Titre: Title:	Dynamic facility location model: static reduction and application to telephone network planning
Auteurs: Authors:	Pierre Evrard, & Maurice Queyranne
Date:	1979
Type:	Rapport / Report
Référence: Citation:	Evrard, P., & Queyranne, M. (1979). Dynamic facility location model: static reduction and application to telephone network planning. (Rapport technique n° EP-R-79-22). https://publications.polymtl.ca/6001/

Génie Industriel

A DYNAMIC FACILITY LOCATION MODEL: STATIC REDUCTION AND APPLICATION TO TELEPHONE NETWORK PLANNING

PIERRE EVRARD

AND

MAURICE QUEYRANNE

RAPPORT TECHNIQUE NO. EP79-R-22

MAY 1979

Ecole Polytechnique de Montréal

Campus de l'Université
de Montréal
Case postale 6079
Succursale 'A'
Montréal, Québec
H3C 3A7

204

A DYNAMIC FACILITY LOCATION MODEL:

STATIC REDUCTION AND APPLICATION TO TELEPHONE

NETWORK PLANNING

PIERRE EVRARD

AND

MAURICE QUEYRANNE

RAPPORT TECHNIQUE NO. EP79-R-22

MAY 1979.

ABSTRACT

This paper presents a discrete dynamic location model in which no more than one change is allowed in the assignment of every customer and the status of every potential warehouse site. It is shown how this problem can be reduced to a static problem with multiple-choice and side constraints. An algorithm for the dynamic problem is then derived and an application to a planning problem in telephone network evolution is described, along with extensive computational results.

ACKNOWLEDGEMENTS

This research was supported by Recherches Bell-Northern (Verdun, P.Q., Canada). The authors thank A. Vaillancourt and S. Labelle, of RBN, for their collaboration. Particular thanks go to J.-P. Combot (now at SITA, Neuilly, France) who suggested this study and provided assistance during its development.

INTRODUCTION

Practical location problems often present a dynamic aspect: not only do we need to locate new facilities, but we also require the exact scheduling of these events in order to guarantee a service level in the most cost-effective way. Most existing analytical approaches to location problems (see e.g. [14], [24], [25]) have dealt with static problems. A common way to handle dynamic location problems was then a two-step approach: in a first step, these static location models were used to derive a desirable facility configuration, which was then used as a target in a second step. Quite often it is left to managers dealing with a changing environment to carry out this second step, using the desirable configuration as a set of guidelines for their decisions. When such parameters as demands or costs vary significantly with time, it happens that the timing of these decisions cannot be overlooked and the target configuration approach may no longer be applied.

One method attempting to take time into account consists in considering "snapshots" of the problem at various periods and then indepently determining an optimum facility configuration in each case. This approach is well justified when the costs for opening and/or closing facilities are small with respect to operation costs, for instance when these facilities are rented. Otherwise, the dependence between these decisions over different time periods must be considered. Several models and exact and heuristic solution methods have been proposed for these dynamic location problems. In [11], Erlenkotter discusses a general model in this class, the Capacity Expansion Problem, which is particularly relevant to the planning of production facilities, and he provides a comprehensive bibliography.

The model considered here is best described in the context of dynamic warehouse location [29], [30], [23], [35]. It is a deterministic, single-commodity, discrete model with finite horizon, discrete-time periods and single-sourcing constraints for customers. Its main characteristic is the following restriction, which may arise from policy or economic considerations: within the considered horizon, no more than one change is allowed in the assignment of every customer and the status of every (potential) warehouse site. An important practical application of this restricted dynamic location model is a planning problem for the evolution of a telephone network, which arises from the introduction of a new switching technology.

This paper is organized as follows. In section 1 we introduce a static location problem which slightly generalizes the classical Simple Plant (or Uncapacitated Warehouse) Location Problem by allowing for multiple-choice and side constraints. In section 2 we describe a dynamic location problem and show how this dynamic problem can be reduced to a static problem of the previous type. The telephone planning problem is described in section 3 and a program DYPALM which we developped for this telephone application using the results of the previous developments is outlined in section 4. In section 5 we describe some computational experiences with DYPALM and compare this approach with other methods used for planning telephone network evolution.

