



Short communication

Effect of friction on the angle of repose of elongated particles

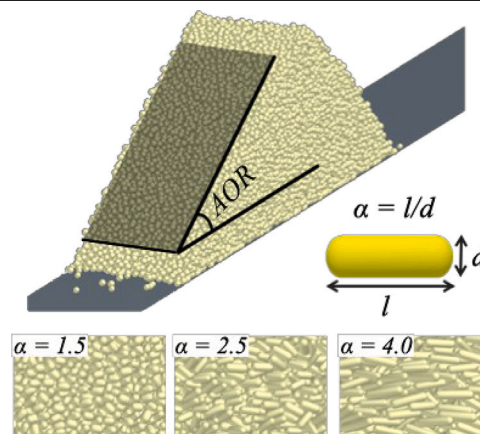
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HIGHLIGHTS

- The AOR exhibits a non-monotonic behavior with particle elongation and friction.
- Higher friction and greater elongation lead to an increase in AOR.
- Critical particle elongation emerges at low friction, beyond which AOR declines.

GRAPHICAL ABSTRACT



ARTICLE INFO

Keywords:

Angle of repose
 Granular matter
 Discrete element modeling
 Particle shape

ABSTRACT

The angle of repose (*AOR*) is a key parameter in powder engineering, primarily influenced by the pouring process, the inter-particle constitutive laws and particle shape. Therefore, the *AOR* of a bulk material is intuitively linked to its macromechanical internal friction angle. Previous studies indicate that more irregular particle shapes, such as angular or elongated particles, increase both shear strength and *AOR*. However, in granular materials with highly irregular shapes and low interparticle friction (or frictionless conditions), this trend reverses for the shear resistance. Despite these results, studies on the *AOR* of such materials are rare and the physical mechanisms of this behavior are unclear. This study investigates the *AOR* of dry, cohesionless granular piles composed of elongated particles using 3D DEM simulations. To represent a wide range of conditions specific to various industrial processes — including pharmaceuticals, food engineering, geotechnics, and mining — we extensively vary particle characteristics, focusing on particle elongation and interparticle friction. The particles, modeled as rounded-cap cylinders, have aspect ratios (length/diameter) ranging from 1 (spheres) to 4. For high interparticle friction ($\mu > 0.2$), the *AOR* increases systematically with elongation. However, at low friction, a critical aspect ratio of 1.5 emerges, beyond which the tendency changes and the *AOR* decreases. We show that this counterintuitive behavior is related to the solid fraction, which depends on particle shape, and can be traced to purely geometrical characteristics such as coordination number and statistical particle orientation. Non-intuitively, the elongation of the particles does not influence the force distributions within the piles.

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1. Introduction

Granular piles are commonly found in various engineering applications, including silos in the food industry, ore stockpiles in mining, earth embankments in geotechnics, landslides in geology, and powder heaps in pharmaceuticals. The primary characteristic parameter of a granular pile is its angle of repose (*AOR*), defined as the angle formed between the pile's surface and the horizontal plane, at which the pile remains mechanically stable. The most frequently observed *AOR* in industrial applications is the one formed by granular sprinkling, where particles accumulate to build a pile on a flat surface.

The *AOR* mainly depends on interparticle friction and cohesion [1–3], particle shape [4] and material humidity [5]. However, the *AOR* is not an intrinsic bulk property, but also depends on the structural (or experimental) boundary conditions, such as the roughness of the basal surface [6,7], the presence of walls in a laterally confined pile [8] and the pile formation process or manner of deposition [9–11].

Intuitively, the *AOR* is closely related to the macromechanical internal friction angle of the bulk material, and numerous studies have suggested that both angles are equivalent [12–14]. However, while their values may be similar, it has been demonstrated that a granular pile can be tilted to an angle greater than the *AOR* before sliding occurs, at which point the internal friction angle is assumed to be fully mobilized [8,15]. Nevertheless, the higher the macromechanical shear strength of the material, the greater the *AOR*.

The effect of particle shape is evident, with rounded ones exhibiting a lower *AOR* compared to irregular shaped ones [16,17], which is consistent with the evolution of the macromechanical strength [18–21]. However, recent studies suggest that the correlation between the macromechanical strength and particle shape could reverse in bulk materials with low interparticle friction [22,23]. Despite this, such a non-monotonic trend has not yet been reported for repose angles. In fact, the few existing studies continue to support the idea that the *AOR* increases for non-spherical particles, approaching an asymptotic value for highly angular or elongated particles [4,24,25]. However, these studies do not systematically investigate the specific implications of interparticle friction on the angle of repose. Moreover, in the case of elongated particles, they primarily consider aspect ratios close to those of spherical particles, limiting the ability to observe the reversal behavior in the *AOR*-elongation relationship.

Motivated by the significance of particle elongation in several industrial applications, such as pharmaceutical powder manufacturing [26,27], biomass conversion processes [28], geotechnics of fossilized soils [29], comminution and grinding of sedimentary [30] or metamorphic [20] rocks, among others, this paper assesses the combined influence of particle elongation and interparticle friction on the angle of repose of granular materials. Particle elongation can vary significantly across these diverse applications. Likewise, contact friction can range from highly rough materials, such as rock clasts or dry biochar, to nearly frictionless scenarios, like smooth pills in pharmaceuticals or submarine landslides due to soil liquefaction in geology [31]. Therefore, with the aim of covering this large diversity of industrial processes in powder technology, our study covers wide ranges of particle elongations and interparticle friction coefficients. We performed 3D Discrete Element Method (DEM) simulations to build virtual, dry granular piles composed of cohesionless, elongated particles. The particles are modeled as cylinders with rounded caps, with aspect ratios (length/diameter) ranging from 1 (spheres) to 4 (elongated particles). The interparticle friction coefficient varied from 0.01 to 1. Granular piles are formed by pouring a constant flow of particles from a fixed domain, and the resulting *AOR* is measured.

We analyze the statistical properties of particle interactions to correlate the macromechanical observations (i.e., *AOR*) with the microstructural features.

