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UNIVERSITÉ DE MONTRÉAL

A PARAMETRIC STUDY OF TUBE-TO-TUBESHEET JOINTS

par

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DÉPARTEMENT DE GÉNIE MÉCANIQUE

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A PARAMETRIC STUDY OF TUBE-TO-TUBESHEET JOINTS

présentée par Hewu Ma

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a été dûment acceptée par le jury d'examen constitué de:

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Les joints plaque-tubes sont critiques pour la performance des échangeurs de chaleur tubulaires tels que ceux des générateurs de vapeur, des refroidisseurs industriels ainsi que des condenseurs. Dans le processus de fabrication des échangeurs de chaleur, les trous de la plaque sont percés à un diamètre légèrement supérieur au diamètre extérieur des tubes dans le but de faciliter le processus d'installation. Les tubes sont alors fixés à la plaque par un processus d'expansion qui crée une interférence entre eux.

La jonction résultante doit être suffisamment serrée pour prévenir toute fuite et empêcher tout mouvement des tubes. L'expansion hydraulique est de plus en plus utilisée pour effectuer un serrage par interférence entre les tubes et la plaque. Le processus d'expansion crée des contraintes résiduelles de valeurs élevées dans la paroi des tubes. Lorsqu'elles sont en traction, ces contraintes augmentent la susceptibilité des tubes à la rupture par corrosion sous tension qui est une cause majeure de rupture des échangeurs de chaleur. Ce mécanisme de rupture est amplifié par la présence d'une crevasse entre la paroi extérieure du tube et le trou dans le plaque.

Par ailleurs, la stabilité des tubes dépend énormément du niveau de leur force d'arrachement. Ainsi, une combinaison d'un niveau minimum de contraintes résiduelles de traction avec un niveau maximum de pression d'interférence résiduelle entre les tubes et la plaque constitue la combinaison la plus désirable.

Dans la présente thèse , la méthode des éléments finis non-linéaires est utilisée pour étudier le problème. À cause des nombreux paramètres reliés aux dimensions, à la fabrication et aux matériaux, la méthode de conception orthogonale est utilisée pour minimiser le nombre d'analyses tout en assurant une compréhension assez précise de l'influence de tous les paramètres impliqués. En se basant sur les résultats obtenus, des équations empiriques sont proposées pour déterminer les contraintes résiduelles maximales et la pression de contact résiduelle. Ces équations peuvent être d'une grande utilité pour les concepteurs et les fabricant d'échangeurs de chaleur.

ABSTRACT

Tube-to-tubesheet joints are critical to the reliability of tubular heat exchangers such as steam generators, industrial coolers and condensers. In the fabrication of heat exchangers, the holes in the tubesheet are drilled slightly larger than the outside diameter of the tube in order to ease the installation process. The tubes are then attached to the tubesheet by an expansion process which creates an interference fit between them. The resulting joint must be tight enough to prevent leakage and restrain tube movement. Hydraulic expansion is becoming one of the most common ways of achieving the interference fit between the tubes and the tubesheet. Due to the expansion process, a high level of residual stresses is created in the tube wall. When tensile, these stresses increase the susceptibility of the tube to Stress-Corrosion Cracking (SCC) which is a major cause of failure of heat exchangers. SCC is further enhanced by the presence of a crevice between the outer tube wall and the hole in the tubesheet, near the shell-side surface.

On the other hand, the stability of the tubes depends greatly on the level of their pull-out force. Therefore, a combination of minimum tensile residual stresses with a maximum residual interference pressure between the tubes and the tubesheet would be the most desirable design.

In this thesis, the nonlinear finite element method was used to investigate the problem. Due to the many dimension-, fabrication-and material- related parameters, the orthogonal design method was used to minimize the number of analyses while providing an accurate understanding of the influence of all parameters involved. Based on the results, some empirical equations for determining maximum residual stresses, residual contact pressure and apparent wall reduction values, are proposed. These equations could provide guidance to the designers and manufacturers of the tube-tubesheet assemblies.

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LIST OF SYMBOLS

a	Outer diameter of tube
a_i	Inner diameter of tube
a'_i	Final inner diameter of tube after expansion process
a_o	Inner diameter of hole in tubesheet
c	Initial clearance between tube and tubesheet
t	Thickness of tube
t'	Final Thickness of tube after expansion process
s	Tube pitch
P	Expansion pressure
Y_{st}	Yield strength of tube
Y_{ss}	Yield strength of tubesheet
E_t	Young's modulus of tube
E_s	Young's modulus of tubesheet
S^*_z	Maximum tensile residual axial stress in transition zone
S^*_h	Maximum tensile residual hoop stress in transition zone
k	Apparent tubewall reduction
k'	Real tubewall reduction
f	Frictional coefficient between the tube and tubesheet
P^*	Average value of residual contact pressure at the outer surface of tube

P_i^*	Average value of the residual contact pressure at the outer surface of central tube, after its own expansion, "Two-Step Method"
P_j^*	Average value of the residual contact pressure at the outer surface of central tube after the expansion of adjacent hole, "Two-Step Method"
R_e	Equivalent radius of tubesheet, "Two-Step Method"
P_e	Equivalent load, "Two-Step Method"
R_o	Outer radius of tubesheet (for single tube surrounded by an annulus representing the tubesheet)
$L_r(m^n)$	Orthogonal array
	where r number of calculations to be performed
	m number of levels for each parameter
	n maximum number of parameters to be analyzed
I_j, II_j	Summation of the results obtained at levels 1 and 2 respectively, of the parameter in column j
V	Sum of squares
ν	Number of degrees of freedom
S^*_z	Maximum tensile residual axial stress
S^*_h	Maximum tensile residual hoop stress
y	Response factor
x	Predictor factor
y_i	ith observation of response factor

x_{ij} ith observation of the jth predictor factor

E(y) Expected value of y for a given x, regression analysis

ß Coefficient

ε Random error term, regression analysis

$F(\nu_1, \nu_2)$ F distribution with ν_1 and ν_2 degrees of freedom

$F_\alpha(\nu_1, \nu_2)$ Critical value of F

F^* Sample value of F test statistic

H_o Null hypothesis

t( ) t distribution with degrees of freedom

$t\alpha\{\ \}$ Critical value of t distribution with degrees of freedom

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CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

1.1 INTRODUCTION

Heat exchangers are used to transfer thermal energy between fluids at different temperatures. Their applications are important in an extremely wide range of industrial plants. They are mostly used in process, power, automotive, air conditioning, refrigeration, cryogenics, heat recovery, manufacturing, etc...[1]. In general, heat exchangers represent about eleven per cent (11%) of total investment in chemical plants and about forty per cent (40%) in oil refineries[2]. Their advance, rationality and reliability have influence over the quality, quantity and costs of production. So they are key components of many products available in the marketplace.

Heat exchangers may be classified according to the transfer processes, degree of surface compactness, construction features, flow arrangements, number of fluids, and fluid phase changes or process function. But they are frequently characterized by the construction features. Examples described below are some major construction types: tubular, plate, extended surface, and regenerative exchangers[3].

Tubular heat exchangers are used widely because of their high reliability, large suitability and ripe experience of design and fabrication. They are, in their various construction forms, the most widespread and commonly used basic heat exchanger configuration in the process industries. They are also used in conventional energy production as condensers, feedwater heaters, and steam generators for pressurized water reactor plants. They are proposed for many alternative energy applications including ocean thermal and geothermal. They are also used in some refrigeration and air conditioning services. One of the reasons for this near-universal acceptance is that tubular heat exchangers provide a relatively large ratio of heat transfer area to volume and weight. They provide this surface in a form which is relatively easy to construct in a wide range of sizes and which is rugged enough mechanically to withstand normal shop fabrication stresses, shipping and field erection stresses, and normal operating conditions. The tubular heat exchangers can be reasonably easily cleaned, and those components which are the most subject to failure such as gaskets and tubes, can be easily replaced. They offer great flexibility of mechanical features to meet almost any service requirements. Finally, good design methods are available, and the expertise and shop facilities for their successful construction are widespread.

There are considerable combinations of heat exchanger designs because the core geometry can be varied easily by changing the tube diameter, length and arrangement. Tubular exchangers can be designed for both high and low pressures relative to the environment and/or relative to the internal fluids[4]. Typical tubular heat exchangers are shown in Figure 1.1. They are built of round tubes mounted in a cylindrical shell with the tube axis parallel to that of the shell[5]. One fluid flows inside the tubes, the other flows inside the shell across and along the tubes. The major components of the exchanger are the tubes, the shell, the front and rear end heads, the baffles, and the tubesheets. The energy transfer between fluids occurs by conduction and convection across the tubes wall. The tubes are therefore the basic component of the tubular heat exchanger. They are generally drawn or extruded seamless metal. The metal is usually low-carbon steel, low alloy steel, copper, Admiralty, cupronickel, inconel, aluminum (in the form of various alloys), or titanium, though many other materials may be specified for special applications.

The two tubesheets shown in Figure 1.1 (No. 6 and 15) support the tubes and separate the two fluids. To prevent mixing of the fluids, the tubes must fit snugly in the tubesheet holes.

Figure 1.2 shows schematically a typical single tube-tubesheet joint.

These joints are the most critical elements of a tubular heat exchanger due to their direct effect on its reliability. There are literally thousands of joints in a typical heat exchanger and each joint must be free of defects. However, the part of the tube near the tubesheet, known as the transition zone (See Figure 1.2) is stressed more severely than the main body of the tube due to the expansion process; this point will be discussed in detail later. Yet the configuration of the joint allows only limited nondestructive examination, which makes the tube-to-tubesheet joint a cause of many heat exchanger failures. Therefore this region must be studied carefully[6].

A secure tube-to-tubesheet joint can be obtained by mechanically expanding the tube in the tubesheet and/or by welding the tubes end to the tubesheet metal around the hole. The joining technique must lend itself to mass production and to uniformity of quality.

For decades the common method for obtaining a tube-to-tubesheet joint was by mechanical expansion through rolling of tubes. Introduced during the mid-nineteenth century [7], this method of tube-to-tubesheet fastening continues to be the dominant technique to this day. The roller expander (Figure 1.3)[8] consists of a cylindrical cage which loosely holds a cluster (3 to 7) of hardened tapered steel rollers. A similarly tapered mandrel is inserted through the cage causing the rollers to make line contact with the tube surface on the outer side and with the mandrel on the inside. A pneumatic or electric drive turns the mandrel (usually in the range of 400 to 1000 rpm) which, in turn, causes the roller to rotate. The axis of the rollers is set at a small angle with respect to the mandrel's axis of rotation. This causes the rotational motion of the mandrel to produce an axial force in addition to rotatory torque on the rollers. Since the rollers are kept from moving axially by the thrust collar, the reactive force makes the mandrel "force-feed."

In pneumatic drives, a pre-set limit torque switch controls the extent of rolling. When rolling tubes, an expander is inserted into the tube end and the tapered mandrel rotated. Feeding the mandrel inward causes the expander rollers to be forced apart and, by rolling over the inside tube surface, cold-work the tube metal. The tube is enlarged and contacts the tube-hole surface; then, because the tube hole is a restraining barrier, further expanding deforms the tube metal and forces it into more intimate contact with the metal of the tube hole. Since all displaced tube metal cannot escape radially, it flows from the centre to each end of the rolled joint. The tube-hole metal is also affected, and the tube hole is slightly enlarged.

Roller expanding method was repeatedly well received by the users. But the high local stresses and deformations which are generated as a result of metal contact between the rolling block in the tube and the tube itself, make stress corrosion cracking in the tube easier to occur. Also, it is not possible to completely remove the initial gap between the tube and the tubesheet because rolling beyond the thickness of the plate is inconceivable. This unsealed gap can then easily develop a corrosion spot and thereby exposes the tube to damage.

In the case of straight tubes welded to two rigid tubesheets, rolling means incorporating high axial stresses in the tubes. These stresses whose magnitude is dependent upon the setting of the rolling operation as well as the rolling length and gap between the tube and bore hole can no longer be relieved. The heat insulating air gap between the tube and the tubesheet where no rolling takes place results in an additional undesirable load on the seam or the rolled joint during transient thermal loading. This can lead to leakage by frequent plant start-up and shutdown[9].

A basic improvement in the rolling process cannot be expected because of the mechanical principle involved. Thus the stated undesirable characteristics and disadvantages of a tube to tubesheet joint can only be eliminated by a fundamental modification in the process. This has led to the development of the hydraulic expansion process.

Figure 1.4 shows a simplified arrangement of the expansion equipment. In order to be able to expand with water or any other liquid, two separate circuits are necessary. The pump first conveys the oil to the medium separator "5". The expansion chamber of probe "3" is then filled with water between the two sealing elements and the piston of the booster "6" is turned to the starting position. When the solenoid control valve "7" changes into the second operating position, the expansion operation of the tube in the tube plate takes place. The large piston in the booster is pushed forward during this process. The liquid pressure in the probe is then greater than the oil pressure in the booster by a factor equal to the ratio of the two piston surface areas.

At the same time, the pistons of the medium separator are pushed back and the water is sucked from the water reservoir "4" into the medium separator. The desired expansion pressure can be set on the overflow valve and it can be read on the pressure gauge during the expansion operation. The third position on the control valve is idling, where no load is applied to the probe, the medium separator, and the booster.

The principle of hydraulic expansion is explained schematically in Figure 1.5 [9]. As the hydraulic pressure in the expanded zone increases, the tube is first deformed elastically (From A to B) until the yield point (B) is reached and then plastically deformed in the expanded region until it has bridged the gap between the outside diameter of the tube and the wall of the bore hole (point C). By increasing the pressure further, the tube is pressed against the wall of the bore hole (From C to I) and simultaneously the tubesheet deforms elastically and then plastically (From F to E). The maximum applied expansion pressure will then be released causing an elastic recovery of tubesheet (From E to G), and of tube (From I to H). The different slopes of the elastic curves indicate the different material properties of the tube and tubesheet.

Due to the unequal permanent circumferential deformations during the expansion phase, residual contact pressure between the tube and tubesheet is created during unloading. The magnitude of elastic recovery is dependent on the expansion pressure level, the initial gap and the material properties of both tube and tubesheet.

In addition to the residual contact pressure, residual stresses are introduced simultaneously in the transition zone. If these stresses are tensile and above a certain threshold value (typically 100 MPa), the tube becomes particularly susceptible to Stress-Corrosion-Cracking (SCC)[10].

On the other hand, in order to improve the thermal efficiency of heat exchangers in chemical and oil refineries, their dimension and the velocity of the fluid are regularly increased causing vibration of tubes. Therefore they are commonly equipped with baffles (Figure 1.1 No.28 and 29) which produce a cross flow around the tube bundles which while favourable for the heat transport are conducive to tube vibrations. If the amplitudes of the vibration become too high, fretting corrosion and erosion of the tubes at their roots and at their transition zone may occur. The higher level of tensile residual stresses will lead to fatigue failure of the joint [11].

A number of papers have investigated the strength of the tube-to-tubesheet joint, with emphasis on contact pressure, holding power, and tightness against leakage. However, residual stresses in the transition zone of each tube have received limited attention, yet they constitute an important aspect of overall strength of the tube-to-tubesheet joint.

1.2 GENERAL BIBLIOGRAPHIC REVIEW

The earlier contributions were basically about rolling expansion technique. Progress in the solution of tube-rolling problems was started in the years 1920's. In 1935, Fisher and Cope[12] dealt with "Entrance End of Rollers" in detail. The shape of the entrance end of expander rollers has a definite influence on the strength and stability of rolled joints. They gave a recommendation value for the entrance ends of all roller used for the rolling-in of tubes. In 1930, Thum and Jantscha [13] studied the rolling speed and feed angle. They proved by test on over 5000 rolled joints that fast machine rolling gave superior results over slower-rolling methods.

Only by 1940, when a new expander was developed, had the weakness of the standard expander been overcome, making the expansion of tubes in seats of unlimited length possible. In 1943 Maxwell [14] mentioned some practical aspects of producing optimum roller expanded joints. One of his important conclusions was that the optimal point of expansion is reached when the metal of the tubesheet surrounding the tube exerts a spring-back measure slightly below the elastic limit of the metal. Maxwell studied a number rollers and recommends 3-roller expanders for use on tubes having nonuniform wall thickness.

Goodier and Schoessow [15] studied the distribution of contact pressure and deformation of the tube during and after expanding, and compared their results with test results. They discussed the variation of residual contact pressure with the thickness of tube, effect of different yield stress of tube and tubesheet.

In 1943 Grimison and Lee [16] provided some results of an experimental investigation to determine the fundamentals involved in tube expanding, the various practical methods of measuring the degree of expansion, the optimum degree of expanding and the ultimate strengths of expanded joints under various conditions.

The plastic states of stress were investigated for various types of material stress-strain characteristics of tube and tubesheet by Nadai [17] using the elasto-plastic theory.

Fisher and Cope [18], in 1943, developed extensive investigations on the procedures of rolling-in small tubes, devised an entirely new rolling expansion technique. A detailed account of controlling rolling and the improvements made in its design can be found in the paper.

Fisher and Brown [19], in 1954 described the experiences gained by expanding tubes into various type of powerhouse equipment. They have assembled all of the information available to them on the art of tube rolling and combine this information with their experience.

In the 1970's, progress in the expansion techniques was achieved by the introduction of hydraulic expansion. In 1976, Krips and Podhorsky [20] described this new method for the anchoring of tubes, and provided the means of calculating contact pressure on the basis of a simple cylindrical model. They believed that the conventional method of mechanical rolling for expanding tubes into the tubesheet had for some time been the target of justified criticism both from engineers supplying the chemical industry and from those supplying equipment for nuclear plants. Any improvement to this mechanical method can only be partially successful and thus one is compelled to look for a different method. Indeed, the mechanical deformation produced by rolling caused stresses which could not be accurately determined because of the inherent irregularities of the method; such stresses increased the tube susceptibility to corrosion; the end gap could not be closed by rolling without risking shearing of the tube. The advantage of the hydraulic method rests in the fact that the working pressure of the hydraulic fluid can be accurately determined producing a consistent and repetitive process, thus increasing the reliability of the equipment.

In 1979, Podhorsky and Krips [21] discussed the advantages of the hydraulic expansion process and the computation of tube fastening. In order to improve the accuracy of their formula, a correction coefficient was used. This coefficient had to be determined by performing tests of different tubes. So it was inconvenient for the calculation of residual contact pressure.

In 1983, Singh and Soler [22] tackled the design of tube-to-tubesheet joints in detail, and in 1984 Soler and Hong [23] studied the influences of geometry, materials, and loading on the final tube-to-tubesheet contact pressure. They developed a special purpose computer solution of the two dimensional rolling problem including elastic-plastic behavior and large deformation to establish the residual contact pressure. The motivation was to present a modern analysis tool to predict the residual contact pressure between tube and tubesheet. They did not include the computation of residual stresses in their solution.

That same year, Druez and Bazergui [24] developed an experimental procedure for the determination of through-thickness residual stresses in straight tubes. The method involved the use of a small number of strain gauges which measure the strains released by controlled material removal during chemical etching. Details were given on the technique and its theoretical principles and results were presented for as-received and for stress-relieved tube samples.

In 1984, Scott, Wolgemuth and Aikin [10] did experimental and theoretical work to determine residual stresses in transition zone of tube-to-tubesheet joints. X-ray diffraction, stress corrosion cracking test and strain gauging were the measuring techniques used. Extensive use of finite element analysis was also made. They concluded that only hydraulic expansion could produce a low-stress joint. Their orientation of investigation was correct, but they didn't do quantitative analysis to predict residual stresses.

In 1985, Druez and Bazergui [25] used this approach to determine the residual stresses in roller-expanded thin tubes. They presented an experimental technique for the detailed determination of the state of residual stresses in the roller-expanded zone of heat-exchanger tubes. They also determined the stresses caused by the interference between the tube and the tubesheet. The same year Bazergui and Marchand [26] published the results of a comparative study on the residual stresses and residual contact pressure created by several methods of tube expansion. The merits of each of these methods were discussed with regards to: the level of residual contact pressure and the level of tensile residual stresses.

In a more recent paper (1987), Jawad, Clarkin and Schuessler [27] investigated the effect of some parameters, including properties and method of attachment, on the strength of tube-to-tubesheet joints.

Aufaure, Boudot, Zacharie, and Proix [28] reported the presence of in-service cracks in the transition zone from the expanded to the nonexpanded portions of the tubes and presented theoretical and experimental results of these residual stresses.

In 1987, Weinstock, Reinis and Soler [29] refined their previous analysis [23] by including strain hardening and temperature-dependent properties and examined the effects of these additional inclusions on the theoretical prediction of the residual tube-tubesheet contact pressure. They used the two dimensional simple annulus model and ignored the nonsymmetry of the real model. But they were conscious that ignoring the non symmetry may cause error. Wang and Soler [30] investigated the residual contact pressure between the tubes and the tubesheet by modifying their previous single tube-to-tubesheet analytical technique. They discussed the effect of adjacent holes and the effect of boundary conditions on the tube-to-tubesheet joint annulus model by using the finite element method.

Chaaban et al [31] have studied the influence of ligament thickness, material strain hardening, sequence of the tube expansion and the level of applied expansion pressure on the interference fit. The effect of the initial clearance on residual stresses that are introduced in the transition zone was also analyzed. For the first time the effect of adjacent holes and that of sequential expansion were investigated. But the analyses were limited to some specific cases and could not be generalized.

In 1989, Updike and Kalnins [32] introduced a simplified axisymmetric model of a rolled tube-to-tubesheet joint. Their objective was to determine the residual stresses in the transition zone and residual contact pressure in the expanded zone. They ignored the effect of adjacent holes and the effect of expansion sequence.

In 1991, Martin [33] summarized the effect of several factors that influence the degree of integrity of the tube-to-tubesheet joints. Middlebrooks [34] summarized the results of recent analytical studies related to the residual stresses in the expanded tube.

Recently, many authors used the finite element method to analyze the tube-to-tubesheet joints, namely: Weinstock, Reinis[30], Wang[31], Hong, Soler[22,23], Jawad, Clarkin and Schuessler[28]. However, in the cases above, the problems have been simplified to a two-dimensional elastic-plastic analysis, and only residual contact pressure was mentioned. The effect of adjacent holes and the effect of sequential expansion were ignored. More of the non-symmetrical effects were not considered. The effect of individual parameters was investigated one at a time and not in a systematic overall approach. Such an approach would provide designers with a more effective, easy-to-use and reasonable accurate method to predict residual contact pressure and residual stresses as a function of the various geometric and material parameters. This is the major purpose of this thesis.

1.3 OUTLINE OF INVESTIGATION

As stated earlier, the strength of a tube-to-tubesheet joint is influenced by many factors such as: method of attachment, material properties and details of construction. Perhaps one of the basic criteria for an optimum design would be to increase the residual contact pressure between the tubes and tubesheet while keeping the tensile residual stresses in the transition zone as low as possible. In order to achieve this optimum, a comprehensive study of the problem is necessary.

The objective of the present research work, therefore, is to study the influence of every parameter involved in the hydraulic joint process using analytical and numerical methods (Finite Element Method, FEM) in order to obtain a simplified design procedure for making tube-tubesheet joints stronger and more reliable. In particular, the construction geometry of the joint, the matching of tube and plate materials, and the expansion pressure level will be investigated in detail.

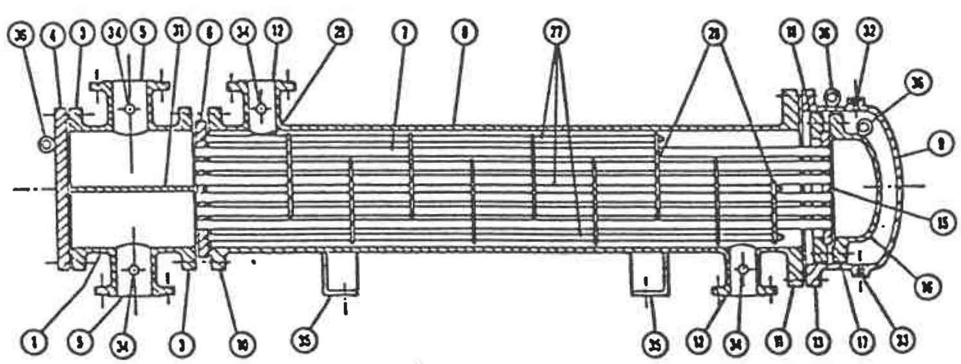
Table 1.1 shows the design parameters that will be considered in the present work. Results will be combined into empirical design equations and graphs.

The proposed FEM models will be presented in Chapter 2. Chapter 3 will present the statistical approach used to analyse the numerical results. The final results, design proposal, and discussion are presented in Chapter 4. Finally, Chapter 5 will include the conclusions and recommendations.

Dimension related parameters	Fabrication parameters	Material parameters
1. Thickness of tube, t 2. Outer diameter of tube, a 3. Tube pitch, s 4. Initial clearance, c	5. Expansion pressure level, P 6. Sequence of expansion process	7. Yield strength of tube, Y_{st} 8. Yield strength of tubesheet, Y_{ss} 9. Young's modulus of tube, E_t 10. Young's modulus of tubesheet, E_s

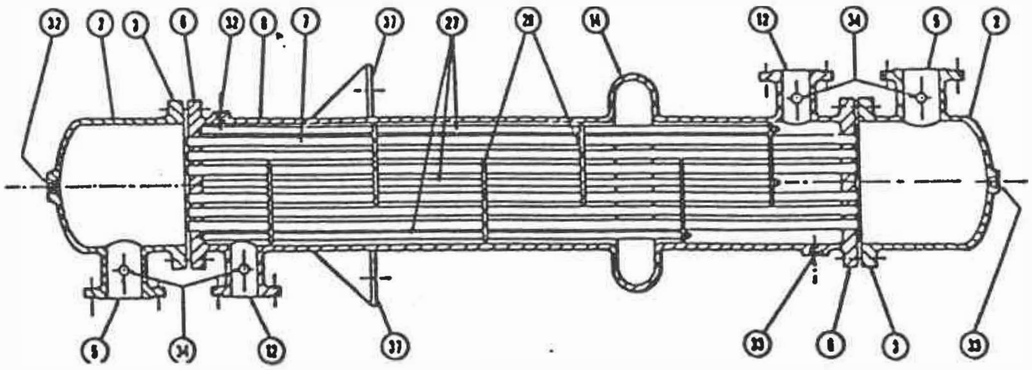
Table 1.1 Parameters involved in the design of tube-tubesheet joints

1. Stationary Head—Channel
2. Stationary Head—Bonnet
3. Stationary Head Flange—Channel or Bonnet
4. Channel Cover
5. Stationary Head Nozzle
6. Stationary Tubesheet
7. Tubes
8. Shell
9. Shell Cover
10. Shell Flange—Stationary Head End
11. Shell Flange—Rear Head End
12. Shell Nozzle
13. Shell Cover Flange
14. Expansion Joint
15. Floating Tubesheet
16. Floating Head Cover
17. Floating Head Flange
18. Floating Head Backing Device
19. Split Shear Ring
20. Slip-on Backing Flange
21. Floating Head Cover—External
22. Floating Tubesheet Skirt
23. Packing Box Flange
24. Packing
25. Packing Follower Ring
26. Lantern Ring
27. Tie Rods and Spacers
28. Transverse Baffles or Support Plates
29. Impingement Baffle
30. Longitudinal Baffle
31. Pass Partition
32. Vent Connection
33. Drain Connection
34. Instrument Connection
35. Support Saddle
36. Lifting Lug
37. Support Bracket
38. Weir
39. Liquid Level Connection

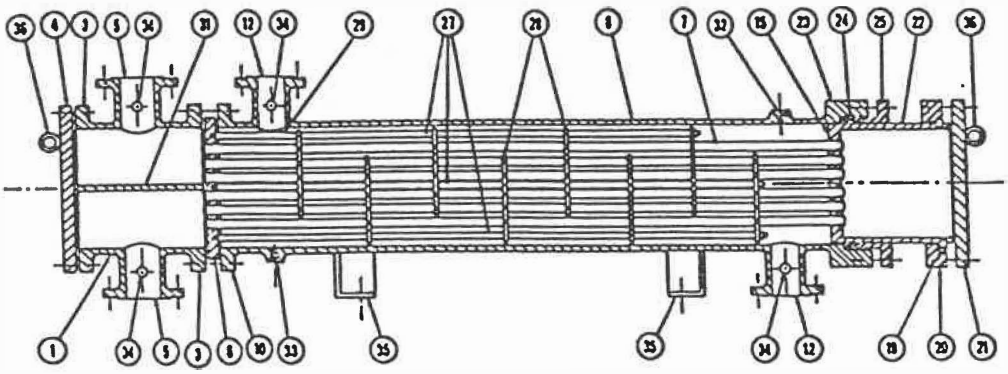


Split-ring floating head exchanger with removable channel and cover, single pass shell

Figure 1.1 Typical heat exchanger [5]



Fixed tubesheet exchanger with bonnet, single pass shell



Outside packed floating head exchanger, single pass shell

Figure 1.1 Typical heat exchanger (con't)

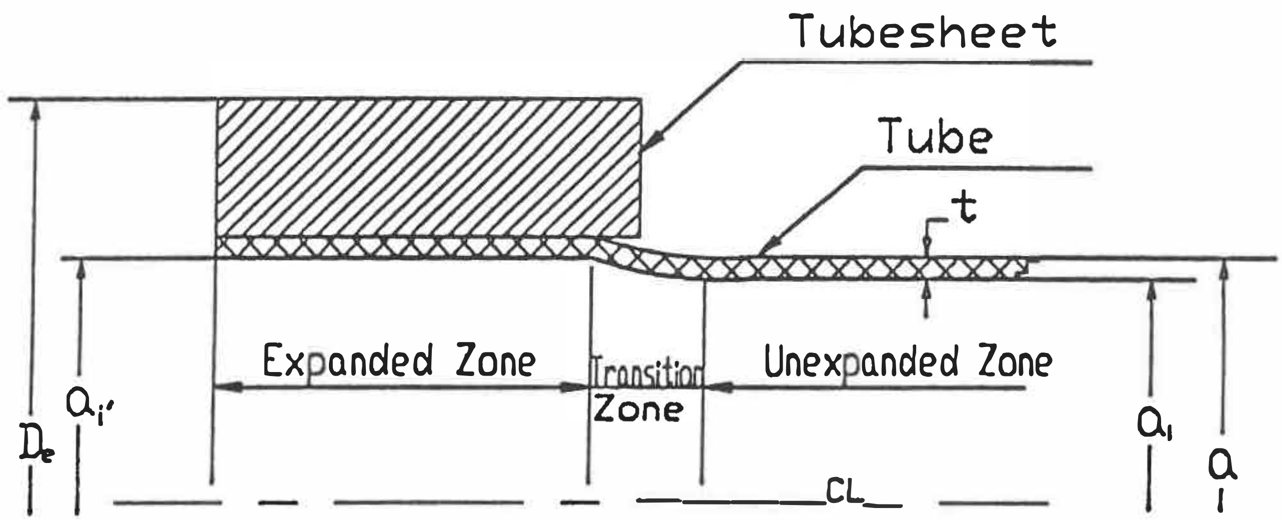


Figure 1.2 Geometry of model of single tube-to-tubesheet joint

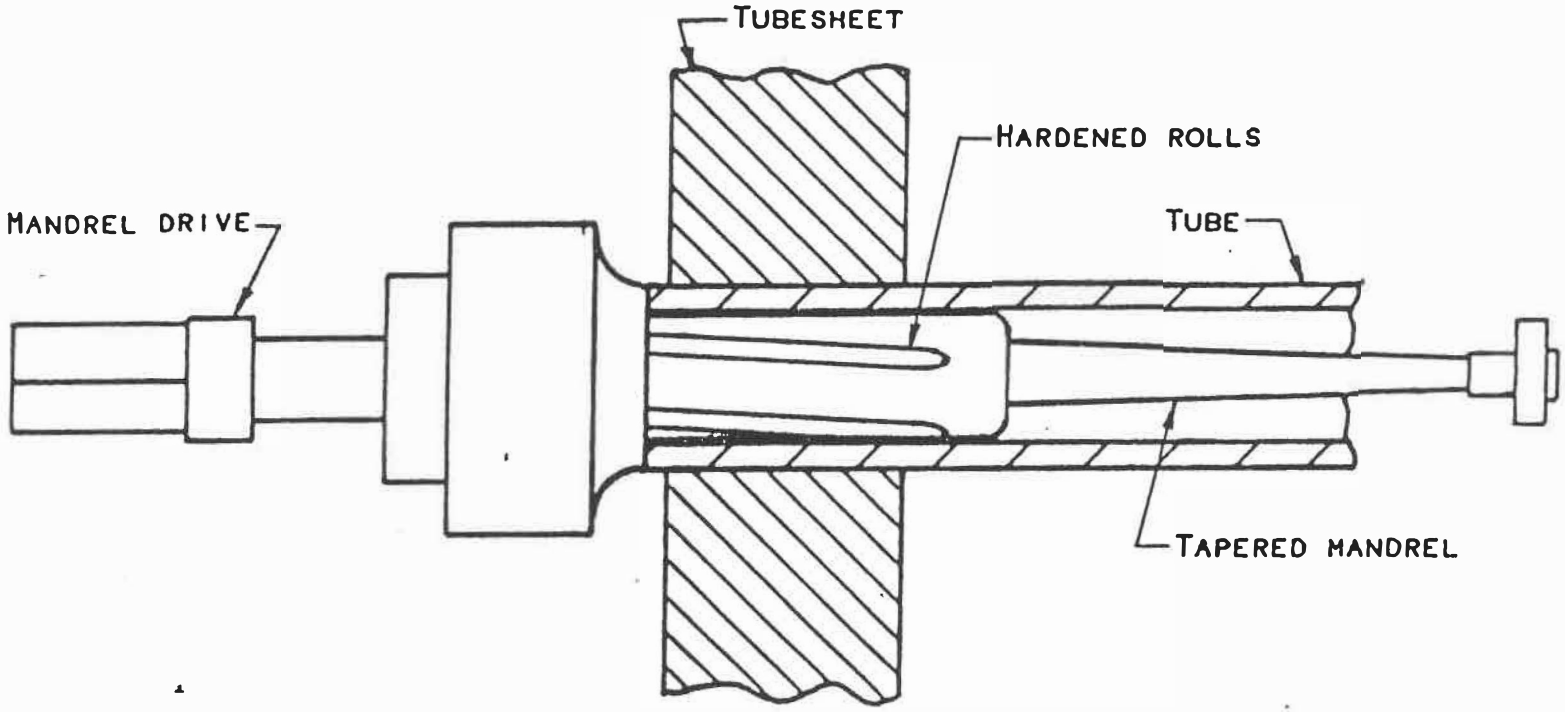


Figure 1.3 Typical roller expander [8]

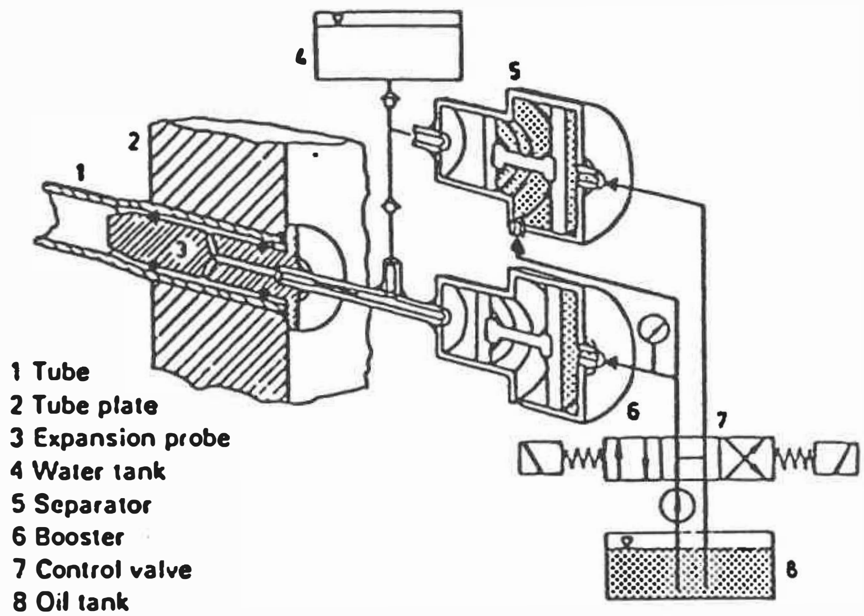


Figure 1.4 Layout of hydraulic expander [21]

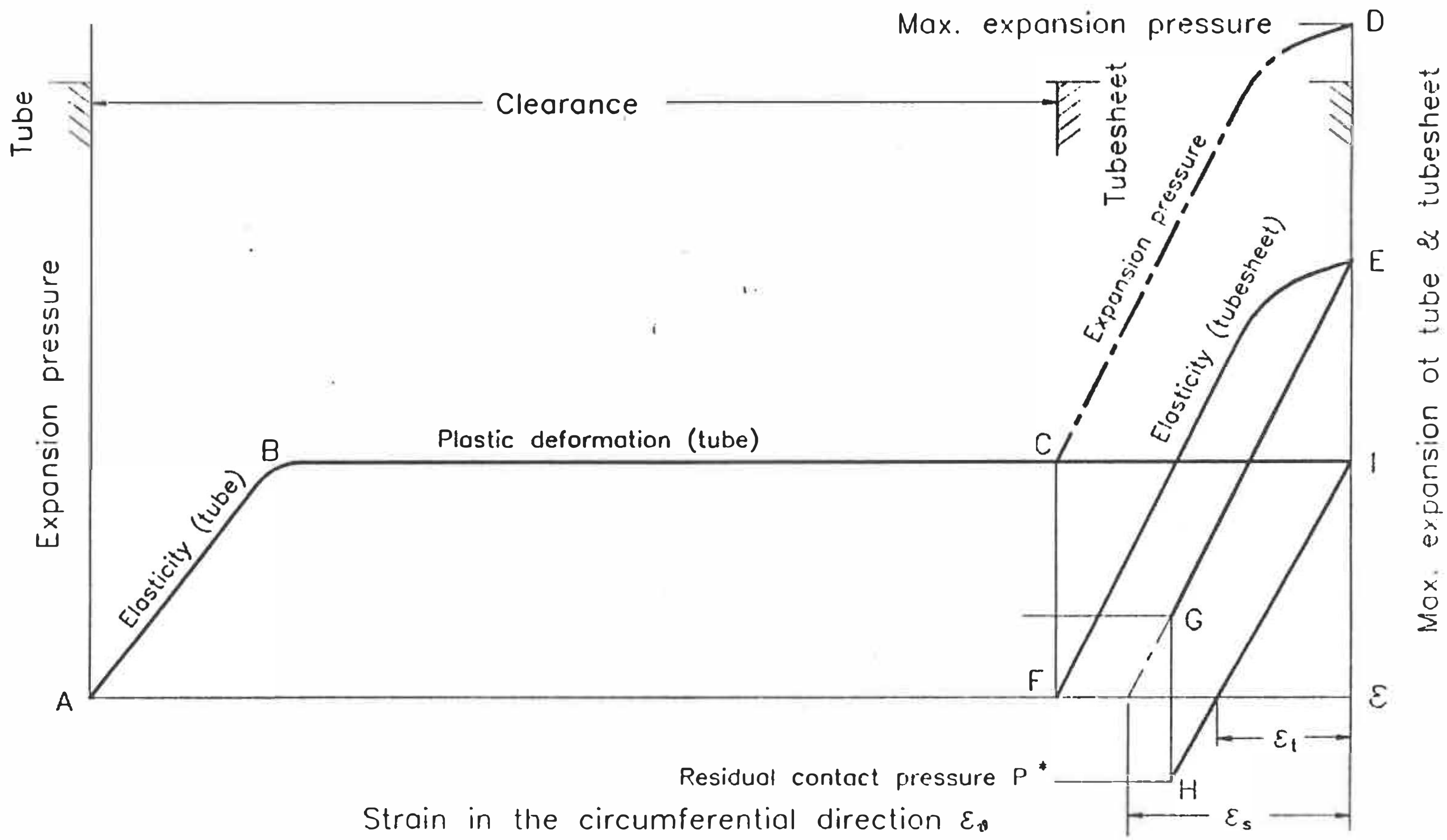


Figure 1.5 The principle of hydraulic expansion

CHAPTER 2

PROPOSED FINITE ELEMENT MODEL FOR THE STRESS ANALYSIS OF TUBE-TO-TUBESHEET JOINTS

2.1 INTRODUCTION

As stated in chapter 1, Stress-Corrosion-Cracking (SCC) has a strong effect on the strength of tube-to-tubesheet joint. Sensitivity to SCC depends on the level of tensile stresses in the material which induce operating and residual stresses. But in our study we will only consider residual stresses. So the stress analysis of tube-to-tubesheet joint is of great importance. So far, most analyses have involved a single tube surrounded by an annulus representing the tubesheet [23] [33] (Figure 2.1). When using this simplified axisymmetric model, however, there is a question concerning the appropriate tubesheet annulus diameter to be considered and the appropriate outer boundary conditions to be applied in light of the fact that there are surrounding tube holes. From [31] and [32], it appears that the simple annulus model is not a valid representation of the real assembly, since both geometry and loading are not consistent particularly when the sequence of tube expansion is considered.

This problem would be solved using the 3-D nonlinear finite element method, but this would be very costly and time consuming because of the need to elasto-plastically load and unload the structure. The objective of our research is to introduce a simplified, yet accurate, approach which will be referred to as the "Two-Step Analysis Method".

2.2 FINITE ELEMENT ANALYSIS

The finite element method is a powerful tool for predicting stress distributions at the surface of the tube and through its thickness. It is also useful for performing a parametric study, due to the flexibility in varying geometric, material and loading parameters with various kinematic boundary conditions.

The elastic/ perfectly plastic analyses have been done using the ABAQUS General Purpose Finite Element Program [35]. ABAQUS is a large code designed for linear and nonlinear analyses of structures in both static and dynamic regimes. For this work the tube-to-tubesheet system is modelled as a static, nonlinear elastic-plastic problem.

2.2.1 Limitation of the 3-D finite element analysis

The choice of the finite element model will have a direct influence on:

1. The accuracy of the results.
2. The required CPU time i.e. the computer cost.

The accuracy of results depends on a suitable degree of the density of the model mesh and the type of element used. When using the plane stress element (CPS8) to analyze the seven tube model (to be described in the following section), the model is divided into 282 elements and 948 nodes. The number of degrees of freedom of the model is 1896 (Fig. 2.2). The CPU time for one run is 9 minutes and 50 seconds (on our IBM ES-9000 computer). When using the axisymmetric element (CAX8) to analyze the single tube model, the model is divided into 116 elements and 415 nodes. The number of degrees of freedom of the model is 830 (Fig. 2.3). The CPU time for one run is 32 seconds.

If, in comparison, a full 3D model was considered, we would have used the C3D20 type of element (20 node, quadratic displacement brick). The density of the mesh of the model would be similar to both the seven tube plane stress model and to the single tube axisymmetric model. The resulting seven tube 3-D model would be divided into about 4700 elements. The number of degrees of freedom of the model will be increased dramatically. According to the theory of F.E.M., computer CPU time is proportional to square of number of degrees of freedom of F.E.M. model. The required CPU time for one run could reach more than a thousand hours! Clearly not a feasible solution.

2.2.2 Finite Element Simplified Analyses

Fig.2.2 shows a seven-tube plane stress model with a diagonal triangular pitch hole array which is typical for heat exchanger designs. Plane stress eight node quadrilateral elements with reduced integration are used. Plane stress model has been chosen rather than plane strain model, because the thickness of tubesheet is small compared to its diameter.

Fig.2.3 shows the single tube axisymmetric model, using the axisymmetric eight node, reduced integration quadrilateral elements. In addition, interface elements are used between the tube and the tubesheet to model the initial clearance between them. The reduced integration elements are used to improve the fluctuation of the results as it will be shown later.

2.2.3 Boundary Conditions and Loading

Several boundary conditions are applied to both finite element models in order to minimize end effects, restrain rigid body movements and include the effect of the remaining structure.

For the seven-tube model, the outer surface is free to move except at points A, B, C and D. At points A and C the horizontal displacement is restricted, and at points B and D the vertical displacement is restricted, (Fig. 2.2).

For the single-tube axisymmetric model, the left edge of both the tube and the tubesheet are prevented from axial motion. This holds the mesh in place axially and minimizes frictional-force errors.

In any event, the restraint forces calculated by the program at the left edge are very small indicating that the actual unrestrained motion would be small (Fig. 2.3).

The models are loaded by applying a uniform pressure along the inside surface of the tube until a maximum desirable value is reached, then this pressure is removed leaving residual contact pressure and stresses in the structure.

2.3 EARLIER THREE-STEP ANALYSIS METHOD

A first attempt at seeking a simplified, yet accurate, solution consisted in what we required to as "Three-Step-Analysis Method".

Step 1: In the case of simultaneous expansion process, use the seven-tube plane stress model (Fig. 2.2) and get the average value of the residual contact pressure at the outer surface of tube-1, (P_j^*). When the sequential expansion process is used (starting by tube 6 and followed by tubes 7, 5, 1, 2, 4, 3), two average residual contact pressure values around tube-1 must be recorded: the first one is obtained after its own expansion (P_i^*) and the second one is obtained after the expansion of tubes 2, 3 and 4, (P_j^*). This is because, before expanding tube-1, the expansion of tubes 6, 7 and 5 have a negligible influence on the residual contact pressure introduced around tube-1, (See Fig.2.4 and for more details, see Section 2.6.). Also, following the expansion of tube-1, it was observed that expansion of tube-2 had some effects on the existing residual contact pressure around tube-1. However, further expansion of tubes-3 and 4 have had a small additional influence. Therefore, in order to simplify the procedure, it was decided in this case to record only P_i^* and the final average residual contact pressure around tube-1 (P_j^*).

<u>Step 2:</u> Use a simple axisymmetric model for the expanded zone (only a few elements are required), see Fig.2.5. In the case of the simultaneous expansion process, adjust the outer radius of the tubesheet (R_e) in such a way that the residual contact pressure around the tube (P^*) is equal to the one obtained in Step-1, P_j^*. When sequential expansion process is used, R_e must be adjusted according to P_i^* and an external pressure P_o, which is equal to "n" times the expansion pressure P, must be applied on the external face of the tubesheet and then removed. This pressure causes further plastic deformations through the thickness and hence affects the residual contact pressure around the tube. The value of factor "n" is chosen in such a way that the new average value of the residual contact pressure around the tube is equal to P_j^*. Here the R_e is called equivalent radius and P_o is called equivalent load.

Step 3: Use the axisymmetric single tube model shown in Fig.2.3, with the equivalent dimensions and loadings that have been found in Steps 1 and 2. For example, in order to determine the residual stresses in the transition zone of the tube using the sequential expansion process, apply first the expansion pressure (P) in the tube of Fig.2.3 and remove it, then apply P_o on the external face of the tubesheet and remove it. The axisymmetric single tube model is called equivalent model.

2.4 THE VALIDITY OF THE THREE-STEP TECHNIQUE

The validity of the three-step technique has been checked against the results obtained, for a typical case, (Case-1 in Table 2.1), using a 3-D elasto-plastic finite element analysis. Only the simultaneous expansion process was considered in order to minimize computer expenses. In this case, due to the symmetry of the geometry and loading, only 1/12 of the entire body was analyzed (see Fig.2.6) using the 20 nodes full integration type of elements. The following paragraphs summarize the results obtained using the "Three-Step Technique", compared to those obtained from the 3-D analysis.

Step 1: Using the seven-tube plane stress model (Fig.2.2), we get the average value of the residual contact pressure around tube-1. This value was found to be 7.3% of the tube's material yield strength, i.e. is $(P_j^*/S_{yt}) = 0.073$.

Step 2: Using the simple axisymmetric model for the expanded zone, (Fig.2.5), the outer radius of the tubesheet (R_e) was adjusted so that the residual contact pressure around the tube (P^*) became equal to the one obtained in Step 1, (i.e. $P^* = P_j^*$). This equivalent radius R_e was found to be 2.4 times the internal radius of the tubesheet; $(R_e / a) = 2.4$.

Step 3: The axisymmetric single-tube model shown in Fig.2.3 was then used with the equivalent radius found in Step 2. After the application and removal of the expansion pressure P, the residual stresses in the transition zone were obtained. These results and those obtained from the 3-D analysis are summarized in Fig.2.7: it may be concluded that, in this particular case, the values and tendency of the residual contact pressure in the expanded zone, and the residual stresses in the transition zone, are in good agreement. The maximum differences are : 0.04 times S_{yt} (for the residual axial stress) and 0.07 times S_{yt} (for the residual hoop stress).

The "Three Step Method" has been further tested against the data published by Updike et al [32]; this is reference Case-2 (Table 2.1). Updike did not consider the sequence of the expansion process. Suffice it, however, to compare the tendency between both results. The graphical comparison is presented in Fig.2.8. Both the simultaneous and sequential processes were considered: One may conclude that the "Three Step Method" and the "Updike Method" are in fair agreement. For the simultaneous case, the following intermediate results have been obtained:

P_j^*/S_{yt} = 0.1068; and R_e/a = 2.1439. For the sequential case, the following intermediate results have been recorded:

P_i^*/S_{yt} = 0.0886, and P_j^*/S_{yt} =0.0841; R_e/a = 2.8585 and P_o/S_{yt} =0.2045.

The simultaneous expansion process produces higher average values of residual contact pressure. However, the residual stress levels introduced in the transition zone are almost independent of the expansion process used. This is indicated clearly in Fig.2.9 where the residual axial and hoop stresses are presented for the simultaneous and sequential processes.

2.5 TWO-STEP ANALYSIS METHOD

The "Three-Step Analysis Method" was simplified further to two steps only by eliminating the second step. The equivalent dimensions and loadings as well as the stress distribution have been found simultaneously in the axisymmetric single tube model:

<u>Step 1:</u> In the case of a simultaneous expansion process, use the plane stress seven-tube model (Fig. 2.2) and get the average value of the residual contact pressure at the outer surface of tube-1, (P_j^*). When a sequential expansion process is used (starting from tube 6 and followed by tubes 7, 5, 1, 2, 4, 3), the final average residual contact pressure value around tube-1 must be recorded: this value

is obtained after the expansion of tubes 1, 2, 4 and 3 only, (P_j^*). Indeed, it was indicated in step 1 of the "Three-Step-Method" that tube 7 and 5 had no influence on the results of tube 1.

Step 2: Use the axisymmetric single tube model shown in Fig.2.3. In both cases, simultaneous expansion and sequential expansion processes, adjust the outer diameter of the tubesheet (D_e) so that the residual contact pressure around the tube (P^*) is equal to P_j^* obtained in Step 1. About three iterations are needed to get a close answer of D_e.

2.6 DISTRIBUTION OF STRESSES ALONG TUBE LENGTH

Typical stress results are shown in Fig.2.10 and Figs.2.11 (a-d). These stresses were calculated using the "Two-step method". They give the stress distributions as a function of the axial position along the tube. The results show that the maximum residual stresses at the inner surface of tube (Figs.2.11 a and b) are higher than at the outer surface of the tube (Figs. 2.11 c,d) in the transition zone. Also, the local bending moment is very large in the transition zone and induces residual stresses with steep gradients. The hoop and axial stresses are of the same order of magnitude. According to the results, both maximum hoop and axial stresses are tensile on the inner surface of tube in the transition zone. For example, Fig.2.11(a) shows a maximum value of tensile axial stress of about 80% of yield stress located at the inner surface of the tube. Under such conditions the tube would be very susceptible to stress corrosion cracking. Fig 2.11(b) shows the hoop residual stress at the inner surface of the tube which seem to be less severe than those introduced in the axial direction. No experimental data are available, to prove that.

2.7 TUBE DEFORMATION IN THE TRANSITION ZONE

The real tubewall reduction in transition zone was studied. The real tubewall reduction is defined:

$$k' = (t - t')/t \ (\%).$$

Figure 2.12 shows the change of distribution of real tubewall reduction along the tube length. It shows that the tubewall reduction sharply increases at a spot when its value increases by about 33%. This is a very significant increase because of the large deformation of the tube in the transition zone which introduces a region of stress concentration. The increase of tubewall reduction and the existence of tensile residual stresses in the transition zone are the primary reason for failure of tube-to-tubesheet joints.

2.8 EFFECT OF SEQUENTIAL EXPANSION PROCESS

Due to the fact that most of the tube-to-tubesheet expansion technique are accomplished in a sequential way, it was felt important to study the effect of this parameter on the distribution of the residual contact pressure and residual stresses around the surface of tube. The sequential expansion process consists of expanding the tubes one after the other in certain order such as, for example (6, 7, 5, 1, 2, 4, 3), see Fig.2.2.

As mentioned in Chapter 1, Chaaban et al. [31] have studied the effect of sequential expansion first using a seven tube plane stress model (Fig.2.2). Fig.2.13 shows the effect of the sequential expansion process. (The dimension and material properties of the model are shown in Table 2.1, reference case 1). Figure 2.13 indicates that the average value of residual contact pressure level around the central tube decreases as its neighbouring tubes are expanded. More specifically, as the surrounding tubes are expanded, the average residual contact pressure level of the central tube decreases to as little as 86% (for the 7-tube model); a rather significant drop.

Figure 2.13, includes some results obtained by using larger finite element methods such as the 13- and 19- tube models. These models are explained in the following section.

2.9 THE 13- AND 19 - TUBE MODELS

In order to study the influence of the external boundary on the results, we have analyzed the same problem using the seven tube plane stress model (Fig.2.2) and 13 and 19-tube plane stress models (Figs.2.14, 2.15). The results obtained from the three models are shown on Fig.2.13. The different results seem to be in good agreement: The average value of residual contact pressure level around each tube decreases as its neighbouring tubes are expanded. The results also indicate that the level of contact pressure drop around the central tube using the 19-tube model is less than the one obtained by using the 7-tube model. The difference is caused by the different outer diameter of the model, i.e. the different stiffness of model of tubesheet. It is however less than about 5%. Therefore, the seven tube plane stress model has enough accuracy for calculations.

The "Two-Step Method" may be used to investigate residual stresses in the transition zone of tube-to-tubesheet joints. The reliability of the method was verified. Therefore it will adopted in the following analyses.

