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## Workload equity in multiperiod vehicle routing problems

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**Abstract**

An equitable distribution of workload is essential when deploying vehicle routing solutions in practice. For this reason, previous studies have formulated vehicle routing problems with workload-balance objectives or constraints, leading to trade-off solutions between routing costs and workload equity. These methods consider a single planning period; however, in practice, equity is often sought over several days. In this work, we show that workload equity over multiple periods can be achieved without impact on transportation costs when the planning horizon is sufficiently large. This is demonstrated in the context of a generic multiperiod vehicle routing problem, using a simple two-phase method. In the first phase, solutions of minimal distance are produced for each period. Next, the resulting routes are allocated to drivers to obtain equitable workloads over the planning horizon. We conducted extensive numerical experiments to measure the performance of the proposed approach and the level of workload equity achieved for different planning-horizon lengths. For horizons of five days or more, we observed that quasi-optimal workload equity and optimal routing costs can be jointly achievable.

**Keywords:** vehicle routing; multiple periods; workload equity

**1. Introduction**

Competition between companies in the transportation and distribution sectors has created a need to improve many business practices. Although minimizing distribution costs is essential, drivers'

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and clients' satisfaction should also not be neglected. Equitable workload assignments, especially, are fundamental to maintaining employee satisfaction at the workplace and can significantly impact the quality of services provided to clients. In recent works on vehicle routing problems with equity considerations, workload balancing is often treated as a second objective—along with cost minimization—in a bi-objective problem. In this context, Matl et al. (2019) reported that the marginal cost of balancing routes is reasonable in most cases: nearly 40% of Pareto-optimal solutions have an additional cost that does not exceed 10% of the minimum-cost solution. Solutions that account for equity also appear more robust against unexpected events, such as a sudden increase in demand, because bottlenecks are reduced (Mourgaya and Vanderbeck, 2007; Matl et al., 2019). Yet, such extra costs remain significant given that the transportation sector operates with very tight profit margins.

In most previous studies, we noted that the definition of workload equity was unnecessarily restrictive, as it focused on the balance between different drivers' workloads for each day separately. In practical situations, however, workload differences among drivers can be acceptable on each given day as long as the total workload over a longer time horizon (e.g., a week) remains equitable. To capture this notion, we formally define and study a multiperiod VRP with workload balance (MVRPB). In this problem, client requests are known on a longer planning horizon, and the goal is to find routes that optimize distance and workload balance over the entire planning horizon. To solve the MVRPB, we propose a two-phase solution approach. In the first phase, a solution with optimal distance is found for each period by optimally solving the corresponding VRPs. The resulting routes are then combined in a second phase to obtain equitable workloads among drivers based on the total distance traveled over the periods. We use this methodology to evaluate to which extent the length of the planning horizon can impact equitable solutions. In this work, we purposely chose to address a canonical multiperiod VRP, which can be easily adapted to a particular context by introducing additional constraints in the VRPs that are solved in the first phase of our method, like a minimum or a maximum number of working hours during a day. Several applications linked to this problem, like the routing of meter readers, are discussed in Section 2.

Therefore, this work makes the following contributions:

- We study workload balance among drivers over an extended planning horizon. With this viewpoint, we show that workload equity can be achieved with a very limited impact on economic efficiency.
- We formally introduce the MVRPB and a simple and efficient two-phase method for its solution. This approach also permits obtaining bounds on the best possible workload equity.
- Through extensive numerical experiments, we demonstrate the performance of the proposed solution approach. We also measure (i) the gap between the equity level achieved by the two-phase method and perfect equity and (ii) the benefits of considering a longer planning horizon. In particular, we observe that our simple two-phase method already provides near-optimal equity and optimal routing costs for five periods or more. So, there is no need to resort to complicated integrated problem-solving methodologies for most practical purposes, since a granularity of five or multiples of five days (e.g., working days in a week) is often used in practice.

The remainder of this paper is organized as follows. Section 2 reviews related works for single- and multiperiod VRPs. Section 3 defines the MVRPB, and Section 4 describes the proposed

two-phase solution approach. Section 5 presents a computational study based on test instances derived from well-known capacitated VRP benchmark instances. Finally, Section 6 concludes the paper and discusses research perspectives.

## 2. Literature review

In this section, we review related studies on workload equity in vehicle routing for both single-period and multiperiod planning.

### 2.1. Single-period planning

The classical capacitated VRP (CVRP) seeks a set of routes starting and ending at a central depot and visiting a given set of clients (Vidal et al., 2020). Each client is characterized by a demand quantity and must be serviced in a single visit. The total demand of the clients over each route should not exceed the vehicle's capacity (considered to be identical for all vehicles). The objective of the problem is to minimize the total traveled distance. The CVRP is NP-hard as it generalizes the traveling salesman problem (TSP). Consequently, no optimal (i.e., exact) polynomial-time algorithm is known for its solution. Moreover, the best existing exact approaches can only solve medium-sized problems counting a few hundred clients in less than a few hours. Accordingly, heuristic approaches are widely used in practical settings (Ribeiro and Lourenço, 2001; Vidal et al., 2013).

To the best of our knowledge, Sutcliffe and Board (1990) were the first to consider equity in a VRP by imposing lower and upper bound on the travel time and number of occupied seats of each route, in a context where mentally handicapped people are transported to a training center. In a related school bus routing application, Bowerman et al. (1995) used a variance measure to balance the length of the drivers' routes and the number of students transported. In addition, they considered two additional criteria within a multiobjective framework. Since these early works, numerous papers have considered workload balance objectives in single-period VRPs with different equity measures (see, e.g., Golden et al., 1997; Lee and Ueng, 1999; Jozefowiez et al., 2009; Lopez et al., 2014; Oyola and Løkketangen, 2014; Bertazzi et al., 2015; Galindres Guancha et al., 2018; Va et al., 2018; Vega-Mejia et al., 2019; Zhang et al., 2019; Lehuédé et al., 2020; Londono et al., 2021). The MIN-MAX measure and the difference between the maximum and minimum workloads (called RANGE by Matl et al., 2018) are most commonly used. When considering distance as the workload metric, MIN-MAX minimizes the longest route in a solution (see, e.g., Lopez et al., 2014), whereas RANGE minimizes the difference between the longest and shortest route (see, e.g., Londono et al., 2021). We refer the reader to Halvorsen-Weare and Savelsbergh (2016), Lozano et al. (2016), and Matl et al. (2018) for a comprehensive list of equity measures.

No solution simultaneously optimizes cost and equity in most situations, especially when considering equity within a single period. Consequently, equity is often considered within multiobjective approaches, in which trade-off solutions have to be found between equity and other objectives such as distance. Many studies along this line refer to the resulting problem as the VRP with 'route balancing' (Jozefowiez et al., 2007, 2009). Some studies have examined how equity objectives affect the shape of the Pareto front in a bi-objective context (Baños et al., 2013; Schwarze and Voss,

2013; Halvorsen-Weare and Savelsbergh, 2016; Lozano et al., 2016; Zhang et al., 2019). Matl et al. (2018, 2019) extensively discussed six equity measures based on eight desirable properties. They claim that no measure satisfies every property or is strictly better than the others for all relevant properties. They also show that a monotonic equity measure such as MIN-MAX, which is based on a variable-sum metric such as distance, avoids non-TSP-optimal solutions and inconsistent solutions and therefore should be an objective of choice. A solution is non-TSP optimal if at least one route can be rearranged while improving its distance. Inconsistency occurs when a given solution is preferred over another solution even though all of its routes are longer. In both cases, the distance of one or more routes has been artificially increased to improve the equity objective.

