(y_2 - x^*)$ 
18:      else
19:         $f_{\text{current}} = f_{\text{current}} + \frac{dh_{i_k}^{-1}(y_1)}{dy^+}(y_2 - y_1)$ 
20:       $y_1 = y_2$ 
21:       $\text{Result.append}((f_{\text{current}}, y_1))$ 
22:   return Result

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Appendix C: Proof of Theorem 2

Function $w_k^{\text{Charge}}(y)$, as given in (13), can be expressed as:

$$w_k^{\text{Charge}}(y) = \max_{y_1 \in [y, B]} \{w_k(y_1) - (h_{i_k}^{-1}(y_1) - h_{i_k}^{-1}(y))\}. \quad (74)$$

Let us express the same function at $y - \Delta$ for some $\Delta > 0$:

$$w_k^{\text{Charge}}(y - \Delta) = \max_{y_1 \in [y - \Delta, B]} \{w_k(y_1) - h_{i_k}^{-1}(y_1) + h_{i_k}^{-1}(y - \Delta)\} \quad (75)$$

$$= \max \left\{ \max_{y_1 \in [y - \Delta, y]} \{w_k(y_1) - h_{i_k}^{-1}(y_1) + h_{i_k}^{-1}(y - \Delta)\}, \right. \\ \left. \max_{y_1 \in [y, B]} \{w_k(y_1) - h_{i_k}^{-1}(y_1) + h_{i_k}^{-1}(y - \Delta)\} \right\} \quad (76)$$

$$= \max \left\{ \max_{x \in [0, \Delta]} \{w_k(y - x) - h_{i_k}^{-1}(y - x) + h_{i_k}^{-1}(y - \Delta)\}, \right. \\ \left. \max_{y_1 \in [y, B]} \{w_k(y_1) - h_{i_k}^{-1}(y_1) + h_{i_k}^{-1}(y)\} - h_{i_k}^{-1}(y) + h_{i_k}^{-1}(y - \Delta) \right\} \quad (77)$$

$$= \max \left\{ \max_{x \in [0, \Delta]} \{w_k(y - x) - h_{i_k}^{-1}(y - x) + h_{i_k}^{-1}(y - \Delta)\}, \right. \\ \left. w_k^{\text{Charge}}(y) - h_{i_k}^{-1}(y) + h_{i_k}^{-1}(y - \Delta) \right\}. \quad (78)$$

Subtracting $w_k^{\text{Charge}}(y)$ on both sides and dividing by Δ , we obtain:

$$\frac{w_k^{\text{Charge}}(y - \Delta) - w_k^{\text{Charge}}(y)}{\Delta} \quad (79)$$

$$= \max \left\{ \frac{w_k(y) - w_k^{\text{Charge}}(y)}{\Delta} + \frac{h_{i_k}^{-1}(y - \Delta) - h_{i_k}^{-1}(y)}{\Delta} + \max_{x \in [0, \Delta]} \left\{ \frac{w_k(y - x) - w_k(y)}{\Delta} - \frac{h_{i_k}^{-1}(y - x) - h_{i_k}^{-1}(y)}{\Delta} \right\} \right. \\ \left. \frac{h_{i_k}^{-1}(y - \Delta) - h_{i_k}^{-1}(y)}{\Delta} \right\} \quad (80)$$

$$= \max \left\{ \frac{w_k(y) - w_k^{\text{Charge}}(y)}{\Delta} + \frac{h_{i_k}^{-1}(y - \Delta) - h_{i_k}^{-1}(y)}{\Delta} + \max_{x \in [0, \Delta]} \left\{ \frac{w_k(y - x) - w_k(y)}{x} \frac{x}{\Delta} - \frac{h_{i_k}^{-1}(y - x) - h_{i_k}^{-1}(y)}{x} \frac{x}{\Delta} \right\} \right. \\ \left. \frac{h_{i_k}^{-1}(y - \Delta) - h_{i_k}^{-1}(y)}{\Delta} \right\}. \quad (81)$$

Now let $\Delta \rightarrow 0^+$, which also implies $x \rightarrow 0^+$ since $x \leq \Delta$. Then, by definition:

$$\lim_{\Delta \rightarrow 0^+} \frac{w_k^{\text{Charge}}(y - \Delta) - w_k^{\text{Charge}}(y)}{\Delta} = -\frac{dw_k^{\text{Charge}}(y)}{dy^-} \quad (82)$$

$$\lim_{\Delta \rightarrow 0^+} \frac{h_{i_k}^{-1}(y - \Delta) - h_{i_k}^{-1}(y)}{\Delta} = -\frac{dh_{i_k}^{-1}(y)}{dy^-}, \quad (83)$$

$$\lim_{x \rightarrow 0^+} \frac{h_{i_k}^{-1}(y - x) - h_{i_k}^{-1}(y)}{x} = -\frac{dh_{i_k}^{-1}(y)}{dy^-}, \quad (84)$$

$$\lim_{x \rightarrow 0^+} \frac{w_k(y - x) - w_k(y)}{x} = -\frac{dw_k(y)}{dy^-} \quad (85)$$

$$\lim_{\Delta \rightarrow 0^+} \frac{w_k(y) - w_k^{\text{Charge}}(y)}{\Delta} = \begin{cases} 0 & \text{if } w_k(y) = w_k^{\text{Charge}}(y) \\ -\infty & \text{otherwise,} \end{cases} \quad (86)$$

and we get (using arguments similar to those used in the proof of Theorem 1):

$$-\frac{dw_k^{\text{Charge}}(y)}{dy^-} = \max \left\{ -\frac{dh_{i_k}^{-1}(y)}{dy^-}, \right. \\ \left. \lim_{\Delta \rightarrow 0^+} \left(\frac{w_k(y) - w_k^{\text{Charge}}(y)}{\Delta} - \frac{dh_{i_k}^{-1}(y)}{dy^-} + \max_{x \in [0, \Delta]} \left\{ -\frac{dw_k(y)}{dy^-} \frac{x}{\Delta} + \frac{dh_{i_k}^{-1}(y)}{dy^-} \frac{x}{\Delta} \right\} \right) \right\} \quad (87)$$

$$= \max \left\{ -\frac{dh_{i_k}^{-1}(y)}{dy^-}, \right. \\ \left. \lim_{\Delta \rightarrow 0^+} \frac{w_k(y) - w_k^{\text{Charge}}(y)}{\Delta} - \min \left\{ \frac{dw_k(y)}{dy^-}, \frac{dh_{i_k}^{-1}(y)}{dy^-} \right\} \right\} \quad (88)$$

$$= \begin{cases} -\frac{dw_k(y)}{dy^-} & \text{if } w_k(y) = w_k^{\text{Charge}}(y), \frac{dw_k(y)}{dy^-} \leq \frac{dh_{i_k}^{-1}(y)}{dy^-} \\ -\frac{dh_{i_k}^{-1}(y)}{dy^-} & \text{otherwise.} \end{cases} \quad (89)$$

