



**Titre:** On the Simultaneous Computation of Target Inventories and Intervals for Bimodal Bike-Sharing Systems

**Auteurs:** Maria Clara Martins Silva, Daniel Aloise, & Sanjay Dominik Jena

**Date:** 2024

**Type:** Article de revue / Article

**Référence:** Silva, M. C. M., Aloise, D., & Jena, S. D. (2024). On the Simultaneous Computation of Target Inventories and Intervals for Bimodal Bike-Sharing Systems. *Transportation Engineering*, 16, 100226 (9 pages).  
Citation: <https://doi.org/10.1016/j.treng.2024.100226>

 **Document en libre accès dans PolyPublie**  
Open Access document in PolyPublie

**URL de PolyPublie:** <https://publications.polymtl.ca/57340/>  
PolyPublie URL:

**Version:** Version officielle de l'éditeur / Published version  
Révisé par les pairs / Refereed

**Conditions d'utilisation:** CC BY-NC-ND  
Terms of Use:

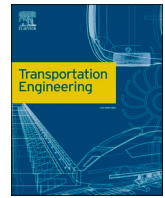
 **Document publié chez l'éditeur officiel**  
Document issued by the official publisher

**Titre de la revue:** Transportation Engineering (vol. 16)  
Journal Title:

**Maison d'édition:** Elsevier BV  
Publisher:

**URL officiel:** <https://doi.org/10.1016/j.treng.2024.100226>  
Official URL:

**Mention légale:** Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license  
Legal notice: (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).



# On the simultaneous computation of target inventories and intervals for bimodal bike-sharing systems

Maria Clara Martins Silva <sup>a</sup>, Daniel Aloise <sup>\*,a</sup>, Sanjay Dominik Jena <sup>b</sup>

<sup>a</sup> GERAD and Polytechnique Montréal, 2500 Chem. de Polytechnique, Montréal, H3T 1J4, Canada

<sup>b</sup> CIRRELT and School of Management, Université du Québec à Montréal, 405 Rue Sainte-Catherine Est, Montréal, H2L 2C4, Canada

## ARTICLE INFO

**Keywords:**  
Bike-sharing  
Rebalancing  
Electric bikes  
Inventory management

## ABSTRACT

The emerging demand for electric bicycles in recent years has prompted several Bike-Sharing Systems around the world to adapt their service to a new wave of commuters. Many of these systems have incorporated electric bikes into their network while still maintaining the use of regular mechanical bicycles. However, the presence of two types of bikes in a Bike-Sharing network may impact how rebalancing operations should be conducted in the system. Regular and electric bikes may exhibit distinct demand patterns throughout the day, which can hinder efficient planning of such operations. In this paper, we propose a new model that provides rebalancing recommendations based on the demand prediction for each type of bike. Additionally, we simulate the performance of our model under different scenarios, considering commuters' varying inclination to substitute their preferred bike with one of a different type. Our empirical experiments indicate the potential of our model to improve user satisfaction, reducing the total lost demand by approximately 10%, while reducing the lost demand for electric bikes by around 30%, on average, when compared to the existing rebalancing strategy used by the real-world Bike-Sharing System under study. Remarkably, this was accomplished while maintaining an almost identical average hourly count of rebalancing operations.

## 1. Introduction

In the last years, bike-sharing systems (BSS) have gained traction as an alternative transportation mode due to their numerous advantages such as the absence of greenhouse gases emission, the promotion of a healthy lifestyle, as well as easy and facilitated access. The history of this mobility service dates back to the 60s and has continuously evolved over time [34]. The fourth BSS generation, which we are currently experiencing, is marked by the inclusion of solar-powered docking stations, real-time system data, mobile apps, flexible parking, and electric bikes, also known as e-bikes [20].

In comparison with regular bikes, e-bikes are faster, easier to ride, especially on hilly paths, and overall they cause less fatigue [18]. To make BSSs more attractive to a group of commuters who are mainly interested in these advantages, BSSs around the world have introduced e-bikes into their networks – e.g. BIXI (Montréal), Citi Bike (New York). Nonetheless, regular bikes still please loyal commuters who search for health benefits or for a cheaper transportation mode. In [39], it is shown that the introduction of e-bikes in BSSs, alongside regular bikes, contributed significantly to the augmentation of BSSs revenues.

However, a network with two types of bikes indeed introduces new challenges at every step of the service's logistics. This is especially the case for dock-based BSSs, where the docks at the stations must be shared by both types of bikes. As such, too many bikes of a given type may lead to lost demand of the second type, and vice-versa, given that the number of docks is limited at each station. Hence, at the moment of rebalancing the inventory of a station, it is important to dynamically determine the number of ideal available bikes of each type in the stations of the system to guarantee its effective service.

Fig. 1 presents the hourly average number of bikes rented on BIXI-Montreal in July 2022, where we can observe that the demand for regular and electric bike trips bounces over the day. Fig. 2 highlights the considerable variation in e-bike demand per station at BIXI. While we can identify areas with high demand, it is notable that high-demand stations can be found adjacent to low-demand stations. This shows that understanding bike demand at station level is a complex task – even more in the presence of a heterogeneous bike offer. Additionally, the demand for bikes has undergone a significant transformation in recent years, driven by the shifts in working habits brought about by the COVID-19 pandemic [14,30].

\* Corresponding author.

E-mail address: [daniel.aloise@polymtl.ca](mailto:daniel.aloise@polymtl.ca) (D. Aloise).

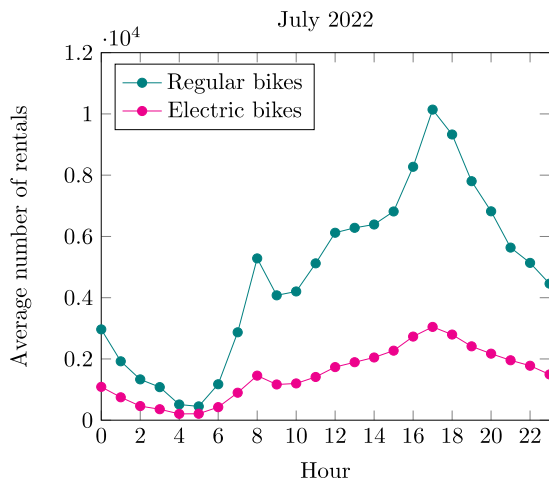


Fig. 1. Hourly average number of rentals on BIXI-Montreal BSS in July 2022.

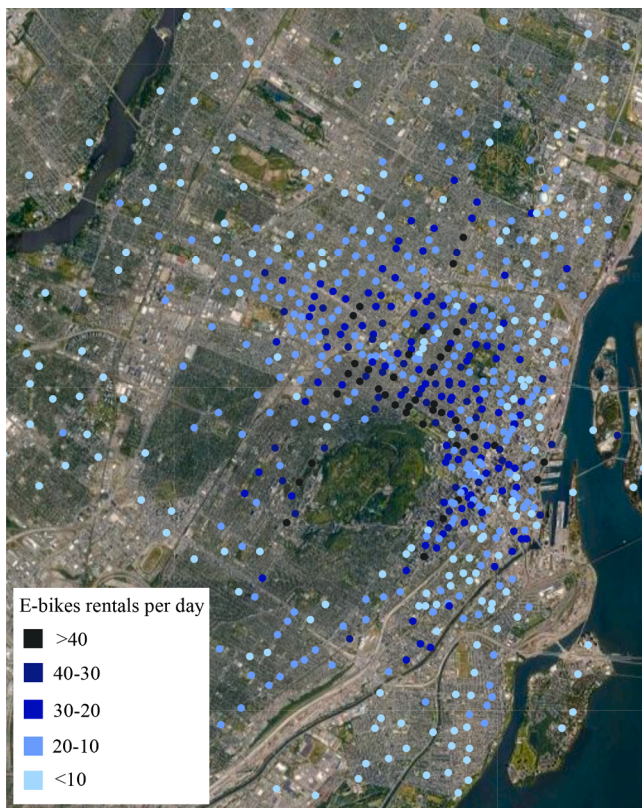


Fig. 2. Average number of rented e-bikes per day at BIXI in July 2022.

