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Online Convex Optimization for On-Board Routing in High-Throughput Satellites

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Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
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Online Convex Optimization for On-Board Routing in High-Throughput Satellites

présenté par **Olivier BÉLANGER**
en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
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DEDICATION

Ad astra per aspera

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RÉSUMÉ

Ces dernières années ont vu une augmentation marquée du nombre d'utilisateurs de services Internet par constellation de satellites. Ces services sont particulièrement nécessaires dans les zones reculées où les réseaux terrestres traditionnels sont inexistants ou trop coûteux à mettre en place, améliorant ainsi la connectivité mondiale. Alors que les principales entreprises de services Internet par satellites se concentrent sur le lancement de satellites supplémentaires comme solution à court terme, une approche plus durable consiste à concevoir des satellites plus performants et à plus haute capacité : les satellites à ultra-haut débit (*extremely high-throughput satellites*, EHTS). Malgré des recherches approfondies sur l'amélioration du transfert de données entre les satellites de ces constellations, il subsiste une lacune importante dans la compréhension et l'optimisation du routage interne au sein des EHTS, en particulier entre les modems. Une telle optimisation est cruciale pour garantir le niveau de qualité de service désiré aux utilisateurs. Ce mémoire aborde le défi de minimiser les coûts de perte de paquets dans un système satellitaire, en tenant compte des ressources de calcul limitées disponibles à bord. Nous modélisons ce problème comme un problème d'optimisation convexe sujet à des contraintes sur le système et physiques. Notre solution repose sur la commande prédictive (*model predictive control*, MPC) pour optimiser le routage de données entre les modems. Nous améliorons ensuite cette approche avec le concept d'optimisation convexe en temps réel (*online convex optimization*, OCO), en utilisant spécifiquement l'algorithme `OIPM-TEC`. Cette nouvelle combinaison, appelée OCO-MPC, offre un équilibre entre performance et efficacité de calcul. La méthodologie basée sur MPC apporte une réactivité face à l'incertitude, tandis que l'OCO permet une adaptation en temps réel avec des besoins de calcul réduits. Cette approche rend notre solution adaptée à une mise en œuvre dans les systèmes matériels satellitaires actuels, où la puissance de traitement est limitée. Nous évaluons les performances de notre approche à l'aide de simulations numériques par rapport aux méthodes traditionnelles qui sont soit irréalistes, soit peu performantes dans des scénarios incertains. Notre méthode s'avère très efficace dans des conditions hautement incertaines tout en étant adaptative et implémentable sur du matériel satellitaire.

ABSTRACT

Recent years have witnessed a significant increase in users of satellite constellation Internet connectivity services. These services are particularly valuable in remote areas where traditional terrestrial networks are unavailable or prohibitively expensive to implement, thus enhancing global connectivity. While current popular satellite Internet service companies have focused on launching more satellites as a short-term solution, a more sustainable approach involves designing better, higher-capacity satellites: extremely high-throughput satellites (EHTS). Despite substantial research on improving data transfer between satellites in such constellations, there remains a significant gap in understanding and optimizing internal routing within EHTS, specifically between modems. Such optimization is crucial for ensuring the desired quality of service to users. This Master's thesis addresses the challenge of minimizing packet loss costs in a satellite system, considering the limited computational resources available on-board. We model this issue as a convex optimization problem subject to system and physical constraints. Our solution is based on model predictive control (MPC) to optimize packet routing between modems. We then enhance this approach utilizing online convex optimization (OCO), specifically using the `OIPM-TEC` algorithm. This new combination, termed OCO-MPC, offers a balance between performance and computational efficiency. MPC provides responsiveness to uncertainty, while OCO enables real-time adaptation with reduced computational requirements. This approach makes our solution suitable for implementation in current satellite hardware systems, where processing power is limited. We assess the performance of our approach in numerical simulations against traditional methods that are either unrealistic or poorly performing in uncertain scenarios. Our method proves highly efficient under highly uncertain conditions while remaining scalable and feasible for satellite hardware implementation.

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LIST OF SYMBOLS AND ABBREVIATIONS

EHTS	Extremely High-Throughput Satellites
HTS	High-Throughput Satellites
LEO	Low Earth Orbit
MMPP	Markov-Modulated Poisson Process
MPC	Model Predictive Control
OCO	Online Convex Optimization
QoS	Quality of Service
VHTS	Very High-Throughput Satellites
OIPM-TEC	Online Interior-point Method for Time-varying Equality Constraints
εOIPM-TEC	Epsilon-Online Interior-point Method for Time-varying Equality Constraints

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CHAPTER 1 INTRODUCTION

Satellites have long been a cornerstone of global communication networks, facilitating a range of applications that have revolutionized how information is shared across the globe. Initially, one of the primary commercial uses of satellites was television broadcasting [1]. By transmitting signals from space, satellites enabled the delivery of television content to vast and geographically dispersed audiences, overcoming the limitations of terrestrial broadcast infrastructure [2]. This capability not only transformed the television industry but also demonstrated the immense potential of satellite technology to bridge communication gaps internationally.

Over the past few decades, the role of satellites has expanded beyond television broadcasting. One of the most significant advancements has been in the area of Internet connectivity. The advent of high-throughput satellites (HTS) and beyond marked a pivotal shift, providing significantly higher data transfer rates compared to traditional satellites [3]. HTS have the capacity to deliver high-speed Internet to regions where terrestrial infrastructure is lacking or non-existent [4].

This expansion in the capabilities of satellites has profound implications for global connectivity. In many parts of the world, especially in remote and rural areas, access to the Internet remains limited or unreliable [5, 6]. This digital divide exacerbates socio-economic inequalities, as access to information and communication technologies is increasingly critical for education, healthcare, economic opportunities, and social inclusion. By providing reliable Internet access to these underserved regions, HTS play a vital role in reducing location-based societal divisions.

The increasing demand for satellite Internet services, driven by the need for global connectivity, presents new challenges and opportunities for optimizing satellite communication networks [4]. One of the primary challenges is efficiently managing the internal routing of data within these networks to ensure optimal performance and resource utilization. Effective internal routing is essential for maintaining a high quality of service (QoS), minimizing latency, and maximizing the throughput of satellite systems.

The complexity of modern satellite networks necessitates innovative approaches to routing and resource management. Traditional routing methods, while effective for simpler systems, often fall short in the dynamic and resource-constrained environment of satellite networks, particularly in HTS [7]. To address these challenges, there is a growing interest in leveraging advanced control and optimization techniques that can dynamically adapt to changing network conditions and demands [8, 9]. However, it is important to note that while significant research exists on constellation-level routing, i.e. between satellites, there remains a gap in the literature specifically addressing internal routing within a single satellite.

This Master's thesis explores the development of a framework for the on-board online routing within the recently introduced extremely high-throughput satellites (EHTS) [10], aiming to enhance the efficiency and reliability of satellite communication networks while taking into account the limited computational resources available on-board. By integrating optimization algorithms and advanced control methodologies tailored for resource-constrained environments, the proposed framework seeks to optimize data flow within satellite networks, ensuring that resources are utilized effectively and service quality is maintained. Ensuring high QoS in EHTS involves complex decision-making processes to manage limited resources efficiently. Optimization techniques can enhance QoS by dynamically routing information based on real-time demand and system load, and balancing traffic appropriately to prevent congestion. This research focuses on developing lightweight, computationally efficient solutions that can operate within the constraints of satellite hardware, balancing the need for sophisticated routing strategies with the practical limitations of on-board processing capabilities while maintaining high QoS.

The importance of this research lies not only in its technical contributions but also in its potential to impact global connectivity. As the demand for high-speed Internet continues to grow, especially in remote and underserved areas, the ability to provide efficient and reliable satellite-based Internet services will be crucial. This work aims to contribute to this goal by offering innovative solutions to the challenges of satellite internal routing, ultimately supporting the broader objective of bridging the digital divide and fostering a more connected world.

The following sections introduce the basic concepts that will be covered in this thesis, namely routing, optimization, and model predictive control. Next, the research objectives and the thesis plan will be presented.

1.1 Routing

Routing is a fundamental concept in network communications, encompassing the process of selecting optimal paths for data transmission across interconnected nodes. In traditional networks, routing algorithms determine the most efficient routes based on various metrics such as distance, cost, or network congestion. These algorithms must be scalable to accommodate networks of varying sizes and adaptable to changing network conditions. Additionally, they often incorporate QoS considerations to meet specific performance requirements for different types of traffic.

General routing protocols, such as Open Shortest Path First [11] or Border Gateway Protocol [12], are designed to handle the complexities of large-scale, distributed networks. These protocols maintain routing tables, exchange network topology information, and continuously update their path selections to ensure efficient data delivery across dynamic network environments. As explained in [13], routing in such contexts must balance the need for optimal path selection with the overhead of maintaining up-to-date network information.

In contrast, internal routing within EHTS presents a unique set of challenges and requirements. Unlike traditional network routing, which deals with interconnected nodes spread across geographical areas, EHTS internal routing focuses on managing data flow between modem banks (or on-board processors) within a single satellite. This specialized form of routing operates under severe constraints in terms of computing power, memory, and energy availability, as highlighted in [7].

The key distinction between general routing and EHTS internal routing lies in their operational context and objectives. While general routing aims to find optimal paths through vast, dynamic networks, EHTS internal routing is concerned with maximizing throughput and minimizing packet loss within the fixed hardware configuration of a satellite. Effective internal routing is crucial to minimize packet losses and maintain high QoS, ensuring reliable data transmission and optimal performance. As [14] explains, the internal architecture of these satellites requires sophisticated routing mechanisms to handle the increasing complexity of on-board digital processing.

HTS internal routing must adhere to strict real-time requirements and QoS constraints while managing potentially conflicting priorities for different data streams. This necessitates the development of specialized algorithms that can make rapid decisions to maintain high throughput and low latency. Furthermore, these algorithms must be highly efficient due to the limited on-board resources, a challenge not typically faced in terrestrial network routing where more powerful hardware is often available, with virtually unlimited access to energy.

In brief, while general routing principles provide a foundation for understanding network data flow, the specific demands of internal routing in EHTS call for innovative approaches tailored to the unique characteristics of satellite communications. This specialized field of routing represents an important area of ongoing research and development in satellite technology, as the industry strives to meet the ever-increasing demand for high-capacity, low-latency satellite communications.

1.2 Optimization

Optimization is a fundamental concept in mathematics and engineering, focusing on finding the best solution from a set of possible alternatives. In the context of satellite communications and network routing, optimization plays a crucial role in maximizing system performance while adhering to various constraints.

The field of optimization encompasses a wide range of techniques and algorithms designed to solve complex problems efficiently. These methods can be broadly categorized into linear, nonlinear, integer, stochastic, dynamic, and convex programming, among others [15]. Each category is suited to different types of problems and constraints, allowing for flexibility in addressing diverse challenges in satellite network management.

In the field of satellite internal routing, particularly within EHTS, optimization techniques are critical for resource allocation, traffic management, and QoS maintenance. These methods allow for the formulation of routing problems as mathematical models, which can then be solved to find optimal or near-optimal solutions [16]. Optimization enables efficient allocation of limited on-board resources, balances traffic across multiple modems to prevent congestion, and ensures strict QoS, all within the tight computational constraints of satellite hardware. The application of these techniques aims to maximize throughput, thus enhancing QoS.

However, the dynamic nature of satellite networks and the real-time requirements of data routing present unique challenges for optimization. This has led to increased interest in online and adaptive optimization techniques that can respond quickly to changing network conditions while maintaining high performance [17].

1.3 Model Predictive Control

Model predictive control (MPC) is an advanced control method that has gained significant traction in various fields, including satellite communications. MPC uses a dynamic model of the system to predict its future behaviour and optimize control actions accordingly [18, 19].

In the context of internal routing within EHTS, MPC can be particularly beneficial. It allows for proactive decision-making based on predicted traffic patterns, enabling more efficient resource allocation and QoS maintenance. By optimizing routing decisions over a given time horizon, MPC can help balance the load across multiple modems and adapt to uncertainty.

The core principle of MPC involves solving an optimal control problem over a rolling horizon at each time step. This approach allows the controller to anticipate future events and take appropriate actions, making it particularly suitable for systems with complex dynamics and constraints [19]. In the MPC framework, only the decision for the first time step of the prediction window is actually implemented. After this implementation, the horizon is shifted forward by one time step, the system state is updated, and the optimization process is repeated. This rolling horizon strategy forms the backbone of MPC, allowing for continuous adaptation to changing conditions while maintaining a forward-looking perspective. In the context of satellite internal routing, MPC can be used to optimize routing decisions as a function of predicted traffic patterns, implementing only the immediate routing decision before re-evaluating the network state and re-optimizing for the next time step.

One of the key advantages of MPC is its ability to handle multi-variable systems and incorporate constraints explicitly in the control formulation. This makes it well-suited for managing the complex, interconnected nature of satellite communication networks, where multiple objectives and constraints must be considered simultaneously [20].

The application of MPC to satellite internal routing presents opportunities for improving network performance and resource utilization. By predicting future network states and optimizing routing decisions over a rolling time horizon, MPC can potentially lead to more efficient and precise routing strategies compared to traditional, reactive approaches [21].

However, implementing MPC in satellite systems presents several challenges, including computational complexity and the need for accurate system models. Addressing these issues is an objective of this thesis.

1.4 Research Objectives

This Master's thesis aims to address the critical challenge of designing an efficient routing framework for EHTS within the constraints of limited on-board computational resources. Our primary objective is to design and optimize an on-board internal routing scheme to enhance satellite performance and capacity. To do so, we first build a convex optimization-based satellite representation with time-varying equality constraints and inequality constraints. Then, to be adaptive and practical, we develop an MPC framework to adapt this optimization problem. Next, we tackle the challenge of limited on-board resources by presenting an online convex optimization-based MPC approach, making it implementable in real-time and compatible with satellite hardware.

Ultimately, this work aspires to contribute to the advancement of EHTS design. By optimizing internal routing, we aim to increase the number of users that can be served per satellite, thereby addressing a crucial aspect of the global connectivity challenge. Our overarching goal is to help bridge the connectivity gap in remote areas across the world.

1.5 Structure of the Master's Thesis

The remainder of this Master's thesis is organized into six chapters. Chapter 2 provides a review of the relevant literature. Chapter 3 presents the details of our optimization and MPC framework. Chapter 4 presents the numerical results. Chapter 5 offers a discussion and conclusion, along with suggestions for future work.

1.6 Contributions

This section summarizes the publications that resulted from this Master's thesis.

As first author

Journal paper

J1. **Olivier Bélanger**, Jean-Luc Lupien, Olfa Ben Yahia, Stéphane Martel, Antoine Lesage-Landry, and Gunes Karabulut Kurt. *Online Convex Optimization for On-Board Routing in High-Throughput Satellites*, IEEE Wireless Communications Letters. August 2024. Submitted.

Conference paper

C1. **Olivier Bélanger**, Olfa Ben Yahia, Stéphane Martel, Antoine Lesage-Landry, and Gunes Karabulut Kurt. *Quality of Service-Constrained Online Routing in High Throughput Satellites.* In: 2024 IEEE Aerospace Conference.

As contributor

Journal paper

J2. Olfa Ben Yahia, Zineb Garroussi, **Olivier Bélanger**, Brunilde Sansò, Jean-François Frigon, Stéphane Martel, Antoine Lesage-Landry, and Gunes Karabulut Kurt. *Evolution of High Throughput Satellite Systems: A Vision of Programmable Regenerative Payload*, IEEE Communications Surveys & Tutorials. August 2024 (early access).

Presentations

P1. *Online Convex Optimization for On-Board Routing in High-Throughput Satellites*, 25th International Symposium on Mathematical Programming, Montréal, QC, Canada. July 2024.

P2. *Model Predictive Control-Based Routing in High-Throughput Satellites*, Optimization Days, Montréal, QC, Canada. May 2024.

P3. *Quality of Service-Constrained Routing in High Throughput Satellites*, CORS/Optimization Days, Montréal, QC, Canada. May 2023.

