Experimental and numerical study on the earth pressure coefficient in a vertical backfilled opening

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Abstract. Determining lateral earth pressure coefficient (EPC) K is a classic problem in geotechnical engineering. It is a key parameter for estimating the stresses in backfilled openings. For backfilled openings with rigid and immobile walls, some suggested using the Jaky's at-rest earth pressure coefficient K_0 while other suggested taking the Rankine's active earth pressure coefficient K_a. A single value was proposed for the entire backfilled opening. To better understand the distributions of stresses and K in a backfilled opening, a series of laboratory tests have been conducted. The horizontal and vertical normal stresses at the center and near the wall of the opening were measured. The values of K at the center and near the wall were then calculated with the measured horizontal and vertical normal stresses. The results show that the values of K are close to K_a at the center and close to K_0 near the wall. Furthermore, the experimental results show that the horizontal stress is almost the same at the center and near the wall, indicating a uniform distribution from the center to the wall. It can be estimated by analytical solutions using either K_a or K_0. The vertical stress is higher near the center than near the wall. Its analytical estimation can only be done by using K_a at the center and K_0 near the wall. Finally, the test results were used to calibrate a numerical model of FLAC2D, which was then used to analyze the influence of column size on the stresses and K in the backfilled opening.

Keywords: backfill; laboratory tests; simultaneous measurement of vertical and horizontal stresses; earth pressure coefficient; arching

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1. Introduction

In geotechnical engineering, it is a critical task to evaluate the stresses in a backfill placed in a rigid confining structure, such as silos, municipal trenches, retaining walls, and underground mine stopes (Keshavarz and Pooresmaeil 2016, Lee 2019, Arefnia *et al*. 2020). This requires the knowledge of an important parameter, called lateral earth pressure coefficient. When the confining structures are stiff and immobile, some researchers (Take and Valsangkar 2001, Blight 2006, Winch 1999, Pirapakaran and Sivakugan 2007a, 2007b, Fahey *et al*. 2009, Singh *et al*. 2010, Ting *et al*. 2011) recommend the use of the Jaky's at-rest earth pressure coefficient K_0 as follows (Jaky 1948):

$$K_0 = 1 - \sin\phi \quad (1)$$

where ϕ (°) is the internal friction angle of the retained soil. Meanwhile, some others (Pirapakaran and Sivakugan 2007b, Zheng and Yu 2015, Hong *et al*. 2016) propose to use the Rankine's active earth pressure coefficient K_a as follows (McCarthy 1988, CGS 2006, Das 2010):

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45° - \frac{\phi}{2}) \quad (2)$$

It is interesting to note that some researchers (Mesri and Hayat 1993, Wanatowski and Chu 2007, Lee *et al*. 2013) suggested using Eq. (2) but still with the name of at-rest earth pressure coefficient.

Those who propose to use at-rest earth pressure coefficient are mainly based on a theory in which the backfill is in at-rest state as long as the confining walls are stiff and immobile. This is however an incomplete understanding and poor interpretation of the earth pressure coefficient theory, omni-presented in any textbooks of soil mechanics (McCarthy 1988, Terzaghi *et al*. 1996). One recalls that lateral earth pressure coefficient K is defined as the ratio of horizontal to vertical principal effective stresses (Coulomb 1776, Rankine 1857, Jaky 1944, Terzaghi *et al*. 1996). It is related to the movement of a stiff and zero thickness retaining wall by considering a pre-existing natural soil, which is initially in an at-rest state. The situation is completely different than the backfilled openings, by which the confining structures (silos, trenches, retaining walls, mine stopes) exist first, followed by the placement of a backfill. The state of the backfill placed in the confining structures is unknown. The immobilization of the confining walls cannot be considered as the necessary and sufficient conditions for the backfill to be considered as in an at-rest state. Rather, Sobhi *et al*. (2017) have shown through numerical modeling that the backfill along the vertical center line of a mine backfilled stope can be in yield state and the value of the horizontal principal stress over the vertical principal stress is close to the Rankine's active earth pressure coefficient. This confirms the theory of active failure or yielding of placed materials in confining structures (Walker 1966, Walters 1973).

Another argument used to support the use of at-rest earth pressure coefficient associated with immobile walls comes from some experimental results obtained by K_0 tests on normally consolidated soils, a name coming from the fact that the confining walls are stiff and bear little deformation during the tests. The first commonly conducted K_0 test is through oedometer designed for one-dimensional consolidation tests (Komornik and Zeilten 1965, Brooker and Ireland 1965, Zhu and Clark 1994, Vardhanabhuti and Mesri 2007, Lirer *et al*. 2011, Zhu *et al*. 2019). The vertical stress is applied through a rigid plate on top of the soil sample (Brooker and Ireland 1965, Mesri and Hayat 1993, Zhu and Clark 1994, Yamamuro *et al*. 1996, Lirer *et al*. 2011, Gao and Wang 2014). The result interpretation assumes that the vertical stress is uniformly distributed across the whole sample. It is probably not the case as the possible arching effect between the soil sample and sidewall can render a uniform vertical stress distribution more unlikely (Li and Aubertin 2009a). Moreover, the tests only involve the measurement of the horizontal stress at sidewall. The result interpretation needs once again an assumption of uniform horizontal stress distribution across the whole sample. This assumption, however, has never been validated by experimental results.

Another type of K_0 test is triaxial compression tests (Bishop and Henkel 1957, Feda 1984, Watabe *et al*. 2003, Shogaki and Nochikawa 2004). Axial stress is applied and lateral stress is measured while lateral deformation is not allowed for the tested soil. This test suffers the same limitations as the oedometer K_0 test with the possible non-uniform distribution of horizontal and vertical stresses within the tested soil associated with the loading system and unavoidable arching effect along the soil sample and sidewall of the triaxial cell.

Gao and Wang (2013) conducted K_0 test by using direct shear box with tactile pressure sensors to measure the vertical and lateral stresses. Their test results showed that the applied vertical pressure at the top of the plate can be significantly different than the measured one at the bottom of the soil sample, due probably to the non-uniform distribution of stresses and arching effect between the soil and shear box walls.

It should be mentioned that the friction along the soil-wall interfaces is commonly considered as negligible, especially with the application of lubricant materials. Recently, Zheng *et al*. (2021) have shown that the commonly used lubricant materials (e.g., Teflon and grease) cannot significantly reduce sidewall friction. Arching is not a negligible phenomenon and the stresses are not uniformly distributed in the tested sample.

The above analysis indicates that the result interpretation of K_0 tests involves some nonrealistic hypotheses, which overly simplify the mechanical response of the backfill confined by rigid and immobile walls. The unique at-rest earth pressure coefficient based on K_0 tests is questionable.

Yang et al. (2017, 2018) conducted numerical modeling to study the stresses in backfilled openings. Their results showed that the value of the horizontal principal stress over the vertical principal stress along the vertical center line can be close to the Rankine's active earth pressure coefficient K_a as long as the value of the Poisson's ratio μ or ϕ of the backfill is smaller than their respective critical value or the at-rest earth pressure coefficient due to Poisson's ratio $(K_0)_\mu$ [$= \mu/(1-\mu)$] when the value of μ or ϕ is higher than their respective critical value. Near the walls, the backfill yields and the minor principal stress over the major principal stress ratio is close to the Rankine's K_a, while the horizontal normal stress over the vertical normal stress ratio is close to the at-rest earth pressure coefficient $(K_0)_\mu$ when an interface element between the backfill and rock wall is introduced.