Besides, reallocating bikes through rebalancing operations can be quite expensive since it involves fuel costs, truck maintenance, driver’s salary, etc. The trucks used for rebalancing are also responsible for CO<sub>2</sub> emissions and other polluting gases, which contradicts the BSS’s commitment to sustainability. Therefore, it is important to optimize the effectiveness of rebalancing operations, updating the inventories of stations that yield minimal lost demand in the system. Nonetheless, leaving to the operator alone the task of understanding both demands and their correlations, while assuring that the rebalancing operations respect the BSS’s limited resources, may lead to suboptimal rebalancing decisions.

The primary objective of our paper is to underscore the significance of incorporating demand predictions for both regular and electric bikes

Table 1

Summary of strategies to improve the service level in EBSS.

Strategy	Research article	Network	Parking	Methodology
Operational rebalancing	Fukushige et al. [8]	Unimodal	Free-floating	User-based approach for rebalancing
	Tan et al. [36]	Unimodal	Parking locations	Optimization of routes for battery exchange
	Zhou et al. [37]	Unimodal	Free-floating	Optimization of routes for battery exchange
Network planning	Zhu [39]	Bimodal	Station-based	Optimization of bikes and e-bikes fleet
	Chen et al. [5]	Unimodal	Station-based	Optimization of bikes fleet and number of docks
	Zhou et al. [38]	Unimodal	Parking locations	Optimization of parking locations
	Martinez et al. [26]	Bimodal	Station-based	Optimization of stations locations
	Hosseini et al. [15]	Unimodal	Station-based	Optimization of stations performance
	Soriguera [35]	Unimodal	Station-based	Optimization of e-bikes fleet, number of stations, number of docks, and rebalancing rate

when making dynamic rebalancing decisions within a station-based Electric Bike-sharing System (EBSS), i.e., a bimodal BSS that has both regular and electric bikes. In light of this, we propose a model that identifies imbalanced stations and determines the target inventory for each bike type. Essentially, our model provides recommendations for when and how many bikes of each type should be added or removed during the rebalancing process. By considering the expected demand for each bike type within a specific time period and accounting for station capacity, our model aims to optimize the rebalancing performance.

To evaluate our model effectiveness, we collected data from BIXI-Montreal<sup>1</sup> and conducted simulations to compare the inventory response between our proposed rebalancing strategy and the one currently employed by BIXI. Furthermore, our simulations explore various policies regarding the option of replacing one type of bike with another for a trip.

The paper is organized as follows. Section 2 reviews relevant literature on our research topic. Section 3 describes the proposed model to compute inventory intervals and target inventory for both regular and electric bikes. Section 4 presents the data, the tuning process, the inventory simulator, and the results of our experiments. Finally, our final remarks are given in Section 5.

## 2. Related works

One key performance indicator to assess a BSS is its *service level*, which is computed as the ratio between the number of satisfied trips and the total number of demanded trips [29]. It indicates whether a BSS is able to meet the commuters’ demand, which is paramount for customer satisfaction.

The strategies to improve the service level in BSSs can be grouped into two main categories: network planning and operational rebalancing. The first consists of designing an ideal network configuration by optimizing the number of stations, docks and bikes, the location of the stations, initial inventories, etc., to reduce the total lost demand in the system [6,29,33]. The second represents an intervention, that can be performed either by the BSS operator or by the commuters, to redistribute the network’s assets, e.g. bikes or batteries, among the stations [2,7,23,28,36].

<sup>1</sup> [www.bixi.com](http://www.bixi.com).

Many studies in the literature have proposed strategies to improve the service level in BSSs with regular bikes. However, works that consider systems with shared e-bikes, hereafter denoted EBSSs, have just recently emerged. Table 1 summarizes the main works in the literature whose goal is to improve the service level in EBSSs. They are classified according to the strategy to improve the service level (operational rebalancing or network planning), the BSS network composition (unimodal or bimodal), the parking configuration (capacitated station-based, uncapacitated parking locations or free-floating), and their methodology.

Fukushige et al. [8], Tan et al. [36] and Zhou et al. [37] propose operational rebalancing strategies since they both deal with the reallocation of resources in the system. Fukushige et al. [8] study a user-based approach to better understand in which scenarios BSS commuters are stimulated to return bikes to a desired location under financial incentives. Tan et al. [36] and Zhou et al. [37] present models that propose travelling routes for exchanging discharged batteries for charged batteries among e-bikes in the system. Battery recharging is directly related to the performance of the service provided since e-bikes with discharged batteries remain unused in the system. In our work, we assume that battery recharging is conducted independently of the rebalancing process. This assumption is based on information provided by BIXI operators. At Bixi-Montreal, almost 20% of the stations are powered, allowing battery recharging on site.

The remaining references in the table address network planning strategies, i.e., they approach the problem of fleet dimensioning [5,35,39], dock dimensioning [5,35]), locating the stations [26,38], rebalancing rate, i.e., reallocated bikes per hour [35], and the correlation between the performance of the stations and external factors, such as the weather, population characteristics and availability of nearby public transportation [15].

Our proposed model proposes to optimize the service level of a bimodal and station-based EBSS using target inventory values and inventory intervals to assist in the rebalancing process. The target inventory value represents the ideal number of bikes for a station and it is often used to establish how many bikes that station should have in a given time period in order to improve its performance [6,12,13,16,17,19,33]. Works proposing the computation of target inventory values vary depending on the metric used to assess the stations performance. For example, the performance of a station can be evaluated based on the expected lost demand [19,33], the expected satisfied demand [12,16,17] or the commuters waiting times [6]. The inventory interval consists of an acceptable range in which the inventory can fluctuate while still meeting the expected demand. They are usually used to select which stations from the network need to be rebalanced [3,17,29].

### 3. Proposed model

In this Section, we present the model developed to automatically generate target inventory values and inventory intervals for bimodal BSS based on demand prediction.

#### 3.1. Target inventory values

Target values refer to the ideal fill level to which the operator may want to set the station inventory for each bike type during the rebalancing process. Addressing the challenge of generating target values for two different types of demand that share station docks requires careful consideration to ensure that both demands are met without exceeding the station's capacity. So, to ensure the feasibility of the rebalancing recommendations provided by our target inventory values, our model divides the number of docks for each demand based on their respective service levels. Additionally, it establishes the optimal initial inventory within the allocated number of reserved docks for each demand.