Reference Case	$\frac{t}{a}$	$\frac{b}{a}$	$\frac{h}{a}$	$\frac{c}{a}$	$\frac{w}{h}$	$\frac{E_t}{S_{yt}}$	$\frac{E'_t}{E_t}$	$\frac{E_s}{E_t}$	$\frac{E'_s}{E_s}$	$\frac{S_{ys}}{S_{yt}}$
No.1	0.0374	2.40	1.33	0.000	1	1222.2	0.00	1.32	0.00	1.6667
No.2	0.0667	2.00	1.00	0.020	1	1000.0	0.01	1.00	0.01	1.0000
No.3	0.0711	2.39	1.00	0.001	1	545.5	0.00	0.98	0.00	0.5540

Table 2.1: Parameters for reference Cases 1, 2 and 3
No.1 $P / S_{yt} = 1.154$
No.2 $k = 5\%$
No.3 $P / S_{yt} = 0.71$

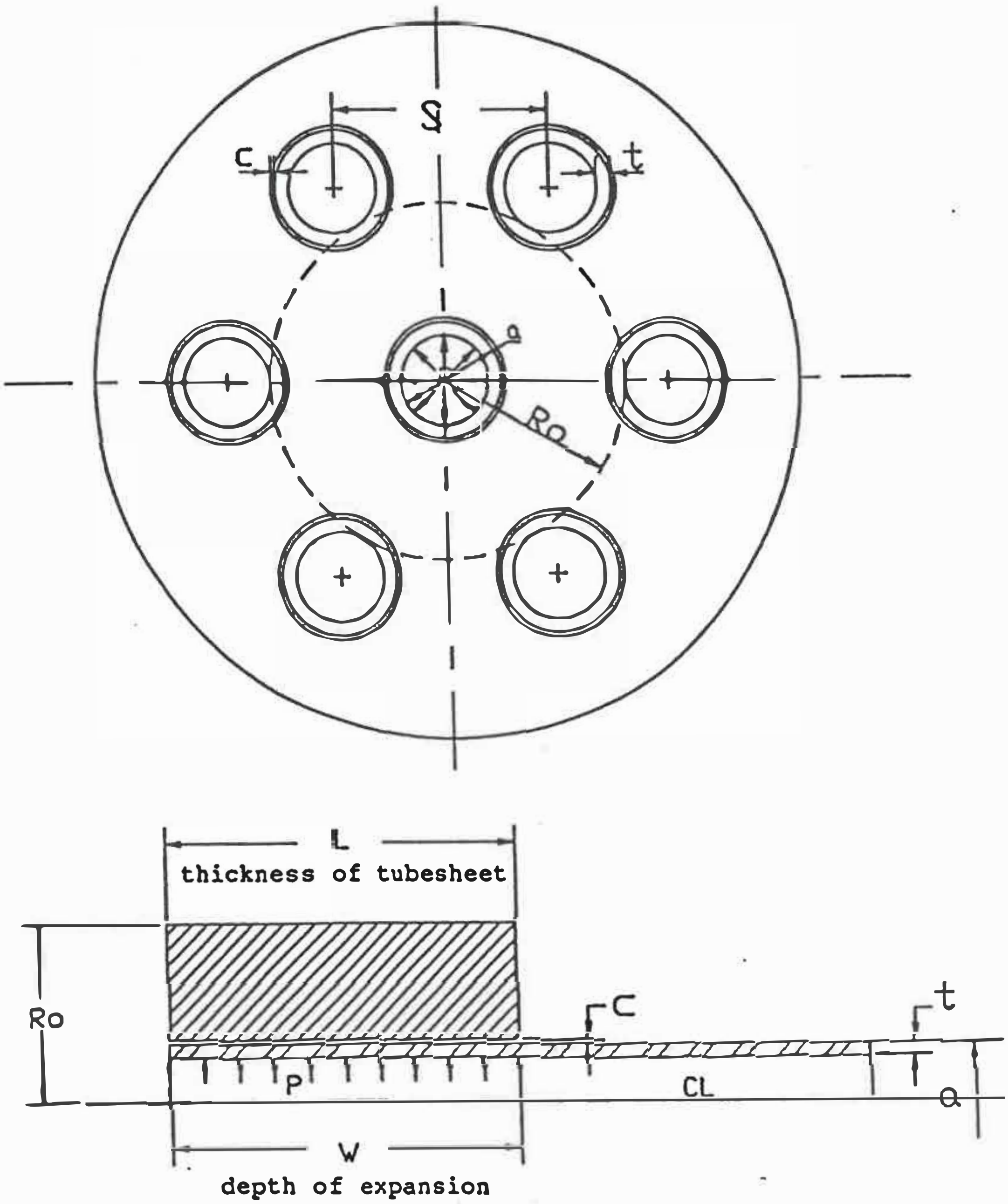


Figure 2.1 Single Tube Surrounded by an Annulus

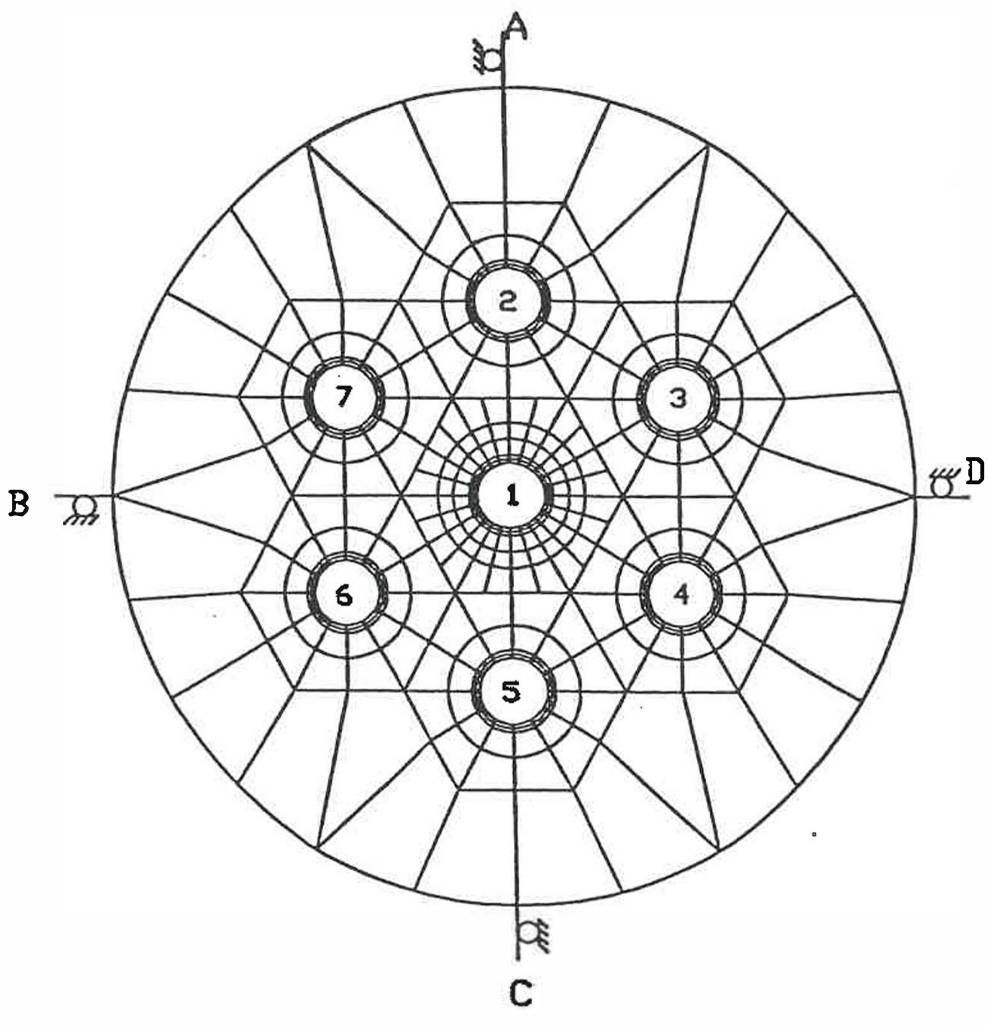


Figure 2.2 Seven tube plane stress model

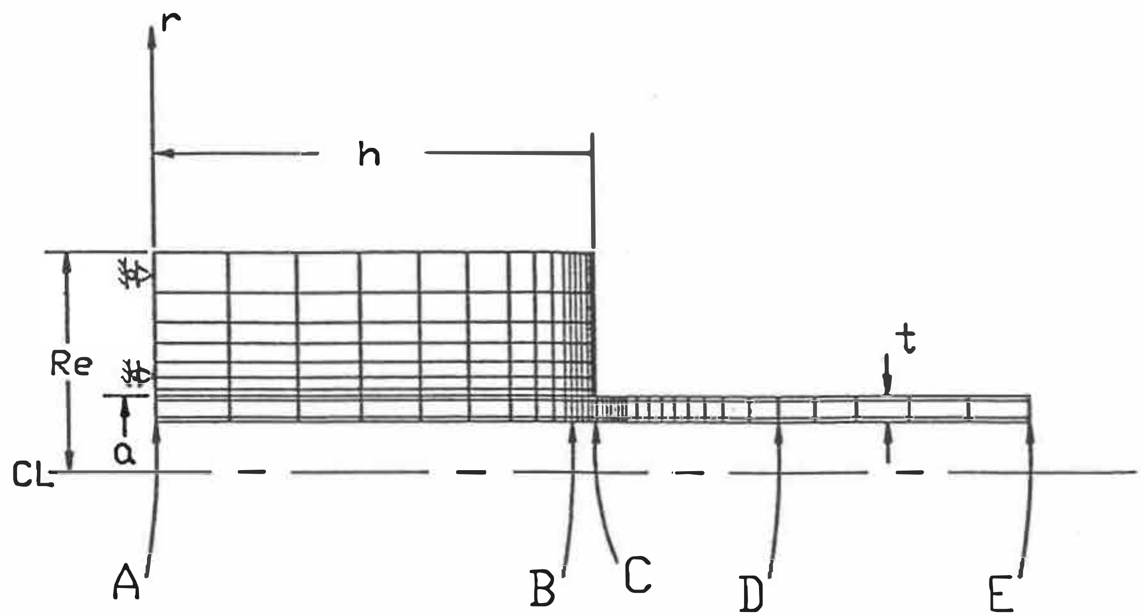


Figure 2.3 Single tube axisymmetric model

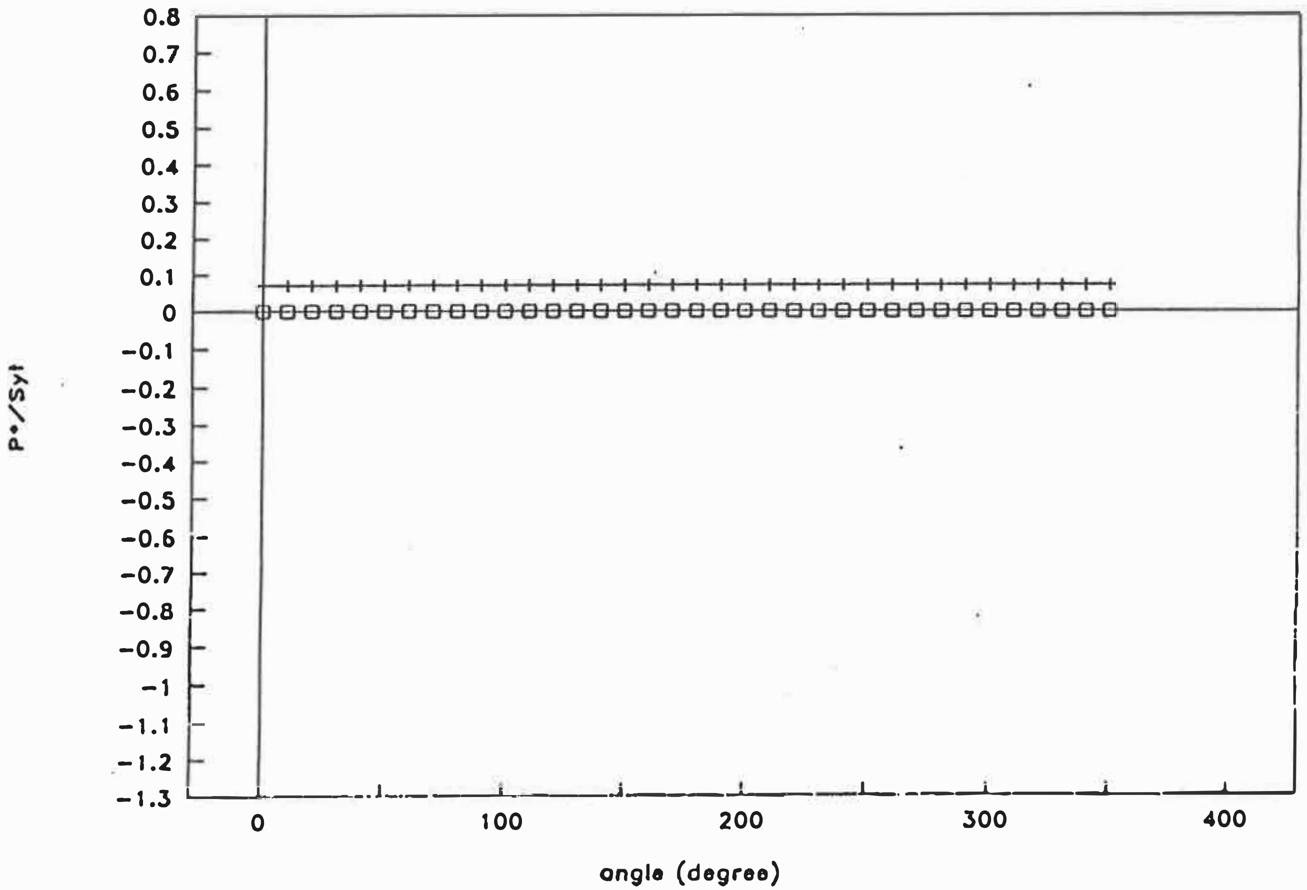


Figure 2.4 Change in distribution of residual contact pressure of central tube (Tube 1, see Fig.2.2)
□ effect after expansion process of tubes 6, 7 and 5 (prior to Tube 1)
+ effect after expansion process of tubes 6, 7, 5 (after Tube 1)

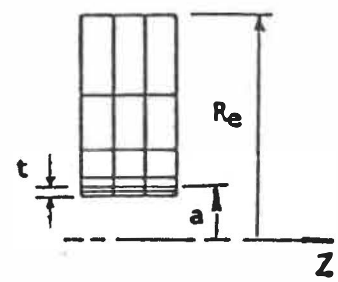


Figure 2.5 Simple axisymmetric model

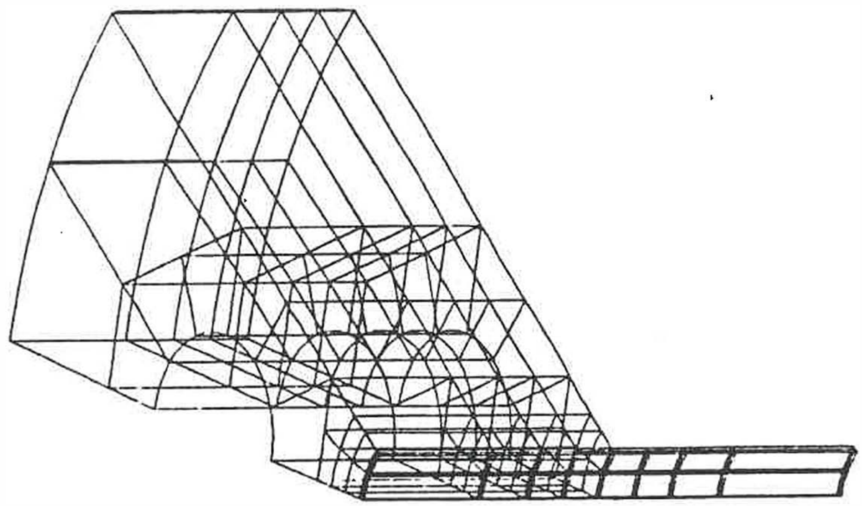
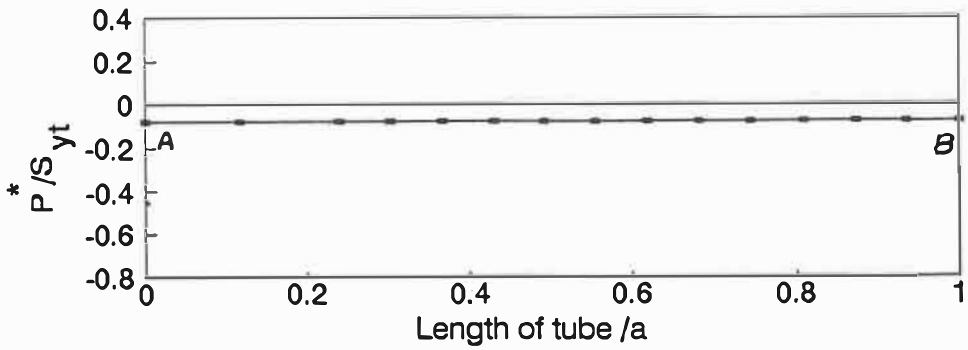
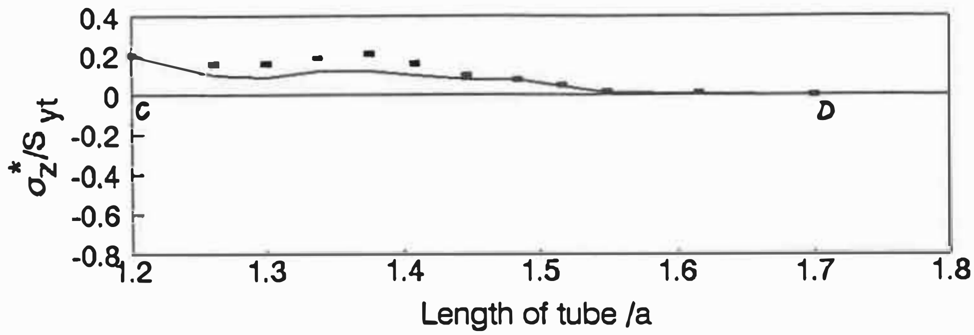


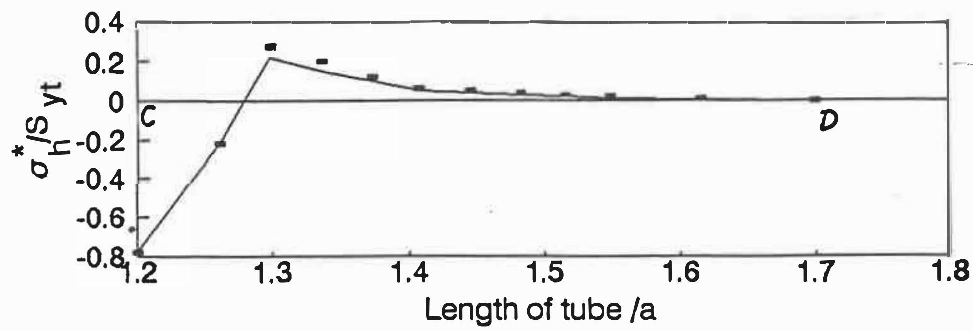
Figure 2.6 3-D elasto-plastic finite element model



Residual contact pressure in expanded zone



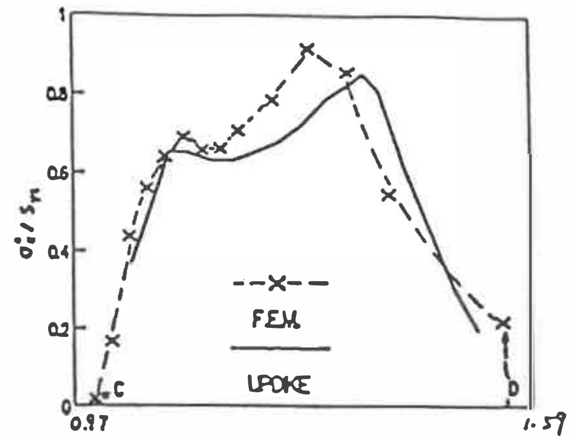
Residual axial stress in transion zone



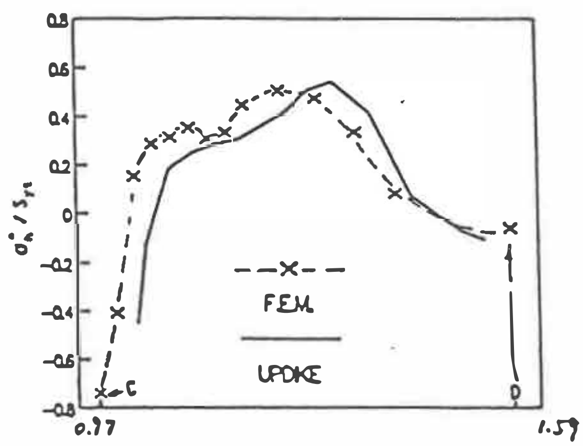
Residual hoop stress in transion zone

Figure 2.7 Comparison between the "Three-Step Method" and a 3-D elasto-plastic finite element analysis (see Fig. 2.3)

- 3-D __ AXI.



(a) Length of tube / a



Length of tube / a

Figure 2.8 Comparison of the "Three-Step Method" with Updike's [32] results (see Fig.2.3)
(a) Residual axial stress in transition zone
(b) Residual hoop stress in transition zone

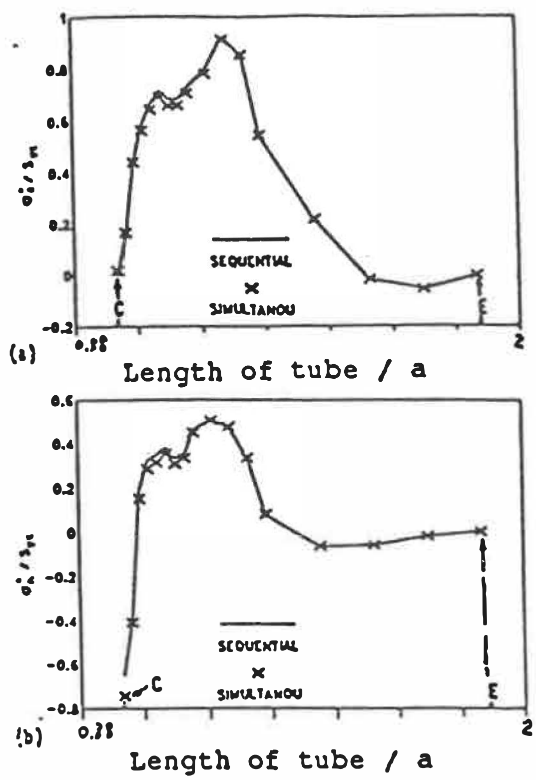


Figure 2.9 Sequential expansion versus simultaneous expansion process (see Fig.2.3)
(a) Residual axial stress in transition zone
(b) Residual hoop stress in transition zone

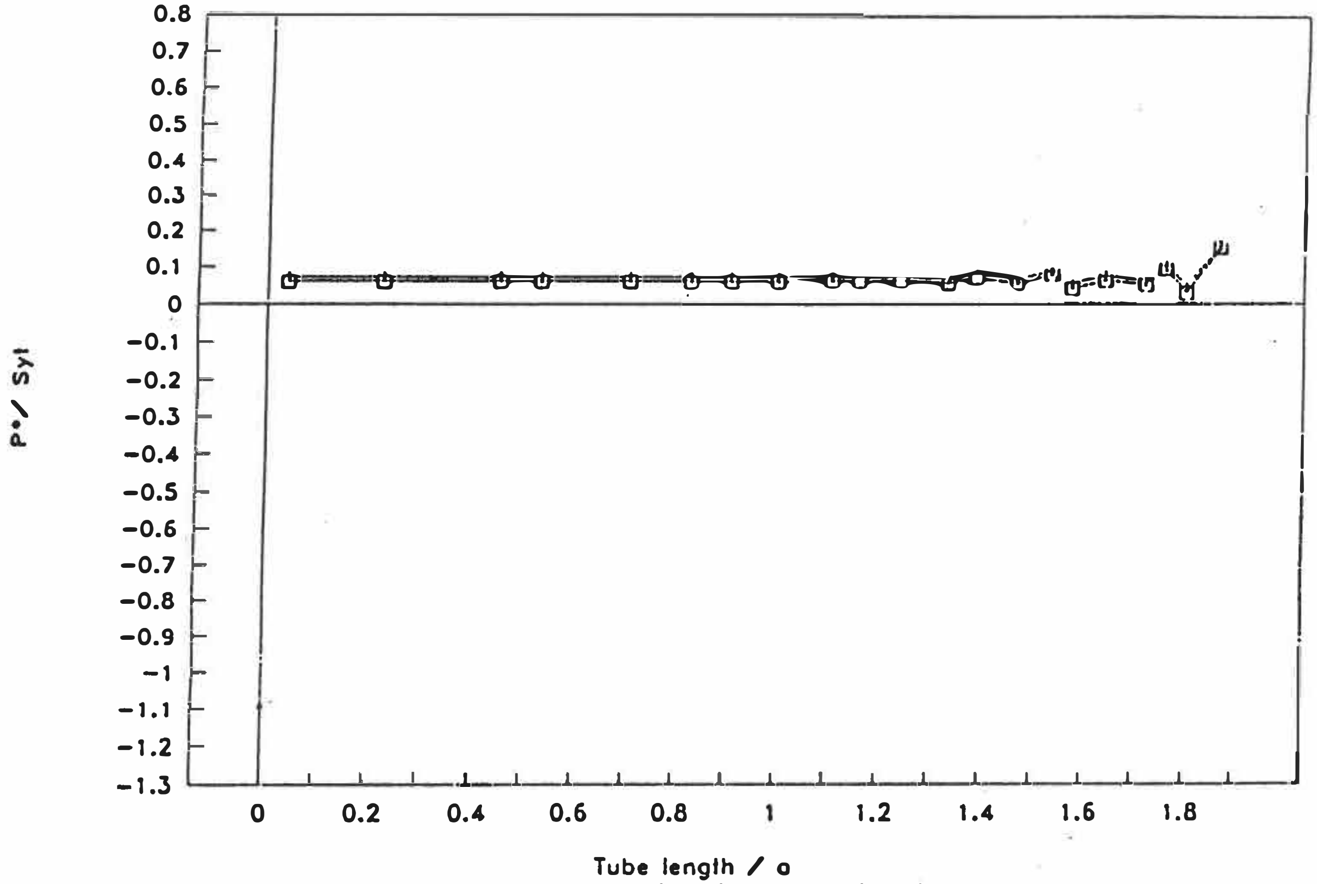


Figure 2.10 Change in distribution of residual contact pressure at inner surface of central tube (Tube 1) along the axis
+ effect after expansion process of tubes 6, 7, 5 and 1
□ effect after expansion process of tubes 6, 7, 5, 1, 2, 4 and 3 (complete expansion process)

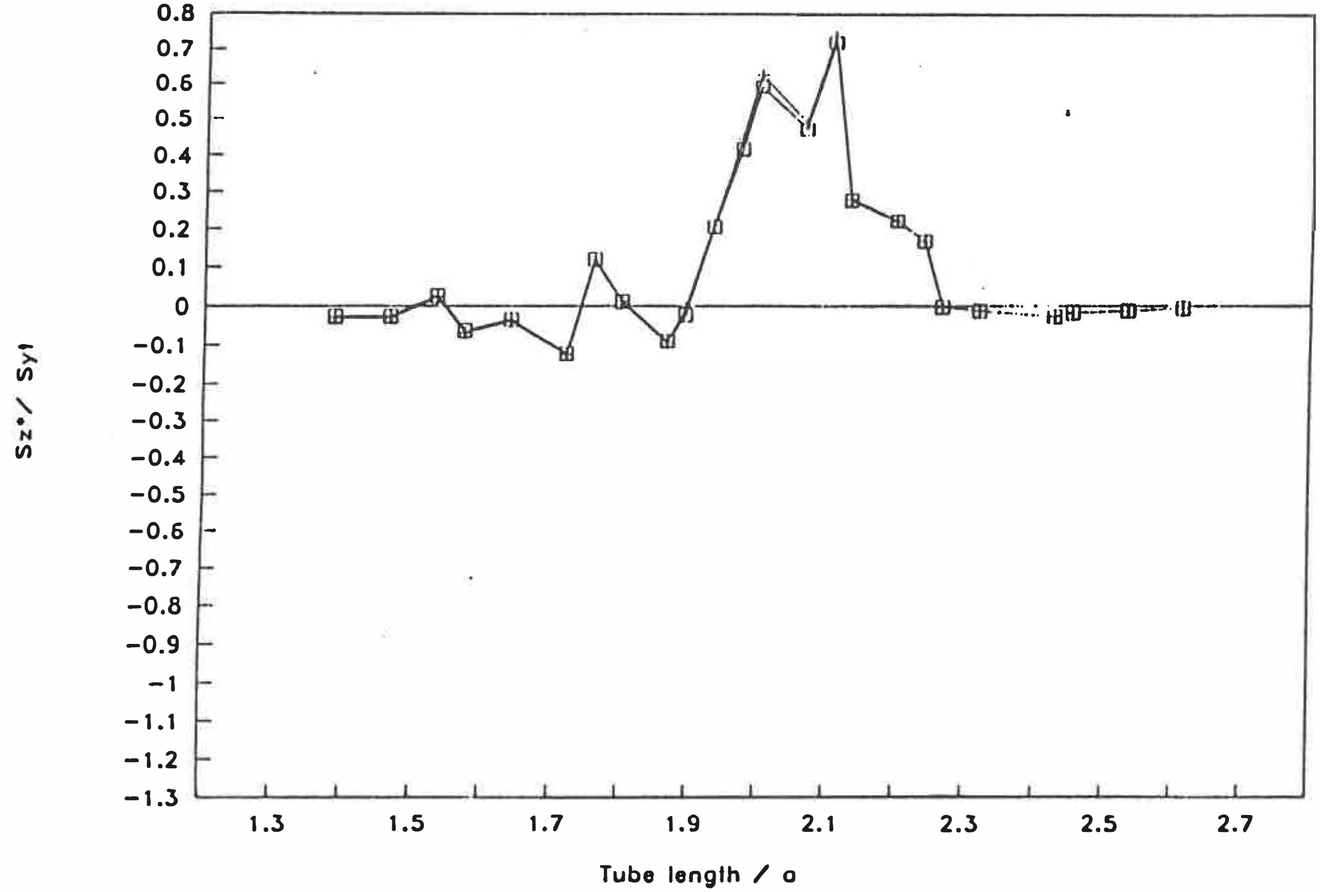


Figure 2.11 a) Residual axial stress (inner surface of tube)

+ effect after expansion process of tubes 6, 7, 5 and 1
□ effect after expansion process of tubes 6, 7, 5, 1, 2, 4 and 3 (complete expansion process)

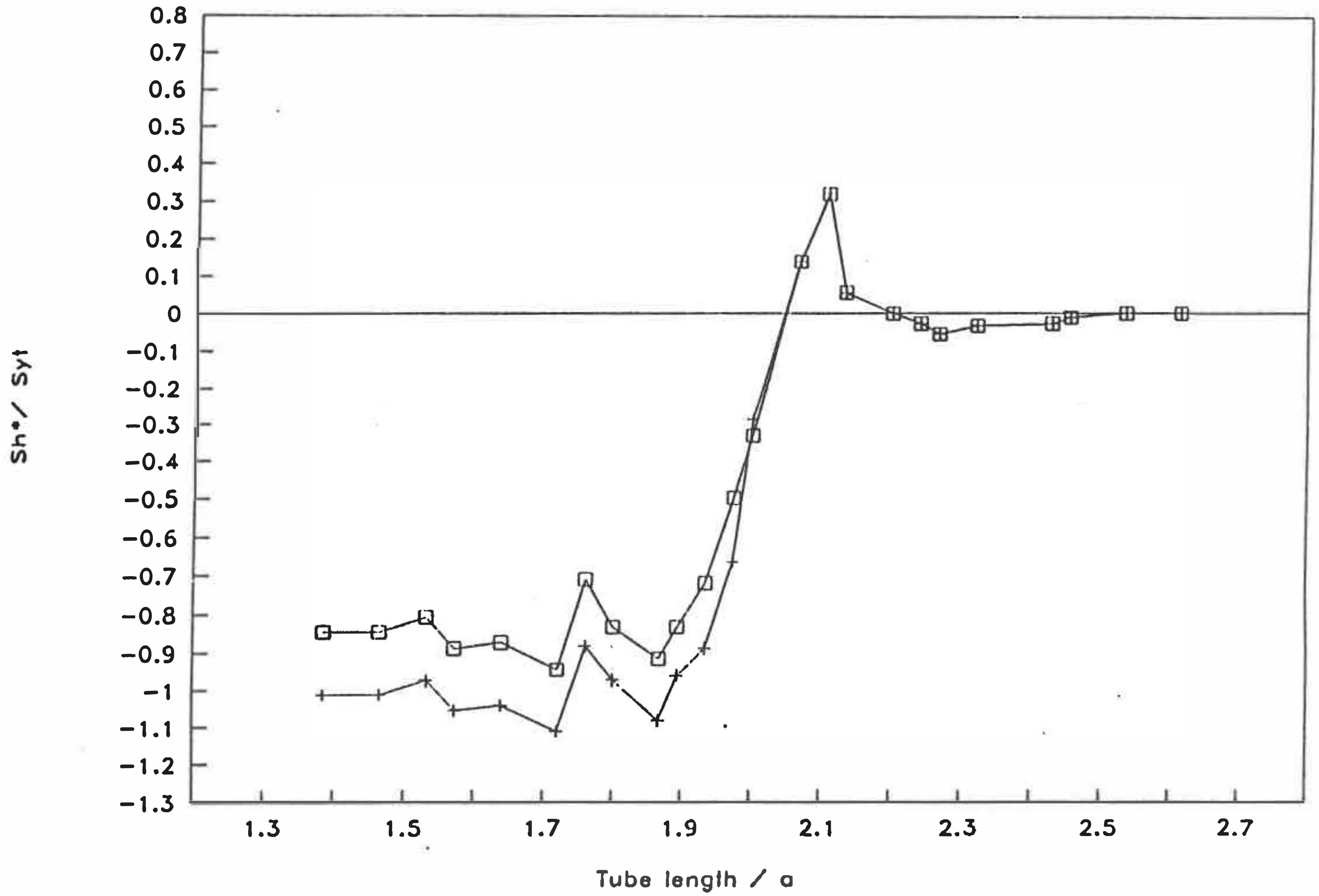


Figure 2.11 b) Residual hoop stress (inner surface of tube)

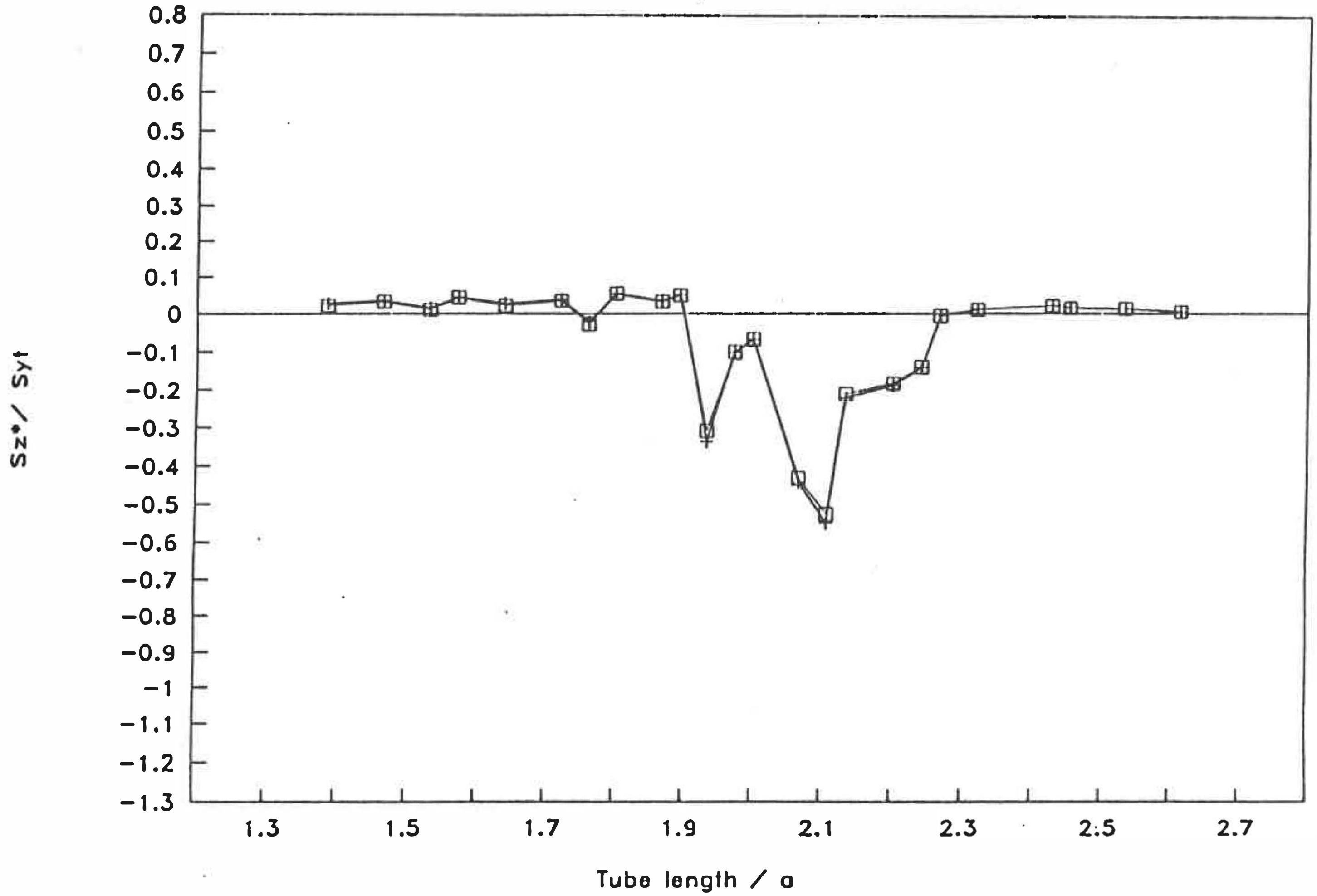


Figure 2.11 c) Residual axial stress (outer surface of tube)

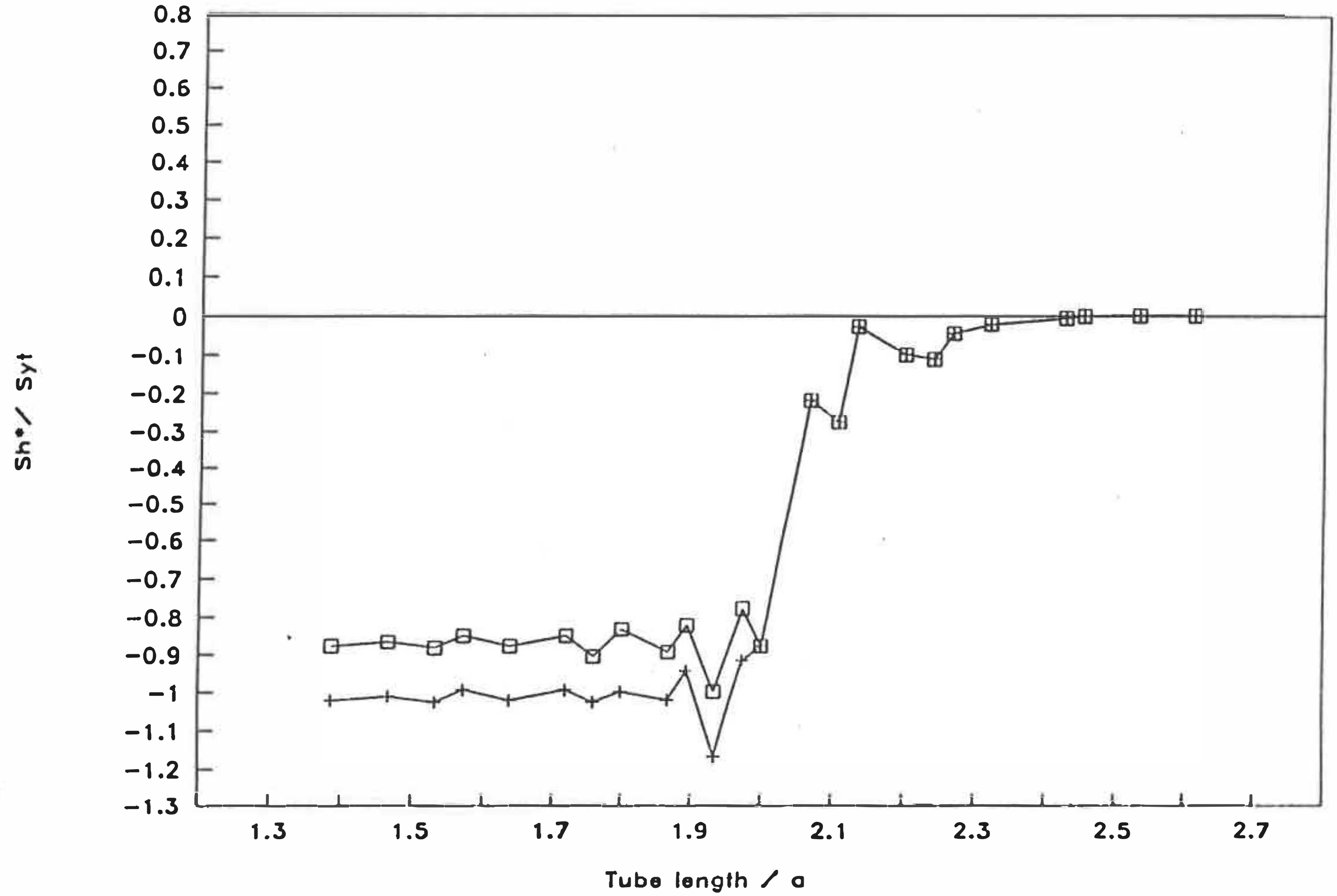


Figure 2.11 d) Residual hoop stress (outer surface of tube)

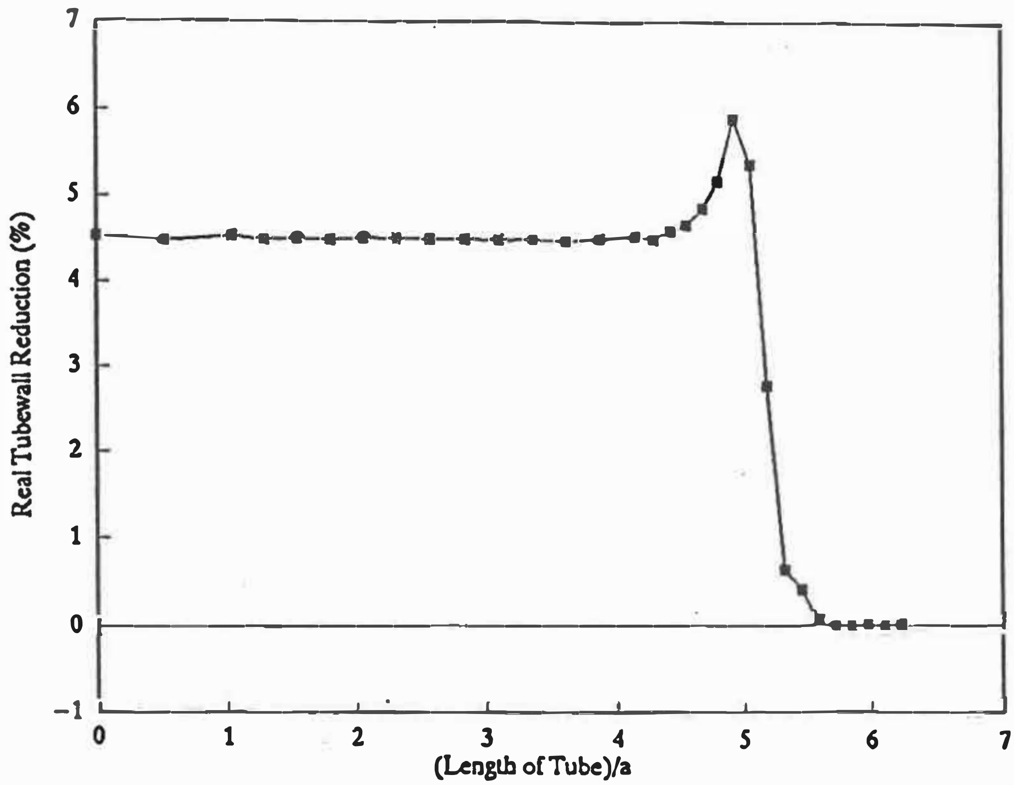


Figure 2.12 Change in distribution of tubewall reduction
(See Table 3.7 (1) Case T4)

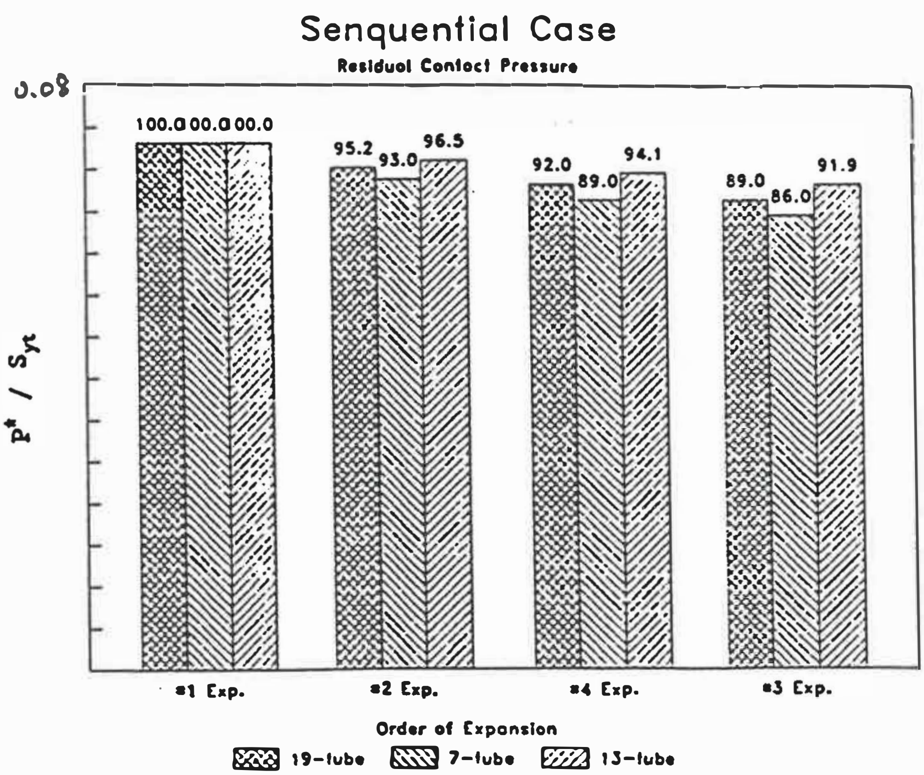


Figure 2.13 Comparison of average residual contact pressure
#1 Expansion process (tube 1)
#2 Expansion process (tubes 1 and 2)
#4 Expansion process (tubes 1, 2 and 3)
#3 Expansion process (tubes 1, 2, 4 and 3)

* See Figs.2.2, 2.14, 2.15 for the three models considered.

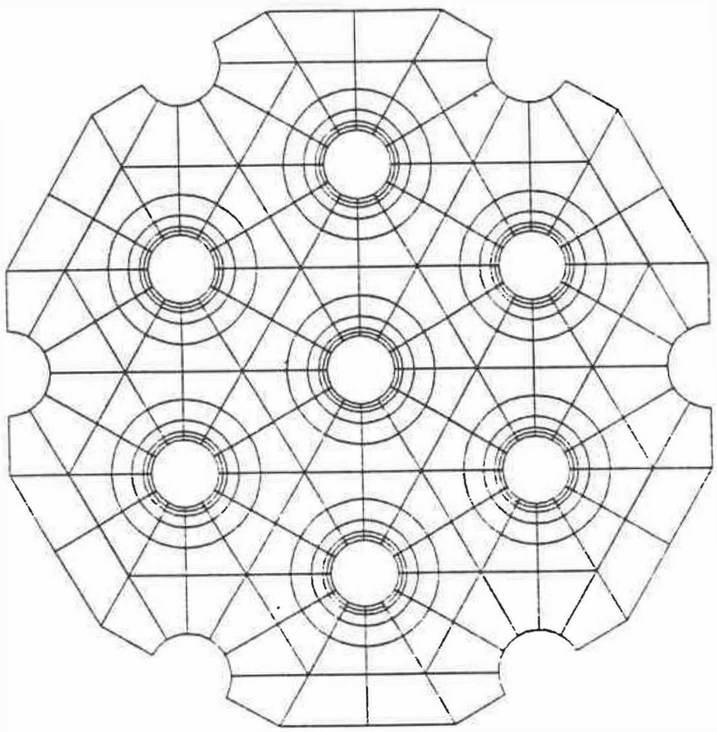


Figure 2.14 13-tube plane stress model

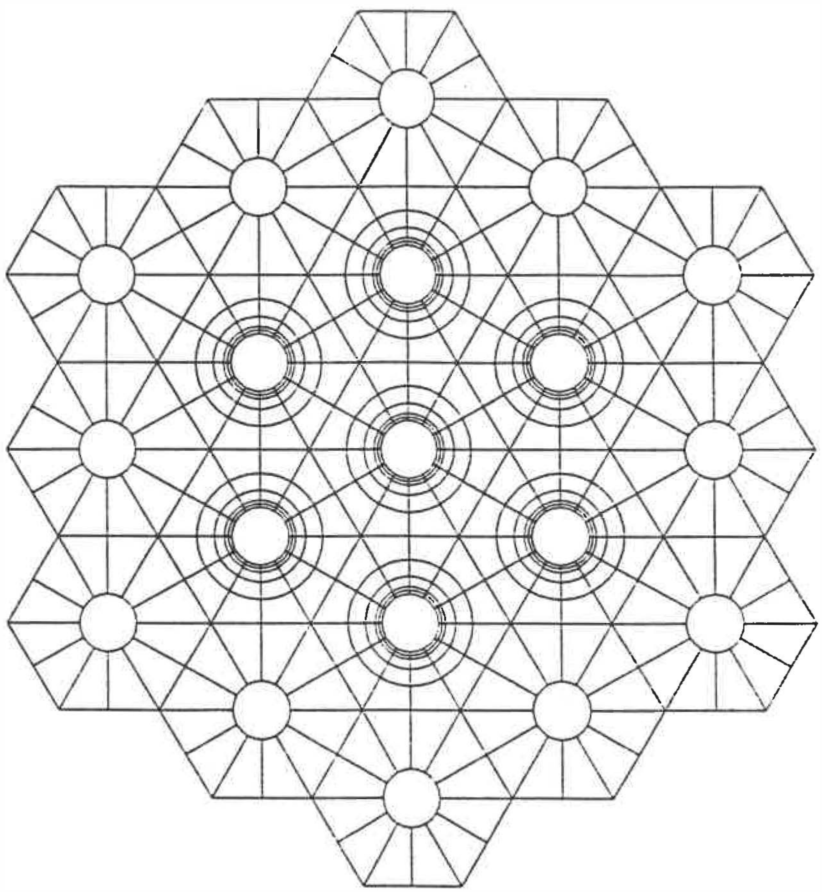


Figure 2.15 19-tube plane stress model

CHAPTER 3

STATISTICAL APPROACH

3.1 INTRODUCTION

As mentioned in Chapter 1, the strength of a tube-to-tubesheet joint is influenced by many parameters. It becomes impractical, however, to perform a complete set of calculations to cover all of the parameters involved. Consequently, it was felt necessary and convenient to use some statistical approach to reduce the cost of the calculations and, at the same time, increase the reliability and universality of the results.

The orthogonal design method [36][37][38][39][40], which may be applied to both numerical calculations and experiments where multiple parameters at several levels are involved, is the statistical approach adopted in the present work. This technique consists in selecting sets of parameters for the calculations using standard tables, called orthogonal arrays.

This chapter will first introduce the results from the investigation of individual parameters. Second, a preliminary investigation involving the orthogonal design approach will be presented. It covers the determination of contact pressure using the procedure developed in [23] for the single tube model. Third, the same approach will be applied to the full investigation.

3.2 INVESTIGATION OF INDIVIDUAL PARAMETERS

Before preceding with a systematic parametric study, a study of the effect of individual parameters was carried out. The usual way of doing an individual parameter study is to fix all of the parameters except one and study its effect on the residual stresses and residual contact pressure by varying its value. For this purpose, using the two-step approach, the study will only consider the effect of the expansion of the central tube. The basic conditions are those of case 3 (Table 2.1). Figure 2.1 recalls the geometry considered.

3.2.1 Effect of Initial Clearance Between Tube and Tubesheet on Residual Stresses

Typical calculated stresses on the inner and outer surfaces of the tube are presented in Figure 3.1(a-d). They give the stress distributions as a function of the axial position along the tube. As it was noticed in Chapter 2, and according to the present results, both the hoop and axial stresses are high and are tensile on the inner surface of tube in the transition zone. Fig.3.1(a) shows that the axial residual stress at the inner surface of the tube increases significantly with the increase of initial radial clearance, it is also tensile and its maximum value may reach 90% of yield stress. The hoop residual stress (Fig.3.1(b)) at the inner surface of the tube increases with the increase of the initial radial clearance but not as significantly as for the axial stress. It is thus concluded that residual stresses may be reduced by increasing the accuracy of fit of tube in the hole, i.e. decreasing the initial radial clearance. Figure 3.2 summarizes the effect of initial clearance on maximum tensile residual stresses in the transition zone. For the closest fit (c/a= 0.0), the residual axial stresses at the inner surface of the tube is about $0.5S_{yt}$. For the loosest fit (c/a= 0.02), it increases to $0.9S_{yt}$.

3.2.2 Effect of Tube Thickness

The analysis has been carried out using t/a ratios of 0.05 to 0.1. Fig.3.3(a-e) show that the contact pressure and the maximum tensile residual stresses at the inner and outer surfaces of the tube decrease, as expected, by increasing the thickness of the tube.

3.2.3 Effect of Depth of Expansion

Figure 3.4(a-e) shows that the depth of expansion has little effect on the residual contact pressure, but a large effect on the maximum residual stresses.

When the depth of expansion is the same as the thickness of the tubesheet, the value of the maximum residual stresses is 2 to 3 times the one when the depth of expansion is 90% of the thickness of the tubesheet. This is due to the stress concentration in the transition zone of the expanded tube caused by the sharp edge of the tubesheet.

3.2.4 Effect of Tube Pitch

According to the simplified model (Fig.2.1), the outer diameter of the tubesheet for the single-tube axisymmetric model is increased by increasing the tube pitch. Figure 3.5(a-e) shows that the contact pressure increases and the maximum tensile residual stresses at the surface of tube decrease when the tube pitch is increased. This was to be expected because, as the tube pitch increases, the stiffness of tubesheet increases, thus the plastic deformation of tube decreases.

3.2.5 Effect of Frictional Force Between Tube and Tubesheet

The frictional force between tube and tubesheet is an important factor with regard to the pull-out strength.

Two cases were analyzed:

1. Dry friction between tube and tubesheet: f=0.3
2. Full lubrication between tube and tubesheet: f=0.0

The results of the analyses are presented in Fig.3.6(a-c). It turns out that "f" has no effect on either the residual contact pressure or residual stresses in the tube. The reason is that most of the axial contraction occurs before contact. The effect on the external surface has been ignored.

3.3 THE PRELIMINARY PARAMETRIC STUDY

3.3.1 Description of Orthogonal Arrays

Orthogonal arrays can be traced back to Euler's Graeco-Latin squares. The idea of using orthogonal arrays for the design of experiments was studied independently in the United States and Japan during World War II. Later, studies on orthogonal functions corresponding to orthogonal arrays were also conducted, and orthogonal arrays rapidly began to be used for expressing functions and assigning calculations and experiments. By using the orthogonal design it is possible to analyze the influence of every factor on the results, and find the most important factors[41]. And by using the orthogonal design method, it is possible to analyze the influence of the various parameters, using variance analysis, in order to come up with some simplified empirical equations for design purposes.

The orthogonal arrays are identified by a code of the form L_a (b^c), where "a" is the number of calculations to be performed, "b" is the number of levels to be considered for each parameter and "c" is the maximum number of parameters to be analyzed. Table 3.1, [37], shows a typical orthogonal array L_{16} (2^{15}). Using this Table, the numerical analysis can be arranged for a maximum of 15 parameters at two levels each, when ignoring their coupled interactions. By involving only 8 parameters (A to H), on the other hand, coupled interactions may be included in the analysis as indicated in Table 3.1. The numbers in the left most column are called the assignment numbers, and run from 1 to 16. The other columns are termed the orthogonal array columns, and contain either the numerals 1 or 2. There are four possible combinations of the numerals of any column and those of any other column (taken two by two), these are (1 1), (1 2), (2 1), and (2 2). When these combinations appear with equal frequency, the two columns are same to be orthogonal. If we select any two columns from among the fifteen columns of the L_{16} (2^{15}) orthogonal array and count the number of combinations of (1 1), (1 2), (2 1), and (2 2), we will find that all of them are orthogonal.

3.3.2 Set of Calculations

In order to explain the use of the orthogonal array, let us consider eight out of 15 parameters. These eight parameters have been chosen based on their relative importance. Each parameter will be considered at two levels (See Table 3.2). Only the effect on residual contact pressure is considered in this investigation. Mathematically, in this case, there are 2^8 possible combinations. Therefore, 256 calcuruns would be required in order to determine the complete effects of the eight parameters on the residual contact pressure. However, by using the orthogonal array $L_{16}(2^{15})$, only 16 out of the 256 calculations are required. These 16 calculations were chosen according to the orthogonal array of Table 3.1 as follows:

Assume the following shorthand descriptions: "A" for outer radius of the tube, "B" for the tubewall thickness, "C" for the initial clearance between tube and tubesheet, etc... (see Table 3.2). Each of these parameters represents one column in Table 3.1.

Some columns represent an interaction between parameters (AxB, etc...). The numbers 1 and 2 in each column indicate the two different levels of the parameter involved. For example, line 8 in Table 3.1, which represent the 8th calculation, must include the parameter A at level 1 (a = 0.01905 m), B at level 2 (t = 0.00277 m), C at level 2 (c = 0.0000762 m), D at level 1 (P = 206.84 Mpa), E at level 2 (E_t = 206.84 Gpa), F at level 1 (S_{yt} = 206.84 Mpa), G at level 1 (E_s = 206.84 Gpa), and H at level 2 (S_{ys} = 275.79 Mpa). All interactions AxB, BxC, etc... have been ignored in this investigation. In Table 3.3, there are 7 columns (3, 5, 6, 9, 10, 12 and 15) for the interaction of every parameter, but the effect of every interaction of parameters mixed together cannot be distinguished in this Table. If we want to separate the interaction we must use a larger orthogonal array and increase the number of calculations accordingly. With these eight parameters which correspond to columns 1, 2, 4, 7, 8, 11, 13 and 14 of the orthogonal array, we perform sixteen runs of calculations in succession.

The results (residual contact pressure) obtained from the 16 calculations are shown in the last column of Table 3.3. A statistical analysis is now required. Assume that I_j and II_j are the summations of the results obtained at levels 1 and 2 respectively of the parameter in column j (j varies from 1 to 15). For instance, parameter E in column 8 has the following summation for level 1:

$$I_8 = R_1 + R_3 + R_5 + R_7 + R_9 + R_{11} + R_{13} + R_{15}$$

$= 47.8 + 21.1 + 0.32 + 6.43 + 4.27 + 24.74 + 41.92 + 33.01$

$= 179.56$

Similarly, one can obtain for level 2:

$II_8 = 252.47$

In Table 3.3, S_j is defined as follows:

$$S_j = 8 \text{ x } (\bar{I}_j - \bar{R})^2 + 8 \text{ x } (\overline{II}_j - \bar{R})^2$$

$$= (I_j^2 + II_j^2)/8 - G^2/16$$

where

$\bar{I}_j = I_j/8$, $\overline{II}_j = II_j/8$,

$G = \Sigma R_j = 432.03$

$\bar{R} = G/16.$

In the above equations, $\bar{I}_j$ and $\overline{II}_j$ are the variety-means, S_j is the sum of squares which corresponds to the variation of the variety-means, and $\bar{R}$ is the mean of all results.

Hence, one can get

$$S_8 = (I_8^2 + II_8^2)/8 - G^2/16 = 332.24$$

Therefore, in comparing the residual contact pressure of $\bar{I}_j$ and $\overline{II}_j$ for parameter E (Young's modulus of tube),

$\bar{I}_j = 22.44$

$\overline{II}_j = 31.56$

This shows that if Young's modulus of the tube is increased from 0.3E8 psi to 0.4E8 psi, the residual contact pressure increases from 3255.38 psi to 4577.13 psi. To compare level 1 and level 2 of parameter B, we need only to compare the mean of calculations Nos.1, 2, 3, 4, 9, 10, 11 and 12, which were performed with level 1 of parameter B, and the mean of calculations Nos.5, 6, 7, 8, 13, 14, 15 and 16, which were performed with level 2 of parameter B. The same logic applies for the other parameters. The calculation results are given in Table 3.3.

Details about the orthogonal arrays can be found in Appendix(A).

3.3.3 Analysis of Variance

A summary of the statistical analysis of the data presented in Table 3.3 is shown in Table 3.4.

In Table 3.4, V_j is the mean square and defined as $V_j = S_j/f_j$, where S_j is sum of squares and f_j is the number of degrees of freedom, [37][38]. $F_j = V_j/V_{error}$. The significance test is based on the statistical F distribution. α is the significance level and is defined as a probability.

In the analysis of variance, the mean square, V_j, indicates the relative significance of each effect of the parameters including interactions. The larger the value of the mean square(V_j), the more significant is the corresponding effect or interaction.

Consequently, it can be seen on Table 3.4 that the yield stress of the tube material seems to be the most significant parameter (F_j =1100). The second most important parameter would be the expansion pressure level(F_j = 521). The third parameter is Young's modulus of the tubesheet material (F_k = 264), and the fourth parameter is Young's modulus of tube material (F_j = 175). In Table 3.3, the values of $\overline{I}_j$ and $\overline{II}_j$ reflect the degree of the effects at levels 1 and 2 of each parameter in column j. The smallest value between $\overline{I}_j$ and $\overline{II}_j$ corresponds to the level which induces the smallest residual contact pressure. For example in column 11 of Table 3.3, the value of $\overline{II}_{11}$ for parameter F is smaller than $\overline{I}_{11}$. Thus level 1 of the parameter F is provided a higher residual contact pressure.

3.3.4 Preliminary Conclusions

Based on the above example, the Orthogonal Design Method may be considered as a very useful technique for the arrangement of calculations and the corresponding analysis of results. The main advantages of this technique are:

1) To reduce the number of calculations and consequently save on computer time;

2) To analyze statistically the significance of the separate main effects of all parameters and their coupled interactions;

3) To determine statistically the optimal combination of the parameters.

