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affiliée à l'Université de Montréal

**Modeling Of Machining Process Damping For Arbitrary Cutting Edge
Geometry**

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Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*

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Geometry**

présenté par **Jonathan THERAROS**

en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*

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RÉSUMÉ

L'amortissement du processus d'usinage est dépendant de la géométrie de l'outil de coupe, des paramètres de vibration du système d'usinage, et des propriétés du matériau de la pièce. Les vibrations instables (broutage) durant l'usinage provoquent une mauvaise qualité de surface, des défaillances prématurées de la machine et de l'outil, et causent une faible productivité. En présence de vibrations, l'outil indente dynamiquement le matériau de travail, causant des déformations plastiques qui amortissent le processus d'usinage et améliorent sa stabilité. La géométrie de l'outil a été modifiée afin d'augmenter la stabilité de l'usinage, et ainsi, la productivité. Cette étude développe un nouveau modèle analytique généralisé pour prédire l'amortissement du processus pour toute géométrie d'outil de coupe bidimensionnelle. Le modèle proposé utilise les paramètres de vibration, les propriétés du matériau, et les conditions de coupe en tant que donnée, et discrétise le contact outil-pièce par une série de ressorts en parallèle pour s'adapter à toute géométrie d'outil. Un modèle constitutif de matériau a été utilisé pour décrire le comportement des ressorts au-delà du domaine élastique. L'indentation de l'outil dans le matériau à usiner provoque la compression des ressorts, et génère une pression de contact à l'interface entre les deux. La force d'amortissement du processus dynamique est obtenue pour un cas d'amplitude, de fréquence et de longueur d'onde de vibration donnée, et est linéarisée pour être utilisée dans des modèles de prédiction de la stabilité de chatter dans le domaine fréquentiel. Ce modèle général introduit évite les identifications expérimentales ou numériques des paramètres d'amortissement du processus, qui sont fastidieuses et chronophages.

Le modèle proposé a été vérifié expérimentalement et numériquement pour plusieurs géométries d'outils de coupe différentes. Il a été démontré que le modèle développé peut être utilisé comme un outil d'évaluation rapide des performances de l'outil de coupe sans avoir le prototype physique de l'outil en main lors de sa phase de conception. Le modèle a également montré que la modification de la géométrie de l'arête de coupe a un impact direct sur l'amortissement du processus et, par conséquent, sur la stabilité de l'usinage.

ABSTRACT

Machining process damping depends on the cutting tool geometry, vibration parameters of the machining system, and material properties of the workpiece. Unstable (chatter) vibrations during machining lead to poor surface quality, premature tool and machine failures, and result in low productivity. In the presence of vibrations, the tool dynamically indents the work material and causes plastic deformations that damp the machining process and improve its stability. The tool geometry has evolved to enhance machining stability, and in turn, the productivity. This study develops a new, generalized analytical model to predict the process damping for any two-dimensional cutting tool geometries. The proposed model takes the vibration parameters, material properties, and cutting conditions as inputs, and discretizes the tool-workpiece contact by a series of parallel springs to accommodate any cutting edge geometries. A material constitutive model has been used to describe the behavior of the springs beyond the elastic range. The indentation of the tool into the work material causes compression of the springs, and generates contact pressure at the interface between the two. The dynamic process damping force is obtained for a given vibration amplitude, frequency, and wavelength, and linearized to be used in the frequency domain chatter stability prediction models. The introduced general analytical model avoids the tedious and time-consuming experimental or numerical identification of process damping parameters.

The proposed model has been verified experimentally and numerically for different cutting tool geometries. It has been shown that the developed model can be used as a rapid cutting tool performance evaluator without having the physical prototype in hand in the tool design phase. Also, the model has shown that modifying the cutting edge geometry has a direct impact on the machining process damping, and therefore on the machining stability.

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LISTE OF SYMBOLS AND ABBREVIATIONS

Abbreviations:

AISI	American iron and steel institute
ALE	Arbitrary Lagrangian-Eulerian
CNC	Computer numerical control
FD	Finite difference
FE	Finite element
JC	Johnson-Cook
PP	Perfectly plastic
p - y	Pressure-Displacement
SLD	Stability lobes diagram
SSI	Soil structure interaction

Symbols:

a	Thermal diffusivity
A, B, C, n, m	Johnson-Cook material constitutive model parameters
$A_{IJ}(i, j)$	Temperature rise at l_i at the interface I due to the heat source element $[l_j, l_{j+1}]$ in the heat source J and its corresponding equations
a_s, b_s	Parameters of the linear equation for a symmetry axis
A_0	Vibration amplitude
b	Half length of the contact in cavity models
$B(l_i)$	Heat partition of a heat source at l_i
b_f	Chamfer length
$B_I(i)$	Heat partition at l_i in the interface I of two corresponding heat sources

$B_q, (1 - B_q)$	Heat partition of two heat source elements $[l_j, l_{j+1}]$ on both sides of the interface q
c	Constraint factor
c_{eq}	Equivalent viscous damping coefficient
$C_I(i)$	Temperature rise at l_i at the interface I of all heat sources being the constant terms of the linear equation
c_p	Specific heat
c_1, c_2	Constants defining the slopes of the texture
D	Workpiece or milling tool diameter
dx, dy	Offset of line heat source in the tangential and radial directions
$d\beta_q$	Angle between two successive tilted line heat sources approximating a circle
E	Elastic modulus
E_{diss}	Dissipated energy by the process damping force
E_{eq}	Dissipated energy by an equivalent viscous damper
f	Reaction force
$F_{dynamic,r}(x)$	Dynamic radial plastic contact force per unit contact length at x
$F_n(x)$	Normal force to the contact at x
$F_{pd}(x)$	Process damping force at x
F_{pr}, F_{pt}	Radial and tangential ploughing forces
$F_{static,r}(x)$	Static radial plastic contact force per unit contact length at x
F_{tc}, F_{fc}	Tangential and feed cutting forces
$F_{total,r}(x)$	Total radial plastic contact force per unit contact length at x
h	Uncut chip thickness
h_c	Cut chip thickness
$H^i(x, t)$	Heaviside function defining the domain of definition section i

h_{text}	Texture height
h_w, k	Winkler foundation depth and stiffness
$K_{pd,r}$	Process damping coefficient in frequency domain
K_{sp}	Ploughing force coefficient
$K_0(x)$	Bessel function of second kind of order zero
L	Heat source length
l_d, l_{tx}	Texture distance and length
l_i	Position in a heat source
L_q	Length of the heat source q
l_{right}, l_{left}	Length of the right and left texture slopes
L_w	Maximum flank wear
$L_Y(x)$	Length of a compressed spring at its elastic limit at x
$L_0(x)$	Unloaded length of the spring at x
n	Number of discretized heat source elements
n_d, n_u	Number of linear heat sources approximating the lower and upper region of the honed edge
n_s	Spindle speed
$n_{springs}$	Number of springs discretizing the workpiece surface
$p_{plastic}$	Plastic contact pressure
$P(t)$	Point P at time t with its abscissa and ordinate
$P_x(t), P_y(t)$	x and y -coordinates of a point P at time t
$\dot{P}_x(t), \dot{P}_y(t)$	Time derivative of the equation defining x and y -coordinates of a point P at time t
$q(l_i)$	Heat intensity of the heat source at l_i
$q_{f,r}(l_i)$	Non-uniform heat intensity of the rake-chip interface

q_r	Uniform heat intensity of the rake-chip interface
r, t	Radial and tangential directions of the cut
R, R'	Distances between a point of interest $M(x, y, z)$ and the position at l_i in the original and imaginary heat sources
R_v, R'_v	Projection of R and R' in the direction of motion
r_c	Radius of the circular elastic-plastic boundary in the cavity model
R_e	Honed edge radius
r_s	Constant defining the stick and slide portion on the rake-chip interface
T, T_0, T_m	Material, melting and room temperatures, respectively
T_{init}	Initial temperature of the work material
V	Cutting speed
V_h	Heat source velocity
V_{indent}	Indented volume
w	Cutting width
w_d	Deflection of a beam in the Winkler foundation
x_1, y_1	Coordinate of the point at l_i in a heat source
x_2, y_2	Coordinate of the point at l_i in an imaginary heat source
γ_q	Angle of the tilted heat source q
$y^q(x, t)$	Profile at the position x of the element q in the cutting edge geometry at time t
$y^{wave}(x)$	Vibratory trajectory of the cutting edge
α, α_r	Clearance and rake angles
α_e	Effective clearance angle (texture tool)
α_c	Chamfer angle (chamfered tool)
β	Cone angle

β_a	Average friction angle
$\beta_{d,i}$	Angle of line of the i^{th} tilted heat source approximating a circle, with respect to the tangent direction
$\gamma_{\text{right}}, \gamma_{\text{left}}$	Angle of the right and left texture slopes
$\varepsilon, \varepsilon_{\text{true}}$	Engineering and true strains
$\varepsilon(x, t)$	Contact strain at x and t of the full cutting edge profile
$\varepsilon^i(x, t)$	Contact strain at x and t of section i of the cutting edge profile
$\varepsilon_p, \varepsilon_e$	Plastic and elastic strains
$\varepsilon_{\text{rep}}^i(x, t)$	Representative strain at x and t of section i of the cutting edge profile
$\dot{\varepsilon}_{\text{true}}, \dot{\varepsilon}_0$	True strain rate and its reference
$\theta(x, y, z)$	Temperature rise at a point P defined in the Cartesian coordinates x, y, z
$\theta_{bq}(l_i)$	Temperature rise at l_i on the side b from the heat source at the interface q
κ	Thermal conductivity
λ	Vibration wavelength
μ	Average friction coefficient
ν	Poisson's ratio
ρ	Density
$\sigma, \sigma_p, \sigma_{\text{true}}$	Engineering, engineering plastic and true stresses
$\sigma_{\text{true},p}, \varepsilon_{\text{true},p}$	True plastic stress and strain
σ_Y, ε_Y	Yield stress and strain
$\tau(x)$	Normal angle to the cutting profile at x
ϕ	Shear angle
χ	Separation angle
ω_c	Chatter frequency

Δx_i	Spring discretization step
ΔT	Phase delay between two successive wavy surfaces

CHAPTER 1 INTRODUCTION

1.1 General context

Machining processes play a crucial role in the field of manufacturing. They ensure the production of complex and critical parts in the aerospace, automotive, naval, and biomedical industries that must respect tight tolerances and high surface quality. These requirements and process productivity are directly impacted by the vibrations occurring during the machining process. The vibrations can be either forced (caused by the cutting force excitation) or regenerative (caused by regeneration of the chip thickness). The regenerative (chatter) vibrations, which is one of the main productivity offenders in machining, are unstable and can cause poor surface quality, and damage the machines and the cutting tools. Stability lobe diagrams (SLD) are used to predict and avoid the chatter vibrations while maximizing the productivity. Extensive knowledge of the physical phenomena occurring during cutting is required to accurately predict the stability limit in SLD such as machining system dynamics, work and cutting tool material properties, cutting parameters and cutting tool geometry.

Currently, analytical prediction of the impact of the cutting tool geometry on the stability is only possible for standard tool profiles. That does not align with the increasing complexity of the geometries produced by the tool manufacturers to maximize the productivity. This limitation leads to an underestimation of the stability limit of the machining process, hindering the potential productivity gains, especially for the case of difficult-to-cut materials such as alloy steels, titanium and nickel alloys. Numerical or experimental identification of the effect of the cutting tool geometry can enhance the prediction accuracy, but at the expense of considerable time and laborious efforts, making it impractical for industrial applications. Thus, it is crucial to have an analytical model to predict the impact of the tool geometry, i.e., the process damping phenomenon caused by the dynamic interaction between the cutting edge and the wavy workpiece surface in the presence of vibrations, on the stability predictions. Such an approach can avoid the aforementioned difficulties inherent to numerical or experimental work while motivating its implementation in an industrial setting.

1.2 Objectives

The primary objective of this work is to develop a novel analytical tool for cutting tool manufacturers to replace the currently practiced time-consuming and costly machining tests with fast simulations at early stages of cutter design. This preliminary design can then be verified and refined by conventional methods, optimizing the tool development process. The sub-objectives are (i) to eliminate any geometrical limitations in the process damping prediction to fully embrace the concept of the tool design, and (ii) to accurately predict the stability limit with the effect of process damping.

A secondary objective is to accurately describe the material behavior as it undergoes elastic-plastic deformations in a cutting process. This description is crucial since it impacts the process damping prediction by controlling the forces occurring at the tool-workpiece contact.

1.3 Methodology

The approach to accomplish the objectives involves the following procedures:

- **Mathematical Description of the Tool's Profile:** That is to define the dynamic indentation of the tool into the workpiece.
- **Discretization of the Workpiece Surface:** That is to create the foundation of springs describing the material surface, which will be compressed due to the tool indentation.
- **Material Behavior Description:** That is to describe the springs behavior as they experience deformations, in order to link contact stress to contact strain. That includes temperature prediction during the cutting process and its effect on the material behavior.
- **Calculation of the Process Damping Forces:** That is to calculate the contact forces and extract the dynamic forces impacting the stability.
- **Extraction of the Equivalent Process Damping:** That is to linearize the process damping forces and extract a damping constant useable in stability predictions.

To validate the prediction of the model, the following steps have been employed:

- **Numerical Comparison:** That is to verify both the process damping and temperature predictions of the model with Finite Element (FE) or Finite Difference (FD) simulations from literature.

- **Experimental Comparison:** That is to verify the process damping and cutting temperature predictions of the model with experimental results.
- **Previous Analytical Model Comparison:** That is to validate the proposed model's results for standard cutting tool geometries by comparing it with a previously developed analytical model [1].

1.4 Organisation of the thesis

The thesis is organized as follows:

CHAPTER 1 introduces the research problem and objectives, as well as briefly presenting the general context and the rationale behind the research problem.

CHAPTER 2 presents a literature review that describes the physical phenomena impacting the process damping and positions the proposed method within the existing models to emphasize its contribution.

CHAPTER 3 describes the main model to predict the machining process damping for an arbitrary cutting tool geometry. It involves the description of a non-linear Winkler model which is the foundation of the proposed method. Additionally, material constitutive laws are presented along with the computation of their inputs, including the calculation of the strain and strain-rate of the deformed material surface.

CHAPTER 4 describes the prediction of the temperature rise in the contact surface between the tool and the workpiece. Temperature prediction is an important factor that directly influences the process damping model. Improvements to existing models are proposed to integrate them into the process damping model.

CHAPTER 5 presents the experimental and numerical validation of the newly introduced process damping model. That includes numerical and experimental verification of the workpiece temperature prediction and its integration into the process damping model. Also, the newly proposed process damping prediction method is compared with the experimental, numerical, and analytical results for a standard cutting edge geometry, and with numerical results for a more complex geometry.

CHAPTER 6 concludes the work, expresses the limitations of the different models and proposes future research directions to address them.

CHAPTER 2 LITERATURE REVIEW

2.1 Machining chatter

Machining vibrations, forced (stable) or regenerative (unstable) [2], impact the surface quality and productivity of machining operations. Regenerative (chatter) vibrations can cause poor surface quality, excessive wear or breakage of the cutting tool [3], [4], and hence must be avoided at all cost. Forced vibrations are usually non-destructive and mainly caused by the periodicity of the milling forces, as well as the tool runout [5], [6]. Chatter vibrations are caused by a phase-shift between two successive wavy surfaces [7]. When the cutting tool vibrates, it creates a wavy surface on the workpiece. Each subsequent cut (one spindle period or tooth-passing period after) also produces a wavy surface beneath the previous one. A phase delay (ΔT) between these waves can cause an exponential increase in the uncut chip thickness (regenerative phenomenon), leading to higher cutting forces and poor surface quality as shown Figure 2.1, where $h(t)$ is the regenerated uncut chip thickness.

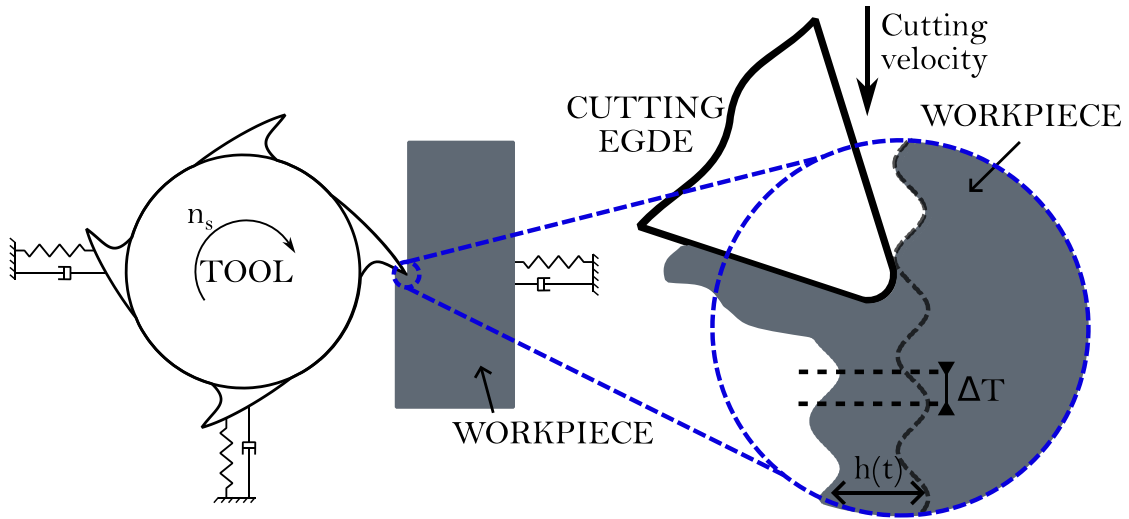


Figure 2.1: Schematic of regenerative chatter

It is then imperative to prevent the chatter vibrations to respect machining tolerances. As mentioned in a review [8], chatter avoidance and suppression has been the subject of numerous researches in the last decades. The main methods include the perturbation or control of the phase-delay (with the modification of the tool geometry, or with the spindle speed variation), the active cancelation of the vibrations (with piezoelectric actuators), or the avoidance of instability with Stability Lobe

Diagram (SLD) [9]. SLD is used in this work since it does not impose any constraints on the tool or on the CNC machine.

2.2 Stability lobe diagram

Stability lobe diagram (SLD) describes the stability limit for a given workpiece material and spindle, or cutting speed range and allows process engineers to select an optimum pair of spindle/cutting speed and depth of cut in the stable region for maximum the productivity. SLD is dependent on various parameters, including the material properties, the system dynamics, the cutting parameters and the tool geometry [10].

The accuracy of the SLD is very limited at low cutting speeds since the stability is very dependent on the tool geometry and tool wear in this region (due to the process damping phenomenon) [9] as shown in Figure 2.2.

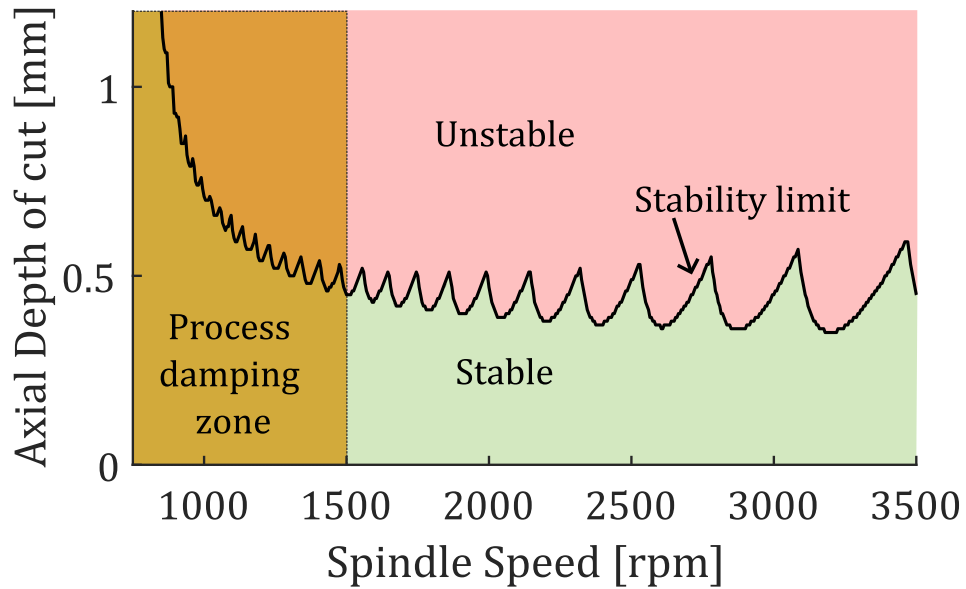


Figure 2.2: Schematic of a stability lobe diagram (SLD)

Figure 2.2 illustrates a predicted stability limit curve separating the stable and unstable regions for a range of spindle speeds. The process damping zone is highlighted towards low cutting speeds and describes the increased contribution of an additional damping as the spindle speed decreases. Materials with low machinability, such as titanium and nickel alloys, are extensively used in the aerospace industry and are restrained to low speeds due to tool life [9]. Therefore, improving the prediction accuracy of SLD in the process damping region is essential to meet the strict

requirements of critical aerospace parts and to prevent costly losses. This work focuses on integrating the tool geometry into the process damping modeling to improve stability prediction in the SLD.

2.3 Machining process damping

Process damping refers to the additional damping forces induced by the dynamic indentation of the two-dimensional (2D) cutting edge of the tool, i.e. its flank face and honed edge, into the workpiece in the presence of vibrations [11] as illustrated in Figure 2.3.

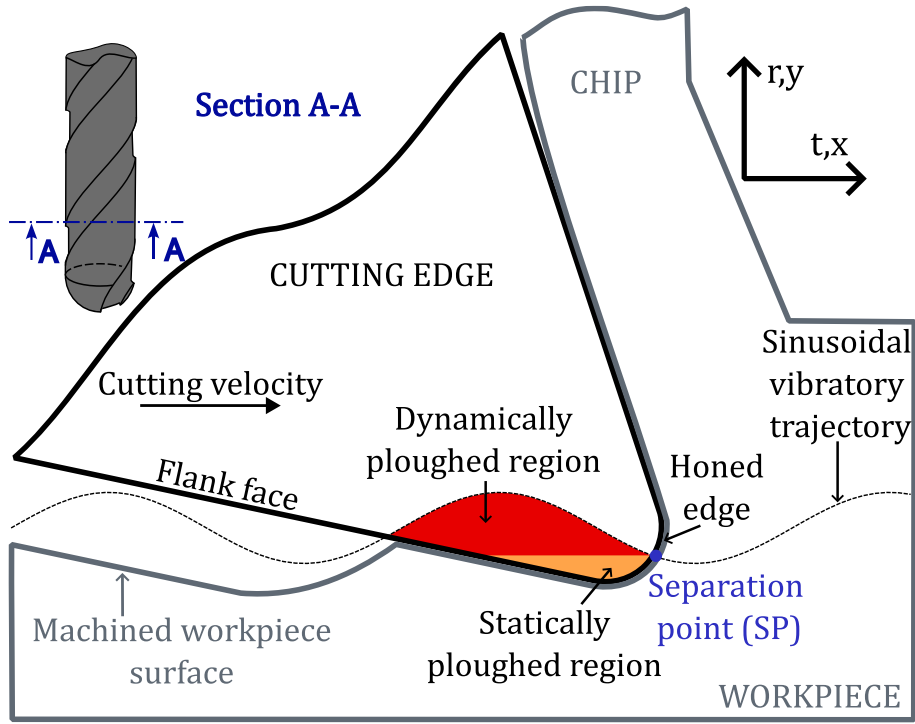


Figure 2.3: 2D indentation of the cutting edge into the workpiece material

The ploughing effect causes plastic deformations which result in heat generation and energy dissipation [12]. Wu [13] first proposed a proportional relation between the ploughing forces (F_{pr} , F_{pt}) and the indented volume (V_{indent}):

$$F_{pr} = K_{sp} V_{indent} \quad (2.1)$$

$$F_{pt} = \mu F_{pr} \quad (2.2)$$

where F_{pr} and F_{pt} are the radial and tangential dynamic forces, respectively, μ is the average friction coefficient, and K_{sp} is the experimentally identified ploughing force coefficient. The indented volume can be estimated analytically by approximating the indented geometry [14], [15] or calculated numerically, but at the cost of higher computation time [15], [16]. Several experimental [15-21] and numerical [24], [25] methods have been proposed to identify K_{sp} . As an alternative, an analytical method using contact mechanics has been proposed to directly predict the dynamic ploughing forces [1] by avoiding the costly numerical and experimental tests. However, it is limited to tools with straight flank (standard) geometry only. All these methods assume full recovery of the ploughed region as it was first done by Wu [11], [13] for simplicity. In an early work [26], the indented volume has been separated into two components: the static ploughing in vibration-free conditions due to the presence of a honed edge, and the dynamic ploughing during vibrations as shown in Figure 2.4. This separation has been extensively used by researchers in subsequent studies [7], [11], [17], [22], [23], [27] to extract the process damping force, F_{pd} , from the dynamically ploughed volume.

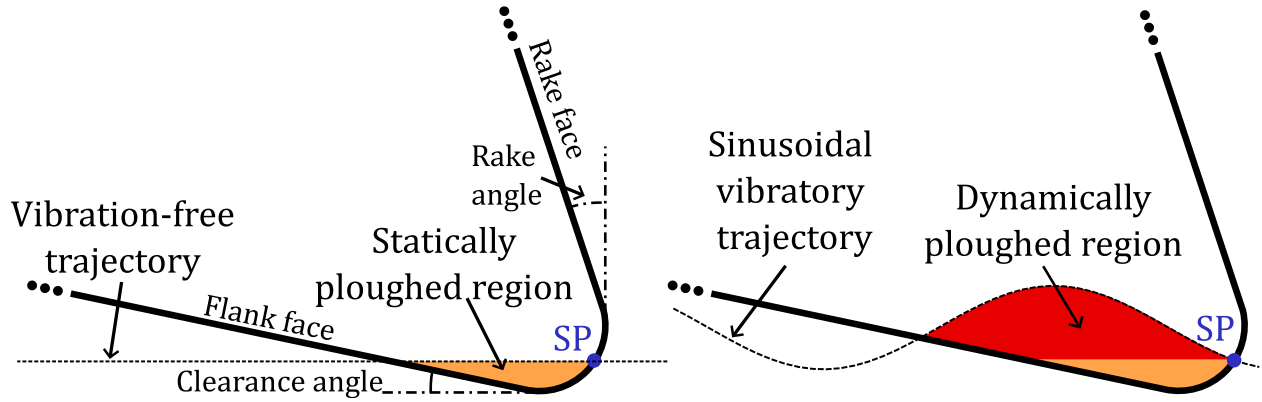


Figure 2.4: Static and dynamic ploughing in machining

It is known that the process damping is a non-linear phenomenon that depends on the vibration amplitude [24] and only occurs during the downward motion of the tool along the vibration wavelength [1], [18]. Hence, the process damping force is linearized over the full wavelength (λ) by replacing it by an equivalent viscous damper [1], [12], [15], [17], [21] to be used in frequency domain SLD calculations [15]. Using the energy dissipation principle [16], [18], [22], an equivalent viscous damper coefficient, c_{eq} , is calculated by equating the energy dissipated by the process damping force, E_{diss} , to the energy dissipated by the equivalent viscous damper, E_{eq} .

$$E_{diss} = \int_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}} F_{pd} dy \quad (2.3)$$

$$E_{eq} = \int_0^{\lambda} c_{eq} \frac{dy}{dt} dy \quad (2.4)$$

In this thesis, focus is on the analytical modeling of process damping to avoid costly numerical and experimental identifications. Moreover, the only analytical model is applicable solely for straight flank geometries. This limitation excludes a critical parameter from the process damping prediction since the cutting edge geometry directly affects the indented volume and the process stability [28]. Also, this restriction contradicts the tendency to modify the tool geometries to suppress the chatter vibrations.

2.4 Impact of the cutting edge geometry on process damping

It is well accepted that cutting edge geometry has a direct influence on the ploughing forces. Decreasing clearance angle and increasing honed edge radius tends to improve the process damping and stability [11], [28], which makes it difficult to predict the stability due to progressive tool wear during machining [28]. Cutting edge geometries have been modified from being standard to chamfered [29], [30], [31], cylindrical [21] and textured [32], [33] as presented in Figure 2.5.