CHAPTER 2 LITERATURE REVIEW

In this chapter, we discuss satellite on-board routing, optimization, model predictive control, and Markov-modulated Poisson processes.

2.1 Satellite On-Board Routing

Satellites play an increasingly vital role in global communications [22], connecting vast and remote territories and bridging communication gaps in regions where deploying terrestrial infrastructure for both cellular and fiber optic networks remains challenging [23]. While remote areas have traditionally relied on satellites for communications and television broadcasting [24], in recent years, these services have expanded to include Internet connectivity [25]. This expansion underlines the immense potential and importance of satellite networks, especially when considering the growth of direct cell-to-satellite connectivity [26]. As a high number of users with varying needs directly engage with satellite resources, the implementation of resilient and reliable QoS mechanisms becomes essential [27]. Prioritizing traffic, ensuring data delivery, and managing congestion become crucial in such scenarios.

With the growing reach and appeal of these services, there is a notable shift from traditional single-beam satellites to the more agile multi-beam HTS [28]. The movement of low Earth orbit (LEO) satellites adds a layer of uncertainty, necessitating rapid adaptation in satellite network management. Unlike their predecessors, HTS are engineered to handle high data rates, addressing the digital requirements of modern society [29]. As global demand for high data rate connectivity grows, the need for rapid and QoS-reliable HTS intensifies.

The intricacies of HTS, coupled with their enhanced capabilities, necessitate specialized components for data management. A pivotal part of this internal architecture is the modem banks, as shown in [10, 30], and depicted in Figure 2.1. They act as the central processing units, ensuring efficient handling, routing, and distribution of the massive influx of data within the satellite system. Software-based, multi-modem banks architectures, as presented in [31], aim to increase total throughput and improve efficiency in flow allocation. This approach is particularly crucial for the recently introduced EHTS, which demand unprecedented levels of data processing and routing capabilities [10].

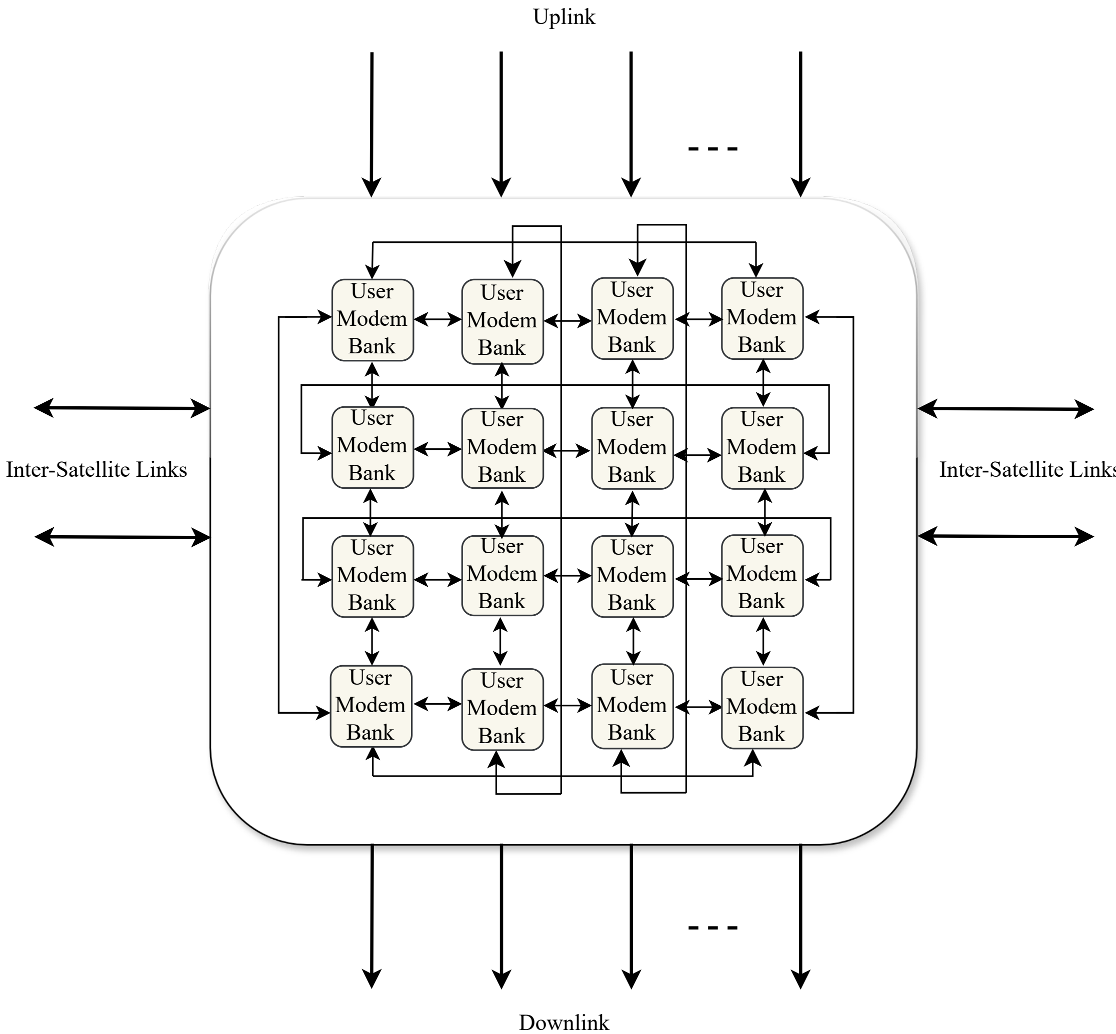


Figure 2.1 Multi-modem EHTS internal architecture [10]

The interconnections between modem banks bring forward new routing and load balancing challenges. Properly addressing these challenges is of high importance because it can significantly enhance data throughput and reduce latency. In the high-stake field of satellite communication, ensuring these efficiencies directly translates to improved user experience and service reliability.

An important aspect of managing satellite communication involves the prioritization of packets and the incoming information rate. Packet priorities determine the processing and transmission order, ensuring high-priority data receives the necessary bandwidth. The incoming information rate represents the rate at which data packets arrive at the satellite, essential for effective traffic management and congestion control.

With the global demand for high-quality communication, enhancing the QoS and expanding satellite accessibility to a broader user base is crucial. A large range of research has explored satellite communication, with substantial literature dedicated to enhancing the performance of both traditional satellites and the more advanced HTS, as well as the networks they create [32–35]. Load balancing strategies, indispensable for evenly distributing incoming data and maintaining peak performance, have gained attention in works like [36]. These strategies are crucial for ensuring that no single modem bank or processing unit is overwhelmed, leading to potential data losses or delays. Efficient load balancing allows for optimal resource utilization, minimizes response time, and prevents system overload. Its relevance to HTS networks, where uneven traffic could lead to significant bottlenecks and degrade QoS, is highlighted in studies such as [37].

However, an important gap emerges when addressing internal routing and load balancing specifically tailored to EHTS. EHTS, given their capability to handle extremely high data rates, inherently require specialized internal routing mechanisms to efficiently manage and distribute the incoming data. The variable and uncertain nature of these incoming flows introduces an additional layer of complexity to the routing challenges. The complexities arise particularly when interconnecting modem banks, amplifying the need for strong routing strategies. While many advancements have been made on the constellation-level (inter-satellite) front [38,39], the challenges inherent to internal EHTS routing remain comparatively overlooked.

Despite the critical need for effective internal packet routing, there is a notable lack of research focusing on routing mechanisms between on-board processors in EHTS. Efficient packet routing is essential to minimize information loss and ensure QoS [27]. While existing literature has begun to explore this area, with some works proposing optimization-based frameworks to tackle the challenge [31], the field remains largely unexplored.

Potential approaches to address this challenge include utilizing multi-commodity flow models to minimize the maximum residual capacity of inter-modem links [31], or employing model predictive control (MPC) strategies to optimize internal routing and packet scheduling. However, implementing MPC or any optimization-based approach in EHTS poses significant challenges due to limited available computational resources. Satellite systems are inherently resource-constrained, limited by low power capabilities, which makes addressing these limitations crucial for the practical deployment of advanced routing solutions.

Improving internal routing in EHTS not only promises more efficient satellite operations but can also catalyze broader socioeconomic advancements by connecting previously under-served and inaccessible communities in remote regions. This, in turn, can pave the way for greater economic opportunities, for example, via improved access to information and education.

2.2 Optimization

Optimization is a branch of applied mathematics that seeks to find the best available solution to minimize an objective function. Constrained optimization extends this goal by incorporating constraints on system parameters, adding complexity to the problem [40]. A general form of a constrained optimization problem can be written as:

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) \\ \text{s.t.}\quad & g_i(\mathbf{x}) \leq 0 \qquad i = 1,\dots,I \\ & h_j(\mathbf{x}) = 0 \qquad j = 1,\dots,J, \end{aligned} \tag{2.1}$$

where $I, J \in \mathbb{N}$ denote, respectively, the number of inequality and equality constraints.

In this formulation, $f(\mathbf{x})$ represents the objective function to be minimized, $g_i(\mathbf{x})$ are inequality constraints that define the feasible region, and $h_j(\mathbf{x})$ are equality constraints that the solution must satisfy.

Convex optimization is a branch of optimization that focuses on problems with a convex objective function and a convex feasible space induced by the constraints, ensuring any local minimum is also a global minimum [15]. This property simplifies the search for an optimum and guarantees global optimality. A typical convex optimization problem is formulated as:

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) \\ \text{s.t.}\quad & g_i(\mathbf{x}) \leq 0 \qquad i = 1,\dots,I \\ & \mathbf{A}^\top\mathbf{x} = \mathbf{b}. \end{aligned} \tag{2.2}$$

In this convex optimization formulation, $f(\mathbf{x})$ is a convex objective function, $g_i(\mathbf{x})$ are convex inequality constraints, and $\mathbf{A}^\top\mathbf{x} = \mathbf{b}$ are affine equality constraints, where $\mathbf{A} \in \mathbb{R}^{n^2}$ and $\mathbf{b} \in \mathbb{R}^m$. The convex nature of the objective function and of the feasible region ensures that any local minimum found is also a global minimum, making the problem more tractable and the solutions more reliable.

However, real-world problems often come with a degree of uncertainty. Parameters may be unpredictable or information may be incomplete. To address these challenges, optimization theory has branched into different subfields: stochastic, robust, online, among others.

Stochastic optimization takes into account the probabilistic nature of problem parameters [41]. It seeks solutions that are optimal in the expected sense, considering the distribution of possible outcomes. This method is valuable when dealing with uncertainties in the system that can be probabilistically modelled.

Robust optimization aims to find solutions that remain feasible under all possible realizations of uncertainty within a system [42]. It provides guarantees even in the worst-case scenarios, making it ideal for applications where reliability is primordial.

Online optimization deals with problems that evolve over time, making decisions sequentially as new data becomes available and iterating quickly instead of solving a problem to optimality [17, 43]. It is particularly suited to environments where information unfolds in real-time, requiring immediate responses.

To address the challenge of internal routing within EHTS, online optimization is chosen as the preferred approach. This decision is driven by the problem's dynamic nature, where conditions and parameters change in real-time, and the need for immediate, adaptive decision-making is critical. Online optimization's ability to iteratively refine solutions as new information becomes available offers a strategic advantage, ensuring that decisions are always based on the most current data. This adaptability, combined with the potential for real-time responsiveness, aligns with the requirements of managing dynamic systems and responding to evolving challenges efficiently. An online optimization approach is designed to be off-the-shelf solver-free, eliminating the need for complex external solvers. This characteristic, coupled with its low computational requirements, makes it particularly suitable for implementation on satellite hardware, where processing power is limited.

2.2.1 EHTS and Optimization

Optimization in EHTS systems establishes a direct link between the physical challenges of satellite communications and the mathematical constructs of objective functions and constraints. The physical limitations, such as bandwidth restrictions, link and queue capacities, and signal variability, are translated into constraints within an optimization framework. These constraints represent the boundaries within which the EHTS must operate, ensuring that the solutions proposed by optimization algorithms are practically applicable in the physical system.

The objective function is designed to reflect the main focus of the EHTS, such as minimizing packet loss or maximizing throughput, which is rooted in the need for reliable, efficient, and QoS-ensuring data transmission. By formulating these physical and operational goals as mathematical objectives, optimization techniques can find a balance between the physical realities of LEO satellites with the desired network performance outcomes, driving the EHTS toward operational excellence that adheres to the demands of high-quality global communication services.

2.2.2 Online Convex Optimization

Online Convex Optimization (OCO) is a subset of online optimization that deals specifically with convex objective and constraint functions. It provides a framework for decision-making in dynamic environments where the objective function may change over time [43]. In OCO, at each time step t, the algorithm chooses a point $\mathbf{x}_t \in \mathbb{R}^n$ from a convex set $\mathcal{X} \in \mathbb{R}^n$, then observes a convex loss function $f_t : \mathbb{R}^n \mapsto \mathbb{R}$ and incurs a loss $f_t(\mathbf{x}_t)$. In this thesis, we work with the specific case of a time-invariant objective function f. The goal is to minimize regret.

In problems where the optimal decision changes over time, the dynamic regret can be used:

$$R_{\mathrm{d}}(T) = \sum_{t=1}^{T} \left(f(\mathbf{x}_t) - f(\mathbf{x}_t^*) \right), \tag{2.3}$$

where $\mathbf{x}_t^*$ is the optimal decision at time t and T is the total number of time steps.

For the internal routing within EHTS, where a certain tolerance to optimality is deemed reasonable, we use the epsilon-dynamic regret, defined as:

$$R_{\varepsilon}(T) = \sum_{t=1}^{T} \left[f(\mathbf{x}_t) - f(\mathbf{x}_t^*) - \varepsilon \right]^+, \tag{2.4}$$

where $\varepsilon > 0$ is the desired tolerance, and where $[.]^+$ is the positive part of the argument, i.e. $[a]^+ \triangleq \max(0, a)$.

OCO has found applications in various fields, including machine learning [17], finance [44], network routing [45], and power systems [46]. Its ability to adapt to changing environments makes it particularly suitable for satellite communication systems, where network conditions and data flows can vary rapidly [17].

2.2.3 Newton Methods in Optimization

Newton methods are a class of iterative optimization algorithms that use second-order derivatives to find the minimum of a function. These methods are known for their rapid convergence rates, especially near the optimum [40].

In the context of convex optimization, Newton methods can be particularly effective due to their ability to take advantage of the curvature information provided by the Hessian matrix. This allows them to make more informed steps towards the optimum compared to first-order methods [15]. First-order methods, such as gradient descent, use only the first derivative (gradient) information and are generally simpler and less computationally intensive but may converge more slowly. In contrast, second-order methods like Newton's method use both the first and second derivatives (Hessian), allowing for faster convergence but at the cost of increased computational complexity.

Interior point methods, a subset of Newton methods, are particularly useful for solving large-scale linear and nonlinear convex optimization problems by iteratively moving within the interior of the feasible region [47]. This approach can handle complex constraints efficiently, making it a powerful tool in optimization.

A specific technique within Newton methods is the infeasible Newton step, which allows for steps that do not necessarily satisfy all constraints at each iteration but are directed towards feasibility and optimality [15]. This approach can be particularly useful in complex optimization problems where strict adherence to constraints at every step is impractical.

2.2.3.1 Newton Methods in Online Convex Optimization

Recent research has explored the integration of Newton methods into the online convex optimization framework [46, 48, 49]. These online Newton methods aim to combine the adaptive nature of OCO with the rapid convergence of Newton's method [50].

For satellite communication systems, this combination could potentially offer significant advantages. The ability to quickly adapt to changing network conditions (provided by OCO) coupled with fast convergence to optimal solutions (provided by Newton methods) could lead to more efficient and responsive routing strategies in EHTS.

However, implementing Newton methods in resource-constrained environments like satellites presents challenges, primarily due to the computational complexity of calculating and inverting the Hessian matrix. Recent work has focused on developing approximations and modifications to make these methods more computationally tractable in online settings [51].