1- A STATIC LOCATION PROBLEM WITH MULTIPLE-CHOICE AND SIDE CONSTRAINTS

Consider the following formulation [1], [10], [24] of the classical Simple Plant Location Problem (SPLP):

$$\left[\begin{array}{llll}
\text{minimize} \ \sum_j \sum_i c_{ij}x_{ij} + \sum_i f_i y_i & & & (1\text{-}1) \\
\text{s.t.} \quad \sum_i x_{ij} = 1 & & \text{all } j & (1\text{-}2) \\
\quad x_{ij} \leqslant y_i & & \text{all } i,j & (1\text{-}3) \\
\quad x_{ij} \geqslant 0 & & \text{all } i,j & (1\text{-}4) \\
\quad y_i = 0 \text{ or } 1 & & \text{all } i & (1\text{-}5)
\end{array}\right.$$

where y_i = 1 if it is decided to open a warehouse, at cost f_i, in location i

and x_{ij} is the fraction of the total demand of customer j which is serviced at unit cost c_{ij} by a warehouse located in i.

The constraints (1-3), which are called Balinski's constraints, prevent a customer j from being serviced by a warehouse which has not been constructed (see [10], [24] for a discussion of other possible formulations of this restriction). Very efficient solution methods exist for the SPLP [22], [33], [10], [24], [27].

We will consider several static generalizations of this model which will enable us to deal with dynamic versions of the location problem. The first such generalization arises when some of the coefficients in (1-1) are allowed to take on negative values. For instance consider a profit maximization version of the SPLP [36]: if the demand of customer j, when serviced by a warehouse at location i, is d_{ij} and it is satisfied at price Π_{ij}, then the net profit associated with decision $x_{ij} = 1$ is $p_{ij} = \Pi_{ij} d_{ij} - c_{ij}$, and the objective (1-1) is replaced by:

$$\text{maximize} \quad \sum_j \sum_i p_{ij} x_{ij} - \sum_i f_i y_i \qquad (1\text{-}1')$$

When we reduce it to a minimization problem (by multiplying (1-1') by minus one), we obtain a problem of type (1-1)-(1-5) with negative coefficients in the objective function. Such problems can be handled by any method designed to solve SPLP's with nonnegative costs by the simple addition of a constant

c'_j (taking $c'_j = -\min_i c_{ij}$ is sufficient) to all costs corresponding to customer j; this translation is justified by constraint (1-2). The algorithms can also be easily adapted to handle negative fixed costs f_i (resulting, for instance, from other activities or from the impact of marketing) by simply observing that, in any optimal solution, the corresponding y_i's will all be equal to unity.

Another extension of the SPLP, rich in modelling capabilities, is to allow for multiple options, as well for the warehouses as for the "service" provided to customers. Assume that, at location i, we have the choice of several types of warehouses, with fixed costs f_i^k, for $k = 1,\dots,K_i$; in addition, we may also select among several levels of service for customer j, with service costs c_{ij}^h, for $h = 1,\dots,H_{ij}$. The resulting problem is

$$\underset{x,y}{\text{minimize}} \quad \sum_j \sum_i \sum_h c_{ij}^h x_{ij}^h + \sum_i \sum_k f_i^k y_i^k \tag{1-1a}$$

$$\text{s.t.} \quad \sum_i \sum_h x_{ij}^h = 1 \quad \text{all } j \tag{1-2a}$$

$$x_{ij}^h \leqslant \sum_k y_i^k \quad \text{all } i,j,h \tag{1-3a}$$

$$x_{ij}^h \geqslant 0 \quad \text{all } i,j,h \tag{1-4a}$$

$$y_i^k = 0 \text{ or } 1 \quad \text{all } i,\, k \tag{1-5a}$$

$$\sum_k y_i^k \leqslant 1 \quad \text{all } i \tag{1-6a}$$

The last constraint (1-6a) rules out more than one warehouse in location i. Obviously, without additional constraints, problem (1-1a)-(1-6a) reduces to (1-1)-(1-5) by merely selecting, for every variable, the cheapest option.

Side constraints of the following linear type

$$Ax + By \geqslant b \tag{1-7}$$

where A, B are matrices and b a vector of conformable dimensions, are useful to model very general additional restrictions which almost invariably arise in practice; for instance a budget constraint, configuration constraints imposing bounds on the number of warehousees in a region [8], and so on, see [15], [18] for a more detailed discussion; they are also useful for generating efficient solutions in multicriteria location problems for public facilities [32]. Most notable are capacity constraints, which define the Capacitated Warehouse Location Problem [7]. The different types of warehouses k mentioned above are usually characterized by different capacities M_i^k ; when only their fixed costs differ, they can be modelled as multiple-choice options as in (1-1a)-(1-6a); when in addition their variable cost (transportation from supply point, inventory costs,...), which is usually incorporated with the service cost c_{ij} varies, the standard modelling practice [15] is to define additional fictitious locations (and also, in some cases, add a configuration constraint). Similarly, the levels of service h appearing in (1-1a)-(1-6a) are often characterized by different service costs c_{ij}^{h} and different demands d_{ij}^{h}. The resulting capacity constraints are, for location i:

$$\sum_{j} \sum_{h} d_{ij}^{h} x_{ij}^{h} \leqslant \sum_{k} M_{i}^{k} y_{i}^{k} \tag{1-8}$$

These levels of service may be used to model additional decision variables, such as frequency of service or response time [4], various marketing strategies [34], [28] or prices [19], [9]; this is particularly useful when "customer" actually denotes a retail store. In all these cases, we assume a finite number of options and no interaction between the demands of customers (or retail stores).