Our model starts by training a machine-learning model to predict hourly rentals and returns at station-level for both types of bikes. Several

studies in the literature focus on demand prediction of BSSs [see, e.g. [1, 4,9,22,24,25,27]]. In this work, we use a predictive model based on a Gradient-boosted tree, introduced in [17] which considers historical data as well as exogenous features such as weather conditions or holidays.

After forecasting the demand at each station of the BSS, we calculate their respective service levels. In our study, considering the availability of two types of bikes, we chose to compute the proportion of satisfied trips independently for each bike type. This approach allows us to assess the service level for each type of bike individually, taking into account their specific demand patterns and availability.

To calculate the service level of a station, we model its inventory as a queue with a single server. The capacity of this queue is set to match the number of available docks at that particular station. Similar to other works that address the random rentals and returns of commuters in a BSS, such as [10,11,17,21,29,31,32], we assumed that the trips follow a Poisson distribution, so that the times between rentals and returns follow exponential distributions.

For a station  $s$  with an initial inventory of  $f$  and a specific number of docks, denoted as  $C^{\mathcal{R}}$ , allocated for regular bikes out of the total capacity of  $C_s$  docks, the expected service level for regular bikes during the time period  $[0, T]$  can be computed as follows:

$$SL_s^{\mathcal{R}}(f, T, C^{\mathcal{R}}) = \frac{\int_0^T \mu_s^{\mathcal{R}}(t)(1 - p_s^{\mathcal{R}}(f, 0, t)) + \lambda_s^{\mathcal{R}}(t)(1 - p_s^{\mathcal{R}}(f, C^{\mathcal{R}}, t)) dt}{\int_0^T \mu_s^{\mathcal{R}}(t) + \lambda_s^{\mathcal{R}}(t) dt}, \quad (1)$$

where  $p_s^{\mathcal{R}}(f, N, t)$  is the probability that the station  $s$  stores  $N$  regular bikes at hour  $t$ , knowing that its initial inventory is equal to  $f$  at time 0;  $\mu_s^{\mathcal{R}}(t)$  and  $\lambda_s^{\mathcal{R}}(t)$  represent the predicted rental and return for regular bikes at hour  $t$  and station  $s$ . Here, the superscript  $\mathcal{R}$  refers to values that are specific to regular bikes. Likewise, the service levels for e-bikes are computed as in Eq. (1) by replacing  $\mu_s^{\mathcal{R}}(t)$ ,  $\lambda_s^{\mathcal{R}}(t)$ ,  $p_s^{\mathcal{R}}(f, N, t)$  and  $C^{\mathcal{R}}$  by  $\mu_s^{\mathcal{E}}(t)$ ,  $\lambda_s^{\mathcal{E}}(t)$ ,  $p_s^{\mathcal{E}}(f, N, t)$  and  $C^{\mathcal{E}}$ , respectively, where the superscript  $\mathcal{E}$  refers to values regarding e-bikes only.

Indeed, Eq. (1) depends on the number of docks  $C^{\mathcal{R}}$  allocated to regular bikes at the analyzed time period. This allocation may vary to optimize the performance of the system based on the anticipated trip demand. In our model, the number of docks allotted for regular and electric bikes at station  $s$  for time period  $[0, T]$ , denoted  $C_s^{\mathcal{R}}(T)$  and  $C_s^{\mathcal{E}}(T)$ , respectively, are determined as:

$$C_s^{\mathcal{R}}(T) = \underset{\bar{C} \in \{0, \dots, C_s\}}{\operatorname{argmax}} \{ \Lambda_s^{\mathcal{R}}(T, \bar{C}) + \Lambda_s^{\mathcal{E}}(T, \bar{C} - C_s) \}, \quad (2)$$

and

$$C_s^{\mathcal{E}}(T) = C_s - C_s^{\mathcal{R}}(T), \quad (3)$$

where function  $\Lambda_s^{\mathcal{R}}(T, x)$  (resp.  $\Lambda_s^{\mathcal{E}}(T, x)$ ) is chosen as the maximum or the average value of  $SL_s^{\mathcal{R}}(f, T, x)$  (resp.  $SL_s^{\mathcal{E}}(f, T, x)$ ) for  $f \in \{0, \dots, x\}$ . The choice of the function as max or avg. influences the service level we want to optimize (the best or the average-case, respectively).

Once the number of docks reserved for regular bikes and e-bikes are determined, our model proceeds to compute the target inventory values for regular and e-bikes for time period  $[0, T]$  as:

$$\mathcal{F}_s^{\mathcal{R}}(T) = \underset{f \in \{0, \dots, C_s^{\mathcal{R}}(T)\}}{\operatorname{argmax}} \{ SL_s^{\mathcal{R}}(f, T, C_s^{\mathcal{R}}(T)) \}, \quad (4)$$

and

$$\mathcal{F}_s^{\mathcal{E}}(T) = \underset{f \in \{0, \dots, C_s^{\mathcal{E}}(T)\}}{\operatorname{argmax}} \{ SL_s^{\mathcal{E}}(f, T, C_s^{\mathcal{E}}(T)) \}. \quad (5)$$

The above equations compute the target values as the inventory fill levels that maximize the service-level for the corresponding bike type.

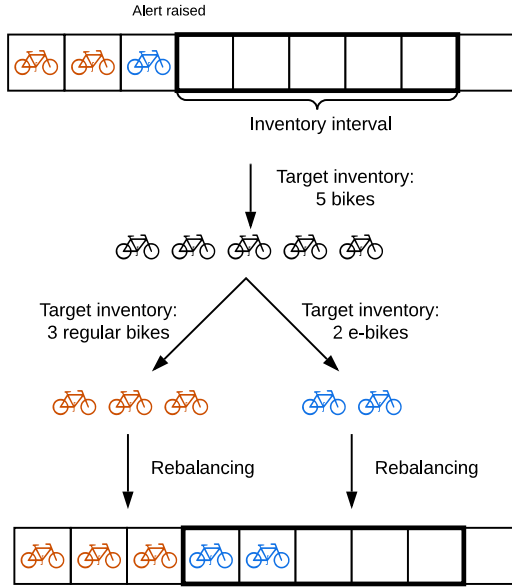


Fig. 3. Simulation of the inventory and rebalancing process in the model B0. Orange bikes represent regular bikes whereas blue bikes represent e-bikes.

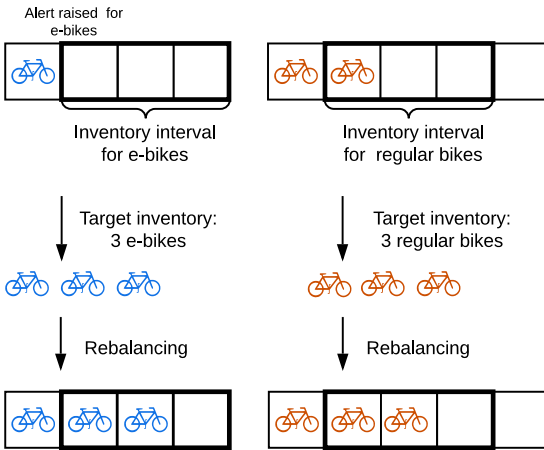


Fig. 4. Simulation of the inventory and rebalancing process in our model. Orange bikes represent regular bikes whereas blue bikes represent e-bikes.

Table 2  
Optimized hyperparameters values used in the tests.