2.3 Model Predictive Control

MPC has evolved significantly since its inception in the 1970s [52]. At its core, MPC is an advanced optimization-based control strategy that solves a finite-horizon optimal control problem at each time step. The general formulation of an MPC problem can be expressed as:

$$\begin{aligned} &\min_{x_{t:t+W}} \sum_{k=t}^{t+W-1} f(\mathbf{x}_k, \mathbf{u}_k) \\ \text{s.t.} \quad & g_i(\mathbf{x}_k, \mathbf{u}_k) \leq 0 \qquad i = 1, \ldots, m, \quad k = t, \ldots, t+W-1 \\ & h_j(\mathbf{x}_k, \mathbf{u}_k) = 0 \qquad j = 1, \ldots, p, \quad k = t, \ldots, t+W-1 \\ & \mathbf{x}_{k+1} = d(\mathbf{x}_k, \mathbf{u}_k) \quad k = t, \ldots, t+W-1, \end{aligned} \tag{2.5}$$

where $\mathbf{x}_k$ and $\mathbf{u}_k$ represent, respectively, the state and control of the system, and $d(\mathbf{x}_k, \mathbf{u}_k)$, the system dynamics. In this formulation, we optimize over a finite time window W, starting from the current time t. The objective function is a sum over this window, and the constraints are applied for each time step within the window. The system dynamics constraint represents the evolution of the state over time.

Initially developed for process control in petrochemical industries, MPC has since found applications in diverse fields. In chemical process control, it has been used to maintain optimal operating conditions in complex refinery operations [53]. The automotive industry has adopted MPC for advanced driver assistance systems and autonomous vehicle control, particularly in trajectory planning and collision avoidance [54]. In energy management, MPC has been crucial in optimizing power grid operations, balancing supply and demand while minimizing operational costs [55].

Transitioning to aerospace applications, MPC has shown promise in various aspects of spacecraft control. For attitude control and station-keeping of satellites, MPC has demonstrated improved performance over traditional control methods [56]. In a recent study, researchers proposed an MPC policy for simultaneous station keeping, attitude control, and momentum management of low-thrust satellites, showcasing the versatility of MPC in handling multiple objectives [57].

2.3.1 Challenges in Applying MPC to Satellite Communications

While MPC offers significant potential for satellite communications, several challenges need to be addressed:

- *Computational Complexity*: Satellite systems have limited on-board computational resources, making the implementation of complex MPC algorithms challenging [58]. Research is needed to develop computationally efficient MPC formulations suitable for satellite hardware.

- *Model Accuracy*: The effectiveness of MPC heavily relies on the accuracy of the system model. While it can tolerate some model inaccuracies for the future, having a model that closely represents reality allows us to build a more realistic and effective algorithm. An accurate model will enhance the predictive capability of MPC and lead to better control performance. As underlined in [59], "for MPC to be successful, developing an accurate model [...] is critical." Moreover, if a model is too far from reality, we will suffer from feasibility problems. In satellite communications, developing accurate models that capture the dynamic nature of network traffic and satellite movement is a significant challenge [23].

- *Real-time Implementation*: The fast-changing nature of satellite communications requires real-time optimization. Developing MPC strategies that can operate in real-time within the constraints of satellite systems is an active area of research [21].

The application of MPC to internal routing in EHTS remains a largely unexplored area. This Master's thesis notably focuses on developing an MPC formulation specifically tailored to the unique challenges of EHTS internal routing. Our approach aims to address the trade-off between MPC performance and computational efficiency in resource-constrained satellite environments. By tackling the computational complexity, enhancing model accuracy, and ensuring real-time implementation, this research seeks to provide an effective solution for optimizing internal routing in EHTS systems.

2.4 Markov-Modulated Poisson Processes

Markov-modulated Poisson processes (MMPPs) are a powerful tool for modelling traffic patterns in various communication networks, including satellite networks. An MMPP is characterized by a Poisson process whose rate parameter $\lambda(t)$ is governed by an underlying continuous-time Markov chain. The state of the Markov chain at any time t determines the rate $\lambda(t)$, allowing the model to switch between different traffic intensities based on state probabilities.

MMPPs are particularly well-suited for capturing the bursty and irregular nature of traffic, which is a common characteristic in satellite communications. The ability of MMPPs to model both predictable periodic patterns and unpredictable bursts makes them ideal for applications where traffic patterns exhibit significant variability.

In the context of satellite networks, MMPPs have been used effectively for Internet Protocol traffic prediction. For instance, in [60], MMPPs were utilized to predict traffic in satellite networks, demonstrating their efficiency in capturing the bursty nature of satellite traffic.

Similarly, [61] highlighted the adaptability of MMPPs for processes with irregular bursts of activity combined with predictable patterns, aligning well with the characteristics of satellite traffic.

Given the similar traffic patterns observed in terrestrial and other satellite networks, the application of MMPPs to EHTS has potential. The inherent ability of MMPPs to capture both predictable periodic patterns and unpredictable bursts in traffic makes them suitable for modelling LEO EHTS traffic. By leveraging MMPPs, more accurate traffic predictions can be achieved, enhancing the reliability of models and bringing them closer to real-world implementation.

We provided a literature review on satellite on-board routing, mathematical optimization, MPC, and MMPPs to justify the challenge we tackle and our approach. Building on the insights gained from this review, we now turn our attention to the development of the MPC and OCO-MPC frameworks, which are designed to address the specific challenges identified in optimizing satellite communication systems.

CHAPTER 3 DETAILS OF THE SOLUTION

This chapter first presents the development of our optimization and MPC framework for internal routing within EHTS, before introducing the more implementable OCO-MPC approach.

3.1 Optimization and Model Predictive Control Approaches

This section presents our optimization-based MPC framework developed for EHTS internal routing. It is based on our published work [62].

3.1.1 EHTS Packet Management Model

In EHTS, there are multiple processing units referred to as modem banks. We consider a satellite equipped with $M \in \mathbb{N}$ modem banks, with each bank managing $P \in \mathbb{N}$ distinct priorities.

Queues play a crucial role in managing the flow of data packets. Each queue, associated with a specific priority within a modem bank, temporarily holds incoming data packets until they can be processed and routed. Complementing the role of the queues are the scheduler weights, $0 \leq w_p^m(t) \leq 1$. These weights are normalized such that they sum to 1 for a given module. They determine the fraction of packets of priority $p \in \{1, 2, ..., P\}$ that are selected from each queue from module $m \in \{1, 2, ..., M\}$ to be processed and subsequently transmitted. Traditionally, routing techniques relied on fixed weights, which often do not accommodate varying traffic demands and patterns effectively. By dynamically adjusting these weights, the satellite can adapt to the incoming flows and the current state of the queues.

We formulate the problem as a multi-commodity flow instance [63], where the data flows represent the commodities. In this context, the uplink beam serves as the source, and the sinks encompass both the downlink beam and lost packets. Figure 3.1 presents the simplified case where $M = P = 2$. In Figure 3.1, the uplink beam is depicted as two distinct beams for clarity of representation. However, it is essential to understand that these are virtual nodes. There is in reality a single uplink beam that contains all the different priorities. Additionally, Figure 3.2 represents an expanded view of the queuing processes within a given module m for a specific priority p. We consider the EHTS internal problem over a time horizon $T \in \mathbb{N}$. We discretize the horizon in time steps and index them by $t \in \{1, 2, ..., T\}$.

The routing scheme of EHTS is characterized by several constraints, as detailed below.

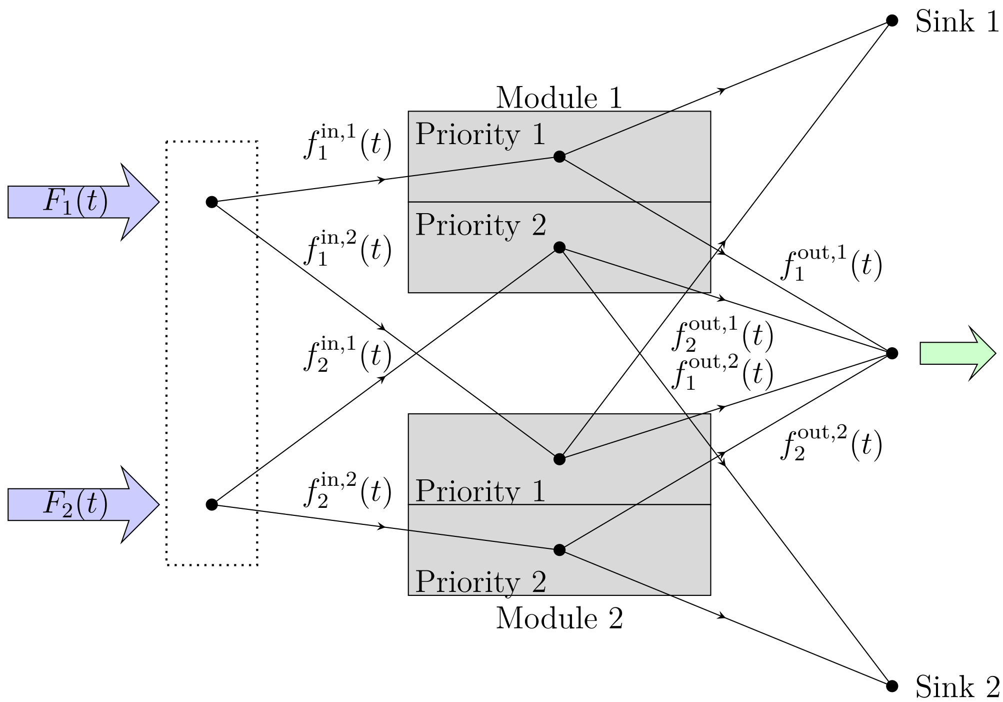


Figure 3.1 EHTS internal routing model for $M = 2$ and $P = 2$

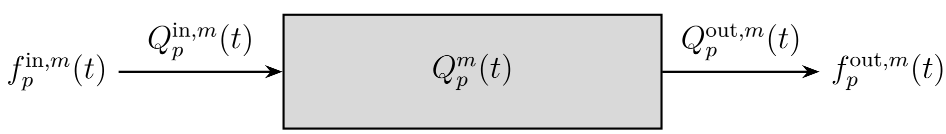


Figure 3.2 EHTS internal queuing model

At each time step t, the net inflow $f_p^{\text{in},m}(t) \in \mathbb{R}^+$ and outflow $f_p^{\text{out},m}(t) \in \mathbb{R}^+$ of commodity $p \in \{1, 2, ..., P\}$ at module $m \in \{1, 2, ..., M\}$ are equilibrated by considering the queue's inflow $Q_p^{\text{in},m}(t) \in \mathbb{R}^+$ and outflow $Q_p^{\text{out},m}(t) \in \mathbb{R}^+$ balance, $\Delta Q_p^m(t) \in \mathbb{R}$, and the packet loss $\mathcal{L}_p^m(t) \in \mathbb{R}^+$:

$$f_p^{\text{in},m}(t) - f_p^{\text{out},m}(t) - \Delta Q_p^m(t) - \mathcal{L}_p^m(t) = 0. \tag{3.1}$$

For every module m, the scheduler weight at a time step t represents the percentage of processed packets from queue of priority p:

$$0 \leq w_p^m(t) \leq 1. \tag{3.2}$$

The scheduler weights $w_p^m(t)$ across all priorities $p \in \{1, 2, ..., P\}$ are normalized to 1:

$$\sum_{p=1}^{P} w_p^m(t) = 1. \tag{3.3}$$

To avoid high disruptions in the system, we introduce a ramping constraint. For a given module and priority, the weight of a scheduler at a time step t is constrained by the maximum deviation from the previous time step:

$$\left| w_p^m(t) - w_p^m(t-1) \right| \leq \overline{\Delta w}, \tag{3.4}$$

where $\overline{\Delta w} \geq 0$ is the maximum allowed change in weight between consecutive time steps.

At each time step t, the outflow $f_p^{\text{out},m}(t)$ from module m represents the fraction of packets processed by that module. It is constrained by the unitless scheduler weights normalized over their respective operation periods, $\Delta s > 0$:

$$f_p^{\text{out},m}(t) \leq \frac{w_p^m(t)}{\Delta s}, \tag{3.5}$$

where Δs is a EHTS parameter acting as the scheduler clock, serving to translate the fraction of packets set by $w_p^m(t) \leq 1$ to a number of processed packets for a given time step t.

Let $F_p(t) \in \mathbb{R}^+$ be the stochastic process modelling the incoming flow of a given priority p. The cumulative incoming flow for a specific priority p on all modules $m \in \{1, 2, ..., M\}$ matches the observed demand $F_p(t)$:

$$\sum_{m=1}^{M} f_p^{\text{in},m}(t) = F_p(t). \tag{3.6}$$

Let $Q_p^m(t)$ be the occupancy of a specific queue at time t. The occupancy at the next time step, $Q_p^m(t+1)$, is determined by its current occupancy $Q_p^m(t)$, to which the net incoming (outgoing) packet balance $\Delta Q_p^m(t)$ is added:

$$Q_p^m(t+1) = Q_p^m(t) + \Delta Q_p^m(t). \tag{3.7}$$

To ensure continuity across successive optimization horizons, each queue starts and concludes with a specific, predefined occupancy $Q_0 \in \mathbb{R}^+$:

$$Q_p^m(0) = Q_p^m(T-1) = Q_0. \tag{3.8}$$

Every queue has a total capacity bounded by $\overline{Q}^m \in \mathbb{R}^+$, induced by the hardware limitations of the modem banks in satellite communication subsystems:

$$\sum_{p=1}^{P} Q_p^m(t) \leq \overline{Q}^m. \tag{3.9}$$

All module output links have a maximum capacity $\overline{C}^m \in \mathbb{R}^+$, representing the transmission bandwidth constraints of the satellite communication channels:

$$\sum_{p=1}^{P} f_p^{\text{out},m}(t) \leq \overline{C}^m. \tag{3.10}$$

Let $k_p > 0$ be the cost incurred when losing a packet of priority $p \in \{1, 2, ..., P\}$. In the batch optimization problem, informed by $F_p(t)$ in hindsight, we seek to minimize the total packet loss cost over all priorities and time steps:

$$\begin{aligned} &\min_{w_p^m(t), f_p^{\text{in},m}(t)} \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{m=1}^{M} \mathcal{L}_p^m(t) k_p \\ &\text{subject to } (3.1) - (3.10). \end{aligned} \tag{3.11}$$

In (3.11), $\sum_{m=1}^{M} \mathcal{L}_p^m(t)$ denotes the total number of lost packets for priority p across all modules at time t. The incurred cost when a packet of priority p is lost is k_p, with higher priorities corresponding to greater costs. The total cost, which we seek to minimize, is the aggregated sum of these costs over all time steps and priorities.

3.1.2 Proposed Approaches

In this section, we present the base proposed approaches, starting with batch optimization, proceeding to MPC, and concluding with alternative methods. The practical, implementable OCO-MPC solution is detailed in Section 3.2.

3.1.2.1 Batch Optimization

In addressing the task of optimizing flow allocation in EHTS, our initial approach is *batch optimization with hindsight* information on the incoming flow. This method processes data from the entire time horizon to derive an optimal solution [64]. This means that the incoming flows are predetermined for the whole horizon and processed as a single batch, i.e., there is no underlying uncertainty.

This approach offers several advantages. Access to the complete data set for the full time horizon allows for the generation of the optimal solution by accounting for future information. This broader perspective enables the system to anticipate and adjust for potential bottlenecks or surges in demand. It ensures that the derived solution optimizes flow allocation throughout the entire time horizon.

However, the comprehensive nature of this method, while it is a strength, can also be viewed as a limitation. Requiring complete information means that the system might fail in dynamic environments with sudden demand shifts. In these circumstances, batch optimization lacks the flexibility to respond to these changes. The method can be computationally demanding, especially when faced with longer horizons or larger data sets. This may result in longer processing times, which is unsuitable for real-time applications, especially given the restricted on-board computing capacity of satellites. As a result, we will consider *batch optimization with hindsight* as the gold standard against which we benchmark our results.