## 2.2. Multiperiod planning

Multiperiod VRPs are defined over a planning horizon of several periods (see, e.g., the survey in Mor and Speranza, 2022). Although they are found in different contexts, for example, the multiperiod extension of the technician routing and scheduling problem (Guastaroba et al., 2021), they naturally occur when considering the periodic VRP (PVRP). In this problem, each client is characterized by a visit frequency representing how many visits are requested over the planning horizon with a list of patterns representing acceptable visit-day combinations. Solving this problem requires selecting a visit pattern for each client and generating the routes for each period. As a consequence, the decisions made at each period become interdependent (Coene et al., 2010). Mourgaya and Vanderbeck (2006) presented several PVRP variants and classified them based on their objectives, constraints, and solution methods. Equity is one of the objectives discussed in that study.

Some studies measured equity within each period. Papers in this category generally arise from real-life case studies and involve different workload equity objectives. Blakeley et al. (2003) studied the problem of an elevator-maintenance company that assigns technicians to clients. The objective was to minimize a weighted sum of travel time, overtime, and unbalanced workload in each period. To reach equity, the authors designed heuristics that optimize a RANGE measure of the travel times. Groër et al. (2009) studied a multiperiod VRP to balance the workload of meter readers over a month for a utility company. To achieve a good balance, they set a lower and upper bound on the number of clients and the length of each daily route. They also constrain the deviation of a client's bill from one month to the next. Their three-stage methodology combines heuristics and integer programming. Gulczynski et al. (2011) presented a PVRP in which an equal workload is sought within each period. The overall objective is a weighted sum of distance and RANGE measures over the number of clients served. The problem was solved with an integer-programming-based heuristic.

Mourgaya and Vanderbeck (2006) designed a hierarchical heuristic to solve an industrial PVRP application with 16,658 visits over a time horizon of 20 days. At a tactical level, the method allocates each client to a combination of days and vehicles. The objective considered at this stage involves minimizing the maximal workload over the days and vehicles subject to additional geographical restrictions. At the operational level, the method minimizes the traveled distance.

Mourgaya and Vanderbeck (2007) then studied a tactical planning problem that required choosing visit days for clients subject to service-level constraints. Once the visit days were selected, the clients were assigned to vehicles to achieve equity among drivers. At this stage, equity was

modeled by a constraint that bounds the workload (served demand) associated with each cluster of clients. In another work, Linfati et al. (2018) developed a two-phase heuristic to balance the number of medication deliveries to patients among the drivers for each day of the planning horizon. They considered a RANGE measure within a weighted-sum objective that also accounts for extra hours, extra capacity, as well as daily and client clustering costs. Finally, Messaoudi et al. (2019) introduced a PVRP with different possible visit frequencies for each client. They relied on objectives that minimized the total route cost and the number of stops to the same client and included a workload balance component between vehicles for each period. This component of the objective minimizes the maximum service time over the weeks of the planning horizon and the days of the week. Furthermore, they imposed a maximum route duration for each vehicle. A decomposition approach was used first to assign clients to weeks and then to assign them to a day within the selected week. A variant of the classical VRP was finally solved for each day using a three-phase adaptive large-neighborhood search. This algorithm was applied in a case study for a hygiene service company performing more than 69,951 visits for 6000 clients over 12 weeks, leading to significant practical savings.

Liu et al. (2013) studied a PVRP for home health care in which three types of patients require services. The primary objective was to reach equity by minimizing the maximum route time for all vehicle routes over the week. The solution method combined tabu search and local search. Liu et al. (2020) balanced the workload among routes for a periodic home health care assignment problem by partitioning the service area into regions. Finally, Schönberger (2016) set upper bounds on route duration to ensure daily workload balance in a multiperiod VRP.

Ribeiro and Lourenço (2001) were, to our knowledge, one of the very few studies considering a multiperiod VRP in which equity is evaluated over multiple periods. The problem is solved with a weighted sum objective considering cost, equity, and market share. The equity objective minimizes the standard deviation of the workloads over many periods, where the workload corresponds to the demand served by each driver. Small instances were solved with a commercial solver. Huang et al. (2019) studied a PVRP for which the objective function minimizes the total workload of all drivers, and the maximum workload difference between two drivers cannot exceed a threshold. The equity metric, in this case, corresponds to the sum of travel and service times. The results show that workload equity among drivers can be achieved at a reasonable cost. Finally, Mancini et al. (2021) recently considered workload balance and service consistency in a collaborative multiperiod VRP, where the number of clients assigned to a carrier over the planning horizon is constrained. They also examined how workload balance and service consistency impact the total solution cost.

As seen in this review, very few studies have focused on multiperiod VRPs that account for equity, and even fewer studies have considered equity over multiple periods. The present study fills this critical methodological gap. It proposes a simple solution approach for an equitable VRP defined over multiple periods and analyzes the resulting equity depending on the planning horizon.

### 3. Problem statement

The MVRPB can be formally defined on a complete graph  $G = (V, E)$ , where  $V$  is the set of vertices and  $E$  is the set of edges. Let  $V = \{0\} \cup C$ , where  $C$  is a set of vertices representing clients and vertex 0 is the depot. The cost  $d_{ij}$  corresponds to the length of edge  $(i, j) \in E$ , representing a

direct trip from  $i$  to  $j$ . For simplicity, we assume that all distances  $d_{ij}$  are integers. Each client  $i \in C$  is characterized by a list of visit days (i.e., periods) within a planning horizon of  $T$  periods and by a demand quantity  $q_i^t$  on each of these days (i.e., demand quantities can differ between days). Finally,  $m$  drivers are available through the planning horizon to perform the visits, and we are given a fleet of  $m$  homogeneous vehicles with capacity  $Q$  located at the depot.

Solving the MVRPB amounts to finding routes for each day in such a way that (i) each client is visited on each requested day, (ii) each route for a given day is assigned to a single driver, (iii) no driver operates more than one route in a day, and (iv) no route exceeds the vehicle capacity. Note that drivers may not necessarily work during each day of the planning horizon. The objectives are to optimize distance and balance the drivers' workloads over the planning horizon.

Matl et al. (2018) discussed different *equity metrics* (e.g., travel time, distance, demand quantity served, number of clients) and *equity measures* (e.g., the maximum workload of a driver, the difference between the smallest and largest workloads, and the standard deviation of workloads). In this work, we focus on distance-based workload equity among drivers. Therefore, the workload of a driver is the total distance traveled by the driver over the planning horizon. We use the MIN-MAX equity measure, which aims to achieve equity by minimizing the maximum total distance of the routes traveled by any driver over the planning horizon.

## 4. Solution approach

To solve the MVRPB, we design a two-phase solution approach. As seen in the remainder of this section, our approach follows a hierarchical objective: it first guarantees a routing solution with minimum cost (i.e., with minimum total distance over the planning horizon) and then optimizes workload balance. This permits us to evaluate the extent to which workload equity can be ensured over a longer planning horizon without sacrificing economic efficiency. Moreover, as seen in the following, the cost-optimal routes found in the first phase will permit us to calculate a bound on the best possible workload equity for any solution.

Our method unfolds in two stages. First, it solves a CVRP for each period considering only the deliveries of this period and minimizing cost (total distance). Then, it solves an allocation problem to assign routes to drivers on each period, intending to optimize workload balance. The remainder of this section details the techniques designed to perform each step efficiently.

### 4.1. First phase: distance optimization

The first phase consists of solving one CVRP per period  $t \in T$  to obtain high-quality routing plans minimizing distance. We rely on mixed-integer linear programming (MILP) techniques to solve these problems. We first present a compact mathematical formulation of the problem and then discuss its solution by branch and price.