3.4 SYSTEMATIC STATISTICAL ANALYSIS OF PARAMETERS

3.4.1 Analysis Method

The Two-Step simplified method presented in Chapter 2 will be adopted for the remaining investigations. Different combinations of common material properties and geometries are utilized (see Table 3.5). In order to generalize the results, nondimensional parameters are used; the ratios t/a and s/a characterize the geometry of the seven-tube model. Three nondimensional parameters, E_t/Y_{st}, E_t/E_s and Y_{ss}/Y_{st}, characterize the stress-strain curve for the materials of the tube-tubesheet joint. The calculated expansion pressure and residual stresses are normalized with respect to the yield stress of the tube, Y_{st}. Generally, the tube wall reduction is a significant indicator of the degree of expansion[6]. Since it is impractical to determine the actual wall reduction, the more appropriately termed "apparent wall reduction" [15], will be determined as follows:

$$k= [(a'_i-a_i)-2c]/2t \ \%$$

(see Fig.1.2)

3.4.2 Calculation Procedure

The calculations are done using three levels of t/a, s/a and c/a for material sets (T1, T2, T3) and two levels for sets (T4, T5, T6). These are common values as shown in the TEMA heat exchanger Standards [5]. On the other hand, six combinations of tube and tubesheet materials are selected based on practical cases (Tab.3.5).

Table 3.6 shows typical orthogonal arrays $L_9(3^4)$ and $L_4(2^3)$. Using array $L_9(3^4)$, the numerical analysis can be arranged for a maximum of 4 parameters at three levels each. This is used for sets T1 to T3 of Table 3.5. Using array $L_4(2^3)$, the numerical analysis can be arranged for a maximum of 3 parameters at two levels each. This is used for sets T4 to T6 of Table 3.5. Two levels have been used for T4, T5 and T6 because these cases have been added lately to improve the precision of Eqn. (6). The 39 observation points (9 for each of the three sets (T1 to T3) and 4 for each of the another three sets (T4 to T6)) are chosen and investigated for the two types of expansion processes, sequential and simultaneous. The results are summarized in Table 3.7. For example, for observation point No.1 (sequential expansion case), the material set is T1, and the other parameters are t/a=0.065, s/a=1.5, c/a=0.032 and P/Y_{st}=0.7; the finite element results were: P^*/Y_{st}=0.02, S^*_z/Y_{st}=1.54, S^*_h/Y_{st}=1.44 and k=4.44%.

3.4.3 Correlation Analysis

The correlation analysis was performed using SAS 6.02 (Statistical Analysis System) [42]. The 6 sets of data for the 39 observation points (Tab. 3.7) were analyzed. A correlation analysis was performed in order to show the strength of the relationship between any two variables. The correlation coefficient is a number that ranges from -1 to +1. A positive correlation means that, as the value of one variable increases, the value of the other variable will also tend to increase. A correlation coefficient near zero means there is little correlation between the two variables. Table 3.8 presents the results of the correlation analysis. For example, in the case of the simultaneous expansion process, the correlation coefficients between P^*/Y_{st} and t/a is 0.215.

In general, two procedures are used to study the relationship between parameters, namely: correlation analysis and regression analysis[43]. Correlation analysis is primarily useful in cases where the relationship between the parameters is not predictive but merely associative. Regression analysis is most useful when values of one parameter can be predicted from changes in other parameters [44]. In the following Chapter, we will use regression analysis to study the relationship between the various parameters.

No.Cal. (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	A	B	AXB CXD DXF GXH	C	AXC BXD EXG FXH	BXC AXD FXG EXH	D	E	AXE BXF CXG DXH	BXE AXF DXG CXH	F	CXE DXF AXG BXH	G	H	DXE CXF BXG AXH
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

Table 3.1 Orthogonal array $L_{16}(2^{15})$

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
	Outer radius of tube	Tube wall thickness	Initial clearance	Expansion pressure	Young's modulus of tube	Yield stress of tube	Young's modulus of tubesheet	Yield stress of tubesheet
Level	(m)	(m)	(m)	(Mpa)	(Gpa)	(Mpa)	(Gpa)	(Mpa)
1	0.01905	0.0021	0.0000254	238.56	206.844	206.844	206.844	206.844
2	0.0254	0.00277	0.0000762	206.844	275.792	275.792	275.792	275.792

Table 3.2 Parameter of calculations with two levels

No. Cal. (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	RESULT (Residual Contact Pressure) (Mpa)
	A	B	AXB CXD DXF GXH	C	AXC BXD EXG FXH	BXC AXD FXG EXH	D	E	AXE BXF CXG DXH	BXE AXF DXG CXH	F	CXE DXF AXG BXH	G	H	DXE CXF BXG AXH	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	47.829
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	21.981
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	21.050
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	18.526
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	0.317
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	29.579
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	6.433
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	63.094
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	4.268
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	35.556
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	24.739
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	35.122
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	41.920
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	37.666
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	33.005
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	10.880
I_j	208.87	209.07	232.86	219.18	203.51	223.05	278.78	179.56	194.85	239.17	307.22	221.96	260.73	212.49	215.52	
II_j	223.16	222.96	199.17	212.85	228.51	208.98	153.24	252.47	237.17	192.86	124.81	210.07	171.29	219.54	216.50	
$\overline{I}_j$	26.11	26.13	29.11	27.40	25.44	27.88	34.85	22.44	24.36	29.90	38.40	27.74	32.59	26.56	26.94	
$\overline{II}_j$	27.90	27.87	24.90	26.61	28.56	26.12	19.15	31.56	29.65	24.11	15.60	26.26	21.42	27.44	27.06	
S_j	12.84	11.88	71.31	2.52	38.98	12.36	998.03	332.77	114.09	133.11	2091.68	9.03	522.92	3.09	0.06	

Table 3.3 The 16 calculations designed by using orthogonal array $L_{16}(2^{15})$ and the results(residual contact pressure)

Column number (k)	Factors and interactions (k)	Sum of squares (S_k)	Degree of freedom (f_k)	Mean squares ($V_k=S_k/f_k$)	F_k (V_k/V_{error})	Significance
1	A	12.84	1	12.84	6.8	*
2	B	11.88	1	11.88	6.4	*
3	AxB+CxD+ExF+ +GxH	71.31	1	71.31	37.5	
4	C	2.52	1	2.52	1.3	–
5	AxC+BxD+ExG+ +FxH	38.98	1	38.98	20.7	
6	BxC+AxD+FxG+ +ExH	12.36	1	12.36	6.5	
7	D	998.03	1	998.03	521.4	**
8	E	332.77	1	332.77	175.9	**
9	AxE+BxF+CxG+ +DxH	114.09	1	114.09	59.3	
10	BxE+AxF+DxG+ +CxH	133.11	1	133.11	70.3	
11	F	2091.68	1	2091.68	1100.9	**
12	CxE+DxF+AxG+ +BxH	9.03	1	9.03	4.7	
13	G	522.92	1	522.92	264.7	**
14	H	3.09	1	3.09	1.6	–
15	DxE+CxF+BxG+ +AxH	0.06	1	0.06	–	
	Error	5.70	3	1.90 (V_{error})		

Note. Where $S_A=S_1$, $S_B=S_2$, $S_C=S_4$, $S_D=S_7$, $S_E=S_8$, $S_F=S_{11}$, $S_G=S_{13}$, $S_H=S_{14}$,
$S_{error}=S_C+S_H+S_{DxE+CxF+BxG+AxH}=S_4+S_{14}+S_{15}$

The significance tests are based on the statistical F distribution.

(**)very significant with α=0.001;

(*)relatively significant with α=0.10

(–)less significant with α near or less than 0.10.

α is so-called the significant level and defined as a probability
$\alpha = P\ (F_K > F_\alpha\ )$

Table 3.4 Analysis of Variance and Significance Test

No.		MATERIAL PROPERTY	E_t/Y_{st} (x 10^5)	E_s/E_t	Y_{ss}/Y_{st}
T1	Tube Tubesheet	Steel Steel	0.9333	1.0357	1.2667
T2	Tube Tubesheet	I-800 Steel	0.5534	0.9310	0.5344
T3	Tube Tubesheet	70:30 Cu-Ni Steel	1.2222	1.3182	1.6667
T4	Tube Tubesheet	70:30 Cu-Ni 90:10 Cu-Ni	1.2222	0.8182	0.8333
T5	Tube Tubesheet	Admiralty Steel	1.0667	1.8125	2.5333
T6	Tube Tubesheet	Steel Muntz Metal	0.75	0.5102	0.5153

Table 3.5: Material properties

No.Cal. (i)	1 t/a	2 s/a	3 P/Y_{st}	4 c/a
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	2	3
9	3	3	1	1

(1)

No.Cal. (i)	1 t/a	2 s/a	3 c/a
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

(2)

Table 3.6 (1) The calculations designed by using orthogonal array $L_9(3^4)$

(2) The calculations designed by using orthogonal array $L_4(2^3)$

OBS	MATERIAL	t/a	s/a	c/a	P/Y_{st}	P^*/Y_{st}	S^*_z/Y_{st}	S^*_h/Y_{st}	k %
1	T1	0.065	1.5	0.032	0.700	0.021	1.540	1.437	4.440
2		0.065	2.5	0.003	0.800	0.033	0.470	0.300	0.484
3		0.065	3.5	0.010	0.900	0.057	0.487	0.480	1.390
4		0.083	1.5	0.010	0.800	0.053	1.107	0.783	1.450
5		0.083	2.5	0.032	0.900	0.064	1.230	1.293	4.380
6		0.083	3.5	0.001	0.700	0.001	0.303	0.133	1.250
7		0.109	1.5	0.003	0.900	0.086	0.713	0.513	0.620
8		0.109	2.5	0.000	0.700	0.009	0.014	0.030	0.011
9		0.109	3.5	0.032	0.800	0.028	1.960	2.123	4.750
10	T2	0.065	1.9	0.032	0.700	0.021	1.315	1.387	5.150
11		0.065	2.5	0.003	0.689	0.014	0.399	0.221	0.544
12		0.065	1.9	0.010	0.689	0.018	0.779	0.714	1.960
13		0.083	1.5	0.010	0.689	0.020	0.807	0.716	2.330
14		0.083	2.5	0.032	0.689	0.013	1.384	1.496	5.180
15		0.083	1.9	0.003	0.700	0.018	0.781	0.574	0.789
16		0.109	1.5	0.003	0.689	0.009	0.853	0.634	0.604
17		0.109	2.5	0.010	0.700	0.005	0.853	0.800	1.640
18		0.065	1.5	0.032	0.616	0.008	1.214	1.132	2.160
19	T3	0.065	1.5	0.032	1.000	0.083	1.500	1.450	4.290
20		0.065	2.5	0.003	1.055	0.077	0.689	0.517	0.529
21		0.065	3.5	0.010	1.111	0.093	1.183	1.011	1.450
22		0.083	1.5	0.010	1.055	0.113	1.178	1.100	1.580
23		0.083	2.5	0.032	1.111	0.121	2.022	2.144	5.210
24		0.083	3.5	0.003	1.000	0.065	0.528	0.389	0.513
25		0.109	1.5	0.003	1.111	0.152	0.867	0.611	0.663
26		0.109	2.5	0.010	1.000	0.089	1.322	1.333	1.850
27		0.109	3.5	0.032	1.051	0.091	1.150	1.250	4.800
28	T4	0.098	1.6	0.032	0.750	0.044	2.189	2.383	4.540
29		0.098	1.6	0.003	0.917	0.113	0.828	0.728	0.622
30		0.130	1.6	0.032	0.917	0.103	2.217	2.383	5.240
31		0.130	2.5	0.003	0.750	0.020	0.672	0.450	0.526
32	T5	0.049	1.8	0.032	1.000	0.035	1.407	1.220	3.970
33		0.049	2.8	0.003	1.100	0.047	0.720	0.460	0.507
34		0.166	1.8	0.032	1.100	0.100	1.100	0.740	6.070
35		0.166	2.8	0.001	1.000	0.008	0.547	0.187	0.261
36	T6	0.065	1.4	0.032	0.485	0.027	1.112	0.709	4.150
37		0.065	2.3	0.003	0.612	0.052	0.365	0.212	0.597
38		0.110	1.4	0.032	0.612	0.040	0.913	0.819	4.810
39		0.110	2.3	0.000	0.485	0.002	0.004	0.025	0.006

Table 3.7 Results of finite element analysis
(1) Sequential Expansion Case

OBS	MATERIAL	t/a	s/a	c/a	P/Y_{st}	P^*/Y_{st}	S^*_z/Y_{st}	S^*_h/Y_{st}	k %
1	T1	0.065	1.5	0.032	0.700	0.043	0.987	0.813	4.130
2		0.065	2.5	0.003	0.800	0.041	0.897	0.577	0.515
3		0.065	3.5	0.010	0.900	0.064	0.853	0.777	1.430
4		0.083	1.5	0.010	0.800	0.078	0.960	0.637	1.510
5		0.083	2.5	0.032	0.900	0.078	1.753	1.853	4.820
6		0.083	3.5	0.001	0.700	0.006	0.510	0.363	0.175
7		0.109	1.5	0.003	0.900	0.116	0.847	0.627	0.672
8		0.109	2.5	0.000	0.700	0.011	0.014	0.039	0.014
9		0.109	3.5	0.032	0.800	0.037	1.733	1.773	5.460
10	T2	0.065	3.5	0.032	0.700	0.023	1.342	1.408	5.090
11		0.065	2.5	0.003	0.689	0.021	0.796	0.552	1.020
12		0.065	3.5	0.010	0.689	0.018	0.758	0.689	1.940
13		0.083	2.0	0.010	0.689	0.025	0.933	0.613	1.890
14		0.083	2.5	0.032	0.689	0.022	1.219	1.158	5.050
15		0.083	3.5	0.003	0.700	0.017	0.716	0.536	0.786
16		0.109	2.0	0.003	0.689	0.016	0.782	0.615	0.646
17		0.109	2.5	0.010	0.700	0.014	0.905	0.504	1.690
18		0.065	2.0	0.032	0.616	0.011	1.166	1.025	4.380
19	T3	0.065	1.5	0.032	1.000	0.100	1.356	1.139	4.270
20		0.065	2.5	0.003	1.055	0.091	0.833	0.606	0.620
21		0.065	3.5	0.010	1.111	0.104	1.111	0.994	1.440
22		0.083	1.5	0.010	1.055	0.139	1.000	0.667	1.630
23		0.083	2.5	0.032	1.111	0.131	1.906	2.044	5.050
24		0.083	3.5	0.003	1.000	0.074	0.678	0.489	0.551
25		0.109	1.5	0.010	1.111	0.093	1.239	1.217	1.780
26		0.109	2.5	0.003	1.000	0.176	0.839	0.622	0.702
27		0.109	3.5	0.032	1.051	0.120	2.150	2.272	6.530
28	T4	0.098	1.6	0.010	0.750	0.073	0.956	0.750	1.570
29		0.098	1.6	0.003	0.917	0.147	0.944	0.672	0.903
30		0.130	1.6	0.032	0.917	0.166	1.011	0.711	5.480
31		0.130	2.5	0.003	0.750	0.034	0.739	0.589	0.535
32	T5	0.049	1.8	0.032	1.000	0.043	1.387	1.253	3.960
33		0.049	2.8	0.003	1.100	0.057	0.893	0.580	0.568
34		0.166	1.8	0.010	1.100	0.112	0.953	0.687	2.050
35		0.166	2.8	0.001	1.000	0.042	0.813	0.760	0.257
36	T6	0.065	1.4	0.032	0.485	0.042	0.987	0.699	4.410
37		0.065	2.3	0.003	0.612	0.062	0.852	0.579	0.989
38		0.110	1.4	0.032	0.612	0.078	0.972	0.719	5.280
39		0.110	2.3	0.000	0.485	0.004	0.004	0.027	0.011

Table 3.7 Results of finite element analysis
(2) Simultaneous Expansion Case

	t/a	s/a	c/a	P/Y_{st}	E_t/Y_{st}	E_s/E_t	Y_{ss}/Y_{st}
	(1) Sequential Expansion Case						
P*/Yst	0.120	-0.090	0.105	0.764	0.716	0.332	0.434
S*z/Yst	0.009	-0.239	0.790	0.254	0.276	0.106	0.085
S*h/Yst	-0.001	-0.160	0.785	0.205	0.248	0.027	0.006
k	0.060	-0.177	0.957	0.056	0.043	0.028	0.031
	(2) Simultaneous Expanion Case						
P*/Yst	0.215	-0.300	0.104	0.688	0.734	0.272	0.400
S*z/Yst	-0.123	0.044	0.728	0.409	0.213	0.255	0.219
S*h/Yst	-0.082	0.172	0.717	0.376	0.196	0.243	0.212
k	-0.097	-0.096	0.977	-0.005	-0.048	-0.083	-0.091

Table 3.8 Correlation coefficients

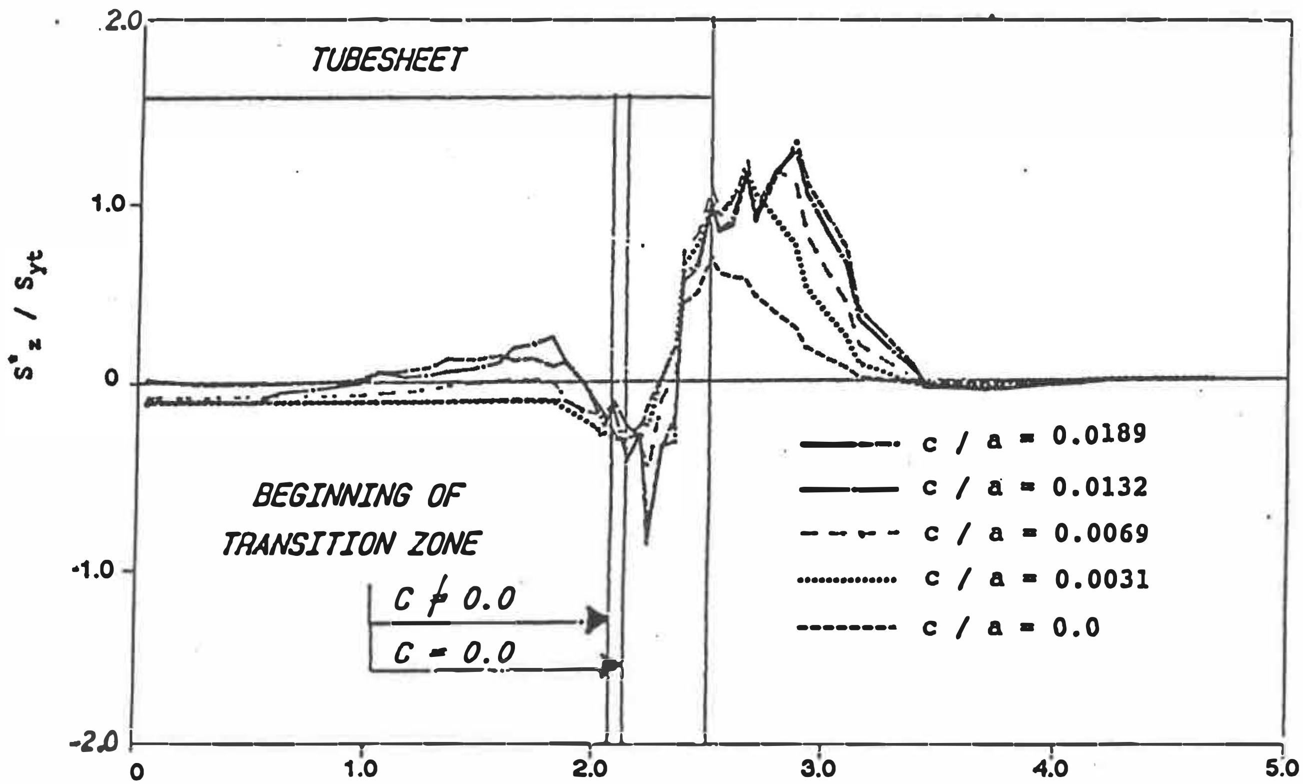


Figure 3.1 Effect of initial clearance between tube and tubesheet on residual stresses

a) Axial residual stress at inner surface of the tube.

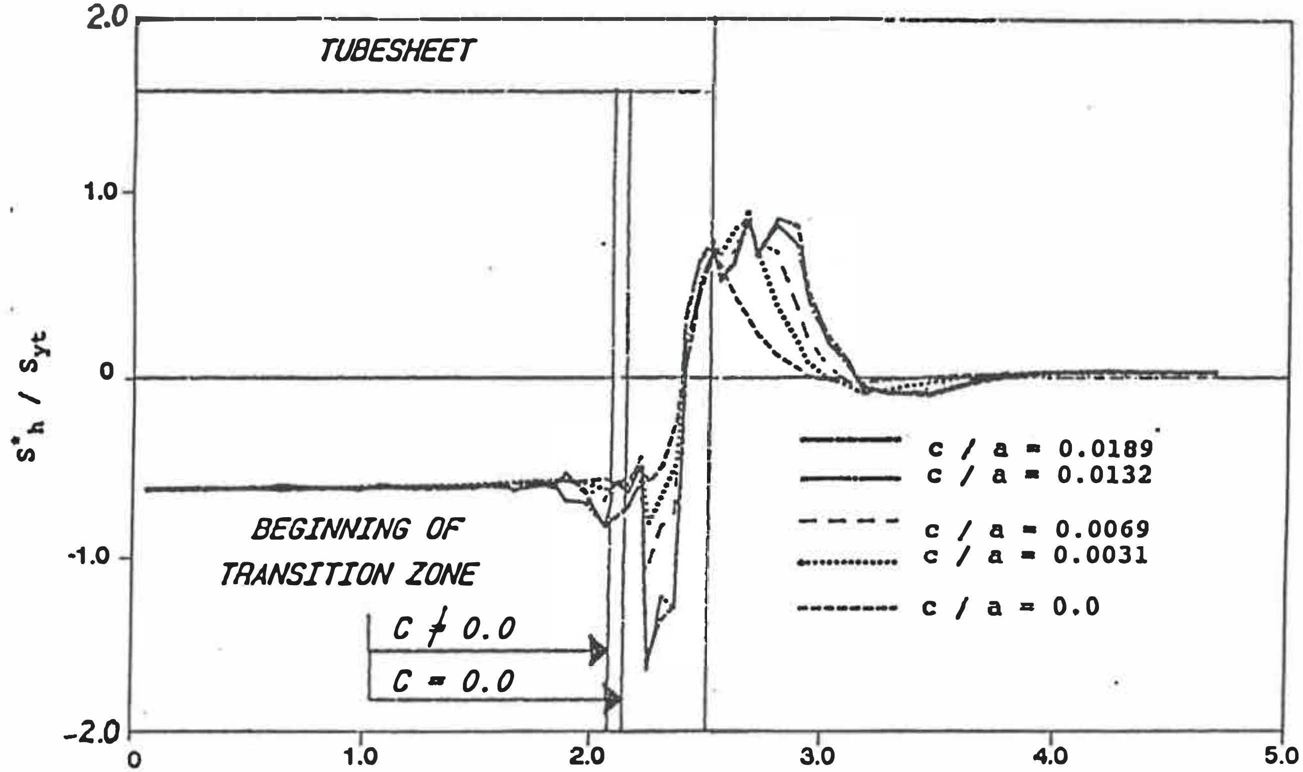


Figure 3.1 (con't)
b) Hoop residual stress at inner surface of the tube.

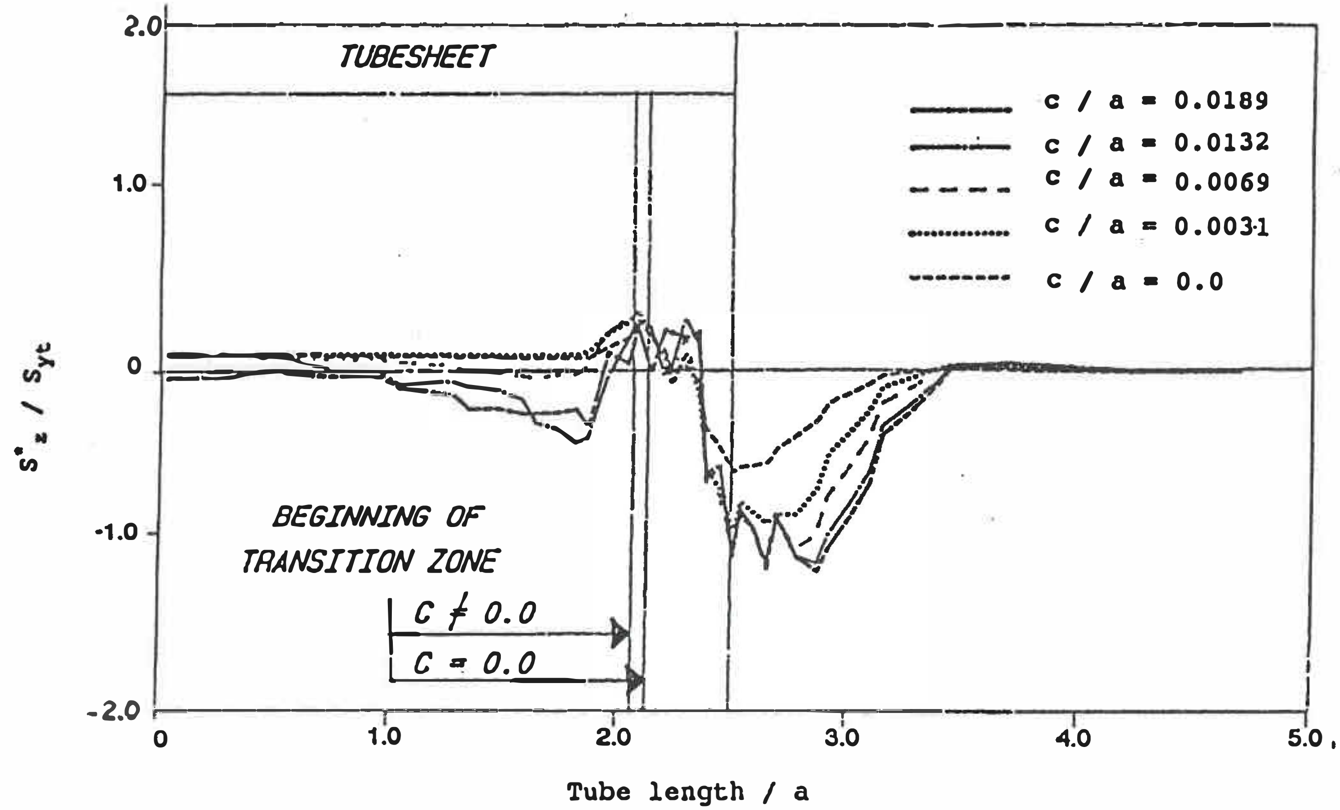


Figure 3.1 (con't)
c) Axial residual stress at outer surface of the tube.

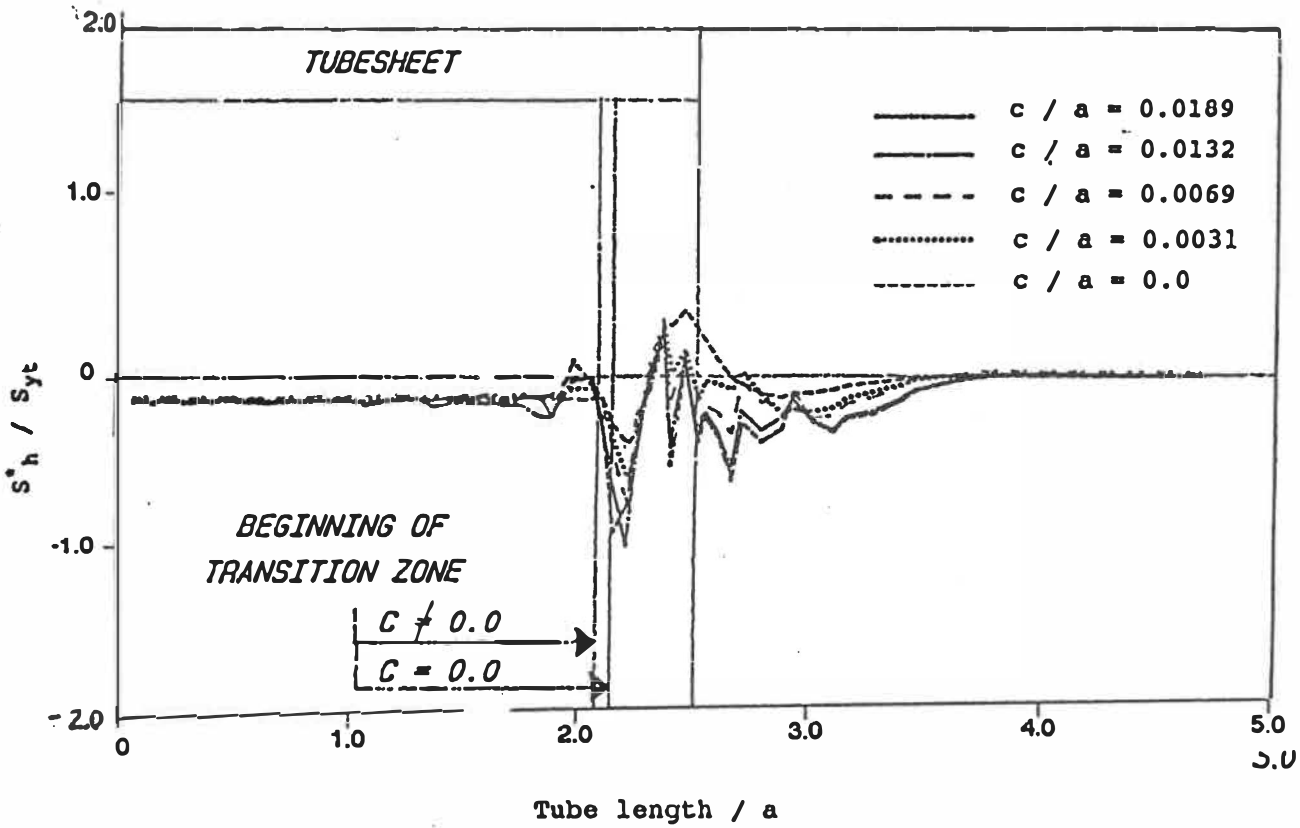


Figure 3.1 (con't)
d) Hoop residual stress at outer surface of the tube.

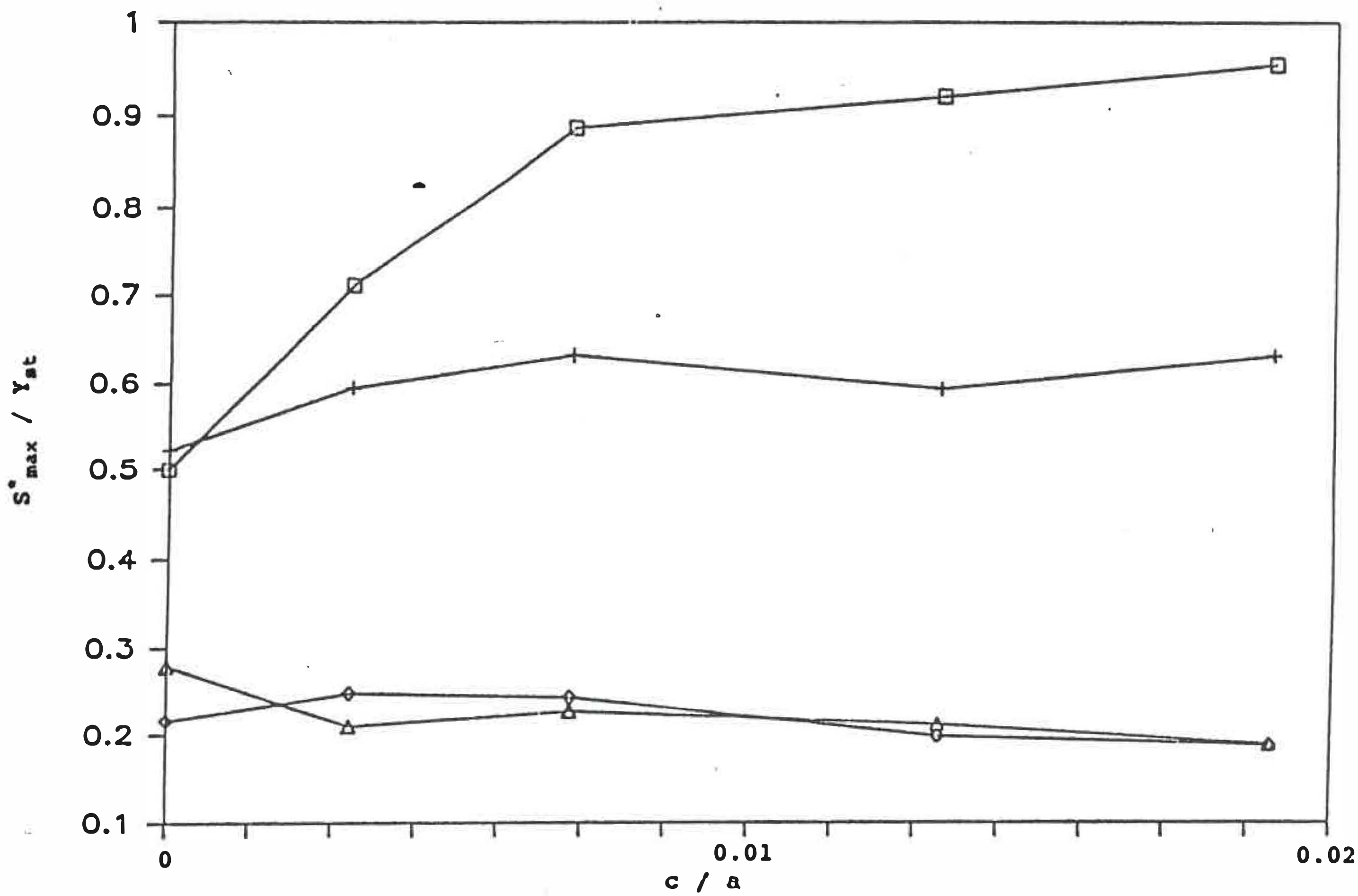


Figure 3.2 Effect of initial clearance on maximum tensile residual stresses in transition zone

□ Axial residual stress at inner surface
+ Hoop residual stress at inner surface
◊ Axial residual stress at outer surface
△ Hoop residual stress at outer surface

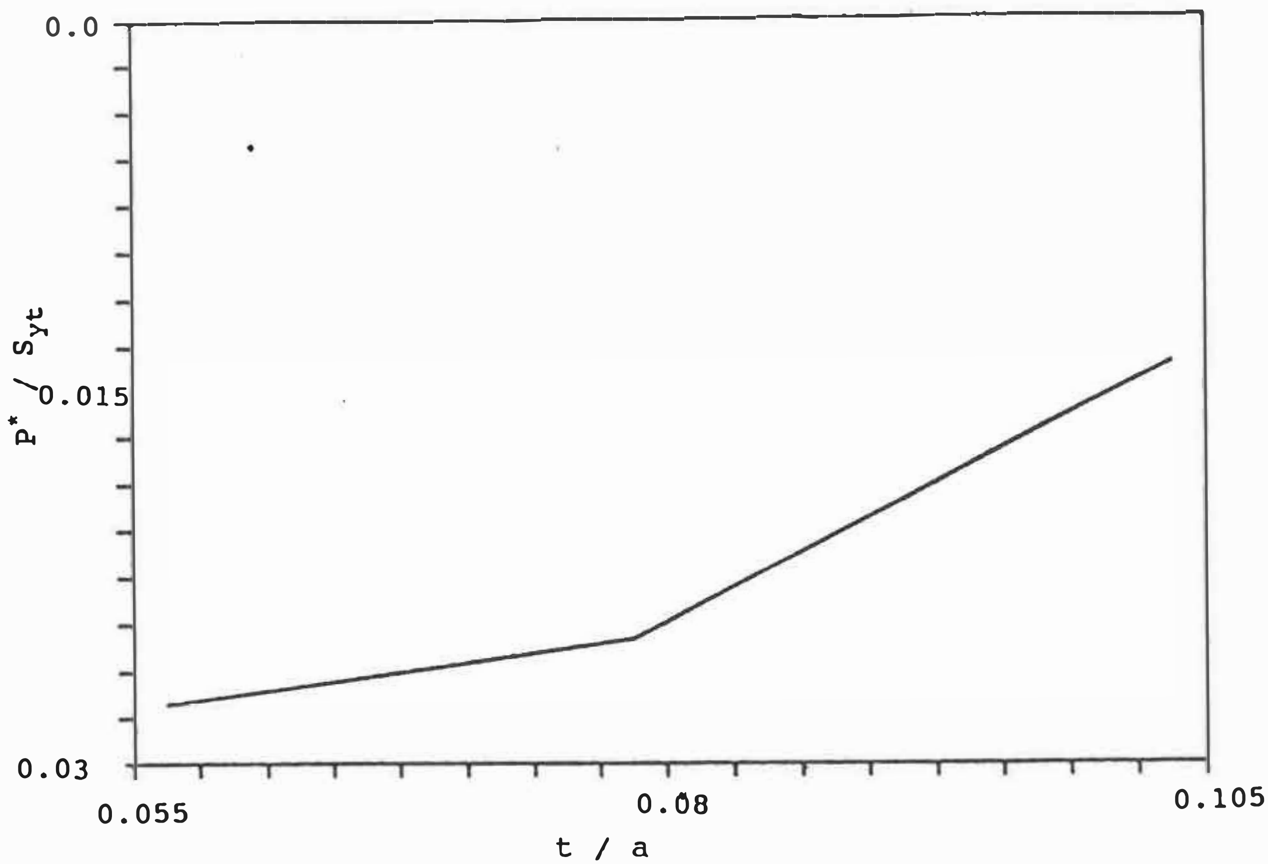


Figure 3.3 Effect of tube thickness
a) Residual contact pressure

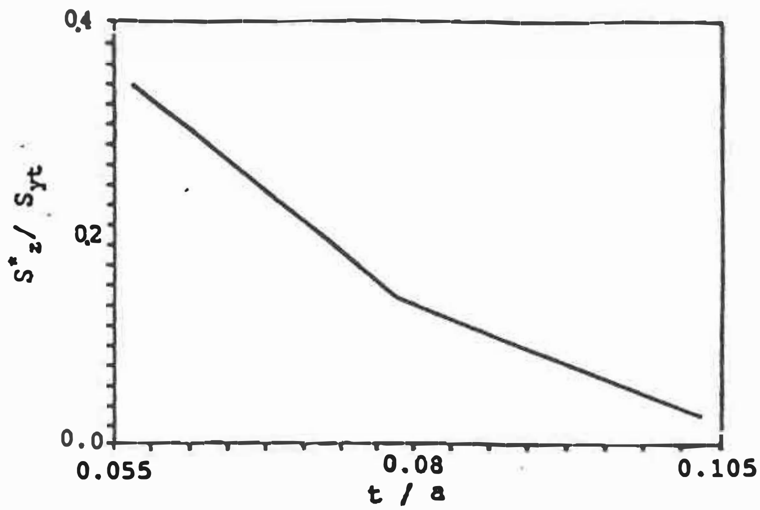


Figure 3.3 (con't)
b) Axial residual stress at inner surface
(maximum tensile)

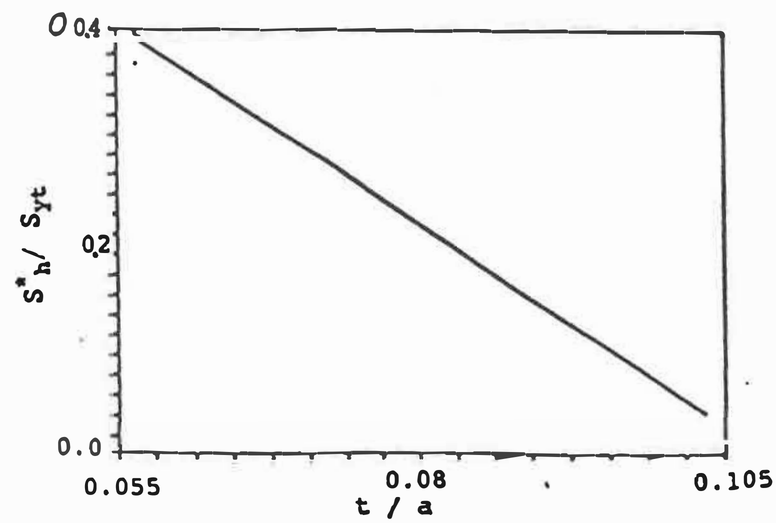


Figure 3.3 (con't)
c) Hoop residual stress at inner surface
(maximum tensile)

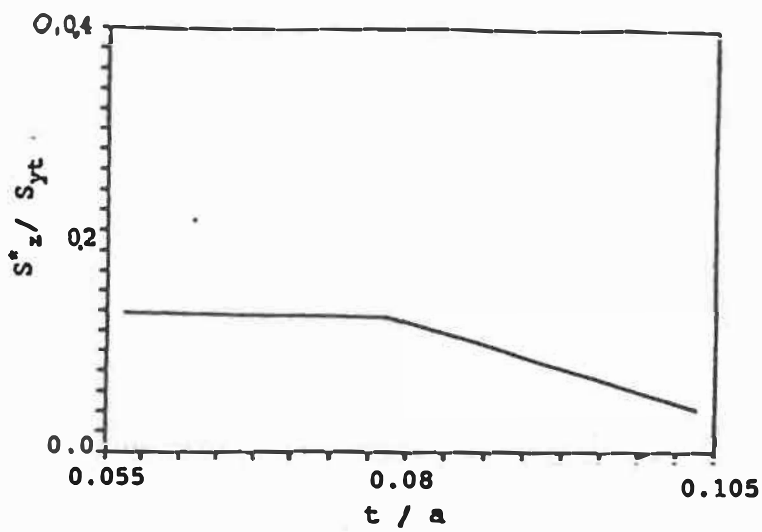


Figure 3.3 (con't)
d) Axial residual stress at outer surface
(maximum tensile)

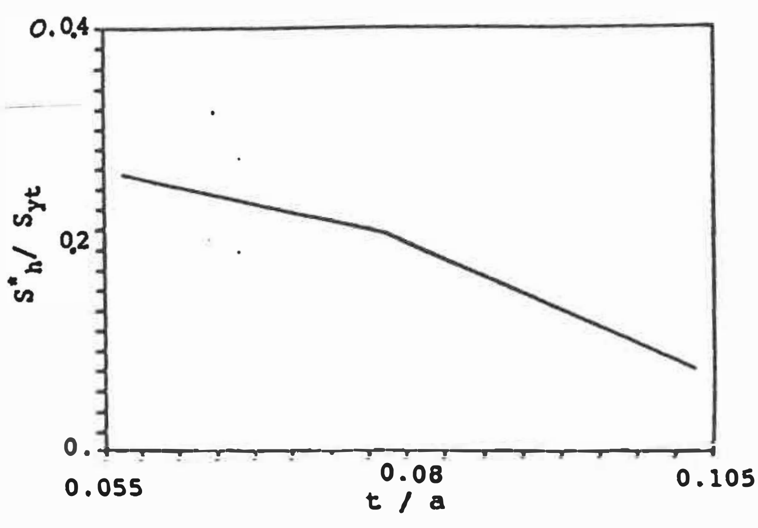


Figure 3.3 (con't)
e) Hoop residual stress at outer surface
(maximum tensile)

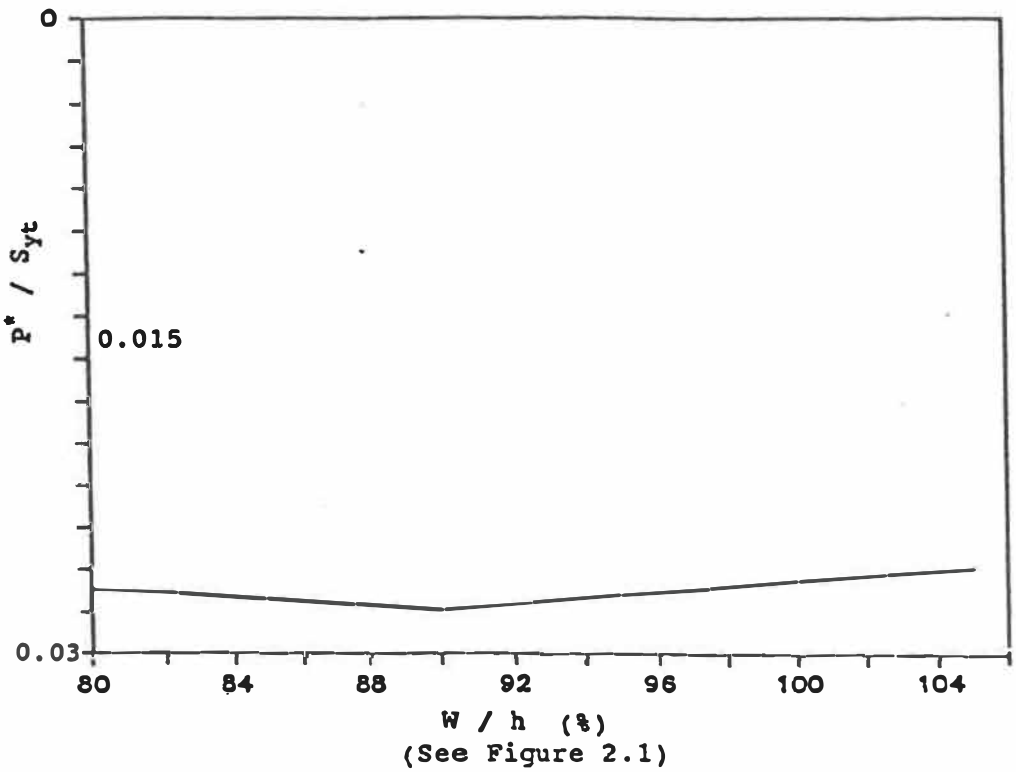
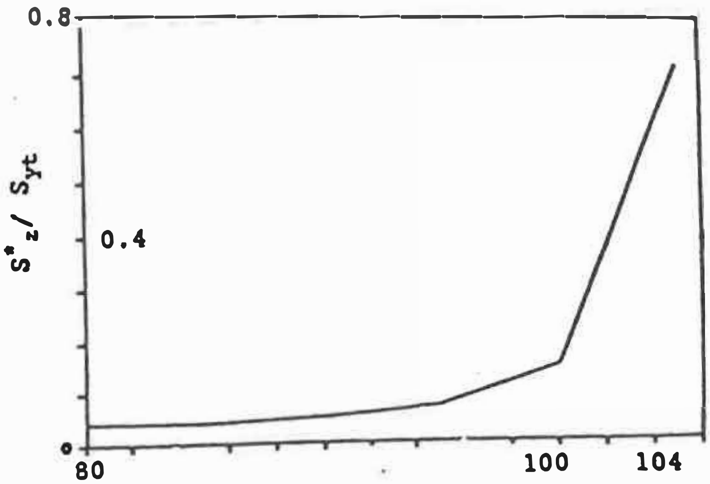
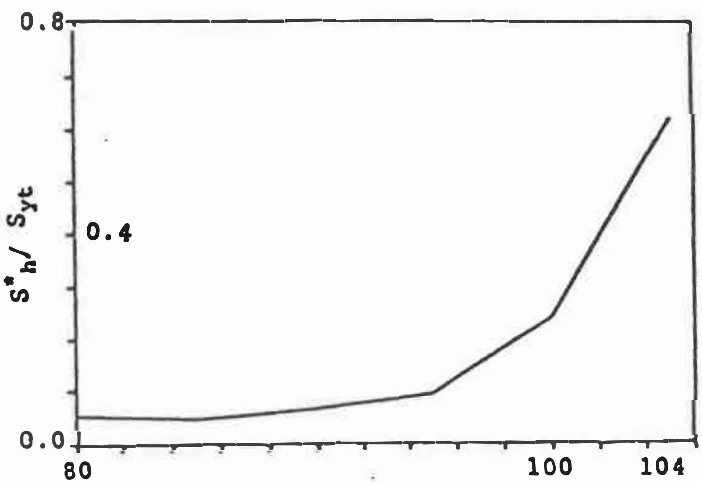


Figure 3.4 Effect of depth of expansion
a) Residual contact pressure



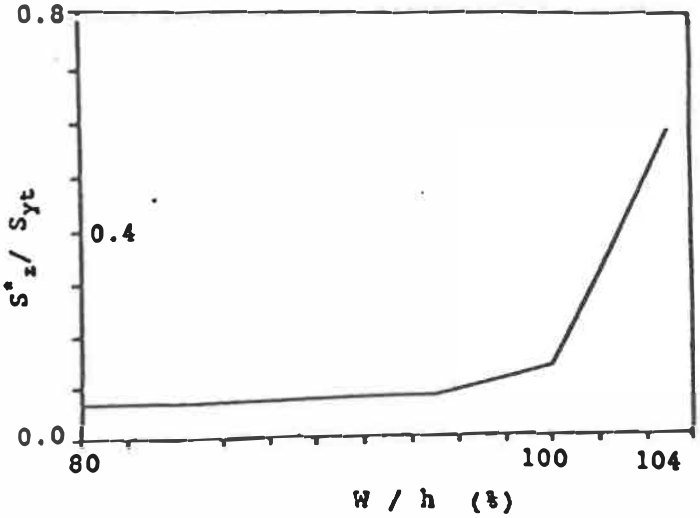
W / h (%)
(See Figure 2.1)

Figure 3.4 (con't)
b) Axial residual stress at inner surface
(maximum tensile)



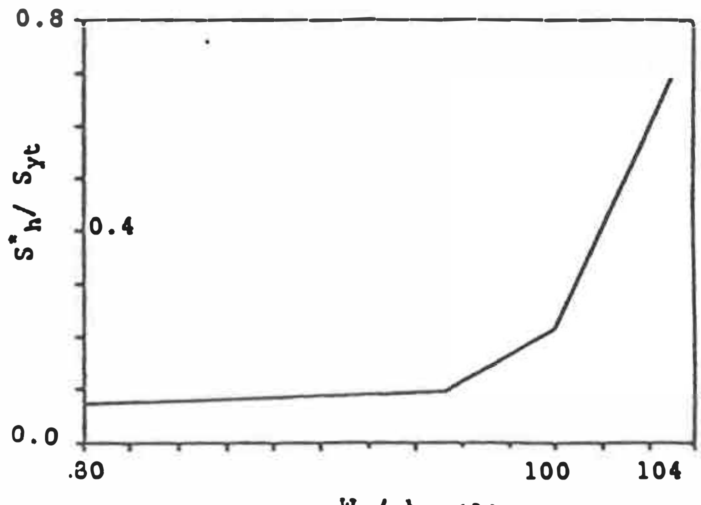
W / h (%)
(See Figure 2.1)

Figure 3.4 (con't)
c) Hoop residual stress at inner surface
(maximum tensile)



(See Figure 2.1)

Figure 3.4 (con't)
d) Axial residual stress at outer surface (maximum tensile)



(See Figure 2.1)

Figure 3.4 (con't)
e) Hoop residual stress at outer surface (maximum tensile)

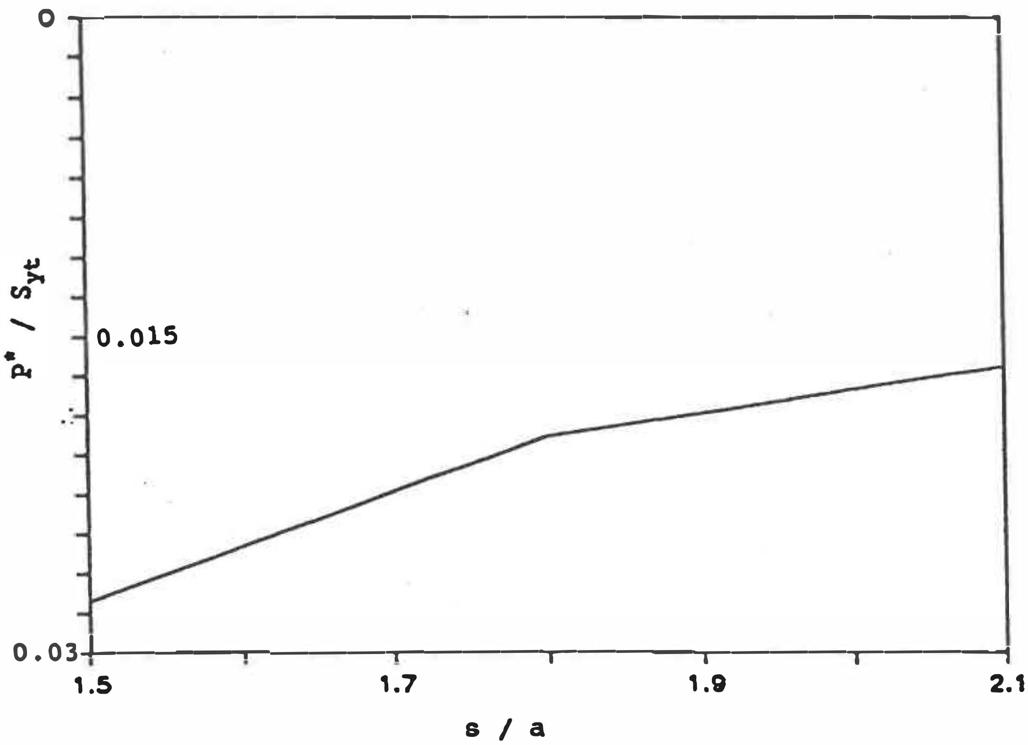


Figure 3.5 Effect of tube pitch
a) Residual contact pressure

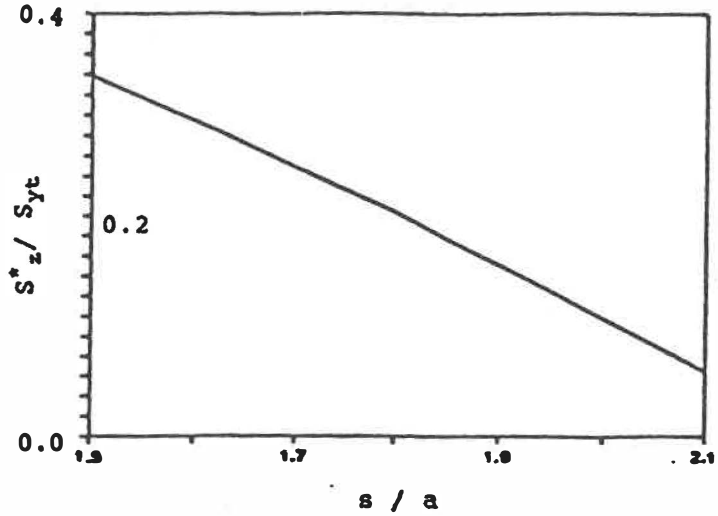


Figure 3.5 (con't)
b) Axial residual stress at inner surface
(maximum tensile)

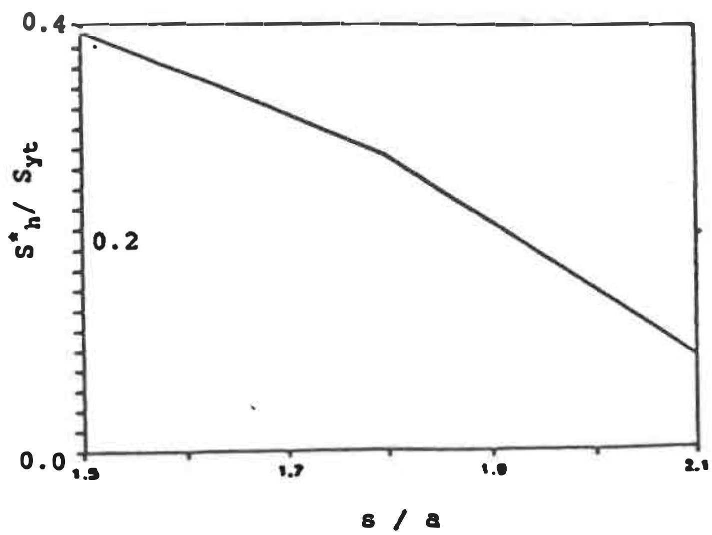


Figure 3.5(con't)
c) Hoop residual stress at inner surface
(maximum tensile)

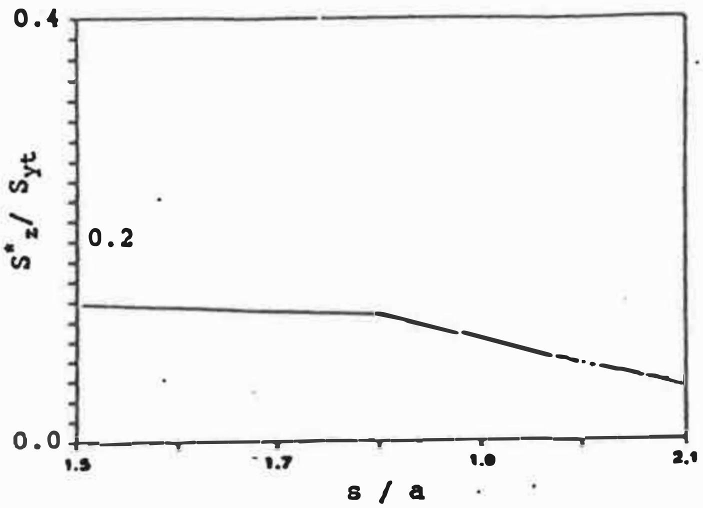


Figure 3.5 (con't)
d) Axial residual stress at outer surface
(maximum tensile)

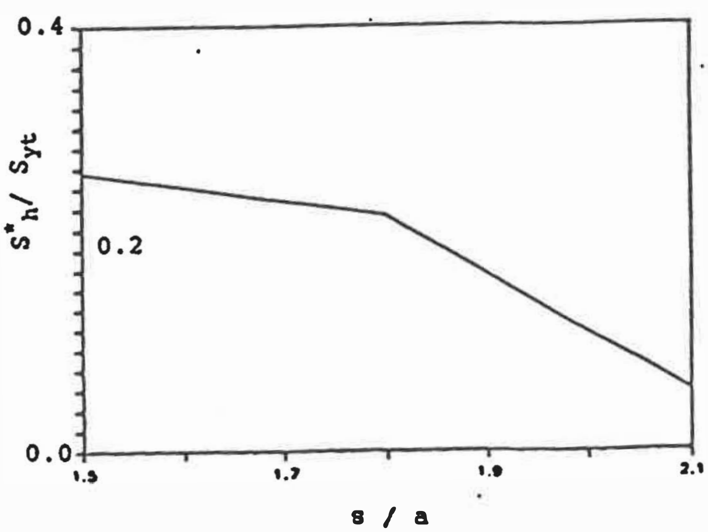


Figure 3.5(con't)
e) Hoop residual stress at outer surface
(maximum tensile)

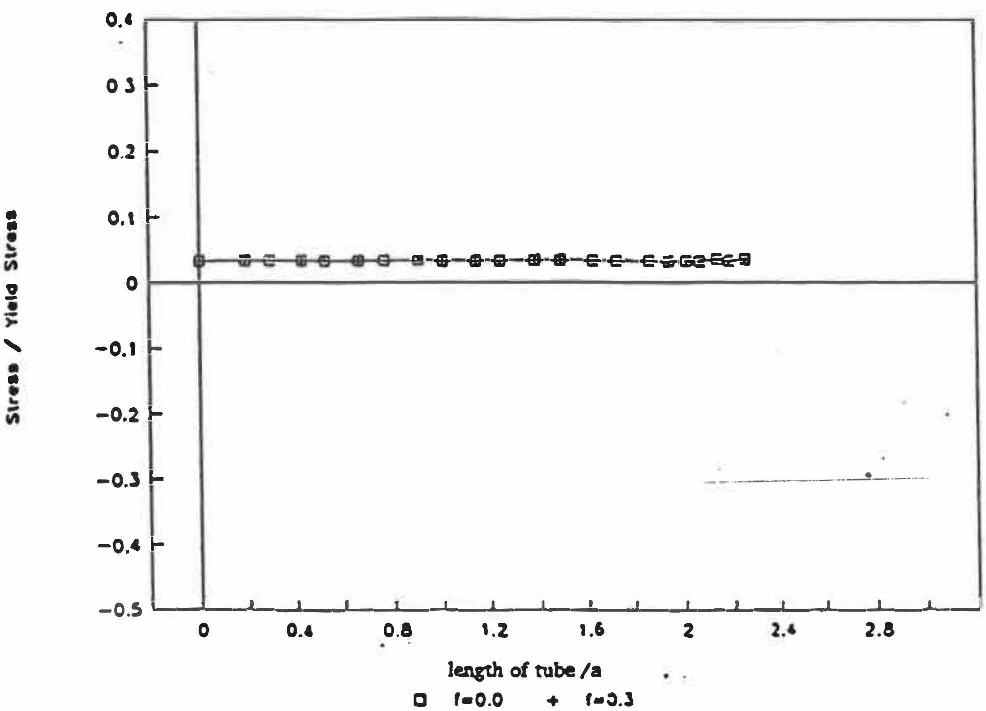


Figure 3.6 Effect of frictional force between tube and tubesheet
a) Residual contact pressure

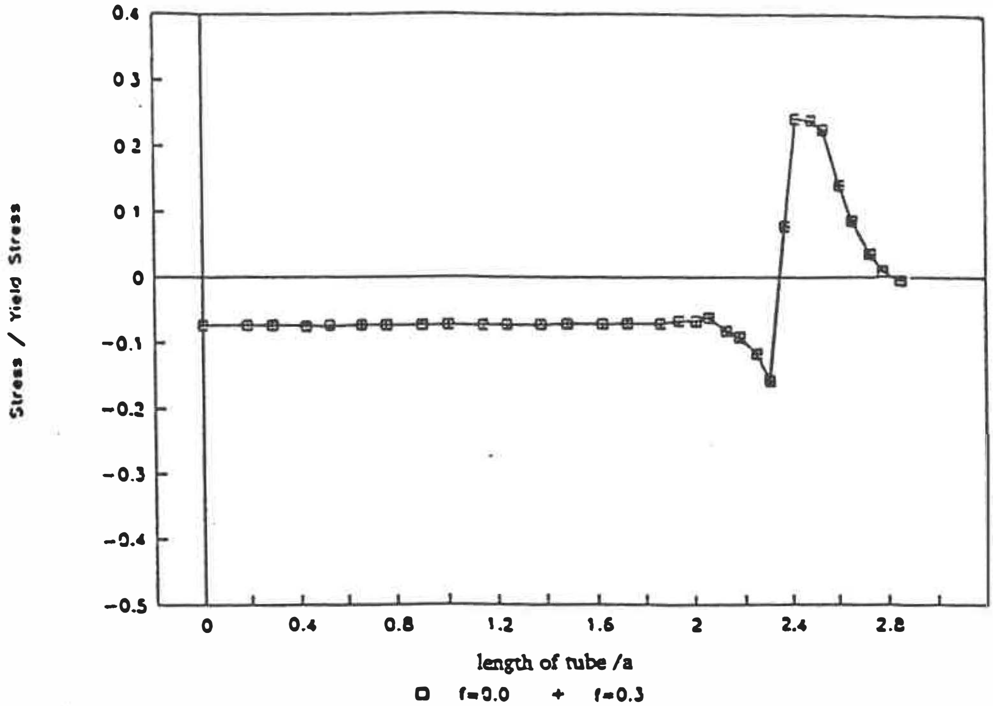


Figure 3.6(con't)
b) Axial residual stress at inner surface

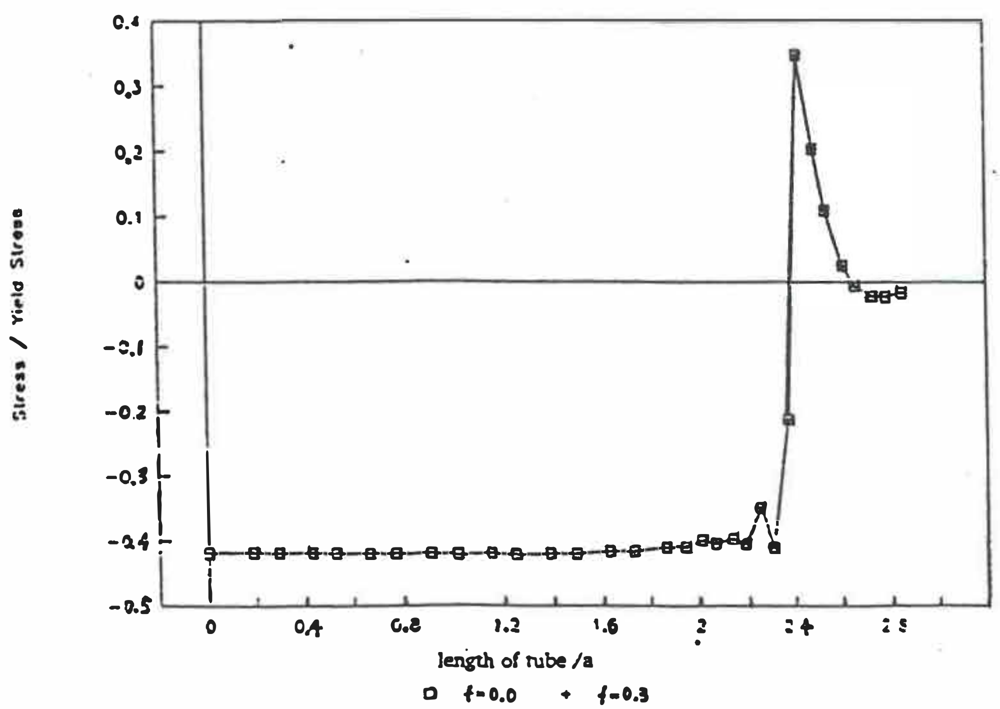


Figure 3.6 (con't)
c) Hoop residual stress at inner surface

CHAPTER 4

REGRESSION ANALYSIS AND FINAL RESULTS

4.1 INTRODUCTION

The finite element analysis of tube-to-tubesheet joints using the "Two-step analysis method" was introduced in Chapter 2. In Chapter 3, the systematic statistical approach was presented. In this Chapter, the empirical equations are developed using regression analysis. The final results and numerical examples are then presented.

