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**A Study on Modelling of Tool Tip Disturbance Forces and Volumetric Errors
Based on The Machine Tool's Compliance Behaviour**

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Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*

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Ce mémoire intitulé :

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Based on The Machine Tool's Compliance Behaviour**

présenté par **Tianliang ZHUANG**

en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*

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RÉSUMÉ

L'erreur volumétrique de machine-outil influence la justesse dimensionnelle et géométrique de la pièce usinée. Beaucoup de méthodologies ont été proposées pour la mesure de l'erreur volumétrique. Cependant, la plupart d'entre elles sont exécutées sous la condition sans charge. Les études sur l'erreur volumétrique, sous la condition avec charge, devraient être proposées pour connaître le comportement de machine-outil avec l'effet de la compliance de la machine-outil. Ce projet vise à identifier la compliance de machine-outil, par laquelle l'erreur volumétrique et la force sur l'outil ont été calculées. La compliance volumétrique translationnelle statique a été mesurée par la méthode proposée par la norme d'ISO 230-1 dans les directions X, Y et Z. Cette compliance a été modélisée mathématiquement par une équation de la somme d'un terme linéaire et d'un terme tangente hyperbolique en fonction de l'erreur volumétrique. La variation non linéaire de cette compliance a été détectée. Le lien direct entre l'erreur volumétrique et le torque de moteur a été établi. La prédiction de l'erreur volumétrique à partir du torque de moteur, en utilisant la méthode d'itération avec la meilleure estimation initiale, a les racines de l'erreur quadratique moyenne (REQM) de 4.9 μm , 6.6 μm et 2.8 μm dans les directions X, Y et Z, respectivement. La compliance dans le test dynamique a été mesurée par le Ball-Roller Bearing Device (BRBD). Un modèle physique et deux modèles de réseau de neurones (NNs_1 et NNs_2) ont été construits pour prévoir la force sur l'outil, générée par la motion relative entre l'outil et la table en utilisant les commandes de la machine, par les signaux de sortie du moteur. Le modèle de NNs_2 a prévu au mieux les forces d'outil avec les REQMs de 47.25 N, 37.12 N et 35.44 N pour la force sur l'outil dans les directions X, Y et radiale, respectivement. L'erreur volumétrique radiale a été calculée par le produit de la force radiale prévue et la compliance radiale mesurée. Le REQM de l'erreur volumétrique radiale est de 4.91 μm dans l'ensemble de données de test. Même si les résultats de cette étude sont soumis à l'appareil spécifique utilisé dans le test, ce test propose une méthodologie générale pour surveiller virtuellement l'erreur volumétrique et la force sur l'outil par les données vient du moteur avec l'aide du modèle de compliance.

ABSTRACT

The machine tool volumetric error affects the machined parts' dimensional and geometrical accuracy. Many different methodologies have been proposed to measure the volumetric error. However, most of them are under no-load conditions. Studies on the volumetric error under loaded conditions should be carried out to consider the machine tool behaviour due to compliance. This project aims to identify the machine tool volumetric compliance, with which the volumetric errors and tool tip forces were further calculated. The static translational volumetric compliance was measured as proposed in ISO 230-1, in the X, Y and Z directions. It was modelled mathematically by an equation as the sum of a linear term and a hyperbolic tangent term of volumetric errors. The nonlinear compliance variation was detected. The direct connection from the feed motor torque to the volumetric errors was constructed. The predictions of the volumetric error from feed motor torque, using iteration with the best starting estimates on the testing dataset, had RMSEs of 4.9 μm , 6.6 μm and 2.8 μm in the X, Y and Z directions, respectively. The radial compliance in the dynamic test was measured by the Ball-Roller Bearing Device (BRBD). One physical model and two neural network models (NNs_1 and NNs_2) were constructed to predict the tool tip forces, generated by relative motion between tool side and table side using machine commands, from the feed motor data. NNs_2 best predicted the tool tip forces with RMSEs of 47.25 N, 37.12 N and 35.44 N for the forces in the X, Y, and radial directions, respectively. The predicted radial volumetric errors were calculated by the product of predicted radial forces and measured radial compliances. The RMSE of the predicted radial volumetric errors was 4.91 μm on the testing dataset. Even though this study's results are subjected to the specific tools used, this test proposes a general methodology to virtually monitor the volumetric errors and tool tip forces from feed motor data with the help of the compliance model.

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LISTE OF SYMBOLS AND ABBREVIATIONS

C_l	Machine tool static translational local compliance
C_g	Machine tool static translational global compliance
C_r	Radial compliance
E_v	Volumetric error
E_{XYZ}	Static translational volumetric error
F_d	Measured tool tip disturbance force, calculated from group mean
$F_{d,pred}$	Predicted tool tip disturbance force
$F_{x,y}$	Measured tool tip disturbance force in X and Y direction
$F_{x,y,pred}$	Predicted tool tip disturbance force in X and Y direction
F_f	Friction force in machine tool
F_r	Radial force
F_t	Tangential force
J_e	Equivalent inertia
K_d	Tool tip disturbance force coefficient
K_f	Friction torque coefficient
R^2_{adj}	Adjusted R Squared
SA	Scale A (OHAUS Trooper)
SB	Scale B (FG-200K AL)
T_d	Tool tip disturbance torque
T_f	Friction torque in machine tool
$T_{f,coul}$	Maximum Coulomb friction torque in machine tool
T_m	Feed motor torque, calculated from group mean
ζ	Constant

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CHAPTER 1 INTRODUCTION

1.1 General introduction

Computer numerical control (CNC) machining centre with five axes improves machining productivity. The numerical unit commands the multi-axes motion with rapidness and precision. The tool's positions and orientations with respect to the workpiece are controlled in a tri-dimensional space. The machining of complexed-shape parts becomes easier with the help of CNC five axes machining centre.

However, the increased complexity of machine results in more error and failure sources from the mechanical and electrical components in the machine tool. These errors can be, for example, static, dynamic, thermal, etc. All the errors listed contribute to volumetric errors that describe the deviation of the tool tip's actual position to its nominal position with respect to the workpiece. They result in the dimensional and geometrical imprecisions of machined parts. Studies on the modelling and compensation of volumetric errors have been carried out. Devices such as laser interferometer, double ball bar and straight edges have been used to measure machine tool errors. However, many of these devices work under no-load condition, which is not a real machining condition where cutting forces are exerted at the tool tip.

Under loaded conditions, the machine deflects as it is a compliant mechanism. If the machine volumetric compliance behaviour can be modelled, the volumetric error can be linked to a given tool tip load. Then, if the tool tip forces are further linked to feed motor data, the connections among the feed motor torque, the tool tip force and the volumetric error can be obtained. The volumetric compliance behaviour is an intermediary that could facilitate the understanding of the machine tool's performance under the loaded condition.

1.2 Problematic

The problems exist in modelling the volumetric compliance and how to monitor the tool tip disturbance force and volumetric errors caused by compliance from the feed motor data. Hence, the project should answer the questions below concerning the problematics:

- How to measure compliance of machine tool under different testing conditions?
- What is/are the model(s) connecting feed motor data, tool tip forces and volumetric error?
- What could influence the accuracy of the model(s)?

1.3 Objectives

The project's main objective is to *model the tool tip forces and volumetric errors based on the volumetric compliance of a five-axis machine-tool*. For achieving the main objective, the following sub-objectives should be attained:

- Obtain the translational volumetric compliances under static testing conditions;
- Obtain the radial volumetric compliances under dynamic testing conditions;
- Identify the connections among feed motor data, tool tip forces and volumetric errors;
- Identify the limitations of model(s) proposed.

1.4 Thesis structure

Apart from the introduction in CHAPTER 1, the literature review in CHAPTER 2, and the methodology listed in CHAPTER 3, the body structure of this work generally contains two parts: the static test in CHAPTER 4 and the dynamic test in CHAPTER 5. Both parts start with constructing the compliance model necessary for calculating tool tip forces and volumetric errors caused by compliance. Next, the relationship between the feed motor data, tool tip disturbance forces and volumetric errors is established. In the static test, the total friction is constructed as a function of the static translational volumetric errors inspired by Dahl's theory, which gives a function to describe the derivate of friction force to relative displacement between contacting surfaces. Then, the direct link from the feed motor torque to the static translational volumetric error is found. The dynamic test compares one commonly used physical model and two neural network models for predicting the tool tip forces from feed motor data. The product of the predicted radial forces in the XY plane and the radial compliance measured then calculates the predicted volumetric errors. The conclusion is drawn in CHAPTER 6. A recommendation on future work is also given in CHAPTER 6. The overall thesis structure is shown as in Figure 1-1.

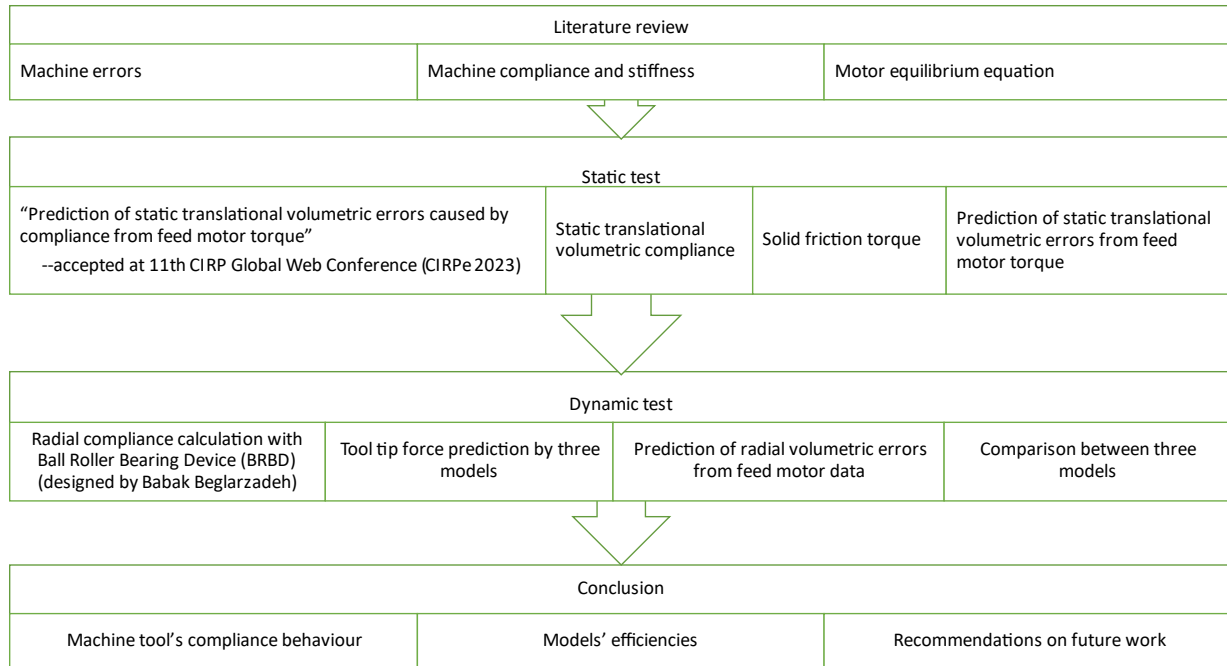


Figure 1-1: Thesis structure

CHAPTER 2 LITERATURE REVIEW

In this chapter, the literature review in the field of methodologies on machine-tool errors identification, machine tool compliances and mathematical equilibrium equations for the feed motor is presented.

2.1 Machine errors

Different errors exist in machine tools, such as geometric, joint kinematic, thermal, dynamic and servo errors. They all affect the machining accuracy. It is important to be familiar with different errors.

Geometric errors are generally errors due to the general shape and assembly imperfections of the axis and machine structures, while joint kinematic errors are caused by the error motions of machine axis carriage [1]. They are also called inter- and intra- errors or link errors and motion errors. As shown in Figure 2-1, each axis could have six motion errors projected in the six degree-of-freedom directions, which can be modeled as a function of the axis position. There are eight link errors, excluding the errors of the spindle axis [2]. The link errors are independent of the axis positions. The inter-errors can be measured by the devices such as straight edges. The intra-errors can be measured using later interferometry devices. The intra errors can be modeled as polynomial functions of joint coordinates with proper degree terms [3].

