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Constitutive Modeling of Flow Liquefaction of Tailings

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Département des génies civil, géologique et des mines

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en vue de l'obtention du diplôme de *Philosophiae Doctor*

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DEDICATION

To My Family...

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RESUME

La liquéfaction par écoulement est la perte rapide de la résistance au cisaillement des sols granulaires saturés et lâche pendant un chargement non drainé. Ce phénomène est à l'origine de certaines des défaillances catastrophiques les plus récentes survenues dans les barrages de résidus miniers. Le potentiel de développement de ce mécanisme est difficile à évaluer en pratique car il est complexe et nécessite l'utilisation de modèles constitutifs avancés afin de le modéliser correctement. Cette recherche se concentre sur le développement d'un modèle constitutif pratique pour simuler la liquéfaction statique, compte tenu des caractéristiques des résidus miniers. En particulier, la teneur en fines des résidus miniers à l'intérieur des barrages à résidus présente une importante variation spatiale. Dans la littérature, des efforts ont été faits pour adapter les lois de comportement pour sable existants afin d'intégrer l'influence de la teneur en fines sur le comportement mécanique des mélanges sable-silt. Cependant, l'efficacité de cette approche dépend principalement de la performance du modèle pour sable choisi.

Cette thèse se concentre sur le modèle NorSand, qui constitue un choix privilégié parmi les professionnels pour simuler le comportement du sable. Il se distingue par l'utilisation efficace de seulement trois ou quatre paramètres. Les paramètres restants du modèle, qui sont utilisés pour la définition du module d'élasticité et de l'état critique, partagent des concepts fondamentaux avec ceux utilisés par d'autres modèles de sol avancés basés sur la mécanique des sols à l'état critique. L'objectif principal de cette thèse est d'étendre les fonctionnalités du modèle NorSand pour couvrir la simulation de la liquéfaction statique des résidus caractérisés par une teneur variable en fines. Tout au long de ce processus, l'intention demeure de préserver la simplicité et la praticité inhérentes au modèle.

Cependant, en raison des limitations majeures inhérentes au modèle NorSand, la thèse poursuit une double approche. Premièrement, elle cherche à améliorer la capacité du modèle à simuler avec précision le comportement non drainé des matériaux granulaires. Cela a abouti à la formulation d'un modèle constitutif qui offre aux utilisateurs la flexibilité de contrôler directement le comportement non drainé des matériaux granulaires, présentant un comportement d'écoulement, d'écoulement limité et de non-écoulement. Par la suite, en tirant parti de cette version améliorée,

l'accent est mis sur l'extension du modèle pour répondre aux défis spécifiques liés aux résidus, en particulier quant à la variation en teneur en fines, et son effet sur le comportement des résidus.

Cet effort de recherche a abouti à une compréhension globale du modèle NorSand, de ses avantages et de ses limites (chapitre 3 et chapitre 4), conduisant à la proposition d'une nouvelle loi non linéaire de dilatation pour le modèle NorSand (chapitre 4). En utilisant la nouvelle loi, une nouvelle version modifiée appelée "PolySand" a été développée (Chapitre 5) pour contrôler directement les résultats de simulation du comportement en écoulement et hors écoulement d'échantillons meubles et denses. De plus, une nouvelle approche a été intégrée au modèle pour permettre de simuler efficacement le comportement d'un écoulement limité, une fonctionnalité qui était absente du modèle NorSand original (chapitre 5). Par la suite, une version étendue nommée « PolySand-Wfines » a été développée pour gérer efficacement le comportement monotone du sable et des mélanges sable-silt (Chapitre 6), contribuant ainsi à l'avancement de la modélisation constitutive pratique des matériaux de résidus.

ABSTRACT

Flow liquefaction is the rapid loss of shear strength of loose saturated granular soils under undrained loading. This phenomenon has been responsible for some of the most recent catastrophic failures that happened in tailings dams. The potential for this mechanism to develop is hard to evaluate in practice because it is complex and requires the use of advanced constitutive models. This research focuses on developing a practical constitutive model for proper simulation of static liquefaction, considering the unique characteristics of tailings. Particularly, one prominent characteristic of tailings within a tailings dam is the spatial variation in fines content throughout the dam structure. In the literature, efforts have been made to adapt existing sand models to incorporate the influence of non-plastic fines content on the mechanical behavior of sand-silt mixtures. Yet, the effectiveness of the proposed framework mainly depends on the selection of an appropriate sand-based model for extension.

This thesis notably focuses on the NorSand model, which is a preferred choice among professionals for simulating sand behavior. It stands out due to its efficient utilization of just three to four material constants. The model's remaining parameters, which are used in the elastic modulus and critical state definition, share fundamental concepts as those employed by other advanced critical state-based soil models. The primary objective is to expand the functionality of the NorSand model to cover the simulation of static liquefaction in tailings materials characterized by varying fines content. Throughout this extension process, the intention remains to preserve the model's inherent simplicity and practicality.

However, due to major limitations inherent in the NorSand model, the thesis pursues a two-fold approach. Firstly, it seeks to enhance the model's capability to accurately simulate the undrained behavior of granular materials. This result in formulating a constitutive model that offers users the flexibility to directly control the undrained behavior of granular materials, involving flow, limited flow, and non-flow behavior. Subsequently, leveraging this refined version, the focus shifts to the extension of the model to address the specific challenges related to tailings materials.

As a result, this research effort has resulted in a comprehensive understanding of the NorSand model, its advantages, and limitations (Chapter 3 and Chapter 4), leading to the proposal of a new nonlinear state-dilatancy law for the NorSand model (Chapter 4). Using the new law, a new modified version referred to as "PolySand" has been developed (Chapter 5) to directly control the simulation results of flow and non-flow behavior of loose and dense samples. Moreover, a novel approach has been integrated into the model to effectively enable the simulation of limited flow behavior, a feature that was absent in the original NorSand model (Chapter 5). Subsequently, an extended version named "PolySand-Wfines" has been developed to effectively manage the monotonic behavior of both sand and sand- non plastic silt mixtures (Chapter 6), thus contributing to the advancement of practical constitutive modeling of tailings materials.

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CHAPTER 1 INTRODUCTION

1.1 Background and Motivation

Flow liquefaction occurs when loose, saturated granular soils undergo undrained shear loading. Mine tailings are particularly susceptible to static liquefaction because they are typically deposited in a saturated and loose state. The consequence of flow liquefaction is the release of a large volume of tailings, leading to catastrophic outcomes (Byrne et al. 2015; Santamarina et al., 2019; Sadrekarimi and Riveros, 2020; Arroyo and Gens, 2021). For example, Feijao tailings dams at Brumadinho, Brazil failed on January 25, 2019, (Robertson et al., 2019) resulting in a massive mudslide, claiming the lives of at least 232 individuals.

The physical properties of Mine tailings vary spatially in the field due to the deposition method, mill process, and ore type (Torres-Cruz and Santamarina, 2020). They could exist as fine-grained, coarse-grained, or a mix of both. In the same tailings facility, tailings could be coarse-grained near the slurry discharge point to fine-grained in regions distant from the discharge point. A change in fines content changes the structure of the coarse-fine tailings from a coarse governed structure to a fine governed structure, which affects the undrained response of such mixtures considerably (Lade and Yamamuro, 1997; Lade et al., 1998; Thevanayagam et al., 2002; Rahman and Lo, 2012; Rahman et al., 2014; Yang et al., 2015; Torres-Cruz and Santamarina, 2020; Yilmaz et al., 2023).

Therefore, analysing the stress-strain behaviour of tailings in a tailings dams, requires a soil model that is able to not only to simulate the behaviour of loose granular soils properly, but also to consider the effect of fines content on its behaviour. In cases where sand-silt mixtures share similar mineralogy and angularity, certain frameworks have been devised to simulate their monotonic behavior by extending existing sand models (for example the equivalent critical state theory developed by Rahman and Lo, 2014). Consequently, the effectiveness of these extended models heavily relies on the capabilities of the original model in simulating sand behavior, and thus, the advantages and disadvantages of the original model are carried over to the newly extended model. Therefore, when dealing with tailings containing varying percentages of fines, it is crucial to first develop a reliable soil model and then considering its extension to accurately simulate the behavior of tailings materials.

In the last thirty years, significant efforts have been dedicated to constitutive modeling aimed at simulating the behavior of granular soils, especially sand (Jefferies, 1993; Wood et al., 1994; Manzari and Dafalias, 1997; Dafalias and Manzari, 2004; Petalas et al., 2020). Among the various models developed, those based on the critical state theory have gained widespread acceptance. The reason for their popularity lies in the framework's ability to effectively simulate soil behavior across a broad range of initial conditions, including different void ratios and confining pressures. Depending on the specific formulation of the models, three distinct critical state-based categories have emerged, classified according to their chronological development in geomechanics: single yield surface plasticity with an associated flow rule (called Cambridge type models), single yield surface plasticity with a non-associated flow rule (e.g. Wood et al. (1994)), and bounding surface plasticity (e.g. Manzari and Dafalias (1997)) typically used for modeling the cyclic behavior of sands. A pioneering Cambridge type model for simulating the monotonic behavior of sand is NorSand (Jefferies, 1993), and it has gained significant traction in geotechnical engineering applications (Jefferies et al., 2019; Shuttle et al., 2021; Pezeshki and Ahmadi, 2022; Ng et al., 2023) due to its straightforward implementations and having few material properties that possess a clear physical meaning (Jefferies and Been, 2015). Apart from critical state parameters, only four to five drained tests conducted on dense samples are required to calibrate NorSand's material properties (Cheng and Jefferies, 2020; Macedo and Vergaray, 2022).

While the model exhibits simplicity and is attractive for its potential for simulating flow liquefaction, it has several limitations. Firstly, the method employed by the model to attain its final formulation limits modifications of its fundamental concepts, including the shape of the critical state line and yield surface. Secondly, users have limited control over the simulated mechanical behavior of loose samples, as the model's formulations derive from considerations focused on dense samples (Jefferies, 1993). Consequently, the model produces flow behavior in loose states that diverges from measured data. To enhance the precision of simulation outcomes, users must incorporate additional mathematical expressions into its hardening rule (Jefferies and Been, 2015) along with manipulation of the bulk modulus (Shuttle and Jefferies, 2016; Reid and Smith, 2021; Ng et al., 2023). Thirdly, owing to these limitations, attempts to extend its applicability within new frameworks of critical state theory, such as anisotropic critical state theory (Li and Dafalias, 2012), fail to yield desirable results (Castonguay and Konrad, 2018). Consequently, it appears that

extending NorSand using equivalent critical state theory (Rahman et al., 2014) to simulate coarse-fine tailings' behavior would also fail to yield accurate results without revising NorSand's formulations to eliminate its limitations.

The demonstrated capability of the NorSand model in simulating flow liquefaction indicates that there are opportunities for further enhancing its ability to simulate and control the undrained mechanical behavior of loose granular samples. This feature forms the fundamental premise of this thesis, which aims to construct a constitutive model for tailings materials.

1.2 Objectives and Scope of the Thesis

The objective of this thesis is to create a constitutive model for the flow liquefaction of granular materials that is easy to implement and understand, allowing practitioners and researchers to use it effectively. The development of such constitutive model should be driven by three specific objectives: simplicity, suitability for loose saturated granular materials, and consideration of the main characteristics of tailings. Given the element of simplicity inherent in the NorSand model, it has been selected as a basis for modification and enhancement, with a specific focus on improving its ability and eliminate its shortcomings. This thesis will cover the following objectives:

- Finding and identifying the underlying cause for the NorSand limitations in properly simulating the flow liquefaction of loose samples.
- Deriving a new model that overcomes NorSand's limitations, by introducing a formulation capable of directly controlling flow liquefaction, but that retains the ease of calibration of the NorSand model.
- Extending the proposed refined model through the utilization of the equivalent critical state theory (Rahman et al., 2014), to model the influence of fines content on the mechanical response of tailings.

1.3 Organization of the Thesis

The focus of this thesis revolves around the constitutive modeling of flow (static) liquefaction in tailings. The research is organized into seven primary chapters.

Chapter 2: Literature Review:

The literature review has been structured into four main parts, addressing key concepts that will be utilized, considered, or assumed in the development of the proposed model. This approach ensures comprehensive coverage of the relevant literature and facilitates the incorporation of essential elements into the model.

- Literature Review Part 1: understanding the undrained and drained behavior of granular soils using laboratory tests.
- Literature Review Part 2: Investigating the effect of fines content on the microstructure of sand-silt mixtures and their mechanical behavior.
- Literature Review Part 3: Understanding soil constitutive models applicable for granular soils.
- Literature Review Part 4: General properties of mines tailings

Chapter 3: On the numerical issues of the NorSand model in the simulation of undrained behavior of granular soils

Chapter 3 examines the NorSand model, widely employed in simulating the undrained response of granular materials. The comprehensive analysis undertaken in this chapter aims to enhance the understanding of the NorSand model's performance in simulating the undrained behavior of granular materials. It also seeks to identify potential areas for improvement, particularly in modeling states looser than the critical. The chapter details the assumptions and mathematical techniques employed in the NorSand model to modify its response and account for the behavior of looser states. An in-depth evaluation of these approaches is provided, highlighting their limitations in accurately capturing the intricate behavior of granular materials.

Chapter 4: Implementation of a Nonlinear State-Dilatancy Law in the NorSand Model

Chapter 4 concentrates on the NorSand state-dilatancy law, which is a linear equation connecting limiting dilatancy to the limiting state parameter. The implications of employing this law on the NorSand model's ability to simulate the drained and undrained behavior of granular soils will be

explored. To enhance the NorSand model, a novel non-linear state dilatancy is proposed in this chapter, leading to an improvement of the NorSand model.

Chapter 5: PolySand Model, a Modified Cambridge Type NorSand Model to Simulate the Monotonic Behavior of Sand

In this chapter, the focus is on developing an improved constitutive model for granular soils known as the PolySand model. By building upon the existing NorSand model and the newly introduced non-linear state dilatancy law, the new model aims to effectively capture the flow liquefaction and limited flow behavior observed in granular soils. The chapter introduces the model's formulation, calibration process, and examines its validity, with particular emphasis on the model's enhanced flexibility in controlling the undrained mechanical behavior of granular materials, including limited-flow. This increased control holds the potential to facilitate more accurate simulations, offering valuable insights into the practical applications of geotechnical and mining engineering involving such soils.

Chapter 6: PolySand-Wfines Model – Extended PolySand Model to Simulate the Monotonic Behaviour of Sand with Different Fines Content

The primary aim of this study is to create a constitutive model that effectively captures the monotonic behavior observed in sand-silt mixtures. Chapter 6 focuses on introducing the PolySand-Wfines model, an extension of the PolySand model. The PolySand-Wfines model operates within the framework of equivalent critical state theory, aiming to enhance the simulation capabilities for accurately representing the mechanical response of these mixtures. To assess the model's simulation capabilities, a series of undrained triaxial tests on sand-silt mixtures with varying fines content from the literature are simulated. These tests provide valuable insights into the model's performance and its ability to capture the complex behavior of sand-silt mixtures.

Chapter 7 Summary, Discussion, Limitations, Future Avenues of Research

In this chapter, a comprehensive summary of the research conducted is presented, highlighting the obtained results, and drawing meaningful conclusions. Recommendations are also provided. Additionally, important aspects and unresolved issues pertaining to this study are identified, emphasizing the need for further attention and future work in these areas.

CHAPTER 2 MECHANISMS OF FLOW LIQUEFACTION AS UNDERSTOOD FROM LABORATORY TESTS

2.1 Introduction

The main objective of this thesis is to develop a constitutive model for tailings materials, able to simulate their mechanical behavior under drained and undrained shear loading. This chapter presents the relevant literature that has been reviewed and analyzed to achieve this objective. The literature review is divided into four parts. The first part focuses on the mechanical behavior of granular soils under both drained and undrained loading conditions, as understood through laboratory tests. The second part explores the impact of fines content on the microstructure of sand-silt mixtures, defining microstructure parameters that aid in interpreting the mechanical behavior of such mixtures. The third part provides some theoretical background on soil constitutive models, which includes an overview of elastoplasticity, and a discussion on the critical state theory framework, and the soil models used to simulate both sand and sand-silt mixtures. Finally, the last part discusses the characteristics of tailings that should be considered during the development of the constitutive model.

2.2 Part 1: understanding the undrained and drained behavior of granular soils using laboratory tests

In the field, soils are subjected to undrained and drained shearing along various stress paths. In the laboratory, the loading paths are limited by the testing device. As a result, the mechanical behavior of the soils can only be identified under specific controlled loading paths such as triaxial compression, triaxial extension, or direct shear. The observations drawn from the different tests can be utilized to develop constitutive soil models, and by using data under a variety of loading paths, the model's capability and accuracy in simulating any random stress paths can be improved. The aim of this section is to present the behavior of soils under given loading paths and the controlling key concepts.

2.2.1 Basic definitions

Table 2-1 outlines the definitions of various stress and strain quantities, loading parameters, and soil properties, which are determined based on the principal stress values of $\sigma_1, \sigma_2, \sigma_3$ and their corresponding principal strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$.

Table 2-1 Definitions of quantities using principal stresses and strains.

	Quantities	Triaxial Compression	Triaxial Extension
Mean Stress	$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$	$p = \frac{\sigma_1 + 2\sigma_3}{3}$	$p = \frac{\sigma_1 + 2\sigma_3}{3}$
Deviatoric Stress	$q = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}}$	$q = \sigma_1 - \sigma_3$	$q = \sigma_1 - \sigma_3$
Shear mode (Lode Angle)	$-\frac{\pi}{6} \leq \theta = \arctan\left(\frac{1}{\sqrt{3}} \frac{2\sigma_3 - \sigma_1 - \sigma_2}{\sigma_1 - \sigma_2}\right) \leq \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	$\theta = -\frac{\pi}{6}$
Shear mode (b parameter)	$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$	$b = 0$	$b = 1$
Angle between vertical axis and direction of major principal stress	α	$\alpha = 0$	$\alpha = \frac{\pi}{2}$
Loading direction tensor	$n = \frac{1}{\sqrt{6(b^2 - b + 1)}} \begin{bmatrix} 3\cos^2\alpha - 1 - b & 0 & -3\cos\alpha\sin\alpha \\ 0 & 2b - 1 & 0 \\ -3\cos\alpha\sin\alpha & 0 & -3\cos^2\alpha + 2 - b \end{bmatrix}$	$n = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	$n = \frac{1}{\sqrt{6}} \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shear Strain	$\varepsilon_q = \left[\frac{2}{3} \boldsymbol{\varepsilon}_d : \boldsymbol{\varepsilon}_d \right]^{1/2}$	$\varepsilon_q = \frac{2}{3} \left(\varepsilon_1 - \frac{\varepsilon_v}{3} \right)$	$\varepsilon_q = \frac{2}{3} \left(\varepsilon_1 - \frac{\varepsilon_v}{3} \right)$
Volumetric Strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = -\frac{e - e_0}{1 + e_0}$	$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$	$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$
Void ratio	$e = e_0 - (1 + e_0)\varepsilon_v$	$e = e_0 - (1 + e_0)\varepsilon_v$	$e = e_0 - (1 + e_0)\varepsilon_v$

e_0 is the initial void ratio of the sample. $\boldsymbol{\varepsilon}_d = \boldsymbol{\varepsilon} - \frac{1}{3}\varepsilon_v \mathbf{I}$, where $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{\varepsilon}_d$ is the deviatoric strain tensor, and $\mathbf{I}$ is the identify matrix of order 3.

Typically, test results are plotted in three spaces: volumetric strain or void ratio versus mean stress, deviatoric stress versus mean stress, and deviatoric strain versus deviatoric stress. The first space records the volumetric response of the soil under different confining pressures, while the second space illustrates the stress path the soil undergoes during loading in a stress space. The last space shows the deviatoric stress - deviatoric strain of the soil response. For drained isotropic loading (zero deviatoric stress), the volumetric response versus means stress plot shows

consolidated lines. Normally consolidated lines correspond to consolidation lines where the current isotropic loading represents the maximum loading the soil has previously experienced.

The most common test currently available to analyse the mechanical behavior of soils is the triaxial test. In this test, the loading is axisymmetric, and the axisymmetric conditions and direction of principal stresses remain unchanged throughout the testing process. Figure 2-1 shows the loading conditions for triaxial compression and triaxial extension test, where $\sigma_2 = \sigma_3$. Equivalently, the triaxial test values and formulation can be obtained based on the mentioned requirements, as presented in Table 2-1.

In the triaxial compression test, the deviatoric stress increases with an increase in axial stress, as depicted in Figure 2-1a. Conversely, in the triaxial extension test, shearing is accomplished by increasing the confining pressure while holding the axial stress constant (Figure 2-1b). The Lode angle (θ) is used as a loading parameter to distinguish between the triaxial compression and extension shearing mode, and is equal to 30° for triaxial compression and -30° for triaxial extension. An alternative parameter to characterize the shear mode is the b parameter which reflects the impact of the intermediate principal stress and has a value of 0 and 1 for triaxial compression and extension tests, respectively.

In soil mechanics another quantity called α is sometimes used, which represents the direction of the maximum principal stress in relation to the vertical, with a 0° for triaxial compression and 90° for triaxial extension, as illustrated in Figure 2-1. α alongside b can be used to define the loading direction tensor which is a unit tensor showing the loading direction. With respect to the loading direction tensor, there is a fabric direction tensor which shows how soil grains are deposited, making the current structure of the soil. Soils are often deposited on a horizontal plane (or called bedding plane), and, as a result, they generally exhibit a stiffer behavior if loaded vertically than horizontally. In other words, the soil behavior is anisotropic, and the orientation of loading compared to the horizontal bedding plane (or in general term the direction of loading tensor with respect to the fabric tensor) will affect the soil's response considerably. In this regard, the hollow cylinder test was developed to facilitate undrained tests under varying constant values of α , with the aim of comprehending the influence of loading direction in comparison to the bedding

plane, and to examine the effects of soil anisotropy on its mechanical behavior (for more information see Yoshimine et al. 1998).

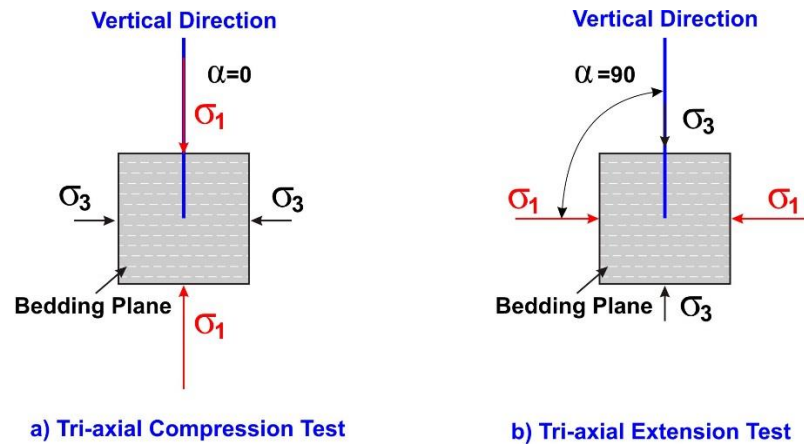


Figure 2-1 definition of α and its corresponding value for a) triaxial compression test, b) triaxial extension test

In the direct simple shear test, the vertical stress is constant while the shear stress varies during loading. This procedure results in a continuous variation of principal stresses and their direction in the sample in comparison with the bedding plane. As a result, it is difficult to investigate the effect of principle stress direction with respect to the bedding plane on the mechanical behavior of granular soils. However, the test provides a stress-strain curves and stress paths that could be used for validation of the soil models developed based on the triaxial formulations and later extended into generalized stress conditions.

2.2.2 Understanding the normally consolidated lines of cohesionless soils

Jefferies and Been (1985) showed that there are infinite normally consolidated lines (NCLs) for sands such as the Erksak Sand, as illustrated in Figure 2-2. Each of these lines is considered a true NCL due to two reasons. Firstly, the samples were prepared under low stresses, either by gentle moist tamping or by pluviation, and were never over-consolidated. Secondly, the unload-reload loops define a classic elastic-plastic response.

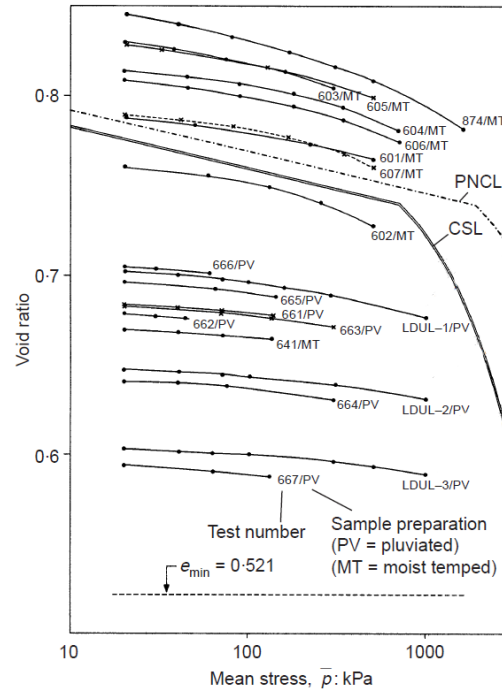


Figure 2-2 Isotropic compression tests on Erksak 330/0.7 Sand (Jefferies and Been, 1985)

Li et al. (2018) conducted multiple one-dimensional consolidation tests on gold tailings with silt-sized particles. The granulometry of the tailings, referred to as "present gold tailings," is illustrated in Figure 2-3. The researchers prepared samples with different initial porosities, ranging from 1 to 10, 11 to 20, and 21 to 28, using different compaction methods: wet compaction, dry compaction, and the slurry method, respectively. The one-dimensional compression behavior of these samples is presented in Figure 2-4. It is observed that all the curves gradually converge towards a specific line as the stresses increase, which the researchers refer to as the unique NCL. Interestingly, Robertson (2017) describes this convergence as the limiting compressing curve (LCC). However, even when the vertical stresses reach 7 MPa, the convergence process is relatively slow. The initial positioning of the NCLs is influenced by the preparation methods employed, but it does not affect the unique NCL or LCC.

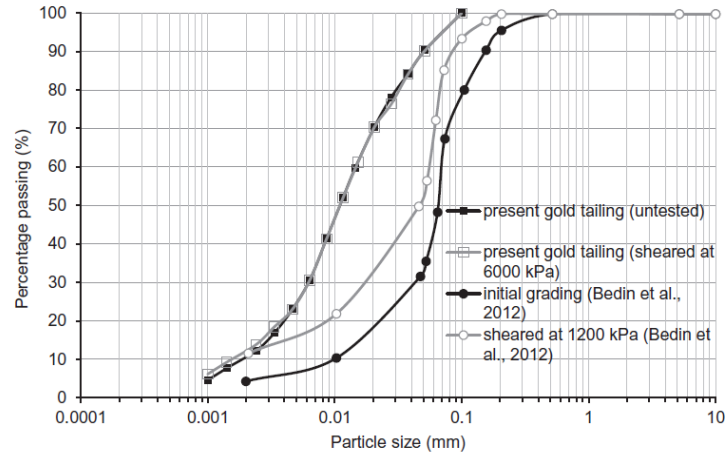


Figure 2-3 Grading curves of tailings materials used in the experimental work of Li et al. (2018) and Bedin et al. (2012)

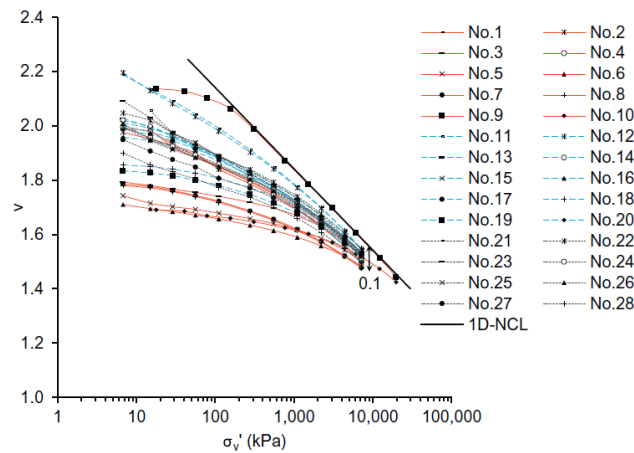


Figure 2-4 One-dimensional *NCLs* for tailings materials with different initial densities and preparation methods (Li et al. 2018)

Table 2-2 compiles the compression index (C_c) for tailings reported by various researchers. The compression index is determined by plotting consolidation curves on a log scale, with void ratio on the y-axis and vertical effective stresses on the x-axis. According to the table, the C_c values for different tailings range from 0.05 to 0.49. It is noteworthy that tailings with higher C_c values exhibit

greater void ratios at lower stresses, as observed in the studies by Grimard et al. (2021) and Bussière (2007).

Table 2-2 Values of compression index (C_c) for tailings reported by different researchers.

Researchers	Tailings type	C_c
Grimard et al. (2021)	Hard rock (Gold)	0.08-0.12
Li et al. (2018)	Gold	0.3
Essayad (2015)	-	0.077-0.1
Bedin et a. (2012)	Gold	0.44
Chang et al. (2011)	Gold (UB)	0.11
Chang et al. (2011)	Gold (MB)	0.3
Chang et al. (2011)	Gold (PO)	0.49
Li (2017)	Copper	0.22
Li (2017)	Iron (UB)	0.38
Li (2017)	Iron (MB)	0.32
Li (2017)	Iron (PO)	0.19
Bussière (2007)	Hard rock mine tailings	0.05-0.3

2.2.3 Understanding the behavior of cohesionless soils under undrained loading using triaxial compression test

Under a given depositional direction, principal stress direction, and stress path, three parameters control the state of the soil: (1) void ratio, (2) confining pressure, and (3) initial fabric.

Influence of void ratio

Figure 2-5 presents the typical undrained responses of sandy soils with different initial states in isotropic compression triaxial tests. Initial confining pressure, direction of principal stresses, fabric, and stress path are the same for the three samples, only the void ratios are different. Three different responses are observed, categorized in three types (Verdugo and Ishihara 1996), as shown in Figure 2-5. The three types are:

- “non-flow behavior” where soils are in a dense state with strain hardening during shear, showing first an increase in pore water pressure, and later a reduction until reaching a final condition so-called “steady (critical) state”. The turning point from

an increase to a diminution in excess pore water pressure is called “phase transformation” by Ishihara (1993).

- “Limited-flow behavior”, which happens in soils with a medium dense state. In such a state, the soil shows a “peak shear strength” followed by a short-term strain softening. After that, the soil recovers its structure, and the strength of the soil increases up to its final condition at a “critical state”. The “phase transformation state” occurs when soil recovers its structure.
- “Flow behavior” where soils are in a loose state, subjected to a continuous excess pore water pressure, reaching a peak shear strength, but losing it quickly upon shearing, and reaching residual shear strength at “critical state” condition. This type of behavior is known as flow liquefaction, responsible for many devastating failures in geotechnical structures.

Influence of confining pressure

In Figure 2-6, the typical undrained responses for samples with the same void ratio but sheared at different confining pressures are presented. Similar pattern responses to the ones mentioned before are observed (Verdugo and Ishihara 1996). Sample 1 shows a non-flow behavior, while samples 2 and 3 illustrate limited-flow and flow behavior, respectively. All three samples reach the same confining pressure and shear strength at critical state.

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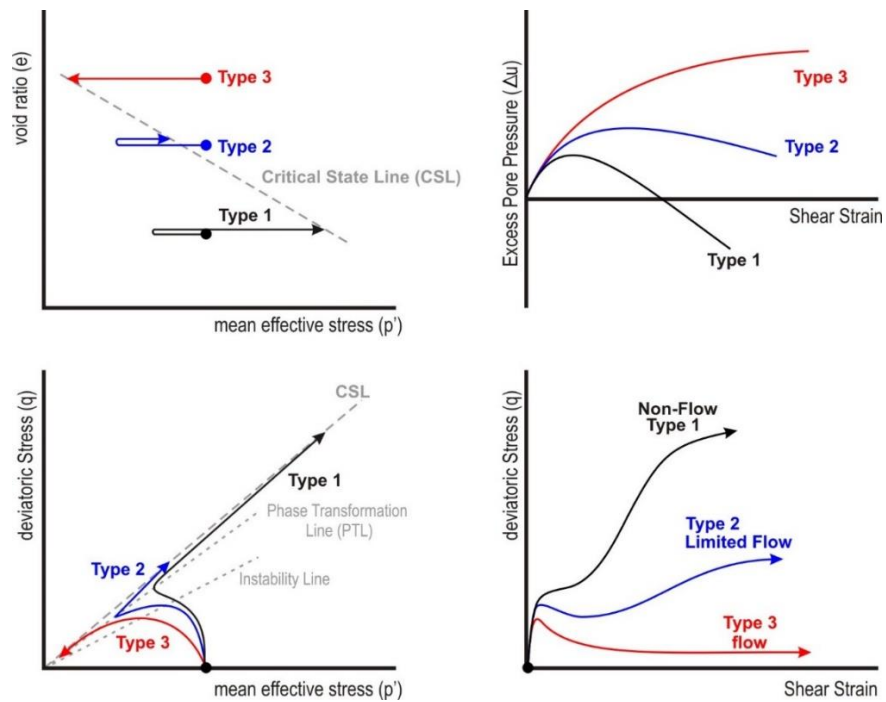


Figure 2-5 Undrained behavior of cohesion-less soils with different initial void ratios and the same confining pressure

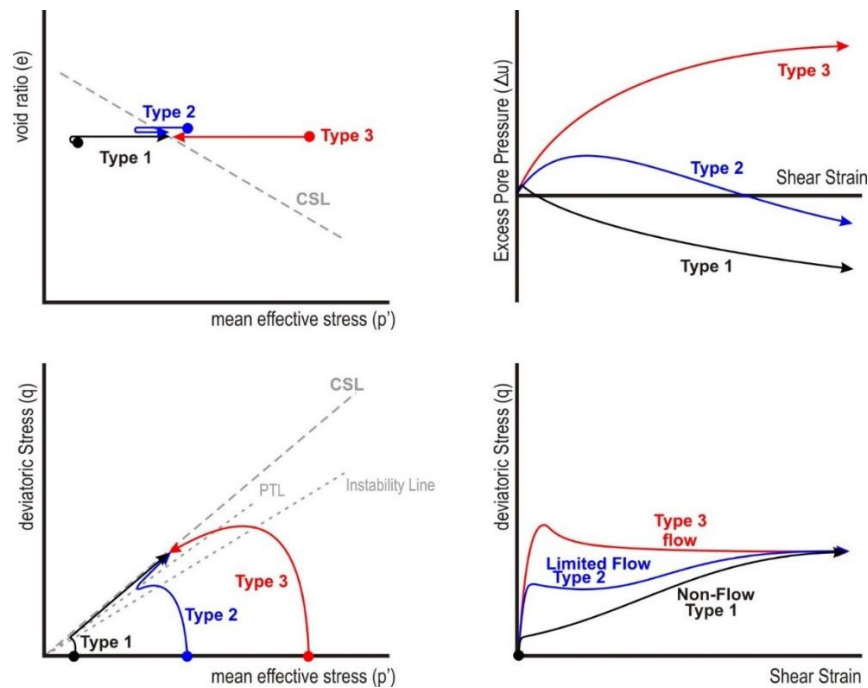


Figure 2-6 Undrained behavior of cohesion-less soils with different initial confining pressures and the same initial void ratio

Influence of initial fabric

Soil is a porous media consisting of discrete soil particles and voids between them. Different deposition methods provide different distributions in the soil mass, affecting its behavior. The term initial fabric is used to describe the distribution of the particles in a soil mass before applying any loading. Some researchers have investigated this feature by changing the deposition method while keeping the soil's initial state and the loading conditions constant. Vaid et al. 1999 showed that samples prepared using the moist-tamped method showed flow behavior (type 3), while the water deposited samples showed a dilative behavior during shear. Figure 2-7 (a) and (b) depict the test results conducted by Riveros and Sadrekarimi (2021) on gold tailings for slurry and moist-tamped deposition methods, respectively. While the initial confining pressures and void ratios are similar for the two samples, the flow behavior is dominant in the samples prepared with the moist tamped method. These results show that the initial fabric alongside the initial confining pressure and void ratio control the soil's response. It must be mentioned that measuring fabric is

quite challenging and requires using some methods such as tessellation method (Li and Li, 2009). From the macroscopic view, a fabric tensor can be described as the statistical indicators that capture the geometric properties of the tiny cells that make up the material (Li and Li, 2009).

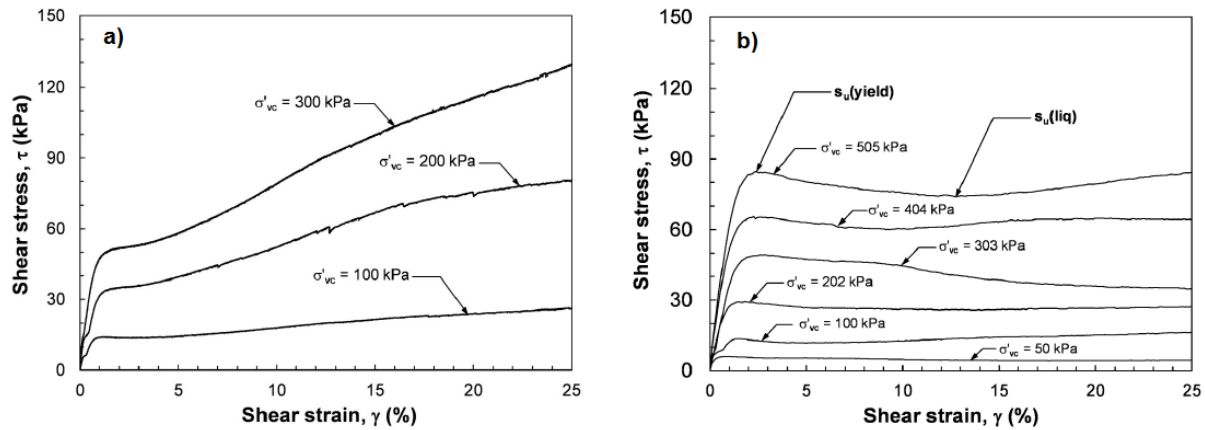


Figure 2-7 Influence of deposition method on the undrained behavior of gold tailings (a) slurry, (b) tamped deposition (Riveros and Sadrekarimi, 2021)

2.2.4 Understanding the behavior of cohesionless soils under undrained loading using hollow cylinder test

Using hollow cylinder test, two loading parameters (α and b) can be controlled. Since the normal vector of the bedding plane is typically parallel to the vertical axis, varying α can provide insights into the effect of loading direction relative to the bedding plane. Yoshimine et al. (1998) conducted undrained tests on Toyoura sand under different α values, with all samples having similar initial states, initial fabric, and a fixed b value of 0.5. As shown in Figure 2-8, increasing α resulted in a shift from non-flow behavior to limited flow behavior.

Shear mode representative parameter b also can be changed while initial states, initial fabric, and α remains constant. Figure 2-9 shows the undrained behavior of Toyoura sand under different b values conducted by Yoshimine et al. (1998). By increasing the b value (changing shear

mode from triaxial compression to triaxial extension), the behavior changes from limited flow to flow.

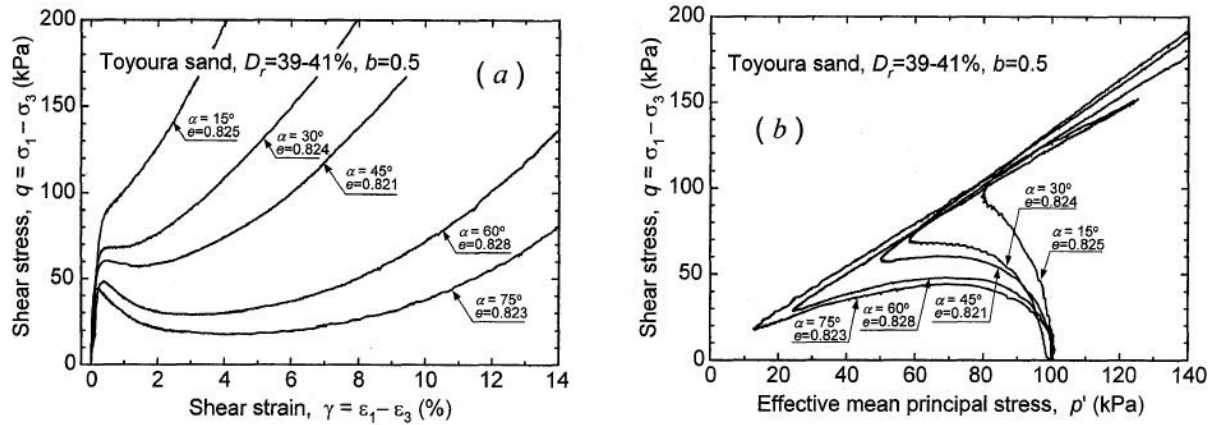


Figure 2-8 Influence of α value on the undrained behavior of cohesionless soils, (a) stress-strain curve, (b) stress path (Yoshimine et al. 1998)

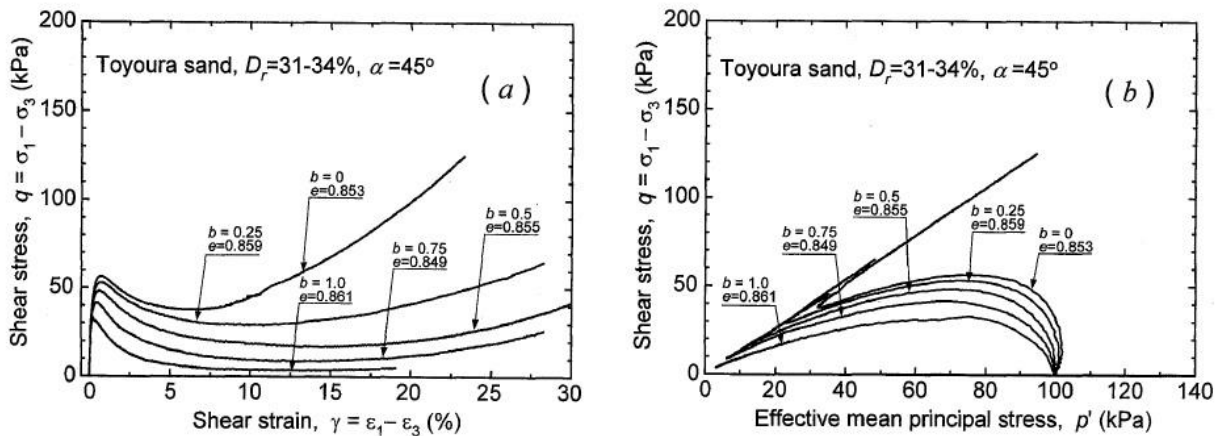


Figure 2-9 Influence of b value (representative of shear mode) on the undrained behavior of cohesionless soils, (a) stress-strain curve, (b) stress path (Yoshimine et al. 1998)

2.2.5 Key concepts of soil's behavior

Information about the soil behavior can be extracted by analyzing the results of undrained tests in terms of stress paths in the q - p space and in the e - $\ln p$ space. Of particular interest are the critical (steady) state or the final condition of the soil, phase transformation, and peak shear stress points for loose states. By drawing these points for different undrained tests and connecting them, some characteristic lines can be defined such as the critical state line (*CSL*), phase transformation line (*PTL*), and instability line (*IL*) or Flow Liquefaction Line (*FLL*). This section presents the characteristics of these lines, the parameters that control them and how they can be used.

2.2.5.1 Critical State Line (*CSL*)

The critical state is the final state that the soil experiences upon further shearing at large shear strains (ε_q). Roscoe et al. (1958) defines it as a state where the soil continues to deform in shear under constant mean effective stress (p), constant deviatoric stress (q), and constant void ratio (e). Mathematically, the critical state has the following conditions:

$$\varepsilon_q \rightarrow \infty \quad \Rightarrow \quad \frac{\partial q}{\partial \varepsilon_q} = \frac{\partial p}{\partial \varepsilon_q} = \frac{\partial e}{\partial \varepsilon_q} = 0 \quad (2-1)$$

Therefore, in the e - q - p space, the critical state is a point for a given stress path. For different initial states, the final state would differ, and all these points are connected, making the critical state a line expressed by the following equation.

$$C(e, q, p) = 0 \quad (2-2)$$

To ensure that this state is definitive and not temporary, the following condition is also required.

$$\frac{\partial^2 e}{\partial \varepsilon_q^2} = 0 \quad (2-3)$$

The three-dimensional shape of the *CSL* in e - q - p space can be mapped on the e - p and q - p space, as shown in Figure 2-10. The *CSL* in the e - $\ln p$ space, for clayey soils, is generally a straight line that can be formulated as follow:

$$e = \Gamma + \lambda \ln(p) \quad (2-4)$$

Where λ is the slope of the linear line, and Γ is the value of void ratio when p is equal to 1 kPa . The linear equation for *CSL* can also be expressed in terms for other references pressures, and p is then normalized by said reference pressure, p_{ref} , and Γ is the value of void ratio when p is equal to p_{ref} . For granular soils, the critical state can also be expressed by the following equation (Li and Wang, 1998):

$$e = \Gamma + \lambda \left(\frac{p}{p_{ref}} \right)^\xi \quad (2-5)$$

Where, Γ , λ , and ξ are material constants. The *CSL* in q - p space is a line passing through the origin that follows the equation:

$$q = Mp \quad (2-6)$$

Where, M is the slope of *CSL* in q - p space.

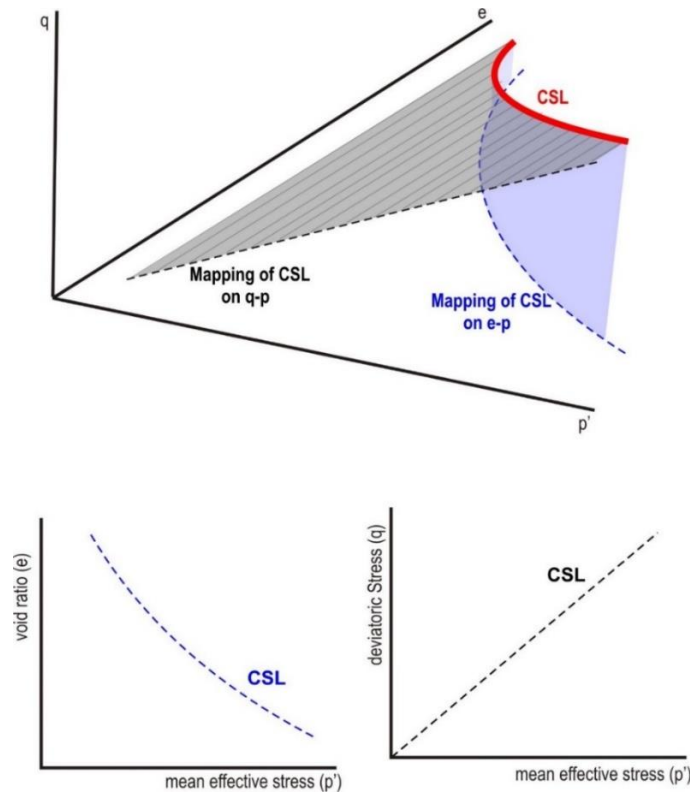


Figure 2-10 Critical state line and its shape on e - p' and q - p' space

Uniqueness of CSL

If the *CSL* is unique, regardless of the stress path, all distortional paths must reach the *CSL* at large strains. Since the *CSL* cannot change its position or shape with increasing shear strain at critical state, the parameters used to define it mathematically are the intrinsic parameters. Intrinsic soil properties are not affected by soil fabric, stress history, void ratio, etc. If the *CSL* is not unique, each shear mode has its own *CSL* affected by loading history or initial fabric. Riemer and Seed (1997) reported two different *CSLs* (or *SSLs*) for extension and compression in triaxial tests, as shown in Figure 2-11. DeGregorio (1990) also reported different *CSLs* for samples prepared using different deposition methods, as shown in Figure 2-12. Li and Dafalias (2012) emphasized that the *CSL* is unique, however it does not mean that it is attainable under all shear test modes. Reaching the *CSL* can sometimes require deformations that are too large to be measured, resulting in a difficult estimation of the location or uniqueness of *CSL*.

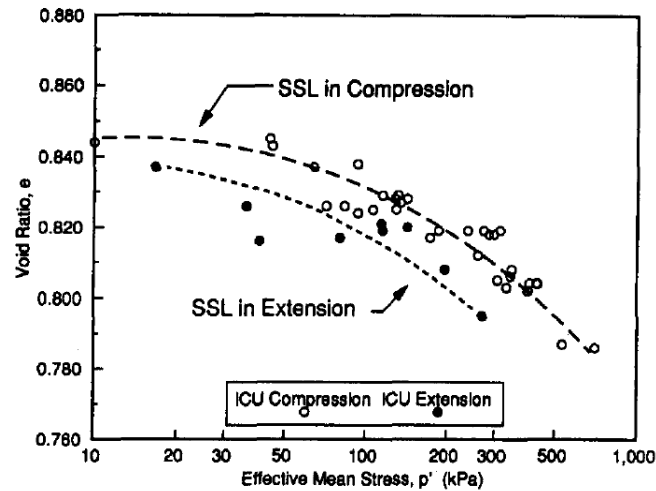


Figure 2-11 Reported SSLs (steady state lines) for compression and extension triaxial test by Riemer and Seed (1997)

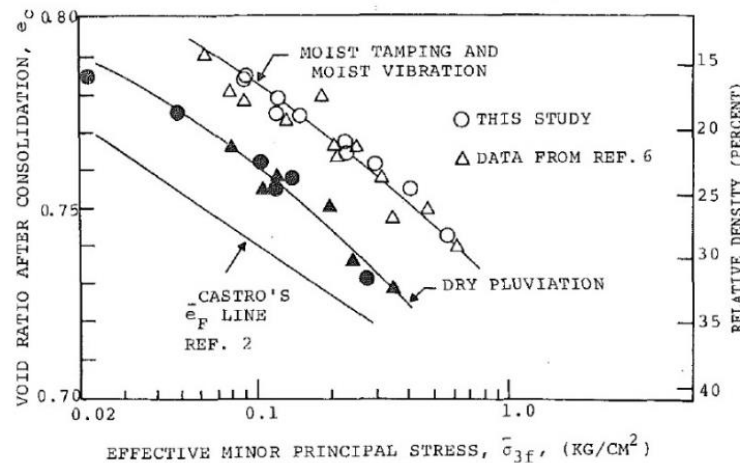


Figure 2-12 Reported CSLs for the samples prepared by different deposition methods by Degregorio (1990)

2.2.5.2 The importance of CSL and the concept of state parameter

Figure 2-5 shows different initial conditions with respect to the CSL in $e - \ln p$ space. Soils with flow behavior type have initial states above the critical state line generally. Other initial states result in a non-flow behavior type. Therefore, the CSL can be used as a criterion to distinguish loose soils from dense ones. This kind of interpretation was reported initially by Casagrande (1936), where he referred the critical void ratio line (CVR) as the final condition of the soil (Figure

2-13). The *CVR* line was used as a boundary for the initial states of the soil that are or are not susceptible to flow liquefaction. The initial states further away from *CVR* line show faster contraction or dilation.

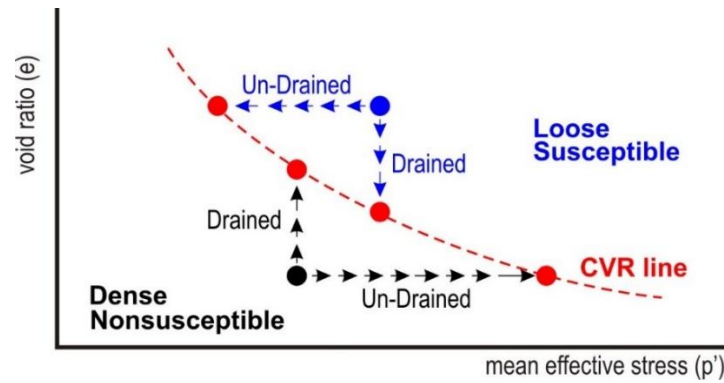


Figure 2-13 Use of *CSL* in determination of loose from dense behavior

Later, Been and Jefferies (1985) introduced the state parameter (ψ). The state parameter is defined as the distance between the current state and the critical state in terms of void ratio using Equation (2-7).

$$\psi = e - e_c \quad (2-7)$$

Where e is the current soil's void ratio at a given mean effective stress (p), and e_c , is the critical state void ratio at the same mean effective stress as shown in Figure 2-14.

The state parameter uses the *CSL* as a reference state to define quantitatively the state of the soil. A positive value means a looser condition, while a negative value represents a denser condition. The state parameter changes during loading because of the changes in void ratio and mean effective stress due to contraction and dilation.

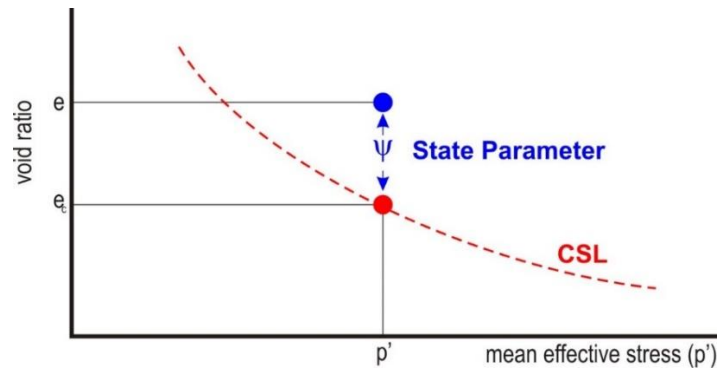


Figure 2-14 Definition of state parameter introduced by Been and Jefferies (1985)

2.2.5.3 Phase Transformation Line (*PTL*)

Phase transformation, defined by Ishihara et al. (1975), is a temporary state where a transition from contraction to dilation occurs in sands. This feature shows itself by a change of direction in the undrained stress path, which occurs when the mean effective stress shifts from a reduction to an increase, as demonstrated in Figure 2-15. In the q - p space, by drawing a line between the origin and the phase transformation point, a line is formed called the phase transformation line (*PTL*). M^d , the symbol used for the slope of *PTL* by many researchers (for example Dafalias and Manzari, 2004), and is called the phase transformation stress ratio.

Soils with different states may exhibit different *PTL*, as illustrated in Figure 2-16 (data from Verdugo and Ishihara, 1996). From the results, as the soil becomes denser (getting lower negative state parameter, ψ), the *PTL* becomes lower. When it gets looser, but its state is denser than critical ($\psi < 0$), the *PTL* shifts up. Finally, soil states that are looser than critical ($\psi > 0$) may not show signs of the phase transformation phenomenon. Therefore, the variation of M^d with the state parameter is likely to follow the schematic graph shown in Figure 2-17.

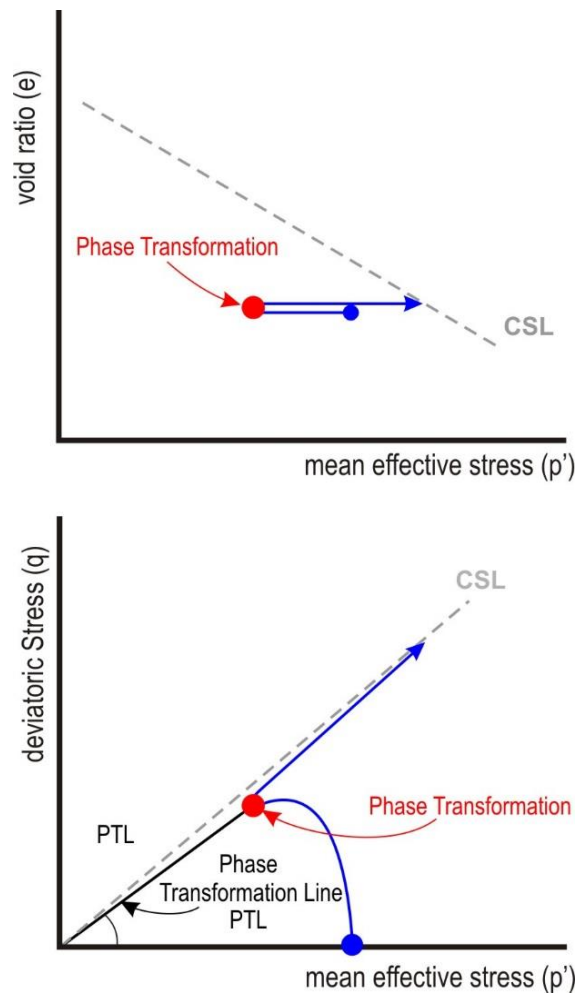


Figure 2-15 Definition of phase transformation state and phase transformation line

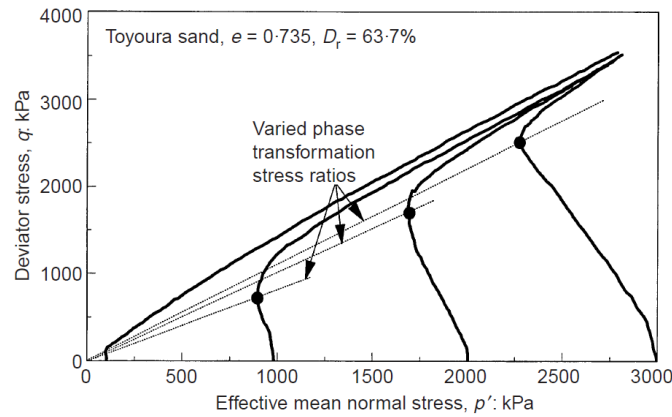


Figure 2-16 Variation in the phase transformation stress ratio with material state (data from Verdugo and Ishihara, 1996)

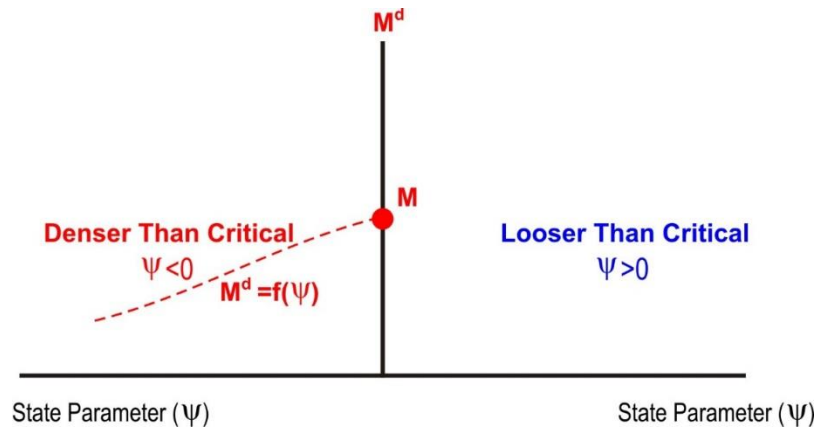


Figure 2-17 Variation of phase transformation state with state parameter

The following equation can be used to express the variation of M^d with the state parameter.

$$\begin{cases} M^d = f(\psi) < M & \psi < 0 \text{ (denser than critical)} \\ M^d = M & \psi \geq 0 \text{ (looser than critical)} \end{cases} \quad (2-8)$$

Different researchers have proposed different equations for the phase transformation state ratio, M^d , when $\psi < 0$.

$$\begin{aligned}
M^d = f(\psi_i) &= M \left(1 + \frac{\chi_i N \psi_i}{M_{tc}} \right) && \text{Jefferies and Been (2015)} \\
M^d = f(\psi) &= M + k^d \psi && \text{Manzari and Dafalias (1997)} \\
M^d = f(\psi) &= M \cdot \exp(m\psi) && \text{Li and Dafalias (2000)}
\end{aligned} \tag{2-9}$$

In all the equations, when ψ equals to zero, $M^d = M$. The first two equations propose a linear relationship, while the last one is an exponential equation. The parameters used in those equations are material constants obtained from the calibration process.

