Appendix A – Supplementary Information for: Fatigue Strength Prediction of 410NiMo Stainless Steel with Surrogate Weld Discontinuities

S1. Sketches of weld bead deposition

Figure S.1(a) shows a sketch of a valley (No.8) formed between two weld beads. Flux residues were distributed in the valley along the weld root, and these residues were subsequently covered by multiple weld beads. **Figure S.1(b)** illustrates a sketch of the weld bead deposition without discontinuities, which was performed on the opposite side of the plate.

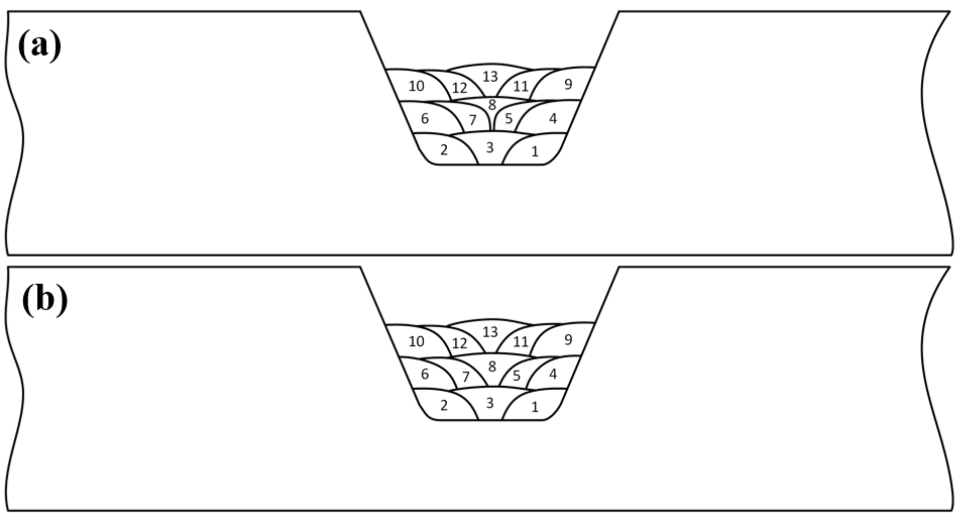

Figure S.1 Weld beads deposition sketch (a) with the discontinuity represented by No.8, and (b) without the discontinuity.

S2. Details of $\Delta K_{\mathrm{th,n}}$ calculation

Table S.1 presents the fatigue strength of a smooth specimen ($\Delta\sigma_0$) and the threshold stress intensity factor range (ΔK_{th}) of the 410NiMo welded material.

Table S.1 Parameters for 410NiMo at $N = 2E^6$ cycles and $R = 0.1$.

$\Delta\sigma_0$ [MPa] [9]	ΔK_{th} [MPa$\sqrt{\mathrm{m}}$] [32]
520	5.16

The experimental results from Akbarian [9] demonstrated that the notch stress intensity factor (NSIF) theory is applicable for microdrilled hemispherical notches (α = 0.18) in stainless steel 410NiMo under conditions of $R = 0.1$ and $N = 2E^6$ cycles. Akbarian obtained an experimental threshold stress intensity factor range for notches ($\Delta K_{\mathrm{th,n}}$) of 178 MPa $\mathrm{m}^{0.18}$.

The parameter $\Delta K_{\mathrm{th,n}}$ can be determined using the general equation that relates ΔK_{th} and α from NSIF theory (**Equation 7**). The same result is obtained using the values from **Table S.1**:

$$\Delta K_{\mathrm{th,n}} = \beta(\Delta\sigma_0)^{(1-2\alpha)}\Delta K_{\mathrm{th}}{}^{2\alpha} = 1.8(520\ \mathrm{MPa})^{(1-0.36)}(5.16\ \mathrm{MPa}\sqrt{\mathrm{m}})^{\ 0.36} = 178\ \mathrm{MPa\ m}^{0.18}.$$

Where the dimensionless variable β is calculated from the point method (PM) (**Equation 8**):

$$\beta_{PM} = \frac{\sqrt{2}}{2^{\alpha}}\pi^{((1/2)-\alpha)} = \frac{\sqrt{2}}{2^{0.18}}\pi^{((1/2)-0.18)} = 1.8$$

S3. Specimen stress distribution

Figure S.2 illustrates the stress distributions of specimens No.3 to No.15 in ascending order of singularity exponent values. The singularity exponents vary between 0.27 and 0.45, and the stress concentration factor varies between 4.06 and 8.31, with one specimen having a much higher k_t of 40.20. The stress distributions of specimens No.1 and No.2 are presented as examples in **Figure 10.**