2. Methodology

We simulated granular piles composed of elongated cylindrical particles built through gravitational deposition, by means of the Discrete Element Method (DEM) based on Molecular Dynamics [32], also called Soft DEM. The models were implemented in the commercial software EDEM [33], which solves the equations of motion in parallel utilizing graphical processing units (GPUs) for enhanced computational efficiency. EDEM has been validated in both academic [34–37] and industrial [34,38–40] contexts. In Soft DEM, the interactions between particles are modeled using a repulsive force law proportional to their overlap volume or depth. In this model, the normal contact force is based on the Hertzian contact law [41], while the tangential contact force is based on the work of Mindlin–Deresiewicz [42].

The particles are cylinders with rounded caps and characterized by the aspect ratio $\alpha = l/d$, where l is the particle length and d is the diameter of the cylinder and rounded caps, as shown in Fig. 1a. The shape of the particles varies from a sphere ($\alpha = 1$) to a thin tube-like shape for large values of the aspect ratio up to $\alpha = 4$, achieved by increasing the length of the particles while keeping the diameter constant. A particle size polydispersity around the mean size ($\bar{d}$) was used ($0.8\bar{d} < d < 1.2\bar{d}$) to prevent the possible formation of self-ordered granular structures [15], while maintaining the aspect ratio. No rolling friction was considered. The static friction coefficient between particles (μ_s), which we will refer to as the friction coefficient for simplicity, varied systematically from 0.01 to 1.00. The physical and geometric parameters used in the simulations are summarized in Table 1. The Young's modulus, Poisson's ratio and density apply to both the particles and the bottom wall of the system. Additionally, the restitution coefficient refers to both particle–particle and particle–bottom wall interactions. These material properties are based on values close to those used in similar studies [24,35]. Regardless, for dry and rigid particles, previous numerical and experimental studies have shown that the angle of repose remains unaffected by damping, Poisson's ratio, Young's modulus, or the restitution coefficient [3,17].

The simulations consist of a rectangular virtual box for the particles to be deposited, with periodic boundary conditions along the x -axis (defined in Fig. 1b). No constraints along the y -axis are imposed, while a rigid wall constraints the base of the pile (ground); the friction coefficient between the bottom wall and the particles was fixed at 1. The periodic length was set to be five times the length of the longest particles (i.e., $W = 5l_{max}$ or $20\bar{d}$). The particles are deposited by gravity ($g = -9.8 \text{ m/s}^2$) from an initial height $H_0 = 140\bar{d}$. They are generated within a fixed cuboid domain in a random manner (both in position and orientation) at a rate of 4×10^3 particles per second. The generation domain is also periodic along the x -axis, with a period W , and has dimensions of $u = 10\bar{d}$ along the y -axis and $h = 40\bar{d}$ along the z -axis. After generation and deposition, the particles form a pyramidal shape, as shown in Fig. 1b, allowing the measurement of the angle of repose (*AOR*) on both sides of the pyramid. A total of 42 simulations were performed, each containing more than 5×10^4 particles. Once the deposition of the particles ended, stability of the pile was assessed by monitoring the ratio of kinetic energy to potential energy, with the system considered stable when this ratio fell below 10^{-5} . Fig. 1b illustrates the simulation setup, including boundary conditions and initial particle placement. Fig. 1c shows internal snapshots of the piles for three different aspect ratios.

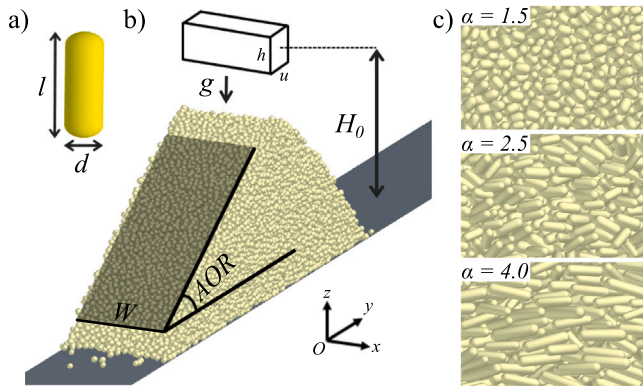


Fig. 1. Numerical pile deposition setup and definition of the parameters: (a) dimensions of the cylindrical particles, showing length (l) and mean diameter (d); (b) final stage of the granular pile; (c) internal snapshots of the piles for aspect ratios $\alpha = 1.5$, 2.5 and 4.0.

Table 1
Simulation parameters.

Parameter	Symbol	Value	Unit
Young modulus	E	$1.3 \cdot 10^8$	Pa
Poisson ratio	ν_p	0.3	–
Density	ρ	$3 \cdot 10^3$	kg/m ³
Particle mean diameter	$\bar{d}$	$2 \cdot 10^{-3}$	m
Restitution coefficient	C_r	0.2	–
Rolling friction coefficient	μ_r	0	–
Width of the box, x-axis periodic condition	W	$20\bar{d}$	m
Initial height	H_0	$140\bar{d}$	m
Gravity acceleration	g	-9.8	m/s ²
Number of Particles	N	$5 \cdot 10^4$	–

3. Results and discussion

3.1. Angle of repose and bulk properties

The AOR is defined as the slope angle formed between the surface of the pile and the x - y plane, at which the pile remains mechanically stable after the deposition (see Fig. 1). Using the SciPy Python library [43], the surface points of the pile are extracted, one point per particle, on both sides of the pile. The AOR is then calculated by fitting these points to a plane equation using the least squares method. Since the surface of the pile is not a perfect smooth plane [44,45], only particles located between 10% and 90% of the maximum height of the pile are considered in the calculation. For a given pile, the mean value of the two slopes is presented in Fig. 2a, with the error bars given by the uncertainty in the fitting plane equation parameters. This method ensures a robust estimate of the angle, minimizing the influence of outliers or irregularities near the base or peak of the pile.