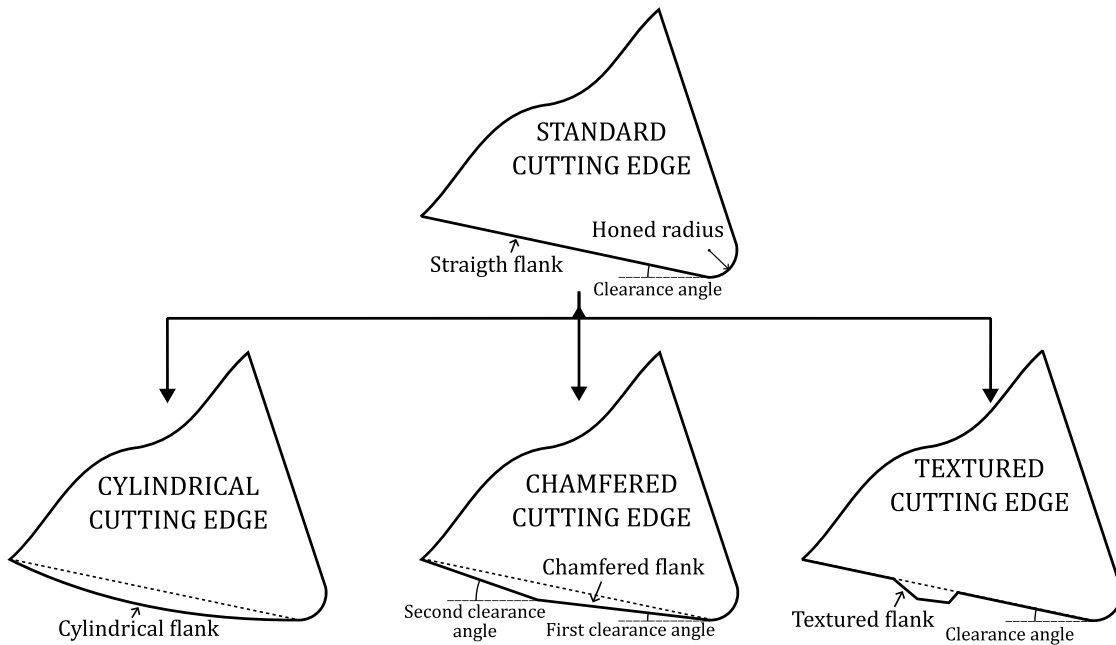


Figure 2.5: Flank face geometries

Modifications of the 2D cutting edge geometry, such as adding a texture or reducing the clearance angle, will increase the machining process damping due to the increased indentation even at higher cutting velocities [33] by expanding the process damping zone shaded in Figure 2.2. That is due to the increased ploughed region, which even occurs at low vibration amplitudes. Improvements in the cutting edge geometry have a direct impact in machining productivity by increasing the stability limit and also by producing better surface quality [33], [34] with reduced tool wear [34].

It is, thus, essential to include variations of the flank geometry in the analytical modeling of process damping for more accurate stability predictions even outside the process damping zone. Additionally, the analytical model is more accessible than complex FE simulations and experimental tests to observe the impact of a given tool geometry on stability, which is a strong argument in tool design.

CHAPTER 3 DEVELOPMENT OF A GENERAL ANALYTICAL PROCESS DAMPING MODEL

3.1 Introduction

Focus is placed on the analytical modelling of process damping to provide an efficient alternative to complex experimental and numerical identification methods. While process damping is directly influenced by the cutting edge geometry, the only analytical model in literature [1] does not allow any geometric divergence from the standard geometry, which limits its applicability in cutting tool design.

In this chapter, a different approach from the existing analytical model has been introduced to remove any limitation on the cutting edge geometry. The proposed method adapts a non-linear Winkler foundation to model the contact mechanics between the arbitrary cutting edge profile and the wavy workpiece surface. A brief description of the contact modelling using Winkler foundations is given first, and the proposed contact model is detailed, including the description of the cutting edge profiles and the contact stress calculation. The proposed model is finally applied to process damping prediction by calculating the process damping forces from the contact stress.

3.2 Winkler foundations

The Winkler foundation is a method widely used to study the soil-structure interaction (SSI) in Civil Engineering [35], [36], [37]. It [38] describes the behavior of a foundation of elastic, parallel, and independent one-dimensional springs that supports a deflected beam. It links the deflection w_d to the reaction force f of the foundation via a foundation stiffness (k) [37] as depicted in Figure 3.1:

$$f = kw_d \quad (3.1)$$

The foundation stiffness (k) is closely linked to the indented material's elastic modulus (E) and the depth of the foundation (h_w) [36], [39], [40], [41], [42]. k conceptually characterizes the material's resistance to deformations:

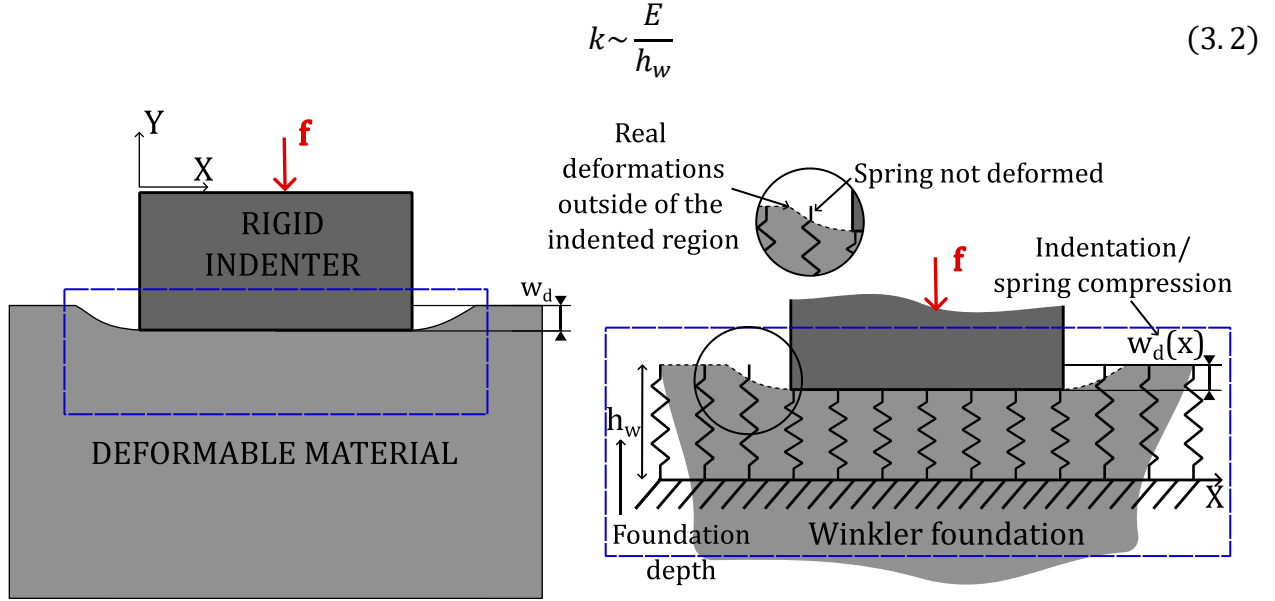


Figure 3.1: Winkler foundation

The definition of h_w varies in literature. It can be calibrated with Hertzian equations [39], [43], it can be set equal to the thickness of the indented body when the latter is thin [36], [40] or usually ignored using experimental [44], [45] or numerical identification [46], [47] of the stiffness in SSI. Still, the depth of the foundation is conceptually linked to the depth of deformation by being related to the thickness of the compressible layer [48] as the indentation causes motion and deformation within a finite depth inside the deformed material [40], [49]. This contact model is fairly simplified and results in some limitations. Notably, it does not consider any interactions between the springs, failing to represent the continuity of the material and its self-interaction [50], [51]. As a result, the deformation does not propagate outside the indented, or loaded, region [51] (Figure 3.1), and the tangential deformations of the surface of a body during indentation are not directly considered [39], [52]. It is worth noting that these limitations can be addressed by using more comprehensive spring foundations, such as the Pasternak foundation [51]. However, this naturally comes with an increase of complexity in its definition.

The Winkler model has been adapted to other contact case outside of SSI, notably to the contact of a rigid sphere [53] or of any axially symmetric body [54]. In its natural form, the foundation only describes elastic deformations of the surface of a body, however the behavior of the deforming body may be nonlinear, it may include plastic deformation and yielding [45] such as in the case of machining process damping. Several authors attempted to integrate this nonlinearity to the

foundation model [44], [45], [55], [56], usually by implementing nonlinear force-displacement, i.e. p - y , curves [57], [58], [59], [60] or stress-strain curves [46], [47], [59], [60] in the context of SSI (referred to as backbone curves).

The approach adopted in this thesis adapts the original Winkler foundation to include elastic-plastic deformations in contact mechanics. Indeed, the objective is to predict the contact forces resulting from the indentation of a rigid body (cutting edge) with an arbitrary profile into an elastic-plastic material (workpiece). The discretization of the contact using the springs removes any limitations to the indenter profile, and a material model can be used in a similar manner to backbone curves in SSI, along with a foundation depth obtained via the deformation depth within the material.

3.3 Two-Dimensional Cutting edge and workpiece material contact model

First, the cutting edge profile is described mathematically in Section 3.3.1, and then the Winkler foundation is defined in Section 3.3.2 and is used to calculate the contact stress at the interface between the cutting edge and the machined surface in Section 3.3.3.

3.3.1 Cutting edge profile description

Three different profiles with a honed edge have been studied in this work:

- Straight flank (standard profile)
- Textured flank
- Chamfered flank

The cutting edge elements are considered as rigid and their position is controlled by the material separation point (SP) (Figure 2.3 and Figure 2.4). SP is on the generated wavy workpiece surface, and the material above SP generates the chip while the material below is rubbed against the workpiece. SP acts as the reference point of the rigid cutting edge, flank face and honed edge radius are defined from this point's coordinates. Though an equivalent wear is assumed, no wear evolution is considered in the model. It should be noted that the profile description is not limited to the examples mentioned above. Basic geometric elements like lines and circles, as mentioned in the following descriptions, can still be applied to complex geometries by breaking them down into multiple segments.

3.3.1.1 Straight flank

A cutting edge with a hone radius and a straight flank is the standard profile used in [1]. The profile geometry is shown in Figure 3.2 along with its geometric parameters.

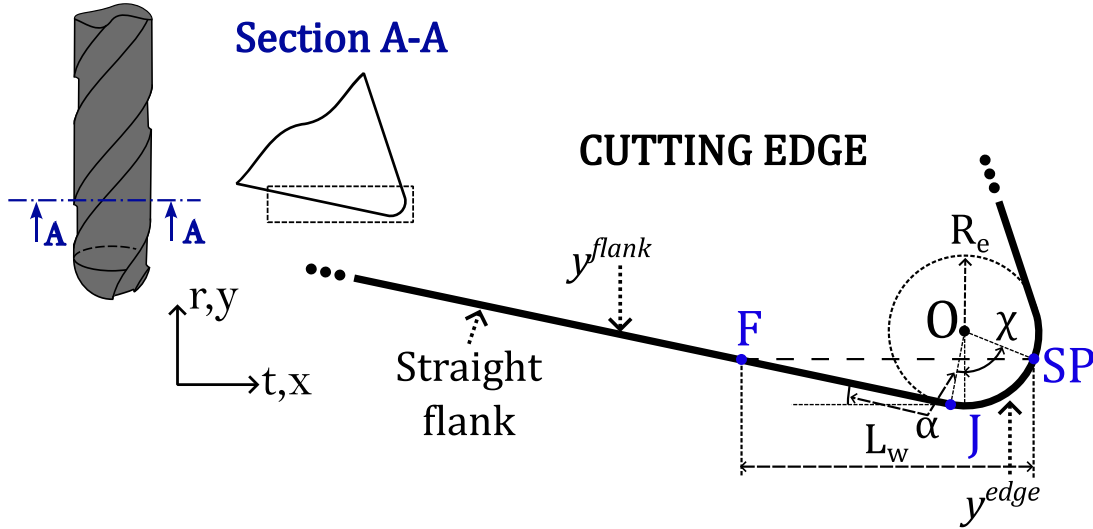


Figure 3.2: Cutting edge geometry with straight flank face (standard)

In Figure 3.2, α and χ define the clearance and separation angles, respectively; R_e is the radius of the honed edge and L_w is the equivalent flank wear.

Mathematical definition of the standard cutting edge profile is already given in [1] for a given set of $SP(t) = (SP_x(t), SP_y(t))$ coordinates as [1]:

$$y^{\text{flank}}(x, t) = \tan(\alpha) (J_x(t) - x) + J_y(t), x < J_x(t) \quad (3.3)$$

$$y^{\text{edge}}(x, t) = \sqrt{R_e^2 - (x - O_x(t))^2} + O_y(t), x \geq J_x(t) \quad (3.4)$$

$$O(t) = (-R_e \sin \chi + SP_x(t), R_e \cos \chi + SP_y(t)) \quad (3.5)$$

$$J(t) = (-R_e \sin \alpha + O_x(t), -R_e \cos \alpha + O_y(t)) \quad (3.6)$$

$$L_w = R_e (\sin \chi + \sin \alpha + \cot(\alpha(\cos \alpha - \cos \chi))) \quad (3.7)$$

$$F(t) = (SP_x(t) - L_w, SP_y(t)) \quad (3.8)$$

where $P_x(t)$ and $P_y(t)$ are the x and y -coordinates of a point P at a time t . The definition of y^{edge} , O and J will remain unchanged for the other profiles.

The coordinates of the separation point $SP(t)$ (also the reference point of the rigid profile) are time dependent and defined as [1]:

$$SP_x(t) = Vt \quad (3.9)$$

$$SP_y(t) = \begin{cases} A_0 \sin\left(\frac{2\pi Vt}{\lambda}\right), & \text{during vibrations} \\ 0, & \text{without vibrations} \end{cases} \quad (3.10)$$

with A_0 and λ are the vibration amplitude and wavelength, respectively and V is the cutting velocity.

3.3.1.2 Textured flank

Textured tools have been used in literature to improve the process stability [24], [32], [33]. An example of textured profile is represented in Figure 3.3 and its geometric parameters are provided in [32].

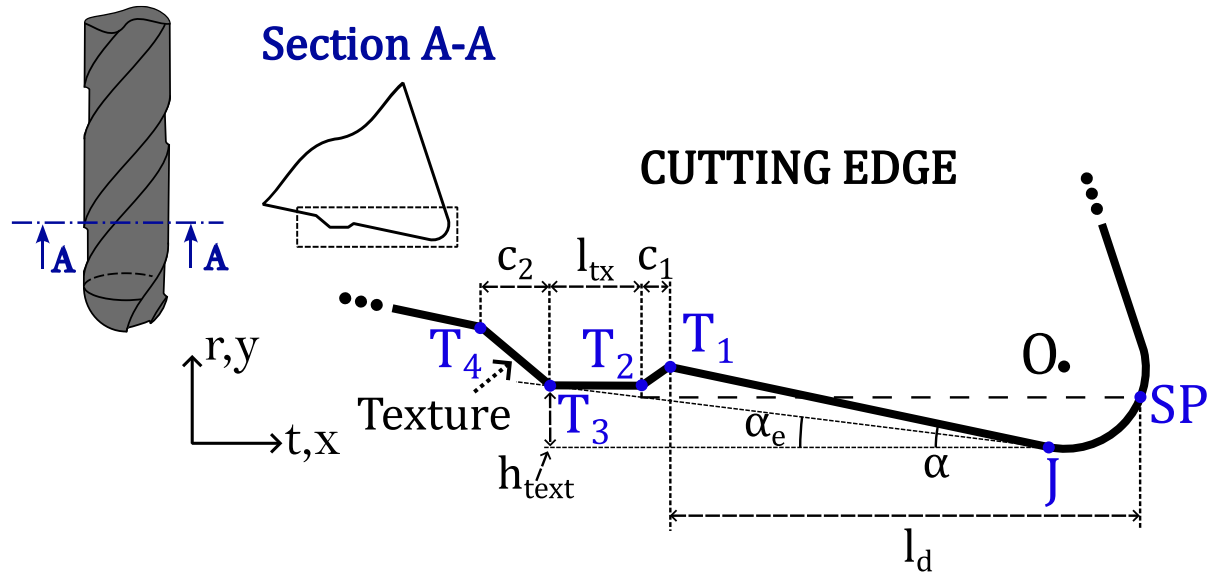


Figure 3.3: Cutting edge geometry with a textured flank face

Slopes of the left and right side of the texture are not given in [32], therefore, additional parameters (c_1 and c_2) are introduced to define these sections. Figure 3.4 describes the intermediate parameters used in the mathematical description of the texture.

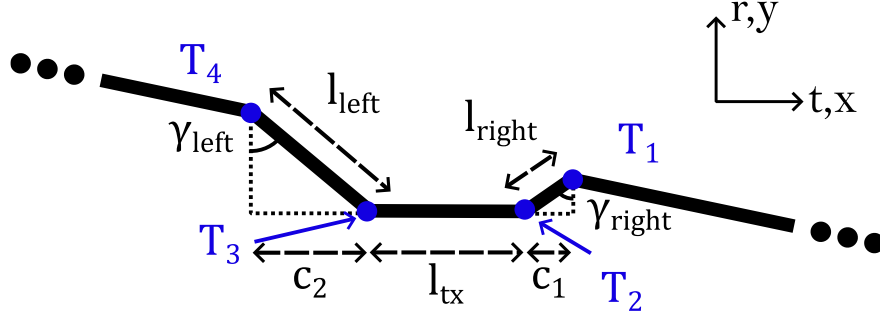


Figure 3.4: Texture geometry

Then, the textured cutting edge profile is mathematically represented using the description from [32], [33], Figure 3.3 and Figure 3.4 as:

$$y^{\text{straight flank}}(x, t) = \tan(\alpha) (J_x(t) - x) + J_y(t), x \in [T_{1,x}(t), J_x(t)] \cup x < T_{4,x} \quad (3.11)$$

$$y^{\text{texture, right}}(x, t) = -\tan(\gamma_{\text{right}})(T_{1,x}(t) - x) + T_{1,y}(t), \quad x \in [T_{2,x}(t), T_{1,x}(t)] \quad (3.12)$$

$$y^{\text{texture, down}}(x, t) = T_{2,y}, x \in [T_{3,x}(t), T_{2,x}(t)] \quad (3.13)$$

$$y^{\text{texture, left}}(x, t) = \tan(\gamma_{\text{left}})(T_{4,x}(t) - x) + T_{4,y}(t), \quad x \in [T_{4,x}(t), T_{3,x}(t)] \quad (3.14)$$

with

$$\gamma_{\text{right}} = \cos^{-1} \frac{c_1}{l_{\text{right}}} \quad (3.15)$$

$$l_{\text{right}} = \sqrt{(T_{1,y}(t) - T_{2,y}(t))^2 + (T_{1,x}(t) - T_{2,x}(t))^2} \quad (3.16)$$

$$\gamma_{\text{left}} = \cos^{-1} \frac{c_2}{l_{\text{left}}} \quad (3.17)$$

$$l_{\text{left}} = \sqrt{(T_{3,y}(t) - T_{4,y}(t))^2 + (T_{3,x}(t) - T_{4,x}(t))^2} \quad (3.18)$$

$$T_1(t) = (SP_x(t) - l_d, \tan(\alpha)(J_x(t) - (SP_x(t) - l_d)) + J_y(t)) \quad (3.19)$$

$$T_3(t) = (T_{1,x}(t) - c_1 - l_{tx}, J_y(t) + h_{\text{text}}) \quad (3.20)$$

$$T_2(t) = (T_{1,x}(t) - c_1, T_{3,x}(t)) \quad (3.21)$$

$$T_4(t) = (T_{3,x}(t) - c_2, \tan(\alpha)(J_x(t) - (T_{3,x}(t) - c_2)) + J_y(t)) \quad (3.22)$$

where l_d is the texture distance; l_{tx} and h_{text} are its length and height, respectively; α and α_e the clearance and effective clearance angles, respectively. h_{text} must respect the following condition [32] to avoid any texture contact with the machined surface in vibration-free conditions:

$$h_{text} > L_w \tan \alpha \quad (3.23)$$

Additionally, the coefficients c_1 and c_2 describe the left and right sides of the texture and help define their slopes γ_{left} and γ_{right} by calculating their respective lengths l_{left} and l_{right} .

3.3.1.3 Chamfered flank

Chamfered tools have also been studied in literature to increase the machining stability [31]. The schematic of an example chamfered cutting edge is displayed in Figure 3.5.

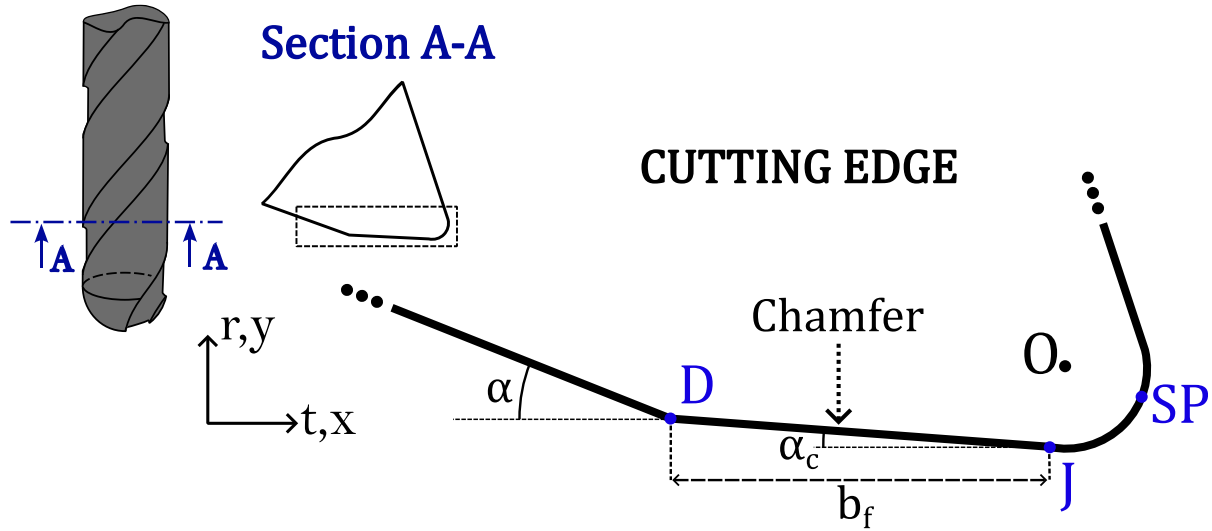


Figure 3.5: Cutting edge geometry with a chamfered flank face

The chamfered cutting edge profile is mathematically represented using geometric description in [30], [31], and Figure 3.5 as:

$$y^{\text{chamfer}}(x, t) = \tan(\alpha_c) (J_x(t) - x) + J_y(t), x \in [D_x(t), J_x(t)] \quad (3.24)$$

$$y^{\text{straight flank}}(x, t) = \tan(\alpha) (D_x(t) - x) + D_y(t), x < D_x(t) \quad (3.25)$$

with

$$D(t) = (-b_f \cos \alpha_c + J_x(t), b_f \sin \alpha_c + J_y(t)) \quad (3.26)$$

where α and α_c are the clearance and chamfer angles, respectively; and b_f is the chamfer length.

3.3.2 Definition of Winkler foundation

Implementing a nonlinear Winkler foundation in contact mechanics require use of p - y or stress-strain curves to describe the springs behavior outside the elastic region. In this study, material constitutive model is used to describe the material behavior as it undergoes deformation. This choice is motivated by the widespread accessibility of analytical material constitutive models such as elastic-perfectly plastic or Johnson-Cook, which are already used in the context of machining.

The Winkler foundation is defined by two elements:

- The position of the discrete springs representing the surface of the machine workpiece,
- The foundation depth, i.e., the unloaded length of the springs, which represents the depth of deformation in the material.

3.3.2.1 Position of discrete springs

The spring foundation discretizes the surface of the workpiece material. In machining, the ploughing effect occurs between the cutting edge and the newly machined surface. As widely assumed in literature [1], [11], [16], [17], [19], [21] the cut surface of the workpiece remains sinusoidal as the cutting edge advances along the vibratory trajectory, which is kept in the proposed model. The sinusoidal vibratory cutting trajectory has usually been expressed [16], [20], [23], [1] with the following equation:

$$y^{\text{wave}}(x) = A_0 \sin\left(\frac{2\pi x}{\lambda}\right) \quad (3.27)$$

The vibration wavelength is linked to the chatter frequency as [1]:

$$\lambda = \frac{V}{\omega_c} \quad (3.28)$$

with V the cutting speed and ω_c the chatter frequency in Hz.

The cut surface of the workpiece, and thus the position of the springs, then depend on the process status: either vibration-free or vibratory as illustrated in Figure 3.6.

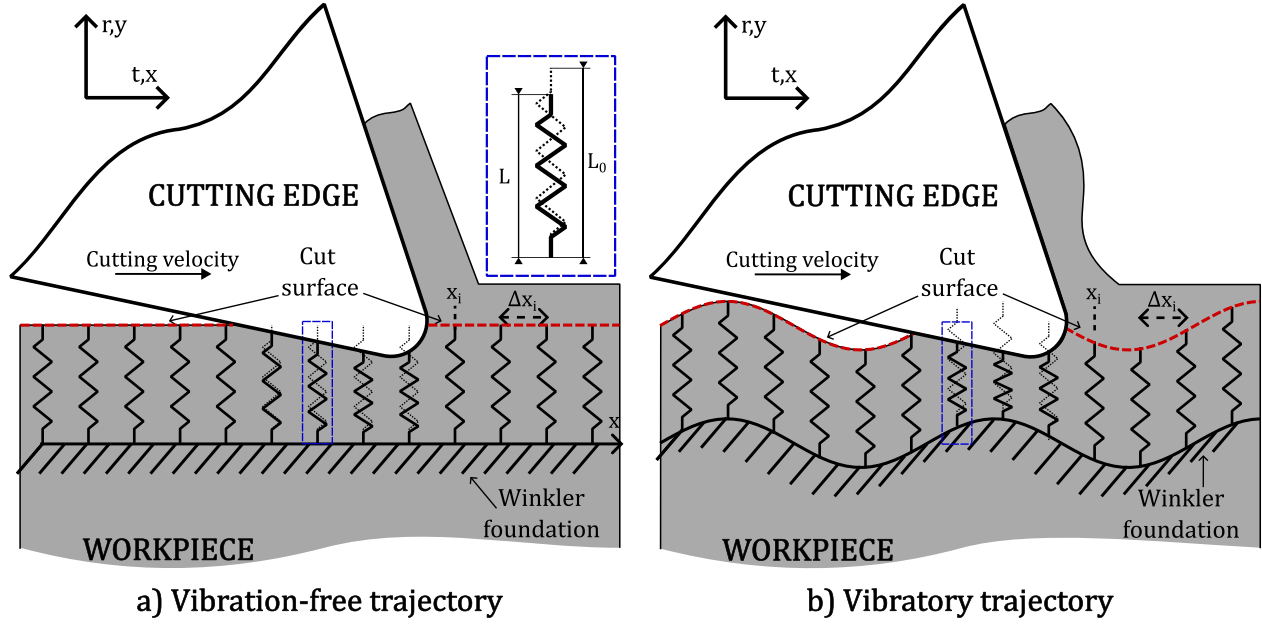


Figure 3.6: Cutting edge trajectory and Winkler's springs positions a) without vibrations, b) with vibrations

3.3.2.2 Foundation depth

Though Winkler foundation depth is challenging to define especially during elastic-plastic deformations, it is linked to the depth of deformation [40], [48], [49] within the deformed body (the workpiece). Hence, the deformation depth is adopted as the foundation depth in this work.

The deformation depth resulting from machining of AISI 1018 steel has been experimentally identified [61] by measuring the residual stress in the material using a layer removal method. The plastic deformations were shown to extend to the depth of 0.508 mm beneath the surface. In addition, the elastic deformations were considered to extend by the same depth beneath the plastic region, resulting in a total deformation zone of 1.016 mm. This method is purely experimental, and the depth of deformation varies depending on the machining operation and material, which makes it difficult to implement in the proposed analytical model.

Another, fully analytical model was studied by Johnson [39]. It predicts the stress distribution in an elastic-perfectly plastic material indented by a conical rigid body. It assumes that the subsurface displacements due to such an indentation are approximately radial (cavity) from the point of first

contact as depicted in Figure 3.7. This model predicts an elastic-plastic boundary within the deformed material, which is used in the Winkler-based approach in the proposed model.

