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Acoustic-structure interaction in disk-disk configurations in water for high head pump-turbines

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Abstract. Francis high-head pumps or pump-turbine runners often have speed and dimension ratios that make acoustic and structural natural frequencies, as well as rotor-stator excitation frequencies, approach each other. When compressibility effect becomes significant, it is not sufficient to only consider the incompressible added mass, which shifts the natural frequencies of the structure. The strong vibroacoustic coupling makes it difficult to distinguish the nature of each mode and the associated effect of the forced response amplification. The project's main aim is to study a disk-disk system behavior in an acoustic cavity filled with water when the acoustic and structural frequencies are close, as a simplified model of a pump-turbine runner in its casing. It is demonstrated here that the compressibility significantly affects vibration modes of the system, particularly when individual acoustic and structural modes have similar frequencies and spatial coherence, resulting in an increase in energy transmission and a decrease in total natural frequency. Additionally, the system's rigidity is affected by the cover's stiffness; when the vibration frequencies of the cover and runner are similar, excitation transmission is divided between them.

1. Introduction and literature review

Hydraulic turbines are subject to various time-varying excitations during operation. Among these, rotor-stator interaction (RSI) is a key deterministic excitation at blade passing frequencies [1].

For turbine manufacturers, understanding the vibration modes of turbine components and their excitation is crucial. The goal is to avoid the proximity of excitation frequencies to natural frequencies, considering similarities in mode shapes. For major vibration modes, predicting vibration amplitudes is essential for system durability. High head turbines and pump-turbines, with large diameters and small channel heights, often display disc-like vibration modes [2]. In pump-turbines, axial disk-like vibrations dominate due to the larger spacing between blades. Higher heads alter the excitation or natural frequencies, increasing the risk of sub-critical excitation nearing natural frequencies.

Currently, hydraulic machinery models are analyzed using modal and forced response analysis in finite element analysis (FEA). However, these complex models have millions of degrees of freedom, making mode interaction analysis challenging. A simplified model that integrates the physics for easier interpretation is missing in the literature.



Water adds inertia to the vibrating structure, known as the added mass effect, modeled using AVMI factors by Lamb [3]. This effect allows for coupling between different structural domains within the system, including rotor-stator coupling, where the runner's vibration modes are influenced by the cover's vibrations through water. This phenomenon has been studied both analytically and experimentally [4, 5, 6, 7].

Rotor rotation leads to the splitting of its vibration modes into a co-rotating mode, which increases frequency, and a counter-rotating mode, which decreases it. This vibration mode splitting must be considered in turbine design [5, 7, 8].

Compressibility has been considered in aeroelastic systems, such as rotating hard disks [9]. Some researchers have explored compressibility effects on hydroelastic systems like liquid-filled tanks [4] or simplified turbine FEA models [10]. However, the precise interaction between acoustics and structure remains unclear.

This paper introduces a model incorporating compressibility in a simple framework suitable for preliminary design, allowing manufacturers to validate complex simulations. The modeling and problem-solving methodology are developed, followed by an analysis of results, focusing on how compressibility affects acoustic-structural interaction in hydraulic systems.

2. Methodology

Considering that high-head pump turbines have a sufficiently high runner ratio R/H , the runner and the cover are modeled as annular disks, as shown in Figure 1. This Figure illustrates a cross-section of the system, indicating the different dimensions: the inner radius R_{in} representing the position where the runner is clamped, the outer radius R_{out} of the runner, the space ΔR_{out} between the outer radius and the rigid wall of the cavity, and the height H_c of the cavity.

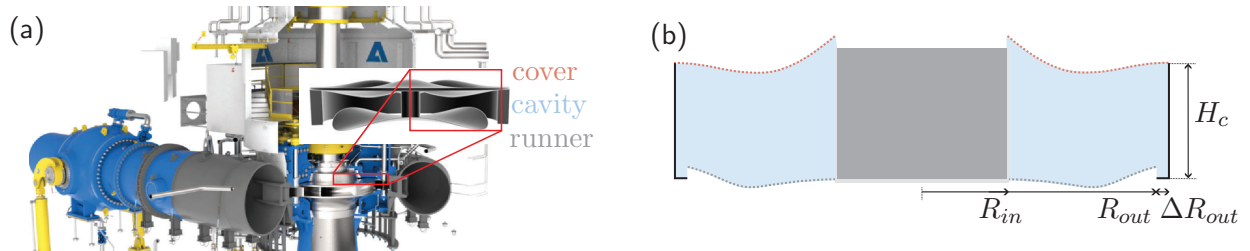


Figure 1: (a) Real geometry idealisation to disks model. (b) Schematic of the geometry and the domains considered

To understand how the interaction between the acoustic cavity and the structures modifies the behavior of the initial system, the eigenmodes are described by the individual eigenshapes of each domain: the dry modes of the disks and the modes with infinitely rigid walls for the acoustic cavity, similarly to Dowell and Voss [11]. The usefulness of such a method is to understand simply how the individual modes couple when the vibration frequencies approach and overlap each other.