Bike substitution	Model	$(\beta^{\mathcal{R}}, \beta^{\mathcal{E}})$
None	shared- $\mathcal{RE}_{max}$	(0.4,0.3)
	shared- $\mathcal{RE}_{avg}$	(0.4,0.5)
All bikes	shared- $\mathcal{RE}_{max}$	(0.4,0.2)
	shared- $\mathcal{RE}_{avg}$	(0.4,0.4)
Reg. bikes → E-bikes	shared- $\mathcal{RE}_{max}$	(0.3,0.5)
	shared- $\mathcal{RE}_{avg}$	(0.3,0.5)
E-bikes → Reg. bikes	shared- $\mathcal{RE}_{max}$	(0.4,0.3)
	shared- $\mathcal{RE}_{avg}$	(0.4,0.3)

Note again that the number of docks allotted to regular and electric bikes,  $C_s^{\mathcal{R}}(T)$  and  $C_s^{\mathcal{E}}(T)$ , respectively, sum to the total dock capacity of the station, as defined in Eq. (3). As such, the computed target values respect the capacity of the station and the number of docks reserved for each type of bike. This ensures that the recommended rebalancing actions based on the target inventory values are practical and feasible to

Table 3  
Results regarding the simulated models: the average number of rebalancing operations per hour and the lost demand (in % with respect to the total served demand).

Simulated model	Bike substitution	Total lost demand %	Regular bikes lost demand %	E-bikes lost demand %	Rebalancing per hour
B0 shared- $\mathcal{RE}_{max}$	None	3.32	2.46	5.89	25.08
		3.07	2.75	4.01	25.77
	shared- $\mathcal{RE}_{avg}$	3.00	2.67	3.98	26.40
B0 shared- $\mathcal{RE}_{max}$	All bikes	2.25	1.84	3.45	23.16
		2.10	1.88	2.80	25.42
	shared- $\mathcal{RE}_{avg}$	2.07	1.83	2.78	26.61
B0 shared- $\mathcal{RE}_{max}$	Reg. bikes → E-bikes	2.89	1.86	5.99	25.63
		2.66	1.97	4.85	26.48
	shared- $\mathcal{RE}_{avg}$	2.64	1.93	4.78	26.86
B0 shared- $\mathcal{RE}_{max}$	E-bikes → Reg. bikes	2.98	2.84	3.42	22.36
		2.81	2.95	2.40	24.50
	shared- $\mathcal{RE}_{avg}$	2.80	2.93	2.43	24.36

implement.

### 3.2. Inventory intervals

Inventory intervals typically serve as an indicator when a station should be rebalanced: this is the case when the station inventory falls outside of the defined interval. Considering only the total rentals or returns without distinguishing between different types of demand can obscure the identification of lost demand for a specific type of bike. Similarly, creating inventory intervals tailored to each demand while assuming that the total number of docks is always available can lead to undesirable situations. For instance, a station may become completely full without triggering any alerts because neither demand (for regular or e-bikes) has exceeded its upper or lower bounds. Therefore, it is crucial to take into account the station capacity when setting inventory intervals to ensure optimal inventory management and prevent potential issues.

To calculate the inventory intervals, we begin by computing the maximum and minimum service levels for each demand for time period  $[0, T]$ , which are given by

$$\underline{SL}_s^{\mathcal{R}}(T) = \min_{f \in \{0, \dots, C_s^{\mathcal{R}}\}} SL_s^{\mathcal{R}}(f, T, C_s^{\mathcal{R}}(T)), \quad (6)$$

and

$$\overline{SL}_s^{\mathcal{R}}(T) = \max_{f \in \{0, \dots, C_s^{\mathcal{R}}\}} SL_s^{\mathcal{R}}(f, T, C_s^{\mathcal{R}}(T)) \quad (7)$$

for regular bikes. These values can be analogously obtained for e-bikes, by replacing the superscript  $\mathcal{R}$  by  $\mathcal{E}$ .

Then, accepted service levels at station  $s$  for time period  $[0, T]$  are calculated for each type of bike as:

$$\Omega_s^{\mathcal{R}}(T) = \underline{SL}_s^{\mathcal{R}}(T) + \beta^{\mathcal{R}} \left( \overline{SL}_s^{\mathcal{R}}(T) - \underline{SL}_s^{\mathcal{R}}(T) \right), \quad (8)$$

and

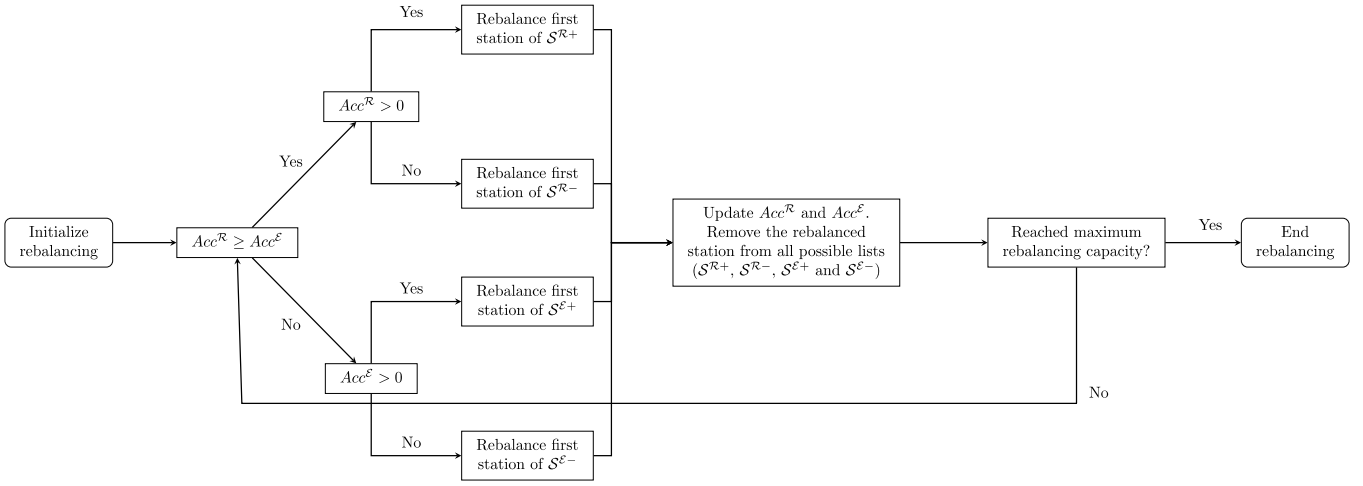


Fig. A.5. Flowchart of the selection procedure of stations for rebalancing.

$$\Omega_s^{\mathcal{R}}(T) = \underline{SL}_s^{\mathcal{R}}(T) + \beta^{\mathcal{R}} \left( \overline{SL}_s^{\mathcal{R}}(T) - \underline{SL}_s^{\mathcal{R}}(T) \right), \quad (9)$$

The model incorporates two hyperparameters,  $\beta^{\mathcal{R}}$  and  $\beta^{\mathcal{E}}$ , which are specific to each type of bike. These hyperparameters provide flexibility for the operator to fine-tune the computed inventory intervals for a station based on the behaviour patterns of its user base. By separately adjusting the values of  $\beta^{\mathcal{R}}$  and  $\beta^{\mathcal{E}}$ , the operator can also customize the inventory intervals to be more or less stringent for each demand type throughout the day.