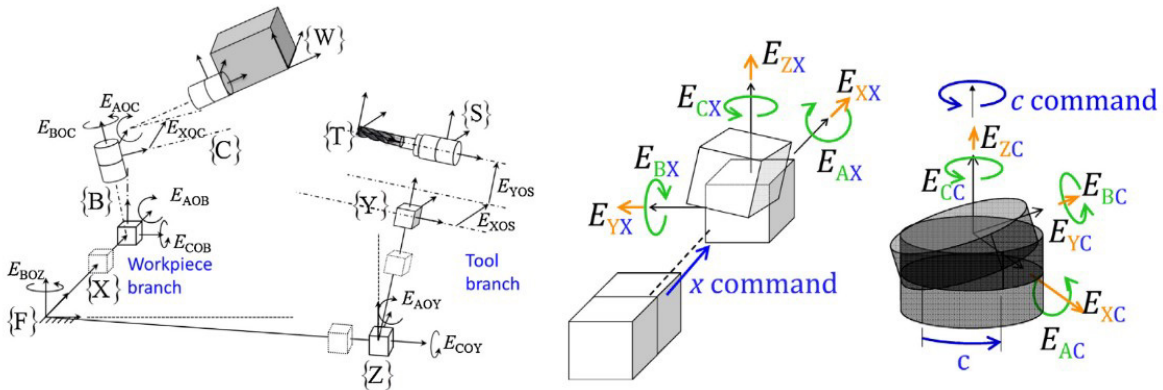


Figure 2-1: Inter-errors (left) and intra-errors (right) illustration [4]

Thermal errors are the errors caused by changes in the temperature of machine components after the machine tool runs for a certain time. The power consumption of machine components, such as the power consumption of the feed motor, was used to model the thermal errors [5-7]. Different

error parameters account for different proportions of the thermal error. For example, in [7], the error parameter EYB0 alone explained 40% of the thermal errors caused by rotary axis activities [7].

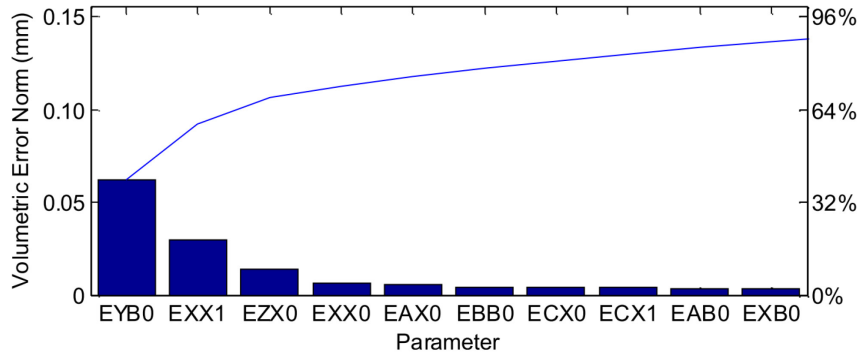


Figure 2-2: Pareto graph of dominant errors for B-axis activity [7]

When more axes move simultaneously, the servo mismatch error appears. The feed rates influence the servo mismatch error. It can be measured as the time in milliseconds at which one axis leads to another under a circular run in a plane [8]. In the clockwise and counterclockwise directions, it is distorted differently in the polar plots along 45° and 135° diagonal directions as in Figure 2-3.

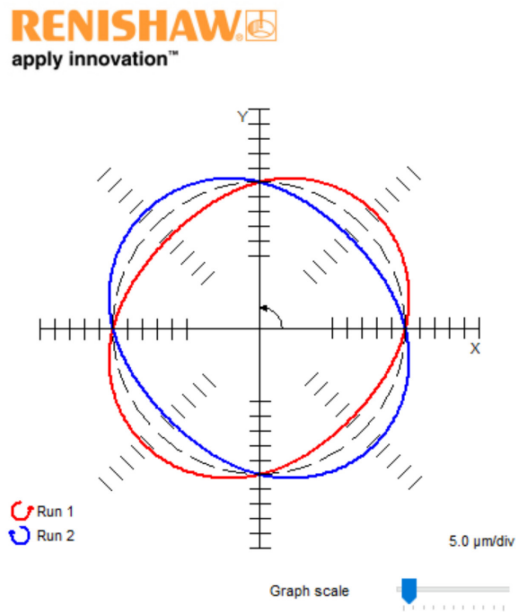


Figure 2-3: Servo mismatch error plot generated by Renishaw Simulator, feed rate 3000 mm/min and test radius 150mm.

The above errors, together with other errors such as backlash errors, reversal spikes, scaling errors, and errors caused by vibrations in the machine, all contribute to the volumetric error describing the deflection of tool tip's actual orientation and position to the nominal with respect to the workpiece [1] as in Figure 2-4. Machine volumetric behaviour can be tested using R-device [9], Capball [2], Scale and Master Balls Artefact (SMABA) method [10], Reconfigurable Uncalibrated Master Ball Artefact (RUMBA) method [11], Double Ball Bar [8], etc. Using Homogenous Transformation Matrix (HTM) [12, 13] and Jacobian Matrix [2, 10, 11, 14, 15], the volumetric errors measured by indirect methods [16], involving multiple axes' movements, can be linked with the motion and geometric errors. The error parameters can be further calculated. The volumetric errors and its uncertainties highly influence the machined part precision as they result in the geometrical and dimensional imperfections of the machined part. They need to be accurately identified for the future compensation on the machine. The volumetric error is the primary analyzed error in this work.

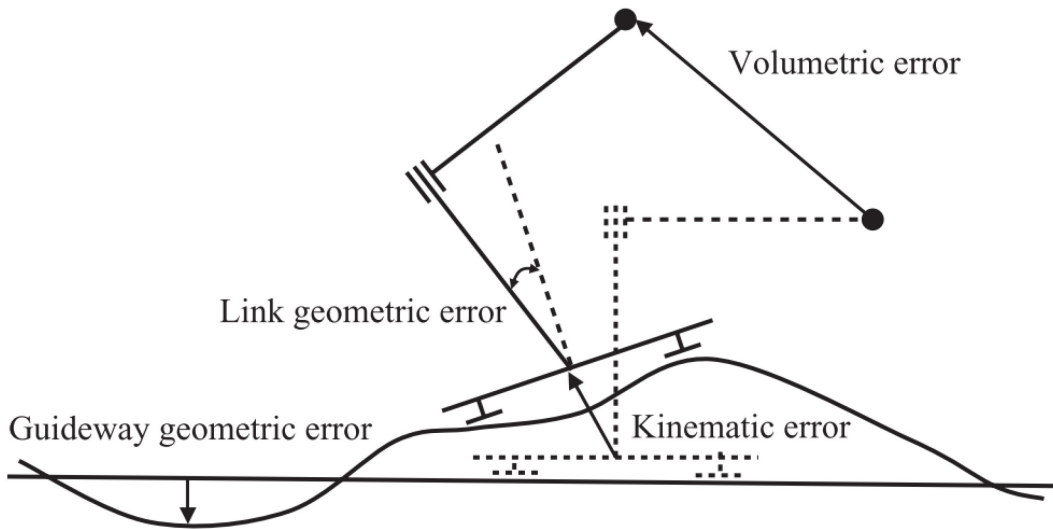
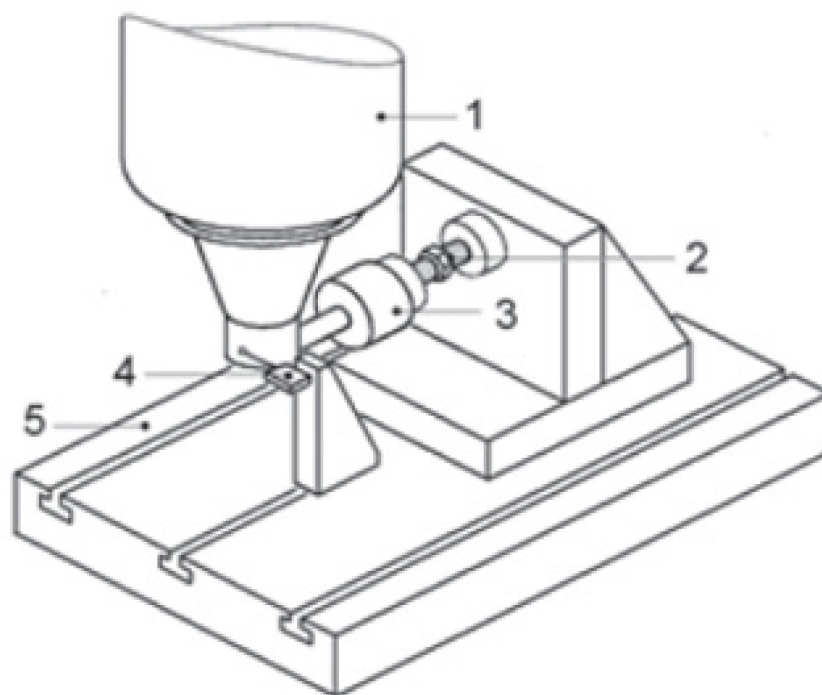


Figure 2-4: Machine error categories illustrated using an X–Y machine with exaggerated errors on the X-axis guideway [1].

2.2 Machine compliance test

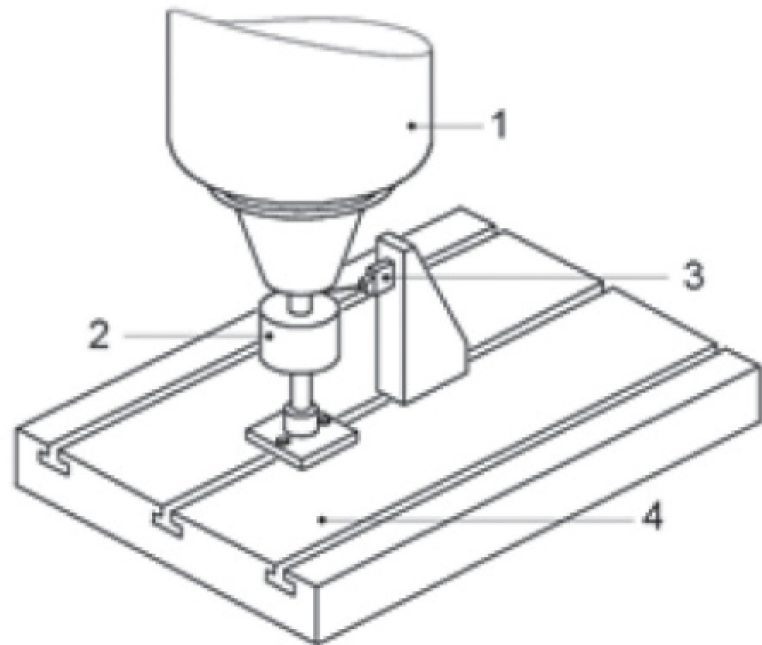
The compliance is the linear (or angular) displacement per unit force (or moment) between two objects [17]. It can describe the error and force relation on the machine tool under loaded conditions. If the compliance or stiffness of the machine is known, given a specific load, the

machine errors can be calculated. The standard of ISO 230-1 [17] and ASME B5.54 [18] provide general methodologies to measure the static machine tool's compliance and hysteresis with forces generated externally and internally. In the test with forces generated externally, the load cell measures the forces generated, and a sensor tests the displacement between the workpiece side and the spindle. In the method of forces generated internally, the command generates the forces, and a linear displacement sensor measures the deflection at the tool tip. The setups are illustrated in Figure 2-5 and Figure 2-6. The final compliance is calculated by the change of forces and variation of displacement from tool side to the workpiece side in a total range, which could be considered as the global compliance.

**Key**

- 1 spindle housing
- 2 differential screw
- 3 load cell
- 4 linear displacement sensor
- 5 machine table surface

Figure 2-5: Machine compliance measurement with force generated externally by load cell [17]



Key

- 1 spindle housing
- 2 load cell
- 3 linear displacement sensor
- 4 machine table surface

Figure 2-6: Machine compliance measurement with force generated internally by commands [17].

The Elastically Linked System (ELS) theory, as in Figure 2-7, considers that the relationship of the tool tip positional deviation to force loads can be modelled as the compliance between the tool tip and the work piece [19] and can be linked to machined parts' accuracy. The Loaded Double Ball Bar (LDBB) was designed and used to measure the radial deviation of the distance from the tool joint to the table joint, given loaded forces in the radial direction [16, 19-21] under a circular run. The compliant behaviour of the machine tool can be identified with the LDBB device. In a later work, using the LDBB device, the researcher built a full translational quasi-static stiffness matrix [20] that describes the force-deviation relationships between tool and workpiece in a tridimensional space. The setup is shown in Figure 2-8. This new device can simultaneously measure the stiffness and cross-stiffness between axes with one device setup, providing many conveniences.