Table 1 summarizes the notations used in the mathematical models. For each period  $t$ , the resulting CVRP <sub>$t$</sub>  subproblem can be mathematically formulated using a three-index undirected

Table 1  
Notations used in the mathematical programs

Sets	$C$	Set of clients
	$E$	Set of edges
	$V$	Set of nodes, $V = C \cup \{0\}$
	$C_t$	Set of clients in period $t \in T$
	$E_t$	Set of edges in period $t \in T$
	$V_t$	Set of vertices in period $t \in T$ , $V_t = C_t \cup \{0\}$
Parameters	$R_t$	Set of routes in period $t \in T$
	$T$	Number of time periods
	$m$	Number of drivers (i.e., maximum number of vehicle routes in each period)
	$Q$	Capacity of each vehicle
	$q_i^t$	Demand of client $i \in C_t$ in period $t \in T$
Variables	$d_{ij}$	Distance of edge $(i, j) \in E$
	$y_{ir}$	Binary variable equal to 1 if client $i$ is on route $r$ , $i \in C$ , $r \in R_t$
	$x_{ijr}$	Binary variable equal to 1 if edge $(i, j) \in E$ is on route $r$ , $i, j \in V$ , $r \in R_t$

formulation, as in Baldacci et al. (2010). In this formulation, the cut set for any  $S \subseteq C_t$  is defined as  $\delta(S) = \{(i, j) \in E_t : i \in S, j \notin S \text{ or } i \notin S, j \in S\}$ :

$$(CVRP_t) \quad \min \sum_{r=1}^m \sum_{(i,j) \in E_t} d_{ij} x_{ijr} \quad (1)$$

$$\text{s.t.} \quad \sum_{r=1}^m y_{ir} = 1 \quad i \in C_t, \quad (2)$$

$$\sum_{i \in C_t} q_i^t y_{ir} \leq Q \quad r \in \{1, \dots, m\}, \quad (3)$$

$$\sum_{(i,j) \in \delta(\{i\})} x_{ijr} = 2y_{ir} \quad i \in C_t, r \in \{1, \dots, m\}, \quad (4)$$

$$\sum_{(i,j) \in \delta(S)} x_{ijr} \geq y_{ir} \quad S \subseteq C_t : |S| \geq 2, i \in S, r \in \{1, \dots, m\}, \quad (5)$$

$$x_{ijr} \in \{0, 1\} \quad (i, j) \in E_t \setminus \{(0, j) : j \in C_t\}, r \in \{1, \dots, m\}, \quad (6)$$

$$x_{0jr} \in \{0, 1, 2\} \quad j \in C_t, r \in \{1, \dots, m\}, \quad (7)$$

$$y_{ir} \in \{0, 1\} \quad i \in C_t, r \in \{1, \dots, m\}. \quad (8)$$

Objective (1) minimizes the total routing cost. Constraints (2) force each client to be served by exactly one route. Constraints (3) ensure that capacity constraints are respected. Constraints (4) are flow-conservation constraints for the routes that also link the  $x$  and  $y$  variables. Constraints (5) are the sub-tour-elimination constraints. These constraints guarantee that each route contains the depot.

Previous studies on integer programming approaches for the CVRP indicate that directly solving Model (1–8) is ineffective for medium instances with more than a few dozen clients and that



combining cuts and column generation generally provides better results. Accordingly, we rely on the VRPSolver framework (Pessoa et al., 2020) for an efficient solution to this problem. This solver provides a generic branch-and-cut-and-price (BCP) framework for many classes of MILPs, including, among others, the considered problem setting. It has achieved a competitive or superior performance on standard test instances compared to specialized VRP algorithms and is currently available at <https://vrpsolver.math.u-bordeaux.fr/>.

This algorithmic framework relies on the solution of successive pricing subproblems that take the form of resource-constrained shortest paths (RCSPs) on a path-generator graph (VRPGraph). A bidirectional-labeling dynamic programming algorithm is then used to solve the RCSPs. VRPSolver relies on the concept of packing sets to generalize well-known cuts. Essentially, a packing set is a subset of arcs such that at most one arc from the given subset appears in the paths that are part of an optimal solution. Packing sets are defined in accordance with the application considered and associated model. In our specific case, the packing sets represent limited memory rank-1 cuts (a generalization of the subset row cuts; Jepsen et al., 2008) and rounded capacity cuts (Laporte and Nobert, 1983). The branching rule in VRPSolver is based on accumulated resource consumption and, if needed to enforce integrality, on a generalization of the branching rule of Ryan and Foster (1981).

Finally, the performance of VRPSolver depends on the availability of a good initial upper bound (UB) to limit the search. To find such an initial solution and bound, we use the hybrid genetic search (HGS; Vidal et al., 2012; Vidal, 2022), a state-of-the-art metaheuristic for the CVRP. HGS could be used as a stand-alone approach for the first phase in time-critical applications, or if the scale of the problems becomes too large for an exact solution. Still, we opted to additionally rely on the exact algorithm in this phase, as this will allow us to obtain lower bounds on the best achievable workload equity (see the next section).

#### 4.2. Second phase: equitable workload allocation

The second phase of the algorithm takes as input the solution  $R^*$  found in the previous phase, represented as the set of cost-optimal routes  $R_t^*$  for each period. It seeks to achieve a fair distribution of these routes among  $m$  identical drivers. In this stage, we use a MIN-MAX objective to minimize the maximum workload of any driver (i.e., the maximum total distance driven by a driver over all periods).

##### 4.2.1. MILP formulation and bounds

This allocation problem can also be mathematically formulated as a MILP. Let  $d_r$  represent the distance driven on each route  $r \in R_t^*$  found in the first phase, and let  $z_{rk}^t$  be a binary variable equal to 1 if route  $r$  is assigned to driver  $k$  in time period  $t$ . Finally, let  $\Delta$  be a continuous variable capturing the maximum distance for a driver over the planning horizon. The best possible workload equity for the considered set of routes can be found by solving the following model:

$$\min \Delta \quad (9)$$

$$\text{s.t. } \sum_{t=1}^T \sum_{r \in R_t^*} d_r z_{rk}^t \leq \Delta \quad k \in \{1, \dots, m\}, \quad (10)$$

$$\sum_{r \in R_t^*} z_{rk}^t \leq 1 \quad t \in \{1, \dots, T\}, k \in \{1, \dots, m\}, \quad (11)$$

$$\sum_{k=1}^m z_{rk}^t = 1 \quad t \in \{1, \dots, T\}, r \in R_t^*, \quad (12)$$

$$z_{rk}^t \in \{0, 1\} \quad t \in \{1, \dots, T\}, r \in R_t^*, k \in \{1, \dots, m\}, \quad (13)$$

$$\Delta \in \mathbb{R}^+. \quad (14)$$

Objective (9) and Constraints (10) model the minimization of the maximum workload over all drivers. Constraints (11) ensure that each driver serves at most one route in each period, whereas Constraints (12) ensure that each route is assigned to exactly one driver in each period. Finally, Constraints (13) and (14) define the domain of the decision variables. Note that the inequality in Constraints (11) can be replaced by an equality if the number of drivers matches the number of routes found in the first phase in any given period  $t$ . The resulting formulation can be viewed as a variant of the bin packing problem (BPP) with conflicts (Capua et al., 2018).