Let y_1 be a point where $w_k(y_1) = w_k^{\text{Charge}}(y_1)$ and $\frac{dw_k(y_1)}{dy^-} \leq \frac{dh_{i_k}^{-1}(y_1)}{dy^-}$. Further let y_0 be the maximum between 0 and the least value less than y_1 for which $\frac{dw_k(y)}{dy^-} \leq \frac{dh_{i_k}^{-1}(y)}{dy^-}$, $\forall y \in [y_0, y_1]$, i.e., $y_0 = \max \{0, \sup \{y' | y' < y_1, \frac{dw_k(y')}{dy^-} > \frac{dh_{i_k}^{-1}(y')}{dy^-}\}$. Then, from (89), we can define $w_k^{\text{Charge}}(y)$ as follows:

$$\int_y^{y_1} -\frac{dw_k^{\text{Charge}}(v)}{dv^-} dv = \int_y^{y_1} -\frac{dw_k(v)}{dv^-} dv \quad (90)$$

$$\Rightarrow -w_k^{\text{Charge}}(y_1) + w_k^{\text{Charge}}(y) = -w_k(y_1) + w_k(y) \quad (91)$$

$$\Rightarrow w_k^{\text{Charge}}(y) = w_k(y), \quad \forall y \in [y_0, y_1]. \quad (92)$$

Conversely, let y_1 be a point such that $w_k(y_1) > w_k^{\text{Charge}}(y_1)$ or $\frac{dw_k(y_1)}{dy^-} > \frac{dh_{i_k}^{-1}(y_1)}{dy^-}$, and let y_0 be the maximum between 0 and the largest value less than y_1 for which $w_k(y_0) = w_k^{\text{Charge}}(y_0)$ and $\frac{dw_k(y')}{dy^-} \leq \frac{dh_{i_k}^{-1}(y')}{dy^-}$, i.e., $y_0 = \max \{0, \max \{y' | y' < y_1, w_k(y') = w_k^{\text{Charge}}(y'), \frac{dw_k(y')}{dy^-} \leq \frac{dh_{i_k}^{-1}(y')}{dy^-}\}$. Note that $w_k^{\text{Charge}}(y) \leq w_k(y)$, $\forall y \in [y_0, y_1]$, by the definition of y_0 and the assumption that $w_k(y_1) > w_k^{\text{Charge}}(y_1)$ or $\frac{dw_k(y_1)}{dy^-} > \frac{dh_{i_k}^{-1}(y_1)}{dy^-}$. Then, from (89), $w_k^{\text{Charge}}(y)$ can be derived as follows:

$$\int_y^{y_1} -\frac{dw_k^{\text{Charge}}(v)}{dv^-} dv = \int_y^{y_1} -\frac{dh_{i_k}^{-1}(v)}{dv^-} dv \quad (93)$$

$$\Rightarrow w_k^{\text{Charge}}(y) - w_k^{\text{Charge}}(y_1) = h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_1) \quad (94)$$

$$\Rightarrow w_k^{\text{Charge}}(y) = w_k^{\text{Charge}}(y_1) + h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_1) \quad (95)$$

$$\Rightarrow w_k^{\text{Charge}}(y) = w_k^{\text{Charge}}(y_1) + h_{i_k}^{-1}(y) - h_{i_k}^{-1}(y_1), \quad \forall y \in [y_0, y_1]. \quad (96)$$

Since we now have an expression for $w_k^{\text{Charge}}(y)$, we can rewrite the definition of y_0 as $y_0 = \max \{0, \max \{y' | y' < y_1, w_k(y') = w_k^{\text{Charge}}(y_1) + h_{i_k}^{-1}(y') - h_{i_k}^{-1}(y_1), \frac{dw_k(y')}{dy^-} \leq \frac{dh_{i_k}^{-1}(y')}{dy^-}\}$.

□

Appendix D: Charging procedure for piecewise linear recharging functions in backward recursion

This section presents a more detailed version of Algorithm 2 for the special case where all recharging functions are piecewise linear. Assume that y_1 and y_2 are two break points such that $y_1 < y_2$ and $(y_1, y_2) \cap K(h_{i_k}, w_k) = \emptyset$, i.e., there are no break points between them. We use Theorem 2 to express the following cases for $w_k^{\text{Charge}}(y)$:

$$\begin{aligned} \text{Case 1: } \frac{dh_{i_k}^{-1}(y_2)}{dy^-} &\leq \frac{dw_k(y_2)}{dy^-} \\ \Rightarrow w_k^{\text{Charge}}(x) &= w_k^{\text{Charge}}(y_2) - \frac{dh_{i_k}^{-1}(y_2)}{dy^-}(y_2 - x), \forall x \in [y_1, y_2], \end{aligned} \quad (97)$$

$$\begin{aligned} \text{Case 2: } \left(\frac{dh_{i_k}^{-1}(y_2)}{dy^-} > \frac{dw_k(y_2)}{dy^-} \right), (w_k^{\text{Charge}}(y_2) &= w_k(y_2)) \\ \Rightarrow w_k^{\text{Charge}}(x) &= w_k^{\text{Charge}}(y_2) - \frac{dw_k(y_2)}{dy^-}(y_2 - x), \forall x \in [y_1, y_2], \end{aligned} \quad (98)$$

$$\begin{aligned} \text{Case 3: } \left(\frac{dh_{i_k}^{-1}(y_2)}{dy^-} > \frac{dw_k(y_2)}{dy^-} \right), (w_k^{\text{Charge}}(y_2) > w_k(y_2)) \\ \Rightarrow \begin{cases} w_k^{\text{Charge}}(x) = w_k^{\text{Charge}}(y_2) - \frac{dh_{i_k}^{-1}(y_2)}{dy^-}(y_2 - x), \forall x \in [y_1, x^*), \\ w_k^{\text{Charge}}(x) = w_k(y_2) - \frac{dw_k(y_2)}{dy^-}(y_2 - x), \forall x \in [x^*, y_2], \end{cases} \end{aligned} \quad (99)$$

where $x^* = y_2 - \max \left\{ y_2 - y_1, \frac{w_k^{\text{Charge}}(y_2) - w_k(y_2)}{\frac{dh_{i_k}^{-1}(y_2)}{dy^-} - \frac{dw_k(y_2)}{dy^-}} \right\}$.