Within EHTS, the batch optimization framework can certainly be of use for stable, predictable scenarios. However, considering the dynamic environment of satellite communication, a more adaptive strategy is essential. This dynamism is primarily driven by two factors: the rapid movement of LEO satellites and the uncertain, time-varying nature of user demand. The inherent time-varying nature of the system, where routing decisions depend on the current queue occupancy, adds another layer of complexity. These factors together create a highly dynamic environment that requires real-time adaptability. This leads to our proposed MPC approach.

3.1.2.2 Model Predictive Control

MPC is an advanced control technique that uses a model of the system to predict its future behaviour over a given time horizon [65]. These predictions guide the system in computing controls that optimize specific performance criteria, such as reducing packet loss cost in our case, while taking into account constraints on the system's inputs, states, and outputs.

In the field of EHTS, the dynamic nature of satellite communication and packet arrival poses challenges that traditional optimization approaches may struggle to address. The ever-changing environment, caused by the varying, uncertain demands and movement of the satellites, requires a method that can adapt swiftly. This is where MPC proves valuable.

The core idea behind applying MPC to EHTS is its inherent adaptability. Unlike batch optimization, which needs the full incoming flow information for the entire time horizon prior to computations, MPC adjusts to new information as it becomes available. By doing so, it continually refines its routing and scheduling decisions, ensuring close to optimal response to evolving scenarios. Instead of working with the realized stochastic process, we work with the expected flow only. In practice, this value can be empirically estimated from historical data. The expression of the flow then becomes:

$$\hat{F}_p(t) = \mathbb{E}[F_p(t)], \tag{3.12}$$

where $\mathbb{E}[\cdot]$ represents the expectation operator. Constraint (3.6) is then substituted by:

$$\sum_{m=1}^{M} f_p^{\mathrm{in},m}(t) = \hat{F}_p(t). \tag{3.13}$$

The formulation of our MPC approach mirrors the constraints and objectives mentioned in Subsection 3.1.2.1. However, its execution differs. For every time step, the MPC method evaluates the optimization problem over a moving time window $1 \leq W < T$. Once the solution is determined, the initial control action from the time window corresponding to the current round is implemented in the satellite. Following this decision, the resulting states of the satellite, such as the outflows and queue occupancies, are observed. With this refreshed state information and new flow data, the time window advances while maintaining its fixed duration. The optimization problem is then revisited and solved for the subsequent time step. As each time step progresses, the system continues this iterative process, leveraging both the current state of the system and, when available, revised flow forecasts. The main difference lies in the optimization function and the online implementation. Problem (3.11) becomes, at time $t \in \{1, 2, ..., T\}$:

$$\begin{aligned} \min_{\substack{w_p^m(\tau), f_p^{\mathrm{in},m}(\tau), \\ \tau \in \{t, t+1, \ldots, t+W\}}} \quad & \sum_{\tau=t}^{t+W} \sum_{p=1}^{P} \sum_{m=1}^{M} \mathcal{L}_p^m(\tau) k_p. \\ \text{subject to} \quad & (3.1) - (3.5), (3.7) - (3.10), (3.13) \end{aligned} \tag{3.14}$$

where $w_p^m(t)$ and $f_p^{\mathrm{in},m}(t)$ are then implemented.

Our simulation results, detailed in the subsequent sections, illustrate MPC's ability to tackle uncertain incoming flow in this setting. The comparison with the batch optimization benchmark reveals that while batch optimization might produce theoretically optimal solutions for static scenarios, MPC excels in dynamic, uncertain conditions typical of satellite communications. Not only does MPC achieve near-optimal results, but it also maintains consistent performance under these ever-changing conditions. The MPC process for online routing and scheduling in an EHTS is summarized in Algorithm 1.

Algorithm 1 MPC online routing and scheduling process

Parameter: W.
for t in $0, 1, \ldots, T-1$ **do**
  Initialize: Given $\hat{F}_p(t)$
  Observe the states $Q_p^m(t), \Delta Q_p^m(t), \mathcal{L}_p^m(t)$ and $f_p^{\text{out},m}(t)$.
  Solve (3.14) considering $\hat{F}_p(t)$.
  Implement $w_p^m(t)$ and $f_p^{\text{in},m}(t)$.

The process and interactions defining our MPC framework for EHTS internal routing are illustrated in Figure 3.3.

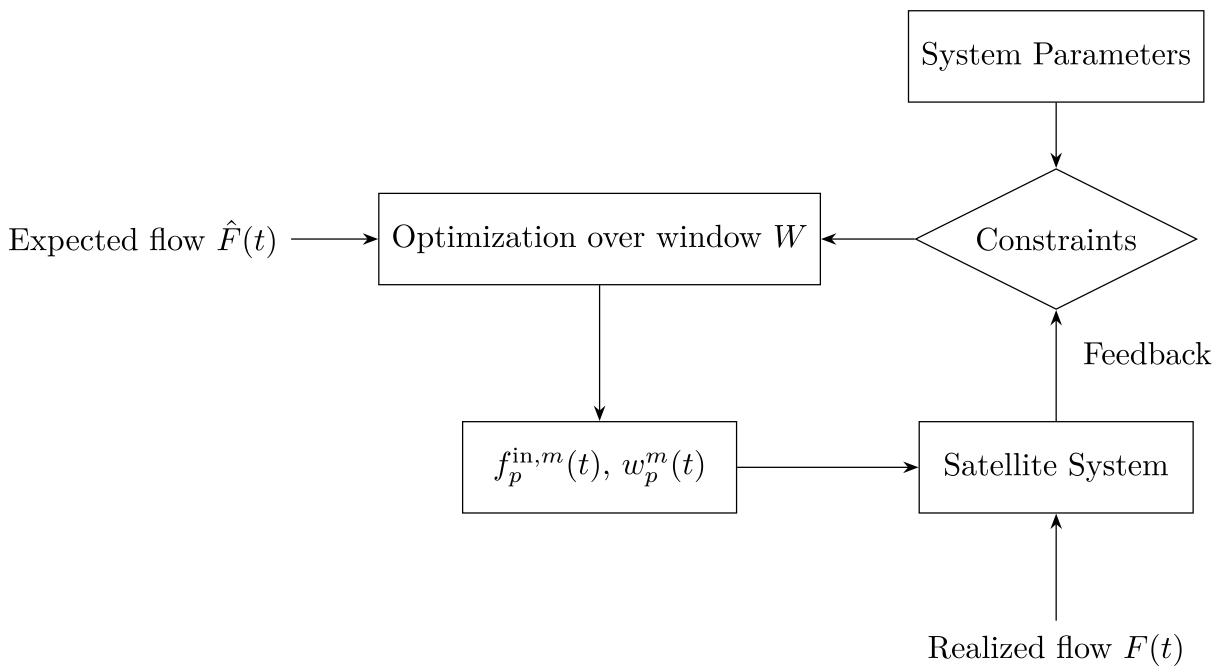


Figure 3.3 MPC framework for EHTS internal routing

3.1.2.3 Alternative Benchmarking Methods

Apart from batch optimization, the first alternative method is termed *static batch optimization with hindsight*. This method removes a degree of freedom by calculating the optimal weights at the initial time step and maintaining them consistently over time. It still uses hindsight information, making it impractical. It is used as a second benchmark for our approach. It can be represented by adding the following constraint to (3.11):

$$w_p^m(t) = w_p^m(1) \quad \forall\, t \in \{2, 3, ..., T\}. \tag{3.15}$$

Another method used for comparison is the *windowless MPC* approach. In this method, we employ an MPC with a window of size $W = 1$. The objective behind this is to ascertain if the added design variable of window length in our original MPC approach is beneficial.

The last comparison method is the *cost-proportional* method, where the following constraint is added to (3.11):

$$w_p^m(t) = \frac{k_p}{\sum_{p=1}^{P} k_p} \quad \forall\, t, p. \tag{3.16}$$

It is a rule-based controller that sets the scheduler weights proportional to the associated cost k_p. While being very computationally lightweight, it is a greedy approach that is not in the least adaptive to incoming flow.

3.2 Online Convex Optimization Framework

Building on Section 3.1, this section presents our OCO-MPC framework. It is based on work soon to be submitted to IEEE Wireless Communication Letters. The main motivation behind this framework is to alleviate the heavy computational burden associated with traditional MPC, which makes it almost impossible to implement on real-world satellite hardware.

3.2.1 Problem Statement

We recall the optimization problem previously introduced in Section 3.1 concisely. For clarity and ease of reference in the present subsection, we have regrouped the equality constraints together, followed by the inequality constraints, using the same parameters as introduced in Section 3.1.

$$\min_{\substack{w_p^m(\tau), f_p^{\text{in},m}(\tau),\\ \tau\in\{t,t+1,\ldots,t+W\}}} \sum_{\tau=t}^{t+W}\sum_{p=1}^{P}\sum_{m=1}^{M} \mathcal{L}_p^m(\tau)k_p \tag{3.17a}$$

$$\text{s.t.} \quad f_p^{\text{in},m}(t) - f_p^{\text{out},m}(t) - \Delta Q_p^m(t) - \mathcal{L}_p^m(t) = 0, \tag{3.17b}$$

$$\sum_{p=1}^{P} w_p^m(t) = 1, \tag{3.17c}$$

$$\sum_{m=1}^{M} f_p^{\text{in},m}(t) = \hat{F}_p(t), \tag{3.17d}$$

$$Q_p^m(t+1) = Q_p^m(t) + \Delta Q_p^m(t), \tag{3.17e}$$

$$Q_p^m(0) = Q_p^m(T-1) = Q_0, \tag{3.17f}$$

$$0 \leq w_p^m(t) \leq 1, \tag{3.17g}$$

$$\left| w_p^m(t) - w_p^m(t-1) \right| \leq \overline{\Delta w}, \tag{3.17h}$$

$$f_p^{\text{out},m}(t) \leq \frac{w_p^m(t)}{\Delta s}, \tag{3.17i}$$

$$\sum_{p=1}^{P} Q_p^m(t) \leq \overline{Q}^m, \tag{3.17j}$$

$$\sum_{p=1}^{P} f_p^{\text{out},m}(t) \leq \overline{C}^m, \tag{3.17k}$$

where (3.17a) is a convex packet loss cost minimization objective function, (3.17b) is the packet balance equation, (3.17c) ensures the scheduler weights are normalized, (3.17d) matches the routed incoming flow to the observed demand per priority, (3.17e) updates the queue occupancy across time, (3.17f) sets the initial and final queue occupancy, (3.17g) bounds the scheduler weights, (3.17h) imposes ramp constraints on the scheduler weights, (3.17i) denormalizes the scheduler weights to associate an actual number of packets processed based on the design parameter Δs, (3.17j) limits the total queue occupancy, and (3.17k) restricts the outflow by the transmission bandwidth.

Problem (3.17) entails solving a multi-period optimization problem to optimality with $P \times M \times W$ constraints at each decision round. This amounts to an important computational load relative to the on-board available resources.

3.2.2 Incoming Flow

In this updated model, we represent the incoming flow $F_p(t)$ as a MMPP. This approach allows for a more realistic representation of the temporal fluctuations in satellite Internet traffic. The MMPP is characterized by a discrete-time Markov chain with state space $\mathcal{S} = \{1, 2, 3\}$, representing three different traffic states. The transition probability matrix P_λ is defined as:

$$P_\lambda = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}.$$

Each state $i \in \mathcal{S}$ is associated with a Poisson process characterized by a rate parameter λ_i packets per time increment.

We make the assumption that given EHTS always know their position relative to Earth and can therefore estimate which traffic intensity λ_i it will experience at any given time. This means that at each time step t, the expected incoming flow $\hat{F}_p(t)$ in (3.17d) will be one of the three λ_i values, i.e. $\hat{F}_p(t) \in \{\lambda_1, \lambda_2, \lambda_3\}$.

3.2.3 Online Convex Optimization

To alleviate the computational burden of MPC, we use OCO [43]. OCO is a framework for providing a sequence of decisions in a dynamic environment where the problem changes over time, and is fully observed only after one commits to a decision [17].

In the context of satellite systems, OCO is particularly advantageous because it enables real-time adjustments to routing and scheduling decisions based on current network conditions, in addition to reducing computational overhead. Our approach does not require solving (3.17) to optimality at each time step; instead, we adopt a strategy where only a single infeasible start Newton step is taken, balancing computational efficiency with the quality of the decision as established by the OCO algorithm performance guarantees.

We utilize the epsilon-online interior-point method for time-varying equality-constrained (ε`OIPM-TEC`) optimization [46] for efficient on-board satellite routing. `OIPM-TEC` guarantees time-averaged optimal decisions on inequality-constrained convex problems with time-varying equality constraints, making it well-suited for our problem. ε`OIPM-TEC` is a more streamlined version of `OIPM-TEC`, providing an even lighter framework while still offering performance guarantees within an ε-tolerance of the round optima.

As presented in [46], the optimization problem is formulated in a matrix form. To align more closely with constraints (3.17b) − (3.17k), we express (3.17) as (3.18), where $\mathbf{x}_t \in \mathbb{R}^{n\times P\times M\times W}$ contains the $n = 6$ vectorized variables $f_p^{\text{in},m}(t)$, $w_p^m(t)$, $\mathcal{L}_p^m(t)$, $f_p^{\text{out},m}(t)$, $Q_p^m(t)$, and $\Delta Q_p^m(t)$ for each pair (p, m) and each time step in $\{t, t+1, ..., t+W\}$:

$$\begin{aligned} &\min_{\mathbf{x}_t} \mathbf{c}^\top \mathbf{x}_t \\ \text{s.t.} \quad &\mathbf{A}\mathbf{x}_t - \mathbf{b}_t = \mathbf{0} \\ &\mathbf{C}\mathbf{x}_t - \mathbf{d} \ \leq \mathbf{0}. \end{aligned} \tag{3.18}$$

We remark that $\mathbf{x}_t$ includes intermediary variables used to ensure all system constraints are respected, i.e., $\mathcal{L}_p^m(t)$, $f_p^{\text{out},m}(t)$, $Q_p^m(t)$, and $\Delta Q_p^m(t)$. The parameters $\mathbf{A}$ and $\mathbf{b}_t$, respectively, represent the multipliers and coefficients of equality constraints (3.17b) − (3.17f), while $\mathbf{C}$ and $\mathbf{d}$, respectively, represent the multipliers and coefficients of the inequality constraints (3.17g) − (3.17k). This formulation is directly applicable to our EHTS on-board routing problem, where the time-varying constraints, particularly (3.17d) and (3.17e), are expressed as equality constraints. It is important to note that we only know the *expected* trajectory of b_t on the MPC window W, while the actual value of b_t is only known *a posteriori*, i.e., at $t + 1$.

3.2.3.1 Matrix Formulation of the MPC problem

A few modifications need to be made to Problem (3.17) to be written in the form of (3.18).

First, the term $w_p^m(t-1)$ from (3.17h) in $\mathbf{C}\mathbf{x}_t - \mathbf{d}$ cannot be included in $\mathbf{C}$ nor $\mathbf{d}$ because neither admit time-varying components. Therefore, we make some adjustments to meet `OIPM-TEC` assumptions. We do this in two steps: (i) we embed all information pertaining to time step $(t-1)$ in $\mathbf{x}_t$, and (ii), we add the following equality constraint:

$$w_p^m(t-1) = \hat{w}_p^m(t-1), \tag{3.19}$$

to $\mathbf{A}\mathbf{x}_t - \mathbf{b}_t = \mathbf{0}$, where $\hat{w}_p^m(t-1)$ is the realized weight for the previous time step. At time t, it can thus be considered as a parameter.

Second, we rewrite (3.17h) as two separate constraints to remove the absolute value, as follows:

$$w_p^m(t) - w_p^m(t-1) - \overline{\Delta w} \leq 0, \tag{3.20}$$

$$-w_p^m(t) + w_p^m(t-1) - \overline{\Delta w} \leq 0. \tag{3.21}$$

Third, to ensure consistency between the satellite system and the optimization problem, i.e., to incorporate feedback from observations, we add the constraint:

$$Q_p^m(t) = Q_{p,\text{sys}}^m(t), \tag{3.22}$$

where $Q_{p,\text{sys}}^m(t)$ is the state of a given queue at the beginning of the time step t, after implementing our previous decision in the satellite system. This state will be of value Q_0 if $t = 0$ or $t = T - 1$.