Let  $\Delta_{\text{BEST}}(R)$  be the workload produced by optimally solving the allocation problem for a given first-phase routing solution  $R$ , and let  $D(R)$  be the total distance of all routes of a solution  $R$  over all periods. The following bounds are valid:

**Lemma 1.** *The best possible workload allocation for a given routing solution  $R$  is such that*

$$\Delta_{\text{BEST}}(R) \geq \left\lceil \frac{D(R)}{m} \right\rceil. \quad (15)$$

*Proof.* The proof directly derives from the formulation of the assignment problem. Summing Constraint (10) over  $k \in \{1, \dots, m\}$  gives

$$\sum_{k=1}^m \sum_{t=1}^T \sum_{r \in R_t} d_r z_{rk}^t \leq m \Delta_{\text{BEST}}(R). \quad (16)$$

Next, using Equation (12) leads to

$$\sum_{t=1}^T \sum_{r \in R_t} d_r \leq m \Delta_{\text{BEST}}(R) \implies \Delta_{\text{BEST}}(R) \geq \frac{D(R)}{m} \quad (17)$$

Finally, since the distances  $d_{ij}$  are integer, then  $\Delta_{\text{BEST}}(R)$  is also an integer, and the right-hand side of the inequality can be rounded up, giving the announced result.  $\square$

**Property 1.** *Let  $\Delta_{\text{opt}}$  be the optimal workload equity achievable in any solution of the MVRPB (including the first-stage solutions that are not optimal in terms of cost), then*

$$\Delta_{\text{opt}} \geq \left\lceil \frac{D(R^*)}{m} \right\rceil. \quad (18)$$

*Proof.* As a consequence of Property 1, the best possible workload equity over all possible routing solutions  $R$  satisfies the following relation:

$$\Delta_{\text{OPT}} = \min_R \Delta_{\text{BEST}}(R) \geq \min_R \left\lceil \frac{D(R)}{m} \right\rceil = \left\lceil \frac{\min_R D(R)}{m} \right\rceil = \left\lceil \frac{D(R^*)}{m} \right\rceil. \quad (19)$$

This relation gives us a lower bound  $\text{LB} = \lceil D(R^*)/m \rceil$  calculated by assuming that the total amount of workload from distance-optimal routing solutions is evenly distributed among drivers. It permits us to evaluate how far our MVRPB solutions are from the optimal workload equity. It is important to remark that  $\Delta_{\text{OPT}}$  can be smaller than  $\Delta_{\text{BEST}}(R^*)$  since the optimal workload balance over multiple periods can involve routes that do not belong to any cost-optimal CVRP solution of a given period. Nevertheless, the bound announced in Property 1 holds for any MVRPB solution.  $\square$

#### 4.2.2. Set-partitioning reformulation and solution approach

A direct solution of Formulation (9–14) using standard MILP solvers such as CPLEX is not effective when the number of periods grows (see the results reported in Section 5.2). This is partly due to symmetry, given that all drivers are considered identical, and renumbering them produces equivalent solutions. One way to circumvent this issue is to solve this problem as a sequence of set-partitioning feasibility problems for different values of  $\Delta$  as defined in model (20–22).

$$\sum_{\sigma \in \Omega_\Delta} \lambda_\sigma = m, \quad (20)$$

$$\sum_{\sigma \in \Omega_\Delta} a_{\sigma t} \lambda_\sigma = 1 \quad t \in \{1, \dots, T\}, r \in R_t^*, \quad (21)$$

$$\lambda_\sigma \in \{0, 1\} \quad \sigma \in \Omega_\Delta. \quad (22)$$

In this formulation, each element  $\sigma \in \Omega_\Delta$  represents an admissible combination of routes (i.e., a schedule) that a driver can operate over the planning horizon without exceeding a workload of  $\Delta$ . Each constant  $a_{\sigma t}$  takes value 1 if and only if  $r$  belongs to  $\sigma$ . Each binary variable  $\lambda_\sigma$  takes value 1 if this schedule is selected for one driver. Constraints (20) ensure that work schedules are created for the  $m$  drivers, and Constraints (21) ensure that each route appears exactly in one schedule.

*Binary search strategy.* Finding a feasible solution of Model (20–22) for a given  $\Delta$  means that there exists a feasible allocation of the routes to drivers in such a way that the maximum workload over the planning horizon does not exceed  $\Delta$ . Therefore, the best workload balance using the routes of  $R^*$  is such that  $\Delta_{\text{BEST}}(R^*) \leq \Delta$ . In contrast, proving that this model is infeasible would imply that  $\Delta_{\text{BEST}}(R^*) > \Delta$ .

With this, we develop a strategy based on a binary search to locate  $\Delta_{\text{BEST}}(R^*)$ . Initially, we start with  $\Delta = \text{LB} = \lceil D(R^*)/m \rceil$  and solve Model (20–22). If this model is feasible, then we have attained the best possible workload equity for  $R^*$ , which is also optimal for the MVRPB (since the LB of Property 1 holds for any MVRPB solution). Otherwise, this means there is no solution with a workload balance of  $\Delta = \lceil D(R^*)/m \rceil$ . In this case, we use a *construction heuristic* to find an initial feasible solution and therefore a UB value for  $\Delta$ . This heuristic consists of ordering the items (i.e.,

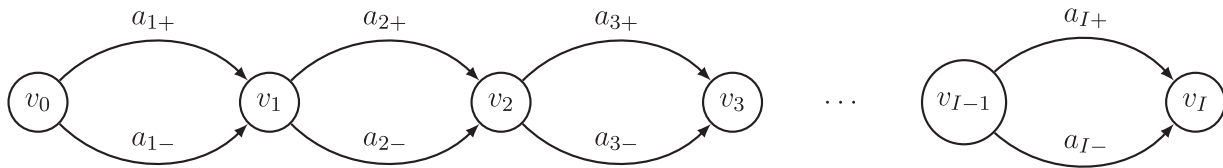


Fig. 1. Path-generator graph for the BPP.

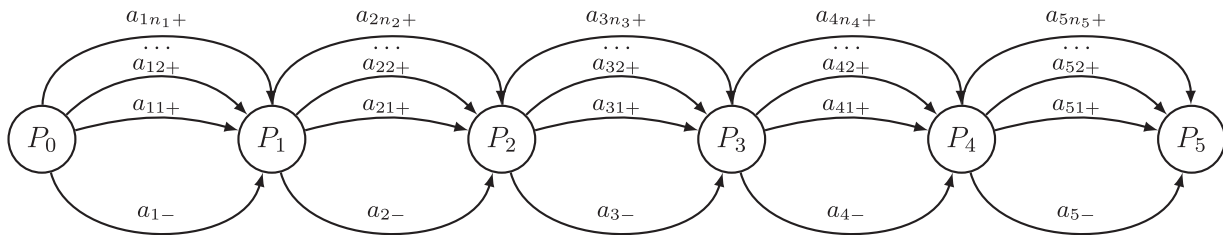


Fig. 2. Path-generator graph for the second phase.

all the routes over the planning horizon) by decreasing workload (distance) and then following this order to insert them iteratively into a compatible bin (i.e., a driver that does not already operate a route on that day) that has the current smallest workload. At this point, we have a range for the best (integer) workload value and locate it by binary search, using Model (20–22) to determine feasibility at each step. This process stops when the model for  $\Delta_{\text{BEST}}(R^*) - 1$  is infeasible and the model for  $\Delta_{\text{BEST}}(R^*)$  is feasible.

**Solution of each subproblem.** Model (20–22) contains an exponential number of variables  $\sigma \in \Omega_{\Delta}$ ; therefore, a direct solution approach is impractical. To solve this problem, we rely once again on the branch-and-price framework provided by VRPSolver. Pessoa et al. (2021) provide adaptations of VRPSolver to the classical BPP and other variants such as vector packing, variable-sized BPP, and variable-sized BPP with optional items. To solve our problem with VRPSolver, we essentially need to redefine the path-generator graph (VRPGraph).

Figure 1 represents the VRPGraph for the classical BPP, assuming that  $I$  items need to be packed. Each node in this graph corresponds to one item, except node  $v_0$ , which represents a starting point. Item  $i$  of weight  $w_i$  is loaded in the bin each time we use arc  $a_{i+}$  along the path from the start node to the end node (conversely, item  $i$  is not loaded in the bin if arc  $a_{i-}$  is used). Next, each path generated in this graph that does not exceed the bin capacity defines a new packing (column) for the column-generation algorithm. Capacity is the only resource consumed in VRPGraph, and a packing set is made of a subset of arcs with nonzero consumption of the resource.