Fig. 2a presents the AOR as a function of α for all the values of μ_s used. In all the cases, the AOR of slightly elongated particles (i.e., $\alpha = 1.5$) increases compared to spherical particles ($\alpha = 1$). Interestingly, for $\mu_s = 0.2$, the AOR does not exhibit significant variation as a function of α , maintaining a value of approximately 22° for all α from 1.5 to 4.0. For $\alpha > 1.5$, the AOR of piles with relatively high friction ($\mu_s > 0.2$) continues to increase asymptotically to $\sim 30^\circ$ and 35° for μ_s of 0.4 and 1.0, respectively. Most remarkable is that for $\alpha > 1.5$, the AOR decreases in all cases with low friction ($\mu_s < 0.2$). Thus, a local maximum is observed at $\alpha = 1.5$, which contradicts previous works that hypothesized an asymptotically increasing AOR with α , regardless of the friction coefficient [4,24]. For $\mu_s = 0.01$, the AOR of $\alpha = 4.0$ reaches as low as $\sim 5^\circ$. Furthermore, in Fig. 2b the AOR is normalized by the reference repose angle of piles composed by spherical particles ($AOR_{\alpha=1.0}$) for each friction coefficient. Two distinct behaviors are evident: (i) low μ_s (< 0.2): decreasing AOR below the value of spherical

particles for high particle elongation; (ii) high μ_s (> 0.2): increasing AOR with α , always above that of spherical particles.

Although the AOR measures the static stability of a granular system, it is closely related to the Mohr–Coulomb critical friction angle (ϕ_{cr}) of a sheared system, which quantifies its dynamic resistance during flow [12,13]. Previous studies on ϕ_{cr} of elongated particles under quasi-static shear reported a behavior similar to that observed in this study [23,46], with a local maximum at $\alpha \approx 1.5$. Moreover, this non-monotonic shear strength behavior is also observed for other particle shapes, such as angular particles [22], which quantifies the sharpness of particles based on the number of edges and corners. For particles with increasing angularity (i.e., from spherical to highly angular), a peak in ϕ_{cr} is observed, followed by either a decrease or an increase, depending on the friction coefficient. These findings are analogous to our results on AOR and indicate that particle shape plays a crucial role in determining the mechanical stability of granular systems, with optimal particle shape values existing for maximum resistance.

This counterintuitive trend in AOR may have a predominantly geometric origin and is highly correlated with the solid fraction of the piles. Fig. 3 shows the solid fraction (ν) of the piles as a function of α . A maximum ν is observed at $\alpha \approx 1.5$, with the pile progressively becoming looser as particle elongation increases. This trend holds for all friction coefficients studied, with the expected observation that higher friction coefficients lead to lower ν . For the most elongated particles under high-friction conditions, ν decreases to approximately 80% of the value observed for spherical particles. This behavior is a well-established phenomenon in the packing of elongated particles [47,48]. For instance, the maximum random packing density of ellipsoids occurs at an aspect ratio of approximately 1.5, followed by a decrease in density as elongation increases. A higher packing density enhances pile stability, leading to larger angles of repose, which also reach a maximum around $\alpha = 1.5$, consistent with the trend of ν .

3.2. Microstructural description

To gain insight into the observations at the pile scale, we analyze the geometrical and structural properties at the particle scale. First, we examine the coordination number (Z), defined as the mean number of contacts per particle. Fig. 4 shows its variation as a function of α . Interestingly, Z increases asymptotically to values that depend on μ_s , despite the reduction in ν . As particles become more elongated, their arrangement during deposition changes, allowing them to come into contact with a greater number of neighbors. This structural reorganization enables higher coordination numbers even in looser packings.

A second parameter describing the microstructure is the orientation angle of the major axis of a particle relative to the ground (i.e., the x - y plane, using the same global coordinate system as in Fig. 1). Fig. 5a shows a sketch of this orientation angle (Ψ) ranging from 0° to 90° , where 0° corresponds to the major axis of the particle being parallel to the ground, and 90° corresponds to the major axis being aligned with the z -axis. The Probability Density Functions (PDFs) of Ψ for three values of α are shown in Fig. 6a: $\alpha = 1.5$ (the smallest value studied at which orientation can be measured), $\alpha = 4.0$ (the largest one studied), and $\alpha = 2.5$ as an intermediate value. Logically, Ψ cannot be evaluated in spherical particles ($\alpha = 1.0$). With $\alpha = 1.5$, the PDFs are broad and show long tails, especially for high friction coefficients. Some particles reach an orientation of even 90° , meaning they are parallel to the z -axis. For lower friction values, the tails of the distributions decrease, with a greater number of particles oriented parallel to the ground, which is also related to a lower AOR. As α increases in Fig. 6a (i.e., $\alpha = 2.5$ and 4), a greater number of particles are oriented parallel to the ground, and this is inversely proportional to μ_s . As the particles become more elongated, they tend to rotate parallel to the ground, where the only force preventing the rotation is the frictional force between the particles. Thus, as this force decreases and elongation

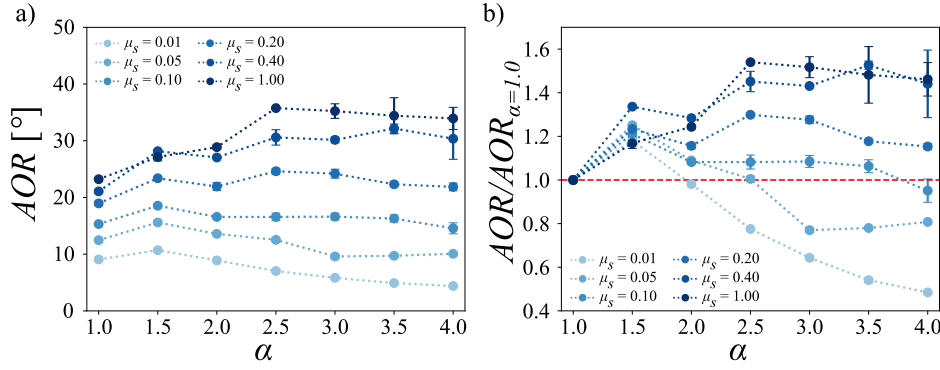


Fig. 2. Repose angle (AOR) as a function of the aspect ratio for different friction coefficients (μ_s). (b) Ratio between the AOR and the repose angle for spherical particles ($AOR_{\alpha=1.0}$) as a function of α . Both plots are shown for different μ_s .