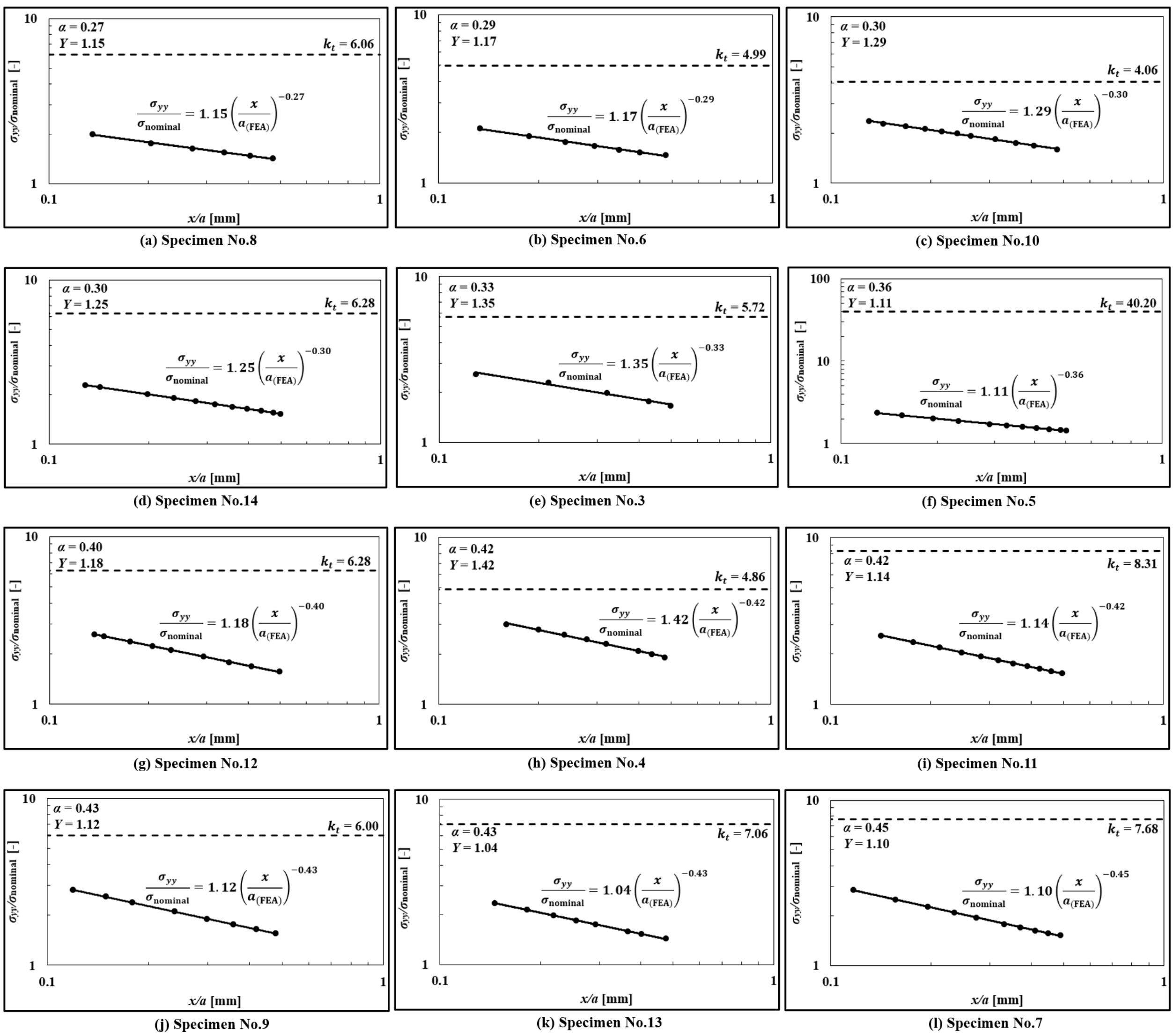

Figure S.2 Normalized stress distribution at the notch tip, bounded by the localized stress concentration factor, k_t.