One possible way to deal with side constraints (1-7) (and (1-8)) is to introduce them, with suitable Lagrange multipliers in the objective function. The resulting problem is of type (1-1a)-(1-6a), with possibly negative costs. As was mentionned above, such a problem can be reduced to a classical SPLP. The algorithm PALM described in [12] implements this approach, coupled with subgradient optimization for the Lagrange multipliers, as one relaxation within a branch-and-bound algorithm designed to solve problem (1-1a)-(1-6a), (1-7), (1-8) with the additional restriction of binary x_{ij}^{h} 's. We will outline in a later section the dynamic version of this algorithm and its application to a practical problem of the evolution of a telephone network [5], which motivated its development.

2- A DYNAMIC LOCATION PROBLEM

In a dynamic location model, we seek to determine not only which warehouses to open (and of what type) and how to assign to them the customers demands (and what will be the levels of service), but also at what time to make these changes in order to assure maximum discounted profit or minimum cost. In the model considered here we first select a finite horizon and discrete time periods (in contrast with several models considered in [11]). We assume nondecreasing demands for every customer and we do not consider the possibility [29] of closing warehouses. More restrictive is our following assumption: at initial time $t = 0$, every customer is serviced by an existing facility (which may be an existing warehouse, or a direct supply point) and we allow at most one change in assignment of demand of customer j within the consider horizon. This may be justified by prohibitive change costs, or by administrative or marketing restrictions; this is also justified when the decisions are only considered for the next few years within which the demand forecasts are reliable enough, and the model is periodically used with updated information.

We introduce the following notation:

let T denote the number of decision times, denoted by $t=1, 2, \ldots, T$

ρ denote the (continuous) discounting rate

y_i^t = 1 if it is decided to open a new warehouse in location i, at time t

= 0 otherwise.

f_i^t is the discounted cost for opening a warehouse in location i at time t, and keeping it open until horizon T, i.e.

$$f_i(t) = F_i(t) + \int_{\tau=t}^{T} g_i(\tau) e^{-\rho\tau} d\tau$$

where $F_i(t)$ is the set-up cost at time t, and

$g_i(\tau)$ is the marginal operating cost at time τ for a warehouse in location i.

x_{ij}^t = 1 if the service of customer j is changed at time t to warehouse i

= 0 otherwise

c_{ij}^t is the cost of such a decision, i.e.

$$c_{ij}^{t} = \int_{\tau=0}^{t} c_{oj}(\tau)e^{-\rho\tau}\,d\tau \;+\; C_{ij}(t) \;+\; \int_{\tau=t}^{T} c_{ij}(\tau)e^{-\rho\tau}\,d\tau$$

where $c_{oj}(\tau)$ is the marginal cost of servicing j with the original facility, at time τ.

$c_{ij}(\tau)$ is the marginal cost of servicing j with the new facility in location i, at time τ

$C_{ij}(t)$ is the cost of this change occuring at time t.

The major cause of variation of service costs $c_{oj}(\tau)$ and $c_{ij}(\tau)$ with time τ is the fact that demand from j increases with time. In addition we use a 0-1 decision variable x_{oj}^{t} equal to one iff we decide not to change at all the assignment of customer j; the corresponding cost is c_{oj}^{t}.

The resulting model is:

$$\text{minimize} \quad \sum_{j}\sum_{i}\sum_{t=1}^{T} c_{ij}^{t}x_{ij}^{t} + \sum_{i\neq 0}\sum_{t=1}^{T} f_{i}^{t}\,y_{i}^{t} \qquad (2\text{-}1)$$

$$\text{s.t.} \quad \sum_{i}\sum_{t=1}^{T} x_{ij}^{t} = 1 \text{ all } j \qquad (2\text{-}2)$$

$$x_{ij}^t \leqslant \sum_{\tau=1}^{t} y_i^\tau \quad \text{all } i \neq 0,\ j,\ t \qquad (2\text{-}3)$$

$$x_{ij}^t = 0 \text{ or } 1 \quad \text{all } i,\ j,\ t \qquad (2\text{-}4)$$

$$y_i^t = 0 \text{ or } 1 \quad \text{all } i,\ t \qquad (2\text{-}5)$$

$$\sum_{t=1}^{T} y_i^t \leqslant 1 \quad \text{all } i \qquad (2\text{-}6)$$

This problem (2-1)-(2-6) is closely related to (1a)-(6a) except for the chronological constraints (2-3) which prohibit assigning a customer j to a warehouse i which has not been constructed at time t. We propose to relax these constraints (2-3) to

$$x_{ij}^t \leqslant \sum_{\tau=1}^{T} y_i^\tau \quad \text{all } i \neq 0,\ j,\ t \qquad (2\text{-}3\text{-}1)$$

