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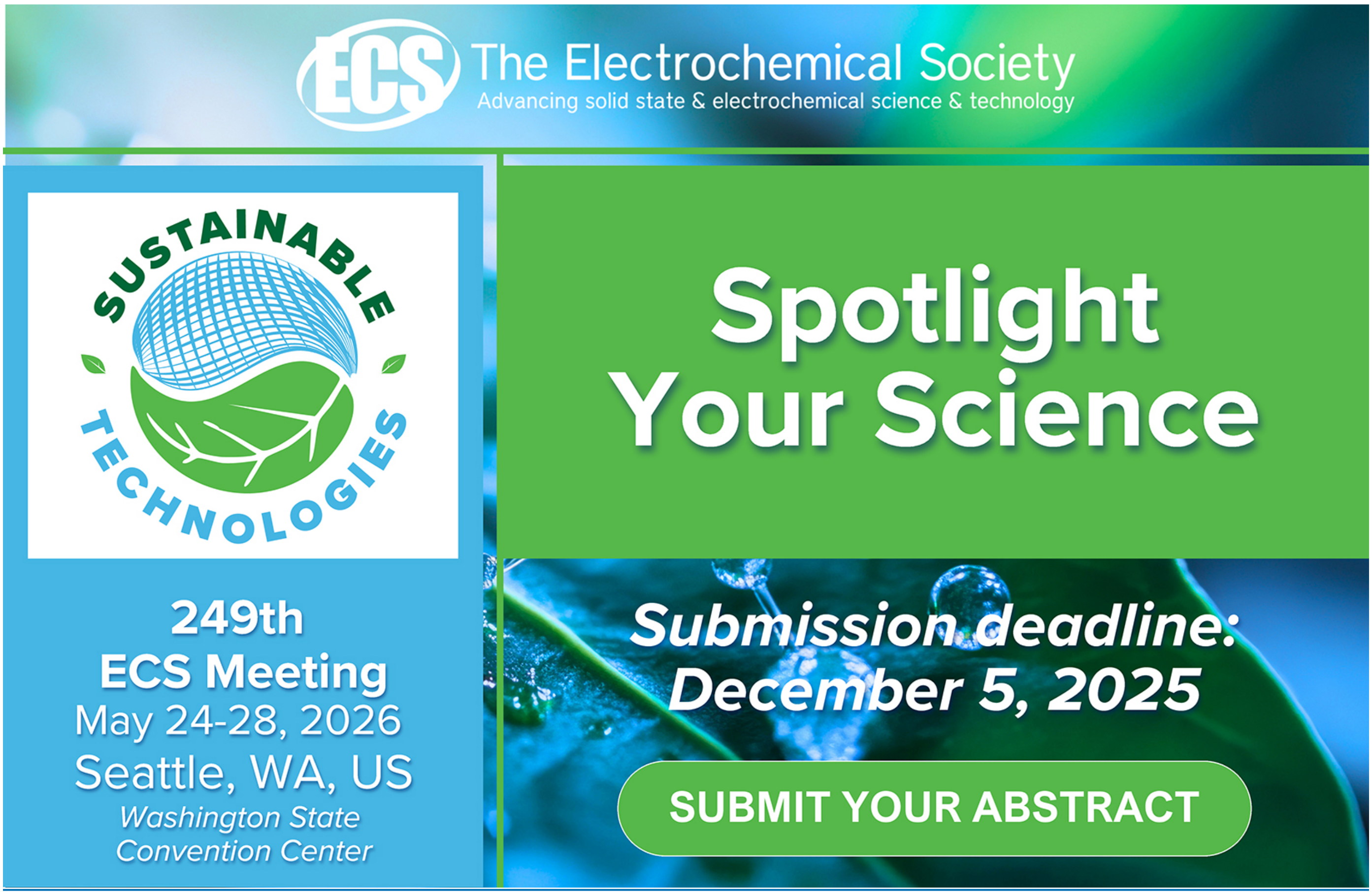

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Numerical and Analytical Estimation of the Hydrodynamic Coefficients of a Radial Seal under Whirling Vibrations