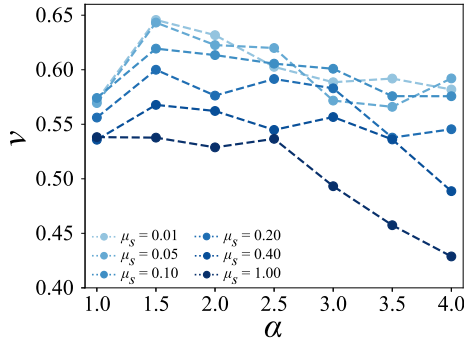


Fig. 3. Solid fraction of the piles, v , as a function of the aspect ratio, α , for different interparticle friction coefficients, μ_s .

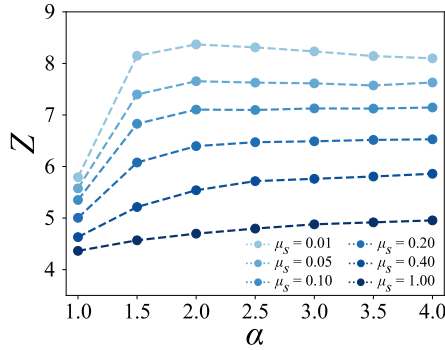


Fig. 4. Mean coordination number, Z , as a function of the aspect ratio, α , for different interparticle friction coefficients, μ_s .

increases, it becomes more likely to find the particles oriented in this manner. In contrast, high-friction conditions allow the particles to resist this natural rotation, resulting in a more stable pyramid-like structure that exhibits a higher AOR. Interestingly, as shown in Fig. 6b, which presents the mean particle orientation angle ($\langle\psi\rangle$) as a function of α for different μ_s , this trend in the orientation of the particles decreases rapidly with increasing α , reaching an asymptotic minimum that depends mainly on the friction coefficient rather than α .

These observations reflect the interplay between frictional locking and particle shape, which jointly contribute to the pile's ability to sustain steeper angles or, conversely, collapse under its own weight. Notably, the difference between the AOR and $\langle\psi\rangle$ follows a quasi-linear dependency on α (Fig. 6c). This relationship is particularly insightful

at low friction values, where the difference is negative, indicating that the particles lie flatter on average than the overall AOR suggests. The contrast explains why the piles formed at low μ_s result in looser packings compared to less elongated particles, as seen in Fig. 3. With increasing μ_s , the difference between the mean particle orientation and AOR increases, suggesting a direct link between particle elongation, friction coefficient and packing stability.

To track the intrinsic behavior of the AOR of elongated particles, here we analyze the micromechanics of the granular system. We examine the behavior of all the interparticle contact forces ($\vec{f}^{(i,j)} = -\vec{f}^{(j,i)}$, where i and j are the particles in contact) through their two components: the normal contact force (f_n) and the tangential contact force (f_t). Fig. 7a and b show the PDFs of f_n and f_t , respectively, both normalized by their respective mean values, for the same three aspect ratios as in Fig. 6a and across the different friction coefficients studied. A key observation is that these distributions do not show significant dependence on μ_s or, counterintuitively, on α . Instead, all the PDFs remain consistent with the typical force distributions observed in piles of spherical particles [4,24,25]. This invariance suggests that, despite the geometric complexity introduced by particle elongation, the force transmission mechanisms remain unaffected by changes in shape or friction. In other words, the system appears to rely on collective force chains that maintain similar statistical properties, regardless of the specific micromechanical details.

An alternative perspective comes from the statistical analysis of the orientation of the interparticle contact force vector, characterized by its polar angle θ . Following the same global reference frame shown in Fig. 1, θ is defined as the angle between $\vec{f}^{(i,j)}$ and the z-axis, which serves as the polar axis in the physics convention (see Fig. 5b). Fig. 8a presents the PDFs of θ for $\mu_s = 0.01$, 0.10, and 1.0 across all α . These friction values were chosen to provide a global comparison of the behavior: $\mu_s = 0.01$ and $\mu_s = 1.00$ are the minimum and maximum friction values studied, while $\mu_s = 0.10$ exhibits an intermediate behavior. In Fig. 8a, note that the angle θ ranges from 0° to 90° , as θ is symmetric at $\theta = 90^\circ$ considering that $\vec{f}^{(i,j)} = -\vec{f}^{(j,i)}$. For a given μ_s , the distributions remain remarkably similar, showing a small influence of α . For low friction values, the contact angle distribution peaks near zero degrees, indicating a predominantly horizontal alignment of the particles in contact. As μ_s increases, this peak shifts toward larger θ . As illustrated in Fig. 8b, the mean value of θ is directly proportional to μ_s and exhibits an asymptotic behavior with respect to α . This trend suggests that higher friction promotes more angular contact configurations, hindering horizontal alignment and enabling the formation of steeper and structurally robust arrangements. The observed asymptotic relationship between α and the mean contact angle indicates that, beyond a certain particle elongation, the system's angular rearrangements become less sensitive to further changes in α , stabilizing the contact structure.

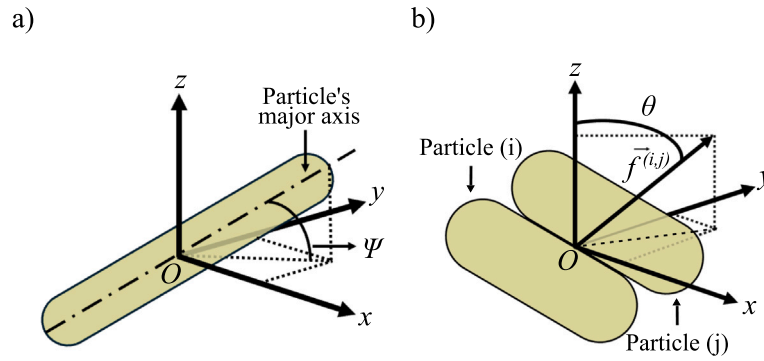


Fig. 5. Sketch defining (a) the orientation angle (Ψ) of the particle's major axis respect to the ground (i.e., $x-y$ plane), and (b) defining the polar angle (θ) of the interparticle contact force ($\hat{f}^{(i,j)}$) between particle i and particle j . Both definitions utilize the same global reference frame $x-y-z$ shown in Fig. 1.