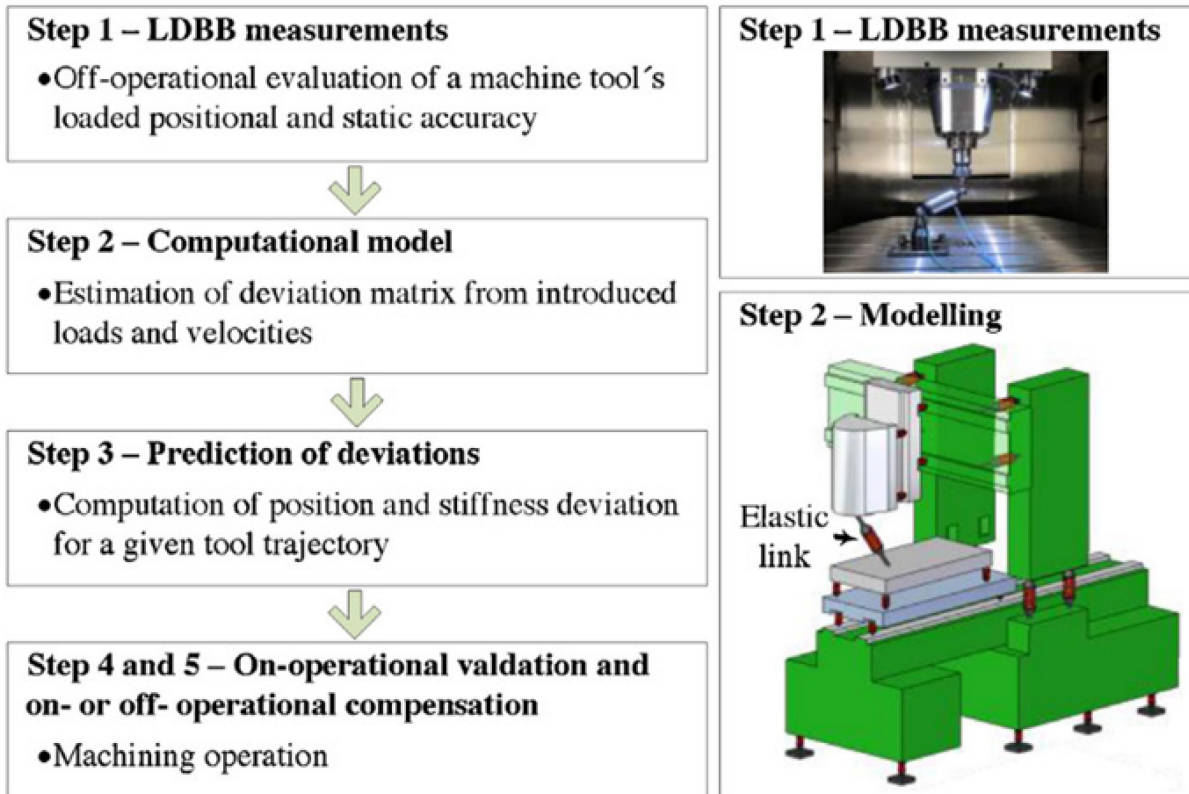


Figure 2-7: Elastically linked System (ELS) theory illustration [16]

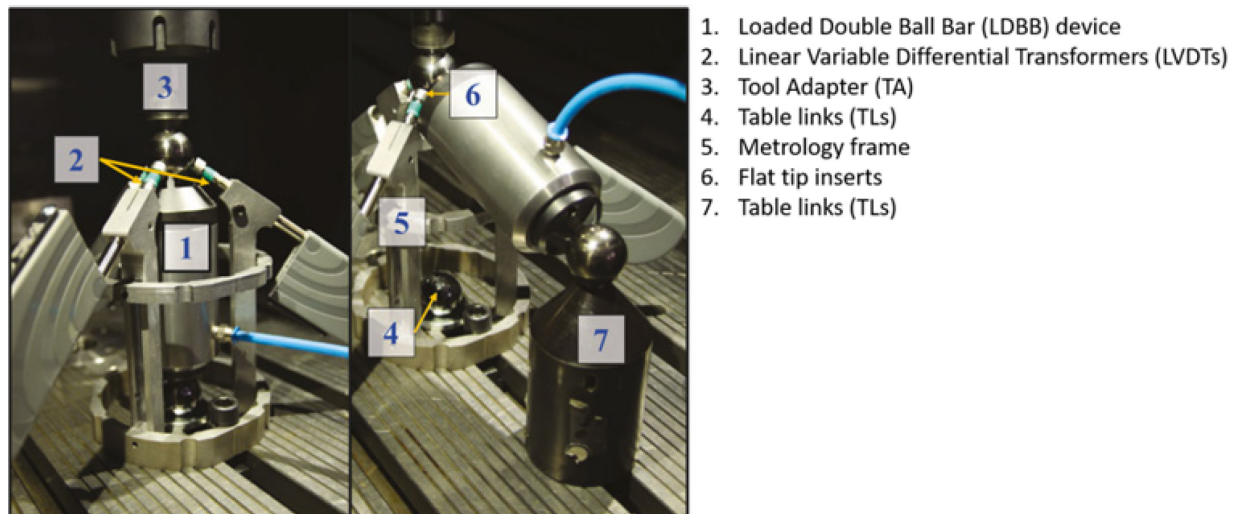


Figure 2-8: Setups for measuring machine quasi-static stiffness using LDBB [20]

Beglarzadeh et al. [14] constructed an equation linking joint compliance to the force and deviation of the LDBB's length, using the Jacobian matrix of a complete circular test in the XY plane. The joint compliances were calculated. The author also compared geometric errors and

errors due to machine compliance under different radial force levels using an elasto-geometric model in [12] later. The analytical structure of this study is illustrated in Figure 2-9.

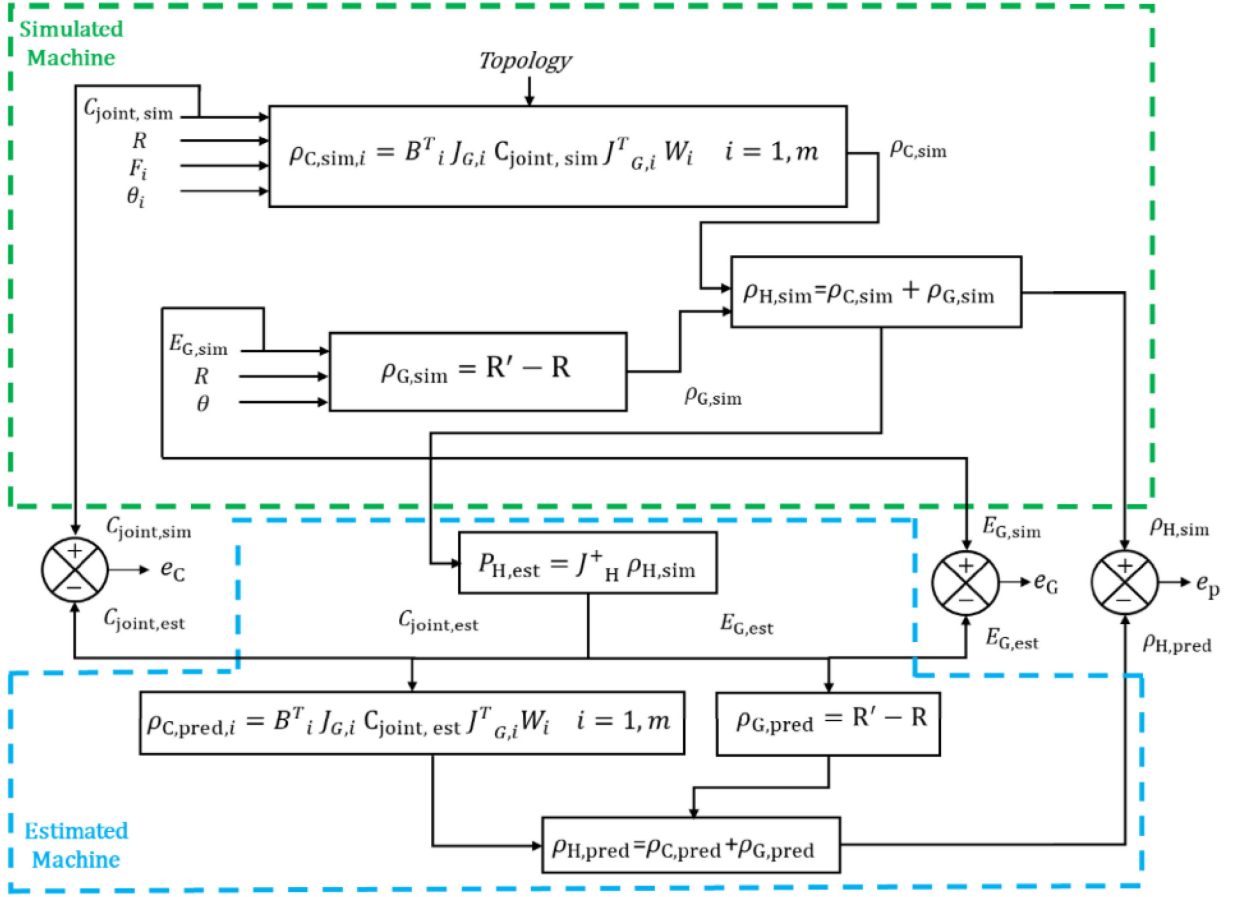


Figure 2-9: Data flow for the model simulation in the elasto-geometric model [12]

Besides the load magnitude, the tool tip force loading frequency could also influence machine compliance under dynamic cutting condition. In [22], a swept signal was fed to the actuator attached to the testing tool. The tool tip forces and accelerations were measured by the dynamometer and accelerometer as in Figure 2-10. The test examined the frequency dependence of compliance. It also gave the directional dependence of compliance on the tested machine.

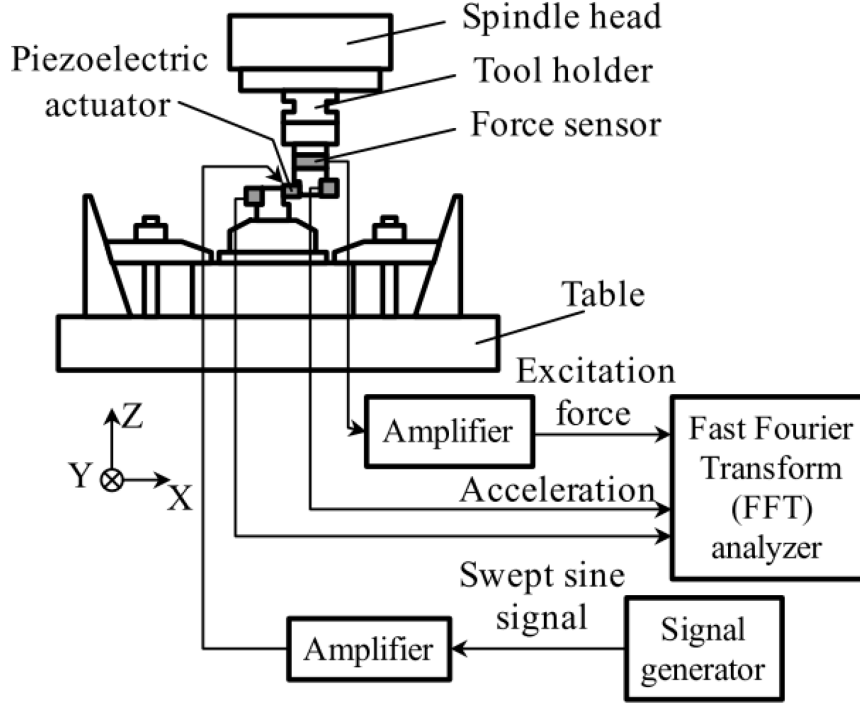


Figure 2-10: Setup for measuring machine compliance with swept sine signal [22]

2.3 Motor equilibrium equation and friction theory

To monitor the tool tip forces and volumetric error caused by compliance using feed motor data, it is necessary to build a link between the tool tip disturbance force and the machine motor data. Altintas built in [23] an equilibrium equation as in Eq (2.1) linking the feed torque T_m , mechanism inertia J_e , viscous damping ($B\omega$), coulomb friction torque T_f and the tool tip disturbance torque T_d . T_m is calculated from torque constant (K_t) and armature current (I). The current and torque exported from the feed drives, forces from the dynamometer and rotary speed (ω) from the feed drive, together with the parameters given by the manufacturer, were used to regress the physical model and to predict the tooltip forces. The cutting forces were predicted. However, a more precise friction model will be needed under precision light milling as concluded in this work[23].