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Abstract. Turbulent leakage flow passes through the radial seals and spiral cases of turbine runners. A small deviation of the runner changes the radial seal flow, inducing a strong pressure gradient. Reaction fluid forces act on the runner which hence whirls. These forces are modeled by dynamic force coefficients on the shaft-line dynamics. We determine those coefficients to predict whirl vibrations. The radial and tangential forces are written as quadratic and linear polynomials of the whirl frequency. The polynomial coefficients correspond to the runner dynamic coefficients. A computational fluid dynamics (CFD) simulation computes these forces for different whirl frequencies of a disk-like structure. Harmonic analyses on a vibroacoustic finite element model (FEM) yield the same forces, which are interpolated to obtain the dynamic coefficients. These numerical models are compared with an analytical potential flow theory model for a plain annular seal geometry. CFD considers more physics than the other methods as it includes the fluid shear. Parametric studies on a clearance-and-side-chamber geometry highlight that hydrodynamic coefficients vary with runner frequency, axial mass flow rate, runner angular frequency and radial seal clearance. The resulting runner dynamic coefficients improve the shaft line analysis, as more physical phenomena are considered on a real geometry.

1. Introduction

In Francis turbines, undesirable and sometimes unacceptable self-excited vibrations can occur due to the flow in hydrodynamic seals [1]. Staubli [2] points out that hydrodynamic seals are designed to reduce leakage flow, to minimize its deleterious effect on efficiency in large machines such as Francis and pump-turbines. A narrow radial seal surrounds the runner band in the radial spiral cases, through which a turbulent flow passes axially in addition to a circumferential shear flow driven by the runner rotation.

Under external excitations the runner is off-centered by a distance of ϵ, and the deformed seal has a smaller and a larger gap of width $C_0 - \epsilon$ and $C_0 + \epsilon$ respectively, as shown in Figure 1.a. Nishimura et al. [3] explain that the flow is squeezed in the narrow passage, generating a strong pressure gradient that causes radial and tangential excitation forces on the runner. The perturbation in the circumferential flow which stems from the runner rotation and from the whirling motion, and perturbation of the inertia of the axial leakage flow, both contribute to the pressure gradient. These forces can promote runner instability. A radial force in the direction of eccentricity increases ϵ, and a tangential force in the direction of the whirl motion increases the motion. The circular orbit of the whirl motion being increased by the fluid forces, it may contribute to a self-excited whirl vibration. Designing turbomachines requires to evaluate those vibrations which are directly affected by the runner seal direct and cross-coupled dynamic mass, damping and stiffness. Those dynamic coefficients are thus required as input data for predicting the shaft line dynamics.

Blevins [4] determines the reaction forces on the runner for two coaxial rotating cylinders without axial flow using the potential flow theory. However, the forces they find only depend on the added mass and do not have tangential shear components, as the fluid is inviscid and irrotational. To calculate all dynamic coefficients, Childs et al. [5] proposes a bulk-flow model for turbulent flow of plain annular seals with a strong axial pressure gradient, based on Hirs' lubrication equations [6]. This bulk flow model is not applicable to more complex geometries, as some required constants are based on experimental data from annular seals. Moreover recirculation and separation are not considered although they are present in turbine seals.

Staubli et al. [7] express the fluid forces as functions of the whirl frequency and the runner dynamic coefficients, and use a computational fluid dynamics (CFD) model to calculate those forces, whose expression yields the dynamic coefficients. This numerical method is appliable on a real turbomachine complex seal geometry. Although CFD includes most of the fluid dynamics, it is numerically expensive. Acoustic-structural models based on the Finite Element Method (FEM) are less complex in calculation [8]. Simplicity of the model lies in the fluid acoustic property, which is considered ideal, hence inviscid. Structural-acoustic FSI problems based en FEM were performed by Milkessa et al. [9], who studied the bending vibration modes of immersed cylinders. Mohd Azman et al. [10] determined the whirling vibrations of a shaft with the same method.

Here we present a method to calculate a disk dynamic coefficients, extendable to real turbine seals. This method is based on the methodology of Staubli et al. [7]. Two numerical models, using CFD and FEM respectively, are set up to evaluate the fluid forces which induce whirl vibrations. Those forces are required to obtain the dynamic coefficients of the runner. An analytical model based on the potential flow theory of Blevins [4] is developed to compare with the numerical models.

The methodology section presents two seal geometries that we investigate. Then it introduces the FEM and CFD, FEM and analytical models. The result section compares the predictions of these models in terms of force coefficients. The paper is concluded with a discussion on the sensitivity of the runner stability to these dynamic coefficients.