All the above literature review indicates that the consideration of an at-rest earth pressure coefficient associated with immobile confining walls is based on incomplete elements. An active state along the vertical center line is also possible. In addition, a unique and constant value of horizontal normal stress over vertical normal stress ratio K with a backfilled opening seems to be discussable. Experimental work with simultaneous measurement of the vertical and horizontal normal stresses not only in the center but also close to the wall of confining structure is necessary. This is however absent in the literature. For instance, Frydman and Keissar (1987) conducted centrifuge tests to measure the stresses within a backfill placed between a retaining wall and rockface. Only horizontal stress was measured. A similar work was later done by Take and Valsangkar (2001), with once again the only measurement of horizontal stresses on unyielding retaining walls. Pirapakaran and Sivakugan (2007b) made an instrumentation with only the measurement of the average vertical stress at the base of backfilled openings. Hong et al. (2016) conducted a series of backfilling tests in a model trench with also the measurement of only the vertical stress at the bottom of the trench. Clearly, these experimental works cannot be used to fully understand the distribution of K within a backfilled opening. More experimental work is necessary with the simultaneous measurement of the vertical and horizontal stresses both at the center and near the wall of an opening.

To fill this gap, a series of backfilling tests were performed in laboratory using a cohesionless sand and an instrumented column with the simultaneous measurements of horizontal and vertical stresses at the center and near the wall of a confining structure. The distribution of earth pressure coefficient K was, for the first time, obtained by laboratory tests as a function of the fill thickness from the center to the wall. The uniform horizontal stress distribution across the width of a backfilled opening shown by numerical modeling (Li and Aubertin 2008) is for the first time approved by the obtained experimental results. An analytical solution based on arching theory for estimating the stresses in a circular backfilled opening is validated by the laboratory test results. The choice of K value by using an analytical arching solution for estimating the stresses at the center or near the wall is for the first time proposed. The experimental results were used to calibrate the numerical model in FLAC2D, which was then used for the sensitivity analysis of the opening geometry and interface properties on the stress and K in the backfilled opening.

2. Laboratory tests

2.1. Tested material

The used backfill material is a sand from an open pit mine in Quebec, Canada. Fig. 1 presents the particle size distribution of the used backfill with a coefficient of uniformity $C_u = 2.6$ and a coefficient of curvature $C_c = 1.1$. The measured specific gravity is $G_s = 2.82$. At the loosest state, the unit weight is $\gamma = 16.8$ kN/m^3. This value will be used in the stress estimation in the backfilled opening because the sand placed in the testing column is mostly close to the loosest state.

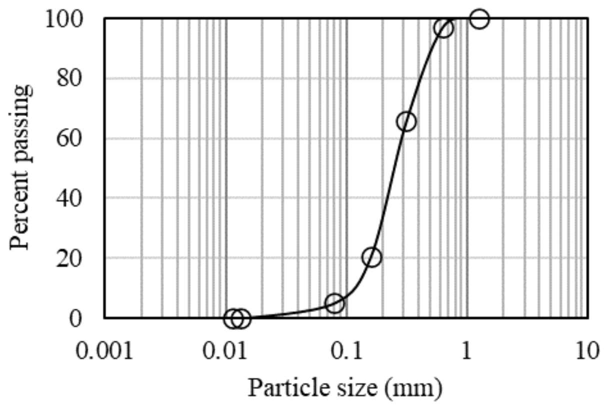


Fig. 1. Grain size distribution curve of the tested sand

Direct shear tests were conducted by following the standard ASTM D3080 (2011) to obtain the internal friction angle ϕ of the sand and the interface friction angle δ between the sand and a piece of Plexiglas, the sidewall material of the used column. To consider the low stress level during the column tests, the applied normal stresses were 10.6, 20, 30, and 50 kPa. When testing the internal friction angle ϕ of the sand, the shear tests were performed by slowly and uniformly placing the sand fill material in the shear box (consists of an upper and lower shear box with the dimension of 60 mm×60 mm×20 mm) to avoid any compact in order for the sand placed in the shear box to be close to the loosest state. When testing the interface friction angle δ between the sand and column inner wall, the lower shear box was placed with metal plates and a Plexiglas, with the top surface at the same level of the lower shear box, the upper shear box was then uniformly filled with air-dried sand to avoid any compaction. The measured ϕ is 35°, while the measured δ is 21°, which is nearly two thirds of ϕ (CGS 2006, Das 2010).

2.2. Instrumentation and testing procedure

In this study, the backfilling and stress measurements tests were conducted in a Plexiglas column with the height of 50 cm and inner diameter of 15.5 cm. Four miniature stress sensors (model: PDA-200 kPa) were mounted at the bottom of the column to monitor the vertical and horizontal stresses of the sand filled in the column from the column center to the wall during the placement. This type of sensor has been widely used in the stress measurements in geotechnical and mining engineering (Jardine *et al*. 2009, Zheng and Li 2020, Jaouhar *et al*. 2021). The diameter and thickness of the sensors were measured as 6.5 and 1 mm, respectively. More detail characteristics of the sensor can be found in Table 1. Fig. 2 presents a diagrammatic sketch of the testing instrumentation (Fig. 2a) and a photo indicating the layout of the four stress sensors in the testing column (Fig. 2b).

Table 1. Characterisits of the TML miniature stress sensor used for the stress measurement

Characteristics	Values
Type	PDA-PB-200 kPa
Capacity	200 kPa
Rated output	1mV/V(2000×10^{-6} strain)
Non-linearity	1%RO
Hysteresis	1%RO
Temperature effect on zero	1%RO/°C
Temperature effect on span	1%/°C
Compensated temperature range	-10 ~ +60°C (no icing)
Allowable temperature range	-20 ~ +70°C (no icing)
Input/Output resistance	350Ω
Allowable exciting voltage	5V
Recommended exciting voltage	2V or less

Two stress sensors (No. 2 and 4) were fixed closely around the center, separated each other by a distance of about 1 cm due to a hole at the base center of the column. This separate distance is also desirable to minimize their mutual interference. The other two stress sensors (No. 1 and 3) were fixed near the wall at two different positions in order to take the advantage of the axis symmetry of the cylindrical column and avoid their mutual interference. Sensors 1 and 2 were glued at the bottom to ensure a tight contact between the sensors and the column bottom.

The input voltage for the used miniature stress sensors was provided by a power supply, which was set as 2 volts. The output voltage from the sensors was too small to be accurately measured. An amplifier was thus first used to augment the output voltage, which was then measured by a multimeter. The stress sensors were calibrated before and after the backfilling and stress measurement tests, with the linear calibration curve shown in Fig. 3. The detail information of the sensor calibration was given in Zheng and Li (2020).