Since the residual contact pressure, residual stresses and apparent tubewall reduction are influenced by dimension-, fabrication- and material- parameters, they are called the dependent or response factors. The dimension-, fabrication- and material parameters are called the independent or predictor factors. The study of the individual parameters (in Chapter 3, Section 3.2), indicates that the relationship between one predictor factor and one response factor is non linear. We thus have to use a multiple nonlinear regression model to describe their relationship.

4.2 MULTIPLE REGRESSION MODELS [45]

Suppose we have reason to believe that a response factor (y) is influenced by (m-1) predictor factor $x_1, x_2, ..., x_{m-1}$. A single observation consists of one value of each predictor factor and the response factor. We denote the number of predictor factor by (m-1) rather than m so that the number of population parameters is m rather than (m+1). A linear statistical relationship between the predictor factors and the response factors would yield the following relationship for each observation of y:

$$y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + ... + \beta_{m-1} x_{i,m-1} + \varepsilon_i \qquad (1)$$

Where X_{ij} is the ith observation of the jth predictor factor

y_i is the ith observation of the response factor

β_0, β_j are parameters

ε_i are random error terms

$i = 1, 2, 3, ... , n$

$j = 1, 2, 3, ... , m-1$

The regression function would be

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_{m-1} x_{m-1} \qquad (2)$$

The method of least squares would then be used to obtain the sample regression equation:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_{m-1} x_{m-1} \qquad (3)$$

As mentioned above, however, the problem is nonlinear. In order to simplify the procedure, the linearization of nonlinear equations will be made by using natural logarithms. This approach is simple and reliable,[45][46].

$$y' = \ln(y), \quad x'_1 = \ln(x_1), \; x'_2 = \ln(x_2), \; \ldots, \; x'_i = \ln(x_i)$$

The method of least squares provides the new linear regression equation, as above:

$$\hat{y} = b_0 + b_1 x'_1 + b_2 x'_2 + \ldots + b_{m-1} x'_{m-1} \qquad (4)$$

and the new regression equation is as follows:

$$\hat{y} = e^{b0} \; x_1^{\,b1} \; x_2^{\,b2} \; \ldots \; x_{m-1}^{\,bm-1} \qquad (5)$$

4.3 EMPIRICAL EQUATIONS

On the basis of Eqn (5), our model has the following general form:

$$I = e^{a_0} \left(\frac{t}{a}\right)^{a_1} \left(\frac{s}{a}\right)^{a_2} \left(\frac{c+10^{-6}}{a}\right)^{a_3} \left(\frac{P}{Y_{st}}\right)^{a_4} \left(\frac{E_t}{Y_{st}}\right)^{a_5} \left(\frac{E_s}{E_t}\right)^{a_6} \left(\frac{Y_{ss}}{Y_{st}}\right)^{a_7} \qquad (6)$$

Where "I" might be either P^*/Y_{st} ,S^*_z/Y_{st}, S^*_h/Y_{st} or k, for each of the simultaneous and sequential expansion processes; in other words, Eqn. (6) actually represents eight equations.

The term $(c+10^{-r})$ is introduced to avoid problems for the case where c=0. In order to choose the appropriate r exponent, six cases $(c+10^{-1}$, $c+10^{-2}$, ... $c+10^{-6})$ for the sequential expansion process and three cases $(c+10^{-4}$, $c+10^{-5}$, $c+10^{-6})$ for the simultaneous expansion process were studied. Figure 4.1 (a and b) show the variation of the R-SQUARE versus the regression equation results and suggests that $(c+10^{-6})$ is the most appropriate. More analyses are needed to justify the proposed form and the parameter $(c+10^{-6})$ of Eqn.(6) particularly.

Eqn.(6) can be analysed by the REG procedure in SAS. The REG procedure fits least-squares estimates to linear regression models. The specific values of every coefficient in Equation (6) are given in Table 4.1 which also shows MODEL F and R-SQUARE for each equation. The value of MODEL F tests how well the model as a whole accounts for the dependent variable's behavior. If the significance probability, labelled PR>F, is small, it indicates significance. R-SQUARE measures how much variation in the dependent variable can be accounted for by the model. In general, the larger the value of R-SQUARE, the better the model's fit [47]. These results will be substantiated and detailed in the following Sections.

The meaning of the individual items in the SAS output and the meaning of "VARIABLE"s which are used in the SAS procedure are shown in a note at the end of Chapter 4.

4.4 VERIFICATION OF THE EMPIRICAL EQUATIONS

The reliability of the equations will be verified based on the analysis of variance and residual analysis.

4.4.1 Analysis of variance

Table 4.2 was produced by the REG procedure in SAS. We can use the F test, where F^* is formed by dividing the mean square for MODEL(MSR) by the mean square for ERROR(MSE), to verify the hypothesis that there is an association between at least one of the predictor factors and the response factor. That is, at least one of the β_j $(j>0)$ is not equal to zero. If all β_j are zero, changes in predictor factors do not have an effect on the response factor. The test is frequently called an "overall F-test" since it simultaneously tests the hypotheses that each $\beta_j (j > 0)$ may be equal to zero [48].

Suppose we obtain a set of n observations $(x_1, x_2, ..., x_{m-1}, y)$ from a population (here n= 39) for which the relationship between the factors is described by the regression function

$$E(y)= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 +...+ \beta_{m-1} x_{m-1} \text{ [here (m-1) =7].}$$

Then we may test the null hypothesis, H_0: $\beta_1 = \beta_2 = ... = \beta_{m-1} = 0$. The sample value of the test statistic used to test the hypothesis is F^*. The decision rule is dictated by the level of significance α chosen for the test. If $F^* > F_\alpha\{m-1,n-m\}$, (here (n-m)= 39-8 =31), we reject H_0 above. We use "F VALUE" F^* =22.173, see Table 4.2 for factor LRCPR (take the logarithm to base e of the ratio of residual contact pressure to the yield stress of the tube, i.e. $\ln(P^*/Y_{st})$), we chose the level of significance α=0.01, since $F_{0.01}\{7,31\}$= 3.28 and $F^* > F_{0.01}$, we can conclude that there is an association between at least one of the predictor factors and the response factor.

In Table 4.2, the value listed under the heading "PROB > F" is the observed significance level or "P value" (P for probability) of the results. The "P VALUE"s are equal to 0.0001; thus we can reject the null hypothesis at any level of significance greater than 0.0001.

"R-SQUARE" will reflect the variation in the dependent factor accounted for by a linear combination of all the independent factors. In Table 4.2, "R-SQUARE" is 0.8335, which indicates that we can account for over 83.35% of residual contact pressure by knowing the values of the parameters in Eqn.(6). As mentioned above, in general the larger the value of R-SQUARE, the better is the model fit. This indicates a good correlation between the empirical equations and the 39 observation points.

4.4.2 Residual Analysis

The assumptions underlying the linear regression model must be met if inferences and predictions are to be valid. The following four assumptions about the model are satisfied [49]:

1. The association between the predictor factors and the response factors is linear.
2. The error terms (ε_j) are independent of each other.
3. The distribution of the error terms is normal.
4. The variance of the error terms is constant for all values of predictor factors.

These assumptions may be tested by using a residual plot. For any observation ($x_{i,1}$, $x_{i,2}$, ..., $x_{i,m-1}$, y_i), the predicted value is $\hat{y}_i$, the difference between observation value y_i and predicted value $\hat{y}_i$ is the residual e_i, i.e. $y_i - \hat{y}_i = e_i$. The residual plot for the linear regression model is a graph of residuals plotted against $\hat{y}$. We proceed as follows to evaluate whether the above four assumptions are satisfied:

1. Computer-generated plots of the residuals against each predictor factor are presented in Appendix D. Figure 4.2 shows a typical residual plot as an example, the residual against the predictor factor LRYS ($\ln(Y_{ss}/Y_{st})$). They show that the points scatter at random above and below the zero line. We can conclude that the association between the response factors and the predictor factors is linear.

2. Computer-generated plots of the residuals against the time order, that is, the order in which we obtained the observations, are given in Appendix D. Figure 4.3 shows the typical residual plot as an example. They show that they are a random pattern. We can conclude that the error terms are independent, at least with respect to time. If they are not independent, in most cases successive residuals will be fairly close together[50].

3. Computer-generated plots of the residuals against each response factor are shown in Appendix D. Figure 4.4 shows a typical plot of residual against the response factor LRPCR (i.e. $\ln(P^*/Y_{st})$). In Table 4.2(a), the standard error of estimate s_e =0.53124 is shown beside ROOT MSE. The s_e is quite important because it estimates the variability of the error term. Figure 4.4 shows that about two-thirds of residuals will lie within s_e of zero and about 95% will lie within $2s_e$ of zero. We can conclude that the distribution of the error term is normal.

4. In the above computer-generated plots, no serious deviations of error variances are noticeable. We can conclude that the variance σ^2 is constant throughout the data, [51].

Thus no assumptions appear to be violated and, therefore, we can conclude that the regression equations are reliable.

4.5 SIGNIFICANCE OF THE PREDICTOR FACTORS

Even when the F test shows that there is some association between at least one predictor factor and the response factor, there is not necessarily an association between each of the predictor factors and the response factor. Perhaps, therefore, some predictor factors are needed in the model while the others can be discarded.

As in linear regression, each β_j is estimated by b_j. Each b_j is normally distributed with mean β_j and standard error σ_{bj}. In multiple regression, $(b_j - \beta_j)/s_{bj}$ has a t distribution with (n-m) degrees of freedom (see Eqn.(1)) for each value of j. Thus we can obtain confidence intervals for β_j and test hypotheses concerning specific values of β_j. Table 4.2 is a computer printout of analysis of variance. In this Table, under the heading "STANDARD ERROR", are listed the standard errors of the factors which are listed under the heading "VARIABLE". In particular, for factor LRPCR (sequential case), s_{b1} = 0.3081 (for LTWT), ... , s_{b7}= 0.6199 (for LRYS). We can test an individual hypothesis that $\beta_j = 0$ by using the usual t test. The P values for each of these tests are listed under the heading "PROB>|T|", [52].

The values of $t^* = b_j / s_{bj}$ for each j are also given in the computer printout under heading "T FOR H0: PARAMETER = 0"; note that $b_2 / s_{b2} = -3.330$ (LPIT), $b_4 / s_{b4} = 6.849$ (LEXPR), $b_6 / s_{b6} = -4.329$ (LRYM). The P values are given in each case. For LPIT, the level of significance is 0.0023; for LEXPR, the level of significance is 0.0001; for LRYM, the level of significance is 0.0001. Thus the predictor factors LPIT, LEXPR and LRYM have an effect on the response factor LRCPR.

4.6 CONFIDENCE AND PREDICTION INTERVALS

A primary purpose for using regression analysis is to estimate a value or values of the response factor from the predictor factor. Two different types of estimation are usually important. For a particular value of predictor factor, we can estimate the overall mean value of response factor, that is, $E(y_p)$. If we are interested in the value of the response factor for a value of a particular predictor factor rather than for all values of a predictor factor, we would want to estimate the value of the response factor for a value of an individual predictor factor. Thus we can estimate a single value y_p. The SAS program gives confidence and prediction intervals as part of the output. In Appendix D, the 95% confidence intervals for

$E(y_p)$ and prediction intervals for y_p are given for each observation. Table 4.3 shows the confidence intervals for $E(y_p)$ and the prediction intervals for individual values of y_p for "LRCPR".

4.7 STEPWISE REGRESSION ——— DETERMINING THE "BEST" MODEL

Clearly if there are several predictor factors, many different models are possible using linear terms only. If there are (m-1) predictor factors, there are two possibilities for each predictor factor ——— it can be included in the model or left out. Thus the number of possible ways to include or leave out each factor is 2^{m-1}. This also includes the possibility that all factors are left out, which is not a possible model, so the number of possible models using m-1 predictor factors is equal to 2^{m-1}-1. If there are a number of predictor factors to consider, coming up with the best subset can be difficult. Stepwise regression was developed to assist in arriving at this optimal subset. Stepwise regression examines a number of different regression equations. Basically, the goal of stepwise techniques is to select a set of predictor factors and put them into a regression one at a time in a specified manner until all factors have been added or until a specified criterion has been met. The criterion is usually one of statistical significance such as: there are no more regressors that would be significant if entered or improvement in variance too small to bother with.

- Short of using the stepwise procedure, the simplest way to obtain the best-fitting model is probably the following.

- Use all factors and select a level of significance (usually 0.1).

- Perform the regression analysis and check the output. If all factors are significant at the chosen level of significance, this is probably the best model.

- If one or more factors are not significant at the chosen level, remove the factor with the highest level of significance (the one for which $|t^*|$ is the smallest) and perform the regression procedure again.

- If the MSE (Mean Square for Error) of the resulting analysis is not smaller than that of the prior model, the first model is the best fit.

If the MSE is smaller and all factors are significant, this is the best-fitting model.

If one or more factors are not significant, repeat the procedure until you obtain the model with the smallest MSE. Thus with seven predictor factors, there are 2^7-1= 127 possible models, [53].

The SAS procedure can be used STEPWISE to determine the best model. The output of the STEPWISE procedure using the FORWARD option for 39 observation is shown in Appendix D. The SAS software allows for "FORWARD" stepwise technique. "FORWARD" starts with the best single regressor, then finds the best one to add to what exists, the next best, etc. One of the "best" models is show in Table 4.4.

4.8 APPLICATIONS

The residual contact pressure, maximum tensile residual axial and hoop stresses and tubewall reduction have been analyzed for the sequential and simultaneous cases. Throughout the regression analysis and variance analysis reported in this chapter, the main effective factors are determined. They are shown in Table 4.5 which summarises some useful results.

The residual contact pressure depends primarily on the expansion pressure level. The Young's modulus ratio of the tube and tubesheet has a secondary effect. From Figure 4.5(a and b), it will be noticed that the residual contact pressure increases when the expansion pressure increases and when the ratio E_s/E_t decreases. The residual contact pressure is particularly sensitive to the latter ratio: when E_s/E_t is changed from 0.5 to 1.0, the residual contact pressure increased by as much as ten times (under the same expansion pressure level).

The initial clearance between tube and tubesheet is the most important parameter affecting the maximum tensile residual axial stress. Figure 4.6(a) shows that the maximum tensile residual axial stress increases significantly with the increase of initial clearance. The second most important parameter is, again, the ratio E_s/E_t which is dependent on material properties of the joints for sequential cases. From Figure 4.6(b), it is noticed that the level of residual axial stresses increases with the expansion pressure level for the simultaneous expansion case.

For the maximum tensile residual hoop stress, the most important parameter is also the initial clearance. Figure 4.7 shows the maximum tensile residual hoop stress increasing with increasing initial clearance. The second most important parameter is the ratio E_t/Y_{st} and the expansion pressure level P for the sequential and simultaneous cases respectively.

Tubewall reduction depends on initial clearance only for the sequential case, but it is also affected by expansion pressure level for the simultaneous case. Figure 4.8 (a and b) shows the tubewall reduction increasing with initial clearance.

4.9 LIMITS OF EQUATION (6)

The applicable range of Equation (6) should be considered carefully because a limited range of values of the various parameters was selected. Equation (6) is quite accurate, nevertheless, for predicting residual contact pressure, residual stresses and apparent wall reduction for cases similar to those investigated in the present work and which were selected on the basis of typical usage values.

Upper and lower limits of the expansion pressure level must be studied. First, the maximum value of P/Y_{st} will be discussed. The simplest case would be the one where there is no clearance (c=0), and the tube and tubesheet are of the same material. In this particular case, the problem is simplified to that of an infinite plate with a hole of a diameter equal to the inner tube diameter. When pressure is applied inside the hole, plastic flow begins at the inside surface of the hole. For a pressure higher than $Y_{st}/\sqrt{3}$, the plastic zone spreads outward from the hole. Under the assumption of plane stress, this may not continue indefinitely: For a pressure value of $2*Y_{st}/\sqrt{3}$ ($1.15*Y_{st}$), the radius of the plastic zone reaches 1.75 times the radius of the hole. Further increase of pressure introduces thickening of the internal surface of the tube which makes the pressure to drop [54]. For convenience, the maximum value of P/Y_{st} = 1.15 will be accepted as a rough guess, [55] (see Appendix B for more details). The minimum expansion pressure is the one for which the recovery of the tube and tubesheet is equal. It is dependent on both the yield point and the geometries of tube $U_t=a/a_i$ and tubesheet $U_s=2R_o/a_o$; and can be expressed by the following equation, [56]:

$$\frac{P}{Y_{st}} = \frac{2\left(U_s^2 - 1\right)}{\sqrt{\left(3U_t^4 + 1\right)\left(1.3U_s^2 + 0.7\right)}} + \frac{\left(U_t^2 - 1\right)}{2} \tag{7}$$

Equation (7) is simple and easy to use, however it is quite a rough estimate and does not include initial clearance. The theoretical solution for the tube-to-tubesheet joint is given in detail in Appendix C, including the upper and lower limits of the expansion pressure level.

4.10 NUMERICAL EXAMPLE

In order to show the application of the Equation (6), let us consider a practical case[57]:

The material of the tube is Titanium (ASTM B-338, Gr.2; ASME SB-338), with the following properties:

E_t = 110 Gpa (16 * 10^6 Psi) Y_{st} = 276 Mpa (40 * 10^3 Psi)

The material of the tubesheet is Aluminum Brass (Alloy C68700, ASTM B-111, ASME SB-111):

E_s = 110 Gpa (16 * 10^6 Psi) Y_{ss} = 124 Mpa (18 * 10^3 Psi)

So we have:

$E_t / Y_{st} = 400$

$E_s / E_t = 1.0$

$Y_{ss} / Y_{st} = 0.45$

$t / a = 0.065$

$s / a = 1.5$

$c / a = 0.032$

The maximum value of the expansion pressure level is $P/Y_{st}=1.15$. Using Equation (6), the calculated residual contact pressure level would be: $P^{*} / Y_{st} = 0.799$

Using equation (7), the minimum value of the expansion pressure level is found to be: $P/Y_{st}= 0.272635$

Again using Egn.(6), $P^{*}/Y_{st}= 0.000012$

Choose the expansion pressure $P/Y_{st} = 0.65$

The results of the empirical equation (6) and those obtained by FEM, are shown in Table 4.6. Both results agree with each other.

In Chapter 4 the regression analysis and the final results were presented. In Chapter 5, we will present our final conclusions.

(1) Sequential Expansion Case

	P^*/Y_{st}	S^*_z/Y_{st}	S^*_h/Y_{st}	k(%)
a0	4.4086	-3.7551	-7.1555	3.8585
a1	-0.4947	0.1205	0.0685	0.3285
a2	-1.0120	-0.2255	-0.2770	0.1030
a3	-0.0879	0.4762	0.3918	0.7027
a4	7.7767	0.3272	0.3129	-1.0618
a5	-0.9126	0.9398	1.3451	0.0667
a6	-4.2044	1.2837	1.2068	0.1547
a7	0.8550	-0.8960	-1.1823	0.2226
Model				
F value	22.17	71.61	44.50	90.90
PROB>F	0.0001	0.0001	0.0001	0.0001
R-SQUARE	0.83	0.94	0.91	0.95

(2) Simultaneous Expansion Case

	P^*/Y_{st}	S^*_z/Y_{st}	S^*_h/Y_{st}	k(%)
a0	0.9133	3.5323	2.6895	7.3228
a1	0.0567	0.0879	0.1527	0.1145
a2	-0.5769	0.2482	0.3927	-0.1288
a3	-0.1222	0.4619	0.3511	0.6438
a4	4.9709	1.2123	0.7749	0.6117
a5	-0.2481	-0.1682	-0.1566	-0.4772
a6	-2.8381	0.0441	-0.1980	-0.6138
a7	0.6765	-0.0801	0.1442	0.1294
Model				
F value	35.64	31.98	49.98	179.39
PROB>F	0.0001	0.0001	0.0001	0.0001
R-SQUARE	0.89	0.89	0.92	0.98

Table 4.1 Coefficients of Equation (6)

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MODEL: MODEL1
DEPENDENT VARIABLE: LRCPR

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	43.80254	6.25751	22.173	0.0001
ERROR	31	8.74874	0.28222		
C TOTAL	38	52.55128			

ROOT MSE	0.53124	R-SQUARE	0.8335
DEP MEAN	-3.46254	ADJ R-SQ	0.7959
C.V.	-15.34252		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	4.408686	6.04003739	0.730	0.4709
LTWT	1	-0.494767	0.30809805	-1.606	0.1184
LPIT	1	-1.012032	0.30391594	-3.330	0.0023
LCLEA	1	0.087917	0.04610431	1.907	0.0658
LEXPR	1	7.776771	1.13539899	6.849	0.0001
LRYMT	1	-0.912606	0.83885485	-1.088	0.2850
LRYM	1	-4.204410	0.97117394	-4.329	0.0001
LRYS	1	0.855029	0.61992148	1.379	0.1777

MODEL: MODEL2
DEPENDENT VARIABLE: LRSZ

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	50.73866	7.24838	71.608	0.0001
ERROR	31	3.13790	0.10122		
C TOTAL	38	53.87655			

ROOT MSE	0.31815	R-SQUARE	0.9418
DEP MEAN	-0.31609	ADJ R-SQ	0.9286
C.V.	-100.65251		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	-3.755139	3.61731495	-1.038	0.3073
LTWT	1	0.120504	0.18451669	0.653	0.5185
LPIT	1	-0.225571	0.18201206	-1.239	0.2245
LCLEA	1	0.476297	0.02761139	17.250	0.0001
LEXPR	1	0.327213	0.67997853	0.481	0.6337
LRYMT	1	0.939816	0.50238136	1.871	0.0709
LRYM	1	1.283758	0.58162587	2.207	0.0348
LRYS	1	-0.896036	0.37126446	-2.413	0.0219

Table 4.2(a) SAS Printout of Analysis of Variance Table for 39 observations (Sequential Expansion Case)

MODEL: MODEL3
DEPENDENT VARIABLE: LRSH

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	35.81529	5.11647	44.498	0.0001
ERROR	31	3.56443	0.11498		
C TOTAL	38	39.37972			

ROOT MSE	0.33909	R-SQUARE	0.9095
DEP MEAN	-0.45183	ADJ R-SQ	0.8890
C.V.	-75.04727		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	-7.155502	3.85533534	-1.856	0.0730
LTWT	1	0.068566	0.19665794	0.349	0.7297
LPIT	1	-0.277073	0.19398851	-1.428	0.1632
LCLEA	1	0.391868	0.02942822	13.316	0.0001
LEXPR	1	0.312995	0.72472132	0.432	0.6688
LRYMT	1	1.345188	0.53543820	2.512	0.0174
LRYM	1	1.206897	0.61989703	1.947	0.0607
LRYS	1	-1.182301	0.39569378	-2.988	0.0055

MODEL: MODEL4
DEPENDENT VARIABLE: LTWR

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	83.16627	11.88090	90.900	0.0001
ERROR	31	4.05178	0.13070		
C TOTAL	38	87.21805			

ROOT MSE	0.36153	R-SQUARE	0.9535
DEP MEAN	0.24154	ADJ R-SQ	0.9431
C.V.	149.67686		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	3.858524	4.11045605	0.939	0.3551
LTWT	1	0.328508	0.20967147	1.567	0.1273
LPIT	1	0.103083	0.20682539	0.498	0.6217
LCLEA	1	0.702734	0.03137559	22.397	0.0001
LEXPR	1	-1.061807	0.77267860	-1.374	0.1792
LRYMT	1	0.066746	0.57086998	0.117	0.9077
LRYM	1	0.154750	0.66091773	0.234	0.8164
LRYS	1	0.222692	0.42187818	0.528	0.6014

Table 4.2(a) (Con't)

Definitions statistical terms notes for Table 4.2

1. MODEL for the fitted regression,

   ERROR for the residual error,

   C TOTAL for the total variation after correcting for the mean

2. Degrees of freedom (DF)(associated for the source), a number reflecting the number of ways observations are free to vary once the mean is known

3. SUM OF SQUARES (for the term), the numerator of a variance estimate; measure of variation defined in terms of the sum of the squared deviations from the mean

4. MEAN SQUARE, the sum of squares divided by the degrees of freedom

5. the F VALUE for testing the hypothesis that all parameters are zero except for the intercept. This is formed by dividing the mean square for MODEL by the mean square for ERROR.

6. the PROB>F, the probability of getting a greater F statistic than that observed if the hypothesis is true. This is the significance probability.

7. ROOT MSE is an estimate of the standard deviation of the error term. It is calculated as the square root of the mean square error.

8. DEP MEAN is the sample mean of the dependent variable

9. C.V. is the coefficient of variation, computed as 100 times ROOT MSE divided by DEP MEAN. This expresses the variation in unitless values.

10. R-SQUARE is a measure between 0 and 1 that indicates the portion of the (corrected) total variation that is attributed to the fit rather than left to residual error. It is calculated as SS(MODEL) divided by SS(TOTAL). It is also called the coefficient of determination. It is the square of the multiple correlation, in other words, the square of the correlation between the dependent variable and the predicted values.

11. ADJ R-SQ, the adjusted R^2, is a version of R^2 that has been adjusted for degrees of freedom. It is calculated:

$$R^2 = 1 - (l - R^2)(n - 1)/dfe$$

where dfe is the degrees of freedom for error.

12. the VARIABLE used as the regressor, including the name INTERCEP to estimate the intercept parameter

LTWT : ln(k)
LPIT : ln(s/a)
LCLEA: ln(c/a)
LEXPR: $\ln(P/Y_{st})$
LRYMT: $\ln(E_t/Y_{st})$
LRYM : $\ln(E_s/E_t)$
LRYS : $\ln(Y_{ss}/Y_{st})$

13. the STANDARD ERROR, the estimate of the standard deviation of the parameter estimate

14. T FOR HO: PARAMETER=0, the t test that the parameter is zero. This is computed as PARAMETER ESTIMATE divided by the STANDARD ERROR.

15. the PROB>¦T¦, the probability that a t statistic would obtain a greater absolute value than that observed given that the true parameter is zero. This is the two-tailed significance probability.

16. Confidence interval, a range about a given statistical estimate within which the actual index is said to be located with some specified degree of confidence.

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MODEL: MODEL1
DEPENDENT VARIABLE: LRCPR

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	30.22761	4.31823	36.326	0.0001
ERROR	31	3.68511	0.11887		
C TOTAL	38	33.91272			

ROOT MSE	0.34478	R-SQUARE	0.8913
DEP MEAN	-3.08419	ADJ R-SQ	0.8668
C.V.	-11.17902		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	0.913333	3.93731090	0.232	0.8181
LTWT	1	0.056766	0.20411700	0.278	0.7828
LPIT	1	-0.576924	0.19634819	-2.938	0.0062
LCLEA	1	0.122267	0.03141819	3.892	0.0005
LEXPR	1	4.970933	0.75407467	6.592	0.0001
LRYMT	1	-0.248197	0.54846688	-0.453	0.6540
LRYM	1	-2.838171	0.63287073	-4.485	0.0001
LRYS	1	0.676520	0.39957898	1.693	0.1005

MODEL: MODEL2
DEPENDENT VARIABLE: LRSZ

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	44.49974	6.35711	43.097	0.0001
ERROR	31	4.57273	0.14751		
C TOTAL	38	49.07247			

ROOT MSE	0.38407	R-SQUARE	0.9068
DEP MEAN	-0.25000	ADJ R-SQ	0.8858
C.V.	-153.62535		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > ¦T¦
INTERCEP	1	3.532348	4.38593614	0.805	0.4267
LTWT	1	0.087988	0.22737450	0.387	0.7014
LPIT	1	0.248208	0.21872051	1.135	0.2652
LCLEA	1	0.461977	0.03499804	13.200	0.0001
LEXPR	1	1.212380	0.83999548	1.443	0.1590
LRYMT	1	-0.168236	0.61096031	-0.275	0.7849
LRYM	1	0.044107	0.70498131	0.063	0.9505
LRYS	1	-0.080080	0.44510783	-0.180	0.8584

Table 4.2(b) SAS printout of Analysis of Variance Table for 39 observations (Simultaneous Expansion Case)

MODEL: MODEL3
DEPENDENT VARIABLE: LRSH

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	24.14442	3.44920	55.195	0.0001
ERROR	31	1.93724	0.06249		
C TOTAL	38	26.08167			

ROOT MSE	0.24998	R-SQUARE	0.9257
DEP MEAN	-0.39300	ADJ R-SQ	0.9090
C.V.	-63.60849		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	2.689513	2.85474134	0.942	0.3534
LTWT	1	0.152700	0.14799472	1.032	0.3102
LPIT	1	0.392700	0.14236196	2.758	0.0097
LCLEA	1	0.351132	0.02277971	15.414	0.0001
LEXPR	1	0.774999	0.54674071	1.417	0.1663
LRYMT	1	-0.156624	0.39766508	-0.394	0.6964
LRYM	1	-0.198058	0.45886197	-0.432	0.6690
LRYS	1	0.144240	0.28971414	0.498	0.6221

MODEL: MODEL4
DEPENDENT VARIABLE: LTWR

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	75.48174	10.78311	104.566	0.0001
ERROR	31	3.19680	0.10312		
C TOTAL	38	78.67854			

ROOT MSE	0.32113	R-SQUARE	0.9594
DEP MEAN	0.24339	ADJ R-SQ	0.9502
C.V.	131.93999		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	7.322858	3.66717904	1.997	0.0547
LTWT	1	0.114557	0.19011289	0.603	0.5512
LPIT	1	-0.128829	0.18287710	-0.704	0.4864
LCLEA	1	0.643875	0.02926264	22.003	0.0001
LEXPR	1	0.611782	0.70233896	0.871	0.3904
LRYMT	1	-0.477217	0.51083755	-0.934	0.3574
LRYM	1	-0.613823	0.58945060	-1.041	0.3058
LRYS	1	0.129418	0.37216458	0.348	0.7304

Table 4.2(b) (Con't)

OBS	DEP VAR LRCPR	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	LOWER95% PREDICT	UPPER95% PREDICT
1	-3.8663	-3.9122	0.255	-4.4332	-3.3911	-5.1144	-2.7099
2	-3.4148	-3.5988	0.148	-3.9014	-3.2962	-4.7237	-2.4739
3	-2.8705	-2.9175	0.229	-3.3840	-2.4510	-4.0971	-1.7379
4	-2.9308	-3.0969	0.169	-3.4415	-2.7524	-4.2339	-1.9600
5	-2.7434	-2.5957	0.188	-2.9800	-2.2114	-3.7453	-1.4461
6	-7.1870	-5.1952	0.234	-5.6731	-4.7173	-6.3794	-4.0110
7	-2.4531	-2.4216	0.234	-2.8998	-1.9435	-3.6059	-1.2373
8	-4.7603	-5.5360	0.337	-6.2223	-4.8497	-6.8185	-4.2534
9	-3.5760	-3.9870	0.253	-4.5031	-3.4709	-5.1871	-2.7869
10	-3.8468	-3.9645	0.191	-4.3533	-3.5756	-5.1156	-2.8133
11	-4.2591	-4.5735	0.209	-4.9988	-4.1481	-5.7374	-3.4095
12	-4.0129	-4.1899	0.188	-4.5733	-3.8064	-5.3392	-3.0406
13	-3.8959	-4.0716	0.193	-4.4644	-3.6788	-5.2241	-2.9191
14	-4.3360	-4.4863	0.208	-4.9114	-4.0612	-5.6502	-3.3224
15	-3.9961	-4.2935	0.190	-4.6804	-3.9066	-5.4440	-3.1430
16	-4.6746	-4.3123	0.220	-4.7601	-3.8644	-5.4846	-3.1399
17	-5.3418	-4.6002	0.220	-5.0499	-4.1505	-5.7733	-3.4271
18	-4.8080	-4.7194	0.236	-5.2012	-4.2376	-5.9051	-3.5336
19	-2.4905	-2.1638	0.206	-2.5841	-1.7434	-3.3259	-1.0016
20	-2.5592	-2.4725	0.189	-2.8584	-2.0865	-3.6226	-1.3223
21	-2.3768	-2.3049	0.225	-2.7646	-1.8453	-3.4819	-1.1280
22	-2.1772	-1.9706	0.189	-2.3567	-1.5845	-3.1208	-0.8204
23	-2.1130	-1.9824	0.165	-2.3183	-1.6465	-3.1168	-0.8481
24	-2.7356	-3.3503	0.188	-3.7337	-2.9669	-4.4996	-2.2010
25	-1.8855	-1.8084	0.231	-2.2790	-1.3377	-2.9896	-0.6271
26	-2.4156	-3.0388	0.150	-3.3446	-2.7330	-4.1646	-1.9130
27	-2.3954	-2.8939	0.216	-3.3350	-2.4529	-4.0637	-1.7241
28	-3.1272	-3.2570	0.285	-3.8378	-2.6761	-4.4863	-2.0276
29	-2.1818	-1.9042	0.258	-2.4304	-1.3780	-3.1087	-0.6997
30	-2.2696	-1.8359	0.252	-2.3507	-1.3211	-3.0354	-0.6364
31	-3.9154	-4.0565	0.290	-4.6480	-3.4650	-5.2909	-2.8221
32	-3.3497	-3.0652	0.279	-3.6343	-2.4961	-4.2890	-1.8413
33	-3.0674	-2.9792	0.256	-3.5017	-2.4567	-4.1821	-1.7763
34	-2.2985	-2.9277	0.318	-3.5760	-2.2793	-4.1903	-1.6650
35	-4.8562	-4.4206	0.299	-5.0299	-3.8113	-5.6637	-3.1776
36	-3.6156	-4.2955	0.316	-4.9393	-3.6518	-5.5558	-3.0353
37	-2.9612	-3.1899	0.286	-3.7739	-2.6059	-4.4208	-1.9591
38	-3.2280	-2.7401	0.307	-3.3668	-2.1135	-3.9918	-1.4885
39	-6.0464	-5.9097	0.357	-6.6375	-5.1819	-7.2149	-4.6045

OBS	RESIDUAL						
1	0.0459	11	0.3143	21	-0.0719	31	0.1411
2	0.1840	12	0.1770	22	-0.2066	32	-0.2845
3	0.0471	13	0.1757	23	-0.1306	33	-0.0882
4	0.1661	14	0.1503	24	0.6147	34	0.6292
5	-0.1477	15	0.2974	25	-0.0771	35	-0.4356
6	-1.9918	16	-0.3623	26	0.6232	36	0.6799
7	-0.0315	17	-0.7415	27	0.4985	37	0.2287
8	0.7757	18	-0.0886	28	0.1297	38	-0.4878
9	0.4110	19	-0.3267	29	-0.2777	39	-0.1367
10	0.1176	20	-0.0867	30	-0.4337		

SUM OF RESIDUALS	0
SUM OF SQUARED RESIDUALS	8.74
PREDICTED RESID SS (PRESS)	15.0011

Table 4.3 SAS Printout for Confidence Intervals for $E(y_p)$ and Prediction Intervals for Individual y_p (Sequential Expansion Case)

```
        FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LTWR
STEP 1   VARIABLE LCLEA ENTERED     R-SQUARE = 0.97007380   C(P) =  3.50679157
                DF          SUM OF SQUARES        MEAN SQUARE          F   PROB>F
 REGRESSION      1             76.32398975        76.32398975    1199.37   0.0001
 ERROR          37              2.35454973         0.06363648
 TOTAL          38             78.67853948

                PARAMETER         STANDARD            TYPE II
 VARIABLE        ESTIMATE            ERROR     SUM OF SQUARES          F   PROB>F

 INTERCEP      3.91315956       0.11340299        75.77261596    1190.71   0.0001
 LCLEA         0.72161752       0.02083673        76.32398975    1199.37   0.0001

BOUNDS ON CONDITION NUMBER:            1,             1
------------------------------------------------------------------------------
STEP 2   VARIABLE LRYS ENTERED      R-SQUARE = 0.97331355   C(P) =  1.33812290
                DF          SUM OF SQUARES        MEAN SQUARE          F   PROB>F

 REGRESSION      2             76.57888863        38.28944431     656.50   0.0001
 ERROR          36              2.09965086         0.05832363
 TOTAL          38             78.67853948

                PARAMETER         STANDARD            TYPE II
 VARIABLE        ESTIMATE            ERROR     SUM OF SQUARES          F   PROB>F

 INTERCEP      3.92307630       0.10866955        76.01203485    1303.28   0.0001
 LCLEA         0.72250857       0.01995253        76.47768053    1311.26   0.0001
 LRYS         -0.14775056       0.07067527         0.25489887       4.37   0.0437
BOUNDS ON CONDITION NUMBER:     1.000457,      4.001826
------------------------------------------------------------------------------
STEP 3   VARIABLE LTWT ENTERED      R-SQUARE = 0.97422770   C(P) =  2.16186515
                DF          SUM OF SQUARES        MEAN SQUARE          F   PROB>F
 REGRESSION      3             76.65081249        25.55027083     441.02   0.0001
 ERROR          35              2.02772699         0.05793506
 TOTAL          38             78.67853948
               PARAMETER         STANDARD            TYPE II
 VARIABLE        ESTIMATE            ERROR     SUM OF SQUARES          F   PROB>F

 INTERCEP      4.32134684       0.37349589         7.75546838     133.86   0.0001
 LTWT          0.15045253       0.13503107         0.07192387       1.24   0.2728
 LCLEA         0.72821452       0.02053476        72.85857380    1257.59   0.0001
 LRYS         -0.15211585       0.07054831         0.26935007       4.65   0.0380
BOUNDS ON CONDITION NUMBER:     1.068831,      9.417562
------------------------------------------------------------------------------
NO OTHER VARIABLE MET THE 0.5000 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.
1                                THE SAS SYSTEM  13:35 SATURDAY, JANUARY 4, 1992
                                                                              8
     SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LTWR

        VARIABLE   NUMBER   PARTIAL     MODEL
 STEP   ENTERED        IN      R**2      R**2         C(P)           F   PROB>F

   1    LCLEA           1    0.9701    0.9701       3.5068   1199.3748   0.0001
   2    LRYS            2    0.0032    0.9733       1.3381      4.3704   0.0437
   3    LTWT            3    0.0009    0.9742       2.1619      1.2415   0.2728
```

Table 4.4 SAS Printout for Stepwise Regression
Performed on 39 Observations (Simultaneous case)

		Residual Contact Pressure P^*/S_{yt}	Maximum Tensile Residual Axial Stress S_z^*/S_{yt}	Maximum Tensile Residual Hoop Stress S_h^*/S_{yt}	Tubewall Reduction k (%)
Sequential Case	Main Effect	1. P	1. c	1. c	1. c
		2. E_s/E_t	2. E_s/E_t	2. E_t/Y_{st}	
Simultaneous Case	Main Effect	1. P	1. c	1. c	1. c
		2. E_s/E_t	2. P	2. P	2. P

Table 4.5 Main Parameters

	Sequential Expansion Case		Simultaneous Expansion Case	
	Eqn. (6)	FEM	Eqn. (6)	FEM
P^*/Y_{st}	0.007327	0.0075	0.01146	0.012
S_z^*/Y_{st}	1.41	1.43	1.38	1.41
S_h^*/Y_{st}	1.00	1.05	1.09	1.14
k (%)	3.8	4.05	4.24	4.66

Table 4.6 Comparison of Results of Eqn.(6) with Results of FEM for Example Case

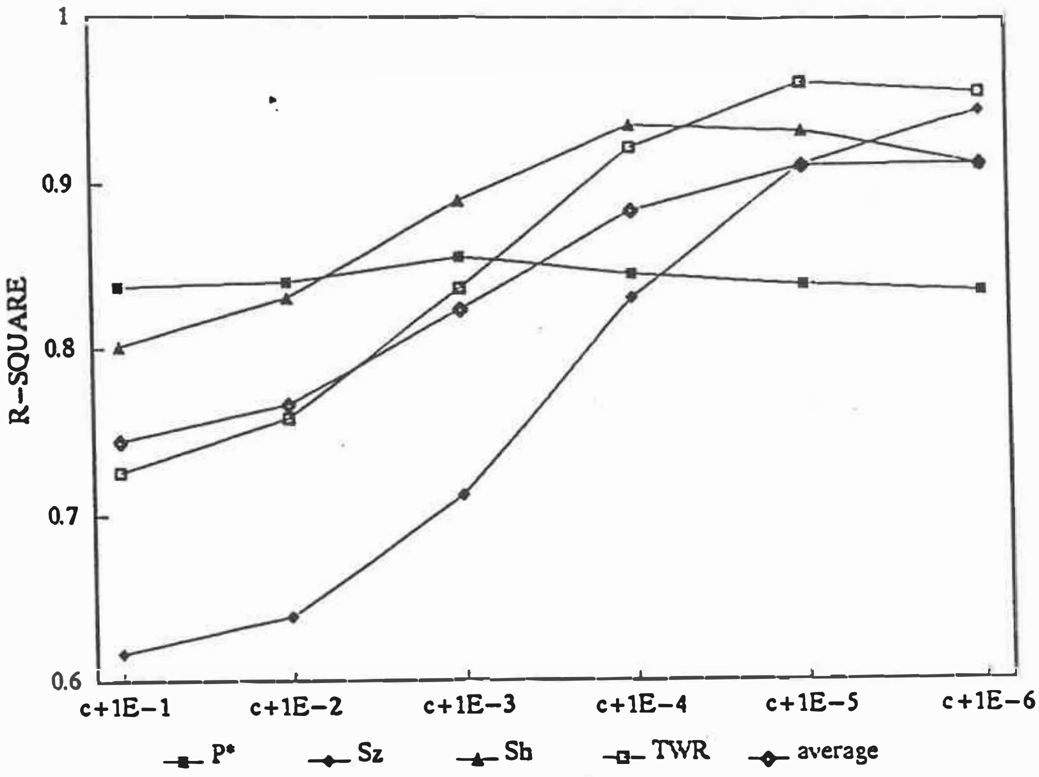

Figure 4.1(a) The influence of term ($c+10^{-r}$) on the R-SQUARE of Eqn.(6) (Sequential Expansion Case)

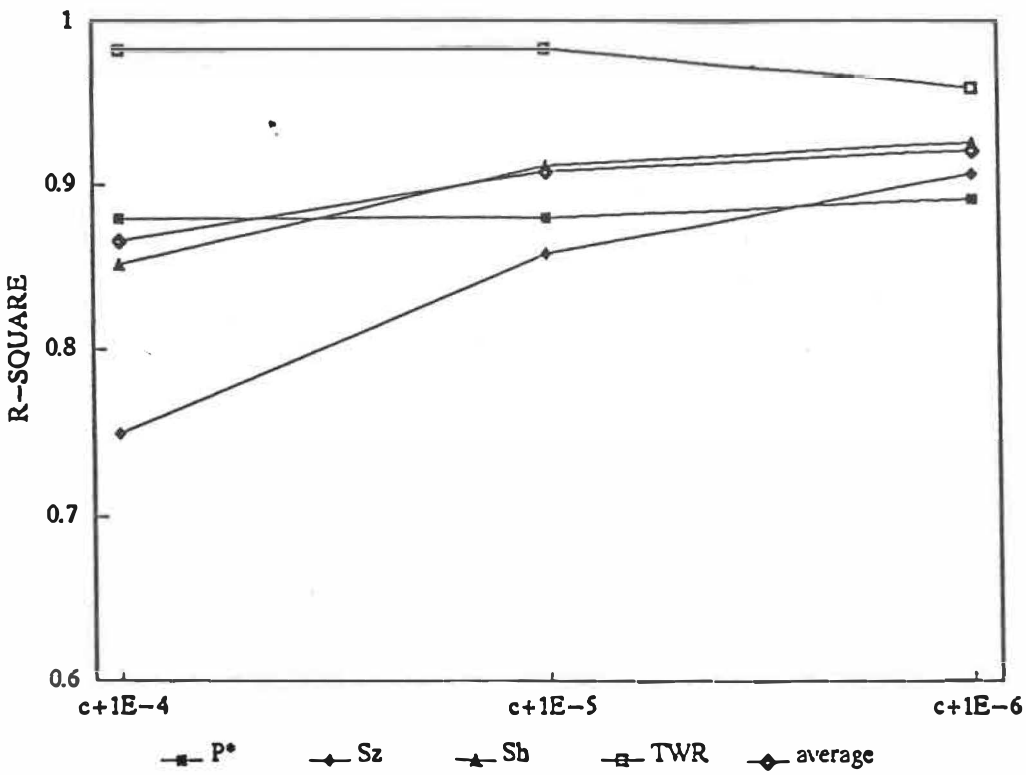

Figure 4.1(b) The influence of term $(c+10^{-r})$ on the R-SQUARE of Eqn.(6) (Simultaneous Expansion Case)

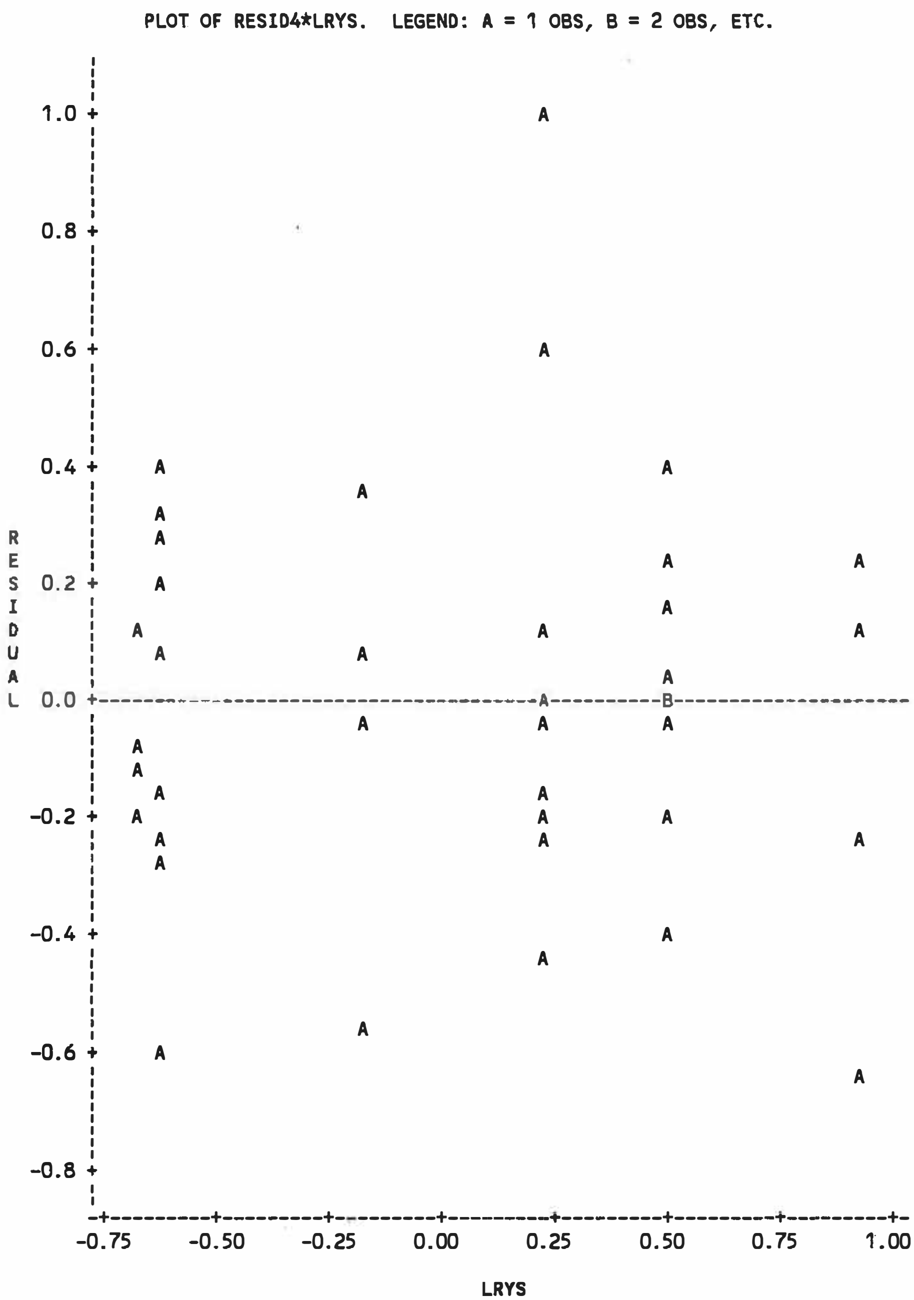

Figure 4.2 SAS printout of the plot of residuals Against LRYS [$\ln(Y_{ss}/Y_{st})$] For sequential Expansion Case

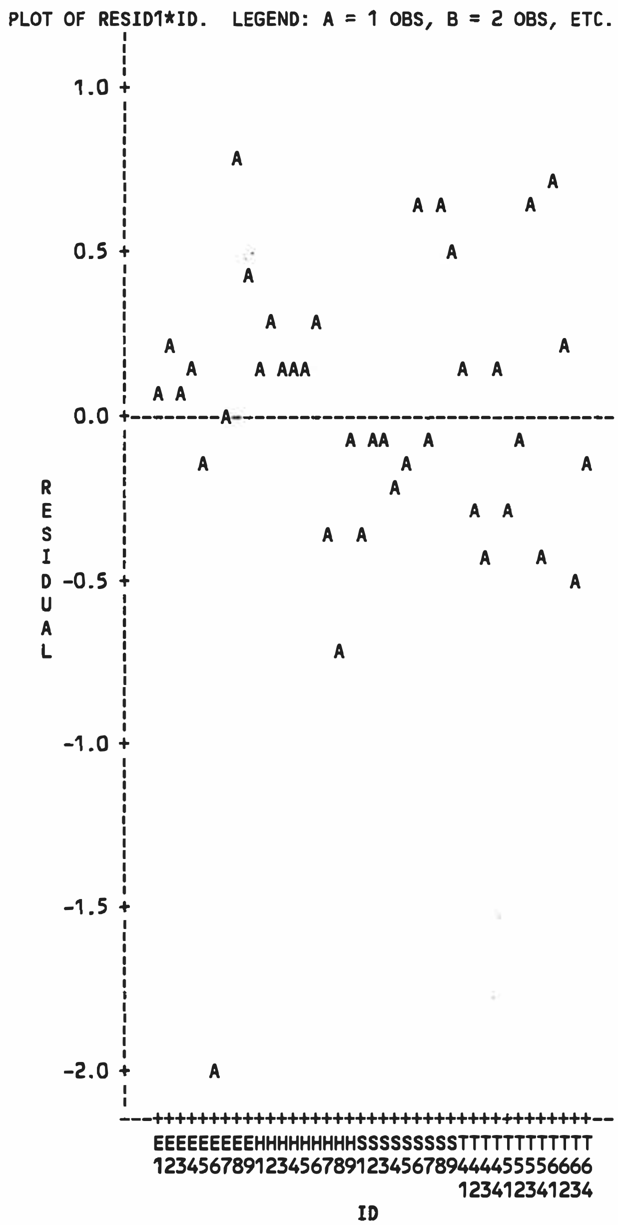

Figure 4.3 SAS Printout of the plot of the Residual Against Time Order for LRCPR (Sequential Case) [E1-9, H1-9, S1-9, T41-44, T51-54, T61-64 Correspond with Material Set. See Table 3.5]