2. Methodology

Two seal geometries are studied. First, Figure 1.b shows a simple plain annular seal of length $L = 10\ mm$ and width $1\ mm$ around a runner of radius $2012\ mm$ off-centred by $\epsilon = 0.6\ mm$. For a better consideration of the flow dynamics in runner dynamic seals, further studies are run on a geometry reproduced from Nishimura et al. [3], represented in Figure 1.c. The fluid enters at the inlet, passes through a radial seal of width C_0 and a side chamber of width C_{ch}. The geometric dimensions are similar of those in Nishimura et al. [3].

The following methodology is based on the work by Staubli et al. [7]. Inside the seals, consider a runner rotating at frequency Ω_R and drawing a circular orbit of eccentricity ϵ at whirl frequency ω as depicted in Figure 1.a. In the relative coordinate system of reference (x, y) attached to the runner center motion and whirling at whirling frequency ω, the force components F_x and F_y fit the radial and tangential forces, respectively. They result from the pressure field acting on the runner surface. Their expression in the rotating frame of reference is:

$$F_x = F_r = \int_0^L \int_0^{2\pi} p_{runner} \cos\varphi R d\varphi dz, \quad F_y = F_t = \int_0^L \int_0^{2\pi} p_{runner} \sin\varphi R d\varphi dz. \qquad (1)$$

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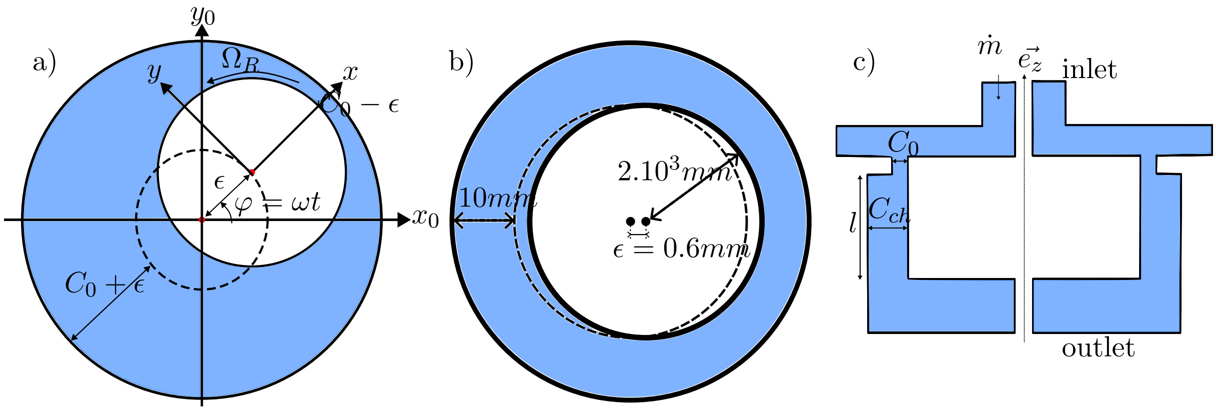

Figure 1: Schematic of the deformed radial seal under the runner eccentricity (a) and of the studied seal geometries: a plain annular seal (a), and radial gaps reproduced from Ref. [3] (b)

To relate the fluid forces to the runner dynamic coefficients like Staubli et al. [7], we assume a linear relationship between the fluid forces and the runner displacement (x_R, y_R), velocity and acceleration in the reference frame (x_0, y_0), and the cross-coupled mass terms are neglected. In the assumption of small deformation and for a rotational symmetry, the dynamic equation of the runner-seal structure is:

$$\left\{ \begin{array}{c} -F_x(t) \\ -F_y(t) \end{array} \right\} = \left[\begin{array}{cc} K & k \\ -k & K \end{array} \right] \left\{ \begin{array}{c} x_R \\ y_R \end{array} \right\} + \left[\begin{array}{cc} C & c \\ -c & C \end{array} \right] \left\{ \begin{array}{c} \dot{x}_R \\ \dot{y}_R \end{array} \right\} + \left[\begin{array}{cc} M & 0 \\ 0 & M \end{array} \right] \left\{ \begin{array}{c} \ddot{x}_R \\ \ddot{y}_R \end{array} \right\}, \quad (2)$$

which depends on the runner dynamic direct mass M, direct damping C, direct stiffness K, cross-coupled damping c and cross-coupled stiffness k. Those dynamic coefficients determine the runner stability. Based on the system characteristic equation, the Routh-Hurwitz method explained in [11] yields three criteria of stability that the hydrodynamic coefficients must fulfill at the same time to be in a stable state:

$$C > 0; \quad \gamma_1 = \frac{C}{M} + \frac{KC - kc}{C^2 + c^2} > 0; \quad \gamma_2 = \frac{C}{M} - \frac{k^2}{KC + kc} > 0. \quad (3)$$