Finally, the inventory intervals for regular bikes and e-bikes at station  $s$  for time period  $[0, T]$  are computed as:

$$\mathcal{J}_s^{\mathcal{R}}(T) = \{f \in \{0, \dots, C_s^{\mathcal{R}}(T)\} \mid SL_s^{\mathcal{R}}(f, T, C_s^{\mathcal{R}}(T)) \geq \Omega_s^{\mathcal{R}}(T)\} \quad (10)$$

and,

$$\mathcal{J}_s^{\mathcal{E}}(T) = \{f \in \{0, \dots, C_s^{\mathcal{E}}(T)\} \mid SL_s^{\mathcal{E}}(f, T, C_s^{\mathcal{E}}(T)) \geq \Omega_s^{\mathcal{E}}(T)\} \quad (11)$$

Remark that, by definition, inventory intervals contain target inventory values. This avoids that a station could be categorized as unbalanced immediately after a rebalancing operation has taken place.

Finally, because the availability of e-bikes is compromised due to battery discharging, there might be an underestimation of the actual demand for e-bikes used in training our Gradient-boosted tree. In response to this concern, we increased the predicted hourly demand for e-bikes by 10%, aligning with observations made by BIXI operators who reported us that, on average, approximately 10% of e-bikes are uncharged per hour in the system. Thus, we aim to obtain a more accurate computation inventory intervals and target inventory values, by identifying additional rebalancing demand for e-bikes.

#### 4. Computational experiments

In this section, we assess our model in comparison with the approach used by BIXI in 2022. First, we will present the data used in our experiments. Then, we briefly explain our simulations to emulate the inventories based on the rebalancing strategy applied. Next, we discuss the process for selecting the best hyperparameters,  $\beta^{\mathcal{R}}$  and  $\beta^{\mathcal{E}}$ , for our model. At last, we present the results collected from our experiments.

##### 4.1. Data

In light of the fact that BIXI added a considerable amount of e-bikes to its network in 2022, we opted to collect only the data from the aforementioned year. Thus, the data used in our experiments contain hourly information from April to September 2022, being grouped into

three categories: temporal, weather, and trip data. The first category includes time features, such as hour, day of the month, day of the week, month, and holidays. The second category contains data describing the weather, such as temperature, humidity, rain, and wind speed. Both temporal and weather data were collected from the official website of the Government of Canada<sup>2</sup> (except for the holiday feature which was manually noted). The trip data is composed of the number of rentals and returns at each station and it was provided by BIXI<sup>3</sup>. In addition to the data mentioned before, BIXI also provided the inventory intervals used in 2022 and network information that includes the capacity of the 745 stations.

The collected data was divided between train, validation, and test datasets. Given that we have access to the inventory intervals manually computed by BIXI operators for August and September 2022, we assign these months to the validation and test dataset, allocating the first 15 days of August and September to the validation dataset and the remaining days of these months to the test dataset. This distribution was chosen to evaluate the rebalancing strategies under varying weather conditions, as these months can present contrasting temperatures and rain features. The training dataset, meanwhile, contains data from April to July 2022.

The proportion of trips made with regular bikes and e-bikes is consistent across all datasets. In the training dataset, approximately 76% of trips were made using regular bikes whereas 24% of the trips used e-bikes. In the validation and test datasets, the proportion of trips using regular bikes and e-bikes is 75% and 25%, respectively.

##### 4.2. Experiment

Our proposed model, denoted hereafter shared- $\mathcal{RE}$ , is compared against a baseline approach, namely B0, that corresponds to the strategy applied by BIXI operators in 2022. The inventory intervals and target inventory values used to assist the rebalancing operations at BIXI were manually determined by their operators. These decisions were based on historical trip data at each station, without differentiation between regular bikes and e-bikes.

###### 4.2.1. Simulation of B0

Simulation B0 begins by initializing the inventories of regular and electric bikes at the stations with their respective target inventory values. However, it is worth noting that BIXI employs a unique target inventory value for each station during specific time periods. To

<sup>2</sup> <https://climate.weather.gc.ca/>.

<sup>3</sup> <https://bixi.com>.

decouple this single target inventory value between regular bikes and e-bikes, a straightforward approach is to distribute it based on the proportion of each bike type rented at the station during the observed period. This distribution can be determined by analyzing the training dataset, which ensures that the distribution of bikes aligns with the observed rental patterns.

Then, at each simulated hour, the inventory of each station is updated with the historical rentals and returns from BIXI data. At this point, the simulation verifies which stations trigger a rebalancing alert, that is, which stations surpass the bounds of their inventory interval. The simulation also keeps track of the lost demand, i.e., how many bikes were missing during the rentals and how many bikes could not be returned due to full stations by assuming that all rentals and returns happen simultaneously at every simulated hour. Thus, we stress the network to capture its possible failures. We note that uncharged bikes still occupy docks for our simulation.

Due to constrained rebalancing capacity, the alerted stations are prioritized based on their degree of imbalance, computed as the deviation of their inventory from the target value. Consequently, only a limited subset of stations, within the capacity threshold, undergo rebalancing procedures per hour.

Fig. 3 illustrates the simulation of the bike inventory of a station using B0. In the illustration, the station raised an alert due to the shortage of bikes, i.e. its inventory (3) is below the inventory lower bound (4). Assuming that the station is selected to be rebalanced, the inventory of regular and e-bikes is updated according to the target inventory value and the station's historical demand observed in the training data.

#### 4.2.2. Simulation of our model

shared- $\mathcal{RE}$  provides inventory intervals and target inventory values for each type of bike used in the EBSS. Therefore, alerts are individually raised for each type of bike demand, and stations are rebalanced according to the associated target inventory values.

Similar to the simulation of B0, the simulation of shared- $\mathcal{RE}$  emulates the inventory based on rentals, returns, and rebalancing operations conducted hourly accounting for the rebalanced capacity of the system. However, in this scenario, the target inventory and inventory intervals are customized for each demand. This allows the rebalancing alerts to identify and address inventory deficiencies or excesses specific to each demand, and calculate target inventory values accordingly.

During the simulation of our model, the rebalancing process is conducted to replenish the inventory of a station, taking into account the target values for regular and electric bikes. These target values are separately computed based on the demand for each type of bike, as explained in Section 3.2. This approach enables the rebalancing process to be triggered by the demand of a specific bike type. In Fig. 4, for example, only the inventory of e-bikes drops below the lower bound of its inventory interval. Subsequently, the rebalancing process focuses on restoring both the regular and e-bike inventories to their respective target values. This reflects a more realistic operational scenario, as an employee is already dispatched to the station for replenishment. In the given example, two e-bikes and one regular bike are added to the station during the rebalancing process.

#### 4.2.3. Additional simulation remarks

This section presents remarks that are applicable to both simulations detailed in Sections 4.2.1 and 4.2.2.

First of all, the inventory of the stations is initialized at the beginning of the simulations by distributing the total number of bikes in the system (i.e., 6377 regular bikes and 2015 electric bikes for the considered simulated period) proportionally to the target inventory values of each bike demand at the first simulated hour. More precisely, the initial inventory of regular bikes at each station  $s$  is made equal to:

$$b_{\text{regular}} \times \frac{\mathcal{F}_s^{\mathcal{R}}(1)}{\sum_{s' \in \mathcal{S}} \mathcal{F}_{s'}^{\mathcal{R}}(1)},$$

where  $b_{\text{regular}}$  corresponds to the total number of regular bikes in the system and  $\mathcal{S}$  to the set of stations. The initial inventory of e-bikes at each station is defined likewise.