2.2.5.4 Instability line (IL)

Flow liquefaction is the phenomenon that happens in soils when their states are looser than critical, and they exhibit a strength softening under continuous rise of pore water pressure resulting in continuous reduction of mean effective stress. An example of this behavior is shown in Figure 2-18, schematically. The deviatoric stress presents a hardening behavior that reaches a maximum value, called undrained peak shear strength, followed by a rapid strain softening, ending in shear strength at large strains, called residual shear strength. Flow liquefaction refers to the strain softening part.

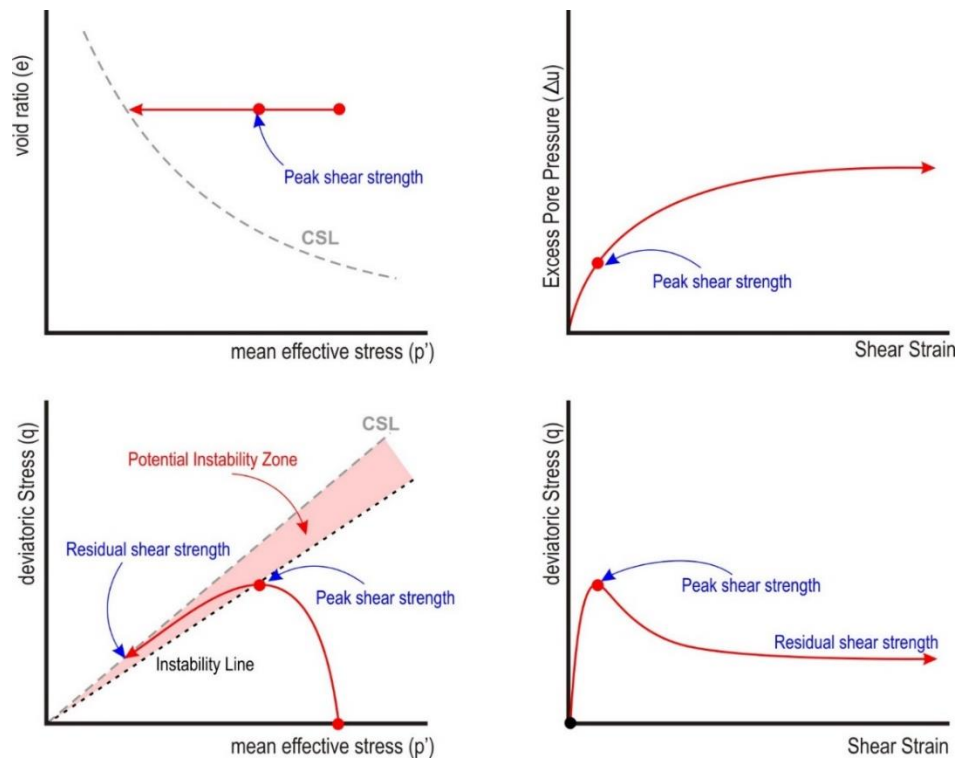


Figure 2-18 Undrained behavior of loose cohesionless soils and the concept of instability line

Many researchers have focused on the triggering point where the flow liquefaction initiates. Sladen and Oswell (1989) introduced a straight line passing through all the peak points of undrained stress paths in the normalized stress space. In the new stress space, the mean effective stress and deviatoric stress are normalized with p_c and $q_c = M \cdot p_c$ (mean effective and deviatoric stress at critical state), respectively. Accordingly, the *CSL* turns into a point with coordinates of (1,1), as shown in Figure 2-19. They drew all the undrained stress paths in this space and connected their peak points. The developed line is called collapse surface, which passes through the *CSL* as a point (See Figure 2-15). In the q - p space, the collapse line is a straight line, as shown in Figure 2-20.

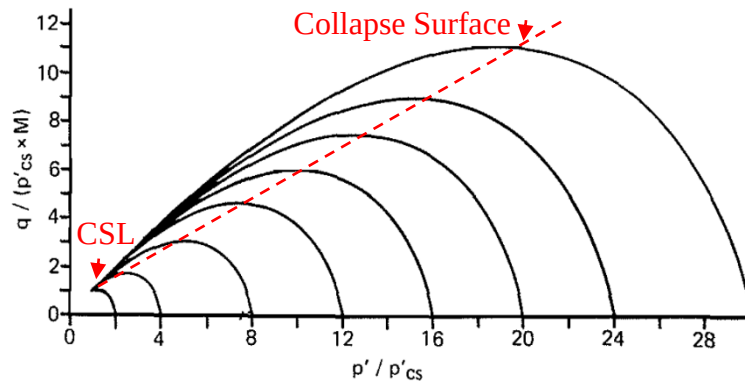


Figure 2-19 Concept of collapse surface in the normalized stress space introduced by Sladen and Oswell (1989)

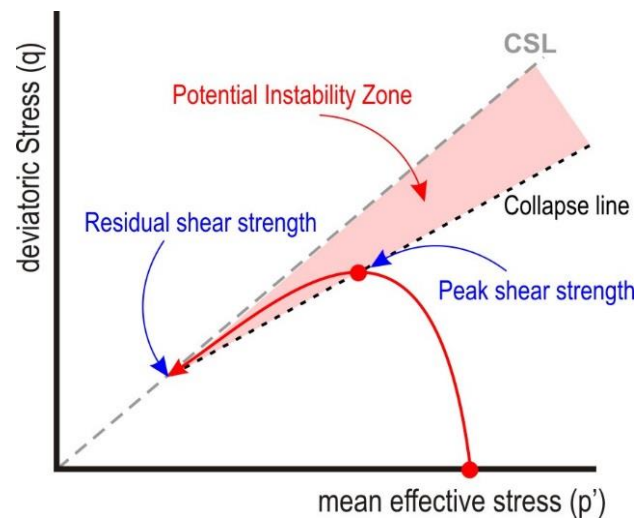


Figure 2-20 Concept of collapse line in the q-p space

Vaid and Chern (1985) used another line starting from the origin to the peak shear strength. They referred to this line as the critical stress ratio (*CSR*) line, which is more straightforward to use in numerical formulations. The region between the *IL* (or *CSR* line) and *CSL* is called the potential instability zone (Yang et al. 2021). If the initial stress in the field is near the instability zone, flow liquefaction can be triggered by a minor disturbance that can be caused by different types of loading. As shown in Figure 2-21, three different loading conditions bring the initial stress

condition of the soil (point A) on the *IL* (Points B, D, and E). AB, AD, and AE are for static, undrained creep, and cyclic loading. By touching the *IL*, the flow liquefaction happens in the BC, DC, and EC paths.

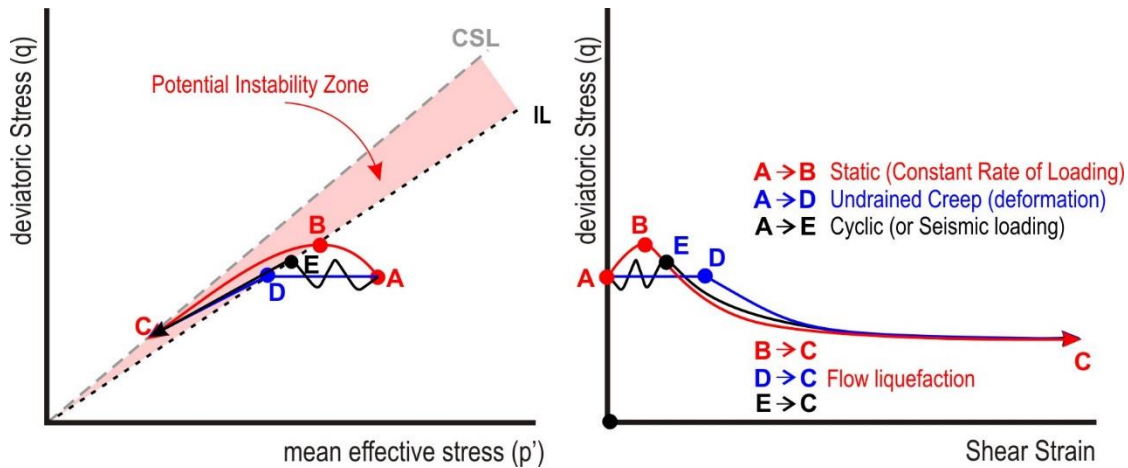


Figure 2-21 The loading scenarios leading to initiate the flow liquefaction

Konrad (1985) stated that *IL* is unique for a sand, and it is independent from the void ratio. However, Yang (2002) reported that the slope of *IL* in the q - p space (M^{IL}) is different depending on the initial soil state. He proposed the following equation based on the published data of four different sands (Banding No. 6, Leighton Buzzard, Nerlerk, and Toyoura sands), where A and B are fitting parameters:

$$M^{IL} = \frac{M}{B} \exp(-A\psi) \quad (2-10)$$

2.2.5.5 Brittleness Index

The brittleness Index (I_B) was introduced by Bishop (1971) to show the intensity of a stress drop from the peak shear strength to the residual shear strength.

$$I_B = \frac{s_u - s_r}{s_u} \quad (2-11)$$

Where s_u and s_r are the peak and residual shear strength in an undrained test conducted on a loose soil. From this definition, it appears that the I_B is affected by the soil initial state, initial fabric, and the shear mode conducted in a laboratory test. In this section, the I_B is only reviewed based on results of compression triaxial undrained test.

An I_B value of one and zero means zero residual shear strength and residual shear strength equal to peak shear strength, respectively. Sadrekarimi (2014) portrayed in Figure 2-22 the variation of normalized s_u and s_r with I_B .

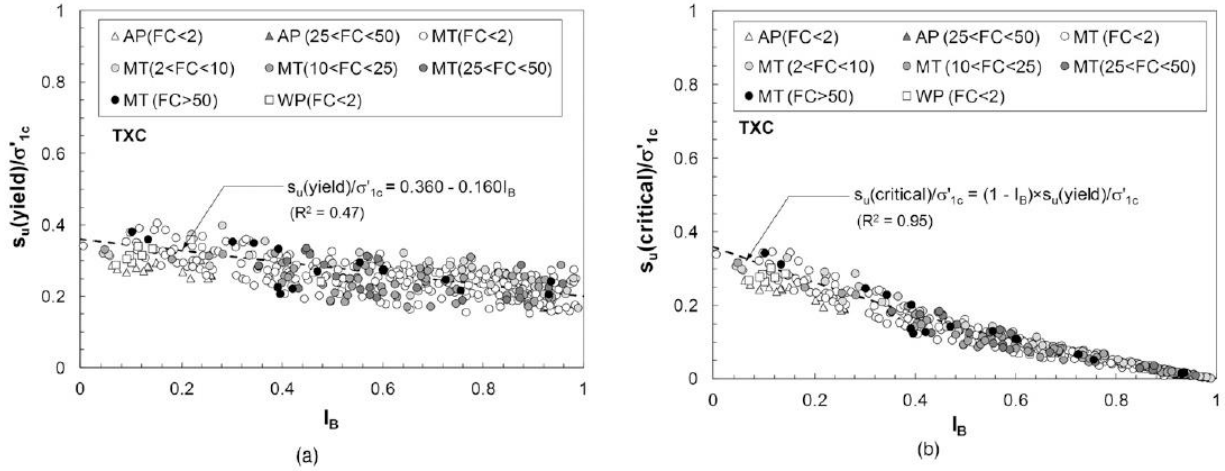


Figure 2-22 Variation of (a) peak undrained shear strength, and (b) residual shear strength with I_B (Sadrekarimi, 2014)

Sladen and Oswell (1989) drew the variation of I_B with the ratio p'_o/p'_{cs} , as shown in Figure 2-23. Where p'_o and p'_{cs} are the initial confining pressure and critical confining pressure at a constant void ratio. Higher p'_o/p'_{cs} means that soil is initially far away from the critical state and has a higher state parameter. By increasing the ratio (soil getting initially looser), I_B reaches unity, meaning zero residual shear strength.

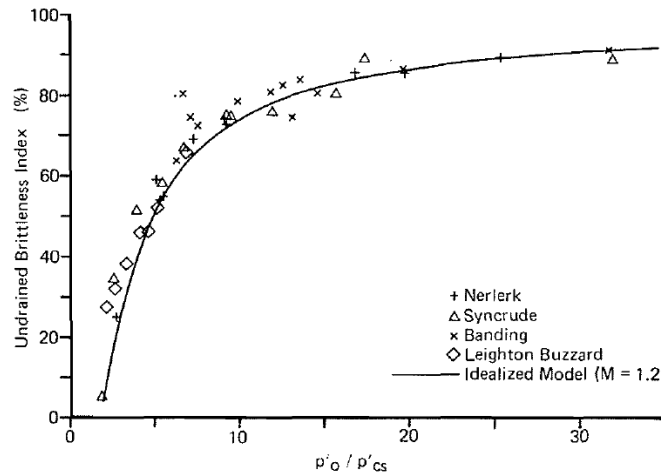


Figure 2-23 Variation of I_B with p'_o / p'_{cs} as a proxy for the looseness of soil (Sladen and Oswell, 1989)

2.2.6 Understanding the Behavior of Cohesionless Soils under Drained Loading

In constitutive modeling of soils, the volumetric behavior of soils during shear loading under drained conditions will affect the undrained behavior. The undrained response is a secondary effect of the tendency of a soil to contract or dilate during shear loading. This tendency shows itself in the evolution of pore water pressure. Therefore, understanding the volumetric behavior of soils during drained shear loading is an important subject to study the soil behavior.

2.2.6.1 Definition of Dilatancy

Soils in loose or dense state show contraction or dilation (volumetric deformation) during shearing. This feature shows a shear-volume coupling that is not in the elastic range behavior. In the elastic concept, during pure shear deformation, the volume change is zero. Upon yielding, the framework of plasticity couples shear and volumetric plastic deformations, using a flow rule. For a given plastic shear strain increment ($d\varepsilon_q^p$), there is an associated plastic volumetric strain increment ($d\varepsilon_v^p$). The ratio of $d\varepsilon_v^p / d\varepsilon_q^p$ shows dilatancy (D). This ratio is the slope of a curve

measured through the variation of volumetric strain versus the variation of shear strain, shown in Figure 2-24.

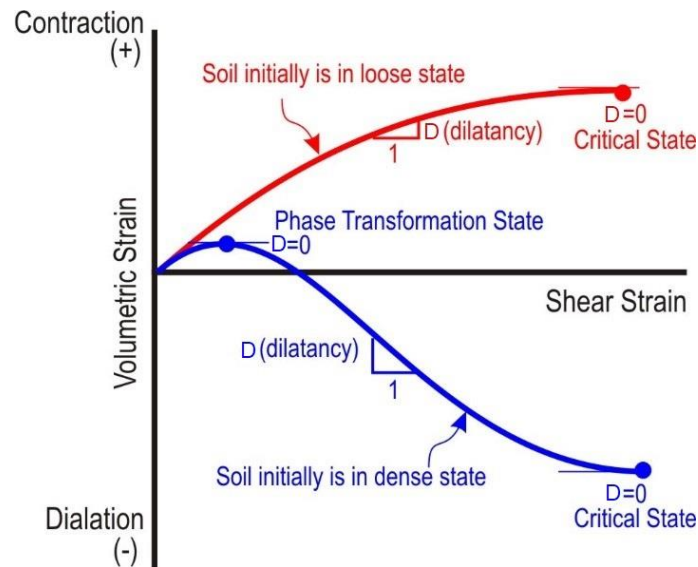


Figure 2-24 Concept of dilatancy (volume deformation during shear loading)

2.2.6.2 Stress-Dilatancy

Dilatancy evolves during shear loading, reaching zero at critical state. The dilatancy can be related to the soil mobilized stress ratio (η), equal to the current deviatoric stress (q) divided by the current mean confining pressure (p). The relationship between the mobilized stress ratio and dilatancy is called the stress-dilatancy rule. This relationship is valid for the entire stress - strain history, and therefore, the behavior of the soil is controlled through it. The importance of this rule is its use as a flow rule in some well-known constitutive models to establish the potential surface (Such as the original and modified Cam-Clay model, NorSand model, or the model presented by Lu and Huang, 2015, see Section 2.4.2).

This relationship can be established based on the plastic work dissipation rate ($\dot{W}^p$), obtained through the following equation:

$$\dot{W}^p = p \cdot d\varepsilon_v^p + q \cdot d\varepsilon_q^p \quad (2-12)$$

By dividing by the mean effective stress and the plastic shear strain increment the following equation is obtained, where the dilatancy and mobilized stress ratio are related to each other:

$$\frac{\dot{W}^p}{p d\varepsilon_q^p} = \frac{d\varepsilon_v^p}{d\varepsilon_q^p} + \frac{q}{p} = D + \eta \quad (2-13)$$

This is a general relationship between the dilatancy and the mobilized stress ratio, not specific to a given constitutive model.

2.2.6.3 Stress-Dilatancy Behavior in Loose Cohesionless Soils

Been and Jefferies (2004) reported drained triaxial compression test results conducted on very loose samples. Figure 2-25 shows mobilized stress ratio against the dilatancy (D^p in the figure). The mobilized stress ratio constantly increases, reaching the value of M , at critical state, when the dilatancy becomes zero. By ignoring the stress-dilatancy behavior for mobilized stress ratio less than 0.5, a linear relationship can be suggested as follows:

$$\frac{\dot{W}^p}{p d\varepsilon_q^p} = D + \eta = M \quad (2-14)$$

The line has a slope of -1 passing through the critical state at ($D=0$, $\eta=M$). This equation is the same equation that was used to build the original Cam-Clay model by Schofield and Wroth (1968).

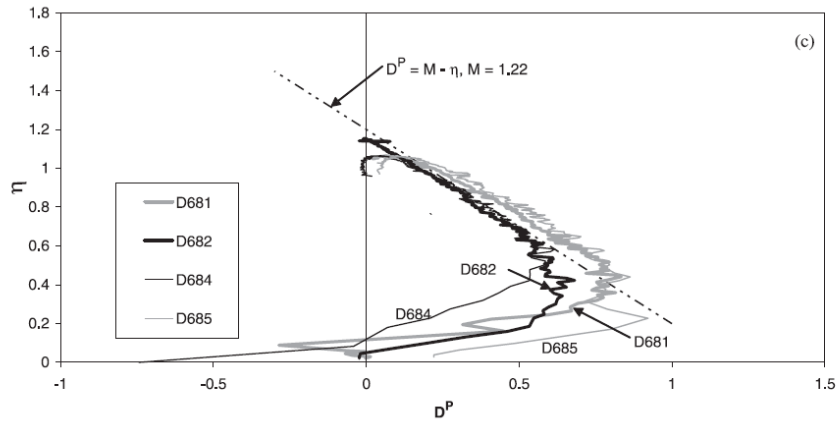


Figure 2-25 Stress-dilatancy behavior of loose cohesion less soils reported by Been and Jefferies (2004)

2.2.6.4 Stress-dilatancy behavior in dense cohesion-less soils

Figure 2-26 depicts schematically the variation of dilatancy with shear strain in a drained test for a dense sand. In the beginning of the test, there is a positive dilatancy, decreasing to reach a zero value. This is where the phase transformation state happens, and soil starts showing negative dilatancy thereafter. The reduction continues until it reaches a minimum value called D_{min} . After that, the dilatancy goes back to zero at critical state.

The stress-dilatancy behavior for drained triaxial compression test conducted on dense Erksak sand is shown in Figure 2-27 (Been and Jefferies, 2004). The locations where the phase transformation, D_{min} , and critical state happen are also shown in the figure. The following linear equation can be proposed to match the experimental data:

$$\frac{\dot{W}^p}{p d\varepsilon_q^p} = D + \eta = M^d \quad (2-15)$$

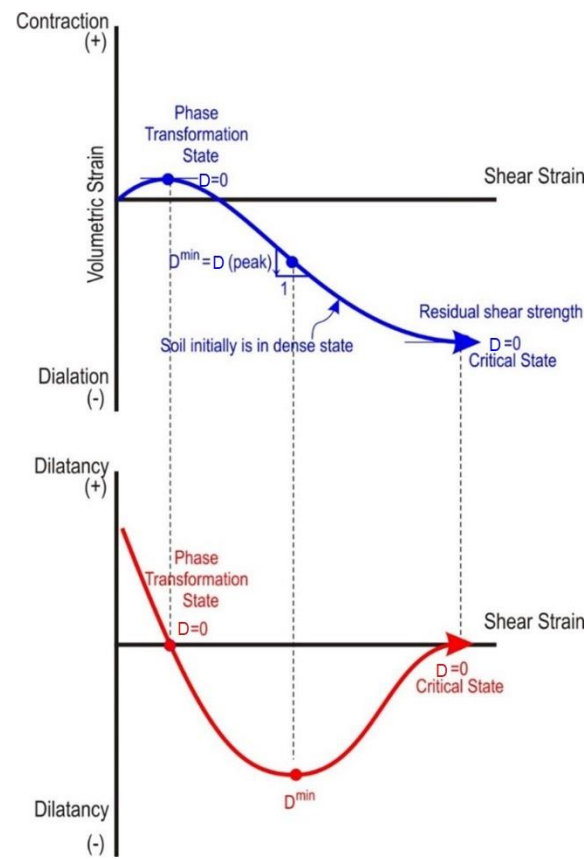


Figure 2-26 Variation of dilatancy in a drained shear test conducted on the dense cohesionless soil

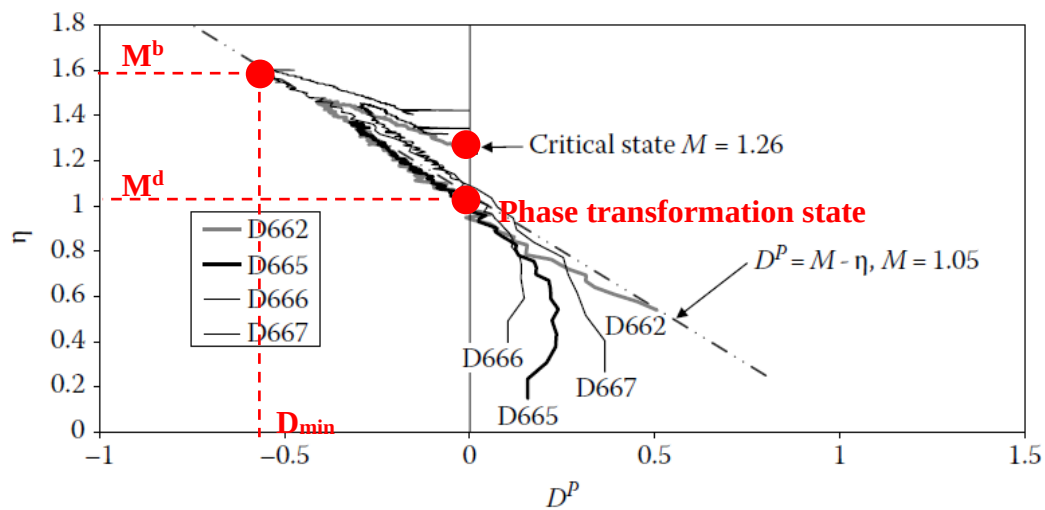


Figure 2-27 Stress-dilatancy behavior of dense cohesion less soil reported by Been and Jefferies (2004)

It is obvious that the proposed equation is a line passing through the phase transformation state. However, in Section 2.2.5.3, it is shown that the phase transformation stress ratio, M^d , is a function of the state parameter. By replacing Equation (2-9) in Equation (2-15) the following relationships are obtained.

$$\begin{aligned}
 D + \eta = M^d &= M \left(1 + \frac{\chi_i N \psi_i}{M_{tc}} \right) && \text{Jefferies and Been (2015)} \\
 D + \eta = M^d &= M + k^d \psi && \text{Manzari and Dafalias (1997)} \\
 D + \eta = M^d &= M \exp(m\psi) && \text{Li and Dafalias (2000)}
 \end{aligned} \tag{2-16}$$

Dense soils tend to reach critical state by increasing the state parameter from a negative value to zero. As a result, M^d is a variable, and the line matching experimental data changes its position. At critical state, the line will pass through the critical state point. In Equation (2-16), another parameter from the strain history, state parameter, has been added. These relationships are called state-dependent stress-dilatancy rules (Li and Dafalias, 2000).

2.2.6.5 State-dilatancy law

According to Jefferies and Been (2015), there is a linear relationship between D_{min} and the state parameter at D_{min} . They called it state-dilatancy law, obeying the following equation:

$$D_{min} = \chi_{tc} \psi_{min} \tag{2-17}$$

Where, χ_{tc} is a parameter obtained from triaxial compression tests conducted on drained tests with different initial state parameters. In Figure 2-28, the relationship for some sands is presented.

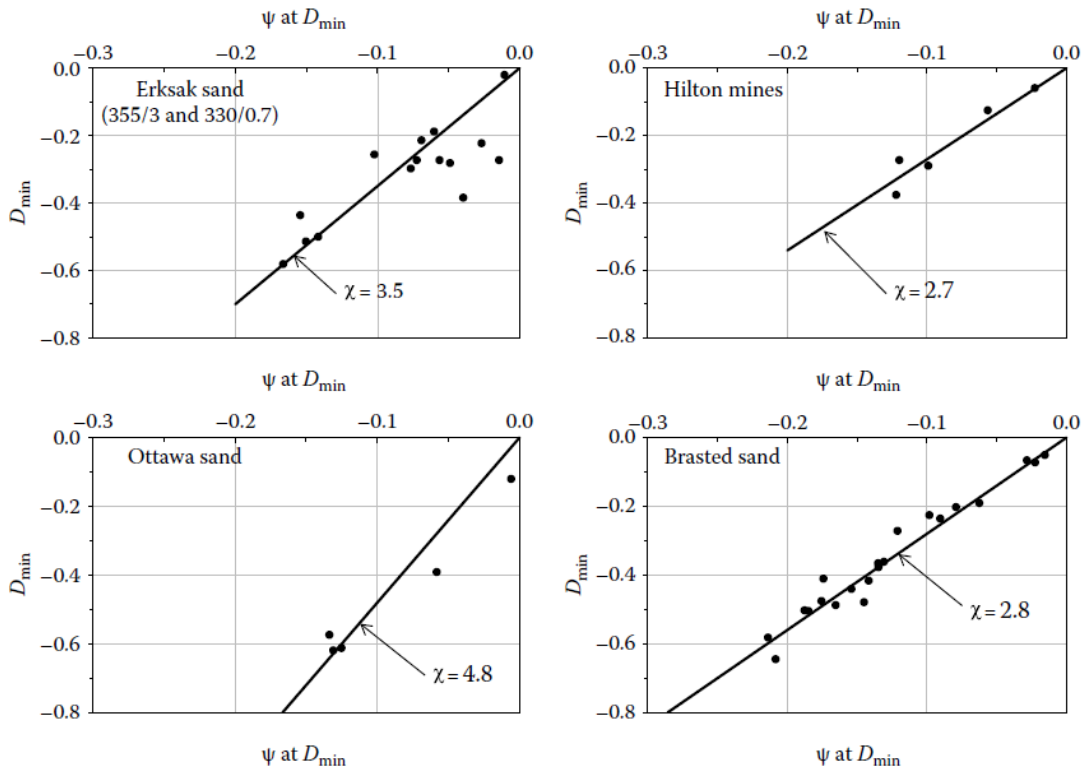


Figure 2-28 Relationship between D_{min} and ψ at D_{min} for different granular soils (Jefferies and Been, 2015)

2.2.6.6 Relationship between the peak mobilized stress ratio and minimum dilatancy in drain tests

Nova and Wood (1982) expressed that the following relationship exists between the peak mobilized stress ratio and minimum dilatancy:

$$D_{min} = \frac{M - M^b}{1 - N} \quad (2-18)$$

Where N is called volumetric coupling parameter. Nova and Wood (1982) used this equation as a flow rule in their constitutive modeling for sand. Figure 2-29 shows the trend calculated from Equation (2-18) with the measured data (Jefferies and Been, 2015) for Fraser River sand (FRS).

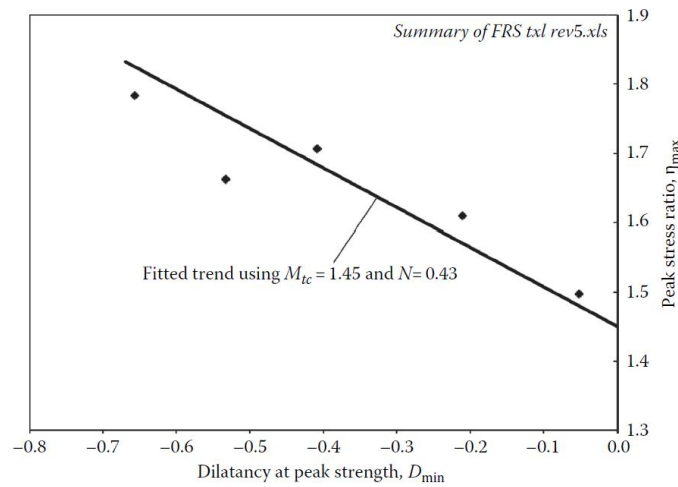


Figure 2-29 Relationship between peak mobilized stress ratio (η_{max} or M^b) and minimum dilatancy (D_{min}), for Fraser River sand (FRS), (Jefferies and Been, 2015)

2.3 Effect of fines content on the microstructure of sand-silt mixtures and their mechanical behavior.

As discussed in Section 2.2, to simulate the behavior of a granular soil, we need information on its initial state (including fabric, void ratio, and confining pressure) and critical state. By comparing these two states using the state parameter, we can evaluate whether the soil is going to show non-flow or flow behavior type during undrained loading. However, a crucial factor affecting the undrained mechanical response of granular soils was not previously addressed. This factor is the microstructure of sand-silt mixture with different fines content. The aim of this section is to explore this concept by providing a literature review on this topic.

2.3.1 Effect of fines content on the microstructure of sand-silt mixtures

By adding fines to a sand matrix, the structure of the soil changes (Figure 2-30). Below a certain fines content, the threshold fines content (FC_{th}), the soil structure is described as “silt within in a sand matrix” (Lashkari, 2016, Thevanayagam et al. 2002). Rahman and Lo (2008) proposed the following experimental equation to estimate FC_{th} .

$$FC_{th} = 0.4 \left(\frac{1}{1 + \exp(0.5 - 0.13\chi)} + \frac{1}{\chi} \right) \quad (2-19)$$

Where χ is the particle diameter ratio equal to (D_{10}/d_{50}) , D_{10} and d_{50} are the effective particle size and the median size of the particles, respectively. D_z and d_z are grain size for coarse and fine grains, respectively, where $z\%$ of the soil particles are finer than that. Figure 2-31 shows the variation of FC_{th} by χ .

For $FC > FC_{th}$, the percent of fine grain increases, however after a certain point called limiting fines content (FC_L), the fines control the shear behavior of matrix. Between FC_{th} and FC_L there is a transition zone before the behavior is completely controlled by fines (Thevanayagam et al. 2002). For $FC > FC_L$, there are enough fines in the matrix to reduce the contacts of sand grains with each other. In this case, the soil structure is depicted as “sands in a silt matrix”.

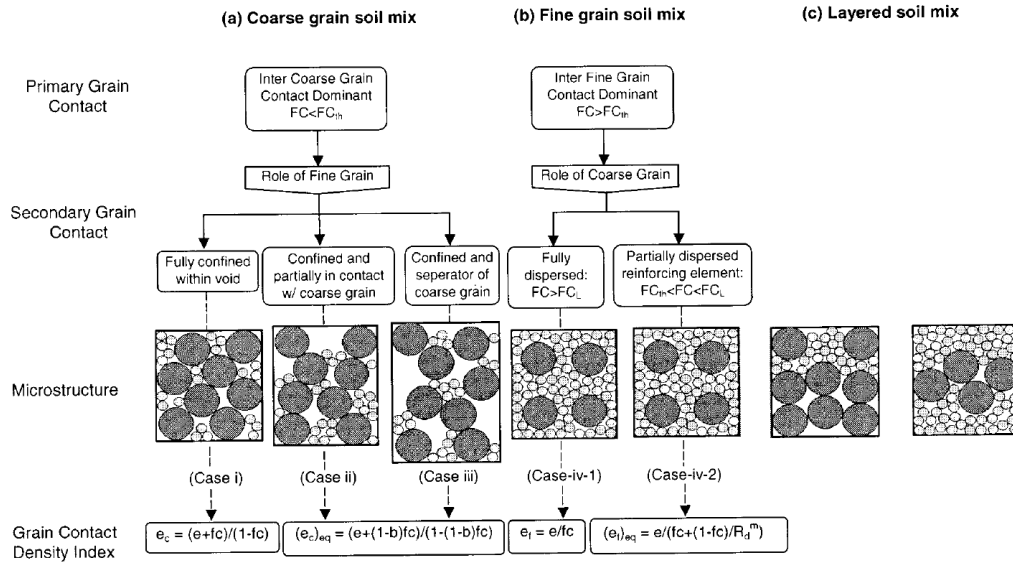


Figure 2-30 changing the microstructure of a coarse-fine mixture by increasing the fine content (Thevanayagam et al. 2002)

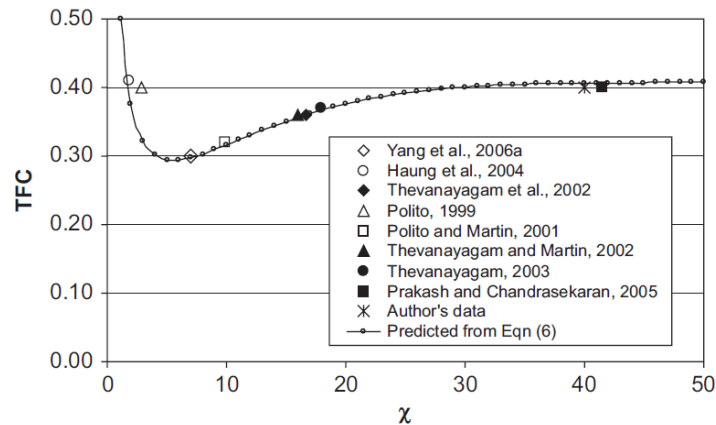


Figure 2-31 Variation of threshold fine content (FC_{th}) with particle diameter ratio (χ) (Rahman and Lo, 2008)

2.3.2 Void ratio and equivalent void ratio

According to Thevanayagam et al. (2002), five definitions of void ratios can be made, void ratio (e), intergranular void ratio (e_c) and equivalent void ratio $(e_c)_{eq}$ for $FC < FC_{th}$, interfine void ratio (e_f) and equivalent void ratio $(e_f)_{eq}$ for $FC > FC_L$.

The reason to define an equivalent void ratio instead of using the typical void ratio is that in a silt-sand matrix with $FC < FC_{th}$, some particles act as filler and have no role in the contact forces between other particles. The volume of these particles can be considered as void spaces from a mechanical behavior aspect.

$$e_c = \frac{e + fc}{1 - fc} \quad (2-20)$$

$$(e_c)_{eq} = \frac{e + (1 - b)fc}{1 - (1 - b)fc}$$

Where, b considers the portion of fine grains that are active in the transmission of forces between particles. $b=0$ means all the fines grains act as fillers, while $b=1$ represent a full activity

of fines in supporting the coarse grain skeleton. The semi empirical equations below have been presented by different researchers to calculate the b value:

$$b = \left[1 - \exp \left(-0.3 \frac{\left(\frac{FC}{FC_{th}} \right)}{1 - \chi^{-0.25}} \right) \right] \left(\frac{1}{\chi} \frac{FC}{FC_{th}} \right)^{\frac{1}{\chi}} \quad \text{Rahman and Lu (2008)} \quad (2-21)$$

$$b = 1.93FC \times \chi^{-0.29} \quad \text{Lashkari (2014)} \quad (2-22)$$

$$b = 0.26 + \frac{1}{1.22} \chi \quad \text{Mohammadi and Qadimi (2015)} \quad (2-23)$$

For a silt-sand matrix with $FC > FC_L$, coarse grains act as reinforcement elements for the fine matrix, and this effect must be considered through an equivalent void ratio. Thevanayagam et al. (2002) proposed the following equations to obtain new void ratios applicable for $FC > FC_L$.

$$e_f = \frac{e}{fc} \quad (2-24)$$

$$(e_f)_{eq} = \frac{e}{fc + \frac{1 - fc}{R_d^m}}$$

Where, R_d is the ratio of the mean size of coarse grains (D_{50}) to the mean size of fine grains (d_{50}), and m is a coefficient that varies between 0 and 1, depending on grain characteristics and fine grain packing.

2.3.3 Effect of fines content on the mechanical behavior of sand-silt mixture

With the aim of investigating the impact of fines on the undrained shear behavior in a silt-sand matrix, Thevanayagam et al. (2002) conducted various test series, namely A, B1, B2, B3, and B4.

The test series A focused on the primary effect of inter-grain contact friction at a nearly constant void ratio of around $e=0.6$. In this regard, silt-sand mixtures were produced with varying FC content, ranging from 0 to 100%, as detailed in Table 2-3.

For FC values below FC_{th} , the maximum shear strength decreases as the fines content increases, and the soil behavior shifts from non-flow type to limited flow (Figure 2-32a and b). The researchers attributed these findings to the value of e_c , which increases with increasing FC, leading to a looser packing compared to the sample with zero fines content (as shown in Table 2-3).

Table 2-3 Characteristics of the samples used by Thevanayagam et al. (2002) to investigate the effect of fines on the undrained behavior of silt-sand matrix

Test Series	Samples Name	Fines (%)	e	e_c	e_f	Flow Behavior type
A *=specimens were too weak and could not be prepared at $e=0.6$	00-598	0	0.598	0.598	n/a	Non-Flow (Dilative)
	00-604	0	0.604	0.604	n/a	Non-Flow (Dilative)
	07-596	7	0.596	0.716	8.51	Non-Flow (Dilative)
	15-584	15	0.584	0.864	3.89	Limited Flow (Contractive-Dilative)
	15-595	15	0.595	0.876	3.97	Limited Flow (Contractive-Dilative)
	25-460*	25	0.460	0.947	1.84	Flow liquefaction (Contractive)
	40-516*	40	0.425	1.375	1.06	Flow liquefaction (Contractive)
	60-516*	60	0.516	2.79	0.86	Limited Flow (Contractive-Dilative))
	100-770	100	0.770	n/a	0.77	Non-Flow (Dilative)

For FC values up to 25%, the specimens were too fragile and could not be prepared at the same void ratio, indicating that the soil continues to weaken as FC increases (Figure 2-32c and d). As a result, samples with FC values of 25% and 40% were prepared at a lower void ratio. However, in terms of e_c , they are looser than the sample with FC=15%.

For higher fines content (FC=60% and 100%), fines have a greater influence on the behavior of the specimens. This can be seen from the changes in e_f . Sample 60-516 (FC=60% and $e=0.516$) is less dense than the sample 40-425 in terms of void ratio, while it tends to be denser (more dilative) in terms of e_f (as shown in Table 2-3). Figure 2-32e and f depict the undrained behavior of the samples with FC values of 60% and 100%.

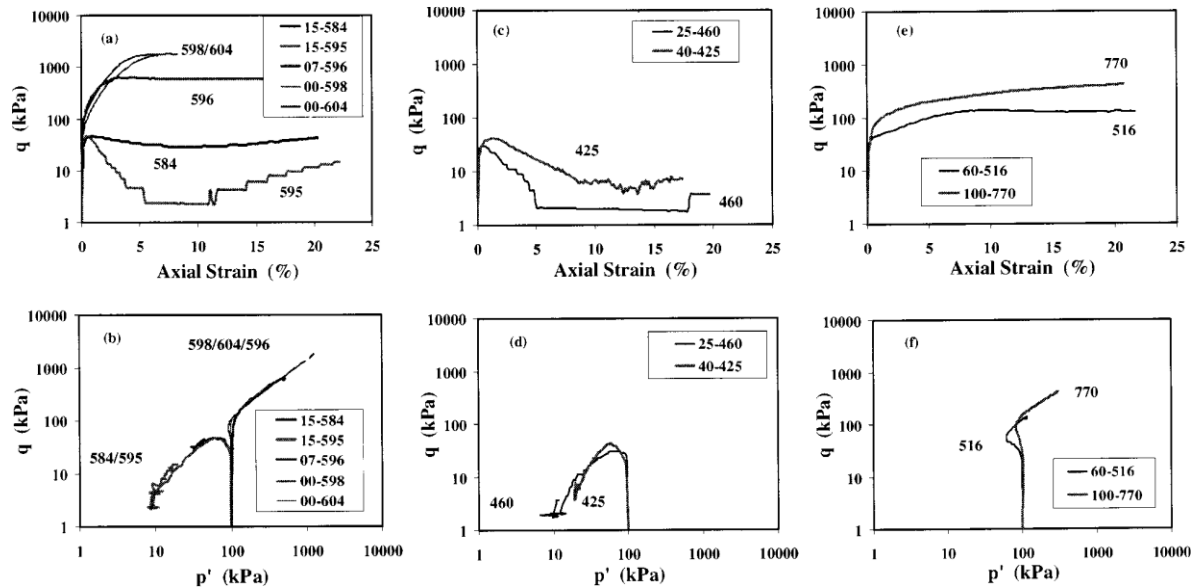


Figure 2-32 Stress-strain and stress path of the samples of A series, (a) and (b) for $FC=0, 7$, and 15% , (c) and (d) for $FC=25$ and 40% , (d) and (e) for $FC=60$ and 100% (Thevanayagam et al. 2002)

2.3.4 Effect of fines content on the critical stress ratio (M)

For fines with the similar angularity of coarse grains, it has been shown that the fine content has little influence on M (the mobilized critical stress ratio) as shown in Figure 2-33 (Torres and Santamarina, 2019, Rahman et al. 2014). Polania et al. (2023) through experimental test results showed that for similar grain shapes, the grain size distribution does not affect the residual shear strength of granular soils. Zhu et al. (2020) also analyzed the effect of the size ratio and fines content (ranging from zero to 100%) on the shear behavior of binary mixtures using discrete element method (DEM). They showed that for circular granular shapes, size ratio and FC have no significant effect on the critical friction angle.

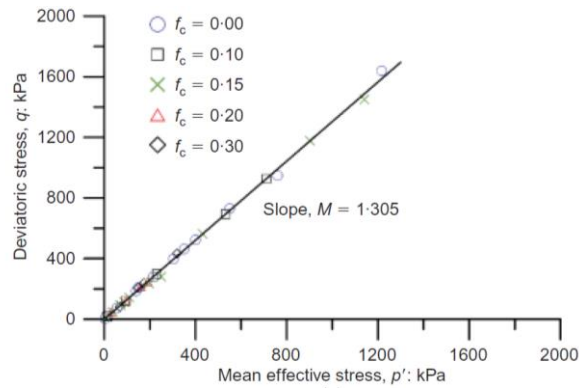


Figure 2-33 Effect of fines content on the critical stress ratio (Rahman et al. 2014)

2.3.5 Effect of fines content on the CSL of sand-silt mixture

Equation (2-4) and Equation (2-5) introduced a linear and non-linear critical state line in $e-\ln(p)$ space, respectively. Recall that for a linear CSL (Equation (2-4)), λ determines the slope of the CSL, while Γ is the void ratio at a confining pressure of 1 kPa and controls the location of the linear CSL.

Sadrekarami (2013) investigated the impact of the angularity of non-plastic fines on the slope of CSL. If the non-plastic fines have a similar particle shape as the sand base soil, the presence of fines does not significantly affect the slope or shape of the CSL. Figure 2-34a shows the CSL of Hokksund sand with different contents of Chengbeinon plastic silt. Both silt and sand particles were made of similar minerals (quartz, feldspar, and mica) with more-or-less similar shapes. As it can be seen, until 15% fines content the slope of CSL remains constant. After that, CSL shows slightly higher slope for additional fines content. Otherwise, when the fines have more angular shapes than the base sand soil, they create a mixture that tends to have higher slope of CSL with increase of fines content. Figure 2-34b shows the results with fines with more angularity, whereby increasing the fines content, the CSL rotates clockwise.

Macedo and Vergaray (2021) investigated the effect of fines content on the λ and Γ of the linear CSL for many tailings. They have shown that λ is not affected considerably by the variation of fine content, while Γ decreases and then increases by fines content (See Figure 2-35).

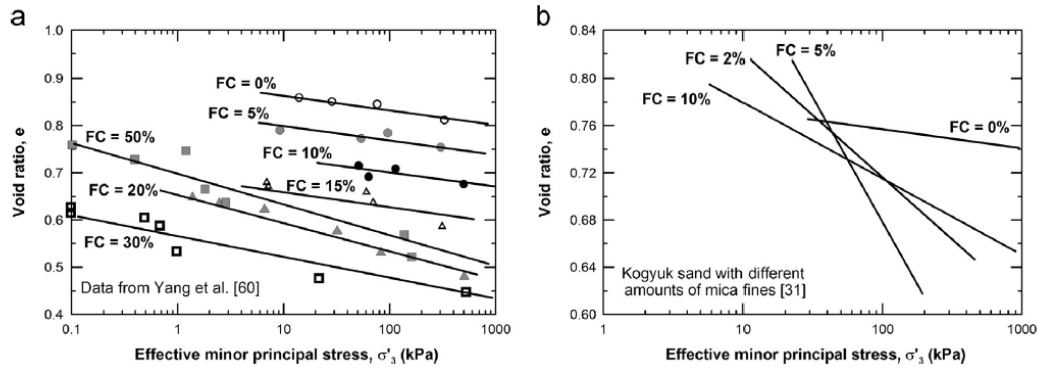


Figure 2-34 Effect of non-plastic fines on the CSL: (a) for fines with particle shapes and mineralogy similar to those of the bases, and (b) for fines with more angular particle shapes (Sadrekarimi, 2013)

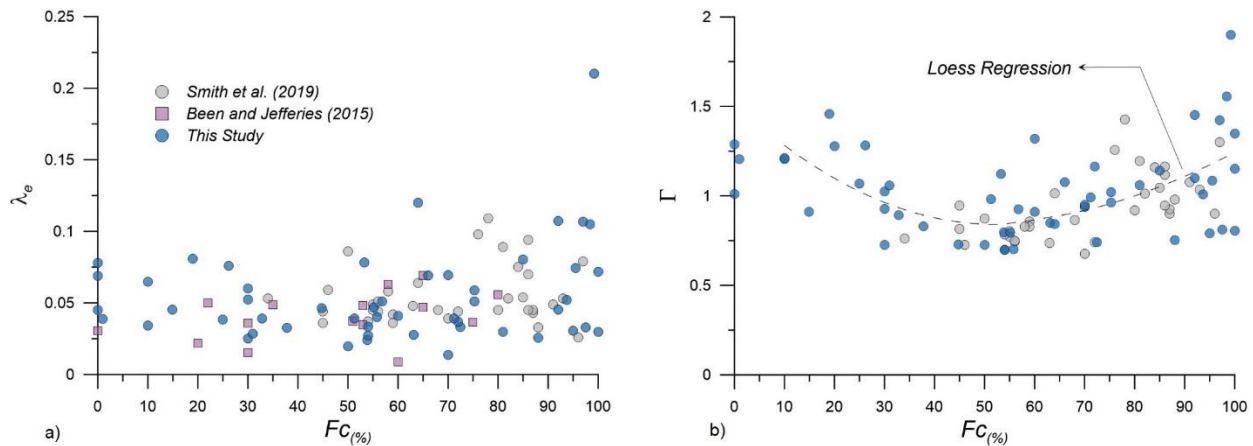


Figure 2-35 Variation of linear CSL parameter with fines content (a) slope of CSL (b) void ratio at confining pressure of 1 kPa (Macedo and Vergaray, 2021)

This pattern of changing the location of CSL with the fines content has also been reported by other researchers (e.g. Murthy et al. 2007, Papadopoulou and Tika, 2008, Rahman et al. 2011, Rahman et al. 2014). They showed that, an increase in non-plastic fines content with similar angularity with base sand tends to move the CSL in $e-\ln p'$ space downward initially. Further increase reverses the trend. Torres and Santamarina (2019) reported similar results for non-plastic tailings, as shown in Figure 2-36.

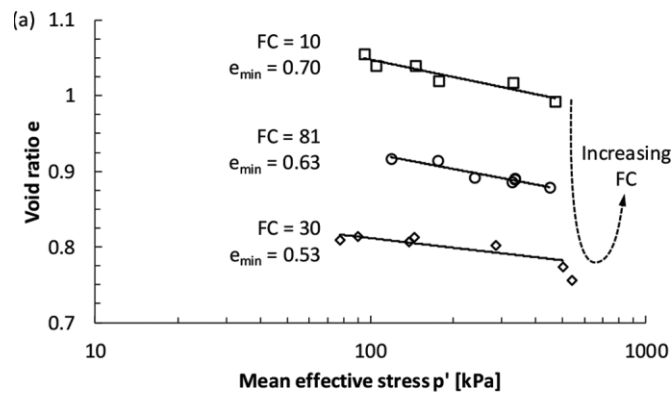


Figure 2-36 Critical state lines for all mixtures of platinum tailings (Torres and Santamarina, 2019)

2.3.6 Effect of fines content on the CSL of sand-silt mixture in equivalent void ratio -confining pressure space

Rahman et al. (2011) stated that a single equivalent critical state line can be found for the “silt in sand” matrix. For each fines content, the *CSL* (or *steady state line – SSL*) in e - $\log p'$ must be redrawn in e^* - $\log p'$. e^* is the equivalent void ratio calculated using Equation (2-20) and Equation (2-24). Figure 2-37 shows that the *SSL* with different fines content builds a single equivalent *SSL* in e^* - $\log p'$ space. The concept of equivalent *CSL* in e^* - $\log p'$ also has been reported by other researchers (Lashkari, 2016, Rahman and Lo, 2012, Thevanayagam et al. 2002).

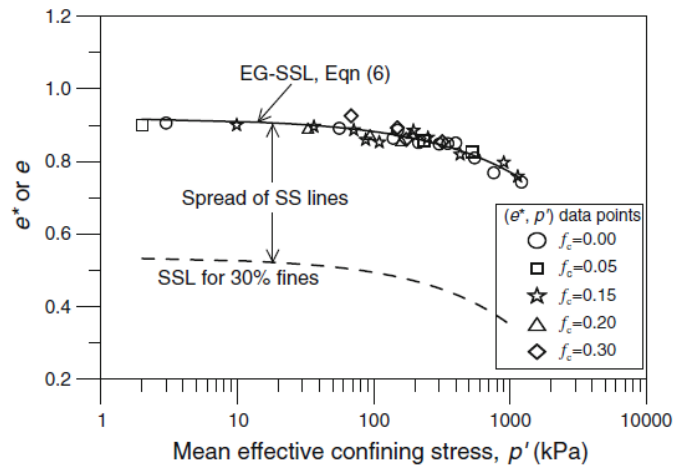


Figure 2-37 Equivalent SSL for sand with fines in e^* - $\log p'$ space (Rahman et al. 2011)

2.3.7 Concept of equivalent state parameter

The use of equivalent CSL in e^* - p' space introduces an appealing concept. Defining the state parameter in this space as the equivalent state parameter makes the concept of equivalent void ratio more interesting. The equivalent state parameter can be used as a normalization factor, such that the sand-silt mixture with similar equivalent state parameter should show similar behaviors. Rahman and Lo (2012) examined this concept using undrained triaxial tests conducted on loose samples with different fines content. Figure 2-38 (Rahman and Lo, 2012) shows the instability stress ratio (Flow liquefaction line) as a function of the equivalent state parameter for Hokksund sand samples with up to 30% fines contents. At a constant equivalent state parameter, samples with different fines content show approximately similar instability stress ratio, showing that equivalent state parameter can be used as a normalization factor. The overall trend agrees with the previously mentioned statement.

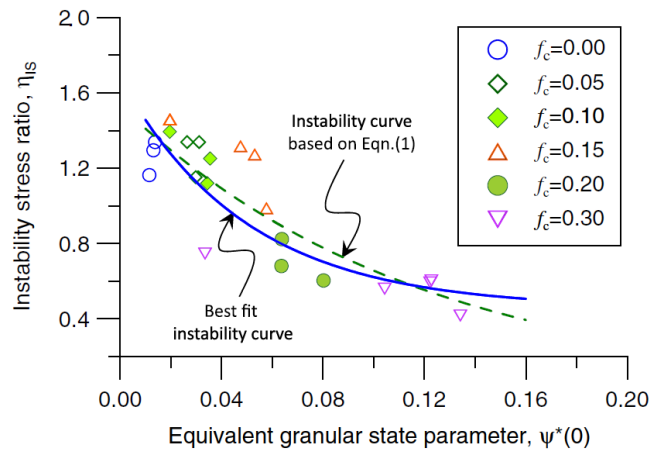


Figure 2-38 Relationship between instability stress ratio and equivalent granular state parameter for Hokksund sand with up to 30% fines contents (Rahman and Lo, 2012)

2.4 Literature Review Part 3: Understanding soil constitutive models - Applicable for granular materials

Previous sections outlined the patterns of mechanical behavior of granular soils under different shearing modes. There are several methods and frameworks to effectively simulate these patterns. For this purpose, the concept of incremental elastoplastic theory will be explained, which is a powerful tool for modeling the mechanical behavior of granular materials. This theory will be combined with the critical state framework to develop suitable soil constitutive models, which will be classified into three categories based on their formulation in terms of flow rule and the number of yield surfaces (single or double). After analyzing the advantages and disadvantages of each approach, the methodology to extend soil models to simulate the sand-silt mixtures will be explained.

2.4.1 Elements of incremental elastoplastic theory

2.4.1.1 Yield Surface and hardening laws

In the framework of elastoplasticity, it is assumed that materials have an elastic region surrounded by a boundary in stress space. The mathematical formulation (f) to define the boundary

of the elastic region is called yield point (in 1D stress space), yield boundary (in 2D stress space), or yield surface (in 3D stress space).

$$f(\sigma_{ij}, K_1, K_2, \dots) = 0 \quad (2-25)$$

Where σ_{ij} are the stress variables, and K_α are parameters used to define the shape and position of the yield surface in stress space. According to this theory, a stress point must be inside or on the yield surface. It cannot exist outside of the yield surface.

When the stress state is inside the yield surface, all deformations are elastic. This means that unloading to the initial stress conditions will result in a return to the original shape with no remaining deformations. A change in stress state inside the elastic region does not affect the boundary of the yield surface.

If the stress conditions are on the yield surface, any additional loading toward the outside of the elastic region, results in (1) a change in the boundaries of the yield surface, and (2) elastoplastic deformations. The first rule is necessary because the stress conditions in the elastoplastic theory are not allowed to exist outside of the yield surface. Therefore, the yield surface must change to be consistent with the new stress conditions. These changes could be done by modifying the shape, size, or position of the yield surface in the stress space. The parameters controlling the current situation of yield surface are K_α parameters in Equation (2-25). These parameters are also called hardening parameters. They are usually a function of the plastic shear strain (ε^p).

$$K_\alpha = K_\alpha(\varepsilon^p) \quad (2-26)$$

Three examples in the changes of the boundaries of the elastic region in stress space are shown in Figure 2-39. They are (i) Isotropic, (ii) Kinematic, and (iii) Mix hardening. In the isotropic hardening, the size of yield surface is increased isotropically in all directions. In the kinematic version, the yield surface moves to a new location without changing its size. In the mix method,

the size and location of the yield surface can change if the stress conditions move toward the outside of the elastic region.

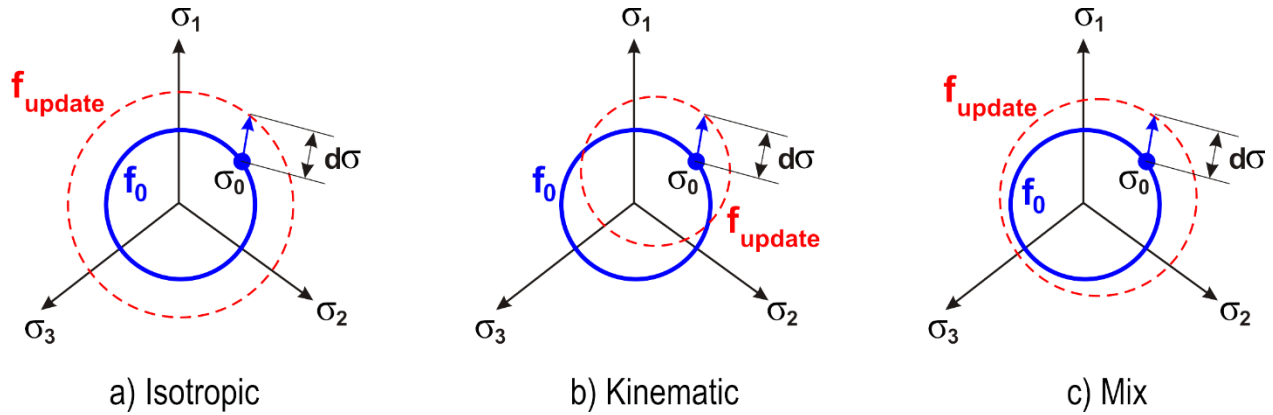


Figure 2-39 Different hardening mechanisms to modify the yield surface with new stress conditions

The mathematical equations for these three types of hardening are shown in the following equations:

$$f(\sigma_{ij}, K_\alpha) = f(\sigma_{ij}) - K_\alpha = 0 ; \text{ isotropic hardening } (a)$$

$$f(\sigma_{ij}, \alpha_{ij}) = F(\sigma_{ij} - \alpha_{ij}) = 0 ; \text{ kinematic hardening } (b) \quad (2-27)$$

$$f(\sigma_{ij}, K_\alpha, \alpha_{ij}) = F(\sigma_{ij} - \alpha_{ij}) - K_\alpha = 0 ; \text{ mixed hardening } (c)$$

2.4.1.2 Consistency Rule

The rule that ensures that the stress conditions remain on the yield surface during the elastoplastic loading is called consistency rule, which requires the following mathematical equation:

$$df(\sigma_{ij}, K_1, K_2, \dots) = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial K_\alpha} \frac{dK_\alpha}{d\varepsilon_{ij}^p} d\varepsilon_{ij}^p = 0 \quad (2-28)$$

2.4.1.3 Elastoplastic loading

Elastoplastic loading happens when the following mathematical conditions are satisfied:

$$f(\sigma_{ij}, K_1, K_2, \dots) = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} > 0 \quad (2-29)$$

In the elastoplastic loading, the stress point moves on the yield surface with the yield surface resulting in elastoplastic deformation. This means that if unloading occurs, the material cannot restore its full original shape before being subjected to elastoplastic loading. The differences between the original shape and new shape in the same stress conditions are the total amount of plastic deformations.