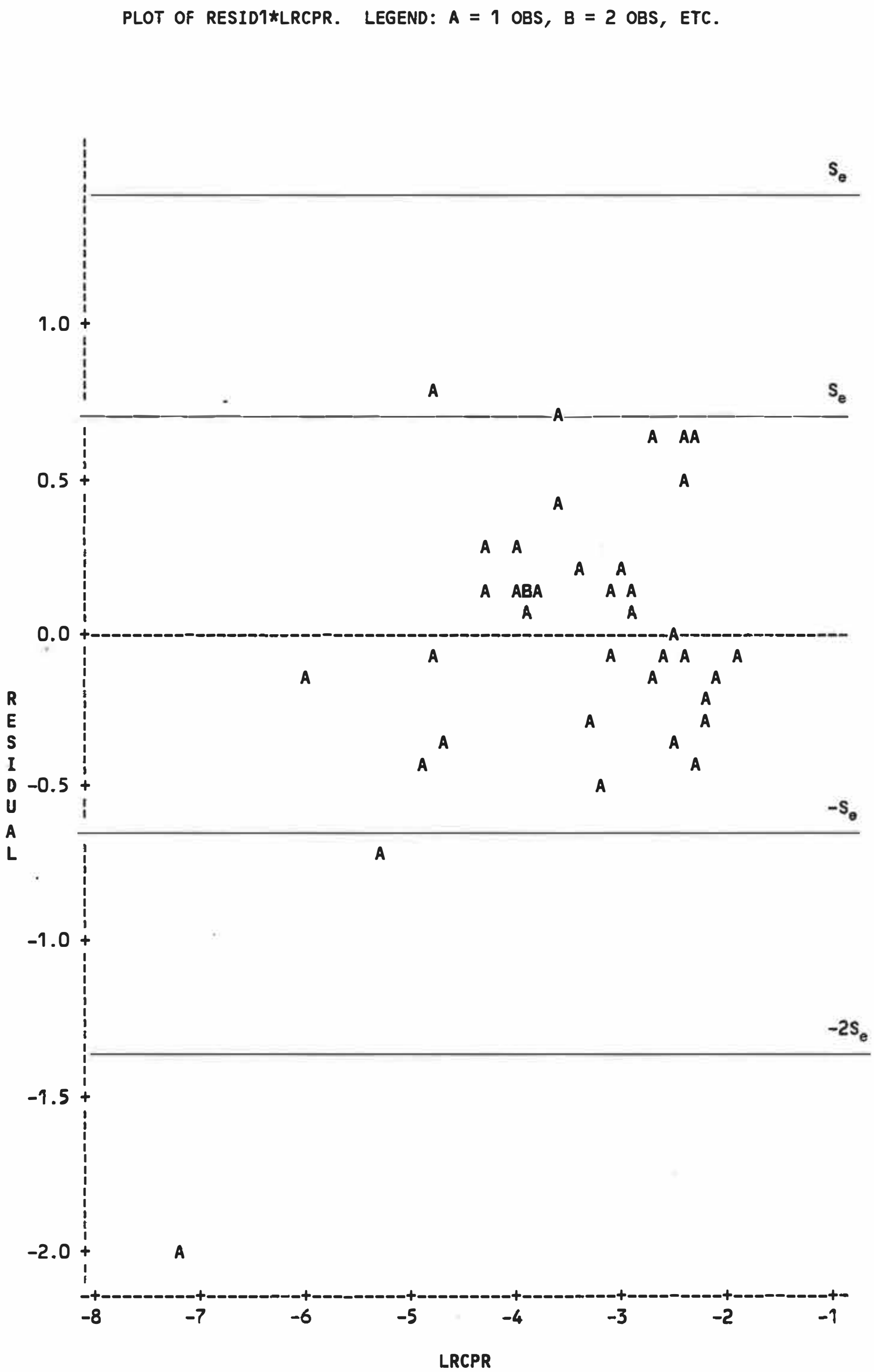

Figure 4.4 SAS Printout of the Plot of the residual Against response parameter LRPCR [ln(P^*/Y_{st})] for sequential Expansion Case

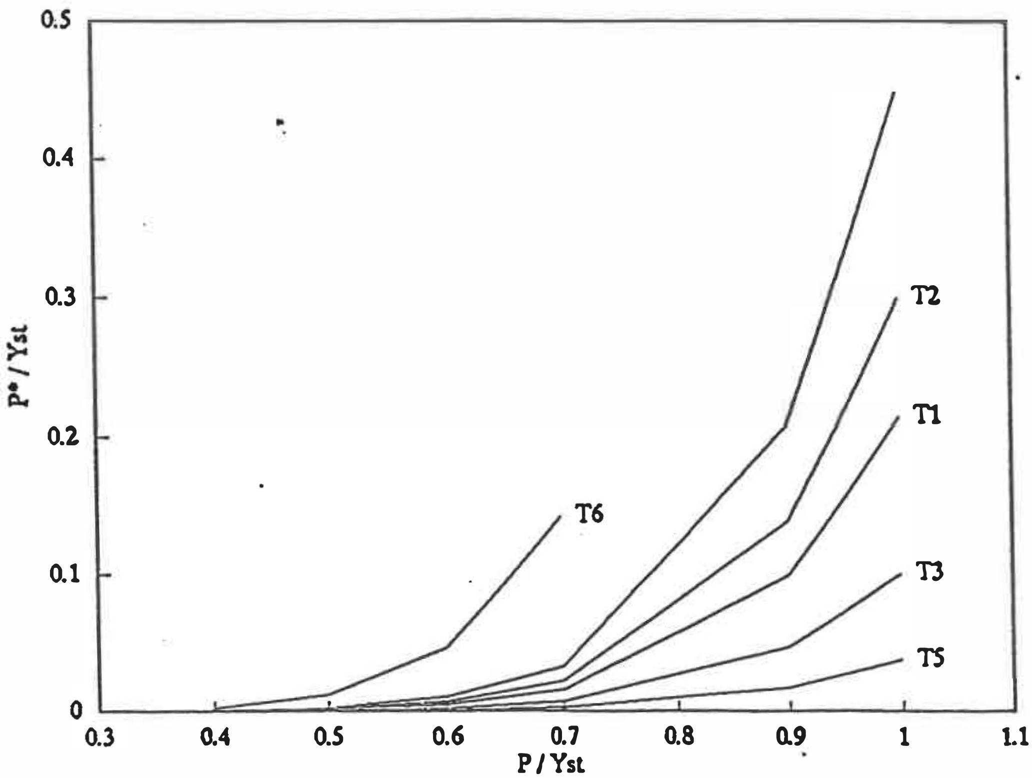

Figure 4.5(a) Residual Contact Pressure Level (P^*/Y_{st}) Against Material Sets (T1-6, See Table 3.5) and Expansion Pressure Level (P/Y_{st}) (from Eqn.(6)) for Sequential Expansion Case

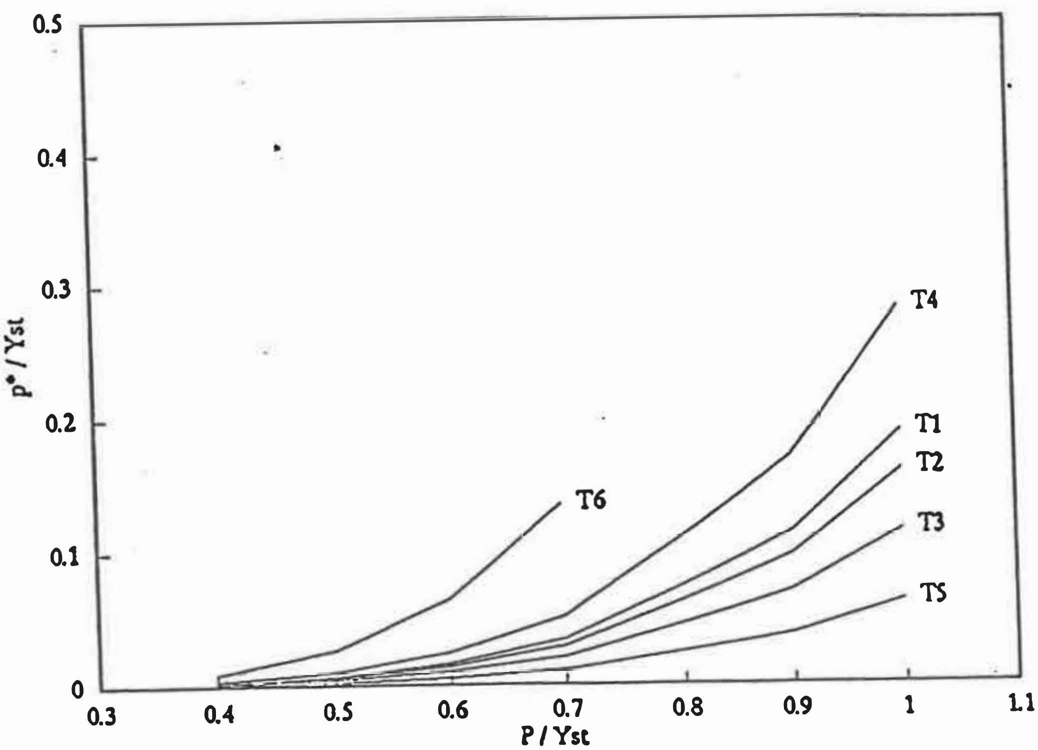

Figure 4.5(b) Residual Contact Pressure Level (P^*/Y_{st}) Against Material Sets (T1-6, See Table 3.5) and Expansion Pressure Level (P/Y_{st}) (from Eqn.(6)) for Simultaneous Expansion Case

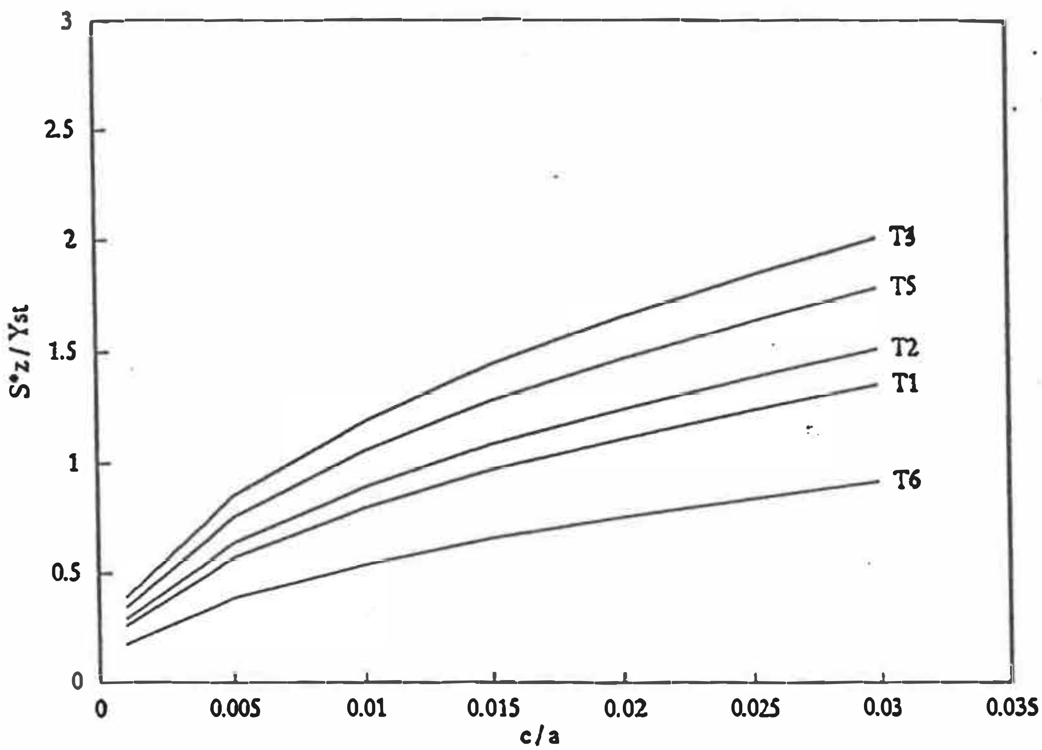

Figure 4.6(a) Maximum Tensile Residual Axial Stress Level (S^{*}_{z}/Y_{st}) Against Material Sets (T1-6, See Table 3.5) and Initial Clearance (c/a) (from Eqn.(6)) for Sequential Expansion Case

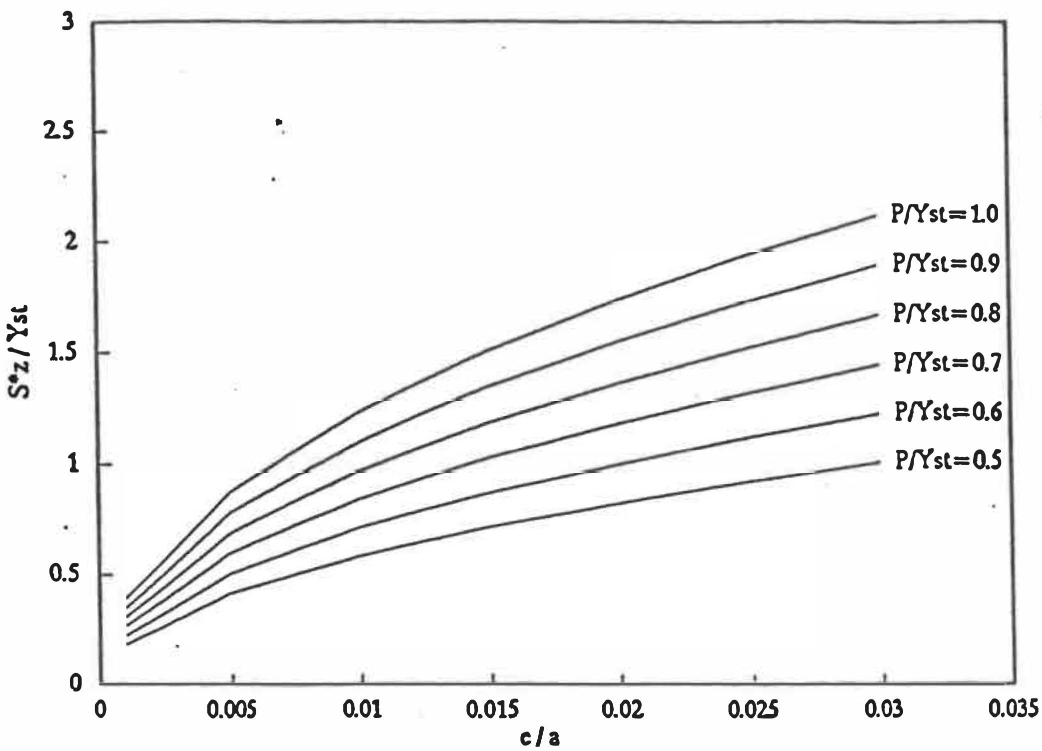

Figure 4.6(b) Maximum Tensile Residual Axial Stress Level (S^{*}_{z}/Y_{st}) Against Expansion Pressure Level (P/Y_{st}) and Initial Clearance (c/a) (from Eqn.(6)) for Simultaneous Expansion Case

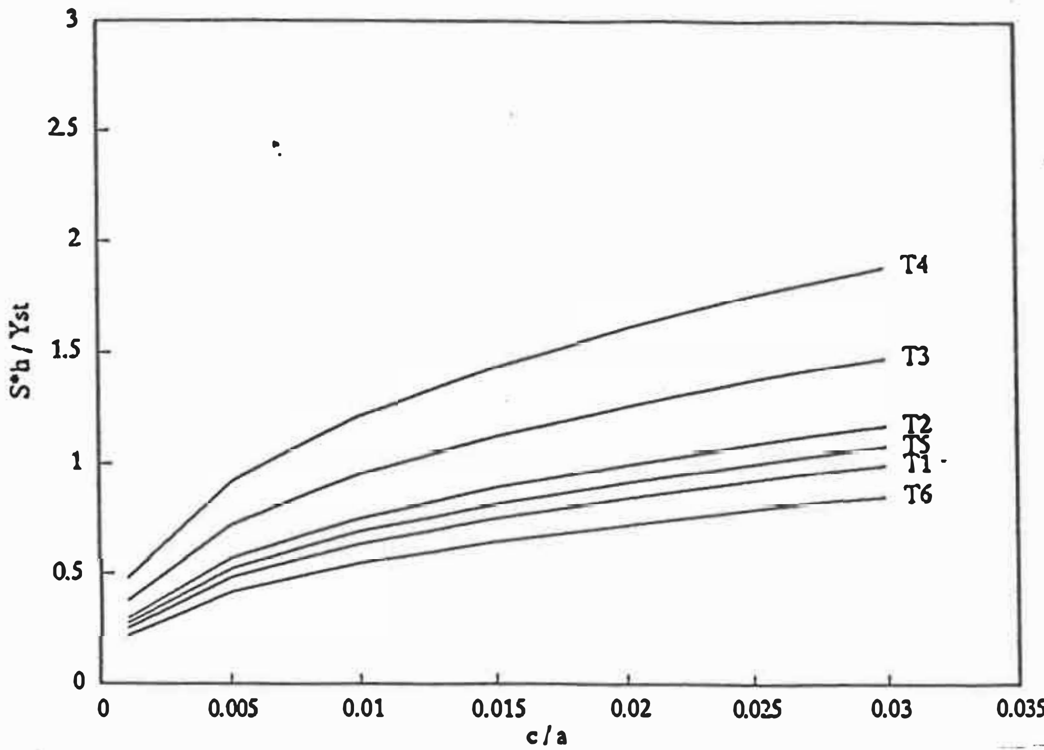

Figure 4.7(a) Maximum Tensile Residual Hoop Stress Level (S^*_h/Y_{st}) Against Material Sets (T1-6, See Table 3.5) and Initial Clearance (c/a) (from Eqn.(6)) for Sequential Expansion Case

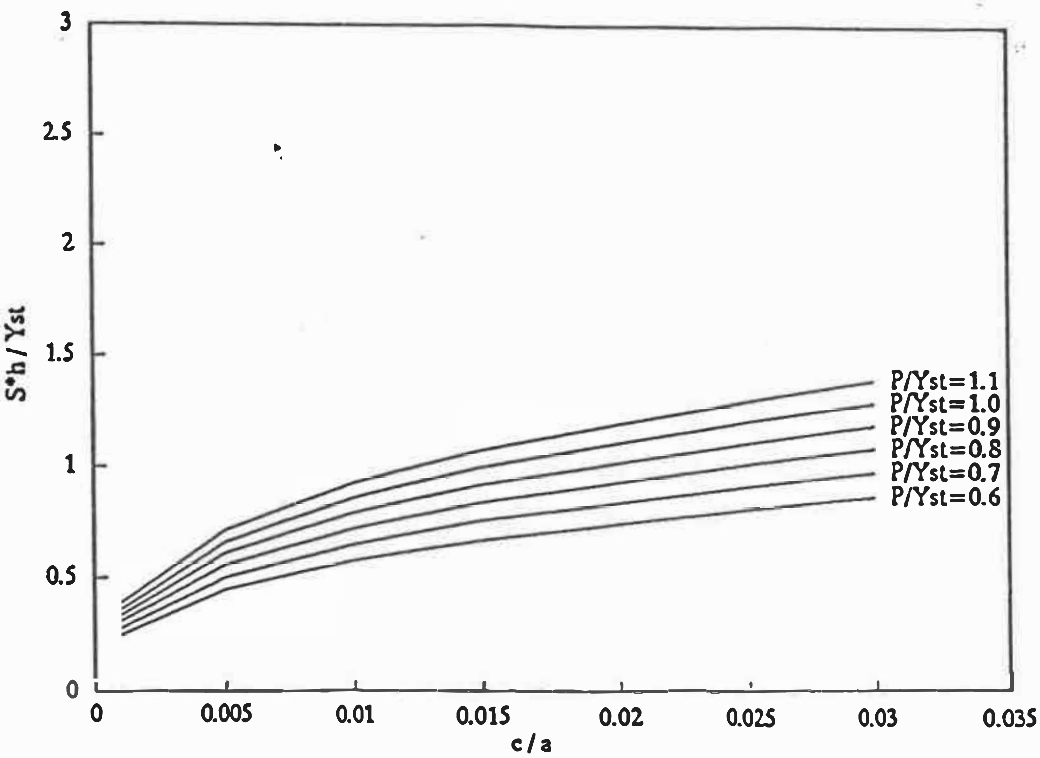

Figure 4.7(b) Maximum Tensile Residual Hoop Stress Level (S^*_h/Y_{st}) Against Initial clearance and Expansion Pressure Level (P/Y_{st}) (from Eqn.(6)) for Simultaneous Expansion Case

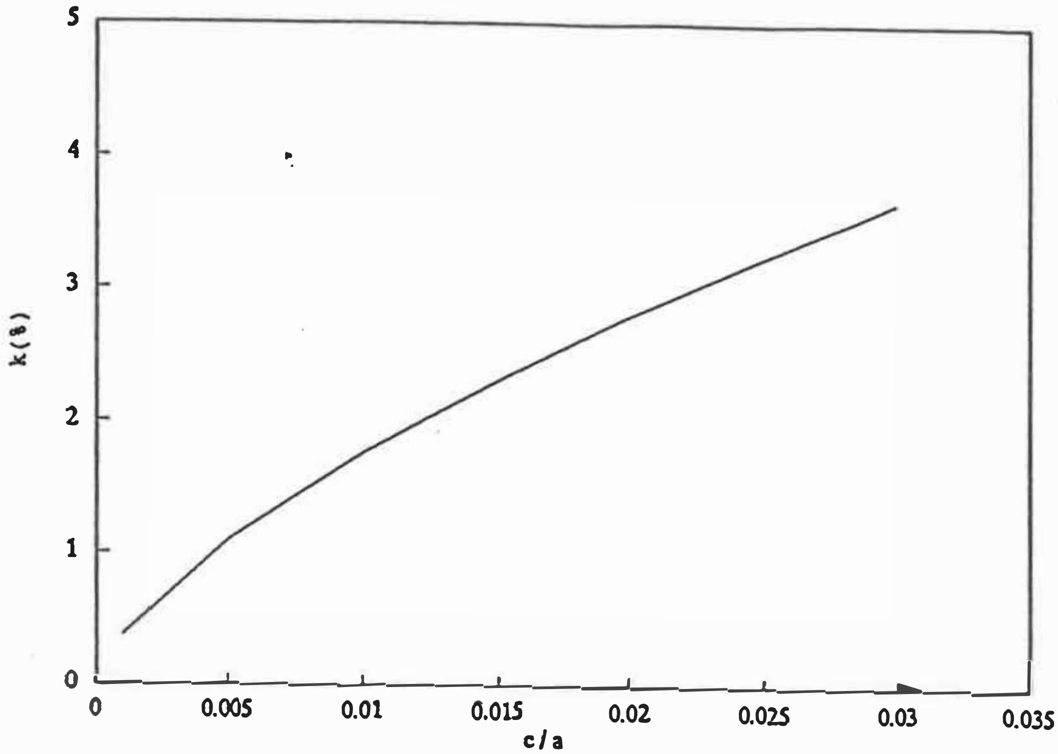

Figure 4.8(a) Apparent Tubewall Reduction (k) Against Initial Clearance (c/a) (from Eqn.(6)) for Sequential Expansion Case

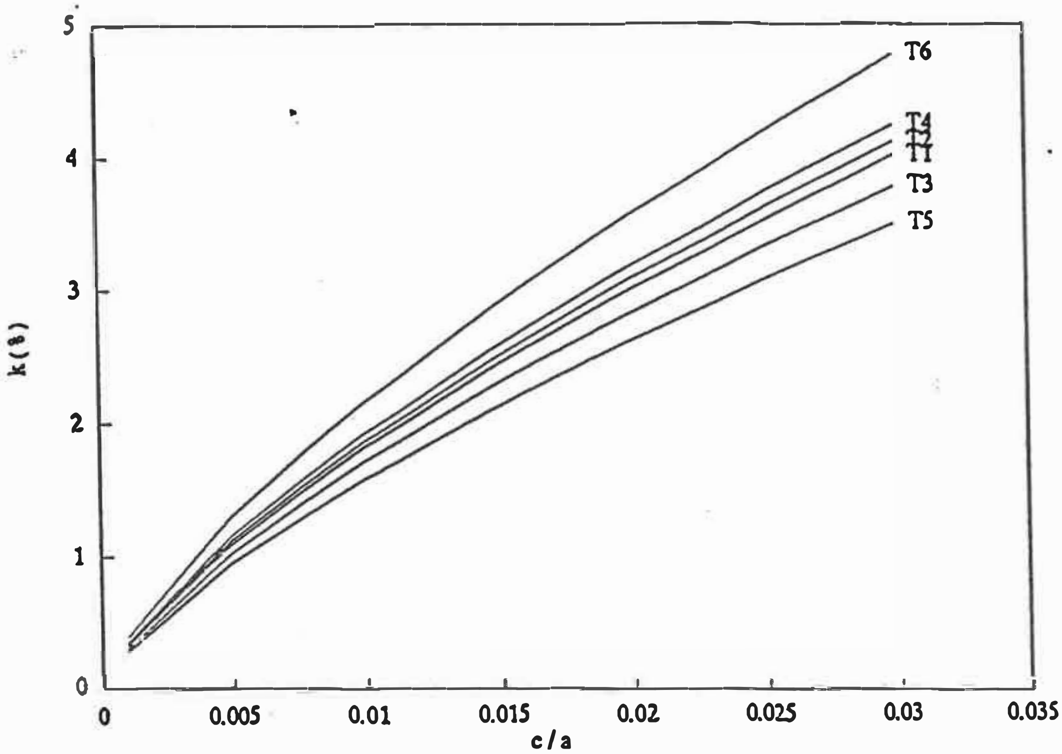

Figure 4.8(b) Apparent Tubewall Reduction (k) Against Material Sets (T1-6, See Table 3.5) and Initial Clearance (c/a) (from Eqn.(6)) for Simultaneous Expansion Case

CHAPTER 5

CONCLUSIONS

Tube-to-tubesheet joints are critical to the reliability of tubular heat exchangers such as steam generators, industrial coolers and condensers. In the fabrication of heat exchangers, the tubes are attached to the tubesheet by means of an expansion process. Hydraulic expansion is one of the most common ways of achieving an interference fit between the tube and the tubesheet. Due to the expansion process, a high level of residual stresses in the tube wall is created. The tensile part of these stresses increases the susceptibility of the tube to Stress-Corrosion Cracking (SCC).

In this thesis, the strength of tube-to-tubesheet joints in heat exchangers has been studied. Due to the complexity of the geometry and the loading conditions, particularly in the transition zone of the tube, stress analyses were performed using the elasto-plastic finite element method. Due to the many dimension-, fabrication- and material- related parameters, the orthogonal design method was used to minimize the number of analyses while providing an accurate understanding of the influence of all parameters involved. The parametric study was based on the analysis of variance of orthogonal design and regression analysis. Two types of tube expansion, sequential and simultaneous, were considered. Empirical equations have been developed for determining the residual contact pressure, maximal tensile residual axial and hoop stresses and tubewall reduction. These equations could provide guidance for the design and manufacture of tube-tubesheet assemblies.

The following are specific conclusions.

5.1 EXPANSION PROCESS AND STRESS DISTRIBUTIONS

The first part (Chapters 2 and 3) involved stress analysis. The most important conclusions may be summarized thus:

1. A simplified 2-D technique named the "Two-Step Method" was developed to investigate residual stresses in the transition zone of tube-to-tubesheet joints, and the reliability of this technique was verified by comparing its results with those obtained using the 3-D finite element elasto-plastic analysis.

2. In all cases, the simultaneous expansion process produces higher average values of residual contact pressure than the sequential one. However, residual stress levels introduced in the transition zone are almost similar. The simultaneous expansion process produces a relatively stronger joint.

3. The real tubewall reduction sharply increases in a small region of the transition zone. This is due to the large deformation of the tube in this zone. The

combinations of tubewall reduction and tensile residual stresses in the transition zone become the primary reason for the failure of tube-to-tubesheet joints. Not enough been paid attention to this phenomenon in previous publications.

5.2 STATISTICAL ANALYSIS AND PARAMETRIC STUDY

The parametric study is based on the statistical analysis which is discussed in Chapters 3 and 4. Some conclusions are summarized below.

1. The Orthogonal Design Method is a useful technique for the arrangement of the calculations and the analysis of the results. The orthogonal design reduces the number of calculations and saves time; and permits the analysis of the significance of the separate main effects of each parameter.

2. The residual contact pressure depends primarily on the expansion pressure level. The Young's modulus ratio (E_s/E_t) of the tubesheet and the tube has also a significant effect. The residual contact pressure increases as the expansion pressure level increases and the ratio E_s/E_t decreases. Typically when E_s/E_t is reduced from 1.0 to 0.5, the residual contact pressure increases by as much as ten times (under the same expansion pressure level). Therefore it is suggested to choose a higher expansion pressure or a material combinations yielding lower E_s/E_t ratio.

3. The initial clearance between the tube and the tubesheet is the most important parameter affecting the maximum tensile axial residual stresses. The maximum tensile residual axial stress increases significantly with the increase of initial clearance. The second most important parameter is, again, the ratio E_s/E_t, but this applies to the sequential expansion case only. Similarly the level of residual axial stresses increases with the increase in expansion pressure level only for the simultaneous expansion case.

4. For the maximum tensile residual hoop stress, the most important parameter is also the initial clearance. The maximum tensile residual hoop stress increased with increasing of the initial clearance. The second most important parameter is the ratio E_t/Y_{st} and the expansion pressure level P for sequential and simultaneous cases respectively.

5. Apparent tubewall reduction depends on initial clearance under both sequential and simultaneous cases. For the simultaneous case, it is also affected by the expansion pressure level.

6. The depth of expansion has little effect on the residual contact pressure, but a large effect on the maximum residual stresses. When the depth of expansion is the same as the thickness of the tubesheet the value of maximum residual stresses

is 2 to 3 times the one when the depth of expansion is 90% of the thickness of the tubesheet. This is due to the additional effect of the sharp edges of the tubesheet.

7. The frictional coefficient between the tube and tubesheet contact surfaces has no effect on either the residual contact pressure or residual stresses in the tube. The reason is that most of the axial contraction occurs before contact.

5.3 RECOMMENDATIONS

1) This research was based on an elastic perfectly plastic material behavior of both the tube and tubesheet. The empirical equations are valid therefore only for this type of materials. Thus, it is suggested to include the strain hardening of the material in a similar investigation of the joint.

2) It is suggested to program the theoretical solution of tube-to-tubesheet presented in Appendix C. This solution gives the residual contact pressure versus the expansion pressure level and other parameters. By doing so, it will be convenient to compare the numerical results to the theoretical predictions which is much easier to perform. Unfortunately, due to time limitation, it was not possible to program this appendix and validate the formulation.

3) Because the real tubewall reduction sharply increases locally in the transition zone. It is suggested to investigate the influence of thermal shock and vibration on the strength of this area.

4) It is suggested to investigate the fatigue strength of the tube-to-tubesheet joint in the operation case.

5) It is suggested to investigate the relaxation of residual stresses.

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APPENDIX A

INTRODUCTION ORTHOGONAL DESIGN[58]

A.A.1 ORTHOGONAL ARRAY AND ORTHOGONAL DESIGN

The orthogonal array is constructed on the base of combinatorial theory and is widely used in experimental designs. The orthogonal experiment is that in which the experiment strategies are arranged and the experiment results are analysed according to the orthogonal array. It is applied to multi-factor and multi-index experiments in which interactions across the factors and random errors exist.

The effects of factors and interactions across factors on testing index can be surveyed with the orthogonal experiment method, so as to determine the optimal technological conditions for each experiment index and to order the priorities of the factors. For orthogonal experiments every factor under consideration should be controllable. The number of values that one factor can take is defined as the factor level.

Define $L_a(b^c)$ as the orthogonal array where L represents the array, subscript a is the number of experiments, superscript c is the number of columns which represents the maximum number of factors allowable in the experiment, b is the number of different values in the array which represents level number of each factor. For example, $L_8(2^7)$ represents an orthogonal array which possesses 8 rows (8 times of testings), 7 columns (at most 7 factors allowed in the test), 2 levels (every factor has two levels).This kind of orthogonal array is called 2-level array.

$L_8(2^7)$

column level experi. No	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Another example is $L_{12}(3x2^3)$ which has 12 rows, and 4 columns in which one is 3-level and 3 columns are 2-level. This kind of array is called the mixed-type array which can be used to arrange the experiments with different factor levels.

A.A.2 THE INTERACTION COLUMNS OF AN ORTHOGONAL ARRAY

When two columns are assigned to any two factors, the interaction between the two factors can be expressed with another column which is called the interaction column. There is only one interaction column in the 2-level array while two interaction columns in the 3-level array, e.g. $L_9(3^4)$, where the interaction columns of any two columns are an additional two columns. Usually one orthogonal array of low level (level number 2 or 3) contains a special array of interaction columns. For example, in the following interaction array of $L_8(2^7)$, it indicates that the interaction column for columns 3 and 5 is the column 6, etc. Some orthogonal arrays, for example $L_{12}(2^{11})$, do not include any interaction columns in it and in this case the interactions will not be considered.

The interaction array of $L_8(2^7)$

1	2	3	4	5	6	7	column
(1)	3	2	5	4	7	6	1
	(2)	1	6	7	4	5	2
		(3)	7	6	5	4	3
			(4)	1	2	3	4
				(5)	3	2	5
					(6)	1	6
						(7)	7

A.A.3 THE ORTHOGONALITY OF THE ARRAY

The array has the following orthogonality properties:

1. the repeated times of each level are equal in a column, for example, in $L_8(2^7)$ each level repeats 4 times in every column.

2. In any two columns the pairs composed of values (levels) of the same rows include all possible combinations of the levels and the repeated times of each pair are the same. For example, in $L_9(3^4)$, the pairs for any two columns contain all possible combinations under 3 levels: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3). Thus orthogonality of the experimental design can be arranged uniformly.

A.A.4 THE SCHEME OF THE EXPERIMENT

1. Steps

a. determine the number of factors in an experiment and the levels of each factor.

b. Analyse the interactions across factors and determine the factors to be considered and those to be temporarily omitted.

c. Determine approximately the possible times of testings according to the available experimental equipment, time and funds.

d. Choose the proper orthogonal array to arrange the experiment.

2. Arrangement

a. Without considering interactions, assign each factor to a column of the array, then the experimental conditions (levels each factor should take) for every testing (corresponding to a row in the array) are determined by levels of columns of arranged factors.

For example, in the experiment where four factors are selected, with three levels for each factor, the following table applies.

The table of factor levels

factor / level	A	B	C	D
1	A1	B1	C1	D1
2	A2	B2	C2	D2
3	A3	B3	C3	D3

Without considering interactions, the scheme is obtained by using the orthogonal array $L_9(3^4)$,

A 4-factor experiment scheme arranged by $L_9(3^4)$

column / testing No	1(A)	2(B)	3(C)	4(D)
1	1(A1)	1(B1)	1(C1)	1(D1)
2	1(A1)	2(B2)	2(C2)	2(D2)
3	1(A1)	3(B3)	3(C3)	3(D3)
4	2(A2)	1(B1)	2(C2)	3(D3)
5	2(A2)	2(B2)	3(C3)	1(D1)
6	2(A2)	3(B3)	1(C1)	2(D2)
7	3(A3)	1(B1)	3(C3)	2(D2)
8	3(A3)	2(B2)	1(C1)	3(D3)
9	3(A3)	3(B3)	2(C2)	1(D1)

As indicated in the table, the conditions for the 1st testing are A1B1C1D1, the conditions for the 2nd experiment are A1B2C2D2,..., the conditions for the 9th experiment are A3B3C2D1.

b. When interactions are to be considered, the factors cannot be arranged arbitrarily. They should be arranged according to proper designs of the table. Different factors (including interactions) cannot be in one column because different effects in one column cannot be separated, otherwise a bigger orthogonal array will need to be used. For example, to arrange a 4-factor experiment with interactions AxB and AxC while omitting other effects, the design should be according to $L_8(2^7)$.

column factor	1	2	3	4	5	6	7
3	A	B	AxB	C	AxC	BxC	
4	A	B	AxB CxD	C	AxC BxD	BxC AxD	D
4	A	B CxD	AxB	C BxD	AxC	D BxC	AxD
5	A DxE	B CxD	AxB CxE	C BxD	AxC BxE	D AxE BxC	E AxD

For the 4-factor experiment, A,B,C,D can be arranged in columns 1,2,4,7, AxB and AxC in columns 3,5. If the interactions across A,B,C,D are to be considered, a bigger array such as $L_{16}(2^{15})$ is required.

A.A.5 THE ANALYSIS OF ORTHOGONAL ARRAY

1. Calculating the level sum K_i in the ith level and level mean of k_i, e.g. according to a scheme of 4-factors at 3-levels arranged using $L_9(3^4)$, the analytical table is as follows:

Column Testing No.	1(A)	2(B)	3(C)	4(D)	Testing Index y
1	1	1	1	1	y1
2	1	2	2	2	y2
3	1	3	3	3	y3
4	2	1	2	3	y4
5	2	2	3	1	y5
6	2	3	1	2	y6
7	3	1	3	2	y7
8	3	2	1	3	y8
9	3	3	2	1	y9
K1	$K_1^{(1)}$	$K_1^{(2)}$	$K_1^{(3)}$	$K_1^{(4)}$	
K2	$K_2^{(1)}$	$K_2^{(2)}$	$K_2^{(3)}$	$K_2^{(4)}$	
K3	$K_3^{(1)}$	$K_3^{(2)}$	$K_3^{(3)}$	$K_3^{(4)}$	
k1	$k_1^{(1)}$	$k_1^{(2)}$	$k_1^{(3)}$	$k_1^{(4)}$	
k2	$k_2^{(1)}$	$k_2^{(2)}$	$k_2^{(3)}$	$k_2^{(4)}$	
k3	$k_3^{(1)}$	$k_3^{(2)}$	$k_3^{(3)}$	$k_3^{(4)}$	
Extreme difference					

where

$K_i^{(j)}$ is a testing index sum (or sum of levels) of the ith level in the jth column ,

$k_i^{(j)}$ is a testing index mean (or level mean) of the ith level in jth column,

$R^{(j)}$ is the extreme difference of k_1,k_2,k_3 in jth column.

For example

$$K_1^{(1)} = y_1 + y_2 + y_3 \; ; \; k_1^{(1)} = \frac{1}{3}K_1^{(1)}$$

$$K_2^{(1)} = y_4 + y_5 + y_6 \; ; \; k_2^{(1)} = \frac{1}{3}K_2^{(1)}$$

$$K_3^{(1)} = y_7 + y_8 + y_9 \; ; \; k_3^{(1)} = \frac{1}{3}K_3^{(1)}$$

$$K_3^{(2)} = y_3 + y_6 + y_9 \; ; \; k_3^{(2)} = \frac{1}{3}K_3^{(2)}$$

....

$$R^{(1)} = \max\{k_1^{(1)}, k_2^{(1)}, k_3^{(1)}\} - \min\{k_1^{(1)}, k_2^{(1)}, k_3^{(1)}\}$$

....

2. Determine the order of factor priorities

Listing the order of factor priorities according to the magnitude of extreme differences where the greater extreme difference means more importance of the factor.

3. Make a graph for the relation between factors and indices

After finding $k_i^{(j)}$, make a graph for each j by using level value i as the horizontal coordinate and k_i as the vertical coordinate. This graph relates the jth factor and the testing index. The greater the change of magnitude of k_i, the more the effect of the corresponding factor is. When the points of the graph are very widespread, the factor is main; when the points are relatively concentrated, the factor is less important.

When the interaction are considered, the k_i of an interaction column is introduced by that interaction. Similarly, the graph can be pictured to illustrate the relation of factors and testing index.

The unassigned columns can be used for estimation of experimental errors. The k_i calculated in the same way can be regarded as testing errors. The change in magnitude of k_i reflects the range of the experimental errors. A graph showing the relation between experimental errors and the testing index can also be plotted.

4. Determine the optimal technological conditions

When interactions are omitted, find the optimal points of the level in the graph of each factor according to the requirements of the testing index, then the combination of optimal levels of the factors is the optimal technological conditions.

A.A.6 The Variance Analysis

Assume the factor A in the array is arranged in the jth column, the level number is b_j, the repeat times of each level is r_j, the experiment number is n (number of columns), then, S_A, the sum of squares of A (or S_j ,the sum of jth square) is

$$S_A = S_j = r_j \sum_{l=1}^{B_j} (K_l^{(j)} - k^{(j)})^2$$
$$= \frac{1}{r_j} \sum_{l=1}^{b_j} (K_l^{(j)})^2 - \frac{1}{n} (\sum_{i=1}^{n} y_i)^2$$

The total sum of square S_{total} is

$$S_{total} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$
$$= \sum_{i=1}^{n} y_i^2 - \frac{1}{n} (\sum_{i=1}^{n} y_i)^2$$

The sum of squares of interaction, S_{int}, is calculated according to the corresponding column with the same formula as used for S_A.

The sum of squares of the errors, S_e, equals to the difference between S_{total} and sum of squares of all the columns where there are factors or interactions, i.e.

$$S_e = S_{total} - \sum S_j - \sum S_{inter}$$

TABLE OF VARIANCE ANALYSE

Souse of Deviation	Sun of Square	Degree of Freedom	Mean Square	Statistic	Confidence Limit	Inference
A	S_A	b_A-1	$s_A = \frac{S_A}{b_A - 1}$	$F_A = \frac{s_A}{s_e}$	$F_\alpha(b_A-1,n_e)$	If $F>F_\alpha$, the effect of factor is signifi-cant; If $F<F_\alpha$, the effect is insignificant.
B	S_B	b_B-1	$s_B = \frac{S_B}{b_B - 1}$	$F_B = \frac{s_B}{s_e}$	$F_\alpha(b_B-1,n_e)$	
A X B	S_{AXB}	$(b_A-1)(b_B-1)$	$s_{A \times B} = \frac{S_{A \times B}}{(b_A - 1)(b_B - 1)}$	$F_{A \times B} = \frac{s_{A \times B}}{s_e}$	$F_\alpha((b_A-1)(b_B-1),n_e)$	
ERROR	S_e	n_e	$s_e = \frac{S_e}{n_e}$			
Total Sum of Square	S_{Total}	n-1				

where

$$n_e = n-1 - \sum_1 (b_j - 1) - \sum_2 (b_j-1)(b_i-1)$$

$\sum_1$ for columns which is arranged for factors

$\sum_2$ for columns which is arranged for interactions

The priorities of factors can be quantitatively determined through the variance analysis of the array. It is possible to determine the principal factors and secondary factors so that it is only needed to consider the principal factors for the optimal technological conditions. As for the levels of the secondary factors, they can be determined through other conditions.

APPENDIX B

EXPANSION OF A CIRCULAR HOLE IN A PLATE [54][59]

Let us consider an infinite plate with a hole on it. The radius of hole is a. A uniform pressure P acts on the inner surface of the hole.(Figure B.1)

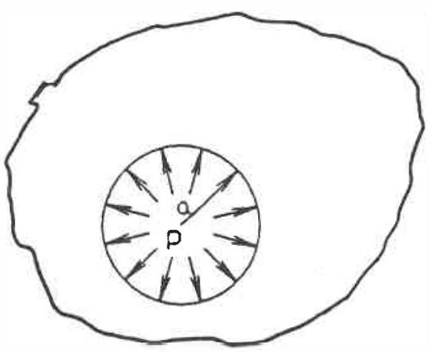


Figure B.1 Infinite plate with hole

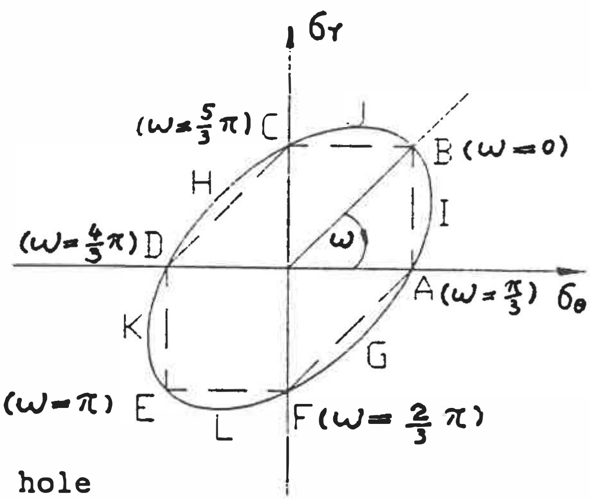


Figure B.2 Relationship between state of stress and ω

Assuming that the plate is under a plane stress state, thus $\sigma_z = 0$.

From the theory of elasticity we have:

$$\sigma_r = -p\frac{a^2}{r^2}, \qquad \sigma_\theta = p\frac{a^2}{r^2} \tag{1}$$

where r and Θ are polar coordinates.

Using Mises yield criterion, yielding takes place when

$$\sqrt{\sigma_\theta^2 - \sigma_\theta\sigma_r + \sigma_r^2} = \sigma_s$$

where σ_r is radial stress and σ_θ is hoop stress in the plate.

According to the elastic solution (1), the stress at the inner surface of the hole is higher than at the outer surface. The inner surface of hole will thus yield first.

at r = a, σ_r = -p, σ_θ = p

$$\sqrt{p^2 - p(-p) + (-p)^2} = \sqrt{3}\ p = \sigma_s$$

When the inner pressure p= $p_e = \sigma_s / \sqrt{3}$, the inner surface of hole yields, but other parts of the infinite plate are still in the elastic state.

Using the parameter equation of the Mises yield criterion

$$\sigma_r = \frac{2\sigma_s}{\sqrt{3}} \cos(\omega + \frac{\pi}{6})$$
$$\sigma_\theta = \frac{2\sigma_s}{\sqrt{3}} \cos(\omega - \frac{\pi}{6}) \qquad (2)$$

Mises yield criterion $\sigma_\theta^2 - \sigma_r\sigma_\theta + \sigma_r^2 = \sigma_s^2$ is a standard ellipse in new system of coordinates whose x-axis is rotated $\pi/4$(Fig.B.2). In the standard ellipse, the semi-major axis is a = $\sqrt{2}\ \sigma_s$, while the semi-minor axis is b = $(\sqrt{2}/3)\ \sigma_s$

In the parametric equations, σ_r, σ_θ vary with ω as follow:

	B	I	A	G	F	L	E	K	D	H	C	J
ω	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{11}{6}\pi$
σ_θ	σ_s	$\frac{2}{\sqrt{3}}\sigma_s$	σ_s	$\frac{1}{\sqrt{3}}\sigma_s$	0	$-\frac{1}{\sqrt{3}}\sigma_s$	$-\sigma_s$	$-\frac{2}{\sqrt{3}}\sigma_s$	$-\sigma_s$	$-\frac{1}{\sqrt{3}}\sigma_s$	0	$\frac{1}{\sqrt{3}}\sigma_s$
σ_r	σ_s	$\frac{1}{\sqrt{3}}\sigma_s$	0	$-\frac{1}{\sqrt{3}}\sigma_s$	$-\sigma_s$	$-\frac{2}{\sqrt{3}}\sigma_s$	$-\sigma_s$	$-\frac{1}{\sqrt{3}}\sigma_s$	0	$\frac{1}{\sqrt{3}}\sigma_s$	σ_s	$\frac{2}{\sqrt{3}}\sigma_s$

According to the Mises criterion, the plastic zone can be determined, At r = a, just enters plastic state,

$$\sigma_r = -p_e = -\frac{\sigma_s}{\sqrt{3}}$$

$$\sigma_\theta = p_e = \frac{\sigma_s}{\sqrt{3}}$$

The corresponding point G, is

$$\omega = \frac{\pi}{2}$$

The equilibrium equation for an axisymmetric body under inner pressure is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

Substituting (2) into the equilibrium equation, we obtain

$$\sin(\omega + \frac{\pi}{6})\frac{d\omega}{dr} + \frac{1}{r}\left[\cos(\omega - \frac{\pi}{6}) - \cos(\omega + \frac{\pi}{6})\right] = 0$$

rearranging it,

$$\frac{dr}{r} = -\frac{\sin(\omega + \frac{\pi}{6})}{\sin\omega}d\omega = -\frac{1}{2}(\sqrt{3} + \cot\omega)d\omega$$

or

$$2\frac{dr}{r} = (-\sqrt{3} - \cot\omega)d\omega \tag{3}$$

Integrating

$$2\ln r = -\sqrt{3}\omega - \int \cot\omega d\omega$$

Sine,

$$\int \cot\omega d\omega = \int \frac{\cos\omega}{\sin\omega} d\omega = \int \frac{d\sin\omega}{\sin\omega} = \ln\ \sin x + C$$

$$\ln r^2 = -\sqrt{3}\omega - \ln\ \sin\omega + \ln C^2$$

$$\ln\left(\frac{r^2\sin\omega}{C^2}\right) = -\sqrt{3}\omega$$

Therefore,

$$\left(\frac{C}{r}\right)^2 = e^{\sqrt{3}\omega}\ \sin\omega \tag{4}$$

Equation (4) is valid in the plastic zone only. r_s is the radius of boundary separating the elastic and plastic zones.

Thus, at $r = r_s$,

$$\sigma_\theta = -\sigma_r = \frac{\sigma_s}{\sqrt{3}}$$

Therefore $=\pi/2$ (corresponding to point G in Fig.B2).

$$(\frac{C}{r_s})^2 = e^{\sqrt{3}\frac{\pi}{2}} \sin\frac{\pi}{2} = e^{\sqrt{3}\frac{\pi}{2}} \quad (5)$$

Dividing (4) by (5),

$$(\frac{r_s}{r})^2 = e^{-\sqrt{3}(\frac{\pi}{2}-\omega)} \sin\omega$$

As the inner pressure P increases, the range of the plastic zone expands. More and more points enter reach the plastic state.

When the inner pressure P increases, the absolute value of σ_r at $r = a$ increases too. However, the increase of absolute value of σ_r is limited by yield criterion. Under the plane stress condition,

$$|\sigma_r| \leq \frac{2}{\sqrt{3}}\sigma_s$$

If $\sigma_r=-(2/\sqrt{3})\sigma_s$, then the state of stress at r = a corresponds point L, i.e.

$$\sigma_r = -\frac{2}{\sqrt{3}}\sigma_s, \quad \sigma_\theta = -\frac{1}{\sqrt{3}}\sigma_s, \quad \omega=\frac{5}{6}\pi$$

therefore

$$(\frac{C}{a})^2 = e^{\sqrt{3}\frac{5}{6}\pi}\sin\frac{5\pi}{6}$$

Now the inner pressure $P=(2/\sqrt{3})\sigma_s=1.15\sigma_s$ reaches its limit. It is impossible to increase more due to the thicking effect of the plate.

Therefore

$$(\frac{r_s}{a})^2 = e^{-\sqrt{3}(\frac{\pi}{2} - \frac{5\pi}{6})}\sin\frac{5\pi}{6} = \frac{1}{2}e^{1.82} = 3.06$$

Therefore, r_c = 1.75a, i.e. according to the Mises criterion, the maximum radius of the plastic zone is r_s = 1.75a.

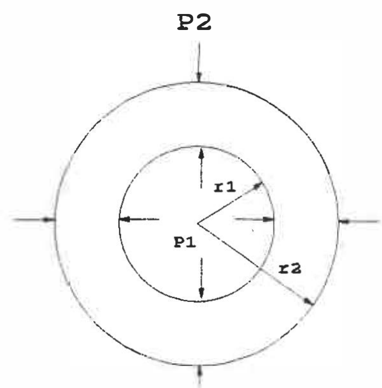
If Tresca criterion is used instead of Mises, the result will be as follow: r_s = 1.65a.

APPENDIX C

THEORETICAL SOLUTION OF TUBE-TO-TUBESHEET JOINT THROUGH EXPANSION [17][60]

A.C.1 ELASTIC SOLUTION

Axisymmetrical plane stress elastic solution:



$$u=u(r),\ v=0,\ \sigma_z=0$$

$$\begin{cases} \varepsilon_r = \dfrac{1}{E}(\sigma_r - \nu\sigma_\theta) = \dfrac{du}{dr} \\ \varepsilon_\theta = \dfrac{1}{E}(\sigma_\theta - \nu\sigma_r) = \dfrac{u}{r} \\ \gamma_{r\theta} = 0 \end{cases}$$

boundary conditions:

$$\begin{cases} \sigma_r\big|_{r=r_1} = -p_1 \\ \sigma_r\big|_{r=r_2} = -p_2 \\ \tau_{r\theta}\big|_{r=r_1} = \tau_{r\theta}\big|_{r=r_2} = 0 \end{cases}$$

$$\begin{cases} \sigma_r = -\dfrac{\dfrac{r_2^2}{r^2}-1}{\dfrac{r_2^2}{r_1^2}-1} p_1 - \dfrac{1-\dfrac{r_1^2}{r^2}}{1-\dfrac{r_1^2}{r_2^2}} p_2 \\ \sigma_\theta = \dfrac{\dfrac{r_2^2}{r^2}+1}{\dfrac{r_2^2}{r_1^2}-1} p_1 - \dfrac{1+\dfrac{r_1^2}{r^2}}{1-\dfrac{r_1^2}{r_2^2}} p_2 \\ \tau_{r\theta} = 0 \end{cases} \tag{1}$$

Given: $n=r_2/r_1$

$$\begin{aligned} u &= r\varepsilon_\theta \\ &= \frac{r}{E}(\sigma_\theta - \nu\sigma_r) \\ &= \frac{1+\nu}{E}\frac{r}{n^2-1}\left[\left(\frac{r_2^2}{r^2}+\frac{1-\nu}{1+\nu}\right)p_1 - n^2\left(\frac{r_1^2}{r^2}+\frac{1-\nu}{1+\nu}\right)p_2\right] \end{aligned} \tag{2}$$

For the case of $p_2=0$, only inner pressure p_1

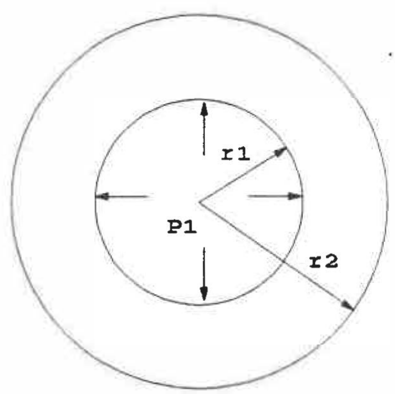
$$\begin{cases} \sigma_r = -\dfrac{1}{n^2-1}\left(\dfrac{r_2^2}{r^2}-1\right)p_1 \\ \sigma_\theta = \dfrac{1}{n^2-1}\left(\dfrac{r_2^2}{r^2}+1\right)p_1 \\ u = \dfrac{1+\nu}{E}\dfrac{r}{n^2-1}\left(\dfrac{r_2^2}{r^2}+\dfrac{1-\nu}{1+\nu}\right)p_1 \end{cases} \tag{3'}$$

For the case of $p_1=0$, only outer pressure p_2

$$\begin{cases} \sigma_r = -\dfrac{n^2}{n^2-1}\left(1 - \dfrac{r_1^2}{r^2}\right)p_2 \\ \sigma_\theta = -\dfrac{n^2}{n^2-1}\left(1 + \dfrac{r_1^2}{r^2}\right)p_2 \\ u = -\dfrac{1+\nu}{E}\dfrac{n^2 r}{n^2-1}\left(\dfrac{r_1^2}{r^2} + \dfrac{1-\nu}{1+\nu}\right)p_2 \end{cases} \qquad (4)'$$

A.C.2 ELASTIC LIMIT PRESSURE OF TUBE

Since there is a gap c between the tube and the bore of the tubesheet, the tube is initially loaded upon the inner pressure p_1 only ($p_2=0$).



We have

$$\begin{cases} \sigma_r = -\dfrac{1}{n^2-1}\left(\dfrac{r_2^2}{r^2} - 1\right) p_1 \\ \sigma_\theta = \dfrac{1}{n^2-1}\left(\dfrac{r_2^2}{r^2} + 1\right) p_1 \\ u = \dfrac{1+\nu_t}{E_t}\dfrac{r}{n^2-1}\left(\dfrac{r_2^2}{r^2} + \dfrac{1-\nu_t}{1+\nu_t}\right) p_1 \end{cases} \qquad (3)$$

This elastic solution is valid until the initial yield of the tube at the inner circumference $r = r_1$ and $c>0$.

During loading process, directions of the principal stresses are not changed, so assuming the material obey Tresca yield criterion,

$$\tau_{max} = \tau_s = \frac{1}{2}\sigma_s = \frac{1}{2}(\sigma_1 - \sigma_3) \qquad (5)$$

So we can get

$$\begin{aligned} \sigma_{st} &= (\sigma_\theta - \sigma_r) \\ &= \frac{2p_1}{n^2-1}\frac{r_2^2}{r^2} \end{aligned}$$

The minimum value of p_1 is at $r=r_1$

$$p_1|_{r=r_1} = p_1^e = \frac{n^2-1}{2n^2}\,\sigma_{st} \tag{6}$$

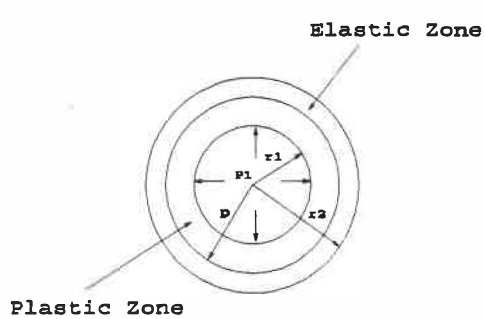
The p_1^e is elastic limit pressure of the tube.

Upon p_1^e, the displacement u at $r=r_2$ from (3)

$$u^e = u|_{r=r_2} = \frac{\sigma_{st}}{E_t}\,\frac{r_2}{n^2} \leq c \tag{7}$$

Otherwise, the outer circumference has been contacted the bore of the tubesheet before the initial yielding of the tube at the inner circumference. The contact problem of two elastic bodies will be discussed later.