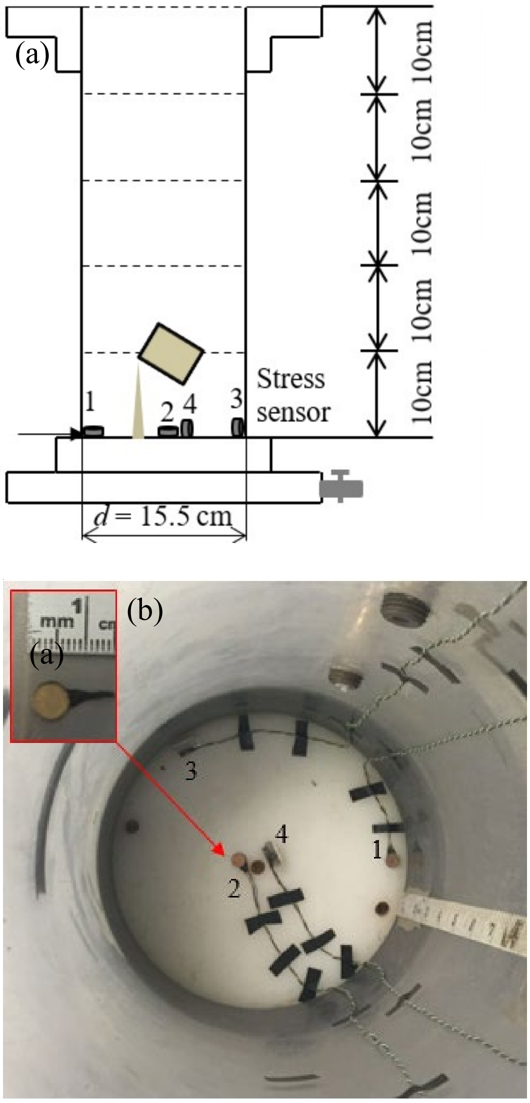


Fig. 2. Testing instrumentation: (a) schematic diagram of the column; (b) photo showing the layout of the four stress sensors

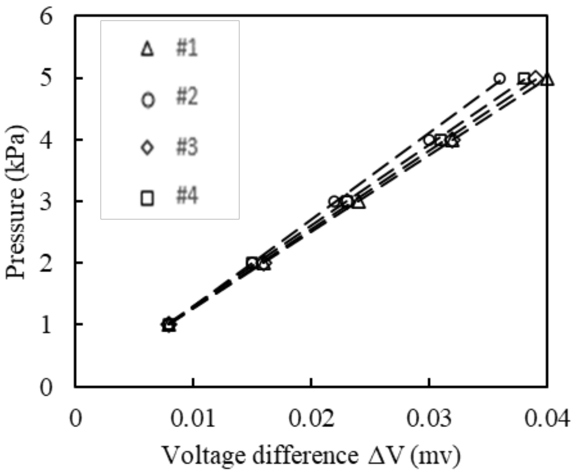


Fig. 3. Calibration results of the four miniature stress sensors

The backfilling and stress measurement tests were started by slowly placing the first layer of backfill 10 cm thick. To minimize the impact of vibration induced by the sand placement, the used sand was first poured in a glass beaker. The glass beaker was then kept down to the opening as close as possible to the bottom of the opening (for the initial backfilling) or the top surface of the previously placed sand layer. The pouring of the sand was realized by keeping an almost constant falling height to result in a uniform backfill in the opening. Data readings were made when the fill height reached 10, 20, 30, 40 and 50 cm, respectively. The backfilling and stress measurements were repeated five times to see the repeatability of the experimental results.

3. Test results and interpretation

Fig. 4 illustrates the variation of the measured σ_v (Fig. 4a) and σ_h (Fig. 4b) at the opening center and close to the opening wall with the sand fill thickness varying from 10 cm to 50 cm. The vertical stress (γh) and horizontal stress ($K_0\gamma h$) estimated by the overburden solution are also included on the figure. Despite the more or less dispersion, the repeatability of the stress measurements can be seen for the measured σ_v and σ_h, indicating the validity of the tests presented in this study.

When the thickness of the backfill is small (i.e. less than 10 cm), the measured σ_v and σ_h at the two positions at the base level of the opening follow almost the vertical stress and horizontal stresses estimated by the overburden solution. This can be considered as a good validation on the accuracy and reliability of the used stress sensors because arching effect can be considered as negligible for backfill with a small thickness. The value of σ_v and σ_h should thus be close to the vertical and horizontal overburden pressures, respectively.

When the backfill thickness exceeds 10 cm, the measured vertical stresses tend to become constant and much smaller than the overburden pressure, indicating that arching effect takes place in the opening. From Fig. 4a, one sees also that the measured σ_v at the opening center are generally higher than those measured close to the opening wall, resulting in non-uniformly distributed vertical stress across the opening width. This is because the sidewall friction induced by the arching effect is along the vertical direction, which can thus reduce the vertical stress applied on the stress sensor near the column wall. This trend has also been illustrated by the numerical modeling performed by several researchers (Li *et al*. 2003, Jaouhar *et al*. 2018, Li and Aubertin 2008, 2009a, 2010, Sivakugan and Sankha 2013).

Regarding the horizontal stress σ_h, Fig. 4b shows that it increases nonlinearly with the backfill thickness. It tends to become constant and always smaller than the horizontal stress calculated by the overburden solution. This shows again that arching effect occurs in the filled opening. Despite the dispersion of the results, one clearly sees the very close horizontal stresses measured at the center and near the wall. These results conform for the first time the uniform horizontal stress distribution shown by numerical modeling of Li et al. (Li *et al*. 2003, Li and Aubertin 2008, 2009a). This behavior can be explained by the fact that the arching effect is only along the fill-wall interface in the vertical direction, which has no influence on the horizontal stress distribution in the backfilled opening.

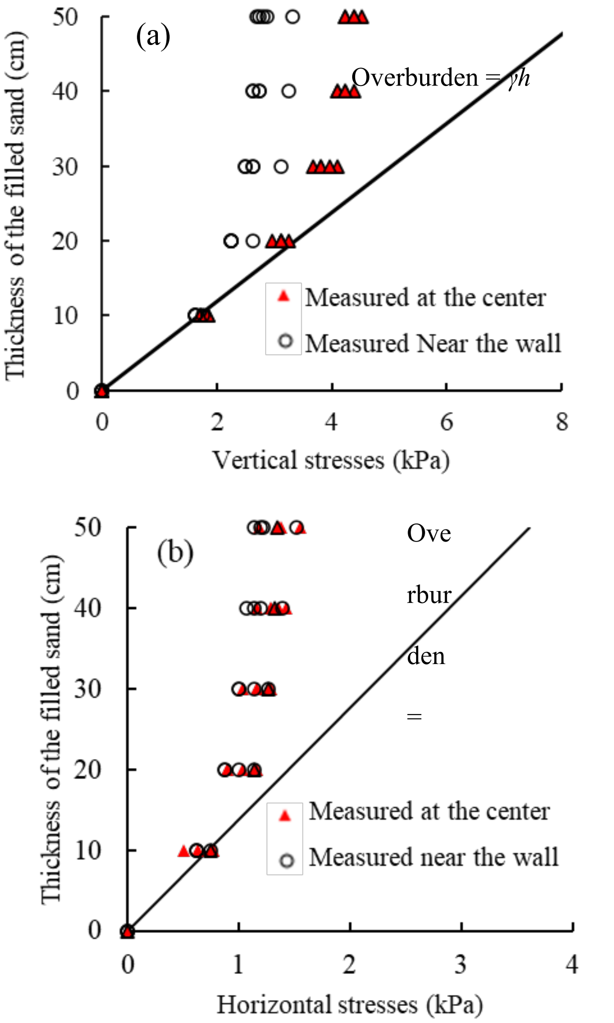


Fig. 4. Measured (a) σ_v and (b) σ_h at the two positions of the base level of the opening filled with backfill thickness varying from 10 cm to 50 cm; plotting of the overburden solution with $\sigma_v = \gamma h$ and $\sigma_h = K_0\gamma h$ [K_0 = 0. 43 based on Eq. (1) using ϕ = 35°]