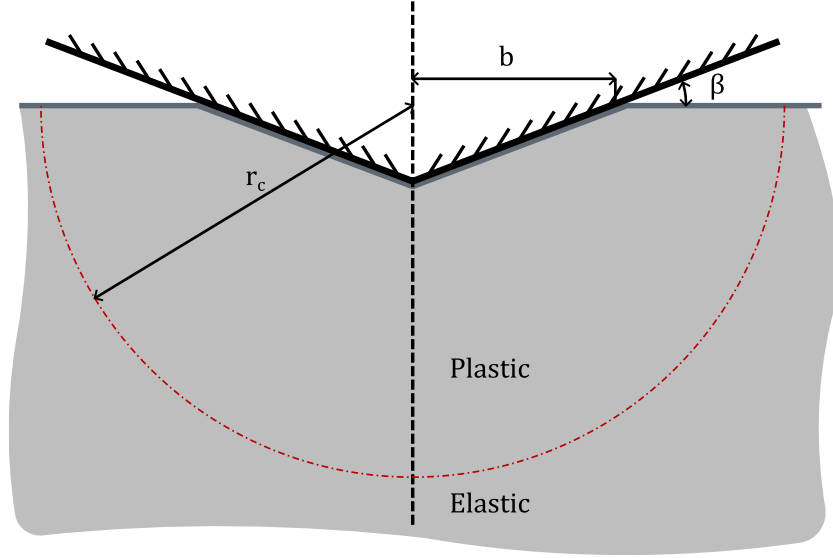


Figure 3.7: Cavity model

From the cavity model [62], [63], two radii of elastic-plastic interface can be deduced for two different material models as:

- Elastic-perfectly plastic [39]:

$$r_c = b \left[\left(\frac{E \tan \beta}{\sigma_Y} + 4(1 - 2\nu) \right) \frac{1}{6(1 - \nu)} \right]^{\frac{1}{3}} \quad (3.29)$$

- Elastic-strain hardening [62]:

$$r_c = b \left(\frac{E}{3\sigma_Y} \cot \left(\frac{\pi}{2} - \beta \right) \right)^{\frac{1}{3}} \quad (3.30)$$

with E , σ_Y and ν the elastic modulus, yield strength and Poisson's ratio of the material, respectively; b the half length of the contact, β the cone's angle as shown in Figure 3.7.

These models represent the stress distribution in the material caused by the indentation of a rigid cone or sphere. They do not account for tangential displacement, changes in geometry, or multiple contacts. Additionally, the strain rate in these indentation tests differs significantly from that produced by the ploughing during machining, which may impact the stress distribution and the elastic-plastic boundary. Moreover, in machining operations, the deformation zone is greatly

influenced by the shearing that occurs in front of the cutting edge. Despite these differences, results from the cavity model are used to predict the elastic-plastic boundary in machining due to its simplicity and reliable accuracy as demonstrated in the experimental results in CHAPTER 6.

The half length of the contact (b) is set to be equal to half the equivalent flank wear in Eq.(3.7) in vibration-free conditions and numerically calculated for the vibratory trajectory of the 2D cutting edge as illustrated in Figure 3.8 by solving:

$$y^{\text{flank}}(x, t) = y^{\text{wave}}(x, t) \quad (3.31)$$

using the mathematical equation y^{flank} describing the flank geometry as presented in Section 3.3.1.

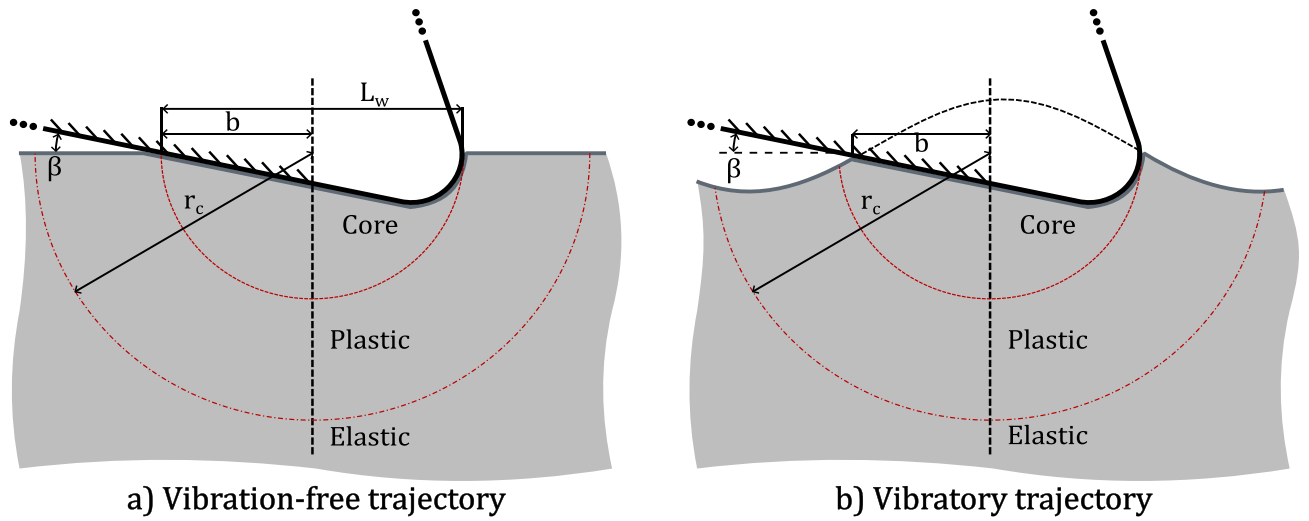


Figure 3.8: Application of the cavity model for the 2D cutting edge a) without vibrations, b) with vibrations

Each spring in the Winkler model includes both elastic and plastic deformations, which makes the link between the unloaded length of the springs, L_0 , and the elastic-plastic boundary defined by r_c unclear. The circular plastic boundary (Figure 3.8) corresponds to the length of the contacting springs, L_Y , at which they begin to undergo plastic deformation once their elastic limit is exceeded. In this case, the yield strain is:

$$\varepsilon_Y = -\frac{L_Y - L_0}{L_0} = \frac{\sigma_Y}{E} \quad (3.32)$$

Considering the circular plastic boundary to be the loaded springs length L_Y , the latter can be mathematically expressed by:

$$L_Y(x) = \sqrt{r_c^2 - (x - SP_x(t) + b)^2} \quad (3.33)$$

Inserting Eq.(3.33) into Eq.(3.32) yields:

$$L_0(x) = \frac{\sqrt{r_c^2 - (x - SP_x(t) + b)^2}}{1 - \sigma_Y/E} \quad (3.34)$$

The position dependent spring unloaded length $L_0(x)$ and loaded springs length $L_Y(x)$ are illustrated in Figure 3.9.

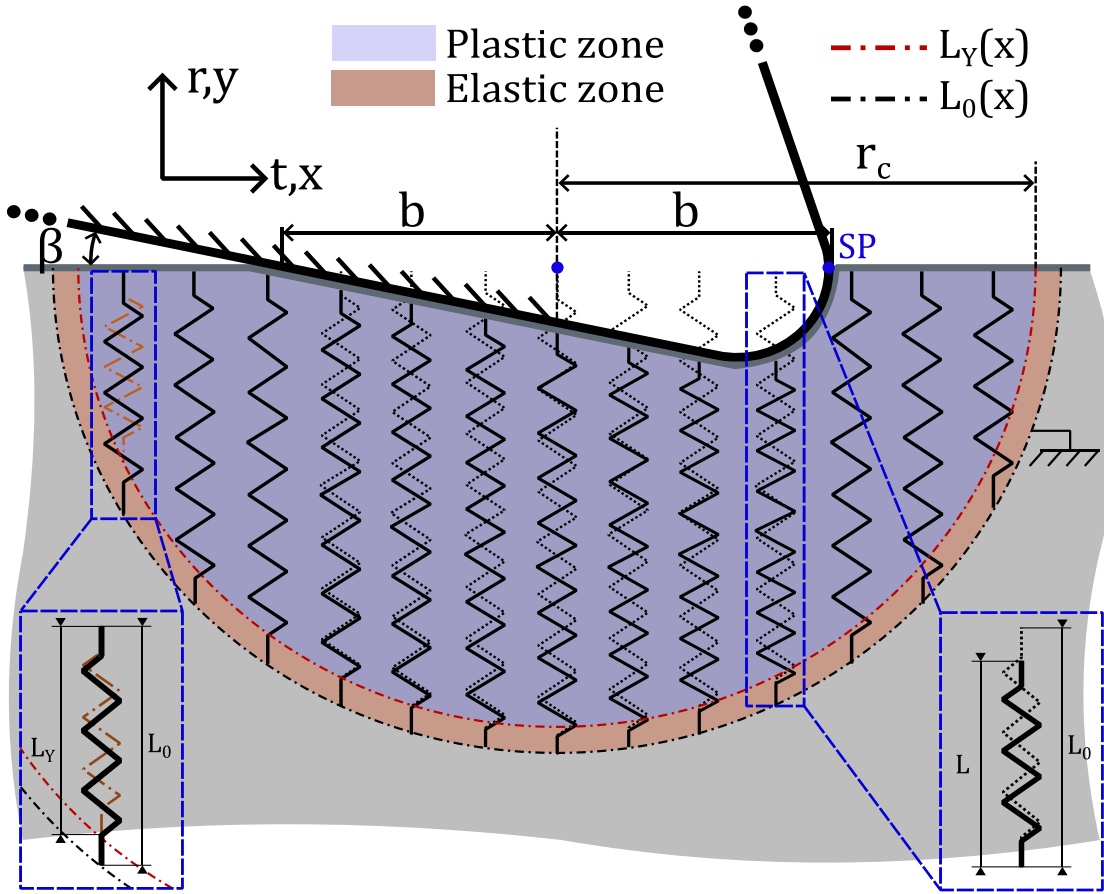


Figure 3.9: Insertion of the cavity model into the Winkler model

Equations (3.29), (3.30) and (3.34) include the material properties (E, σ_Y, ν), the tool geometry (β, a), and the vibration parameters (b , influenced by the vibrations as shown in Figure 3.8) in the prediction of the depth of deformation.

Multiple contacts could occur while the cutting edge generates the wavy machined surface depending on the cutting edge geometry (notably for a textured flank face). In this case, each contact is considered to have its own cavity model, following the procedure described from Eq.(3.29) to Eq.(3.34).

3.3.3 Contact stress calculation

Once the spring foundation is defined, the required cutting edge position-dependent inputs (strain, strain rate and temperature) are evaluated using the material constitutive model of the workpiece surface represented by discrete springs.

3.3.3.1 Material constitutive models

Several material models exist in the literature to represent the material behavior. Some of the most common models [64], [65], [66] are:

Table 3.1: List of material constitutive laws

$\sigma = K_1 \varepsilon^{n_1}$	Hollomon
$\sigma = \sigma_0 + K_2 \varepsilon^{n_2}$	Ludwick
$\sigma = K_3 (\varepsilon + \varepsilon_0)^{n_3}$	Swift
$\sigma = \sigma_0 - \sigma_0 K_4 e^{-n_5 \varepsilon}$	Voce
$\sigma = \frac{\varepsilon}{E} + \left(\frac{\varepsilon}{K_5} \right)^{n_5}$	Ramberg-Osgood
$\sigma = \begin{cases} E\varepsilon, & \sigma \leq \sigma_Y \\ \sigma_Y, & \sigma > \sigma_Y \end{cases}$	Elastic-perfectly plastic
$\sigma = (A + B\varepsilon^n) \left(1 + C \ln \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \left(1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right)$	Johnson-Cook

where σ and ε are the stress and strain, $K_q, n_q, \sigma_0, A, B, C, m$ and n are material-dependent parameters of the corresponding models. The Hollomon, Ludwick, Swift, Voce and Ramberg-Osgood models include strain-hardening in their constitutive model, however, their accuracy

depend greatly on the material used and on the strain range they model [64], [65], [67], [68]. A significant drawback of these models is that their parameters are not commonly found in literature, limiting their use to a few materials. On the other hand, the elastic-perfectly plastic material model is widely used due to its simplicity and the accessibility of its parameters and has already been applied in analytical modeling of machining process damping [1]. While the previous models are usually used to describe tensile or compressive tests, the Johnson-Cook constitutive model can describe high strain rate deformations in machining operations [66], [69]. Its widespread use in FE simulations of machining processes has resulted in a database of parameters for various materials [12], [70]. Hence, elastic-perfectly plastic and the Johnson-Cook material constitutive models are employed in this work. The elastic-perfectly plastic material model does not take into account the effects of strain hardening and thermal softening of the material, however, it is used as a comparison with Johnson-Cook material model to identify the impact of these effects in the process damping forces.

In order to ensure a fair comparison, the yield strength (σ_Y) used in the elastic-perfectly plastic constitutive law is taken as equal to that of the Johnson-Cook constitutive law.

$$\sigma_{eng} = \begin{cases} E \varepsilon_{eng}, & \varepsilon_{eng} \leq \varepsilon_{eng,Y} \\ \sigma_{eng,Y}, & \varepsilon_{eng} > \varepsilon_{eng,Y} \end{cases} \quad (3.35)$$

with σ_{eng} and ε_{eng} the engineering stress and strain, respectively, $\varepsilon_{eng,Y}$ the engineering yield strain and E the elastic modulus. For simplicity, the engineering stress and strain will be referred to as σ and ε . When required, the true stress and strain will be specified as σ_{true} and ε_{true} .

Johnson-Cook material model describes the plastic stress resulting from plastic deformations as:

$$\sigma_{true,p} = (A + B \varepsilon_{true,p}^n) \left(1 + C \ln \left(\frac{\dot{\varepsilon}_{true}}{\dot{\varepsilon}_0} \right) \right) \left(1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right) \quad (3.36)$$

where $\sigma_{true,p}$ and $\varepsilon_{true,p}$ are the equivalent true plastic stress and strain, respectively, A is the material yield strength, B, C, n and m are additional parameters describing the strain hardening and the thermal softening, $\dot{\varepsilon}_{true}$ and $\dot{\varepsilon}_0$ are the equivalent plastic strain rate and its reference, and T, T_m and T_0 are the actual material, melting and room temperatures, respectively.

Since Eq.(3.36) does not describe the elastic region, Hooke's law is added to include the contribution of the elastic deformations. Also, the true stress and strain are converted to their engineering equivalent to describe the springs behavior using the following equations:

$$\varepsilon_{\text{true}} = \tan(1 + \varepsilon) \quad (3.37)$$

$$\sigma = \frac{\sigma_{\text{true}}}{1 + \varepsilon} \quad (3.38)$$

The engineering plastic ε_p and elastic ε_e strains can be calculated with the elastic modulus E and the yield strength σ_Y :

$$\varepsilon_e = \begin{cases} \varepsilon, & \text{if } \varepsilon \leq \varepsilon_Y \\ \varepsilon_Y, & \text{if } \varepsilon > \varepsilon_Y \end{cases} \quad (3.39)$$

$$\varepsilon_p = \begin{cases} 0, & \text{if } \varepsilon \leq \varepsilon_Y \\ \varepsilon - \varepsilon_Y, & \text{if } \varepsilon > \varepsilon_Y \end{cases} \quad (3.40)$$

where $\varepsilon_Y = \frac{\sigma_Y}{E}$ is the yield strain.

Using Eq.(3.36) to Eq.(3.40), the engineering stress can be calculated in both the elastic and plastic domain with the Johnson-Cook constitutive law:

$$\sigma = \begin{cases} E \varepsilon, & \sigma \leq \sigma_Y \\ (A + B \tan(1 + \varepsilon_p)^n) \left(1 + C \ln \left(\frac{\tan(1 + \varepsilon_p)}{\varepsilon_0} \right) \right) \left(1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right) \frac{1}{1 + \varepsilon}, & \sigma > \sigma_Y \end{cases} \quad (3.41)$$

Three parameters need to be calculated to use Eq.(3.41) and a fortiori Eq.(3.35): the strain (ε), strain rate ($\dot{\varepsilon}$) and temperature (T). The calculation of the strain and strain rate will be discussed in the following sections, while the temperature calculation will be described in CHAPTER 4.

3.3.3.2 Strain calculation

The cutting edge is defined by its position and profile. Its position is ruled by the position of SP (Figure 2.3), which is time dependent following Eqs.(3.9) and (3.10). The cutting edge profiles presented in Section 3.3.1 are mathematically described by their honed radius ($y^{\text{edge}}(x, t)$) in Eq.(3.4), and their flank face that is defined by a series of equations ($y^{\text{straight flank}}(x, t)$, $y^{\text{texture,down}}(x, t)$). To provide a general equation of the strain, those equations are referred as

$y^i(x, t)$, with i the i^{th} equation describing the cutting edge profile as illustrated in Figure 3.10 for a chamfered tool.

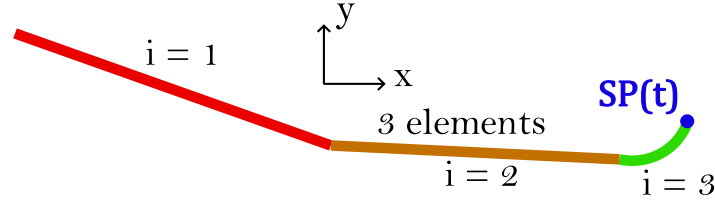


Figure 3.10: Dividing the cutting edge profile into line and circle elements

For each of the edge profiles, a representative strain, $\varepsilon_{rep}^i(x, t)$, can be calculated between the corresponding section and the wavy surface defined by Eq.(3.27) as:

$$\varepsilon_{rep}^i(x, t) = \frac{y^i(x, t) - y^{wave}(x)}{L_0(x)} \quad (3.42)$$

where $L_0(x)$ is the unloaded length of the spring at position x .

This strain is representative since it calculates the strain regardless of whether the i^{th} profile element is above or below the wavy workpiece surface (even when there are no indentations). This is solved by isolating the strain caused by the indentation of the i^{th} section in the wavy surface:

$$\varepsilon^i(x, t) = \frac{|\varepsilon_{rep}^i(x, t)| - \varepsilon_{rep}^i(x, t)}{2} \quad (3.43)$$

The strain caused by the entire cutting edge can then be calculated as:

$$\varepsilon(x, t) = \sum_i^n \varepsilon^i(x, t) H^i(x) \quad (3.44)$$

where n is the number of the segments on the flank face, $H^i(x, t)$ the Heaviside function defining the domain of definition of $y^i(x, t)$. An example can be given for the textured tool geometry presented in Section 3.3.1.2 as:

$$\begin{aligned}
\varepsilon(x, t) = & \varepsilon^{\text{edge}}(x, t)H(1 - J_x(t)) + \\
& \varepsilon^{\text{straight flank}}(x, t) \left(H(1 - T_{1,x}(t)) - H(1 - J_x(t)) + 1 - H(1 - T_{4,x}(t)) \right) + \\
& \varepsilon^{\text{texture, right}}(x, t) \left(H(1 - T_{2,x}(t)) - H(1 - T_{1,x}(t)) \right) + \\
& \varepsilon^{\text{texture, down}}(x, t) \left(H(1 - T_{3,x}(t)) - H(1 - T_{2,x}(t)) \right) + \\
& \varepsilon^{\text{texture, left}}(x, t) \left(H(1 - T_{4,x}(t)) - H(1 - T_{3,x}(t)) \right)
\end{aligned} \tag{3.45}$$

which analytically and fully define the strains of the cutting edge interference with the workpiece material.

3.3.3.3 Strain-rate calculation

The strain-rate is only necessary for the Johnson-Cook constitutive law (Eq.(3.36)). By definition, the strain rate is obtained by derivative of the strain over time:

$$\dot{\varepsilon}(x, t) = \frac{\partial \varepsilon}{\partial t}(x, t) \tag{3.46}$$

Inserting Eq.(3.43) into Eq.(3.46) yields:

$$\varepsilon^i(x, t) = \frac{\dot{\varepsilon}_{\text{rep}}^i(x, t)}{2} \left[\frac{\varepsilon_{\text{rep}}^i(x, t)}{|\varepsilon_{\text{rep}}^i(x, t)|} - 1 \right] \tag{3.47}$$

$$\dot{\varepsilon}_{\text{rep}}^i(x, t) = \frac{\dot{y}^i(x, t)}{L_0(x)} \tag{3.48}$$

Line and circular elements are used to describe the flank and edge geometries in Section 3.3.1 as:

$$y^{\text{line}}(x, t) = \tan(\gamma) (P_x(t) - x) + P_y(t) \tag{3.49}$$

$$y^{\text{circle}}(x, t) = \sqrt{r^2 - (x - O_x(t))^2} + O_y(t) \tag{3.50}$$

where $P(t) = (P_x(t), P_y(t))$ is a representative point on the given line, γ is an angle defining the line's slope, r and $O(t) = (O_x(t), O_y(t))$ are the circle's radius and center point. Time derivatives of Eqs.(3.49) and (3.50) yields:

$$\dot{y}^{\text{line}}(x, t) = \tan(\gamma) \dot{P}_x(t) + \dot{P}_y(t) \quad (3.51)$$

$$\dot{y}^{\text{circle}}(x, t) = -\frac{\dot{O}_x(t)(x - O_x(t))}{\sqrt{r^2 - (x - O_x(t))^2}} + \dot{O}_y(t) \quad (3.52)$$

All of the points on the cutting edge profile are defined relative to the reference point SP , which simplifies their time derivative to the derivative of point SP defined by Eq.(3.9) and Eq.(3.10):

$$S\dot{P}_x(t) = V \quad (3.53)$$

$$S\dot{P}_y(t) = \begin{cases} \frac{2A_0\pi V}{\lambda} \cos\left(\frac{2\pi V t}{\lambda}\right), & \text{along vibratory trajectory} \\ 0, & \text{without vibrations} \end{cases} \quad (3.54)$$

3.4 Calculation of process damping forces

The material constitutive law coupled with the Winkler foundation provides the contact stress for each spring. A constraint factor representing the yielding of the deformed workpiece is used to calculate the contact pressure from the contact stress. The total radial contact force is then calculated by integration over the contact area. The static component of the contact force is subtracted from the total contact force to obtain the dynamic, process damping force. The process damping force is linearized by an equivalent viscous damping coefficient dissipating the same amount of energy over one full vibration cycle and used in the machining stability predictions.

3.4.1 Determination of the constraint factor

During an elastic-plastic contact, the plastic zone first appears beneath the contact surface [39]. An increase in the contact force (and thus of contact pressure) expands the subsurface plastic zone until it reaches the contact surface. This causes the contact pressure to exceed the material's yield flow during plastic deformations. The location of the first yield zone and the relation between the contact pressure and the yield flow is related to the contact geometry, the material properties, and the indentation depth [39], [71]. A coefficient describes the relationship between the plastic contact pressure (p_{plastic}) and the calculated plastic flow stress (σ_p) described in Eq.(3.41). This coefficient is commonly called the constraint factor (c) and is usually used to describe the material's hardness indentation [72].

$$p_{plastic} = c\sigma_p \quad (3.55)$$

A first coefficient value of $c = 3.3$ was proposed by Tabor [73] for Vickers indentation of mild steel and copper. Several authors proposed constraint factor values for different material and indentation type, all of which were contained within a range of 1 to 3.3 [74]. However, it was shown that the coefficient could increase well beyond the values of 3, especially for rough surfaces [75]. The majority of these values could be obtained from experimental or numerical indentation tests of pyramid or ball indenters with small strain rate as compared to those encountered in machining. Alternatively, a constraint value obtained from slip-line theory has already been used to describe the relation between the plastic contact pressure and the flow stress in the context of machining [1]. This coefficient describes the relation between the flow stress and the plastic contact pressure [1], [39], [76], [77]:

$$c = (2.57 - 0.023\alpha), \alpha \in [0, 30]^\circ \quad (3.56)$$

where α is the clearance angle of the cutting edge. Eq.(3.56) is adopted in the current work to link the contact pressure applied on the springs during plastic deformation, to the flow stress obtained from Eq.(3.41).

Springs in the elastic deformation regime are not used in the process damping force calculation since they do not contribute to the energy dissipation. The status (elastic or plastic) of a spring deformation is obtained from its calculated strain Eq.(3.44) as:

$$\text{status} = \begin{cases} \text{elastic,} & \text{if } \varepsilon \leq \varepsilon_Y \\ \text{plastic,} & \text{if } \varepsilon > \varepsilon_Y \end{cases} \quad (3.57)$$

3.4.2 Calculation of the dynamic contact (process damping) force

The plastic contact pressure of the springs is calculated for each cutting edge position, i.e. time step, and the total radial plastic contact force per unit contact length $F_{\text{total},r}$ is evaluated by integrating the contact pressure over the contact area. The integral is computed by a Riemann sum due to the discontinuity of the springs:

$$F_{\text{total},r} = \lim_{n \rightarrow \infty} \sum_{i=1}^{n_{\text{springs}}} p_{\text{plastic}}(x_i) \Delta x_i \quad (3.58)$$

with $p_{\text{plastic}}(x_i)$ is the plastic contact pressure of the i^{th} spring, n_{springs} is the number of springs undergoing plastic deformation, and Δx_i is the spatial discretization step along x-direction as shown in Figure 3.6.

The static ploughing force occurring in vibration-free conditions does not contribute to the machining dynamics and the process damping [1], [7], [11]. Thus, the static radial plastic contact force $F_{\text{static},r}$ is obtained by removing $y^{\text{wave}}(x)$ from the Eqs.(3.31) and (3.42). The dynamic contact, process damping $F_{\text{pd},r}(x)$, force is then obtained by removing the static component of the total contact force:

$$F_{\text{dynamic},r}(x) = F_{\text{total},r}(x) - F_{\text{static},r}(x) \quad (3.59)$$

It must be noted that the forces in Eq.(3.59) are expressed in N/m . The actual force in N can be obtained by multiplying $F_{\text{dynamic},r}$ by the width of cut during an orthogonal cutting, or by the discretized height in milling operations as illustrated in Figure 3.11.

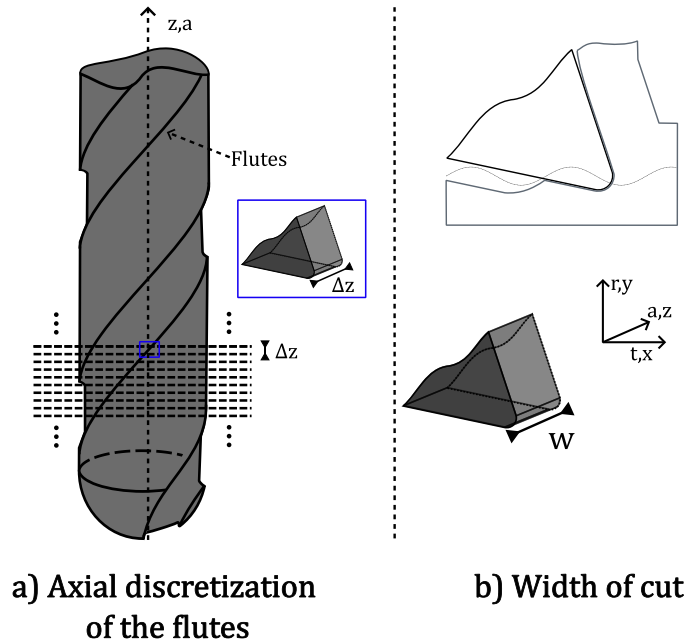


Figure 3.11: Axial dimension of the cutting edge a) milling operation b) orthogonal cutting

3.4.3 Linearization of the process damping force

The process damping force only occurs on half the vibratory wavelength [1]. The ploughed region is equal or less than the statically ploughed region and process damping is not active while the cutting edge moves upward along the wave. Hence, process damping force only takes place during the downward motion of the cutting edge along the wave $\left[\frac{\lambda}{4}, \frac{3\lambda}{4}\right]$ as demonstrated in Figure 3.12, which makes it a non-linear force.

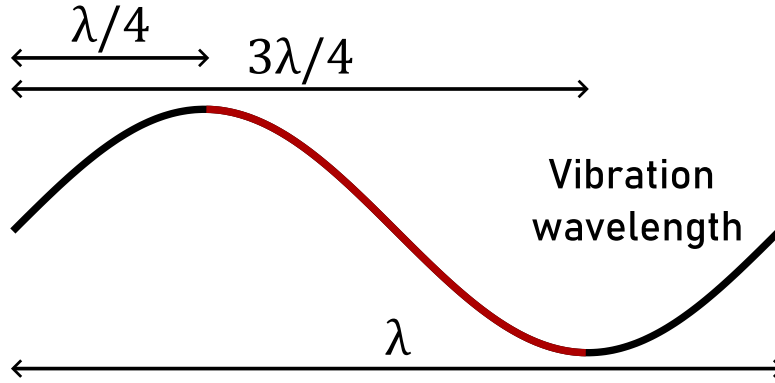


Figure 3.12: Process damping zone (shown in red)

In order to be used in the frequency domain machining stability prediction models, the process damping force is linearized by representing it by an equivalent viscous damper [1], [15], [21] dissipating the same amount of energy over one full wavelength [16], [18], [22]. If E_{diss} is the dissipated energy by the process damping force along half the wavelength $\left[\frac{\lambda}{4}, \frac{3\lambda}{4}\right]$, and E_{eq} is the dissipated energy by an equivalent viscous damper over the entire wavelength, then Eq.(2.3) and Eq.(2.4) are set equal to calculate the equivalent viscous damping coefficient, c_{eq} :

$$E_{diss} = E_{eq} \quad (3.60)$$

$$\int_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}} F_{pd,r}(x) dy^{wave} = \int_0^{\lambda} c_{eq} \frac{dy^{wave}}{dt}(x) dy^{wave}(x) \quad (3.61)$$

Substitution of Riemann sum into E_{diss} (Eq.(2.3)) yields:

$$E_{diss} = \lim_{n \rightarrow \infty} \frac{A_0 2\pi}{\lambda} \sum_{i=1}^{n_{springs}} F_{pd,r}(x_i) \cos \frac{2\pi x_i}{\lambda} \Delta x_i \quad (3.62)$$

Also, the equivalent viscous damper integral can be simplified [1] as:

$$E_{eq} = \frac{2(A_0\pi)^2}{\lambda} c_{eq} V_c \quad (3.63)$$

Using Eqs.(3.60)-(3.63), the equivalent viscous damping coefficient (c_{eq}) can be obtained as:

$$c_{eq} = \frac{\lim_{n \rightarrow \infty} \sum_{i=1}^{n_{springs}} F_{pd,r}(x_i) \cos \frac{2\pi x_i}{\lambda} \Delta x_i}{A_0 V_c \pi} \quad (3.64)$$

3.5 Summary

In this chapter, a new analytical process damping model is proposed to remove geometric limitations on the cutting edge profile. This is achieved by defining a non-linear Winkler foundation to describe the contact between the honed edge and flank face of the cutting edge, and the work material. This foundation discretizes the workpiece surface by a series of one-dimensional parallel springs for any arbitrary cutting edge geometry with the limitation of ignored. The simplicity of the model brings several limitations, notably the removal of any shear interaction between the springs, failing to describe the continuity of the material. Still, the Winkler model is used to satisfy the necessity to describe the contact forces for an arbitrary cutting edge geometry.