2.4.1.4 Incremental Elastoplasticity

One specific elastoplastic material could be subjected to a variety of loading and unloading stress paths. To capture the behavior of the material during an entire loading history with multiple loading/unloading directions, the elastoplastic theory is formulated in an incremental form; it is referred to as the incremental elastoplastic theory.

Loading can be applied using the known incremental stress ($d\sigma_{ij}$) or known incremental strain ($d\varepsilon_{ij}$). In the latter case, the aim of incremental elastoplasticity is to find (1) the new stress rate ($d\sigma_{ij}$), (2) the new elastic domain (or new yield surface), and (3) the amount of elastic and plastic strains. The known parameters are the current stress condition (σ_{ij}), the current yield surface (f_0), and the total strain rate ($d\varepsilon_{ij}$) as a sign of loading/unloading applied to the material.

In this theory, it is assumed that $d\varepsilon_{ij}$ can be divided into elastic ($d\varepsilon_{ij}^e$) and plastic strains ($d\varepsilon_{ij}^p$) components:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (2-30)$$

The second assumption is that the stress-strain behavior for the elastic region can be found from the following equation:

$$d\sigma_{ij} = D_{ijkl}(d\varepsilon_{kl} - d\varepsilon_{kl}^p) = D_{ijkl}d\varepsilon - D_{ijkl}d\varepsilon_{kl}^p \quad (2-31)$$

Where, D_{ijkl} is the constitutive matrix for elastic behavior, and its function is known. The only unknown parameter to find $d\sigma_{ij}$ in Equation (2-31) is $d\varepsilon_{ij}^p$.

2.4.1.5 Flow rule and plastic potential surface

The incremental elastoplastic theory assumes that the direction of $d\varepsilon_{ij}^p$ is aligned with the direction of a normal vector to a surface in the stress space, called the potential surface (Equation (2-32)). This assumption is known as a flow rule. The potential surface could have a similar shape as the yield surface, i.e., be identical to it, or different.

$$g(\sigma_{ij}) = 0 \quad (2-32)$$

In the case that the yield and potential surfaces are the same, the flow rule is called associated, otherwise it is called a non-associated flow rule. Based on the flow rule, the plastic strain rate can be calculated using the following formula:

$$d\varepsilon_{ij}^p = \lambda \frac{\partial g}{\partial \sigma_{ij}} \quad (2-33)$$

Where λ is a positive value called scalar plastic consistency multiplier (λ).

2.4.1.6 Stress-strain relationship

Isotropic Hardening

For isotropic hardening material, the yield function can be described using Equation (2-27)a. By combining Equations(2-28), (2-31), and (2-33), the scalar multiplier (λ) can be computed as:

$$\lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} d\varepsilon_{ij}}{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial g}{\partial \sigma_{kl}} - \frac{\partial f}{\partial K_\alpha} \frac{\partial K_\alpha}{\partial \varepsilon_{ij}^p} \frac{\partial g}{\partial \sigma_{ij}}} \quad (2-34)$$

Inserting Equations (2-33) and (2-34) into Equation (2-31) results in the final stress-strain relationship as follows:

$$d\sigma_{ij} = D_{ijkl}(d\varepsilon_{kl} - d\varepsilon_{kl}^p) = \left(D_{ijkl} - \frac{D_{ijmn} \frac{\partial g}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} D_{pqkl}}{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial g}{\partial \sigma_{kl}} - \frac{\partial f}{\partial K_\alpha} \frac{\partial K_\alpha}{\partial \varepsilon_{ij}^p} \frac{\partial g}{\partial \sigma_{ij}}} \right) d\varepsilon_{kl} \quad (2-35)$$

Kinematic Hardening

In kinematic hardening, the size and shape of the yield surface is constant. The yield surface in this case only shows a movement in stress space. In Equation (2-27)b, this movement is controlled by α_{ij} , which is called back stress tensor and represents the center of the yield surface in stress space. In the kinematic hardening models, another rule must be considered about the direction of yield surface movement. One of such rules is the Melan-Prager rule (Ottosen and Ristinmaa, 2005), where the direction of movement is in the direction of plastic strain vector (see Figure 2-40).

Therefore, the following equation can be considered based on the Melan-Prager rule for yield surface:

$$d\alpha_{ij} = c \cdot d\varepsilon_{ij}^p \quad (2-36)$$

The consistency rule (Equation (2-28)) is rewritten as follows for kinematic hardening of Equation (2-27)b.

$$df(\sigma_{ij}, \alpha_{ij}) = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \alpha_{ij}} d\alpha_{ij} = 0 \quad (2-37)$$

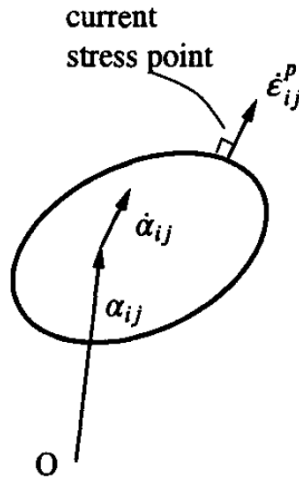


Figure 2-40 Definition of the direction of yield surface movement in kinematic hardening using Melan-Prager rule (Ottosen and Ristinmaa, 2005)

According to the assumed form of the yield function (2-27)b, the following relationship is valid:

$$\frac{\partial f}{\partial \sigma_{ij}} = -\frac{\partial f}{\partial \alpha_{ij}} \quad (2-38)$$

Using Equation (2-38), (2-36), and (2-37), the scalar multiplier can be found:

$$\lambda = -\frac{1 \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}}{c \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial g}{\partial \sigma_{ij}}} \quad (2-39)$$

The increments of back stress tensor and plastic strains can be determined by Equations (2-36) and (2-33):

$$d\alpha_{ij} = \frac{\frac{\partial g}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}}}{\frac{\partial f}{\partial \sigma_{ij}} \frac{\partial g}{\partial \sigma_{ij}}} d\sigma_{kl} \quad (2-40)$$

$$d\varepsilon_{ij}^p = \frac{1 \frac{\partial g}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}}}{c \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial g}{\partial \sigma_{ij}}} d\sigma_{kl} \quad (2-41)$$

Therefore, the total strain rate can be calculated as:

$$\begin{aligned} d\varepsilon &= d\varepsilon^e + d\varepsilon^p = [D_{ijkl}]^{-1} d\sigma_{kl} + \frac{1 \frac{\partial g}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}}}{c \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial g}{\partial \sigma_{ij}}} d\sigma_{kl} \\ &= \left([D_{ijkl}]^{-1} + \frac{1 \frac{\partial g}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}}}{c \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial g}{\partial \sigma_{ij}}} \right) d\sigma_{kl} \end{aligned} \quad (2-42)$$

By inversing Equation (2-42), the final stress-strain relationship can be obtained.

2.4.1.7 The concept of bounding surface plasticity

In the case of cyclic loading, the models discussed previously cannot simulate the response of materials properly. In these models, the elastic region surrounded by the yield surface is usually too large. As a result, when unloading happens, the stress point goes back into the elastic region, and shows elastic behavior. As long as the stress point is inside that region, any cyclic loading will be simulated with an elastic behavior, without plastic deformation, which is not in agreement with the behavior of soils. In addition, energy dissipation and the damping behavior of materials during cyclic loading will be ignored. One solution is to limit the size of the elastic region. Upon unloading, the stress point goes back inside the yield surface and can then reach the other side of the yield surface, where plastic deformations will begin by changing the location of yield surface. This is similar to the kinematic hardening discussed in 2.4.1.6. In these sections, the focus was on monotonic loading, where the hardening laws defined to guide the translation of the yield surface to failure are straightforward. In cyclic loading, since we are dealing with unloading/reloading in any direction, the hardening laws need to be defined in a more controlled, restricted way to get better results. One way is to introduce another surface in stress space called the bounding surface. The small yield surface is inside the bounding surface and is only allowed to move in that region.

The concept of bounding surface plasticity (or two surface plasticity) was proposed by Krieg (1975) and Dafalias and Popov (1975). As shown in Figure 2-41, this theory requires two surfaces, an inner surface (loading or yield surface, $f=0$) and an outer surface (bounding or limit surface, $F=0$).

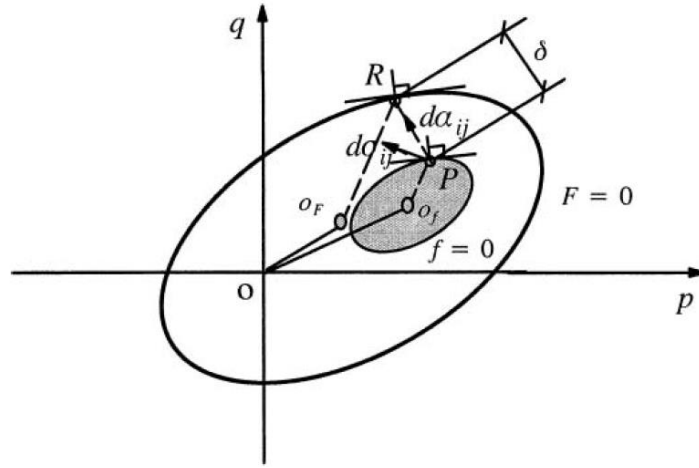


Figure 2-41 Bounding surface plasticity (Yu, 2007): loading surface (f), Bounding surface (F), current stress point (P), image stress point (R)

Both surfaces can follow mixed hardening rules, meaning that they can change their size and position. Therefore, the following equations can be derived for both.

$$\begin{aligned}
 f &= F(\sigma_{ij} - \alpha_{ij}) - (\sigma_y)^p = 0 && \text{loading surface} \\
 F &= F(\sigma_{ij} - \beta_{ij}) - (\sigma_b)^p = 0 && \text{bounding surface}
 \end{aligned} \tag{2-43}$$

The hardening rule of the loading surface depends on a distance (δ) from the current stress state (point P in Figure 2-41) to the bounding surface (Point R in Figure 2-41). When δ is high, the material is stiff. In the other hand, when δ is getting close to zero, the material's stiffness drops, reaching the corresponding value on the bounding surface. This allows a smooth transition from a high stiffness on the loading surface to a low stiffness on the bounding surface.

Point R on the bounding surface is called the mapping stress point or image point, which needs to be determined. One of the common rules to define it is called Mroz translation rule (Mroz, 1967). First a line is drawn from the center of the loading surface to the current stress point. Second, from the center of the bounding surface, a parallel line is drawn to intersect the bounding surface.

The intersection point is the image point (see figure Figure 2-41). Other rules such as Iwans translation rule (Iwan, 1976) have been used as well,.

2.4.1.8 Conclusion

Based on this basic description of incremental elastoplastic theory, the main elements that need to be defined for one specific constitutive model are:

- Yield surface (f) and bounding surface (F)
- Isotropic or kinematic hardening laws (K_α)
- Flow Rule
- Potential Surface (g):
- Elastic constitutive matrix (D)

Note that if the models are based on a single yield surface, there is no need to define the bounding surface.

2.4.2 Elastoplasticity in the concept of critical state theory for soils

Critical state theory declares that all soil samples regardless of their initial state conditions (i.e. for any value of the state parameter), must reach critical state when sheared at large strains. This theory states that by further shearing, no changes should occur in deviatoric stress, mean stress, and volumetric strain. All the elastoplastic models respecting these requirements are set in the framework of critical state theory, such as Cam-Clay (Roscoe et al. 1963), NorSand (Jefferies, 1993), Dafalias and Manzari (2004), PM4Sand (Boulanger and Ziotopoulou, 2013), Wood et al. (1994), etc. The mentioned models can be categorised, as shown in table Table 2-4, based on their formulation in terms of flow rule and the number of yield surfaces (single or double)

Table 2-4 Classification of different soil models in the framework of critical state theory

Associated flow rules	Non-Associated flow rules	
Single yield surface (Cambridge type models)	Single yield surface	Bounding surface
• Cam-Clay (Roscoe et al. 1963) • NorSand (Jefferies, 1993)	• Wood et al. (1994) • Lu and Huang (2015) • Yin et al. (2016)	• Manzari and Dafalias (1997) • Dafalias and Manzari (2004) • PM4Sand (Boulanger and Ziotopoulou, 2013)

All the above models can simulate the behavior of soils under various initial states, ranging from loose to dense. One advantage of the bounding surface plasticity models is their ability to accurately simulate soil behavior during unloading and reloading cycles, which requires complex formulation and additional material parameters. However, if only monotonic loading is of interest, the single yield surface plasticity models may simulate accurately the soil behavior with fewer material parameters. Among the single yield surface models, the NorSand model has gained popularity due to its simplicity and straightforward approach in its calibration. However, its simplicity does not necessarily guarantee flexibility in controlling the obtained soil behavior and produces some shortcomings. A thorough discussion on the concept of the NorSand model and its performance on the simulation of undrained behavior of granular soils will be presented in Chapter 3.

2.4.2.1 Cambridge type models

These types of models such as NorSand and Cam-Clay generally follow the following steps to derive their formulations:

Step 1: Find an appropriate stress-dilatancy rule, valid for the entire stress path.

Step 2: Integrate the stress-dilatancy rule in step 1 to define the yield surface in the concept of normality or associated flow rule.

Step 3: Define the hardening rule. The size of the yield surface is usually controlled by a parameter defined as a mean confining pressure (p_0). The hardening rule is written in the framework of critical state theory to modify (p_0) to soften or harden the yield surface.

The differences between the Cam-Clay and NorSand model is the latter's capability in controlling the amount of dilatancy by using a concept called limiting dilatancy. The other difference is the stress-dilatancy they are using to develop their yield surfaces. In the Cam-Clay, the yield surface is attached to the CSL, while in the NorSand the yield surface only attaches CSL at large strains.

2.4.2.2 Soil models with single yield surface and the concept of non-associated flow rules

To develop a soil model in this framework the following steps are usually necessary.

Step 1: The yield function in these models usually takes the following format:

$$F = q - \eta p = 0 \quad (2-44)$$

Where η is the stress ratio controlling the evolution of the yield surface.

Step 2: A hardening rule is defined to control the evolution of the yield surface (η) with shear loading (plastic deviatoric strain, ε_q^p). For example, Lu and Huang (2015) and Wood et al. (1994) relate η to ε_q^p using simple hyperbolic functions as follows.

$$\begin{aligned} \eta &= M \cdot \exp(-n^b \psi) \frac{\varepsilon_q^p}{B + \varepsilon_q^p} \quad (\text{Lu and Huang, 2015}) \\ \eta &= (M - k\psi) \frac{\varepsilon_q^p}{B + \varepsilon_q^p} \quad (\text{Wood et al. 1994}) \end{aligned} \quad (2-45)$$

The derivations of above formulations give the hardening rules. These rules guarantee that at critical state $\psi = 0$ (at large shear strains), the stress ratio reaches the critical stress ratio (M), following critical state theory. n^b , k and B are material constants.

Step 3: To control the dilatancy, these models can rely on the definition of a plastic potential surface, or they can directly use the stress-dilatancy rule.

$$\begin{aligned} D &= A \cdot M \cdot (\exp(n^d \psi) - 1) \quad (\text{Lu and Huang, 2015}) \\ D &= A(M - \eta) \quad (\text{Wood et al. 1994}) \end{aligned} \quad (2-46)$$

As it can be seen, the dilatancy rule presented by Lu and Huang (2014) is state dependent, which can be more precise. At large strains, since $\eta = M$ and $\psi = 0$, the dilatancy value would be zero satisfying the requirements of critical state theory.

Figure 2-42 shows the measured and predicted undrained stress path for isotropically consolidated undrained triaxial tests of the model presented by Lu and Huang (2014). However, the application of the model is only analyzed for the triaxial compression tests, and not for other shearing modes that could occur in a boundary value problem.

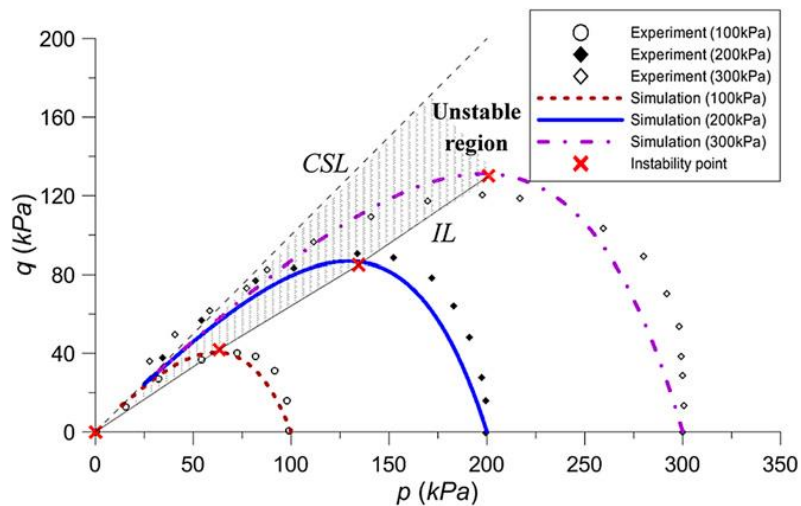


Figure 2-42 Undrained stress path, experimental data vs. simulations made by the model presented by Lu and Huang (2015)

2.4.2.3 Soil models with bounding surface plasticity

The soil models in the context of bounding surface plasticity can have quite complex and extensive formulations. In this section, for the sake of simplicity, only one model, the Dafalias and Manzari (2004) model, under the triaxial formulation without considering the effect of fabric will be explained. The model introduces four types of lines (in q - p plane) to build its mathematical equations, yield, dilatancy, critical and bounding lines. These lines are shown schematically in Figure 2-43.

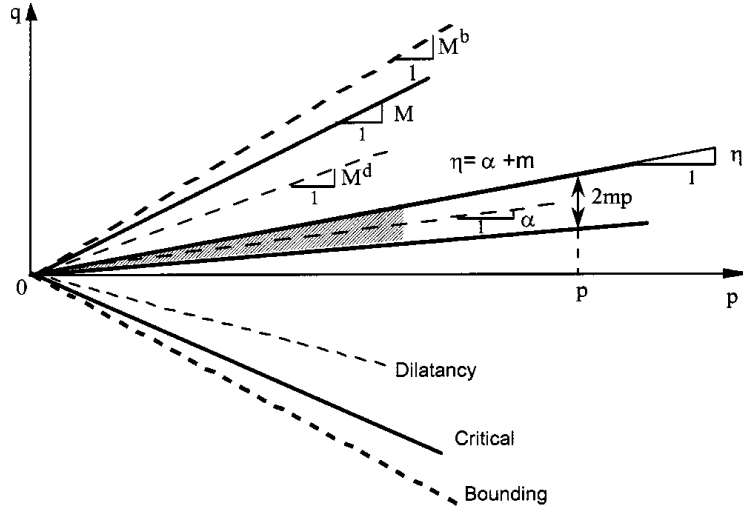


Figure 2-43 Schematic of the yield, critical, dilatancy, and bounding lines (Dafalias and Manzari, 2004)

The yield surface is given by the following equation:

$$f = |\eta - \alpha| - m = 0 \quad (2-47)$$

This equation represents a wedge in q - p space, which bisector has a slope of α , and the wedge opening is $2mp$, as shown in Figure 2-43. The yield surface can show both isotropic and kinematic hardening during yielding. By taking m a constant value, only kinematic hardening is allowed in the model. The elementary stress-strain relationships in the model are given by:

$$d\varepsilon_q^e = \frac{dq}{3G} \quad d\varepsilon_v^e = \frac{dp}{K} \quad (2-48)$$

$$d\varepsilon_q^p = \frac{d\eta}{H} \quad d\varepsilon_v^p = D|d\varepsilon_q^p| \quad (2-49)$$

The superscripts e and p are for elastic and plastic strains, respectively. G and K are the elastic shear and bulk moduli, respectively. Based on the equations, the variation in plastic shear strain happens when the stress ratio changes. H is the plastic hardening modulus and is obtained from the following equation:

$$H = h(M^b - \eta) \quad (2-50)$$

Where h can be a constant or a function of the state variables. The bounding line represented by M^b is given by the following expression.

$$M^b = M \cdot \exp(n^b \psi) \quad (2-51)$$

Here, n^b is a positive material constant, and M is the slope of the CSL in q - p space. The model is developed in the framework of critical state theory which means that upon shearing at large strains the critical state is reached, the state parameter is 0 and M^b is equal to M , and the bounding line collapses onto the CSL. The void ratio at critical state and at a given confining pressure is given in the e - $\ln p'$ by the following expression:

$$e_c = (e_c)_{ref} - \lambda \ln \left(\frac{p}{p_{ref}} \right) \quad (2-52)$$

Where $(e_c)_{ref}$, λ , and p_{ref} are material constants.

The model uses the following state dependent dilatancy rule to determine the contractive or dilative behavior of soil on the yield surface:

$$D = A_d(M^d - \eta), \quad A_d = A_0(1 + \langle sz \rangle) \quad (2-53)$$

Where A_d is a function of the state. A_0 is a constant value, $s = \pm 1$, considering the direction of loading, and $\mathbf{z}$ is the fabric dilatancy internal variable. The dilatancy line or phase transformation line has a slope of M^d in q - p space, and it varies with the soil current state parameter:

$$M^d = M \cdot \exp(n^d \psi) \quad (2-54)$$

Where n^d is a positive material constant. Based on these equations for loose states with $\psi > 0$, $M^b < M < M^d$, and vice versa for dense states, while $M^b = M = M^d$ at critical state or for $\psi = 0$.

Figure 2-44 shows the response of the model for an undrained compression test conducted on Toyoura sand with different initial mean confining pressures. The results are in a good agreement with the test data, although the model slightly underestimate the undrained peak shear strength.

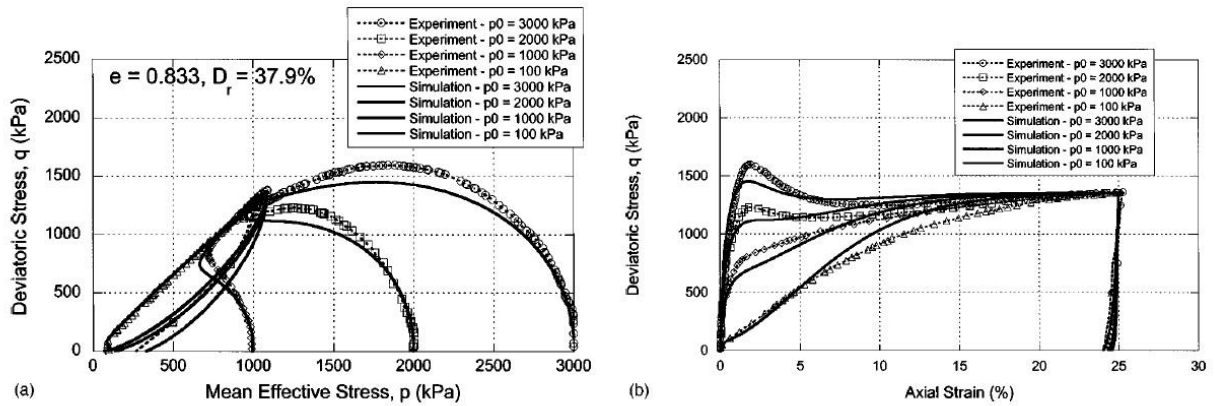


Figure 2-44 Simulations versus experiments in undrained compression made by Dafalias and Manzari (2004)

2.4.3 Soil models considering the effect of fines content.

There are two main approaches to consider the effect of fines content on the mechanical behavior of sand-silt mixtures: the Global void ratio approach and the Equivalent void ratio approach. These two approaches use a base model for granular soils and extend its formulation to simulate the behavior of sand-silt mixtures.

2.4.3.1 Global Void Ratio approach

This approach states that there is no need to change the formulation of the base model and that the material properties can also be valid when the fines content also increases, and that the effect of fines content should only be considered by changing the position of the critical state line. Recalling

Equation (2-4) and (2-5), the position of the CSL is controlled by Γ which is the critical void ratio when $p = 1\text{kPa}$. To obtain the variation of Γ with fc the following steps are used by Yin et al. (2016).

Step 1: Reverse Equations (2-20) and (2-24) to obtain the variation of void ratio with fine content.

$$e = (e_c)_{eq} - (1 - b)(1 + (e_c)_{eq})fc \quad FC < FC_{th} \quad (2-55)$$

$$e = (e_f)_{eq} \left(fc + \frac{1 - fc}{\chi^m} \right) \quad FC > FC_L \quad (2-56)$$

Step 2 : Replace e with Γ .

$$\Gamma = (\Gamma_c)_{eq} - (1 - b)(1 + (\Gamma_c)_{eq})fc \quad FC < FC_{th} \quad (2-57)$$

$$\Gamma = (\Gamma_f)_{eq} \left(fc + \frac{1 - fc}{\chi^m} \right) \quad FC > FC_L \quad (2-58)$$

Step 3: It is straightforward to find $(\Gamma_c)_{eq}$ and $(\Gamma_f)_{eq}$ when fc equals to 0 and 1 respectively. when $fc = 0$, $\Gamma = \Gamma_c$, where the sand-silt mixture only contains sand grains and Γ_c is the critical void ratio at $p = 1\text{ kPa}$, for the base sand. At $fc = 1$, $\Gamma = \Gamma_f$, where the sand-silt mixture only contains silt grains and Γ_f is the critical void ratio at $p = 1\text{ kPa}$, for silt. Replacing these, into the above equations yields the following expressions.

$$\Gamma = \Gamma_c - (1 - b)(1 + \Gamma_c)fc \quad FC < FC_{th} \quad (2-59)$$

$$\Gamma = \Gamma_f \left(fc + \frac{1 - fc}{\chi^m} \right) \quad FC > FC_L \quad (2-60)$$

Step 4: Use a transition function to change the formulation from Equation (2-57) to Equation (2-58). Yin et al. (2016) proposed a hyperbolic tangent function. As a result, the variation of Γ with fc can be expressed using the following equation:

$$\Gamma = [\Gamma_c - (1 - b)(1 + \Gamma_c)f_c] \frac{1 - \tanh(\xi(f_c - f_{th}))}{2} + \Gamma_f \left(f_c + \frac{1 - f_c}{\chi^m} \right) \frac{1 + \tanh(\xi(f_c - f_{th}))}{2} \quad (2-61)$$

In this equation ξ is a material property controlling the rate of change from Equation (2-59) to Equation (2-60). Yin et al. (2016) used Equation (2-61) to determine the location of critical state line for different sand-silt mixtures (Figure 2-45), (a,e), (b,f), (c,g), and (d,h) presenting the results for Toyoura sand-silt, Foundary sand-silt, Hokksund sand-silt, and Assyros sand silt, respectively. Figure 2-45 e, f, g, and h also show the variation of Γ (in the figures e_{cr0}) with fines content presented by Equation (2-61) in comparison with measured data.

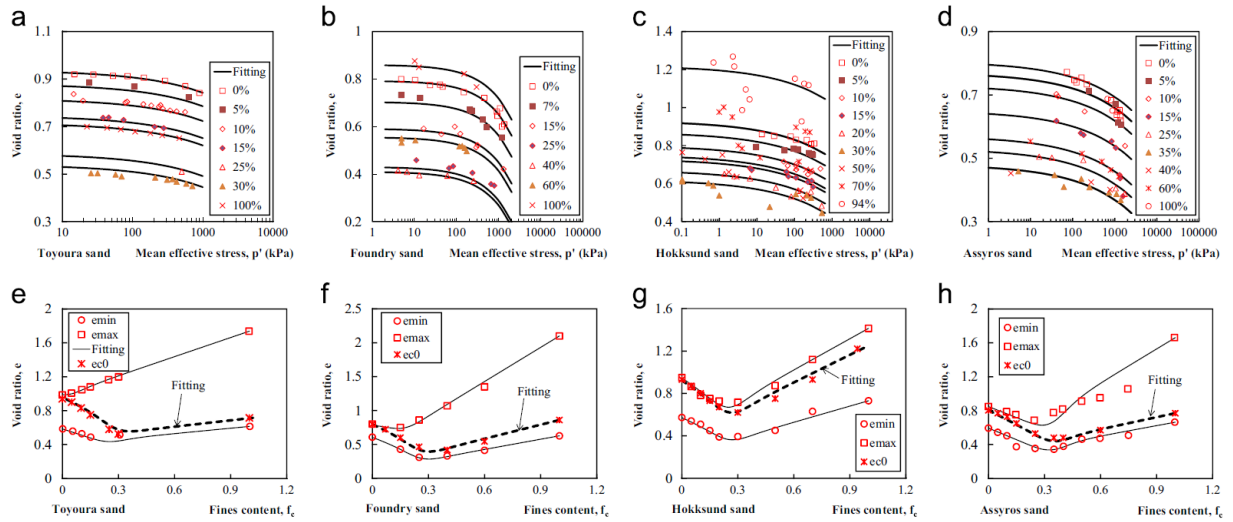


Figure 2-45 (a, b, c, and d) CSLs of different Silt-Sand mixtures, and (e, f, g, and h) critical state void ratio at 1kPa versus fines content for various Sand-Silt mixtures (Yin et al. 2016)

The model used by Yin et al. (2016) was a single yield surface plasticity with a non-associated flow rule, as seen in Table 2-4. In Figure 2-47, their simulation of the undrained behavior of sands with different fines content are shown in comparison with the test data. Both, the measured data and the simulations show that an increase in the fines content increase the possibility for flow liquefaction, even though the relative density of the soil increases from 15% to 41%. These

data for soils with different fines content are inconsistent with the theory that soil show more dilatant behavior when their relative density increases. They related this feature to the creation of a “particle structure” due to the presence of fines. The simulations show that the proposed approach was successful to simulate this behavior.

2.4.3.2 Equivalent void ratio approach

Rahman et al. (2014) tried to simulate the sand-silt mechanical behavior through a new framework called equivalent critical state theory. Their approach only examines the sand-silt mixtures with $FC < FC_{th}$ and contains four steps as follows:

Step 1: Select an appropriate soil model to simulate the behavior of granular materials.

Step 2: Obtain the required material properties of for a clean sand (i.e. without fines).

Step 3: Replace all void ratios in the formulation of the selected soil model with an equivalent void ratio.

Step 4: Replace the CSL in the selected model with the equivalent CSL.

Rahman et al. (2014) selected Manzari and Dafalias (1997) as a base model for sand, and changed its formulation based on the above procedure. The model’s responses for flow behavior and non-flow behavior for different fines content are shown in Figure 2-46. The prediction for flow-behavior was accurate, while for non-flow behavior, only the overall behavior pattern was captured.

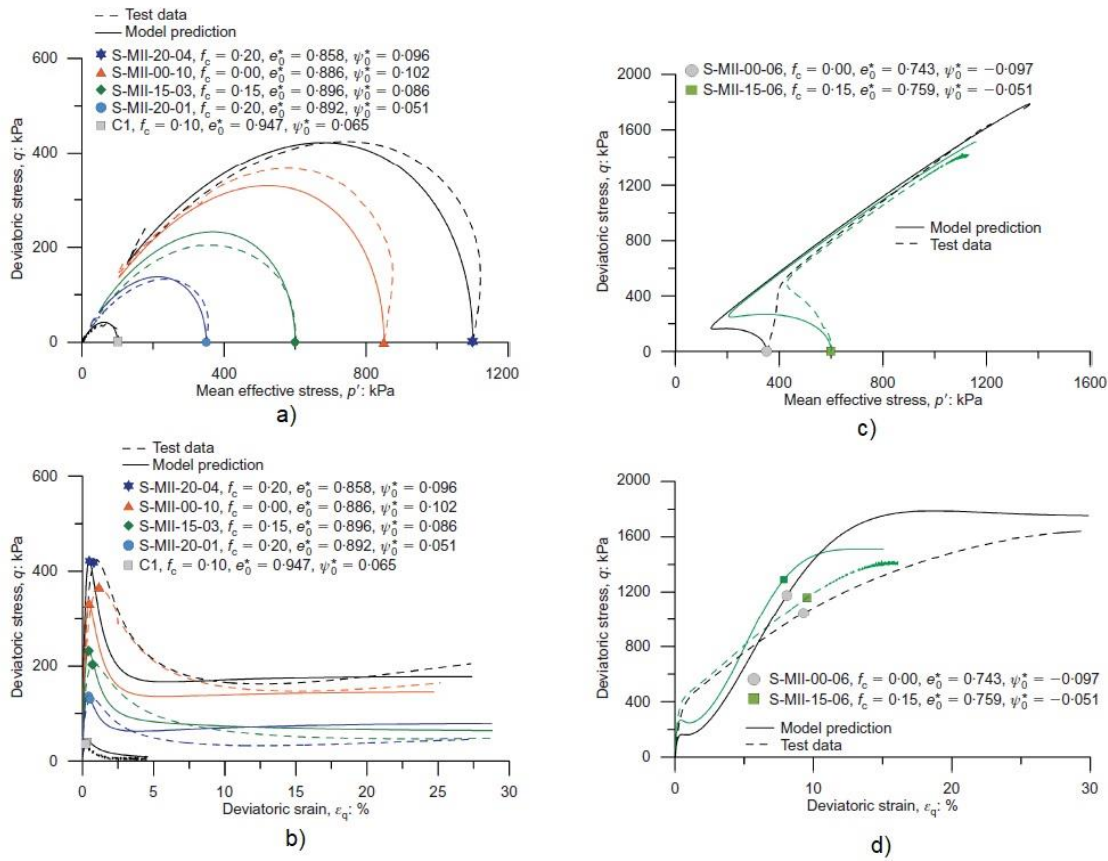


Figure 2-46 Comparison between test results and model predictions for flow behavior (a-effective stress path, b-deviatoric stress-deviatoric strain) and non-flow behavior (c-effective stress path, d-deviatoric stress-deviatoric strain) of Sydney sand (Rahman et al. 2014)

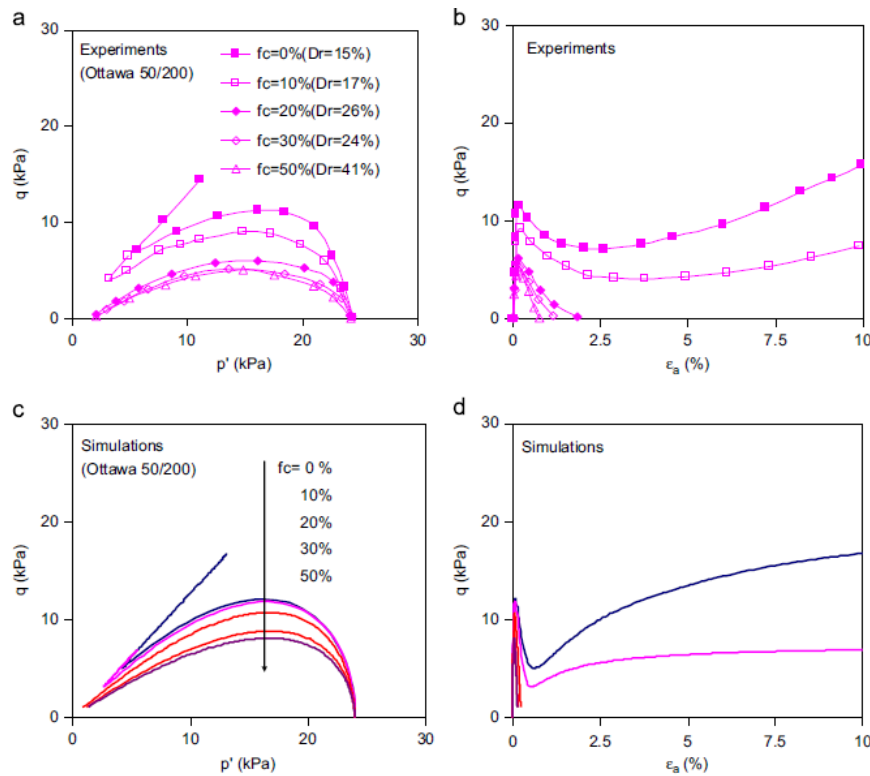


Figure 2-47 Comparison of experimental results and simulations for Ottawa sand with fine contents varying from 0% to 50%. (Yin et al. 2016)

2.5 Literature Review Part 4: Tailings Features

Mine tailings are waste products that remain after the removal of the valuable minerals from the ore. The grain size distribution of the tailings varies depending on the characteristics of the ore and the milling process, ranging from clay-sized to sand-sized particles. As such, there could be similarity in the drained and undrained mechanical behavior of tailings with granular soils. Their behavior is controlled by the following parameters: critical state properties, the deposition method (affecting their initial state such as initial void ratio and fabric), and fines content with similar angularity of fines tailings in comparison with coarse tailings. This section presents the main characteristics that must be considered in the constitutive models.

2.5.1 Flow behavior type in tailings dams

Tailings are most often deposited as a slurry into a reservoir behind a dam, called a tailings dam. The dam can be constructed to its full height before the disposal of tailings or can be built in a staged process in parallel to the disposal of tailings. The latter type, which is more common in the mining industry is typically constructed using waste rock and has a different structure than the usual water retention systems. There are three methods of constructions: upstream, centerline, and downstream. Impoundments constructed using the upstream method are the most fragile and prone to failures, while the downstream method is considered the safest.

Some tailings dams have been subjected to sudden failures where a significant amount of tailings was released in a rapid flow toward downstream, leading to major environmental and human losses. One example of such failure in recent history is the Fundao dam failure that happened on November 5, 2015, at Samarco, Brazil (Sadrekarimi and Riveros, 2020). About 32 *million m³* of iron mine tailings were released, submerging the town of Bento Rodrigues. Another example is the failure at the Mount Polley Mine in British Columbia, that occurred on August 4, 2014. It released about 17 *million m³* of water and 8 *million m³* of tailings materials (Byrne et al. 2015). These two case studies and other examples are listed in Table 2-5 with supplemental information.

Table 2-5 Flow failure in different tailings dams.

Feijao dam, 2019	Location: Minas Gerais, Brazil Final Height: 86 m Period of Construction: 37 years from 1976 Type of tailings dam: Upstream Drainage condition: Poor downward internal drainage system, no drainage system at the starter dam, and the low permeability materials used to construct the dam's face. Layering: The profile obtained through the <i>CPTu</i> test results confirmed interlayer coarse and fine materials. Released volume of tailings: 9.7 million m³ Human Loss: 270 Triggering mechanism by Robertson et al. (2019): flow liquefaction was initiated due to a critical combination of strength loss of the unsaturated zone caused by the intense rainfall towards the end of 2018 and the ongoing internal strains due to creep. Triggering mechanism by VALE (2019): analyses of the dam behavior under sever conditions, including increased precipitation and incorporation of creep effects showed no large displacement and no signs of approaching failure. VALE suggested the effect of a local borehole overpressure as a possible scenario to trigger liquefaction. The local borehole was the only significant activity taking place in the dam at the time of failure. According to VALE (2021), the drilling borehole involves the application of a water overpressure. This increase of pore water pressure may result in flow liquefaction of loose granular material.
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Table 2 – 5 Flow failure in different tailings dams (cont'd)

Cadia Valley Operations (CVO), 2018	Location: New South Wales, Australia Height at the time of event: 94 m Period of Construction: 19 years from 1998 Type of tailings dam: Upstream Saturation: While there was a drop in water level in some areas, the measured water pressure near the location of failure confirmed the existence of high pore water pressure. Layering: The in-situ state parameter estimated through the <i>CPTu</i> tests showed loose tailings material. The foundation profile in the vicinity of the event was characterized as weak and compressible with a strain softening response under loading, and with a low relative density. Its presence was not identified until the failure. Triggering mechanism by Jefferies et al. (2019): three scenarios as a leading mechanism for the failure were investigated: (1) progressive failure under increasing static load, (2) increasing water pressure causing excessive yielding and (3) external shock. The first scenario is reported as the possible answer for the event. By rising the height of the dam, a progressive failure happened in the foundation resulting in an outward horizontal movement. This movement reduced the required support for the tailings, increasing yielding in tailings. By reaching yielding in the tailings, up to the instability line, a rapid change from drained to undrained condition happened resulting in flow liquefaction. This mechanism was similar to the failure mechanism that happened in the Aznalcóllar Tailings Dam in Spain reported by Gens and Alonso (2006).
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Table 2 – 5 Flow failure in different tailings dams (cont'd)

Fundao dam, 2015	Location: Minas Gerais, Brazil Height at the time of event: 80 m Period of Construction: 19 years from 1998 Type of tailing dam: Upstream Saturation: The presence of more compressible slimes at the left abutments in comparison to the right or center impeded the free drainage conditions, causing high piezometric values reported for that area, indicating more saturation in overlying sand tailings. Layering: Morgenstern et al. (2016) reported high state parameters in the tailings at the left and the right abutments. Released volume of tailings: 32 <i>million m³</i> of iron mine tailings Human Loss: 19 People Triggering mechanism: A lateral extrusion mechanism developed in the slime, due to the increase in dam height. Consequently, the overlying sand faced a reduction in confining pressure, a mechanism that triggered flow liquefaction.
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Table 2 – 5 Flow failure in different tailings dams (cont'd)

Merriespruit Harmony Mine, 1994	Location: South Africa Height at the time of event: 31 m Period of Construction: 16 years from 1978 Type of tailings dam: Ring-dykes, in this method, the reservoir for tailings deposition is created using a dyke along the perimeter of a rectangular or circular shape. The reason to use such a structure is the flat terrain near most mines in South Africa. Saturation: Before the event, there was free water on the top of the impoundment followed by a small rainfall causing the water to overtop the dam. Human Loss: 17 People Triggering mechanism by Fourie et al. (2001): The overtopping water eroded the upstream face and induced some local slip failures. By washing away, further over steepening happened, resulted in a sequence of progressive failures. Losing the confinement pressure in tailings pushed their stress conditions toward the instability line and initiated the flow liquefaction.
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All the examples given are a result of specific conditions existing in the tailings dams. These conditions are: (i) A large volume of tailings that are in a loose state due to the deposition method, (ii) high shear stresses within the tailings due to the slope geometry of the dam, and (iii) high levels of saturation within the dam caused by poor drainage or external factors such as rainfall. This kind of loose saturated granular materials will show flow behavior type, a process of sudden strength loss under an undrained shear loading with a rapid buildup of pore water pressure (See Sections 2.2.3 and 2.2.5.4). Although the mechanism of failure is similar for most cases, the triggering mechanism to activate the flow liquefaction is different for each case. Those mechanisms could be:

- Existence of weak layers or weak foundations resulting in a reduction in confinement pressure inside the tailings
- Water level rising and overtopping
- Strength loss of the unsaturated zone above the water level due to intense rainfall
- Ongoing internal strains due to creep
- Rapid construction
- Rapid cyclic loading
- Internal erosion and piping
- Fatigue loading

2.5.2 Critical state of tailings

Many researchers have conducted tests to analyze the critical state line of tailings, and have found a high nonlinearity of the CSL, starting from a nearly flat line at low confining pressure, and steepening at high confining pressure (Bedin et al., 2012). Bedin et al. (2012) reported grain crushing during shear for the samples with high confining pressure. This shows a tendency to reach a dense state by increasing the slope of *CSL* at high confining pressure. Li et al (2018) states that regardless of the steepening of *CSL* at high confining pressure, there is no sign of grain crushing in the silt-sized gold tailings they tested.

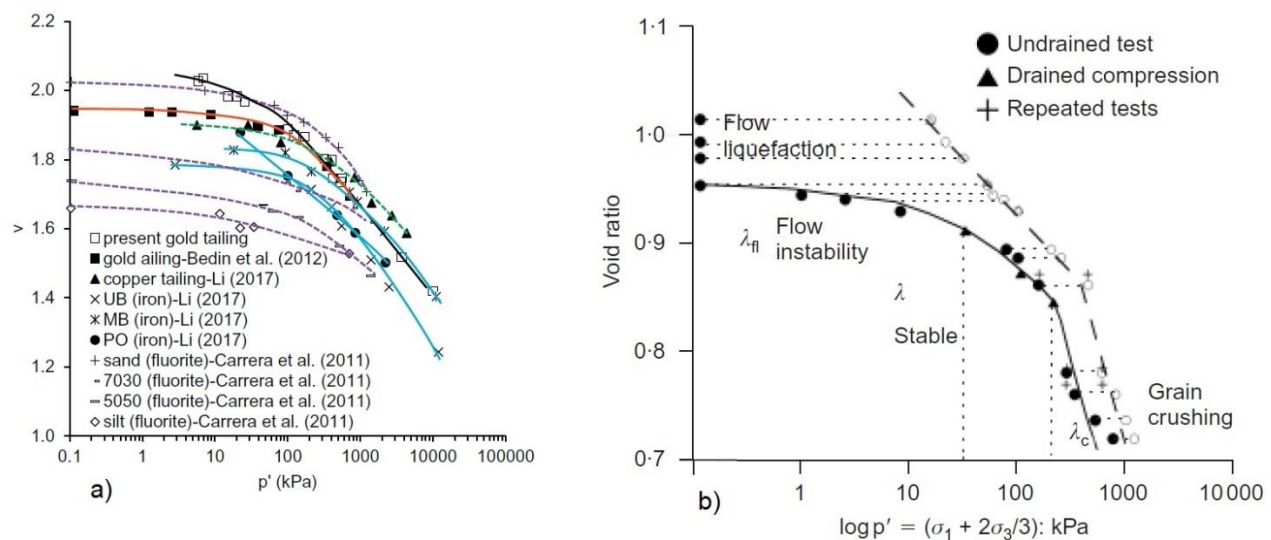


Figure 2-48 CSL in tailings a) Li et al (2018), b) Bedin et al. (2012)

2.5.3 Fines content of tailings in the impoundment

Mine tailings have physical characteristics varying in the field due to the deposition method, mill process, and ore type. They could exist as fine-grained, coarse-grained, or a mix of both. In the same tailings facility, tailings could be coarse-grained near the slurry discharge point to fine-grained in regions distant from the discharge point. This kind of spatial variability in the grading has been reported for different tailings dams (Hue et al. 2017, Wijewickreme et al. 2005), one of them being the Feijao Dam.

For the Feijao dam, the profile obtained through the *CPTu* test results confirmed interlayer coarse and fine materials, as shown in Figure 2-49. Despite this clean profile, it is difficult to recognize a boundary between tailings with different grading size distributions (VALE, 2021). The true profile considers the spatial variability due to the percent of fines. Table 2-6 shows the fines content in Feijao dam at different alignments, i.e. at different vertical profiles at different distances from the dam outer surface.

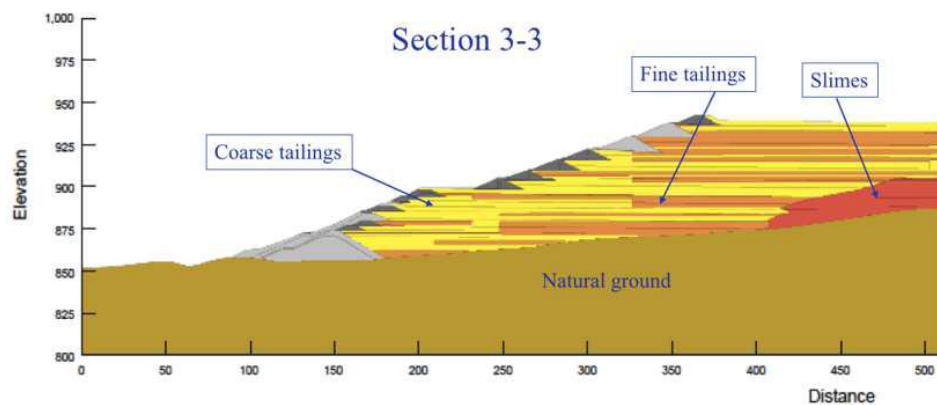


Figure 2-49 Cross-section of Dam B1, showing the locations of slimes, fine tailings, and coarse tailings (Robertson et al. 2019)

Table 2-6 Fines content in different alignments and in different stages (VALE, 2021)

		ALIGNMENT	ALIGNMENT	ALIGNMENT	ALIGNMENT	ALIGNMENT	ALIGNMENT
		1	2	3	4	5	6
STAGE	15					16%	
STAGE	14					20%	
STAGE	13				15%	69%	
STAGE	12				16%	54%	47%
STAGE	11				20%	44%	31%
STAGE	10				24%	38%	45%
STAGE	9	0%	0%	20%	11%	12%	20%
STAGE	8	0%	6%	23%	37%	41%	
STAGE	7	6%	4%	12%	50%	89%	
STAGE	6	2%	11%	36%	51%	72%	
STAGE	5	0%	11%	9%	43%	43%	
STAGE	4	9%	28%	19%	60%		
STAGE	3	51%	55%	23%	99%		
STAGE	2	17%	71%	46%	100%		
STAGE	1	37%	62%	71%	63%		

In practice, tailings with different fines content (FC) are treated as materials with different mechanical properties. However, it is not practical to define properties for each tailing materials with a specific fines content, and it is preferable to define a limited number of tailings materials based on their fines content. Therefore, Robertson et al. (2019) and VALE (2021) selected coarse ($FC < 10\%$), fine ($FC > 50\%$), and average tailings ($10\% < FC < 50\%$) based on the grading distributions in the dam, to perform stress-deformation analyses to investigate the triggering mechanism of flow liquefaction. This classification is arbitrary, and, as it has shown in Section 2.3.3, the mixtures with different fines content can have completely different behavior under undrained loading.

Figure 2-50a, b, and c show the shape of CSL for the coarse, average and fine tailings of Feijao Dam (Robertson et al. 2019), Cadia Valley Operations (CVO, Jefferies et al. 2019), and Merriespruit Harmony Mine (Fourie and Papageorgiou, 2001). From the figure, the CSLs are parallel when the amount of fines changes, meaning that the fines must have similar angularity with coarse grains, and the concept of equivalent void ratio can be used for these tailings.

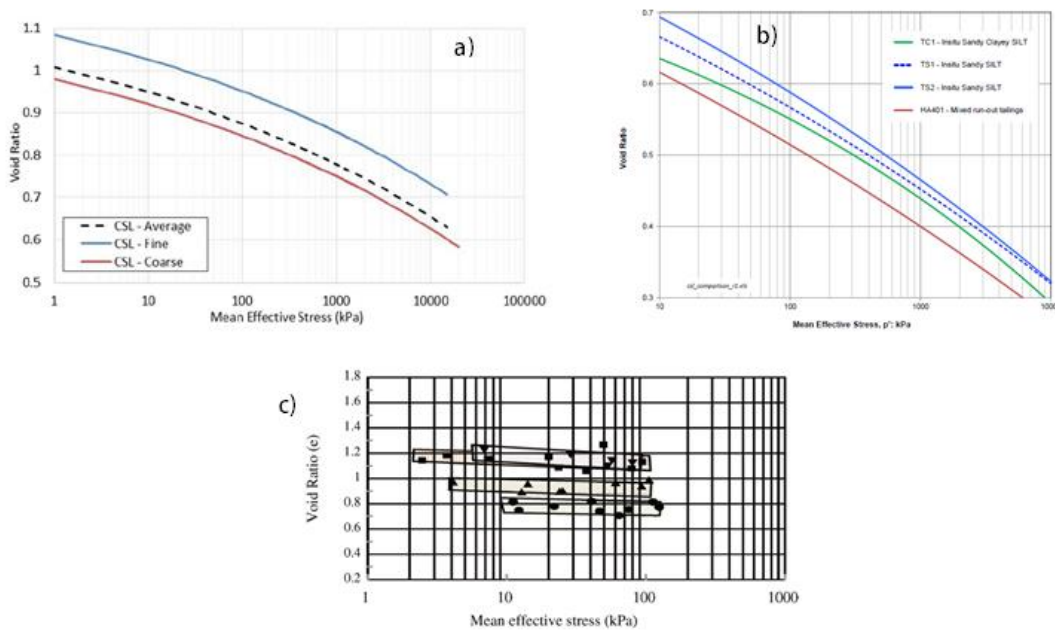


Figure 2-50 *CSL* for tailings with different fines content in; (a) Feijao Dam (Robertson et al. 2019), (b) CVO (Jefferies et al. 2019), and (c) Merriespruit Harmony Mine (Fourie and Papageorgiou, 2001)

2.6 Conclusions

The first part of the literature review (Section 2.2) focused on the behavior of granular soils subjected to drained and undrained loading using laboratory tests. It identified three types of undrained behavior, non-flow, limited flow, and flow, which were affected by several factors, such as the initial void ratio, initial confining pressure, initial fabric, shear mode, and loading direction to the bedding plane. The concept of critical state and state parameter was introduced to assess the looseness or denseness of a sample and to determine its potential behavior under drained and undrained loading. The stress path of granular soils was also analyzed, and the phase transformation and instability lines were discussed. Dilatancy, stress-dilatancy, state-dilatancy, and Nova's rule were explained using data from drained tests. These concepts will be incorporated into soil models to control the mechanical behavior of granular soils.

The second part of literature review (Section 2.3) briefly investigated the effect of non-plastic fines content on the microstructure and undrained mechanical behavior of sand-silt

mixtures. It introduces new definitions for void ratio named as intergranular, interfine, and equivalent which serve as useful tools in analyzing and inferring sand-silt mixtures. The concept of equivalent void ratio is particularly noteworthy, as it enables multiple CSLs associated with different fines content to be transformed into a unique equivalent CSL. These definitions offer useful characteristics that can be used to develop a soil constitutive model for sand-silt mixtures.

The third part (Section 2.4) discussed the constitutive models in the framework of elastoplasticity and the importance of critical state theory to simulate soil behavior. Several models presented by different researchers were evaluated and some approaches to extend those models to simulate the behavior of sand-silt mixtures were discussed.

The final section (Section 2.5) showed that the tailings are mostly loose in the field, susceptible to high risk of flow liquefaction. Also, tailings exhibit high nonlinearity in CSL and a wide range of fines content from 0 to 100. Consequently, the most suitable constitutive model for tailings must possess the capability to simulate static liquefaction with high accuracy, account for the nonlinearity of CSL, and integrate the effect of fine content into its formulation.

CHAPTER 3 ARTICLE 1: ON THE NUMERICAL ISSUES OF THE NORSAND MODEL IN THE SIMULATION OF UNDRAINED BEHAVIOUR OF GRANULAR SOILS

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3.1 Abstract

The NorSand model is one of the most used soil models to simulate the undrained and drained response of granular materials. The model requires the following parameters: two elastic material properties, three to four parameters to define the critical state in $e-p$ and $q-p$ space, two parameters to define the image stress ratio and limiting dilatancy, and two hardening parameters. These few material properties, a straightforward calibration procedure to determine them, and its availability in commercial software such as FLAC and PLAXIS are the main reasons for the considerable growth of the NorSand application in the simulation of the stress-strain behavior of granular materials. However, the material properties are obtained mainly based on drained test results conducted on dense samples. Therefore, the model uses some assumptions and mathematical techniques to modify its response to simulate the undrained behavior of soil states looser than the critical state. This paper explains how the NorSand model employs this mathematical technique and discusses the effectiveness of this approach. Additionally, the paper looks at what is the model's main shortcomings in the simulation of the undrained behaviour of granular materials.

3.2 Introduction

It is essential to study in depth the performance of a constitutive soil model in order to understand its principles, underlying concepts, and limitations, and be able to develop novel ideas to improve said model. Jefferies (1993), for instance, conducted a thorough analysis of traditional

Cambridge-type models (Roscoe et al. 1963) to reveal why they fail to simulate accurately the behavior of granular soils. His research showed that the attachment of the yield surface to the critical state was the root cause of the issue, resulting in a single normally consolidated line. This, however, is inconsistent with the behavior of granular soils, which exhibit infinite normally consolidated lines. To address this problem, Jefferies (1993) proposed that the yield surface should be separated from the critical state line and only reach it at large strains. This necessitated that the location and size of the yield surface be controlled by the state parameter (Been and Jefferies, 1985). This added feature enhanced the capability of Cambridge-type models to simulate the behavior of granular soils. The influence of the state parameter on the size and location of the yield surface and the use of an associated flow rule implies that the dilatancy is also a state parameter-dependent quantity. The concept of state-dependent dilatancy later has been recognized widely and is used in models with non-associated flow rules (e.g. Li et al. 1999, Dafalias and Manzari, 2004, Lu and Huang, 2014), resulting in the development of models well capable of simulating sand's behavior.

Likewise, understanding the performance of the NorSand model in simulating the drained and undrained behavior of granular soils is necessary to identify its limitations. The principal drawback of the original NorSand model (Jefferies, 1993) is its inability to accurately simulate the undrained behavior of loose states, such as flow liquefaction (Chang and Jefferies, 2020, Jefferies and Been, 2015). To overcome this limitation, Jefferies and Been (2015) proposed modifying the hardening rule by adding a new term known as the cap-softening term, where the user is only able to turn this feature on and off. This paper tries to explain the reasons behind the need for this modification and evaluates the effectiveness of the proposed approach.

3.3 NorSand Yield Surface

The formulation of the NorSand model presented by Jefferies and Been (2015) is considered here to analyse its general formulation. NorSand uses an associated flow rule to define its yield surface, by integrating the stress-dilatancy rule (Equation (3-1)). Depending on this rule, the NorSand yield surface (Equation (3-2)) takes a bullet-type shape, similar in shape to the yield surface of the original Cam-Clay model (Figure 3-1).

$$D = M_i - \eta \quad (3-1)$$

$$\eta = M_i \left(1 - \ln \left(\frac{p}{p_i} \right) \right) \quad (3-2)$$

The point on the yield surface that exhibits zero dilatancy is referred to as the image state (Figure 3-1), which is characterized by three parameters: image stress ratio (M_i), image confining pressure (p_i), and image state parameter (ψ_i), which is the state parameter corresponding to p_i . In order to determine the image stress ratio, the NorSand model combines various concepts and assumptions, including the State-Dilatancy law, Nova's rule, and the Stress-Dilatancy rule. The State-Dilatancy law describes the relationship between limiting dilatancy (D_L) and its corresponding state parameter (ψ_L).

$$D_L = f(\psi_L) \quad (3-3)$$

During a drained triaxial compression (DTC) test, there is a specific point on the stress-strain path where the soil experiences minimal dilatancy (D_{tc-min}) and maximum stress ratio (η_{tc-max}) (For example Figure 2-26). At this point, the state parameter can be defined as ψ_{tc-min} . NorSand considers D_{tc-min} and ψ_{tc-min} as limiting dilatancy (D_L) and its corresponding state parameter (ψ_L), respectively. Moreover, the stress ratio at the limiting dilatancy (η_L) is equal to η_{tc-max} . In most cases, the following linear equation fits well with experimental data:

$$D_{tc-min} = \chi_{tc} \cdot \psi_{tc-min} \quad (3-4)$$

where, χ_{tc} is a material constant called the state-dilatancy parameter. For a general drained path conducted on dense samples, state-dilatancy law has been modified as follows:

$$D_L = \frac{M}{M_{tc}} \chi_{tc} \psi_L \quad (3-5)$$

where M_{tc} is the critical stress ratio for triaxial compression test, and M is the critical stress ratio obtained from the following equation:

$$M = M_{tc} - \frac{M_{tc}^2}{3+M_{tc}} \cos\left(\frac{3}{2}\theta + \frac{\pi}{4}\right) \quad (3-6)$$

Here θ is the Lode Angle used to consider the shear mode effect (see Section 2.2.1 and Table 3-1 to find its formulation). D_{min} is used to control the NorSand yield surface expansion. It must be noted that for the loose states under drained condition, there is no measurable quantity as minimum dilatancy. Therefore, NorSand assumes that Equation (3-5) can be extended in its linear format for states looser than critical.