$$T_m = J_e \frac{d\omega}{dt} + B\omega + T_f + T_d \quad (2.1)$$

In [24], the torque requirement of the feed motor is calculated by the equation as Eq (2.2). The feed torque requirement was deemed as the sum of torques reflected on the motor due to

mechanism inertia, static load, and dynamic load. This model partitioned the static load requirement (T_{sr}) into the friction in guideways, the losses of friction in the bearings and cutting forces. It gave a relatively more complete equation describing the feed torque requirements. However, it does not consider the transient friction change at the moving direction change point because, in this equation, it gives only the maximum requirement of feed motor torque. The friction torque is calculated using friction coefficient and normal forces, which is similar to Coulomb friction, and it does not describe the friction variation at the initial motion starting stage.

$$T_m = J_e \frac{d\omega}{dt} + B\omega + T_{sr} \quad (2.2)$$

In [15], Aslan and Altintas constructed a new motor equilibrium equation, of which the feed motor torque equals the sum of torque requirement due to inertia, friction torque and cutting torque. The friction torque, in the equilibrium equation, was modelled under the LuGre Model as in Eq (2.3-2.5). The friction torque (τ_f) in this model considers the rotary speed (v), the deflection of bristles (z) and its first derivative, micro and macro aspects of the relative motion using parameters of σ_0 , σ_1 , σ_2 and Coulomb friction torque (τ_{coul}) and maximum static friction torque (τ_{stat}). The Stribeck effect in this study was captured with the help of Stribeck velocity (v_s), and a full friction model was built. It further compensated the structural modes and current variation. This study proposed that cutting forces prediction from the machine's feed motor current output without extra sensor is possible. The prediction accuracy in this study could be influenced by the modelling of friction, kinematics, motor current signal compensation, and other compensations in the machine tool.

$$\tau_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \quad (2.3)$$

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} \sigma_0 z \quad (2.4)$$

$$g(v) = \tau_{coul} + (\tau_{stat} - \tau_{coul}) e^{-\left(\frac{v}{v_s}\right)^2} \quad (2.5)$$

The modelling of friction is one of the necessary parts of the feed motor equilibrium equation. Apart from modelling the friction into Coulomb and viscous part as introduced in [23], different friction theories could also be used in the equilibrium equation under different friction statuses

and for different test objectives. Sören Andersson et al. compared different sliding friction models concerning the viscous friction, coulomb friction, and micro-slip of small displacements under different conditions. It is suggested in the paper that if the final position is important in the experiment, a combined Stribeck and Dankowicz friction model should be considered [25]. Besides, Dahl proposed that the solid friction F_f could be modelled as an empirical slope function of friction F_f to displacement X as in Eq (2.6). X is the angular or linear displacement between contact surfaces [26, 27], F_c is the Coulomb friction level parameter, and S is a stabilizing factor. The solid friction function was constructed, and the solid friction hysteresis was captured as in Figure 2-11. The friction-displacement slope function controls the variation of friction in the differential function. After the relative displacement increases above a specific value, the friction reaches the maximum. It can be added with the dX/dt term to include the velocity influence on friction.

$$\frac{dF_f}{dX} = \sigma \left| 1 - \frac{F_f}{F_c} \text{sign}\dot{X} \right|^i S \quad (2.6)$$

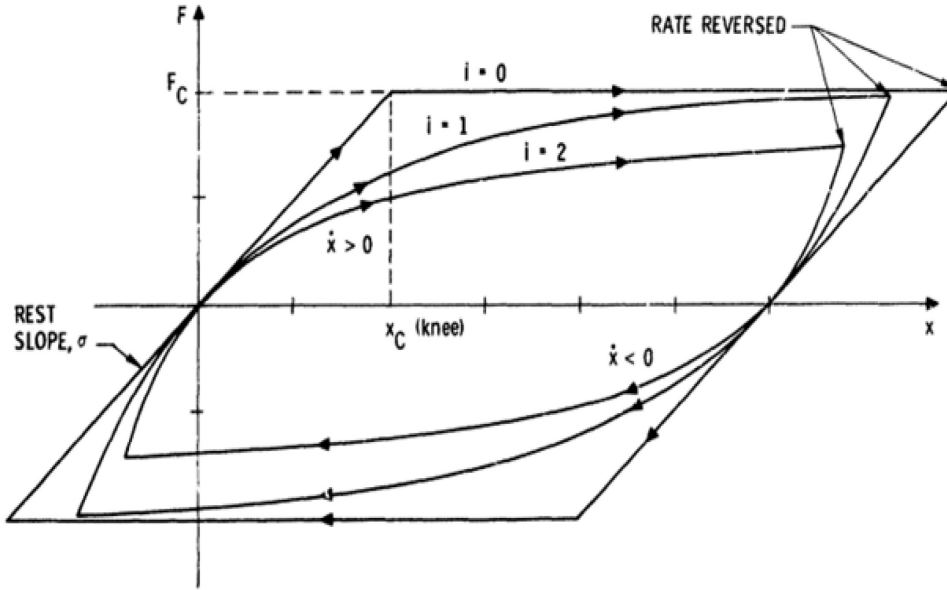


Figure 2-11: Solid friction deflection hysteresis loops with different shape factor i under Dahl's theory[27].

CHAPTER 3 METHODOLOGY

Two different setups were used for the static translational test and dynamic circular test. Two tests were both on the machine KX8-five of Huron Graffenstaden and used dynamometer Kistler 9255B for force measurements. The following methodologies were used.

3.1 Uncertainties of dynamometer measurements

As the dynamometer is the reference for measuring the forces at the tool tip, its accuracy in measuring the force needs to be estimated. The calibration of the dynamometer is composed of the following parts. The methodology and analysis of this part are listed in APPENDIX A and APPENDIX B.

- a) Calibration of scale SA (OHAUS Trooper) with certified precision weights;
- b) Calibration of scale SB (FG-200K AL) with weights calibrated by SA;
- c) Measurements of iron weights with SB and used for dynamometer calibration;
- d) Calculation of the estimated reading uncertainties of the dynamometer.

3.2 Static test

The test, adjusted from ISO 230-1, measured machine static translational volumetric compliance from internal forces generated by commanding the tool tip to press against the dynamometer surface. In this test, commands generated the internal deflections inside the system and the forces on the tool. These deflections include deflections from everywhere in the system, including deflections in the spindle, in the tool, in the setup, inside the measuring system, etc. It was assumed that the same deflections from tool to workpiece will be induced if the same equivalent forces are exerted on the tool, but in the opposite direction of commands. Hence, the volumetric errors resulted from the deflections caused by external forces, have the same absolute value as commands, but in the opposite direction. The methodology is as follows. They are explained in CHAPTER 4.

- a) Generation of the tool tip forces by G code, commanding tool pressing against the dynamometer table with an increment of 1 μm till 200 μm , 200 μm , 50 μm along X, Y and Z directions;

- b) Measurement of the tool tip forces by the dynamometer, and feed torques and position by the CNC machine;
- c) Calculation of local and global compliance;
- d) Construction of the feed motor static equilibrium equation that links the feed motor torque, tool tip forces and volumetric errors;
- e) Evaluation of the model's capability.

3.3 Dynamic test

The test is conducted in the XY plane with the Ball-Roller Bearing Device (BRBD) designed by Babak Beglarzadeh. The methodology is composed of the following parts and is fully detailed in CHAPTER 5.

- a) Generating tool tip forces by commanding bearing pressing against sphere;
- b) Measurement of the tool tip forces in the X and Y direction using the dynamometer;
- c) Calculation of radial and tangential forces using trigonometry;
- d) Calculation of the radial volumetric compliance;
- e) Construction of model(s) linking the feed motor, tool tip forces and volumetric errors;
- f) Evaluation of the model's efficiency.

CHAPTER 4 ARTICLE 1: PREDICTION OF MACHINE TOOL'S STATIC TRANSLATIONAL VOLUMETRIC ERROR CAUSED BY COMPLIANCE FROM FEED MOTOR TORQUE

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The corresponding author contributed around 75% to this work.

4.1 Abstract

Machine tool volumetric compliance quantifies the deflection of the tool tip relative to the workpiece under the effect of a mutual force. Its knowledge can help to virtually monitor tool path errors. This study, conducted under static conditions, aims to establish a model for directly predicting static translational volumetric errors caused by compliance from machine tool CNC feed motor torque outputs without the need to calculate the tool tip disturbance force. Static translational volumetric compliance was measured as proposed in ISO 230-1 in the X, Y and Z directions. The friction torques in each direction were modelled as a function of volumetric errors using Dahl's theory with some adjustments. The friction torque variations were detected. The final model links feed motor torques directly to static translational volumetric errors. With the help of an exponential term, the model proposed has a better performance on capturing the friction variations at the motion starting and movement reversal points. The static translational volumetric compliance models had R^2_{adj} values of 0.999, 0.997, and 0.997 in the X, Y and Z directions, respectively. The RMSE with the best starting estimates on the testing dataset for predicting the static translational volumetric errors directly from the feed motor torques along the X-, Y- and Z-axis were 4.9 μm , 6.6 μm and 2.8 μm under maximum absolute volumetric errors values of 200 μm , 200 μm , and 50 μm , respectively.

Keywords: machine tool; errors; compliances; friction torque.

4.2 Introduction

The volumetric errors describe the deviation between the tool tip's actual and nominal position with respect to the workpiece. They reflect the machining system's accuracy, which influences

the machined parts' quality. One of its causes is the tool, workpiece and machine's internal deflection under loaded conditions. Procedures to measure the machine's compliance can be found in ISO and ASME standards [17, 18]. If the equivalent volumetric compliance between the tool tip and workpiece sides is obtained, given tool tip disturbance forces, the volumetric errors can be calculated.

The friction can be modelled as a function of relative displacement [26-28] or of the relative velocity [15, 29] under different friction status. The tool tip forces can be further connected with the volumetric errors by machine volumetric stiffness [20, 30] or, its inverse, compliance [14, 22]. If the friction and tool tip disturbance torque on the drives are modelled as a function of volumetric errors, the volumetric errors can, in principle, be predicted directly from feed motor torque without using the tool tip forces as an intermediary, which is the novelty of this work.

The work is under static testing conditions. It starts with constructing the static translational volumetric compliance model. Next, the tool tip disturbance torque is calculated. Then, the friction torque function of translational volumetric errors is proposed. Finally, the direct connection from static translational volumetric errors to the feed motor torque is found with the help of the motor static equilibrium equation. The static translational volumetric errors are predicted directly from feed motor torque by solving the equation using `fsolve` of the SciPy package in Python. The model is reconstructed and re-analyzed with the weights term added to discuss the influences of dynamometer weights on the model. The statistical method is used to explain the influence of the dynamometer self-weight.

• Nomenclature

T_m	feed motor torque, calculated from the group mean
T_d	tool tip disturbance torque
T_f	friction torque
F_d	tool tip force, calculated from the group mean
F_f	friction force
K_d	tool tip disturbance torque coefficient
K_f	friction torque coefficient

E_{XYZ} translational volumetric error caused by compliance

C_l local static translational volumetric compliance

C_g global static translational volumetric compliance

ζ constant value

4.3 Methodology

The experiment was conducted on a machine tool model KX8-five of Huron Graffenstaden with topology wCAYFXZSt. Tool tip forces, F_d , were produced by machine axis motion commands pressing the tool against the dynamometer table and measured by the dynamometer by Kistler model 9255B as shown in Figure 4-1. The command equals the translational volumetric error, E_{XYZ} , but in the opposite direction, because in reality, the tool does not move relative to the workpiece (dynamometer table) as the command implies.