We therefore have the following optimization problem, where the equality and inequality constraints are presented and regrouped in the form of (3.18):

$$\min_{\substack{w_p^m(\tau), f_p^{\text{in},m}(\tau),\\ \tau \in \{t, t+1, \ldots, t+W\}}} \sum_{\tau=t}^{t+W} \sum_{p=1}^{P} \sum_{m=1}^{M} \mathcal{L}_p^m(\tau) k_p \tag{3.23a}$$

$$\begin{aligned}
\text{s.t.} \quad & f_p^{\text{in},m}(t) - \mathcal{L}_p^m(t) - f_p^{\text{out},m}(t) - \Delta Q_p^m(t) && = 0, && (3.23\text{b})\\
& \sum_{p=1}^{P} w_p^m(t) - 1 && = 0, && (3.23\text{c})\\
& \sum_{m=1}^{M} f_p^{\text{in},m}(t) - \hat{F}_p(t) && = 0, && (3.23\text{d})\\
& - Q_p^m(t) + Q_p^m(t+1) - \Delta Q_p^m(t) && = 0, && (3.23\text{e})\\
& w_p^m(t-1) - \hat{w}_p^m(t-1) && = 0, && (3.23\text{f})\\
& Q_p^m(t) - Q_{p,\text{sys}}^m(t) && = 0, && (3.23\text{g})\\
& - w_p^m(t) && \leq 0, && (3.23\text{h})\\
& w_p^m(t) - 1 && \leq 0, && (3.23\text{i})\\
& w_p^m(t) - w_p^m(t-1) - \overline{\Delta w} && \leq 0, && (3.23\text{j})\\
& - w_p^m(t) + w_p^m(t-1) - \overline{\Delta w} && \leq 0, && (3.23\text{k})\\
& - \frac{w_p^m(t)}{\Delta s} + f_p^{\text{out},m}(t) && \leq 0, && (3.23\text{l})\\
& \sum_{p=1}^{P} Q_p^m(t) - \overline{Q}^m && \leq 0, && (3.23\text{m})\\
& \sum_{p=1}^{P} f_p^{\text{out},m}(t) - \overline{C}^m && \leq 0, && (3.23\text{n})
\end{aligned}$$

To create the decision vector $\mathbf{x}_t \in \mathbb{R}^{n \times P \times M \times (W+1)}$, we introduce a few definitions.

We note that it contains all $n = 6$ optimization variable types used to model the internal dynamics of EHTS: $f_p^{\text{in},m}(t)$, $w_p^m(t)$, $\mathcal{L}_p^m(t)$, $f_p^{\text{out},m}(t)$, $Q_p^m(t)$, and $\Delta Q_p^m(t)$.

Each variable type is grouped in a matrix for a given type step t, e.g.:

$$F_p^{\text{in},m}(t) = \begin{bmatrix} f_1^{\text{in},1}(t) & f_2^{\text{in},1}(t) & \cdots & f_P^{\text{in},1}(t) \\ f_1^{\text{in},2}(t) & f_2^{\text{in},2}(t) & \cdots & f_P^{\text{in},2}(t) \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{\text{in},M}(t) & f_2^{\text{in},M}(t) & \cdots & f_P^{\text{in},M}(t) \end{bmatrix}_{M \times P} . \tag{3.24}$$

We define the operator vec : $\mathbb{R}^{m \times n} \mapsto \mathbb{R}^{mn}$ as follows:

$$\text{vec}\left(F_p^{\text{in},m}(t)\right) = \begin{bmatrix} f_1^{\text{in},1}(t) \\ f_1^{\text{in},2}(t) \\ f_1^{\text{in},M}(t) \\ f_2^{\text{in},1}(t) \\ f_2^{\text{in},2}(t) \\ \vdots \\ f_2^{\text{in},M}(t) \\ \vdots \\ f_P^{\text{in},M}(t) \end{bmatrix}_{MP \times 1} . \tag{3.25}$$

We define the decision vector $\mathbf{x}_t$ for a given time window ranging from time steps $t-1$ to $t+W-1$ as :

$$\mathbf{x}_t = \begin{bmatrix} \mathrm{vec}(F_p^{\mathrm{in},m}(t-1)) \\ \mathrm{vec}(W_p^m(t-1)) \\ \mathrm{vec}(\mathcal{L}_p^m(t-1)) \\ \mathrm{vec}(F_p^{\mathrm{out},m}(t-1)) \\ \mathrm{vec}(Q_p^m(t-1)) \\ \mathrm{vec}(\Delta Q_p^m(t-1)) \\ \mathrm{vec}(F_p^{\mathrm{in},m}(t)) \\ \mathrm{vec}(W_p^m(t)) \\ \mathrm{vec}(\mathcal{L}_p^m(t)) \\ \mathrm{vec}(F_p^{\mathrm{out},m}(t)) \\ \mathrm{vec}(Q_p^m(t)) \\ \mathrm{vec}(\Delta Q_p^m(t)) \\ \mathrm{vec}(F_p^{\mathrm{in},m}(t+1)) \\ \mathrm{vec}(W_p^m(t+1)) \\ \mathrm{vec}(\mathcal{L}_p^m(t+1)) \\ \vdots \\ \mathrm{vec}(F_p^{\mathrm{out},m}(t+W-1)) \\ \mathrm{vec}(Q_p^m(t+W-1)) \\ \mathrm{vec}(\Delta Q_p^m(t+W-1)) \end{bmatrix}. \tag{3.26}$$

Equality constraints (3.23b) $-$ (3.23g) are then embedded in $\mathbf{A}$ and $\mathbf{b}_t$. Let the number of columns of $\mathbf{A}$ be the number of decision variables $nPM(W+1)$. Let the rows of $\mathbf{A}$ be the number of individual constraints. Constraints (3.23b), (3.23e), and (3.23e) lead to a new row for each combination of m and p, whereas (3.23c) leads to M rows, and (3.23d), to P rows. This holds for each time step in $\{t-1,\ldots,t+W-1\}$. The column vector $\mathbf{b}_t$, which has the same number of rows as $\mathbf{A}$, contains the parameters associated to the time-varying equality constraints (3.23d), (3.23f) , and (3.23g), as well as those for the time-invariant equality constraints (3.23b), (3.23c), and (3.23e). Similarly, $\mathbf{C}$ and $\mathbf{d}$ together model the inequality constraints (3.23h) $-$ (3.23n). Like $\mathbf{A}$, $\mathbf{C}$ has $nPM(W+1)$ columns. Constraints (3.23h) $-$ (3.23l), (3.23n) lead to PM rows per time step, and (3.23m) leads to M rows per time step. Meanwhile, the column vector $\mathbf{d}$, which has the same number of rows as $\mathbf{C}$, contains the parameters associated to the time-invariant inequality constraints (3.23h)$-$(3.23n). Finally, to ensure a closed, compact feasible space, we numerically introduce box constraints for each decision variable.

Matrices $\mathbf{A}$ and $\mathbf{C}$ and vectors $\mathbf{b}_t$, $\mathbf{d}$, and $\mathbf{c}^\top$ are presented in detail in Appendix A.

3.2.3.2 `OIPM-TEC`

`OIPM-TEC` solves a problem of the form (3.18) by taking an infeasible start Newton step towards optimality and directly observing the impact of that decision [46]. `OIPM-TEC` ensures feasibility by maintaining strict feasibility for time-invariant inequality constraints and sublinearly bounding the violation of time-varying equality constraints under some assumptions. Integrating this approach into our MPC framework, instead of systematically solving to optimality, allows faster adaptation to the changing conditions reflected in $\mathbf{b}_t$ by reducing the computational overhead.

In this Master's thesis, we use a more streamlined version of `OIPM-TEC`, namely ε`OIPM-TEC`. This provides an even lighter framework while still offering performance guarantees within an ε-tolerance of the optima.

3.2.4 OCO-MPC

In this subsection, we present our methodology in detail. We focus on the algorithmic integration of OCO with MPC for satellite on-board routing.

Let $\phi : \mathbb{R}^{n \times P \times M \times W} \mapsto \mathbb{R}$ be the log-barrier functional as introduced in [15], defined as $\phi(\mathbf{x}) = -\sum_i \log(-f_i(\mathbf{x}))$ where $g_i(\mathbf{x}) \leq 0$ are the inequality constraints, and let $\eta > 0$ be the barrier parameter. As presented in [15], interior-point methods solve a problem in the form of (3.27) instead of directly solving one similar to (3.18):

$$\min_{\mathbf{x}_t} \quad d_\eta(\mathbf{x}_t), \tag{3.27}$$

where $d_\eta(\mathbf{x}_t) = \eta \mathbf{c}^\top \mathbf{x}_t + \phi(\mathbf{x}_t)$ is the log-barrier functional-augmented objective function [46].

From [62], [46], [15, equations 10.21 and 10.22], we define the OCO-MPC in Algorithm 2, where we combine ε`OIPM-TEC` and our MPC framework. The feedback control is performed in steps $5-6$ of Algorithm 2, while the infeasible Newton step is performed in steps $8-9$ to update $\mathbf{x}_{t+1}$.

While ε`OIPM-TEC` is effective in many scenarios, it is only guaranteed to respect equality constraints under conditions more stringent than can be assumed for this application. To mitigate this potential issue, we introduce a feedback correction mechanism that proportionally adjusts the flow allocation $f_p^{\text{in},m}(t)$ for all pairs (p, m) when the inflow is observed.

Algorithm 2 OCO-MPC for online on-board routing

1: **Parameters:** W, $\mathbf{A}$.
2: **Initialization:** Given $\mathbf{x}_0$ and η.
3: **for** t in $\{0, 1, \ldots, T-1\}$ **do**
4: Observe $\hat{F}_p(t)$.
5: Implement the decision $w_p^m(t)$ and $f_p^{\text{in},m}(t)$ from $\mathbf{x}_t$.
6: **if** $\mathbf{x}_t$ is not such that $\sum_{m=1}^{M} f_p^{\text{in},m}(t) = F_p(t)$ $\forall\, p \in \{1, 2, \ldots, P\}$ **then**
7: Apply the feedback correction.
8: Observe the outcome $\mathbf{c}^\top \mathbf{x}_t$, the new constraint $\mathbf{b}_t$, and the new states $Q_p^m(t)$, $\Delta Q_p^m(t)$, $\mathcal{L}_p^m(t)$, and $f_p^{\text{out},m}(t)$.
9: Update decision:
10: $\begin{bmatrix} \mathbf{x}_{t+1} \\ - \end{bmatrix} = \begin{bmatrix} \mathbf{x}_t \\ - \end{bmatrix} - \begin{bmatrix} \nabla^2 \phi(\mathbf{x}_t) & \mathbf{A}^\top \\ \mathbf{A} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \nabla d_\eta(\mathbf{x}_t) \\ \mathbf{A}\mathbf{x}_t - \mathbf{b}_t \end{bmatrix}$

The sequence $\{\mathbf{x}_t\}_{t=1}^{T}$ provided by Algorithm 2 has provable bounds on dynamic regret and constraint violations under certain conditions. Interested readers are referred to [46] for further details.

It is worth noting that the dash in the update of Algorithm 2's line 10 represents the unused dual variable, which we omit for clarity. Our approach, requiring only one interior-point method step per iteration, is inherently more computationally efficient than solving to full optimality with Mosek [66]. While Mosek also uses interior-point methods, it typically requires multiple steps to reach optimality, whereas our method achieves a good approximation with a single step.

Having developed and discussed the MPC and OCO-MPC frameworks for internal routing within EHTS, we now move on to assess their practical performance in numerical settings.

CHAPTER 4 EXPERIMENTAL RESULTS

This chapter begins by detailing the benchmarking process for the MPC approach, followed by an evaluation of the OCO-MPC framework, highlighting their respective performances and effectiveness in optimizing internal routing within EHTS.

4.1 MPC Framework

This section presents our experimental setup, describes the numerical comparison methods, and provides the results along with their analysis for the developed MPC framework.

4.1.1 Simulation Setup

The simulations were constructed to emulate a realistic environment for EHTS. The experimental framework is defined over a time horizon of $T = 100$ time steps, and the MPC approach uses a window $W = 10$. The empirical justification for this choice of window size is provided in Subsection 4.1.3. The satellites are modelled with $M = 16$ modem banks [31], which can each accommodate $P = 3$ distinct priorities of data packets, e.g., high priority ($p = 1$), like voice over Internet Protocol, medium priority ($p = 2$), like instant messaging, and low priority ($p = 3$), like emails. The packet loss costs, corresponding to the significance of these priorities, are set to 10, 4, and 1, respectively. Importantly, smaller numbers were used for readability. The entire experimental setup is designed to be scalable, allowing for the representation of flow values across various orders of magnitude, such as 10^9 packets in the context of EHTS.

The system is constrained such that each queue can hold a maximum of $\overline{Q} = 10$ packets, with both the initial and final states of the queues being empty ($Q_0 = 0$) to promote the continuity of the routing strategy. Lastly, the scheduler weights cannot deviate by more than $\overline{\Delta w} = 10\%$ between consecutive time steps.

We assume that $F_p(t)$ is distributed according to a Poisson process with an average arrival rate of λ_p packets per time step [67]. In this study, the timeline is divided into segments representing progressively higher traffic, characterized by average arrival rates λ_p starting at 10 packets per time step and increasing linearly up to 100. To establish an inversely proportional relationship relative to packet priorities, we normalized λ_p by the packet loss cost k_p.

Table 4.1 Experimental setup parameters, MPC framework

Parameter	Value	Description
T	100 time steps	Time horizon of the simulation
W	10 time steps	MPC window size
M	16	Number of modem banks
P	3	Number of distinct priorities
k_p	10, 4, 1	Packet loss costs for high, medium, and low priority packets
$\overline{Q}$	10 packets	Maximum queue size
Q_0	0	Initial and final queue occupancy
$\overline{\Delta w}$	10%	Maximum deviation of scheduler weights between consecutive time steps
λ_p	$10 - 100$ packets per time step	Average arrival rate, increasing linearly

Table 4.1 summarizes the key parameters and their values used in the experimental setup, providing an overview of the simulation environment and its constraints.

4.1.2 Comparison Methods

The proposed MPC approach is rigorously tested against other methods to demonstrate its potential benefits and capabilities. The first benchmark is the *batch optimization with hindsight* method. This is the "gold standard", as it utilizes *ex post* information for the total time horizon, optimizing weights and routing incoming flow based on the actual, realized flow for every time step. However, while it offers a theoretical optimum, its applicability is limited as it uses information not readily available when routing decisions are made.

The second benchmark is a variant of the gold standard, known as the *static batch optimization with hindsight* method. It closely resembles its predecessor, with one added constraint: the weights remain constant over time. This emphasizes consistency in allocation decisions and leads to streamlined computations, but at the expense of adaptability, as it removes one degree of freedom.

The third benchmark, *windowless MPC*, simplifies the standard MPC approach by using a window of only one time step. This eliminates the potential benefit of considering future states, a key advantage of traditional MPC. However, this simplification translates to increased computational efficiency.