Figure 2 provides an adapted path-generator graph for Problem (20–22). In this graph, nodes correspond to periods, except the first node  $P_0$ , which represents a starting point. An arc of type  $a_{ij+}$  then goes from  $P_{i-1}$  to  $P_i$  for each possible route  $j \in \{1, 2, 3, \dots, n_i\}$  in period  $P_i$ , where  $n_i$  is the number of routes in period  $P_i$ . If one of these parallel arcs is used, the corresponding route is selected and contributes to the total workload (total distance limited to  $\Delta$ , which stands as the bin's capacity). An arc of type  $a_{i-}$  is also available to represent the possibility of bypassing period  $P_i$  (i.e.,

no route is selected for that period). With these conventions, a path from the start node to the end node that does not exceed the workload limit  $\Delta$  corresponds to a feasible assignment of routes to a driver over the planning horizon.

We use the standard parameter setting of VRPSolver with just a minor modification to its diving heuristic. VRPSolver typically uses a diving heuristic before branching to improve the primal solution (Sadykov et al., 2019), but only at the root node. In contrast, our implementation allows strong diving at each node to quickly locate feasible solutions. If such a solution is found, then the solver can be immediately stopped since the model is known to be feasible.

## 5. Computational study

The goal of our experimental study is twofold: (i) evaluating the performance of the proposed solution approach and especially the computational effort needed for each of its steps and (ii) measuring to which extent workload equity can be achieved over multiple periods without sacrificing economic efficiency, by simply allocating cost-optimal routes to drivers in an equitable fashion as done in our two-phase approach.

Our experiments are conducted on a 2.4-GHz Intel Gold 6148 Skylake processor with 8 GB of RAM. The VRPSolver interface is implemented in Julia v1.4.2 with JuMP v0.18. VRPSolver uses BaPCod, a C++ library for implementing a generic BCP, and CPLEX 12.8 to solve the linear and mixed-integer linear programs. All experiments have been conducted on a single thread.

### 5.1. Test instances

To construct test instances for the MVRPB, we rely on a subset of the CVRP instances of Uchoa et al. (2017), as they include diverse characteristics: distribution and number of clients, depot location, and average route length. The complete set contains 100 Euclidean instances with 100 to 1000 clients. The distances are rounded to the nearest integer as in the original instances.

Since the MVRPB is defined in a multiperiod context, we had to modify the original instances. Therefore, we selected the instances X-n200-k36, X-n204-k19, X-n209-k16, X-n214-k11, X-n219-k73, X-n223-k34, X-n228-k23, X-n233-k16, X-n237-k14, and X-n242-k48 including between 199 and 241 clients from Uchoa et al. (2017). We generated three different 10-period MVRPB configurations for each of these 10 instances by randomly selecting 50, 75, or 100 clients in each period. Finally, for each period and client with demand  $d$  in the original instance, we randomly selected a new demand realization from a uniform integer distribution in  $\{\lceil 0.5 \times d \rceil, \dots, \lceil 1.5 \times d \rceil\}$ . This way, clients can have different demands at different periods. We repeated this generation (clients and demands selection) 10 times for each configuration, leading to  $10 \times 3 \times 10 = 300$  MVRPB instances defined over 10 periods. Finally, to obtain instances with fewer periods  $T \in \{2, 3, 5, 7\}$ , we retained the first  $T$  periods of each 10-period instance. The number of drivers in each 10-period instance was set to the maximum number of routes from optimal CVRP solutions over the 10 periods. Consequently, some drivers may be idle for a given period. The number of drivers in the 2-, 3-, 5-, and 7-period instances is kept identical to the number of drivers in the corresponding 10-period instance.

Table 2

Performance of the route-optimization step, for MVRPB instances with 50 clients per period

Instance	Distance		T (seconds)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	16284.45	16281.48	15.8	614.1	10.0
X-n204-k19	7200.85	7200.85	14.0	7.8	5.3
X-n209-k16	9699.69	9699.69	14.2	14.2	4.8
X-n214-k11	3782.88	3782.88	14.2	52.1	3.4
X-n219-k73	28562.15	28562.15	13.4	2.3	17.0
X-n223-k34	11464.69	11464.17	13.1	7.3	9.5
X-n228-k23	7652.85	7652.72	14.1	25.5	7.3
X-n233-k16	6704.64	6704.64	12.8	12.9	4.5
X-n237-k14	7980.45	7980.45	13.1	31.4	3.0
X-n242-k48	19970.99	19970.05	15.0	7.6	12.1

Table 3

Performance of the route-optimization step, for MVRPB instances with 75 clients per period

Instance	Distance		T (seconds)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	23388.65	23379.47	28.7	185.5	14.7
X-n204-k19	9294.97	9294.97	22.0	31.6	7.9
X-n209-k16	13272.78	13272.68	24.5	60.1	6.6
X-n214-k11	4881.22	4881.2	25.2	268.2	5.0
X-n219-k73	41826.76	41826.76	20.3	2.8	25.0
X-n223-k34	16035.39	16034.47	20.8	23.4	13.3
X-n228-k23	10379.97	10379.84	21.8	239.4	10.2
X-n233-k16	8480.25	8479.83	19.6	253.3	6.3
X-n237-k14	10870.43	10870.43	21.7	34.9	5.0
X-n242-k48	28644.96	28640.61	27.3	123.6	17.2

## 5.2. Computational performance

We first evaluate the performance of each of the two steps of the proposed approach: the computational effort needed to find cost-optimal CVRP solutions in each period, and the effort to find an equitable workload allocation in the second step. We refer to these steps as (1) route optimization and (2) multiperiod workload balancing.

### 5.2.1. Route optimization

Tables 2–4 report the performance of the route-optimization step for the 10-period instances with 50, 75, and 100 clients in each period. For brevity, the results are presented in aggregated form, with one line for each original instance of Uchoa et al. (2017), by averaging over the 10 corresponding MVRPB instances and 10 periods. From left to right, the columns report the names of the associated original instances, the average traveled distance per period in the solutions found by HGS and VRPSolver, the average computational time in seconds of these two methods, and finally, the number of drivers #k.

Table 4

Performance of the route-optimization step, for MVRPB instances with 100 clients per period

Instance	Distance		T (seconds)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	30780.5	30758.06	47.6	979.1	19.6
X-n204-k19	11535.58	11534.75	32.5	470.4	10.0
X-n209-k16	16724.49	16722.15	39.9	420.6	8.3
X-n214-k11	6076.82	6076.64	39.2	1041.6	6.1
X-n219-k73	55473.58	55473.58	27.6	2.7	34.0
X-n223-k34	20522.69	20519.9	31.8	69.3	17.1
X-n228-k23	12888.06	12887.6	32.2	950.6	12.5
X-n233-k16	10288.46	10287.92	27.0	1296.1	8.0
X-n237-k14	13518.71	13518.58	35.0	697.4	6.0
X-n242-k48	37262.13	37247.82	40.9	324.4	22.4

As seen in these experiments, the computational time used by VRPSolver to optimally solve the underlying CVRP problem for each period is generally small, with a median value of 25.0 seconds. However, in a handful of cases, the computational time may be long, reaching 12.8 hours in the worst case (one exceptional case in 3000 CVRP single-period subproblems). In contrast, HGS has a more controllable computational time, ranging from 11.4 seconds to 2.43 minutes, with a median value of 21.8 seconds. We observe that the initial solutions found by HGS were almost optimal in terms of cost, with an average distance over all instances of 16,715.0 compared to 16,712.9 for VRPSolver, that is, with an average gap error of only 0.013%. Given this, we recommend using HGS as the underlying solution approach for the route optimization step in practical time-critical applications. In the context of this study, we complete the solution process to achieve cost-optimal solutions with VRPSolver for each period, as this permits us to derive bounds on the best possible workload balance through Equation (18) from Property 1.