As per [7], in the relative coordinate system (x, y) the forces along the x- and y-axes are actually the radial and tangential reaction forces. The circular displacement of the runner in the reference frame (x_0, y_0) is:

$$\left\{ \begin{array}{c} x_R(t) \\ y_R(t) \end{array} \right\} = \left\{ \begin{array}{c} \epsilon cos(\omega t) \\ \epsilon sin(\omega t) \end{array} \right\}. \quad (4)$$

Introducing Eq. 4 into Eq. 2, F_x/ϵ and F_y/ϵ are equal to:

$$\frac{F_x}{\epsilon} = M\omega^2 - c\omega - K, \quad \frac{F_y}{\epsilon} = -C\omega + k. \quad (5)$$

Linear and quadratic regressions of F_x/ϵ and F_y/ϵ over the whirl frequency ω yield the polynomial coefficients which are the direct and cross-coupled dynamic coefficients. For a fixed ϵ, F_x and F_y must be known for several ω values. Two numerical models aim to calculate those force components. They are compared to an analytical model for the plain annular seal.

2.1. Computational Fluid Dynamics (CFD) model

A CFD model of the runner-seal system is set up in ANSYS CFX 2022 R1. Based on the work by Staubli et al. [7], the axisymmetric geometry is coupled with a circular runner orbit about the concentric position, hence steady-state calculations can be run in the whirling frame of reference (x, y). No-slip conditions are applied on the runner and stator surfaces, with angular frequencies $\Omega_{runner} = \Omega_R - \omega$ and $\Omega_{stator} = -\omega$ in the relative frame of reference. An axial mass flow rate $\dot{m}$ is applied at the inlet, and the static pressure is set to 0 at the outlet. A shear-stress transport (SST) model of turbulence closes the turbulence equations.

2.2. Vibroacoustic FEM model

A numerical acoustic-structural FEM model is also implemented for the simple annular seal of Figure 1.b, in Ansys Mechanical APDL 2022 R1. The cylindrical runner is modeled with a centered point mass linked with rigid connections to an annular surface of outer radius R_{runner}, which interacts with the acoustic elements. This mass is attached by radial and tangential springs. The surrounding confined fluid domain is inviscid and irrotational. Simulations are run in the runner frame of reference, where the fluid rotates relative to the runner, at the angular velocity is $\Omega_{fluid/runner} = \Omega_F - \Omega_R = (1/2 - 1)\Omega_R = -1/2\Omega_R$ as did Berthet et al. [8]. The circular motion $(x_R, y_R) = (\epsilon cos(\omega t), \epsilon sin(\omega t))$ is applied to the runner punctual mass. Harmonic analyses are run for different values of ω. The integration of the pressure gradient yields the radial force $F_x = \pi L R_R \Delta p$, with R_R the runner radius and L its length. There is no phase shift between the force and runner displacement hence $F_y = 0$.

2.3. Analytical model based on potential flow theory

An analytical potential flow model is developed for the plain annular seal. It stems from the potential flow theory for two coaxial cylinders of Blevins [4]. The outer cylinder is fixed, and the inner cylinder motion is $(x_R, y_R) = (\epsilon cos(\omega t), \epsilon sin(\omega t))$. The flow is inviscid and irrotational. The base flow is considered steady-state and purely circumferential, with a uniform magnitude of angular frequency $\Omega_f = 1/2\Omega_R$. The deduced runner radial velocity is $v_{R_r} = \dot{\epsilon} cos\theta - 1/2\Omega_R \epsilon sin\theta$. Non penetration boundary conditions constrain the fluid radial velocity on the runner surface $(v_{f_r}(R_R) = v_{R_r})$ and stator surface $(v_{f_r}(R_S) = 0)$, with R_R, R_S the runner and stator radii. The flow potential ϕ verifies the Laplace equation and the boundary conditions. The Euler equation expresses the pressure distribution p as a function of ϕ. Integration of p in the relative frame of reference (x, y) along the x- and y-axis yields:

$$F_x = \int_0^L \int_0^{2\pi} \cos\theta R_R d\theta dz = \rho \pi L R_R^2 \frac{R_R^2 + R_S^2}{R_S^2 - R_R^2} \left(-\omega^2 + \frac{1}{2}\omega\Omega_R\right)\epsilon, \quad F_y = 0. \tag{6}$$

3. Results and discussion

To verify both numerical models, Figure 2 shows the comparison of F_x/ϵ from the analytical with the results from the numerical methods for the plain annular seal. For all models F_x/ϵ follows the same quadratic trend, although they start to deviate from each other for approximately $\omega \geq 30\ rad/s$. Overall, the similar tendency of F_x/ϵ verifies the numerical models. However both analytical and vibroacoustic models consider an irrotational flow thus the circumferential shear stress, which causes the pressure distribution to generate F_y, is not included. Hence only the CFD model evaluates F_y/ϵ.

Parametric studies are performed on the geometry of Figure 1.c using the CFD model. Figure 3 shows the variation F_x/ϵ and F_y/ϵ in function of ω for different mass flow rates $\dot{m}$, runner angular frequencies Ω_R and radial seal widths C_0. For an increasing $\dot{m}$, F_x/ϵ keeps the same trend while the slope of F_y/ϵ increases. When Ω_R increases, F_x/ϵ and F_y/ϵ decrease, but F_x/ϵ has a more significant curvature while F_y/ϵ approximately keeps the same slope. Finally when

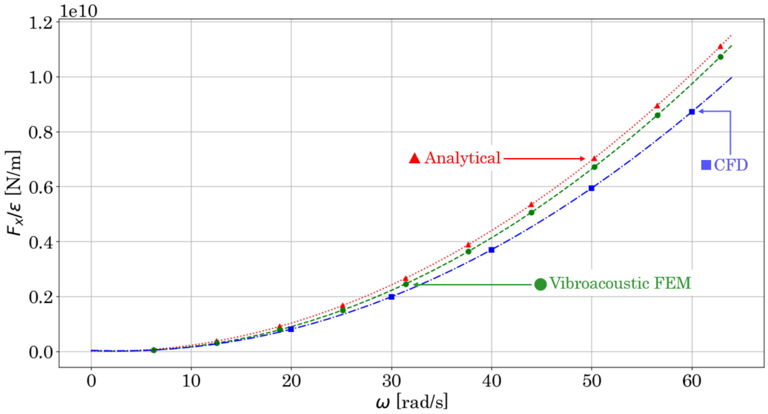

Figure 2: F_x/ϵ from the three models, on the plain annular seal at $\Omega_R = 5$ rad/s, $\dot{m} = 0$ kg/s