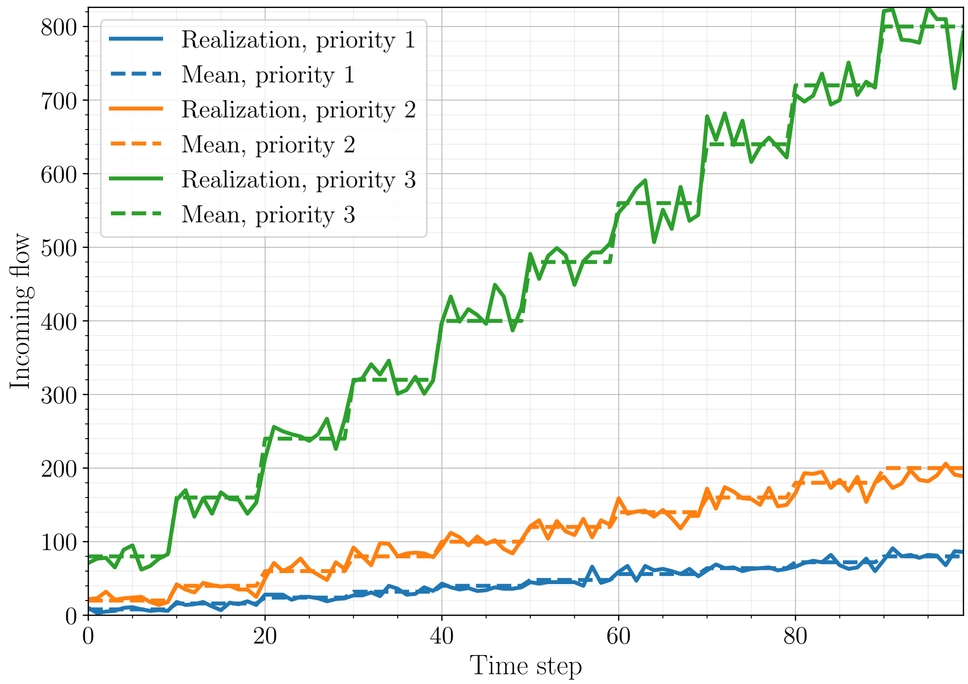
Lastly, the *cost proportional* allocation method, in contrast, presents a simpler and more direct approach. Instead of unpacking the complexities of the incoming flows, it focuses on proportional weight allocations based on costs, ensuring rapid decision-making. While it prioritizes computational efficiency over precision, this method remains an important consideration.

The next subsection details the numerical assessment of the MPC method against these four benchmarks, shedding light on the advantages the MPC approach brings to packet routing in EHTS.

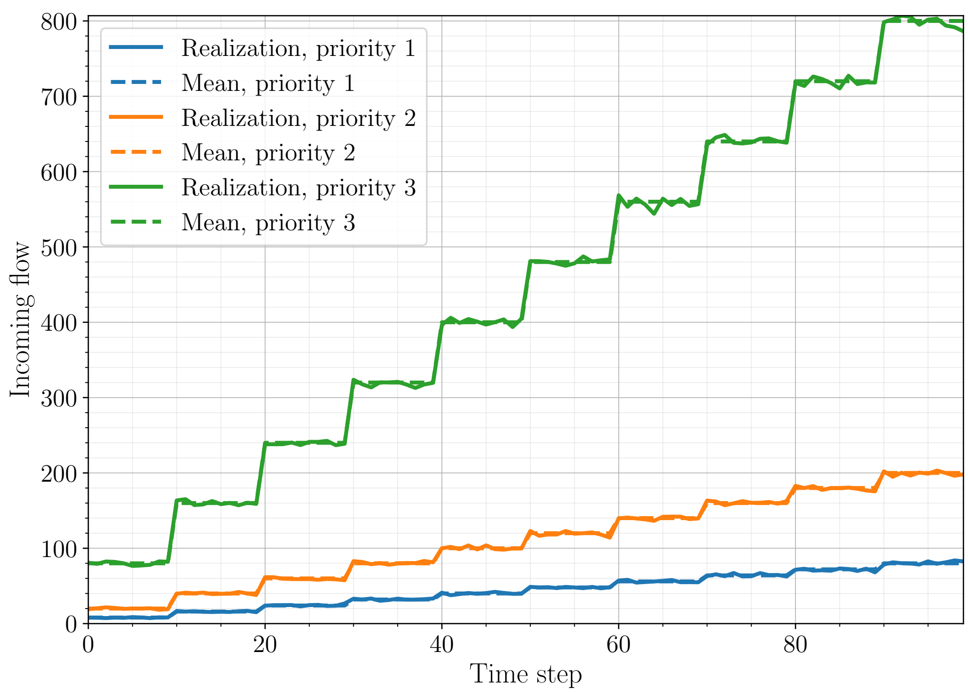
4.1.3 Results and Analysis

Having outlined the experimental framework and benchmarking methods, this subsection presents the quantitative results of our MPC simulations. To ensure consistency in our findings, all simulations were conducted 100 times using a Monte Carlo approach. These results offer insights into the efficacy of the MPC technique.

Initially, understanding the flow dynamics is essential as they form the basis of the system's operation. The incoming flow distribution is depicted in Figure 4.1. Figure 4.1a displays the incoming flows for a single run, highlighting the system's inherent uncertainty. In contrast, Figure 4.1b showcases the incoming flows over time, averaged across 100 Monte Carlo runs.



(a) For a single run



(b) Averaged over 100 Monte Carlo runs

Figure 4.1 Incoming flows across time

Next, we analyze the aggregated outflows across all M modules for each priority level. The results, specific to our MPC approach, are illustrated in Figure 4.2.

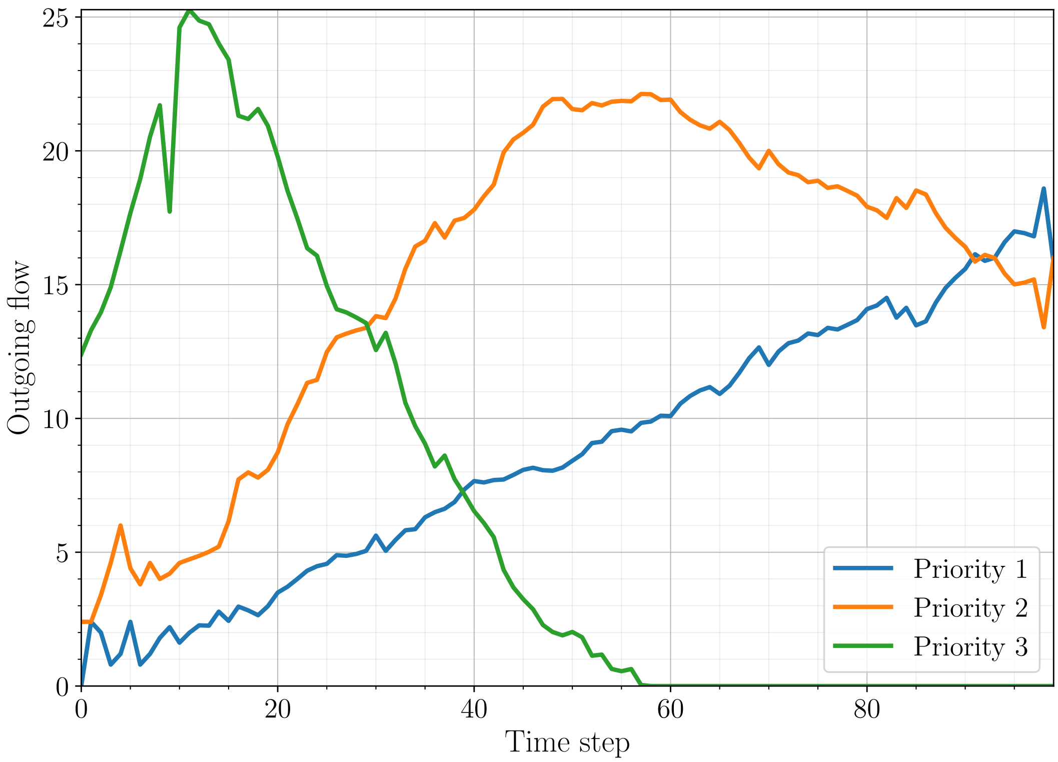


Figure 4.2 Outgoing flows across time averaged over 100 Monte Carlo runs

As illustrated in Figure 4.2, when the system tends towards saturation, the MPC approach prioritizes transmitting packets of the highest priorities, discarding the lowest one.

Figure 4.3 presents the lost packets across time, delineated by priorities, for different scenarios. It shows that the *batch optimization with hindsight* generally discards fewer packets of Priority 1 than MPC but more of Priorities 2 and 3, which leads to a lower cost.

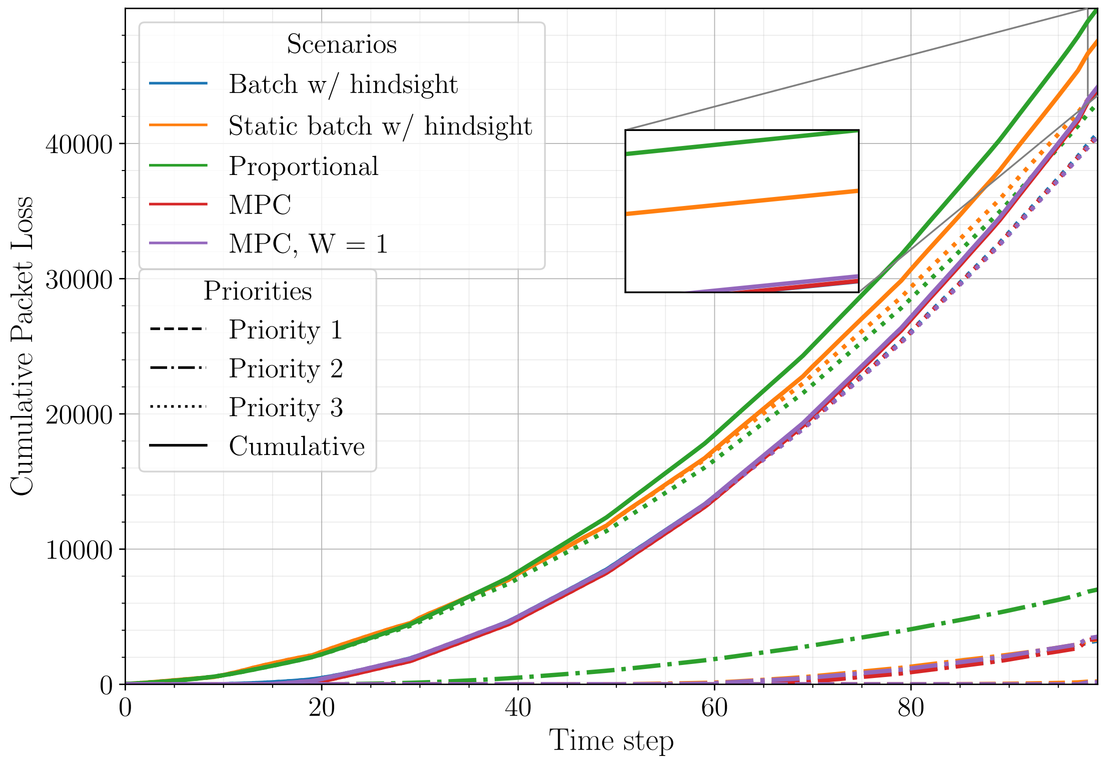


Figure 4.3 Lost packets across time averaged over 100 Monte Carlo runs

Building on Figure 4.3, we now focus on the central aspect of this study: packet loss costs. Minimizing packet loss cost is the primary objective of these optimization efforts. Figure 4.4 illustrates the comparative performance of various methods in minimizing packet loss costs over time. The results align with initial expectations. The *batch with hindsight* method unsurprisingly emerged as the best technique. As previously highlighted, this method leverages hindsight information about the actual realization of the stochastic process modeling the flows, providing a rationale for its superior performance. Following closely, our MPC approach demonstrated its capabilities, with cumulative averaged costs only 1.05% greater than the *batch with hindsight* method. Considering that the MPC method operates with the expected information rather than comprehensive data, this marginal increase underscores its effectiveness. Moreover, losses from the MPC method were 3.54% inferior to those of the *windowless MPC* method, which registered 4.59% more costs than the hindsight optimum. The *static batch with hindsight* method engendered costs 10.78% higher than its dynamic *batch with hindsight* counterpart and 9.73% higher than the MPC approach. In contrast, the *proportional approach* registered the highest costs with a total of 32.35% more than the

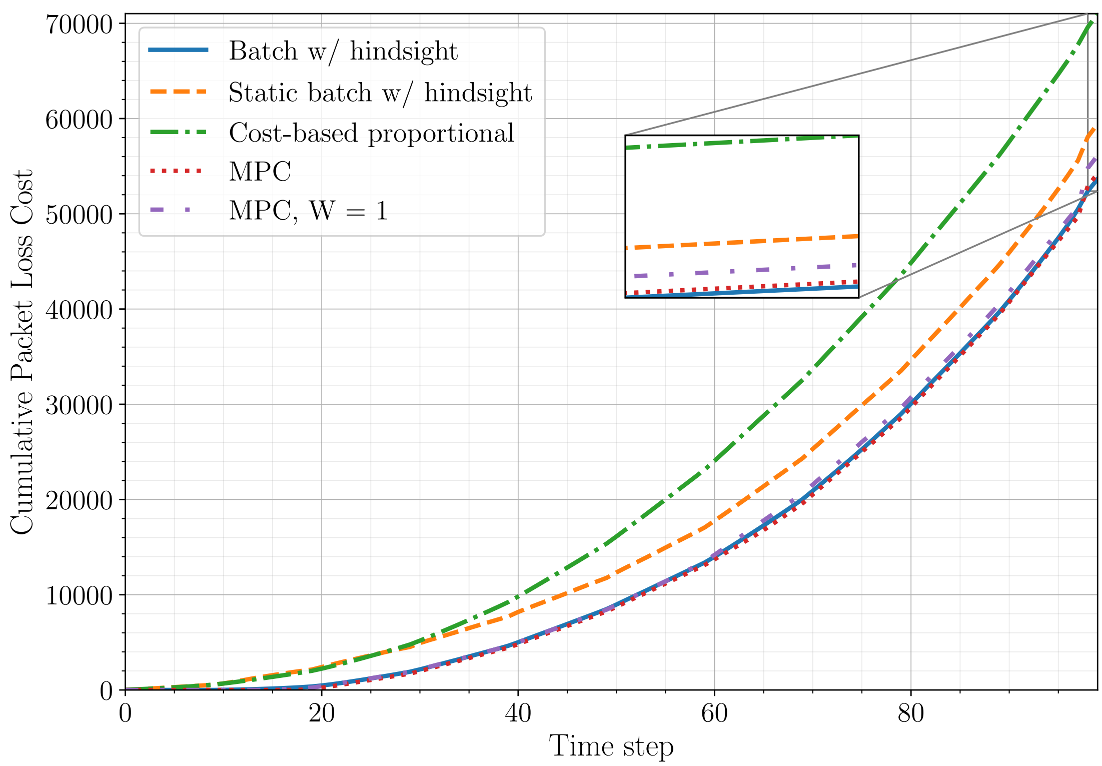


Figure 4.4 Packet loss cost across time averaged over 100 Monte Carlo runs

hindsight optimum. The significant cost disparity between our approach and the *proportional* method highlights the benefits of the MPC and of a comprehensive scheduling mechanism.