Finally, we must observe that the problems associated with each period are independent and can be solved in parallel. We used this observation in our experiments, as the multicore structure of our processor permitted us to independently solve the CVRPs associated with each period on a different core, therefore maximizing our utilization of available computational resources and reducing the total time needed to conduct our experiments.

### 5.2.2. Multiperiod workload balancing

In the workload balancing step, the routes of the optimal CVRP solution for each period are assigned to drivers to minimize the maximum (MIN-MAX) total distance traveled by each driver over the entire planning horizon.

Table 5 first illustrates the performance of CPLEX when directly solving model (9–14), based on a maximum allocated time of 7200 seconds or two hours. These results were obtained with the CPLEX setting: *extremely aggressive level of symmetry breaking (level 5)*. This setting proved to be better or as good as classical symmetry-breaking constraints, like ordering the routes based on their routing costs. Each entry in the table is an average over the 10 MVRPB instances associated with each one of the 10 original instances taken from Uchoa et al. (2017) for 50, 75, or 100 clients per

Table 5  
CPLEX performance

# clients	2 periods		3 periods		5 periods		7 periods		10 periods	
	T (seconds)	# Sol	T (seconds)	# Sol	T (seconds)	# Sol	T (seconds)	# Sol	T (seconds)	# Sol
50	0.04	100	108.1	99	2781.7	62	3535.9	59	3244.1	66
75	0.09	100	726.8	90	3897.7	52	5237.3	35	4728.9	47
100	0.19	100	928.2	90	4818.9	38	5680.5	32	5195.2	38

period, respectively (100 instances in each case). Apart from the average solution time in seconds, we also indicate the number of instances over 100 that were solved optimally by CPLEX within the allocated time (# Sol).

Although CPLEX is very fast on instances with only two periods, a sharp increase in solution time and number of unsolved CVRP instances is observed when three periods and five periods are considered. This large number of unsolved instances led us to the development of the alternative methodology presented in Section 4.2.2 where a number of set-partitioning feasibility problems are solved. In the following, our analysis of its performance is based on three key workload measurements:

- UB – The workload produced by the constructive approach described in Section 4.2.2.
- BEST – The best workload obtained after completing the binary search.
- LB – The MVRPB lower bound of Equation (18). It assumes that distance is optimal and workload equity is perfect (although no solution might exist actually achieving this bound).

Tables 6–8 report the workload values of UB, BEST, and LB for the different instances, with a varying number of periods and with 50, 75, and 100 clients in each period, respectively. Each line in the tables corresponds to an average value over 10 different instances. These tables also indicate the average number of routes  $\#r$ , the average number of binary-search operations in our algorithm  $\#It$  and the average solution time in seconds.

As seen in the reported experiments, only a few seconds are required in most cases to complete the multiperiod workload balancing step. This confirms the efficiency of our two-stage solution approach. Generally, instances involving a larger number of routes and drivers (e.g., X-n219-k73) lead to more complex workload balancing problems. Again, there is also inherent variability, given that MILP approaches can exhibit substantially different computational times when solving instances of similar sizes. Overall, the computational time of the second step ranges from 4.40 to 7600.99 seconds with a median value of 8.53 seconds.

The workload allocation created by the construction approach (described in Section 4.2.2) is the best possible for the considered first-stage routes (i.e.,  $UB = BEST$ ) for 298 out of 300 MVRPB instances with two periods, as well as for 15 out of 300 MVRPB instances with three periods. In contrast, as soon as the number of periods becomes greater than five, the initial construction approach is unlikely to lead to the best possible workload allocation and the underlying mathematical model produces much better solutions.

When the planning horizon contains five days or more, we frequently notice cases of ‘perfect’ workload equity, where the obtained workload balance matches the theoretical lower bound (i.e.,  $BEST = LB$ ) and, therefore, is optimal for the MVRPB. The ability to achieve perfect balance comes



**Table 6**  
**Performance of the multiperiod workload balancing step: MVRPB instances with 50 clients per period**

Tests	2 periods						3 periods						5 periods						7 periods						10 periods					
	LB	UB	Bistr	#r	#lt	T (seconds)	LB	UB	Bistr	#r	#lt	T (seconds)	LB	UB	Bistr	#r	#lt	T (seconds)	LB	UB	Bistr	#r	#lt	T (seconds)	LB	UB	Bistr	#r	#lt	T (seconds)
X-200-k-36	3199.6	3551.5	3551.5	18.5	10.0	6.46	4858.2	5291.1	5033.9	27.9	9.7	7.18	8084.7	8630.9	8090.3	46.7	10.0	56.35	11372.8	11989.7	11372.8	65.8	1.0	8.47	16274.3	16907.1	16274.3	84.6	3.0	279.05
X-n204-k-19	2735.3	3185.4	3185.4	9.9	10.1	6.54	4107.5	4570.4	4320.7	15.0	9.8	6.80	6810.6	7095.7	6822.5	25.1	9.1	5.82	9584.2	9721.3	9585.0	35.3	7.0	18.75	13663.1	13883.5	13663.1	50.4	1.0	9.39
X-209-k-16	4077.9	4846.0	4846.0	8.3	11.0	6.65	6167.3	7088.4	6421.1	12.5	10.9	7.36	10276.2	10574.6	10304.9	20.9	9.1	5.78	14297.5	14690.3	14300.5	29.4	9.4	23.59	20353.8	20725.2	20353.8	41.8	1.0	8.15
X-214-k-11	2256.0	2644.0	2644.0	6.3	10.1	6.39	3403.7	3643.4	3546.0	9.3	8.9	5.98	5608.7	5886.0	5652.8	15.6	9.2	5.89	7939.2	8186.8	7946.5	19.9	8.7	14.93	11333.4	11498.9	11333.5	28.3	1.6	8.62
X-219-k-73	3378.8	3696.6	3696.6	34.0	9.9	6.68	5012.3	5469.3	5035.8	51.0	9.9	6.03	8377.7	8650.5	8377.7	85.0	1.0	740.10	11778.3	12115.5	11778.3	119.0	2.2	17.72	16801.8	17059.7	16801.8	170.0	1.0	23.96
X-223-k-34	2436.9	2909.9	2909.9	16.7	10.3	6.19	3630.5	4052.6	3708.1	24.9	9.6	5.34	5991.6	6285.4	5994.4	45.7	9.3	49.34	8438.0	8702.3	8438.0	57.8	1.0	7.77	12101.1	12276.0	12101.2	83.1	1.7	439.17
X-228-k-23	2084.1	2705.4	2705.4	10.8	10.7	6.30	3156.1	3665.2	3412.0	16.5	10.0	4.98	5254.3	5630.0	5271.0	27.8	9.3	12.61	7291.4	7592.9	7291.5	38.8	1.9	11.91	10513.6	10815.5	10513.6	50.9	1.3	9.87
X-233-k-16	2998.2	3541.6	3541.6	8.2	10.5	6.73	4524.8	5116.2	4785.1	12.2	9.9	6.18	7560.4	8115.5	7594.1	20.5	9.7	6.62	10527.6	11028.3	10530.8	28.4	9.8	20.70	15072.1	15367.5	15072.1	40.7	1.0	7.69
X-237-k-14	5336.4	5718.7	5718.7	6.0	9.7	6.39	7995.7	8451.0	8196.9	9.0	9.9	5.63	13326.9	13647.9	13384.9	15.0	8.4	6.71	18636.1	18856.6	18646.0	18.9	8.2	8.15	26601.9	26985.1	26602.1	27.0	2.6	8.58
X-242-k-48	329.5	3875.0	3875.0	12.4	10.6	6.71	4957.2	5833.6	5063.6	35.2	10.2	5.53	7224.7	8669.3	826.3	53.1	9.7	52.25	11365.1	11899.5	11509.3	74.1	2.1	10.54	26001.9	26999.3	26001.9	106.1	1.0	18.65
average	3183.3	3696.4	3696.4	14.0	10.3	6.50	4781.3	5293.1	4952.3	21.4	9.9	6.10	7956.6	8320.6	7667.6	35.5	8.5	94.15	11233.0	11478.3	11139.9	48.7	5.1	14.25	15624.3	16231.8	15624.3	68.3	1.5	81.31