Common non-standard cutting edge profiles have been mathematically described to be used with the Winkler foundation based process damping model. A cavity model describes the depth of deformation resulting from the contact and has been used to define the unloaded length of the springs. Two material constitutive models have been selected to describe the material behavior outside of the elastic region. The time and position dependant strain and strain rate are calculated analytically and used to calculate the contact stress. A constraint factor is used to link the contact stress in the plastic region to the plastic contact pressure. The process damping force is calculated from the contact pressure and it is linearized using an equivalent viscous coefficient to be used in the stability predictions.

CHAPTER 4 WORKPIECE TEMPERATURE PREDICTION

4.1 Introduction

As mentioned in Section 3.3.3.1, workpiece temperature is required to use the Johnson-Cook constitutive law (Eq.(3.41)), and to use temperature dependent mechanical properties of the work material (the elastic modulus E and yield strength σ_Y) for the elastic perfectly plastic material model. Especially when high deformations and forces are involved, temperature is an important factor to take into consideration [78].

In this chapter, an existing analytical workpiece temperature prediction model is improved and integrated into the process damping prediction model presented in CHAPTER 3. First, Section 4.2 provides a brief literature review on analytical temperature prediction. The original form of the adopted temperature prediction model is detailed in Section 4.3, along with its limitations. Section 4.4 presents the improvements made to the model, while its integration into the developed process damping model is described in Section 4.5.

4.2 Thermal modeling of machining in its original form

Temperature prediction and measurement have been the subject of extensive research in machining in the last two decades due to its effect on the surface quality, tool life and built-up edge effect [79], [80]. Several experimental methods [81] using thermal cameras [82] or thermocouples [83], [84] have been developed. However detailed temperature measurements require complex and expensive set-ups [79], [80]. Numerical prediction using FE and FD can provide detailed temperature results with good accuracy [85], [86], [87] and is therefore the most widespread prediction method in literature. Halfway between experimental and numerical methods, first analytical models were developed in the early 1950s by Jaeger, Rosenthal, Chao and Trigger [88], [89], [90] and have been the basis of most of the subsequent analytical models. Komanduri and Hou [91], [92], [93] refined these models and predicted the temperature rise in orthogonal cutting in the two deformation zones (shear zone and chip-rake interface) as illustrated in Figure 4.1. Some significant improvements have been lately proposed:

- Chou and Song [94] added the contribution of the tertiary deformation zone in Figure 4.1 (flank rubbing) to the heat generation. This was also realised later by Yan et. al. [95] using an empirical heat partition from infrared thermal camera in this zone.

- Huang and Yang [96] introduced heating time, allowing to take into account the workpiece's cooling for intermittent cutting.

The majority of analytical models is only applicable in orthogonal machining, but more recent studies are capable of predicting the cutting temperature during milling operations [97], [98].

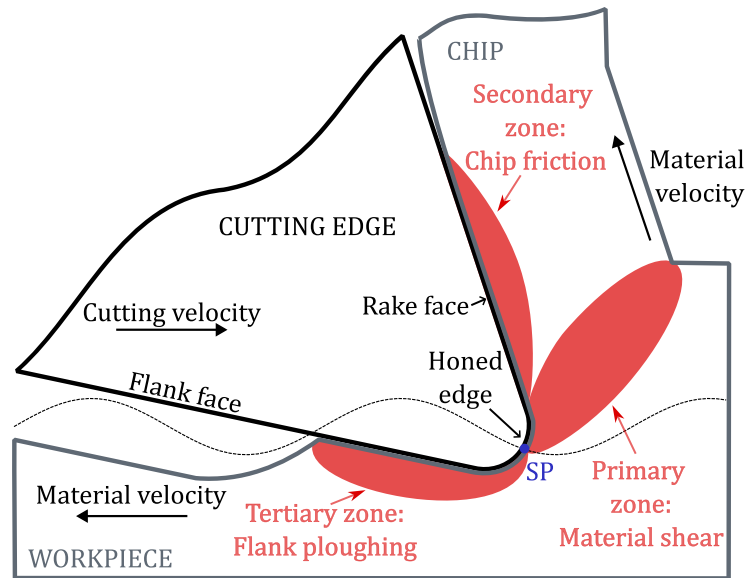


Figure 4.1: Deformation zones in machining

The improved workpiece temperature prediction model presented in this chapter is based upon the model proposed by Komanduri and Hou [91], [92], [93] to predict the temperature beneath the flank face and the hone radius of the cutting edge for the process damping phenomenon.

4.3 Temperature rise model description and limitations

The original form of the temperature model described in this section and used in [94], [95], [96] is valid under the following assumptions [99], [100], [101]:

- The cutting is 2D (orthogonal cutting),
- Temperature distribution is in steady-state,
- All plastic deformations and frictions are converted into heat without any loss,
- No heat loss is considered by convection (free surfaces are adiabatic),
- The tool is sharp (no honed edge) and cuts without any vibrations,

- The material is homogenous.

4.3.1 Heat source definition

Two deformation zones are considered in the majority of the studies, which are the rake face-chip interface and the shear zone. The chip and the workpiece are considered as two separate bodies with the shear plane as interface [91]. The third deformation (flank ploughing) zone is considered for chamfered tools [95], [96]. Each deformation zone is an interface between two bodies (including the shear plane as the workpiece/chip interface) is modeled by two heat sources (one in each body) of specific heat intensity and length as shown in Figure 4.2. Two type of heat sources are used depending on the body it describes: a moving line for the moving material on the workpiece and chip side, and a stationary rectangle on the tool side [93], as displayed in Figure 4.2.

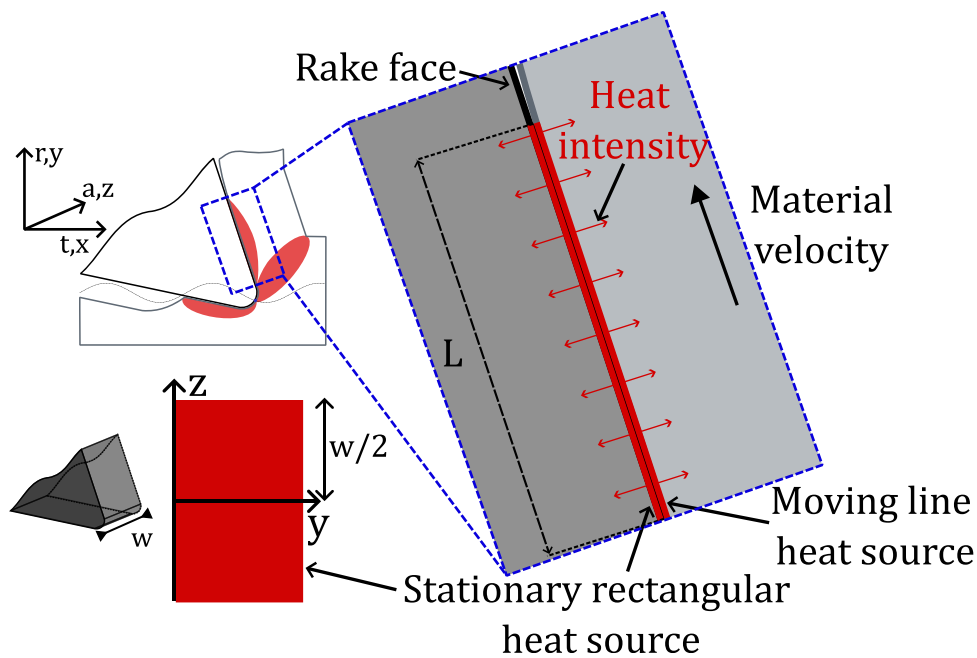


Figure 4.2: Example of heat sources at the chip-rake interface

4.3.2 Temperature rise calculation

The temperature distribution around the heat sources is ruled by Fourier's heat transfer equation [89], [99], [102]:

$$\frac{\partial \theta}{\partial t} = a \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (4.1)$$

with

$$a = \frac{\kappa}{\rho c_p} \quad (4.2)$$

where $\theta = T - T_{init}$ is the temperature rise from an initial temperature T_{init} , a is the thermal diffusivity of the material, defined by its thermal conductivity κ , density ρ , and specific heat c_p .

Under the assumption of steady-state, and the geometries of the heat sources, two equations describing the temperature rise θ at a point of interest $M(x, y, z)$ can be deduced for a moving tilted line and a stationary rectangular heat source [89], [99], [102]:

$$\theta(x, y) = \int_0^L \frac{q(l_i)}{2\pi\kappa} e^{-(x-l_i \sin \gamma)V_h/2a} K_0 \left(\frac{V_h \sqrt{(x-x')^2 + (y-y')^2}}{2a} \right) dl_i \quad (4.3)$$

for a line heat source, and

$$\theta(x, y, z) = \int_0^L \frac{q(l_i)}{2\pi\kappa} \int_{-w/2}^{w/2} \frac{dz_i dl_i}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \quad (4.4)$$

for a rectangular heat source. L is the length of the heat sources, w is their width (for the rectangular heat sources), γ is their tilt angle, V_h is the material flow velocity at the interface (for the moving heat source), $q(l_i)$ is the heat intensity at l_i , κ and a are the material's thermal conductivity and diffusivity, respectively, x', y', z' are the cartesian coordinates of the point at l_i in the line heat source and $K_0(x)$ is the Bessel function of second kind of order zero.

A general definition of heat intensity is [99], [100], [103]:

$$q = \frac{FV_h}{wL} \quad (4.5)$$

with F the force occurring at the studied interface, V_h the material flow velocity at this interface, w the width of cut and L the length of this interface, i.e. of the heat source. The heat intensity is usually defined as constant for the shear and flank interfaces, but has been modified as non-uniform to represent the stick and slide behavior of the chip on the rake face of the tool [99], [104]:

$$q_{f,r}(l_i) = \begin{cases} q_r \frac{2}{1+r}, & 0 < l_i < rL_c \\ q_r \frac{2(L_c - l_i)}{(1-r^2)L_c}, & r_s L_c < l_i < L_c \end{cases} \quad (4.6)$$

where q_r is the heat intensity of the rake-chip interface defined by Eq.(4.5), l_i is the position on the heat source, L_c is the chip-rake contact length and also length of the heat source, r_s defines the stick and slide portion of the interface and is usually taken as 0 or 0.5.

It has been shown [91], [105] that surfaces in contact with the air (free surfaces) can be considered as adiabatic in machining. The method of images is used to simplify the boundary conditions by introducing an imaginary heat source defined in Figure 4.3, symmetric to the original one, with the adiabatic surface as symmetry plane [91], [99], [105].

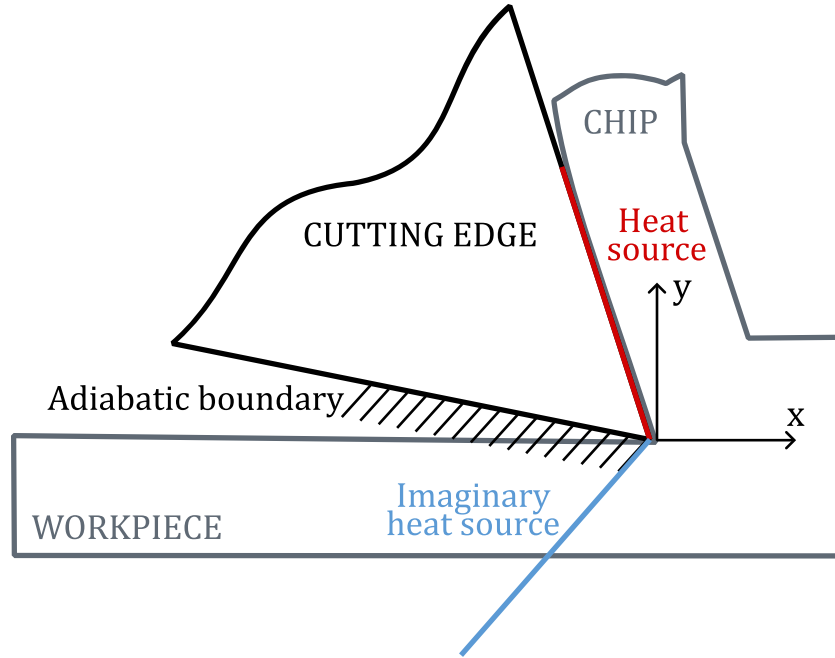


Figure 4.3: Example of an imaginary heat source in machining

Then Eq.(4.3) and Eq.(4.4) become (example of one imaginary heat source for a general case):

$$\theta(x, y) = \int_0^L \frac{q(l_i)}{2\pi\kappa} e^{-R_v V_h / 2a} \left[K_0 \left(\frac{V_h R}{2a} \right) + K_0 \left(\frac{V_h R'}{2a} \right) \right] dl_i \quad (4.7)$$

$$\theta(x, y, z) = \int_0^L \frac{q(l_i)}{2\pi\kappa} \int_{-w/2}^{w/2} \left(\frac{1}{R} + \frac{1}{R'} \right) dz_i dl_i \quad (4.8)$$

where R and R' are the distances between the point of interest $M(x, y, z)$ and the position at l_i in the original and imaginary heat sources, respectively, R_v is the projection of R in the direction of the heat source motion. A visual representation of the geometric parameters of Eq.(4.7) is given in Figure 4.4, which can also represent Eq.(4.8) by adding a third dimension, z .

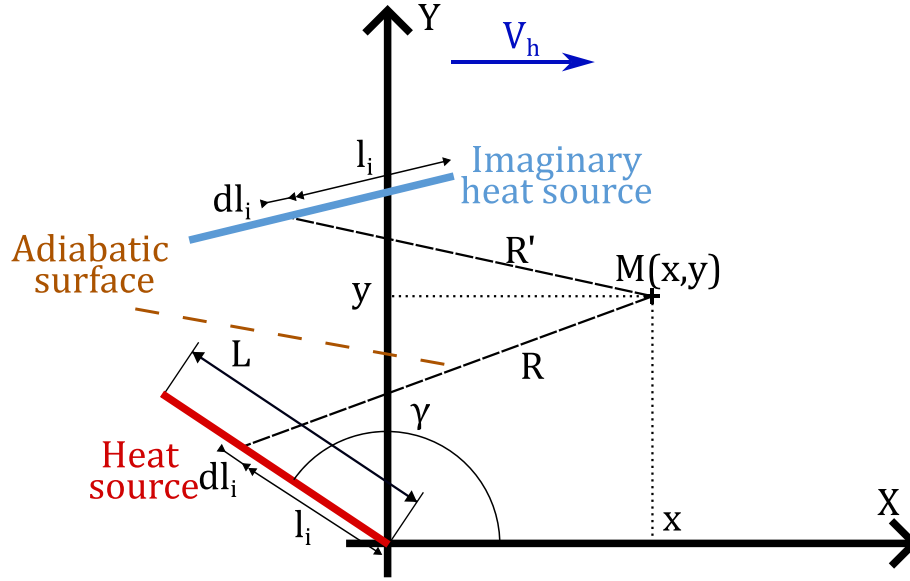


Figure 4.4: Visual representation of geometrical parameters of temperature calculation using a moving line heat source

4.3.3 Heat partition calculation

When two heat sources are considered at both side of an interface, the temperature at that interface can be discontinuous due to the difference of heat source geometry (moving line and stationary rectangular on the rake-chip interface) and due to the contribution of other heat sources. This discontinuity should cause heat flow at this interface, breaking the assumption of steady-state temperature [99]. A heat partition is therefore necessary to equalize the temperature at all contacting interfaces to ensure steady-state conditions. While some constant heat partition values were first considered, non-uniform distribution were quickly adopted due to the complexity of the temperature field [91]. Integration of the heat partition in Eq.(4.7) and Eq.(4.8) yields:

$$\theta(x, y) = \int_0^L B(l_i) \frac{q(l_i)}{2\pi\kappa} e^{-R_v V_{h/2a}} \left[K_0 \left(\frac{V_h R}{2a} \right) + K_0 \left(\frac{V_h R'}{2a} \right) \right] dl_i \quad (4.9)$$

$$\theta(x, y, z) = \int_0^L B(l_i) \frac{q(l_i)}{2\pi\kappa} \int_{-w/2}^{w/2} \left(\frac{1}{R} + \frac{1}{R'} \right) dz_i dl_i \quad (4.10)$$

where $B(l_i)$ is the heat partition of the heat source at l_i .

Non-uniform heat partition has been adopted by numerous researchers [99], [100], [101], [104], [105], [106], [107], and two main analytical methods are used to define it. The first [93] one is formulated as [92]:

$$B_{q_1}(l_i) = B_{0,q_1} - \Delta B + 2\Delta B \left(\frac{l_i}{L} \right)^m + C\Delta B \left(\frac{l_i}{L} \right)^k \quad (4.11)$$

$$B_{q_2}(l_i) = B_{0,q_2} + \Delta B - 2\Delta B \left(\frac{l_i}{L} \right)^m - C\Delta B \left(\frac{l_i}{L} \right)^k \quad (4.12)$$

where q_1 and q_2 are the two sides of an interface (e.g., q_1 = chip and q_2 = tool), $B_{q_1}(l_i)$ and $B_{q_2}(l_i)$ are the position-dependant heat partitions of the heat sources on each side, l_i is the position on the heat sources of length L , and $B_{0,q_1}, B_{0,q_2}, \Delta B, C, m, k$ are constants that must be chosen to match the temperatures on both side of the interface and calculated [104], [108] via an optimization method.

Huang and Liang [107] proposed a second method that involves discretizing the heat sources into infinitesimal elements. In this method, an arbitrary heat partition distribution is chosen at all interfaces, so that the sum equals 1, for example, for the rake-chip interface, the heat partition will be B_f on the chip side and $1 - B_f$ on the tool side. The non-uniform heat partition is then assumed to be constant within each infinitesimal element. By ensuring that the temperature equality at each infinitesimal element (taking into account the contributions from all other heat sources contributing to that temperature), a linear system of equations is created in matrix form. Solving this system provides the heat partition for each infinitesimal element, thereby recreating the non-uniform heat distribution. This method is the most common in literature [99], [100], [101], [107]. For example, for the case represented in Figure 4.5, at the rake-chip interface, on the chip side, the temperature from both the tool side and chip side must be equal. That brings:

$$\theta_{cs}(l_i) + \theta_{cf}(l_i) = \theta_{tp}(l_i) + \theta_{tf}(l_i) \quad (4.13)$$

where $\theta_{bq}(l_i)$ being the temperature rise at l_i on the rake-chip interface, on the side $b \in \{c, t\}$ from the heat source at the interface $q \in \{p, f, s\}$, c and t refer to the chip and the tool, respectively, and p, f and s refer to the ploughed, friction (chip-rake), and shear interfaces. After discretizing all the heat sources, Eq.(4.13) becomes:

$$B_s \theta'_{cs}(l_i) + B_f \theta'_{cf}(l_i) = (1 - B_p) \theta'_{tp}(l_i) + (1 - B_f) \theta'_{tf}(l_i) \quad (4.14)$$

where $\theta'_{bq}(l_i)$ being the temperature rise at l_i on the rake-chip interface, on the side $b \in \{c, t\}$ from the infinitesimal heat source element dl_i at the interface $q \in \{p, f, s\}$, B_q and $(1 - B_q)$ refer to the heat partition on both side of the interface q .

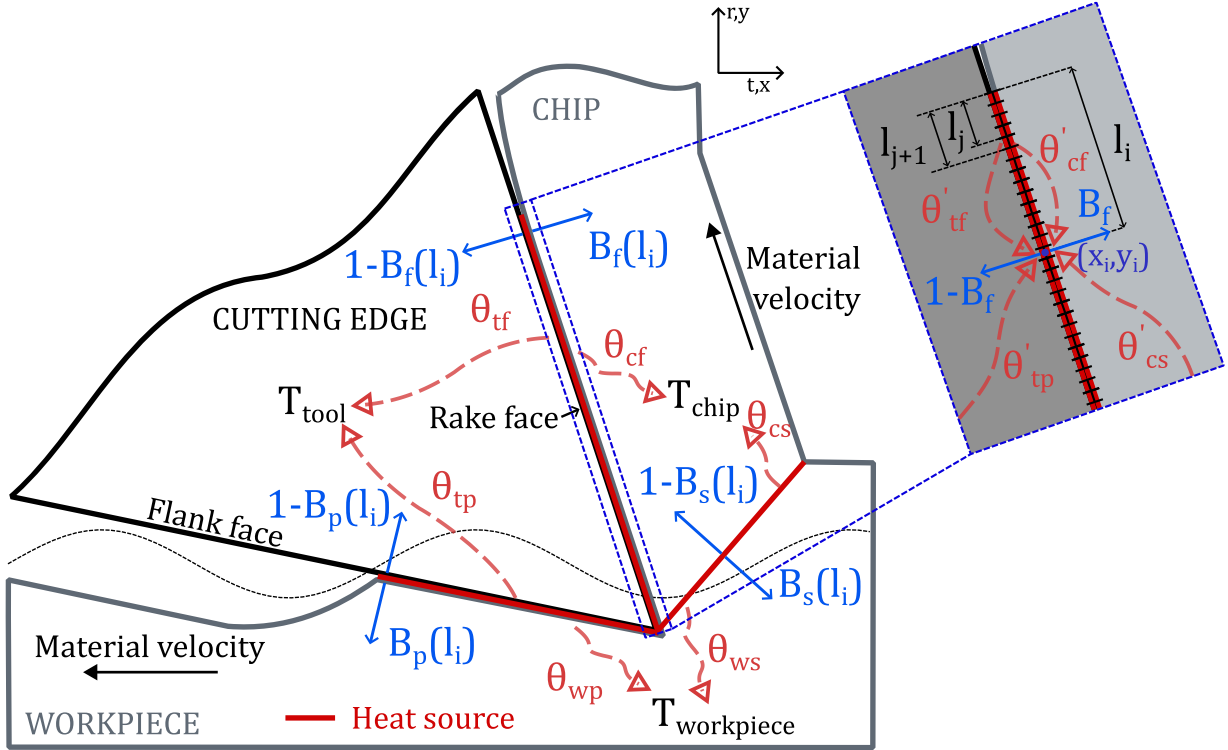


Figure 4.5: Example of heat distribution on the rake-chip interface of a sharp tool

An example of $\theta'_{bq}(l_i)$ is given here, with (x_i, y_i) the coordinates of the point at l_i in the rake-chip interface, and $[l_j, l_{j+1}]$ the infinitesimal element of the heat source at the interface q :

$$\theta'_{bq}(x_i, y_i) = B_q \int_{l_j}^{l_{j+1}} \frac{q(l_i)}{2\pi\kappa} e^{-R_v V_h / 2a} \left[K_0 \left(\frac{V_h R}{2a} \right) + K_0 \left(\frac{V_h R'}{2a} \right) \right] dl_i \quad (4.15)$$

Similar example can be given for a stationary rectangular heat source by adding the third dimension as:

$$\theta'_{bq}(x_i, y_i, z_i) = B_q \int_{l_j}^{l_{j+1}} \frac{q(l_i)}{2\pi\kappa} \int_{-w/2}^{w/2} \left(\frac{1}{R} + \frac{1}{R'} \right) dz_i dl_i \quad (4.16)$$

Isolating the constant heat partitions yields:

$$B_p \theta'_{tp}(l_i) + B_s \theta'_{cs}(l_i) + B_f \left(\theta'_{cf}(l_i) + \theta_{tf}(l_i) \right) = \theta'_{tp}(l_i) + \theta'_{tf}(l_i) \quad (4.17)$$

Finally, the system can be put in matrix form as:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}_{3n \times 3n}, B = \begin{bmatrix} B_p \\ B_f \\ B_s \end{bmatrix}_{3n \times 1}, C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}_{3n \times 1}$$

where $A_{IJ}(i, j)$ corresponds to the temperature rise at l_i in the interface I due to the heat source element $[l_j, l_{j+1}]$ in the heat source J and its corresponding equations. $C_I(i)$ corresponds to temperature rise at l_i in the interface I of all the heat sources being the constant terms of the linear equation. $B_I(i)$ corresponds to the heat partition at l_i in the interface I of the two corresponding heat sources and n corresponds to the number of discretized heat source elements.

The calculation of the heat partition is crucial to have an accurate prediction of workpiece temperature rise as it introduces the influence of all heat sources on another. It can be noted that, as the number of heat source increases as well as their discretization, the condition number of the matrix A goes up drastically reaching values up to 10^{20} . With such a condition number, the results are prone to errors. Several verifications have been implemented to ensure accuracy of the results, that includes checking of the consistency of the system and the singularity of the matrix, equilibrating it and tracking of the residual error when solving the linear system as exemplified in Figure 4.6.

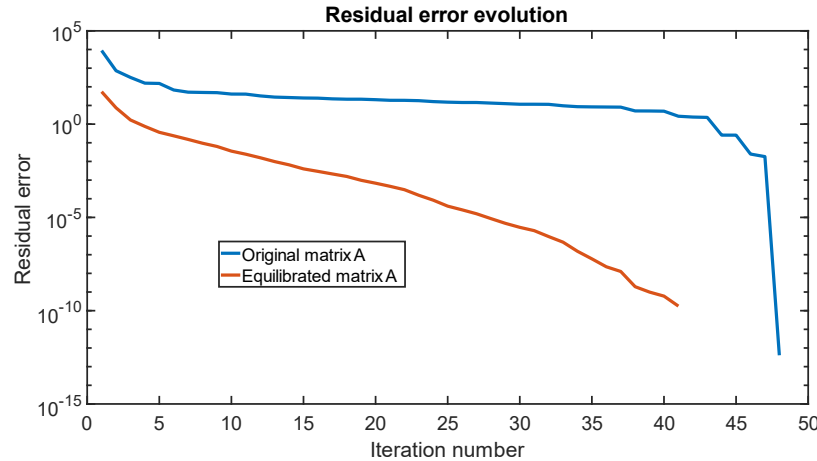


Figure 4.6: Example of residual error tracking for un-equilibrated and equilibrated matrix A

4.3.4 Limitations of the existing model

Though there is a great deal of work in literature, the existing model has only been used for sharp tools without hone radius [93], [96], [99]. However, estimation of the temperature under the hone radius region is necessary for process damping calculations since it takes part in ploughing. A method to add the contribution of the honed edge in the temperature prediction is proposed in this study.

Also, more general temperature prediction equations are introduced using a global coordinate system rather than a separate one for each heat source, incorporating the variation in heat source velocity direction.

Furthermore, no published literature has included varying flank-workpiece contact in the presence of vibrations. The cases that included the flank face were exclusively due to the use of a chamfered tool with a 0° clearance angle and no vibration [94], [95]. On the other hand, the only vibration case found in the literature did not consider the contributions of the flank face and the honed edge [105]. A method to include the varying flank-workpiece contact to the temperature prediction is introduced as well.

4.4 Improvements to the existing workpiece temperature prediction model

4.4.1 Modeling of the honed edge

Although the circular honed edge impacts the ploughing forces, there are no studies having included its effect in the analytical prediction of workpiece temperature during machining. This gap has been filled with two proposed modifications as follows:

4.4.1.1 Linear approximation of the honed edge

A first approach consists of approximating the circular honed edge by a set of lines to be able to use Eq.(4.9) and Eq.(4.10). The honed edge is divided into two main zones: the honed edge below the separation point (SP) and the portion above it to differentiate the material velocity and direction in both regions. These two regions are respectively discretized into n_d and n_u elements as depicted in Figure 4.7.