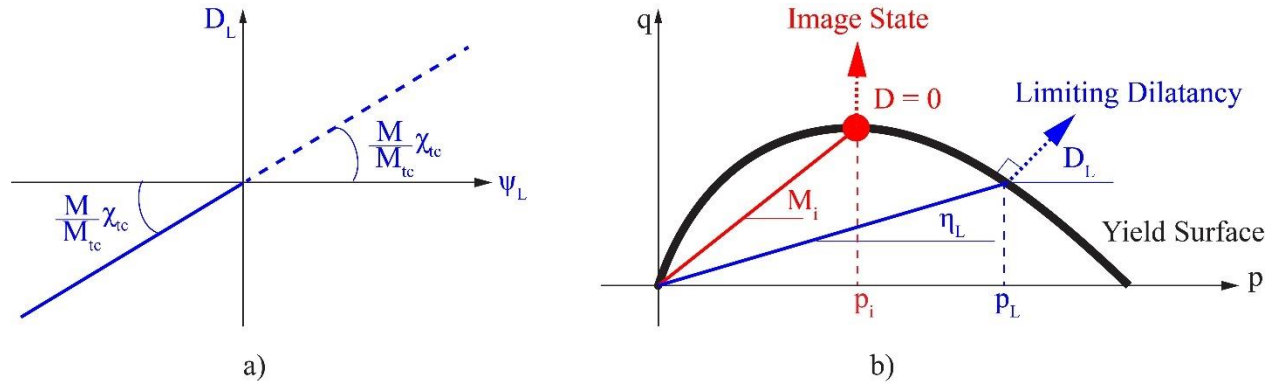


Figure 3-1 a) Relationship between limiting Dilatancy (D_L) and its corresponding state parameter, b) the concept of limiting Dilatancy on the NorSand yield surface.

According to Equation (3-5), ψ_L is required to calculate D_L . Therefore, NorSand presents the following relationship to calculate ψ_L from the image state parameter ψ_i .

$$D_L = \frac{M}{M_{tc}} \chi_{tc} \psi_L = \frac{M}{M_{tc}} \chi_i \psi_i \quad (3-7)$$

where, χ_i is called the NorSand state-dilatancy parameter and is a positive value. This expression forces D_L to have the same sign as ψ_i , and take a zero value if $\psi_i = 0$. Using the critical state line (CSL) equation and stress dilatancy rule, a unique relationship can be found between ψ_i and ψ_L , as follows:

$$\psi_L = g(\psi_i) = \psi_i + \lambda_l \frac{D_L}{M_i} \quad (3-8)$$

By combining Equations (3-8), and (3-7), χ_i , and, as a result, D_L , can be calculated. If the CSL is defined with Equation (3-9), then χ_i would be computed with Equation (3-10):

$$e_c = \Gamma - \lambda_l \ln(p) \quad (3-9)$$

$$\chi_i = \frac{\chi_{tc}}{1 - \lambda_l \frac{\chi_{tc}}{M_{i,tc}}} \quad (3-10)$$

here, Γ is the critical void ratio at $p=1$ kPa, and λ_l is the slope of *CSL* in $e-\ln p$ space. In Equation (3-10), $M_{i,tc}$ is the image stress ratio under the triaxial compression loading.

As mentioned above, for a drained triaxial compression (DTC) test conducted on dense states, there is a point where the soil experiences a minimum dilatancy (D_{tc-min}) with maximum stress ratio (η_{max}). Nova's rule relates D_{tc-min} to η_{tc-max} through the following formulation:

$$D_{tc-min} = \frac{M_{tc} - \eta_{tc-max}}{1 - N} \quad (3-11)$$

where, N is a material constant called shear-volume coupling factor. The original NorSand formulation assumes that it can be applied for the whole stress path as follows:

$$D_L = \frac{M - \eta_L}{1 - N} \quad (3-12)$$

By combining Equations (3-12), (3-7) and (3-1), the image stress ratio can be obtained:

$$M_i = M \left(1 + N \frac{\chi_i \psi_i}{M_{tc}} \right) \quad \text{for } \psi_i < 0 \quad (3-13)$$

For loose states (ψ_i), however, Equation (3-13) is not valid, because in that case, M_i becomes greater than M . The formulation of the classic NorSand model assumes that a symmetric version of Equation (3-13) can be used for loose states.

$$M_i = M \left(1 - N \frac{\chi_i \psi_i}{M_{tc}} \right) \quad \text{for } \psi_i > 0 \quad (3-14)$$

Subsequently, the image stress ratio can be written in its general format as follows:

$$M_i = M \left(1 - N \frac{\chi_i |\psi_i|}{M_{tc}} \right) \quad (3-15)$$

And $M_{i,tc}$ can be obtained:

$$M_{i,tc} = M_{tc} \left(1 - N \frac{\chi_i |\psi_i|}{M_{tc}} \right) \quad (3-16)$$

For a non-linear CSL defined with Equation (3-17), Equation (3-10) is no longer valid.

$$e_c = \Gamma - \lambda \left(\frac{p}{p_{ref}} \right)^\xi \quad (3-17)$$

In Equation (3-17), Γ is the critical void ratio at $p=1$ kPa, and λ and ξ are material constants used to define the nonlinear CSL. p_{ref} is the reference pressure equal to 1 MPa. For nonlinear CSL, it can be proved that in Equation (3-10), λ_l should be calculated from the following equation to use a non-linear CSL in the NorSand formulation.

$$\lambda_l = \lambda \xi \left(\frac{p}{p_{ref}} \right)^\xi \quad (3-18)$$

The formulation of the NorSand model considering the CSL, stress-dilatancy rule, yield surface, and image stress ratio is summarized in Table 3-1.

Table 3-1 – General formulation used to obtain the yield surface of NorSand model.

CSL	$e_c = \Gamma - \lambda_l \ln(p)$
Stress – dilatancy rule	$D = M_i - \eta$
Yield surface	$\eta = M_i \left(1 - \ln \left(\frac{p}{p_i} \right) \right)$
Image Stress ratio	$M_i = M \left(1 - \frac{\chi_i N \psi_i }{M_{tc}} \right)$ where $\chi_i = \frac{\chi_{tc}}{1 - \lambda_l \frac{\chi_{tc}}{M_{i,tc}}}$
Nova's rule	$D_L = \frac{M - \eta_L}{1 - N}$
State-Dilatancy law	$D_L = \frac{M}{M_{tc}} \chi_{tc} \psi_L$

3.4 NorSand Hardening Rule in the Framework of Critical State Theory (CST)

The classic CST framework imagines that critical state can be achieved at large strains only when two conditions are met: (i) volumetric strain rate ($d\varepsilon_v$) or dilatancy (D) should be zero, and (ii) there should be continuous zero dilatancy under continuous shearing at large strains ($d(d\varepsilon_v) = 0$). In the case of the NorSand model with an associated flow rule, zero dilatancy occurs when the current confining pressure (p) reaches the image confining pressure (p_i). Therefore, to be on the critical state at large strains, both p and p_i should reach p_c , the confining pressure at the critical state. Additionally, q should also reach q_c , the deviatoric shear stress at critical state.

$$\varepsilon_q^p \rightarrow \infty \Rightarrow \begin{cases} p \rightarrow p_i \rightarrow p_c & \text{or } \psi \rightarrow \psi_i \rightarrow 0 \\ q \rightarrow q_c & \text{or } \eta \rightarrow M \end{cases} \quad (3-19)$$

The second requirement of CST forces that at large strain $d\psi_i = 0$. This means that at large strains the state parameter after reaching critical state should not change.

$$\psi_i = e - (\Gamma - \lambda_l \ln(p_i)) \quad (3-20)$$

$$\frac{d\psi_i}{d\varepsilon_q^p} = \frac{de}{d\varepsilon_q^p} + \lambda_l \frac{\frac{dp_i}{d\varepsilon_q^p}}{\frac{p_i}{d\varepsilon_q^p}} = 0 \quad (3-21)$$

de at critical state is zero, therefore the second term in the above equation must reach zero at critical state. The second term can be used as a hardening rule to control the size of yield surface as follows:

$$h = \frac{dp_i}{d\varepsilon_q^p} = H(p_{i,max} - p_i); \quad H = (H_0 - H_y \psi) \frac{p}{p_i} \quad (3-22)$$

where H_0 and H_y are the hardening parameters. $p_{i,max}$, obtained from the following equation, is controlled by limiting dilatancy (D_L) to limit the expansion of yield surface.

$$p_{i,max} = p \cdot \exp\left(\frac{D_L}{M_i}\right) = p \cdot \exp\left(\frac{\frac{M}{M_{tc}} \chi_i \psi_i}{M_i}\right) \quad (3-23)$$

At critical state, $\psi_i \rightarrow 0$, $D_L \rightarrow 0$, and as a result $p_{i,max} \rightarrow p \rightarrow p_i$. Therefore, $dp_i = 0$ and the second rule of CST is satisfied.

By combining Equation (3-1) and Equation (3-2), it can be easily proved that the following expression is valid:

$$\exp\left(\frac{D_L}{M_i}\right) = \frac{p_i}{p_L} \quad (3-24)$$

Where p_L is the confining pressure at $D = D_L$. By replacing the above term in Equation (3-23), the hardening rule can be rewritten as follows:

$$h = \frac{dp_i}{d\varepsilon_q^p} = H \cdot p \cdot \left(\frac{p_i}{p_L} - \frac{p_i}{p}\right) \quad (3-25)$$

Therefore, whenever $p_L < p$, the size of the yield surface will increase, otherwise, the yield surface will shrink. For $\psi_i \geq 0$, D_L is positive and $p_L \geq p_i$, and for $\psi_i \leq 0$ the opposite condition will be applied. These conditions, along with the NorSand hardening concept, are shown schematically in Figure 3-2, for ($\psi_i < 0$), $\eta_L = \eta_{max}$ (maximum stress ratio that can be developed) and $D_L = D_{min}$ (minimum dilatancy) observed under drained tests.

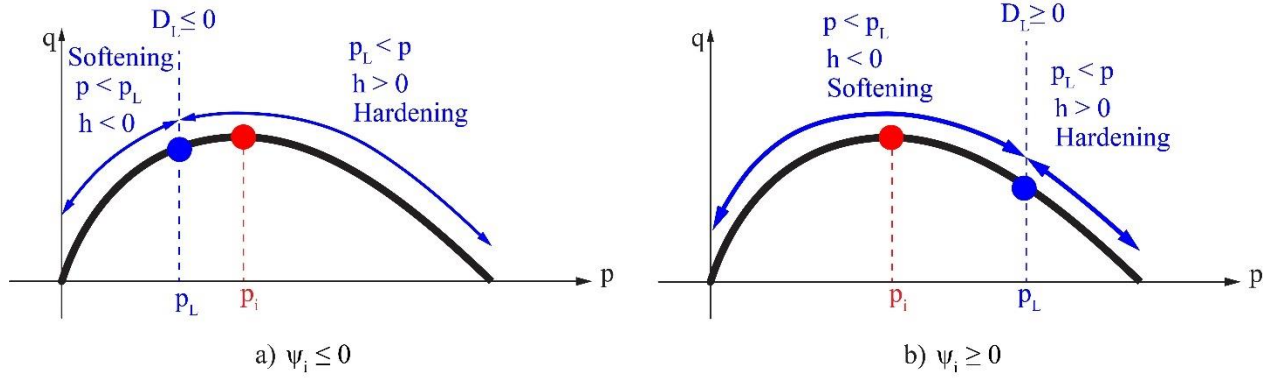


Figure 3-2 the concept of hardening rule used in the NorSand model

3.5 Range of Applicability of the NorSand Model

From Equation (3-16), the image stress ratio is a linear function of the image state parameter. It is obvious that for $|\psi_i| \geq \frac{M}{N\chi_i}$, $M_i \leq 0$, and the model loses its applicability. It must be noted that near to zero image stress ratio also affects the volumetric response of the NorSand model, and it is recommended that M_i should not be lower than a certain value (Jefferies and Been, 2015):

$$\frac{M}{1+N} \leq M_i \quad (3-26)$$

By using this condition in Equation (3-16), the range of state parameter where the NorSand model is applicable can be obtained:

$$|\psi_i| \leq \frac{M}{(1+N)\chi_i} \quad (3-27)$$

Beyond this range, the NorSand model reaches critical state at higher mean confining pressure because of lower image stress ratio.

3.6 Classic NorSand Model and Undrained Loading

Here, the formulation of the NorSand model presented in the previous sections is referred to as the classic or original NorSand model. This version has two main limitations in the simulation of granular soils under undrained loading:

- Inability of simulating flow liquefaction properly (In order to enhance the simulation results, adjustments were required either in the hardening rule or the bulk modulus, as noted by Jefferies and Been (2015) and Ng et al. (2023))
- Inability of simulating limited flow feature (Jefferies, 1993)

3.6.1 Classic NorSand Model and Flow Liquefaction

Flow liquefaction is a type of cohesionless soil behavior that occurs at states initially looser than critical state ($\psi_i > 0$). According to the explanations presented in Section 3.4, and visually depicted in Figure 3-2b, a positive image state parameter yields $p_i < p_L$ and $M_i < \eta_L$. At the test's inception, $p_L < p$, hence $h > 0$, and the yield surface expands. Given that $p_i < p$, normality rules mandate a reduction in mean confining pressure during the yield surface expansion, this indicates that p and p_i approach each other (with p_i increasing due to hardening and p declining due to normality). Zone 1 in Figure 3-3 shows this part of the stress path. Once $p_L = p$, the yield surface expansion ceases. Subsequently, when $p < p_L$, normality, and hardening rules work in tandem, leading to a reduction in p and p_i towards the critical state. This section of the stress path is represented by zone 2 in Figure 3-3. Flow liquefaction emerges at a particular stress ratio (η_{FL}) in zone 2, ranging from η_L to M_i .

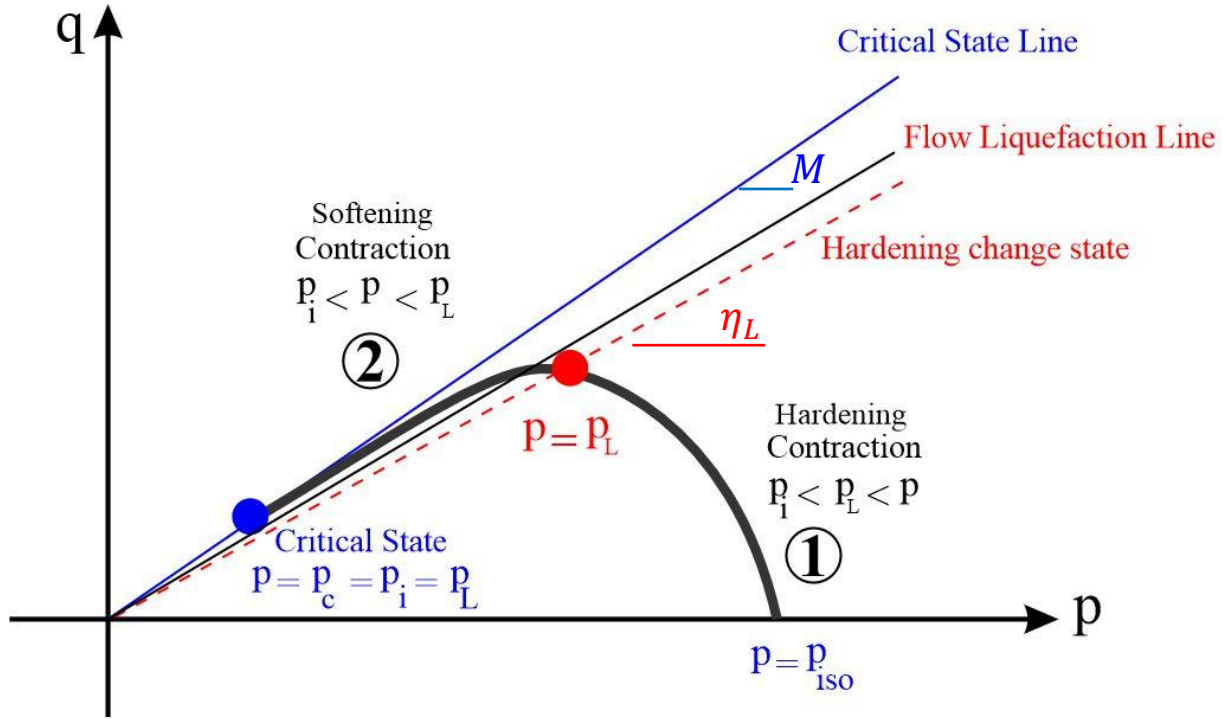


Figure 3-3 schematic view of the location where static liquefaction has been initiated with the NorSand model

Based on the above discussion, the classic NorSand model can simulate flow liquefaction. However, the major limitation is the accuracy of the simulation results in comparison with measured data. By using the stress dilatancy rule and applying the limiting dilatancy, the following expression can be derived:

$$\eta_L = M_i + D_L = M_i + \frac{M}{M_{tc}} \chi_i \psi_i \quad (3-28)$$

χ_i is the material property that is obtained from tests conducted on dense states ($\psi_i < 0$); And then it is used to simulate the mechanical behavior of loose states. This assumption produces η_L that is slightly lower than $M_i \cong M$ for loose states. As explained earlier, flow liquefaction happens somewhere between η_L and M (see Figure 3-3.). meaning that η_{FL} takes place at high stress ratio close to M . This is not in agreement with the observed loose soil behavior, where measured η_{FL} is much lower (Jefferies and Been, 2015). researchers such as Shuttle and Jefferies (2016), Reid and Smith (2021), and Ng et al. (2023) have attempted to manipulate the bulk modulus to enable NorSand to achieve a better η_{FL} that aligns with experimental data.

3.6.2 Classic NorSand Model and Limited Flow Behavior

Granular soils can show another type of mechanical behaviour under undrained loading called limited flow behaviour. During limited flow behavior, the soil shows a loss of strength similar to flow liquefaction, but at large strains the soil can start gaining strength and exhibit strain hardening behaviour. The classic NorSand model cannot simulate the limited flow behavior due to its hardening rule. Equation (3-25) is only capable to simulate flow liquefaction shown in Figure 3-3. To simulate the limited flow behavior, the hardening rule should be modified in such a way that produces the desired stress-path drawn schematically in Figure 3-4. As it can be seen the path contains three zones. Zone 1, increase in q by decreasing in p , Zone 2, decreases in q and p , and Zone 3, increases in q and p . The new modified hardening rule must be able to simulate strain hardening (zone 1), strain softening (zone 2), and strain hardening (zone 3). However, the hardening rule in the classic NorSand model only allows for the simulation of zone 1 and zone 2 (strain hardening followed by strain softening before reaching critical state) for loose states.

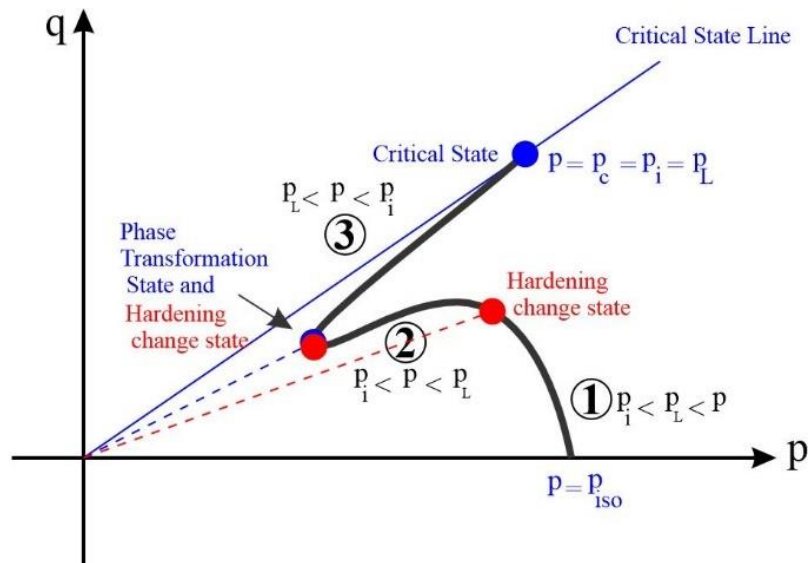


Figure 3-4 Ideal simulation of limited flow feature through the concept of hardening rule and normality rule in Cambridge type models

3.7 NorSand Model with Cap-Softening Feature (Jefferies and Been, 2015)

To solve the classic NorSand limitations under undrained conditions, developers of the NorSand model tried to modify the hardening rule by adding a new term called the cap-softening term. This additional term is only applied for undrained response and is not considered for the drained response. Although, this kind of modification improves the model's response, it implies that the model is unable to reach a unique formulation to use under drained and undrained condition. The modified hardening rule for undrained condition is written as follows:

$$h_{undrained} = \frac{dp_i}{d\varepsilon_q^p} = H \left(p \frac{p_i}{p_L} - p_i \right) + S \frac{\eta}{\eta_L} \frac{\partial \left(p \frac{p_i}{p_L} \right)}{\partial \varepsilon_q^p} \quad (3-29)$$

where S is a switch, 0 for drained loading and 1 for undrained loading. By expanding the cap-softening term in Equation (3-29) and considering derivations, the undrained hardening rule is turned into Equation (3-30):

$$h_{undrained} = \frac{dp_i}{d\varepsilon_q^p} = \frac{H \left(p \frac{p_i}{p_L} - p_i \right) - S \frac{\eta}{\eta_L} \left(\frac{p_i}{p_L} \right) KD}{1 + p \frac{\eta}{\eta_L} \frac{d \left(\frac{p_i}{p_L} \right)}{d\psi_i}} \quad (3-30)$$

where K is the elastic bulk modulus. In the new hardening rule, p_L and everything associated with it, including D_L and η_L , lose their meaning as limiting quantities. This fact can be proven by Equation (3-30), where at $p = p_L$, it is not necessary for $h_{undrained}$ to be zero. In other words, the cap-softening term shifts the point where the hardening regime reverses the expansion of the yield surface to shrinkage (Figure 3-5). The new point ($p = p_{L-cap}$) can be obtained by finding the root of Equation (3-30).

$$p_{L-cap} = p_L + S \frac{\eta}{\eta_L} \frac{K}{H} D \quad (3-31)$$

Therefore, in the new hardening rule, the location of p in comparison with p_{L-cap} , controls the hardening-softening behavior of the NorSand yield surface. Figure 3-5 shows schematically the effect of cap-softening term on the location of the limiting hardening quantity. It must be mentioned that, at the beginning of any simulation with initial isotropic condition, $p_{L-cap} = p_L$ due to the term $\frac{\eta}{\eta_L} = 0$. At critical state, $p_{L-cap} \rightarrow p_L$, since $D \rightarrow 0$.

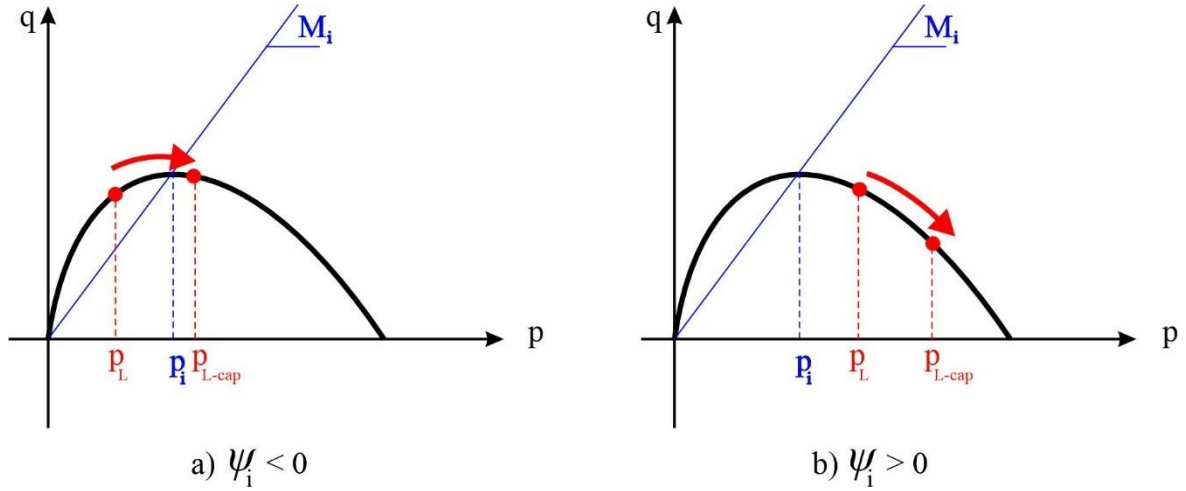


Figure 3-5 Effect of cap-softening term on the location of limiting dilatancy by shifting it to a new location

3.7.1 NorSand Model with Cap-Softening and Flow Liquefaction (Jefferies and Been, 2015)

As shown schematically in Figure 3-5b, for loose states, the presence of cap-softening results in shifting of limiting hardness quantity from p_L to p_{L-cap} . Due to the shape of the yield surface, $\eta_{L-cap} < \eta_L$. Based on the new hardening rule, flow liquefaction (η_{FL}) must occur between η_{L-cap} and M instead of η_L and M . As a result, the new hardening rule reduces the simulated η_{FL} , and improved the agreement of the simulations with real data. Figure 3-6, shows

examples of simulations using the NorSand model, with and without the effect of cap-softening term.

3.7.2 NorSand Model with Cap-Softening and Limited Flow Feature

As explained in Section 3.6.2, the limited flow behavior requires a strain hardening-softening-hardening feature. This fact means that NorSand should experience at least two changes of hardening regime before reaching critical state. The undrained hardening regime change takes place when $p = p_{L-cap}$ and as a result $dp_i = 0$. The presence of cap softening term causes that the undrained hardening regime change ($dp_i = 0$) appears several times at $p = p_i = p_{L-cap} \neq p_c$ or at $\psi = \psi_i = \psi_{L-cap} \neq 0$. However, at large strains, the NorSand reaches the critical state when dp_i and ψ_i both reach zero.

The developers of the NorSand model restricted the value of S in Equation (3-30) to 0 or 1, which limits the number of times where undrained hardening regime change would occur. Figure 3-6, presents examples of simulations using the NorSand model, where the value of S varies. The material properties and initial state condition (p_0, e_0) are shown in Table 3-2. $S = 0$ corresponds to the classic NorSand model where the cap-softening has not been applied, $S = 1$ is with the cap-softening term suggested by the developers of the NorSand model. An example of simulation with $S = 1.5$ is presented to illustrate the rigidity of the formulation. As it can be seen, by increasing S outside of the recommended value, the model starts bouncing around the critical state, gets instable and unable to simulate the soil behavior properly, not giving the necessary flexibility to adjust the behavior of the model.

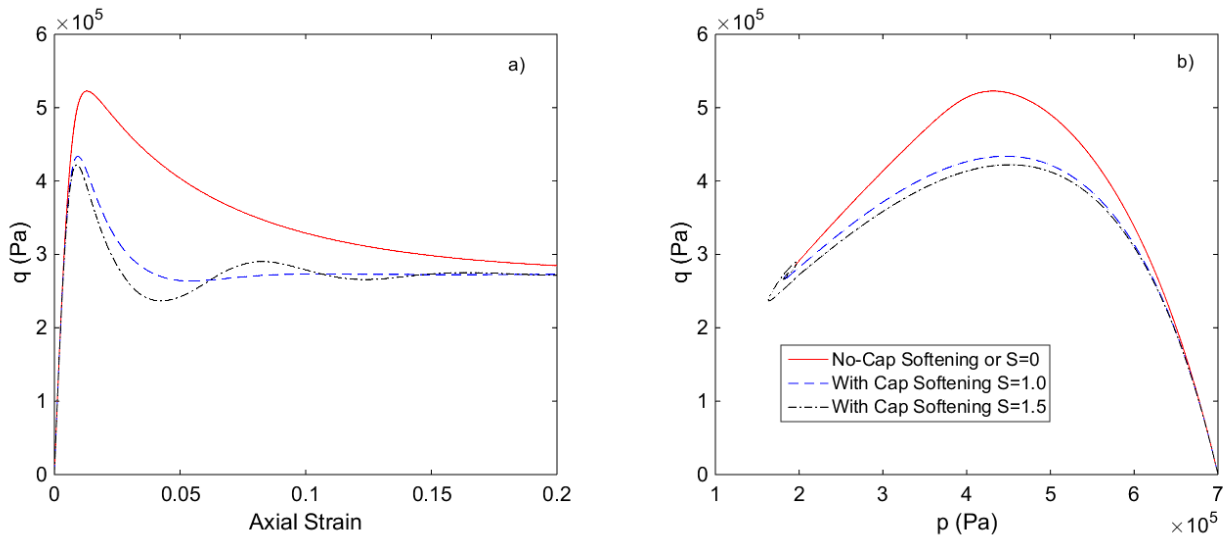


Figure 3-6 Effect of the cap-softening term on the undrained behavior of loose states a) deviatoric stress-axial strain, b) deviatoric stress-confining pressure

Table 3-2 Material properties of Fraser River Sand used to simulate the undrained behavior of a loose sample with the NorSand model.

M	Γ	λ	ξ	χ_{tc}	N	H_0	H_y	p_0	e_0
1.47	1.01	0.087	0.38	5	0.4	120	400	700e3	0.9

3.7.3 Outcomes of the Cap-Softening Feature

It appears that the NorSand model, in its current formulation may lack the flexibility to effectively reproduce a wide range of undrained response in loose states, instead, it replicates an undrained behavior pattern. To study the capability of their model to match the behavior of different soils, the developers of the NorSand model have compared its response to soil data using a wide range of material properties (Jefferies and Been, 2015). For instance, in Figure 3-7, the normalized undrained peak shear strength of loose samples ($\frac{s_u}{p_0}$) is illustrated alongside their initial state parameters (ψ_0) for various soils. The results obtained from the NorSand model for different values

of $(\frac{H}{I_r})$ are also displayed in the figure. Here, I_r denotes the shear rigidity and can be derived using the following equation:

$$I_r = \frac{G}{p} \quad (3-32)$$

Where G is the shear modulus of the soil. The graph reveals that for initially loose samples having high initial state parameters, ψ_0 , the model's efficacy in accurately predicting undrained peak responses weakens. Additionally, the model may require different $(\frac{H}{I_r})$ values to match the behavior of a specific soil under different loading conditions and initial state parameter. The H value selected must be applicable not only for the undrained condition but also for the drained response.

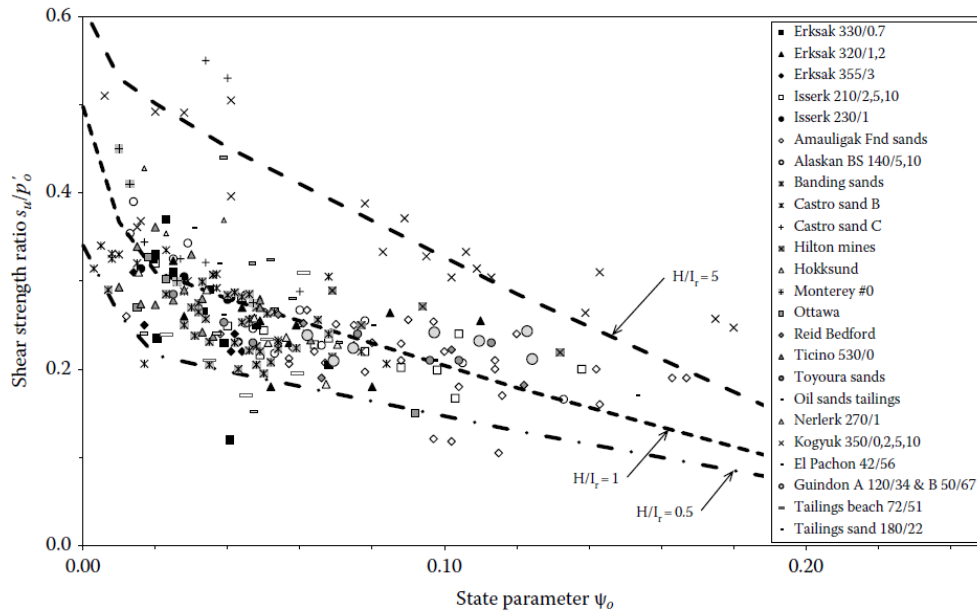


Figure 3-7 Effect of $\frac{H}{I_r}$ on the normalized undrained peak shear strength simulated through the NorSand model in comparison with measured data (Jefferies and Been, 2015)

3.8 Conclusion

According to the above discussion, the NorSand model, while widely used for simulating the drained and undrained behavior of granular materials, has several limitations to capture the behavior of granular materials. Firstly, it can only simulate soil response within a particular range of initial state parameters. Secondly, in order to capture the flow behavior of loose material, the model employs a cap-softening term to modify its undrained hardening rule that does not provide flexibility in its response. This term is chosen to satisfy the NorSand approach to the critical state and avoid adding additional parameters to the model. By altering the limiting hardening point, the cap-softening term enhances NorSand's ability to simulate flow liquefaction and limited flow behavior in the undrained response. However, the value of the cap-softening term is either 1 or 0, which limits the capacity of a user to calibrate the model to match a target behavior.

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CHAPTER 4 ARTICLE 2: IMPLEMENTATION OF A NONLINEAR STATE-DILATANCY LAW IN THE NORSAND MODEL

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4.1 Abstract

The original NorSand (ONS) state-dilatancy law (ONS-SDL) presents challenges when trying to make changes in the formulation of the critical state line or stress-dilatancy rule and the associated yield surface. Furthermore, utilizing this law restricts the applicability of the model within a certain range of state parameter. Because of its limitations, using a higher state-dilatancy parameter for loose samples to improve the undrained response is often not possible. This paper provides a review of the ONS-SDL, and the requirements of a state-dilatancy law in Cambridge-type models in a more explicit way. A comprehensive analysis is then conducted to examine the effects of the chosen law on the model's formulation, its capability to simulate drained responses of both loose and dense samples, as well as its effectiveness in simulating undrained responses of loose samples. To address the aforementioned limitations, a novel nonlinear state-dilatancy law is introduced, offering improved responses and enhanced flexibility. This not only improves the model's performance but also allows for the utilization of different stress-dilatancy rules in future models, providing greater flexibility and adaptability. The model with new law has been verified using drained and undrained triaxial data existing in the literature.

4.2 Introduction

The introduction of the original NorSand (ONS) model by Jefferies (1993) marked a significant advancement in enhancing traditional Cambridge-type models (Roscoe et al. 1963) to simulate the behavior of granular materials. This progress was achieved through two key features:

(i) decoupling the yield surface from the critical state line using the concept of state parameter (ψ), defined by Been and Jefferies (1985) as the difference between the current void ratio and the critical void ratio at the same mean effective mean stress, and (ii) introducing a new law known as the state-dilatancy law. In the state-dilatancy law, state refers to the state parameter (ψ) and dilatancy (D) represents the ratio of the incremental plastic volumetric strain versus the incremental plastic deviatoric strain. The state-dilatancy law establishes a relationship between the minimum observed dilatancy ($D = D_{min} = D_L$) during drained shear loading of a dense sample and the corresponding state parameter associated with this minimum dilatancy ($\psi = \psi_{min} = \psi_L$). Figure 4-1 shows the typical mechanical response of a drained sample undergoing drained triaxial compression loading. The ONS specific state-dilatancy law employs a linear correlation between two variables (D_L and ψ_L) passing through the origin. This law is used in the hardening rule to control changes in the model's hardening regime, from expansion of the yield surface to its shrinkage.

Since the model's initial release, multiple versions of the model have been developed to enhance its accuracy in simulating the drained and undrained mechanical behavior of sand (Jefferies and Been, 2015; Jefferies et al. 2015). However, all subsequent versions have primarily focused on modifying the hardening rule by adding new terms such as a Cap-Softening term or making its image stress ratio (M_i) state-dependent (the image stress ratio representing the stress ratio corresponding to the stress point on the yield surface with zero dilatancy), and the state-dilatancy law of the model has received less attention.

This paper first defines the concept of state-dilatancy law, its application, and the requirements for following critical state theory before deriving its mathematical function. It then discusses the linear state-dilatancy law chosen by Jefferies (1993) for the ONS formulation, exploring challenges in altering the critical state line (CSL) or the yield surface shape in the ONS model. The paper examines the drained and undrained responses of the ONS model through a few theoretical examples to study the limitations of the state-dilatancy law. Subsequently, a new approach to formulating alternative state-dilatancy laws is presented. This approach empowers users to select their preferred CSL equation and yield surface shape. By employing this proposed approach, a new nonlinear state-dilatancy law is suggested to better model the behavior of granular materials. The modified version is referred to as the Modified NorSand (MNS) model throughout the paper, and its performance in the simulation of drained and undrained triaxial response is

discussed. A Matlab code, available upon request, has been developed to produce the drained and undrained responses of both ONS and MNS for triaxial compression tests.

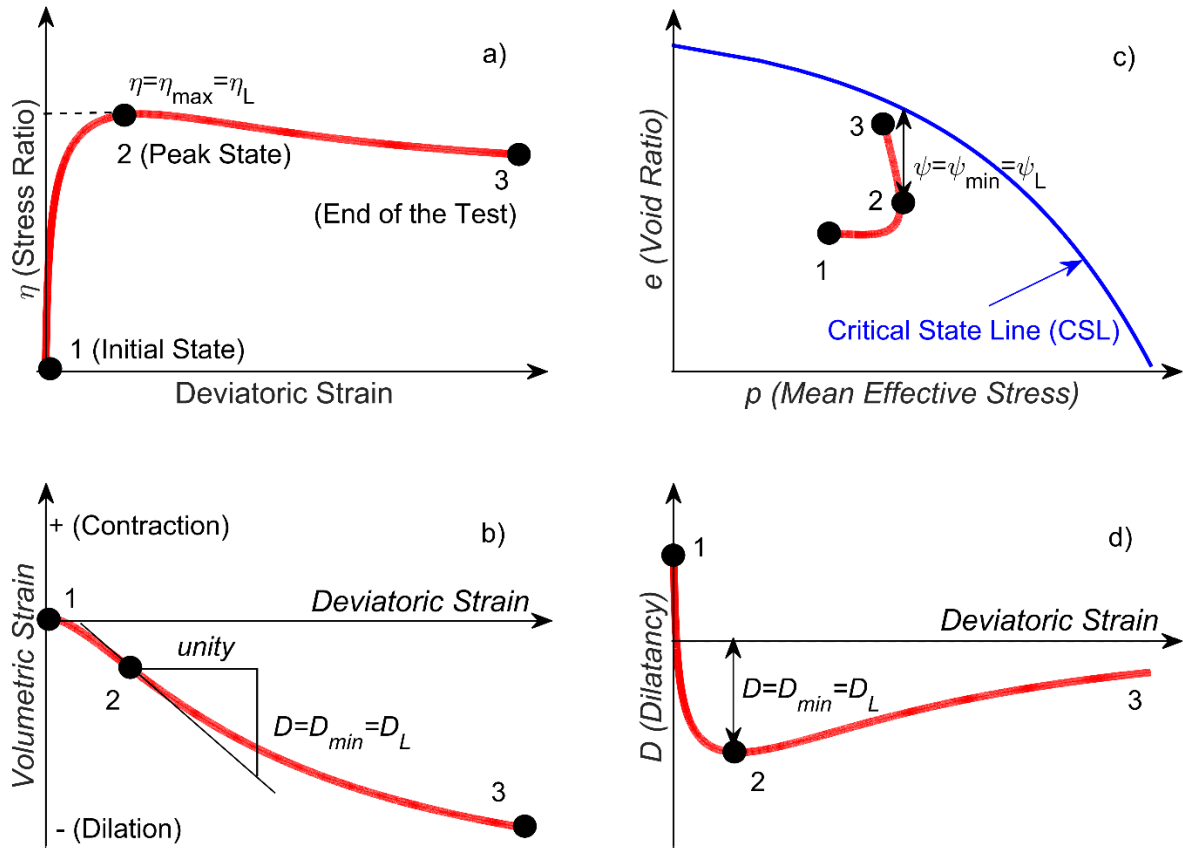


Figure 4-1 Typical response of a dense sand under drained triaxial compression test in different spaces

4.3 Original NorSand Model

4.3.1 Distinct Features of ONS Compared to Traditional Cambridge Type Models

If all plastic work is assumed to dissipate entirely in friction (Thurairajah, 1962), the following stress-dilatancy rule (Rowe, 1962; Collins and Muhunthan, 2003) can be obtained by

considering the plastic work per unit volume of a triaxial sample subjected to externally applied mean stress ($p = (\sigma_1 + 2\sigma_3)/3$) and deviatoric shear stress ($q = \sigma_1 - \sigma_3$).

$$D = M_i - \eta \quad (4-1)$$

Here, σ_1 and σ_3 are major and minor principal stresses, respectively. D represents dilatancy, which is the ratio of incremental plastic volumetric strain to incremental plastic deviatoric strain (See Figure 4-1b, to understand the definition visually). In this study, compressive stress is defined as positive, while contraction (reduction in volume) is a negative change in volume. M is the ratio of (q/p) at critical state, and η is the current ratio of (q/p) . Models of the Cambridge type employ an associated flow rule and integrate the stress-dilatancy rule to obtain their yield surface. The resulting yield surface from Equation (4-1) takes the form of a bullet shape, as illustrated in Figure 4-2. The point corresponding to zero dilatancy is referred to as the image state, as named by Jefferies (1993). The mean stress associated with the image state is denoted as p_i . The slope of the line connecting the origin to the image state is $M_i = M$. The maximum mean stress on the yield surface corresponds to a zero-stress ratio and is denoted as p_{iso} , and called isotropic state. The ratio between p_{iso} and p_i for the obtained yield surface remains constant, and equal to $r = \exp(1)$. Additionally, there are two other points of interest on the yield surface: one represents the current state, indicating the current stress condition, and the other represents the limiting state, which will be discussed later.

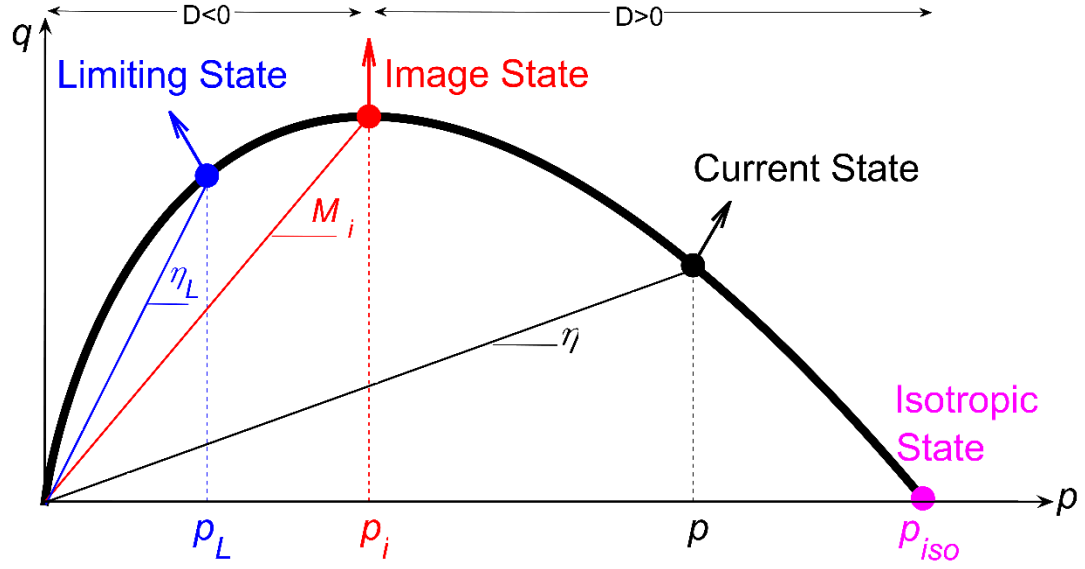


Figure 4-2 ONS and OCC yield surface and defined states on the yield surface

The constants in the integrated form of the stress-dilatancy equation can be determined by using the conditions on the isotropic state. The obtained original Cam-Clay (OCC) yield surface can be described using the following expression (Roscoe et al. 1963):

$$\eta = -M \cdot \ln\left(\frac{p}{p_{iso}}\right) \quad (4-2)$$

To obtain the ONS yield surface as proposed by Jefferies (1993), Equation (4-2) can be reformulated by substituting (p/p_{iso}) with (p/p_i) . (p_i/p_{iso}) with (p_i/p_{iso}) being equal to $1/\exp(1)$.

$$\eta = M \left(1 - \ln\left(\frac{p}{p_i}\right)\right) \quad (4-3)$$

The OCC considers the evolution of its yield surface using p_{iso} , whereas ONS considers p_i to track this evolution. Therefore, the stress-strain relationship for these models would be as follows:

$$d\varepsilon_q^e = \frac{dq}{3G}; \quad d\varepsilon_v^e = \frac{dp}{K}; \quad d\varepsilon_v^p = D \cdot d\varepsilon_q^p \quad (4-4)$$

$$\begin{cases} dp_{iso} = Hd\varepsilon_v^p; & \text{For OCC} \\ dp_i = Hd\varepsilon_q^p; & \text{For ONS} \end{cases} \quad (4-5)$$

Where, G , K , and H are elastic shear, elastic bulk, and plastic modulus, respectively. ε denotes strain, with subscripts q and v indicating the deviatoric and volumetric strain components. The superscripts e and p are used to distinguish between the elastic and plastic parts of the strain, respectively.

The hardening rule must be defined to ensure that the model adheres to the critical state theory. In the case of the OCC, adherence to the critical state theory is achieved by connecting the yield surface, to the critical state line and allowing it to evolve along this line, as shown schematically in Figure 4-3. As a result, p_{iso} consistently follows a trajectory in the $e - p$ space (e is void ratio) known as the isotropic normally consolidated line, which is parallel to the mapping of the critical state line in the $e - p$ space. The spacing ratio between these two parallel lines remains constant and is equal to $r = \exp(1)$. These distinctive features of the OCC model are visually depicted in Figure 4-3.

On the other hand, ONS does not enforce the yield surface to attach to the critical state line and allows it to move and evolve more freely. The location of the ONS yield surface in the $e - q - p$ space is determined by the concept of state parameter - ψ (Been and Jefferies, 1985), which is the difference between the current void ratio and the critical void ratio at the same mean stress. To track the location of the ONS yield surface, only one mean stress point on the yield surface is needed to calculate the state parameter. The selected point in ONS is the image state, along with its corresponding image confining pressure (p_i) and image state parameter (ψ_i). It is important to note that for ONS to align with the framework of critical state theory, the yield surface should come into contact with the critical state line through the image state at large strains. In other words, p_i should approach the critical mean stress (p_c), resulting in ψ_i approaching zero at large strains. This flexibility in the movement and evolution of the yield surface distinguishes ONS from the OCC, and the OCC yield surfaces depicted in Figure 4-3 can be regarded as the ultimate yield surfaces for the ONS under large strains.

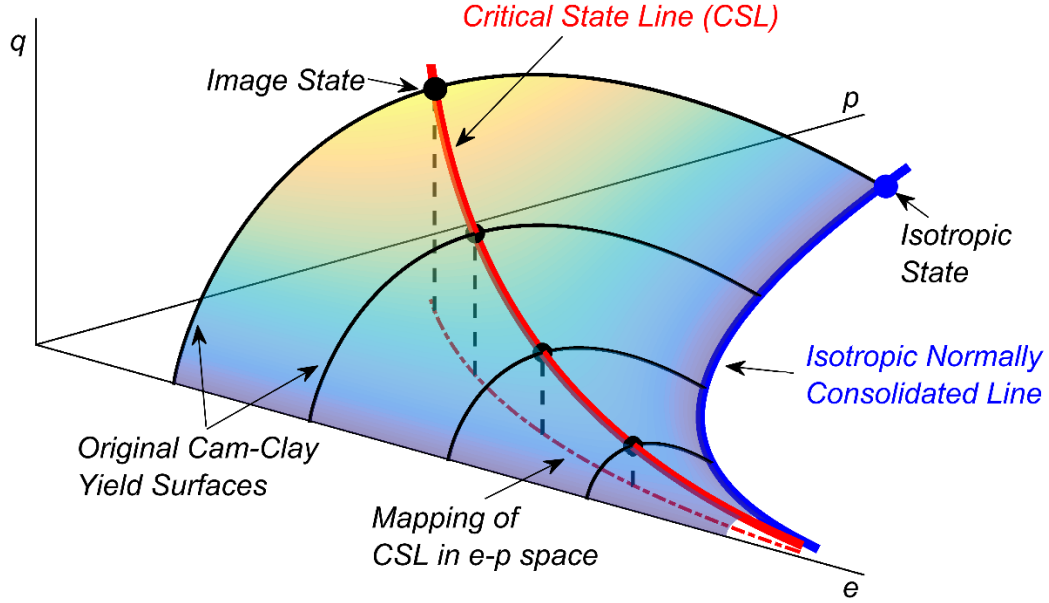


Figure 4-3 schematic view of the attachment of the traditional Cambridge type models to the CSL through the image state

4.3.2 Concept and Application of Limiting Dilatancy (D_L)

The term "limiting maximum hardness" used by Jefferies (1993) is similar to the concept of "limiting state", although different names are used for some of the parameters in Jefferies (1993). These specific parameters are limiting confining pressure or limiting mean stress (p_L), limiting dilatancy (D_L) (minimum dilatancy in Jefferies' paper, D_{min}), limiting stress ratio (η_L) (maximum stress ratio in Jefferies' paper, η_{max}), and limiting state parameter (ψ_L) (state parameter corresponding with D_{min} in Jefferies' paper, ψ_{min}). However, for the sake of simplicity, we will label the limiting state quantities with a subscript L in the present publication.

To discuss the concept of limiting state, the ONS approach for simulating the drained behavior of a dense sample subjected to shear loading at a constant mean confining pressure is presented. The isotropic pressure at the initial state (p_0) is regarded as the isotropic state on the

ONS yield surface (p_{iso}) and a dense sample has an initial negative state parameter ($\psi_0 < 0$). Consequently, the initial conditions of the sample and initial yield surface of the ONS model would be similar to phase 0 as depicted in Figure 4-4.

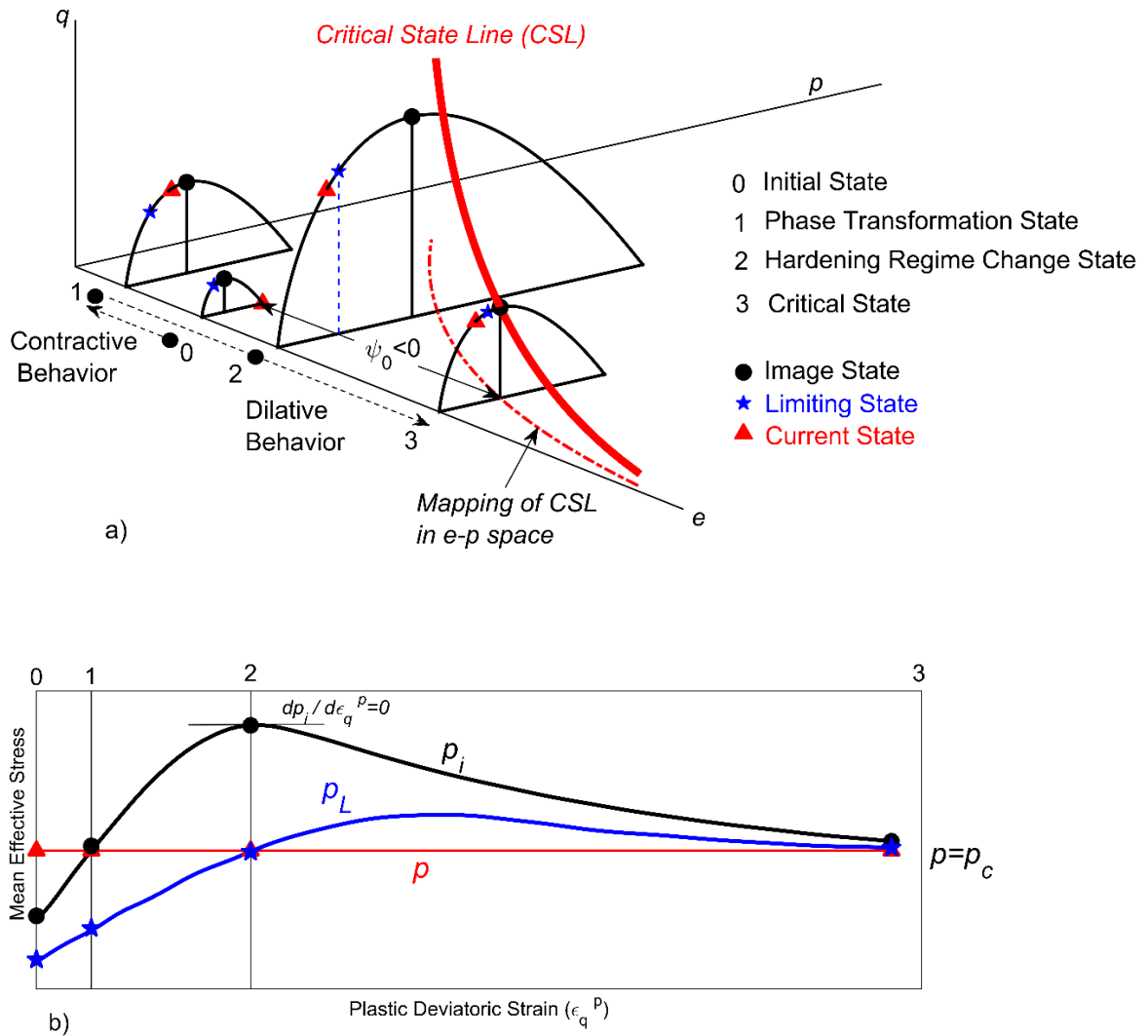


Figure 4-4 Schematic view of the stress path of an initially dense sample under shearing and at a constant mean stress with the ONS approach

Since the initial stress condition lies on the contractive side of the yield surface, normality requires a volume reduction, resulting in a decrease in void ratio. Thus, the stress path progresses from phase 0 to phase 1 in Figure 4-4. The ONS yield surface must always expand unless a change

of hardening regime is detected by the model. As the yield surface expands and reaches phase 1, the current stress condition passes through the image state. Consequently, the current stress moves to the dilative side of the ONS yield surface. Due to normality, the model undergoes a change in its path and begins to exhibit dilation, as shown in Figure 4-4, as a phase transformation state.

However, the expansion of the ONS yield surface must continue until reaching phase 2. At phase 2, as the current state passes through the limiting state, the ONS hardening rule forces a change in the hardening regime of its yield surface. Therefore, the limiting state serves as a trigger where the model detects when to change from expansion of its yield surface to its shrinkage. While the current state remains on the dilative side of the yield surface, it continues to exhibit a dilative behavior. Simultaneously, the yield surface starts to contract, approaching phase 3, which represents the critical state. At critical state, the current state tends to converge towards the image state.

The hardening rule of ONS can be expressed by incorporating the parameters related to the limiting state, as follows so that the simulation process of a dense sample is consistent with the above discussion:

$$dp_i = Hd\varepsilon_q^p; \quad H = (H_0 - H_y\psi)h\left(\frac{p}{p_i}\right)^2\left(\frac{p_i}{p_L} - \frac{p_i}{p}\right)p_i \quad (4-6)$$

Where H_0 and H_y are plastic hardening constants. Note that in the ONS formulation (Jefferies and Shuttle, 2005), a term denoted as $(p_{i,max}/p)$ is used, which is equal to (p_i/p_L) . Based on this equation, whenever p is greater than p_L , the yield surface will expand, and conversely, when p is less than p_L , the yield surface will contract. By utilizing Equations (4-2) and (4-1), a relationship between the limiting dilatancy (D_L) and the ratio (p_L/p_i) used in Equation (4-6) can be derived as follows:

$$\frac{p_L}{p_i} = \exp\left(\frac{D_L}{M}\right) \quad (4-7)$$

The limiting dilatancy (D_L) plays a central role in determining the behavior of ONS, and it should be defined such that it enables the accurate simulation of the soil's behavior.

4.3.3 Requirements of Limiting Dilatancy (D_L)

According to critical state theory, one of the requirements is that critical state should be reached on the yield surface at the image state with zero dilatancy, and the yield surface should have a constant size even at large strains. For this to occur, the change in image confining pressure (dp_i) must be zero as shear strains increase. Therefore, based on the ONS hardening rule (Equation (4-6)), (p/p_i) should be equal to (p_L/p_i) . Both p and p_i tend to approach the critical mean confining pressure (p_c), resulting in (p/p_i) approaching 1 at the critical state. As a result, (p_L/p_i) must also be 1 at large strain (See Figure 4-4b). Applying this value in Equation (4-7), it can be determined that D_L (limiting dilatancy) must be zero at large strains. However, prior to reaching large strains, dense samples should have a negative value for D_L , so that the limiting state be positioned on the dilative side of the yield surface. These conditions ensure that the corresponding soil behavior, illustrated in Figure 4-4, can be accurately simulated. Therefore, D_L is a state dependent parameter, which must be negative for dense samples, reaching a zero value at large strains, when state parameter is zero. Therefore, from a mathematical perspective, the limiting dilatancy should be positive when dealing with loose samples exhibiting positive state parameters. This assumption proves to be highly effective in simulating the mechanical behavior of loose samples (Jefferies and Been, 2015; Cheng and Jefferies, 2020; Razavi and Yniesta, 2023).