Table 7  
Performance of the multiperiod workload balancing step: MVRPB instances with 75 clients per period

Tests	2 periods						3 periods						5 periods						7 periods						10 periods					
	LB	UB	Bst	#r	#It	T (seconds)	LB	UB	Bst	#r	#It	T (seconds)	LB	UB	Bst	#r	#It	T (seconds)	LB	UB	Bst	#r	#It	T (seconds)	LB	UB	Bst	#r	#It	T (seconds)
X-200-k-36	3201.3	3463.1	3463.1	28.1	9.6	6.67	4788.9	5201.0	4916.3	41.9	9.8	7.60	7954.8	8459.4	7955.3	69.4	6.0	23.64	11132.6	11608.2	11132.6	97.0	1.0	11.97	15919.8	16472.8	15919.8	138.5	1.0	20.89
X-204-k-19	2345.8	2695.6	2695.6	14.5	9.8	6.78	3540.7	3841.5	3622.5	21.7	9.4	6.78	5895.7	6219.6	5898.7	36.0	9.6	27.47	8257.0	8633.6	8257.0	50.2	1.0	7.93	11786.4	11941.7	11786.4	71.8	1.0	13.86
X-n209-k-16	4089.0	4606.9	4606.9	12.5	10.4	6.62	6136.5	6724.1	6284.7	18.7	10.1	6.68	10188.3	10686.7	10196.7	31.0	9.9	16.26	14216.4	14493.9	14216.8	43.1	4.2	22.23	20224.5	20496.7	20224.5	61.2	1.0	15.97
X-n214-k-11	1951.2	2292.2	2292.2	8.5	9.7	6.29	2918.5	3289.1	3090.3	12.9	9.7	6.40	4846.5	5115.8	4859.4	21.9	9.1	8.21	6805.0	7026.2	6805.5	30.4	5.1	13.41	9762.8	9945.0	9762.8	43.9	1.0	9.21
X-n219-k-73	3349.5	3604.7	3604.7	50.0	9.6	7.28	5002.9	5498.0	5014.2	75.0	9.7	11.77	8337.8	8553.9	8337.9	100.0	1.8	1705.40	11696.1	11979.5	11696.1	150.0	1.0	20.36	16731.2	17034.0	16731.3	225.0	1.9	52.37
X-n223-k-34	2424.1	2770.6	2770.6	24.6	9.9	6.71	3604.4	3978.5	3653.8	36.8	9.5	7.57	5983.5	6219.6	5984.0	61.5	5.2	132.47	8404.2	8601.1	8404.2	86.5	1.0	10.84	12071.3	12249.7	12071.3	123.7	1.0	22.66
X-228-k-23	2033.6	2466.4	2466.4	16.3	10.0	6.82	3053.9	3622.9	3188.7	24.5	10.1	7.28	5017.0	5348.4	5020.6	40.0	9.7	51.00	7103.5	7431.2	7103.5	57.0	1.0	7.72	10217.8	10484.5	10217.8	82.0	1.0	17.36
X-233-k-16	2709.8	3103.9	3103.9	11.5	10.0	6.62	4053.7	4570.3	4251.0	17.1	9.9	7.38	6818.4	7142.7	6827.9	28.8	9.5	11.17	9532.3	9847.3	9532.4	40.5	1.9	11.33	13525.2	13792.1	13525.2	57.7	1.0	13.70
X-237-k-14	4313.7	4815.3	4815.3	10.0	11.1	7.14	4472.5	4757.0	4665.4	15.0	10.9	7.04	10835.0	11452.6	10854.5	25.0	9.2	7.68	15187.6	15520.8	15189.3	35.0	9.4	20.20	21741.2	21826.9	21741.2	50.0	1.0	8.68
X-242-k-48	4111.4	3886.9	3886.9	20.8	10.1	6.97	5107.4	5707.5	5171.3	47.5	10.1	8.16	8470.0	8984.9	8470.2	79.3	2.8	194.67	11752.5	12041.8	11752.5	110.4	1.0	12.82	16688.9	16882.6	16658.9	156.4	1.0	22.91
Ave.	2982.9	3403.9	3403.5	20.8	10.1	6.79	4468.1	4990.8	4615.8	31.1	9.9	7.67	7434.7	7818.4	7440.5	49.3	7.3	217.80	10408.7	10718.4	10409.0	70.0	2.7	13.88	14863.9	15112.6	14863.9	101.0	1.1	19.76

**Table 8**  
Performance of the multiperiod workload balancing step – MVRPB instances with 100 clients per period

Tests	2 periods						3 periods						5 periods						7 periods						10 periods					
	LB	UB	Bst	#r	#lt	T (seconds)	LB	UB	Bst	#r	#lt	T (seconds)	LB	UB	Bst	#r	#lt	T (seconds)	LB	UB	Bst	#r	#lt	T (seconds)	LB	UB	Bst	#r	#lt	T (seconds)
X-200-k-36	3161.3	3439.1	3407.4	37.2	9.4	7.61	4727.0	5311.2	4834.0	55.9	10.0	8.86	7865.5	8477.5	7865.5	92.7	1.0	76.79	10982.5	11470.7	10982.5	116.0	1.0	13.99	15703.2	16244.7	15703.2	184.6	1.0	25.56
X-204-k-19	2309.4	2607.1	2607.1	19.4	9.6	7.21	3460.6	3731.8	3507.1	29.1	9.1	7.99	5763.0	6048.3	5763.0	43.2	9.5	90.53	8071.2	8337.0	8071.2	67.1	1.0	10.42	11527.4	11775.4	11527.4	86.3	1.0	12.66
X-209-k-16	4043.6	4448.2	4448.2	16.0	10.1	8.00	6062.4	6610.1	6173.2	24.2	10.1	6.33	10137.2	10570.4	10140.8	40.3	9.8	35.44	14176.1	14523.5	14176.1	56.4	1.0	8.28	20204.7	20512.9	20204.7	80.4	1.0	11.06
X-214-k-11	1969.8	2176.2	2176.2	11.7	9.2	7.96	2967.3	3311.4	3053.0	17.3	9.2	5.91	4987.2	5239.6	4994.1	28.7	8.9	10.83	7026.2	7299.6	7026.4	40.1	2.5	10.80	9986.3	10105.7	9986.3	57.5	1.0	9.27
X-219-k-73	3239.3	3486.3	3486.3	68.0	9.5	7.36	4879.9	5296.2	4886.1	91.8	9.8	21.87	8159.9	8477.6	8159.9	170.0	1.0	15.04	11397.0	11765.1	11397.0	238.0	1.0	31.32	16316.2	16654.4	16316.2	340.0	1.0	102.17
X-223-k-34	2384.1	2708.7	2708.7	32.7	9.7	6.95	3603.1	3871.1	3631.7	49.1	9.1	9.55	6054.6	6243.3	6054.6	81.4	1.0	43.94	8413.6	8631.7	8413.6	113.2	1.0	12.68	12001.9	12232.6	12001.9	162.1	1.0	20.40
X-228-k-23	2012.2	2479.0	2479.0	20.3	10.3	7.32	3064.6	3598.2	3180.6	31.0	10.0	8.19	5112.6	5417.1	5114.5	51.6	7.4	29.32	7169.0	7410.0	7169.0	72.5	1.0	9.67	10325.3	10654.1	10325.3	105.1	1.0	12.57
X-233-k-16	2524.4	2955.2	2923.2	15.4	10.0	7.58	3754.3	4308.4	3885.5	23.0	10.0	7.79	6307.1	6874.5	6310.2	38.9	9.9	28.01	8849.7	9410.8	8849.7	54.5	1.0	11.70	12602.4	12996.6	12602.4	77.3	1.0	10.34
X-237-k-14	4487.4	4942.2	4942.2	12.0	10.3	7.41	6733.5	7343.4	6919.2	18.0	10.3	7.22	11209.6	11801.3	11218.6	30.0	10.1	11.40	15710.0	15927.3	15710.3	42.0	3.4	14.09	22531.5	22764.5	22531.5	60.0	1.0	9.66
X-242-k-48	3386.4	3832.8	3832.8	41.9	10.3	7.17	5049.7	5845.3	5072.4	63.4	9.8	10.15	8254.0	8804.4	8254.0	103.7	1.0	1467.53	11637.7	12000.6	11637.9	146.1	2.9	552.76	16633.5	16834.3	16633.5	208.5	1.0	32.24
av	2956.2	3307.5	3301.1	27.5	9.6	7.46	4430.2	4892.7	4514.3	40.3	9.7	6.39	7385.1	7795.4	7385.1	68.1	9.0	219.88	10343.3	10677.6	10343.4	94.6	1.6	67.57	14783.2	15077.5	14783.2	136.2	1.0	24.59