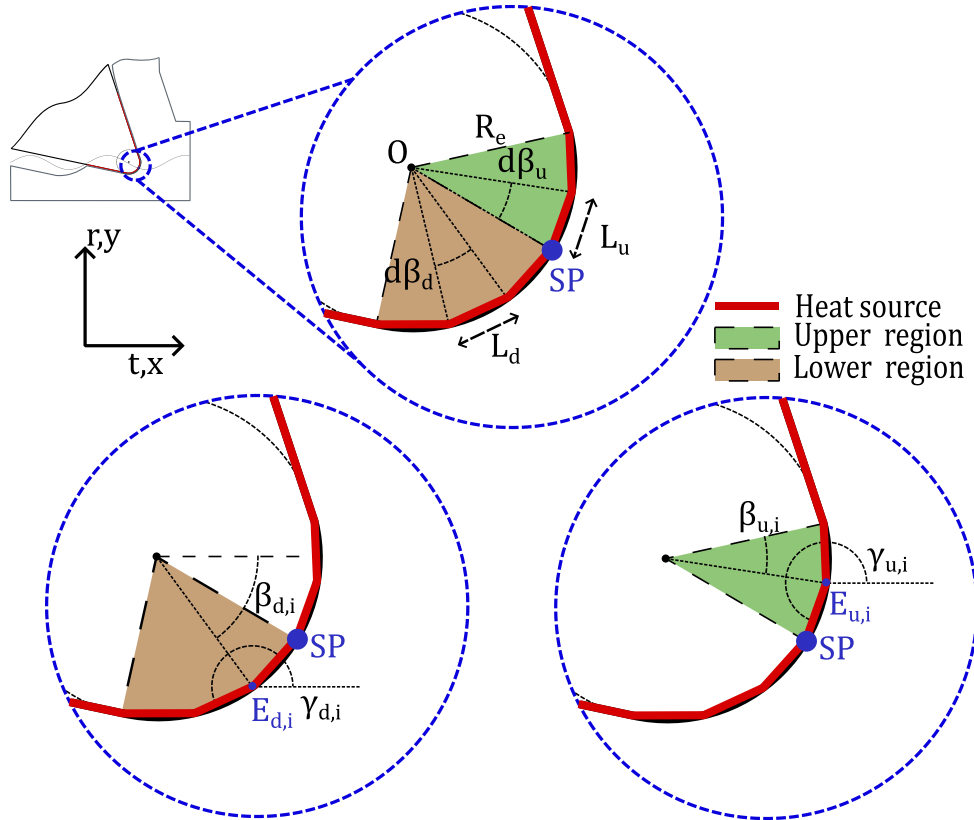


Figure 4.7: Discretization of the honed edge

In order to use Eq.(4.9) and Eq.(4.10), the length and angle of each tilted line are defined in the upper and lower regions. The i^{th} tilted line in the lower region is defined by $\gamma_{d,i}$ and L_d as:

$$\gamma_{d,i} = \frac{3\pi}{2} - \beta_{d,i} - \frac{d\beta_d}{2} \quad (4.18)$$

$$L_d = \sqrt{(-R_e \sin(\beta_{d,i} + d\beta_d) + R_e \sin(\beta_{d,i}))^2 + (R_e \cos(\beta_{d,i} + d\beta_d) - R_e \cos(\beta_{d,i}))^2} \quad (4.19)$$

with

$$d\beta_d = \frac{\chi + \alpha}{n_d} \quad (4.20)$$

$$\beta_{d,i} = \left(\frac{\pi}{2} - \chi\right) + d\beta_d(i - 1) \quad (4.21)$$

$$E_{d,i} = (R_e \cos \beta_{d,i} + O_x, -R_e \sin \beta_{d,i} + O_y) \quad (4.22)$$

where α and χ are the clearance and separation angles and R_e is the radius of the honed edge.

The i^{th} tilted line in the upper region is similarly defined by $\gamma_{u,i}$ and L_u . Eqs.(4.18), (4.19) and (4.22) can be reused, and Eqs.(4.20)-(4.21) become:

$$d\beta_u = \frac{\pi/2 - \chi + \alpha_r}{n_u} \quad (4.23)$$

$$\beta_{u,i} = -\alpha_r + d\beta_u(i - 1) \quad (4.24)$$

It must be noted that a sufficient number of linear heat sources should be used to approximate the honed edge, else use of an insufficient number of linear heat sources (< 3) might cause a singularity in the temperature rise as exemplified in Figure 4.8.

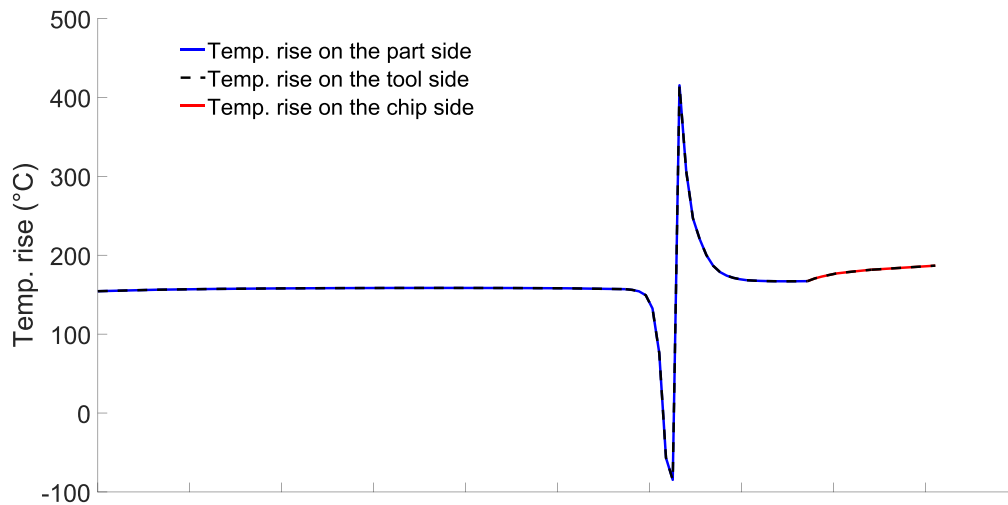


Figure 4.8: Visualization of the singularity in workpiece temperature rise calculations due to insufficient number of linear heat sources around the honed edge

Figure 4.8 displays side by side the temperature rise in the flank, lower honed edge and upper honed edge regions. Though the exact reason for the presence of this singularity could not be identified, it is attributed to the numerical errors in computations. Hence, the problem has been addressed by increasing the number heat sources in the circular region until disappearance of the singularity as seen in Figure 4.9.

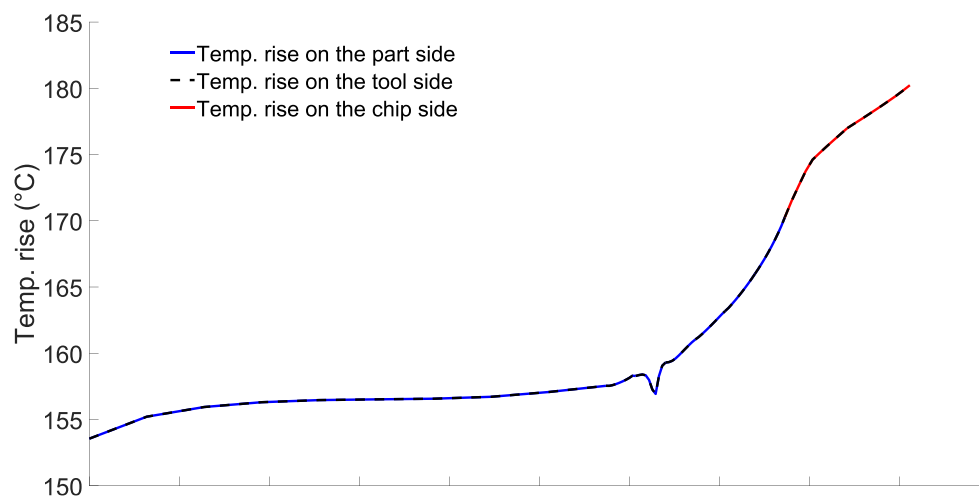


Figure 4.9: Visualization of the diminution of the temperature singularity

4.4.1.2 Circular heat source for the honed edge

Alternatively, a moving circular heat source is proposed instead of approximating the honed edge with multiple linear heat sources, which are prone to have temperature singularity problem (Figure 4.8) as shown in Figure 4.10.

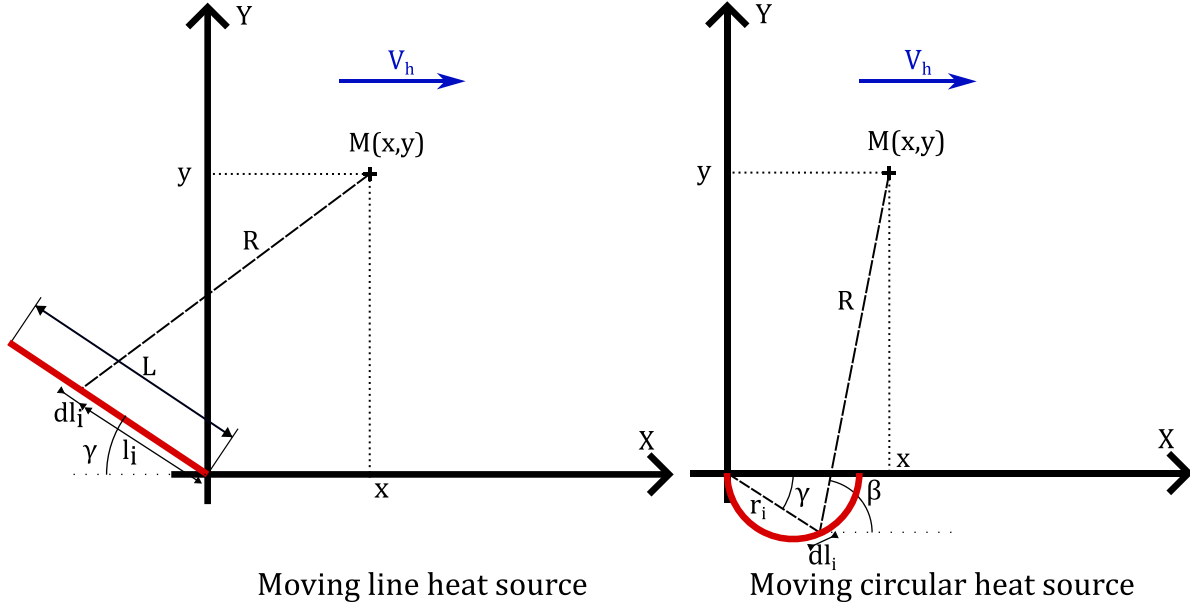


Figure 4.10: Comparison of moving line heat source and the introduced moving circular heat source

The following equation is derived to calculate the temperature rise at a point of interest $M(x, y)$ due to a moving half-circle heat source.

$$\theta(x, y) = \int_0^{\pi/2} \frac{q(\gamma)}{2\pi\kappa} e^{-R_v(\gamma)V_h/2a} K_0\left(\frac{V_h R(\gamma)}{2a}\right) 2R_e d\gamma \quad (4.25)$$

with

$$R(\gamma) = \sqrt{(y + r_i(\gamma) \sin \gamma)^2 + (x - r_i(\gamma) \cos \gamma)^2} \quad (4.26)$$

$$r_i(\gamma) = \frac{R_e}{(1 + \tan^2 \gamma) \cos \gamma} \quad (4.27)$$

$$\beta(\gamma) = \tan^{-1} \frac{y + r_i(\gamma) \sin \gamma}{x - r_i(\gamma) \cos \gamma} \quad (4.28)$$

$$R_v(\gamma) = R(\gamma)\beta(\gamma) \quad (4.29)$$

where V_h is the heat source velocity, $q(\gamma)$ is the heat intensity at γ , κ and α are the work material's thermal conductivity and diffusivity, respectively and R_e is the honed edge radius.

The results of temperature prediction of an arbitrary half circle heat source are compared against those of the linearly approximated honed edge heat source in Figure 4.11. As seen both introduced methods approached the same results.

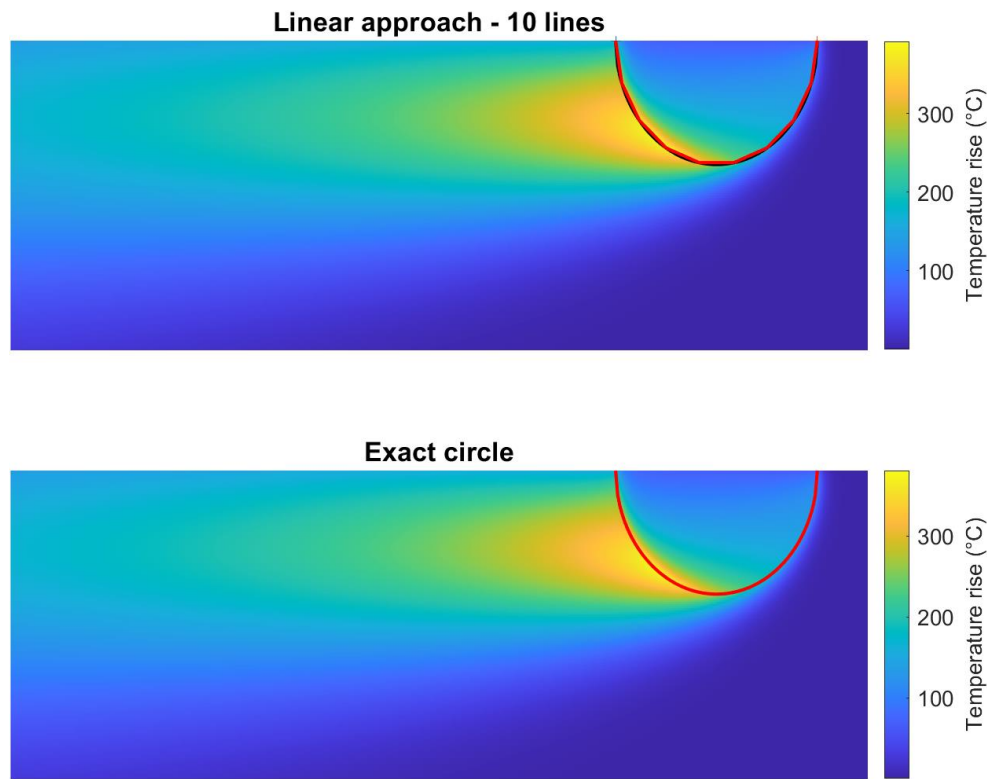


Figure 4.11: Method comparison - temperature field calculation of half circle heat source

This equation is successfully developed late in the research process. Hence it has not been adopted in the following work but is mentioned here for its potential to replace the linear approximation of the circular honed edge since it proposes as simpler, faster, and more stable way of predicting the workpiece temperature directly using a circular heat source.

4.4.2 Modification of the temperature rise equations

In literature, a local coordinate system is defined for each heat source and used to compute the temperature rise. It is useful when the number of heat sources is limited but rather complicated to apply when the number, position and orientation of the heat sources vary greatly, which is the case

in this study due to the linear approximation of the circular heat source. A global coordinate system for all heat sources is defined in this section.

In addition, existing studies define the heat source velocity direction in the direction of an axis of the local coordinate system and do not allow changing it. However, the use of a global coordinate system can define the heat source velocity direction. It should be kept in mind that variation of velocity direction should be allowed in the proposed model because the cutting edge follows a sinusoidal vibratory path. This is justified because only the downward motion of the tool explains the significant contact and process damping effect with the flank face, which in turn justifies the presence of a heat source in this area. The temperature rise equations are thus modified to include the variation of the heat source velocity direction.

4.4.2.1 Introduction of a global coordinate system

The global coordinate system is introduced by adding an offset on the heat source as shown in Figure 4.12. The offset dx, dy impact the calculation of the point at l_i in the heat source (point $P(x_1, y_1)$ and its image $P'(x_2, y_2)$ in Figure 4.12) along with the distances R and R' .

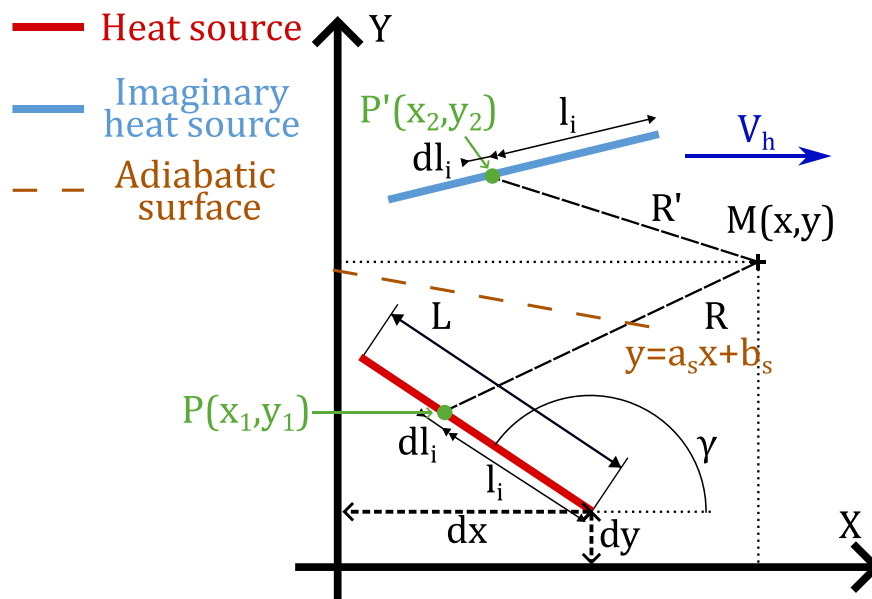


Figure 4.12: Tilted and shifted moving line heat source

Calculation of the distances R and R' with an adiabatic surface of equation $y = a_s x + b_s$ is given as:

$$x_1(l_i, \gamma, dx) = l_i \cos \gamma + dx \quad (4.30)$$

$$y_1(l_i, \gamma, dy) = l_i \sin \gamma + dy \quad (4.31)$$

$$R(x, y, l_i, \gamma, dx, dy) = \sqrt{(x - x_1(l_i, \gamma, dx))^2 + (y - y_1(l_i, \gamma, dy))^2} \quad (4.32)$$

$$x_2(l_i, \gamma, dx, dy) = \frac{2(x_1(l_i, \gamma, dx) + y_1(l_i, \gamma, dy)a_s - a_sb_s)}{1 + a_s^2} - x_1(l_i, \gamma, dx) \quad (4.33)$$

$$y_2(l_i, \gamma, dx, dy) = 2 \left(a_s \frac{(x_1(l_i, \gamma, dx) + y_1(l_i, \gamma, dy)a_s - a_sb_s)}{1 + a_s^2} + b_s \right) - y_1(l_i, \gamma, dy) \quad (4.34)$$

$$R'(x, y, l_i, \gamma, dx, dy) = \sqrt{(x - x_2(l_i, \gamma, dx, dy))^2 + (y - y_2(l_i, \gamma, dx, dy))^2} \quad (4.35)$$

where l_i is the position of the differential heat element dl_i on the line heat source, γ is the slope of the line heat source, dx, dy are the offset of the line heat source and a_s, b_s are the coefficients of the symmetry axis.

In case the symmetry axis is vertical, i.e. $x = a_s$, the following equations apply:

$$x_2(l_i, \gamma, dx, dy) = 2a_s - x_1(l_i, \gamma, dx) \quad (4.36)$$

$$y_2(l_i, \gamma, dx, dy) = y_1(l_i, \gamma, dy) \quad (4.37)$$

Similar equations can be obtained for the stationary rectangular heat source by adding the third dimension (z) in the calculation of the distances R and R' as:

$$R(x, y, z, l_i, z_i, \gamma, dx, dy) = \sqrt{(x - x_1(l_i, \gamma, dx))^2 + (y - y_1(l_i, \gamma, dy))^2 + (z - z_i)^2} \quad (4.38)$$

$$R'(x, y, z, l_i, z_i, \gamma, dx, dy) = \sqrt{(x - x_2(l_i, \gamma, dx, dy))^2 + (y - y_2(l_i, \gamma, dx, dy))^2 + (z - z_i)^2} \quad (4.39)$$

4.4.2.2 Integration of the velocity direction

The velocity direction in existing studies cannot be changed because only the distance R between the point of interest $M(x, y, z)$ and the position at l_i in the heat source is projected in the direction of heat source motion. It means that the contribution of the velocity on the moving imaginary heat

source is not included in the temperature field, which provide inconsistent results. This can be solved by separating the contribution of the original and imaginary heat sources as:

$$\theta(x, y) = \int_0^L B(l_i) \frac{q(l_i)}{2\pi\kappa} e^{-R_v V_h / 2a} K_0 \left(\frac{V_h R}{2a} \right) dl_i + \int_0^L B(l_i) \frac{q(l_i)}{2\pi\kappa} e^{-R'_v V_h / 2a} K_0 \left(\frac{V_h R'}{2a} \right) dl_i \quad (4.40)$$

where R_v and R'_v are the projection of R and R' in the direction of motion, respectively.

A comparison between the results obtained from the standard method with local coordinate system and the proposed method with a global coordinate system including the varying velocity direction is given for a moving line heat source with two imaginary heat sources in Figure 4.13.

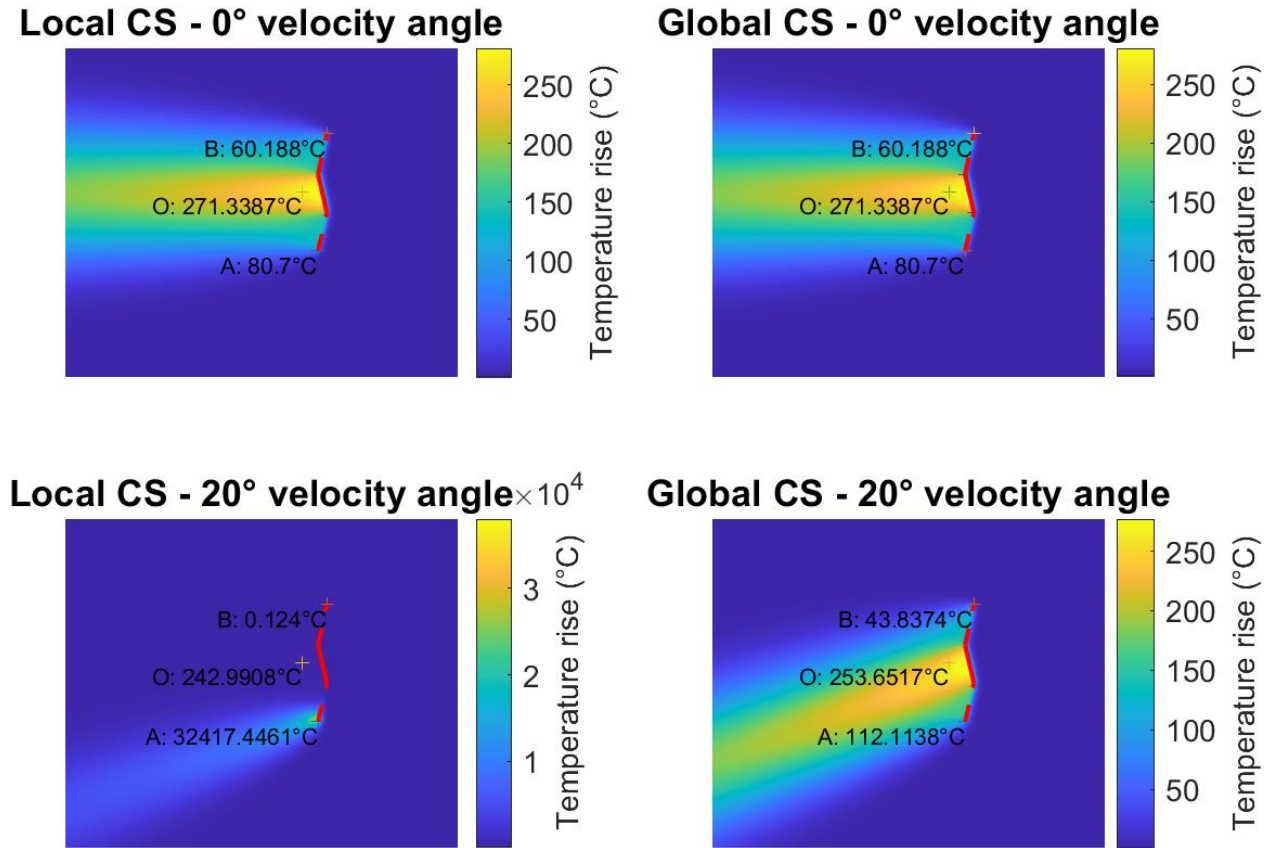


Figure 4.13: Temperature field calculation of moving line heat source with two images – coordinate system (CS) and velocity angle comparison

The two approaches provide the same results when no velocity angle is considered, but it is clear that the equation given in [91] cannot include a variation in the velocity direction due to the aberrant results. However, the modified equations can include any velocity angle.

4.5 Integration into the process damping model

In this section, a temperature equation is defined in the three following bodies: workpiece, chip and the cutting tool. The temperature rise in these bodies is calculated for each cutting edge position, for varying flank-contact length in the presence of vibrations.

According to Eq.(4.10) and Eq.(4.40), the temperature rise equation of one heat source is defined by 5 elements, namely its length, angle, offset, heat intensity and symmetry axes (additionally, the width of cut for the stationary rectangular heat source in machining). The heat intensity is shared by both heat sources at an interface, and is defined by the force at the interface, the material flow velocity, material flow angle and the width of cut. A set of equations are obtained to describe these elements for all heat source in their respective bodies.

Once the parameters of the temperature rise equations defined, the heat partition at all the interfaces are calculated following the procedure described in Section 4.3.3.

4.5.1 Temperature rise prediction within the workpiece

A visual representation of the temperature prediction in the workpiece is given in Figure 4.14. The adiabatic boundary considered in the workpiece is the uncut workpiece surface that is assumed to be flat and serves as symmetry axis for the heat sources [104]. Only moving line heat sources are considered at three heat generating interfaces: the flank-workpiece contact, the lower honed edge-workpiece contact and the shear plane. A heat source is placed on the flank contact, another one on the shear plane, and n_d heat sources approximate the lower part of the honed edge, as discussed in Section 4.4.1.

The symmetry axis is defined by $a_s = 0$ and $b_s = SP_y + h$:

$$y(x) = SP_y + h \quad (4.41)$$

where h is the uncut chip thickness. The heat source velocity angle is considered to be 0° (parallel to the tangential direction) for all heat sources.

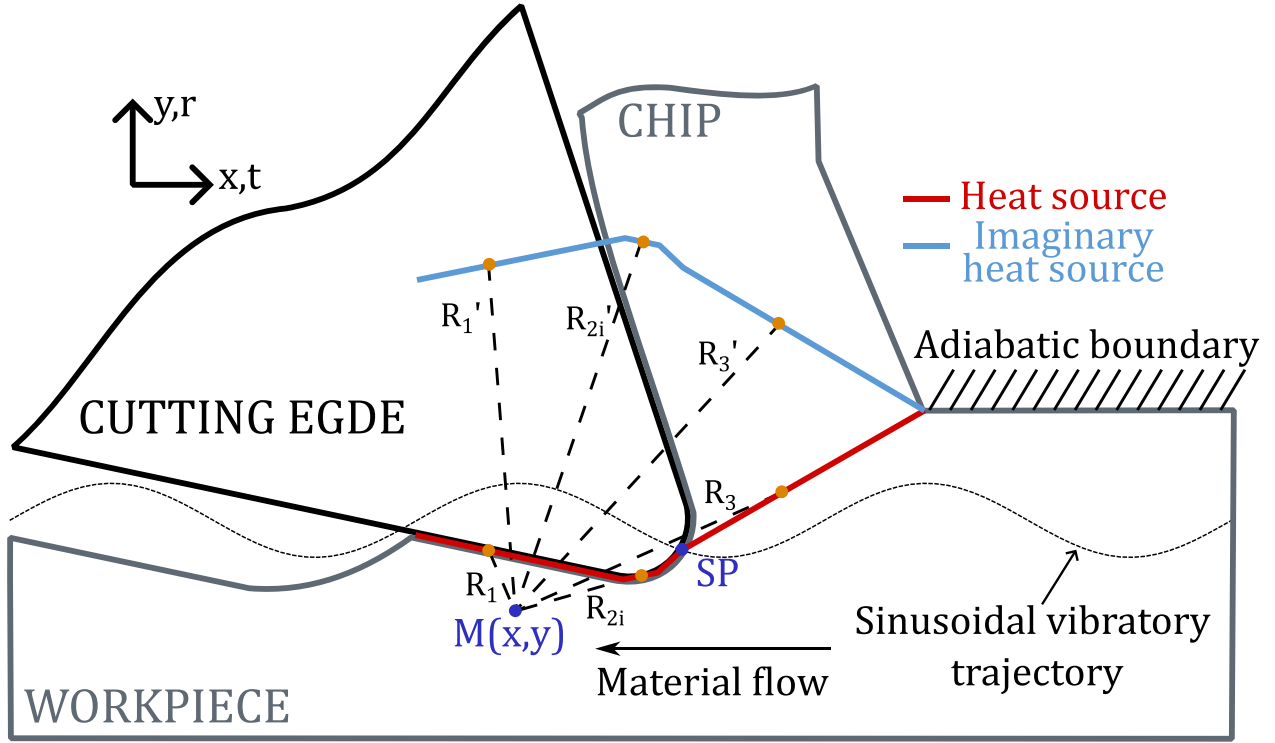


Figure 4.14: Model of temperature rise in the workpiece

The parameters of Eq.(4.10) and Eq.(4.40) are given in Table 4.1.