From the measured σ_v and σ_h, the distribution of K can be obtained at the opening center (Fig. 5a) and near the wall (Fig. 5b). On the figure, the Rankine's active K_a = 0.27 (Eq. (2) using ϕ = 35°) and Jaky's at-rest K_0 = 0. 43 (Eq. (1) using ϕ = 35°) are also plotted. Despite the dispersion of the results, it can be clearly seen that the value of K obtained by the measured σ_h and σ_v is close to the Rankine's K_a at the center and close to Jaky's K_0 near the wall. However, when the thickness of the backfill is small, the average value of K is between K_a and K_0, somewhat closer to K_0. This is because the placement of the backfill in layers could initially generate higher vertical and horizontal stresses than the overburden pressure (Sobhi *et al*. 2017). For backfill with a small thickness (e.g., 10 cm for the first layer), the excess vertical stress in the backfill can be released in the vertical direction while the horizontal stress is locked in the horizontal direction due to the confining walls, resulting in higher value of K. With the increased thickness, the backfill can reach an active (yield) state due to the variation of the deviatoric stress of the backfill, especially at a large depth. These experimental results confirm the numerical results presented by Li and co-workers (Sobhi *et al*. 2017, Yang *et al*. 2017, 2018), who showed that the backfill placed in a confining structure can yield and attain an active state at the center of the opening even though the confining walls are immobile during the filling operation.

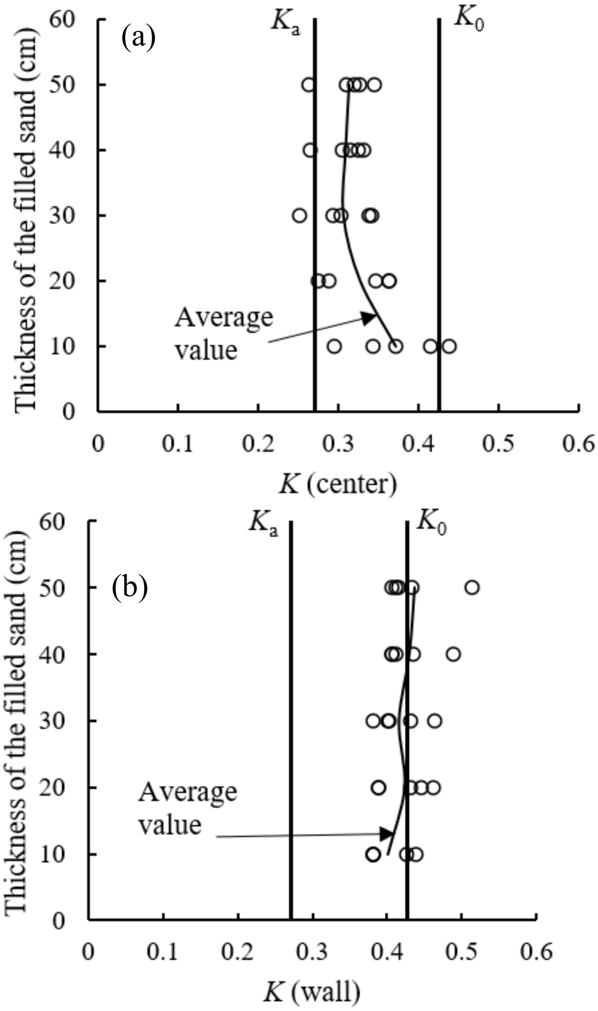


Fig. 5. Distribution of the earth pressure coefficient (EPC) K at the base level of the opening filled with backfill thickness varying from 10 cm to 50 cm, calculated from the measured σ_h and σ_v (a) at the center and (b) close to the wall, plotted with the Rankine's (K_a) and Jaky's (K_0)

4. Validation of arching analytical solution by the experimental results

A number of arching analytical solutions have been proposed over the years with different shapes of cross section of confining structures, such as vertical or inclined trenches (Marston 1930; Li et al. 2012), vertical column (e.g. Pirapakaran and Sivakugan, 2007a, 2007b), 2D or 3D vertical stopes (Aubertin et al. 2003; Li et al. 2003, 2005, 2009b, 2009c), and 2D or 3D inclined stopes

(Caceres 2005; Ting et al. 2011, 2012; Jahanbakhshzadeh et al. 2018a, 2018b, 2019). When a granular material is placed in a confining structure, it tends to move downward due to its gravity and the addition of new layers. As the walls of confining structure are stiff, shear stresses occur along the interfaces between the backfill and the confining walls. This leads to a load transfer from the backfill to the confining walls, resulting in smaller stresses in the backfill compared to those calculated based on the overburden solution. This phenomenon is well known as arching effect (Janssen 1895, Ahmadi and Hosseininia 2018, Chen et al. 2020, Li et al. 2023).

For an opening with a circular cross section as shown in this study, Li *et al*. (2014) applied the arching theory and proposed the following analytical solution to estimate the vertical (σ_v) and horizontal (σ_h) stresses at the base of the backfilled opening:

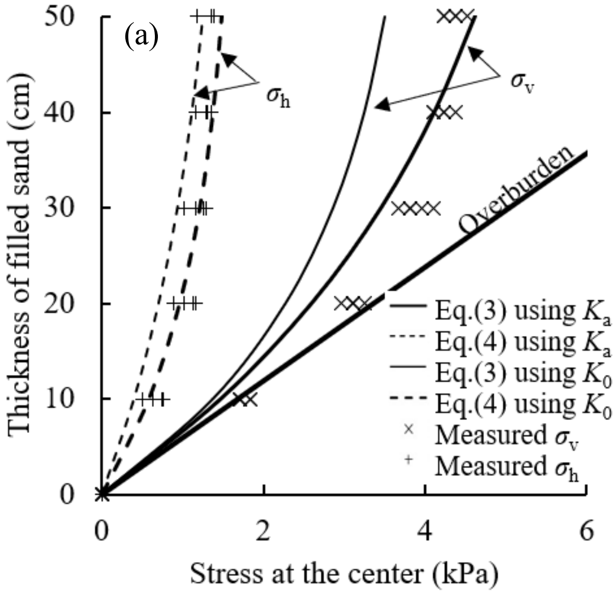
$$\sigma_v = \frac{\gamma d}{4K\tan\delta}\left(1 - exp\left(\frac{-4K\tan\delta}{d}h\right)\right) \quad (3)$$

$$\sigma_h = \frac{\gamma d}{4\tan\delta}\left(1 - exp\left(\frac{-4K\tan\delta}{d}h\right)\right) \quad (4)$$

where γ (kN/m^3) is the unit weight of the backfill; d (m) is the diameter of the column; δ (°) is the friction angle along the interface between the backfill and column inner wall; h (m) is the thickness of the backfill; K is the earth pressure coefficient.