A.C.3 ELASTICO-PLASTIC DEFORMATION OF THE TUBE



Increasing the inner pressure

$$p_1 > p_1^e$$

the tube will be experienced elasto-plastic deformation. Let represent the interface,

$$\rho \le r \le r_2 \qquad \text{elastic region,}$$

The elastic solutions (3) and (6) are available with the replacement of

$$r_1 \Rightarrow \rho\ , \quad n \Rightarrow n_\rho = \frac{r_2}{\rho}\ , \; p_1 \Rightarrow p_\rho$$

We have

$$\begin{cases} \sigma_r = -\dfrac{1}{n_\rho^2-1}\left(\dfrac{r_2^2}{r^2} - 1\right) p_\rho \\ \sigma_\theta = \dfrac{1}{n_\rho^2-1}\left(\dfrac{r_2^2}{r^2} + 1\right) p_\rho \\ u = \dfrac{1+\nu_t}{E_t}\dfrac{r}{n_\rho^2-1}\left(\dfrac{r_2^2}{r^2} + \dfrac{1-\nu_t}{1+\nu_t}\right) p_\rho \end{cases}$$

We have

$$\begin{cases} \sigma_r = -\dfrac{\sigma_{st}}{2n_\rho^2}\left(\dfrac{r_2^2}{r^2} - 1\right) \\ \sigma_\theta = \dfrac{\sigma_{st}}{2n_\rho^2}\left(\dfrac{r_2^2}{r^2} + 1\right) \qquad (8) \\ u = \dfrac{1+\nu_t}{2E_t}\dfrac{\sigma_{st}}{n_\rho^2}\, r\left(\dfrac{r_2^2}{r^2} + \dfrac{1-\nu_t}{1+\nu_t}\right) \qquad (9) \end{cases}$$

$$p_\rho = \frac{n_\rho^2 - 1}{2n_\rho^2}\,\sigma_{st} \tag{10}$$

For $r_1 \le r \le \rho$ **plastic zone:**

The material is considered perfectly plastic, the stresses must satisfy the equilibrium and the yield criterion (Tresca)

$$\begin{cases} \dfrac{d\sigma_r}{dr} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0 \\ \sigma_\theta - \sigma_r = \sigma_{st} \end{cases}$$

with the boundary conditions

$$\begin{cases} \sigma_r|_{r=r_1} = -p_1 \\ \sigma_r|_{r=\rho} = -p_\rho \end{cases}$$

With the first boundary condition

$$\begin{cases} \sigma_r = -p_1 + \sigma_{st} \ln\dfrac{r}{r_1} \\ \sigma_\theta = -p_1 + \sigma_{st}\left(1 + \ln\dfrac{r}{r_1}\right) \end{cases} \tag{11}$$

With the second boundary condition (interface)

$$\begin{aligned} \sigma_r|_{r=\rho} &= -p_1 + \sigma_{st} \ln \frac{\rho}{r_1} \\ &= -p_\rho = -\frac{n_\rho^2 - 1}{2n_\rho^2}\sigma_{st} \end{aligned}$$

$$p_1 = p_1(\rho) = \sigma_{st}\left[\ln\frac{\rho}{r_1} + \frac{1}{2}\left(1-\frac{\rho^2}{r_2^2}\right)\right] \tag{12}$$

Calculation of the displacement u in the plastic zone.

From Tresca yield function

$$f = \sigma_\theta - \sigma_r - \sigma_{st} = 0$$

the associated flow rule

$$\begin{cases} d\varepsilon_r^p = d\lambda\dfrac{\partial f}{\partial \sigma_r} = -d\lambda \\ d\varepsilon_\theta^p = d\lambda\dfrac{\partial f}{\partial \sigma_\theta} = d\lambda \\ d\varepsilon_z^p = d\lambda\dfrac{\partial f}{\partial \sigma_z} = 0 \end{cases}$$

From the equations we get

$$\varepsilon_z^p = 0 \qquad \text{zero plastic axial strain}$$

$$d\varepsilon_v^p = d\varepsilon_r^p + d\varepsilon_\theta^p + d\varepsilon_z^p = 0$$
$$\Rightarrow \varepsilon_v^p = 0$$

Zero plastic volume strain.

$$\begin{cases} \varepsilon_z = \varepsilon_z^e + \varepsilon_z^p = \varepsilon_z^e = \dfrac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right] \\ \varepsilon_v = \varepsilon_r + \varepsilon_\theta + \varepsilon_z = \varepsilon_v^e = \dfrac{1-2\nu}{E}\left[(\sigma_r + \sigma_\theta + \sigma_z)\right] \end{cases} \tag{13}$$

For plane stress $\sigma_z = 0$

$$\varepsilon_z = -\frac{\nu}{E}\,(\sigma_r + \sigma_\theta)$$

$$\varepsilon_r + \varepsilon_\theta = \frac{1-2\nu}{E}(\sigma_r + \sigma_\theta) - \varepsilon_z$$

$$\frac{du}{dr} + \frac{u}{r} = \frac{1-\nu}{E}(\sigma_r + \sigma_\theta)$$

$$\begin{aligned}
\frac{1}{r}\frac{d}{dr}(ru) &= \frac{1-\nu_t}{E_t}\,(\sigma_r + \sigma_\theta) \\
&= \frac{1-\nu_t}{E_t}\left[-2p_1 + \sigma_{st}(1+2\ln\frac{r}{r_1})\right] \\
&= \frac{1-\nu_t}{E_t}\,\sigma_{st}\left(2\ln\frac{r}{\rho} + \frac{\rho^2}{r_2^2}\right)
\end{aligned}$$

Integral

$$ru = \frac{1-\nu_t}{E_t}\sigma_{st}\left[r^2\ln r - \frac{1}{2}r^2 - \left(2\ln\rho - \frac{\rho^2}{r_2^2}\right)\frac{r^2}{2}\right] + D$$

at r=ρ

$$\begin{aligned}
\rho u|_{r=\rho} &= \frac{1-\nu_t}{E_t}\sigma_{st}\left(-\frac{1}{2}\rho^2 + \frac{1}{2}\frac{\rho^4}{r_2^2}\right) + D \\
&= \rho\frac{1+\nu_t}{2E_t}\frac{\sigma_{st}}{n_\rho^2}\left(\frac{r_2^2}{\rho^2} + \frac{1-\nu_t}{1+\nu_t}\right)
\end{aligned}$$

$$D = \frac{\sigma_{st}}{E_t}\rho^2$$

Thus for $r_1 \le r \le \rho$

We have

$$u = \frac{1-\nu_t}{E_t}\sigma_{st}\left[r\ln r - \frac{1}{2}r - \left(2\ln\rho - \frac{\rho^2}{r_2^2}\right)\frac{r}{2}\right] + \frac{D}{r}$$

$$= \frac{\sigma_{st}}{E_t}\left\{(1-\nu_t)\left[r\ln\frac{r}{\rho} - \frac{1}{2}r\left(1 - \frac{1}{n_\rho^2}\right)\right] + \frac{\rho^2}{r}\right\} \tag{14}$$

When

$$\rho = r_2, \quad n_\rho = \frac{r_2}{\rho} = 1,$$

the tube is fully plastic which is not desirable.

$$p_1^t = \sigma_{st}\ln\frac{r_2}{r_1} = \sigma_{st}\ln(n) \tag{15}$$

This is plastic collapse pressure of the tube.

The tube becomes to contact with the bore of the tubesheet at $r=r_2$ while the stress in this region of the tube is still in elastic state.

$$u\big|_{r=r_2} = \frac{1+\nu_t}{2E_t}\frac{\sigma_{st}}{n_\rho^2}r_2\left(1+\frac{1-\nu_t}{1+\nu_t}\right) = C$$

$$\frac{\sigma_{st}}{E_t}\frac{r_2}{n_\rho^2} = C$$

$$\rho^* = \sqrt{\frac{E_t}{\sigma_{st}}r_2 c} < r_2 \tag{16}$$

The value of gap c must be chose to satisfy

$$\rho^* < r_2 \tag{17}$$

$$p_1^* = p_1(\rho^*) = \sigma_{st}\left[\ln\frac{\rho^*}{r_1} + \frac{1}{2}\left(1-\frac{\rho^{*2}}{r_2^2}\right)\right] \tag{18}$$

The elasto-plastic solutions

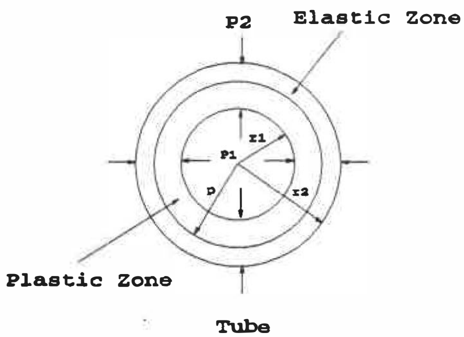
(11), (12) and (14)	$r_1 \le r \le \rho$	**plastic region**
(8), (9) and (10)	$\rho \le r \le r_2$	**elastic region**

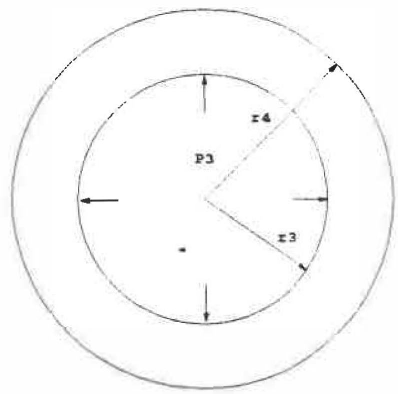
are only valid when

$$p_1^e \le p_1 \le p_1^*$$

A.C.4 JOINT OF TUBE AND TUBESHEET

When $p_1 > P_1^*$, the tubesheet begins to be loaded.





$$n = \frac{r_2}{r_1} \quad n_\rho = \frac{r_2}{\rho} \qquad\qquad m = \frac{r_4}{r_3}$$

tube $r_1 \le r \le \rho$ plastic   tubesheet $r_3 = r_2 + c$

$\rho \le r \le r_2$ elastic $p_3 = p_2$

A.C.4.1

For the tubesheet, the elastic solution (3) is available with substituting

$$\begin{aligned} r_2 &\Rightarrow r_4 \\ n = \frac{r_2}{r_1} &\Rightarrow m = \frac{r_4}{r_3} \\ p_1 &\Rightarrow p_3 \\ E_t &\Rightarrow E_s \\ \nu_t &\Rightarrow \nu_s \end{aligned}$$

We get

$$\begin{cases} \sigma_r = -\dfrac{1}{m^2 - 1}\left(\dfrac{r_4^2}{r^2} - 1\right) p_3 \\ \sigma_\theta = \dfrac{1}{m^2 - 1}\left(\dfrac{r_4^2}{r^2} + 1\right) p_3 \\ u = \dfrac{1+\nu_s}{E_s}\dfrac{r}{m^2-1}\left(\dfrac{r_4^2}{r^2} + \dfrac{1-\nu_s}{1+\nu_s}\right) p_3 \end{cases} \qquad (19)$$

A.C.4.2 For tube, elastic region

$$\rho \le r \le r_2$$

the elastic solutions (1) and (2) are available with substituting

$$\begin{aligned} r_1 &\Rightarrow \rho \\ n = \frac{r_2}{r_1} &\Rightarrow n_\rho = \frac{r_2}{\rho} \\ E &\Rightarrow E_t \\ \nu &\Rightarrow \nu_t \\ p_1 &\Rightarrow p_\rho \end{aligned}$$

We get

$$\begin{cases} \sigma_r = -\dfrac{\dfrac{r_2^2}{r^2} - 1}{n_\rho^2 - 1} p_\rho - \dfrac{1 - \dfrac{\rho^2}{r^2}}{1 - \dfrac{1}{n_\rho^2}} p_2 \\ \sigma_\theta = \dfrac{\dfrac{r_2^2}{r^2} + 1}{n_\rho^2 - 1} p_\rho - \dfrac{1 + \dfrac{\rho^2}{r^2}}{1 - \dfrac{1}{n_\rho^2}} p_2 \end{cases} \qquad (21)'$$

By Tresca yield criterion (5) and (21)'

$$\sigma_{st} = (\sigma_\theta - \sigma_r) = \frac{2p_\rho}{n_\rho^2 - 1} \frac{r_2^2}{r^2} - \frac{2p_2}{1 - \dfrac{1}{n_\rho^2}} \frac{\rho^2}{r^2}$$

at r=ρ

the stress satisfies the above condition, therefore

$$p_\rho = \frac{n_\rho^2 - 1}{2n_\rho^2} \sigma_{st} + p_2 \qquad (22)$$

substituting (22) into (21)'

$$\begin{cases} \sigma_r = -\dfrac{\sigma_{st}}{n_\rho^2}\left(\dfrac{r_2^2}{r^2} - 1\right) - \left(\dfrac{\dfrac{r_2^2}{r^2} - 1}{n_\rho^2 - 1} + \dfrac{1-\dfrac{p^2}{r^2}}{1 - \dfrac{1}{n_\rho^2}}\right) p_2 \\[2ex] \sigma_\theta = \dfrac{\sigma_{st}}{n_\rho^2}\left(\dfrac{r_2^2}{r^2} + 1\right) - \left(\dfrac{\dfrac{r_2^2}{r^2} - 1}{n_\rho^2 - 1} - \dfrac{1-\dfrac{p^2}{r^2}}{1 - \dfrac{1}{n_\rho^2}}\right) p_2 \end{cases} \tag{21}$$

$$\begin{aligned} u = {} & \frac{1 + \nu_t}{2E_t}\,\frac{\sigma_{st}}{n_\rho^2} r \left(\frac{r_2^2}{r^2} + \frac{1-\nu_t}{1+\nu_t}\right) \\ & + \frac{1+\nu_t}{E_t}\,\frac{p_2}{n_\rho^2 - 1}\, r \left[\left(\frac{r_2^2}{r^2} + \frac{1-\nu_t}{1+\nu_t}\right) - n_\rho^2\left(\frac{r_1^2}{r^2} + \frac{1-\nu_t}{1+\nu_t}\right)\right] \end{aligned} \tag{23}$$

A.C.4.3

For tube, plastic region

$$r_1 \leq r \leq \rho$$

the plastic stresses (11) are still valid.

$$\begin{cases} \sigma_r = -p_1 + \sigma_{st} \ln\dfrac{r}{r_1} \\[2ex] \sigma_\theta = -p_1 + \sigma_{st}\left(1 + \ln\dfrac{r}{r_1}\right) \end{cases} \tag{24}$$

From the interface condition, from (24) and (22):

$$\sigma_r|_{r=\rho} = -p_1 + \sigma_{st}\ln\frac{\rho}{r_1} = -p_\rho$$
$$= -\left(\frac{n_\rho^2 - 1}{2n_\rho^2}\sigma_{st} + p_2\right)$$

We have

$$p_1 = p_1(\rho) = \sigma_{st}\left[\ln\frac{\rho}{r_1} + \frac{1}{2}\left(1-\frac{\rho^2}{r_2^2}\right)\right] + p_2 \qquad (25)$$

The displacement u could be derived as before

$$\frac{1}{r}\frac{d}{dr}(ru) = \frac{1-\nu_t}{E_t}\sigma_{st}\left(2\ln\frac{r}{\rho} + \frac{\rho^2}{r_2^2}\right) + \frac{1-\nu_t}{E_t}2p_2$$

Integral, we have

$$ru = \frac{1-\nu_t}{E_t}\sigma_{st}\left[r^2\ln r-\frac{1}{2}r^2 - \left(2\ln\rho - \frac{\rho^2}{r_2^2}\right)\frac{r^2}{2}\right] + \frac{1-\nu_t}{E_t}p_2 r^2 + D$$

The value of D can be determined as follow:

at $r = \rho$ interface

$$\rho u|_{r=\rho} = \frac{1-\nu_t}{E_t}\sigma_{st}\left(-\frac{1}{2}\rho^2 + \frac{1}{2}\frac{\rho^4}{r_2^2}\right) + \frac{1-\nu_t}{E_t}p_2\rho^2 + D$$

$$= \rho\frac{1+\nu_t}{2E_t}\frac{\sigma_{st}}{n_\rho^2}\rho\left(\frac{r_2^2}{\rho^2} + \frac{1-\nu_t}{1+\nu_t}\right)$$

$$+ \rho\frac{1+\nu_t}{E_t}p_2\rho\left[\frac{1}{n_\rho^2-1}\left(n_\rho^2 + \frac{1-\nu_t}{1+\nu_t}\right) - \frac{n_\rho^2}{n_\rho^2-1}\left(\frac{r_1^2}{\rho^2} + \frac{1-\nu_t}{1+\nu_t}\right)\right]$$

So

$r_1 \leq r \leq \rho$

$$u = \frac{1-\nu_t}{E_t}\sigma_{st}\left[r\ln\frac{r}{\rho} - \frac{1}{2}r\left(1 - \frac{1}{n_\rho^2}\right)\right] + \frac{1-\nu_t}{E_t}p_2 r + \frac{D}{r} \qquad (26)$$

A.C.4.4

Contact conditions for the tube and tubesheet joint

at

$r = r_3$ $\qquad r_3 = r_2 + c$

From (20)

$$u|_{r=r_3} = \frac{1+\nu_s}{E_s}\frac{r_3}{m^2-1}\left(\frac{r_4^2}{r_3^2} + \frac{1-\nu_s}{1+\nu_s}\right)p_3$$

at $r=r_2$, from (23)

$$u\big|_{r=r_2} = \frac{1+\nu_t}{2E_t}\frac{\sigma_{st}}{n_\rho^2} r_2 \left(1 + \frac{1-\nu_t}{1+\nu_t} \right) + \frac{1+\nu_t}{E_t}\frac{P_2}{n_\rho^2 - 1} r_2 \left[\left(1 + \frac{1-\nu_t}{1+\nu_t} \right) - n_\rho^2 \left(\frac{1}{n^2} + \frac{1-\nu_t}{1+\nu_t} \right) \right]$$

The contact conditions are

$$p_3 = p_2 , \quad u\big|_{r=r_3} + c = u\big|_{r=r_2}$$

$$\frac{1+\nu_s}{E_s}\frac{r_3}{m^2-1}\left(m^2 + \frac{1-\nu_s}{1+\nu_s}\right)p_2 + c = \frac{\sigma_{st}}{E_t}\frac{r_2}{n_\rho^2} + \frac{2p_2}{E_t}\frac{r_2}{n_\rho^2 - 1} - \frac{(1+\nu_t)P_2}{E_t}\frac{r_2 n_\rho^2}{n_\rho^2 - 1}\left(\frac{1}{n^2} + \frac{1-\nu_t}{1+\nu_t}\right)$$

So

$$\begin{aligned} p_2 &= p_2(\rho) = \frac{1}{A}\left(\frac{\sigma_{st}}{E_t}\frac{\rho^2}{r_2} - c\right) \\ A &= \frac{1+\nu_s}{E_s}\frac{r_3}{m^2+1}\left(m^2 + \frac{1-\nu_s}{1+\nu_s}\right) \\ &\quad + \frac{1+\nu_t}{E_t}\frac{n_\rho^2 r_2}{n_\rho^2-1}\left(\frac{1}{n^2} + \frac{1-\nu_t}{1+\nu_t}\right) - \frac{2}{E_t}\frac{r_2}{n_\rho^2-1} \end{aligned} \tag{27}$$

Substituting (27) into (25) to eliminate p_2

$$p_1 = p_1(\rho) = \sigma_{st}\left[\ln\frac{\rho}{r_1} + \frac{1}{2}\left(1 - \frac{\rho^2}{r_2^2}\right)\right] + \frac{1}{A}\left(\frac{\sigma_{st}}{E_t}\frac{\rho^2}{r^2} - c\right) \tag{28}$$

A.C.4.5 Elastic limit pressure of the tubesheet

Eqn.(6) can be used

$$\begin{aligned} p_1^e &\Rightarrow p_3^e \\ n &\Rightarrow m \\ \sigma_{st} &\Rightarrow \sigma_{ss} \end{aligned}$$

We get

$$p_3^e = \frac{m^2-1}{2m^2}\sigma_{ss} \tag{29}$$

The tubesheet reaches initial yield at $r=r_3$ upon pressure p_3^e. The corresponding elasto-plastic interface in the tube is at

$$r = \rho^{**}$$

which is determined by (27) and (29).

$$\begin{aligned} p_2^e = p_2(\rho^{**}) &= \frac{1}{A}\left(\frac{\sigma_{st}}{E_t}\frac{(\rho^{**})^2}{r^2} - c\right) \\ &= \frac{m^2 - 1}{2m^2}\,\sigma_{SS} = p_3^e \end{aligned} \tag{30}$$

ρ^{**} is solved by (30). Again this interference radius ρ^{**} must be smaller than r_2:

$$\rho^{**} < r_2$$

Substituting ρ^{**} in to (28), we get

$$p_1^{**} = p_1(\rho^{**}) = \sigma_{st}\left[\ln\frac{\rho^{**}}{r_1} + \frac{1}{2}\left(1 - \frac{(\rho^{**})^2}{r_2^2}\right)\right] + \frac{m^2-1}{2m^2}\sigma_{ss} \tag{31}$$

$$\text{If} \qquad \rho^{**} \geq r_2$$

it means that the tube became fully plastic before the tubesheet yielding. This is no desirable. The geometrical or material parameters of the tube and tubesheet may be adjusted to make $\rho^{**} < r_2$. Or if $\rho \rightarrow r_2$, $n_\rho \rightarrow 1$, then from (27) $A \rightarrow \infty$, from (28)

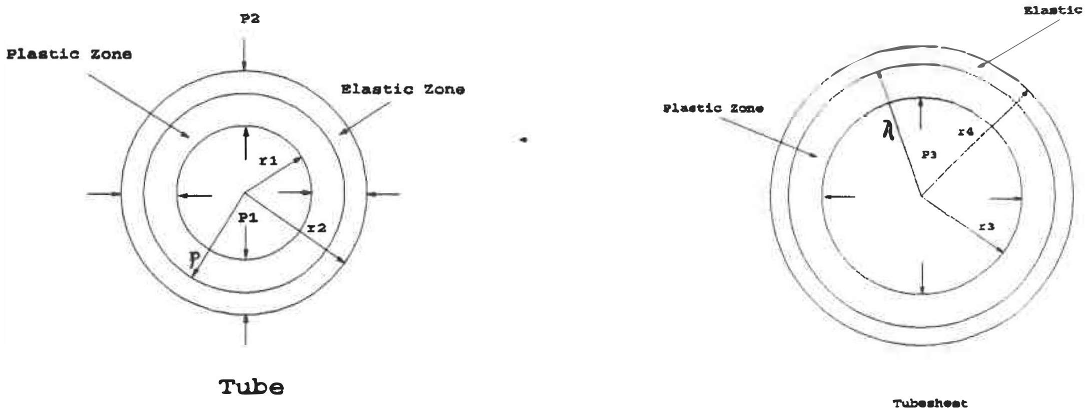
$$p_1 \rightarrow \sigma_{st}\ln(n)$$

Let the loading process to terminate at

$$p_1^t = \sigma_{st}\ln(n) \tag{15}$$

A.C.5 BOTH THE TUBE AND TUBESHEET ARE IN ELASTO-PLASTIC STATES

When $p_1 > p_1^{**}$, the tubesheet will deform elasto-plastically through a radius λ:



$$\frac{r_2}{r_1} = n \qquad \frac{r_4}{r_3} = m$$

$$\frac{r_2}{\rho} = n_\rho \qquad \frac{r_4}{\lambda} = m_\lambda$$

tube		tubesheet	
$r_1 \le r \le \rho$	plastic	$r_3 \le r \le \lambda$	plastic
$\rho \le r \le r_2$	elastic	$\lambda \le r \le r_4$	elastic

A.C.5.1. For the tubesheet, the solutions (8), (9) and (10) for the elastic region; (11), (12) and (14) for the plastic region are available with the replacement of

$$r_1 \Rightarrow r_3 \quad r_2 \Rightarrow r_4 \quad n \Rightarrow m \quad \rho \Rightarrow \lambda \quad n_\rho \Rightarrow m_\lambda$$

$$p_1 \Rightarrow p_3 \quad p_\rho \Rightarrow p_\lambda \quad E_t \Rightarrow E_s \quad \nu_t \Rightarrow \nu_s \quad \sigma_{st} \Rightarrow \sigma_{ss}$$

$\lambda \leq r \leq r_4$, elastic region

We have

$$\begin{cases} \sigma_r = -\dfrac{\sigma_{ss}}{2m_\lambda^2}\left(\dfrac{r_4^2}{r^2}-1\right) \\ \sigma_\theta = \dfrac{\sigma_{ss}}{2m_\lambda^2}\left(\dfrac{r_4^2}{r^2}+1\right) \end{cases} \tag{32}$$

$$u = \frac{1+\nu_s}{2E_s}\frac{\sigma_{ss}}{m_\lambda^2} r\left(\frac{r_4^2}{r^2} + \frac{1-\nu_s}{1+\nu_s}\right) \tag{33}$$

$$p_\lambda = \frac{m_\lambda^2-1}{2m_\lambda^2}\sigma_{ss} \tag{34}$$

$r_3 \leq r \leq \lambda$ plastic region

$$\begin{cases} \sigma_r = -p_3 + \sigma_{ss}\ln\dfrac{r}{r_3} \\ \sigma_\theta = -p_3 + \sigma_{ss}\left(1+\ln\dfrac{r}{r_3}\right) \end{cases} \tag{35}$$

$$p_3 = p_3(\lambda) = \sigma_{ss}\left[\ln\frac{\lambda}{r_3} + \frac{1}{2}\left(1-\frac{\lambda^2}{r_4^2}\right)\right] \tag{36}$$

$$u = \frac{\sigma_{ss}}{E_s}\left\{(1-\nu_s)\left[r\ln\frac{r}{\lambda} - \frac{1}{2}r\left(1 - \frac{1}{m_\lambda^2}\right)\right] + \frac{\lambda^2}{r}\right\} \tag{37}$$

A.C.5.2. For the tube, the solutions for the elastic region,

$$\rho \le r \le r_2$$

are the same as (21) and (23).

For the plastic region,

$$r_1 \le r \le \rho$$

are the same as (24), (25) and (26).

Let

$$\rho \Rightarrow r_2$$

$$\text{then} \qquad p_1 \Rightarrow p_1^t = \sigma_{st} \ln(n)$$

A.C.5.3. Contact conditions for the tube and tubesheet

$$p_2 = p_3, \qquad u\big|_{r=r_3} + c = u\big|_{r=r_2}$$

at $r=r_3$ $(r_3=r_2+c)$ from (37) and at $r=r_2$ from (23)

$$\begin{aligned} &\frac{\sigma_{ss}}{E_s}\left\{(1-\nu_s)\left[r_3 \ln\frac{r_3}{\lambda} - \frac{1}{2}r_3\left(1 - \frac{1}{m_\lambda^2}\right)\right] + \frac{\lambda^2}{r_3}\right\} + c \\ &= \frac{\sigma_{st}}{E_t}\frac{r_2}{n_\rho^2} + \frac{2\sigma_{ss}}{\sqrt{3}E_t}\frac{r^2}{n_\rho^2 - 1}\left[2 - n_\rho^2\left(\frac{1+\nu_t}{n^2} + 1 - \nu_t\right)\right] \\ &\left[\ln\frac{\lambda}{r_3} + \frac{1}{2}\left(1 - \frac{\lambda^2}{r_4^2}\right)\right] \end{aligned} \tag{38}$$

This equation gives the relationship between ρ and λ.

Let

$$\lambda \rightarrow r_4 \qquad m_\lambda \rightarrow 1$$

then from (38) ρ^P is obtained. From (36), $p_3^P(\rho = r_4) = \sigma_{ss}\ln(m)$ substituting ρ^P and p_3^P into (25), the plastic limit pressure of the tubesheet p_1^P is derived.

A.C.6 Residual contact pressure

When the inner pressure p_1 is reduced to zero after the expansion of the tube, there is residual contact pressure in the joint due to a permanent deformation. The unloading process is elastic. Therefore, an elastic stress distribution due to Δp_1 and Δp_2 could be superimposed on the tube and tubesheet system. $\Delta p_1 = -p_1$ will just reduce the pressure to zero at the inner surface $r = r_1$ of the tube. $\Delta p_2 = \Delta p_3$ is determined by $\Delta u_2 = \Delta u_3$ to keep the tube and tubesheet in contact.

In the tube $r_1 \leq r \leq r_2$, using (1) and (2) with

$$p_1 \Rightarrow \Delta p_1,\ p_2 \Rightarrow \Delta p_2$$
$$E \rightarrow E_t\ ,\quad \nu \rightarrow \nu_t$$

$$\begin{cases} \Delta\sigma_r = \dfrac{\dfrac{r_2^2}{r^2} - 1}{n^2-1}p_1 - \dfrac{1-\dfrac{r_1^2}{r^2}}{1-\dfrac{1}{n^2}}\Delta p_2 \\ \Delta\sigma_\theta = -\dfrac{\dfrac{r_2^2}{r^2} + 1}{n^2-1}p_1 - \dfrac{1+\dfrac{r_1^2}{r^2}}{1-\dfrac{1}{n^2}}\Delta p_2 \end{cases} \tag{39}$$

$$\Delta u = \frac{1+\nu_t}{E_t}\frac{r}{n^2-1}\left[-\left(\frac{r_2^2}{r^2}+\frac{1-\nu_t}{1+\nu_t}\right)p_1 - n^2\left(\frac{r_1^2}{r^2}+\frac{1-\nu_t}{1+\nu_t}\right)\Delta p_2\right] \tag{40}$$

In the tubesheet $r_3 \leq r \leq r_4$, using (3) with

$$p_1 \rightarrow \Delta p_2,\ r_2 \rightarrow r_4,\ n \rightarrow m,\ E \rightarrow E_s,\ \nu \rightarrow \nu_s,\ \Delta p_3 = \Delta p_2$$

$$\begin{cases} \Delta\sigma_r = -\dfrac{1}{m^2-1}\left(\dfrac{r_4^2}{r^2}-1\right)\Delta p_2 \\ \Delta\sigma_\theta = \dfrac{1}{m^2-1}\left(\dfrac{r_4^2}{r^2}+1\right)\Delta p_2 \end{cases} \tag{41}$$

$$\Delta u = \frac{1+\nu_s}{E_s}\frac{r}{m^2-1}\left(\frac{r_4^2}{r^2}+\frac{1-\nu_s}{1+\nu_s}\right)\Delta p_2 \tag{42}$$

From (40), $r=r_2$

$$\Delta u_2=\frac{1+\nu_t}{E_t}\frac{r_2}{n^2-1}\left[-\left(1+\frac{1-\nu_t}{1+\nu_t}\right)p_1 - \left(1+n^2\frac{1-\nu_t}{1+\nu_t}\right)\Delta p_2\right]$$

From (42) $r=r_3$

$$\Delta u_3=\frac{1+\nu_s}{E_s}\frac{r_3}{m^2-1}\left(m^2+\frac{1-\nu_s}{1+\nu_s}\right)\Delta p_2$$

let $\Delta u_2=\Delta u_3$

$$\Delta p_2=-\frac{\frac{1+\nu_t}{E_t}\frac{r_2}{n^2-1}\left(1+\frac{1-\nu_t}{1+\nu_t}\right)}{\frac{1+\nu_t}{E_t}\frac{r_2}{n^2-1}\left(1+n^2\frac{1-\nu_t}{1+\nu_t}\right)+\frac{1+\nu_s}{E_s}\frac{r_3}{m^2-1}\left(m^2+\frac{1-\nu_s}{1+\nu_s}\right)}p_1 \tag{43}$$

The residual stresses and permanent deformation after unloading are

$$\begin{cases}(\sigma_r)_o=\sigma_r+\Delta\sigma_r \\ (\sigma_\theta)_o=\sigma_\theta+\Delta\sigma_\theta\end{cases} \tag{44}$$

$$u_o=u+\Delta u \tag{45}$$

The residual contact pressure is

$$(p_2)_o=(p_3)_o=p_2+\Delta p_2 \tag{46}$$

The above solutions are only valid if the tube and tubesheet do not enter the reverse plastic yield during the unloading, i.e. there is no new plastic deformation which is also desirable.

$$\begin{aligned} \sigma_{st} &> [(\sigma_r)_o - (\sigma_\theta)_o] \quad \text{in tube} \\ \sigma_{ss} &> [(\sigma_r)_o - (\sigma_\theta)_o] \quad \text{in tubesheet} \end{aligned} \tag{47}$$

From (47), a valid of p_1^r is determined by (47),

Let

$$p_1 < p_1^r \tag{48}$$

to insure the elastic unloading (without reverse yielding).

A.C.7 DISCUSSION

The above solutions are derived upon

$$u^e = u|_{r=r_2} = \frac{\sigma_{st}}{E_t}\frac{r_2}{n^2} \le c \tag{7}$$

It is possible that when the initial gap c=0 or very small, at

$$p_1 = p_1^e = \frac{n^2-1}{2n^2}\sigma_{st} \tag{6}$$

$$u^e = u|_{r=r_2} = \frac{\sigma_{st}}{E_t}\frac{r_2}{n_2} > c \tag{49}$$

It means that the tube contacted to the tubesheet before yielding. Let p_1^c, the inner pressure of the tube, at which the tube and tubesheet contact to each other, from (3)

$$u\big|_{r=r_2} = \frac{1+\nu_t}{E_t}\frac{r_2}{n^2-1}\left(\frac{r_2^2}{r_2^2} + \frac{1-\nu_t}{1+\nu_t}\right)p_1^c = c$$

$$\Rightarrow p_1^c = \frac{E_t}{2r_2}(n^2-1)c \tag{50}$$

The solution could be obtained step by step in the same way as the previous one.

APPENDIX D

COMPUTER OUTPUT FOR STATISTICAL ANALYSIS

```
1                             THE SAS SYSTEM                                 1
                                            10:07 WEDNESDAY, OCTOBER 23, 1991
 MODEL: MODEL1
 DEPENDENT VARIABLE: LRCPR

                           ANALYSIS OF VARIANCE

                              SUM OF         MEAN
        SOURCE        DF     SQUARES       SQUARE      F VALUE      PROB>F

        MODEL          7    43.80254      6.25751       22.173      0.0001
        ERROR         31     8.74874      0.28222
        C TOTAL       38    52.55128

            ROOT MSE       0.53124     R-SQUARE       0.8335
            DEP MEAN      -3.46254     ADJ R-SQ       0.7959
            C.V.         -15.34252

                           PARAMETER ESTIMATES

                     PARAMETER        STANDARD    T FOR H0:
      VARIABLE  DF    ESTIMATE           ERROR   PARAMETER=0     PROB > |T|

      INTERCEP   1    4.408686      6.04003739         0.730         0.4709
      LTWT       1   -0.494767      0.30809805        -1.606         0.1184
      LPIT       1   -1.012032      0.30391594        -3.330         0.0023
      LCLEA      1    0.087917      0.04610431         1.907         0.0658
      LEXPR      1    7.776771      1.13539899         6.849         0.0001
      LRYMT      1   -0.912606      0.83885485        -1.088         0.2850
      LRYM       1   -4.204410      0.97117394        -4.329         0.0001
      LRYS       1    0.855029      0.61992148         1.379         0.1777

1                             THE SAS SYSTEM                                 2
                                            10:07 WEDNESDAY, OCTOBER 23, 1991

                        DEP VAR   PREDICT
                  OBS     LRCPR     VALUE  RESIDUAL

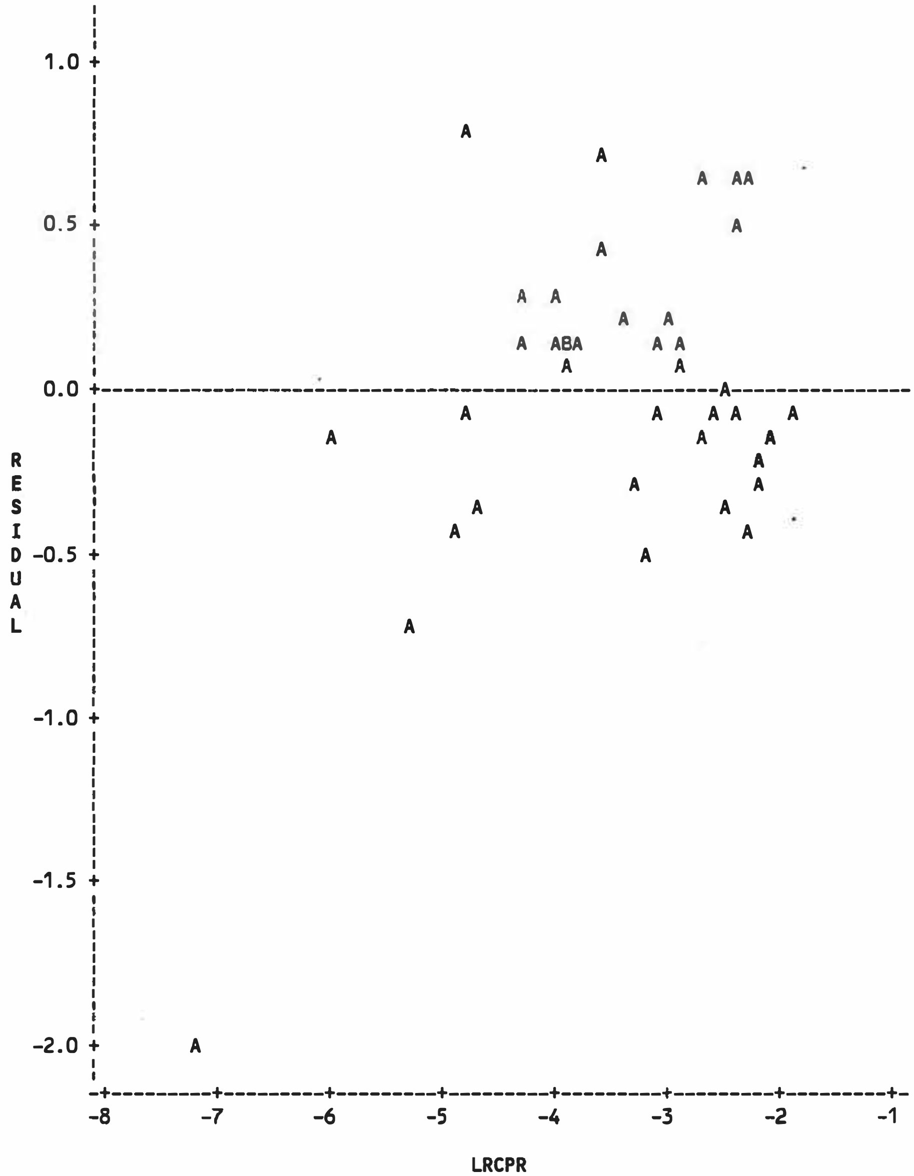
                    1   -3.8663   -3.9122    0.0459
                    2   -3.4148   -3.5988    0.1840
                    3   -2.8705   -2.9175    0.0471
                    4   -2.9308   -3.0969    0.1661
                    5   -2.7434   -2.5957   -0.1477
                    6   -7.1870   -5.1952   -1.9918
                    7   -2.4531   -2.4216   -0.0315
                    8   -4.7603   -5.5360    0.7757
                    9   -3.5760   -3.9870    0.4110
                   10   -3.8468   -3.9645    0.1176
                   11   -4.2591   -4.5735    0.3143
                   12   -4.0129   -4.1899    0.1770
                   13   -3.8959   -4.0716    0.1757
                   14   -4.3360   -4.4863    0.1503
                   15   -3.9961   -4.2935    0.2974
                   16   -4.6746   -4.3123   -0.3623
                   17   -5.3418   -4.6002   -0.7415
                   18   -4.8080   -4.7194   -0.0886
                   19   -2.4905   -2.1638   -0.3267
                   20   -2.5592   -2.4725   -0.0867
                   21   -2.3768   -2.3049   -0.0719
                   22   -2.1772   -1.9706   -0.2066
                   23   -2.1130   -1.9824   -0.1306
```

24	-2.7356	-3.3503	0.6147
25	-1.8855	-1.8084	-0.0771
26	-2.4156	-3.0388	0.6232
27	-2.3954	-2.8939	0.4985
28	-3.1272	-3.2570	0.1297
29	-2.1818	-1.9042	-0.2777
30	-2.2696	-1.8359	-0.4337
31	-3.9154	-4.0565	0.1411
32	-3.3497	-3.0652	-0.2845
33	-3.0674	-2.9792	-0.0882
34	-2.2985	-2.9277	0.6292
35	-4.8562	-4.4206	-0.4356
36	-3.6156	-4.2955	0.6799
37	-2.9612	-3.1899	0.2287
38	-3.2280	-2.7401	-0.4878
39	-6.0464	-5.9097	-0.1367

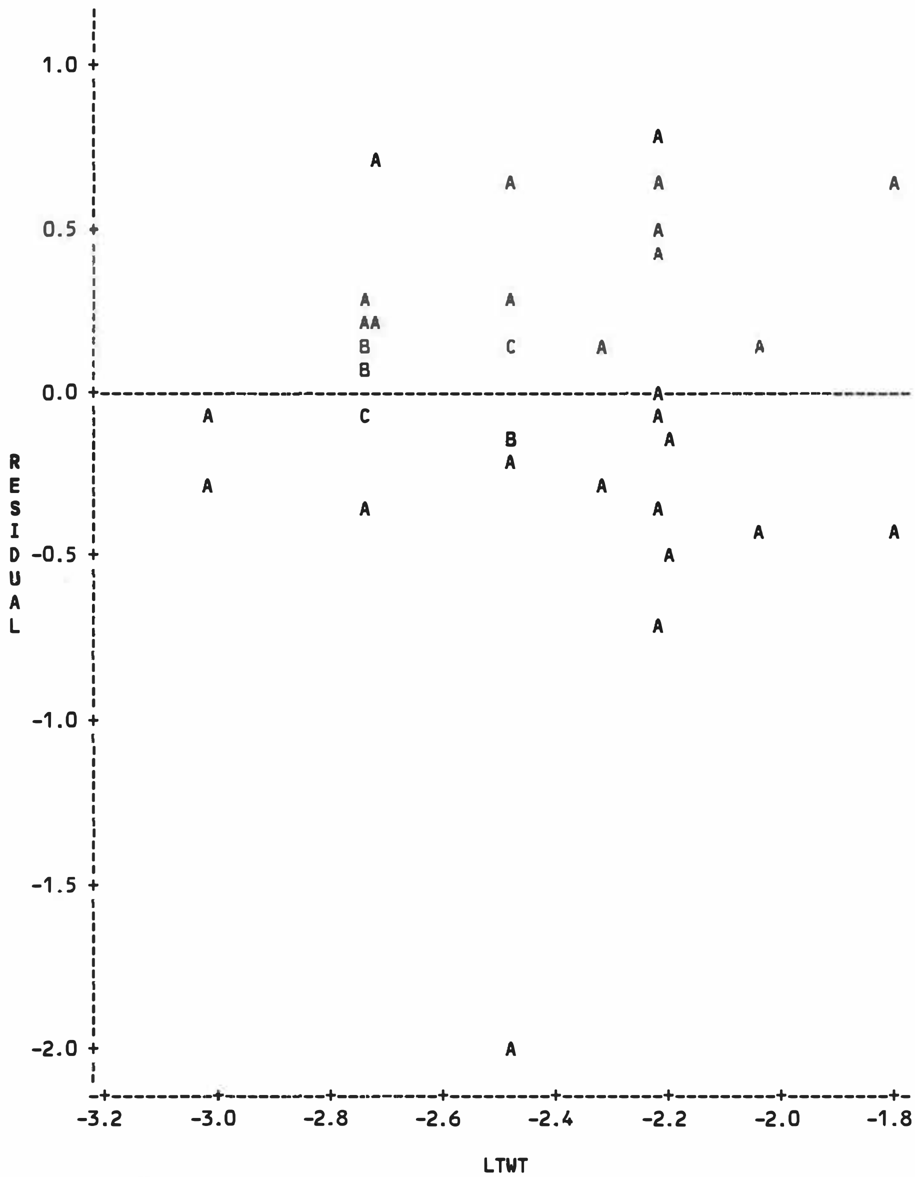
SUM OF RESIDUALS	0
SUM OF SQUARED RESIDUALS	8.7487
PREDICTED RESID SS (PRESS)	15.0011

1 THE SAS SYSTEM 23:58 SUNDAY, JANUARY 19, 1992
4

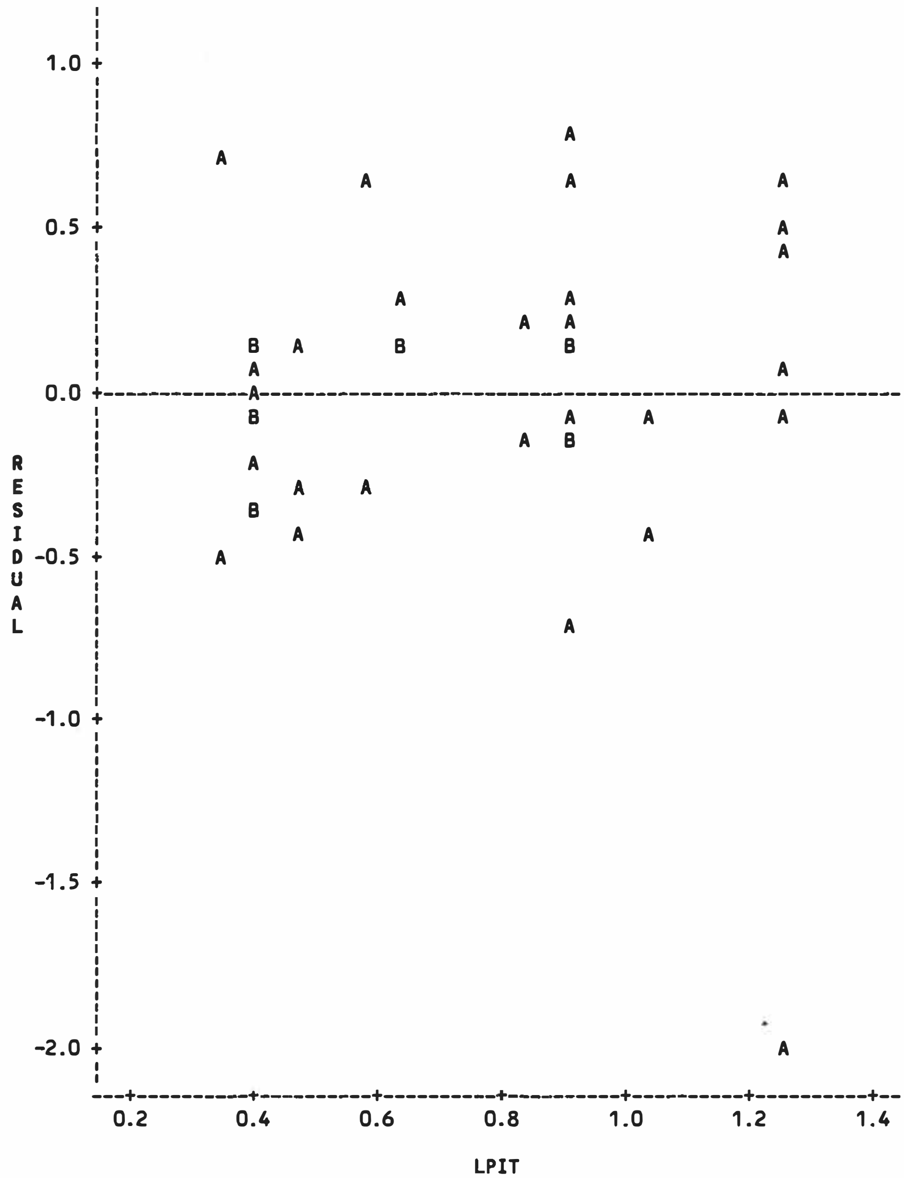
PLOT OF RESID1*LRCPR. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



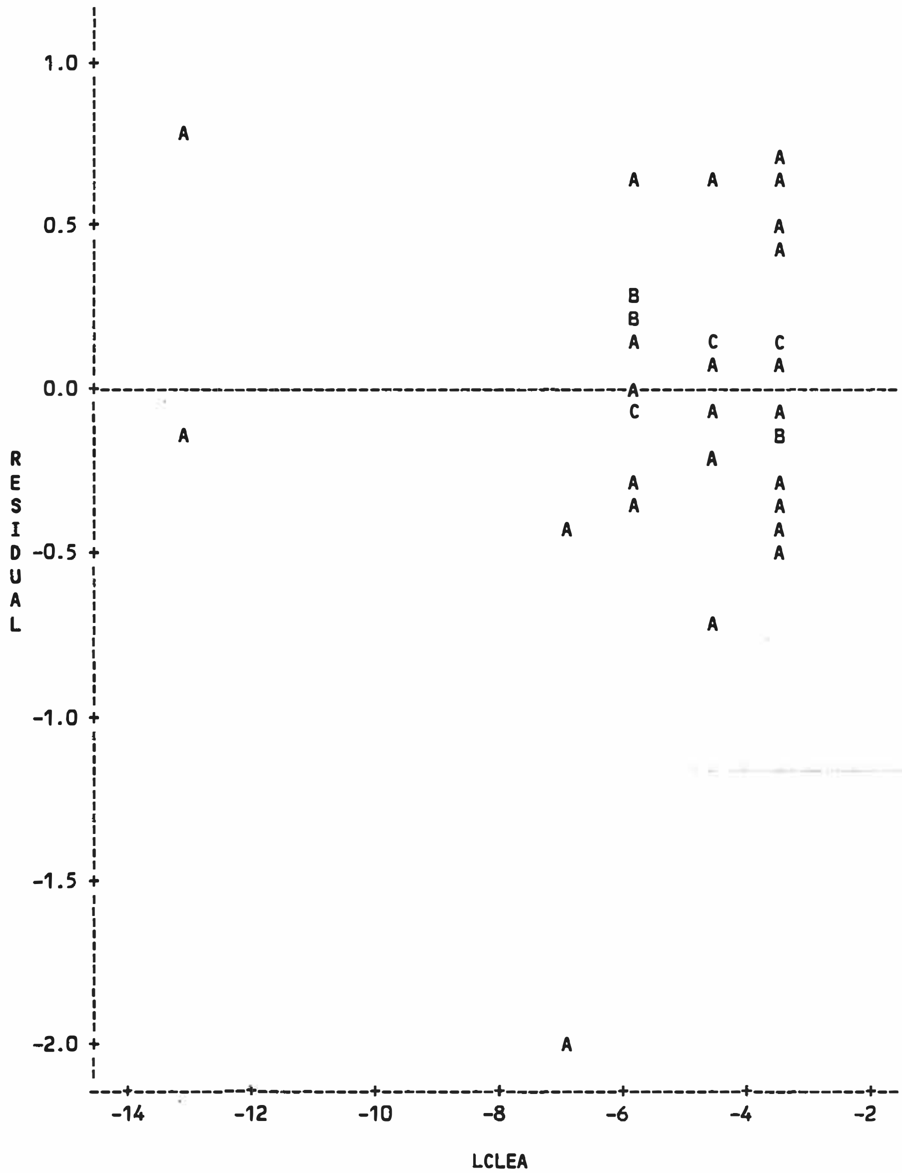
PLOT OF RESID1*LTWT. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



PLOT OF RESID1*LPIT. LEGEND: A = 1 OBS, B = 2 OBS, ETC.