The structural modes considered in the description are, therefore, solutions of the plate equation [12].

$$D\nabla^4 w - \frac{\partial^2}{\partial t^2} \rho h w = 0, \quad (1)$$

where w is the displacement variable, D is the flexural rigidity of the plate, ρ is its density, and h is its thickness, along with the proper boundary conditions. The runner disk is clamped at R_{in} and free at R_{out} , while the cover disk is clamped at $R_{out} + \Delta R_{out}$ and free at R_{in} . The solutions of such a system are separated according to circular symmetry. In each domain, the solutions $w_{n,p}$ take the form:

$$w_{n,p}(r, \theta) = (\cos(n\theta) \vee \sin(n\theta)) W_p(r), \quad (2)$$

where $W_p(r)$ is decomposed into functions of the first and second kind of Bessel functions, specifically J_n , Y_n , I_n , and K_n .

The variable used to describe the acoustic domain is the velocity potential ϕ , as it allows condensation of the acoustic-structure interaction into a single gyroscopic-type matrix. The acoustic potential ϕ then satisfies the Helmholtz equation:

$$\nabla^2 \phi + \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \phi = 0,$$

where c_0 is the speed of sound in the fluid domain and the boundary conditions $\partial\phi/\partial n = 0$ on the walls are used.

Due to the cylindrical symmetry of the domain, the solutions of the equation are separated according to r, θ, z :

$$\phi_{n,p,q}(r, \theta) = (\cos(n\theta) \vee \sin(n\theta)) \Phi_p(r) \cos\left(\frac{q\pi}{H_c} z\right), \quad (3)$$

where Φ_q is decomposed into Bessel functions of the first kind.

However, these modes alone are not sufficient to model the acoustic/structure interaction because for these modes the fluid velocity at the walls is zero, i.e., $\partial\phi/\partial n = 0$. However, during the acoustic-structure interaction, as the structure can vibrate, this boundary condition on the fluid velocity can no longer be satisfied, since the structure sets the fluid particles in motion. This reasoning was discussed in Ginsberg [13], which shows that to improve the convergence of the model, additional mathematical corrective modes must be added to account for the degree of freedom $\partial\phi/\partial n$. Polynomial functions are used here for this purpose.

To model the acoustic/structure interaction, it is natural to non-dimensionalize the problem in order to understand where the model lies between predominantly acoustic, structural, or intermediate vibration behaviors. Four dimensionless numbers are used to model the interaction between acoustic field and two structures. The reference structure to non-dimensionalize the problem is the runner.

The acoustic-runner frequency ratio, denoted as $F_a = f_{a,0}/f_{s,0}$, is the ratio of acoustic stiffness to structural stiffness. A low value of F_a represents situations such as an air duct with a compressible fluid, while a high value of F_a applies to blood vessels with incompressible fluid. The acoustic-runner mass ratio, represented as $\mathcal{M}_a = \rho_a R_r / \rho_s e_r$, is calculated by the formula. This ratio is typically low for air applications and high for applications involving water. The cover-runner frequency ratio, marked as $F_c = f_{c,0}/f_{r,0}$, is the ratio of cover stiffness to runner stiffness. A flexible cover corresponds to a low value of F_c , while an infinitely rigid cover results in a high value of F_c . Lastly, the cover-runner mass ratio is defined by $\mathcal{M}_c = \rho_c e_c R_r / \rho_s e_r R_c$. In instances with a dense and/or thick cover, $\mathcal{M}_c$ is low, and conversely, a thin and/or light cover produces a high $\mathcal{M}_c$.

The system of coupled equations A1 is written in weak form in Eq. A2 (in appendix). Then, the system is discretized using the Ritz-Galerkin method. The modes used for decomposition in the various domains are: the structural modes, which are solutions to the dry problem, and the acoustic modes, which are solutions to the problem with an infinitely rigid structure.

The discretized system of equations takes the form of a classical second-order system in mechanics:

$$M\ddot{X} + G\dot{X} + KX = 0. \quad (4)$$

The mass and stiffness terms contain information about the natural vibration of each domain, while the matrix G contains the acoustic-structure interaction. This matrix is antisymmetric and relates the first temporal derivatives, giving it the characteristics of a gyroscopic matrix.