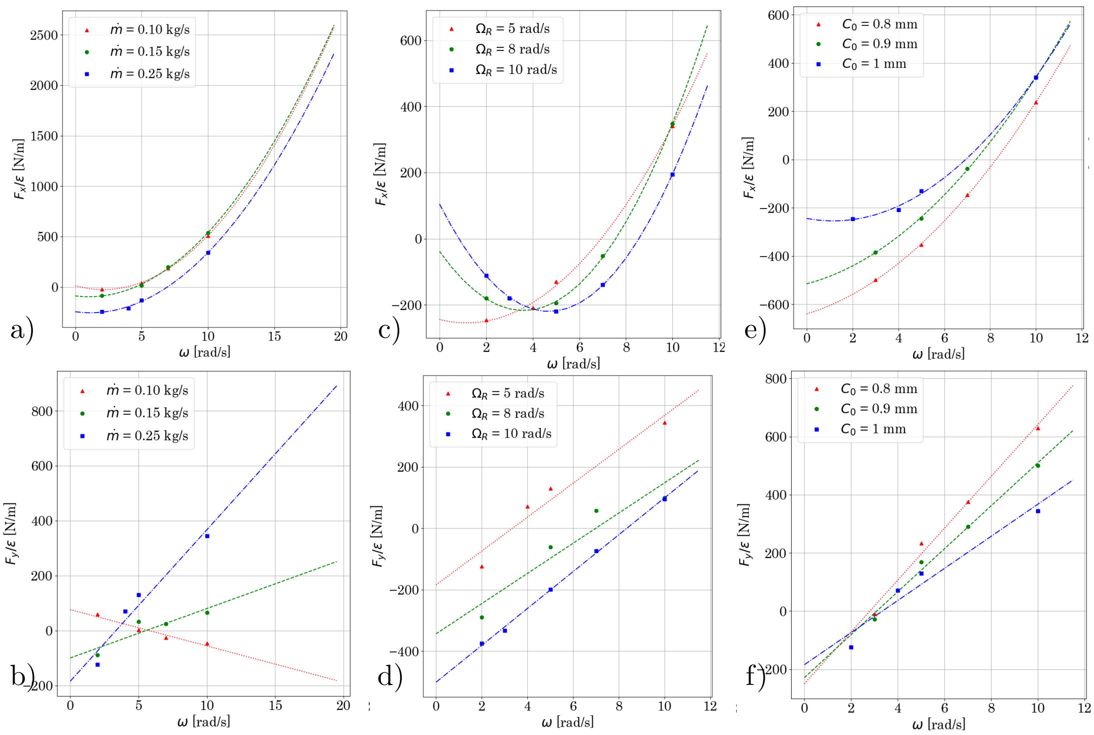

Figure 3: F_x/ϵ and F_y/ϵ as functions of mass flow rate $\dot{m}$ (a)(b), runner angular frequency Ω_R (c)(d), and radial seal width C_0 (e)(f) on the geometry according to Figure 1.c

C_0 increases, F_x/ϵ increases too and F_y/ϵ has a more horizontal slope. Referring to Eq. 5 these tendencies correspond to variations of the dynamic coefficients found by regressions of F_x/ϵ and F_y/ϵ, and summarized in Table 1, Table 2, Table 3. Those tables also include γ_1 and γ_2 defining the stability criteria of Eq. 5. Increasing $\dot{m}$ promotes instability as C, γ_1 and γ_2 become even more negative. The influence of Ω_R is hard to evaluate as C decreases in negative values when Ω_R increases, but γ_1 and γ_2 increase almost up to 0. Finally a larger seal width C_0 reduces the runner instability. The three tables also state that all tested combinations of the various parametric studies are unstable by at least two of the criteria from Eq. 3 C, γ_1 and γ_2.

Table 1: Variation of the runner dynamic coefficients with $\dot{m}$, evaluated with the CFD model

$\dot{m}$ (kg/s)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)	γ_1	γ_2
0.10	8.61	13.23	-12.99	36.37	76.34	-0.43	-0.70
0.15	7.88	-17.99	87.96	16.01	-99.62	-2.26	0.84
0.25	7.65	-55.16	244.54	17.80	-184.34	-10.25	-5.18

Table 2: Variation of the runner dynamic coefficients with Ω_R, evaluated with the CFD model

Ω_R (rad/s)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)	γ_1	γ_2
5	7.65	-55.16	244.54	17.80	-184.34	-10.24	-5.18
10	13.83	-48.67	38.03	99.56	-341.69	-0.90	-0.26
15	14.75	-60.00	-105.03	138.45	-501.00	-0.74	-0.09

Table 3: Variation of the runner dynamic coefficients with C_0, evaluated with the CFD model

C_0 (mm)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)	γ_1	γ_2
0.8	5.92	-89.08	640.18	-28.64	-249.66	-22.38	-13.80
0.9	6.06	-73.84	515.67	-25.00	-228.84	-19.39	-10.57
1.0	7.65	-55.16	244.54	17.80	-184.34	-10.25	-5.18

4. Conclusion

Calculation of F_x/ϵ and F_y/ϵ in function of ω allows for the determination of the runner seal dynamic coefficients by their quadratic and linear regression. Three models of calculation give similar radial forces on a plain annular seal. Decreasing $\dot{m}$, and increasing C_0 improves the runner stability. Further parametric studies against the spiral case width and length are to be performed in the future. Comparisons of the flow in the current seal geometry and in a real Francis turbine geometry will take place, to study the sensitivity of the dynamic coefficients to the geometric simplifications.

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