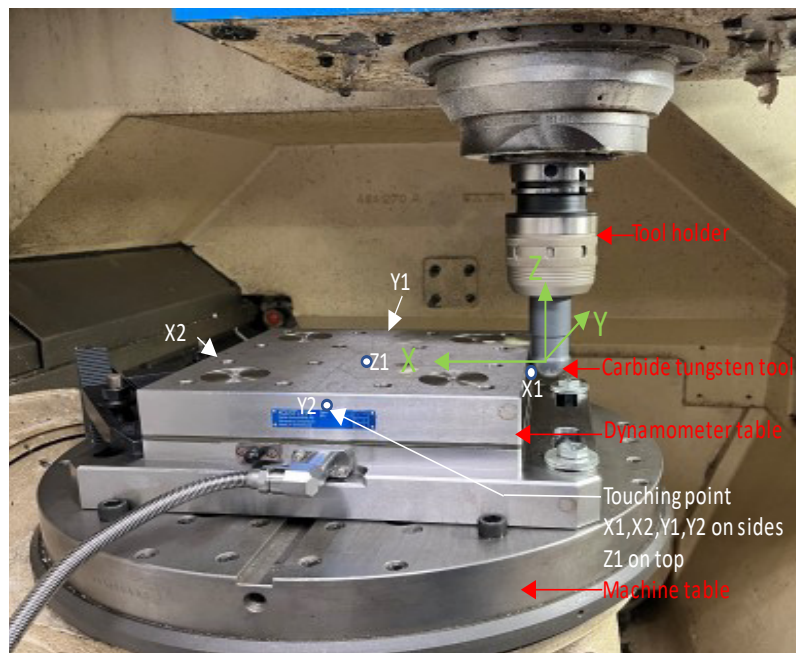


Figure 4-1: Test setup. Tool tip forces are generated by gradually commanding the tool to move inwardly into the dynamometer from zero to 200 μm , 200 μm , and 50 μm in X, Y, and Z directions, respectively, and then outward, resulting in 200, 200 and 50 subpoints, respectively. Motions at each one μm are suspended for 2 s. The test at each position is repeated twice more. Forces and feed motor torques during the motion are discarded to retain static data. Positive directions of tool tip are chosen opposite to that of the dynamometer.

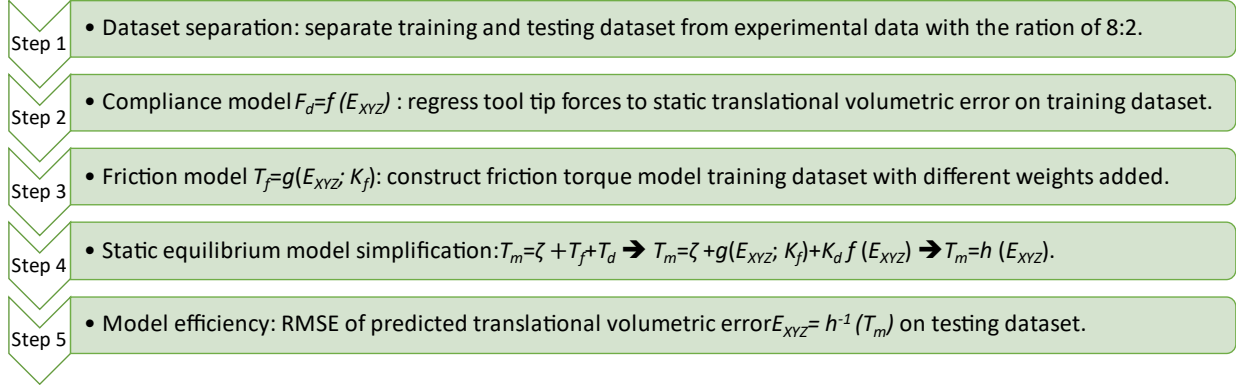


Figure 4-2: General methodology flow. Step 4 adds a constant ζ to keep the model residual following the normal distribution. ζ also deals with the Z-axis gravity's influence and the unknown initial friction status.

4.4 Data analysis

4.4.1 Experimental measurement of the static translational volumetric compliance

The machine local static translational volumetric compliance C_l at subpoint i of each testing point was calculated by the variation of E_{XYZ} to the variation of F_d as in Eq (4.1), where j is half of the distance between two interpolated subpoints (7 μm for X and Y direction, 3 μm for Z direction). i is numerically the same as $|E_{XYZ}|$. The global static translational volumetric compliance C_g was calculated by the maximum variation of E_{XYZ} and F_d as in Eq (4.2) at each testing point. The local compliances around the zero and maximum command points were not calculated due to a lack of data points used for interpolation. The expanded local compliance uncertainties were calculated from the combined local compliance uncertainties from measured force uncertainties as per the GUM standard [31] with the coverage factor $k=2$ as in Eq (4.3), where ΔE_{XYZ} is a constant distance as in Eq (4.1). The dynamometer was pre-calibrated using weights with estimated uncertainties. The uncertainties sources are listed in Figure 4-3. The results of local and global compliance are shown in Figure 4-4, Figure 4-5 and Figure 4-6. The Z direction is the least compliant with global compliance of 0.024 $\mu\text{m/N}$, followed by the Y axis, and the X axis has the greatest compliance with the global compliance of 0.146 $\mu\text{m/N}$. This conclusion conforms to the machine's structure, with Z perpendicular to the working table, Y linked directly with the foundation and X suspended on the top of the machine. The global compliances along Y+ and Y-

directions show asymmetric behaviour. The local compliances are generally constant after the $|E_{XYZ}|$ increases above a threshold where the tool tip forces are higher.

$$C_{l,i} = \frac{\Delta E_{XYZ}}{\Delta F_d} = \frac{E_{XYZ,i+j} - E_{XYZ,i-j}}{F_{d,i+j} - F_{d,i-j}} \quad (4.1)$$

$$C_g = \frac{E_{XYZ,max} - E_{XYZ,min}}{F_{d,max} - F_{d,min}} \quad (4.2)$$

$$U_{c_{l,i}} = 2 \sqrt{u_{F,i+j}^2 \left(\frac{\Delta E_{XYZ}}{\Delta F_d} \right)^2 + u_{F,i-j}^2 \left(\frac{\Delta E_{XYZ}}{\Delta F_d} \right)^2} \quad (4.3)$$

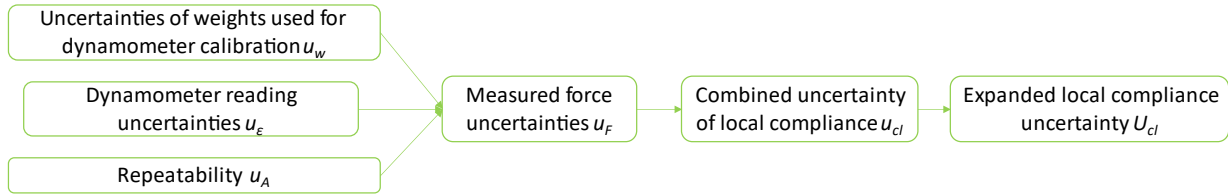


Figure 4-3: Uncertainties sources of local compliance. The positions exported from the encoder have negligible uncertainties and they are not included in the uncertainty sources.

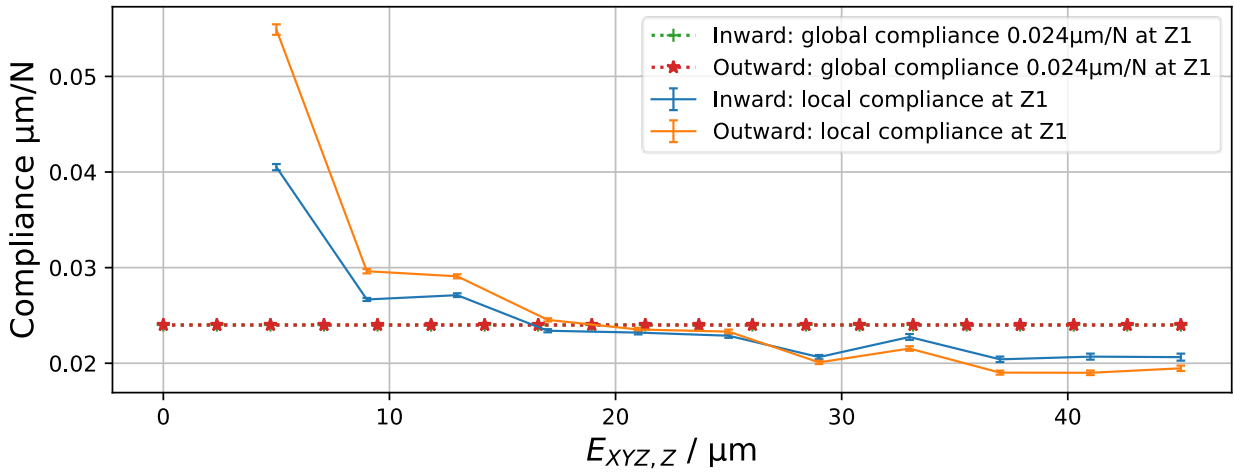


Figure 4-4: Local compliance plots with uncertainties in Z direction

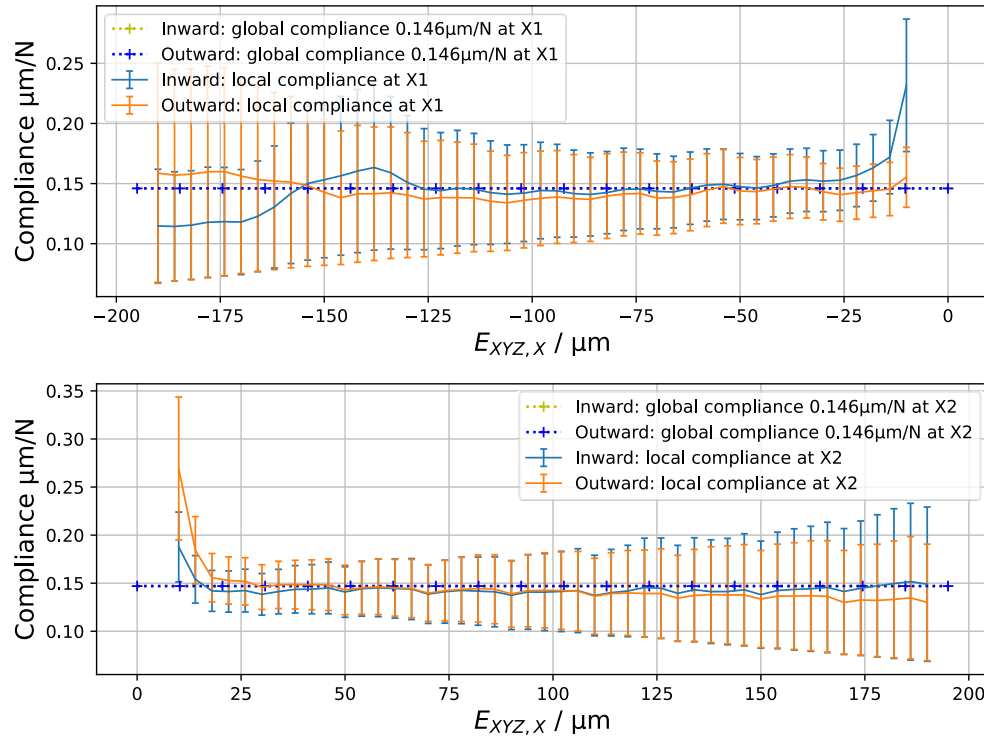


Figure 4-5: Local compliance plots with uncertainties in X direction

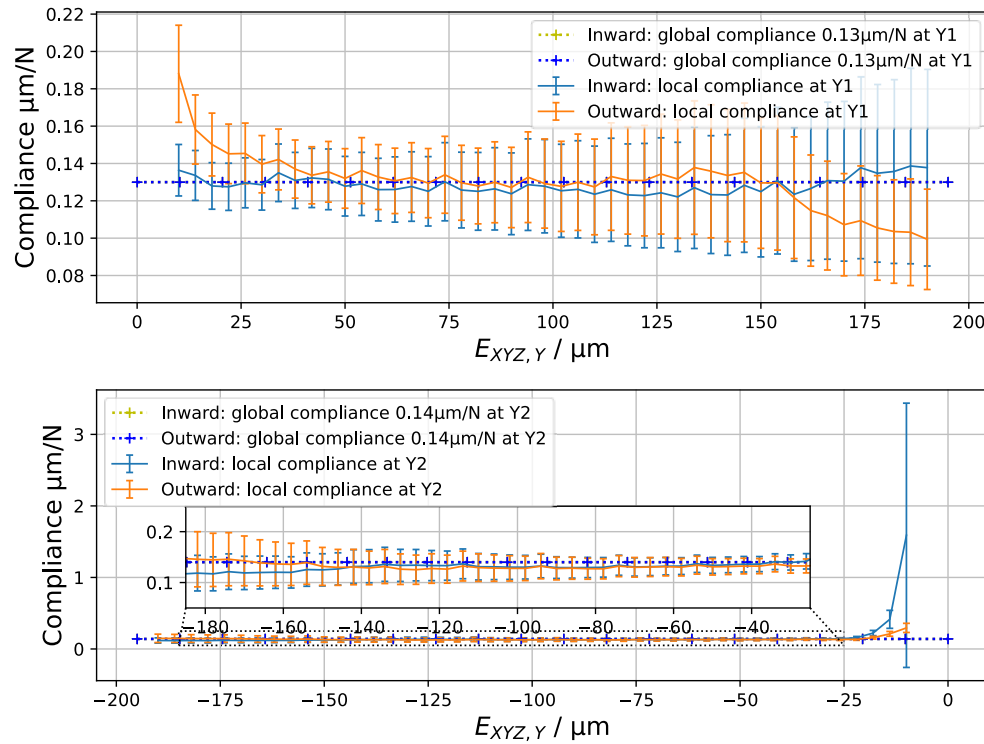


Figure 4-6: Local compliance plots with uncertainties in Y direction

4.4.2 Static translational volumetric compliance model

The compliance was modelled by Eq. (4.4), where a , b and c were coefficients obtained on the training data set by the `cftool` function in Matlab which automatically estimated the regression coefficients of a given regression function. The model was subsequently tested on the testing dataset. The coefficients obtained are shown in Table 4-1. The F_d versus E_{XYZ} plots of all experimental data and model prediction for the three test directions are shown in Figure 4-7. The linear term in Eq. (4.4) shows the constant compliance behaviour; the $\tanh$ function controls nonlinear behaviour at smaller commands and lower tool tip forces range. The tool tip disturbance torque T_d in Eq (4.5) can be calculated by multiplying the tool tip disturbance force with coefficient K_d found after establishing the motor static equilibrium equation of Eq (4.9) in section 4.4.4 Motor static equilibrium equation.