Fig. 6a presents the distribution of the σ_v and σ_h at the center measured by the stress sensors and calculated by applying the analytical solution (Eqs. (3) and (4)) with γ = 16.8 kN/m^3, d = 0.155 m, δ = 21° and considering K as the Rankine's active ($K_a = \tan^2(45° - \phi/2) = 0.27$) and Jaky's at-rest ($K_0 = 1 - \sin\phi = 0.43$) earth pressure coefficients, respectively.

One can see that for the σ_h at the two positions of the opening, good agreements are attained between the experimental measurements and analytical results calculated by Eqs. (3) and (4) using both K_a and K_0. These results are straightforward by observing Eq. (4), which show that the influence of K may become negligible at very shallow and very deep depths. For the σ_v at the center (Fig. 6a), good agreements are attained between the experimental measurements and analytical results calculated by Eq. (3) using K_a. Poor agreements are attained between the experimental measurements and analytical results calculated by Eq. (3) using the Jaky's K_0. The results indicate again that the backfill can reach an active (yielding) state even though the confining walls are immobile during the placement of the backfill as shown by the numerical simulations reported by Sobhi *et al*. (2017) and part of the column test results reported by Li *et al*. (2014). For the σ_v close to the wall (Fig. 6b), good agreements are attained between the experimental measurements and analytical results calculated by Eq. (3) using the Jaky's K_0. Poor agreements are attained between the experimental results and analytical results calculated by Eq. (3) using the Rankine's K_a. This is the first time that the arching solution has been calibrated by the experimental results with the simultaneously measured vertical and horizontal stresses at the center and near the wall. Moreover, it is also the first time that that recommendation is proposed for the use of the arching analytical solution with K_a at the center and K_0 along the wall for calculating the vertical stress at these two positions.



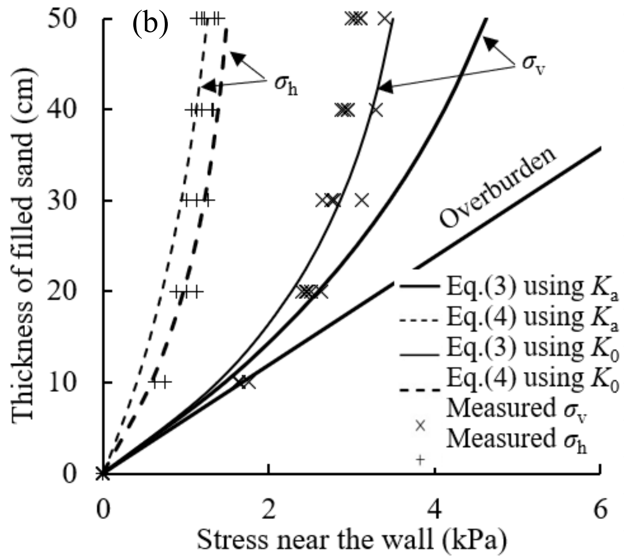


Fig. 6. Comparison between the distribution of the σ_v and σ_h at the base level (a) at the opening center; (b) near the opening wall, measured by the stress sensors and predicted by the arching analytical solution (Eqs. (3) and (4)) using Rankine's (K_a) and Jaky's (K_0) as well as estimated by the overburden solution, for the opening filled with backfill thickness varying from 10 cm to 50 cm

These experimental results also confirm the theoretical and numerical results reported by Li *et al*. (2003), who conducted numerical simulation with FLAC2D and showed that the vertical stresses at the stope center calculated by FLAC2D agreed well with those calculated by the Marston (1930) solution using K_a while the σ_v close to the stope wall calculated by FLAC2D agreed fairly well with those predicted by the Marston solution using K_0. The σ_h at the two positions obtained by FLAC2D agreed relatively well with those calculated by the Marston solution using either K_a or K_0 (somewhat better with K_0).

5. Comparison between numerical modeling and experimental results

In this section, the stress state of the backfill in the opening during the placement were investigated using the commercially available finite-difference program FLAC2D. The numerical model was first validated and calibrated by the experimental results and so they can then be used to study the stress distribution and K in large-scale openings (e.g., mine stopes), which is more representative in field observations.

5.1. Numerical model

Numerical modeling was performed using the two-dimensional FLAC with the option of axisymmetric configuration to simulate the filled opening, as presented in Fig. 2. Fig. 7 presents the numerical model of a radial cross section of the filled opening, built in FLAC2D using the axis of symmetry (AOS).

The opening wall in Plexiglas is modeled as linearly elastic and has the properties of γ_c (unit weight) = 11.9 kN/m^3, μ_c (Poisson's ratio) = 0.3, and, E_c (Young's modulus) = 3.2 GPa (Ting et al., 2012). The tested sand fill is considered as elasto-plastic with the yield function obeying the Mohr-Coulomb criterion. The geotechnical properties of the backfill are measured as unit weight γ =16.8 kN/m^3, dilation angle ψ = 0°, internal friction angle ϕ = 35°, and cohesion c = 0 kPa. The Poisson's ratio μ and Young's modulus E of the backfill are unknow, which will be calibrated to obtain a good fit for the stress measurement results. Interface elements were introduced in the numerical models with interface properties of c_i = 0 kPa (cohesion), δ_i = 21° (friction angle). The value of δ_i was taken as δ_i (= 2/3ϕ), which is a common practice in soil mechanics (CGS 2006, Liu *et al*. 2016). As recommended in the FLAC2D manual (Itasca 2015), the normal stiffness k_n and shear stiffness k_s of the fill-wall interface can be determined from the bulk and shear modulus of the backfill, which are related to its calibrated values of μ and E.

The upper boundary and right (column wall) outer boundary are allowed to have free movement in all the directions, while the bottom boundary is not allowed to have displacement in all the directions. The left outer boundary (AOS) is not allowed to have horizontal displacement but can have free vertical movement. Sensitivity analyses were performed to ensure stable results of numerical modeling by examining the variation of stresses as function of mesh size and layer number. The sensitivity analyses resulted in 25 layers (i.e. a filling rate of 2 cm/layer), backfill elements of 0.155 cm × 0.5 cm and column wall elements of 0.125 cm × 0.5 cm.