Based on these three cases, we propose Algorithm 4 to compute $w_k^{\text{Charge}}(y)$ for all feasible values of y . This algorithm is similar to Algorithm 3.

Algorithm 4 Compute w_k^{Charge}

```

1: function CHARGE( $w_k, h_{i_k}$ )
2:    $Y_{\text{values}} = K(h_{i_k}, w_k) \triangleright$  ordered vector of decreasing  $y$ -values where  $h_{i_k}$  or  $w_k$  has a break or
   end point
3:    $y_2 = B$ 
4:    $w^{\text{current}} = w_k(y_2)$ 
5:    $\text{Result} = [(w^{\text{current}}, y_2)] \triangleright$  vector of (time, SoC) points
6:   for  $i = |Y_{\text{values}}| - 2, \dots, 0$  do
7:      $y_1 = Y_{\text{values}}[i]$ 
8:     if  $\frac{dw_k(y_2)}{dy^-} \geq \frac{dh_{i_k}^{-1}(y_2)}{dy^-}$  then  $\triangleright$  (Case 1)
9:        $w^{\text{current}} = w^{\text{current}} - \frac{dh_{i_k}^{-1}(y_2)}{dy^-}(y_2 - y_1)$ 
10:    else if  $\frac{dw_k(y_2)}{dy^-} < \frac{dh_{i_k}^{-1}(y_2)}{dy^-}, w^{\text{current}} = w_k(y_2)$  then  $\triangleright$  (Case 2)
11:       $w^{\text{current}} = w^{\text{current}} - \frac{dw_k(y_2)}{dy^-}(y_2 - y_1)$ 
12:    else  $\triangleright$  (Case 3)
13:       $x^* = y_1 - (w^{\text{current}} - w_k(y_2)) / (\frac{dh_{i_k}^{-1}(y_2)}{dy^-} - \frac{dw_k(y_2)}{dy^-})$ 
14:      if  $y_1 < x^*$  then
15:         $w^{\text{current}} = w^{\text{current}} - \frac{dh_{i_k}^{-1}(y_2)}{dy^-}(y_2 - x^*)$ 
16:         $\text{Result.append}((w^{\text{current}}, x^*))$ 
17:         $w^{\text{current}} = w^{\text{current}} - \frac{dw_k(y_2)}{dy^-}(x^* - y_1)$ 
18:      else
19:         $w^{\text{current}} = w^{\text{current}} - \frac{dh_{i_k}^{-1}(y_2)}{dy^-}(y_2 - y_1)$ 
20:       $y_2 = y_1$ 
21:       $\text{Result.append}((w^{\text{current}}, y_2))$ 
22:   return Result

```

Appendix E: Time window preprocessing

Lera-Romero, Miranda Bront, and Soullignac (2024) state that they “consider as input for the TDEVPTW the preprocessed TDVRPTW instances” and refer the readers to Desrosiers et al. (1995) for an explanation of the time window preprocessing method used. Consequently, recharging stations are excluded from consideration during this preprocessing. Figures 12 and 13 illustrate an example network, both with and without the inclusion of a charging node. The number on each arc represents both travel time and energy consumption. Assume that the vehicle’s battery capacity is 14 and the charger’s recharging rate is 1.

The only feasible path through the network is $0 \rightarrow 1 \rightarrow \text{Charger} \rightarrow 2 \rightarrow 3$, where the vehicle charges 4 units at the charger. The corresponding start of service times along this path are 0, 10, 13, 20, and 26, respectively. However, when time window preprocessing is applied to the network without the charger, following the

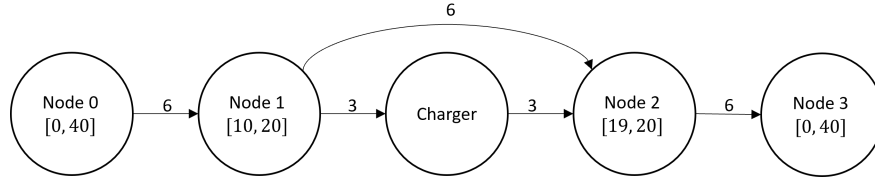


Figure 12 Example network with the charger included.

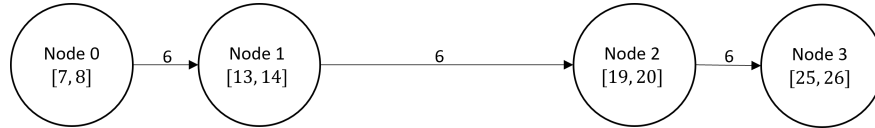


Figure 13 Example network without the charger included, and with invalid preprocessed time windows.

approach described by Lera-Romero, Miranda Bront, and Soullignac (2024), the time windows for nodes 0 through 3 become $[7, 8]$, $[13, 14]$, $[19, 20]$, and $[25, 26]$, respectively. As a result, the previously feasible path is no longer valid because the vehicle would not be able to arrive at node 2 within its preprocessed time window after the necessary recharge. This shows that time window preprocessing for the E-VRPTW is invalid if recharging nodes are not considered.

Appendix F: Detailed results

Tables 8-15 give detailed results for each instance solved. The column *Root* gives the lower bound at the root node of the Branch-and-Bound tree, after applying cuts and decremental state space relaxation. *DualBnd* and *Primal* gives the final dual and primal bounds found, respectively. The computation time in seconds is provided by the column indicated by *Time*, while *Gap* gives the optimality gap in percentage based on the dual and primal bounds. The number of Branch-and-Bound nodes solved completely, meaning there are no improving columns and the cut separation and decremental state space procedure are finished, is given by the column *#Nodes*. This implies that a dual bound might have been found even if the reported number of nodes is zero. The number of subset row cuts added to the RMP is given by the column *#SRC*.