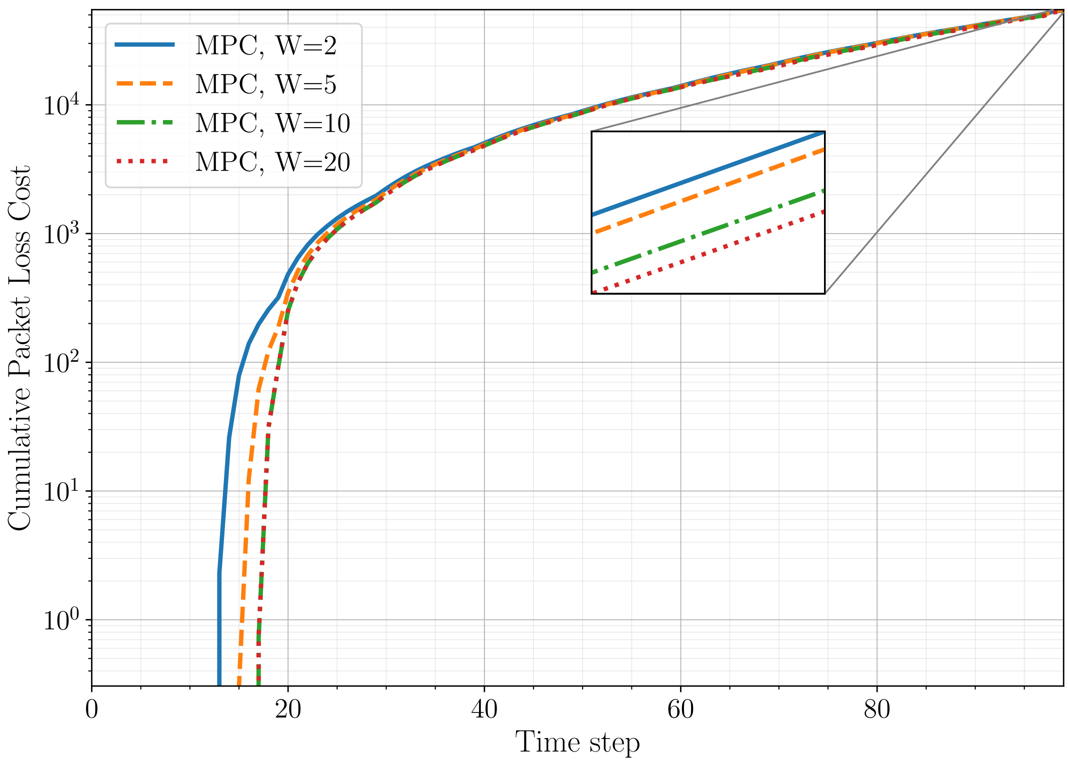


Figure 4.5 Packet loss cost across time averaged over 100 Monte Carlo runs for different time windows

Lastly, in our MPC method, we utilized a window size of 10. This choice proved to be advantageous, as it resulted in lower losses compared to smaller window sizes, as shown in Figure 4.5. Specifically, the losses were 1.71% and 1.19% higher with window sizes of 2 and 5, respectively, when compared to the window size of 10. Additionally, we found that a window size of 10 entails only 0.61% more packet loss costs than a window size of 20, while requiring approximately half the computation time for our current implementation. This highlights the importance of balancing performance with computational efficiency, as the smaller window size of 10 achieves similar results to the window size of 20, but with significantly reduced computation time. These findings highlight the effectiveness of our chosen window size in optimizing the performance of our MPC approach.

4.2 OCO-MPC Framework

This section presents the numerical settings and results for the developed OCO-MPC framework.

4.2.1 Numerical Setting

Let $F(t)$ be the incoming flow, now modelled as a continuous-time Markov chain with state space $\mathcal{S} = \{1, 2, 3\}$, representing three different traffic states. Consider the transition probability matrix P_λ defined as:

$$P_\lambda = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.05 & 0.2 & 0.75 \end{bmatrix}.$$

Each state $i \in \mathcal{S}$ is associated with a Poisson process characterized by a rate parameter λ_i packets per time increment, where $\lambda_1 = 20$, $\lambda_2 = 25$, and $\lambda_3 = 30$. At any given time t, the traffic intensity follows a Poisson distribution with rate $\lambda_{F(t)}$, modulated by the current state $F(t)$ of the Markov chain.

This approach allows us to realistically model the temporal fluctuations in satellite Internet traffic. Simulating the traffic as an MMPP lets us capture the inherent stochasticity and time-dependent behaviour of the system. Compared to the previous numerical setting, where the incoming flow was simply increasing linearly across time, this new setting is more realistic and uses fewer assumptions, making it better suited for real-world applications.

This numerical setup closely follows the one presented in Table 4.1, with a few key differences. We maintain the same time horizon of $T = 100$ time steps, but reduce the MPC window to $W = 5$. This reduction is primarily due to computational requirements (reducing the matrix sizes by half), while still maintaining excellent performance, as shown in Figure 4.6. The number of modem banks ($M = 16$) and priorities ($P = 3$) remain the same as in the previous setup. The most significant change is in the modeling of the incoming flow: instead of a linearly increasing arrival rate, we now use a MMPP to represent the incoming flow, allowing for more realistic temporal fluctuations in satellite Internet traffic. All other parameters, including the maximum queue size $\overline{Q}$, initial and final queue states Q_0, and the maximum deviation of scheduler weights $\overline{\Delta w}$, remain the same as in the previous setup. The use of ε`OIPM-TEC` requires the setting of additional parameters. We initialize the initial guess $\mathbf{x}_0$ randomly and verify its feasibility before applying the Newton step. Additionally, we set the barrier parameter η to 10^4, which translates to $\varepsilon \sim \frac{O(N)}{\eta}$, where $N = 6(W+1)MP$ is the dimension of the decision variable $\mathbf{x}_t$ [46, Theorem 2]. This modified setup allows us to evaluate the performance of our OCO-MPC framework under more dynamic and realistic traffic conditions while maintaining comparability with our previous results. Simulation parameters are summarized in Table 4.2.

We conduct 100 Monte Carlo simulations to ensure consistency and to account for the stochastic nature of the MMPP-based incoming flow.

Table 4.2 Experimental setup parameters, OCO-MPC framework

Parameter	Value	Description
T	100 time steps	Time horizon of the simulation
W	5 time steps	MPC window size
M	16	Number of modem banks
P	3	Number of distinct priorities
k_p	10, 4, 1	Packet loss costs for high, medium, and low priority packets
$\overline{Q}$	10 packets	Maximum queue size
Q_0	0	Initial and final queue occupancy
$\overline{\Delta w}$	10%	Maximum deviation of scheduler weights between consecutive time steps
λ_p	$\{20, 25, 30\}$ packets per time step	Average arrival rate, following an MMPP
η	10^4	Barrier parameter for the ε`OIPM-TEC` algorithm

We use the following methods to benchmark our OCO-MPC algorithm:

- *Batch with hindsight*: Problem (3.17) solved with hindsight information on the incoming flow, serving as the best yet unreachable performance;
- *MPC*: Our MPC framework solved to optimality at each round, providing a reference for our online algorithm, though being computationally expensive;
- *Cost-proportional allocation*: A rule-based controller that sets the scheduler weights proportional to the associated cost k_p.

4.2.2 Numerical Results

The incoming flow distribution, depicted in Figure 4.6, illustrates the flows across priorities, highlighting the system's inherent uncertainty when modelled with a Markov-modulated

Poisson process. This figure shows the results from a single Monte Carlo run to showcase the variability and uncertainty in the system.

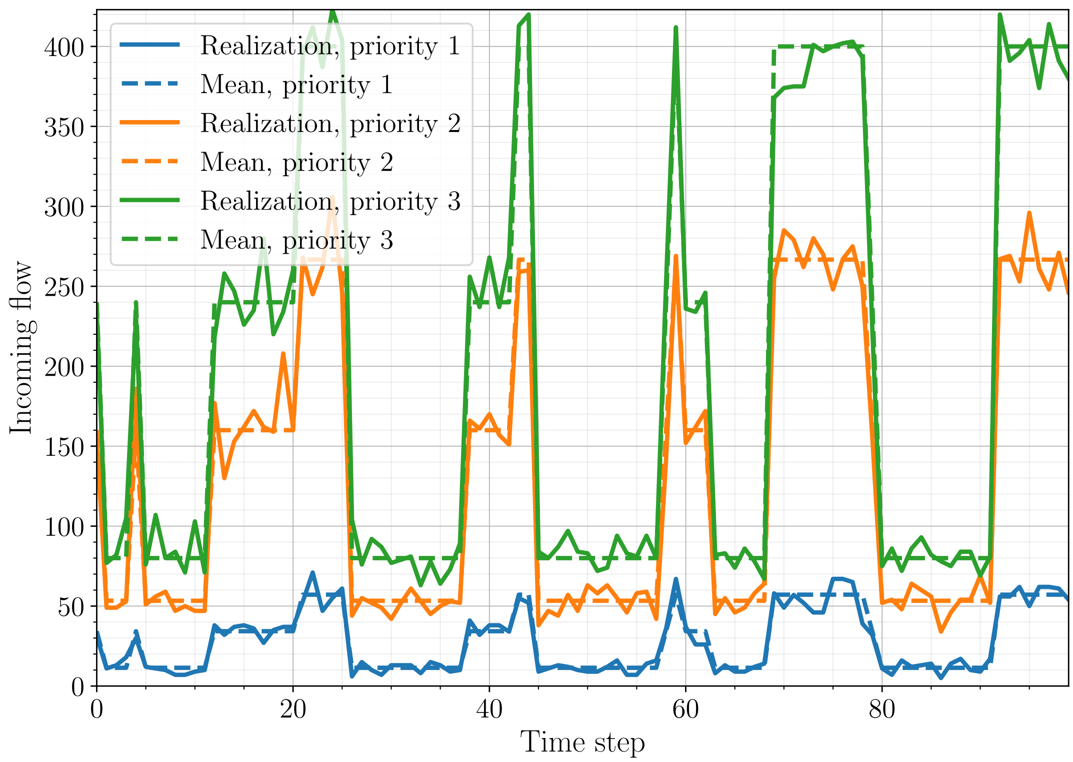


Figure 4.6 Incoming flows across time

Figure 4.7 illustrates the performance of our OCO-MPC algorithm against the other benchmarks. The cumulative packet loss costs are averaged over 100 Monte Carlo runs to ensure statistical significance. The shadowed regions depict the 95^{th} percentile of the data, highlighting the range of variability and uncertainty in the system's performance across the Monte Carlo simulations.

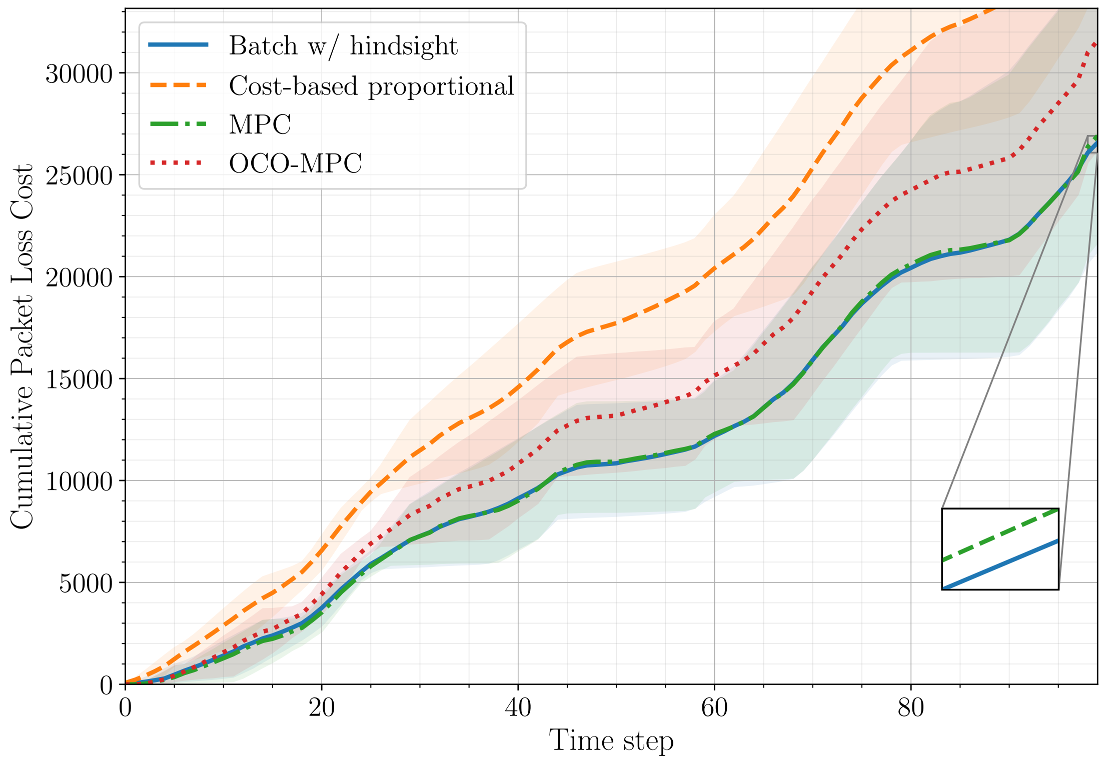


Figure 4.7 Cumulative packet loss cost across time

By design, the *batch with hindsight* method performs best because it uses full information on the incoming flow. The MPC framework performs closely, suffering only 1.22% more packet loss costs than the *batch with hindsight* method. This shows once again the relevance of applying such a framework to the problem of flow routing on-board EHTS. Our OCO-MPC approach also performs well, with 18.80% more packet loss costs than the *batch with hindsight* method and 17.37% more than the comparative MPC method. It, however, uses a single Newton step per iteration towards the optima and does not use a full-on solver like [66], making it readily implementable while still showing good performance. It also performs much better than the *proportional approach*, which registered the highest costs with a total of 48.71% more than the hindsight optimum. While the *proportional approach* is greedy and also computationally lightweight, it performs poorly. This contrast highlights the advantage of utilizing the limited on-board processing capabilities by implementing the OCO-MPC approach.

CHAPTER 5 CONCLUSION

This Master's thesis presents a novel mathematical framework for modelling and solving the internal routing problem within multi-modem EHTS. Our research makes significant contributions to the field of satellite communications.

We first develop a comprehensive multi-period optimization problem that minimizes packet loss costs across time, subject to system and physical constraints. Building upon this formulation, we introduce an MPC framework that operates with the expected incoming flow, enhancing the realism and applicability of our approach.

Recognizing the computational challenges inherent to MPC, we refine our methodology by presenting the OCO-MPC framework. This innovative approach employs a single Newton step towards the optimum, eliminating the need for a full optimization solver while providing adequate performance and significantly reducing computational complexity.

Our research demonstrates the effectiveness of these approaches through rigorous numerical simulations and comparisons. We illustrate the close-to-optimal performance of our MPC approach against other methods, showcasing its high effectiveness while operating solely on expected values. We further validate the performance of our OCO-MPC approach using a more realistic incoming flow model based on an MMPP. This analysis not only confirms the approach's effectiveness but also, for the first time, shows its potential implementability in the context of internal routing withing EHTS. This analysis not only confirms the approach's effectiveness but also, for the first time, shows its potential implementability in the context of internal routing within EHTS.

To the best of our knowledge, this work represents the first MPC-based approach in this context. As such, it has the potential to make significant contributions to the field of telecommunications, particularly in satellite network optimization.

In summary, this Master's thesis presents a comprehensive mathematical framework for addressing the complex problem of internal routing within EHTS. The proposed OCO-MPC approach offers a powerful and computationally efficient solution, bridging the gap between theoretical optimization and practical implementation in satellite systems. Our work lays a solid foundation for future research in this critical area of satellite communications and network optimization.

5.1 Limitations and Future Work

The first area for improvement in this work is the internal topology of the satellites used in our experiments. The model, as presented in Figure 3.1, only contains one layer of modems, without any interconnection between them. To make our model more realistic, we could use a toroidal topology, as suggested in [31]. This approach aligns with current research trends in satellite network modelling and would increase the complexity of our system, requiring, for example, additional constraints in our optimization problem.

Next, it would be interesting to exploit the fast computation provided by ε`OIPM-TEC` to further decrease the duration of the time steps to closely track incoming flow and, therefore, further enhance performance while maintaining computational efficiency. This approach could improve our ability to monitor the incoming flow by processing fewer packets per time step while increasing the number of time steps (and conserving the same total time), potentially leading to improved overall performance. An interesting study on the trade-off between the added computational time and improved performance may prove valuable.

Finally, a significant improvement could be made on the implementation side by designing the code in C/C++ instead of Python, as it is currently done. This change could provide more accurate run-times to evaluate the computation time of our approaches and further showcase the advantage of OCO-MPC in a realistic satellite environment. It would bring our implementation closer to what could be actually deployed on satellite hardware, enhancing the practical relevance of our research [68, 69].

REFERENCES

[1] D. Gerbarg, *Television Goes Digital*. New York, NY: Springer, 2009.

[2] NWT Timeline. (2024) Anik A-1. [Online]. Available: https://www.nwttimeline.ca/stories/anik-a1/

[3] C. Sacchi, F. Granelli, R. Bassoli, F. H. P. Fitzek, and M. Ruggieri, *A Roadmap to Future Space Connectivity*. Cham, Switzerland: Springer, 2023.