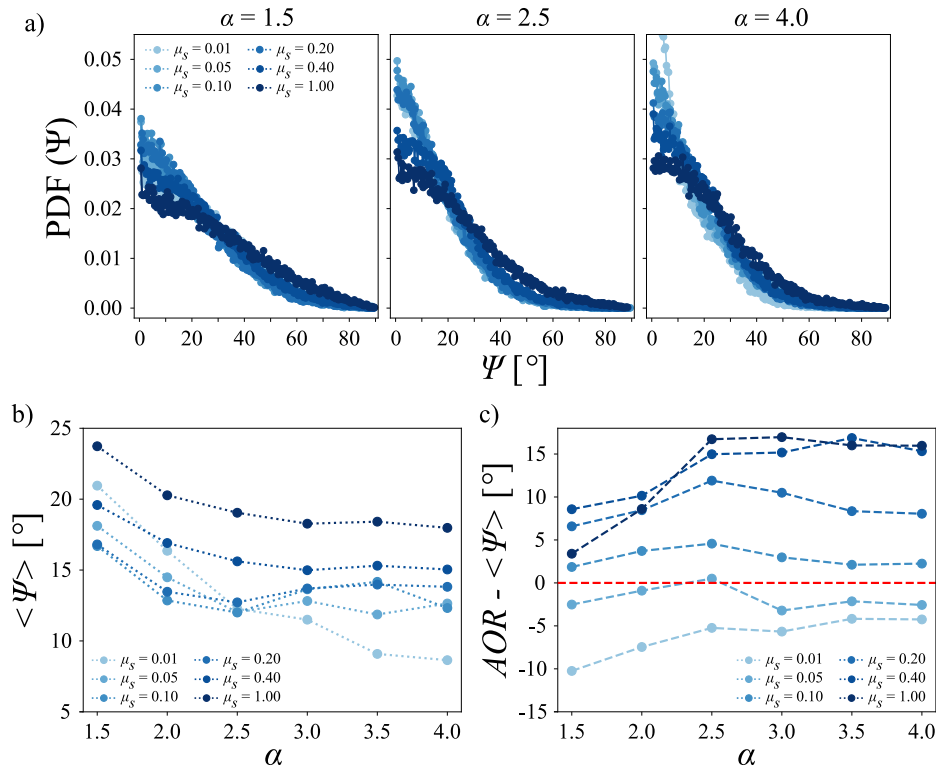


Fig. 6. (a) Probability density function, PDF, of the orientation angle of the particles, Ψ , for different friction coefficients, μ_s , and aspect ratios $\alpha = 1.5, 2.5$, and 4.0 . (b) Mean orientation angle of the particles as a function of the aspect ratio. (c) Difference between the repose angle, AOR , and the mean orientation angle as a function of the aspect ratio. $\langle \cdot \rangle$ stands for the mean value.

4. Conclusions

We investigated the angle of repose (AOR) of frictional, dry, cohesionless granular piles composed of elongated particles using 3D Discrete Element Method simulations. The particles were modeled as cylinders with rounded caps, varying in aspect ratio from nearly spherical to elongated tube-like shapes.

This study highlights the complex interplay between particle elongation, interparticle friction and granular pile stability. The AOR showed a nonmonotonic behavior: while it systematically increases with particle elongation for high interparticle friction, a critical aspect ratio of ~ 1.5 emerges for low interparticle friction, beyond which the AOR decreases, leading to flatter piles. This critical value may originate from the optimal packing of the elongated particles, which also reaches a maximum at ~ 1.5 .

Under low friction conditions, elongated particles referentially align horizontally, reducing the AOR while forming a denser packing with

higher coordination numbers. Conversely, high friction conditions restrict rotational mobility, resulting in steeper pile structures with higher particle orientation angles. Surprisingly, the overall force distribution remains unaffected by variations in particle elongation or friction coefficients.

These findings provide deeper insights into how particle shape and interparticle friction jointly influence granular stability, with potential applications in industries handling irregular shaped particles or powders. Our results could encompass a wide range of conditions relevant to various processes, including pharmaceuticals, food engineering, geotechnics and mining, where diverse particle shapes and interparticle friction are frequently observed. Further studies should explore additional factors such as cohesion, irregular shaped particles, composite-shaped particles and nonconvex particle geometries, to better understand granular stability in real-world scenarios.