For Tables 11-13, note that the code used by Desaulniers et al. (2016) requires integer values. Hence they preprocess the data in the instances in a way that ensures integer values, while simultaneously ensuring that the triangle inequality is satisfied. Our code does not have this requirement, and hence the preprocessing steps are not performed, which may lead to slight discrepancies in the numerical results.

Table 8 Detailed results for the new E-VRPTW-NL instances with 25 customers.

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	621.40	621.40	621.40	0.70	0.0	1	0
C102	512.70	512.70	512.70	1.47	0.0	1	0
C103	342.50	342.50	342.50	14.17	0.0	1	5
C104	430.00	430.00	430.00	11.00	0.0	1	0
C105	520.00	520.00	520.00	0.74	0.0	1	0
C106	555.00	555.00	555.00	2.13	0.0	1	18
C107	503.60	503.60	503.60	1.66	0.0	1	7
C108	490.40	490.40	490.40	2.17	0.0	1	0
C109	439.14	441.10	441.10	32.62	0.0	3	6
R101	658.60	658.60	658.60	0.43	0.0	1	0
R102	436.10	436.10	436.10	0.67	0.0	1	0
R103	490.90	490.90	490.90	1.60	0.0	1	4
R104	337.70	348.70	348.70	136.95	0.0	9	35
R105	537.50	537.50	537.50	0.66	0.0	1	3
R106	465.70	465.70	465.70	1.43	0.0	1	0
R107	409.90	409.90	409.90	6.00	0.0	1	21
R108	419.60	419.60	419.60	3.29	0.0	1	0
R109	454.82	455.10	455.10	2.52	0.0	3	17
R110	401.30	401.30	401.30	3.49	0.0	1	4
R111	376.70	376.70	376.70	35.30	0.0	1	36
R112	380.60	380.60	380.60	12.07	0.0	1	28
RC101	726.50	726.50	726.50	0.75	0.0	1	12
RC102	631.20	631.20	631.20	0.98	0.0	1	13
RC103	543.70	543.70	543.70	1.90	0.0	1	9
RC104	499.20	499.20	499.20	4.56	0.0	1	7
RC105	588.50	588.50	588.50	0.81	0.0	1	0
RC106	545.00	545.00	545.00	1.26	0.0	1	4
RC107	492.70	492.70	492.70	1.13	0.0	1	0
RC108	454.10	454.10	454.10	2.86	0.0	1	0
C201	415.20	415.20	415.20	1.41	0.0	1	0
C202	403.50	403.50	403.50	9.34	0.0	1	0
C203	390.70	390.70	390.70	27.18	0.0	1	0
C204	360.90	360.90	360.90	23.31	0.0	1	0
C205	407.80	407.80	407.80	1.95	0.0	1	0
C206	387.80	387.80	387.80	9.54	0.0	1	0
C207	396.20	396.20	396.20	4.20	0.0	1	0
C208	393.40	393.40	393.40	7.23	0.0	1	0
R201	372.90	372.90	372.90	1.85	0.0	1	0
R202	374.10	374.10	374.10	3.62	0.0	1	0
R203	381.60	381.60	381.60	25.86	0.0	1	17
R204	314.20	314.20	314.20	40.89	0.0	1	0
R205	372.30	372.30	372.30	4.83	0.0	1	0
R206	423.10	423.10	423.10	16.97	0.0	1	4
R207	329.60	329.60	329.60	42.23	0.0	1	0
R208	314.30	314.30	314.30	592.87	0.0	1	0
R209	421.10	421.10	421.10	15.87	0.0	1	19
R210	314.10	314.10	314.10	23.82	0.0	1	0
R211	382.40	382.40	382.40	43.73	0.0	1	5
RC201	593.60	593.60	593.60	2.62	0.0	1	0
RC202	539.40	539.40	539.40	5.17	0.0	1	0
RC203	442.00	442.00	442.00	65.95	0.0	1	29
RC204	444.20	444.20	444.20	44.96	0.0	1	5
RC205	535.20	535.20	535.20	6.65	0.0	1	8
RC206	587.90	587.90	587.90	3.65	0.0	1	0
RC207	484.70	484.70	484.70	21.99	0.0	1	5
RC208	437.20	437.20	437.20	31.01	0.0	1	0

Table 9 Detailed results for the new E-VRPTW-NL instances with 50 customers.