(namely the one for the horizon T),

and to append the exact constraints (2-3) as side constraints of the type $Ax + By \geqslant b$ (1-7). In order to keep the number of side constraints manageable, we can use surrogate constraints of the Efroymson-Ray type:

$$\sum_j x_{ij}^t \leqslant n \sum_{\tau=1}^{t} y_i^\tau \quad \text{all } i \neq 0,\ t \qquad (2\text{-}3\text{-}2)$$

where n is the total number of customers (of course we can replace it with the maximum number n_i^t of customers assignable to i at time t whenever such information is available). Other forms of surrogate constraints are possible, e.g.

$$\sum_j \sum_{\tau=1}^{t} x_{ij}^{\tau} \leqslant n(1 - y_i^{t+1}) \quad \text{all } i \neq 0,\ t=1,\ldots,T-1 \qquad (2\text{-}3\text{-}3)$$

provided that they are equivalent, in 0-1 variables, to the chronological constraints (2-3).

Several other comments about the formulation (2-1)-(2-6) are appropriate. First, we note that our decision variables y_i^t and x_{ij}^t have a different meaning from those used by Roodman and Schwarz [29]: their variables Y_{it} and X_{ijt} represent the <u>state</u> of the system at period t, as in a "snapshot" approach, while our variables y_i^t and x_{ij}^t represent the <u>changes</u> occuring in period t. Also we do not consider the opportunity of closing any facility. Indeed it is precisely these features, and also of course the assumption of at most one change in assignment for every customer, which allow us to solve a <u>unique</u> SPLP in one relaxation step, instead of solving one SPLP for every time-period as in [29]. Thus the limitation in generality of our model implies a much greater ease in solution, which is far from negligible when this model applies. In addition our model takes into account the costs of changing the customer assignments, which are not included in the Roodman-Schwarz model. Another remark concerns the possibility of including both multiple service levels and multiple types of warehouses for every year; this is a straightforward extension of the model which was not included in the formulation (2-1)-(2-6) in order to avoid complicated notation. Indeed the program DYPALM, reported on in the next section, includes these multiple-choice options.

Capacity constraints may also be included in the dynamic model. When the demands are nondecreasing, only one capacity constraint per warehouse location is necessary, namely the one for the horizon T:

$$\sum_j \sum_{\tau=1}^{T} d_{ij}^{T} x_{ij}^{\tau} \leq \sum_{\tau=1}^{T} M_i^T y_i^{\tau} \quad \text{all } i \neq 0 \tag{2-8}$$

On the other hand the time-expanded form of these capacity constraints, namely

$$\sum_j \sum_{\tau=1}^{t} d_{ij}^{t} x_{ij}^{\tau} \leq \sum_{\tau=1}^{t} M_i^t y_i^{\tau} \quad \text{all } t \text{ and } i \neq 0 \tag{2-8-1}$$

make the chronological constraints unnecessary (in 0-1 variables); thus, we may use them instead of the chronological constraints (2-3). In addition these time-expanded capacity constraints apply for any evolution (not necessarily nondecreasing) of the demands.

Finally we note that the side constraints can be used to model time-dependent restrictions which appear in dynamic models, such as a limited budget or a limited supply of new equipment for every year. In any case, we have reduced the dynamic location problem to a static problem with multiple-choice and side constraints.

3- APPLICATION TO A PROBLEM OF OPTIMAL EVOLUTION OF A TELEPHONE NETWORK

The model described in the previous sections has been applied in a study of the evolution of a telephone network, brought about the introduction of a new switching technology. The existing analog switches, which are basically electro-mechanical devices with a limited capacity, are being replaced by digital switches which offer, among other advantages, much larger capacity and the possibility to use remote concentrators. Thus several existing analog switches could be replaced by a single digital switch and several concentrators connected to it. Given forecasts for the future demands in every location, the problem is to determine which facilities to place, in which locations and at what time, in order to satisfy this demand in a most cost-effective manner. This planning problem involves investments of millions of dollars in the near future.