Throughout the simulation, the amount of bikes in the system is kept consistent by employing an algorithm that selects stations while ensuring an equilibrium between the bikes taken from and added to these stations. A detailed explanation of this procedure is provided in Appendix A.

Finally, the simulations take into account that 10% of the electric bikes are uncharged per hour. These bikes are randomly chosen at the beginning of each simulated hour, making them unavailable in the system, and hence, incurring lost demand. The code for both simulations, as well as the code of the proposed model, can be accessed in the repository.<sup>4</sup>

#### 4.3. Analysis of commuters preferences

Based on the commuters' preferences, four different scenarios are emulated regarding bike substitutions:

- None: in this scenario, users never replace their desired bike with another type.
- All bikes: in this scenario, users are flexible in their preferences and will always accept any available bike, regardless of their initial choice.
- Reg. bike → E-bikes: only users who seek regular bikes accept an e-bike if the first is unavailable.
- E-bike → Reg. bikes: only users who seek for an e-bike accept a regular bike if the first is unavailable.

By simulating these different scenarios, we can analyze the impact of bike substitution preferences on the overall bike availability and system performance. This provides insights into the feasibility and desirability of allowing bike substitutions in a bike-sharing system and helps optimize the allocation and utilization of bikes based on customer preferences.

#### 4.4. Tuning

The values of  $\beta^{\mathcal{R}}$  and  $\beta^{\mathcal{E}}$  were tuned through simulations with the validation set with the objective of minimizing the lost demand. Table 2 presents the optimized hyperparameter values, where shared- $\mathcal{RE}_{\text{max}}$  and shared- $\mathcal{RE}_{\text{avg}}$  refer to the use of  $\Lambda_s(\cdot)$  as the maximum or the average service level obtained for different values of the initial inventory (see Section 3.1 - Eq. (2)).

The analysis of Table 2 reveals a discernible pattern in the values of the hyperparameters  $\beta^{\mathcal{R}}$  and  $\beta^{\mathcal{E}}$  based on the bike substitution policy implemented. When there are no bike substitutions or restrictions on bike types, the values of both hyperparameters are fairly similar. However, in scenarios where regular bikes can be substituted with e-bikes, the inventory intervals display greater stringency towards e-bikes, leading to lower values of  $\beta^{\mathcal{R}}$  and higher values of  $\beta^{\mathcal{E}}$ . Conversely, in the scenario where only electric bikes can be replaced by regular bikes, the hyperparameter results exhibit the opposite trend. This demonstrates the model's ability to prioritize each demand independently, as well as its capacity to adapt to the users' preferences.

<sup>4</sup> [https://github.com/datascientistbss/Paper\\_Journal](https://github.com/datascientistbss/Paper_Journal).

#### 4.5. Results

Our results compile the number of rebalancing operations and lost demand computed from the simulations of the baseline and our proposed models. The inventory intervals and the target values for B0 were provided by BIXI for the following time periods in a day: from 6 am to 9 am, from 9 am to 11 am, from 11 am to 4 pm, from 4 pm to 7 pm, and from 10 pm to 6 am. These same time periods were used by shared- $\mathcal{RE}$  to compute inventory intervals and target values. Besides, the root mean squared error (RMSE) of the Gradient-boosted tree demand prediction on the test data was 1.70.

Our experiments considered an hourly rebalancing capacity of 50 stations. This value was provided to us by BIXI as the valid capacity for the year 2022.

Table 3 presents the results regarding the number of rebalancing operations per hour and the percentage of lost demand over the total served demand from the simulations of models B0, and shared- $\mathcal{RE}$  and its different settings of bike substitution.

In summary, the results show that:

- The performance of both shared- $\mathcal{RE}_{max}$  and shared- $\mathcal{RE}_{avg}$  indicates a minimal difference in terms of lost demand and rebalancing operations when implementing the dock division at a station based on either the best or average service level provided.
- Our model consistently outperforms the baseline model, reducing the total lost demand by up to 10% (bike substitution = 'None', shared- $\mathcal{RE}_{avg}$ ). More specifically, the inventory intervals and target inventory values generated by our model have proven to be highly effective in decreasing lost demand for e-bikes when compared to BIXI's rebalancing strategy, reducing the lost demand for e-bikes by up to 32%. This demonstrates that shared- $\mathcal{RE}_{max}$  and shared- $\mathcal{RE}_{avg}$  effectively identify and adapt to the increasing demand for e-bikes better than B0.
- When comparing the various configurations of user preferences, the results align with our expectations across all models. The scenario where no bike substitution is allowed generates the highest lost demand, while the scenario where any substitution is accepted yields the lowest demand loss. The remaining scenarios fall somewhere in between these extremes. Notably, the scenario where only regular bikes can be replaced by electric bikes results in less lost demand than the reverse scenario. This can be attributed to the considerably higher demand for regular bikes observed in our simulation data, which can be partially attributed to the fact that 76% of BIXI's bikes are regular ones. Consequently, the majority of lost demand cases computed in our simulations involve regular bikes.
- By introducing the option to replace regular bikes with electric bikes, the occurrences of lost demand can be significantly reduced. This is due to the potential to fulfill the demand for regular bikes with available electric bikes, thereby mitigating lost opportunities for riders.
- Overall, B0 requires fewer rebalancing interventions than our model. This result is expected as the simulation shared- $\mathcal{RE}$  can detect imbalances of each type of demand, leading to a higher number of alerts and, consequently, a higher number of rebalancing activities. Nevertheless, the rebalancing operations carried by shared- $\mathcal{RE}$  respect BIXI's maximum rebalancing capacity.
- We observe that the average number of rebalancing operations per hour in all simulations is significantly lower than the hourly rebalancing capacity of 50 stations. This is attributed to the reduced demand typically observed from 10 pm to 7 am, as illustrated in Fig. 1. During these off-peak hours, the number of alerts and, consequently, rebalancing operations, is considerably lower than 50. In contrast, in periods of high demand the number of alerts is often greater than the rebalancing capacity – in a single simulated hour, the maximum number of observed alerts was 398.

## 5. Conclusion

### 5.1. General discussion

Rebalancing bike-sharing systems is a multifaceted task that encompasses numerous factors such as demand variability, time sensitivity, and user preferences. As electric bikes become more popular in existing systems, accounting for such additional demand adds complexity to the rebalancing planning. We propose a model capable of providing targeted rebalancing recommendations for station-based Electric Bike Sharing Systems, comprising both regular and e-bikes. Our model leverages predicted demand for the upcoming hours to tailor recommendations specific to the demand of each bike type.