4.3.4 ONS State-Dilatancy Law

Given that the location of the yield surface in the ONS model varies with the image state parameter (ψ_i), it is better to establish the state dependency of D_L based on ψ_i . A simple relationship to fulfill its requirements (see Section 4.3.3) is a linear approach used by Jefferies (1993) as demonstrated below:

$$D_L = \chi_i \psi_i \quad (4-8)$$

Here Equation (4-8) will be referred to as the ONS image state-dilatancy law. In this context, χ_i is referred to as the ONS state dilatancy parameter. Nevertheless, a significant challenge arises when it comes to calibrating the χ_i parameter through the approach employed by the model. When conducting drained tests on dense samples, we can only observe D_L (limiting dilatancy) and its corresponding state parameter (ψ_L), which can be utilized to derive an equation that aligns with the data. The relationship between limiting dilatancy (D_L) and the limiting state parameter (ψ_L) is known as the state-dilatancy law. Jefferies (1993) showed that utilizing similar expressions as shown in Equation (4-8) provides a suitable formulation for the state-dilatancy law. Consequently, the following ONS-SDL was introduced:

$$D_L = \chi_{tc} \psi_L \quad (4-9)$$

χ_{tc} is called state dilatancy parameter and can be easily calibrated using laboratory data from tests conducted on dense samples. This calibration requires at least two drained tests on dense samples under varying initial confining pressures. These samples are expected to exhibit a peak in the deviatoric stress-deviatoric strain space (See Figure 4-1). The values of dilatancy and state parameter evaluated at the peak are plotted. Equation (4-9) is fitted to the data by changing the χ_{tc} parameter. An example of the calibration is presented in Figure 4-5 for Fraser River Sand (FRS) using four dense samples (Jefferies, and Been, 2015), and Toyoura Sand (Verdugo and Ishihara, 1996) using two dense samples.

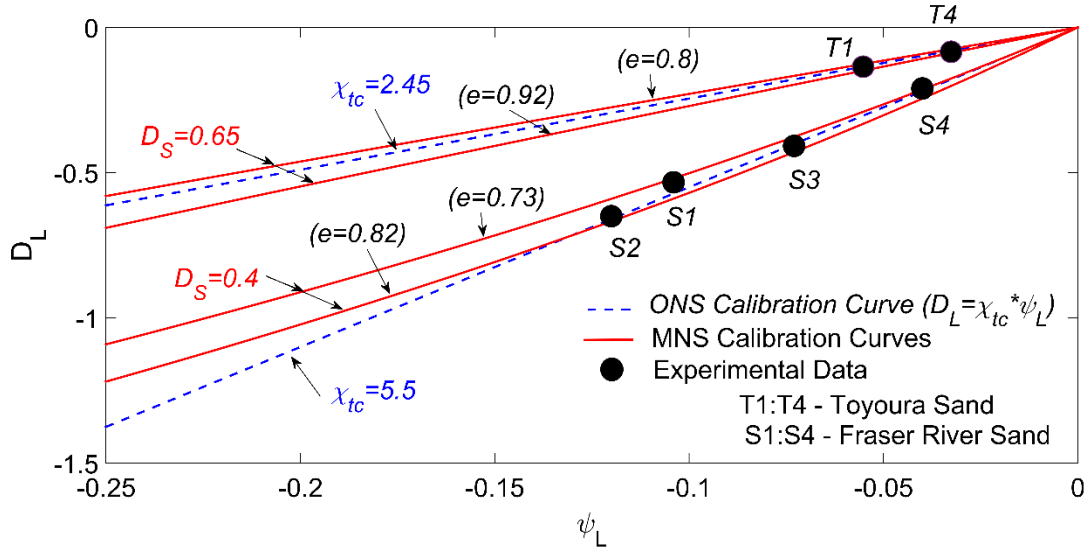


Figure 4-5 Calibration process for the dilatancy state parameter in the ONS (χ_{tc}) and MNS (D_s) model

To determine the ONS state dilatancy parameter (χ_i), the ONS model requires equating Equation (4-8) with Equation (4-9).

$$D_L = \chi_i \psi_i = \chi_{tc} \psi_L \quad (4-10)$$

Solving this equation involves incorporating two additional equations, namely the stress-dilatancy rule and the critical state line. Consequently, χ_i is expected to vary if there are any changes to these equations. Thus far, the conventional approach for the ONS model has been to utilize Equation (4-1) as the stress-dilatancy rule and the subsequent linear equation as its critical state line.

$$e_c = \Gamma - \lambda_l \ln(p) \quad (4-11)$$

In this equation, Γ represents the critical void ratio at a pressure of 1 kPa, while λ_l is the slope of the CSL in the $e - \ln(p)$ space. Using Equation (4-9) and the definition of the state parameter (Been and Jefferies, 1985), the relationship between (ψ_i) , (ψ_L) , and their corresponding mean confining pressures on the yield surface (p_i) and (p_L) can be expressed as follow:

$$\psi_i = \psi_L + \lambda_l \ln\left(\frac{p_i}{p_L}\right) \quad (4-12)$$

By replacing (p_L/p_i) ratio from Equation (4-7) in Equation (4-12), Equation (4-13) can be derived:

$$\psi_i = \psi_L - \lambda_l \frac{D_L}{M} = \psi_L - \lambda_l \frac{\chi_{tc} \psi_L}{M} \quad (4-13)$$

Substituting of Equation (4-13) in Equation (4-10) will yield in the following expression:

$$\chi_i = \frac{\chi_{tc}}{1 - \lambda_l \frac{\chi_{tc}}{M}} \quad (4-14)$$

4.3.5 Rigidity of the ONS Model

When the critical state line is replaced with the subsequent non-linear critical state line, Equation (4-15) (Li and Wang, 1998), it becomes challenging to establish a straightforward method for determining χ_i using the Equation (4-10).

$$e_c = a - b \left(\frac{p}{p_{ref}} \right)^c \quad (4-15)$$

Where Γ , λ , p_{ref} and ξ are material constants. To solve this problem and integrate the nonlinear critical state line, a solution is to ensure that the differential of e_c with respect to p is equalized in both Equations (4-15) and (4-11)(Jefferies and Been, 2015). This will result in the following equation:

$$\lambda = b.c. \left(\frac{p}{p_{ref}} \right)^c \quad (4-16)$$

Therefore, to incorporate a nonlinear critical state line, λ_l in Equation (4-14) needs to be substituted by Equation (4-16), as suggested by Jefferies and Been (2015).

Additionally, modifying the stress-dilatancy rule to accommodate different shapes of the yield surface poses a significant challenge in the ONS model. This is because employing alternative stress-dilatancy rules may not yield a direct term for χ_i as presented in Equation (4-10).

4.3.6 Applicability of the ONS Model for a Limited Range of Initial State Parameters for Loose Samples

By examining the linear image state dilatancy law in the ONS model (Equation (4-8)), we can conclude that when ψ_i is negative, p_L should lie on the left side of p_i on the yield surface. Similarly, when ψ_i is positive, p_L should be positioned on the right side of p_i on the yield surface. These concepts are shown schematically on Figure 4-6.

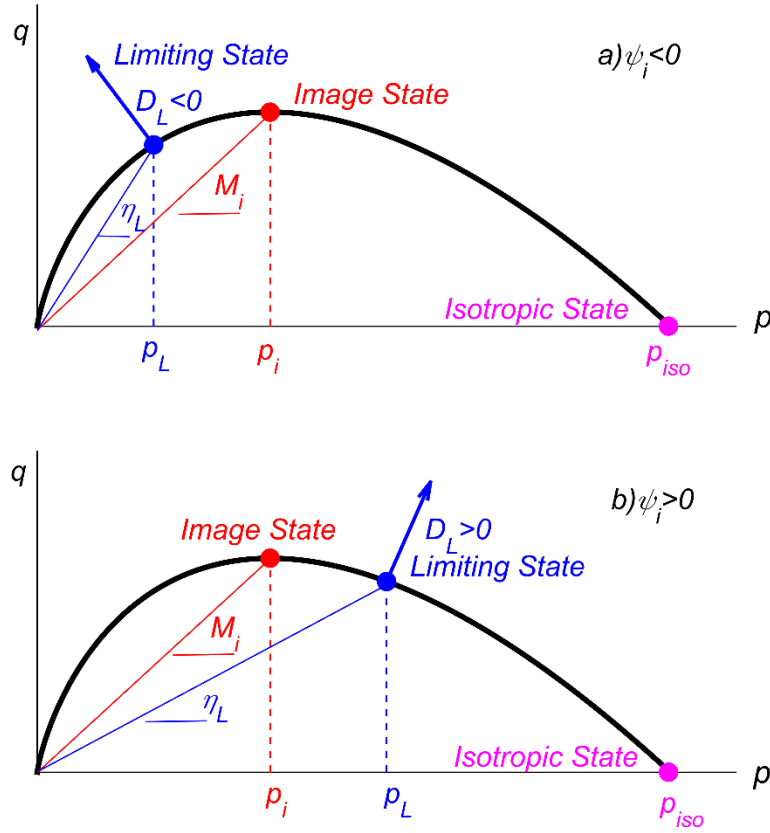


Figure 4-6 Concept of limiting dilatancy used for a) dense samples, and b) loose samples

As a result, in the case of dense samples, p_L should vary between 0 to p_i , while for loose samples, p_L should vary between p_i to p_{iso} . However, the ONS model does not adhere to these restrictions, and the linear state-dilatancy law may cause p_L to fall outside of these ranges. It's crucial to highlight that the limiting state cannot exceed the yield surface boundaries. In such situations, the model fails to provide stress-strain simulations. Hence, the model remains valid as long as its limiting dilatancy $D_L = \chi_i \psi_i$ doesn't surpass the dilatancy value at $p = p_{iso}$. At p_{iso} , dilatancy has its maximum positive value M , thereby making the model applicable only when $(D_L < M)$. Stated differently, the model is applicable solely when ψ_i is lower than the subsequent value (Jefferies and Been, 2015):

$$\psi_{i,max} \leq \frac{M}{\chi_i} \quad (4-17)$$

To illustrate this limitation, an example involving Fraser River Sand (Jefferies and Been, 2015) is used, with the following parameters: CSL parameters: $a = 1.01$, $b = 0.087$, $c = 0.38$ and $M = 1.45$, state dilatancy parameter: $\chi_{tc} = 5.5$ (see Figure 4-5), and hardening parameters: $H_0 = 60$, $H_y = 280$. The hardening parameters have been selected to provide the best match between the ONS simulation results and the triaxial compression test results. The validation of these chosen parameters for FRS, as per the work of Jefferies et al. (2015), is presented in the upcoming sections. The ONS model uses the following expressions to calculate the elastic moduli for granular soils:

$$G = \frac{G_0 \cdot p_{ref}}{e - e_{min}} \cdot \left(\frac{p}{p_{ref}} \right)^m, \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} \quad (4-18)$$

Here, G_0 , m , and e_{min} are material constants, and p_{ref} is the reference pressure equal to atmospheric pressure 101.325 kPa . e and ν are void ratio and Poisson's ratio. For the FRS, these values have been selected as $G_0 = 375$, $e_{min} = 0.344$, $m = 0.466$, and $\nu = 0.1$ based on the work of Jefferies et al. (2015).

Based on these properties, taking $p_0 = 0.1 \text{ MPa}$ as an example, χ_i has been determined as 8.21 using Equation (4-14) and Equation (4-16). Consequently, $\psi_{i,max}$ would amount to 0.176. For $p_0 = 0.1 \text{ MPa}$, the initial image confining pressure is calculated by dividing p_0 by $\exp(1)$, resulting in $p_{i,0} = p_0 / \exp(1) = 0.0367 \text{ MPa}$. Applying the definition of the curved CSL in Equation (4-15), the critical void ratio at the image state is determined to be 0.95. By adding $\psi_{i,max}$ to this value, the maximum void ratio that ONS can simulate at $p_0 = 0.1 \text{ MPa}$ would be 1.126. Figure 4-7a and Figure 4-7b illustrate the triaxial drained behavior of loose samples, with an initial mean stress of 0.1 MPa and initial void ratios of 1 and 1.12.

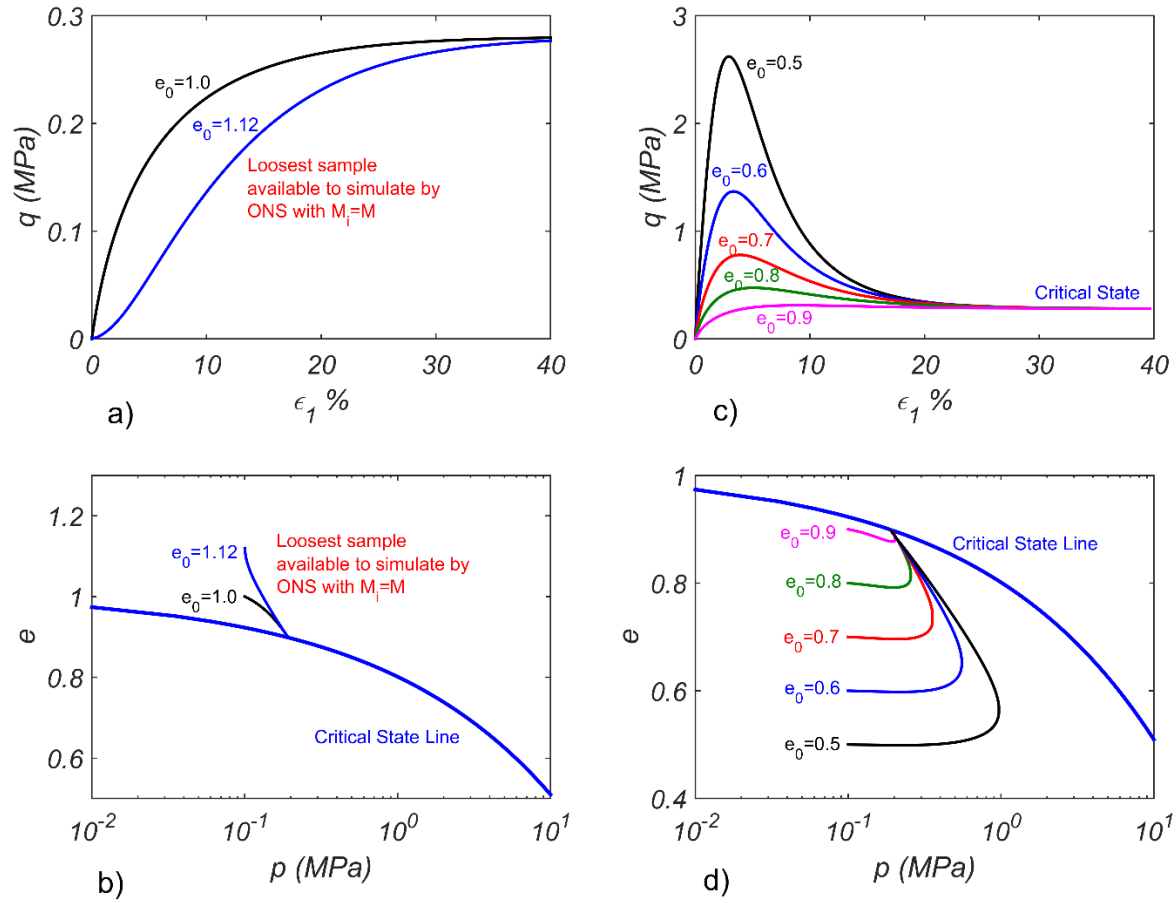


Figure 4-7 Typical simulations of the ONS model with $M_i = M$ for the (a,b) loose, and (c,d) dense samples with the same initial confining pressure (100 kPa) and different initial void ratios

4.3.7 Concerns of the ONS model on the simulation of drained behavior of dense samples

According to Equation (4-8), in the case of dense samples, a higher negative initial image state parameter will result in a more negative limiting dilatancy with higher limiting stress ratio. As a result, the yield surface will keep expanding until it reaches its limit state. This expansion will cause the stress-strain curve to exhibit a high peak shear strength. Figure 4-7c and Figure 4-7d present typical triaxial drained simulations of the ONS model for dense samples with the material

properties for the FRS. The simulations were conducted with an initial mean stress of 0.1 MPa and initial void ratios of 0.9, 0.8, 0.7, 0.6 and 0.5. Notably, the sample with $e_0=0.5$ exhibits a higher negative state parameter, leading to an unusually high peak shear strength.

To control the peak shear strength of dense samples under drained condition, it is suggested that M_i should not be a constant value equal to M , instead it should be state-dependent. To this end, the image stress ratio is modified by applying Nova's rule and introducing a new material property, which is applicable for dense samples (Jefferies and Been, 2015). Nova's rule (Nova and Wood, 1982) states that there is a linear relationship between the limiting dilatancy (D_L) and the limiting stress ratio (η_L) or peak strength ratio (η_{max}) on the limiting state, as expressed in the following formulation (See Figure 4-1 to see the variables).

$$D_L = \frac{M - \eta_L}{1 - N} \quad (4-19)$$

In this equation, N is the shear-volume coupling factor, which will be used to control the peak drained stress ratio under drained loading. To calibrate N , Equation (4-19) will be fitted to the experimental data extracted from drained triaxial compression tests conducted on dense samples. These experimental data include dilatancy (D_L) and stress ratio (η_L) values for different samples at the peak point (See Figure 4-1). For example, for Toyoura Sand and FRS the experimental data and the corresponding N value are shown in Figure 4-8.

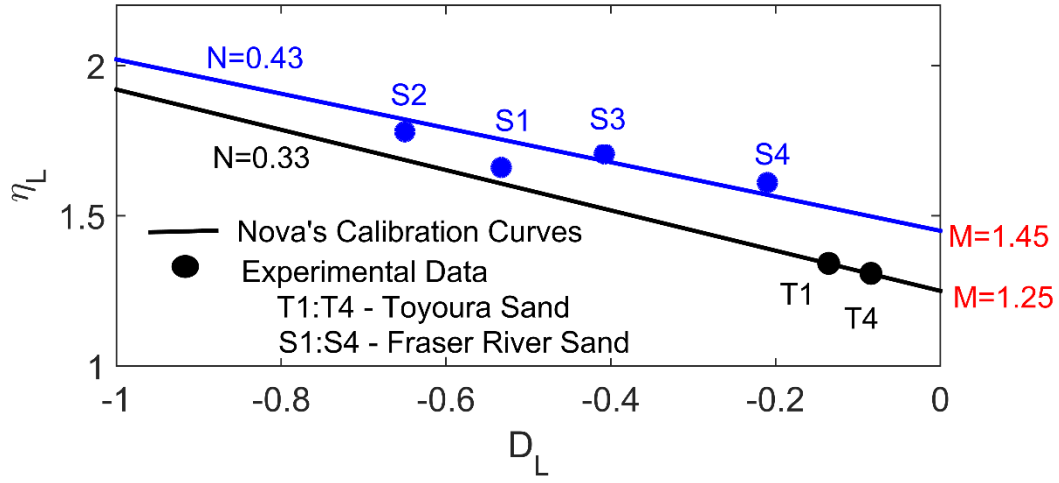


Figure 4-8 Calibration process for the shear volume coupling factor using Nova's rule

By substituting η_L with $(M_i - D_L)$, see the stress-dilatancy rule, Equation (4-1), and subsequently replacing D_L with Equation (4-7), Equation (4-26) can be rearranged to obtain the image stress ratio as follows:

$$M_i = \frac{M}{1 + N \cdot \ln\left(\frac{p_L}{p_i}\right)} \quad (4-20)$$

Equation (4-27) is a general equation, and, for the ONS model, Equation (4-7) can be replaced into Equation (4-27) to obtain the ONS image stress ratio as follows.

$$M_i = M \left(1 + N \frac{\chi_i \psi_i}{M} \right) \quad (4-21)$$

Nova's rule and as a result Equation (4-21) are only valid for dens samples with $\psi_i < 0$. For loose samples, it is assumed that the symmetric version of Equation (4-21) yields satisfactory results for the ONS model (Jefferies and Been, 2015).

$$M_i = M \left(1 - N \frac{\chi_i |\psi_i|}{M} \right) \quad (4-22)$$

The state-dependent image stress ratio and other constitutive equations of the ONS is presented in Table 4-1. It is important to note that M_i can be defined as a constant by setting $N = 0$. Table 4-2 presents the constants of the ONS model with a state-dependent M_i for two different sands, Fraser River and Toyoura. It should be noted that, for FRS, the elastic and critical state material constants are adopted from the research of Jefferies et al. (2015), while for Toyoura Sand, they are derived from the work of Manzari and Dafalias (1997).

Table 4-1 ONS and MNS formulations

	MNS	ONS (Jefferies and Been, 2015) with the state-dependent image state ratio
Limiting confining pressure ratio $\left(\frac{p_L}{p_i}\right)$	$\left(\frac{p_L}{p_i}\right)^c = \begin{cases} 1 + (\exp(c) - 1) \frac{\psi_i}{D_s(\exp(c) - 1) + \psi_i}, & \psi_i > 0 \\ 1 + \frac{\psi_i}{D_s - \psi_i} & \psi_i < 0 \end{cases}$	$\frac{p_L}{p_i} = \exp\left(\frac{\chi_i \psi_i}{M_i}\right)$ $\chi_i = \frac{\chi_{tc}}{1 - b \cdot c \cdot \left(\frac{p}{p_{ref}}\right)^c \frac{\chi_{tc}}{M_i}}$
Image Stress ratio	$M_i = \frac{M}{1 + N \left \ln\left(\frac{p_L}{p_i}\right) \right }$	$M_i = \frac{M}{1 + N \left \ln\left(\frac{p_L}{p_i}\right) \right }$ or $M_i = M \left(1 - \frac{\chi_i N \psi_i }{M} \right)$
CSL	$e_c = a - b \left(\frac{p}{p_{ref}} \right)^c$	
Stress – dilatancy rule	$D = M_i - \eta$	
Yield surface	$\eta = M_i \left(1 - \ln\left(\frac{p}{p_i}\right) \right)$	
State-Dilatancy law	$D_L = M_i \ln\left(\frac{p_L}{p_i}\right)$	
Nova's rule	$D_L = \frac{M - \eta_L}{1 - N}$	
Hardening rule	$dp_i = h \left(\frac{p}{p_i} \right)^2 \left(\frac{p_i}{p_L} - \frac{p_i}{p} \right) p_i d\varepsilon_q^p; h = H_0 - H_y \psi$	
Elastic moduli	$G = \frac{G_0 \cdot p_{ref}}{e - e_{min}} \cdot \left(\frac{p}{p_{ref}} \right)^m, \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)}$	

Table 4-2 Material properties of the ONS model with state dependent M_i for FRS and Toyoura Sand

	constants	FRS	Toyoura Sand
Elastic material properties	G_0	375	125
	e_{min}	0.344	0.62
	m	0.466	0.5
	ν	0.1	0.05
Critical state parameters	M	1.45	1.25
	a	1.01	0.934
	b	0.087	0.019
	c	0.38	0.7
State-dilatancy parameter	χ_{tc}	5.5	2.45
Shear-volume coupling factor	N	0.43	0.33
Plastic hardening parameters	H_0	60	80
	H_y	280	500

The variations of M_i , D_L , and η_L with ψ_i can be plotted for the ONS model with state-dependent M_i based on the formulations presented in Table 4-1 (Figure 4-9). As χ_i in the ONS model is pressure-dependent for the curved CSL, M_i , D_L , and η_L become pressure-dependent. Consequently, in Figure 4-9a, the curves are plotted for $p_0 = 1 \text{ MPa}$, while in Figure 4-9b the curve corresponding to $p_0 = 0.01 \text{ MPa}$ is also included.

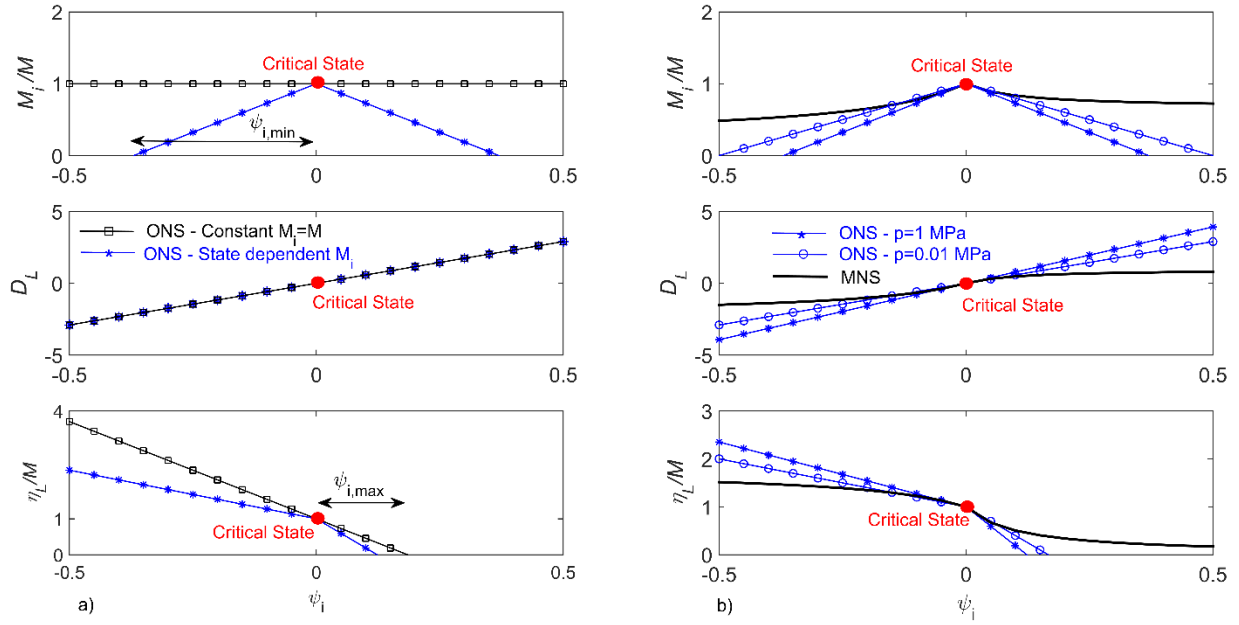


Figure 4-9 Variation of M_i , D_L , and η_L with ψ_i a) in the ONS with constant and state-dependent M_i , b) in the ONS with state-dependent M_i and MNS model

According to Figure 4-9a, the application of Nova's rule has led to a reduction in η_L compared to the scenario with a constant $M_i = M$. In Figure 4-9a, it is observed that the state-dependent M_i can reach zero for samples exhibiting both high negative and positive image state parameters. Therefore, the ONS model with state dependent M_i cannot simulate the monotonic behaviour for the samples with $\psi_i < \psi_{i,min}$. $\psi_{i,min}$ can be calculated by computing $\psi_{i,min}$ so that M_i is zero.

$$\text{For } \psi_i < 0, \quad M_i = M \left(1 + N \frac{\chi_i \psi_i}{M} \right) = 0 \rightarrow \psi_{i,min} = -\frac{M}{N \chi_i} \quad (4-23)$$

Furthermore, the reduced M_i associated with the state parameter narrows $\psi_{i,max}$, further limiting the applicable range of the ONS model for loose samples. $\psi_{i,max}$ is obtained with $\eta_L = M_i - D_L$ equal to zero.

$$\begin{aligned}
\text{For } \psi_i > 0, \quad \eta_L = M_i - D_L = M \left(1 - N \frac{\chi_i \psi_i}{M} \right) - \chi_i \psi_i = 0 \quad \rightarrow \quad \psi_{i,max} \\
= \frac{M}{(1 + N)\chi_i}
\end{aligned} \tag{4-24}$$

Consequently, the ONS model with a state-dependent M_i is only applicable for $\psi_{i,min} < \psi_i < \psi_{i,max}$. Using Equations (4-23) and (4-24) and considering $p_0 = 0.1 \text{ MPa}$, the restricted range would be $-0.353 < \psi_i < 0.117$ using the material parameters considered for the FRS. The corresponding void ratio for this restriction is obtained by adding these values to the critical void ratio at $p_{i0} = p_0 / \exp(1)$ which is $e_c(p_{i0}) = 0.95$.

Figure 4-10 illustrates how the ONS model, incorporating state-dependent M_i , simulates the drained behavior of both loose and dense samples corresponding to $\psi_{i,min}$ and $\psi_{i,max}$, or their equivalent void ratios $e_0=0.6$ and $e_0=1.06$, respectively. The figure also includes simulation results of the ONS model with a constant M_i for comparison. Despite the improved ONS response facilitated by the application of Nova's rule, effectively controlling the peak drained shear strength, it introduces an additional constraint on the dense side and further restrict the range of applicability on the loose side.

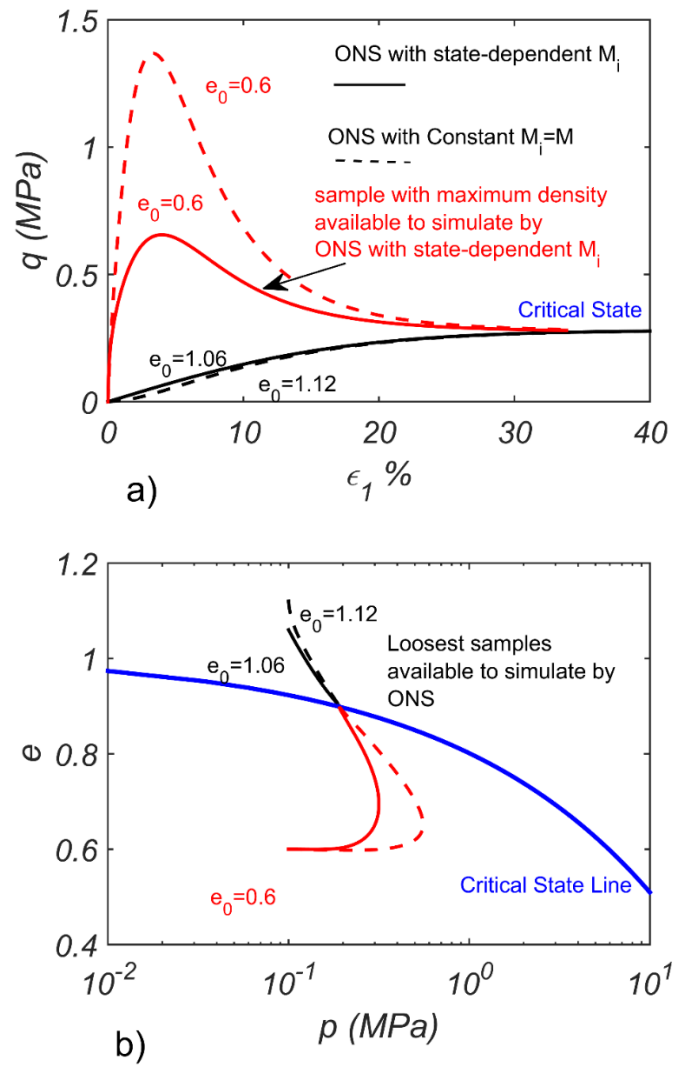


Figure 4-10 simulation results of the ONS model with state-dependent and constant M_i

4.4 Modified NorSand Model

4.4.1 Introduction of a New Nonlinear State Dilatancy Law

The modified version introduces a different method for establishing a connection between limiting dilatancy and the limiting state parameter. In contrast to the ONS approach outlined in Section 4.3, this modified procedure involves the following steps:

Step (1): A new relationship is formulated to express the ratio (p_L/p_i) as a function of (ψ_i) . The relationship will introduce a new material property (χ) to control this function. Here, this equation will be called limiting state law.

Step (2): The relationship between the limiting dilatancy (D_L) and the ratio (p_L/p_i) is obtained by combining the stress-dilatancy equation and the yield surface equation. For example, when using the stress-dilatancy rule described in Equation (4-1) and the yield surface from equation (4-3), the relationship between D_L and (p_L/p_i) can be expressed as shown in Equation (4-7). Using this procedure, it becomes easy to change the stress-dilatancy rule and as a result the yield surface. To illustrate this through an example, let's consider the stress-dilatancy rule proposed by Burland (1965) and Roscoe and Burland (1968) as follows:

$$D = \frac{M^2 - \eta^2}{2\eta} \quad (4-25)$$

The integration form of Equation (4-25) corresponds to a yield surface employed in the modified Cam-Clay model (Wood, 1990).

$$\eta^2 = M^2 \left(2 \frac{p_i}{p} - 1 \right) \quad (4-26)$$

By substituting Equation (4-25) into Equation (4-26) and utilizing the limiting state quantities, we can derive the following relationship between the (p_L/p_i) ratio and the limiting dilatancy (D_L) .

$$D_L = M \frac{1 - \frac{p_i}{p_L}}{\sqrt{2 \frac{p_i}{p_L} - 1}} \quad (4-27)$$

Note that, the ONS hardening rule (Equation (4-6)) is inclined towards employing stress-dilatancy rules that possess nonzero incremental plastic deviatoric strain at ($\eta = 0$). However, here using Equation (4-25) is presented to illustrate how the model can utilize different complex equation forms of stress-dilatancy with greater flexibility.

Step (3): By substituting the (p_L/p_i) ratio obtained in Step 1 into the equation from Step 2, the limiting dilatancy (D_L) is determined as a function of (ψ_i).

Step (4): The relationship between (ψ_i), (ψ_L), and their respective mean confining pressures on the yield surface (p_i and p_L) can be established using the definition of any desired critical state line. By incorporating the (p_L/p_i) ratio found in Step 1 into the obtained equation in Step 4, the limiting state parameter is also expressed as a function of (ψ_i). For example, for the linear critical state line of Equation (4-11), this relationship is shown in Equation (4-12). For the nonlinear critical state line of Equation (4-15), the following relationship is derived:

$$\psi_L = \psi_i + \lambda \left(\frac{p_i}{p_{ref}} \right)^\xi \left(\left(\frac{p_L}{p_i} \right)^\xi - 1 \right) \quad (4-28)$$

Where $\lambda \left(\frac{p_i}{p_{ref}} \right)^\xi$ in the Equation (4-28) can be replaced with $(\psi_i + \Gamma - e)$ using the nonlinear CSL of Equation (4-15).

Step (5): Consequently, both D_L and ψ_L are functions of ψ_i , forming a pair ($D_L(\psi_i), \psi_L(\psi_i)$), which can be regarded as a novel state-dilatancy law. Both D_L and ψ_L are determined by ψ_i and χ . Each ψ_i corresponds to a distinct set of D_L and ψ_L values based on the chosen constant material

property (χ). Gathering these data points for various ψ_i values allow for the construction of a (D_L, ψ_L) curve. χ can be calibrated by matching the curve with experimental data.

The introduced procedure enables users to have the flexibility of selecting any desired critical state line and stress-dilatancy rule for implementation in the model. The primary focus is now on defining an appropriate function for the ratio (p_L/p_i) as a function of (ψ_i) . To establish this relationship, the requirements of limiting dilatancy highlighted in Section 4.3.3 are applied to the (p_L/p_i) ratio using Equation (4-7). According to these requirements, for dense samples ($\psi_i < 0$) with negative D_L , the ratio should be less than 1. Conversely, for loose samples ($\psi_i > 0$) with positive D_L , the ratio should be greater than 1. For $\psi_i = 0$, the ratio should be 1.

Here additional constraints also are added since the ratio should not exceed $r = \exp(1)$ or 0 on the yield surface as the magnitude of ψ_i increases for loose and dense samples, respectively. For theoretical purposes, the highest value of $|\psi_i|$ is considered infinite, thus, as ψ_i approaches $+\infty$, the ratio should approach $r = \exp(1)$, and as ψ_i approaches $-\infty$, the ratio should approach zero. To satisfy all these conditions, the following nonlinear relationship is proposed.

$$\left(\frac{p_L}{p_i}\right)^\xi = 1 + \begin{cases} (r^\xi - 1) \frac{\psi_i}{\chi(r^\xi - 1) + \psi_i} & \psi_i > 0 \\ \frac{\psi_i}{\chi - \psi_i} & \psi_i < 0 \end{cases} \quad (4-29)$$

In the equation, χ is called dilatancy state parameter and will serve as a replacement for χ_i in the new model. This formulation in its current form is designed in such a way that it could be compatible with the nonlinear critical state line of Equation (4-15). For linear critical state line, ξ can be set to 1 in the formulation. For other forms of CSLs, a similar approach can be used. Finally, by replacing Equation (4-29) into Equation (4-7), and Equation (4-28), the limiting dilatancy and limiting state parameter can be obtained.

4.4.2 Formulation of the Modified NorSand Model

Integrating Equation (4-22) into the ONS formulation is a relatively straightforward task. The only adjustment needed is to substitute the (p_L/p_i) ratio from Equation (4-29) with its corresponding term in the ONS. The formulations of the MNS are presented in Table 4-1.

To determine the value of D_S , the only different material property in the modified version, the following procedure can be followed. First, the variations of M_i , D_L , and η_L with ψ_i are plotted for the ONS model based on the formulations presented in Table 4-1 (Figure 4-9). For the MNS model, D_S is selected in a manner that ensures the resulting M_i , D_L , and η_L curve align perfectly with the corresponding segments of the curves falling within the applicable range of the ONS model. Using this procedure, for the Fraser River Sand, D_S is selected as 0.4.

D_S can be independently calibrated from the ONS by utilizing the data of D_L and ψ_L obtained from various drained triaxial tests on samples denser than critical as presented in Figure 4-5 the samples of FRS (S1 to S4) and Toyoura Sand (T1 and T4). For the MNS model, the step-by-step procedure outlined in section (4.4.1) can be followed to obtain D_L and ψ_L as functions of ψ_i for dense samples ($\psi_i \leq 0$):

$$D_L = M \frac{\ln \left(1 + \frac{\psi_i}{D_S - \psi_i} \right)}{c + N \left| \ln \left(1 + \frac{\psi_i}{D_S - \psi_i} \right) \right|} \quad \psi_i \leq 0 \quad (4-30)$$

$$\psi_L = \psi_i + (\psi_i + a - e) \left(\frac{\psi_i}{D_S - \psi_i} \right) \quad \psi_i \leq 0 \quad (4-31)$$

In the above equations, D_S is chosen so that the (D_L, ψ_L) curves align with experimental data for various void ratios (e). The dense FRS samples were tested with an initial void ratio ranging from 0.7 to 0.8, reaching a void ratio at the peak point between 0.73 and 0.82. The calibrated (D_L, ψ_L) curves for the MNS at the peak void ratio are presented in the Figure 4-5 for FRS. $D_S = 0.4$ yields well-calibrated curves for the dense FRS samples, and using a similar approach for the Toyoura Sand, D_S has been obtained as 0.65.

4.4.3 Verification of the MNS versus the experimental Data

Figure 4-11 and Figure 4-12 show the triaxial compression drained responses of FRS (S1 to S10) and Toyoura Sand samples (T1 to T6). All samples initially have a range of void ratios where the ONS and MNS calibration curves match with each other (See Figure 4-5). Consequently, the responses generated by the ONS and MNS closely align, as illustrated in Figure 4-11 and Figure 4-12. These figures show that the MNS can simulate the same response as the ONS under drained response. Data beyond the applicable range for the ONS was not available. The undrained response of loose samples showing flow liquefaction is investigated in the upcoming sections for the Toyoura sand and FRS.

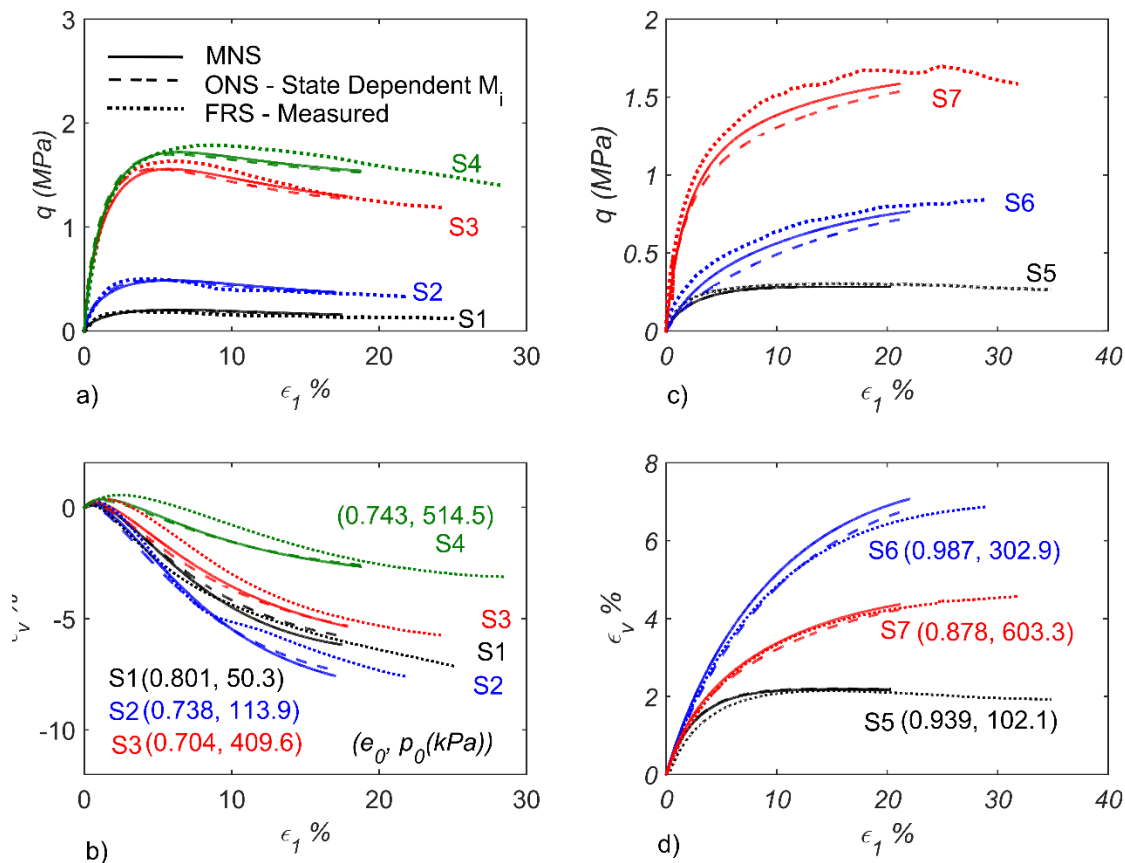


Figure 4-11 Simulation results using ONS and MNS versus measured data in drained compression triaxial tests on Fraser River Sand (Jefferies and Been, 2015), (a,b) S1 to S4, dense samples (c,d) S5 to S7, loose samples

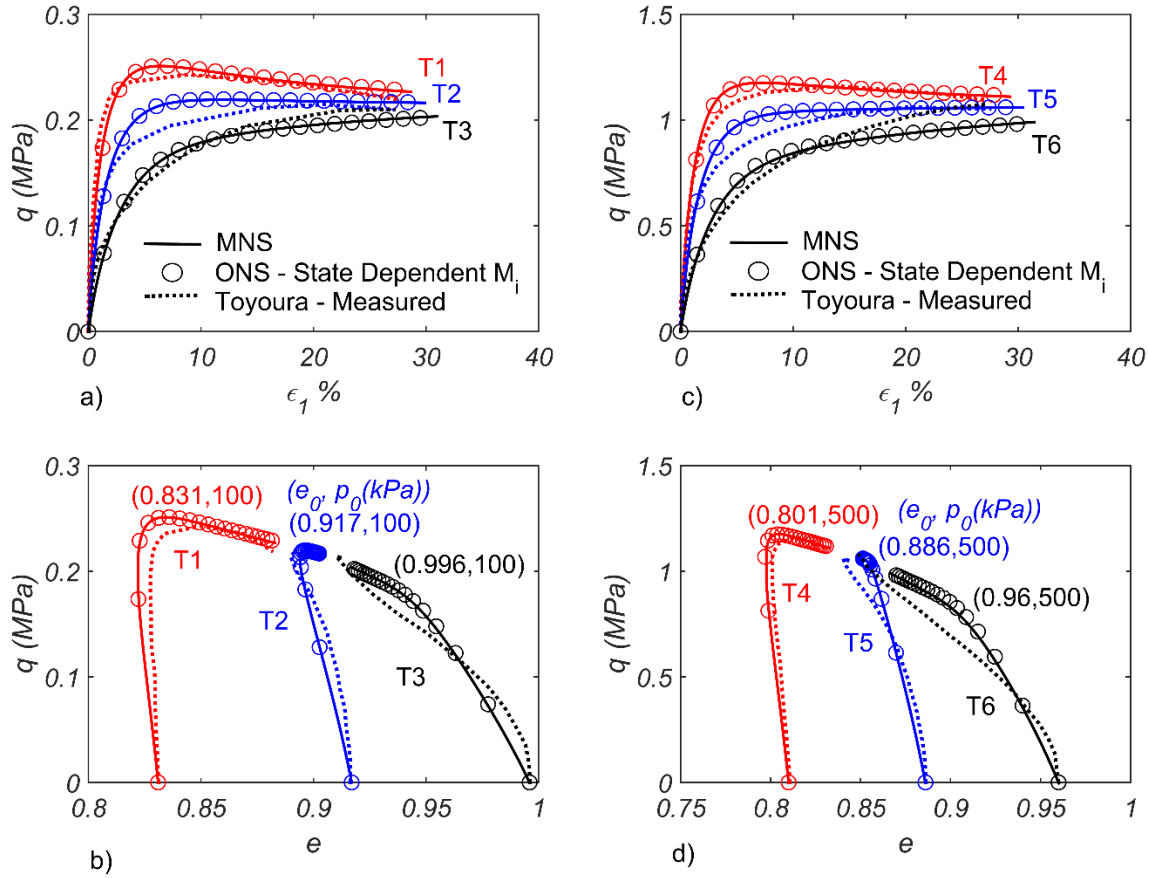


Figure 4-12 Simulation results using ONS and MNS versus measured data in drained compression triaxial tests on Toyoura Sand (Verdugo and Ishihara, 1996), (a,b) T1 to T3, samples with initial confining pressure of 100 kPa (c,d) T4 to T6, , samples with initial confining pressure of 500 kPa

4.4.4 Achievements of the New Nonlinear State-Dilatancy Law

4.4.4.1 Applicability for All States:

Figure 4-9b illustrated the relationship between ψ_i and the behavior of the updated M_i , D_L , and η_L parameters in the MNS model. The figure plainly shows that the MNS model prevents stress ratios from reaching zero as the positive image state parameter increases. This suggests that the limiting state will never extend beyond the yield surface. Additionally, on the negative side, the nonlinearity of the new law prevents the image stress ratio from being zero.

Figure 4-13 presents the same simulations as Figure 4-7 but using the MNS model. Notably, the ONS's simulations of the samples with $e_0=0.6$ and $e_0=1.1$ are also included for comparison. The figure highlights that the MNS model can now simulate loose samples with $e_0>1.06$ and dense samples with $e_0<0.6$ for $p_0=0.1$ MPa in the case of Fraser River sand. These outcomes suggest that the new model is applicable to ranges beyond those restricted by Equation (4-23) and Equation (4-24).

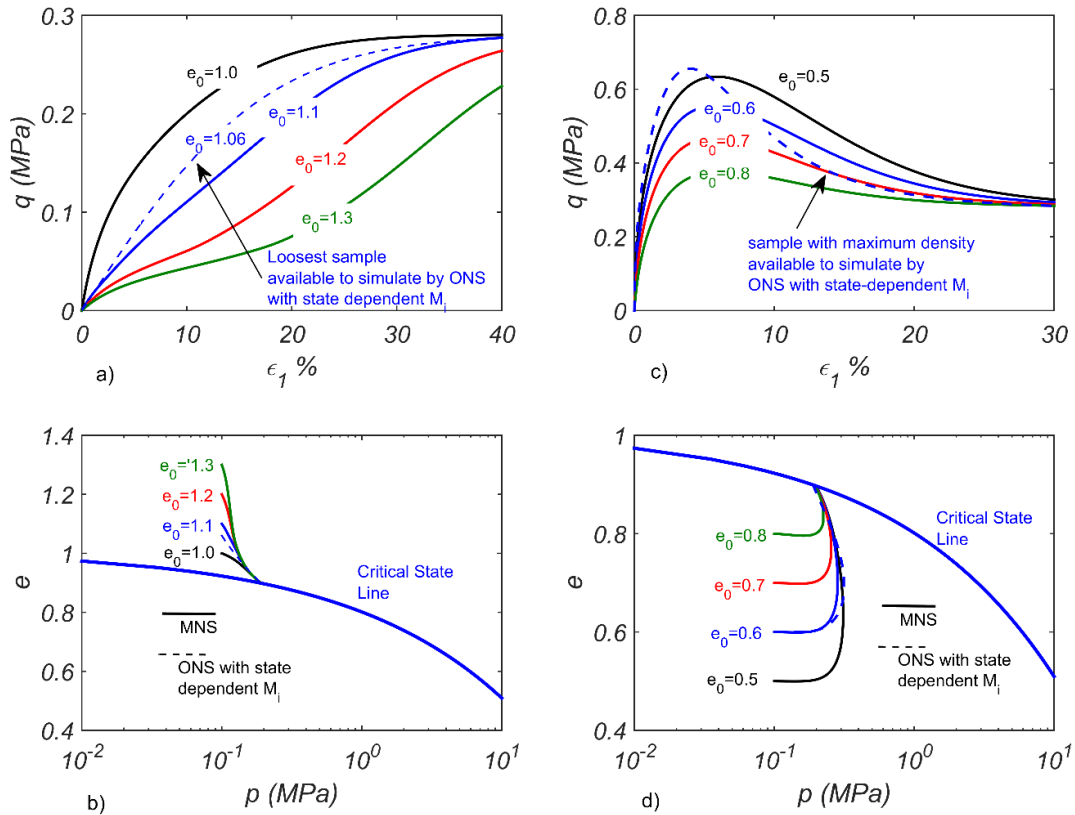


Figure 4-13 Typical simulations of the MNS model for the (a,b) loose, and (c,d) dense samples with the same initial confining pressure (100 kPa) and different initial void ratios

4.4.4.2 Suitable for Choosing Distinct Material Properties for Loose and Dense Samples:

To evaluate the performance of flow liquefaction simulation by the ONS and its modified version, undrained triaxial simulations of loose samples of FRS and Toyoura Sand are compared with test data provided by Jefferies and Been (2015) for FRS and Verdugo and Ishihara (1996) for

Toyoura Sand. The FRS samples, labeled as S8 and S9, had initial void ratios and initial confining pressures of (0.973, 201 kPa) and (0.966, 301 kPa), respectively. Toyoura Sand samples, denoted as T7 and T8, had initial void ratios and initial confining pressures of (0.907, 1000 kPa) and (0.907, 2000 kPa), respectively. All the samples are initially loose, meaning that their initial state are located above their corresponding CSL.

Figure 4-14 shows the simulation results in comparison with measured data for the FRS samples. As it can be seen, the ONS and MNS provide similar responses and yield acceptable simulation results in comparison with experimental data. Figure 4-15 (a,b) displays the simulation results compared with measured data for the Toyoura Sand samples. Interestingly, while both models yield similar results when compared to each other, their responses significantly differ from the measured data. The fact the ONS model provides good results for drained response, lacking to provide the same level of accuracy for undrained response has been recognised by the developers of the model. To solve this issue, one method could be proposing different dilatancy state parameters for the drained and undrained response. In the case of Toyoura Sand, χ_{tc} can be increased to get better fit with the experimental data. However, increasing χ_{tc} further limits the range of applicability ($\psi_{i,min} < \psi_i < \psi_{i,max}$). These findings provide the basis for the recommendation by the developers of ONS (Jefferies and Been, 2015) to maintain χ_{tc} values between 2 and 5.5. Figure 4-15 (c,d) shows the simulation results compared with the ONS model with $\chi_{tc} = 5.5$. Although the simulation results have been improved, but there is still a discrepancy between the simulation results of the ONS model and the experimental data.

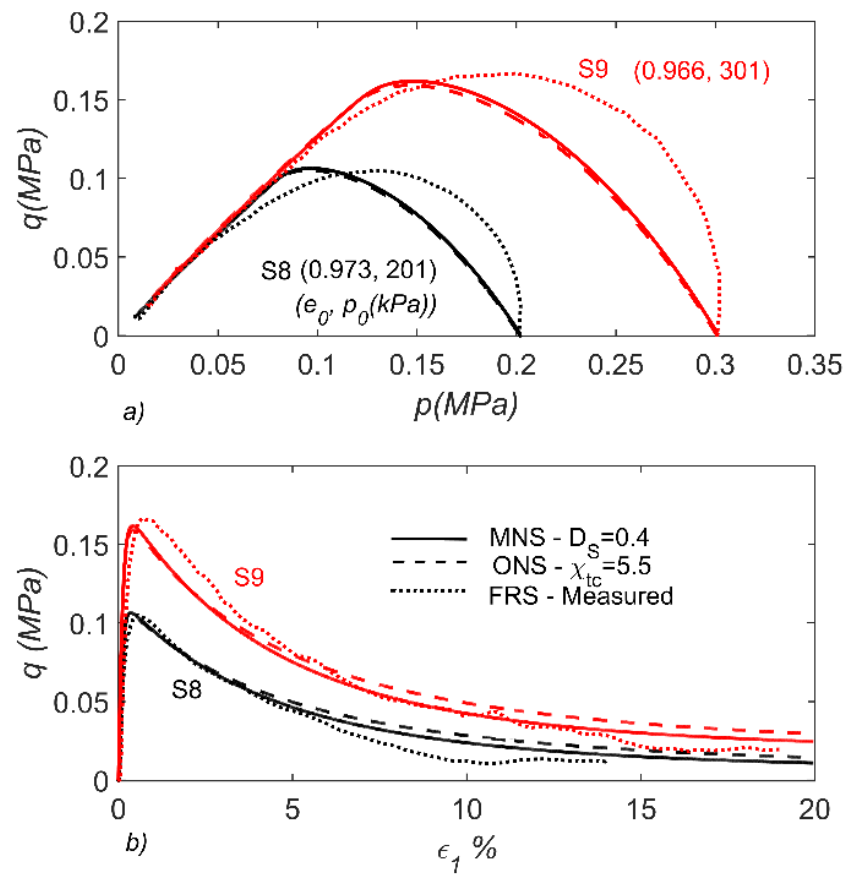


Figure 4-14 Simulation results using ONS and MNS versus measured data in undrained compression triaxial tests on Fraser River Sand (Jefferies and Been, 2015), S8 and S9, loose samples showing flow behavior.

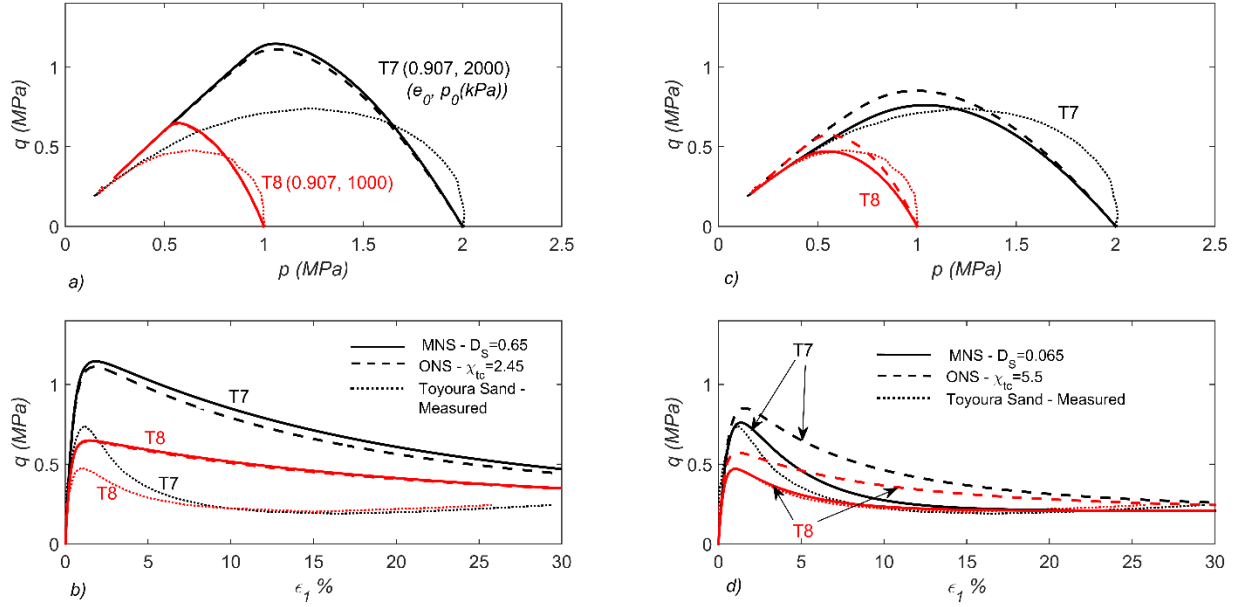


Figure 4-15 Simulation results (a,b) without adjusting the dilatancy state parameters (c,d) with adjusting the dilatancy state parameters using ONS and MNS versus measured data in undrained compression triaxial tests on Toyoura Sand (Verdugo and Ishihara, 2015)

By opposition, the MNS model can accommodate the selection of different dilatancy state parameters under drained and undrained conditions without further restricting the model's response. Figure 4-15 (c,d) illustrates that by altering D_s from 0.65 to 0.065, the model achieves high accuracy in simulating the flow liquefaction of Toyoura samples.

To comprehend the overall capabilities of the ONS in simulating flow liquefaction, beyond a comparison limited to just two samples of Toyoura Sand and Fraser River Sand, we can examine the trends that the ONS and MNS can generate for loose samples under undrained loading. Figure 4-16 shows the brittleness index (I_B) of different loose tailings materials versus their initial state pressure index (Macedo and Vergaray, 2022). The brittleness Index (I_B) was introduced by Bishop (1971) to show the intensity of a stress drop from the peak shear strength to the residual shear strength.

$$I_B = \frac{q_p - q_{cr}}{q_p} \quad (4-32)$$

Where q_p and q_{cr} are the peak and residual shear strength in an undrained test conducted on a loose soil.

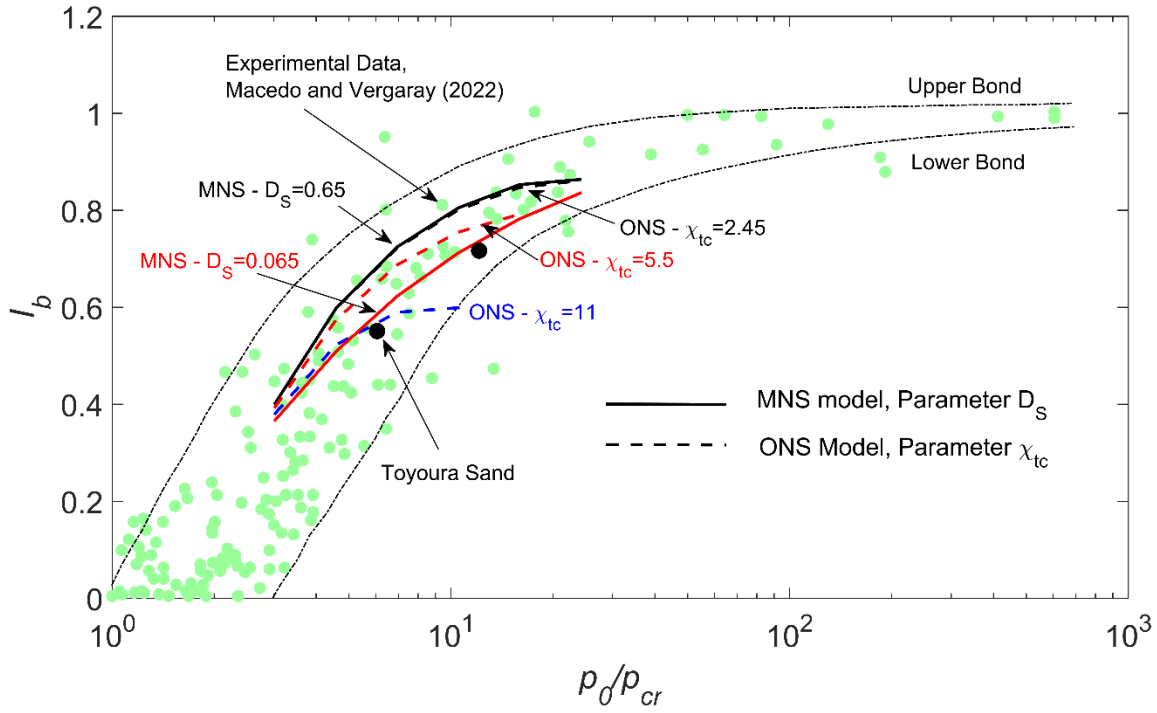


Figure 4-16 Comparison with measured data in terms of brittleness index (I_B) and state pressure index extracted from Macedo and Vergaray (2022) and the typical response of ONS and MNS for loose samples using different set of material properties

The state pressure index (p/p_{cr}), as defined by Wang et al. (2002), compares the current mean stress (p) to the critical mean stress (p_{cr}) at the same void ratio. It serves as an alternative measure to quantify the density or looseness of a sample. When the ratio (p/p_{cr}) exceeds 1, the sample is in a loose condition, and a higher value indicates increased looseness. In Figure 4-16,

upper and lower bounds are illustrated, the upper bound representing samples with a higher undrained peak shear strength compared to the samples following the lower bound trend. For a soil model to effectively simulate the undrained behavior of loose samples, the obtained brittleness index for different initial state pressure indices should exhibit a similar trend between the upper and lower bounds. In the figure, the brittleness index (I_B) of Toyoura Sand Samples (T7 and T8) also has been shown.