We outline the major aspects of this planning problem in connection with the dynamic location problem described in section 2. A more detailled description can be found in [5] and [12]. The set of "customers" is here the set of existing switch locations, with the associated demands, in number of lines per year. The set of potential locations is identical, since any existing analog switch could be replaced by either a digital switch or a concentrator and the increased capacity of the new equipment makes it unnecessary to invest in new buildings. In a given location, the options are:

(i) to install a digital switch, corresponding to a warehouse in the previous model; there are several types of digital switches, with different costs and capacities, and there is also the option of overlay, which is described later.

(ii) to install a concentrator which must be connected to a digital switch in a different location; this corresponds to servicing this "customer" by a warehouse in a different location; there are also different types of concentrators which may or may not be connected to the different types of digital switches; again there is the overlay option; the per-line cost for the remote option depends on the distance between this location and the digital switch on which it is homed.

(iii) to keep an analog switch; this option corresponds to direct transportation from the plant to the customer in the physical distribution context, and can be modelled by using an additional "dummy" location.

The option of overlay consists in keeping, at a given decision time t (not necessarily t=0), the existing analog lines with the corresponding switching equipment, and to install new equipment, concentrator or digital switch, which will be used to meet the future demand. This option can be economically attractive, and it doubles the number of types in cases (i) and (ii).

Economic considerations and also administrative policies rule out the possibility of performing several changes at any given location. Thus the existing analog equipment will be kept and provisioned until either overlay or replacement is performed (if any) and no further modification will be considered. So the assumptions of section 2 are verified.

In a previous work [5] it has been observed that the various costs are strictly concave functions of time. As a consequence, it can be shown that in an optimum solution, all changes must be made either immediatly (i.e. at time t=0) or at the end of the period of study, that is the horizon. In that case the problem reduces to a simple static SPLP [5]. However there are conditions under which this reduction is not valid. First the previous result assumes infinite capacities; while this is sometimes a safe practical assumption, this is no longer true for large cities or for areas with a demand increasing at a fast rate. Second, it has also been observed [26] that cost functions may not be concave at all. Finally there are also time-dependent constraints such as a limit on budget per year, and also limited supplies of new equipment. For all these reasons a simple SPLP model is no longer sufficient.

4- OUTLINE OF THE PROGRAM DYPALM

We have developped a program named DYPALM which solves an extended version of the dynamic problem (2-1)-(2-6) with:

(i) multiple-choice options

(ii) capacity constraints of the type (2-8)

(iii) general side constraints expressed as linear inequations of type (1-7).

DYPALM is an extension of a program PALM which solves the static problem (1-1a)-(1-6a) with the above additional constraints (ii) and (iii). We briefly outline below the basic structure of PALM and then comment on its modification for more efficiently solving the dynamic problem.

The program PALM is basically a branch-and-bound algorithm and a detailed description can be found in [12]. The branching scheme is a multiple branching on the variables y_i^k based on the constraint (1-6a). The bounding scheme is twofold. The first relaxation used is a multiple-choice SPLP of type (1-1a)-(1-6a); it is obtained from Lagrangian relaxation of the capacity and side constraints. The resulting SPLP is solved by Erlenkotter's method [10] without resorting to branch-and-bound unless the primal-dual gap is larger than a given value. The Lagrange multipliers are updated using subgradient optimization [20], [21], [2]. We have observed that this relaxation is often sufficient for solving static problems when the capacities are not too tight. When it appears that this first relaxation is not sufficient, the program uses a second relaxation, similar to one used by Ross and Soland [31] for the Generalized Assignment Problem and by Geoffrion and McBride [18] for the Capacitated Plant Location Problem without single-sourcing constraints: constraints (1-3a) and side constraints are relaxed, using again Lagrange multipliers derived from the first relaxation, and the resulting problem can be decomposed in several disjoint multiple-choice knapsack problems with a choice constraint (1-6a) on the right-hand side.

When the branching process has assigned 0-1 values to all y_i^k variables, the remaining problem is a Generalized Assignment Problem with multiple-choice and side constraints. It is solved again by branch-and-bound, using multiple branching on the x_{ij}^h's based on (2-2), a multiple-choice knapsack problem relaxation similar to the above second relaxation, and subgradient optimization. It appeared in our experiments that, in the presence of additional constraints, the use of a few subgradient iterations is faster than both the pure branch-and-bound approach of Ross and Soland [31] and the pure subgradient approach of Chalmet and Gelders [3].