From a theoretical perspective, our work extends existing works on the definition and computation of service-levels [see, e.g. 29], inventory target values and inventory intervals [see, e.g. 17] for regular bikes to bimodal systems with both regular and electric bikes. Here, a simultaneous computation is required to ensure that the total station capacity is not exceeded when considering both types of resources. Our model offers an automated division of docks per station based on predicted demand while allowing for customization according to the operator's requirements for each demand. One significant advantage is that our model independently adjusts the inventory intervals for each bike type. This flexibility is crucial, as it accommodates the varying preferences of BSS users. One proposed model variant, namely shared- $\mathcal{RE}_{max}$ , computes the inventory intervals and target inventory values aiming to maximize the ratio of satisfied trips while dividing the available docks between regular and electric bikes. Conversely, the alternate variant, denoted shared- $\mathcal{RE}_{avg}$ , aims to maximize the average of this ratio across varying initial inventory values. Both variants are compared against B0, a rebalancing strategy emulating the current practice at BIXI Montreal.

From a practical perspective, the mechanisms of inventory intervals and target inventory values are often already an essential part of the rebalancing process. A major advantage of our approach is hence its relative simple deployment in existing decision-making processes and its minimal requirement of resources: a computer with moderate processing power and a database to feed the prediction model. Furthermore, the manual computation of inventory targets and intervals, although common in BSSs, may result in suboptimal performance. This issue is particularly pronounced in the context of bimodal EBSSs, where the nuances of each demand type may be overlooked. Such manual computation can hence be easily replaced by an automatized computation of such parameters, without the need of restructuring the existing information and decision-making process.

Our empirical experiments show that our proposed model is able to reduce the amount of total lost demand in all simulated scenarios, demonstrating its ability to adapt to diverse trip patterns and commuters' preferences. In the scenario with the greatest difference in performance, our model managed to reduce the total lost demand by up to 10% and the lost demand for electric bikes by up to 32% over the baseline model B0.

The results further demonstrate the importance of comprehending commuters' preferences and their willingness to substitute their initial bike choice when designing a rebalancing strategy. This understanding enables operators to make informed decisions regarding the supply of each bike type, ensuring the provision of a high-quality service. Additionally, our results show that actively encouraging commuters to consider alternative bikes when their desired option is unavailable can have a significant impact on reducing the lost demand. This effect was particularly pronounced when the initially preferred bike type exhibits higher demand compared to the other. By promoting bike substitution, operators can effectively mitigate the occurrence of lost demand, leading to improved service reliability and user satisfaction.

Finally, from a financial perspective, the predictive models used within our method make use of open-source libraries, therefore circumventing the need for licensing costly optimization solvers, which



would be required when using classical rebalancing optimization models. However, it is important to note that our algorithm proposes rebalancing recommendations on a tactical level, i.e., it identifies the ideal station inventories without explicitly proposing vehicle routes. Operators are therefore required to design rebalancing routes based on these recommendations, either by manual planning or using routing algorithms.

### 5.2. Limitations and future work

Due to the use of actual trip data in our experiments, we have not considered information about trips that were not undertaken due to the unavailability of bikes or docks, also referred to as unobserved demand. In future research, we plan to address this limitation by conducting experiments using synthetic data. This will enable us to explore a wider range of scenarios and accurately quantify demand losses, allowing for a more comprehensive evaluation of the rebalancing recommendations.

Finally, the rebalancing recommendations provided by our model can be seamlessly integrated into optimization routing models. This integration would allow for the optimization of the entire rebalancing process in a unified manner, maximizing the effectiveness and efficiency of the system as a whole. This direction holds promise for future research and offers potential for further improvements in the field of bimodal bike-sharing system management.

## Appendix A. Maintaining equilibrium between the number of bikes picked up and dropped off

This step is crucial as it ensures that the number of bikes in the network remains stable through the simulation.

Let us define four distinct lists that categorize the stations inventory requirements: the lists  $\mathcal{S}^{R+}$  and  $\mathcal{S}^{E+}$  contain alerted stations that require additional regular and electric bikes, respectively; whereas the lists  $\mathcal{S}^{R-}$  and  $\mathcal{S}^{E-}$  contain the stations that require regular and electric bikes to be removed from their inventory. The rebalancing process starts by assigning the stations that raised rebalancing alerts to one list associated to regular bikes ( $\mathcal{S}^{R+}$  or  $\mathcal{S}^{R-}$ ) and/or to another list associated to electric bikes ( $\mathcal{S}^{E+}$  or  $\mathcal{S}^{E-}$ ). Stations on all lists are sorted based on their deviation from the target inventory value, ensuring that those with the greatest difference have a higher priority and, therefore, a higher chance of being selected for rebalancing.

The stations are iteratively selected to be rebalanced according to the value of the variables  $accumulator^R$  and  $accumulator^E$ . These variables store the number of regular and electric bikes either exceeding or missing in the network after a sequence of rebalancing operations have occurred – note that negative values signalize missing bikes whereas positive values signalize exceeding bikes in the system. The stations are chosen for rebalancing with the goal of consistently driving the variables  $accumulator^R$  and  $accumulator^E$  towards zero.

Fig. A.5 presents the flowchart of the process of selecting the stations to be rebalanced. The procedure starts by identifying which accumulator ( $accumulator^R$  or  $accumulator^E$ ) has the highest absolute value and if its value is positive or not. This step is critical as it dictates from which list ( $\mathcal{S}^{R+}$ ,  $\mathcal{S}^{R-}$ ,  $\mathcal{S}^{E+}$  or  $\mathcal{S}^{E-}$ ) the next station to be rebalanced will be drawn. After the list is selected, the first station of the list, i.e. the station with the highest priority, is selected to be rebalanced regarding both regular and electric bikes. This results in updating  $accumulator^R$  and  $accumulator^E$ , and on removing the selected station from all lists where it is present. That procedure continues until that the number of selected stations reaches the maximum rebalancing capacity. Besides, the procedure is halted whenever it selects, according to the accumulator variables, a list of stations that is empty.

## References

- [1] N. Boufidis, A. Nikiforiadis, K. Chrysostomou, G. Aifadopoulou, Development of a station-level demand prediction and visualization tool to support bike-sharing systems' operators, *Transp. Res. Procedia* 47 (2020) 51–58.
- [2] T. Bulhões, A. Subramanian, G. Erdoğan, G. Laporte, The static bike relocation problem with multiple vehicles and visits, *Eur. J. Oper. Res.* 264 (2) (2018) 508–523.
- [3] J. Brinkmann, M.W. Ulmer, D.C. Mattfeld, Short-term strategies for stochastic inventory routing in bike sharing systems, *Transp. Res. Procedia* 10 (2015) 364–373.
- [4] L. Chen, D. Zhang, L. Wang, D. Yang, X. Ma, S. Li, Z. Wu, G. Pan, T.-M.-T. Nguyen, J. Jakubowicz, Dynamic cluster-based over-demand prediction in bike sharing systems. *Proceedings of the 2016 ACM International Joint Conference on Pervasive and Ubiquitous Computing*, 2016, pp. 841–852.
- [5] Z. Chen, Y. Hu, J. Li, X. Wu, Optimal deployment of electric bicycle sharing stations: model formulation and solution technique, *Netw. Spatial Econ.* 20 (1) (2020) 99–136.
- [6] S. Datner, T. Raviv, M. Tzur, D. Chemla, Setting inventory levels in a bike sharing network, *Transp. Sci.* 53 (1) (2019) 62–76.
- [7] I.A. Forma, T. Raviv, M. Tzur, A 3-step math heuristic for the static repositioning problem in bike-sharing systems, *Transp. Res. Part B Methodol.* 71 (2015) 230–247.
- [8] T. Fukushima, D.T. Fitch, S. Handy, Can an incentive-based approach to rebalancing a dock-less bike-share system work? Evidence from Sacramento, California, *Transp. Res. Part A Policy Pract.* 163 (2022) 181–194.
- [9] D. Gammelli, Y. Wang, D. Prak, F. Rodrigues, S. Minner, F.C. Pereira, Predictive and prescriptive performance of bike-sharing demand forecasts for inventory management, *Transp. Res. Part C Emerg. Technol.* 138 (2022) 103571.
- [10] D.K. George, C.H. Xia, Fleet-sizing and service availability for a vehicle rental system via closed queueing networks, *Eur. J. Oper. Res.* 211 (1) (2011) 198–207.
- [11] S. Ghosh, P. Varakantham, Y. Adulyasak, P. Jaillet, Dynamic repositioning to reduce lost demand in bike sharing systems, *J. Artif. Intell. Res.* 58 (2017) 387–430.
- [12] M.D. Gleditsch, K. Hagen, H. Andersson, S.J. Bakker, K. Fagerholt, A column generation heuristic for the dynamic bicycle rebalancing problem, *Eur. J. Oper. Res.* (2022).
- [13] H.R. Gómez Márquez, R. López Bracho, A. Ramírez-Nafarrate, A simulation-optimization study of the inventory of a bike-sharing system: the case of Mexico City Ecobici's system, *Case Stud. Transp. Policy* 9 (3) (2021) 1059–1072.