To account for the rotation of the runner relative to the static cover, we consider that the runner rotates at a rate of Ω_r . We also model the fluid as rotating at a rate of $\Omega_f = K\Omega_r$,

where $K = 0.45$ is the entrainment factor for $10^6 < Re < 4.5 \times 10^6$ [5, 8, 14]. This implies relative velocities of the fluid with respect to each structure: $\Omega_{f/c} = K\Omega_r$ and $\Omega_{f/r} = (1 - K)\Omega_r$. Then, for each contact between the acoustic domain and the structure, the use of the material derivative introduces a new added stiffness term K_Ω proportional to the rotational velocity of the system. For a nodal diameter n , $K_\Omega = n\Omega_r K_0$. Taking into consideration the fact that the excitation frequency of Rotor Stator Interaction (RSI) is directly proportional to the runner's rotational speed [15], this added stiffness can be integrated to calculate the harmonic response from RSI excitation. Here, we denote the number of blades as Z_b and the number of guide vanes as Z_0 . We further define m and p as integers that determine the number of nodal diameters n , given by $n = mZ_b - pZ_0$. Additionally, the relation $\omega = mZ_b\Omega_r$ is established. With these definitions, we can then reformulate the modal problem into a new harmonic system of equations, incorporating the presence of F_n , the second member generated by RSI excitations with n nodal diameter:

$$-\omega^2 MX + \omega(jG + \frac{n}{mZ_b}K_0)X + KX = F_n \exp(j\omega t). \quad (5)$$

3. Results

The analysis focuses on two dimensionless parameters: the frequency ratios F_a between the runner and the acoustics, and F_c between the runner and the cover. Three specific values have been evaluated for each parameter, representing different scenarios of relative stiffness between the runner and the cover, as well as between the runner and the acoustic compressibility of water. Table 1 provides a summary of the different scenarios explored.

Table 1: Cover and runner stiffness ratio values considered in different scenarios

F_c (Cover vs. Runner Stiffness)		F_a (Acoustic vs. Runner Stiffness)	
Scenario	Value	Scenario	Value
Cover stiffness = runner stiffness	1	Stiff runner, optimal energy	0.3
Cover moderately rigid	2	Runner flexibility = acoustic compressibility	1
Cover infinitely rigid	∞	Incompressible water	∞

Figure 2 analyzes the model's eigenmode evolution with the dimensionless speed of sound. As F_a is varied from 0 to large values, the modeled fluid change from highly compressible towards incompressible in comparison to the runner compliance. In Figure 2 (a), given F_c , the system's two lowest frequencies for three nodal diameters are displayed. Near 0, one frequency linearly increases with F_a , representing a pure acoustic frequency, and continuously rises to an asymptote for $F_a \gg 1$, signifying added incompressible mass. The second frequency, beginning at the runner's natural frequency, veers towards the increasing frequency when $F_a > 0.15$. Figure 2 (b) allows observing the evolution of the power transmitted between the acoustic domain and the structural domains. For very low F_a , this transmitted power tends towards 0, then in the region where the eigenvalues veer $F_a \simeq 0.3$, the transmitted power reaches a maximum before finally decreasing towards an asymptotic value in the incompressible regime. The region of the maximum is known as the "veering region" [16] in the literature and is the region where the acoustic and structural modes mix together. In Figure 2 (a), as F_c lowers from ∞ to 1 with high enough F_a , the asymptotic frequency is observed to decline. This decline is minor between $F_c = \infty$ and 2, suggesting a swift progression towards an asymptotic regime with increasing F_c . In Figure 2 (b), the power conveyed to the cover grows while the power to the runner lessens as F_c decreases. Therefore, as the cover's stiffness nears that of the runner, the power exchange becomes equally spread across the entire structure. We also note that the veering region size reduces in figures (a) and (b) as F_c decreases.

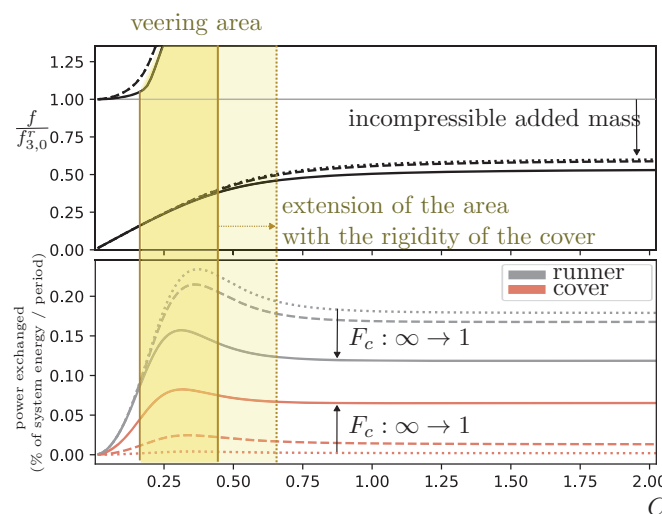


Figure 2: effect of the dimensionless acoustic stiffness F_a parameter : (a) on the evolution of the frequencies; (b) and on evolution of the power of the first mode exchanged between acoustic and structure; (—): $F_c = 1$ flexible cover, (---): $F_c = 2$ stiffer cover, (.....): $F_c = \infty$ infinitely rigid cover