[4] Fraunhofer Institute for Integrated Circuits IIS. High Throughput Satellite Systems. [Online]. Available: https://www.iis.fraunhofer.de/en/ff/kom/satkom/hts.html

[5] J. Garrity and A. Husar, Asian Development Bank. (2021) Digital Connectivity and Low Earth Orbit Satellite Constellations: Opportunities for Asia and the Pacific. [Online]. Available: https://www.adb.org/publications/digital-connectivity-low-earth-orbit-satellite-opportunities

[6] National Research Council Canada. (2021) Satellite communications bridge the digital divide between urban and remote areas. [Online]. Available: https://nrc.canada.ca/en/stories/satellite-communications-bridge-digital-divide-between-urban-remote-areas

[7] G. Maral and M. Bousquet, *Satellite Communications Systems: Systems, Techniques and Technology*, 6th ed. Wiley, 2020.

[8] L. Liu, T. Zhang, and Y. Lu, "A Novel Adaptive Routing Algorithm for Delay-Sensitive Service in Multihop LEO Satellite Network," *KSII Transactions on Internet and Information Systems*, vol. 10, no. 8, pp. 3551–3567, 2016.

[9] C. Dai, G. Liao, and Q. Chen, "Service-Oriented Routing with Markov Space-Time Graph in Low Earth Orbit Satellite Networks," *Transactions on Emerging Telecommunications Technologies*, vol. 32, no. 2, p. e4072, 2021.

[10] O. Ben Yahia, Z. Garroussi, O. Bélanger, B. Sansò, J.-F. Frigon, S. Martel, A. Lesage-Landry, and G. Karabulut Kurt, "Evolution of High-Throughput Satellite Systems: A Vision of Programmable Regenerative Payload," *IEEE Communications Surveys & Tutorials*, pp. 1–34, 2024 (early access).

[11] J. Moy, "OSPF Version 2," IETF, RFC 2328, 1998.

[12] Y. Rekhter, T. Li, and S. Hares, "A Border Gateway Protocol 4 (BGP-4)," IETF, RFC 4271, 2006.

[13] A. S. Tanenbaum and D. J. Wetherall, *Computer Networks*, 5th ed. Pearson, 2011.

[14] H. Fenech, S. Amos, A. Tomatis, and V. Soumpholphakdy, "High Throughput Satellite Systems: An Analytical Approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 1, pp. 192–202, 2015.

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[16] D. P. Bertsekas, *Network Optimization: Continuous and Discrete Models*. Belmont, MA: Athena Scientific, 1998.

[17] S. Shalev-Shwartz, "Online Learning and Online Convex Optimization," *Foundations and Trends in Machine Learning*, vol. 4, no. 2, p. 107–194, Feb 2012.

[18] E. F. Camacho and C. Bordons, *Model Predictive Control*, 2nd ed. London: Springer, 2007.

[19] J. B. Rawlings, D. Q. Mayne, and M. Diehl, *Model Predictive Control: Theory, Computation, and Design*, 2nd ed. Madison, WI: Nob Hill Publishing, 2017.

[20] A. Bemporad, F. Borrelli, and M. Morari, "Model Predictive Control Based on Linear Programming - The Explicit Solution," *IEEE Transactions on Automatic Control*, vol. 47, no. 12, pp. 1974–1985, 2002.

[21] L. Wang, *Model Predictive Control System Design and Implementation Using MATLAB*. London: Springer-Verlag London Limited, 2009.

[22] M. Richharia and L. D. Westbrook, *Satellite Systems for Personal Applications: Concepts and Technology*. Wiley, Aug 2010.

[23] J. Irekvist and P. Linder. (2022) Technology's Important Role in Bridging Canada's Digital Divide. [Online]. Available: https://www.ericsson.com/en/blog/6/2022/ericsson-bridging-canadas-digital-divide

[24] T. Pratt and J. E. Allnutt, *Satellite Communications*. John Wiley & Sons, 2019.

[25] M. Graydon and L. Parks, "'Connecting the Unconnected': A Critical Assessment of US Satellite Internet Services," *Media, Culture & Society*, vol. 42, no. 2, p. 260–276, 2020.

[26] D. Tuzi, T. Delamotte, and A. Knopp, “Satellite Swarm-Based Antenna Arrays for 6G Direct-to-Cell Connectivity,” *IEEE Access*, vol. 11, pp. 36 907–36 928, 2023.

[27] K.-H. Lee and K. Y. Park, “Overall Design of Satellite Networks for Internet Services with QoS Support,” *Electronics*, vol. 8, no. 6, 2019.

[28] M. Luglio, S. Romano, C. Roseti, and F. Zampognaro, “Satellite Multi-Beam Multicast Support for an Efficient Community-Based CDN,” *Computer Networks*, vol. 217, p. 109352, 2022.

[29] C. Zhang, Y. Zhou, P. Qin, and Z. Zhao, “High-Throughput Satellite Flexibility Design and Modeling,” *Journal of Physics: Conference Series*, vol. 2469, no. 1, p. 012006, Mar. 2023.

[30] Kuiper Systems LLC, “Application for Authority to Launch and Operate a Non-Geostationary Satellite Orbit System in Ka-band Frequencies: Technical Appendix,” Tech. Rep., 2019.

[31] O. Ben Yahia, Z. Garroussi, B. Sansò, J.-F. Frigon, S. Martel, A. Lesage-Landry, and G. Karabulut Kurt, “A Scalable Architecture for Future Regenerative Satellite Payloads,” 2024, arXiv:2407.06075.

[32] International Telecommunication Union, “Key Elements for Integration of Satellite Systems into Next Generation Access Technologies,” Tech. Rep. ITU-R M.2460-0, Jul. 2019.

[33] M. M. Azari, S. Solanki, S. Chatzinotas, O. Kodheli, H. Sallouha, A. Colpaert, J. F. Mendoza Montoya, S. Pollin, A. Haqiqatnejad, A. Mostaani, E. Lagunas, and B. Ottersten, “Evolution of Non-Terrestrial Networks From 5G to 6G: A Survey,” *IEEE Communications Surveys & Tutorials*, vol. 24, no. 4, pp. 2633–2672, 2022.

[34] M. Cello, M. Marchese, and F. Patrone, “HotSel: A Hot Spot Selection Algorithm for Internet Access in Rural Areas through Nanosatellite Networks,” in *IEEE Global Communications Conference (GLOBECOM)*, 2015, pp. 1–6.

[35] S. Agnelli, P. Feltz, P.-F. Griffiths, and D. Roth, “Satellite’s Role in the Penetration of Broadband Connectivity within the European Union,” in *7th Advanced Satellite Multimedia Systems Conference and the 13th Signal Processing for Space Communications Workshop (ASMS/SPSC)*, 2014, pp. 306–311.

[36] K. Suresh, A. Alqahtani, T. Rajasekaran, M. S. Kumar, V. Ranjith, R. Kannadasan, N. Alqahtani, and A. A. Khan, “Enhanced Metaheuristic Algorithm-Based Load Balancing in a 5G Cloud Radio Access Network,” *Electronics*, vol. 11, no. 21, 2022.

[37] N. Pachler, E. F. Crawley, and B. G. Cameron, “Beam-to-Satellite Scheduling for High Throughput Satellite Constellations Using Particle Swarm Optimization,” in *IEEE Aerospace Conference (AERO)*, 2022, pp. 1–9.

[38] Z. Qu, G. Zhang, H. Cao, and J. Xie, “LEO Satellite Constellation for Internet of Things,” *IEEE Access*, vol. 5, pp. 18 391–18 401, 2017.

[39] O. Markovitz and M. Segal, “Advanced Routing Algorithms for Low Orbit Satellite Constellations,” in *IEEE International Conference on Communications*, 2021, pp. 1–6.

[40] J. Nocedal and S. Wright, *Numerical Optimization*. Springer, 2006.

[41] A. Shapiro, D. Dentcheva, and A. Ruszczynski, *Lectures on Stochastic Programming: Modeling and Theory*. Society for Industrial and Applied Mathematics, 2021.

[42] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton University Press, 2009, vol. 28.

[43] E. Hazan, *Introduction to Online Convex Optimization*, 2nd ed. Cambridge, MA, USA: The MIT Press, 2022.

[44] A. Agarwal, E. Hazan, S. Kale, and R. E. Schapire, “Algorithms for Portfolio Management Based on the Newton Method,” in *Proceedings of the 23rd international conference on Machine learning*, 2006, pp. 9–16.

[45] B. Awerbuch and R. Kleinberg, “Online Linear Optimization and Adaptive Routing,” *Journal of Computer and System Sciences*, vol. 74, no. 1, pp. 97–114, 2008.

[46] J.-L. Lupien, I. Shames, and A. Lesage-Landry, “Online Interior-point Methods for Time-varying Equality-constrained Optimization,” 2024, arXiv:2307.16128.

[47] S. Wright, *Primal-Dual Interior-Point Methods*. SIAM, 1997.

[48] J.-L. Lupien and A. Lesage-Landry, “An Online Newton’s Method for Time-Varying Linear Equality Constraints,” *IEEE Control Systems Letters*, vol. 7, p. 1423–1428, 2023.

[49] M. Zinkevich, “Online Convex Programming and Generalized Infinitesimal Gradient Ascent,” in *Proceedings of the 20th International Conference on Machine Learning (ICML-03)*. AAAI Press, 2003, pp. 928–936.

[50] E. Hazan, A. Agarwal, and S. Kale, “Logarithmic Regret Algorithms for Online Convex Optimization,” in *Machine Learning*, vol. 69, no. 2-3. Springer, 2007, pp. 169–192.

[51] A. Mokhtari and A. Ribeiro, "Adaptive Newton Method for Empirical Risk Minimization to Statistical Accuracy," *Advances in Neural Information Processing Systems*, vol. 29, 2016.

[52] J. Richalet, A. Rault, J.-L. Testud, and J. Papon, "Model Predictive Heuristic Control: Applications to Industrial Processes," *Automatica*, vol. 14, no. 5, pp. 413–428, 1978.

[53] S. J. Qin and T. A. Badgwell, "A Survey of Industrial Model Predictive Control Technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

[54] P. Falcone, F. Borrelli, J. Asgari, H. P. Tseng, and D. Hrovat, "Predictive Active Steering Control for Autonomous Vehicle Systems," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 3, pp. 566–580, 2007.

[55] A. Parisio, E. Rikos, and L. Glielmo, "Model Predictive Control of a Smart Grid: An Overview," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 1803–1812, 2014.

[56] S. Gupta, R. Sharma, and R. Kumar, "Model Predictive Control for Spacecraft Rendezvous and Docking with a Rotating Platform," *Acta Astronautica*, vol. 109, pp. 121–133, 2015.

[57] J. Scharnagl, P. Kremmydas, and K. Schilling, "Electric Satellite Station Keeping, Attitude Control, and Momentum Management by MPC," *IFAC-PapersOnLine*, vol. 53, no. 9, pp. 1–6, 2020.

[58] M. Schwenzer, M. Ay, T. Bergs, and D. Abel, "Review on Model Predictive Control: an Engineering Perspective," *The International Journal of Advanced Manufacturing Technology*, vol. 117, no. 5, pp. 1327–1349, Nov 2021.

[59] Monolithic Power Systems. Model Predictive Control (MPC) in Power Electronic Systems. [Online]. Available: https://www.monolithicpower.com/en/learning/mpscholar/power-electronics/control-of-power-electronic-systems/model-predictive-control-mpc

[60] F. Delli Priscoli and D. Pompili, "A Demand-Assignment Algorithm Based on a Markov Modulated Chain Prediction Model for Satellite Bandwidth Allocation," *Wireless Networks*, vol. 15, pp. 999–1012, 2009.

[61] S. Scott and P. Smyth, "The Markov Modulated Poisson Process and Markov Poisson Cascade With Applications to Web Traffic Modelling," *Bayesian Statistics*, vol. 7, pp. 0–0, Jan. 2003.

[62] O. Bélanger, O. Ben Yahia, S. Martel, A. Lesage-Landry, and G. Karabulut Kurt, "Quality of Service-Constrained Online Routing in High Throughput Satellites," in *IEEE Aerospace Conference*, 2024, pp. 1–9.

[63] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications.* Upper Saddle River, NJ, USA: Prentice Hall, 1993.

[64] D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming.* Springer Science & Business Media, 2008.

[65] J. B. Rawlings, D. Q. Mayne, and M. M. Diehl, *Model Predictive Control: Theory, Computation, and Design*, 2nd ed. Nob Hill Publishing, LLC, 2017.

[66] MOSEK ApS, *The MOSEK Optimization Software*, 2023, version 10.1 [Software]. Available from https://www.mosek.com/.

[67] D. Medhi and K. Ramasamy, *Network Routing: Algorithms, Protocols, and Architectures*, 2nd ed. Morgan Kaufmann, 2018.

[68] A. Burns and A. Wellings, *Real-Time Systems and Programming Languages: Ada 95, Real-Time Java and Real-Time POSIX.* Addison-Wesley Longman Publishing Co., Inc., 2016.

[69] J. W. Valvano, *Embedded Systems: Introduction to ARM Cortex-M Microcontrollers.* CreateSpace Independent Publishing Platform, 2012.

APPENDIX A DEFINITIONS OF MATRICES

We present here a detailed definition of the matrices **A** and **C** and the vectors **b** and **d** used in Problem (3.18).

For simplicity, all elements of A are represented for $W = 2$.

We define **A** as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{v_m} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{v_p} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} \\ \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

where $\delta_{\mathbf{mm'}}$ and $\delta_{\mathbf{pp'}}$ are Kronecker deltas such that

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

We also define

$$\mathbf{v}_m = \underbrace{\begin{bmatrix} \mathbf{0}_{m-1} & 1 & \mathbf{0}_{M-m} \end{bmatrix}}_{P \text{ times}},$$

and

$$\mathbf{v}_p = \begin{bmatrix} \mathbf{0}_{(p-1)M} & \mathbf{1}_M & \mathbf{0}_{(P-p)M} \end{bmatrix},$$

where $\mathbf{0}_n \in \mathbb{R}^n$ and $\mathbf{1}_n \in \mathbb{R}^n$ represent vectors of length n filled with zeros and ones, respectively.

We define $\mathbf{b}_t$ as:

$$\mathbf{b}_t = \begin{bmatrix} \mathbf{0}_{MP} \\ \mathbf{1}_M \\ \hat{\mathbf{F}}_p \\ \mathbf{0}_{MP} \\ \hat{\mathbf{w}}_p^m(t-1) \\ \mathbf{Q}_{\text{sys},MP} \end{bmatrix},$$

where $\mathbf{0}_{MP}$ is a zero vector of size MP, $\mathbf{1}_M$ is a vector of ones of size M,

$$\hat{\mathbf{F}}_p = \begin{bmatrix} \hat{F}_1(t) \\ \hat{F}_2(t) \\ \vdots \\ \hat{F}_P(t) \end{bmatrix}_{P \times 1},$$

and

$$\hat{\mathbf{w}}_p^m(t-1) = \begin{bmatrix} \hat{w}_0^0(t-1) \\ \hat{w}_0^1(t-1) \\ \vdots \\ \hat{w}_P^M(t-1) \end{bmatrix}_{P \times M}.$$

We define $\mathbf{C}$ as:

$$\mathbf{C} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{-1}{\Delta s}\delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{v_m} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta_{\mathbf{mm'}}\delta_{\mathbf{pp'}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

We define $\mathbf{d}$ as:

$$\mathbf{d} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \overline{\mathbf{\Delta w}} \\ \overline{\mathbf{\Delta w}} \\ \mathbf{0} \\ \overline{\mathbf{Q}} \\ \overline{\mathbf{C}} \end{bmatrix}.$$

We define $\mathbf{c}^\top$ as

$$\mathbf{c}^\top = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{v}_k & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{v}_k & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{A.1}$$

such that

$$\mathbf{v}_k = \begin{bmatrix} \underbrace{k_1}_{1\times M} & \underbrace{k_2}_{1\times M} & \cdots & \underbrace{k_P}_{1\times M} \end{bmatrix},$$

and where k_p is the cost of losing a packet of priority p.