## CRediT authorship contribution statement

**Maria Clara Martins Silva:** Conceptualization, Data curation, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Daniel Aloise:** Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Supervision, Validation, Writing – original draft, Writing – review & editing. **Sanjay Dominik Jena:** Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Supervision, Validation, Writing – original draft, Writing – review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgments

The authors are grateful to BIXI Montreal who provided resources throughout the project. The authors also thank the Natural Sciences and Engineering Research Council of Canada (NSERC) for its financial support.

- [14] S. Hossain, M.A. Islam, M.S. Akther, COVID-19 impact on travel and work habits of office workers in Bangladesh, *Trans. Eng.* 11 (2023) 100162.
- [15] K. Hosseini, A. Stefaniec, M. O'Mahony, B. Caulfield, Optimising shared electric mobility hubs: Insights from performance analysis and factors influencing riding demand, *Case Stud. Transp. Policy* 13 (2023) 101052.
- [16] J. Huang, H. Sun, H. Li, L. Huang, A. Li, X. Wang, Central station-based demand prediction for determining target inventory in a bike-sharing system, *Comput. J.* 63 (3) (2020) 573–588.
- [17] P. Hulot, D. Aloise, S.D. Jena, Towards station-level demand prediction for effective rebalancing in bike-sharing systems. *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, 2018, pp. 378–386.
- [18] S. Ji, C.R. Cherry, L.D. Han, D.A. Jordan, L. Lee, Electric bike sharing: simulation of user demand and system availability, *J. Clean. Prod.* 85 (2014) 250–257.
- [19] N. Jian, S.G. Henderson, An introduction to simulation optimization. *2015 Winter Simulation Conference (WSC)*, 2015, pp. 1780–1794.
- [20] R. Julio, A. Monzon, Long term assessment of a successful e-bike-sharing system. *Key drivers and impact on travel behaviour*, *Case Stud. Transp. Policy* (2022).
- [21] A. Kabra, E. Belavina, K. Girotra, Bike-share systems: accessibility and availability, *Manage. Sci.* 66 (9) (2020) 3803–3824.
- [22] X. Li, Y. Xu, X. Zhang, W. Shi, Y. Yue, Q. Li, Improving short-term bike sharing demand forecast through an irregular convolutional neural network, *Transp. Res. Part C Emerg. Technol.* 147 (2023) 103984.
- [23] M. Lowalekar, P. Varakantham, S. Ghosh, S.D. Jena, P. Jaillet, Online repositioning in bike sharing systems. *Twenty-seventh International Conference on Automated Planning and Scheduling*, 2017.
- [24] A. Lozano, J.F. De Paz, G. Villarrubia Gonzalez, I.D.H. De La, J. Bajo, Multi-agent system for demand prediction and trip visualization in bike sharing systems, *Appl. Sci.* 8 (1) (2018) 67.
- [25] E.H.-C. Lu, Z.Q. Lin, Rental prediction in bicycle-sharing system using recurrent neural network, *IEEE Access* 8 (2020) 92262–92274.
- [26] L.M. Martinez, L. Caetano, T. Eiró, F. Cruz, An optimisation algorithm to establish the location of stations of a mixed fleet biking system: an application to the city of Lisbon, *Procedia-Social Behav. Sci.* 54 (2012) 513–524.
- [27] P. Mrazovic, J.L. Larriba-Pey, M. Matskin, A deep learning approach for estimating inventory rebalancing demand in bicycle sharing systems. *2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC) 2*, 2018, pp. 110–115.
- [28] E. Possani, E. Castillo, Optimizing the inventory and routing decisions in a bike-sharing system: a linear programming and stochastic approach, *Case Stud. Transp. Policy* 9 (4) (2021) 1495–1502.
- [29] J. Schuijbroek, R.C. Hampshire, W.J. Van Hoes, Inventory rebalancing and vehicle routing in bike sharing systems, *Eur. J. Oper. Res.* 257 (3) (2017) 992–1004.
- [30] M.E. Shaik, S. Ahmed, An overview of the impact of COVID-19 on road traffic safety and travel behavior, *Trans. Eng.* 9 (2022) 100119.
- [31] J. Shu, M.C. Chou, Q. Liu, C.-P. Teo, I.L. Wang, Models for effective deployment and redistribution of bicycles within public bicycle-sharing systems, *Oper. Res.* 61 (6) (2013) 1346–1359.
- [32] T. Raviv, M. Tzur, I.A. Forma, Static repositioning in a bike-sharing system: models and solution approaches, *EURO J. Transp. Logist.* 2 (3) (2013) 187–229.
- [33] T. Raviv, O. Kolka, Optimal inventory management of a bike-sharing station, *IIE Trans.* 45 (10) (2013) 1077–1093.
- [34] H. Si, J.-g. Shi, G. Wu, J. Chen, X. Zhao, Mapping the bike sharing research published from 2010 to 2018: a scientometric review, *J. Clean. Prod.* 213 (2019) 415–427.
- [35] F. Soriguera, A continuous approximation model for the optimal design of public bike-sharing systems, *Sustain. Cities Soc.* 52 (2020) 101826.
- [36] S. Tan, Z. Li, N. Xie, Dynamic capacitated arc routing problem in e-bike sharing system: a Monte Carlo tree search approach, *J. Adv. Transp.* 2021 (2021).
- [37] Y. Zhou, Z. Lin, R. Guan, J.B. Sheu, Dynamic battery swapping strategies for e-bike sharing systems with electric fences, Available at SSRN 4267760 (2022).
- [38] P. Zhou, C. Wang, Y. Yang, X. Wei, E-sharing: data-driven online optimization of parking location placement for dockless electric bike sharing. *2020 IEEE 40th International Conference on Distributed Computing Systems (ICDCS)*, 2020, pp. 474–484.
- [39] S. Zhu, Optimal fleet deployment strategy: Model the effect of shared e-bikes on bike-sharing system, *J. Adv. Transp.* 2021 (2021).