S4. Specimens fatigue strength prediction

Figure S.3 shows the fatigue results of specimens No.3 to No.15, arranged in ascending order of their singularity exponent values. The red point with the horizontal bar represents the experimental fatigue strength reported over the discontinuity size range measured on the fracture surfaces. The point is positioned at $a_{(\text{mean})} = \left(a_{(\text{min})} + a_{(\text{max})}\right)/2$. These results are superimposed on their respective LENM diagrams, with a comparison slope from the LEFM (Kitagawa-Takahashi) approach also included.

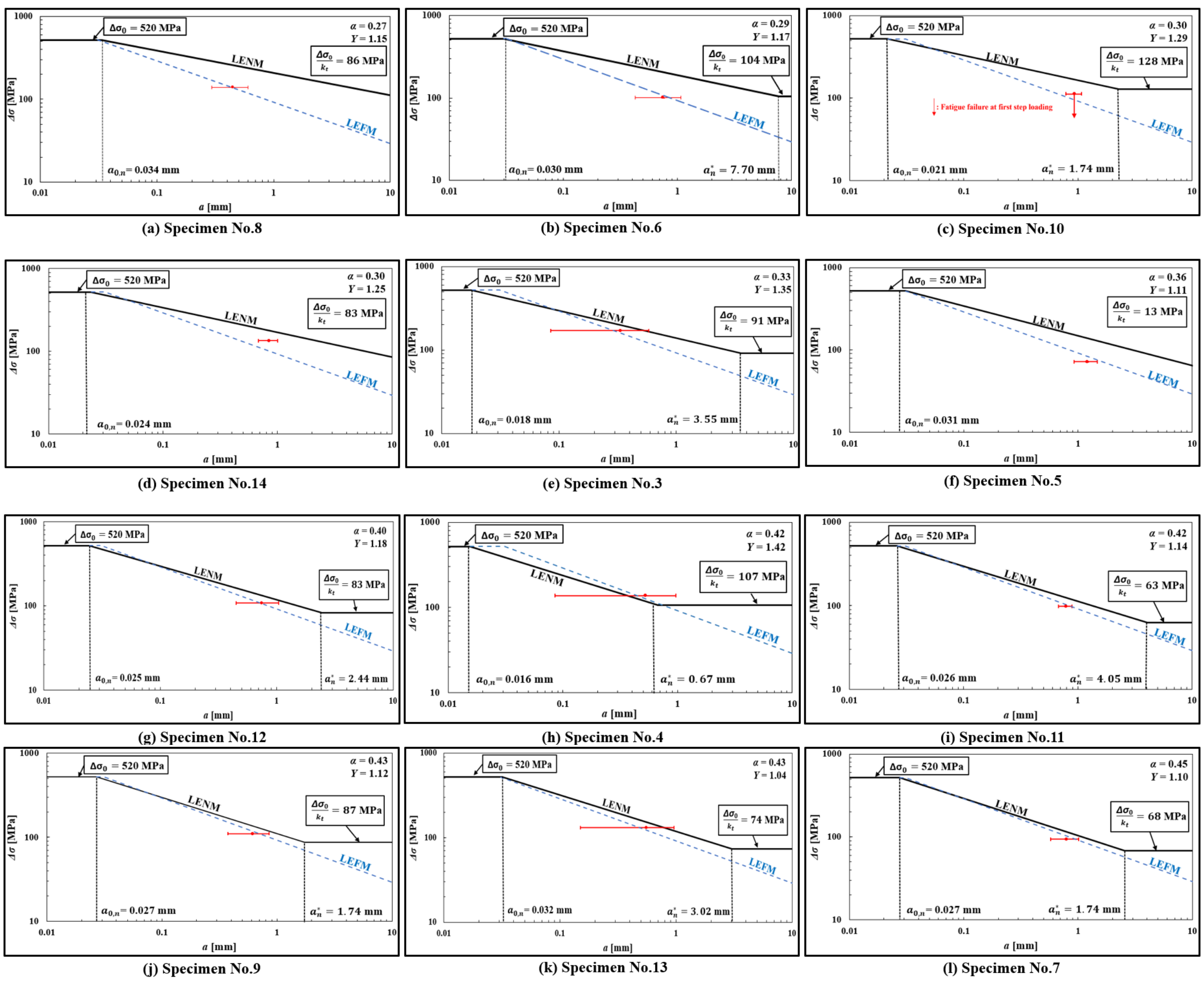

Figure S.3 Fatigue strength of elongated discontinuity in the 410NiMo welded material for $N = 2E^6$ cycles and $R = 0.1$. Predictions from (black line) Notch theory, (blue line) Linear elastic fracture mechanics and (red bar) Experimental data.