Table 4.1: Workpiece temperature rise parameters

Heat source	L	γ	dx	dy	V_h	F
Flank	Varying	$\pi - \alpha$	J_x	J_y	V	Varying
i^{th} approx. line below SP	Eq.(4.19)	Eq.(4.18)	Eq.(4.22)		Varying	Varying
Shear plane	$\frac{h}{\sin \phi}$	ϕ	0	0	$V \frac{\cos \alpha_r}{\cos(\phi - \alpha_r)}$	$F_{tc} \cos \phi - F_{fc} \sin \phi$

where h is the uncut chip thickness, ϕ and α_r are the shear and rake angles, F_{tc} and F_{fc} are the tangential and feed cutting forces. The point coordinates J_x and J_y are defined in Section 3.3.1. Calculation of the shear plane length, shear force, and shear velocity are obtained from [109].

It can be noted that the velocities of the honed edge heat sources vary, and it is assumed that they decrease linearly from the cutting velocity V at the end of the flank face to zero at SP where the material stagnant. Moreover, the normal contact forces acting on the flank and the honed edge, and the length of the flank face vary since they are calculated from the process damping model

$$\beta_a = \alpha_r + \tan^{-1} \left(\frac{F_{fc}}{F_{tc}} \right) \quad (4.46)$$

The chip-rake contact length, the friction force, and average friction angle β_a are given in [109]. Similar to the lower portion of the honed edge, the velocities of the upper honed edge heat sources vary from zero at SP to the chip flow velocity at the beginning of the rake face. The point O is defined in Section 3.3.1.

4.5.3 Temperature rise prediction within the tool

Free flank surface of the tool is considered adiabatic [104] as shown in Figure 4.17. All the heat sources are stationary and rectangular, and represent the flank-workpiece contact, the honed edge contact with the workpiece and the chip, and the rake-chip contact.

The symmetry axis is defined by $a_s = -\tan(\alpha)$ and $b_s = \tan(\alpha)J_x + J_y$:

$$y(x) = -\tan(\alpha)x + \tan(\alpha)J_x + J_y \quad (4.47)$$

where the point J is defined in Section 3.3.1.

All the parameters required to calculate the tool temperature rise are given in Table 4.3.

Table 4.3: Tool temperature rise parameters

Heat source	L	γ	dx	dy	V_h	F
Flank	Varying	$\pi - \alpha$	J_x	J_y	V	Varying
i^{th} approx. line below SP	Eq.(4.19)	Eq.(4.18)	Eq.(4.22)		Varying	Varying
i^{th} approx. line above SP					Varying	$F_{tc} \sin \alpha_r - F_{fc} \cos \alpha_r$
Rake	$\frac{h \sin(\phi + \beta_a - \alpha_r)}{\sin(\phi) \cos(\beta_a)}$	$\frac{\pi}{2} + \alpha_r$	$O_x + R_e \cos \alpha_r$	$O_x + R_e \sin \alpha_r$	$V \frac{\sin \phi}{\cos(\phi - \alpha_r)}$	$F_{tc} \sin \alpha_r - F_{fc} \cos \alpha_r$

The only additional parameter to Table 4.1 and Table 4.2 is the width of cut w , and the rest of the parameters are identical.

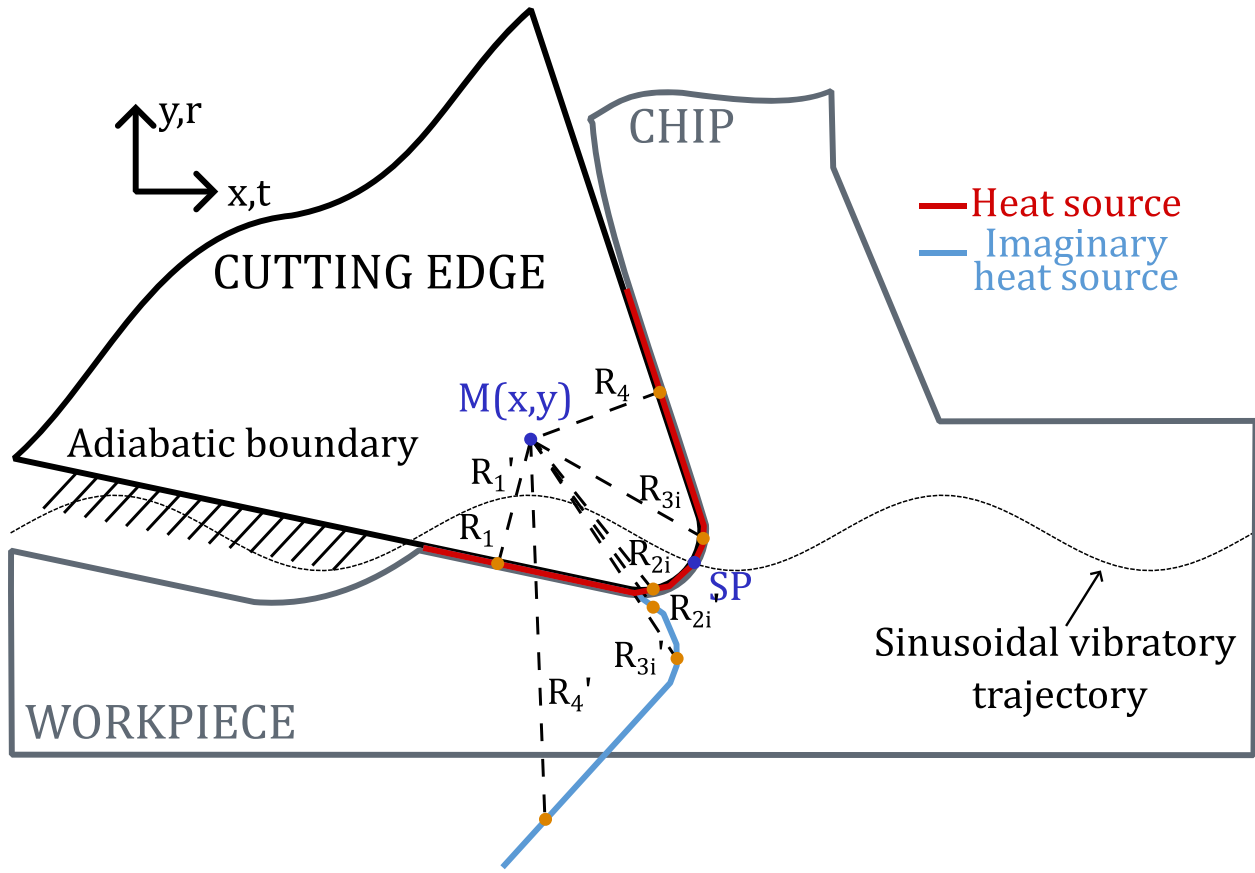


Figure 4.17: Model of temperature rise in the tool

4.5.4 Iterative workpiece temperature rise calculation

The temperature rise caused by the flank-workpiece contact and in the lower honed edge region depends on the contact force in these regions. The contact force can be estimated from the process damping model described in CHAPTER 3. However, due to the typical high temperature rise in machining, especially for difficult to cut materials (steel, titanium and nickel alloys), material properties may significantly vary with the workpiece temperature. In the proposed models, these variations of material properties influence the contact force, which in turn affect the temperature in the tool-workpiece contact area. The proposed iterative algorithm is as follows:

1. Assume an initial workpiece temperature: $T = 20^{\circ}\text{C}$ (room temperature),
2. Calculate the normal contact (process damping) force with process damping model for the workpiece temperature: $F_n(T)$,

3. The temperature on the ploughing interface is calculated with the normal contact force: $T(F_n, x)$, with x the abscissa of a point at the interface, and:

- Either average the temperature over the whole interface
- Or, curve-fit the temperature to obtain position dependant temperature

The first approach is used in this work for simplicity.

4. Repeat Steps 2 and 3 until the workpiece temperature converges. In this work an average workpiece temperature variation of 5°C is as the stop criterion.

Convergence time vary and may be affected by the initial temperature guess depending on the temperature-dependant material properties and on the evaluated cutting forces during machining. Hence, two curves $T(F_n)$ and $F_n(T)$ can be created by imposing a range of forces and initial temperature values in advance. These curves can be fitted by mathematical equations and used to realize steps 1 to 4 of the proposed iterative algorithm. The main advantage is that since the time consuming calculations of $T(F_n)$ are already realised, the iterative process becomes instantaneous and variation of the initial temperature guess can be implemented at a low computational cost.

4.6 Summary

In this chapter, temperature rise in the flank and honed edge regions has been calculated. First, existing literature models have been studied by revealing their limitation of ignored hone radius and its effect on the machining temperature predictions. However, temperature rise at the honed edge-workpiece interface during machining is necessary for the proposed process damping prediction model. A method to include honed-edge effect has been introduced, along with generalization of the existing literature model by integrating a general coordinate system and the possibility to vary the heat source velocity direction along the vibratory cutting edge trajectory. It was observed that a singularity in the temperature rise calculations in the lower honed edge region may occur if the latter is discretized by an insufficient number of linear heat sources. Even though the exact reason for this singularity is still unknown, it is suppressed by increasing the number of heat sources in that region, at the cost of higher computation time. Alternatively, a circular heat source which eliminates the temperature singularity problem is introduced. Using the newly defined temperature rise equations, a temperature rise model has been defined for a cutting tool with a honed edge. That includes an introduced iterative algorithm to obtain the temperature in the

ploughed area. The temperature prediction is then integrated into the Johnson-Cook material model or is used with the elastic-perfectly plastic material model with temperature dependent material properties.

CHAPTER 5 VERIFICATION OF THE PROCESS DAMPING MODEL

5.1 Introduction

Two analytical models have been developed in CHAPTER 3 and CHAPTER 4 for the prediction of machining process damping for an arbitrary cutting edge geometry, and for the prediction of the temperature rise during machining, respectively. This chapter presents verification of these models with numerical and experimental results:

- Validation of the temperature prediction model is realised in two steps. First, the temperature model and the heat partition calculations are verified with numerical and experimental results for a steady-state orthogonal (2D) cutting process using a cutting tool without a honed edge and a flank-workpiece contact. Then, the iterative algorithm is verified with numerical and experimental results, including flank-workpiece contact and a honed edge.
- The predictions of the proposed process damping model is compared against experimental, numerical, and analytical results from available literature for a standard cutting edge geometry. Next, the model is verified for a textured (non-standard) tool with numerical results from literature.
- The cavity model used to describe the deformation depth and the depth of the Winkler foundation is also verified with experimental results.

5.2 Validation of workpiece temperature prediction

5.2.1 Verification of the heat partition calculation

The heat partition calculation is verified by comparison with numerical results given by Ding and Yu [99], as well as numerical and experimental values provided by Aich et. al. [110]. Both machining cases [99], [110] consider two interfaces (rake-chip and shear plane) but do not include a honed edge and a flank-workpiece contact.

5.2.1.1 Comparison with Ding and Yu [99]

The cutting parameters and the cutting tool geometry used by Ding and Yu, [99], as well as the thermal properties of the AL6061-T6 workpiece and the tool are given in Table 5.1 and Table 5.2.

Table 5.1: Machining parameters of the comparison case given in [99]

h [mm]	w [mm]	V [m/min]	α_r [°]	α [°]	F_{tc} [N]	F_{fc} [N]	ϕ [°]	L_c [mm]
0.1	2.15	300	0	7	258.2	184.4	16.2	0.54

where h is the uncut chip thickness; w is the width of cut; V is the cutting velocity; α_r , α and ϕ are the rake, clearance and shear angles, respectively; L_c is the length of the rake-chip contact, and F_{tc} and F_{fc} are the tangential and feed cutting forces.

Table 5.2: Thermal properties of the comparison case given in [99]

Workpiece: Al 6061-T6			Tool: Cemented carbide		
κ [W/(m.K)]	c_p [J/(kg.K)]	ρ [kg/m ³]	κ [W/(m.K)]	c_p [J/(kg.K)]	ρ [kg/m ³]
167	896	2700	46	240	15000

where k is the thermal conductivity, c_p is the specific heat capacity and ρ is the density.

The results obtained from FE simulations in [99] are shown in Figure 5.1. The prediction results of the analytical model developed in this work is given in Figure 5.2.

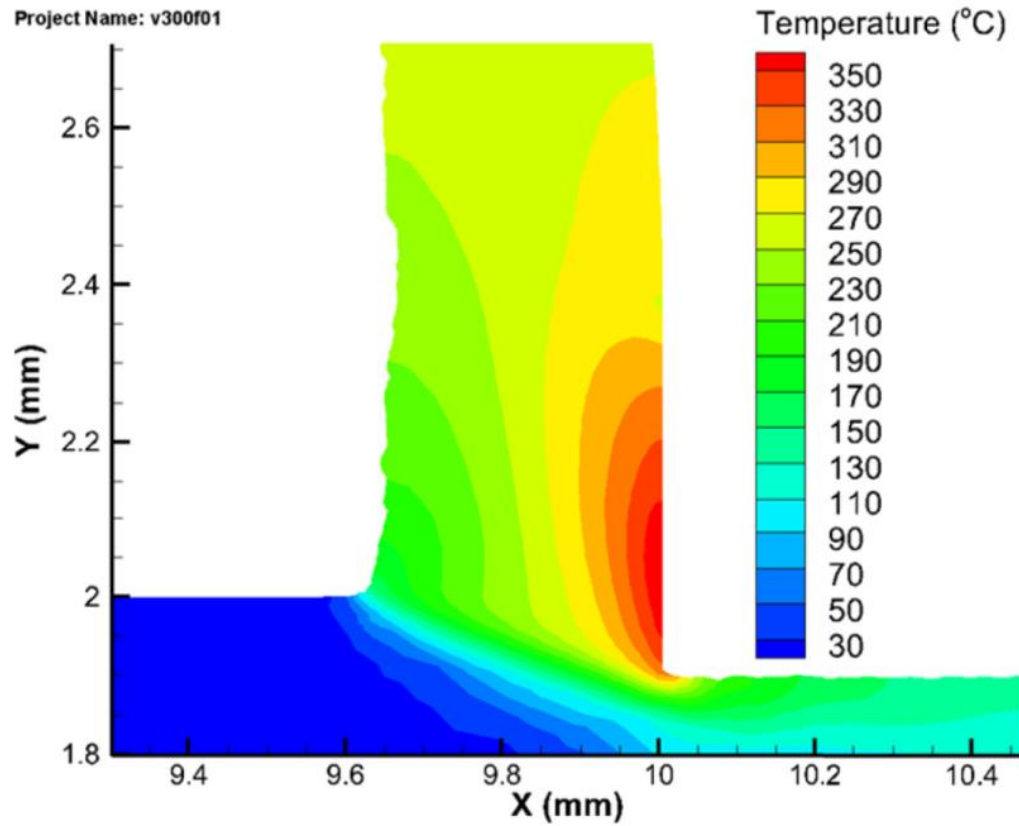


Figure 5.1: Numerical (FE) prediction results of Ding and Yu [99]¹

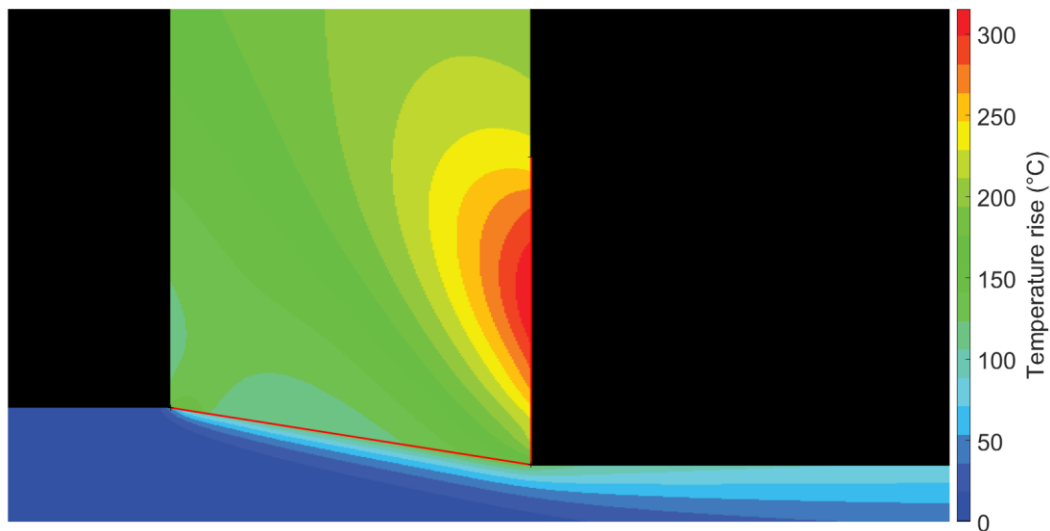


Figure 5.2: Prediction of the proposed analytical model for the machining case in [99]

In Figure 5.2, the predicted tool temperature is hidden to simplify the comparison with Figure 5.1, since the tool temperature is not shown in [99]. The position of the tool in Figure 5.2 is also different

than Figure 5.1 causing a change in x and y -coordinate ranges between the two, but it does not affect the temperature results. It is clear that the temperature field obtained using the proposed model matches well with that of Figure 5.1. "Hot zone" (the red zone in the figures) at the tool-chip interface is closer to the sharp edge in [99] having maximum temperature of 365°C, while the analytically obtained results have a maximum predicted temperature of 316°C (13% discrepancy). These discrepancies can be attributed to the description of the non-uniform heat intensity at the rake-chip interface and the assumed value of 0.5 for r_s in Eq.(4.6), which can influence the position of the "hot zone". Also, the employed constant heat intensity on the shear plane heat source and the introduced heat partition calculations affects the prediction results. Overall, the predicted workpiece temperature shows a good match with the numerical results in [99].

5.2.1.2 Comparison with Aich et. al. results

Aich et. al. [110] carried out two sets of FE simulations using two Arbitrary Lagrangian-Eulerian (ALE) approaches, as well as experiments to model the thermomechanical behavior of AISI 1045 workpiece in orthogonal cutting. The employed machining and thermal parameters are given in Table 5.3 and Table 5.4. The thermal properties of the tool were not provided in the article, they were assumed to be the same as those provided by Ding and Yu [99].

Table 5.3: Machining parameters of the comparison case given in [110]

h [mm]	w [mm]	V [m/min]	α_r [°]	α [°]	F_{tc} [N]	F_{fc} [N]	h_c [mm]
0.15	1.6	200	5	7	558.8	368.2	0.3

where h_c is the cut chip thickness.

Table 5.4: Thermal properties of the comparison case given in [110]

Workpiece: AISI 1045			Tool: Cemented carbide - Assumed		
κ [W/(m.K)]	c_p [J/(kg.K)]	ρ [kg/m ³]	κ [W/(m.K)]	c_p [J/(kg.K)]	ρ [kg/m ³]
48.3	420	7862	46	240	15000

¹ Reprinted from "An improved thermal model for the orthogonal cutting considering non-uniform heat partition of shear heat source", Yuxing Ding, Jianchao Yu, Volume 190, 2023, with permission from Elsevier.

The results obtained in [110] for the two ALE models are given in Figure 5.3, and the analytically predicted workpiece temperature using the proposed model are given in Figure 5.4.

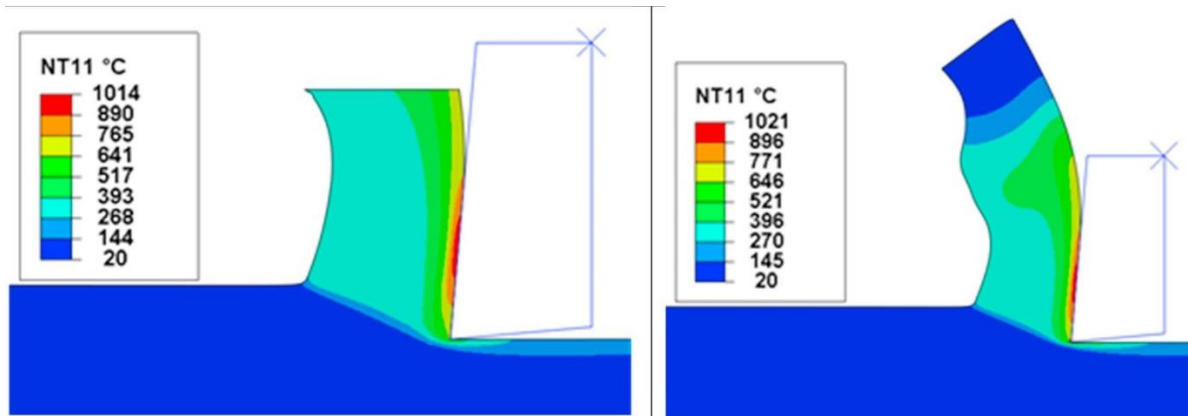


Figure 5.3: Numerical (FE) prediction results of Aich et. al. [110][99] with two ALE models²

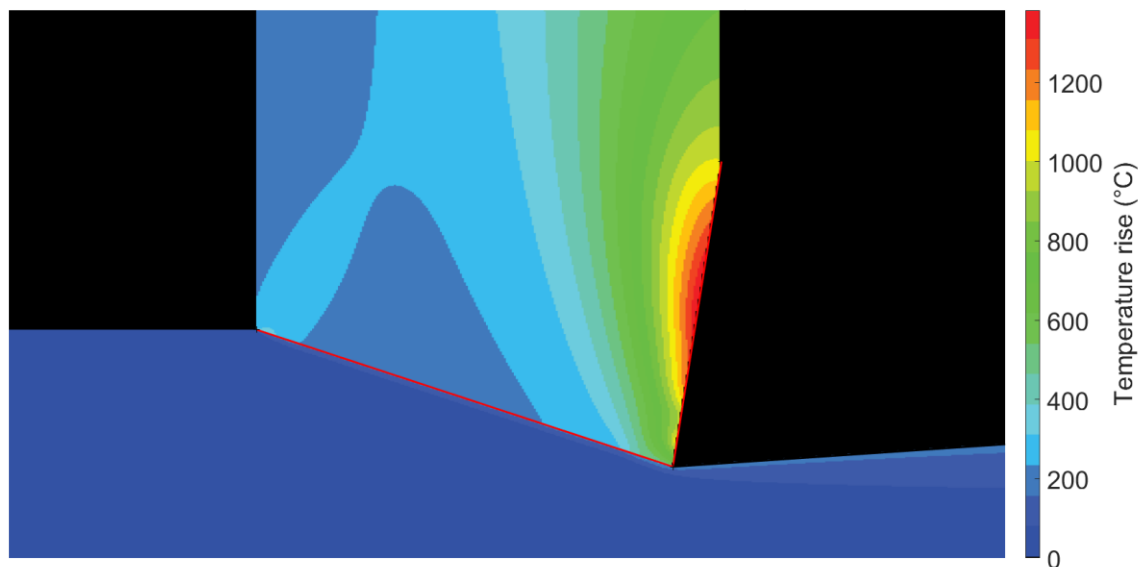


Figure 5.4: Predicted workpiece temperature using the proposed analytical model for the machining case in [110]

Though the analytically predicted temperature distribution and the location of the peak temperature closely follow those of the numerical analysis in [110], the predicted peak temperature (1214°C) has 19.7% discrepancy with the peak workpiece temperature (1014°C) in [110]. However, it must be noted that the experimental results given in [99] for the same case have a maximum temperature

² Reprinted from “An improved thermomechanical modeling for orthogonal cutting of AISI 1045 steel”, Zoubir Aich, Kamel Haddouche, Khaled Djellouli, Abdelhamid Ghezal, Volume 17, 2023, with permission from Elsevier.

of 1240°C on the rake face, which is predicted by a 2% error. The errors are due to assumptions made due to lack of tool's thermal properties and the distribution of the heat intensity on the rake face. Overall, the proposed model matches the experimental and numerical results with a good accuracy.

However, the results could not be verified for a tool with a honed edge, nor including the flank-workpiece contact, which also prevents the verification of the iterative algorithm. However, experimental and numerical results of the flank temperature allow verification of the iterative method next.

5.2.2 Iterative workpiece temperature prediction model verification

The iterative temperature prediction model is the direct result of integrating the improved thermal model into the proposed process damping model. It aims at predicting a set of temperatures and contact forces in the ploughed region as explained in Section 4.5.4.

5.2.2.1 Numerical results

Numerical simulations using the FD method proposed in [85] were realised to predict the temperature at the workpiece surface in contact with the tool. The simulations represent experimental orthogonal machining tests of AISI 1045 in [23]. The cutting, vibration and geometric parameters of the experiments are given in Table 5.5, the cutting parameters used for the iterative workpiece temperature prediction model are given in Table 5.6, and the temperature-dependent thermal properties of AISI 1045 are curve-fitted from [78].

Table 5.5: Geometric, machining and vibration parameters in the experiments [1], [23]

α_r [°]	α [°]	χ [°]	R_e [μm]	D [mm]	w [mm]	V [m/min]	ω_c [Hz]	A_0 [μm]
0	7	50	20	35	1	20-400	120	35

where α_r , α and χ are the rake, clearance and separation angles; R_e is the honed edge radius; D is the workpiece diameter; w is the cutting width; V is the cutting velocity; and ω_c and A_0 are the vibration frequency and amplitude.

Table 5.6: Cutting parameters for the workpiece temperature predictions [23]

h [mm]	V [m/min]	F_{tc} [N]	F_{fc} [N]	ϕ [°]
≈ 0.15	108	≈ 300	≈ 200	≈ 20

where F_{tc} and F_{fc} the tangential and radial cutting forces. It can be noted that in Table 5.6, the uncut chip thickness and the shear angle were not provided in the literature and are thus assumed with representative values.

The temperature obtained with FD model in [111] is 220°C, and the temperature predicted using the iterative model is 234°C, resulting in a 6% error. Even though the results are close, exact comparison is not possible due to the lack of experimental data. Still, it can be seen that the proposed iterative workpiece temperature model can predict the temperature with an acceptable error.

5.2.2.2 Experimental results

Experimental results are obtained and compared against the predictions of the proposed model using the collaborative experimental data provided by Professor Julius Schoop of the University of Kentucky. Orthogonal cutting operations on Inconel 718 material, which is commonly used in aerospace industry, were realised with cutting tools having a honed edge and a flat wear. The flat wear corresponds to a specific chamfered cutting edge geometry with a chamfer angle of 0°. The peak temperature on the flank face was measured using residual stress measurement at the intersection between the flank face and the static ploughing line (see Figure 2.4). The experimental results from the University of Kentucky are compared with the predictions using the proposed iterative model and also using the experimentally measured ploughing forces (F_p) for four different cutting edge geometries as presented in Table 5.7, and their corresponding sets of cutting parameters are given in Table 5.8. The temperature-dependent material properties of Inconel 718 were also provided by Professor Julius Schoop of the University of Kentucky, and they are listed in Table 5.9.

Table 5.7: Geometric parameters of the cutting edge

Geometry	α_r [°]	α [°]	α_c [°]	b_f [μm]	R_e [μm]
1	0	5	0	50	15
2				100	25
3				150	35
4				200	45

Table 5.8: Cutting parameters used in the cutting tests

Geometry	h [μm]	w [mm]	V [m/min]	h_c [m]	F_{tc} [N] (measured)	F_{fc} [N] (measured)	F_p [N] (measured)
1	25	3	30	50	193	115.5	256.5
2					197	125.5	332.5
3					203.5	136	425
4					218	152	684

where h_c is the cut chip thickness and F_p is the radial ploughing force. A representative chip compression ratio (h/h_c), of 0.5 is assumed.

Table 5.9: Temperature dependent mechanical and thermal properties of Inconel 718

Property	Equations (valid for $T \in [0,1000]$ °C)
Elastic modulus: E (GPa)	$E(T) = -1.938 \cdot 10^{-5} \cdot T^2 - 0.04788 \cdot T + 208$
Yield strength: Y (MPa)	$Y(T) = -2.405 \cdot 10^{-6} \cdot T^3 + 0.0018 \cdot T^2 - 0.6146 \cdot T + 1206$
Poisson's ratio: ν	0.28
Density: ρ (kg/m ³)	8220

Thermal conductivity: κ (W/(m·K))	$\kappa(T) = 0.01505 \cdot T + 11.07$
Specific heat: c_p (J/(kg·K))	$c_p(T) = 0.0002339 \cdot T^2 - 0.03916 \cdot T + 428$
Thermal diffusivity: a (m ² /s)	$a(T) = 7.292 \cdot 10^{-16} \cdot T^3 - 3.632 \cdot 10^{-12} \cdot T^2 + 4.296 \cdot 10^{-9} \cdot T + 3.134 \cdot 10^{-6}$

It can be noted that the radial ploughing forces (F_p) in Table 5.8 are identified from the experimentally measured radial (F_f) and tangential (F_t) total forces. F_f and F_t were measured for several uncut-chip thickness (h) values and the radial (F_p) and tangential (F_{te}) ploughing forces are obtained using the orthogonal machining mechanics [109] at $h = 0$ μm as shown in Figure 5.5.