The Program DYPALM is a specialized version of PALM implementing the model described in section 2. In DYPALM the number of options for both "warehouses" (digital switches) and "customers" (concentrators) is multiplied by the number of decision periods, and chronological constraints are introduced among the side constraints. These chronological constraints are not stored explicitly but are generated, under the compact form (2-3-3), when their feasibility is tested and their Lagrange multipliers are to be modified. The chronological constraints are also used in the branching process to eliminate some variables: in the branch defined by $y_i^t=1$, all variables x_{ij}^{τ} with $\tau < t$ are automatically set to zero. Finally the knapsack constraints (2-8) are strengthened as follows: when considering the right-hand side defined by $y_i^t=1$, again all variables x_{ij}^{τ} with $\tau < t$ are set to zero, thus reducing the size of the problem.

The program DYPALM has been coded in FORTRAN and consists of about four thousands FORTRAN instructions. Its detailed structure and user instructions are documented in [13]. It is now being used to perform studies for the planning of the telephone network evolution for Canadian cities.

5- COMPUTATIONAL EXPERIMENTS

We describe below the computational experiments which have been performed as validation tests for DYPALM. The network under study is that of a large city in Ontario and it contains 16 locations. For every location there are two types of "warehouses" (digital switches: replacement and overlay), two types of "service" (concentrators: replacement and overlay) and the option of keeping the "existing service" (analog switch). There are either three or four decision periods for these studies, and thus either 1680 or 2240 binary variables. We note that the input for such a problem consists of the cost coefficients for each one of these variables, plus demands and capacities and possibly the entries for side constraints. This input is produced by another program and stored on a file to be read by DYPALM.

The first set of runs is described in Table 1. In these experiments DYPALM has been compared with PALM used as follows: for every decision variable (digital switch and concentrator, for both replacement and overlay), the optimum decision time has been determined and the corresponding minimum cost is used in (1-1a) for this option. In some cases the local cost is not a concave function of time and the minimum does not occur at time $t=0$ or $t=T$; thus the static SPLP [5] is only an approximation to the dynamic problem. In run 1 (see Table 1) the PALM solution verifies the chronological constraints and thus is optimal. In runs 2 and 3 some of the chronological constraints are violated in the PALM solution, that is a concentrator is installed in a location and homed on a digital switch which does not yet exist. Except for such ill-timed concentrator placements, PALM and DYPALM solutions are also identical in runs 2 and 3. The DYPALM solution obtained with four periods differs from the three-period solution in only the decision time for the placement of one concentrator, which has been delayed by one period. The data for runs 1, 2, and 3 are similar except for different trunking costs.

For every run, Table 1 gives for both PALM and DYPALM the total CPU time and the value of the solution. The CPU times are on an IBM 370/168 and include inputting the data files in open format and producing the output on other files. The objective values of the solutions have been intentionnally altered to preserve the confidentiality of the data, but their relative values have been unchanged.

RUN NUMBER	PALM CPU TIME	PALM SOLUTION VALUE	DYPALM NUMBER OF PERIODS	DYPALM CPU TIME	DYPALM SOLUTION VALUE
1	.33	1940.10	3	3.06	1940.10
2	.33	1572.24[a]	3	.63	1572.30
3	.33	1889.22[a]	3	.78	1889.76
			4	.91	1889.46

[a] Solution violating some chronological constraint

All CPU times are in seconds, on IBM 370/168.

TABLE 1: Comparison between static and dynamic programs.

Further runs (see Table 2) have been made to test the sensitivity of the solution of run 1 to various parameter changes. This solution requires 7 digital switches, allocated as follows: four in period 1, two in period 2 and one in period 3. Runs 4 to 10 differ from run 1 by the addition of one side constraint on the total number of digital switches. In runs 7 and 9 the branch-and-bound algorithm is used to produce an heuristic solution with specified accuracy. Runs 11 to 16 differ by limits on this total number of digital switches for every period; blank entries indicate no limit. Other runs have been made to test the sensitivity of the solution to the capacity of the digital switches. In runs 1 to 16, this capacity is 1 000; different capacities have been used in runs 17 to 20 (these capacity figures have also been altered for confidentiality). It has also been observed that in all these runs the concentrator option with overlay always dominated the concentrator option with replacement in the following sense: overlay costs less and the demand is smaller. Thus in all the previous solutions overlay with concentrators is always preferred to replacement with concentrators. The data for runs 21 to 25 have been artificially constructed as follows: they are derived from data of run 1 by exchanging the demands between the overlay and replacement options for concentrators. In run 21 the capacities are large enough to accomodate the less expensive option. In run 25, the capacities are set to zero and the only solution is to keep analog equipment. The difference in the value of solutions between runs 1 and 25 gives a measure of the savings resulting from the use of new digital equipment.