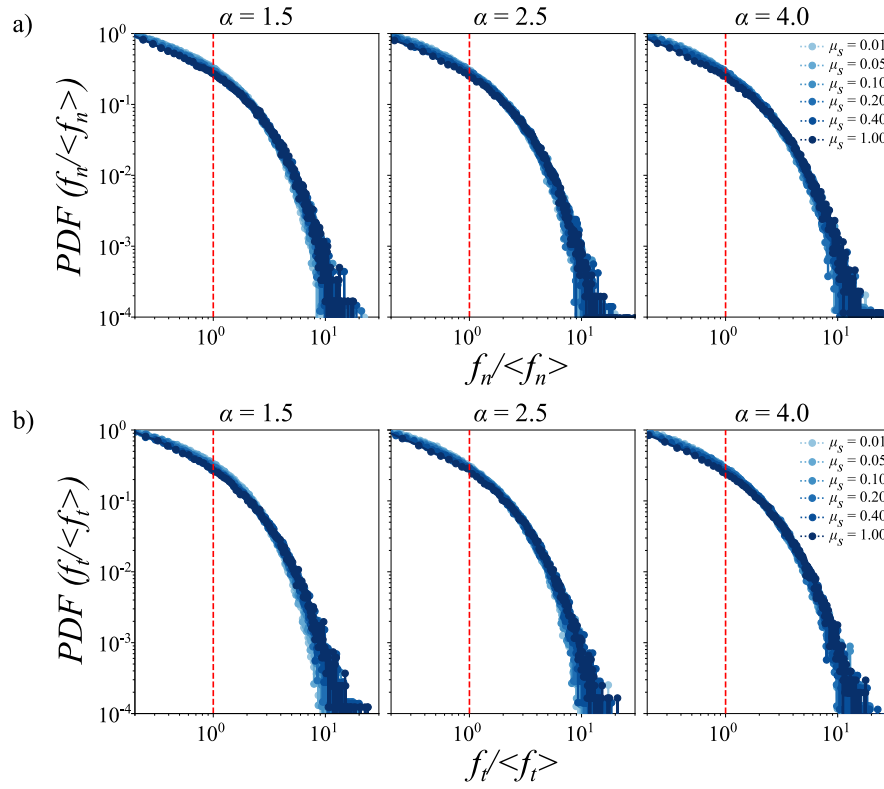


Fig. 7. Probability density functions, PDFs, of (a) the normal contact forces, normalized by their respective mean values, $f_n / \langle f_n \rangle$, and (b) the tangential contact forces, normalized by their respective mean values, $f_t / \langle f_t \rangle$, for different friction coefficients. The same values of aspect ratio, α , as in Fig. 6a are used for comparison.

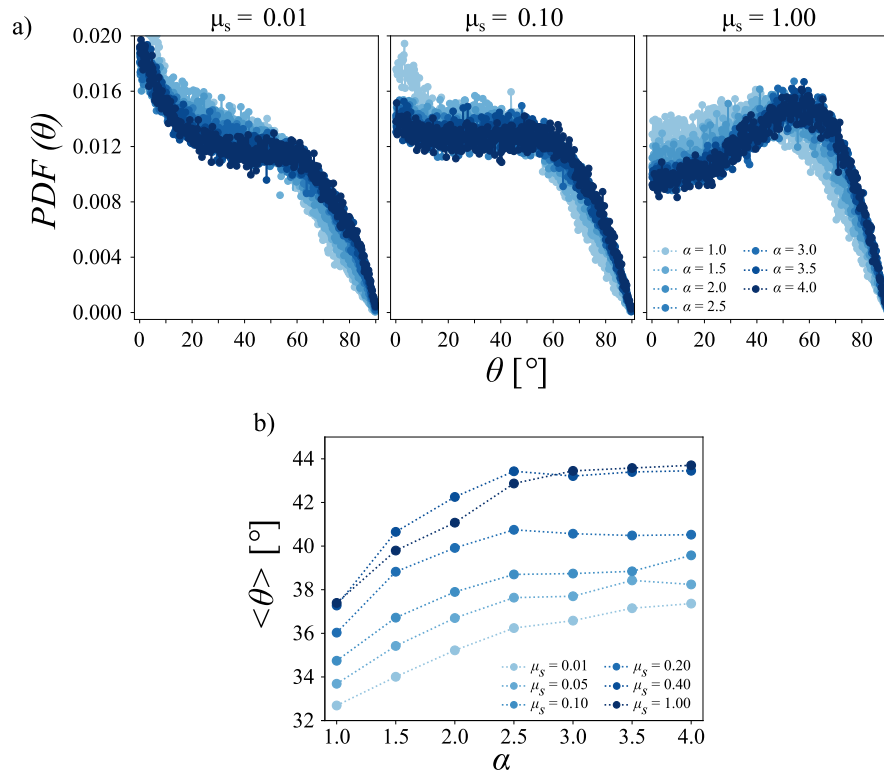


Fig. 8. (a) Probability density function, PDF, of the polar angle θ of the contact force vector for $\mu_s = 0.01, 0.10$, and 1.00 , across different aspect ratios, α . (b) Mean value of θ , $\langle \theta \rangle$, as a function of the aspect ratio for different friction coefficients.

CRediT authorship contribution statement

Manuel Cárdenas-Barrantes: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Carlos Ovalle:** Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve the readability of the text.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This study benefited from financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC) [projects ALLRP 568607 – 21 and ALLRP 590599 - 23], the Consortium de recherche et d'innovation en transformation métallique (CRITM) [project 2021 – 068] and the industrial partners of the Research Institute on Mines and Environment (RIME) UQAT-Polytechnique (irme.ca/en).

Data availability

Data will be made available on request.