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	769.50	769.50	769.50	2.79	0.0	1	0
C102	772.50	772.50	772.50	12.75	0.0	1	0
C103	628.06	628.07	2748.80	3600.00	77.2	2	55
C104	553.00	553.00	553.00	252.24	0.0	1	0
C105	727.60	727.60	727.60	3.46	0.0	1	0
C106	730.80	730.80	730.80	5.49	0.0	1	0
C107	701.30	701.30	701.30	5.42	0.0	1	0
C108	713.70	713.70	713.70	6.73	0.0	1	1
C109	654.50	654.50	654.50	18.08	0.0	1	13
R101	919.90	919.90	919.90	1.24	0.0	1	2
R102	819.00	819.00	819.00	3.79	0.0	1	0
R103	756.00	756.00	756.00	5.39	0.0	1	0
R104	589.81	589.90	589.90	481.31	0.0	7	3
R105	814.24	817.20	817.20	16.57	0.0	19	22
R106	772.46	775.90	775.90	200.24	0.0	23	72
R107	664.00	664.00	664.00	24.40	0.0	1	20
R108	514.47	519.90	519.90	2187.42	0.0	3	11
R109	745.74	750.80	750.80	406.32	0.0	277	52
R110	692.20	697.30	697.30	1206.91	0.0	59	70
R111	701.64	709.30	709.30	432.92	0.0	55	61
R112	577.45	577.45	2016.30	3600.00	71.4	2	28
RC101	1038.42	1043.00	1043.00	62.74	0.0	111	73
RC102	862.00	862.00	862.00	6.42	0.0	1	15
RC103	810.99	812.00	812.00	33.49	0.0	7	13
RC104	672.12	673.20	673.20	124.86	0.0	3	25
RC105	945.73	951.60	951.60	19.49	0.0	15	25
RC106	833.70	833.70	833.70	8.43	0.0	1	19
RC107	758.20	764.30	764.30	446.86	0.0	15	60
RC108	706.56	713.20	713.20	1274.92	0.0	51	74
C201	501.30	501.30	501.30	10.71	0.0	1	0
C202	521.60	521.60	521.60	59.12	0.0	1	0
C203	511.70	511.70	511.70	575.32	0.0	1	0
C204	466.15	466.15	892.80	3600.00	47.8	0	0
C205	511.60	511.60	511.60	39.82	0.0	1	0
C206	514.20	514.20	514.20	363.36	0.0	1	0
C207	529.30	529.30	529.30	56.43	0.0	1	7
C208	526.00	526.00	526.00	130.57	0.0	1	0
R201	672.30	672.30	672.30	15.61	0.0	1	0
R202	644.50	644.50	644.50	114.60	0.0	1	11
R203	568.20	568.20	568.20	749.54	0.0	1	7
R204	-	-	1086.50	3600.00	-	0	0
R205	603.50	603.50	603.50	89.80	0.0	1	7
R206	576.93	577.00	577.00	742.24	0.0	3	37
R207	510.30	510.30	510.30	2756.47	0.0	1	0
R208	-	-	1441.10	3600.00	-	0	0
R209	598.62	599.49	599.50	3600.00	0.0	2	38
R210	517.55	517.70	517.70	1980.00	0.0	3	20
R211	513.70	513.83	514.90	3600.00	0.2	6	32
RC201	809.50	809.50	809.50	10.20	0.0	1	0
RC202	797.40	797.40	797.40	137.19	0.0	1	13
RC203	667.80	667.80	667.80	873.02	0.0	1	11
RC204	601.80	601.80	601.80	940.36	0.0	1	0
RC205	718.69	718.70	718.70	83.43	0.0	3	27
RC206	793.30	793.30	793.30	52.35	0.0	1	3
RC207	644.80	644.80	644.80	203.78	0.0	1	0
RC208	575.20	575.20	575.20	3116.09	0.0	1	15

Table 10 Detailed results for the new E-VRPTW-NL instances with 100 customers.

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	1040.90	1040.90	1040.90	20.59	0.0	1	0
C102	992.32	997.30	997.30	3501.56	0.0	41	100
C103	927.70	927.70	3458.30	3600.00	73.2	0	9
C104	-	-	5791.40	3600.00	-	0	0
C105	1004.68	1007.80	1007.80	584.93	0.0	19	97
C106	992.57	998.48	5333.40	3600.00	81.3	108	127
C107	988.40	994.42	5202.30	3600.00	80.9	71	133
C108	935.57	966.57	5791.40	3600.00	83.3	13	100
C109	888.28	888.81	5791.40	3600.00	84.7	3	79
R101	1554.80	1554.80	1554.80	7.92	0.0	1	6
R102	1408.70	1408.70	1408.70	38.38	0.0	1	7
R103	1140.98	1147.65	1148.20	3600.00	0.0	212	108
R104	-	-	5000.00	3600.00	-	0	0
R105	1277.68	1277.80	1277.80	72.50	0.0	9	41
R106	1185.13	1188.88	4907.20	3600.00	75.8	208	105
R107	1075.95	1075.95	4970.80	3600.00	78.4	2	51
R108	-	-	4716.70	3600.00	-	0	0
R109	1140.23	1148.83	4984.10	3600.00	77.0	92	135
R110	1040.73	1040.73	3897.60	3600.00	73.3	2	50
R111	1035.75	1037.90	2840.00	3600.00	63.5	2	58
R112	-	-	4809.80	3600.00	-	0	0
RC101	1588.94	1606.69	1608.20	3600.00	0.1	983	187
RC102	1453.73	1459.60	1459.60	835.29	0.0	53	101
RC103	1246.45	1250.56	6167.10	3600.00	79.7	21	98
RC104	1118.49	1118.49	5725.00	3600.00	80.5	0	67
RC105	1399.57	1401.20	1401.20	325.36	0.0	5	78
RC106	1328.87	1342.09	6328.10	3600.00	78.8	85	172
RC107	1165.67	1165.67	6120.80	3600.00	81.0	2	100
RC108	1140.61	1142.15	6640.90	3600.00	82.8	2	53
C201	627.50	627.50	627.50	606.21	0.0	1	0
C202	631.74	631.74	643.30	3600.00	1.8	2	0
C203	-	-	3029.80	3600.00	-	0	0
C204	-	-	4049.60	3600.00	-	0	0
C205	627.50	627.50	627.50	965.26	0.0	1	0
C206	-	-	753.00	3600.00	-	0	0
C207	-	-	1615.40	3600.00	-	0	0
C208	-	-	839.50	3600.00	-	0	0
R201	1091.38	1092.50	1092.50	1540.98	0.0	43	32
R202	982.46	982.46	4471.20	3600.00	78.0	0	65
R203	-	-	3726.30	3600.00	-	0	0
R204	-	-	2921.30	3600.00	-	0	0
R205	941.43	941.43	3304.70	3600.00	71.5	0	31
R206	-	-	4257.80	3600.00	-	0	0
R207	-	-	3738.40	3600.00	-	0	0
R208	-	-	2900.40	3600.00	-	0	0
R209	-	-	4379.20	3600.00	-	0	0
R210	-	-	3873.70	3600.00	-	0	0
R211	-	-	3341.00	3600.00	-	0	0
RC201	1253.44	1254.30	1254.30	367.11	0.0	5	25
RC202	1131.77	1133.14	2633.30	3600.00	57.0	5	53
RC203	-	-	1321.50	3600.00	-	0	0
RC204	-	-	4950.90	3600.00	-	0	0
RC205	1067.16	1068.33	4009.70	3600.00	73.4	7	13
RC206	1055.50	1055.87	3210.00	3600.00	67.1	6	29
RC207	-	-	2131.40	3600.00	-	0	0
RC208	-	-	1565.60	3600.00	-	0	0

Table 11 Detailed results for the E-VRPTW-L instances with 25 customers from Desaulniers et al. (2016).