Run number	Limit number of digital switches[a] global	period 1	period 2	period 3	Capacity of digital switches[b]	Maximum deviation from optimum[c]	DYPALM CPU time[d]	Solution value
4	6						4.58	1941.45
5	5						5.06	1944.12
6	4						5.25	1952.01
7	3					2.0%	5.76	2006.28
8	3						39.09	1968.81
9	2					.5%	42.33	2046.90
10	2						> 100	(2046.90)[e]
11		4	1			.5%	2.40	1949.61
12		4	1				7.25	1941.33
13		3	2	2			3.04	1940.76
14		1	1			1.0%	12.22	1997.73
15		1	1			.5%	22.33	1994.61
16		1	1				> 100	(1994.61)[e]
17					1400		4.59	1940.10
18					800		4.84	1940.10
19					700		8.02	1940.79
20					600		11.03	1942.02
21					1200		4.59	1940.10
22					1000		4.24	1942.23
23					200		6.12	2482.59
24					100		1.56	2557.05
25					0		1.02	2571.18

a Blank entry indicates no limit

b Blank entry indicates a capacity of 1000

c Blank entry inditates 0% deviation (exact solution)

d CPU time in seconds on IBM 370/168.

e Value of best solution found before time-limit

TABLE 2 Results of sensitivity tests.

We conclude this section by a comparison between DYPALM and other methods for planning the evolution of a telephone network. First, the model used in DYPALM is a simplification, in which the inter-office trunking costs have not been included; these costs are dependent on the relative locations of digital (and analog) switches, making the problem more similar to a Quadratic Assignment Problem [14]. This is yet a much more difficult combinatorial problem, and the telephone planning problem is still more complicated because the traffic depends on the location of switches and on routing methods, and also because of its dynamic aspect. However it has been observed [5] that the inter-office trunking represents a small part of the overall network cost (switching and trunking). Furthermore, the variations of this cost for different solutions can be considered as negligible with regard to the uncertainties in the demand and cost forecasting. The cost of solutions including this inter-office trunking cost can be computed by using the simulator LNES [6]. This simulator has been used by experienced practitioners to derive an evolution for the same network as used in run 1 of Table 1. They made about two hundred runs with LNES to manually produce this solution in several months. The simulator LNES is also used as a subroutine in a heuristic algorithm LNEO [6]. We give in Table 3 the total costs of the solutions produced by the manual method, the heuristic LNEO and the program DYPALM, and approximate total running times (IBM 370/168) for these three methods.

run number (Table 1)	Manual solution value (CPU time[a])	LNEO solution value (CPU time[a])	DYPALM solution value (CPU time[a,c])
1	2262. (3000.)	2254.5 (140.)	2250. (4.6)
2	b	1800. (140.)	1800.1 (2.1)

a: approximate value, seconds on IBM 370/168

b: no manual attempt has been made on this problem.

c: including one run of the simulator LNES

<u>TABLE 3</u> Comparisons of three methods for planning the evolution of a telephone network.

6- CONCLUSION

In conclusion, we state that it is possible to directly deal with the dynamic aspect of practical location problems in an efficient way. For the planning problem of telephone network evolution reported here, a discrete dynamic model provided the most useful tool: the comparatively limited computer requirements of the branch-and-bound algorithm DYPALM make it attractive to derive an optimum evolution for various scenarios and to achieve a better understanding of the economic implications of the decisions to be made.

REFERENCES

[1] BALINSKI, M.L. On finding integer solutions to linear programs. Mathematica, Princeton, N.J. (1964).

[2] CAMERINI, P.M., FRATTA, L. and MAFFIOLI, F. On improving relaxation methods by modified gradient techniques. Mathematical Programming Study 3 (1975) 26-34.

[3] CHALMET, L.G. and GELDERS, L.F. Lagrangean relaxations for solving a warehousing model, Werkdokument 76-15, Catholic University of Leuven, Belgium (1976).

[4] CHRISTOFIDES, N. and WATSON-GANDY, C.D.T. Improving profits with distribution services. Physical Distribution 3, (1973) 322-331.

[5] COMBOT, J.P. and MASON, L.G. Optimal switch-concentrator location for a telephone network. IEEE Transactions on Comm. 27 (1979) 36-44.

[6] COMBOT, J.P., PARMENTIER, P. and TSUI, M. Optimal digital switching introduction into the local network, to appear in the Proceedings of the International Switching Symposium, Paris, France (1979).

[7] DAVIS, P.S. and RAY, T.L. A branch-and-bound algorithm for the capacitated facilities location problem. Naval Research Logistics Quarterly,16 (1969) 331-334.