To assess the trends of the ONS and its modified version in simulating flow liquefaction, a series of undrained triaxial simulations were conducted on loose samples using the material properties of Toyoura Sand. The samples were initially characterized by a void ratio of 0.907, while the initial state pressure index varied between 3 and the point where the ONS model loses its applicability in properly simulating undrained behavior. Figure 4-16 illustrates that by adjusting D_s , MNS can generate different trends within the upper and lower bounds. Opting for $D_s = 0.065$ yields a good fit with the experimental data of Toyoura Sand. However, the ONS produces reasonable trends only when χ_{tc} is less than 5.5. Further increases in χ_{tc} result in trends that deviate from the expected patterns for granular materials.

To address the ONS capability in proper simulation of flow liquefaction, the model developers (Jefferies and Been, 2015) proposed an alternative approach. They suggested incorporating a new term into the hardening rule of Equation (4-6), known as the cap-softening term. However, the inclusion of this additional feature does not fully resolve the problem (Razavi and Yniesta, 2023). Consequently, several researchers (Shuttle and Jefferies, 2016; Reid and Smith, 2021; Ng et al., 2023) attempted to use the cap-softening term in conjunction with adjusting the values of the bulk modulus for undrained simulations. This adjustment to the bulk modulus has a significant impact on the ONS undrained responses, yielding results that better align with measured data. Despite the improvement in simulation accuracy, this approach has drawbacks. It not only modifies the original and appealing form of the hardening rule but adjusting the elastic modulus for undrained loading (Razavi and Yniesta, 2023) can yield an inaccurate response at low strains for the benefit of higher accuracy at high strains.

4.5 Conclusion

The linear state-dilatancy law within the ONS model presents challenges when dealing with excessively dense or loose samples. Under drained conditions, the ONS model with constant $M_i = M$ results in an unusually high peak shear strength for dense samples. Conversely, for loose samples, the model is not applicable when ψ_i surpasses $\psi_{i,max}$, as the limiting state extends beyond the yield surface.

The initial issue in the model has been addressed in the literature (Been and Jefferies, 2015) by rendering M_i state-dependent through the incorporation of Nova's rule and the model's linear state-dilatancy law. Despite this modification, the model encounters another limitation on the dense side and is applicable only within a specific range of initial image state parameters ($\psi_{i,min} < \psi_i < \psi_{i,max}$). Both $\psi_{i,max}$ and $\psi_{i,min}$ are functions of the dilatancy state parameter, χ_{tc} and N , and opting for higher χ_{tc} further narrows down the applicable range of the model. This reasoning reveals why developers (Been and Jefferies, 2015) recommend χ_{tc} values between 2 and 5.5.

The paper shows that the ONS with state-dependent M_i may not yield accurate simulation results for undrained responses of loose samples in some cases. One solution involves using different χ_{tc} parameters to control the undrained response. However, keeping χ_{tc} within the proposed range causes the model to lose its capability to accurately simulate flow liquefaction. As a result, the model developers (Been and Jefferies, 2015) explored alternative methods, such as modifying the hardening rule or adjusting the elastic modulus, to improve the simulation of flow liquefaction.

This paper illustrated that all these limitations come from the inherent nature of the model's linear state-dilatancy law. Furthermore, the model face difficulties in incorporating different forms of critical state lines or alternative stress-dilatancy rules.

To enhance the model's adaptability in adjusting formulations, a new procedure has been developed to construct state-dilatancy laws with their specified designed dilatancy state parameter (D_S). Based on this approach, a nonlinear state-dilatancy law is proposed for the ONS model, ensuring that the limiting confining pressure remains within the specified ranges imposed by the ONS yield surface. Integrating this law into the ONS formulation is straightforward. The example

modified model termed MNS, permits potential adjustments to the stress-dilatancy rule and the critical state line formulation, providing users the flexibility to choose different yield surface shapes.

Furthermore, by using an alternative state-dilatancy, the MNS is applicable across a wide range of states, producing reasonable results for both dense and loose samples. It permits the selection of distinct material properties for both drained and undrained conditions. By adjusting the dilatancy state parameter (D_s), the model improves its capability to accurately simulate the undrained behavior of loose samples, closely matching measured data.

4.6 Data Availability Statement

Data generated or analyzed during this study are available from the corresponding author upon reasonable request.

4.7 Acknowledgements

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CHAPTER 5 ARTICLE 3: POLYSAND MODEL, A MODIFIED CAMBRIDGE TYPE NORSAND MODEL TO SIMULATE THE MONOTONIC BEHAVIOR OF GRANULAR MATERIALS

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5.1 Abstract

In this paper, the PolySand model, a new modified version of the original NorSand model aimed at simulating the mechanical behavior of granular materials, is introduced. The foundational principles of the original NorSand model, including the utilization of a limiting dilatancy concept and the framework of Cambridge-type models, are retained by the PolySand model. To enhance the model's accuracy in capturing the undrained behavior of granular soils, two additional material properties are introduced through the implementation of a nonlinear state-dilatancy law and image stress ratio. These newly introduced parameters control the flow and non-flow behavior of sand and can be easily obtained from the results of undrained triaxial tests. The concept of a cap-softening term, present in the latest versions of NorSand, is eliminated by the new model, and the model's original plastic modulus is reverted to. Additionally, PolySand is able to simulate limited flow behavior, a feature that was missing in the original NorSand model. With PolySand, users are empowered with the capability to control and readily adjust the simulation of flow (static liquefaction), limited flow, and non-flow behavior, offering a versatile and flexible modeling approach.

5.2 Introduction

Granular soils exhibit three distinct mechanical responses during undrained shear loading (Verdugo and Ishihara, 1996), as depicted schematically in Figure 5-1: flow behavior, limited flow

behavior, and non-flow behavior. Flow behavior is typically observed in loose samples, displaying a consistent reduction in mean confining pressure and a shear strength decrease from a peak value (instability point) to the residual shear strength at large strains. Limited flow behavior is described as strain hardening of the soil upon reaching a quasi-critical state after the instability point. Further shearing results in reaching the true critical state at large strains. In contrast, non-flow behavior occurs in dense samples, manifesting as a minor initial contraction followed by a phase transformation state (PTS), and a continuous dilation and constant strain hardening before reaching the critical state.

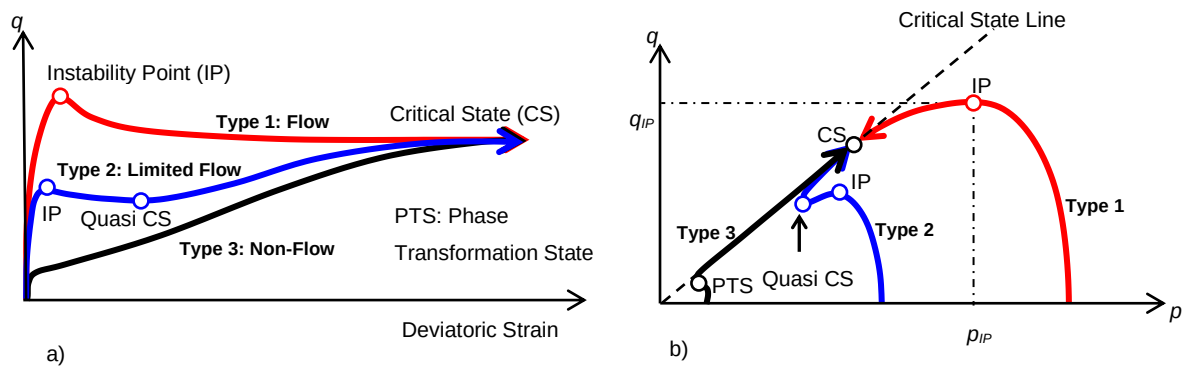


Figure 5-1 Typical undrained mechanical behavior of granular soils under the same void ratio and different initial confining pressures, a) deviatoric stress (q)-deviatoric strain, b) mean effective stress (p)-deviatoric stress (q)

The original NorSand model presented by Jefferies (1993) was an attempt to expand traditional Cambridge-type constitutive models to simulate these behaviors from loose to dense states using the state-parameter concept (Been and Jefferies, 1985). The traditional Cambridge-type models, such as Cam-Clay (Roscoe et al., 1963), modified Cam-Clay (Burland, 1965), and Granta gravel (Schofield and Wroth, 1968), possess four distinctive features: (1) a single-yield surface, (2) an associated-flow rule, (3) compatibility with the critical state theory, and (4) a yield surface that is attached to the critical state line (CSL). Although the first two features make these models easy to use, they fail to predict the volumetric response accurately (Jefferies and Shuttle,

2002). Furthermore, the last feature restricts the capability of traditional Cambridge-type models to properly simulate the elasto-plastic behavior of granular soils for states looser than critical (Jefferies, 1993). The original NorSand addresses these shortcomings by (1) introducing the concept of limiting dilatancy, which constrains the expansion and shrinkage of the yield surface to control the dilatancy, and (2) separating the yield surface from the CSL through the concept of state parameter (Jefferies, 1993).

The NorSand model (Jefferies, 1993, Jefferies and Been, 2015) incorporates the concept of limiting dilatancy based on the results of drained tests conducted on dense samples. Based on the initial definition given by Jefferies (1993), the limiting dilatancy corresponds to the minimum dilatancy (D_{min}) at the highest stress ratio (η_{max}) during drained shear loading on dense samples. The state parameter corresponding to the limiting dilatancy is defined as (ψ_{min}). According to NorSand, a unique relationship between (D_{min}) and (ψ_{min}), known as state-dilatancy law, allows for the estimation of the limiting dilatancy. Consequently, the NorSand state-dilatancy law originates from drained conditions and is subsequently applied directly to simulate the behavior of sand under undrained conditions. As a result, the model demonstrates effectiveness in modeling the soil behavior under drained conditions but has limitations in its capacity to capture undrained behavior, particularly flow behavior (Cheng and Jefferies, 2020; Razavi and Yniesta, 2023). Additionally, the formulation of the original NorSand (Jefferies, 1993) only allows for the simulation of flow and non-flow behavior, and the model lacks the ability to reproduce limited flow.

To improve the NorSand model's capability to simulate undrained mechanical behavior in sand, particularly flow behavior, Jefferies and Been (2015) introduced the concept of a cap-softening term that modifies the hardening rule under undrained conditions. Users have the flexibility to include or exclude this term from the formulation (Cheng and Jefferies, 2020), but are limited in the control of both flow and non-flow behavior (Razavi and Yniesta, 2023). As a result, other researchers (Shuttle and Jefferies, 2016; Reid and Smith, 2021; Ng et al., 2023) have attempted to further adjust the values of the bulk modulus in undrained simulations to achieve a more accurate representation of undrained mechanical behavior.

This paper presents a modified version of NorSand, known as PolySand, to address the shortcomings in the NorSand model's accurate simulation of undrained mechanical behavior in granular materials. Additionally, it offers insights into potential future model enhancements. The paper begins with a detailed explanation of the NorSand model formulations (Jeffereis, 1993; Jefferies and Been, 2015). This section outlines how the model attempts to simulate the undrained behavior of sand and highlights the model's limitations, primarily associated with the NorSand limiting dilatancy function.

Next, the formulation of PolySand is explained. PolySand leverages the nonlinear state-dilatancy law proposed by Razavi and Yniesta (2023) and refines it by introducing two distinct dilatancy state properties: drained dilatancy state parameter (DSP) and undrained DSP. The first parameter, denoted as χ_d , governs the model's behavior under drained conditions, while the second, designated as χ_u , controls the model's response under undrained conditions. Additionally, under undrained conditions, another material property known as the undrained dilatancy parameter (N_u) is incorporated to better control the phase transformation in dense samples.

The calibration of PolySand material constants is discussed and validation examples are provided regarding its capability to simulate both drained and undrained behavior of sand using the well-documented experiments data conducted on Toyoura sand (Verdugo and Ishihara 1996).

Finally, a discussion regarding the incorporation of the limited flow feature into the model's formulation is presented. This paper presents a proof of concept by using Li and Dafalias' work (2012) to illustrate the feasibility of incorporating limited flow behavior in future model versions.

5.3 Symbols used in the formulation of NorSand and PolySand

In this paper, all confining stresses are effective, and the superposed dot implies a rate. The quantities in bold, are tensors, and others are scalar values. Using the stress tensor $\boldsymbol{\sigma}$, the following terms can be defined:

$$p = \frac{1}{3} \text{tr}(\boldsymbol{\sigma}); \quad \mathbf{s} = \boldsymbol{\sigma} - p \cdot \boldsymbol{\delta}; \quad J_2 = \frac{1}{2} \mathbf{s} : \mathbf{s}; \quad q = \sqrt{3J_2}; \quad \eta = \frac{q}{p} \quad (5-1)$$

Where p is the mean effective stress, $\mathbf{s}$ is the deviatoric stress tensor, J_2 is the second invariant of the deviatoric stress tensor, q is the deviatoric shear stress, η is the stress ratio, and $\boldsymbol{\delta}$ is the Kronecker delta (Equation (5-2)), tr is the trace of the tensor. The $(:)$ operator used in Equation (5-1) denotes an inner product of tensors.

$$\boldsymbol{\delta} = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}, \quad i, j = 1, 2, 3 \quad (5-2)$$

By having the strain tensor $\boldsymbol{\varepsilon}$, the following terms can be defined:

$$\varepsilon_v = \text{tr}(\boldsymbol{\varepsilon}); \quad \boldsymbol{\gamma} = \boldsymbol{\varepsilon} - \frac{1}{3}\varepsilon_v \cdot \boldsymbol{\delta}; \quad \varepsilon_\gamma = \frac{1}{q} \mathbf{s} : \boldsymbol{\gamma} \quad (5-3)$$

Where $\boldsymbol{\gamma}$ denotes the deviatoric strain tensor. ε with subscripts v and γ indicate the volumetric strain and work conjugate shear strain respectively. It is worth noting that ε_γ , was first introduced by Resende and Martin (1985). Jefferies and Shuttle (2002) confirmed that this definition of shear strain is necessary when using Cambridge type models to achieve a work contingency in general stress-strain conditions. Strain rate can be decomposed into elastic and plastic rate as follows:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p \quad (5-4)$$

The superscripts e and p will be used to distinguish between the elastic and plastic parts of the strain, respectively. Plastic Dilatancy is defined as follows:

$$D = \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_\gamma^p} \quad (5-5)$$

In the rest of the paper, the term “Dilatancy” is used for the plastic dilatancy.

5.4 Original NorSand Formulation

This section presents the formulation of the original NorSand model (Jefferies, 1993) and its latest version (Jefferies and Been, 2015; Cheng and Jefferies, 2020), as some of these equations are later used for the PolySand model.

5.4.1 Dilatancy and Yield Surface

Rowe's dilatancy theory (Rowe, 1962) asserts a relationship between dilatancy and the difference between the dilatancy stress ratio, or in this study image stress ratio (M_i), and the current stress ratio (η). M_i is also referred to as the slope of the phase transformation line by Ishihara et al. (1975).

$$D = A_i(M_i - \eta) \quad (5-6)$$

A_i can be a constant or a function of the state parameter. The original NorSand model (Jefferies and Been, 2015) sets A_i equal to 1. Using the associated flow rule, Equation (5-6) can be written as:

$$D = \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_\gamma^p} = -\frac{q}{p} = M_i - \eta \quad (5-7)$$

The NorSand yield surface is obtained by integrating Equation (5-7), which results in the following equation (Jefferies, 1993; Jefferies and Been, 2015).

$$f = \eta - M_i \left(1 - \ln \left(\frac{p}{p_i} \right) \right) = 0 \quad (5-8)$$

The obtained yield surface, shown schematically in Figure 5-2, possesses four characteristic state points: the image state, isotropic state, limiting state, and current state. The image state (Jefferies, 1993), located at the zero dilatancy point on the yield surface, is defined by the image stress ratio (M_i) and image mean stress (p_i). Similarly, the limiting state is characterized by the limiting stress ratio (η_L), limiting mean stress (p_L), and limiting dilatancy (D_L). The isotropic state, which is located at the edge of the yield surface and has zero stress ratio, is defined by isotropic mean stress (p_{iso}) and isotropic dilatancy (D_{iso}). Finally, the current state corresponds to the current stress condition on the yield surface. While the location of the image state and isotropic state on the yield surface is constant, limiting state and current state are free to move on the yield surface.

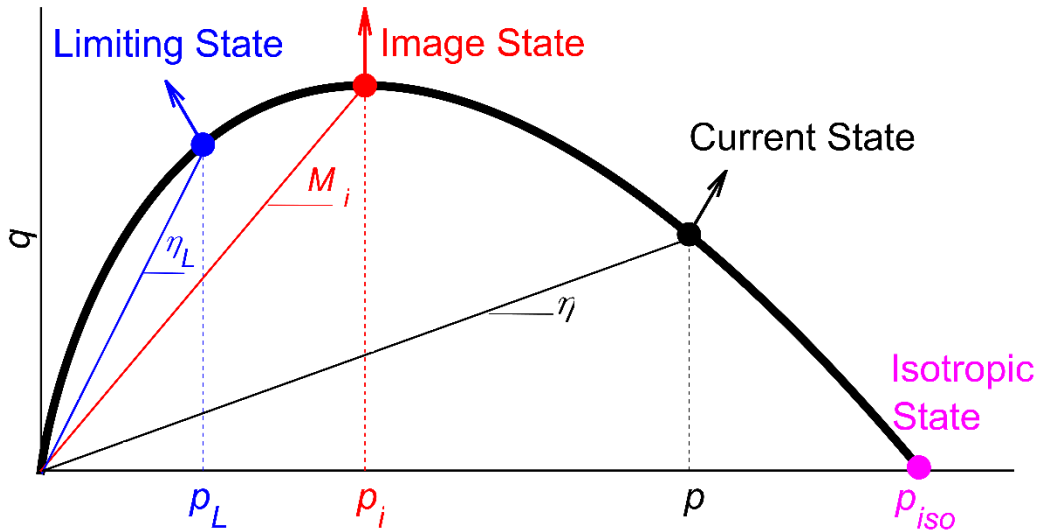


Figure 5-2 Schematic representation of the NorSand yield surface illustrating the current, image, isotropic, and limiting states

Another relationship is obtained using the combination of the stress-dilatancy rule (Equation (5-7)) and the yield surface (Equation (5-8)) as follows:

$$\frac{D}{M_i} = \ln\left(\frac{p}{p_i}\right) \quad (5-9)$$

By inserting the limiting state quantities in Equation (5-9), the limiting dilatancy can be obtained.

$$D_L = M_i \cdot \ln\left(\frac{p_L}{p_i}\right) \quad (5-10)$$

The limiting stress ratio can also be found from the stress-dilatancy rule:

$$\eta_L = M_i - D_L = M_i \left(1 - \ln\left(\frac{p_L}{p_i}\right)\right) \quad (5-11)$$

Following a similar procedure, the isotropic mean stress and isotropic dilatancy can be obtained by setting $\eta_{iso} = 0$, as follows:

$$D_{iso} = M_i \quad (5-12)$$

$$p_{iso} = \exp(1) \cdot p_i = r \cdot p_i \quad (5-13)$$

Equation (5-13) implies that for the NorSand yield surface of Equation (5-8), the ratio (r), equal to p_{iso} divided by p_i is constant, and equal to $exp(1)$ (Roscoe et al., 1963; Jefferies, 1993; Jefferies and Been, 2015).

5.4.2 Critical state and state parameter

In the current version of NorSand, a nonlinear critical state line (CSL) is used in $e - p$ space and is expressed as follows (Li and Wang, 1998):

$$e_c = \Gamma - \lambda \left(\frac{p}{p_{ref}} \right)^\xi \quad (5-14)$$

e is void ratio, and e_c is the critical void ratio. Γ , λ , and ξ are material constants. p_{ref} is the reference pressure equal to the atmospheric pressure 101 kPa.

The critical stress ratio (M) is obtained through the following equation according to the work of Jefferies and Shuttle (2011).

$$M = M_{tc} - \frac{M_{tc}^2}{3 + M_{tc}} \cos \left(\frac{3}{2} \theta + \frac{\pi}{4} \right) \quad (5-15)$$

Where M_{tc} is the critical stress ratio under triaxial compression loading. θ is the Lode angle which is calculated through the following expression:

$$\theta = \frac{1}{3} \sin^{-1} \left(\frac{3\sqrt{3}}{2} \frac{j_3}{j_2^{3/2}} \right); \quad j_3 = \det(\mathbf{s}); \quad j_2 = \frac{1}{2} \mathbf{s} : \mathbf{s} \quad (5-16)$$

With $\theta = -\pi/6$ and $\theta = \pi/6$ for triaxial extension and triaxial compression paths; J_2 and J_3 are the second and third invariant of deviatoric stress tensor ($\mathbf{s}$).

The soil state can be quantitatively defined using the following expression as state parameter, ψ (Been and Jefferies, 1985):

$$\psi = e - e_c \quad (5-17)$$

A positive state parameter indicates that the soil is in a loose state, while a negative value indicates a dense state. The state parameter varies during loading due to changes in void ratio and mean effective stress caused by dilation and contraction. Using Equation (5-17) and Equation (5-14), the following state parameters can be defined:

$$\psi_L = e - \left(\Gamma - \lambda \left(\frac{p_L}{p_{ref}} \right)^\xi \right) \quad (5-18)$$

$$\psi_i = e - \left(\Gamma - \lambda \left(\frac{p_i}{p_{ref}} \right)^\xi \right) \quad (5-19)$$

ψ_L and ψ_i are the limiting and image state parameter, corresponding to the limiting (p_L) and image mean stress (p_i) shown in Figure 1. Combining these two equations yields the following expression:

$$\psi_L = \psi_i + \lambda \left(\left(\frac{p_L}{p_{ref}} \right)^\xi - \left(\frac{p_i}{p_{ref}} \right)^\xi \right) = \psi_i + \lambda \left(\frac{p_i}{p_{ref}} \right)^\xi \left(\left(\frac{p_L}{p_i} \right)^\xi - 1 \right) \quad (5-20)$$

$\lambda \left(\frac{p_i}{p_{ref}} \right)^\xi$ on the left side of the equation can be replaced by $(\psi_i + \Gamma - e)$ using Equation (5-19). As a result, the relationship between ψ_L and ψ_i can be expressed as follows:

$$\psi_L = \psi_i + (\psi_i + \Gamma - e) \left(\left(\frac{p_L}{p_i} \right)^\xi - 1 \right) \quad (5-21)$$

5.4.3 NorSand Isotropic Hardening Law- Deviatoric Hardening

M_i and p_i control the dimensions and position of the yield surface within the stress space (Equation (5-8)). Assuming M_i depends on ψ_i , and considering that ψ_i is a function of p_i (as shown in Equation (5-19)), any alteration in p_i will ultimately impact the size of the yield surface. In NorSand, p_i is assumed to change with the plastic deviatoric shear strain in the following manner:

$$\dot{p}_i = H \cdot \dot{\epsilon}_\gamma^p \quad (5-22)$$

Where H is the plastic modulus. Jefferies (1993) introduced the following plastic modulus for the NorSand model:

$$H = h \left(p \frac{p_i}{p_L} - p_i \right) \quad (5-23)$$

Where h is the hardening parameter obtained through the following equation (Jefferies and Been, 2015):

$$h = \frac{M}{M_{tc}} (H_0 - H_y \psi) \left(\frac{p}{p_i} \right) \quad (5-24)$$

H_0 and H_y are the NorSand hardening parameters (Jefferies and Been, 2015). Equation (5-23) can be rewritten as follows:

$$H = h \cdot p_i \frac{p_i}{p_L} \left(\frac{p}{p_i} - \frac{p_L}{p_i} \right) = h_0 \left(\frac{p}{p_i} - \frac{p_L}{p_i} \right) \quad (5-25)$$

Where h_0 is the hardening parameter obtained through the following equation.

$$h_0 = \frac{M}{M_{tc}} \frac{(H_0 - H_y \psi)}{\frac{p_L}{p_i}} \left(\frac{p}{p_i} \right) p_i \quad (5-26)$$

Updating p_i using Equation (5-22) will, in turn, modify ψ_i according to Equation (5-19). In the NorSand model, ψ_i serves the purpose of determining the position of the NorSand yield surface within the $e - p$ space.

5.4.4 Limiting State Law

According to Equation (5-25), when $p > p_L$, the yield surface will expand, whereas if $p < p_L$, it will lead to the contraction of the yield surface. To determine the limiting state on the yield surface, NorSand introduces a "limiting state law" (Jefferies, 1993). The limiting state law is a link between the limiting state quantities (p_L , η_L , or D_L) and ψ_i . Consequently, the location of the

limiting state on the yield surface is conditional upon the position of the NorSand yield surface within the $(e - p)$ space, as determined by ψ_i .

Jefferies (1993) introduced the following state-dilatancy law to link the limiting state (ψ_i) and dilatancy (D_L).

$$D_L = \chi_i \psi_i \quad (5-27)$$

Where, χ_i is the NorSand dilatancy state parameter, and is obtained using the following equation (Cheng and Jefferies, 2020; Razavi and Yniesta, 2023):

$$\chi_i = \frac{\frac{M}{M_{tc}} \chi_{tc}}{1 - \lambda \xi \left(\frac{p}{p_{ref}} \right)^\xi \frac{M}{M_{tc}} \frac{\chi_{tc}}{M}} \quad (5-28)$$

Where χ_{tc} is called dilatancy state parameter (DSP) (Jefferis, 1993). χ_{tc} is a constant obtained from test results on dense sand subjected to loading under drained triaxial compression. Consequently, it can be regarded as the drained DSP. The remaining parameters in Equation (5-28) have been defined in previous sections.

According to the Equation (5-27), if $\psi_i < 0$, the limiting state must have negative dilatancy and be located on the dilative side of the yield surface. On the other hand, if $\psi_i > 0$, the limiting state must be located on the contractive side of the yield surface. Finally, when $\psi_i = 0$, $D_L = 0$, meaning that limiting state is identical to the image state. Using Equation (5-10) and Equation (5-27), (p_L/p_i) ratio can be calculated as follows:

$$F_L = \frac{p_L}{p_i} = \exp\left(\frac{D_L}{M_i}\right) = \exp\left(\frac{\chi_i \psi_i}{M_i}\right) \quad (5-29)$$

This ratio is used in Equation (5-25) to calculate the NorSand plastic modulus. The ratio (p_L/p_i) can be represented in the $(p/p_i, \psi_i)$ space, as depicted in Figure 5-3, to visually demonstrate the connection between the (p_L/p_i) ratio and the χ_{tc} parameter's variations. The figure also provides insight into how the NorSand model uses Equation (5-29) to locate the limiting state on any random selected yield surface corresponding with ψ_{i0} .

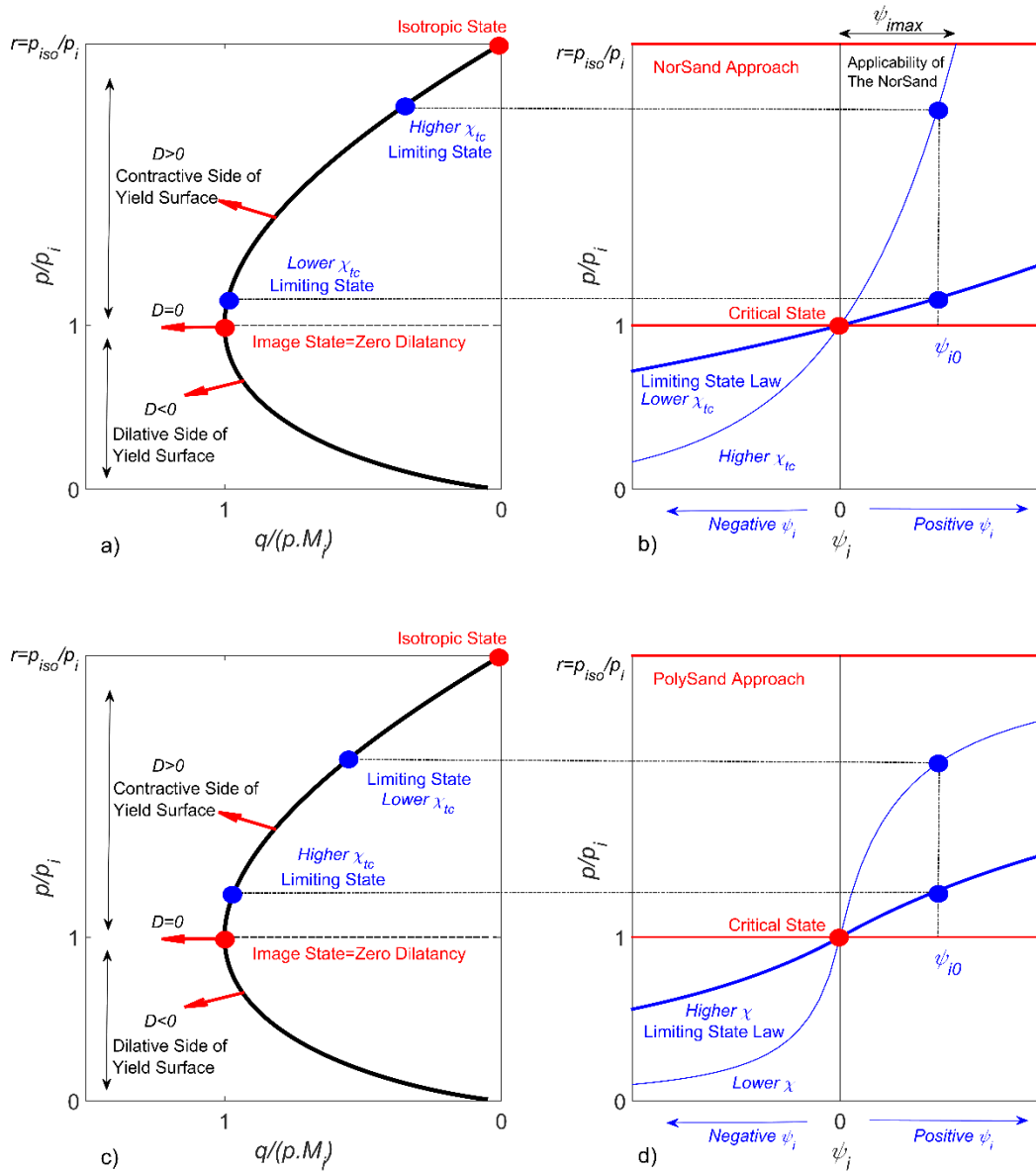


Figure 5-3 Determination of the location of the limiting state on the yield surface (corresponding with ψ_{i0}) using the limiting state law (a, b) NorSand model, (c,d) PolySand model

5.4.5 NorSand Image Stress Ratio

The image stress ratio for dense samples ($\psi_i \leq 0$) can be determined by applying Nova's rule (Nova and Wood, 1982) developed for dense samples under drained conditions. This rule

establishes a relationship between the limiting dilatancy (D_L) and the limiting stress ratio (η_L) on the limiting state:

$$D_L = \frac{M - \eta_L}{1 - N_d} \quad (5-30)$$

N_d is the drained shear-volume coupling factor or drained dilatancy parameter. By substituting η_L and D_L from Equation (5-10) and Equation (5-11), Equation (5-30) can be rearranged to obtain the image stress ratio as follows:

$$M_i = \frac{M}{1 - N_d \cdot \ln\left(\frac{p_L}{p_i}\right)} \quad (5-31)$$

Equation (5-29) can be replaced into Equation (5-31) to obtain the NorSand image stress ratio as follows:

$$M_i = M \left(1 + N_d \frac{\chi_i \psi_i}{M} \right) \quad (5-32)$$

For loose samples ($\psi_i \geq 0$), NorSand assumes that the symmetric version of Equation (5-32) can be used (Jefferies and Been, 2015). Therefore, the NorSand image stress ratio would be as follows:

$$M_i = M \left(1 - N_d \frac{\chi_i |\psi_i|}{M} \right) \quad (5-33)$$

5.4.6 Elastic moduli:

Norsand defines the elastic shear and bulk moduli as follows (Jefferies and Been, 2015; Cheng and Jefferies, 2020):

$$G = G_0 \cdot p_{ref} \cdot \left(\frac{p}{p_{ref}} \right)^{0.5}, \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \quad (5-34)$$

G_0 is a material constant, and p_{ref} is the reference pressure. ν is the Poisson's ratio. Note that the complete stress-strain relationship for this model can be found in other textbooks (Yu, H.S., 2007; Jefferies and Been, 2015). These texts describe an elastoplastic model that incorporates a single yield surface plasticity concept, follows an isotropic hardening rule, and utilizes a flow rule based on normality, also known as the associated flow rule.

5.4.7 Integration of NorSand in the critical state theory

At critical state, the dilatancy value is zero which corresponds with the image state on the NorSand yield surface. As a result, if the image mean stress (p_i) reaches the critical mean stress (p_c), NorSand will follow the framework of the critical state theory (Jefferies, 1993). The combination of the Normality (associated flow rule) and the NorSand hardening rule (Equation (5-22) and Equation (5-25)) always leads to the current confining pressure (p) moving towards the image confining pressure (p_i), and the image confining pressure (p_i) towards the critical confining pressure (p_c). On the other hand, according to Equation (5-29) at critical state ($\psi_i = 0$), $(p_L/p_i) = 1$. This means that H (Equation (5-25)) will reach zero when the current state approaches the critical state. As a result, at critical state, the yield surface will not evolve, and the current state remains unchanged. It is quite clear that for the NorSand model, the location of critical state would be at ($\psi_i = 0$), $(p/p_i) = 1$, as shown in Figure 5-3.

5.4.8 Simulation of undrained behavior with NorSand

This section illustrates how NorSand simulates the undrained mechanical behavior of sand (Figure 5-1), by considering two samples, one being loose ($\psi_i > 0$) and the other dense ($\psi_i < 0$). Initially, both samples are positioned on the isotropic state of their respective yield surfaces with initially zero stress ratio and ($p/p_i = \exp(1)$). The initial states of these two samples are shown in Figure 5-4, in the stress-state space.

As the dense sample approaches the critical state, it undergoes a phase transformation, crossing the threshold of ($p/p_i = 1$) and transitioning from the contractive side to the dilative side of the yield surface. This phase transformation phenomenon is effectively captured by NorSand (Figure 5-4). Then, since (p/p_i) is on the dilative side, the sample starts to show a negative excess pore water pressure, which causes the (p/p_i) ratio to not fall below the (p_L/p_i) ratio. According to the hardening rule of the NorSand (Equation (5-25)), as long as ($p/p_i > p_L/p_i$), the yield surface continues to expand. This constant expansion allows the model to simulate the strain hardening behavior observed in dense samples (known as non-flow behavior) throughout their entire path until they reach critical state.

For the loose sample, (p/p_i) surpasses (p_L/p_i) before reaching critical state (Figure 5-4). This marks a change in the hardening regime, transitioning from the expansion of the yield surface to its shrinkage. This capability allows for the simulation of flow liquefaction. Notably, the entire stress-state path for the loose samples occurs in the contractive side of the yield surface (Figure 5-4), implying that the model will continuously experience the development of excess pore water pressure before reaching the critical state.

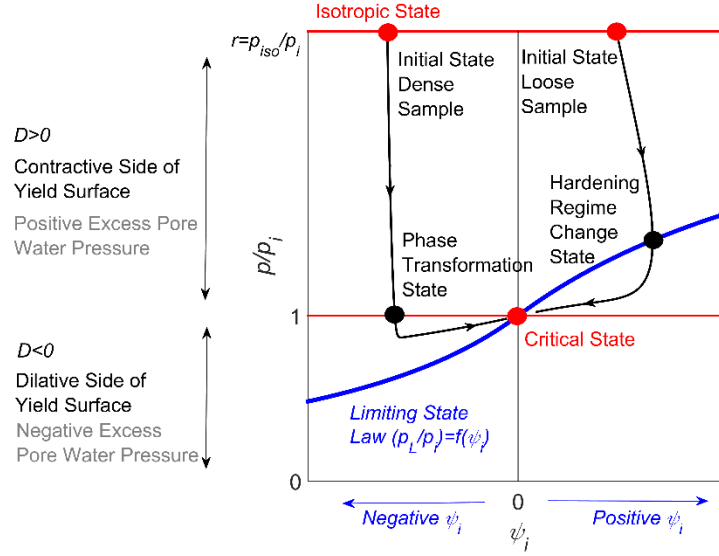


Figure 5-4 Stress-state path for undrained loose and dense samples in comparison with limiting state law

5.4.9 Limitations of the NorSand model

5.4.9.1 Limited Flow Behavior

Based on the discussion made in the previous section, the NorSand model (Jefferies, 1993; Jefferies and Been, 2015) can reproduce the non-flow behavior for dense samples and the flow behavior for loose samples. However, in its current formulation, the model is unable to simulate limited flow behavior.

5.4.9.2 Flow Behavior

Figure 5-3b illustrates the varying (p_L/p_i) ratio in relation to ψ_i for distinct χ_{tc} parameters. The legend associated with the 'lower χ_{tc} ' corresponds to values between 2 and 5, a range proposed by the developers of NorSand for χ_{tc} (Jefferies and Been, 2015). This range was obtained from their calibration process, which involved drained triaxial tests conducted on dense sand samples characterized by negative ψ_i values. Subsequently, the model uses the same χ_{tc} to simulate the undrained monotonic behavior. Under undrained conditions, a lower χ_{tc} for loose samples positions the limiting state on the yield surface near the image state, as shown in Figure 5-3a.

Consequently, loose samples will exhibit flow liquefaction accompanied by an instability stress ratio approximating M_i . This implies that the simulated flow liquefaction will yield a higher instability stress ratio within the suggested range value for χ_{tc} .

One potential solution is to use different values of χ_{tc} for drained and undrained conditions, with the latter requiring a higher value. However, due to Equation (5-29), if a higher χ_{tc} value for undrained conditions is considered, it results in a limiting state too close to the isotropic state. Consequently, a significantly reduced limiting stress ratio is generated by this approach, as demonstrated in Figure 5-3a and Figure 5-3b, resulting in a lower instability stress ratio for loose samples (Razavi and Yniesta, 2023). An additional issue emerges when employing higher χ_{tc} values. Illustrated in Figure 5-3b, at a specific ψ_i denoted as ψ_{imax} , the limiting state aligns with the isotropic state, resulting in a zero limiting stress ratio. Furthermore, beyond the threshold of ψ_{imax} , the NorSand model loses its applicability to simulate any mechanical behavior. ψ_{imax} can be defined as the maximum positive ψ_i , where the limiting state is going to be located on the isotropic state.

Since controlling NorSand limiting state law is not feasible, researchers have proposed different alternatives. One option is to modify the plastic modulus of the model by adding the cap-softening term (Jefferies and Been, 2015; Cheng and Jefferies, 2020) as follows:

$$H = h_0 \left(\frac{p}{p_i} - \frac{p_L}{p_i} \right) - S \frac{K}{P} \frac{\eta}{\eta_L} \frac{D}{1 + \frac{\chi_i}{M_{i,tc}} \lambda} \quad (5-35)$$

Wherein, S represents the Boolean switch (0 for drained and 1 for undrained loading). The only parameter that has been undefined in Equation (5-35) is $M_{i,tc}$, which is the M_i by replacing $M = M_{tc}$ in Equation (5-33). This addition improves the model's response and allows to generate flow behavior that has a lower instability stress ratio. However, one drawback of the cap-softening approach is its impact on the original model's plastic modulus, and its lack of flexibility to control the undrained behavior (Razavi and Yniesta, 2023), as the cap-softening is either activated or not.

Several researchers have suggested to further adjust the value of the bulk modulus as a controlling parameter (Shuttle and Jefferies, 2016; Reid and Smith, 2021; Ng et al., 2023) to enhance the undrained response of the model. The selection of the bulk modulus for this purpose can be explained by referring to the flow liquefaction criterion developed for elastoplastic constitutive models as proposed by Andrade et al. (2013) for a broad family of isotropic elastoplastic constitutive models. They showed that elastic bulk modulus directly affects the undrained mechanical behavior, and adjusting its value can influence the mechanical response.

5.4.9.3 Non-Flow Behavior

NorSand typically exhibits a high phase transformation stress ratio. The image stress ratio (Equation (5-33)) derived from Nova's rule (Nova and Wood, 1982) can effectively replicate the maximum stress ratio observed in dense samples under drained conditions. The reported N_d value for this purpose ranges from 0.3 to 0.5 (Jefferies and Been, 2015). Employing the same value under undrained conditions, however, induces a phase transformation stress ratio close to M . As a result, in the phase transformation state, N should be higher, so that it results in a lower M_i .

5.5 PolySand model formulation

Razavi and Yniesta, (2023) proposed a new limiting state law which makes it possible to introduce separate material properties under drained and undrained conditions. This paper aims to enhance this law to improve the NorSand model's accuracy in simulating the undrained mechanical behavior, limited flow, flow, and non-flow behavior. In this section the formulation of the PolySand model is presented. The same equations as NorSand are retained by PolySand for dilatancy, yield surface, critical state, state parameter, and the stress-strain relationship. As a result, the corresponding equations from Equation (5-6) to Equation (5-23) are also applicable to PolySand, along with Equation (5-34) for the elastic moduli.

5.5.1 PolySand Limiting State Law:

PolySand replace the NorSand limiting state law with the law introduced by Razavi and Yniesta (2023) as follows:

$$F_L = \frac{p_L}{p_i} = \begin{cases} \left(1 + (r^\xi - 1) \frac{\psi_i - \psi_\chi}{\chi(r^\xi - 1) + \psi_i - \psi_\chi} \right)^{\frac{1}{\xi}} & \psi_\chi \leq \psi_i \\ \left(1 + \frac{\psi_i - \psi_\chi}{\chi - (\psi_i - \psi_\chi)} \right)^{\frac{1}{\xi}} & \psi_i \leq \psi_\chi \end{cases} \quad (5-36)$$

Here, the term ψ_χ is the limited-flow state variable, while χ represents the PolySand dilatancy state variable. Other quantities have been defined in the previous sections. ψ_χ allows to model limited-flow but can be set to zero for simplicity.

The variation of the PolySand (p_L/p_i) ratio in relation to the χ parameter, with ψ_χ set to zero is illustrated in Figure 5-3d. This figure also provides insight into how PolySand determines the position of the limiting state on the yield surface, using Equation (5-36). Using the new law, the PolySand model gains the ability to smoothly adjust the position of the limiting state on the yield surface and prevent it from approaching the isotropic state excessively, even when dealing with high positive ψ_i values, by varying χ values in Equation (5-36). This flexibility is clearly demonstrated in Figure 5-3c and Figure 5-3d. Additionally, Equation (5-36) removes the restrictions on the model's applicability, consistently enabling the localization of the limiting state on the yield surface. Within Equation (5-36), ψ_{imax} can go to infinity, as clearly indicated in Figure 5-3.

The calibration of the dilatancy state variable (χ) can be achieved using test results where a transition from a strain-hardening to a strain-softening response is observed during shear loading. The hardening regime change indicates that $dp_i = 0$ which results in the current state and limiting state having the same location on the yield surface (based on Equation (5-25)). Therefore, current stress and state quantities (D , p , η , and ψ) at the hardening regime change, can be considered as (D_L , p_L , η_L , and ψ_L). Using these parameters, the dilatancy state variable (χ) can be calibrated. The tests where the samples show clear hardening regime change can be drained tests on dense samples ($\psi_i < 0$, $\frac{p_L}{p_i} < 1$), or undrained tests on loose samples ($\psi_i > 0$, $\frac{p_L}{p_i} > 1$).

However, under these conditions, the calibrated χ value would vary, and it is reasonable to employ distinct χ values for drained and undrained loading. Here, the calibrated χ values for drained and undrained loading will be denoted as χ_d and χ_u , respectively, and they will be referred to as the drained DSP and undrained DSP. Therefore, χ can be defined as follows:

$$\chi = S \cdot \chi_u + (1 - S) \cdot \chi_d \quad (5-37)$$

S represents a Boolean switch (0 for drained and 1 for undrained loading). Based on these explanations, χ_d in the PolySand is equivalent to χ_{tc} in the NorSand.

5.5.2 PolySand Image Stress Ratio

PolySand considers $M_i = M$ for loose samples ($\psi_i > \psi_\chi = 0$). For dense samples ($\psi_i \leq \psi_\chi = 0$), the model uses Equation (5-32), in which N_d has been replaced with N .

$$M_i = \begin{cases} M & \psi_\chi \leq \psi_i \\ \frac{M}{1 - N \ln(F_d)} & \psi_i \leq \psi_\chi \end{cases} \quad (5-38)$$

$$F_d = \begin{cases} \left(1 + (r^\xi - 1) \frac{\psi_i - \psi_\chi}{\chi_d (r^\xi - 1) + \psi_i - \psi_\chi} \right)^{\frac{1}{\xi}} & \psi_\chi \leq \psi_i \\ \left(1 + \frac{\psi_i - \psi_\chi}{\chi_d - (\psi_i - \psi_\chi)} \right)^{\frac{1}{\xi}} & \psi_i \leq \psi_\chi \end{cases} \quad (5-39)$$

F_d corresponds to $F_L = (p_L/p_i)$ from Equation (5-36) with $\chi = \chi_d$. N in Equation (5-38) is defined as follows.

$$N = S \cdot N_u + (1 - S) \cdot N_d \quad (5-40)$$

N_u and N_d are calibration-based parameters and represent the drained and undrained dilatancy parameters responsible for controlling the dilatancy through the image stress ratio for dense samples ($\psi_i \leq \psi_\chi = 0$). N_u controls the stress ratio during phase transformation under undrained conditions, whereas N_d controls the maximum stress ratio in drained conditions, for dense samples. Notably, N_d serves a comparable role to the drained shear-coupling volume factor in the NorSand model.

5.5.3 PolySand Hardening Parameter

PolySand uses the same plastic modulus of the original NorSand model (Equation (5-25)) with the following hardening parameter (h_0).

$$h_0 = \frac{M}{M_{tc}} \frac{H_0 - H_y \psi_i}{F_d} \left(\frac{p_i}{p_{ref}} \right)^{-0.5} \left(\frac{p}{p_i} \right) p_i \quad (5-41)$$

h_0 has the same equation as Equation (5-26) for the NorSand. However, the (p_L/p_i) ratio in the denominator has been replaced with F_d . Equation (5-41) additionally incorporates the term $(p_i/p_{ref})^{-0.5}$ which results in a reduction of the hardening parameter with increasing confining pressure, improving the ability to simulate the plastic modulus more effectively. This term has also been used by previous researchers (Dafalias and Manzari, 2004).

Finally, the PolySand material properties are summarised in Table 5-1. Compared to NorSand, PolySand includes two additional properties (χ_u , and N_u) while sharing the same material properties for the elastic moduli, critical state, shear-volume coupling factor (N_d), and two hardening parameters (H_0 , H_y). The concept behind χ_d in PolySand aligns with that of χ_{tc} in NorSand, as both will undergo the same calibration through drained tests.

Table 5-1 PolySand material constants and their contribution in the PolySand formulation

	Equation Number	Material constants
Critical state line Critical stress ratio	Equation (5-14) Equation (5-15)	$\Gamma, \lambda, \xi, M_{tc}$
Elastic modulus	Equation (5-34)	G_0, ν
Limiting State Law	Equation (5-36)	χ_d, χ_u
Image Stress Ratio	Equation (5-38)	N_d, N_u
Hardening parameter	Equation (5-41)	H_0, H_y

5.6 Calibration

The triaxial compression test data conducted on the standard Japanese Toyoura sand, provided by Verdugo and Ishihara (1996), is used here to show the capacity of PolySand in simulating the behavior of granular soils. The Toyoura sand tested is characterized by a mean diameter of 0.17mm, a uniformity coefficient of 1.7, maximum void ratio of 0.977, minimum void ratio of 0.597, and a specific gravity of 2.65.

The elastic moduli and critical state line used in this study are similar to previous studies using the data (Li et al., 1999; Li and Dafalias, 2000; Dafalias and Manzari, 2004) The remaining constants are model-specific and calibrated using the data.

5.6.1 Parameters calibrated from drained test results:

NorSand drained shear volume coupling factor or PolySand drained dilatancy parameter (N_d): For the drained test results on dense samples, the drained hardening regime change is characterized by $d\eta = 0$. The point with $d\eta = 0$ is associated with minimum dilatancy ($D = D_{min} < 0$), maximum stress ratio ($\eta = \eta_{max} > M$), and the corresponding state parameter shown with ($\psi = \psi_{min}$). By collecting scattered data points ($D_L = D_{min}$, $\eta_L = \eta_{max}$, and $\psi_L = \psi_{min}$) obtained from various test results, one can calibrate N_d using Nova's rule as outlined in

Equation (5-30). For Toyoura Sand, a value of 0.33 for N_d fits the experimental data well (Figure 5-5a).

Drained dilatancy state parameter for PolySand (χ_d) and NorSand (χ_{tc}): The data utilized for calibrating N_d is also used to calibrate χ_d and χ_{tc} . For NorSand (Jefferies, 1993; Jefferies and Been, 2015) D_L and ψ_L are related as follows:

$$D_L = \chi_{tc} \cdot \psi_L \quad (5-42)$$

However, the PolySand model necessitates a procedure to establish the relationship between D_L and ψ_L . In the case of dense states ($\psi_i < \psi_\chi = 0$), the determination of the limiting dilatancy for the PolySand model can be achieved using Equation (5-10), Equation (5-38), and Equation (5-36) as outlined below:

$$\begin{aligned} D_L &= M_i \cdot \ln\left(\frac{p_L}{p_i}\right) = \frac{M}{1 - N \ln(F_d)} \cdot \ln\left(\frac{p_L}{p_i}\right) \\ &= \frac{M}{1 - \frac{N}{\xi} \ln\left(1 + \frac{\psi_i - \psi_\chi}{\chi_d - (\psi_i - \psi_\chi)}\right)} \cdot \frac{\ln\left(1 + \frac{\psi_i - \psi_\chi}{\chi_d - (\psi_i - \psi_\chi)}\right)}{\xi} \end{aligned} \quad (5-43)$$

On the other hand, the limiting state parameter for ($\psi_i < \psi_\chi = 0$) can be obtained using Equation (5-21) and replacing (p_L/p_i) with from Equation (5-36) and considering $\chi = \chi_d$.

$$\psi_L = \psi_i + (\psi_i + \Gamma - e) \cdot \frac{\psi_i - \psi_\chi}{\chi_d - (\psi_i - \psi_\chi)} \quad (5-44)$$

For a unique ψ_i value, D_L and ψ_L can be calculated using Equation (5-43) and Equation (5-44), respectively, to obtain a new curve in the $(D_L - \psi_L)$ space, which depends on $(\Gamma - e)$, and χ_d .

Figure 5-5b shows the (D_L, ψ_L) scatter data for the Toyoura Sand. By selecting $\chi_d = 0.6$ and $\chi_{tc} = 2.45$, the NorSand and PolySand (D_L, ψ_L) curves can be plotted as shown in Figure 5-5b. The drained tests were performed at $\Gamma - e \cong 0.1$, and with proper calibration, both models should yield similar simulation results under drained conditions.

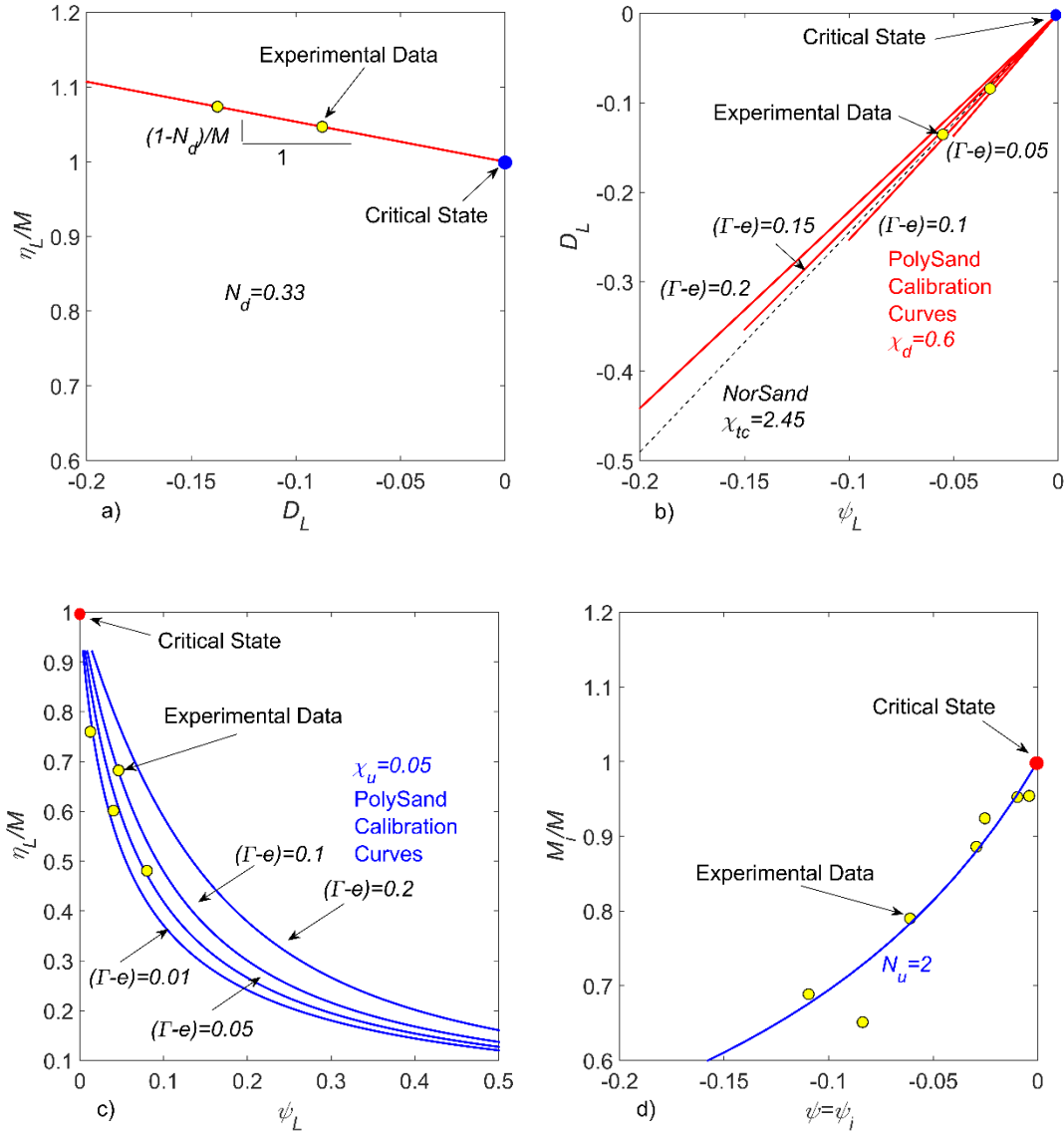


Figure 5-5 Calibration of PolySand and NorSand material constants, drained test data on dense Toyoura sand (a) N_d (b) NorSand (χ_{tc}) and PolySand (χ_d), undrained test data for loose Toyoura sand, (c) PolySand (χ_u), (d) PolySand (N_u), data extracted from Verdugo and Ishihara (1996)

5.6.2 Parameters calibrated from undrained test results:

Undrained DSP (χ_u) for PolySand: For loose samples with ($\psi_i > 0$), identifying the precise point where the undrained hardening regime transition ($dp_i = 0$) occurs is not feasible visually from test data. The only detectable point is the undrained peak shear strength or flow instability point where $dp_i \neq 0$ while ($\frac{dq}{dp} = 0$) (See Figure 5-1). The corresponding state parameter and stress ratio for the instability point (IP) can be represented by (η_{IP}, ψ_{IP}). This point appears slightly after the undrained hardening regime transition initiates. As a result, the initial calibration of χ_u can be done by fitting the (η_L, ψ_L) equation to the (η_{IP}, ψ_{IP}) data collected from undrained tests. If necessary, this value can be adjusted later on to increase the accuracy of the simulations.

By substituting the (p_L/p_i) ratio from Equation (5-36) and $M_i = M$ from Equation (5-38) into Equation (5-11), the following expression can be derived for loose samples ($\psi_i > \psi_\chi = 0$), under undrained conditions:

$$\eta_L = M_i \cdot \left(1 - \ln \left(\frac{p_L}{p_i} \right) \right) = M \cdot \left(1 - \frac{1}{\xi} \ln \left(1 + (r^\xi - 1) \frac{\psi_i - \psi_\chi}{\chi_u(r^\xi - 1) + \psi_i - \psi_\chi} \right) \right) \quad (5-45)$$

Note that under undrained condition $\chi = \chi_u$. The limiting state parameter for loose samples ($\psi_i > \psi_\chi = 0$) can be obtained using Equation (5-21) by replacing (p_L/p_i) from Equation (5-36) and considering $\chi = \chi_u$.

$$\psi_L = \psi_i + (\psi_i + \Gamma - e)(r^\xi - 1) \frac{\psi_i - \psi_\chi}{\chi_u(r^\xi - 1) + \psi_i - \psi_\chi} \quad (5-46)$$

For a unique ψ_i value, η_L and ψ_L can be calculated to obtain a new curve in the $(\eta_L - \psi_L)$ space that depends on $(\Gamma - e)$, and χ_u . Figure 5-5c shows the (η_{IP}, ψ_{IP}) scatter data for the Toyoura Sand. The figure also displays the PolySand calibration curves for $\chi_u = 0.05$ representing different sets of $(\Gamma - e)$ values, along with the experimental data for Toyoura Sand. Note that the undrained drained tests were performed at $\Gamma - e \cong 0.02 - 0.1$.