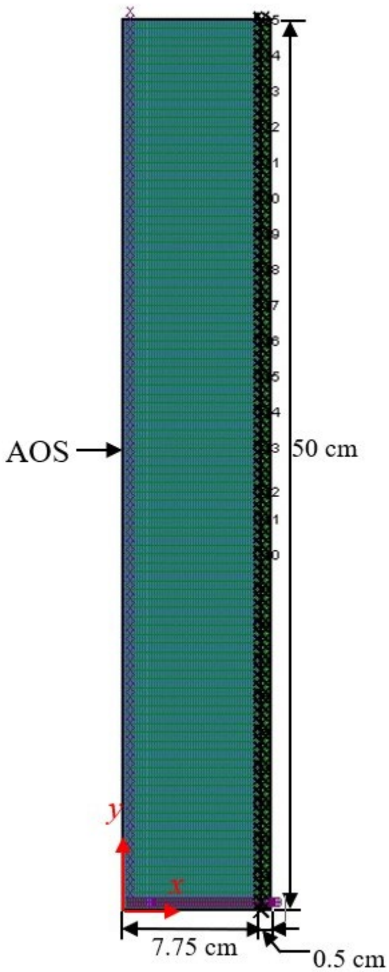


Fig. 7. Numerical model of the radial cross section of the filled opening built in FLAC2D using the axis of symmetry (AOS)

5.2. Comparison with numerical and analytical results

The obtained stress measurement results are divided into two parts. The measured stresses at the center are used to obtain the value of E and μ of the backfill through a calibration process against the numerical models performed with FLAC2D and then the measured stresses near the wall are used to verify the validity of prediction of the numerical model using the calibrated parameters.

Fig. 8a presents the distribution of the σ_v and σ_h at the center measured by the stress sensors and calculated by FLAC2D to calibrate the value of E and μ of the backfill. The good agreements between the experimental and numerical results are obtained when the value of E and μ are taken as 30 MPa and 0.25, respectively. The value of E is in the range of the typical values for sandy materials (Pirapakaran and Sivakugan 2007b) while the value of μ is consistent with that measured by Suwal and Kuwano (2013) using the disk transducer for a sand with a similar grain size distribution curve in the loose state to that used in this study. Then, the calibrated E (= 30 MPa) and μ (= 0.25) are used in FLAC2D to predict the stresses near the wall. Again, good agreements are obtained between the measured stresses and those predicted by FLAC2D, as shown in Fig. 8b. The good agreements between the experimental and numerical results in Fig. 8 indicate that the value of E (= 30 MPa) and μ (= 0.25) can be employed to analyze the stress distribution in the opening during the backfilling. The numerical modeling of the arching effect in a backfilled opening has been for the first time calibrated with experimental results in terms of the vertical and horizontal stresses at the center and near the wall.

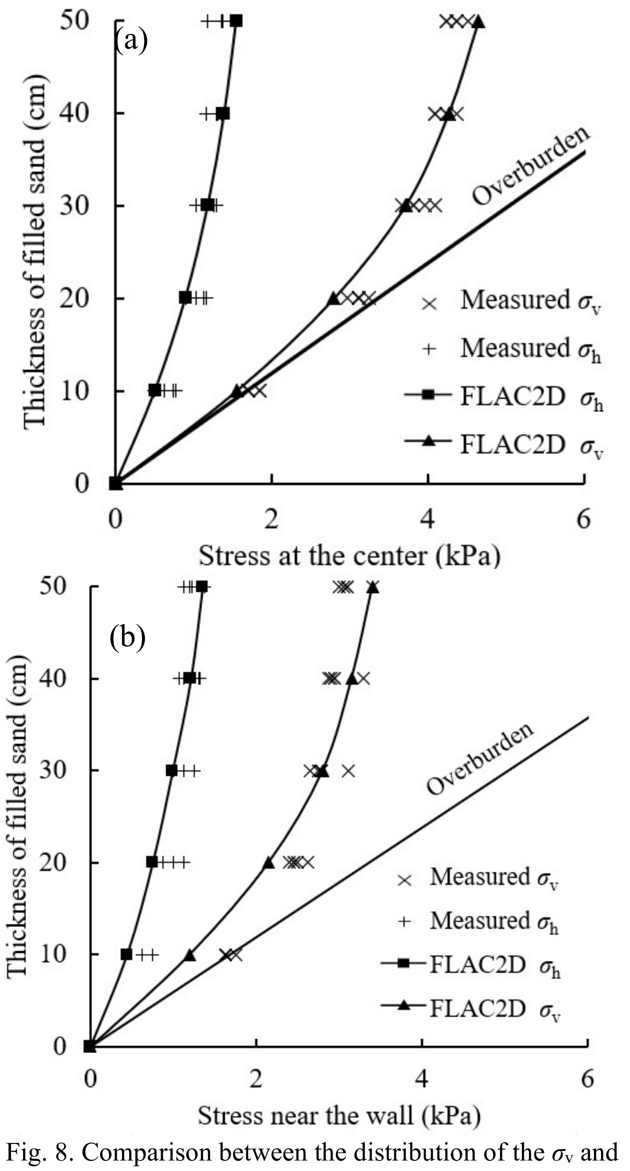


Fig. 8. Comparison between the distribution of the σ_v and σ_h at the base level (a) at the opening center, calculated by FLAC2D using E = 30 MP and μ = 0.25; (b) near the opening wall, predicted by FLAC2D using E = 30 MP and μ = 0.25, for the opening filled with backfill thickness varying from 10 cm to 50 cm; on the figure are also plotted the stresses measured by the stress sensors and estimated by the overburden solution

Fig. 9 presents the distribution of the EPC K at the opening center and close to the opening wall, obtained from the σ_v and σ_h measured by the stress sensor and obtained by FLAC2D. On the figure, Rankine's K_a = 0.27 (Eq. (2) using ϕ = 35°) and Jaky's K_0 = 0. 43 (Eq. (1) using ϕ = 35°) are also plotted. It can be clearly seen that at the opening center, the value of K calculated by the measured σ_v and σ_h agrees very well with that obtained by FLAC2D, both are close to the Rankine's active K_a. These results also confirm the numerical results of Li et al. (Li *et al*. 2003, Sobhi *et al*. 2017, Yang *et al*. 2017, 2018).

Near the wall, Fig. 9b shows that the EPC K obtained by the measured σ_v and σ_h is slightly higher than the value of K obtained by FLAC2D. Both are close to Jaky's at rest coefficient K_0. The numerical results are consistent with those reported by Li et al. in the 2D plane strain numerical model (Li *et al*. 2003, Yang *et al*. 2017).

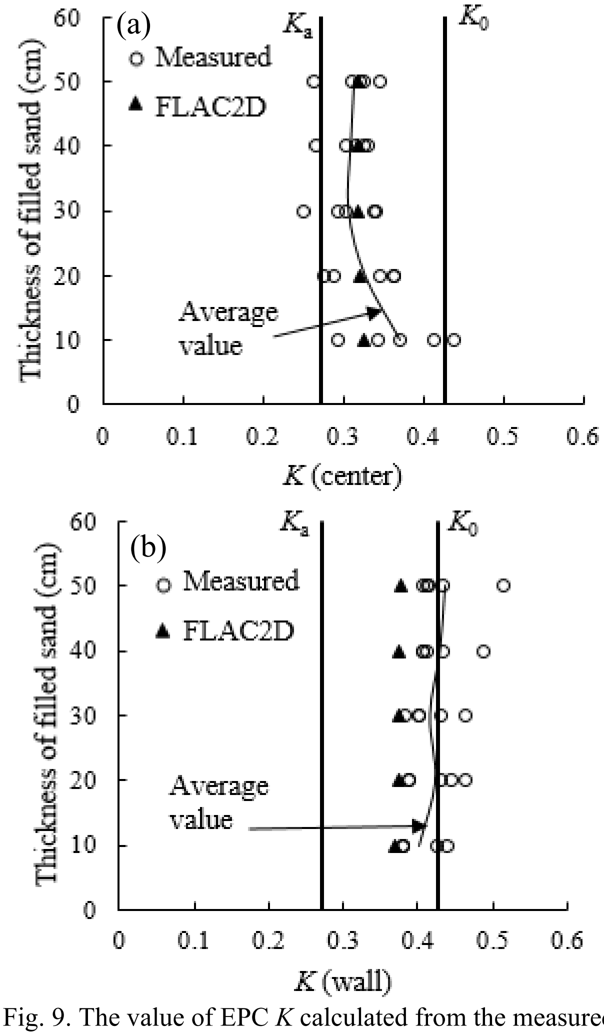


Fig. 9. The value of EPC K calculated from the measured σ_v and σ_h and obtaiend by FLAC2D (a) at the opening center and (b) close to the wall of the base level of the opening, plotted with the Rankine's (K_a) and Jaky's (K_0)