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	621.40	621.40	621.40	0.53	0.0	1	0
C102	520.30	520.30	520.30	1.23	0.0	1	2
C103	344.30	344.30	344.30	7.52	0.0	1	4
C104	430.30	430.30	430.30	6.87	0.0	1	0
C105	523.80	523.80	523.80	0.50	0.0	1	0
C106	555.00	555.00	555.00	1.13	0.0	1	11
C107	503.60	503.60	503.60	0.95	0.0	1	12
C108	493.50	493.50	493.50	0.91	0.0	1	0
C109	441.10	441.10	441.10	2.66	0.0	1	6
R101	659.40	659.40	659.40	0.37	0.0	1	0
R102	436.10	436.10	436.10	0.56	0.0	1	0
R103	490.90	490.90	490.90	1.23	0.0	1	2
R104	347.50	350.60	350.60	85.23	0.0	5	36
R105	541.90	541.90	541.90	0.47	0.0	1	2
R106	478.90	478.90	478.90	2.24	0.0	1	24
R107	409.50	409.50	409.50	2.93	0.0	1	6
R108	422.80	422.80	422.80	1.86	0.0	1	0
R109	451.90	451.90	451.90	1.79	0.0	1	24
R110	404.90	404.90	404.90	4.87	0.0	1	14
R111	380.24	381.10	381.10	76.33	0.0	3	42
R112	384.10	384.10	384.10	6.59	0.0	1	31
RC101	726.50	726.50	726.50	0.54	0.0	1	5
RC102	637.90	637.90	637.90	0.69	0.0	1	10
RC103	550.54	551.00	551.00	2.59	0.0	3	12
RC104	514.80	514.80	514.80	5.85	0.0	1	33
RC105	588.50	588.50	588.50	0.50	0.0	1	0
RC106	547.90	547.90	547.90	0.59	0.0	1	0
RC107	496.20	496.20	496.20	0.86	0.0	1	0
RC108	469.50	469.50	469.50	2.86	0.0	1	13
C201	415.20	415.20	415.20	1.24	0.0	1	0
C202	403.50	403.50	403.50	8.29	0.0	1	0
C203	390.70	390.70	390.70	18.25	0.0	1	0
C204	360.90	360.90	360.90	10.26	0.0	1	0
C205	407.80	407.80	407.80	2.02	0.0	1	0
C206	387.80	387.80	387.80	6.50	0.0	1	0
C207	396.20	396.20	396.20	2.11	0.0	1	0
C208	393.40	393.40	393.40	6.15	0.0	1	0
R201	372.90	372.90	372.90	1.93	0.0	1	0
R202	374.10	374.10	374.10	3.42	0.0	1	0
R203	381.60	381.60	381.60	12.23	0.0	1	4
R204	314.20	314.20	314.20	21.74	0.0	1	0
R205	372.30	372.30	372.30	2.71	0.0	1	0
R206	423.10	423.10	423.10	7.54	0.0	1	4
R207	329.60	329.60	329.60	23.11	0.0	1	0
R208	314.30	314.30	314.30	235.26	0.0	1	0
R209	421.10	421.10	421.10	8.38	0.0	1	11
R210	314.10	314.10	314.10	20.02	0.0	1	0
R211	382.40	382.40	382.40	41.68	0.0	1	18
RC201	593.60	593.60	593.60	2.81	0.0	1	0
RC202	539.40	539.40	539.40	4.74	0.0	1	0
RC203	442.00	442.00	442.00	41.16	0.0	1	23
RC204	444.20	444.20	444.20	37.72	0.0	1	6
RC205	535.20	535.20	535.20	5.50	0.0	1	12
RC206	587.90	587.90	587.90	2.83	0.0	1	0
RC207	484.70	484.70	484.70	13.96	0.0	1	6
RC208	437.20	437.20	437.20	22.08	0.0	1	0

Table 12 Detailed results for the E-VRPTW-L instances with 50 customers from Desaulniers et al. (2016).

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	769.50	769.50	769.50	2.63	0.0	1	0
C102	775.10	775.10	775.10	7.05	0.0	1	0
C103	623.24	628.17	637.10	3600.00	1.4	46	82
C104	548.40	548.40	548.40	161.02	0.0	1	0
C105	727.60	727.60	727.60	3.01	0.0	1	0
C106	732.90	732.90	732.90	4.09	0.0	1	1
C107	701.90	701.90	701.90	3.92	0.0	1	0
C108	716.90	716.90	716.90	5.22	0.0	1	3
C109	661.61	667.00	667.00	640.95	0.0	177	67
R101	921.85	923.00	923.00	2.01	0.0	3	7
R102	845.05	845.40	845.40	9.03	0.0	3	39
R103	774.63	774.80	774.80	11.65	0.0	3	20
R104	603.34	603.34	1607.50	3600.00	62.5	2	57
R105	820.93	825.90	825.90	25.61	0.0	23	76
R106	776.47	786.20	786.20	238.97	0.0	55	85
R107	665.83	666.20	666.20	32.98	0.0	3	24
R108	529.23	529.23	531.50	3600.00	0.4	2	25
R109	751.36	756.60	756.60	192.57	0.0	127	52
R110	694.08	700.10	700.10	680.51	0.0	71	76
R111	715.40	715.40	715.40	14.98	0.0	1	7
R112	583.40	583.40	583.40	204.07	0.0	1	5
RC101	1042.93	1047.70	1047.70	60.79	0.0	157	60
RC102	872.30	872.30	872.30	2.60	0.0	1	0
RC103	816.33	819.30	819.30	90.31	0.0	41	26
RC104	674.80	684.80	684.80	753.73	0.0	47	26
RC105	954.10	962.80	962.80	83.15	0.0	111	48
RC106	845.73	855.70	855.70	105.51	0.0	99	35
RC107	762.11	767.10	767.10	325.10	0.0	7	49
RC108	708.32	712.70	712.70	710.08	0.0	25	55
C201	501.30	501.30	501.30	5.30	0.0	1	0
C202	521.60	521.60	521.60	35.84	0.0	1	0
C203	511.70	511.70	511.70	445.14	0.0	1	0
C204	466.15	466.15	1740.70	3600.00	73.2	0	0
C205	511.60	511.60	511.60	42.53	0.0	1	0
C206	514.20	514.20	514.20	228.88	0.0	1	0
C207	529.30	529.30	529.30	41.16	0.0	1	10
C208	526.00	526.00	526.00	96.44	0.0	1	0
R201	672.30	672.30	672.30	17.14	0.0	1	0
R202	644.50	644.50	644.50	73.01	0.0	1	2
R203	568.20	568.20	568.20	557.84	0.0	1	10
R204	-	-	1747.60	3600.00	-	0	0
R205	603.50	603.50	603.50	98.86	0.0	1	6
R206	576.46	577.00	577.00	719.24	0.0	7	39
R207	510.30	510.30	510.30	2400.97	0.0	1	0
R208	-	-	1525.60	3600.00	-	0	0
R209	598.69	599.50	599.50	3020.06	0.0	3	28
R210	517.55	517.70	517.70	1839.19	0.0	3	31
R211	513.32	513.39	514.90	3600.00	0.3	6	31
RC201	809.50	809.50	809.50	8.08	0.0	1	0
RC202	797.40	797.40	797.40	90.01	0.0	1	17
RC203	667.80	667.80	667.80	652.25	0.0	1	26
RC204	601.90	601.90	601.90	664.09	0.0	1	0
RC205	718.59	718.70	718.70	83.68	0.0	3	20
RC206	793.30	793.30	793.30	60.03	0.0	1	6
RC207	644.80	644.80	644.80	196.34	0.0	1	0
RC208	575.20	575.20	575.20	2530.51	0.0	1	20