$$F_d = f(E_{XYZ}) = aE_{XYZ} + b \tanh\left(-\frac{E_{XYZ}}{c}\right) \quad (4.4)$$

$$T_d = K_d F_d \quad (4.5)$$

Table 4-1: Coefficients of mathematical compliance model in Eq (4.4)

Axis	a	b	c	R^2_{adj}	RMSE	
					Training	Testing
X	7.09	57.88	10.63	0.999	25.01	25.85
Y	7.73	88.57	15.23	0.997	45.87	44.51
Z	47.49	402.36	11.48	0.997	31.38	30.44

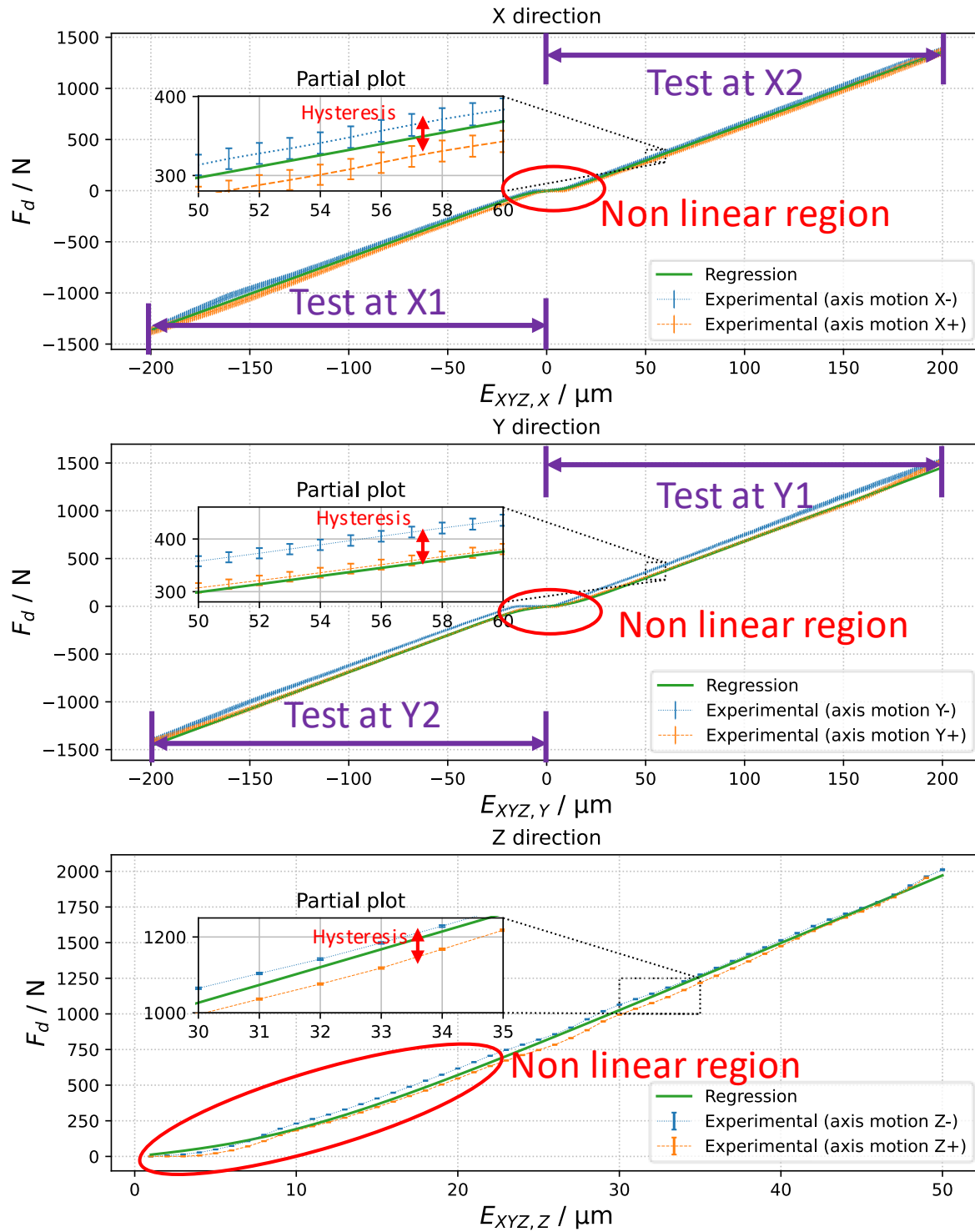
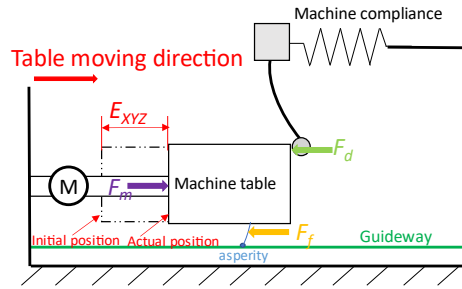


Figure 4-7: Tool tip forces F_d to static translational volumetric errors E_{XYZ} plots in all three directions for experimental data and model regression.

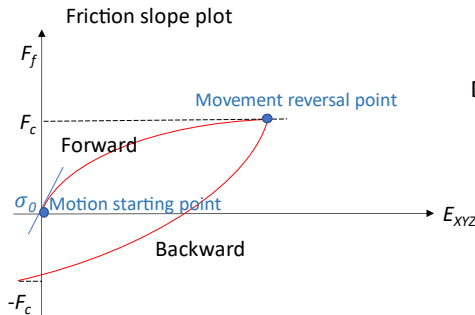
4.4.3 Solid friction model

The relative velocity between contact surfaces did not influence friction in this test, as the data during motion was discarded. Lubrication was considered uniform along the axis in the static test. The friction can be considered similar to the solid friction. The friction can be calculated by the sum of friction losses in the guideways and bearings [24]. Dahl's friction model gives that the friction can be modeled as the function of linear or angular relative displacement for the above two system elements [26-28, 32]. Since the angular relative displacement of the bearing can be calculated from the linear relative displacement of the table to guideways (E_{XYZ}) by multiplying the transmission ratio, the total friction losses of bearing and guideways can be linearly combined as a function of E_{XYZ} , which can be modified by Dahl's theory. The Dahl's theory in the solid friction form can describe a friction function of F_f with respect to the relative displacement E_{XYZ} between the contact surfaces, as in Eq. (4.6) [27, 28]. In Eq (4.6), σ_0 is the rest stiffness parameter, $sgn(\dot{E}_{XYZ})$ is the E_{XYZ} change direction, and F_c is the Coulomb friction level parameter. If the exponent parameter, i , is taken as 1, the stabilizing factor S can be taken as 1[27].

$$\frac{dF_f}{dE_{XYZ}} = \sigma_0 \left| 1 - \frac{F_f}{F_c} sgn(\dot{E}_{XYZ}) \right|^i S \quad (4.6)$$



E_{XYZ} : linear relative displacement between table and guideway.
 F_m : motor pushing force.
 F_d : tooltip reacting disturbance force.
 F_f : total equivalent friction force.
 F_c : Coulomb friction level parameter
 σ_0 : rest stiffness parameter.
 i : exponent parameter.
 S : stabilizing factor, $sgn(1 - \frac{F_f}{F_c} sgn(\dot{E}_{XYZ}))$. When $i=1$, $S=1$ works.



Dahl model: friction slope empirical function.

$$\frac{dF_f}{dE_{XYZ}} = \sigma_0 \left| 1 - \frac{F_f}{F_c} sgn(\dot{E}_{XYZ}) \right|^i S$$

Figure 4-8: Illustrations based on Dahl friction model[26-28]. The kinematic effect of dE_{XYZ}/dt in model and lubrication effect is not considered for the static test.

The solution of the differential equation Eq. (4.6) follows the form of Eq. (4.7); however, in this test, there are different combinations of coefficients α_1 , α_2 , α_3 , since the absolute term in Eq. (4.6), inward and outward motions and the moving direction under predefined coordinate influence the solution form. Besides, the conventionally chosen $i=1$ can also influence the friction-displacement slope. To simplify the solution form, the piecewise function was used in the friction torque. The outward motion was also considered separately from the inward because its initial friction state was not the same as the motion starting points. The outward motion was considered starting from zero relative displacements, but with pre-loaded Coulomb friction torque at the movement reversal points. The friction torque was finally modelled in the form of Eq (4.8) by simplifying Eq. (4.7) with a friction torque coefficient K_f , μ_1, μ_2 and θ , where (a) for the inward and (b) for the outward motion of the tool tip to the dynamometer surface, with $sgn(\dot{E}_{XYZ})$ representing the direction of static translational volumetric error change of the tool tip to the dynamometer surface under predefined coordinates, and K_f numerically the same as the maximum Coulomb friction torque.

$$F_f = \alpha_1 \frac{F_c}{sgn(\dot{E}_{XYZ})} + \alpha_2 e^{\frac{\alpha_3 \sigma_0 sgn(\dot{E}_{XYZ})}{F_c} E_{XYZ}} \quad (4.7)$$

$$T_f = \begin{cases} sgn(\dot{E}_{XYZ})(K_f - \mu_1 e^{-\theta(|E_{XYZ}|)}) & (a) \\ sgn(\dot{E}_{XYZ})(K_f - \mu_2 e^{-\theta(|E_{XYZ}|_{max} - |E_{XYZ}|)}) & (b) \end{cases} \quad (4.8)$$

4.4.4 Motor static equilibrium equation

The feed motor torque T_m overcomes the friction torque T_f and tool tip disturbance torque T_d . Combining T_d and T_f from Eq. (4.5) and Eq. (4.8), and adding a constant term ζ , the motor static equilibrium equation can be written as Eq. (4.9) for inward (a) and outward motion (b). A method similar with the hyperparameter tuning strategy estimates the unknown coefficients K_d , K_f , μ_1 , μ_2 , θ . The hyperparameters yielding the minimum RMSE of T_f were selected as the final coefficients in the model in the tuning procedure because T_f has the greatest influence on the model, and accurate friction models may be needed in precision light milling [23]. 966306 combinations of hyperparameters were searched and compared. The pseudo-code is shown in Figure 4-9. The final results are shown in Table 4-2. The constant ζ in the Z direction is higher because the motor torque needs to overcome the gravity of the Z axis and spindle during the test. It also deals with the unknown initial friction status along the Z direction. In parameter searching, the friction

torque cannot be measured in the test, so the estimated friction torque $T_{f,est}$ was calculated by subtracting the constant and tool tip disturbance torque from feed motor torque. It is compared with the predicted friction torque $T_{f,pred}$ calculated by Eq (4.8). The estimated and predicted friction torques ($T_{f,pred}$, $T_{f,est}$) are plotted in Figure 4-10. RMSE between the estimated and predicted friction torque on the testing dataset is shown in Figure 4-11. The friction torque increased from zero to the maximum Coulomb friction torque at the motion starting point. The nonlinear translational volumetric compliance behaviour could result from the machine mechanism's friction variation at the motion starting stage, because the increasing friction torque works similarly with adding extra friction-induced micro-compliance to the system, so the compliance at those regions is nonlinear. Same for the movement reversal points. Beyond these two regions, the friction torque reaches the limits, and the micro compliance induced by friction cannot change anymore. Hence, the micro-compliance caused by friction does not influence the static translational volumetric compliance and the compliance is almost linear. The friction torque's nonlinear variations between positive and negative maximum Coulomb friction torque at movement reversal points were also captured by Eq (4.8) with the help of the exponential term. The friction model works for the motion starting and movement reversal points as in Figure 4-10, where the friction does not reach the limits.

$$T_m = \begin{cases} \zeta + \text{sgn}(\dot{E}_{XYZ})(K_f - \mu_1 e^{-\theta|E_{XYZ}|}) + K_d f(E_{XYZ}) & (a) \\ \zeta + \text{sgn}(\dot{E}_{XYZ})(K_f - \mu_2 e^{-\theta(|E_{XYZ}|_{max} - |E_{XYZ}|)}) + K_d f(E_{XYZ}) & (b) \end{cases} \quad (4.9)$$

Table 4-2: Hyperparameter tuning results with the coefficients finally identified in the motor static equilibrium function in Eq (4.9) on training dataset.