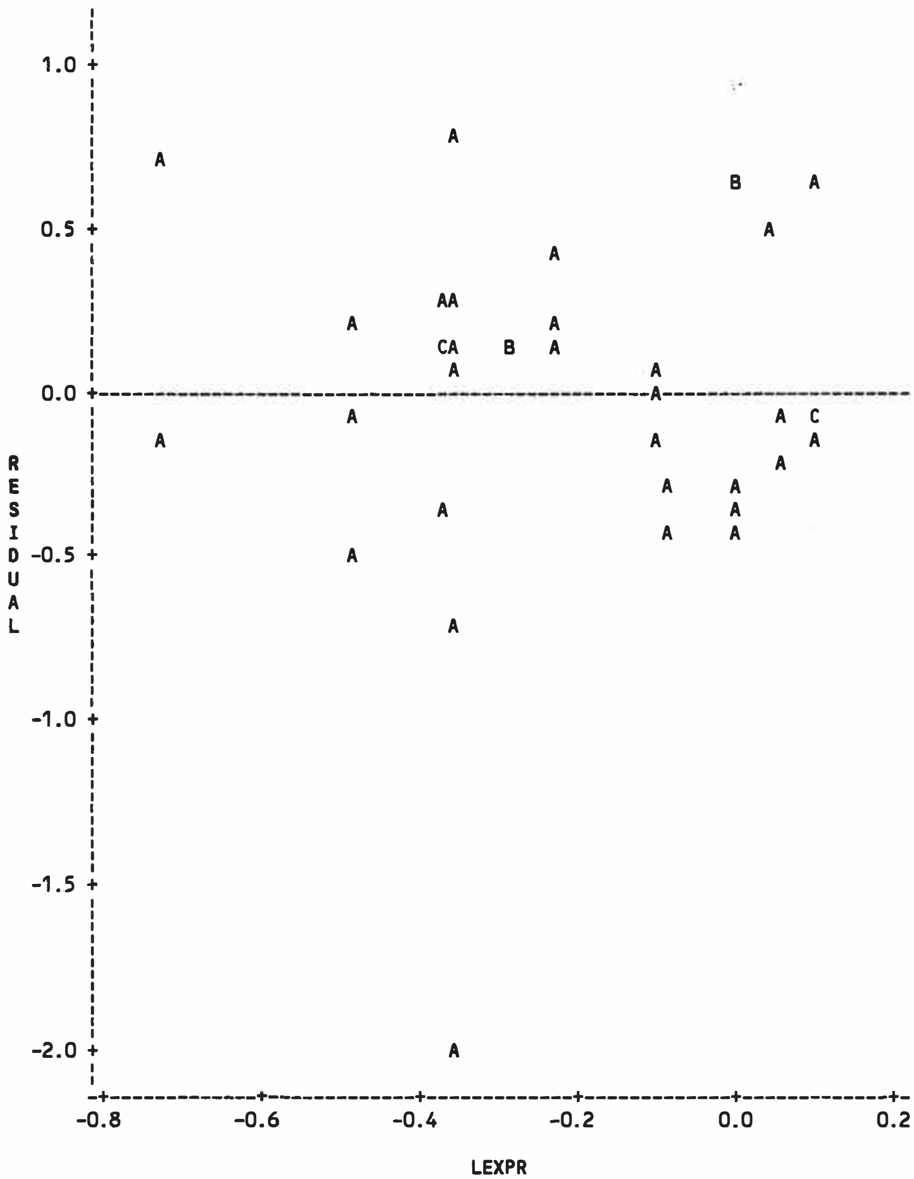


PLOT OF RESID1*LCLEA. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



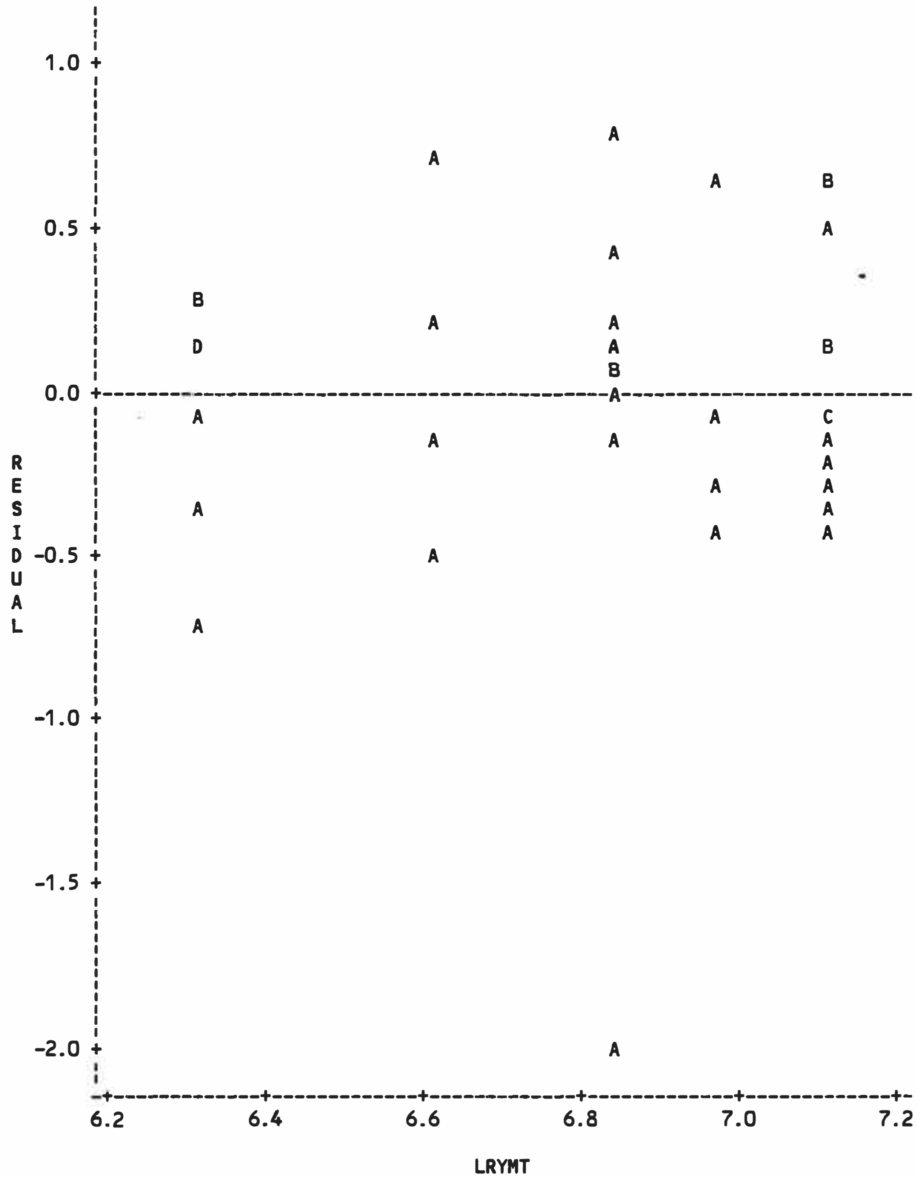
1 THE SAS SYSTEM 23:58 SUNDAY, JANUARY 19, 1992 8

PLOT OF RESID1*LEXPR. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



1 THE SAS SYSTEM 23:58 SUNDAY, JANUARY 19, 1992
9

PLOT OF RESID1*LRYMT. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



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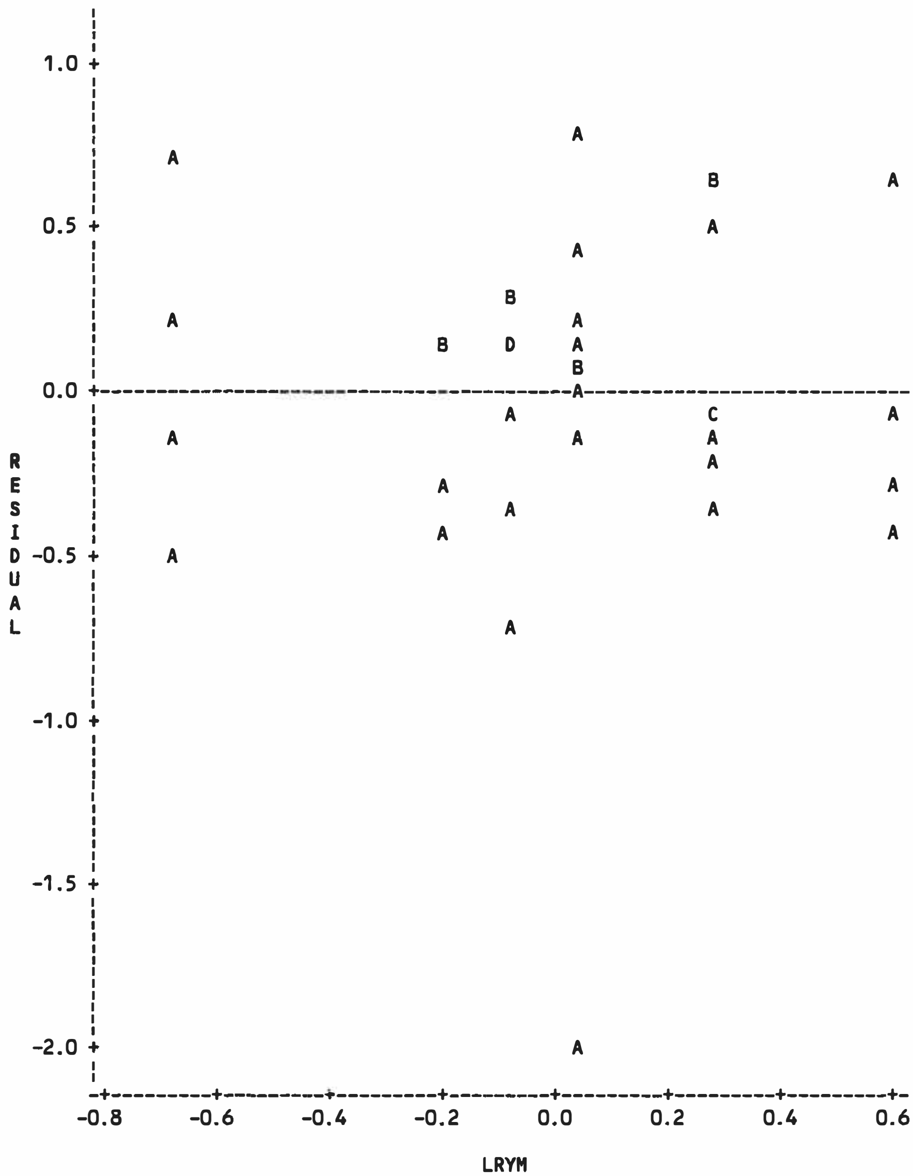
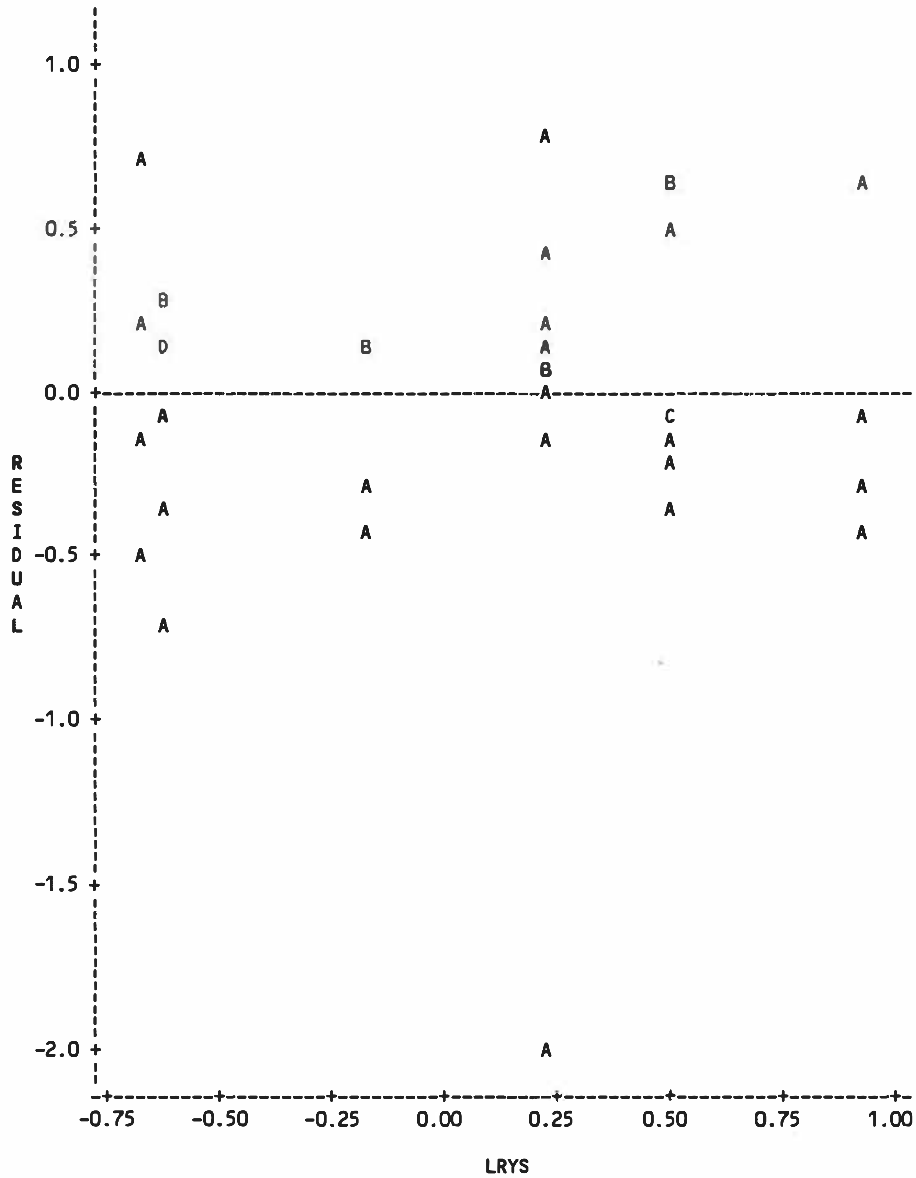
1                                    THE SAS SYSTEM   23:58 SUNDAY, JANUARY 19, 1992
                                                                                  10
          PLOT OF RESID1*LRYM.  LEGEND: A = 1 OBS, B = 2 OBS, ETC.
```

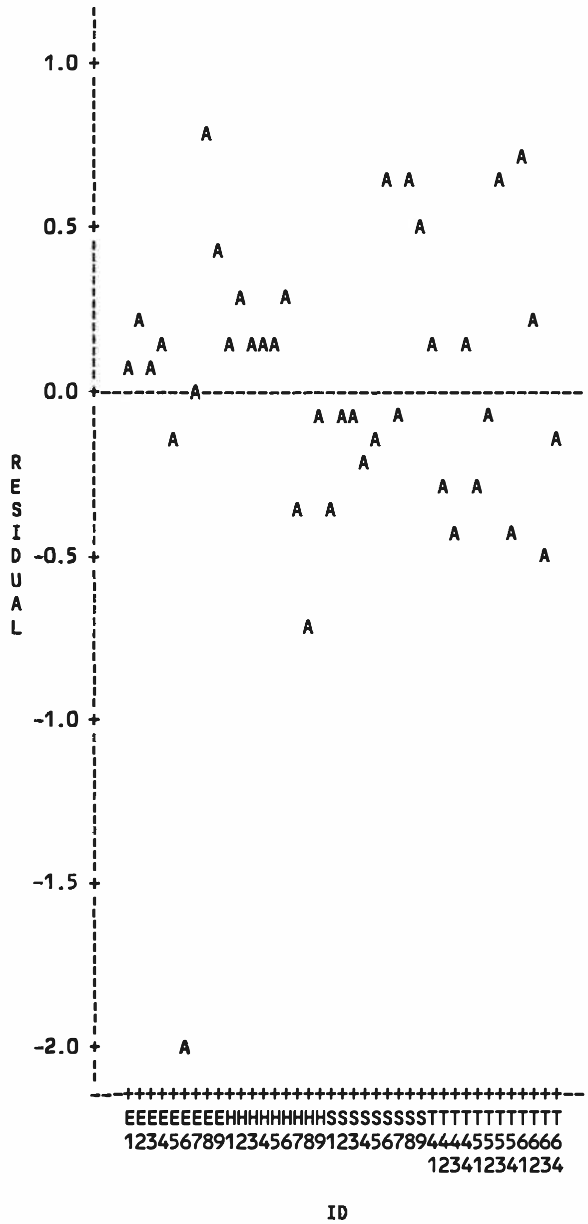


PLOT OF RESID1*LRYS. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



1 THE SAS SYSTEM 23:58 SUNDAY, JANUARY 19, 1992
12

PLOT OF RESID1*ID. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



MODEL: MODEL1
DEPENDENT VARIABLE: LTWR

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	7	76.78300	10.96900	179.389	0.0001
ERROR	31	1.89554	0.06115		
C TOTAL	38	78.67854			

ROOT MSE	0.24728	R-SQUARE	0.9759
DEP MEAN	0.24339	ADJ R-SQ	0.9705
C.V.	101.59798		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	7.563524	2.82329611	2.679	0.0117
LTWT	1	0.136008	0.14644238	0.929	0.3602
LPIT	1	-0.077509	0.14119775	-0.549	0.5870
LCLEA	1	0.717417	0.02478590	28.945	0.0001
LEXPR	1	0.698040	0.53866588	1.296	0.2046
LRYMT	1	-0.458835	0.39336844	-1.166	0.2523
LRYM	1	-0.564053	0.45395044	-1.243	0.2234
LRYS	1	0.088826	0.28630695	0.310	0.7585

1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992
38

OBS	DEP VAR LTWR	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	LOWER95% PREDICT	UPPER95% PREDICT
1	1.4183	1.3055	0.121	1.0588	1.5521	0.7440	1.8669
2	-0.6636	-0.3381	0.069	-0.4795	-0.1966	-0.8618	0.1857
3	0.3577	0.5810	0.105	0.3678	0.7942	0.0335	1.1285
4	0.4121	0.5977	0.078	0.4384	0.7570	0.0688	1.1266
5	1.5728	1.4745	0.088	1.2941	1.6549	0.9389	2.0101
6	-1.7430	-1.2099	0.109	-1.4321	-0.9877	-1.7610	-0.6588
7	-0.3975	-0.1459	0.110	-0.3697	0.0778	-0.6977	0.4058
8	-4.2405	-4.8206	0.151	-5.1292	-4.5120	-5.4119	-4.2294
9	1.6974	1.4033	0.125	1.1485	1.6581	0.8382	1.9684
10	1.6273	1.4630	0.100	1.2592	1.6668	0.9191	2.0070
11	0.0198	-0.2191	0.097	-0.4164	-0.0218	-0.7606	0.3225
12	0.6627	0.6177	0.095	0.4234	0.8121	0.0773	1.1582
13	0.6366	0.6944	0.090	0.5106	0.8781	0.1576	1.2311
14	1.6194	1.5113	0.087	1.3329	1.6896	0.9763	2.0462
15	-0.2408	-0.2009	0.095	-0.3943	-0.00744	-0.7410	0.3393
16	-0.4370	-0.1315	0.103	-0.3406	0.0776	-0.6774	0.4145
17	0.5247	0.7252	0.095	0.5321	0.9183	0.1852	1.2652
18	1.4770	1.4172	0.111	1.1899	1.6444	0.8640	1.9703
19	1.4516	1.3190	0.096	1.1224	1.5157	0.7777	1.8604
20	-0.4780	-0.3803	0.090	-0.5631	-0.1975	-0.9167	0.1561
21	0.3646	0.4926	0.103	0.2817	0.7035	-0.0540	1.0393
22	0.4886	0.5554	0.088	0.3768	0.7341	0.0204	1.0905
23	1.6194	1.3862	0.078	1.2281	1.5443	0.8577	1.9148
24	-0.5960	-0.4105	0.087	-0.5882	-0.2329	-0.9452	0.1242
25	0.5766	0.6287	0.097	0.4313	0.8261	0.0871	1.1703
26	-0.3538	-0.3474	0.069	-0.4881	-0.2067	-0.8710	0.1762
27	1.8764	1.3581	0.107	1.1405	1.5756	0.8088	1.9073
28	0.4511	0.5423	0.128	0.2811	0.8035	-0.0256	1.1102

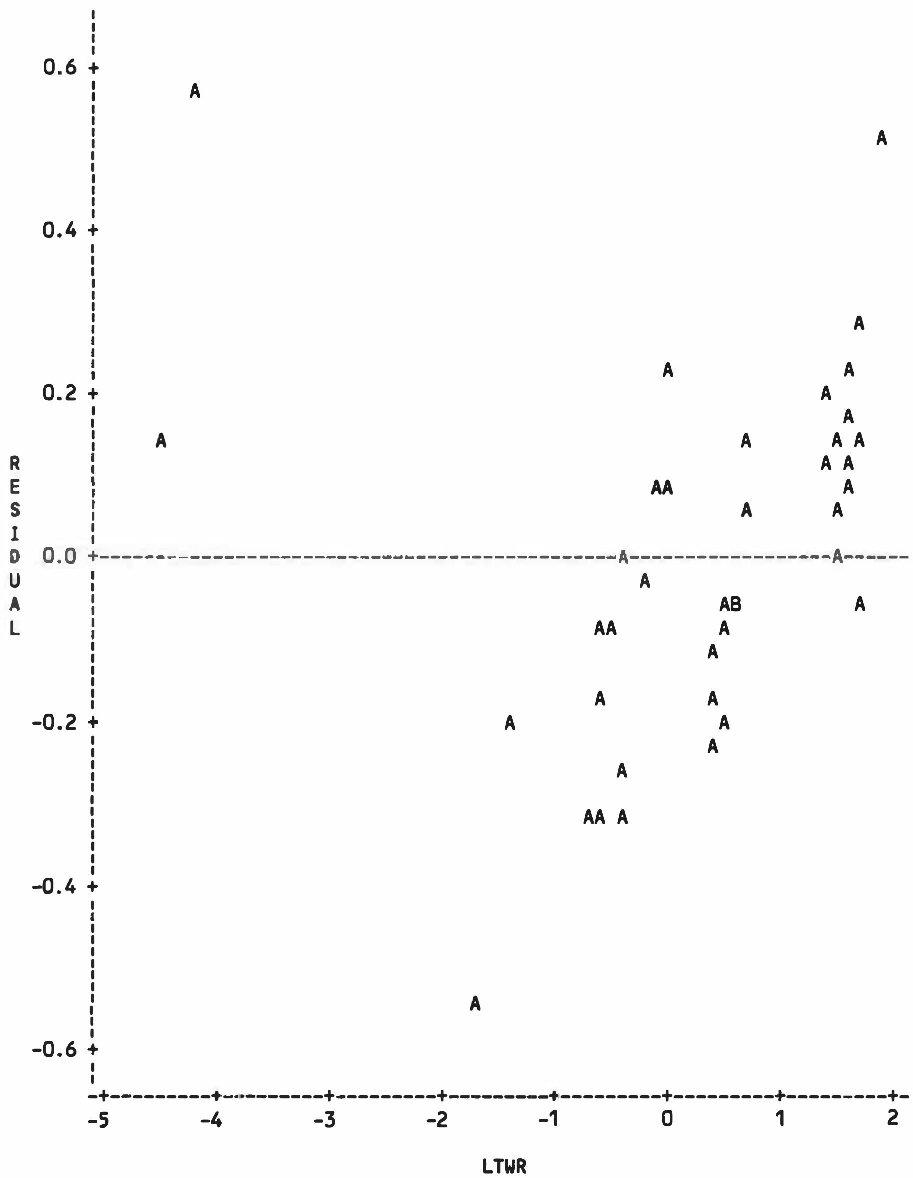
29	-0.1020	-0.1805	0.122	-0.4296	0.0685	-0.7430	0.3819
30	1.7011	1.5550	0.118	1.3137	1.7964	0.9959	2.1142
31	-0.6255	-0.3168	0.136	-0.5945	-0.0391	-0.8925	0.2590
32	1.3762	1.1865	0.132	0.9181	1.4549	0.6152	1.7578
33	-0.5656	-0.4783	0.121	-0.7251	-0.2316	-1.0398	0.0831
34	0.7178	0.5848	0.143	0.2938	0.8757	0.00253	1.1670
35	-1.3587	-1.1647	0.140	-1.4495	-0.8799	-1.7439	-0.5855
36	1.4839	1.4747	0.148	1.1730	1.7764	0.8870	2.0624
37	-0.0111	-0.0979	0.134	-0.3709	0.1750	-0.6714	0.4755
38	1.6639	1.7094	0.143	1.4185	2.0002	1.1272	2.2915
39	-4.4918	-4.6489	0.164	-4.9827	-4.3151	-5.2537	-4.0441

OBS	RESIDUAL
1	0.1128
2	-0.3255
3	-0.2233
4	-0.1856
5	0.0982
6	-0.5331
7	-0.2516
8	0.5801
9	0.2941
10	0.1643
11	0.2389
12	0.0450
13	-0.0578
14	0.1081
15	-0.0399
16	-0.3055
17	-0.2005
18	0.0599
19	0.1326
20	-0.0977
21	-0.1280
22	-0.0669
23	0.2332
24	-0.1855
25	-0.0521
26	-0.00645
27	0.5183
28	-0.0912
29	0.0785
30	0.1461
31	-0.3087
32	0.1897
33	-0.0873
34	0.1331
35	-0.1940
36	0.00919
37	0.0869
38	-0.0454
39	0.1571

SUM OF RESIDUALS	0
SUM OF SQUARED RESIDUALS	1.8955
PREDICTED RESID SS (PRESS)	3.3160

1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992
40

PLOT OF RESID4*LTWR. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



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1                                  THE SAS SYSTEM   00:03 MONDAY, JANUARY 20, 1992

           PLOT OF RESID4*LTWT.  LEGEND: A = 1 OBS, B = 2 OBS, ETC.
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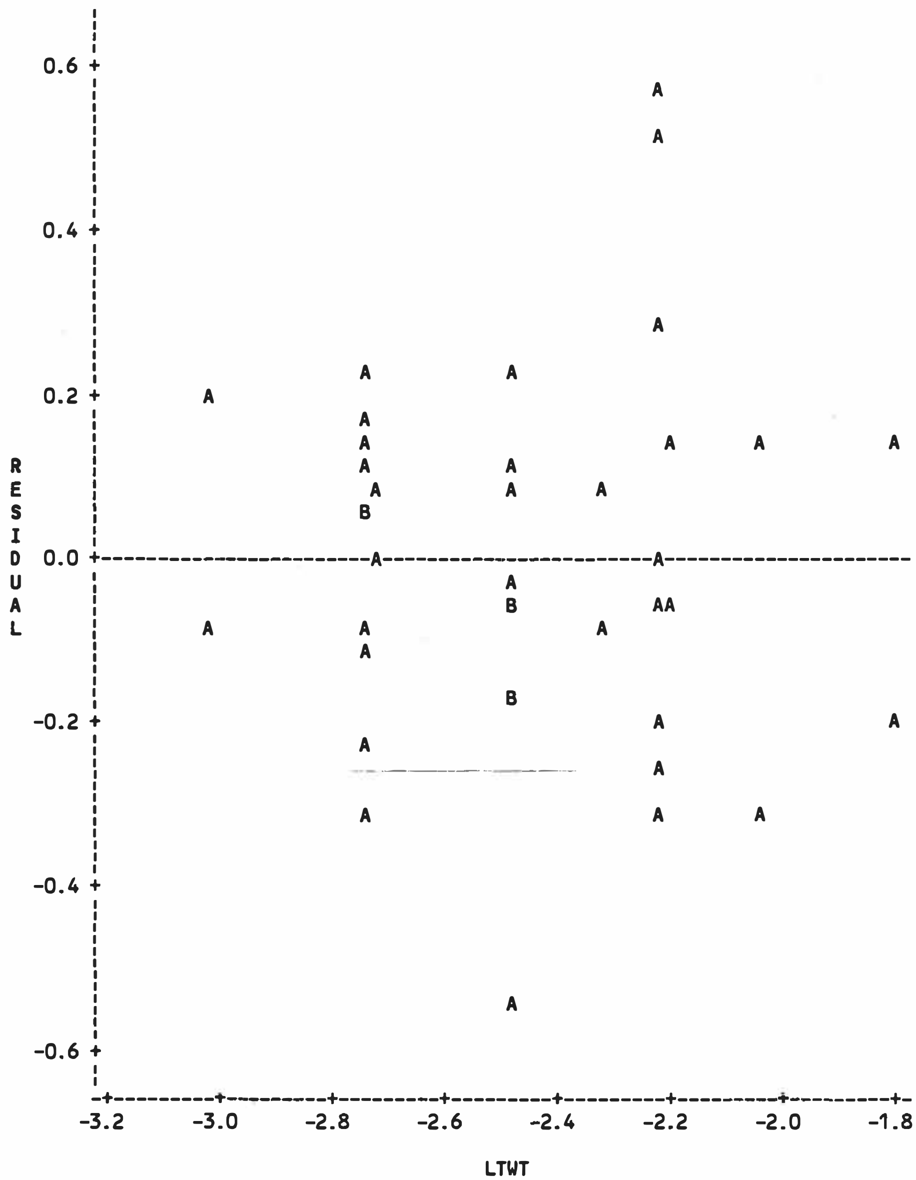
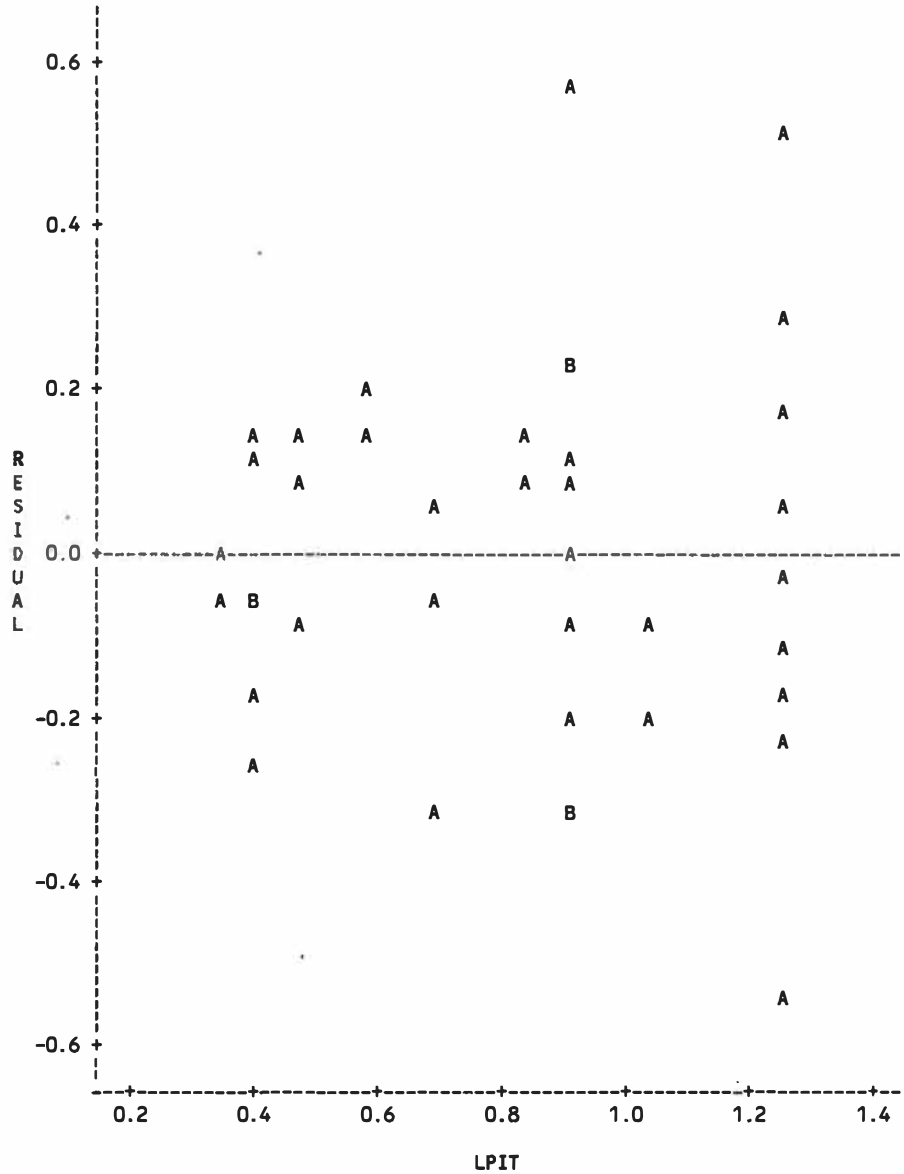


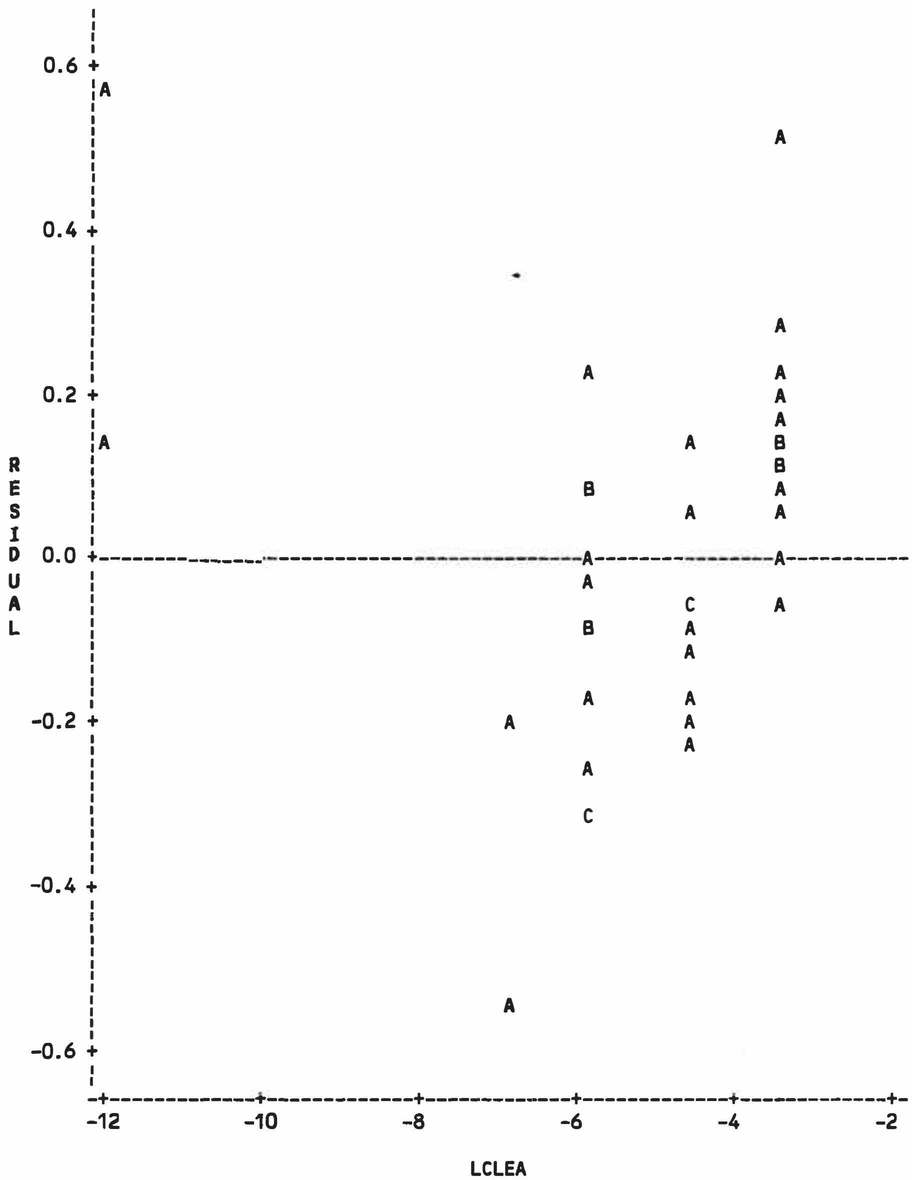
1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992

PLOT OF RESID4*LPIT. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



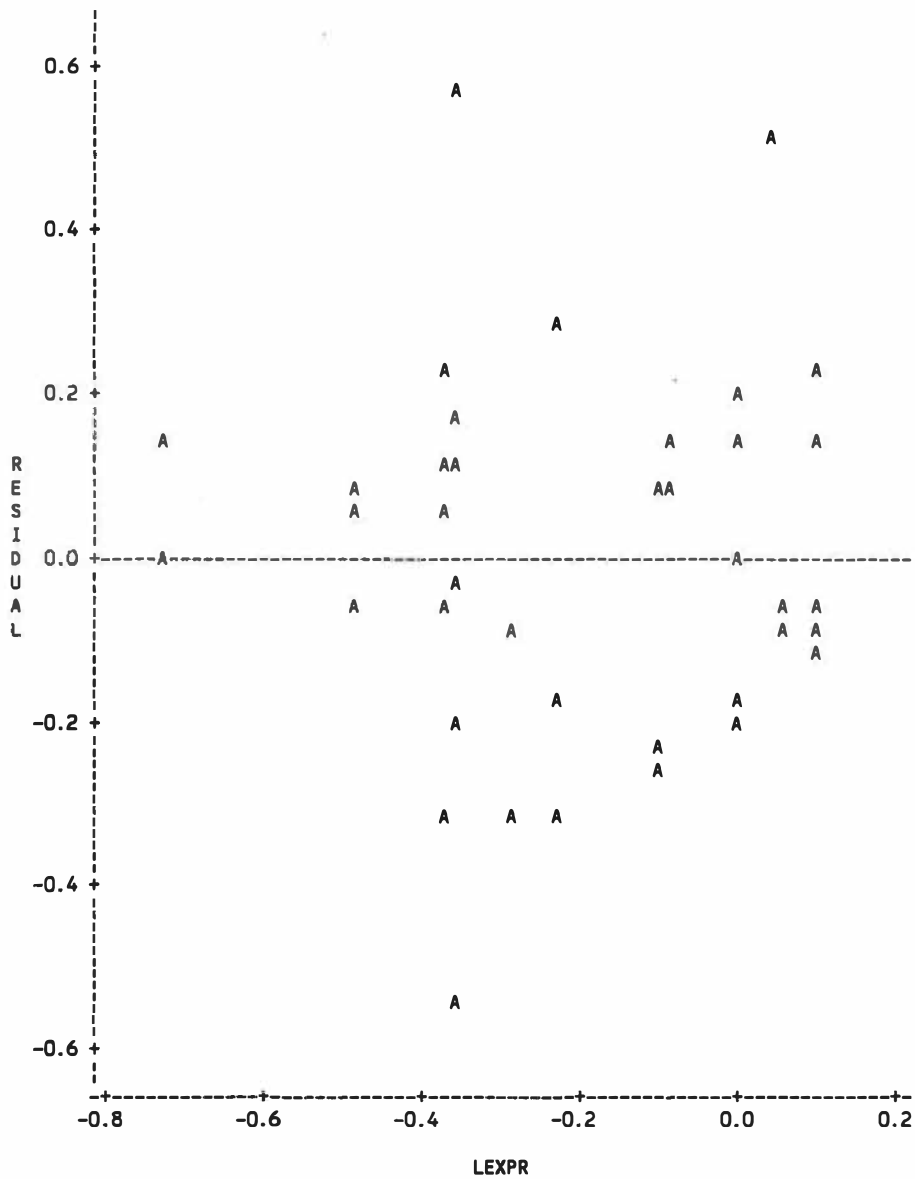
1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992

PLOT OF RESID4*LCLEA. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



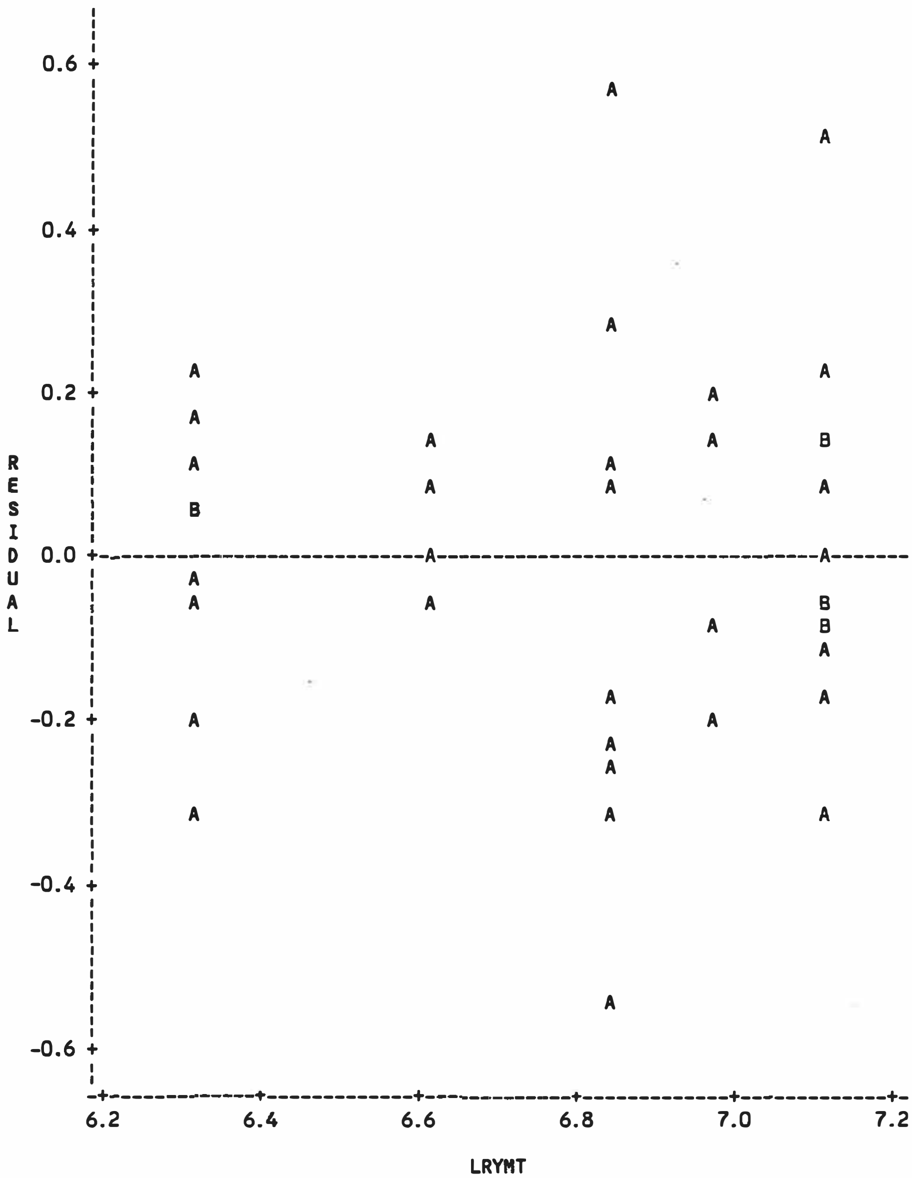
1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992

PLOT OF RESID4*LEXPR. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



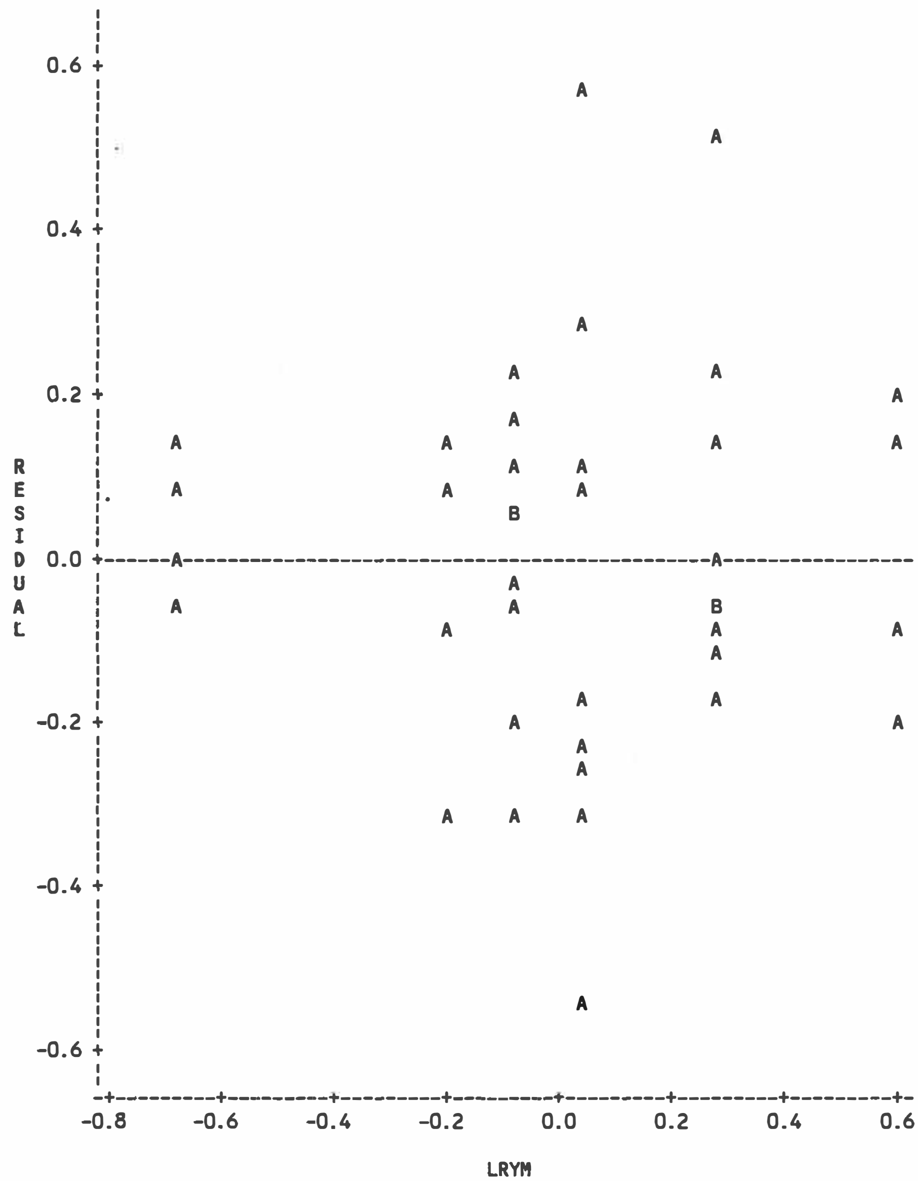
1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992

PLOT OF RESID4*LRYMT. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



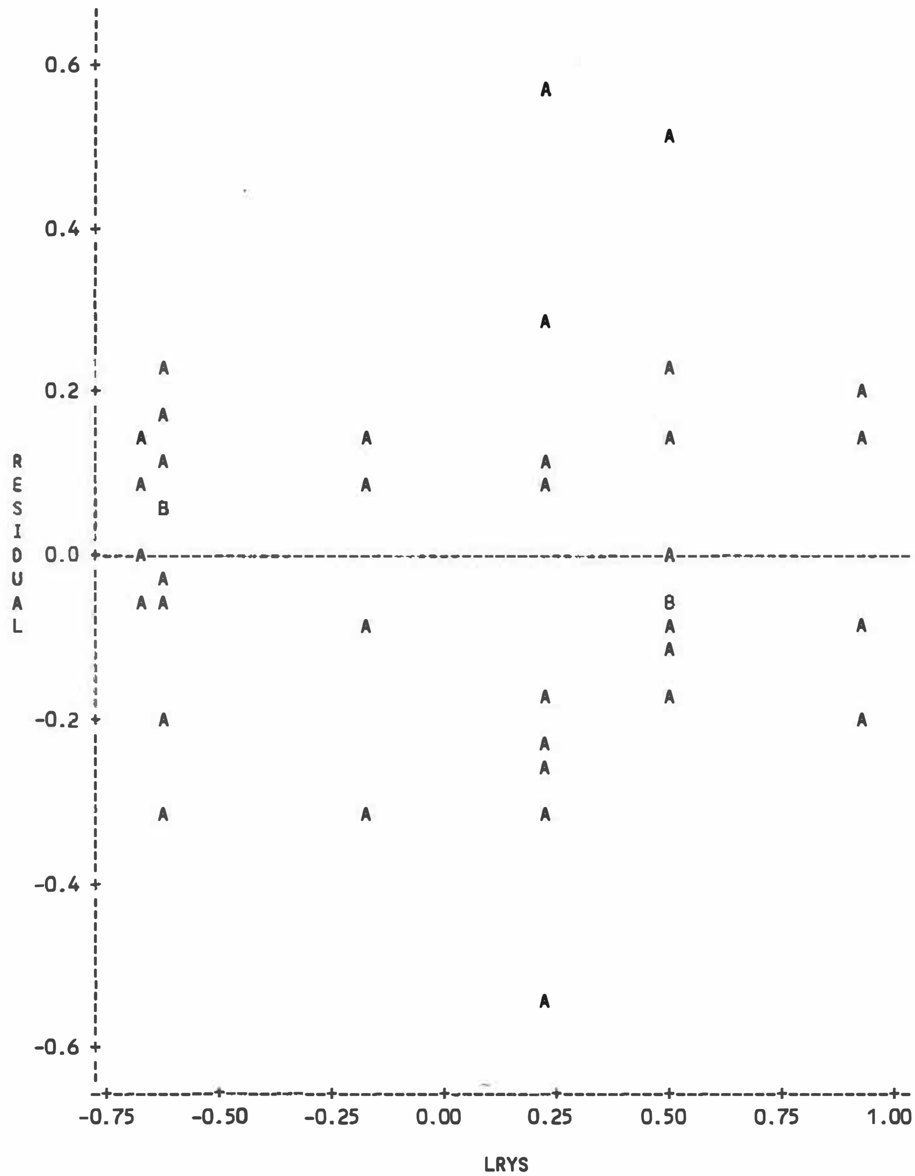
1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992
46

PLOT OF RESID4*LRYM. LEGEND: A = 1 OBS, B = 2 OBS, ETC.

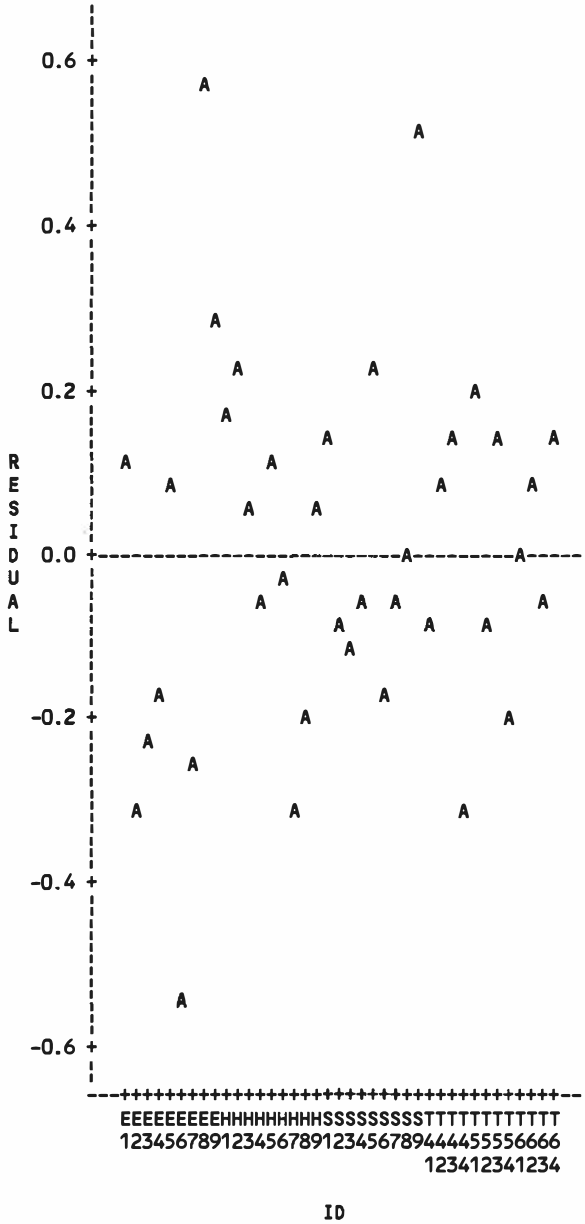


1 THE SAS SYSTEM 00:03 MONDAY, JANUARY 20, 1992
47

PLOT OF RESID4*LRYS. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



PLOT OF RESID4*ID. LEGEND: A = 1 OBS, B = 2 OBS, ETC.



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1                          THE SAS SYSTEM  13:35 SATURDAY, JANUARY 4, 1992
            FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRCPR
STEP 1   VARIABLE LEXPR ENTERED     R-SQUARE = 0.51122222   C(P) =102.10594164
                DF        SUM OF SQUARES      MEAN SQUARE          F    PROB>F

 REGRESSION      1         17.33693578      17.33693578      38.70    0.0001
 ERROR          37         16.57578362       0.44799415
 TOTAL          38         33.91271940
             PARAMETER        STANDARD          TYPE II
 VARIABLE     ESTIMATE           ERROR   SUM OF SQUARES          F    PROB>F

 INTERCEP   -2.48242472     0.14437551     132.44557150     295.64    0.0001
 LEXPR       2.93767477     0.47223015      17.33693578      38.70    0.0001

BOUNDS ON CONDITION NUMBER:           1,            1
------------------------------------------------------------------------------
STEP 2   VARIABLE LRYM ENTERED      R-SQUARE = 0.76183087   C(P) = 33.80828114
                DF        SUM OF SQUARES      MEAN SQUARE          F    PROB>F

 REGRESSION      2         25.83575644      12.91787822      57.58    0.0001
 ERROR          36          8.07696296       0.22436008
 TOTAL          38         33.91271940
             PARAMETER        STANDARD          TYPE II
 VARIABLE     ESTIMATE           ERROR   SUM OF SQUARES          F    PROB>F

 INTERCEP   -1.75577712     0.15613497      28.37160912     126.46    0.0001
 LEXPR       6.13741761     0.61803182      22.12561103      98.62    0.0001
 LRYM       -2.66158341     0.43244765       8.49882066      37.88    0.0001
BOUNDS ON CONDITION NUMBER:     3.420117,      13.68047
------------------------------------------------------------------------------
STEP 3   VARIABLE LPIT ENTERED      R-SQUARE = 0.82913414   C(P) = 16.92919254
                DF        SUM OF SQUARES      MEAN SQUARE          F    PROB>F

 REGRESSION      3         28.11819353       9.37273118      56.61    0.0001
 ERROR          35          5.79452586       0.16555788
 TOTAL          38         33.91271940
             PARAMETER        STANDARD          TYPE II
 VARIABLE     ESTIMATE           ERROR   SUM OF SQUARES          F    PROB>F

 INTERCEP   -1.18903617     0.20319200       5.66926558      34.24    0.0001
 LPIT       -0.79624805     0.21444894       2.28243710      13.79    0.0007
 LEXPR       5.77031648     0.54002801      18.90238261     114.17    0.0001
 LRYM       -2.28640244     0.38497754       5.83961780      35.27    0.0001

BOUNDS ON CONDITION NUMBER:     3.673165,      24.87626
------------------------------------------------------------------------------
STEP 4   VARIABLE LCLEA ENTERED     R-SQUARE = 0.87231265   C(P) =   6.81728912

                DF        SUM OF SQUARES      MEAN SQUARE          F    PROB>F

 REGRESSION      4         29.58249397       7.39562349      58.07    0.0001
 ERROR          34          4.33022542       0.12735957
 TOTAL          38         33.91271940

             PARAMETER        STANDARD          TYPE II
 VARIABLE     ESTIMATE           ERROR   SUM OF SQUARES          F    PROB>F

 INTERCEP   -0.81564124     0.20949375       1.93057994      15.16    0.0004
 LPIT       -0.68307881     0.19102781       1.62847094      12.79    0.0011
 LCLEA       0.10524436     0.03103841       1.46430044      11.50    0.0018
 LEXPR       5.44388255     0.48333430      16.15674401     126.86    0.0001
 LRYM       -2.20075151     0.33860091       5.38018681      42.24    0.0001

BOUNDS ON CONDITION NUMBER:     3.693722,      38.40619
------------------------------------------------------------------------------
STEP 5   VARIABLE LRYS ENTERED      R-SQUARE = 0.88860148   C(P) =   4.24814578
```

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	30.13489258	6.02697852	52.65	0.0001
ERROR	33	3.77782682	0.11447960		
TOTAL	38	33.91271940			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	-0.91013384	0.20322319	2.29610682	20.06	0.0001
LPIT	-0.56923093	0.18838078	1.04527817	9.13	0.0048
LCLEA	0.13162034	0.03178257	1.96334330	17.15	0.0002
LEXPR	4.82078481	0.53893240	9.15999007	80.01	0.0001
LRYM	-2.60559396	0.37016500	5.67219131	49.55	0.0001
LRYS	0.51032882	0.23232067	0.55239861	4.83	0.0352

BOUNDS ON CONDITION NUMBER: 5.507512, 90.07139

NO OTHER VARIABLE MET THE 0.5000 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRCPR

STEP	VARIABLE ENTERED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)	F	PROB>F
1	LEXPR	1	0.5112	0.5112	102.1059	38.6990	0.0001
2	LRYM	2	0.2506	0.7618	33.8083	37.8803	0.0001
3	LPIT	3	0.0673	0.8291	16.9292	13.7863	0.0007
4	LCLEA	4	0.0432	0.8723	6.8173	11.4974	0.0018
5	LRYS	5	0.0163	0.8886	4.2481	4.8253	0.0352

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FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRSZ

STEP 1 VARIABLE LCLEA ENTERED R-SQUARE = 0.82300806 C(P) = 10.11209393

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	40.38704066	40.38704066	172.05	0.0001
ERROR	37	8.68543213	0.23474141		
TOTAL	38	49.07247279			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	2.41949478	0.21780421	28.96720188	123.40	0.0001
LCLEA	0.52492536	0.04001948	40.38704066	172.05	0.0001

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE LEXPR ENTERED R-SQUARE = 0.86766502 C(P) = 0.72983095

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	42.57846793	21.28923396	118.02	0.0001
ERROR	36	6.49400486	0.18038902		
TOTAL	38	49.07247279			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	2.48906989	0.19197165	30.32564679	168.11	0.0001
LCLEA	0.49532131	0.03609533	33.96895476	188.31	0.0001
LEXPR	1.07460897	0.30831310	2.19142726	12.15	0.0013

BOUNDS ON CONDITION NUMBER: 1.058617, 4.234467

STEP 3 VARIABLE LPIT ENTERED R-SQUARE = 0.87494349 C(P) = 0.87467823

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	42.93564051	14.31188017	81.62	0.0001
ERROR	35	6.13683228	0.17533807		
TOTAL	38	49.07247279			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	2.27674145	0.24073408	15.68297622	89.44	0.0001
LPIT	0.31018034	0.21732681	0.35717258	2.04	0.1624
LCLEA	0.50566598	0.03631700	33.99254820	193.87	0.0001
LEXPR	1.02111794	0.30626780	1.94906066	11.12	0.0020

BOUNDS ON CONDITION NUMBER: 1.102531, 9.674211

STEP 4 VARIABLE LRYMT ENTERED R-SQUARE = 0.87736214 C(P) = 2.25820761

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	43.05432960	10.76358240	60.81	0.0001
ERROR	34	6.01814319	0.17700421		
TOTAL	38	49.07247279			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	4.35423696	2.54853895	0.51668418	2.92	0.0967
LPIT	0.23598220	0.23641077	0.17636305	1.00	0.3252
LCLEA	0.49486720	0.03879905	28.79510770	162.68	0.0001
LEXPR	1.33755285	0.49398319	1.29772047	7.33	0.0105
LRYMT	-0.29515323	0.36044078	0.11868910	0.67	0.4186

BOUNDS ON CONDITION NUMBER: 2.769527, 31.561

NO OTHER VARIABLE MET THE 0.5000 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRSZ

STEP	VARIABLE ENTERED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)	F	PROB>F
1	LCLEA	1	0.8230	0.8230	10.1121	172.0491	0.0001
2	LEXPR	2	0.0447	0.8677	0.7298	12.1483	0.0013
3	LPIT	3	0.0073	0.8749	0.8747	2.0371	0.1624
4	LRYMT	4	0.0024	0.8774	2.2582	0.6705	0.4186

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FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRSH

STEP 1 VARIABLE LCLEA ENTERED R-SQUARE = 0.84386879 C(P) = 24.46663247

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	22.00950328	22.00950328	199.98	0.0001
ERROR	37	4.07216185	0.11005843		
TOTAL	38	26.08166513			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	1.57766504	0.14913620	12.31648467	111.91	0.0001

```
 LCLEA          0.38750888      0.02740237      22.00950328      199.98    0.0001

BOUNDS ON CONDITION NUMBER:              1,            1
---------------------------------------------------------------------------------

STEP 2   VARIABLE LEXPR ENTERED      R-SQUARE = 0.89490341    C(P) =  7.02877098

                 DF          SUM OF SQUARES       MEAN SQUARE            F    PROB>F

 REGRESSION       2          23.34057105       11.67028553      153.27    0.0001
 ERROR           36           2.74109407        0.07614150
 TOTAL           38          26.08166513

                 PARAMETER          STANDARD           TYPE II
 VARIABLE         ESTIMATE             ERROR    SUM OF SQUARES            F    PROB>F

 INTERCEP       1.63188890      0.12472192      13.03518106      171.20    0.0001
 LCLEA          0.36443675      0.02345074      18.38876194      241.51    0.0001
 LEXPR          0.83750424      0.20030770       1.33106777       17.48    0.0002

BOUNDS ON CONDITION NUMBER:      1.058617,      4.234467
---------------------------------------------------------------------------------

STEP 3   VARIABLE LPIT ENTERED       R-SQUARE = 0.91583228    C(P) =  1.05746748

                 DF          SUM OF SQUARES       MEAN SQUARE            F    PROB>F

 REGRESSION       3          23.88643075        7.96214358      126.95    0.0001
 ERROR           35           2.19523438        0.06272098
 TOTAL           38          26.08166513

                 PARAMETER          STANDARD           TYPE II
 VARIABLE         ESTIMATE             ERROR    SUM OF SQUARES            F    PROB>F

 INTERCEP       1.36940080      0.14398124       5.67364566       90.46    0.0001
 LPIT           0.38345616      0.12998153       0.54585970        8.70    0.0056
 LCLEA          0.37722520      0.02172092      18.91723489      301.61    0.0001
 LEXPR          0.77137670      0.18317646       1.11226065       17.73    0.0002

BOUNDS ON CONDITION NUMBER:      1.102531,      9.674211
---------------------------------------------------------------------------------

STEP 4   VARIABLE LTWT ENTERED       R-SQUARE = 0.91818861    C(P) =  2.15999691

                 DF          SUM OF SQUARES       MEAN SQUARE            F    PROB>F

 REGRESSION       4          23.94788782        5.98697196       95.40    0.0001
 ERROR           34           2.13377730        0.06275816
 TOTAL           38          26.08166513

                 PARAMETER          STANDARD           TYPE II
 VARIABLE         ESTIMATE             ERROR    SUM OF SQUARES            F    PROB>F

 INTERCEP       1.72828441      0.39021409       1.23110388       19.62    0.0001
 LTWT           0.14305066      0.14455696       0.06145708        0.98    0.3294
 LPIT           0.40954847      0.13266662       0.59807748        9.53    0.0040
 LCLEA          0.38419914      0.02284171      17.75521091      282.91    0.0001
 LEXPR          0.74133871      0.18572798       0.99988223       15.93    0.0003

BOUNDS ON CONDITION NUMBER:      1.218523,      18.17646
NO OTHER VARIABLE MET THE 0.5000 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

     SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LRSH

        VARIABLE    NUMBER    PARTIAL      MODEL
 STEP   ENTERED         IN       R**2       R**2          C(P)             F    PROB>F
```

1	LCLEA	1	0.8439	0.8439	24.4666	199.9802	0.0001
2	LEXPR	2	0.0510	0.8949	7.0288	17.4815	0.0002
3	LPIT	3	0.0209	0.9158	1.0575	8.7030	0.0056
4	LTWT	4	0.0024	0.9182	2.1600	0.9793	0.3294

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FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LTWR

STEP 1 VARIABLE LCLEA ENTERED R-SQUARE = 0.97007380 C(P) = 3.50679157

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	76.32398975	76.32398975	1199.37	0.0001
ERROR	37	2.35454973	0.06363648		
TOTAL	38	78.67853948			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	3.91315956	0.11340299	75.77261596	1190.71	0.0001
LCLEA	0.72161752	0.02083673	76.32398975	1199.37	0.0001

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE LRYS ENTERED R-SQUARE = 0.97331355 C(P) = 1.33812290

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	76.57888863	38.28944431	656.50	0.0001
ERROR	36	2.09965086	0.05832363		
TOTAL	38	78.67853948			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	3.92307630	0.10866955	76.01203485	1303.28	0.0001
LCLEA	0.72250857	0.01995253	76.47768053	1311.26	0.0001
LRYS	-0.14775056	0.07067527	0.25489887	4.37	0.0437

BOUNDS ON CONDITION NUMBER: 1.000457, 4.001826

STEP 3 VARIABLE LTWT ENTERED R-SQUARE = 0.97422770 C(P) = 2.16186515

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	76.65081249	25.55027083	441.02	0.0001
ERROR	35	2.02772699	0.05793506		
TOTAL	38	78.67853948			

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	TYPE II SUM OF SQUARES	F	PROB>F
INTERCEP	4.32134684	0.37349589	7.75546838	133.86	0.0001
LTWT	0.15045253	0.13503107	0.07192387	1.24	0.2728
LCLEA	0.72821452	0.02053476	72.85857380	1257.59	0.0001
LRYS	-0.15211585	0.07054831	0.26935007	4.65	0.0380

BOUNDS ON CONDITION NUMBER: 1.068831, 9.417562
NO OTHER VARIABLE MET THE 0.5000 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE LTWR

STEP	VARIABLE ENTERED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)	F	PROB>F
1	LCLEA	1	0.9701	0.9701	3.5068	1199.3748	0.0001
2	LRYS	2	0.0032	0.9733	1.3381	4.3704	0.0437
3	LTWT	3	0.0009	0.9742	2.1619	1.2415	0.2728