[8] ELLWEIN, L.B. and GRAY, P. Solving fixed-charge location-allocation problems with capacity and configuration constraints. AIEE Transactions 3 (1971) 290-298.

[9] ERLENKOTTER, D. Facility location with price-sensitive demands: private, public and quasi-public. Management Science 24 (1977) 378-386.

[10] ERLENKOTTER, D. A dual-based procedure for uncapacitated facility location. Operations Research 26 (1978) 992-1009.

[11] ERLENKOTTER, D. A comparative study of approaches to dynamic location problems. Paper presented at the International Symposium on Locational Decisions, Banff, Canada (1978).

[12] EVRARD, P. Problème d'affectation et de localisation multiples avec contraintes générales. M.Sc. Thesis, Ecole Polytechnique de Montréal, Canada (1979).

[13] EVRARD, P. Problème d'affectation et de localisation multiples avec contraintes générales: structure et résultats d'exploitation, Recherches Bell-Northern, Verdun, P.Q., Canada (1979).

[14] FRANCIS, R.L. and WHITE, J.A. Facility layout and location: an analytical approach (Prentice-Hall, Englewood-Cliffs, N.J., 1974).

[15] GEOFFRION, A.M. A guide to computer-assisted methods for distribution systems planning. Sloan Management Review 16 (1975) 17-41.

[16] GEOFFRION, A.M. Better distribution planning with computer models. Harvard Business Review, 54 (July-August 1976) 92-99.

[17] GEOFFRION, A.M. Making better use of optimization capability in distribution system planning. Working paper No. 279. Western Management Science Institute, University of California at Los Angeles, CA (1978).

[18] GEOFFRION, A.M., and MCBRIDE, R. Lagrangean relaxation applied to facility location problems. AIIE Transactions 10 (1978) 40-47.

[19] HANSEN, P. and THISSE, J.-F. Multiplant location for profit maximisation. Environment and Planning A, vol. 9 (1977) 63-73.

[20] HELD, M. and KARP, R.M. The traveling salesman problem and minimum spanning trees: part II, Mathematical Programming 1 (1971) 6-25.

[21] HELD, M., WOLFE, P. and CROWDER, H. Validation of subgradient optimization. Mathematical Programming 6 (1974) 62-88.

[22] KHUMAWALA, B.M. An efficient brand and bound algorithm for the warehouse location problem. Management Science 18 (1972), B718-B731.

[23] KHUMAWALA, B.M. and WHYBARK, D.C. Solving the dynamic warehouse location problem. Int. J. of Physical Distribution 6 (1976) 238-251.

[24] KRARUP, J. and PRUZAN, P.M. Selected families of discrete location problems, in P.L. Hammer, E.L. Johnson and B. Korte, eds. Discrete Optimization 77 Surveys (North-Holland, Amsterdam 1979).

[25] LEA, A.C. Location-allocation systems: an annotated bibliography. Discussion Paper 13, Department of Geography, University of Toronto, Canada (1973).

[26] MASON, L.G. and COMBOT, J.P. Optimal modernization policies for telecommunications facilities. Recherches Bell-Northern, Verdun, P.Q., Canada (1979).

[27] MCGINNIS, L.F. A survey of recent results for a class of facilities location problems. AIIE Transactions 9 (1977) 11-18.

[28] MERCER, A. The impact of marketing on branch location. Paper presented at the International Symposium on Locational Decisions, Banff, Canada (1978).

[29] ROODMAN, G.M. and SCHWARZ, L.B. Optimal and heuristic facility phase-out strategies. AIIE Transactions 7 (1975) 177-184.

[30] ROODMAN, G.M. and SCHWARZ, L.B. Extensions of the multi-period facility phase-out model: new procedures and application to a phase-in/phase-out problem. AIIE Transactions 9 (1977) 103-107.

[31] ROSS, G.T. and SOLAND, R.M. A branch and bound algorithm for the generalized assignment problem. Mathematical Programming 8 (1975) 91-103.

[32] ROSS, G.T. and SOLAND, R.M. A multicriteria approach to the location of public facilities. Rapport technique EP78-R-16, Ecole Polytechnique de Montréal, Canada (1978).

[33] SCHRAGE, L. Implicit representation of variable upper bounds in linear programming. Mathematical Programming Study 4 (1975) 118-132.

[34] STOKER, R.B. Incorporating market characteristics into physical distribution models. European J. of Operations Research 2 (1978) 232-245.

[35] SWEENEY, D.J. and TATHAM, R.L. An improved long-run model for multiple warehouse location. Management Science 22 (1976) 748-758.

[36] TRUSCOTT, W.G. The treatment of revenue generation effects of facility location. AIIE Transactions 7 (1975) 63-69.