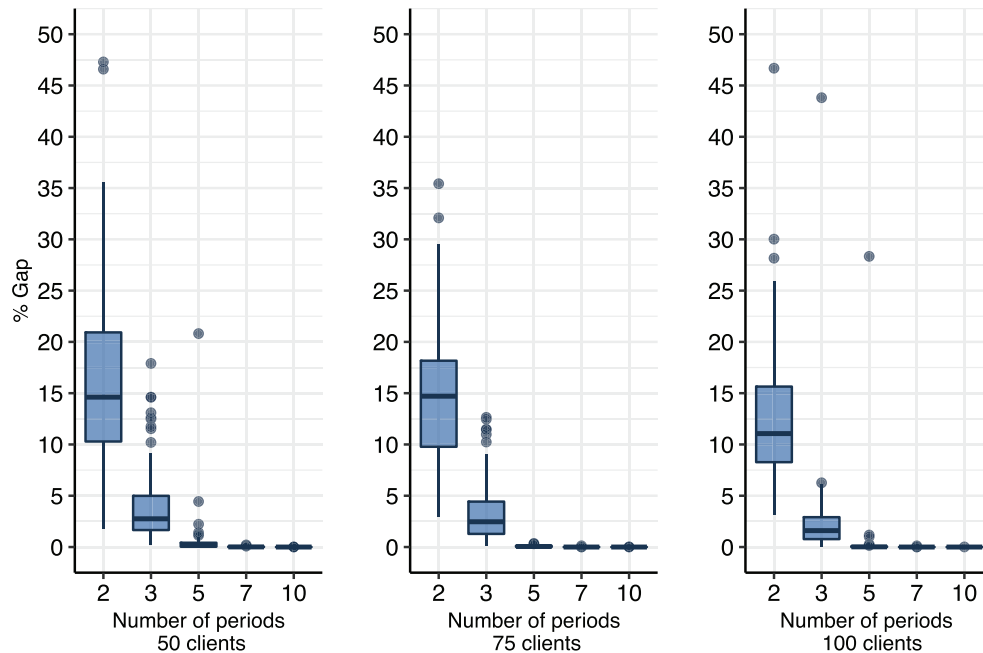


Fig. 3. Convergence of BEST toward LB as the number of periods increases for instances with 50, 75, and 100 clients in each period.

as a consequence of the increased number of possible assignment solutions, which grows exponentially with the number of periods. In practice, in cases with five periods or more, it is sufficient to focus on cost-optimal routing solutions and to create equitable workloads by careful assignment. Finally, as our binary search strategy includes a first step to verify if a solution with perfect balance exists, all the cases for which perfect balance is possible are solved in a single call to the feasibility subproblem. In the other cases, it generally takes between 7 and 11 iterations.

### 5.3. Planning-horizon length and workload equity

The previous section showed that our two-stage approach permits to achieve near-optimal workload equity, in practice, for planning horizons with at least five periods. To visualize more clearly the impact of the number of periods over workload equity, Fig. 3 provides additional boxplot representations of the Gap(%) between the ideal workload (LB from Equation (18)) and the solution value (BEST) of our two-phase approach. We used the following calculation:  $\text{Gap} = 100 \times (\text{BEST} - \text{LB}) / \text{LB}$ . We provide separate plots for the cases with 50, 75, or 100 clients per period. Each boxplot corresponds to the data of a given planning horizon with  $T \in \{2, 3, 5, 7, 10\}$  periods, therefore gathering gap measurements from 100 instances. The whiskers indicate the minimum, first quartile (Q1), median, third quartile (Q3), and maximum. The minimum corresponds to the  $Q1 - 1.5 \times \text{interquartile range}$ , while the maximum corresponds to the  $Q3 + 1.5 \times \text{interquartile range}$ . Outliers that fall beyond the minimum and maximum range are additionally depicted as small circles.

These boxplots give another viewpoint on the convergence toward the best possible equity as the number of periods increases. In the vast majority of the cases (excluding a few outliers), a planning horizon of five days is sufficient to find equitable solutions with workload discrepancies below 1% between drivers. In these situations, there is no need to seek a trade-off between routing costs and workload equity since cost-optimal routing solutions can be used to achieve equity. Another benefit of our two-stage approach is its flexibility since additional constraints, decisions, and objectives (i.e., routing attributes; Vidal et al., 2013) only need to be integrated in the first phase of the solution approach.

Finally, in the cases with very few periods (e.g., two or three days), we observe that focusing the search on optimal routing solutions does not permit achieving the best possible workload equity. In such situations, it would be helpful to consider alternative routing solutions. One possibility would consist of producing multiple routing solutions for each period and extending the workload balancing step to include all these alternatives. Another approach, more complex to develop in practice, would be to solve the routing and driver-allocation problem in an integrated manner, considering distance and workload equity in a bi-objective solution method. However, in both cases, the user would need to specify a trade-off between acceptable extra routing costs and the desired workload equity level. Such a development is left as a research perspective.

## 6. Conclusion

In this work, we have revisited workload equity in vehicle routing with a longer-term perspective, considering a planning horizon of several days. We have shown that a two-phase optimization approach can identify equitable solutions with minimal distance. When the planning horizon exceeds five days, the resulting solutions are optimal in terms of distance and near-optimal (below 1% gap) in terms of equity. Therefore, workload equity appears to be achievable without integrated approaches and trade-off calibration and without any compromise on operational efficiency.

Several important research perspectives are open in connection with this study. First, the two-phase optimization process can integrate constraints and objectives defined at the scope of each period (e.g., visit time windows, single-period workload balance, maximum work duration, or rest periods) in the first phase of the solution process. This can be done by simply adapting the mathematical models or using tailored heuristics. In all these cases, the equitable assignment of routes performed at the second step remains identical and likely effective due to the growth of the number of possible combinations as a function of the number of periods. In a similar fashion, different workload equity measures or compensation schemes for work imbalance could be considered, through adequate adaptations of the second phase allocation model. Next, our work has focused so far on deterministic settings, where the complete client demand is known on the planning horizon. Practical situations often involve dynamically revealed request information, and therefore it is an open question to determine to what extent multiperiod workload equity is achievable in dynamic contexts. Another important aspect of practical delivery systems concerns delivery consistency. When the same driver regularly visits the same areas or clients, the service quality and the satisfaction of drivers and clients generally increase (Kovacs et al., 2014). Our approach toward equitable workload allocation largely benefits from the exponential number of possible route-driver allocation combinations. However, consistency may significantly reduce the number of allocation possibilities,

such that new approaches may be needed to conciliate three key aspects in a multiperiod setting: cost efficiency, workload equity, and delivery consistency.

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