References

- [1] J. Carstensen, P.-C. Chan, Relation between particle size and repose angles of powders, *Powder Technol.* 15 (1) (1976) 129–131, [http://dx.doi.org/10.1016/0032-5910\(76\)80037-X](http://dx.doi.org/10.1016/0032-5910(76)80037-X).
- [2] J. Zegzulka, F. Maršák, J. Nečas, J. Hlosta, D. Žurovec, I. Ivan, Thermodynamic theory of the most energy-efficient natural repose angle, *Contin. Mech. Thermodyn.* 35 (2) (2023) 393–412, <http://dx.doi.org/10.1007/s00161-023-01184-1>.
- [3] Y. Zhou, B. Xu, A. Yu, P. Zulli, An experimental and numerical study of the angle of repose of coarse spheres, *Powder Technol.* 125 (1) (2002) 45–54, [http://dx.doi.org/10.1016/S0032-5910\(01\)00520-4](http://dx.doi.org/10.1016/S0032-5910(01)00520-4).
- [4] Z.Y. Zhou, R.P. Zou, D. Pinson, A.B. Yu, Angle of repose and stress distribution of sandpiles formed with ellipsoidal particles, *Granul. Matter* 16 (5) (2014) 695–709, <http://dx.doi.org/10.1007/s10035-014-0522-4>.
- [5] H. Kalman, Effect of moisture content on flowability: Angle of repose, tilting angle, and Hausner ratio, *Powder Technol.* 393 (2021) 582–596, <http://dx.doi.org/10.1016/j.powtec.2021.08.010>.
- [6] J. Carstensen, T.-F. Lai, P. Toure, J. Sheridan, Repose angles as a function of the supporting surface, *Int. J. Pharm.* 5 (2) (1980) 157–160, [http://dx.doi.org/10.1016/0378-5173\(80\)90020-4](http://dx.doi.org/10.1016/0378-5173(80)90020-4).
- [7] Z. Chik, L.E. Vallejo, Characterization of the angle of repose of binary granular materials, *Can. Geotech. J.* 42 (2) (2005) 683–692, <http://dx.doi.org/10.1139/t04-118>.
- [8] Y. Grasselli, H. Herrmann, On the angles of dry granular heaps, *Phys. A* 246 (3) (1997) 301–312, [http://dx.doi.org/10.1016/S0378-4371\(97\)00326-9](http://dx.doi.org/10.1016/S0378-4371(97)00326-9).
- [9] Y. Nakata, S. Moriguchi, S. Kajiyama, R. Kido, N. Kikkawa, H. Saomoto, D. Takano, Y. Higo, Experimental data of 3D printed granular material for verification of discrete element modeling simulation, *Soils Found.* 62 (4) (2022) 101178, <http://dx.doi.org/10.1016/j.sandf.2022.101178>.
- [10] S. Duverger, V. Angelidakis, S. Nadimi, S. Utili, S. Bonelli, P. Philippe, J. Duriez, Investigation techniques and physical aspects of the angle of repose of granular matter, *Granul. Matter* 26 (1) (2024) 20, <http://dx.doi.org/10.1007/s10035-023-01378-z>.
- [11] B. MacIver, The Formation and Initial Stability of Slopes on Cohesionless Materials, in: AD-a392 609, Army engineer nuclear cratering Group Livermore ca, 1967.
- [12] J. Santamarina, G. Cho, Determination of critical state parameters in sandy soils—Simple procedure, *Geotech. Test. J.* 24 (2) (2001) 185–192, <http://dx.doi.org/10.1520/GTJ11338J>.
- [13] J. Metcalf, Angle of repose and internal friction, *Int. J. Rock Mech. Min. Sci. Géoméch. Abstr.* 3 (2) (1966) 155–161, [http://dx.doi.org/10.1016/0148-9062\(66\)90005-2](http://dx.doi.org/10.1016/0148-9062(66)90005-2).
- [14] J. Lee, H.J. Herrmann, Angle of repose and angle of marginal stability: Molecular dynamics of granular particles, *J. Phys. A Math. Gen.* 26 (2) (1993) 373, <http://dx.doi.org/10.1088/0305-4470/26/2/021>.
- [15] Y.F. Bruno Andreotti, O. Pouliquen, *GRANULAR MEDIA, Between Fluid and Solid*, Cambridge University Press, London, 2013.
- [16] B.-B. Dai, J. Yang, C.-Y. Zhou, Micromechanical origin of angle of repose in granular materials, *Granul. Matter* 19 (2) (2017) 24, <http://dx.doi.org/10.1007/s10035-017-0709-6>.
- [17] H.M. Beakawi Al-Hashemi, O.S. Baghabra Al-Amoudi, A review on the angle of repose of granular materials, *Powder Technol.* 330 (2018) 397–417, <http://dx.doi.org/10.1016/j.powtec.2018.02.003>.
- [18] G.-C. Cho, J. Dodds, J.C. Santamarina, Particle shape effects on packing density, stiffness, and strength: Natural and crushed sands, *J. Geotech. Geoenviron. Eng.* 132 (5) (2006) 591–602, [http://dx.doi.org/10.1061/\(ASCE\)1090-0241\(2006\)132:5\(591\)](http://dx.doi.org/10.1061/(ASCE)1090-0241(2006)132:5(591)).
- [19] Y. Xiao, L. Long, T.M. Evans, H. Zhou, H. Liu, A.W. Stuedlein, Effect of particle shape on stress-dilatancy responses of medium-dense sands, *J. Geotech. Geoenviron. Eng.* 145 (2) (2019) 04018105, [http://dx.doi.org/10.1061/\(ASCE\)GT.1943-5606.0001994](http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0001994).
- [20] C. Ovalle, C. Dano, Effects of particle size–strength and size–shape correlations on parallel grading scaling, *Geotech. Lett.* 10 (2) (2020) 191–197, <http://dx.doi.org/10.1680/jgele.19.00095>.
- [21] S. Carrasco, D. Cantor, C. Ovalle, F. Dubois, Particle shape distribution effects on the critical strength of granular materials, *Comput. Geotech.* 177 (2025) 106896, <http://dx.doi.org/10.1016/j.compgeo.2024.106896>.
- [22] T. Binaree, E. Azéma, N. Estrada, M. Renouf, I. Preehawuttipong, Combined effects of contact friction and particle shape on strength properties and microstructure of sheared granular media, *Phys. Rev. E* 102 (2020) 022901, <http://dx.doi.org/10.1103/PhysRevE.102.022901>.
- [23] U. Ali, M. Kikumoto, Role of particle shape in sheared granular media: roundness and elongation, *Powder Technol.* 449 (2025) 120436, <http://dx.doi.org/10.1016/j.powtec.2024.120436>.
- [24] H. Chen, S. Zhao, X. Zhou, DEM investigation of angle of repose for super-ellipsoidal particles, *Particuology* 50 (2020) 53–66, <http://dx.doi.org/10.1016/j.partic.2019.05.005>.
- [25] H. Zhao, X. An, D. Gou, B. Zhao, R. Yang, Attenuation of pressure dips underneath piles of spherocylinders, *Soft Matter* 14 (2018) 4404–4410, <http://dx.doi.org/10.1039/C8SM00280K>.
- [26] M.-N. Pons, H. Vivier, V. Delcour, J.-R. Authelin, L. Paillères-Hubert, Morphological analysis of pharmaceutical powders, *Powder Technol.* 128 (2) (2002) 276–286, [http://dx.doi.org/10.1016/S0032-5910\(02\)00177-8](http://dx.doi.org/10.1016/S0032-5910(02)00177-8), 3rd French Colloquium on Powder Science & Technology.
- [27] W. Yu, L. Liao, R. Bharadwaj, B.C. Hancock, What is the “typical” particle shape of active pharmaceutical ingredients? *Powder Technol.* 313 (2017) 1–8, <http://dx.doi.org/10.1016/j.powtec.2017.02.043>.
- [28] P.N. Ciesielski, M.F. Crowley, M.R. Nimlos, A.W. Sanders, G.M. Wiggins, D. Robichaud, B.S. Donohoe, T.D. Foust, Biomass particle models with realistic morphology and resolved microstructure for simulations of intraparticle transport phenomena, *Energy Fuels* 29 (1) (2015) 242–254, <http://dx.doi.org/10.1021/ef502204v>.
- [29] C. Ovalle, G. Arenaldi-Perisic, Mechanical behaviour of undisturbed diatomaceous soil, *Mar. Georesources Geotechnol.* 39 (5) (2021) 623–630, <http://dx.doi.org/10.1080/1064119X.2020.1720049>.
- [30] S. Linero, S. Fityus, J. Simmons, A. Lizcano, J. Cassidy, Trends in the evolution of particle morphology with size in colluvial deposits overlying channel iron deposits, *EPJ Web Conf.* 140 (2017) 14005, <http://dx.doi.org/10.1051/epjconf/201714014005>.
- [31] J. Locat, H.J. Lee, Submarine landslides: Advances and challenges, *Can. Geotech. J.* 39 (1) (2002) 193–212, <http://dx.doi.org/10.1139/t01-089>.
- [32] P.A. Cundall, O.D.L. Strack, A discrete numerical model for granular assemblies, *Géotechnique* 29 (1) (1979) 47–65, <http://dx.doi.org/10.1680/geot.1979.29.1.47>.
- [33] Altair Engineering Inc., EDEM, 2025, <https://altair.com/edem>.
- [34] R. Balevičius, R. Kačianauskas, Z. Mróz, I. Sielamowicz, Analysis and DEM simulation of granular material flow patterns in hopper models of different shapes, *Adv. Powder Technol.* 22 (2) (2011) 226–235, <http://dx.doi.org/10.1016/j.appt.2010.12.005>, Special issue of the 6th World Congress on Particle Technology.
- [35] Y. Tan, Y. Yu, J. Fottner, S. Kessler, Automated measurement of the numerical angle of repose (aMAoR) of biomass particles in EDEM with a novel algorithm, *Powder Technol.* 388 (2021) 462–473, <http://dx.doi.org/10.1016/j.powtec.2021.04.062>.
- [36] O. Kirstein, C.M. Wensrich, Evolution of a contact force network in a 2D granular assembly: II—the impact of particle plasticity, *Granul. Matter* 27 (1) (2025) 25, <http://dx.doi.org/10.1007/s10035-025-01504-z>.