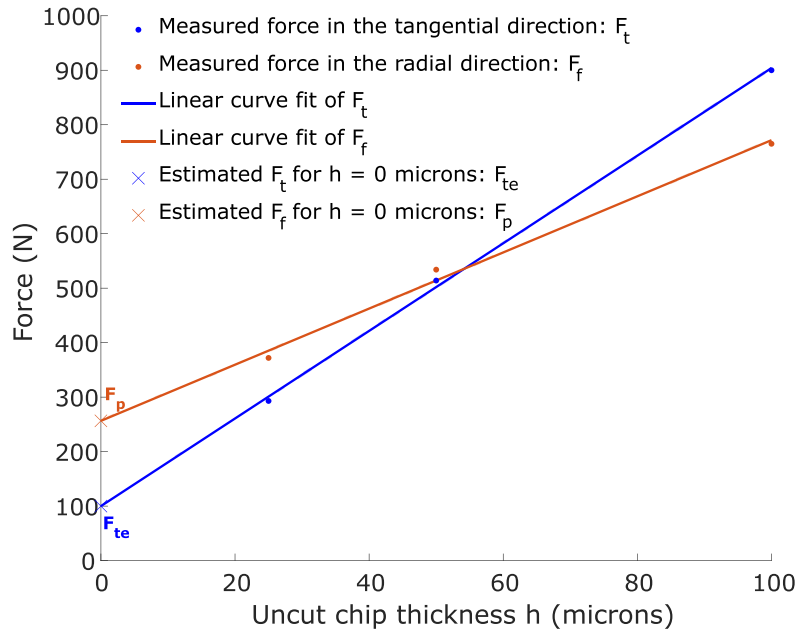


Figure 5.5: Example of radial (F_p) and tangential (F_{te}) ploughing forces identified by extrapolating F_f and F_t to $h = 0$ μm (for $b_f = 50$ μm , $R_e = 15$ μm)

To avoid the singularities in workpiece temperature prediction described in Section 4.5.4, two and six linear heat sources are respectively used to approximate the upper and lower circular sections of the honed edge. In addition, it was observed that the deformations caused by ploughing are close to the elastic limit ($5.7494 \cdot 10^{-3}$) that is 20% of the maximum strain ($2.8363 \cdot 10^{-2}$) and 30% of

the average strain ($1.8754 \cdot 10^{-2}$) in the ploughing regions. That affects the constraint factor (c) in Eq.(3.56), which only applies to fully uncontained plastic deformations. c increases linearly from elastic-plastic deformation regime until the steady-state fully plastic one is reached [72], [74]. As a result, Eq.(3.56) overestimates c that is adjusted to a value of $c = 1.5$ to more accurately represent the deformation occurring in the ploughed region. Comparison of the experimentally measured workpiece temperatures with the temperature prediction model using experimental ploughing forces (Table 5.8) and the iterative model is shown in Figure 5.6.

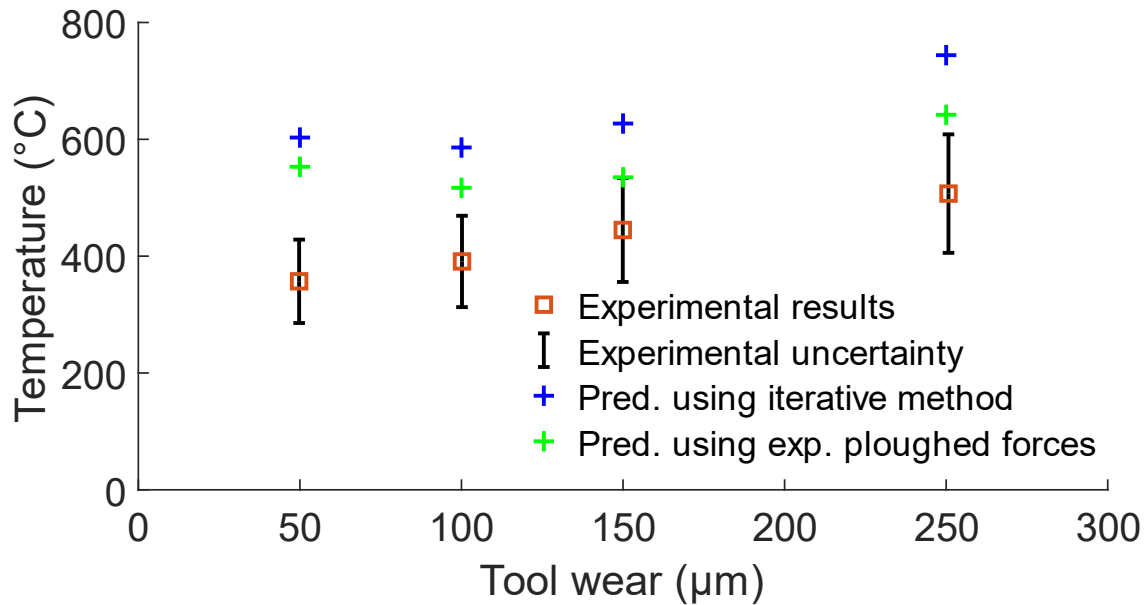


Figure 5.6: Peak temperature on the flank face: comparison between the experimental measurements and the predicted temperature using the iterative approach and the experimentally identified ploughing forces

In Figure 5.6, the experimental temperature has an uncertainty of 20%, and there is an average error of 11% between that uncertainty range and the temperature predicted using experimentally identified ploughing forces. This error represents the inaccuracy only due to the temperature prediction model. An average of 15% error (for a total of 26%) is added due to the ploughing force prediction model. These discrepancies are due to the difficulty in reaching the convergence in the iterative process for that specific case. The contact forces are high for Inconel 718 due to its material properties, which results in elevated workpiece temperatures. The temperature in different regions within the material and/or in the cutting tool inevitably exceeds the 1000°C boundary of the temperature-dependant material properties (Table 5.9) during iterative simulations of. These

properties have therefore been assumed to decrease linearly until reaching a null value at 1500°C. This assumption brings additional inaccuracy into the results. Still, the maximum prediction error is 26%, which is acceptable with the assumption on the temperature-dependent material properties.

5.3 Validation of the predicted deformation depth

The deformation depth directly affects the depth of the Winkler foundation, notably by controlling the amount of indentation necessary to cause plastic deformation of the springs. However, the use of the cavity model in machining is not direct since the latter only describes indentation without any tangential deformation. The depth of deformation obtained from machining experiments are compared with the predicted depth using the cavity model. The experimental depth of deformation is obtained from the collaboration with Dr. Julius Schoop of the University of Kentucky and is measured using Digital Image Correlation (DIC) for two chamfered tools with 0° chamfer angle as shown in Figure 5.7.

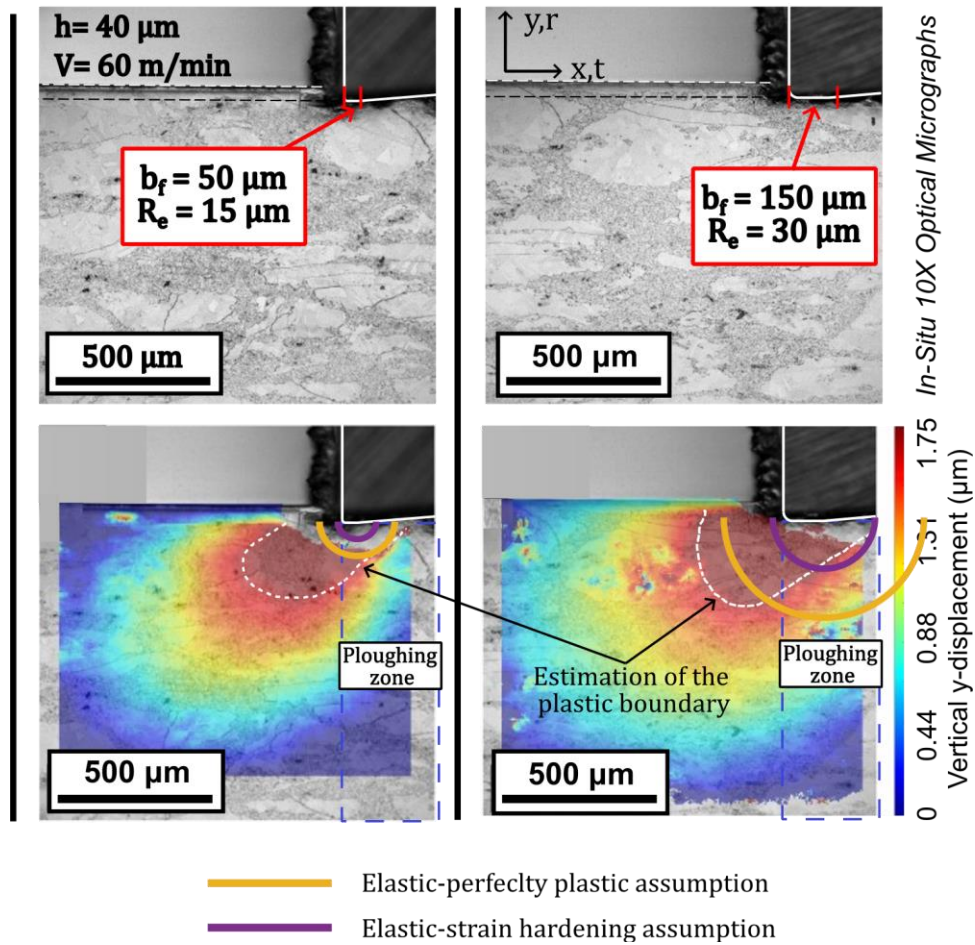


Figure 5.7: Visualization of the plastic deformation boundary of the workpiece under the cutting edge from the cavity model, and the vertical displacement from DIC measurements

In Figure 5.7, the color map of material's displacement was obtained using DIC method that shows the displacement of the material with respect to a reference position (image frame). The illustrated plastic boundary shows the maximum material displacement field caused by combined shearing and ploughing. As seen, the two cavity models (elastic-perfectly plastic and elastic-strain hardening) are in a good agreement with the DIC results and match the plastic boundary caused by cutting edge ploughing under the flank face for the given case. The predicted radii with the strain hardening assumption (83.5 \$\mu\text{m}\$ for \$b_f = 50 \mu\text{m}\$ and 217.7 \$\mu\text{m}\$ for \$b_f = 150 \mu\text{m}\$) are smaller than those of the perfectly plastic solution (163.3 \$\mu\text{m}\$ for \$b_f = 50 \mu\text{m}\$ and 426 \$\mu\text{m}\$ for \$b_f = 150 \mu\text{m}\$). This is expected since the strain hardening adds resistance to plastic deformation as compared to the perfectly plastic constitutive model. As expected, significant material displacement occurs outside the ploughing zone because of the material shearing and chip generation during machining,

however, it does not impact the ploughing phenomenon under the tool and is not taken into account. Still, the tangential deformations occurring in front of the tool impacts the deformations under the tool-workpiece contact. More precisely, it is a function of the uncut chip thickness as illustrated in Figure 5.8.

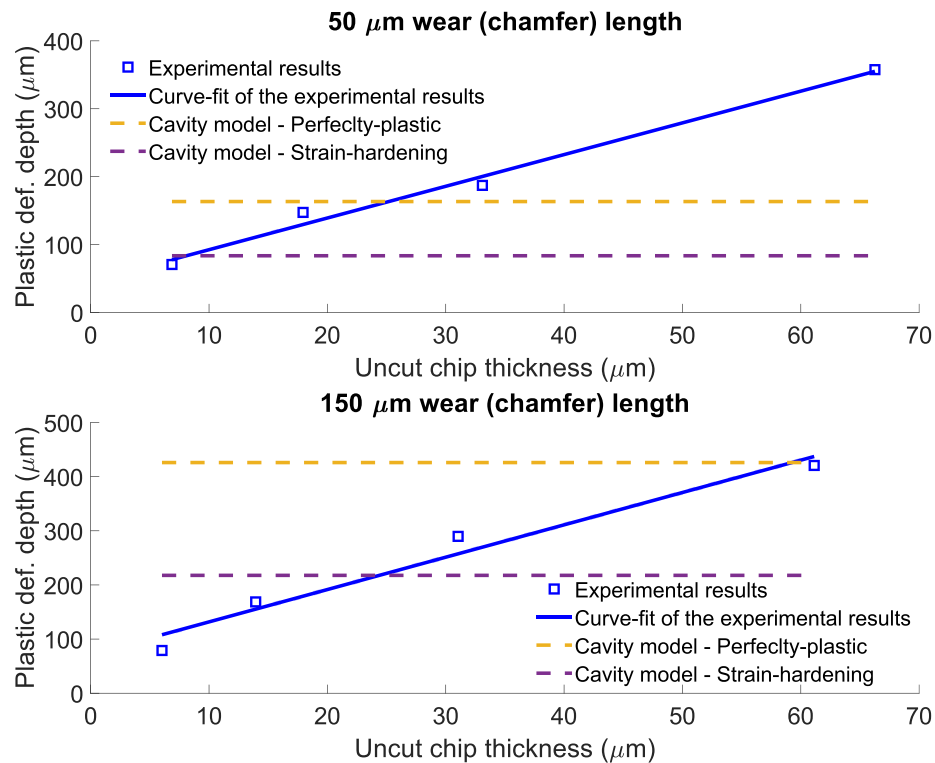


Figure 5.8: Comparison of the experimentally measured plastic deformation depth and the analytically predicted plastic deformation depth

In Figure 5.8, the depth of the plastic deformation under the tool varies linearly with the uncut chip thickness while the cavity model predictions remain constant. This is due to the fact that the cavity model only uses the indentation characteristics to predict the depth of deformation, without including any tangential component of deformation. Still, the constant predictions with both perfectly plastic and strain-hardening assumptions are within the range of depth of plastic deformation for a representative range of uncut chip-thickness. These results bring confidence to employ the cavity models to predict the depth of deformation beneath the cutting edge during a cutting operation. Improvements of the cavity models to include tangential deformations can be studied but are not the subject of the current work.

5.4 Process damping validation

The developed process damping model has been verified for a standard cutting edge geometry with experimental [23] and numerical [25] results as well as the analytical results [1]. Process damping predictions for a complex, textured geometry are compared with numerical results from literature [32] to validate the capacity of the proposed model to be applicable for an arbitrary cutting edge geometry, which is the main contribution of this work.

5.4.1 Standard cutting edge

Process damping predictions for a standard cutting edge are compared with experimental [23], numerical [25] and analytical results [1] in literature. The predicted process damping coefficient (equivalent viscous damping coefficient, c_{eq}) is compared with the identified ones in literature for orthogonal cutting operations with AISI 1045 material. Available literature measures a velocity-dependant process damping coefficient, C_i , which is linked to the proposed equivalent viscous damping coefficient, c_{eq} , as follows [1]:

$$c_{eq} = \frac{C_i}{V} \quad (5.1)$$

where V is the cutting speed.

The vibration wavelength (λ) of the cutting process is dependant on the cutting speed (V). However, there exist no direct link between the machining parameters and the vibration amplitude (A_0), which is a function of the excitation (machining) forces and the machining system's dynamics. The amplitude is then assumed to be constant while wavelength varies and set equal to the imposed value in literature [23], [25]. The wavelength (λ) is linked to the cutting velocity as in Eq.(3.28).

Process damping predictions obtained by the only existing analytical model [1] for standard cutting edge geometry with an elastic-perfectly plastic material model is also compared with the proposed model, for both experimental [23] and numerical [25] cases, to verify the proposed generalized model (CHAPTER 3) for such geometries using the same material properties for a fair comparison. Additionally, the same results are compared with the proposed generalized model by including the Johnson-Cook material constitutive model for a workpiece temperature predicted with the improved temperature model (CHAPTER 4). All of these results are obtained using both the cavity model (varying deformation depth) presented in Eq.(3.29) and the constant deformation depth

(1.016 mm) measured by Wu [61]. Finally, a stability lobe diagram is generated for the experimental case [23] and compared with experimentally identified stable and unstable test points.

5.4.1.1 Numerical results

Process damping coefficients were identified in orthogonal micro-cutting finite element (FE) simulations of AISI 1045 by Jin and Altintas [25]. The cutting, geometric and vibration parameters are listed in Table 5.10.

Table 5.10: Geometric, machining and vibration parameters in the FE simulations [1], [25]

α_r [°]	α [°]	χ [°]	R_e [μm]	D [μm]	w [mm]	V [m/min]	ω_c [Hz]	A_0 [μm]
0	7	50	21.4	0.6	0.1	50-400	1500	≈ 3

with α_r , α and χ are the rake, clearance and separation angles, respectively; R_e is the honed edge radius; D is the diameter of the milling tool; w is the cutting width; V is the cutting velocity; and ω_c and A_0 are respectively the vibration frequency and amplitude. Jin and Altintas defined the dynamics of the tool and allowed the cutting edge to vibrate during the FE simulation. That results in a varying vibration amplitude, which is represented by an average value in the proposed model as in common literature [1], [11], [15].

In order to ensure a fair comparison between the process damping prediction using an elastic-perfectly plastic and Johnson-Cook material models, the yield strength is set equal for both. The elastic modulus and Johnson-Cook parameters for AISI 1045 are given in Table 5.11 [78], [110]:

Table 5.11: Elastic and Johnson-Cook parameters for AISI 1045 [78], [110]

E [GPa]	$A = \sigma_Y$ [MPa]	B [MPa]	C	n	m	$\dot{\epsilon}_0$ [s ⁻¹]	T_m [°C]	T_0 [°C]
199.2	553.1	600.8	0.234	0.01234	0.6	1	1460	20

where E is the elastic modulus, A and σ_Y are the material yield strength, B, C, n and m are the Johnson-Cook parameters, $\dot{\epsilon}_0$ is the reference strain-rate, T_m and T_0 are the melting and room temperatures.

The mean flank face temperature is predicted as 202°C [111] using a finite difference model given in [85]. That temperature value could not be compared with the iterative algorithm proposed in this

work due the lack of inputs data. The temperature value of 202°C is used for simulating the process damping for the case in [25]. Numerically identified process damping coefficient in [25] is $C_i = 1.54 \cdot 10^5$ N/m, and is compared with the proposed and previous analytical models' predictions in Figure 5.9 using Eq.(5.1).

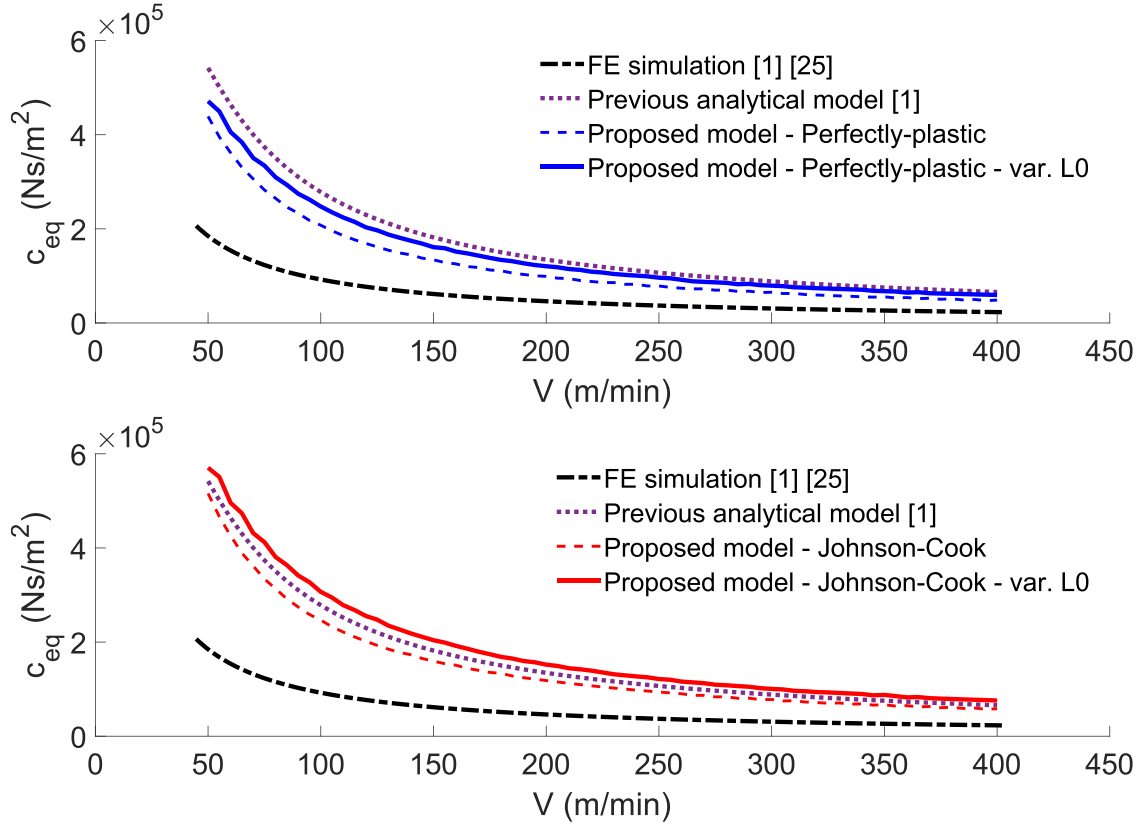


Figure 5.9: Equivalent viscous damping coefficient, c_{eq} , comparison with previous analytical [1], and numerical [25] results

It appears that use of the cavity model to create a varying unloaded length of the springs L_0 increases the process damping values as compared to those obtained with the constant unloaded length of 1.016 mm [61]. This is because within the range of vibration wavelength ($[0.55, 4.44]$ mm) and amplitude, the constant depth of deformation of 1.016 mm does not represent the actual indentation of the tool into the workpiece. Also, the predictions using the Johnson-Cook material model are higher than those obtained with the elastic-perfectly plastic material model, this is due to the strain hardening effect which results in higher contact stress for the same indentation as compared to the elastic-perfectly plastic case. In all cases, the proposed model matches very well the previous analytical model, but then also shares its drawbacks, namely that there is a significant

error when compared with the numerical results. This error is mainly due to two reasons: the constraint factor is assumed to be constant while it should be varying with the indentation depth, and it is assumed that the vibration amplitude remains constant while the cutting velocity increases. The latter affects the c_{eq} results because the vibration amplitude varies in the FE simulation, which presents a different scenario than the constant amplitude assumption in the proposed model. However, the actual slope of the proposed curve with cutting speed matches that of the numerical results.

The main advantage of the proposed model is that it provides such results in less than five minutes without any preliminary experimental work as long as the material parameters are known, while the numerical results require several time-expensive simulations (in the order of days) as each cutting speeds requires a separate FE simulation with laborious preparatory work.

5.4.1.2 Experimental results

Controlled vibrations using a piezo actuator in orthogonal machining of AISI 1045 were realised in [23]. The piezo-actuated experimental set-up resulted in low frequency vibrations (120 Hz) which are different from typical machining vibration frequencies (1000 – 2000 Hz). However, due to the controlled nature of the experiment, process damping still occurs and can be compared with predicted results. The same experiments have been previously used in Section 5.2.2.1 to validate the proposed iterative workpiece temperature predictions against the numerical simulations from literature [111]. The predicted temperature of 220°C is recalled and used in the following calculations. The experimentally identified process damping coefficient $C_i = 0.611 \cdot 10^6$ N/m is directly used in this section to validate the proposed model. The machining parameters are listed in Table 5.5 and the constitutive law parameters are given in Table 5.11, with an updated elastic modulus of 197.6 GPa at 220°C from the temperature-dependant properties of 1045 steel given in [78].

Similar to the previous section, the equivalent viscous damping coefficient (c_{eq}) from the proposed general model is compared with both experimental [23] and previous analytical results [1], with constant and varying springs unloaded length, as well as with elastic-perfectly plastic and Johnson-Cook constitutive laws. The results are shown in Figure 5.10.

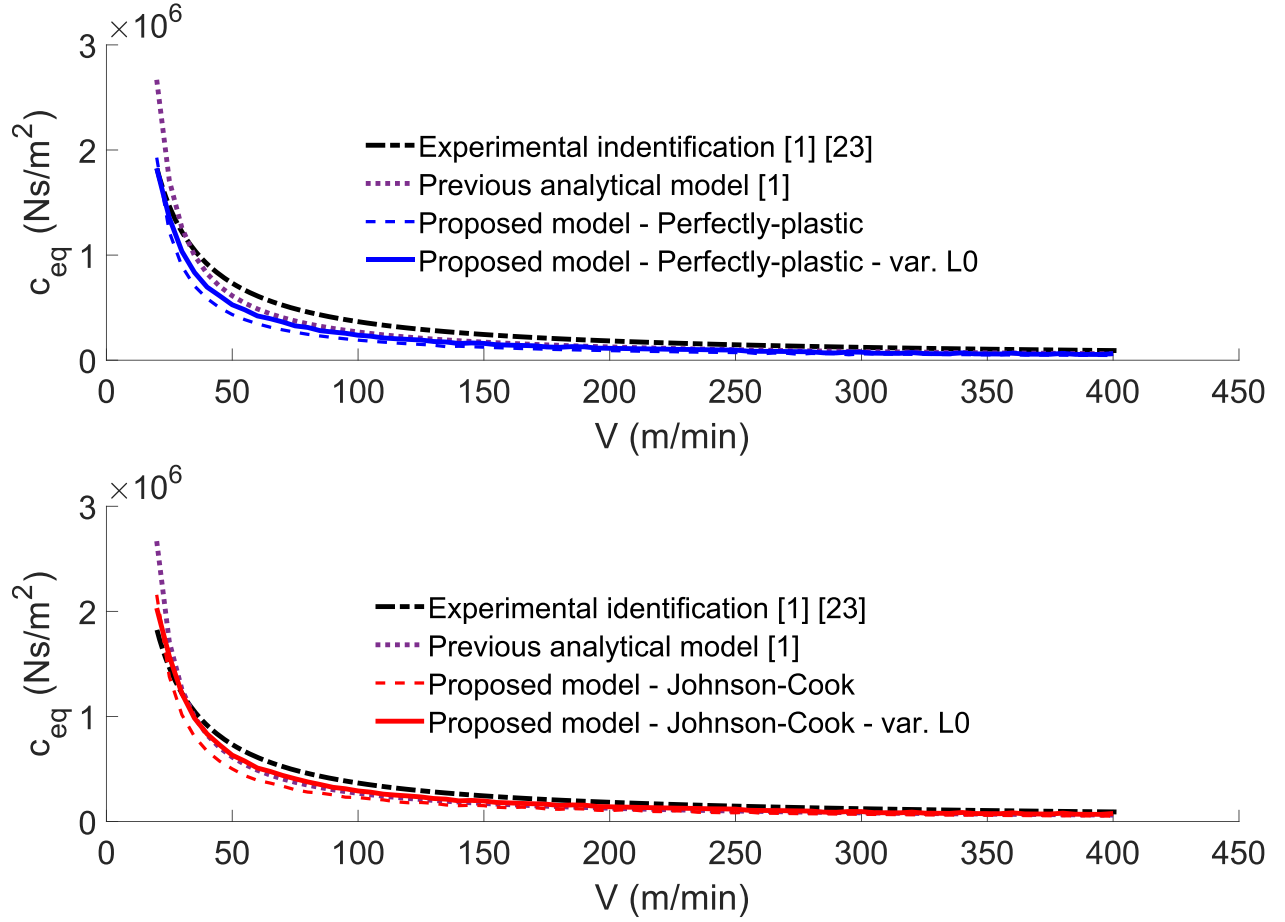


Figure 5.10: Equivalent viscous damping coefficient, c_{eq} , comparison between previous [1] and new analytical and experimental [23] results

The same observations regarding the effect of the cavity model and the material constitutive law as those made in the previous section can be made. The important findings are that there is a close match between the newly proposed model's results and both the experimental and previous analytical model. Also, it can be seen that the use of Johnson-Cook material model brings an increase of accuracy in the results as expected since it is a coupled thermo-mechanical material constitutive model. Again, the slope of the predictions matches very well with the experimental results. Even though the same assumptions, as those used in the previous comparison, are used, it appears that the constraint factor equation was more representative of the real phenomenon as compared to FE simulations. For this case, the results can be generated in 20 minutes, which is negligible compared to the time required to conduct numerous experimental tests while interrupting the production to obtain the experimental curve in Figure 5.10, which is evidently not industry friendly.

Stability lobes diagram (SLD) is also generated using the predicted c_{eq} and compared with the experimentally identified stable and unstable width of cut and spindle pairs from literature [23] in Figure 5.11.