APPENDIX E

PROGRAM FOR AUTOMATIC GENERATION OF MESH

(1) For axisymmetric model

```
/FILE 2 NAME(E1E.ABA) NEW(REPL) SPACE(100)
/LOAD FORTG
      DIMENSION CODI(2,1525)
C*************************************************************
      ET=0.22E+8
      YT=0.18E+5
      POT=0.30
C*************************************************************
      ES=0.18E+8
      YS=0.15E+5
      POS=0.30
C*************************************************************
      BBB1=0.0653
      BBB2=0.1106
      BBB3=0.109
      CCC1=1.5
      CCC2=2.5
      CCC3=3.5
      DDDD1=YT*0.4847
      DDDD2=YT*0.6122
      DDDD3=YT*1.111
C*************************************************************
      T=0.098
      P=0.135E5
      P1=P/YT
      A=1.0
      B=5.2
      C=0.032
      H=B
C*************************************************************
      F=0.15
C*************************************************************
      WRITE(2,5)B,A,C,H,T,ET,YT,POT,ES,YS,POS,P1
    5 FORMAT('*HEADING'/'**B =',E12.4,' A=',E12.4,
     1'   C=',E12.4,' H=',E12.4,
     1' T=',E12.4/'**ET=',E12.4,' YT=',E12.4,' POT=',E12.4/
     1'**ES=',E12.4,' YS=',E12.4,' POS=',E12.4/'** P=',E12.4)
C    1'*DATA CHECK')
      TTH=T
      STH=(B-A)/2.0
      RTI=A/2.0-T
      RTO=A/2.0
      RSO=B/2.0
```

```
      GAP=C
C
WRITE(2,*)B,A,C,H,TTH,STH,ET,EHT,P,RTI,RTO,RSO,POT,POS,GAP,
ES,       1YS,
C    1EHS
      DO 10 I=101,139,1
      CODI(1,I)=RTI
   10 CONTINUE
      DO 20 I=501,539,1
      CODI(1,I)=RTI+TTH*0.75
   20 CONTINUE
      DO 90 I=701,739,1
      CODI(1,I)=RTO
   90 CONTINUE
      DO 95 I=901,923,1
      CODI(1,I)=RTO+C
   95 CONTINUE
      DO 130 I=1501,1523,1
      CODI(1,I)=RSO
  130 CONTINUE
C***********************************************************
      CODI(2,101)=0.0
      CODI(2,501)=0.0
      CODI(2,701)=0.0
      CODI(2,901)=0.0
      CODI(2,1501)=0.0
      CODI(2,103)=0.2*H
      CODI(2,503)=0.2*H
      CODI(2,703)=0.2*H
      CODI(2,903)=0.2*H
      CODI(2,1503)=0.2*H
      CODI(2,115)=0.8*H
      CODI(2,515)=0.8*H
      CODI(2,715)=0.8*H
      CODI(2,915)=0.8*H
      CODI(2,1515)=0.8*H
      CODI(2,923)=H
      CODI(2,1523)=H
      CODI(2,131)=1.2*H
      CODI(2,531)=1.2*H
      CODI(2,731)=1.2*H
      CODI(2,135)=1.4*H
      CODI(2,535)=1.4*H
      CODI(2,735)=1.4*H
      CODI(2,139)=2.0*H
      CODI(2,539)=2.0*H
      CODI(2,739)=2.0*H
      WRITE(2,810)
  810 FORMAT('*NODE')
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=101,103,2)
```

```
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=115,131,16)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=135,139)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=501,503,2)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=515,531,16)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=535,539)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=701,703,2)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=715,731,16)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=735,739)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=901,903,2)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=915,923,8)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=1501,1503,2)
      WRITE(2,820)(L,(CODI(K,L),K=1,2),L=1515,1523,8)
  820 FORMAT(I4,',',E10.4,',',E10.4)
      WRITE(2,830)
  830 FORMAT(
     1'*NGEN,NSET=TII'/'101,103'/'103,115'/
     1'115,131'/'131,135'/'135,139'/
     1'*NGEN,NSET=TI1'/'501,503'/'503,515'/
     1'515,531'/'531,535'/'535,539'/
     1'*NGEN,NSET=TOO'/'701,703'/'703,715'/
     1'715,731'/'731,735'/'735,739'/
     1'*NGEN,NSET=SII'/'901,903'/'903,915'/'915,923'/
     1'*NGEN,NSET=SOO'/'1501,1503'/'1503,1515'/'1515,1523'/
     1'*NSET,NSET=RE,GENERATE'/'101,701,100'/'901,1501,100'/
     1'*NSET,NSET=CLT,GENERATE'/'701,723'/
     1'*NSET,NSET=CLS,GENERATE'/'901,923')
      WRITE(2,835)
  835 FORMAT(
     1'*NSET,NSET=CL011,GENERATE'/'101,111,1'/
     1'*NSET,NSET=CL111,GENERATE'/'701,711,1'/
     1'*NFILL,BIAS=0.5,TWO STEP'/'TII,TI1,4,100'/
     1'*NFILL,TWO STEP'/'TI1,TOO,2,100'/
     1'*NFILL,BIAS=0.6,TWO STEP'/'SII,SOO,6,100')
      WRITE(2,836)
  836 FORMAT(
     1'*ELEMENT,TYPE=CAX8R'/
     1'101,103,101,301,303,102,201,302,203'/
     1'401,903,901,1101,1103,902,1001,1102,1003'/
     1'*ELGEN,ELSET=TUBE'/
     1'101,19,2,1,3,200,100'/
     1'*ELGEN,ELSET=SHEET'/
     1'401,11,2,1,3,200,100'/
     1'*ELSET,ELSET=TUBE111,GENERATE'/'106,117,1'/
     1'*ELSET,ELSET=TUBE333,GENERATE'/'306,317,1'/
     1'*ELSET,ELSET=TUBE11,GENERATE'/'101,111,1')
      WRITE(2,840)F
  840 FORMAT('*ELEMENT,TYPE=INTER3A'/
      1'1001,701,702,703,901,902,903'/
     1'*ELGEN,ELSET=SURFACE'/'1001,11,2,1'/
      1'*INTERFACE,ELSET=SURFACE'/
```

```
     1'*FRICTION'/E10.4,',10000.0'/
     1'*NORMAL'/'SURFACE,CLT,1.0,0.0')
      WRITE(2,860)ET,POT,YT,ES,POS,YS
  860 FORMAT('*SOLID SECTION,ELSET=TUBE,MATERIAL=A'/
     1'*MATERIAL,NAME=A'/
     1'*ELASTIC'/E10.4,',',E10.4/
     1'*PLASTIC'/E10.4/
     1'*SOLID SECTION,ELSET=SHEET,MATERIAL=B'/
     1'*MATERIAL,NAME=B'/
     1'*ELASTIC'/E10.4,',',E10.4/
     1'*PLASTIC'/E10.4/
     1'*BOUNDARY'/'RE,2'/'739,1'
C    1/'*PLOT'/'*DRAW'
     1)
C***********************************************************
      WRITE(2,870)P
 870  FORMAT('*STEP,INC=90,CYCLE=15'/
      1'*STATIC,PTOL=150.0'/'0.15,1.0'/0'/
     1'*DLOAD,OP=NEW'/'TUBE11,P1,',E12.5/
     1'*EL PRINT,ELSET=TUBE111,FREQUENCT=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*NODE PRINT,NSET=CL011,FREQUENCY=40'/'U'/
     1'*NODE PRINT,NSET=CL111,FREQUENCY=40'/'U'/
     1'*END STEP')
      WRITE(2,880)
  880 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=150.0'/
     1'*DLOAD,OP=NEW'/'TUBE11,P1,0.0'/
     1'*EL PRINT,ELSET=TUBE111,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=TUBE333,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CL011,FREQUENCY=40'/'U'/
     1'*NODE PRINT,NSET=CL111'/'U'/
C    1'*PLOT'/'*DISPLACED'/
C    1'*PLOT'/'*DETAIL,ELSET=DETAIL'/'*DISPLACED'/
     1'*END STEP')
C***********************************************************
C     WRITE(2,871)P1
 871  FORMAT('*STEP,INC=20,CYCLE=12'/'*STATIC,PTOL=10.0'/
     1'*DLOAD,OP=NEW'/'SHEET2,P3,',E12.5/
     1'*EL PRINT'/'S'/
     1'*NODE PRINT'/'U'/
     1'*END STEP')
C     WRITE(2,881)
  881 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=1.0'/
```

```
     1'*DLOAD,OP=NEW'/'SHEET2,P1,0.0'/
C    1'*EL PRINT,ELSET=TUBE11'/'2,1,1,1,1'/
     1'*EL PRINT,ELSET=TUBE1'/'S'/
     1'*EL PRINT,ELSET=TUBE4'/'S'/
C    1'*PLOT'/'*DISPLACED'/
C    1'*PLOT'/'*DETAIL,ELSET=DETAIL'/'*DISPLACED'/
     1'*NODE PRINT,NSET=CL01'/'U'/
     1'*NODE PRINT,NSET=CL11'/'U'/
     1'*END STEP')
C***********************************************************
C     WRITE(2,872)P2
 872  FORMAT('*STEP,INC=20,CYCLE=12'/'*STATIC,PTOL=1.0'/
     1'*DLOAD,OP=NEW'/'SHEET2,P3,',E12.5/
     1'*EL PRINT'/'S'/
     1'*NODE PRINT'/'U'/
     1'*END STEP')
C     WRITE(2,882)
  882 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=1.0'/
     1'*DLOAD,OP=NEW'/'SHEET2,P1,0.0'/
C    1'*EL PRINT,ELSET=TUBE11'/'S'/
     1'*EL PRINT,ELSET=TUBE1'/'S'/
     1'*EL PRINT,ELSET=TUBE4'/'S'/
     1'*NODE PRINT,NSET=CL01'/'U'/
     1'*NODE PRINT,NSET=CL11'/'U'/
     1'*END STEP')
C***********************************************************
C     WRITE(2,873)P3
 873  FORMAT('*STEP,INC=20,CYCLE=12'/'*STATIC,PTOL=1.0'/
     1'*DLOAD,OP=NEW'/'SHEET2,P3,',E12.5/
     1'*EL PRINT'/'S'/
     1'*NODE PRINT'/'U'/
     1'*END STEP')
C     WRITE(2,883)
  883 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=1.0'/
     1'*DLOAD,OP=NEW'/'SHEET2,P1,0.0'/
C    1'*EL PRINT,ELSET=TUBE11'/'S'/
     1'*EL PRINT,ELSET=TUBE1'/'S'/
     1'*EL PRINT,ELSET=TUBE4'/'S'/
     1'*NODE PRINT,NSET=CL01'/'U'/
     1'*NODE PRINT,NSET=CL11'/'U'/
     1'*END STEP')
C***********************************************************
C     WRITE(2,890)T
  890 FORMAT('*STEP,INC=20,CYCLE=12'/'*STATIC,PTOL=10.0'/
     1'*TEMPERATURE,OP=NEW'/'ALL,',E10.4/
     1'*EL PRINT,ELSET=TUBE11'/'S'/
     1'*EL PRINT,ELSET=TUBE4'/'S'/
     1'*END STEP')
C     WRITE(2,900)
  900 FORMAT('*STEP,INC=20,CYCLE=12'/'*STATIC,PTOL=10.0'/
```

```
1'*TEMPERATURE,OP=NEW'/'ALL,20.0'/
1'*EL PRINT,ELSET=TUBE11'/'S'/
1'*EL PRINT,ELSET=TUBE4'/'S'/
1'*PLOT'/'*DISPLACED'/'*DRAW'/
1'*PLOT'/'*DETAIL,ELSET=DETAIL'/'*DISPLACED'/'*DRAW'/
1'*END STEP')
 STOP
 END
```

(2) For Seven Tube-Model

```
C/SYS REG=3000
C/FILE 2 NAME(P71MO.ABA) NEW(REPL) SPACE(3000)
C/LOAD FORTG
      DIMENSION CODI(2,12000)
C************************************************************
      A=1.0
      T=0.2
      PITCH=2.00
      C=0.0005
C************************************************************
      P=20780.0
C************************************************************
      ET=0.22E+8
      YT=0.18E+5
      POT=0.3
C************************************************************
      ES=0.29E+8
      YS=0.3E+5
      POS=0.3
C************************************************************
***
      PAI=3.141592654
      S3=SQRT(3.0)
      PIT=PITCH/S3
      PITT=PIT-A*0.5
      PA=PAI/12.0
      PA1=PAI/24.0
      ROU=PITCH*2.0
      WRITE(6,5)ROU,PITCH,A,T,P,ET,YT,POT,ES,YS,POS
    5 FORMAT('*HEADING'/5E12.5/'**',6E12.5/
      1'*PREPRINT,MODEL=NO')
      WRITE(6,10)
   10 FORMAT('*NODE')
      DO 100 J10=101,124,4
      CODI(1,J10)=PIT*COS((J10-101)*PA)
      CODI(2,J10)=PIT*SIN((J10-101)*PA)
      IF(ABS(CODI(1,J10)).LE.1.0E-3) CODI(1,J10)=0.0
      IF(ABS(CODI(2,J10)).LE.1.0E-3) CODI(2,J10)=0.0
      WRITE(6,101)J10,CODI(1,J10),CODI(2,J10)
  101 FORMAT(I5,',',E12.4,',',E12.4)
  100 CONTINUE
      DO 110 J11=103,123,4
      CODI(1,J11)=PITCH*0.5*COS(PAI/6.0+(J11-103)*PA)
      CODI(2,J11)=PITCH*0.5*SIN(PAI/6.0+(J11-103)*PA)
      IF(ABS(CODI(1,J11)).LE.1.0E-3) CODI(1,J11)=0.0
      IF(ABS(CODI(2,J11)).LE.1.0E-3) CODI(2,J11)=0.0
      WRITE(6,101)J11,CODI(1,J11),CODI(2,J11)
  110 CONTINUE
```

```
      DO 120 J12=102,124,2
      CODI(1,J12)=PITCH*0.5/COS(PA)*COS(PA+(J12-102)
     1*PA)
      CODI(2,J12)=PITCH*0.5/COS(PA)*SIN(PA+(J12-102)
     1*PA)
      IF(ABS(CODI(1,J12)).LE.1.0E-3) CODI(1,J12)=0.0
      IF(ABS(CODI(2,J12)).LE.1.0E-3) CODI(2,J12)=0.0
      WRITE(6,101)J12,CODI(1,J12),CODI(2,J12)
  120 CONTINUE
      DO 123 J12=125,147,1
      CODI(1,J12)=(CODI(1,J12-24)+CODI(1,J12-23))*0.5
      CODI(2,J12)=(CODI(2,J12-24)+CODI(2,J12-23))*0.5
      IF(ABS(CODI(1,J12)).LE.1.0E-3) CODI(1,J12)=0.0
      IF(ABS(CODI(2,J12)).LE.1.0E-3) CODI(2,J12)=0.0
      WRITE(6,101)J12,CODI(1,J12),CODI(2,J12)
  123 CONTINUE
      CODI(1,148)=(CODI(1,124)+CODI(1,101))*0.5
      CODI(2,148)=(CODI(2,124)+CODI(2,101))*0.5
      WRITE(6,121)CODI(1,148),CODI(2,148)
  121 FORMAT('148',',',E12.4,',',E12.4)
      DO 1100 I11=1101,1124,1
      CODI(1,I11)=(A*0.5-T)*COS((I11-1101)*PA)
      CODI(2,I11)=(A*0.5-T)*SIN((I11-1101)*PA)
      IF(ABS(CODI(1,I11)).LE.1.0E-3) CODI(1,I11)=0.0
      IF(ABS(CODI(2,I11)).LE.1.0E-3) CODI(2,I11)=0.0
C     WRITE(6,101)I11,CODI(1,I11),CODI(2,I11)
 1100 CONTINUE
      DO 1200 I12=1201,1223,2
      CODI(1,I12)=(A*0.5-T*0.75)*COS((I12-1201)*PA)
      CODI(2,I12)=(A*0.5-T*0.75)*SIN((I12-1201)*PA)
      IF(ABS(CODI(1,I12)).LE.1.0E-3) CODI(1,I12)=0.0
      IF(ABS(CODI(2,I12)).LE.1.0E-3) CODI(2,I12)=0.0
C     WRITE(6,101)I12,CODI(1,I12),CODI(2,I12)
 1200 CONTINUE
      DO 1300 I13=1301,1324,1
      CODI(1,I13)=(A-T)*0.5*COS((I13-1301)*PA)
      CODI(2,I13)=(A-T)*0.5*SIN((I13-1301)*PA)
      IF(ABS(CODI(1,I13)).LE.1.0E-3) CODI(1,I13)=0.0
      IF(ABS(CODI(2,I13)).LE.1.0E-3) CODI(2,I13)=0.0
C     WRITE(6,101)I13,CODI(1,I13),CODI(2,I13)
 1300 CONTINUE
      DO 1400 I14=1401,1423,2
      CODI(1,I14)=(A*0.5-T*0.25)*COS((I14-1401)*PA)
      CODI(2,I14)=(A*0.5-T*0.25)*SIN((I14-1401)*PA)
      IF(ABS(CODI(1,I14)).LE.1.0E-3) CODI(1,I14)=0.0
      IF(ABS(CODI(2,I14)).LE.1.0E-3) CODI(2,I14)=0.0
C     WRITE(6,101)I14,CODI(1,I14),CODI(2,I14)
 1400 CONTINUE
      DO 1500 I15=1501,1524,1
      CODI(1,I15)=A*0.5*COS((I15-1501)*PA)
```

```
      CODI(2,I15)=A*0.5*SIN((I15-1501)*PA)
      IF(ABS(CODI(1,I15)).LE.1.0E-3) CODI(1,I15)=0.0
      IF(ABS(CODI(2,I15)).LE.1.0E-3) CODI(2,I15)=0.0
C     WRITE(6,101)I15,CODI(1,I15),CODI(2,I15)
 1500 CONTINUE
      DO 1600 I16=1601,1623,2
      CODI(1,I16)=(A*0.5+PITT*0.1)*COS((I16-1601)*PA)
      CODI(2,I16)=(A*0.5+PITT*0.1)*SIN((I16-1601)*PA)
      IF(ABS(CODI(1,I16)).LE.1.0E-3) CODI(1,I16)=0.0
      IF(ABS(CODI(2,I16)).LE.1.0E-3) CODI(2,I16)=0.0
C     WRITE(6,101)I16,CODI(1,I16),CODI(2,I16)
 1600 CONTINUE
      DO 1700 I17=1701,1724,1
      CODI(1,I17)=(A*0.5+PITT*0.2)*COS((I17-1701)*PA)
      CODI(2,I17)=(A*0.5+PITT*0.2)*SIN((I17-1701)*PA)
      IF(ABS(CODI(1,I17)).LE.1.0E-3) CODI(1,I17)=0.0
      IF(ABS(CODI(2,I17)).LE.1.0E-3) CODI(2,I17)=0.0
C     WRITE(6,101)I17,CODI(1,I17),CODI(2,I17)
 1700 CONTINUE
      DO 1800 I18=1801,1823,2
      CODI(1,I18)=(A*0.5+PITT*0.6)*COS((I18-1801)*PA)
      CODI(2,I18)=(A*0.5+PITT*0.6)*SIN((I18-1801)*PA)
      IF(ABS(CODI(1,I18)).LE.1.0E-3) CODI(1,I18)=0.0
      IF(ABS(CODI(2,I18)).LE.1.0E-3) CODI(2,I18)=0.0
C     WRITE(6,101)I18,CODI(1,I18),CODI(2,I18)
 1800 CONTINUE
      DO 11100 I111=10101,10148,1
      CODI(1,I111)=(A*0.5-T)*COS((I111-10101)*PA1)
      CODI(2,I111)=(A*0.5-T)*SIN((I111-10101)*PA1)
      IF(ABS(CODI(1,I111)).LE.1.0E-3) CODI(1,I111)=0.0
      IF(ABS(CODI(2,I111)).LE.1.0E-3) CODI(2,I111)=0.0
      WRITE(6,101)I111,CODI(1,I111),CODI(2,I111)
11100 CONTINUE
      DO 11200 I112=10201,10247,2
      CODI(1,I112)=(A*0.5-0.667*T)*COS((I112-10201)*PA1)
      CODI(2,I112)=(A*0.5-0.667*T)*SIN((I112-10201)*PA1)
      IF(ABS(CODI(1,I112)).LE.1.0E-3) CODI(1,I112)=0.0
      IF(ABS(CODI(2,I112)).LE.1.0E-3) CODI(2,I112)=0.0
      WRITE(6,101)I112,CODI(1,I112),CODI(2,I112)
11200 CONTINUE
      DO 11300 I113=10301,10348,1
      CODI(1,I113)=(A*0.5-0.333*T)*COS((I113-10301)*PA1)
      CODI(2,I113)=(A*0.5-0.333*T)*SIN((I113-10301)*PA1)
      IF(ABS(CODI(1,I113)).LE.1.0E-3) CODI(1,I113)=0.0
      IF(ABS(CODI(2,I113)).LE.1.0E-3) CODI(2,I113)=0.0
      WRITE(6,101)I113,CODI(1,I113),CODI(2,I113)
11300 CONTINUE
      DO 11400 I114=10401,10447,2
      CODI(1,I114)=(A*0.5-T/6.)*COS((I114-10401)*PA1)
      CODI(2,I114)=(A*0.5-T/6.)*SIN((I114-10401)*PA1)
```

```
      IF(ABS(CODI(1,I114)).LE.1.0E-3) CODI(1,I114)=0.0
      IF(ABS(CODI(2,I114)).LE.1.0E-3) CODI(2,I114)=0.0
      WRITE(6,101)I114,CODI(1,I114),CODI(2,I114)
11400 CONTINUE
      DO 11500 I115=10501,10548,1
      CODI(1,I115)=(A*0.5-C*0.5)*COS((I115-10501)*PA1)
      CODI(2,I115)=(A*0.5-C*0.5)*SIN((I115-10501)*PA1)
      IF(ABS(CODI(1,I115)).LE.1.0E-3) CODI(1,I115)=0.0
      IF(ABS(CODI(2,I115)).LE.1.0E-3) CODI(2,I115)=0.0
      WRITE(6,101)I115,CODI(1,I115),CODI(2,I115)
11500 CONTINUE
      DO 11700 I117=10701,10748,1
      CODI(1,I117)=(A*0.5+C*0.5)*COS((I117-10701)*PA1)
      CODI(2,I117)=(A*0.5+C*0.5)*SIN((I117-10701)*PA1)
      IF(ABS(CODI(1,I117)).LE.1.0E-3) CODI(1,I117)=0.0
      IF(ABS(CODI(2,I117)).LE.1.0E-3) CODI(2,I117)=0.0
      WRITE(6,101)I117,CODI(1,I117),CODI(2,I117)
11700 CONTINUE
      DO 11800 I118=10801,10847,2
      CODI(1,I118)=(A*0.5+PITT*0.1)*COS((I118-10801)*PA1)
      CODI(2,I118)=(A*0.5+PITT*0.1)*SIN((I118-10801)*PA1)
      IF(ABS(CODI(1,I118)).LE.1.0E-3) CODI(1,I118)=0.0
      IF(ABS(CODI(2,I118)).LE.1.0E-3) CODI(2,I118)=0.0
      WRITE(6,101)I118,CODI(1,I118),CODI(2,I118)
11800 CONTINUE
      DO 11900 I119=10901,10948,1
      CODI(1,I119)=(A*0.5+PITT*0.2)*COS((I119-10901)*PA1)
      CODI(2,I119)=(A*0.5+PITT*0.2)*SIN((I119-10901)*PA1)
      IF(ABS(CODI(1,I119)).LE.1.0E-3) CODI(1,I119)=0.0
      IF(ABS(CODI(2,I119)).LE.1.0E-3) CODI(2,I119)=0.0
      WRITE(6,101)I119,CODI(1,I119),CODI(2,I119)
11900 CONTINUE
      DO 12000 I119=11001,11047,2
      CODI(1,I119)=(A*0.5+PITT*0.35)*COS((I119-11001)*PA1)
      CODI(2,I119)=(A*0.5+PITT*0.35)*SIN((I119-11001)*PA1)
      IF(ABS(CODI(1,I119)).LE.1.0E-3) CODI(1,I119)=0.0
      IF(ABS(CODI(2,I119)).LE.1.0E-3) CODI(2,I119)=0.0
      WRITE(6,101)I119,CODI(1,I119),CODI(2,I119)
12000 CONTINUE
      DO 12100 I119=11101,11148,1
      CODI(1,I119)=(A*0.5+PITT*0.5)*COS((I119-11101)*PA1)
      CODI(2,I119)=(A*0.5+PITT*0.5)*SIN((I119-11101)*PA1)
      IF(ABS(CODI(1,I119)).LE.1.0E-3) CODI(1,I119)=0.0
      IF(ABS(CODI(2,I119)).LE.1.0E-3) CODI(2,I119)=0.0
      WRITE(6,101)I119,CODI(1,I119),CODI(2,I119)
12100 CONTINUE
      I11=11100
      DO 12200 I119=11201,11247,2
      CODI(1,I119)=(CODI(1,I119-100)+CODI(1,I119-I11))*0.5
      CODI(2,I119)=(CODI(2,I119-100)+CODI(2,I119-I11))*0.5
```

```
      IF(ABS(CODI(1,I119)).LE.1.0E-3) CODI(1,I119)=0.0
      IF(ABS(CODI(2,I119)).LE.1.0E-3) CODI(2,I119)=0.0
      WRITE(6,101)I119,CODI(1,I119),CODI(2,I119)
      I11=I11+1
12200 CONTINUE
      DO 200 J21=201,216,1
      CODI(1,J21)=CODI(1,J21-100)
      CODI(2,J21)=CODI(2,J21-100)+PITCH
      IF(ABS(CODI(1,J21)).LE.1.0E-3) CODI(1,J21)=0.0
      IF(ABS(CODI(2,J21)).LE.1.0E-3) CODI(2,J21)=0.0
      WRITE(6,101)J21,CODI(1,J21),CODI(2,J21)
  200 CONTINUE
      DO 201 J22=222,224,1
      CODI(1,J22)=CODI(1,J22-100)
      CODI(2,J22)=CODI(2,J22-100)+PITCH
      IF(ABS(CODI(1,J22)).LE.1.0E-3) CODI(1,J22)=0.0
      IF(ABS(CODI(2,J22)).LE.1.0E-3) CODI(2,J22)=0.0
      WRITE(6,101)J22,CODI(1,J22),CODI(2,J22)
  201 CONTINUE
      DO 2100 I21=2101,2124,1
      CODI1=CODI(1,I21-1000)
      CODI2=CODI(2,I21-1000)+PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI(1,I21)=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI(2,I21)=0.0
      WRITE(6,101)I21,CODI1,CODI2
 2100 CONTINUE
      DO 2200 I=2201,2223,2
      CODI1=CODI(1,I-1000)
      CODI2=CODI(2,I-1000)+PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 2200 CONTINUE
      DO 2300 I23=2301,2324,1
      CODI1=CODI(1,I23-1000)
      CODI2=CODI(2,I23-1000)+PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I23,CODI1,CODI2
 2300 CONTINUE
      DO 2400 I24=2401,2423,2
      CODI1=CODI(1,I24-1000)
      CODI2=CODI(2,I24-1000)+PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I24,CODI1,CODI2
 2400 CONTINUE
      DO 2500 I=2501,2524,1
      CODI1=CODI(1,I-1000)
      CODI2=CODI(2,I-1000)+PITCH
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      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 2500 CONTINUE
      DO 2600 I26=2601,2623,2
      CODI(1,I26)=CODI(1,I26-1000)
      CODI(2,I26)=CODI(2,I26-1000)+PITCH
      IF(ABS(CODI(1,I26)).LE.1.0E-3) CODI(1,I26)=0.0
      IF(ABS(CODI(2,I26)).LE.1.0E-3) CODI(2,I26)=0.0
      WRITE(6,101)I26,CODI(1,I26),CODI(2,I26)
 2600 CONTINUE
      DO 2700 I27=2701,2724,1
      CODI(1,I27)=CODI(1,I27-1000)
      CODI(2,I27)=CODI(2,I27-1000)+PITCH
      IF(ABS(CODI(1,I27)).LE.1.0E-3) CODI(1,I27)=0.0
      IF(ABS(CODI(2,I27)).LE.1.0E-3) CODI(2,I27)=0.0
      WRITE(6,101)I27,CODI(1,I27),CODI(2,I27)
 2700 CONTINUE
      DO 2800 I28=2801,2823,2
      CODI(1,I28)=CODI(1,I28-1000)
      CODI(2,I28)=CODI(2,I28-1000)+PITCH
      IF(ABS(CODI(1,I28)).LE.1.0E-3) CODI(1,I28)=0.0
      IF(ABS(CODI(2,I28)).LE.1.0E-3) CODI(2,I28)=0.0
      WRITE(6,101)I28,CODI(1,I28),CODI(2,I28)
 2800 CONTINUE
      DO 300 J31=301,308,1
      CODI(1,J31)=CODI(1,J31-200)+PITCH*0.5*S3
      CODI(2,J31)=CODI(2,J31-200)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI(1,J31)=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI(2,J31)=0.0
      WRITE(6,101)J31,CODI(1,J31),CODI(2,J31)
  300 CONTINUE
      DO 301 J32=318,324,1
      CODI(1,J32)=CODI(1,J32-200)+PITCH*0.5*S3
      CODI(2,J32)=CODI(2,J32-200)+PITCH*0.5
      IF(ABS(CODI(1,J32)).LE.1.0E-3) CODI(1,J32)=0.0
      IF(ABS(CODI(2,J32)).LE.1.0E-3) CODI(2,J32)=0.0
      WRITE(6,101)J32,CODI(1,J32),CODI(2,J32)
  301 CONTINUE
      DO 3100 I31=3101,3124,1
      CODI1=CODI(1,I31-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I31-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I31,CODI1,CODI2
 3100 CONTINUE
      DO 3200 I=3201,3223,2
      CODI1=CODI(1,I-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
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      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 3200 CONTINUE
      DO 3300 I33=3301,3324,1
      CODI1=CODI(1,I33-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I33-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I33,CODI1,CODI2
 3300 CONTINUE
      DO 3400 I34=3401,3423,2
      CODI1=CODI(1,I34-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I34-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I34,CODI1,CODI2
 3400 CONTINUE
      DO 3500 I35=3501,3524,1
      CODI1=CODI(1,I35-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I35-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I35,CODI1,CODI2
 3500 CONTINUE
      DO 3600 I36=3601,3623,2
      CODI1=CODI(1,I36-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I36-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I36,CODI1,CODI2
 3600 CONTINUE
      DO 3700 I=3701,3724,1
      CODI1=CODI(1,I-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 3700 CONTINUE
      DO 3800 I38=3801,3823,2
      CODI1=CODI(1,I38-2000)+PITCH*0.5*S3
      CODI2=CODI(2,I38-2000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I38,CODI1,CODI2
 3800 CONTINUE
      DO 401 J40=401,404,1
      CODI(1,J40)=CODI(1,J40-300)+PITCH*0.5*S3
      CODI(2,J40)=CODI(2,J40-300)-PITCH*0.5
      IF(ABS(CODI(1,J40)).LE.1.0E-3) CODI(1,J40)=0.0
      IF(ABS(CODI(2,J40)).LE.1.0E-3) CODI(2,J40)=0.0
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      WRITE(6,101)J40,CODI(1,J40),CODI(2,J40)
  401 CONTINUE
      DO 402 J41=414,424,1
      CODI(1,J41)=CODI(1,J41-300)+PITCH*0.5*S3
      CODI(2,J41)=CODI(2,J41-300)-PITCH*0.5
      IF(ABS(CODI(1,J41)).LE.1.0E-3) CODI(1,J41)=0.0
      IF(ABS(CODI(2,J41)).LE.1.0E-3) CODI(2,J41)=0.0
      WRITE(6,101)J41,CODI(1,J41),CODI(2,J41)
  402 CONTINUE
      DO 4100 I41=4101,4124,1
      CODI1=CODI(1,I41-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I41-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I41,CODI1,CODI2
 4100 CONTINUE
      DO 4200 I=4201,4223,2
      CODI1=CODI(1,I-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 4200 CONTINUE
      DO 4300 I43=4301,4324,1
      CODI1=CODI(1,I43-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I43-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I43,CODI1,CODI2
 4300 CONTINUE
      DO 4400 I44=4401,4423,2
      CODI1=CODI(1,I44-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I44-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I44,CODI1,CODI2
 4400 CONTINUE
      DO 4500 I45=4501,4524,1
      CODI1=CODI(1,I45-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I45-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I45,CODI1,CODI2
 4500 CONTINUE
      DO 4600 I46=4601,4623,2
      CODI1=CODI(1,I46-3000)+PITCH*0.5*S3
      CODI2=CODI(2,I46-3000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I46,CODI1,CODI2
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4600 CONTINUE
     DO 4700 I=4701,4724,1
     CODI1=CODI(1,I-3000)+PITCH*0.5*S3
     CODI2=CODI(2,I-3000)-PITCH*0.5
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I,CODI1,CODI2
4700 CONTINUE
     DO 4800 I=4801,4823,2
     CODI1=CODI(1,I-3000)+PITCH*0.5*S3
     CODI2=CODI(2,I-3000)-PITCH*0.5
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I,CODI1,CODI2
4800 CONTINUE
     DO 501 J51=510,524,1
     CODI(1,J51)=CODI(1,J51-400)
     CODI(2,J51)=CODI(2,J51-400)-PITCH
     IF(ABS(CODI(1,J51)).LE.1.0E-3) CODI(1,J51)=0.0
     IF(ABS(CODI(2,J51)).LE.1.0E-3) CODI(2,J51)=0.0
     WRITE(6,101)J51,CODI(1,J51),CODI(2,J51)
 501 CONTINUE
     DO 5100 I51=5101,5124,1
     CODI1=CODI(1,I51-4000)
     CODI2=CODI(2,I51-4000)-PITCH
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I51,CODI1,CODI2
5100 CONTINUE
     DO 5200 I=5201,5223,2
     CODI1=CODI(1,I-4000)
     CODI2=CODI(2,I-4000)-PITCH
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I,CODI1,CODI2
5200 CONTINUE
     DO 5300 I=5301,5324,1
     CODI1=CODI(1,I-4000)
     CODI2=CODI(2,I-4000)-PITCH
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I,CODI1,CODI2
5300 CONTINUE
     DO 5400 I54=5401,5423,2
     CODI1=CODI(1,I54-4000)
     CODI2=CODI(2,I54-4000)-PITCH
     IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
     IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
     WRITE(6,101)I54,CODI1,CODI2
5400 CONTINUE
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      DO 5500 I55=5501,5524,1
      CODI1=CODI(1,I55-4000)
      CODI2=CODI(2,I55-4000)-PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I55,CODI1,CODI2
 5500 CONTINUE
      DO 5600 I56=5601,5623,2
      CODI1=CODI(1,I56-4000)
      CODI2=CODI(2,I56-4000)-PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I56,CODI1,CODI2
 5600 CONTINUE
      DO 5700 I=5701,5724,1
      CODI1=CODI(1,I-4000)
      CODI2=CODI(2,I-4000)-PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 5700 CONTINUE
      DO 5800 I=5801,5823,2
      CODI1=CODI(1,I-4000)
      CODI2=CODI(2,I-4000)-PITCH
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 5800 CONTINUE
      DO 601 J61=606,620,1
      CODI(1,J61)=CODI(1,J61-500)-PITCH*0.5*S3
      CODI(2,J61)=CODI(2,J61-500)-PITCH*0.5
      IF(ABS(CODI(1,J61)).LE.1.0E-3) CODI(1,J61)=0.0
      IF(ABS(CODI(2,J61)).LE.1.0E-3) CODI(2,J61)=0.0
      WRITE(6,101)J61,CODI(1,J61),CODI(2,J61)
  601 CONTINUE
      DO 6100 I61=6101,6124,1
      CODI1=CODI(1,I61-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I61-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I61,CODI1,CODI2
 6100 CONTINUE
      DO 6200 I=6201,6223,2
      CODI1=CODI(1,I-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 6200 CONTINUE
      DO 6300 I63=6301,6324,1
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      CODI1=CODI(1,I63-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I63-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I63,CODI1,CODI2
 6300 CONTINUE
      DO 6400 I64=6401,6423,2
      CODI1=CODI(1,I64-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I64-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I64,CODI1,CODI2
 6400 CONTINUE
      DO 6500 I65=6501,6524,1
      CODI1=CODI(1,I65-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I65-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I65,CODI1,CODI2
 6500 CONTINUE
      DO 6600 I66=6601,6623,2
      CODI1=CODI(1,I66-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I66-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I66,CODI1,CODI2
 6600 CONTINUE
      DO 6700 I=6701,6724,1
      CODI1=CODI(1,I-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 6700 CONTINUE
      DO 6800 I=6801,6823,2
      CODI1=CODI(1,I-5000)-PITCH*0.5*S3
      CODI2=CODI(2,I-5000)-PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 6800 CONTINUE
      DO 700 J71=706,716,1
      CODI(1,J71)=CODI(1,J71-600)-PITCH*0.5*S3
      CODI(2,J71)=CODI(2,J71-600)+PITCH*0.5
      IF(ABS(CODI(1,J71)).LE.1.0E-3) CODI(1,J71)=0.0
      IF(ABS(CODI(2,J71)).LE.1.0E-3) CODI(2,J71)=0.0
      WRITE(6,101)J71,CODI(1,J71),CODI(2,J71)
  700 CONTINUE
      DO 7100 I71=7101,7124,1
      CODI1=CODI(1,I71-6000)-PITCH*0.5*S3
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      CODI2=CODI(2,I71-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I71,CODI1,CODI2
 7100 CONTINUE
      DO 7200 I=7201,7223,2
      CODI1=CODI(1,I-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 7200 CONTINUE
      DO 7300 I73=7301,7324,1
      CODI1=CODI(1,I73-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I73-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I73,CODI1,CODI2
 7300 CONTINUE
      DO 7400 I74=7401,7423,2
      CODI1=CODI(1,I74-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I74-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I74,CODI1,CODI2
 7400 CONTINUE
      DO 7500 I75=7501,7524,1
      CODI1=CODI(1,I75-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I75-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I75,CODI1,CODI2
 7500 CONTINUE
      DO 7600 I76=7601,7623,2
      CODI1=CODI(1,I76-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I76-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I76,CODI1,CODI2
 7600 CONTINUE
      DO 7700 I=7701,7724,1
      CODI1=CODI(1,I-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I-6000)+PITCH*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 7700 CONTINUE
      DO 7800 I=7801,7823,2
      CODI1=CODI(1,I-6000)-PITCH*0.5*S3
      CODI2=CODI(2,I-6000)+PITCH*0.5
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      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
 7800 CONTINUE
      DO 900 J90=901,948,1
      CODI(1,J90)=ROU*COS((J90-901)*PAI/24)
      CODI(2,J90)=ROU*SIN((J90-901)*PAI/24)
      IF(ABS(CODI(1,J90)).LE.1.0E-3) CODI(1,J90)=0.0
      IF(ABS(CODI(2,J90)).LE.1.0E-3) CODI(2,J90)=0.0
      WRITE(6,101)J90,CODI(1,J90),CODI(2,J90)
  900 CONTINUE
      CODI(1,801)=(CODI(1,323)+CODI(1,901))*0.5
      CODI(2,801)=(CODI(2,323)+CODI(2,901))*0.5
      IF(ABS(CODI(1,801)).LE.1.0E-3) CODI(1,801)=0.0
      IF(ABS(CODI(2,801)).LE.1.0E-3) CODI(2,801)=0.0
      WRITE(6,901)CODI(1,801),CODI(2,801)
  901 FORMAT(' 801',',',E11.4,',',E11.4)
      DO 800 J81=803,809,2
      CODI1=(CODI(1,(J81-502))+CODI(1,(J81+100)))*0.5
      CODI2=(CODI(2,(J81-502))+CODI(2,(J81+100)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)J81,CODI1,CODI2
  800 CONTINUE
      DO 810 I=811,819,2
      CODI1=(CODI(1,(I-608))+CODI(1,(I+98)))*0.5
      CODI2=(CODI(2,(I-608))+CODI(2,(I+98)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
  810 CONTINUE
      DO 820 I=821,829,2
      CODI1=(CODI(1,(I-114))+CODI(1,(I+96)))*0.5
      CODI2=(CODI(2,(I-114))+CODI(2,(I+96)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
  820 CONTINUE
      DO 830 I=831,839,2
      CODI1=(CODI(1,(I-220))+CODI(1,(I+94)))*0.5
      CODI2=(CODI(2,(I-220))+CODI(2,(I+94)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
  830 CONTINUE
      DO 840 I=841,849,2
      CODI1=(CODI(1,(I-326))+CODI(1,(I+92)))*0.5
      CODI2=(CODI(2,(I-326))+CODI(2,(I+92)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
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      WRITE(6,101)I,CODI1,CODI2
  840 CONTINUE
      DO 850 I=851,855,2
      CODI1=(CODI(1,(I-432))+CODI(1,(I+90)))*0.5
      CODI2=(CODI(2,(I-432))+CODI(2,(I+90)))*0.5
      IF(ABS(CODI1).LE.1.0E-3) CODI1=0.0
      IF(ABS(CODI2).LE.1.0E-3) CODI2=0.0
      WRITE(6,101)I,CODI1,CODI2
  850 CONTINUE
      CODI1=(CODI(1,401)+CODI(1,947))*0.5
      CODI2=(CODI(2,401)+CODI(2,947))*0.5
      WRITE(6,102)CODI1,CODI2
  102 FORMAT(' 857',',',E11.4,',',E11.4)
      CODI1=(CODI(1,403)+CODI(1,901))*0.5
      CODI2=(CODI(2,403)+CODI(2,901))*0.5
      WRITE(6,103)CODI1,CODI2
  103 FORMAT(' 859',',',E11.4,',',E11.4)
      WRITE(6,8001)
 8001 FORMAT('*NSET,NSET=CI14'/'5709')
      WRITE(6,8100)
 8100 FORMAT('*ELEMENT,TYPE=CPS8R'/

1'10101,10103,10101,10301,10303,10102,10201,10302,10203'/

1'10401,10703,10701,10901,10903,10702,10801,10902,10803'/

1'10124,10101,10147,10347,10301,10148,10247,10348,10201'/

1'10424,10701,10747,10947,10901,10748,10847,10948,10801'/
     1'10601,11103,11101,101,102,11102,11201,125,11203'/
     1'10602,11105,11103,102,103,11104,11203,126,11205'/
     1'10603,11107,11105,103,104,11106,11205,127,11207'/
     1'10604,11109,11107,104,105,11108,11207,128,11209'/
     1'10605,11111,11109,105,106,11110,11209,129,11211'/
     1'10606,11113,11111,106,107,11112,11211,130,11213'/
     1'10607,11115,11113,107,108,11114,11213,131,11215'/
     1'10608,11117,11115,108,109,11116,11215,132,11217'/
     1'10609,11119,11117,109,110,11118,11217,133,11219'/
     1'10610,11121,11119,110,111,11120,11219,134,11221'/
     1'10611,11123,11121,111,112,11122,11221,135,11223'/
     1'10612,11125,11123,112,113,11124,11223,136,11225'/
     1'10613,11127,11125,113,114,11126,11225,137,11227'/
     1'10614,11129,11127,114,115,11128,11227,138,11229')
      WRITE(6,8103)
 8103 FORMAT(
     1'10615,11131,11129,115,116,11130,11229,139,11231'/
     1'10616,11133,11131,116,117,11132,11231,140,11233'/
     1'10617,11135,11133,117,118,11134,11233,141,11235'/
     1'10618,11137,11135,118,119,11136,11235,142,11237'/
     1'10619,11139,11137,119,120,11138,11237,143,11239'/
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     1'10620,11141,11139,120,121,11140,11239,144,11241'/
     1'10621,11143,11141,121,122,11142,11241,145,11243'/
     1'10622,11145,11143,122,123,11144,11243,146,11245'/
     1'10623,11147,11145,123,124,11146,11245,147,11247'/
     1'10624,11101,11147,124,101,11148,11247,148,11201')
      WRITE(6,8104)
8104 FORMAT('2101,2103,2101,2301,2303,2102,2201,2302,2203'/
     1'2112,2101,2123,2323,2301,2124,2223,2324,2201'/
     1'2401,2703,2701,201,203,2702,2801,202,2803'/
     1'2408,2717,2715,215,109,2716,2815,216,2817'/
     1'2409,2719,2717,109,107,2718,2817,108,2819'/
     1'2410,2721,2719,107,105,2720,2819,106,2821'/
     1'2411,2723,2721,105,223,2722,2821,222,2823'/
     1'2412,2701,2723,223,201,2724,2823,224,2801'/
     1'3101,3103,3101,3301,3303,3102,3201,3302,3203'/
     1'3112,3101,3123,3323,3301,3124,3223,3324,3201')
      WRITE(6,8110)
8110  FORMAT('3401,3703,3701,301,303,3702,3801,302,3803'/
     1'3404,3709,3707,307,201,3708,3807,308,3809'/
     1'3405,3711,3709,201,223,3710,3809,224,3811'/
     1'3406,3713,3711,223,105,3712,3811,222,3813'/
     1'3407,3715,3713,105,103,3714,3813,104,3815'/
     1'3408,3717,3715,103,101,3716,3815,102,3817'/
     1'3409,3719,3717,101,319,3718,3817,318,3819'/
     1'3410,3721,3719,319,321,3720,3819,320,3821'/
     1'3411,3723,3721,321,323,3722,3821,322,3823'/
     1'3412,3701,3723,323,301,3724,3823,324,3801')
      WRITE(6,8115)
8115 FORMAT('4101,4103,4101,4301,4303,4102,4201,4302,4203'/
     1'4112,4101,4123,4323,4301,4124,4223,4324,4201'/
     1'4401,4703,4701,401,403,4702,4801,402,4803'/
     1'4402,4705,4703,403,321,4704,4803,404,4805'/
     1'4403,4707,4705,321,319,4706,4805,320,4807'/
     1'4404,4709,4707,319,101,4708,4807,318,4809'/
     1'4405,4711,4709,101,123,4710,4809,124,4811')
      WRITE(6,8120)
8120  FORMAT('4406,4713,4711,123,121,4712,4811,122,4813'/
     1'4407,4715,4713,121,415,4714,4813,414,4815'/
     1'4408,4717,4715,415,417,4716,4815,416,4817'/
     1'4409,4719,4717,417,419,4718,4817,418,4819'/
     1'4410,4721,4719,419,421,4720,4819,420,4821'/
     1'4411,4723,4721,421,423,4722,4821,422,4823'/
     1'4412,4701,4723,423,401,4724,4823,424,4801'/
     1'5101,5103,5101,5301,5303,5102,5201,5302,5203'/
     1'5112,5101,5123,5323,5301,5124,5223,5324,5201'/
     1'5401,5703,5701,417,415,5702,5801,416,5803'/
     1'5402,5705,5703,415,121,5704,5803,414,5805'/
     1'5403,5707,5705,121,119,5706,5805,120,5807')
      WRITE(6,8125)
8125  FORMAT('5404,5709,5707,119,117,5708,5807,118,5809'/
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     1'5405,5711,5709,117,511,5710,5809,510,5811'/
     1'5406,5713,5711,511,513,5712,5811,512,5813'/
     1'5407,5715,5713,513,515,5714,5813,514,5815'/
     1'5408,5717,5715,515,517,5716,5815,516,5817'/
     1'5409,5719,5717,517,519,5718,5817,518,5819'/
     1'5410,5721,5719,519,521,5720,5819,520,5821'/
     1'5411,5723,5721,521,523,5722,5821,522,5823'/
     1'5412,5701,5723,523,417,5724,5823,524,5801'/
     1'6101,6103,6101,6301,6303,6102,6201,6302,6203'/
     1'6112,6101,6123,6323,6301,6124,6223,6324,6201'/
     1'6401,6703,6701,117,115,6702,6801,116,6803'/
     1'6402,6705,6703,115,113,6704,6803,114,6805')
      WRITE(6,8130)
8130 FORMAT('6403,6707,6705,113,607,6706,6805,606,6807'/
     1'6404,6709,6707,607,609,6708,6807,608,6809'/
     1'6405,6711,6709,609,611,6710,6809,610,6811'/
     1'6406,6713,6711,611,613,6712,6811,612,6813'/
     1'6407,6715,6713,613,615,6714,6813,614,6815'/
     1'6408,6717,6715,615,617,6716,6815,616,6817'/
     1'6409,6719,6717,617,619,6718,6817,618,6819'/
     1'6410,6721,6719,619,513,6720,6819,620,6821'/
     1'6411,6723,6721,513,511,6722,6821,512,6823'/
     1'6412,6701,6723,511,117,6724,6823,510,6801'/
     1'7101,7103,7101,7301,7303,7102,7201,7302,7203'/
     1'7112,7101,7123,7323,7301,7124,7223,7324,7201'/
     1'7401,7703,7701,109,215,7702,7801,216,7803'/
     1'7402,7705,7703,215,213,7704,7803,214,7805')
      WRITE(6,8135)
8135 FORMAT('7403,7707,7705,213,707,7706,7805,706,7807'/
     1'7404,7709,7707,707,709,7708,7807,708,7809'/
     1'7405,7711,7709,709,711,7710,7809,710,7811'/
     1'7406,7713,7711,711,713,7712,7811,712,7813'/
     1'7407,7715,7713,713,715,7714,7813,714,7815'/
     1'7408,7717,7715,715,609,7716,7815,716,7817'/
     1'7409,7719,7717,609,607,7718,7817,608,7819'/
     1'7410,7721,7719,607,113,7720,7819,606,7821'/
     1'7411,7723,7721,113,111,7722,7821,112,7823'/
     1'7412,7701,7723,111,109,7724,7823,110,7801'/
     1'9001,301,323,901,903,324,801,902,803'/
     1'9002,303,301,903,905,302,803,904,805'/
     1'9005,201,307,909,203,308,809,811,202'/
     1'9006,205,203,909,911,204,811,910,813')
      WRITE(6,8140)
8140 FORMAT('9010,213,211,917,707,212,819,821,706'/
     1'9011,709,707,917,919,708,821,918,823'/
     1'9015,609,715,925,611,716,829,831,610'/
     1'9016,613,611,925,927,612,831,926,833'/
     1'9020,513,619,933,515,620,839,841,514'/
     1'9021,517,515,933,935,516,841,934,843'/
     1'9025,417,523,941,419,524,849,851,418'/
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     1'9026,421,419,941,943,420,851,942,853'/
     1'9028,401,423,945,947,424,855,946,857'/
     1'9029,403,401,947,901,402,857,948,859'/
     1'9030,321,403,901,323,404,859,801,322')
      WRITE(6,8145)
 8145 FORMAT('*ELEMENT,TYPE=INTER3,ELSET=SURFACE'/
     1'1,10503,10502,10501,10703,10702,10701'/
     1'24,10501,10548,10547,10701,10748,10747'/

1'*ELGEN,ELSET=SURFACE'/'1,23,2,1'/'*INTERFACE,ELSET=SURFACE'
C    1'*FRICTION'/E10.4,',10000.0'/
C    1'*NORMAL'/'SURFACE,CLT,1.0,0.0'
     1)
      WRITE(6,8200)
 8200 FORMAT('*ELGEN'/'1,23,2,1'/
     1'10101,23,2,1,2,200,100'/
     1'10401,23,2,1,2,200,100'/
     1'10124,2,200,100'/
     1'10424,2,200,100'/
     1'2101,11,2,1,3,200,100'/
     1'3101,11,2,1,3,200,100'/
     1'4101,11,2,1,3,200,100'/
     1'5101,11,2,1,3,200,100'/
     1'6101,11,2,1,3,200,100'/
     1'7101,11,2,1,3,200,100'/
     1'2112,3,200,100'/
     1'3112,3,200,100'/
     1'4112,3,200,100'/
     1'5112,3,200,100'/
     1'6112,3,200,100'/
     1'7112,3,200,100')
      WRITE(6,8205)
 8205 FORMAT(
     1'*ELGEN'/'2401,7,2,1'/'3401,3,2,1'/'4408,4,2,1'/
     1'5406,6,2,1'/'6404,6,2,1'/'7404,4,2,1')
      WRITE(6,8210)
 8210 FORMAT('9002,3,2,1'/'9006,4,2,1'/
     1'9011,4,2,1'/'9016,4,2,1'/'9021,4,2,1'/'9026,2,2,1'/
     1'*MPC'/'2,125,101,102,103'/'2,126,101,102,103'/
      1'2,127,103,104,105'/'2,128,103,104,105'/
       1'2,129,105,106,107'/'2,130,105,106,107'/
     1'2,131,107,108,109'/'2,132,107,108,109'/
      1'2,133,109,110,111'/'2,134,109,110,111'/
       1'2,135,111,112,113'/'2,136,111,112,113'/
     1'2,137,113,114,115'/'2,138,113,114,115'/
      1'2,139,115,116,117'/'2,140,115,116,117'/
       1'2,141,117,118,119'/'2,142,117,118,119'/
     1'2,143,119,120,121'/'2,144,119,120,121'/
      1'2,145,121,122,123'/'2,146,121,122,123'/
       1'2,147,123,124,101'/'2,148,123,124,101')
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      WRITE(6,8211)
 8211 FORMAT(
     1'*ELSET,ELSET=TUBE11,GENERATE'/'10101,10124,1'/
     1'*ELSET,ELSET=TUBE21,GENERATE'/'2101,2112,1'/
     1'*ELSET,ELSET=TUBE31,GENERATE'/'3101,3112,1'/
     1'*ELSET,ELSET=TUBE41,GENERATE'/'4101,4112,1'/
     1'*ELSET,ELSET=TUBE51,GENERATE'/'5101,5112,1'/
     1'*ELSET,ELSET=TUBE61,GENERATE'/'6101,6112,1'/
     1'*ELSET,ELSET=TUBE71,GENERATE'/'7101,7112,1'/
     1'*ELSET,ELSET=TUBES1'/
      1'TUBE11,TUBE21,TUBE31,TUBE41,TUBE51,TUBE61,
     1TUBE71'/
     1'*ELSET,ELSET=TUBE12,GENERATE'/'10201,10224,1'/
     1'*ELSET,ELSET=TUBE1,GENERATE'/
      1'10101,10124,1'/'10201,10224,1'/
     1'*ELSET,ELSET=TUBE2,GENERATE'/
      1'2101,2112,1'/'2201,2212,1'/
     1'*ELSET,ELSET=TUBE211'/'2101'/
     1'*ELSET,ELSET=TUBE3,GENERATE'/
      1'3101,3112,1'/'3201,3212,1'/
     1'*ELSET,ELSET=TUBE4,GENERATE'/
      1'4101,4112,1'/'4201,4212,1'/
     1'*ELSET,ELSET=TUBE5,GENERATE'/
      1'5101,5112,1'/'5201,5212,1'/
     1'*ELSET,ELSET=TUBE6,GENERATE'/
      1'6101,6112,1'/'6201,6212,1'/
     1'*ELSET,ELSET=TUBE7,GENERATE'/
      1'7101,7112,1'/'7201,7212,1'/
     1'*ELSET,ELSET=TUBE'/
      1'TUBE1,TUBE2,TUBE3,TUBE4,TUBE5,TUBE6,TUBE7')
      WRITE(6,8215)
 8215 FORMAT(
     1'*ELSET,ELSET=SHEET1,GENERATE'/'10401,10424,1'/
     1'10501,10524,1'/'10601,10624,1'/
     1'*ELSET,ELSET=SHEET2,GENERATE'/
      1'2401,2412,1'/'2301,2312,1'/
     1'*ELSET,ELSET=SHEET3,GENERATE'/
      1'3401,3412,1'/'3301,3312,1'/
     1'*ELSET,ELSET=SHEET4,GENERATE'/
      1'4401,4412,1'/'4301,4312,1')
      WRITE(6,8220)
 8220 FORMAT ('*ELSET,ELSET=SHEET5,GENERATE'/
      1'5401,5412,1'/'5301,5312,1'/
     1'*ELSET,ELSET=SHEET6,GENERATE'/
      1'6401,6412,1'/'6301,6312,1'/
     1'*ELSET,ELSET=SHEET7,GENERATE'/
      1'7401,7412,1'/'7301,7312,1'/
     1'*ELSET,ELSET=SHEET9,GENERATE'/'9001,9030,1'/
     1'*ELSET,ELSET=SHEET'/
      1'SHEET1,SHEET2,SHEET3,SHEET4,SHEET5,SHEET6'/
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     1'SHEET7,SHEET9')
      WRITE(6,8400)ET,POT,YT,ES,POS,YS
 8400 FORMAT('*SOLID SECTION,ELSET=TUBE,MATERIAL=A'/
     1'*MATERIAL,NAME=A'/
     1'*ELASTIC'/E10.4,',',E10.4/
     1'*PLASTIC'/E10.4/
     1'*SOLID SECTION,ELSET=SHEET,MATERIAL=B'/
     1'*MATERIAL,NAME=B'/
     1'*ELASTIC'/E10.4,',',E10.4/
     1'*PLASTIC'/E10.4)
      WRITE(6,8500)
 8500 FORMAT('*BOUNDARY'/'901,2'/'925,2'/'913,1'/'937,1')
C     WRITE(6,8600)
 8600 FORMAT('*PLOT'/'*DRAW')
      WRITE(6,8700)P
 8700 FORMAT

1('*STEP,INC=80,CYCLE=12'/'*STATIC,PTOL=150.0'/'0.15,1.0'
   1/'*DLOAD,OP=NEW'/'TUBE11,P1,',E10.4/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/'U'/
     1'*END STEP')
      WRITE(6,8800)
 8800 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=150.0'/
     1'*DLOAD,OP=NEW'/'TUBE11,P1,0.0'/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/
     1'U'/
C    1'*PLOT'/'*DISPLACED'/
     1'*END STEP')
      WRITE(6,8801)P
 8801 FORMAT('*STEP,INC=80,CYCLE=12'/
      1'*STATIC,PTOL=150.0'/'0.15,1.0'/
     1'*DLOAD,OP=NEW'/'TUBE21,P1,',E10.4/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/'U'/
     1'*END STEP')
      WRITE(6,8802)
 8802 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=150.0'/
     1'*DLOAD,OP=NEW'/'TUBE21,P1,0.0'/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
```

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     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/
C    1'*PLOT'/'*DISPLACED'/
     1'*END STEP')
      WRITE(6,8803)P
 8803 FORMAT('*STEP,INC=80,CYCLE=12'/
      1'*STATIC,PTOL=150.0'/'0.15,1.0'/
     1'*DLOAD,OP=NEW'/'TUBE41,P1,',E10.4/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/1'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/'U'/
     1'*END STEP')
      WRITE(6,8804)
 8804 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=150.0'/
     1'*DLOAD,OP=NEW'/'TUBE41,P1,0.0'/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
AT NODES'
     1/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/
C    1'*PLOT'/'*DISPLACED'/
     1'*END STEP')
      WRITE(6,8805)P
 8805 FORMAT('*STEP,INC=80,CYCLE=12'/
      1'*STATIC,PTOL=150.0'/'0.15,1.0'/
     1'*DLOAD,OP=NEW'/'TUBE31,P1,',E10.4/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/'U'/
     1'*END STEP')
      WRITE(6,8806)
 8806 FORMAT('*STEP,INC=20,CYCLE=6'/'*STATIC,PTOL=150.0'/
     1'*DLOAD,OP=NEW'/'TUBE31,P1,0.0'/
     1'*EL PRINT,ELSET=TUBE211,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
     1'*EL PRINT,ELSET=SURFACE,FREQUENCY=40,POSITION=AVERAGED
     1AT NODES'/'S'/
C    1'*PRINT,CONTACT=YES'/
     1'*NODE PRINT,NSET=CI14,FREQUENCY=40'/
C    1'*PLOT'/'*DISPLACED'/
     1'*END STEP')
      STOP
      END
```