- [37] M. Li, Y. Yu, H. Saxén, Numerical and experimental study of particle shape effect on pile properties based on a 3D sinter model, *Powder Technol.* 452 (2025) 120496, <http://dx.doi.org/10.1016/j.powtec.2024.120496>.
- [38] S.B. Yeom, E.-S. Ha, M.-S. Kim, S.H. Jeong, S.-J. Hwang, D.H. Choi, Application of the discrete element method for manufacturing process simulation in the pharmaceutical industry, *Pharm.* 11 (8) (2019) <http://dx.doi.org/10.3390/pharmaceutics11080414>.
- [39] G. Wang, W. Zhang, M. Ji, H. Miao, Z. Jin, Numerical simulation and parameter optimization of earth auger in hilly area using EDEM software, *Sci. Rep.* 12 (1) (2022) 19526, <http://dx.doi.org/10.1038/s41598-022-23833-2>.
- [40] H. Yuan, S. Liang, J. Wang, Y. Lu, Numerical simulation and analysis of vibrating rice filling based on EDEM software, *Agric.* 12 (12) (2022) <http://dx.doi.org/10.3390/agriculture12122013>.
- [41] H. Hertz, Ueber die Berührung fester elastischer Körper, *J. Reine Angew. Math.* 1882 (92) (1882) 156–171, <http://dx.doi.org/10.1515/crll.1882.92.156>.
- [42] R.D. Mindlin, H. Deresiewicz, Elastic spheres in contact under varying oblique forces, *J. Appl. Mech.* 20 (3) (1953) 327–344, <http://dx.doi.org/10.1115/1.4010702>.
- [43] P. Virtanen, R. Gommers, T.E. Oliphant, M. Haberland, T. Reddy, D. Cournapeau, E. Burovski, P. Peterson, W. Weckesser, J. Bright, S.J. van der Walt, M. Brett, J. Wilson, K.J. Millman, N. Mayorov, A.R.J. Nelson, E. Jones, R. Kern, E. Larson, C.J. Carey, Í. Polat, Y. Feng, E.W. Moore, J. VanderPlas, D. Laxalde, J. Perktold, R. Cimrman, I. Henriksen, E.A. Quintero, C.R. Harris, A.M. Archibald, A.H. Ribeiro, F. Pedregosa, P. van Mulbregt, SciPy 1.0 Contributors, SciPy 1.0: Fundamental algorithms for scientific computing in python, *Nat. Methods* 17 (2020) 261–272, <http://dx.doi.org/10.1038/s41592-019-0686-2>.
- [44] M. Rackl, F.E. Grötsch, M. Rusch, J. Fottner, Qualitative and quantitative assessment of 3D-scanned bulk solid heap data, *Powder Technol.* 321 (2017) 105–118, <http://dx.doi.org/10.1016/j.powtec.2017.08.009>.
- [45] J. Frączek, A. Złobicki, J. Zemanek, Assessment of angle of repose of granular plant material using computer image analysis, *J. Food Eng.* 83 (1) (2007) 17–22, <http://dx.doi.org/10.1016/j.jfoodeng.2006.11.028>, Future of Food Engineering.
- [46] E. Azéma, F. Radjai, Stress-strain behavior and geometrical properties of packings of elongated particles, *Phys. Rev. E* 81 (2010) 051304, <http://dx.doi.org/10.1103/PhysRevE.81.051304>.
- [47] S.R. Williams, A.P. Philipse, Random packings of spheres and spherocylinders simulated by mechanical contraction, *Phys. Rev. E* 67 (2003) 051301, <http://dx.doi.org/10.1103/PhysRevE.67.051301>.
- [48] A. Wouterse, S.R. Williams, A.P. Philipse, Effect of particle shape on the density and microstructure of random packings, *J. Phys. Condens. Matter.* 19 (40) (2007) 406215, <http://dx.doi.org/10.1088/0953-8984/19/40/406215>.