Undrained dilatancy parameter (N_u): This parameter controls the phase transformation that takes place during undrained loading in dense samples ($\psi_i < \psi_\chi = 0$). During phase transformation, the current state aligns with the image state, implying that either $p = p_i$ or $\psi = \psi_i$. By utilizing $N = N_u$ under undrained condition and substituting it into Equation (5-38) for dense samples, the following equation is obtained:

$$M_i = \frac{M}{1 - N_u \ln\left(\frac{p_L}{p_i}\right)} = \frac{M}{1 - N_u \ln\left(\left(1 + \frac{\psi - \psi_\chi}{\chi_d - (\psi - \psi_\chi)}\right)^{\frac{1}{\xi}}\right)} \quad (5-47)$$

This equation can be fitted to the data at phase transformation state, thereby enabling the determination of N_u . Figure 5-5d shows the experimental data and the fitted curve for $N_u = 2$.

5.6.3 Parameters calibrated through trial-and-error methods:

The remaining parameters for both NorSand and PolySand including (H_0 , and H_y) are determined iteratively to achieve a match between simulation results and experimental data. To begin, one can assume that $H_y = 0$. Subsequently, the optimal value for H_0 is obtained through a trial-and-error approach. If needed, adjustments to the values of H_y can be made to yield the best-

fitting results. The calibrated PolySand and NorSand material properties for the Toyoura Sand model are shown in Table 5-2.

Table 5-2: PolySand and NorSand model properties

Elasticity	Critical state	State-dilatancy parameter	Shear-volume coupling factor	Plastic modulus
$G_0 = 294$ $\nu = 0.05$	$M_{tc} = 1.25$ $\Gamma = 0.934$ $\lambda = 0.019$ $\xi = 0.7$	NorSand $\chi_{tc} = 2.45$	NorSand $N_d = 0.33$	NorSand $H_0 = 80$ $H_y = 500$
		PolySand $\chi_d = 0.6$ $\chi_u = 0.05$	PolySand $N_d = 0.33$ $N_u = 2$	PolySand $H_0 = 100$ $H_y = 600$

5.7 Control of flow and non-flow behavior in the PolySand model

In PolySand, the undrained peak shear strength for loose states is controlled by the undrained DSP (χ_u). Figure 5-6a and Figure 5-6b illustrate the impact of different values for χ_u on the stress-strain and stress path of a loose sample of Toyoura Sand. The response for $\chi_u=0.05$ can be considered as the target or desired response, expected to closely align with the measured data for loose Toyoura sand. The inclusion of other χ_u values is intended to highlight the model's ability to generate distinct flow behaviors when different values of χ_u are selected. Lower χ_u values result in lower undrained peak shear strengths at lower instability stress ratios. Furthermore, the Figure 5-6a and Figure 5-6b showcase the responses produced by the NorSand model, both with and without the cap-softening term. While the incorporation of the cap-softening feature improves the NorSand response, it still differs from the expected behavior.

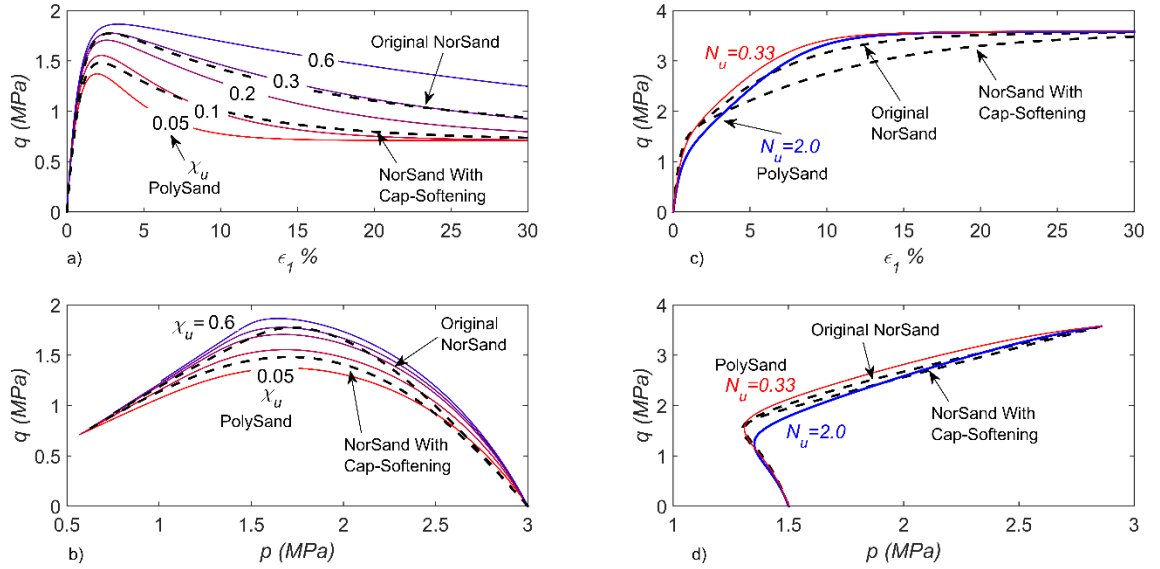


Figure 5-6 Influence of (a,b) χ_u , the undrained DSP, on the flow behavior of loose sample, $e_0=0.87$, $p_0=3$ MPa, (c,d) N_u , the undrained dilatancy, on the non-flow behavior of dense sample, $e_0=0.735$, $p_0=1.5$ MPa

Figure 5-6c and Figure 5-6d illustrate the impact of undrained dilatancy (N_u) on the non-flow behavior of a dense sample. N_u controls the phase transformation stress ratio (PTSR) and higher N_u values result in a reduction of PTSR. The figures also show the responses for the NorSand model, where both the models with and without cap-softening exhibit PTSR values corresponding to $N_u = N_d = 0.33$. Since the calibrated N_u value based on experimental results is 2, this indicates that NorSand yields a higher PTSR than what would be expected. In the rest of the paper, under undrained conditions, only the NorSand with cap-softening model will be compared to PolySand.

5.8 Validation

5.8.1 Drained Response

Figure 5-7 depicts the results of simulations conducted using the PolySand and NorSand (represented by red and blue solid lines, respectively), alongside the corresponding laboratory data

(represented by dashed lines) at two different initial confining pressures (100 and 500 kPa). These results demonstrate the robust capability of both PolySand and NorSand models in accurately simulating the drained behavior of granular soils, even when subjected to a wide range of initial void ratios.

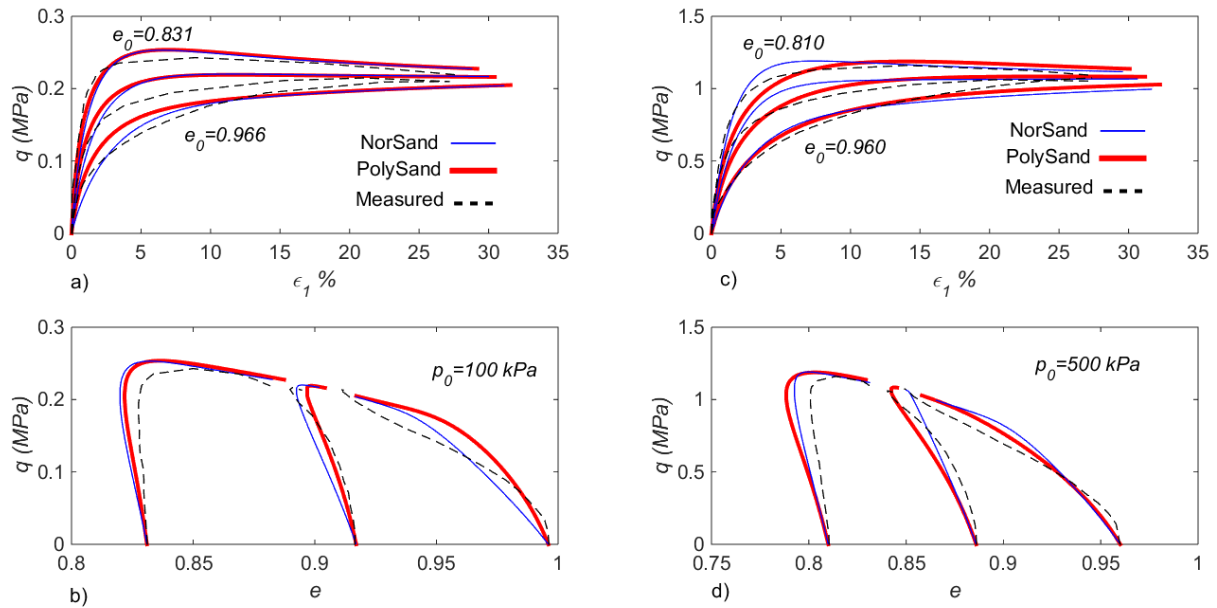


Figure 5-7 Simulation results using PolySand and NorSand versus measured data in drained compression tests on Toyoura sand (Verdugo and Ishihara 1996), (a,b) $p_0 = 100$ kPa, (c,d) $p_0 = 500$ kPa

5.8.2 Undrained Response

Figure 5-8 and Figure 5-9 present the results of simulations conducted using the PolySand and NorSand (with cap-softening) models, compared to experimental data. The samples in Figure 5-8 have an initial void ratio of 0.735, and they all initially exhibit a negative state parameter. Consequently, all the samples should display non-flow behavior. Both models capture the overall the non-flow patterns of soil behavior. However, PolySand has successfully captured the phase transformation stress ratio due to its utilization of the new material property (N_u), while NorSand exhibits a higher phase transformation stress ratio. Regarding the stress-strain behavior, PolySand also demonstrates patterns that are closer to the measured data.

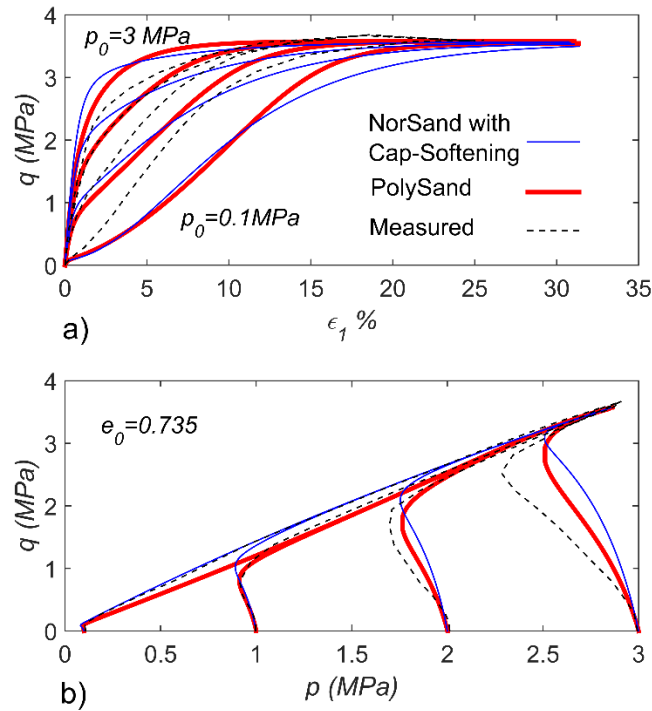


Figure 5-8 Simulation results using PolySand- and NorSand versus measured data in undrained compression tests on Toyoura sand (Verdugo and Ishihara 1996), $e_0=0.735$, a) deviatoric shear stress-axial strain, b) effective stress path in q - p

In Figure 5-9a and Figure 5-9b, the samples initially have a void ratio of 0.833. Among them, two samples (with $p_0=3$ MPa and $p_0=2$ MPa) have a loose behavior, while two samples (with $p_0=1$ MPa and $p_0=0.1$ MPa) exhibit dense behavior. In both cases, whether for loose or dense samples, PolySand emerges as a highly effective model, successfully capturing both flow and non-flow behavior. In contrast, despite the inclusion of a cap-softening term, the NorSand model simulates the corresponding behavior less accurately. A similar trend is observed in Figure 5-9c and Figure 5-9d, which show simulation results for samples with an initial void ratio of 0.907. PolySand closely matches the expected behavior. Overall, these findings highlight PolySand's proficiency in simulating the undrained behavior of sand and the possibility for users to control

both flow and non-flow behavior using two additional material properties, denoted as (χ_u) and (N_u) , respectively.

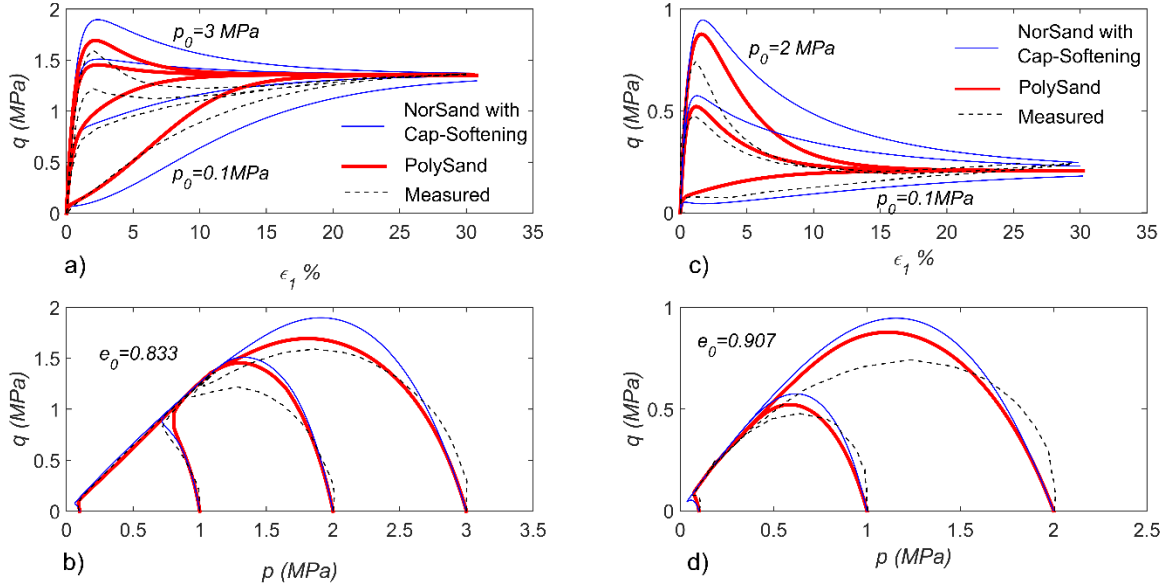


Figure 5-9 Simulation results using PolySand and NorSand versus measured data in undrained compression tests on Toyoura sand (Verdugo and Ishihara 1996), (a,b) $e_0=0.833$, (c,d) $e_0=0.907$

5.9 Adding limited flow behavior into PolySand

As emphasized in Section 5.4.8, both the original NorSand model and PolySand, which incorporates the original NorSand plastic modulus (Equation (5-25)), are bi-state models. This means they can only simulate either flow or non-flow behavior, lacking the capability to reproduce limited flow behavior (Figure 5-1). Figure 5-9 shows that loose samples of Toyoura sand exhibit limited flow behavior to some extent, and PolySand was not able to reproduce such behavior. It is worth noting that NorSand with a cap-softening term, may exhibit this feature, as demonstrated in Figure 5-9c and Figure 5-9d for the sample with $e_0 = 0.907$ and $p_0 = 0.1$ MPa. In this section, the capability of PolySand to simulate the limited flow feature is discussed.

In the previous examples, the limited-flow state variable ψ_χ , integrated in PolySand's limiting state law (Equation (5-36)) was set to zero. If this variable is assigned a negative constant value ($\psi_\chi < 0$), it results in the translation of the PolySand's limiting state law along the ψ_i axis, as illustrated in Figure 5-10. The application of the limiting state law with a nonzero negative ψ_χ and employing the plastic hardening modulus outlined in Equation (5-25) leads to the convergence of all shear modes towards a new point at $(\psi_\chi, p/p_i = 1)$. In other words, all shear modes will reach a line that is parallel to the CSL, shifted downwards by an amount corresponding to ψ_χ . The corresponding line is referred to as quasi-critical state line (QCSL). $\psi_\chi = 0$ means that the QCSL is the same as CSL.

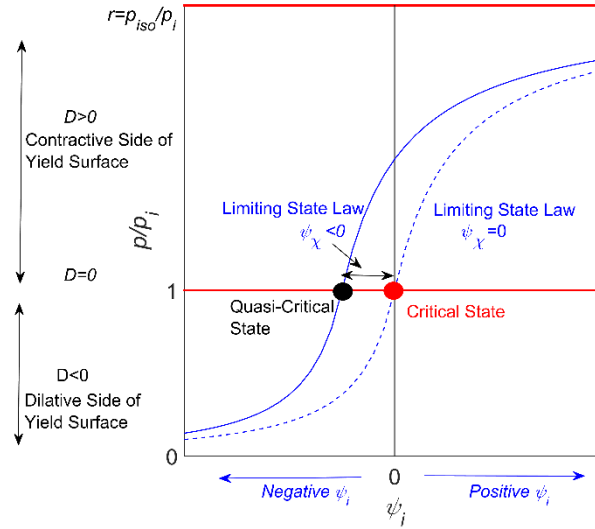


Figure 5-10 Effect of the limited-flow state variable, ψ_χ , on the limiting state law's position in $(p/p_i, \psi_i)$ space

If ψ_χ was to become zero through further shearing, the sample would attain a quasi-critical state near $\psi_i = \psi_\chi$, and with continued shearing until ψ_χ reaches zero, it would ultimately reach the true critical state. To this end, ψ_χ can be defined as follows:

$$\psi_\chi = \psi_A A_{\varepsilon_Y}^p \quad (5-48)$$

Where ψ_A is a constant negative material property, and $A_{\varepsilon_v^p}$ is a transition function transitioning from 1 to zero to drive ψ_χ from ψ_A to zero at large strains. This approach, originally introduced by Li and Dafalias (2012), was aimed at extending critical state-based models to account for the influence of evolving fabric on the mechanical behavior of sand. In this context, to demonstrate how limited flow behavior can be incorporated into the PolySand model, the same function for $A_{\varepsilon_v^p}$ that was employed by Li and Dafalias (2012) for triaxial compression is used.

$$\psi_\chi = \psi_A \cdot \exp(-c \cdot \varepsilon_v^p) \quad (5-49)$$

Where c is a constant controlling the rate of transition from 1 to zero. According to Li and Dafalias (2012), ψ_A is a material property that depends on the confining pressure, void ratio, loading direction tensor, fabric direction tensor, and the initial fabric structure of the soil. However, for the purpose of this study, it is assumed that ψ_A is a constant. This assumption is made solely for the purpose of illustrating how limited flow behavior can be integrated into the PolySand model. The consideration of the influence of all these variables out of the scope of this paper, and should be conducted within the framework of anisotropic critical state theory as proposed by Li and Dafalias (2012).

Figure 5-11a and Figure 5-11b illustrate the influence of ψ_A on the undrained behavior of loose samples. The behavior transitions from flow to limited flow behavior with the selection of higher negative values for ψ_A . Furthermore, Figure 5-11c and Figure 5-11d presents the PolySand simulation results, incorporating nonzero values for ψ_χ , for undrained compression tests conducted on Toyoura sand (Verdugo and Ishihara, 1996), with an initial void ratio of $e_0 = 0.833$. Here, ψ_A and c have been set to -0.01 and 8 , respectively. As observed, the utilization of Equation (5-49) improves PolySand simulation results.

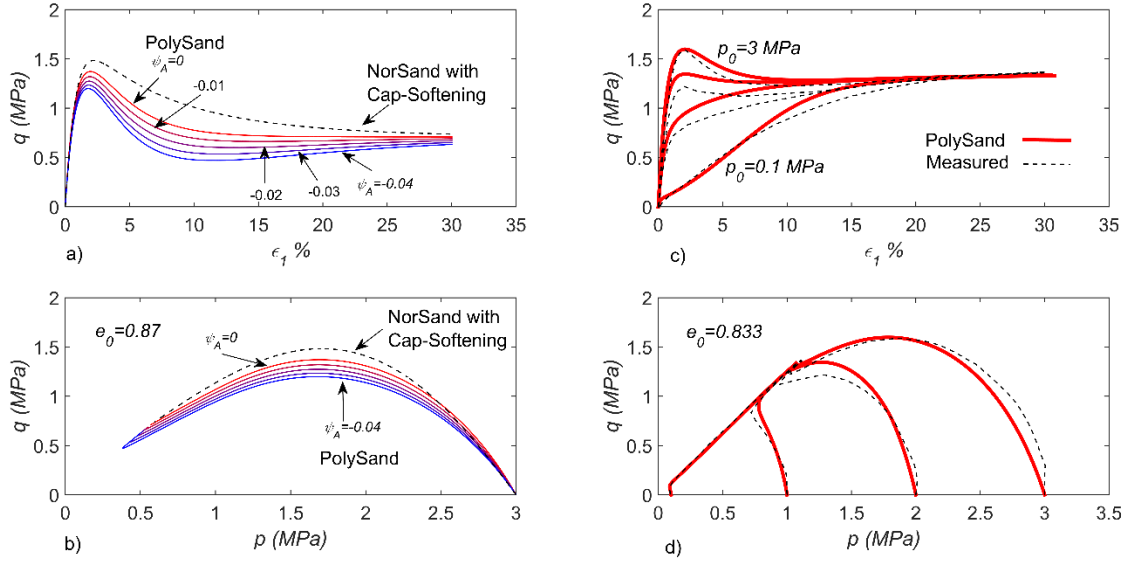


Figure 5-11 (a,b) Influence of ψ_A , the anisotropy parameter enabling the simulation of the limited flow behavior, $e_0=0.87$, $p_0=3\text{MPa}$, (c,d) PolySand with limited flow feature to simulate the undrained compression tests on Toyoura sand (Verdugo and Ishihara 1996), $e_0=0.833$

5.10 Conclusion

This paper introduces a new single-yield surface, critical state-compatible model called PolySand to better capture the undrained behavior of granular soils. It builds upon the original NorSand model (Jefferies, 1993), incorporating novel formulations for the relationship between limiting dilatancy and the limiting state parameter, commonly referred to as state-dilatancy laws.

The introduction of a new limiting state law enables the definition of distinct material properties to control drained and undrained behavior in the PolySand model, achieved through the drained DSP and undrained DSP, respectively. Undrained DSP is a new material property which adjusts the transition of the hardening regime, particularly in loose states under undrained conditions. Consequently, users possess the capability to directly manage the model's initiation of flow (static) liquefaction. The undrained DSP holds clear physical meaning and can be calibrated based on the peak undrained shear strength of loose samples and their corresponding state parameters.

Furthermore, by incorporating the new limiting state law, the PolySand model eliminates the need for cap-softening introduced in the newer versions of the NorSand model. As a result, the plastic modulus of the original NorSand remains unchanged in both drained and undrained loading conditions. Using the original NorSand plastic modulus is appealing, since it makes it possible to add the limited flow feature into the model's formulation through the limiting state law. To illustrate the integration of limited flow behavior into the model, a straightforward approach has been employed in the model's formulation, resulting in improved model performance. However, it is essential to note that this introduced simple approach is intended to demonstrate the concept's feasibility, and further research is needed to finalize its formulation. Nonetheless, the study highlights the model's adaptability and its potential to leverage the anisotropic critical state theory (ACST) framework for integration into its future versions.

To assess the effectiveness, and reliability of the PolySand model, a comparison is conducted between the model's predictions and laboratory test results obtained from well-documented experiments on Toyoura sand. The outcomes of this comparison yield compelling evidence that the PolySand is well-suited to simulate the drained and undrained monotonic behavior of granular materials. In addition, the response of the PolySand model is compared with that of the NorSand model. Both models exhibit similar responses for drained conditions, but, due to the introduction of new controlling material properties for undrained conditions, PolySand produces more accurate results in undrained triaxial tests. This highlights the advantages of PolySand in accurately capturing the undrained behavior of granular material.

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5.12 Data Availability Statement

Data generated or analyzed during this study are available from the corresponding author upon reasonable request.

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CHAPTER 6 ARTICLE 4: POLYSAND-WFINES MODEL – EXTENDED POLYSAND MODEL TO SIMULATE THE MONOTONIC BEHAVIOR OF GRANULAR MATERIALS WITH DIFFERENT FINES CONTENT

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6.1 Abstract

Traditional constitutive models simulate the behavior of sand-silt mixtures by assigning varying material properties based on fines content. An alternative approach is defining their behavior within the equivalent critical state theory (Eq-CST) framework using a unified set of material properties. Within this approach, soil models that incorporate bounding surface plasticity have been mostly tested and validated. While Cambridge-type models are popular in practice, they haven't been updated to represent the monotonic behavior of sand-silt mixtures. To address this gap, a recently developed model in this category, known as PolySand, has been selected for extension, and it will be referred to as PolySand-Wfines. While extended models within the Eq-CST framework are typically applied to sand-silt mixtures dominated by coarse grains, PolySand-Wfines is further tested for mixtures governed by fine grains. The findings suggest that this theory may be applicable to mixtures controlled by fine grains as well. To evaluate the PolySand-Wfines model's simulation capabilities, simulations of undrained triaxial tests of sand-silt mixtures are performed and compared with laboratory data published in the literature. Analysis of the simulation results highlights the good performance of the model in properly simulating the flow and limited flow behavior of sand-silt mixtures with different fines contents.

6.2 Introduction

The monotonic behavior of granular materials under undrained shear loading can be categorized into three main types: Flow behavior, limited flow behavior, and non-flow behavior (Verdugo and Ishihara, 1996) as illustrated in Figure 6-1. Flow behavior is typically observed in loose soil samples, where the samples undergo strain softening after reaching a peak deviatoric stress known as the instability point (IP). Limited flow behavior follows a similar pattern to flow behavior but transitions to strain hardening after the initial strain softening, reaching a state referred to as the quasi-critical state. Non-flow behavior, on the other hand, is characteristic of dense soil samples. In this case, the samples undergo a decrease in mean effective stress followed by a phase transformation state and subsequent increase in mean effective stress. Notably, non-flow behavior does not exhibit strain softening.

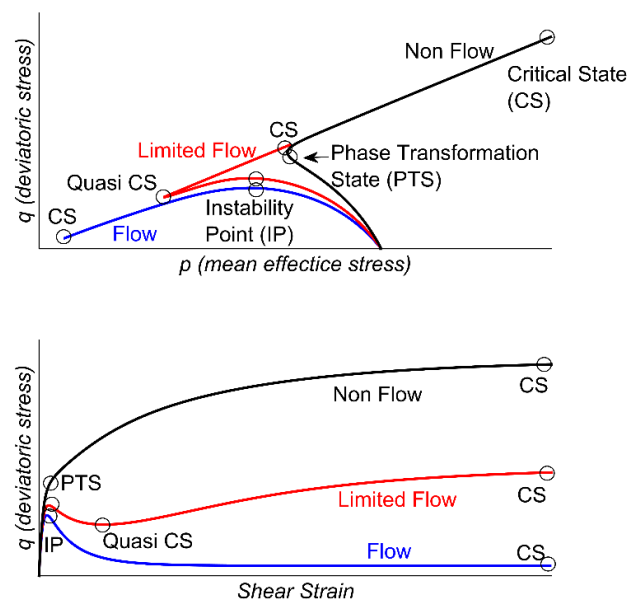


Figure 6-1 Typical undrained behavior of granular materials

Granular materials comprise sands, but also finer soils, and sand-silt mixtures with different fines content. For example, mine tailings can range from fine-grained to coarse-grained or a combination of both. Within the same tailings facility, there is usually spatial variations in both

grain size and fines content, with tailings being coarser near the point of slurry discharge and finer in regions farther away from the discharge point (Torres-Cruz and Santamarina, 2020). This spatial variability in tailings grading can have a significant impact on the overall behavior of tailings (Figure 6-1) and tailings dams (Wijewickreme et al., 2005; Carrera et al., 2011; Jefferies et al., 2019). According to the findings of Arroyo and Gens (2021), the alteration or relocation of tailings layers with varying fines content can influence the level of risk associated with flow liquefaction in tailings dams. This is primarily due to the influence of fines content on the undrained mechanical behavior of sand-silt mixtures, which not only affects the microstructure but also the material properties.

Multiple studies have aimed at understanding the influence of fines content on the undrained and drained behavior of sand-silt mixtures by conducting experimental investigations (e.g. Zlatović and Ishihara, 1995; Lade and Yamamuro, 1997; Lade et al., 1998; Polito and Martin, 2001; Thevanayagam et al., 2002; Yang et al., 2006; Murthy et al., 2007; Belkhatir et al., 2011; Chang et al., 2011; Rahman and Lo, 2012; Yang et al., 2015; Torres-Cruz and Santamarina, 2020; Yilmaz et al., 2023;). Their results have been influential in developing frameworks for interpreting the drained and undrained behavior of sand-silt mixtures characterized by similar mineralogy and grain shape in both the sand and silt fractions (Rahman and Lo, 2008; Rahman et al., 2014; Lashkari, 2014; Yang et al., 2015; Mohammadi and Qadimi, 2015). These frameworks redefine the critical state theory to account for the impact of fines and influence the positioning of the critical state line. As a result, these approaches facilitate the extension of existing sand constitutive models within the critical state theory framework (e.g. Wood et al., 1994; Manzari and Dafalias, 1997; Golchin and Lashkari, 2014) to accurately simulate the behavior of sand-silt mixtures (Rahman et al., 2014; Lashkari, 2016; Yin et al., 2016; Rahman and Dafalias, 2023). The accuracy level of the extended model depends on the selection of the base sand model and how well the impact of fines is incorporated into the extended model. Rahman et al. (2014) adopted the boundary surface model proposed by Manzari and Dafalias (1997) using the concept of equivalent void ratio, which is applicable for low fines content, while Yin et al. (2016) attempted to extend a single yield surface plasticity model with a non-associated flow rule using the global void ratio approach. Lashkari (2016) extended the bounding surface plasticity model initially proposed for sand by Golchin and Lashkari (2014) to simulate sand-silt mixtures.

Despite these advancements and the development of constitutive models specifically for sand-silt mixtures (Lashkari, 2016; Sun et al., 2021; Rahman and Dafalias, 2023), the NorSand model (Jefferies 1993; Jefferies and Been, 2015), a Cambridge type model, is the standard of practice to model the behavior of tailings (Morgenstern et al., 2016; Jefferies et al., 2019). This preference is primarily due to its ease of use, particularly in terms of material property determination, as well as its limited number of material constants. However, a significant drawback of the NorSand model is the limited control it offers users over the undrained behavior of granular materials, particularly regarding flow liquefaction (Shuttle and Jefferies, 2016; Razavi and Yniesta, 2023; Razavi, 2023). Furthermore, the NorSand model has yet to been extended within the mentioned frameworks to simulate the behavior of sand-silt mixtures.

The PolySand model (Razavi and Yniesta, 2023) was introduced with the purpose of expanding upon the NorSand model through the incorporation of a novel nonlinear state-dilatancy law, granting PolySand greater flexibility in simulating the undrained behavior of granular materials. Notably, PolySand enables the utilization of distinct material properties in both drained and undrained loading conditions, providing users enhanced control over the modeling process. Furthermore, PolySand is able to simulate the limited flow behavior. The model maintains the practicality of the NorSand model, as the newly defined material properties have clear physical interpretations and can be obtained through a straightforward procedure (Razavi and Yniesta, 2023).

The primary objective of this paper is to extend the PolySand model using the equivalent void ratio approach, with the aim of providing a practical constitutive model for sand-silt mixtures. Toward this objective, a discussion of the microstructure of sand-silt mixtures, considering varying fines content, is first presented. This provides insights into the characteristics of coarse and fine governed mixtures, which will be utilized within the framework of equivalent critical state theory. Subsequently, the enhanced formulation of PolySand, referred to as PolySand-Wfines, is introduced. The model's calibration process is presented, demonstrating its adaptability and practicality. The model is then employed to simulate the monotonic behavior of two specific sand-silt mixtures: Sydney sand with fines (Rahman et al., 2014) and Foundry sand with fines (Thevanayagam et al., 2002).

6.3 Effect of Fines Content on the Microstructure of Sand-Silt Mixtures

As per the Unified Soil Classification System (USCS), particles with a size range of 0.01 to 0.075 mm are classified as silt (also referred to as fines in the present study), while those within the range of 0.075 to 4.75 mm are classified as sand (referred here as coarse). Consequently, when analyzing a coarse-fine mixture, a grain size distribution curve can be defined for both the coarse and fine fractions. To characterize the grain size distribution, it is typical to use D_{10} and D_{50} , which are known as the effective and mean particle diameter size, respectively. D_x denotes the particle size that $x\%$ of the grains are finer than on the grain size distribution curve. In this context, the symbols D and d are used for the sand and silt part of a mixture, respectively. The particle diameter ratio for a sand-silt mixture can be defined as follows (Thevanayagam et al., 2002; Rahman et al., 2014; Yin et al., 2016):

$$R = \begin{cases} R_c = \frac{D_{10}}{d_{50}} & \text{when the coarse grains control the structure} \\ R_f = \frac{D_{50}}{d_{50}} & \text{when the fine grains control the structure} \end{cases} \quad (6-1)$$

A binary mixture with different proportions of coarse and fine grains can be created which will vary in terms of relative density of the package (D_R) and fines content (FC). The structure of the mixtures can follow two different mechanisms: a filling mechanism, where the fine particles fill part of the voids between the coarse grains but the package's skeleton is controlled by the coarse grains, and an embedment mechanism, where the coarse grains are surrounded by a packing of fine particles, and the fines control the mechanical behavior. The two extreme cases are when the percentage of fine grains is 0 and 100 (Change and Deng, 2020), where their behavior is controlled by the filling mechanism and the embedment mechanism respectively. For mixtures with fines and coarse fractions of similar nature, but different fines content, a change from filling mechanism to embedment mechanism occurs at a given fines content called threshold fines content (FC_{th}) (Thevanayagam et al., 2002).

6.3.1 Filling Mechanism

Assuming a coarse-fine package with different fines content and a fixed relative density and particle diameter ratio, the following equation can be used to determine the package's void ratio (Thevanayagam et al., 2002).

$$e = e_c^* - (1 - b)(1 + e_c^*) \frac{FC}{100} \quad FC < FC_{th} \quad (6-2)$$

The intergranular equivalent void ratio, e_c^* , is the void ratio of the package when $FC = 0$, and is solely dependent on D_R . b is a microstructure parameter called fines participation factor which measures the active fraction of fine grains involved in contact force transfer in a fine-coarse package, and ranges from 0 to 1. A value of zero corresponds to no participation, while a value of one represents full participation of the fine grains (Thevanayagam et al., 2002, Rahman and Lo, 2012). By setting $b = 0$ and $b = 1$ in Equation (6-2), a lower and upper bonds for the variation of void ratio with FC are obtained.

$$e = e_c^* \quad \text{Upper bond, } b = 1 \quad (6-3)$$

$$e = e_c^* - (1 + e_c^*) \frac{FC}{100} \quad \text{Lower bond, } b = 0 \quad (6-4)$$

b is highly dependent on the particle diameter ratio R_c , FC , and D_R as follows.

Effect of R_c on b : When R_c is 1, the coarse and fine grains are the same size, and all the grains are active resulting in $b = 1$. When R_c is significantly larger than 1 ($R_c \gg 1$), fine grains possess a much smaller diameter in comparison to the coarse particles. As a result, they can easily

occupy the void space between the coarse particles, and none of them contribute to supporting the coarse-grain skeleton, leading to $b = 0$.

Effect of FC on b : When FC is close to zero, the percentage of fines is low and they can easily fill void spaces, resulting in a b value close 0. However, as FC increases, fines start to separate coarse grains and take a role in force transmission, leading to an increase in the b value.

Effect of D_R on b : Under the same fines content and R_c value, the relative density (D_R) of a coarse package also influences the value of b . When the coarse package is in its loosest condition, it is easy for the fine grains to fill the voids, and b can approach zero. Conversely, when the coarse package is in its densest condition, b can be higher (Yin et al., 2016).

In a given sand-silt mixture, FC and R_c are known constants. The only variable within a mixture is the soil state. Therefore, to simplify the determination of the b value, it is advisable to select a specific known state, either the loosest, densest, or critical state, as suggested by prior studies (Thevanayagam et al., 2002; Yin et al., 2016; Change and Deng, 2020). Most researchers have chosen the critical state as the preferred option for estimating the b value (Thevanayagam et al., 2002; Rahman and Lo, 2014; Mohammadi and Qadimi, 2015; Lashakri, 2016).

The void ratio at critical state (e_{cr}) for a given sand-silt mixture can be obtained using the following equation (Li and Wang, 1998):

$$e_{cr} = \Gamma - \lambda \left(\frac{p}{p_{ref}} \right)^\xi \quad (6-5)$$

Where, p is the mean effective stress equal to $tr(\boldsymbol{\sigma})/3$, $\boldsymbol{\sigma}$ is the effective stress tensor, λ is the slope of the line, and Γ is the value of void ratio when p is equal to zero. ξ is a constant material property. According to previous studies (Thevanayagam et al., 2002; Rahman and Lo, 2014; Mohammadi and Qadimi, 2015; Lashakri, 2016), for a mixture with similar angularity of fine and coarse grains and a specified value of R_c , a change of FC only affects Γ . This means that the variation of void ratio at critical state (e_{cr}) with FC will have the same variation as Γ with FC . As

a result, and using Equations (6-2) and (6-5), it is possible to obtain the following equation for Γ as a function of FC (Rahman et al, 2014).

$$\Gamma = \Gamma_c^* - (1 - b_{cs})(1 + \Gamma_c^*) \frac{FC}{100} \quad (6-6)$$

Where Γ_c^* is Γ of the sample with zero fines content and b_{cs} is the b value at critical state. By replacing Equation (6-6) into Equation (6-5), the variation of e_{cr} with FC can be obtained. Since Γ_c^* is a constant, by having the variation of Γ with FC through lab tests data, the parameter b_{cs} corresponding with each FC can be easily obtained. In the case of limited experimental data, some researchers (Rahman and Lo, 2008; Lashakri, 2016; Mohammadi and Qadimi, 2015) have proposed experimental equations to estimate b_{cs} instead of calibrating it through test results, such as Equation (6-7) (Lashkari, 2016).

$$b_{cs} = 1.93.FC.R_c^{-0.29} \quad (6-7)$$

Other researchers (Thevanayagam et al., 2002; Yin et al., 2016) used a constant b_{cs} to fit experimental data with Equation (6-6), resulting in Γ varying linearly with FC . However, using expressions such as Equation (6-7) makes the variation of Γ with FC nonlinear which is a better representation of the soil behavior (Rahman et al., 2014; Lashkari, 2016).

6.3.2 Embedment Mechanism

Thevanayagam et al. (2002) proposed the following equation to estimate the void ratio when the fines content is greater than FC_{th} thus indicating that the embedment mechanism dominates.

$$e = e_f^* \left(\frac{FC}{100} + \frac{1 - \frac{FC}{100}}{R_f^s} \right) \quad FC > FC_{th} \quad (6-8)$$

e_f^* is called the interfine equivalent void ratio and is the void ratio when $FC = 100\%$. s is a microstructure parameter called embedment participation factor, similar to b , ranging from zero to 1, which depends on R_f , FC , and D_R . By substituting $s = 0$ and $s = 1$ in Equation (6-8), lower and upper bounds for the variation of void ratio with FC can be obtained.

$$e = e_f^* \quad \text{Upper bond, } s = 0 \quad (6-9)$$

$$e = e_f^* \left(\frac{FC}{100} + \frac{1 - \frac{FC}{100}}{R_f} \right) \quad \text{Lower bond, } s = 1 \quad (6-10)$$

Based on Equations (6-8) and (6-5), the following equation is valid at critical state when the shape of the CSL is independent of FC and FC only affects Γ .

$$\Gamma = \Gamma_f^* \left(\frac{FC}{100} + \frac{1 - \frac{FC}{100}}{R_f^{s_{cs}}} \right) \quad (6-11)$$

Γ_f^* is Γ when the package has $FC = 100\%$, and s_{cs} is the s value at critical state. The parameter s_{cs} corresponding with each FC is selected in a manner that, by altering FC and Γ , Γ_f^* remains constant. Yin et al. (2016) states that using a constant s_{cs} is consistent with experimental tests.

6.3.3 Threshold Fines Content

Equation (6-6) and Equation (6-11) provide the variation of void ratio with fines content for the filling and embedment mechanism at critical state, respectively. The corresponding threshold fines content, separating these two mechanisms from each other, can be estimated by finding the point of intersection of the two curves.

$$(FC_{th})_{cs} = 100 \frac{\Gamma_c^* - \frac{\Gamma_f^*}{\chi_f^{s_{cs}}}}{(1 - b_{cs})(1 + \Gamma_c^*) + \Gamma_f^* - \frac{\Gamma_f^*}{R_f^{s_{cs}}}} \quad (6-12)$$

If no experimental data is available, Rahman and Lo (2008) proposed a semi-experimental equation to estimate FC_{th} value as follows:

$$(FC_{th})_{cs} = 0.4 \left(\frac{1}{1 + \exp(0.5 - 0.13R_c)} + \frac{1}{R_c} \right) \quad (6-13)$$

6.4 Summary of Equivalent Critical State Theory (Eq-CST)

In this section, a summary of the Eq-CST based on the work of Rahman et al., 2014 is presented. Rahman et al. (2014) used the equivalent approach for $FC \leq FC_{th}$, to extend sand-based constitutive models for simulating the behavior of sand-silt mixtures. The chosen sand-based model for the extension should satisfy the following criteria: (1) it should be formulated within the framework of critical state theory, utilizing e and ψ as state variables; (with $\psi = e - e_{cr}$ as defined by Been and Jefferies (1985)); (2) It must be capable of simulating both the loose and dense behaviors of sand under drained and undrained conditions using a single set of material properties; and (3) its parameters should be defined through a relatively simple and objective calibration procedure.

It is important to note that the application of the equivalent approach has been used to situations where $FC \leq FC_{th}$ (Rahman et al., 2014). The present study later explores its applicability in cases where $FC > FC_{th}$, when fines play a significant role in shaping the structure of sand-silt mixtures. In the Eq-CST, the void ratio (e) is replaced with the equivalent void ratio (e^*) as follows (Rahman et al., 2014):

$$e^* = \begin{cases} e_c^* = \frac{e + (1-b)\frac{FC}{100}}{1 - (1-b)\frac{FC}{100}} & FC \leq FC_{th} \\ e_f^* = \frac{e}{\frac{FC}{100} + \frac{1 - \frac{FC}{100}}{R_f^s}} & FC > FC_{th} \end{cases} \quad (6-14)$$

These equations come from reversing Equation (6-2) and Equation (6-8). The critical state line in $e^* - p$ space can be obtained as follows:

$$e_{cr}^* = \Gamma^* - \lambda \left(\frac{p}{p_{ref}} \right)^\xi; \quad \Gamma^* = \begin{cases} \Gamma_c^* & FC \leq FC_{th} \\ \Gamma_f^* & FC > FC_{th} \end{cases} \quad (6-15)$$

The critical void ratios at zero confining pressure, denoted as Γ_c^* and Γ_f^* , correspond to samples with zero and 100 percent fines content, respectively. These constants find application in Equation (6-6) and Equation (6-11) to derive the variations of b_{cs} and s_{cs} with FC by knowing the variation of Γ with FC . It is assumed (Rahman et al., 2014; Lashkari, 2016) that the obtained values of b_{cs} and s_{cs} can be regarded as b and s for the calculation of e^* in Equation (6-14).

The equivalent state parameter (ψ^*) (Eq-state parameter) as a quantifying parameter to determine the degree of looseness or denseness of a sand-silt mixture can be defined as follows (Been, and Jefferies, 1985; Rahman et al., 2014):

$$\psi^* = e^* - e_{cr}^* \quad (6-16)$$

Therefore, to extend a sand-based model within the framework of Eq-CST, the formulation of the chosen model must have e , e_{cr} , and $\psi = e - e_{cr}$ replaced with their respective counterparts, e^* , ψ^* , and e_{cr}^* .

6.5 Formulation of PolySand-Wfines

PolySand, as introduced by Razavi and Yniesta (2023), effectively meets all the criteria outlined by Rahman et al., (2014) for it to be considered a suitable candidate for extension under the Eq-CST to simulate the mechanical behavior of loose coarse-fine tailings mixtures. Additionally, obtaining its material properties is straightforward as they hold clear physical meaning. In this section, a revised formulation of PolySand, namely PolySand-Wfines, will be presented. The influence of fines on the formulation will be examined, emphasizing the parameters that are modified. In PolySand-Wfines, every quantity marked with an asterisk (*) indicates its dependence on the one of the following quantities: either the equivalent void ratio (e^*), equivalent critical state line (e_{cr}^*), or equivalent state parameter (ψ^*).

In the current paper, all the stresses are effective, and the superposed dot implies a rate. By having the effective stress ($\boldsymbol{\sigma}$) and strain tensor ($\boldsymbol{\varepsilon}$), the following quantities can be defined which will be used in the PolySand-Wfines formulation:

$$p = \frac{1}{3} \text{tr}(\boldsymbol{\sigma}); \quad \mathbf{s} = \boldsymbol{\sigma} - p \cdot \boldsymbol{\delta}; \quad J_2 = \frac{1}{2} \mathbf{s} : \mathbf{s}; \quad q = \sqrt{3J_2} \quad (6-17)$$

$$\varepsilon_v = \text{tr}(\boldsymbol{\varepsilon}); \quad \boldsymbol{\gamma} = \boldsymbol{\varepsilon} - \frac{1}{3} \varepsilon_v \cdot \boldsymbol{\delta}; \quad \varepsilon_\gamma = \frac{1}{q} \mathbf{s} : \boldsymbol{\gamma} \quad (6-18)$$

Where p is mean effective stress, and q is the deviatoric stress. η is the stress ratio equal to (q/p) . $\mathbf{s}$ is the deviatoric stress tensor, J_2 is the second invariant of deviatoric stress tensor, δ is the Kronecker delta, $\boldsymbol{\gamma}$ denotes the deviatoric strain tensor. ε with subscripts v and γ indicate the volumetric strain and work conjugate shear strain. The superscripts e and p in the strains will be used to distinguish between the elastic and plastic parts of the strains, respectively.

6.5.1 Critical Stress Ratio

The grain size distribution of a granular soil affects the critical stress ratio (M) only if the shapes of the fine and coarse particles differ significantly, otherwise, it has little influence on the critical state friction angle (Torres and Santamarina, 2019, Zhu et al. 2020, Polania et al. 2023). As such, it can be concluded that M is independent of the fines content for the sand-silt mixtures with similar grain angularity and mineralogy.

The following equation presented in the work of Jefferies and Shuttle (2011) is used to derive the critical stress ratio (M) for PolySand-Wfines.

$$M = M_{tc} - \frac{M_{tc}^2}{3 + M_{tc}} \cos\left(\frac{3}{2}\theta + \frac{\pi}{4}\right) \quad (6-19)$$

Where M_{tc} represents the critical stress ratio determined via triaxial compression testing, while θ denotes the Lode angle, which can be computed using the following expression:

$$\theta = \frac{1}{3} \sin^{-1} \left(\frac{3\sqrt{3}}{2} \frac{j_3}{j_2^{3/2}} \right); \quad j_3 = \det(\mathbf{s}); \quad j_2 = \frac{1}{2} \mathbf{s} : \mathbf{s} \quad (6-20)$$

With $\theta = -\pi/6$ and $\theta = \pi/6$ for triaxial extension and triaxial compression paths; J_2 and J_3 are the second and third invariant of deviatoric stress tensor ($\mathbf{s}$).

6.5.2 Stress-Dilatancy Rule and the Yield Surface

The Stress-Dilatancy rule used in the PolySand-Wfines model is a modified version of the same rule employed in PolySand (Razavi and Yniesta, 2023), where M_i is substituted with M_i^* as follows:

$$D = M_i^* - \eta \quad (6-21)$$

D is the dilatancy, and equal to $(\varepsilon_v^p / \varepsilon_\gamma^p)$. ε_v^p is the incremental plastic volumetric shear strain and ε_γ^p is the incremental plastic deviatoric strain. M_i^* is called image stress ratio. The yield surface is obtained by integrating Equation (6-21), which results in the following yield surface:

$$f = \eta - M_i^* \left(1 - \ln \left(\frac{p}{p_i} \right) \right) = 0 \quad (6-22)$$

The schematic form of the yield surface is shown in Figure 6-2. On the yield surface, four different states are shown as follows: (1) image state corresponding to zero dilatancy, image mean stress p_i , and image stress ratio M_i^* , (2) limiting state corresponding to limiting dilatancy (D_L^*), limiting mean stress (p_L^*), and limiting stress ratio (η_L^*), (3) current state with current dilatancy (D), current mean stress (p), and current stress ratio (η), and (4) the isotropic state (corresponding to zero stress ratio).

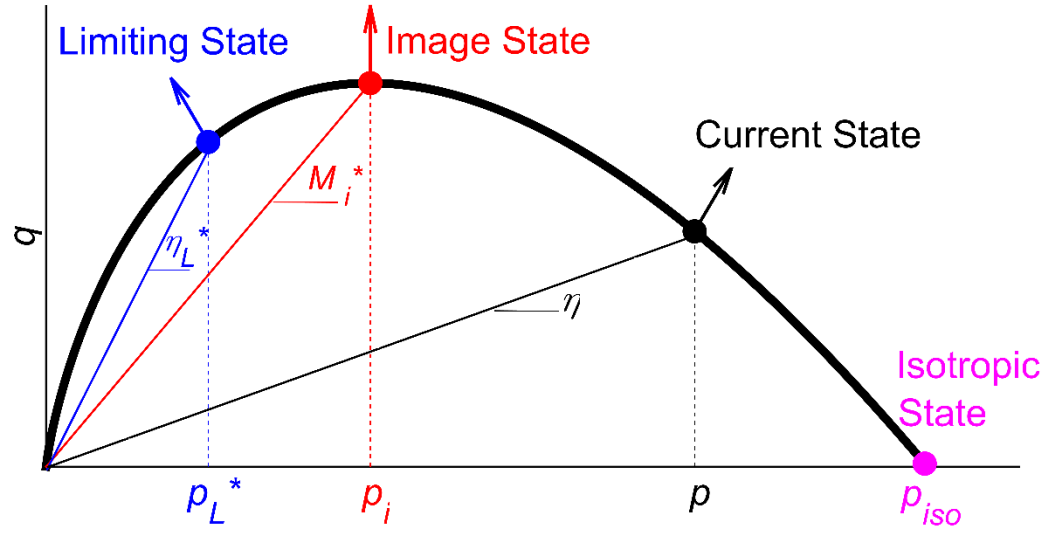


Figure 6-2 Schematic form of the yield surface used in the PolySand-Wfines

For the image and limiting state, the following equivalent state parameters can be defined using Equation (6-16).

$$\psi_i^* = e^* - \left(\Gamma^* - \lambda \left(\frac{p_i}{p_{ref}} \right)^\xi \right) \quad (6-23)$$

$$\psi_L^* = e^* - \left(\Gamma^* - \lambda \left(\frac{p_L^*}{p_{ref}} \right)^\xi \right) \quad (6-24)$$

Where, ψ_i^* and ψ_L^* , are the equivalent image and equivalent limiting state parameters. Combining these two state parameters yields the following expression:

$$\psi_L^* = \psi_i^* + (\psi_i^* + \Gamma^* - e^*) \left(\left(\frac{p_L^*}{p_i} \right)^\xi - 1 \right) \quad (6-25)$$

M_i^* is dependent on the equivalent image state parameter ψ_i^* , and will be defined in the following sections.

6.5.3 PolySand-Wfines Hardening Law

The model's yield surface, Equation (6-22) depends on both M_i^* and p_i . Given the assumption that M_i^* is a function of ψ_i^* , and ψ_i^* is in turn dependent on p_i (See Equation (6-23)), it can be inferred that p_i will ultimately determine the size of the yield surface. In PolySand-Wfines, p_i is assumed to change with the plastic deviatoric shear strain in the following manner:

$$\dot{p}_l = H^* \cdot \varepsilon_\gamma^p \quad (6-26)$$

Where H^* is the plastic modulus calculated from the following equation:

$$H^* = h_0^* \left(\frac{p}{p_i} - \frac{p_L^*}{p_i} \right) \quad (6-27)$$

Here, h_0^* is the hardening parameter, and (p_L^*/p_i) is obtained through the limiting state law.

6.5.3.1 Limiting State law

The limiting state law of the model is established to determine the position of the limiting state on the yield surface by referencing its location in the $e^* - p$ space. This location of the yield surface in $e^* - p$ space is characterized by the parameter ψ_i^* . The connection between (p_L^*/p_i) ratio as a quantity showing the location of limiting state in comparison with the image state on the yield surface with ψ_i^* in the PolySand-Wfines is defined as follows:

$$\frac{p_L^*}{p_i} = \begin{cases} \left(1 + (r^\xi - 1) \frac{\psi_i^* - \psi_\chi}{\chi(r^\xi - 1) + \psi_i^* - \psi_\chi} \right)^{\frac{1}{\xi}} & \psi_\chi \leq \psi_i^* \\ \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi - (\psi_i^* - \psi_\chi)} \right)^{\frac{1}{\xi}} & \psi_i^* \leq \psi_\chi \end{cases} ; r \quad (6-28)$$

$$= \exp(1)$$

Where, χ and ψ_χ are the dilatancy state variable and limited-flow state variable respectively (Razavi and Yniesta, 2023). χ is defined as follows:

$$\chi = S \cdot \chi_u + (1 - S) \cdot \chi_d \quad (6-29)$$

Where S represents a Boolean switch (0 for drained and 1 for undrained loading). χ_u and χ_d are drained and undrained dilatancy state parameters, defined as follows:

$$\chi_u = \begin{cases} \chi_{uc} & FC \leq FC_{th} \\ \chi_{uf} & FC > FC_{th} \end{cases} ; \chi_d = \begin{cases} \chi_{dc} & FC \leq FC_{th} \\ \chi_{df} & FC > FC_{th} \end{cases} \quad (6-30)$$

When considering the filling and embedment mechanisms, it is necessary to determine the respective material quantities χ_u and χ_d separately. The subscripts c and f are employed to distinguish the filling (coarse-dominated) and embedment (fines-dominated) mechanisms, respectively.

The limited-flow state variable in Equation (6-28) is defined as follows (Razavi and Yniesta, 2023):

$$\psi_\chi = \psi_A \exp(-c \cdot \varepsilon_V^p) \quad (6-31)$$

Where, c represents a positive constant that controls the rate at which ψ_χ transitions from ψ_A at low strains to zero at high strains. The material property ψ_A is defined as follows:

$$\psi_A = \begin{cases} \psi_{A_c} & FC \leq FC_{th} \\ \psi_{A_f} & FC > FC_{th} \end{cases} \quad (6-32)$$

ψ_{A_c} and ψ_{A_f} are negative constant parameters controlling the limited flow behavior in the model for the filling and embedment mechanism, respectively.

6.5.3.2 Hardening Parameter

The hardening parameter in Equation (6-27) is defined as follows:

$$h_0^* = \frac{M}{M_{tc}} \frac{H_0 - H_y \psi_i^*}{F_d^*} \left(\frac{p_i}{p_{ref}} \right)^{-0.5} \left(\frac{p}{p_i} \right) p_i \quad (6-33)$$

Where, H_0 and H_y are material constants; and F_d^* is obtained as follows:

$$F_d^* = \begin{cases} \left(1 + (r^\xi - 1) \frac{\psi_i^* - \psi_\chi}{\chi_d (r^\xi - 1) + \psi_i^* - \psi_\chi} \right)^{\frac{1}{\xi}} & \psi_\chi \leq \psi_i^* \\ \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)} \right)^{\frac{1}{\xi}} & \psi_i^* \leq \psi_\chi \end{cases} \quad (6-34)$$

F_d^* is the (p_L^*/p_i) ratio of Equation (6-28) by replacing $\chi = \chi_d$.

6.5.4 Image stress ratio

The image stress ratio in the PolySand-Wfines is defined as follows:

$$M_i^* = \begin{cases} M & \psi_\chi \leq \psi_i^* \\ \frac{M}{1 - N \ln(F_d^*)} & \psi_i^* \leq \psi_\chi \end{cases} \quad (6-35)$$

N is called dilatancy parameter obtained as follows:

$$N = S \cdot N_u + (1 - S) \cdot N_d \quad (6-36)$$

N_u and N_d are undrained and drained dilatancy parameters used in the model (Razavi and Yniesta, 2023). These parameters are assumed to be independent of the filling or embedment mechanism. To provide a validation of this assumption, experimental data from Yilmaz et al. (2023) is examined. According to the PolySand model (Razavi and Yniesta, 2023). N_u controls the phase transformation stress ratio under undrained conditions. For the sake of simplicity, PolySand-Wfines assumes that N_u is independent of the fines content. N_d should be calibrated by fitting the following equation to the lab drained test data conducted on dense samples.

$$D_L^* = \frac{M - \eta_L^*}{1 - N_d} \quad (6-37)$$

This equation is known as Nova's rule (Nova and Wood, 1986). Drained samples under drained conditions present a maximum stress ratio ($\eta = \eta_{max}$) in their stress path where the dilatancy is minimum ($D = D_{min}$), and where a change from hardening to softening occurs. In the model, a hardening regime change occurs when H^* is zero. As a result, at that point $D_L^* = D =$

D_{min} and $\eta_L^* = \eta = \eta_{max}$. Yilmaz et al. (2023) did several drained tests on dense samples with varying fines content. By extracting the (η_{max}, D_{min}) data from their work, Figure 6-3 is drawn. This figure illustrates the influence of FC on the (η_{max}, D_{min}) pairs. As FC increases, until reaching FC_{th} , the (η_{max}, D_{min}) pairs move downward. However, further increase in FC causes the pairs to move upward. The line observed in the figure is derived from Nova's rule (Equation (6-37)), with its slope being $1 - N_d$. The data shows that the value of N_d remains unaffected by changes in FC . In the figure, M corresponds to zero dilatancy and also remains unaffected by a change in fines content.