Table 13 Detailed results for the E-VRPTW-L instances with 100 customers from Desaulniers et al. (2016).

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
C101	1040.90	1040.90	1040.90	17.00	0.0	1	0
C102	995.26	1001.19	5672.70	3600.00	82.4	31	101
C103	934.73	934.73	2811.20	3600.00	66.7	1	16
C104	-	-	5791.40	3600.00	-	0	0
C105	1004.21	1013.00	1013.00	1026.64	0.0	67	96
C106	993.20	999.20	999.20	1021.07	0.0	33	95
C107	988.98	994.56	5116.00	3600.00	80.6	101	146
C108	936.47	970.21	5791.40	3600.00	83.2	12	113
C109	902.10	904.33	4718.90	3600.00	80.8	5	64
R101	1563.80	1563.80	1563.80	6.74	0.0	1	7
R102	1414.99	1415.00	1415.00	49.88	0.0	3	20
R103	1150.29	1152.80	1152.80	679.08	0.0	11	53
R104	-	-	5000.00	3600.00	-	0	0
R105	1295.95	1297.10	1297.10	60.95	0.0	7	49
R106	1196.75	1199.59	1199.90	3600.00	0.0	272	75
R107	1089.04	1091.84	4970.80	3600.00	78.0	20	86
R108	-	-	4716.70	3600.00	-	0	0
R109	1151.28	1159.86	4984.10	3600.00	76.7	121	132
R110	1043.81	1043.81	3739.50	3600.00	72.1	2	55
R111	1048.65	1051.47	4987.60	3600.00	78.9	19	76
R112	-	-	4809.80	3600.00	-	0	0
RC101	1594.35	1606.30	1606.30	2781.69	0.0	975	178
RC102	1468.66	1475.60	1475.60	554.60	0.0	27	109
RC103	1263.42	1271.17	6167.30	3600.00	79.4	34	87
RC104	1115.94	1115.94	5655.60	3600.00	80.3	0	37
RC105	1422.00	1422.00	1422.00	43.06	0.0	1	18
RC106	1339.94	1346.40	1346.40	1158.24	0.0	37	134
RC107	1165.69	1165.69	6044.40	3600.00	80.7	2	105
RC108	1154.78	1154.78	6640.90	3600.00	82.6	2	62
C201	627.50	627.50	627.50	262.29	0.0	1	0
C202	-	-	627.50	3600.00	-	0	0
C203	-	-	1655.30	3600.00	-	0	0
C204	-	-	4419.60	3600.00	-	0	0
C205	627.50	627.50	627.50	774.04	0.0	1	0
C206	-	-	786.50	3600.00	-	0	0
C207	-	-	3074.70	3600.00	-	0	0
C208	-	-	783.50	3600.00	-	0	0
R201	1091.66	1092.50	1092.50	1942.96	0.0	73	33
R202	983.12	983.12	4471.20	3600.00	78.0	0	71
R203	-	-	2927.50	3600.00	-	0	0
R204	-	-	2785.40	3600.00	-	0	0
R205	940.40	940.40	3304.70	3600.00	71.5	1	30
R206	-	-	4257.80	3600.00	-	0	0
R207	-	-	3837.80	3600.00	-	0	0
R208	-	-	2885.00	3600.00	-	0	0
R209	-	-	4379.20	3600.00	-	0	0
R210	-	-	3847.50	3600.00	-	0	0
R211	-	-	3341.00	3600.00	-	0	0
RC201	1253.41	1254.30	1254.30	453.53	0.0	5	38
RC202	1132.88	1133.90	2408.20	3600.00	52.9	8	63
RC203	-	-	1838.60	3600.00	-	0	0
RC204	-	-	5537.80	3600.00	-	0	0
RC205	1067.22	1068.36	1068.50	3600.00	0.0	12	27
RC206	1055.41	1056.27	3103.50	3600.00	66.0	9	33
RC207	-	-	2510.60	3600.00	-	0	0
RC208	-	-	1522.10	3600.00	-	0	0

Table 14 Detailed results for the E-VRP-NL instances from Montoya et al. (2017).