	ζ	K_f	K_d	θ	μ_1	μ_2	RMSE
X	-0.06	-1.3	-0.0031	0.1	-1.2	-1.6	0.119
Y	-0.09	-1.2	-0.0028	0.1	-1.2	-1.7	0.116
Z	-9.4	1.2	0.0029	0.1	1.1	2.0	0.189

Algorithm 1 Find the best performance hyperparameters

```

Create a performance list p=[];
Create hyperparameter range of  $\zeta, K_f, K_d, \theta, \mu_1, \mu_2$ ;
while  $\zeta$  in range do;
  while  $K_f$  in range do;
    while  $K_d$  in range do;
      while  $\theta$  in range do;
        while  $\mu_1$  in range do;
          while  $\mu_2$  in range do;
             $T_{f,est} = T_m - \zeta - T_d$ 
 $T_{f,pred} = \begin{cases} \text{sgn}(E_{XYZ})(K_f - \mu_1 e^{-\theta(|E_{XYZ}|)}) & \text{inward} \\ \text{sgn}(E_{XYZ})(K_f - \mu_2 e^{-\theta(|E_{XYZ}|_{max} - |E_{XYZ}|)}) & \text{outward} \end{cases}$ 
 $RMSE = T_{f,pred} - T_{f,est}$ 
 $p \leftarrow [RMSE, \zeta, K_f, K_d, \theta, \mu_1, \mu_2]$ 
Sort p by RMSE in ascending order
Export p[0]

```

Figure 4-9: Pseudo code of hyperparameters tuning to find model's coefficients

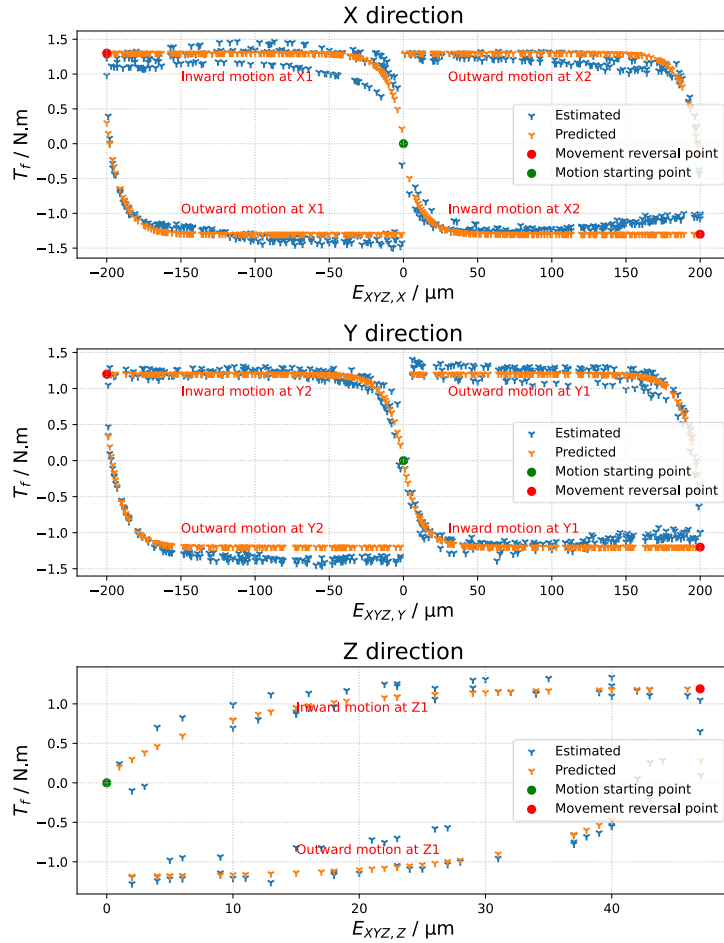


Figure 4-10: Predicted and estimated friction torque plots on testing dataset. Results are from tests repeated with different weights (8.13 kg, 12.93 kg, 45.76 kg) added to the surface of the dynamometer. The direction of friction torque is defined by the direction of feed motor torque output.

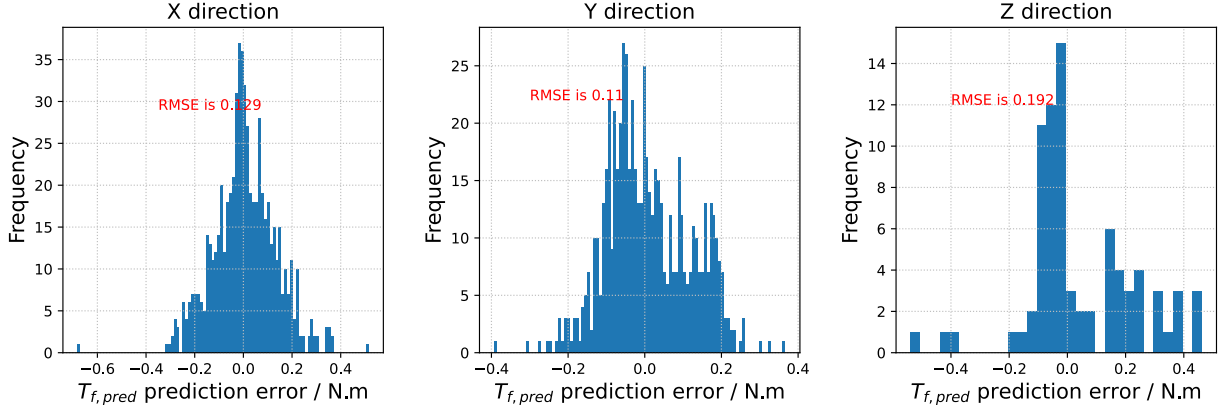


Figure 4-11: Histogram of RMSE between predicted and estimated friction torque on the testing dataset.

4.4.5 Prediction of static translational volumetric errors directly from feed motor torque

For a given T_m , suitable solutions for E_{XYZ} can be calculated using an iterative process, such as using the function `fsolve` in the SciPy package in Python given an initial guess for E_{XYZ} in Eq (4.9). `fsolve` found the E_{XYZ} using MINPACK's algorithm modified from Powell's Method [33] by minimizing the difference between a given experimental feed motor torque and the feed motor torque calculated from Eq (4.9). The RMSE examined the model between the roots found after iterations and the real volumetric errors. The starting estimate was chosen as 1 μm initially together with $\text{sign}(E_{XYZ})$, in this case equivalent with $\pm 1 \mu\text{m}$. Afterwards, the optimized solution of the predicted E_{XYZ} is calculated. The iteration was repeated with the initial starting estimate increased by 1 μm till the maximum engagement. With each different starting estimate, the RMSE of predicted E_{XYZ} was exported. The RMSE of predicted E_{XYZ} with the proposed friction model decreases while the initial starting estimate of E_{XYZ} increases as in Figure 4-12. The minimum RMSE appears at the starting estimate of $\pm 135 \mu\text{m}$, $\pm 139 \mu\text{m}$ and $\pm 27 \mu\text{m}$ in the X, Y and Z direction, where the RMSEs of predicted E_{XYZ} are $4.9 \mu\text{m}$, $6.6 \mu\text{m}$ and $2.8 \mu\text{m}$ respectively. It is almost half of the RMSE of predicted E_{XYZ} only with the Coulomb friction model [25], of which the static motor equilibrium equation is modelled only with Coulomb friction torque $\pm T_{f,coul}$ as shown in Eq (4.10) without considering the friction variation at the motion starting and movement reversal points. When the starting estimate value is chosen above 25 μm in the X, Y

direction, and 1 μm in Z direction, the model proposed could predict the E_{XYZ} with the RMSEs of less than 10 μm .

$$T_m = \zeta + \text{sgn}(\dot{E}_{XYZ})T_{f,coul} + T_d \quad (4.10)$$

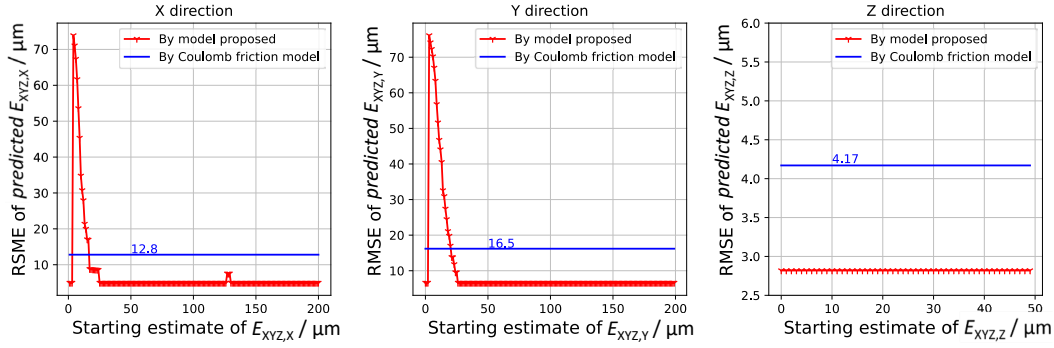


Figure 4-12: Starting estimate's influence on the model's performance

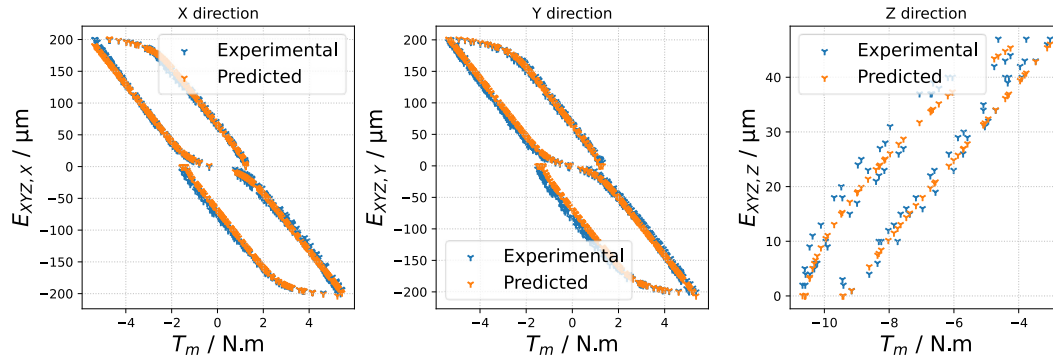


Figure 4-13: Predicted and experimental E_{XYZ} versus T_m plot. E_{XYZ} predicted from Eq (4.9) with the best starting estimate of $\pm 135 \mu\text{m}$, $\pm 139 \mu\text{m}$ and $\pm 27 \mu\text{m}$ in X, Y and Z direction respectively.