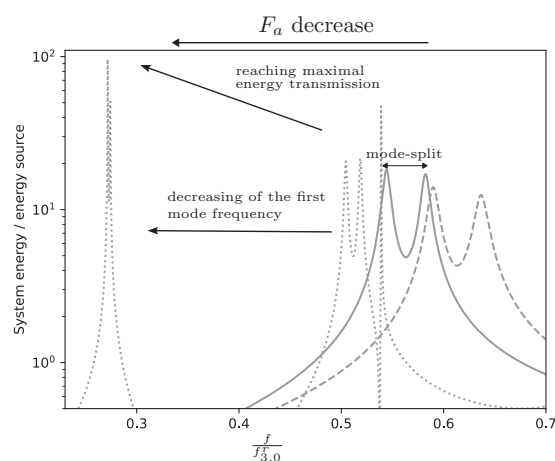


Figure 3: Harmonics response of the system varying the F_a parameter for $F_c = \infty$: (—): $F_a = 1$ moderate rigidity runner; (---): $F_a = \infty$ fluid considered incompressible; (.....): $F_a = 0.3$ maximum exchanged in showed in Figure 2

Lastly, the harmonic analyse presented in Figure 3 illustrates the phenomenon of mode splitting. The Figure demonstrates that the intensity of the mode splitting significantly decreases as it approaches the veering region. Additionally, the harmonic analysis confirms the impact of fluid compressibility observed in the bifurcation diagram (Figure 2): a decrease in mode frequency and an increase in energy exchange as F_a decreases.

4. Conclusion

The compressibility plays a significant role that needs to be considered in the calculation of the vibration modes of the system. When an individual acoustic mode and an individual structural mode are close in frequency and spatially coherent (i.e., their scalar product is high), it is likely that the corresponding mode of the total system is located in the veering or mode-mixing region.

In this region, as the structural natural frequency combines with the acoustic natural frequency, the total natural frequency is reduced. Moreover, in this region, the energy transmission between the acoustic and structural modes is maximized. This can be problematic when the excitation is purely acoustic (e.g., RSI), as a significant portion of the energy is transmitted to the structural vibrations, both to the runner and the cover.

Finally, the stiffness of the cover increases the overall rigidity of the system. When the cover and the runner have similar vibration frequencies, the transmitted energy is distributed between

the two structures, thereby reducing the total energy received by the runner alone. Additionally, in this scenario, the veering region is narrower and more difficult to reach compared to when the cover is much stiffer than the runner.

Acknowledgments

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Appendix A.

The system of equations for the interaction problem is the following :

$$\nabla^4 w_r + \ddot{w}_r = \mathcal{M}_a \dot{\phi}; \quad (\text{A1a})$$

$$\nabla^4 w_c + F_c^2 \ddot{w}_c = -\mathcal{M}_a \mathcal{M}_c \dot{\phi}; \quad (\text{A1b})$$

$$\nabla^2 \phi - \frac{1}{F_a^2} \ddot{\phi} = 0; \quad (\text{A1c})$$

$$\nabla \phi \cdot n_r = \dot{w}_r \text{ on the runner}; \quad (\text{A1d})$$

$$\nabla \phi \cdot n_c = -\dot{w}_c \text{ on the cover}; \quad (\text{A1e})$$

where w_r is the displacement of the runner, w_c is the displacement of the cover, and ϕ is the acoustic velocity potential.

Then, weak form of the system of equations is depicted in :

$$w_r \in H^2, w_c \in H^2, \phi \in H^1; \quad (\text{A2a})$$

$$\forall \tilde{w}_r \in H^2, \frac{1}{\mathcal{M}_a} \int_{\Gamma_r} \nabla^2 w_r \nabla^2 \tilde{w}_r + \ddot{w}_r \tilde{w}_r d\Gamma = \int_{\Gamma_r} \dot{\phi} \tilde{w} d\Gamma \quad (\text{A2b})$$

$$\forall \tilde{w}_c \in H^2, \frac{1}{\mathcal{M}_a \mathcal{M}_c} \int_{\Gamma_c} \nabla^2 w_c \nabla^2 \tilde{w}_c + F_c^2 \ddot{w}_c \tilde{w}_c d\Gamma = - \int_{\Gamma_c} \dot{\phi} \tilde{w} d\Gamma; \quad (\text{A2c})$$

$$\forall \tilde{\phi} \in H^1, \int_{\Omega} \nabla \phi \cdot \nabla \tilde{\phi} + \frac{1}{F_a^2} \ddot{\phi} \tilde{\phi} d\Omega = - \int_{\Gamma_r} \dot{w} \tilde{\phi} d\Gamma; \quad (\text{A2d})$$