5.3. Sensitivity analysis of the geometry and interface properties

In reality, backfilled opening (e.g., mine stopes, silos) can have a much larger plan dimension and height (i.e., as high as 200 m for a mine stope) than the column used in this study. Moreover, the fill-wall interface of a mine stope can be very coarse and different from the column used in this study. Additional numerical simulations were also performed to study the impact of the scale effect and fill-wall interface properties on the state of the backfill placed in a backfilled opening.

These numerical modelings were performed by changing the diameter of the opening or thickness of the placed backfill, or fill-wall interface friction angle at one time while using the same properties for the opening wall and sand (also with the calibrated E and μ) as well as following the same simulation procedures as shown in the section 5.1, including the same material properties used for backfill and column, mesh and mechanical boundary condition.

Figure 10 shows the variation of the K along the center line and near the wall of an opening with the fill-wall interface friction angle of 0°, 21° (= 2/3ϕ), and 35° (= ϕ). The Rankine's K_a = 0.27 (Eq. (2) using ϕ = 35°) and Jaky's K_0 = 0. 43 (Eq. (1) using ϕ = 35°) are also included on the figure. One can see that the value of K along the centerline of the opening is always close to K_a despite the different fill-wall interface friction angles. This result indicates that the fill-wall interface friction angle has negligible influence on the state of the backfill along the centerline of an opening. However, near the opening wall, the fill-wall interface friction angle can significantly influence the state of the placed backfill. When the fill-wall interface friction angle increases from 0° to 35°, the value of K increase from around 3.3 (close to K_a) to that around 4.1 (close to K_0).

Fig. 11 illustrates the variation of K at the opening center (Fig. 11a) and close to the opening wall (Fig. 11b) as a function of backfill thickness, with the opening diameter ranging from 4 to 20 m and final backfill thickness of 40 m. The Rankine's K_a = 0.27 (Eq. (2) using ϕ = 35°) and Jaky's K_0 = 0. 43 (Eq. (1) using ϕ = 35°) are also plotted on the figure. One can see that the K is close to K_a at the opening center. When the opening diameter varies from 4 m to 20 m, the corresponding value of K is 0.32 and 0.33, respectively, which are 15% and 18% higher than K_a, respectively. Near the opening wall, the value of K is close to K_0. When the diameter varies from 4 m to 20 m, the value of obtained K is 0.37 and 0.40 at the thickness of 40 m, respectively, which are 15% and 9% lower than K_0, respectively. The opening diameter has only a slight impact on the K at the opening center and near the opening wall. More specifically, the values of K at the center become slightly closer to K_a with the decreased diameter of the opening while the K close to the wall are slightly closer to K_0 with increased diameter of the opening.

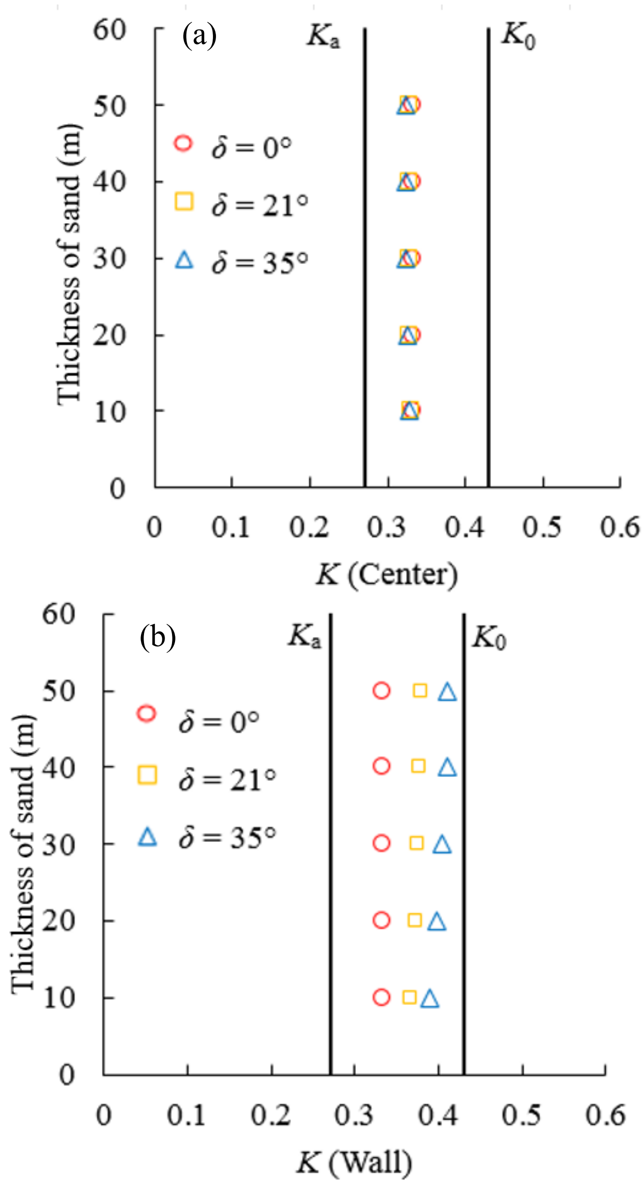


Figure 10. Variation of the EPC K at the base level (a) at the opening center and (b) close to the opening wall of the opening with the fill-wall interface friction angle of 0°, 21°, and 35°