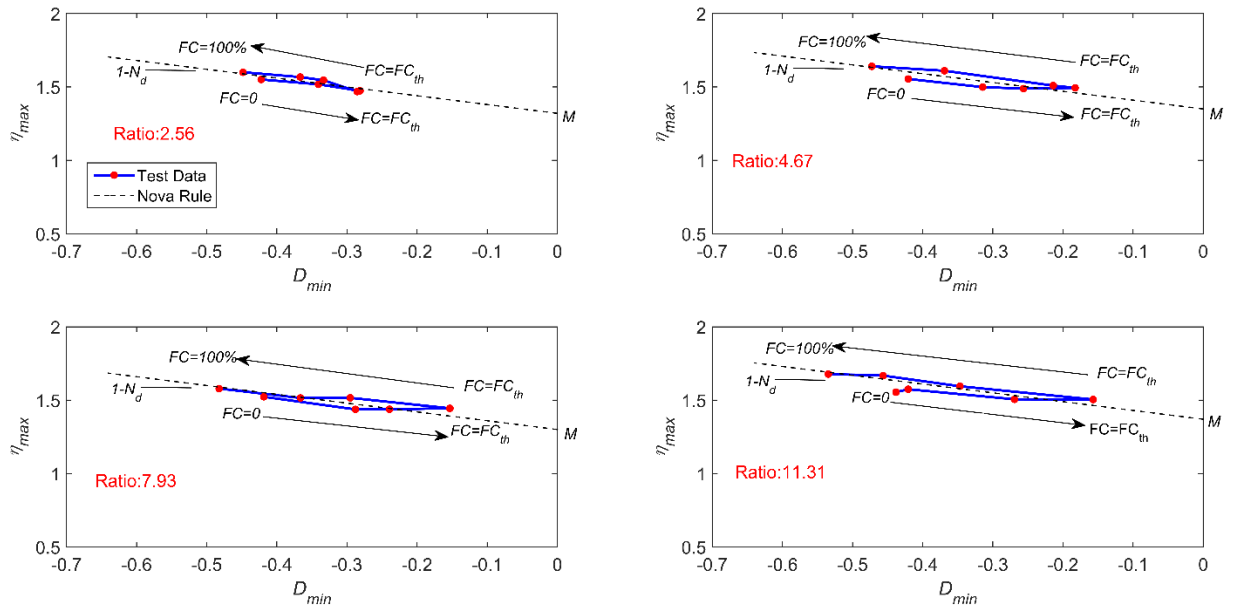


Figure 6-3 Effect of FC on the $(\eta_L^* = \eta_{max}, D_L^* = D_{min})$ pairs extracted from the work of Yilmaz et al. (2023) for different particle diameter ratios.

6.5.5 Stress-Strain Relationship in the PolySand-Wfines

The incremental stress-strain relationship in the PolySand-Wfines is defined as follows:

$$\dot{\sigma} = D^{ep} \cdot \dot{\epsilon} = D^e \cdot \dot{\epsilon}^e = 2G \cdot \dot{\gamma}^e + K \cdot \dot{\epsilon}_v^e \cdot \delta \quad (6-38)$$

Where $\mathbf{D}^{ep}$ and $\mathbf{D}^e$ are the elastoplastic and elastic modulus, $\dot{\boldsymbol{\epsilon}}$ and $\dot{\boldsymbol{\epsilon}}^e$ are the incremental strain tensor and incremental elastic part of the strain tensor. $\dot{\boldsymbol{\gamma}}^e$ and $\dot{\epsilon}_v^e$ are the incremental elastic deviatoric strain tensor, and incremental elastic volumetric strain, respectively. K and G are the elastic bulk and shear moduli, defined as follows (Jefferies and Been, 2015; Cheng and Jefferies, 2020, Razavi and Yniesta, 2023):

$$G = G_0 \cdot p_{ref} \cdot \left(\frac{p}{p_{ref}} \right)^{0.5}, \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \quad (6-39)$$

Here, G_0 is a material constant, and p_{ref} is the reference pressure taken as the atmospheric pressure, ν is the Poisson's ratio. $\mathbf{D}^{ep}$ can be obtained as follows:

$$\mathbf{D}^{ep} = \mathbf{D}^e - \frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}} \frac{\partial f^T}{\partial \boldsymbol{\sigma}} \mathbf{D}^e}{\frac{\partial f^T}{\partial \boldsymbol{\sigma}} \mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}} - \left(\frac{\partial f}{\partial K_\alpha} \frac{\partial K_\alpha}{\partial \boldsymbol{\epsilon}^p} \right)^T \frac{\partial g}{\partial \boldsymbol{\sigma}}} \quad (6-40)$$

T indicates transposition. The yield surface denoted as f is Equation (6-22) of the PolySand-Wfines model, while the plastic surface, equivalent to the yield surface in Cambridge-type models, is taken as $g = f$. The size of the yield surface is determined by the parameters represented as K_α in the yield surface equation. According to Equation (6-22), the K_α parameters for this model are M_i^* and p_i .

6.5.6 Summary of Material Constants in the PolySand-Wfines

Table 6-1 presents the PolySand-Wfines model's input parameters, distinguishing between those independent of the microstructure mechanisms and those dependent on the sand-silt mixture's microstructure. Specifically, for the filling mechanism ($FC \leq FC_{th}$), an additional parameter called the fines participation factor (b) is introduced, complementing the existing

PolySand material parameters. Conversely, for the embedment mechanism ($FC > FC_{th}$), two extra material properties, namely the embedment participation factor (s) and the embedment particle diameter ratio (R_f), are incorporated alongside the existing PolySand material properties.

Table 6-1 Parameters Employed by PolySand-Wfines

		applicable for the filling mechanism $FC \leq FC_{th}$	applicable for the embedment mechanism $FC > FC_{th}$
Elasticity Parameters	Parameters independent of microstructure mechanism	G_0 ν	
Shear-volume coupling factor		N_u, N_d	
Hardening parameters		H_0 H_y	
Critical state Parameters		M_{tc} λ ξ	
Critical state Parameters	Parameters dependent on microstructure mechanism	Γ_c^* b	Γ_f^* s R_f
Dilatancy State Parameters		χ_{uc} χ_{dc}	χ_{uf} χ_{df}
Limited-flow state parameters		ψ_{Ac}, c	ψ_{Af}, c

6.6 Calibration Process for the PolySand-Wfines Parameters

In this section, a step-by-step procedure is presented to obtain the PolySand-Wfines material properties used in this study. To demonstrate the calibration process, the triaxial test data of Sydney Sand-Silt mixture with a filling mechanism ($FC \leq FC_{th}$) is used (Rahman et al., 2014). The base granular soil is Sydney Sand, which is a clean, uniform size, quartz sand with a mean grain size of 0.3mm, and the fines in the mixture consisted of 2/3 natural silt from Majura riverbank deposits and 1/3 of commercial kaolin. The mix of fines is a well-graded, low-plasticity fines. The fines content in the sand-silt mixtures they tested ranges from 0% to 30%, and the threshold fines content

(FC_{th}) for the mixture was found to be 39%. As a result, their mixtures are characterized with a filling mechanism.

Step 1; Calibration of Critical State Parameters: The nonlinear CSL (Equation (6-15)) is used to fit the test data to determine the critical state parameters corresponding to zero fines content.. According to the Rahman et al.'s study (2014), these parameters for the Sydney Sand are $M_{tc} = 1.305$, $\lambda = 0.0375$, $\xi = 0.6$, and $\Gamma_c^* = 0.92$. For the embedment mechanism, Γ_f^* at $FC = 100\%$ must be determined.

Step 2; Calibration of b and s : the second step is to determine the fines participation factor (b) for the sand-silt mixtures with different fines content with the filling mechanism. Rahman et al. (2014) presented the critical state lines (CSL) of mixtures with varying fines content (0%, 10%, 15%, 20%, and 30%). Using Equation (6-15), Rahman et al. (2014) showed that the parameters λ and ξ remain constant, and only Γ exhibited changes with increasing fines content. Considering Equation (6-6) and by having Γ_c^* for $FC = 0$, and Γ for different FC values, the corresponding $b = b_{cs}$ for each FC can be obtained as shown in Table 6-2.

Table 6-2 Variation of Γ and b with FC as reported by Rahman et al. (2014)

FC	Γ	b
0%	0.92	0
10%	0.747	0.103
15%	0.675	0.15
20%	0.613	0.2
30%	0.505	0.28

The equivalent void ratio at critical state is computed using Equation (6-14) and plotted in the equivalent void ratio (e^*) -effective mean stress (p) in Figure 6-4, where the fitted Eq-CSL has the following parameters: $\Gamma_c^* = 0.92$, $\lambda = 0.0375$, $\xi = 0.6$. As can be seen, all the mixtures have a unique Eq-CSL.

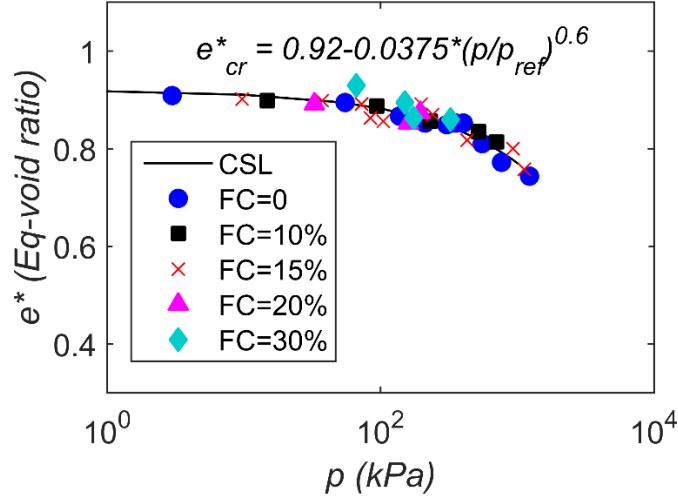


Figure 6-4 Equivalent critical state line in $e^* - \ln p$ space for Sydney sand-silt mixtures with filling mechanism (Rahman et al. 2014),

When dealing with the embedment mechanism, a similar procedure is employed. By utilizing Equation (6-8) and obtaining Γ_f^* for $FC = 100\%$, along with having Γ values for different FC , the corresponding $s = s_{cs}$ for each FC can be derived.

Step 3; Calibration of χ_u and ψ_A : χ_u is employed to identify the location of the undrained hardening regime change during undrained loading of loose samples ($\psi_i^* > \psi_\chi$). For such samples, the exact moment when the undrained hardening regime change ($dp_i = 0$) occurs cannot be detected through visual analysis of the test data. The only apparent point is when the undrained peak shear strength or flow instability occurs, characterized by $dp_i \neq 0$ while $(\frac{dq}{dp} = 0)$. This point can be denoted by (η_{IP}, ψ_{IP}) and is observed after the initiation of the undrained hardening regime transition. Consequently, Razavi and Yniesta (2023) suggest an initial calibration of χ_u by aligning the (η_L^*, ψ_L^*) equation of the model with the (η_{IP}, ψ_{IP}) data obtained from undrained tests. Using Equation (6-22), at the hardening regime change ($\frac{p_L^*}{p_i} = \frac{p}{p_i}$), the following equation can be obtained:

$$\eta = \eta_L^* = M_i^* \cdot \left(1 - \ln \left(\frac{p_L^*}{p_i} \right) \right) \quad (6-41)$$

For loose samples ($\psi_i^* > \psi_\chi$), Equation (6-28) can be turned into the following equation under undrained condition and for $\chi = \chi_u$.

$$\frac{p_L^*}{p_i} = \left(1 + (r^\xi - 1) \frac{\psi_i^* - \psi_\chi}{\chi_u(r^\xi - 1) + \psi_i^* - \psi_\chi} \right)^{\frac{1}{\xi}} \quad (6-42)$$

By replacing Equation (6-42) into Equation (6-41) and Equation (6-25) the following equations can be derived:

$$\eta_L^* = M \left(1 - \frac{1}{\xi} \ln \left(1 + (r^\xi - 1) \frac{\psi_i^* - \psi_\chi}{\chi_u(r^\xi - 1) + \psi_i^* - \psi_\chi} \right) \right) \quad (6-43)$$

$$\psi_L^* = \psi_i^* + (\psi_i^* + \Gamma^* - e^*)(r^\xi - 1) \frac{\psi_i^* - \psi_\chi}{\chi_u(r^\xi - 1) + \psi_i^* - \psi_\chi}$$

Equation (6-43) contains two unknowns (χ_u) and (ψ_χ). Usually, hardening regime change for loose samples occurs at low strains, and ψ_χ can be assumed to be ψ_A according to Equation (6-31). Therefore, by fitting Equation (6-43) with (η_{IP}, ψ_{IP}) data, both (χ_u) and (ψ_A) can be calibrated.

Table 6-3 provides information on the samples which were subjected to undrained triaxial loading as tested by Rahman et al. (2014). The undrained tests were done on loose samples with $(\Gamma_c^* - e_0^*)$ ranging from 0.02 to 0.16. Additionally, Figure 6-5 presents undrained peak stress ratios versus equivalent state parameters retrieved from the test results. Although the range of initial equivalent state parameters for the samples is narrow (ψ_0^* in Table 6-3), the PolySand-Wfines

calibration curves, obtained using $\chi_u = 0.03$ and $\psi_A = -0.01$ through Equation (6-43), match well with experimental data.

Table 6-3 Undrained test samples properties (adapted from Rahman et al. 2014)

Test Name	FC (%)	b	p_0 (kPa)	e_0	e_0^*	ψ_0^*	$\Gamma_c^* - e_0^*$	Observed Behavior
S-MII-00-06	00	0.00	350	0.743	0.743	-0.097	0.177	NF
S-MII-00-07			600	0.850	0.850	0.040	0.07	LF
S-MII-00-10			850	0.875	0.875	0.090	0.045	F
S-MII-15-03	15	0.15	600	0.656	0.897	0.088	0.023	F
S-MII-15-05			600	0.637	0.876	0.066	0.044	LF
S-MII-15-06			600	0.535	0.759	-0.051	0.161	NF
S-MII-15-08			850	0.637	0.876	0.092	0.044	F
S-MII-20-01	20	0.2	350	0.589	0.891	0.051	0.029	LF
S-MII-20-03			850	0.574	0.874	0.089	0.046	LF
S-MII-20-04			1100	0.571	0.870	0.108	0.05	LF
S-MII-30-01	30	0.28	350	0.482	0.890	0.050	0.03	F
S-MII-30-04			1100	0.450	0.850	0.088	0.07	LF

NF= non-flow, F=flow, LF=limited flow

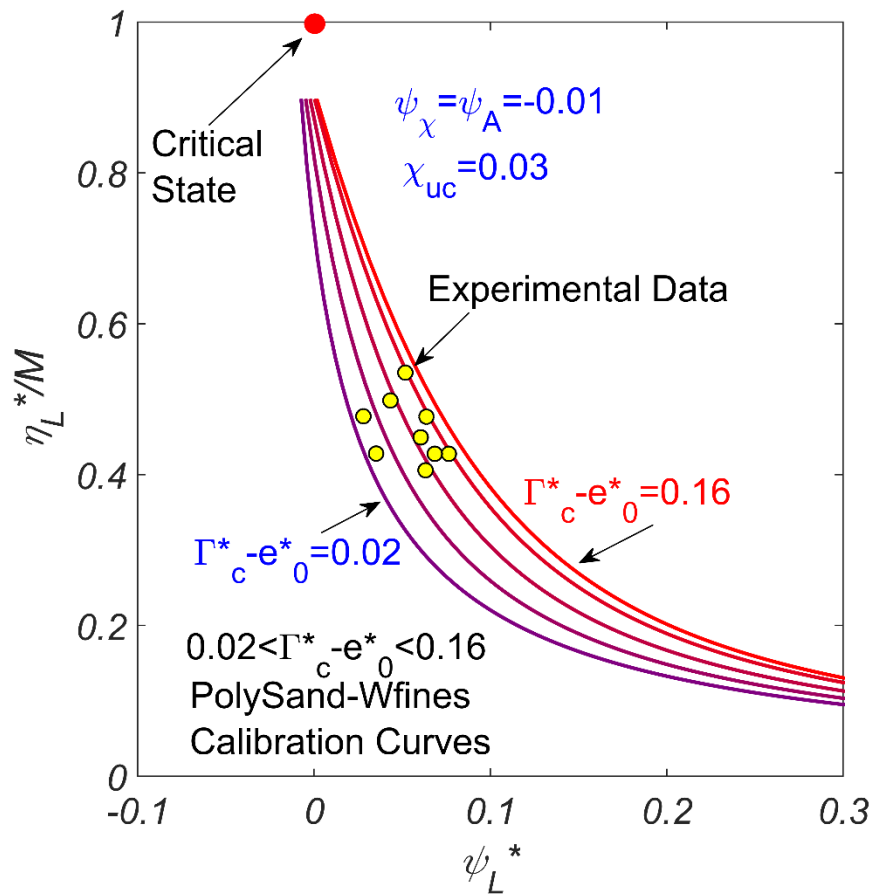


Figure 6-5 Normalized peak undrained stress ratio (η_L^*/M) versus corresponding equivalent state parameters: scatter data from the undrained triaxial tests conducted on loose samples against the PolySand calibration curves.

Note that since the data pertains to sand-silt mixtures with a filling mechanism ($FC \leq FC_{th}$), the calibrated χ_u and ψ_A are χ_{uc} and ψ_{Ac} , otherwise, if the embedment mechanism was at play they would be χ_{uf} and ψ_{Af} .

Step 4; Calibration of N_d : The calibration of this parameter requires the maximum stress ratio ($\eta_L^* = \eta_{max}$) and minimum dilatancy ($D_L^* = D_{min}$) from drained test results conducted on dense samples. N_d is obtained by fitting the data with Equation (6-37). Since, this data was not available from the work of Rahman et al. (2014), a value of 0.4 was chosen for N_d , which falls within the typical range of 0.2 to 0.5 suggested by Jefferies and Been (2015).

Step 5; Calibration of χ_d : χ_d is calibrated by utilizing drained test data conducted on dense samples. Within the PolySand-Wfines model, χ_d serves to identify the occurrence of the change in the hardening regime during drained triaxial loading of dense samples (where $\psi_i^* \leq \psi_\chi$). This transition happens at a specific point along the loading history, characterized by the occurrence of the maximum stress ratio ($\eta = \eta_L^* = \eta_{max}$) and the minimum dilatancy ($D = D_L^* = D_{min}$) when ($\psi^* = \psi_L^* = \psi_{min}$). To determine χ_d , PolySand-Wfines necessitates the fitting of the set of equations (D_L^*, ψ_L^*) with the data (D_{min}, ψ_{min}) derived from experimental data.

By combining Equation (6-21) and Equation (6-22), the following equation can be obtained:

$$D = M_i^* \cdot \ln\left(\frac{p}{p_i}\right) \quad (6-44)$$

At a hardening regime change, $\frac{p_L^*}{p_i} = \frac{p}{p_i}$, and therefore, the following expression is valid:

$$D = D_L^* = M_i^* \cdot \ln\left(\frac{p_L^*}{p_i}\right) \quad (6-45)$$

For ($\psi_i^* \leq \psi_\chi$), using Equation (6-28) and Equation (6-32), the following equation can be obtained.

$$F_d^* = \frac{p_L^*}{p_i} = \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)}\right)^{\frac{1}{\xi}} \quad (6-46)$$

By replacing Equation (6-46) into Equation (6-45) and Equation (6-25) the following expression can be found.

$$D_L^* = \frac{M}{\xi} \frac{\ln \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)} \right)}{1 - \frac{N_d}{\xi} \ln \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)} \right)}; \quad (6-47)$$

$$\psi_L^* = \psi_i^* + (\psi_i^* + \Gamma^* - e^*) \cdot \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)}$$

Typically, under drained conditions, a change in the hardening regime occurs at medium to large strains. As a result, according to Equation (6-31), ψ_χ can be assumed to be zero during this phase. Consequently, in Equation (6-47), the only parameter that remains unknown is χ_d . By fitting the values (D_L^*, ψ_L^*) obtained from this equation with the experimental data's (D_{min}, ψ_{min}) , we can determine the value of χ_d . If the sand-silt mixtures of interest is controlled by a filling mechanism ($FC \leq FC_{th}$), then the calibrated χ_d is denoted as χ_{dc} ; otherwise, it is as χ_{df} .

Because of this absence of experimental drained test results for dense samples in the dataset of Rahman et al. (2014), a typical value of 0.4 within the range proposed by Razavi and Yniesta (2023) has been chosen for χ_{dc} .

Step 6; Calibration of N_u : N_u in the model controls the phase transformation state occurring during the non flow behavior of dense samples ($\psi_i^* \leq \psi_\chi$). When ($\psi_i^* \leq \psi_\chi$), the following equation can be obtained by replacing Equation (6-46) into Equation (6-35) and considering N equal to N_u under undrained conditions:

$$M_i^* = \frac{M}{1 - \frac{N_u}{\xi} \ln \left(1 + \frac{\psi_i^* - \psi_\chi}{\chi_d - (\psi_i^* - \psi_\chi)} \right)} \quad (6-48)$$

In cases of non-flow behavior, the phase transformation state occurs at low strains, leading to the assumption that ψ_χ can be equated to ψ_A . Furthermore, at phase transformation state, the current state aligns with the image state, resulting in p being equal to p_i , ψ^* being equal to ψ_i^* , and η being equal to M_i^* . Consequently, with knowledge of the stress ratio and equivalent state parameter at the phase transformation state for samples exhibiting non-flow behavior, it becomes possible to fit Equation (6-48) to this data to determine N_u . In cases where limited experimental test data is available, N_u can be considered equal to N_d (obtained in step 4) or higher.

Step 7; Calibration of H_0 , H_y , and c : The remaining parameters for PolySand-Wfines, namely (H_0 , H_y , and c), are determined through an iterative process aimed at achieving the best agreement between simulation results and experimental data. The procedure begins by conducting simulations without incorporating the limited flow feature. In this phase, ψ_A , which was calibrated in step 3, is assumed to be zero. This assumption serves to eliminate the influence of the limited-flow state variable from the model's formulations, as well as the impact of the parameter c as defined in Equation (6-31). Subsequently, H_y is set to zero, and the optimal value for H_0 is determined through a trial-and-error methodology. If necessary, adjustments to the values of H_y can be made to improve the model's accuracy and attain the best-fitting results. Once the values of H_0 and H_y have been finalized, ψ_A is then set to its calibrated value, and the parameter c is obtained through another trial-and-error process.

Step 8; Elastic Material Properties: In this study, a modification is introduced to the elastic material properties originally presented by Rahman et al. (2014). This modification is necessary because the equation they employed differs from Equation (6-41).

The calibrated material properties for the PolySand-Wfines for the Sydney sand-silt mixtures are presented in Table 6-4.

Table 6-4 PolySand-Wfines model constants for Sydney sand-silt mixture

	Input parameters for the filling mechanism	Sydney Sand-Silt Mixtures
Elasticity Parameters	G_0 ν	168 0.3
Dilatancy Parameters	N_u N_d	0.4 0.4

Hardening parameters	H_0 H_y	170 500
Critical state Parameters	M_{tc} λ ξ Γ_c^* b	1.305 0.0375 0.6 0.92 0-0.25
Dilatancy state parameters	χ_{uc} χ_{dc}	0.03 0.4
Limited-flow State parameters	ψ_{Ac} c	-0.01 5

6.7 PolySand-Wfines Simulation Results

6.7.1 Sydney Sand-Silt Mixtures with filling mechanism

This section presents simulations of the tests on the Sydney sand-silt mixture from Rahman et al., 2014. Figure 6-6a and Figure 6-6b presents the comparison between model predictions and test results for FC ranging from 0% to 20% and confining pressure ranging from 300 kPa to 1100 kPa. Figure 6-6c and Figure 6-6d shows the simulation results for the samples with FC ranging from 15% to 30% and confining pressure ranging from 300 kPa to 900 kPa. These tests encompass a broad spectrum of initial void ratios, ranging from 0.455 to 0.886 (e_0), while e_0^* falls within a narrower range of 0.87-0.896, and ψ_0^* ranges from 0.051 to 0.108. The tests exhibit flow and limited-flow behavior, which are both properly simulated by PolySand-Wfines.

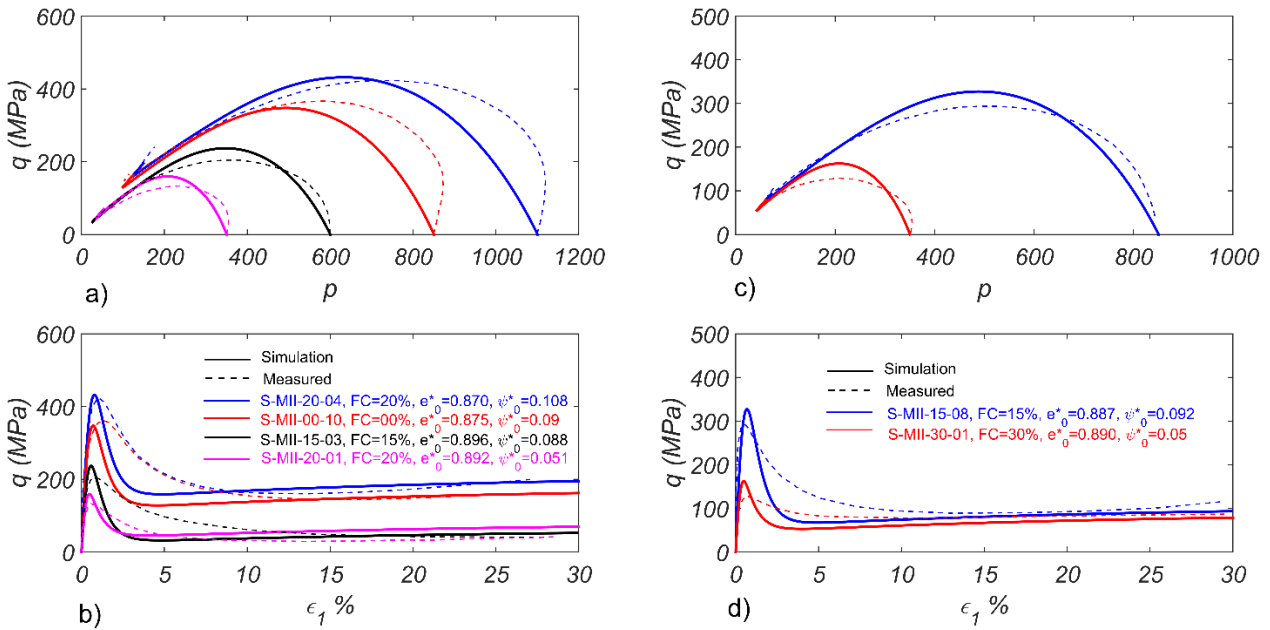


Figure 6-6 Comparison of test results (Rahman et al., 2014) and simulations with the PolySand-WFines model for various initial confining pressures, Fines Contents, and similar initial equivalent void ratio: (a, c) effective stress path, (b, d) deviatoric stress-axial strain

The applicability of PolySand-Wfines to simulate the limited flow behavior is also assessed using five additional test results, as shown in Figure 6-7. The overall quality of the simulations can be considered as good, further supporting the effectiveness of PolySand-Wfines in capturing flow and limited-flow behavior for sand-silt mixtures controlled by a filling mechanism.

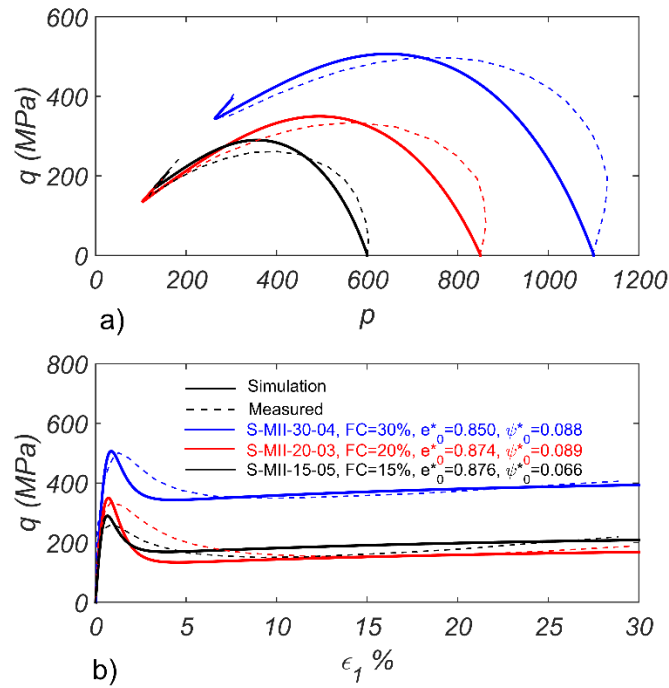


Figure 6-7 Comparison of test results (Rahman et al., 2014) and simulations using the PolySand-Wfines model for limited-flow behavior: a) effective stress path, b) deviatoric stress-axial strain

6.7.2 Foundry Sand-Silt Mixtures

This section focuses on the Foundry Sand-Silt mixtures used by Thevanayagam et al. (2002), where the fines contents ranges from zero to 100%, with a threshold fines content (FC_{th}) of 25%. The Foundry sand used in their tests is referred to as "OS00", and the non-plastic fines are crushed silica fines denoted as "OS100". The soil mixtures are named OS00, OS07, OS15, OS40, OS60, and OS100, with the numbers representing the fines content (FC).

6.7.2.1 Calibration process for Foundry Sand-Silt Mixtures

The calibration procedure employed to derive the corresponding material properties for the Foundry sand-silt mixtures mirrors the process explained in the Section 6.6 for the Sydney sand-silt mixture. Since all tests reported by Thevanayagam et al. (2002) have been conducted under undrained loading conditions, these data can only be used for the calibration of the critical state

parameters, the undrained dilatancy state parameter (χ_u), undrained dilatancy parameter (N_u), and the limited-flow state parameter (ψ_A). Regarding other calibrated-based parameters, including the drained dilatancy state parameter (χ_d), and drained dilatancy parameters (N_d), typical values are assumed. Here, values of $\chi_d = \chi_{dc} = \chi_{df} = 0.5$ and $N_d = 0.4$ are considered, in accordance with the recommendations of Razavi and Yniesta (2023).

Figure 6-8a and Figure 6-8b displays the equivalent critical state lines for the filling and embedment mechanism of the Foundry sand-silt mixtures. These figures show the equivalent critical state points for mixtures with different fines content added to them using Equation (6-14), where ($b = 0.25$) is employed for samples controlled by the filling mechanism, and ($s = 0.65$, and $R_f = 25$) have been considered for the embedment mechanism (Thevanayagam et al., 2002). Thevanayagam et al. (2002) have reported M_{tc} equal 1.2.

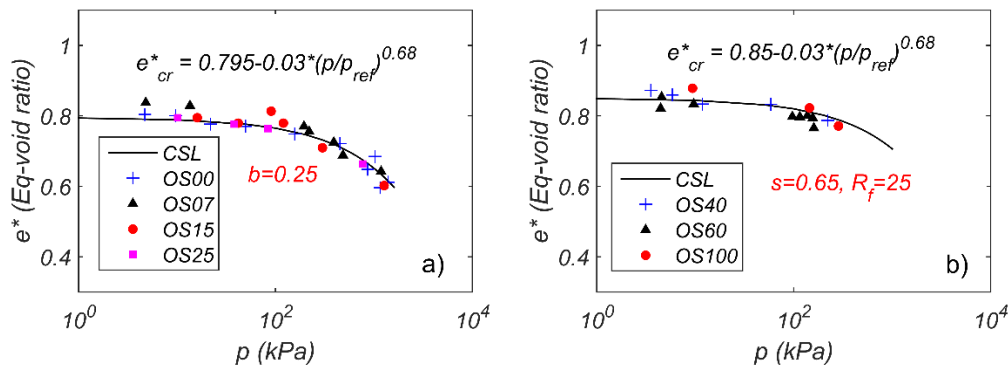


Figure 6-8 Equivalent critical state line in $e^* - \ln p$ space for Foundry sand-silt mixtures, a) samples with filling mechanism, and b) samples with embedment mechanism (Data from Thevanayagam et al., 2002)

According to Step 3 in Section 6.6, the calibration of both $\chi_u(\chi_{uc}, \chi_{uf})$ and $\psi_A(\psi_{Ac}, \psi_{Af})$ is accomplished through the fitting of experimental data from samples exhibiting filling and embedment mechanisms using Equation (6-43). Figure 6-9 illustrates the relationship between undrained peak shear stress ratios and their corresponding equivalent state parameters for samples showing flow and limited flow behavior. These data are extracted from experiments involving samples with filling and embedment mechanisms (Thevanayagam et al., 2002). It is important to

note that the undrained tests were conducted on loosely packed samples, with $(\Gamma_c^* - e_0^*)$ and $(\Gamma_f^* - e_0^*)$ ranging from 0 to 0.01. Consequently, the PolySand-Wfines calibration curves based on Equation (6-43) are shown solely for these values.

The calibrated values for $(\chi_u$ and $\psi_A)$ for the filling mechanism are $(\chi_{uc} = 0.035, \psi_{Ac} = -0.01)$, and for the embedment mechanism, they are $(\chi_{uf} = 0.2, \psi_{Af} = -0.01)$. It is worth noting that the available experimental data is limited, and the tests only cover a narrow range of initial equivalent void ratios close to Γ_c^* and Γ_f^* . Consequently, the accuracy of the calibration process cannot be solely assessed based on Figure 6-9. However, this example serves to demonstrate how the parameters of PolySand-Wfines should be calibrated. With an increased number of data points, the accuracy of the calibration process would significantly improve.

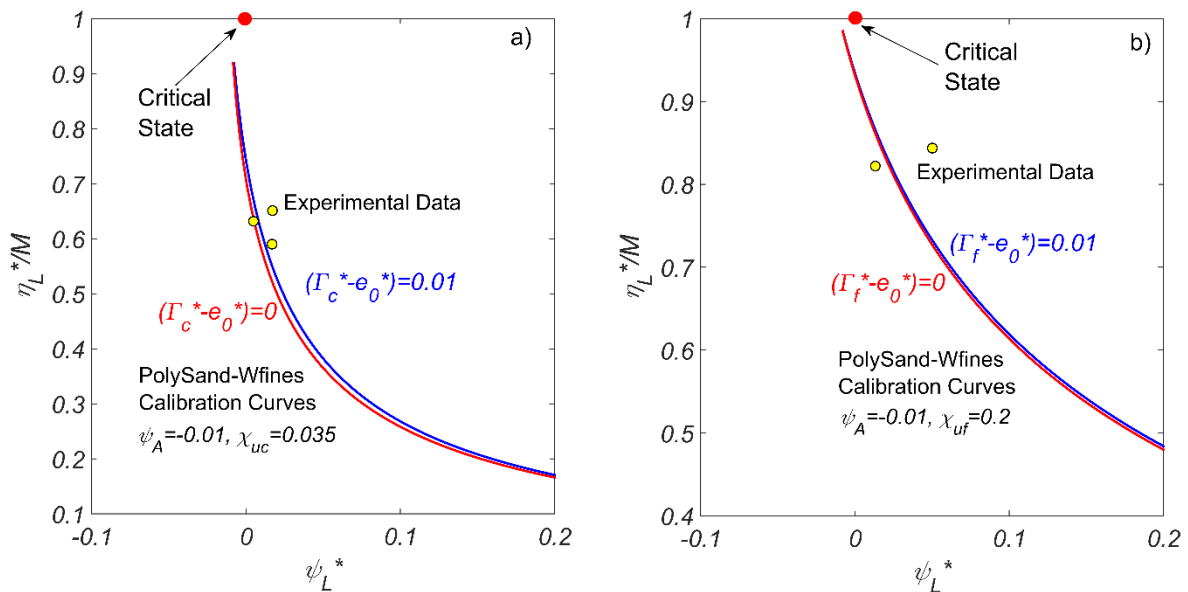


Figure 6-9 Normalized peak undrained stress ratio (η_L^*/M) versus corresponding equivalent limiting state parameters (ψ_L^*) : a) samples with filling mechanism, and b) samples with embedment mechanism, against the PolySand-Wfines calibration curves, (Data from Thevanayagam et al., 2002)

Information on the phase transformation state for the dense samples displaying non-flow behavior is utilized to calibrate the N_u parameter. Figure 6-10 depicts the experimental data extracted from the work of Thevanayagam et al. (2002), where the curve generated through Equation (6-48) is incorporated, with $N_u = 3.5$. The elastic material properties are derived from Thevanayagam et al.'s (2002). The remaining parameters for PolySand-Wfines, (H_0 , H_y , and c), are determined through an iterative process to find the best fit between the simulation and test results.

Table 6-5 presents the calibrated material properties of the PolySand-Wfines in the Foundry sand-silt mixture considering both filling and embedment mechanisms.

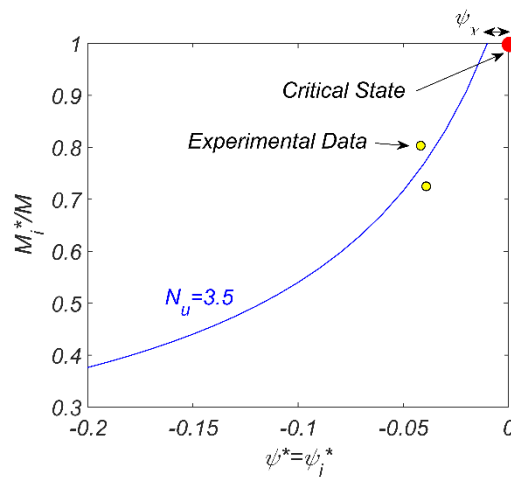


Figure 6-10 Calibration of PolySand-Wfines (N_u) parameter with experimental data showing the phase transformation state on dense samples showing non-flow behavior, data extracted from Thevanayagam et al. (2002)

Table 6-5 PolySand-Wfines model input parameters for Foundry Sand with fines

	Input parameters for the filling mechanism	Input Parameters for the filling mechanism	Input Parameters for the embedment mechanism
Elasticity Parameters	$G_0 = 150$ $\nu = 0.2$		
Dilatancy Parameters	$N_u = 3.5$ $N_d = 0.4$		
Hardening parameters	$H_0 = 90$ $H_y = 100$		
Critical state Parameters	$M_{tc} = 1.2$ $\lambda = 0.03$ $\xi = 0.68$		
Critical state Parameters		$\Gamma_c^* = 0.795$ $b = 0.25$	$\Gamma_f^* = 0.85$ $s = 0.65$
Dilatancy state parameters		$\chi_{uc} = 0.03$ $\chi_{dc} = 0.5$	$\chi_{uf} = 0.2$ $\chi_{df} = 0.5$
Limited-flow State parameters		$\psi_{Ac} = -0.01$	$\psi_{Af} = -0.01$
Limited-flow State parameters	$c = 11$		

6.7.2.2 Simulation results for Foundry Sand-Silt Mixtures

Figure 6-11a and Figure 6-11b present a comparison between the predictions of the model and the test results for FC = 7% and 15%. The initial equivalent void ratios for these mixtures were similar. The simulation results demonstrate the ability of PolySand-Wfines to capture the limited flow characteristic and simulate sand-silt mixtures with varying fines content. Figure 6-11c and Figure 6-11d showcase a comparison between the nonflow and flow behaviors of the Foundry Sand-silt mixture with different fines content and the model's predictions. The results indicate an overall good level of agreement, indicating the model's ability to represent the observed behavior.

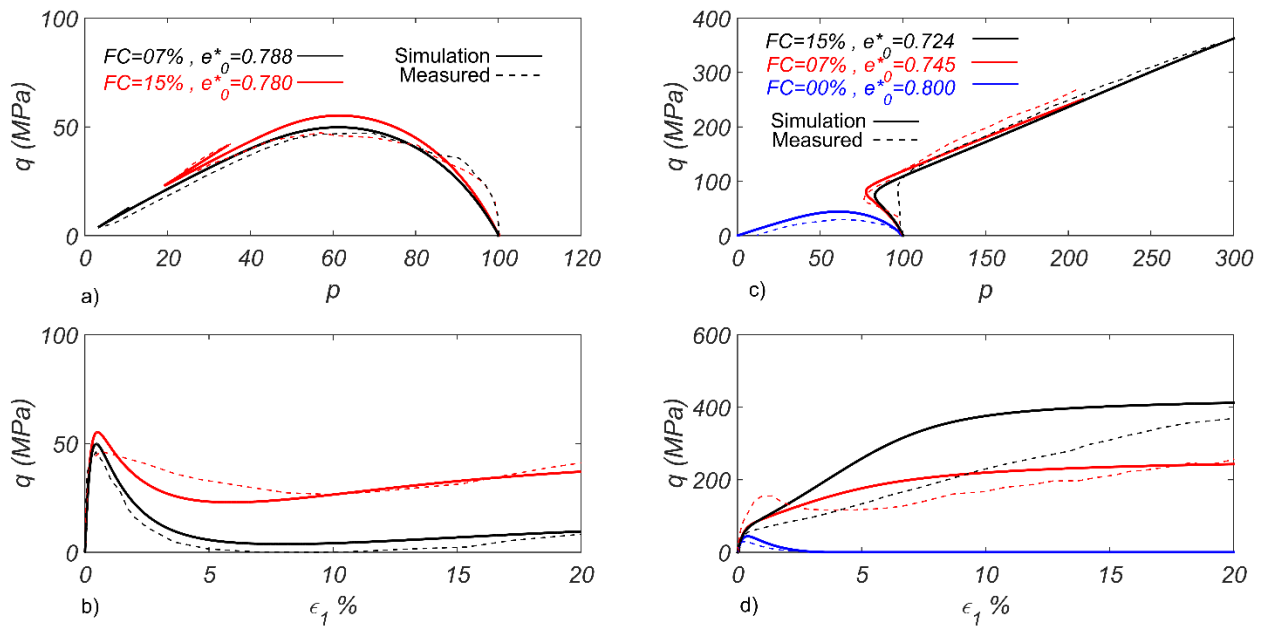


Figure 6-11 Comparison of test results and simulations with the PolySand-WFines model: a, c) effective stress path, b, d) deviatoric stress-axial strain, (Data from Thevanayagam et al., 2002)

Figure 6-12 compares the simulations results and the laboratory tests results, for the samples controlled by an embedment mechanism. The model is able to reproduce the observed pattern, partially capturing the undrained monotonic behavior of sand-silt mixtures with a fines content greater than FC_{th} , controlled by the embedment mechanism. However, there are discrepancies between the measured and simulated values, which may be explained by the limited dataset, which limits the accuracy of the calibration, and the limitation of the equivalent approach. Although the overall trend is acceptable, the application of the equivalent approach to the embedment mechanism necessitates further study and additional experimental research.

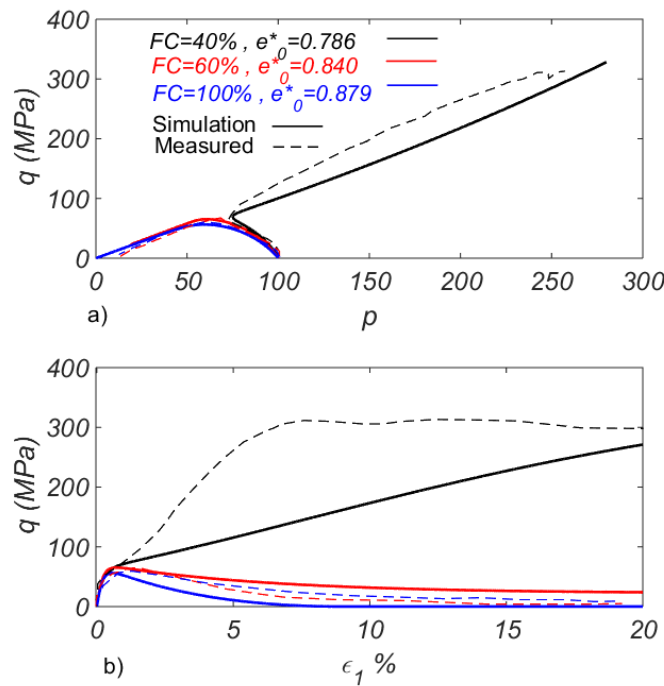


Figure 6-12 Comparison of test results on samples showing an embedment mechanism and simulations with the PolySand-WFines model: a) effective stress path, b) deviatoric stress-axial strain (Data from Thevanayagam et al., 2002)

6.8 Conclusion

This paper presents an extension of the PolySand model, termed PolySand-Wfines, developed to model the behavior of sand-silt mixtures with fines content ranging from 0% to 100%. By incorporating the equivalent critical state theory, the PolySand-Wfines model integrates the influence of fines on the overall behavior of the mixture with a single set of input parameters, regardless of the fines contents. In PolySand-Wfines, the traditional void ratio and state parameter are replaced with their equivalent counterparts, namely the equivalent void ratio and equivalent state parameter, which consider the influence of fines content.

The behavior of sand-silt mixtures and the two distinct mechanisms that control it, i.e. the filling and embedment mechanisms, each associated with different fines content levels, were first discussed. The formulation of the PolySand-Wfines model and the dependency of its material properties on the fines content were studied, and it was determined that the model's dilatancy state

parameters and limited-flow state variables should be determined separately for each mechanism. Other material properties remain unaffected by variations in fines content.

The formulation of the PolySand-Wfines model is developed to effectively control the flow behavior of sand-silt mixtures by introducing a limited number of new parameters. The model proves to be accurate in capturing the intricate flow characteristics exhibited by these mixtures. In cases where limited flow is observed, the limited state parameter is successful in the simulation of the limited flow behavior typically encountered in sand-silt mixtures.

To evaluate the performance and reliability of the PolySand-Wfines model, a comparison between the model predictions and undrained test results obtained from well-documented experiments on Sydney sand-silt mixture and Foundry sand-silt mixture was presented. The results confirm that PolySand-Wfines is well-suited to simulate the undrained monotonic behavior of sand-silt mixtures exhibiting a filling mechanism, thereby validating its effectiveness as a predictive tool for practical applications in geotechnical and mining engineering. However, when considering the embedment mechanism, the simulation results demonstrated a limited accuracy in capturing the observed behavior. The limited accuracy might arise from the limited dataset, and further validation is needed.

6.9 Acknowledgements

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CHAPTER 7 SUMMARY, DISCUSSION, LIMITATIONS, FUTURE AVENUES OF RESEARCH

7.1 Introduction

The primary goal of this thesis was to develop a practical constitutive model capable of directly controlling flow liquefaction in loose granular soils, while also considering the influence of fines content. The motivation behind this research came from the susceptibility of mine tailings to flow liquefaction, due to their loose saturated state, and the spatial variability of their fines content.

To achieve this aim, a thorough examination of the constitutive behavior of granular soils was conducted, leading to the formulation of a specific model in the category of Cambridge type models for sand. Two versions of the model were proposed to predict the behavior of both sand and sand-silt mixtures across a wide range of densities and consolidation pressures.

The thesis was organized as follows: Chapter 1 provided a brief introduction, followed by a comprehensive literature review in Chapter 2. In Chapter 3, the NorSand model, used to simulate the undrained monotonic behavior of sand, was examined to identify areas that required improvement. In Chapter 4, the linear state-dilatancy law of the NorSand model and its implications on mechanical responses were studied, and a novel nonlinear state-dilatancy law was presented. Subsequently, Chapter 5 introduced the PolySand constitutive model to overcome the limitations of NorSand and evaluated its performance under triaxial conditions. Finally, Chapter 6 extended the proposed model to simulate sand-silt mixtures and validated it using two different sets of test results documented in the literature.

7.2 Summary of Findings

Understanding the NorSand Functionality and Limitations: Chapter 3 focused on assessing the effectiveness of the original NorSand model in simulating the undrained mechanical behavior of sand. The study highlighted that the original NorSand model faced challenges in accurately representing flow liquefaction and lacked the ability to simulate limited flow behavior. It was revealed that the original NorSand model produced higher instability stress ratio near the

critical stress ratio than observed in test results, when the material properties were chosen in the typical range proposed by Jefferies and Been (2015).

To overcome these limitations, the developers of NorSand introduced a modification to the hardening rule, which involved incorporating a cap-softening term. This enhanced version demonstrated some capability in reproducing flow and limited flow behavior within specific constraints, and with limited flexibility. Chapter 3 examined the effectiveness of this additional term in modeling the undrained behavior of granular soils and proved that the users have no direct control over the undrained mechanical behavior of loose samples. Instead, they must accept the response provided by the cap-softening term in the modified version. It is essential to note that the results indicated that any attempt to amplify the effect of cap-softening would result in numerical instability within the model. Additionally, it was highlighted that the NorSand applicability was limited to a restricted range of initial image state parameters.

Understanding the Strengths of the NorSand Model: Despite the aforementioned limitations, it is important to acknowledge the notable strengths of the model. One key advantage lies in its material parameters, each of which holds a clear physical meaning and can be easily calibrated. Additionally, the incorporation of limiting dilatancy and the state parameter within the Cambridge type model proved to be powerful tools that deserve recognition (see Chapter 4). To utilize these advantages and address the primary shortcomings of the model, this study endeavored to introduce an alternative model with a similar core concept as NorSand but with different formulations. The goal was to leverage the strengths of the original model while offering improved solutions to its limitations.

Proposing a New Nonlinear State-Dilatancy Law: Chapter 4 concentrated on addressing the problems associated with the NorSand model, with a particular focus on its state-dilatancy law, identified as the root cause of the problem. The study showed that the state-dilatancy law necessitates additional condition that must be satisfied for the model to function properly. This new condition enforces that the limiting state must reside on the yield surface for any potential state of the granular soil. Upon introducing this requirement, a new nonlinear state-dilatancy law was proposed, meeting all the necessary criteria. This new law enabled a change of the dilatancy state parameter to effectively control flow liquefaction and be in better agreement with measured data.

Moreover, the proposed law removed the limitations on applicability, which were present in the original NorSand model, expanding its range of practical use.

Developing PolySand Model: Chapter 5 of the study introduced the PolySand model, which incorporates the proposed nonlinear state-dilatancy law in Chapter 4. PolySand introduces one additional material property for loose states called "undrained-dilatancy state parameter (DSP)" which was designed to control the initiation of static liquefaction produced by the model. This material property holds a clear physical meaning, and its calibration can be straightforwardly achieved using experimental triaxial data from tests done on loose samples. Additionally, the model's formulation incorporated a nonlinear critical state line.

The thesis focused on three main objectives, all of which were addressed through the PolySand model. Firstly, having control on the initiation of flow liquefaction was achieved. Secondly, practicality and simplicity were maintained by adding only one additional material property required for the model. Lastly, the nonlinearity of the critical state line was considered and effectively incorporated. In addition, the PolySand model kept the NorSand's original hardening rule and has been modified to simulate the limited flow behavior, a feature that was missing in the original NorSand model.

Extending PolySand to PolySand-Wfines: Chapter 6 focused on achieving the last objective of the thesis, which was to incorporate the influence of fines content on the proposed model. The aim was to extend the existing PolySand model to a new version called PolySand-Wfines, enabling the simulation of the monotonic behavior of sand-silt mixtures. The investigation in Chapter 6 revealed that the PolySand formulation was suitable for working with the concept of an equivalent critical state theory. Originally, this framework was proposed for mixtures with a specific range of fines content, where the structure was mainly influenced by coarse grains. However, the study also demonstrated that by employing a similar approach with different material properties and further studies, the approach may be applied to mixtures where fines play a dominant role in governing the structures.

7.3 Discussion

Complexity issues in changing the NorSand state-dilatancy law: The research's key objective was to directly control flow liquefaction in tailings materials using the NorSand model,

which motivated the exploration of alternative approaches to address this issue. The NorSand's original state-dilatancy law was defined based on drained test results obtained from dense samples (Jefferies, 1993). However, it became evident that this definition could not provide promising results under undrained loading (See Chapter 3 and Chapter 4). To address this limitation, the model's developers introduced the cap-softening term, which influenced the undrained response of the original NorSand. This addition showed the developers' awareness of the problem, but it did not provide a complete solution, prompting questions about their decision to persist with this approach.

According to the findings in Chapter 4, the reason lies in the extreme complexity involved in transforming the state-dilatancy law (D_L, ψ_L) into NorSand state-dilatancy law (D_L, ψ_i) for use in the model's formulation. This transformation is necessary because the model's formulation is based on (D_L, ψ_i) , while its material properties are derived from fitting of (D_L, ψ_L) to the measured data. However, the process of this transformation through the NorSand approach can become highly challenging, particularly when attempting to modify the formulations of the state-dilatancy law or alter the critical state line or stress-dilatancy rule. In some cases, such changes may prove impossible to achieve effectively.

Owing to these complexities and considering that the linear state-dilatancy law represents the simplest option, the literature has not pursued alterations to this law. As changing the NorSand state-dilatancy law is judged unfeasible, alternative approaches are explored. One such option is to utilize a different state-dilatancy parameter for loose samples to adjust their response. However, as demonstrated in Chapter 4, the NorSand linear state-dilatancy law does not offer significant flexibility to modify its material properties. Consequently, the exclusive practical choice was to incorporate the cap-softening term, with accepting all its consequences as discussed in Chapter 3.

Managing the undrained behavior using the NorSand model: One of the challenges associated with using NorSand is how to effectively utilize the model in its current formulation to control the undrained behavior of sand. NorSand offers a method for simulating undrained behavior through a specific set of procedures (Shuttle and Jefferies, 2016). Initially, in the first step, the cap-softening option needs to be activated while considering undrained conditions. It is evident that the simulation results produced may not align with experimental findings. Consequently, to enhance the agreement between simulation and experimental data, the second step involves adjusting the

bulk modulus. These two options are employed to manage the undrained behavior of sand using the NorSand model, but they are not regarded as an appealing solution for addressing this complex problem. This is because incorporating the cap-softening term modifies NorSand's original hardening rule under undrained conditions, resulting in the neglect of the full potential of this rule and making it challenging to make further adjustments or improvements to the model.

Power of the NorSand's original hardening rule: The original NorSand's hardening rule (introduced by Jefferies, 1993), which lacks a cap-softening term, exhibits significant effectiveness in simulating both the monotonic behavior of dense and loose samples. That rule transforms the model into a binary-state model, where it produces non-flow behavior for dense samples and flow behavior for loose samples. Furthermore, when utilizing the original hardening rule in conjunction with a well-defined state-dilatancy law, it becomes straightforward to determine the transition point for a change in the hardening regime. This transition is identifiable through laboratory tests conducted on granular soils, and by utilizing this data, it becomes possible to calibrate the new parameters employed in the state-dilatancy parameter. Consequently, this allows for the control and adjustment of the model's response through its original hardening rule.

Therefore, with an appropriate state-dilatancy law, these features can be harnessed to enhance the control over the flow behavior of loose samples. One of the primary focus of this thesis has been the introduction of a novel state-dilatancy law and the exploration of new material properties that can yield improved results without altering the model's original hardening rule. Among the state-dilatancy parameters introduced in this research, "undrained-DSP" stands out. This incorporated parameter possesses a clear physical meaning and undergoes calibration by conducting four to five additional undrained loading tests on loose samples. The calibration process in determining the Undrained-DSP proved to be straightforward, and comparison between the numerical simulation of PolySand and measured data demonstrated the model's improved ability to simulate flow liquefaction more accurately in granular soils.

Unlocking the potential of NorSand's original hardening rule and its adaptability for incorporating the limited-flow feature: The PolySand model no longer relies on the restrictive mathematical concept of the cap-softening term in its formulation. However, due to the utilization of the Original NorSand's hardening rule in the PolySand, the model is unable to reproduce the limited-flow feature. To integrate this aspect into the model, this study drew inspiration from the

work of Li and Dafalias (2012) and incorporated it into the PolySand formulation. Through this approach, the PolySand has been granted with the capability to simulate the limited-flow. This formulation introduces a new parameter referred to as the "limited flow state parameter," which, when adjusted, enables control over the limited-flow behavior. These options were absent in the NorSand model, regardless of whether the cap-softening term was present or not. All these enhancements are attributed to returning to the NorSand's original hardening rule and implementing an appropriate state-dilatancy law in the PolySand. It is worth noting that in the NorSand model with cap-softening term, controlling the limited-flow feature through the same approach used in the PolySand becomes challenging. This is because the cap-softening term also introduces a degree of limited-flow behavior which cannot be controlled, as shown in chapter 3.

7.4 Contributions to the Field of Mining Geotechnical Engineering

The NorSand state-dilatancy law and its application were discussed in this research, demonstrating the difficulties faced in accurately representing the undrained behavior of granular soils, particularly loose sands.

To undertake these challenges, the NorSand model was modified, introducing a fresh perspective on the state-dilatancy law used in Cambridge-type models for sand behavior simulation. This new viewpoint led to the proposal of a novel state-dilatancy law, resulting in the development of a distinct model known as PolySand. The PolySand model allowed for direct control of the flow and limited flow behavior of loose, saturated granular soils through the incorporation of two additional material properties. In terms of practical implementation, the newly added materials in PolySand could be easily calibrated, and the model was readily adaptable to programming codes, making it a user-defined model in existing finite element and finite difference geotechnical software.

PolySand-Wfines, an extension of PolySand that employs equivalent critical state theory to simulate the mechanical behavior of sand-silt mixtures, has been validated. This validation not only confirmed the practicality of the proposed model and its ability to yield accurate simulation results when compared to test data but also provided evidence supporting the validity of the framework proposed by Rahman et al. (2014).

7.5 Limitations and Future Avenues of Research

Upon the Completion of PolySand-F: The existing version of PolySand is an isotropic version, and the current study has not taken into consideration the potential influence of different shear modes and principal stress rotations on the sand's behavior, despite their known significant impact. Consequently, the future direction of research aims to investigate how PolySand, along with its newly added material properties, can capture the effects of changing shear modes and principal stress directions on the mechanical behavior of sand. To achieve this, additional laboratory tests should be conducted with the purpose of identifying the relevant physical patterns that occur in the stress-strain behavior of granular materials. These patterns are essential for defining the variation of PolySand's material properties in response to the different shear modes and directions of principal stresses.

Regarding the Utilization of PolySand in Sand-Silt Mixtures with Embedment Mechanism: As emphasized in Chapter 6, conducting experimental studies (drained and undrained tests) on sand-silt mixtures with fines governing the microstructures is essential. By providing such data, it becomes possible to advance the equivalent critical state theory, making it adaptable for these specific microstructures. Although PolySand-W_{fines} can already offer a similar pattern for the embedment mechanism, observing and studying how these mixtures behave can serve as a base foundation for refining both the framework and PolySand-W_{fines} further.

On the Applicability of PolySand for the Seismic Behavior of Soils: PolySand is a model designed specifically to simulate the monotonic behavior of sand. Consequently, it is not well-suited to accurately simulate the plastic volumetric and shear behavior caused by seismic loading, as the unloading/reloading process occurs within the elastic region. This limitation hinders the model's ability to accurately capture the cyclic mobility exhibited by saturated soils under seismic conditions. To address this issue and enhance the model's capability, it can adopt a similar approach used by other models such as UBCSand (Beaty and Byrne, 2011), the Finn-Byrne model (Byrne, 1991), or the approach introduced by Jefferies et al. (2015). By incorporating such modifications, PolySand can gain the power needed to simulate the seismic behavior of saturated soils.

Assessing PolySand's Applicability in the Boundary Value Problems: PolySand can be implemented into finite element and finite difference programs, like Plaxis and FLAC, as a user-defined soil model. By doing so, this additional model becomes valuable in analyzing various

tailings dams prone to static liquefaction-induced failure. The model's exceptional ability to accurately simulate this phenomenon allows for a meaningful comparison of its response with other existing models, such as NorSand, to identify differences. Furthermore, PolySand-Wfines can be utilized to model the behavior of tailings with different fines percent, thereby revealing the true mechanical characteristics of tailings dams.

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