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
tc0c10s2cf1	19.75	19.75	19.75	0.06	0.0	1	0
tc0c10s2ct1	12.30	12.30	12.30	0.06	0.0	1	1
tc0c10s3cf1	19.75	19.75	19.75	0.04	0.0	1	0
tc0c10s3ct1	10.80	10.80	10.80	0.07	0.0	1	0
tc1c10s2cf2	9.03	9.03	9.03	0.34	0.0	1	9
tc1c10s2cf3	16.37	16.37	16.37	0.14	0.0	1	12
tc1c10s2cf4	16.10	16.10	16.10	0.07	0.0	1	0
tc1c10s2ct2	10.08	10.75	10.75	7.25	0.0	9	40
tc1c10s2ct3	13.17	13.17	13.17	0.05	0.0	1	0
tc1c10s2ct4	13.83	13.83	13.83	0.09	0.0	1	0
tc1c10s3cf2	9.03	9.03	9.03	0.29	0.0	1	10
tc1c10s3cf3	16.37	16.37	16.37	0.12	0.0	1	14
tc1c10s3cf4	14.90	14.90	14.90	0.09	0.0	1	1
tc1c10s3ct2	9.20	9.20	9.20	1.15	0.0	1	6
tc1c10s3ct3	13.02	13.02	13.02	0.14	0.0	1	0
tc1c10s3ct4	13.21	13.21	13.21	0.08	0.0	1	0
tc2c10s2cf0	21.77	21.77	21.77	0.05	0.0	1	7
tc2c10s2ct0	12.45	12.45	12.45	0.89	0.0	1	43
tc2c10s3cf0	21.77	21.77	21.77	0.05	0.0	1	7
tc2c10s3ct0	9.22	11.51	11.51	25.82	0.0	19	67
tc0c20s3cf2	26.60	27.47	27.47	37.84	0.0	221	56
tc0c20s3ct2	17.02	17.08	17.08	15.34	0.0	3	48
tc0c20s4cf2	26.56	27.47	27.47	109.04	0.0	535	58
tc0c20s4ct2	16.95	16.99	16.99	55.48	0.0	9	44
tc1c20s3cf1	17.49	17.49	17.49	51.25	0.0	1	15
tc1c20s3cf3	15.34	16.44	16.44	1788.63	0.0	127	110
tc1c20s3cf4	17.00	17.00	17.00	0.99	0.0	1	0
tc1c20s3ct1	18.94	18.94	18.94	13.22	0.0	1	12
tc1c20s3ct3	12.35	12.60	12.60	491.55	0.0	105	0
tc1c20s3ct4	16.21	16.21	16.21	1.60	0.0	1	0
tc1c20s4cf1	16.38	16.38	16.38	80.95	0.0	1	16
tc1c20s4cf3	15.32	16.44	16.44	1364.04	0.0	65	144
tc1c20s4cf4	17.00	17.00	17.00	1.15	0.0	1	0
tc1c20s4ct1	17.80	17.80	17.80	25.56	0.0	1	22
tc1c20s4ct3	14.36	14.43	14.43	245.51	0.0	5	94
tc1c20s4ct4	17.00	17.00	17.00	2.07	0.0	1	0
tc2c20s3cf0	23.46	24.68	24.68	197.38	0.0	157	91
tc2c20s3ct0	24.37	25.06	27.06	10800.00	7.4	1158	343
tc2c20s4cf0	23.52	24.67	24.67	611.54	0.0	417	145
tc2c20s4ct0	25.28	26.02	26.02	8328.36	0.0	2273	151

Table 15 Detailed results for the E-VRP-NL instances from Montoya et al. (2017).

Instance	Root	DualBnd	Primal	Time [s]	Gap [%]	#Nodes	#SRC
tc0c40s5cf0	31.68	32.20	32.20	33.17	0.0	5	27
tc0c40s5cf4	30.10	30.25	30.25	658.20	0.0	17	53
tc0c40s5ct0	27.91	27.91	27.91	10.43	0.0	1	3
tc0c40s5ct4	28.27	28.60	28.64	10800.00	0.1	160	121
tc0c40s8cf0	30.23	30.40	30.40	118.08	0.0	5	45
tc0c40s8cf4	27.84	28.11	28.11	1232.89	0.0	13	37
tc0c40s8ct0	25.99	26.22	26.22	236.20	0.0	3	32
tc0c40s8ct4	28.86	29.03	29.07	10800.00	0.1	85	74
tc1c40s5cf1	63.81	64.51	64.51	2112.61	0.0	351	147
tc1c40s5ct1	52.00	52.33	52.33	2735.13	0.0	213	56
tc1c40s8cf1	40.64	40.64	40.64	78.64	0.0	1	17
tc1c40s8ct1	39.33	39.64	71.95	10800.00	44.9	43	89
tc2c40s5cf2	26.93	27.54	27.54	2557.00	0.0	125	94
tc2c40s5cf3	18.00	19.19	121.39	10800.00	84.2	10	51
tc2c40s5ct2	26.26	26.83	26.93	10800.00	0.4	1058	284
tc2c40s5ct3	21.68	21.68	87.76	10800.00	75.3	2	40
tc2c40s8cf2	26.14	26.53	139.50	10800.00	81.0	141	102
tc2c40s8cf3	17.97	18.76	121.39	10800.00	84.5	10	39
tc2c40s8ct2	24.50	25.49	83.71	10800.00	69.6	32	82
tc2c40s8ct3	21.29	21.29	123.19	10800.00	82.7	3	12
tc0c80s12cf0	33.51	33.51	176.39	10800.00	81.0	0	54
tc0c80s12cf1	39.84	39.84	193.11	10800.00	79.4	0	38
tc0c80s12ct0	36.58	36.58	177.09	10800.00	79.3	0	40
tc0c80s12ct1	38.97	38.97	215.20	10800.00	81.9	0	21
tc0c80s8cf0	38.40	38.40	178.21	10800.00	78.5	2	35
tc0c80s8cf1	43.32	43.32	195.89	10800.00	77.9	0	57
tc0c80s8ct0	40.03	40.03	177.50	10800.00	77.4	2	25
tc0c80s8ct1	43.35	43.35	167.73	10800.00	74.2	0	44
tc1c80s12cf2	-	-	175.86	10800.00	-	0	0
tc1c80s12ct2	-	-	175.43	10800.00	-	0	0
tc1c80s8cf2	-	-	158.94	10800.00	-	0	0
tc1c80s8ct2	-	-	158.90	10800.00	-	0	0
tc2c80s12cf3	-	-	52.45	10800.00	-	0	0
tc2c80s12cf4	40.63	40.63	249.34	10800.00	83.7	0	37
tc2c80s12ct3	-	-	189.17	10800.00	-	0	0
tc2c80s12ct4	-	-	242.19	10800.00	-	0	0
tc2c80s8cf3	-	-	188.27	10800.00	-	0	0
tc2c80s8cf4	45.20	45.20	189.30	10800.00	76.1	0	32
tc2c80s8ct3	-	-	186.72	10800.00	-	0	0
tc2c80s8ct4	42.47	42.47	249.68	10800.00	83.0	0	0

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