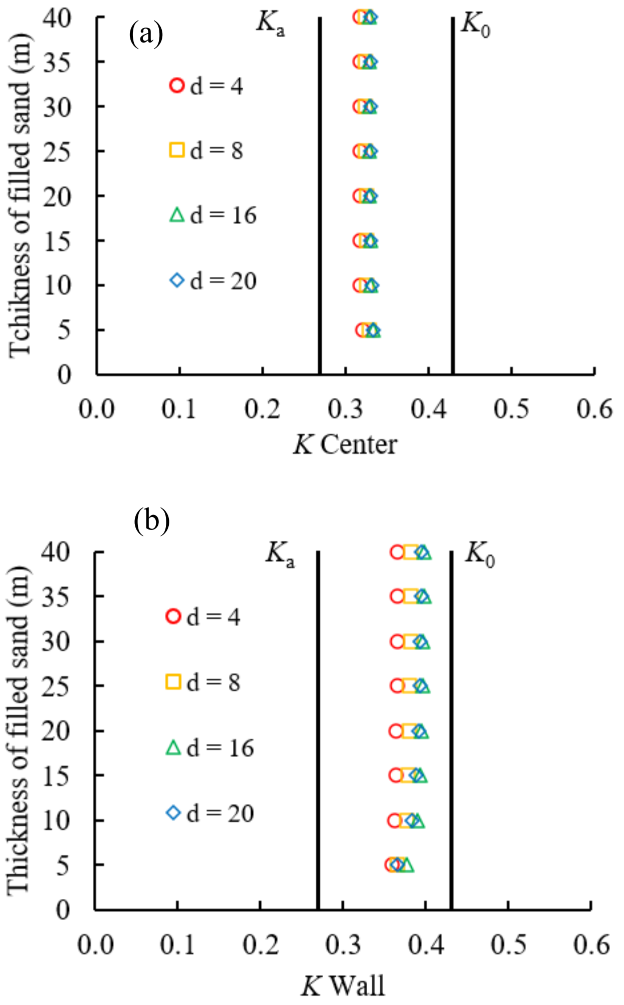


Figure 11. Variation of the K at the base level (a) at the center and (b) close to the wall of the opening filled with different backfill thicknesses and with the opening diameter ranging from 4 to 20 m

6. Discussions

The accurate measurement of the stress sensor is an important concern, which can be mainly influenced by the inclusion and arching effects. The former refers to the installation of a large stiffness object (sensor) tends to alter the natural stress state in the soil. The latter (arching effect) is due to the small deflection caused by the interaction between the sensitive membrane and the surrounding soils. As the miniature stress sensors in this study were installed on a rigid surface at the base of the column, the influence of the inclusion effect can be very small and a high precision of stress measurement can be obtained (Clayton and Bica 1993, Talesnick 2005, Berthoz *et al*. 2013). The arching effect mainly depends on the stiffness of the sensor and soil particles. It can be assessed by the comparison between the measured stress by sensors and the natural stress in the real condition without installing sensors, a ratio also called cell action factor (CAF). The magnitude of CAF is closely related to the flexibility factor (F) calculated as follow (Take and Valsangkar 2001):

$$F = \frac{E_{soil} d_s^3}{E_{sensor} t^3} \tag{5}$$

where E_{soil} (kPa) is the Young's modulus of the filled sand; E_{sensor} (kPa) is the Young's modulus of the miniature stress sensor; d_s (mm) is the diameter of the sensor; t (mm) is the thickness of the sensor. Clayton and Bica (1993) experimentally showed that to ensure a high value of CAF (> 0.9, the measured stress is very close to that under the natural condition under this value), the flexibility factor F calculated by Eq. (5) should be smaller than 0.7. With the Young's modulus of 30 MPa for the sand and 1.23×10^8 kPa for the used sensor, the value of F can be calculated as 0.067, which is much smaller than 0.7, resulting in a very high value of CAF (> 0.9). The arcing effect of the stress sensor can be considered as negligible. The sensor is thus able to give accurate stress measurements. This has also been confirmed by the liner calibration curves as well as the good agreements between the measured horizontal/vertical stresses and overburden stresses for a small sand thickness of 10 cm as shown in Fig. 4. This is because the arching effect can be negligible for this small thickness of backfill and the measured vertical and horizontal stresses should be equal to the overburden pressure.

Despite the encouraging results and findings, several improvements are expected on the instrumentation and testing procedure to increase the accuracy and reduce the dispersion of experimental results. During the filling operation, the falling height was tried to be kept constant through manual control. However, the slight variation was unavoidable. The backfill placed in the opening could be slightly heterogeneous along the height. This heterogeneity of backfill can partly explain the dispersion of the experimental results. In the future, the test instrumentation can be improved by using a more elaborated filling system, such as air pluviation technique (Take and Valsangkar 2001).

In this study, the backfill was delicately placed in the confining structure with a beaker. It is better to design a pouring device that can keep a constant falling height and make a more uniform backfill. This design was not completed due to the laboratory device limitations. Nevertheless, attention had been paid during the pouring operation in order to obtain a backfill as uniform as possible. The beaker was kept down to the opening as close as possible to the bottom of the opening (for the initial backfilling) or the top surface of the previously placed sand layer to minimize the impact of vibration induced by sand placement. Nevertheless, it is noted that the experimental results presented in Figure 4 are quite disperse. This dispersion can be contributed to several aspects. For example, the falling height during the filling operation was tried to be kept constant through manual control. Slight variations might be unavoidable. In addition, the laboratory is close to a construction site where construction has been lasting for more than two years. The minor vibration associated with the unaware neighbor construction and the possible fill heterogeneity associated with falling height can partly explain the dispersion of the experimental results. In the future, more experimental work can be realized by making and using a pluviation device to obtain well controlled and uniform backfill in a laboratory exempt of disturbance.

Another limitation is associated with the placement of the stress sensors, which were all positioned at the bottom level of the model stope. The stress and K variations shown in this study represent only the state of the backfill at the base as a function of the backfill thickness. More work is required and ongoing to study the stresses and K at different heights for a given thickness of backfill. In addition, the backfill is dry and the opening is vertical. More works are needed by considering the presence of water and inclined stopes.

7. Conclusions

In this paper, the horizontal σ_h and vertical σ_v stresses were measured at the base level at the stope center and close to the stope wall of a model stope filled with different thicknesses of backfill. The distribution of the earth pressure coefficient was, for the first time, obtained by laboratory tests as a function of the backfill thickness from the stope center to that close to the stope wall. The main conclusions are drawn as follows:

- The measured σ_h and σ_v follow the stresses calculated by the overburden solution when the backfill thickness is small. The pace of increase decreases and the measured stresses become constant with further increase in the backfill thickness, indicating that arching effect takes place in the backfilled model stope.
- The σ_h is uniformly distributed from the center to the wall of the stope. These measured results are consistent with the numerical results reported by Li and Aubertin (2008, 2009a, 2010). It can be estimated by the analytical solutions using either Rankine's K_a or Jaky's K_0.
- The σ_v is higher at the stope center than that close to the stope wall. It is not uniformly distributed from the center to the wall of the stope. This corresponds to the numerical results reported by Li and Aubertin (2008, 2009a, 2010). The experimental results agree well with those calculated by the analytical solution (Eq. (3)) using the Rankine's K_a at the center and the Jaky's K_0 close to the wall.
- When the fill thickness is small, the ratio of the measured σ_h to the σ_v is close to Jaky's K_0 at the center and near the wall. This result corresponds quite well to the earth pressure coefficient at-rest measured with oedometer tests for cohesionless soils.
- When the fill thickness becomes large, the values of K calculated by the measured stresses become close to K_a at the center, which agree well with the numerical results presented in this and previous studies. The backfill at the stope center can yield and attain an active state even with immobile surrounding walls. Near the wall, the values of K calculated by the measured stresses are close to K_0, which also agree fairly well with the numerical results obtained by FLAC2D.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada (402318), Fonds de recherche du Québec—Nature et Technologies (2015-MI-191676), Mitacs Elevate Postdoctoral Fellowship (IT12573), and the industrial partners of the Research Institute on Mines and Environment (RIME UQAT-Polytechnique; http://rime-irme.ca/).

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