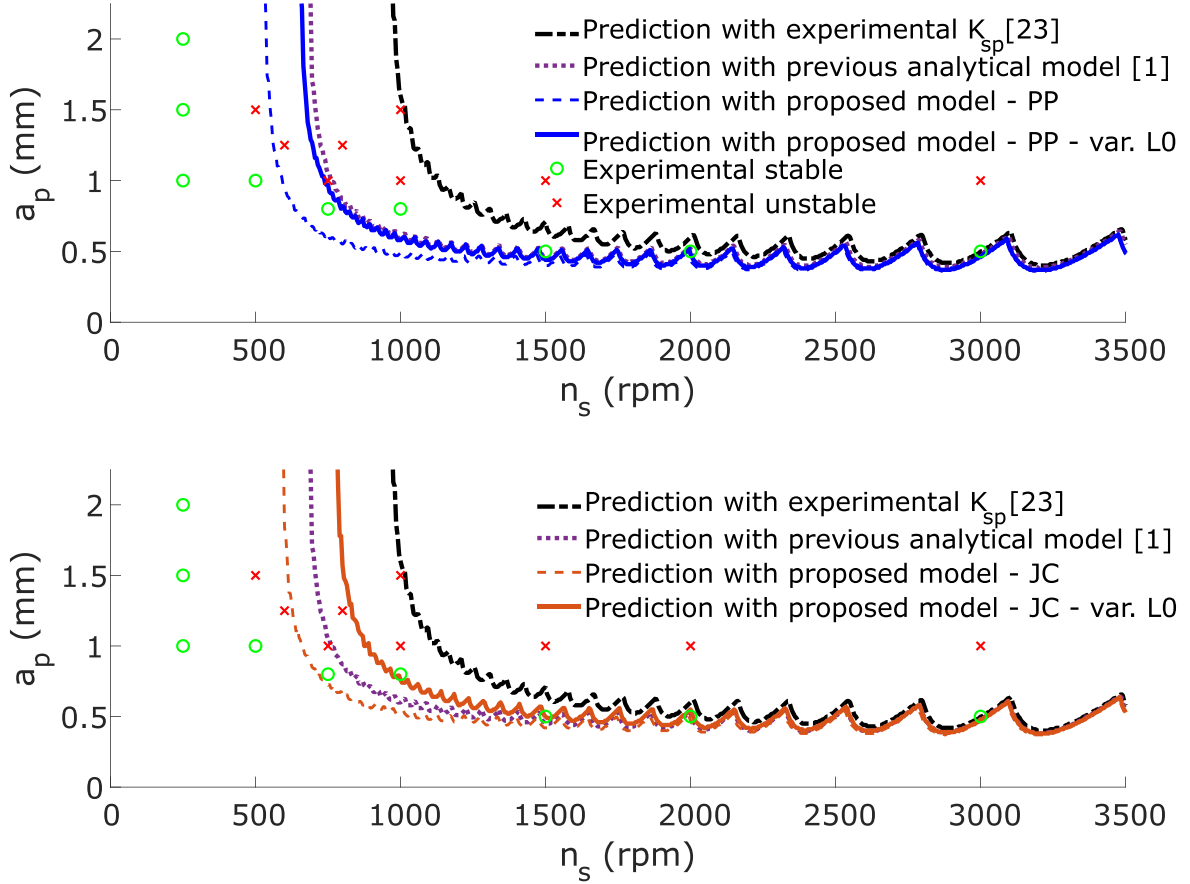


Figure 5.11: Stability lobes diagram prediction comparison for Perfectly-Plastic (PP) and Johnson-Cook (JC) material models, and constant and varying unloaded length of surface discretizing springs (n_s and a_p refer to the spindle speed and the width of cut, respectively)

The impact of the cavity and the Johnson-Cook material models is more clearly reflected in the SLD. Specifically, the cavity model increases the stability prediction by providing a more accurate representation of the tool-workpiece indentation compared to the assumption of a constant deformation depth. Additionally, the inclusion of strain-hardening in the Johnson-Cook constitutive law contributes to the increased the stability. Moreover, it appears clearly that the stability curve predicted using the c_{eq} with Johnson-Cook constitutive law and the cavity model matches more accurately with the experimentally identified stable and unstable points.

The impact of the temperature on the stability prediction can be visualized in Figure 5.12, where the stability limit is generated for one additional temperature (0°C).

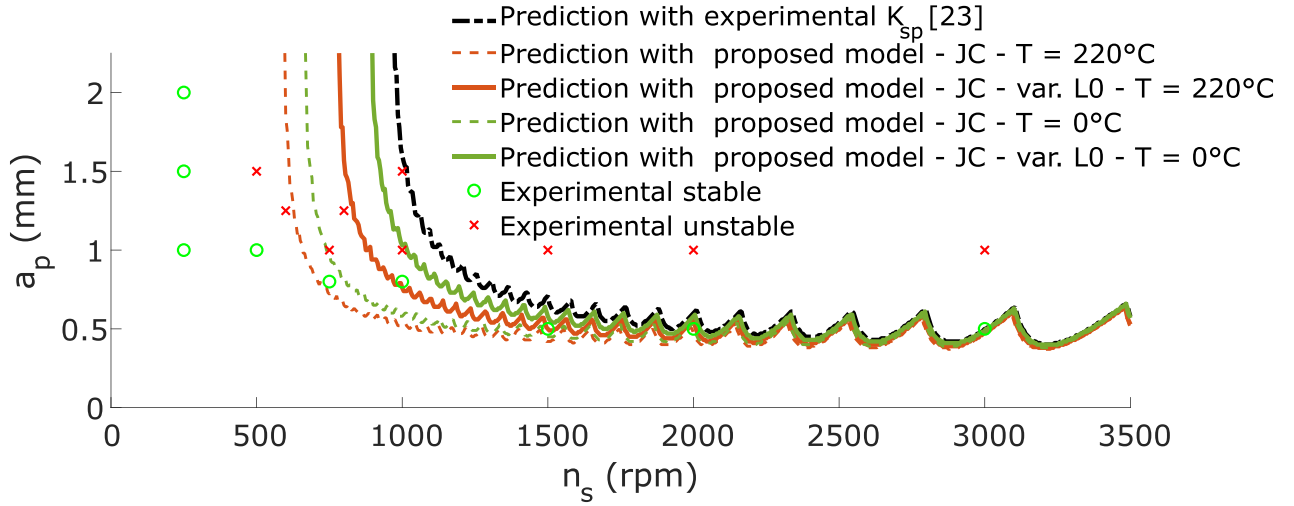


Figure 5.12: Comparison of SLD prediction for two different workpiece temperatures

It appears that the machining stability is sensitive to the workpiece temperature. More specifically, the difference is inversely related to the spindle speed, which proves the critical impact of the temperature in process damping.

5.4.2 Textured cutting edge

Numerical simulations were conducted in [32] to study the effect of a textured tool geometry on the machining process damping. A set of cutting simulations of S45C steel were run for a range of vibration wavelength $\lambda \in [0.3, 6]$ mm and vibration amplitude $A_0 \in [0, 70]$ μm . It was observed that the textured tool could improve the process damping for a wider range of cutting velocities. For each set of (λ, A_0) , the process damping forces were extracted and expressed in frequency domain. The velocity-dependant process damping coefficients were then calculated as:

$$K_{pd,r} = -\frac{1}{w} \text{imag} \left(\frac{F_{pd}(\omega_c)}{A_0} \right) \quad (5.2)$$

where w is the width of cut, $F_{pd}(\omega_c)$ is the process damping force in the frequency domain at the chatter frequency ω_c , A_0 is the vibration amplitude, and $K_{pd,r}$ is the process damping coefficient in Pa. It was observed [32] that the boundary of the effective process damping wavelength (the wavelength at which process damping begins to affect and increases stability) could be approximately estimated using a linear equation. This equation is a function of the nominal

clearance α and effective clearance angle α_e of the textured cutting edge, defined in Section 3.3.1.2:

$$A_0 = \frac{\lambda \tan \alpha}{2\pi} \text{ and } A_0 = \frac{\lambda \tan \alpha_e}{2\pi} \quad (5.3)$$

The same physical quantities representing the process damping in [32] are deduced using the proposed generalized model. Four different figures are compared with the published results³, which are the $K_{pd,r}(\lambda, A_0)$ map, for both straight and textured tool with the cavity model and the constant deformation dept of 1.016 mm [61]. Only the Johnson-Cook constitutive law is used in the predictions with the parameters in Table 5.12 [112]:

Table 5.12: S45C steel material properties

E [GPa]	$A = \sigma_Y$ [MPa]	B [MPa]	C	n	m	$\dot{\epsilon}_0$ [s ⁻¹]	T_m [°C]	T_0 [°C]
190	553	600	0.134	0.0134	1	0.134	1460	1

with E being the elastic modulus, A and σ_Y the material yield strength, B, C, n and m the Johnson-Cook parameters, $\dot{\epsilon}_0$ the reference strain-rate, and T_m and T_0 the melting and room temperatures.

The geometric parameters of the used textured cutting tools (Section 3.3.1.2) are given in Table 5.13 [32]:

Table 5.13: Geometric parameters of the textured tool

α [°]	α_e [°]	l_d [mm]	l_{tx} [mm]	h_{text} [μm]	χ [°] (arbitrary)	R_e [μm] (arbitrary)
7	3.5	0.55	0.1	40	50	20

where α, α_e and χ are the clearance, effective clearance and separation angles, respectively; l_d, l_{tx} and h_{text} are the texture geometric parameters; and R_e is the honed edge radius.

The separation angle is assumed to be 50° [1] as in the previous cases, and the honed edge radius is approximated as 20 μm. The clearance and separation angles, α and χ , are the same for the

³ Reprinted from “Flank face texture design to suppress chatter vibration in cutting”, N. Suzuki, W. Takahashi, H. Igeta, T. Nakanomiya, Volume 69, Issue 1, Pages 93-96, 2020 CIRP, with permission from Elsevier.

standard tool, as well as the honed edge radius R_e . Comparison of the $K_{pd,r}(\lambda, A_0)$ results and the boundary lines are given in Figure 5.13 to Figure 5.16.

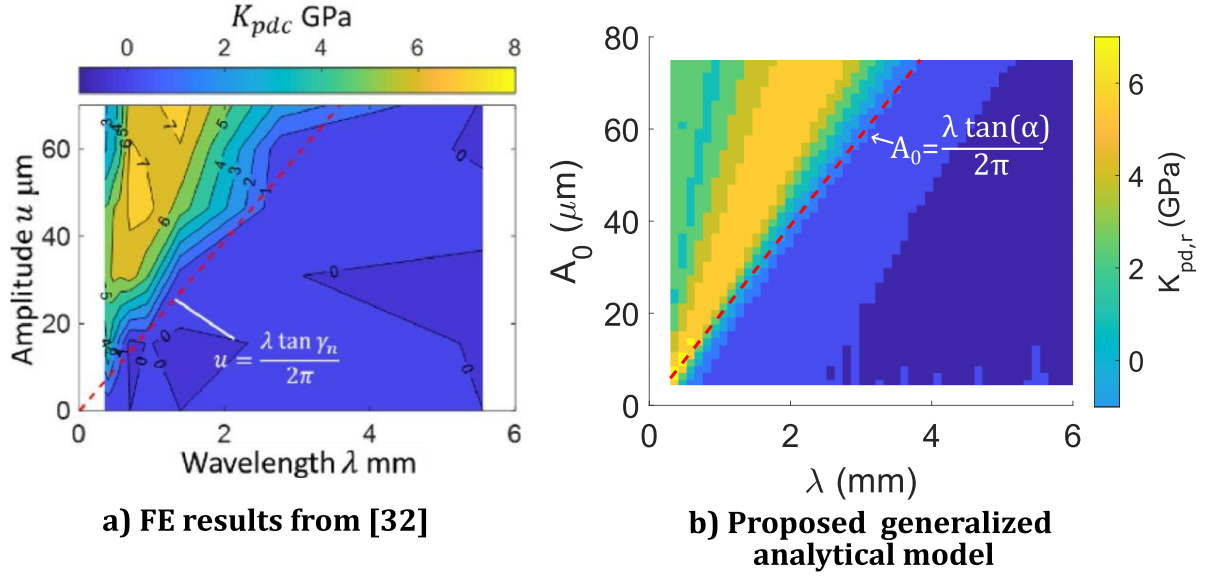


Figure 5.13: Comparison of $K_{pd,r}$ map for a standard tool; a) numerical results³ with u the vibration amplitude, γ_n and γ_e the clearance and effective clearance angles, and $K_{pd,c}$ the process damping coefficient; b) proposed analytical results using constant deformation depth

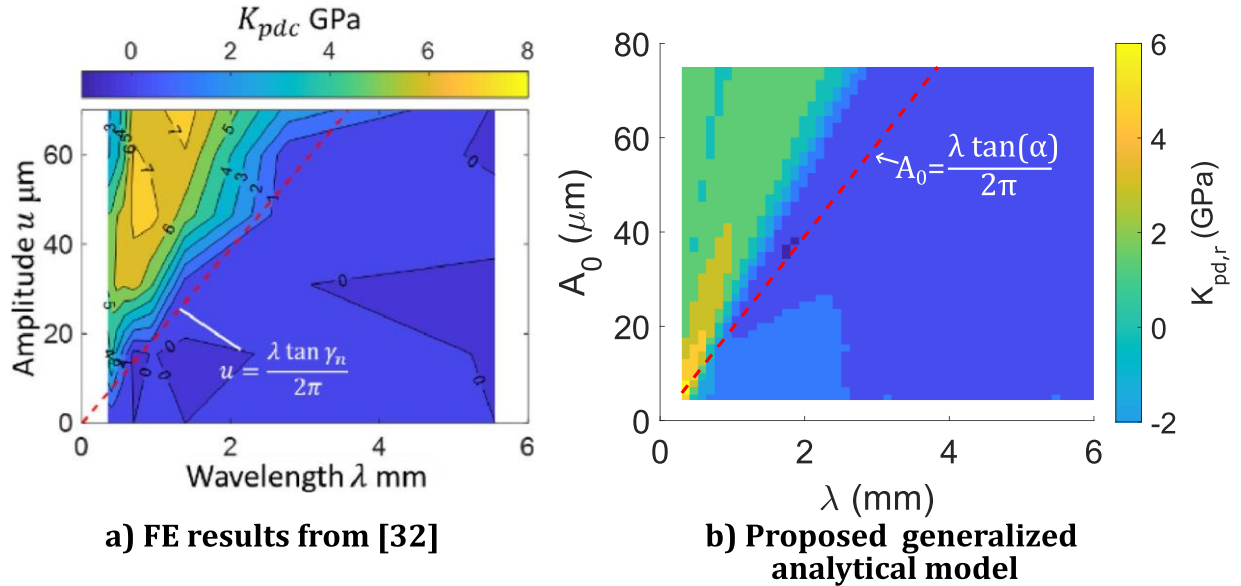


Figure 5.14: Comparison of $K_{pd,r}$ map for a standard tool; a) numerical results³ with u the vibration amplitude, γ_n and γ_e the clearance and effective clearance angles, and $K_{pd,c}$ the

process damping coefficient; b) proposed analytical results using varying deformation depth (cavity model)

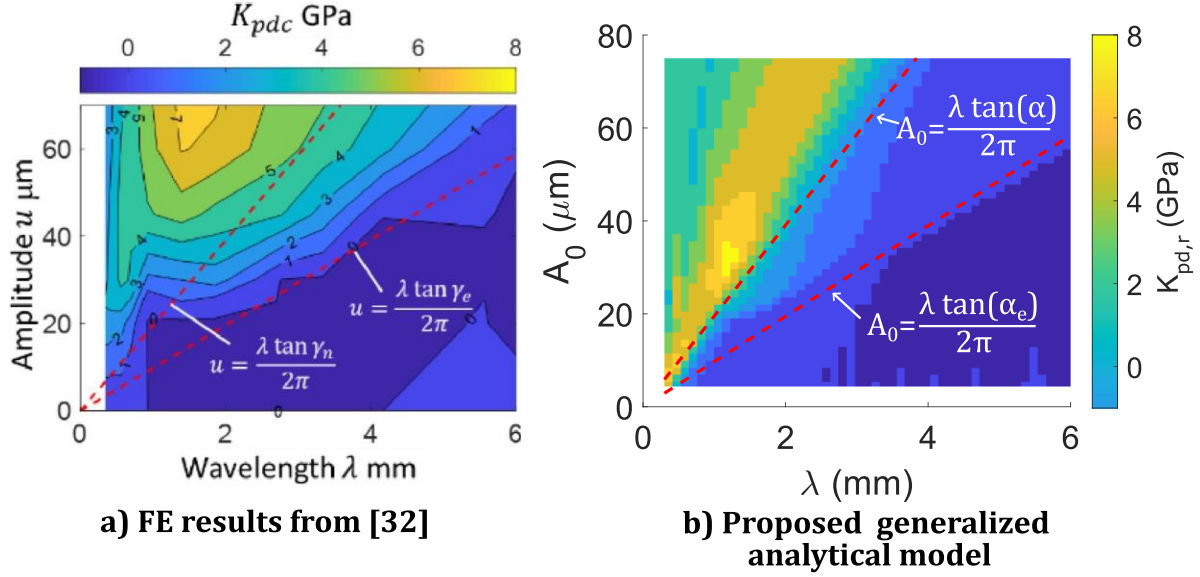


Figure 5.15: Comparison of $K_{pd,r}$ map for a textured tool; a) numerical results³ with u the vibration amplitude, γ_n and γ_e the clearance and effective clearance angles, and K_{pdc} the process damping coefficient; b) proposed analytical results using constant deformation depth

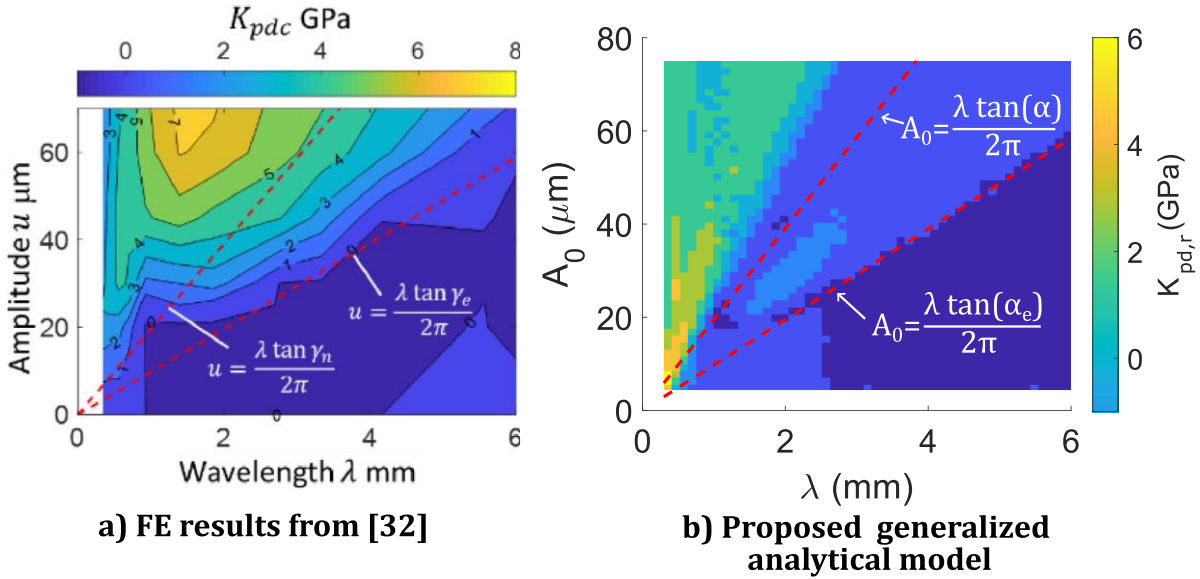


Figure 5.16: Comparison of $K_{pd,r}$ map for a textured tool; a) numerical results³ with u the vibration amplitude, γ_n and γ_e the clearance and effective clearance angles, and K_{pdc} the process damping coefficient; b) proposed analytical results using varying deformation depth (cavity model)

In the numerical results for the standard tool (Figure 5.13 and Figure 5.14), an increase in the process damping is observed as the vibration wavelength decreases and the vibration amplitude increases within the region roughly bounded by Eq.(5.3). The FE results in Figure 5.15 and Figure 5.16 for the textured tool show some differences as compared to those obtained for the standard tool. Specifically, the process damping coefficient becomes high for a wider range of vibration wavelengths and amplitudes, and still follows the boundary defined by Eq.(5.3) for the textured geometry. However, it can be noted that a reduction in the coefficient is observed at low wavelengths (for $\lambda < 1 \text{ mm}$) when compared with the standard tool results. This is theoretically unexpected because the texture should increase the process damping, however FE results state otherwise, for which the reason is unclear and not given in [32]. Still, those results are reproduced with the proposed model, and an important observation on the influence of the cavity model can be made from Figure 5.13-Figure 5.16. It appears that the varying deformation depth model (and thus varying unloaded length of the springs) has a critical impact on process damping and causes inaccurate values especially at high vibration amplitudes. That is because the cavity model is usually used to describe small indentations (e.g., under $40 \text{ }\mu\text{m}$ for Vickers indenters [113]). High vibration amplitudes (up to $80 \text{ }\mu\text{m}$) causes significant indentation of the cutting edge into the workpiece and inaccurate deformation depth value, and hence results in underestimated $K_{pd,r}$ values for high vibration amplitudes. It can also be noted that the wide range of vibration parameters causes unrealistic sets (λ, A_0) , especially for a pair of high amplitude and small wavelength. Such vibrations would be damped in reality or cause breakage of the cutting tool. The predicted process damping with the constant deformation depth and springs unloaded length of 1.016 mm [61] do not have the problem for the high vibration amplitudes, and provide accurate results compared to the numerical values. Overall, it can be seen that the proposed generalized model overpredicts the process damping coefficient for low vibration amplitudes and wavelengths but matches more accurately the linear boundary defined by Eq.(5.3). The range of values for the process damping coefficient matches that of the numerical results when the constant deformation depth is considered and the effect of the texture is easily identified from the results.

Those results validate the ability of the proposed generalized model to analytically predict the process damping phenomenon for an arbitrary cutting edge geometry. Though the depth of deformation has shown to be unusable for high vibration amplitudes, the model manages to predict the process damping by assuming a constant depth value. The results could be generated in less

than an hour for 2500 sets of (λ, A_0) . This provides a decisive advantage as compared to numerical simulations that would have required days long calculation and preparation time.

5.5 Summary

In this chapter, three proposed or improved models of the thesis have been applied to experimental and numerical cases. The temperature prediction model for a sharp tool without flank contact has been compared to numerical and experimental results from literature and has shown good agreement with the numerical and experimental results. Still, the difficulty with temperature prediction is the necessity of complete knowledge of the geometric, cutting and material parameters of the machining process. Assumptions are therefore usually made when some parameters are unknown which brings uncertainty in the prediction results. The introduction of the proposed honed edge heat sources as well as the implementation of the temperature prediction in the process damping model via an iterative algorithm have been verified experimentally and numerically. Though good match occurs with the numerical results, there is a 26% discrepancy in the temperature predictions with the experimental results due to inaccurate description of the temperature dependent material properties above 1000°C for Inconel 718 material. That also illustrates the main drawback of the temperature model, namely its reliance on extensive knowledge of the physical phenomena occurring in the cutting process.

A swift experimental verification of the applicability of the cavity models in machining context is fulfilled, and the predictions roughly match the deformation depth under the tool even though more complex models may be required to include the impact of shear zone deformations and the influence of the uncut chip thickness. Since the deformation depth is not the main focus of the current research, the results are considered sufficient.

The predicted process damping for a standard tool has been verified with numerical, experimental and analytical results. A good match with the existing analytical model is obtained and the effect of the cavity and material models have been observed, namely an increase of process damping for both. An increased error has been observed while comparing with the numerically identified process damping constants, especially at low cutting speeds. This is due to the assumptions of constant vibration amplitude and constraint factor, which should be expected to vary with the vibration wavelength. Again, the slope of the predictions' curve matches that of the numerical results, and predictions only take a few minutes, which is the main advantage of the analytical

model over the numerical and experimental results. The temperature model is used to predict the flank face temperature in the experimental verification and leads to a good match. Stability lobes diagrams have also been generated and compared with the experimental stability evaluations from literature. It is clear that the proposed generalized process damping model accurately predicts the stability limits, and the impact of the workpiece temperature on the stability predictions has also been highlighted.

The ability of the proposed model to be applicable for arbitrary cutting edge geometries is verified numerically with a textured tool and compared with the results obtained for a standard tool. Due to the range of vibration amplitudes used in this verification, the cavity model showed its limitation by causing inaccurate results for high indentations ($> 40 \mu\text{m}$). It is noted that wide ranges of vibrations wavelengths and amplitudes result in unrealistic vibrations, especially at high amplitudes, which puts the limitation of the cavity model into perspective. Still, more accurate results are obtained by assuming a constant experimentally measured deformation depth value from literature, and the effect of the presence of the texture on the process damping has clearly been captured by the proposed model. The advantage of the computation time of the proposed analytical, general model over the finite element simulations and experimental identifications is more pronounced since 2500 process damping calculations have been conducted for each tool geometry in less than an hour on a computer with Intel i7 processor having 4 cores, while the equivalent amount of finite element simulations may require days.

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

6.1 Summary, contributions and limitations

This thesis introduces a new analytical process damping model that eliminates the limitations on the cutting edge geometry. The significant effect of the cutting tool geometry on the machining stability has been well-established in literature. However, considering the effect of arbitrary cutting edge geometry in an efficient generalized analytical model is crucial to propose an alternative to costly numerical and experimental methods, which is the main contribution of this work. Hence, this thesis has paved the way for a virtual cutting tool performance evaluation method, that can replace the currently practiced trial-and-error based strategies with simulation-based ones for cutting tool manufacturers.

The developed process damping model, complemented by an elastic-perfectly plastic material behavior, has been verified with experimental and numerical results in Section 5.4.1. It has additionally been enhanced through the introduction of the Johnson-Cook constitutive law (Section 3.3.3) to include the strain hardening and thermal softening of the workpiece material in the process damping predictions. The inclusion of the effect of honed edge on the workpiece temperature has been introduced (Section 4.4.1) for the first time in literature and an iterative algorithm has been proposed to predict the effect of the temperature on the process damping forces in Section 4.5.4. These modifications have been verified in Section 5.2 with numerical and experimental results. The main downside of the temperature prediction model is the necessity to have complete knowledge of the physical phenomenon occurring during cutting (cutting forces, tool geometry, work material properties), which may be difficult to obtain. A cavity model has been used to predict the deformation depth in the indented workpiece, which is a necessary and yet intricated component of the proposed process damping model. Its applicability in machining context has been verified through experimental measurements in Section 5.3, but still have limitations when the dynamic interference between the cutting edge and the workpiece becomes too severe as shown in Section 5.4.2.

The consideration of the tool geometry as an input for an analytical generalized process damping prediction for the first time in literature is the principal contribution of this work. This is accompanied by the inclusion of strain hardening and thermal softening in the process damping predictions. More precisely, the cutting temperature prediction has been enhanced by introducing

the honed edge contribution and has been directly integrated into the process damping predictions through an iterative algorithm.

The main limitations of the generalized process damping model are:

- The simple contact model using Winkler foundations does not include any tangential deformations, nor continuity in the indented material which can lead to underestimation and discontinuity of the contact stress.
- The prediction of the deformation depth does not include the tangential deformations and can fail for excessive indentations.
- The constraint factor is assumed to be constant but its variation can have a significant impact on the accuracy of the process damping forces by linking the contact stress to contact pressure.
- The temperature calculation requires complete knowledge of the machining process and material properties, which is not accessible easily.
- The temperature calculation is time-costly. It can provide results in under 10 minutes depending on the discretization of the honed edge, which can lead to long computation time in the iterative algorithm.
- The process damping phenomenon is highly dependent on the vibration amplitude which is not easily known a priori.

6.2 Future work

The analytical modeling of the process damping is still in its early stages. Moreover, it is a very complex phenomenon affected by the vibrations, contact mechanics, and thermal modeling. This study attempts to include all these components for the first time with numerous assumptions. Here is a list of future research directions to lift the current assumptions and limitations and to more accurately predict the real phenomenon of process damping:

- An improvement of the Winkler foundation to include tangential deformations should be done, as it is the basis of this proposed model. The use of more complex foundations such as Pasternak's could permit it.
- Direct prediction and use of contact stiffness between the tool and workpiece could avoid the use of the deformation depth in the Winkler foundation.

- A more complex deformation depth description in machining could be used. Finite element simulations and experimental tests using DIC cameras could provide empirical relations.
- Inclusion of a geometry and indentation varying constraint factor could be a valuable improvement to the proposed model.
- Replacement of the discretization of the honed edge by the proposed analytical relation in Section 4.4.1 could address the important computation time of the current temperature model which is mainly due of the discretization of the honed edge radius.
- Inclusion of the prediction of the vibration amplitude at the border of stability could greatly enhance the accuracy and applicability of the model.

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