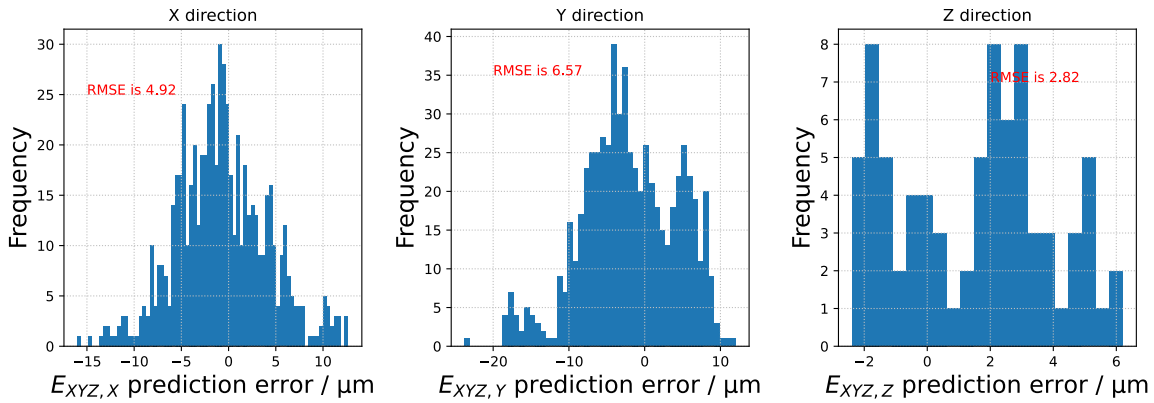


Figure 4-14: Histogram of predicted E_{XYZ} with the best starting estimate of $\pm 135 \mu\text{m}$, $\pm 139 \mu\text{m}$ and $\pm 27 \mu\text{m}$ in the X, Y and Z directions, respectively

The test was repeated with different weights added on the dynamometer surface. The new model, as in Eq (4.11), was used to regress the overall data, where W is a term concerning the weights in kilogram on the machine table, T_d and T_f can be substituted by Eq (4.5) and Eq (4.8). The statistical criterion, such as the p-value of the W term, the model's R^2_{adj} and F-value and the regression condition number are listed in Table 4-3. Along the Z direction, the additional term W in the model from Eq (4.11) is insignificant as the p-value of the term is higher than 0.05. Along the X and Y direction, the additional term W does not increase the model accuracy since the R^2_{adj} has insignificant improvements; however, it increases the model instability as the condition number increases. The dynamometer's self-weight has negligible influences on the model's accuracy as the term W is insignificant. Hence, the model could also work when the dynamometer is removed.

$$T_m = W + \zeta + T_f + T_d \quad (4.11)$$

Table 4-3: Statistical criterion of regression by model from Eq (4.9) and Eq (4.11)

	X	Y	Z
P-value of W , with Eq (11)	0	0	0.275
R^2_{adj} , with Eq (9)	0.998	0.999	0.981
R^2_{adj} , with Eq (11)	0.998	0.998	0.981
F-value, with Eq (9)	9.8E05	1.0E06	9916
F-value, with Eq (11)	6.6E05	5.4E05	6615
Cond Number, with Eq (9)	2.41	2.35	5.12
Cond Number, with Eq (11)	306	306	334

4.5 Conclusion

The static translational volumetric compliance model was constructed with force generated internally in the machine. The Z axis is the least compliant, while the X axis is the most compliant. Local compliance variations and global compliance were detected. The linear and nonlinear compliance behaviour regions were found with the mathematical model established

using the $\tanh()$ function. Under smaller engagements and tool tip forces, the compliance is nonlinear, while at larger engagements and tool tip forces, the compliance is almost linear. Compared with the Coulomb friction model, the solid friction torque model established by the adjusted Dahl theory captured the friction torque variation at the motion starting and movement reversal points. Besides, the feed motor in the Z direction needs around 9.4 N.m torque to overcome the gravity of the Z axis and spindle components. This phenomenon was not found in the X and Y directions.

The static translational volumetric errors were predicted directly from the feed motor torque by the model proposed using the `fsolve` with starting estimates. The minimum RMSEs of predicted E_{XYZ} on the testing dataset are 4.9 μm , 6.6 μm and 2.8 μm , for initial starting estimates of $\pm 135 \mu\text{m}$, $\pm 139 \mu\text{m}$ and $\pm 27 \mu\text{m}$, in the $\pm X$, $\pm Y$ and $\pm Z$ directions, respectively. As long as the absolute value of the initial guess of E_{XYZ} is not chosen less than 25 μm , the model proposed can predict the E_{XYZ} with a RMSE of prediction less than 10 μm . The influence of dynamometer weight on the model was proved negligible by using statistical criteria. The model proposed can be used without the dynamometer added onto the machine table.

The model proposed in this work is subjected to the specific setup, tool used and the static testing condition. The static translational volumetric compliance tested also includes the setup's and tool's compliance. The solid friction model should be modified to be used under dynamic working conditions. Future work can focus on the decoupling of the tool and spindle compliance from the volumetric compliance tested to obtain the machine compliance, and on ameliorating the friction model for various working conditions.

Acknowledgements

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CHAPTER 5 DYNAMIC TEST

The dynamic test was conducted with the Ball Roller Bearing Device (BRBD) designed by Babak Beglarzadeh. It is a simplified device for this test without considering the cutting forces frequency variation of the real cutting tool. This test aims to study and model the tool tip forces and volumetric errors principally due to compliance in a circular test. The setup is illustrated in Figure 5-1. The tests were conducted with different feed rates of 1000 mm/min, 2000 mm/min, 4000 mm/min, 6000 mm/min and 8000 mm/min with the tool moving in the direction in clockwise (CW) and counterclockwise (CCW) direction with respect to the sphere. The radial compliance measured in this test is quasi-static as the tool tip force variations has negligible influence on the test. However, concerning the prediction of tool tip forces, as more noises appeared in the current signal from the motor during the test under higher feed rate, and also the dynamic forces such as inertia and friction forces appeared in the drives, it should be better to be considered as dynamic. The commanded radial engagements were considered the opposite of the volumetric errors for this testing system, under the hypothesis of same magnitude of tool tip forces generating the same deflection in the same direction as the forces in the testing system. The commanded radial engagement was 200 μm . The test was conducted in the XY plane. The forces, feed currents, encoder positions and rotary speeds of the X and Y drives were exported from the dynamometer and CNC machine tool. The feed currents were transformed to feed torques used for the later calculation using the equations in Table 5-1. These equations were identified by regressing the torque and current data exported from circular runs without the engagements between the tool and sphere, with the radius of 50 mm in the XY, XZ and YZ plane.

Table 5-1: Feed motor torque T_m to current I relation

Direction	Torque (T_m) to current (I) equation.
X	$T_m=1.0109 I-0.0038$
Y	$T_m=1.0109 I-0.0037$
Z	$T_m=1.0884 I-0.0099$

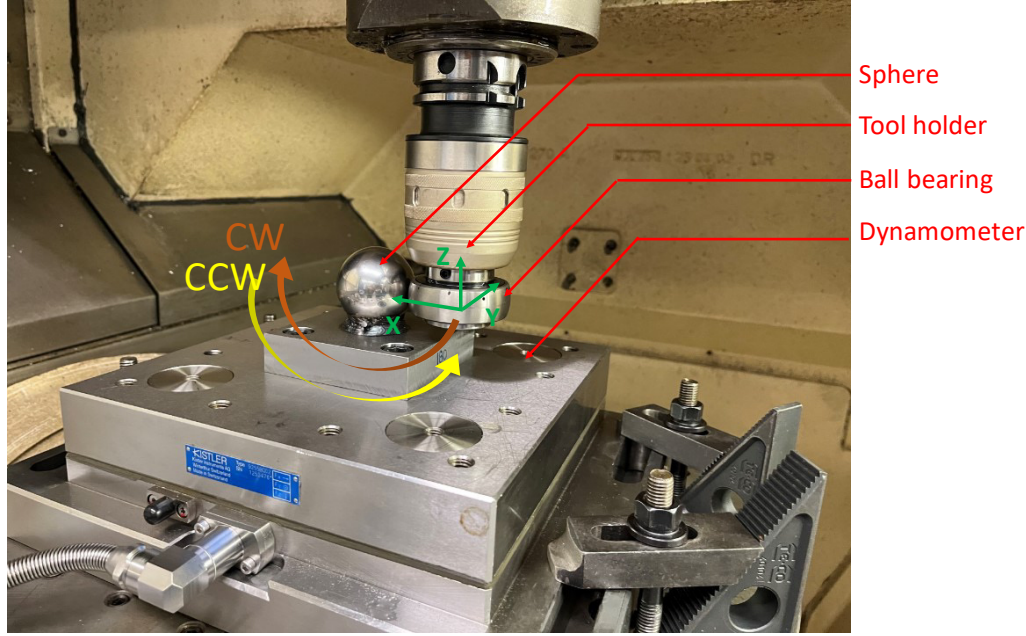


Figure 5-1: Ball Roller Bearing Device (BRBD) setup. Positive directions of tool were selected opposite to that of dynamometer.

5.1 Radial compliance behaviour

The encoder positions (X , Y) were exported from the CNC machine tool. The radial length of the tool path without radial engagement was 54.4 mm. The corrected sphere center (X_0 , Y_0) was calculated by the midpoint of four extreme points, as shown in Figure 5-2. The corrected sphere center (X_0 , Y_0) and the radial engagements $\Delta\rho$ in micrometer were calculated by Eq. (5.1-5.3).

$$X_0 = \frac{X_1 + X_2}{2} \quad (5.1)$$

$$Y_0 = \frac{Y_3 + Y_4}{2} \quad (5.2)$$

$$\Delta\rho = 1000 * (54.4 - \sqrt{(X - X_0)^2 + (Y - Y_0)^2}) \quad (5.3)$$

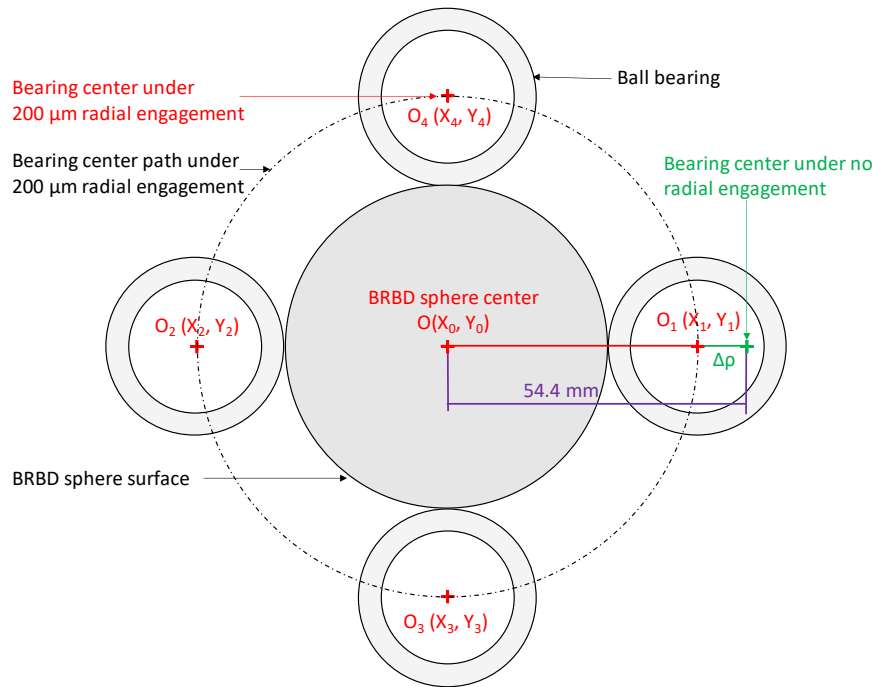


Figure 5-2: Illustration of calculating sphere center coordinates and radial engagements

The asymmetric machine compliance behaviour and the geometric imperfections of the setups result in the amplitude difference between the measured forces in the X and Y direction in the dynamic circular test, as shown in the raw force data plots from Figure C-7 to Figure C-11 in APPENDIX C. The composed equivalent forces F_c , calculated from the dynamometer-measured forces in X and Y directions (F_x, F_y), does not always lie in the radial direction of the path. There is an angular difference between the radial direction and composed equivalent force direction. To analysis the radial compliance behaviour C_r , (F_x, F_y) were projected in the radial and tangential directions, and the radial forces were used to calculated the radial compliances. The angle γ was calculated by the difference between encoder position outputs and sphere center position ($X-X_0, Y-Y_0$) as in Eq. (5.5), using arctan2 code of NumPy package in Python. Using Eq. (5.5), the time at which the tool arrived at the corresponding positions was approximately calculated, which was used later to calculate the derivative of motor rotary speed $d\omega/dt$. As shown in Figure 5-3, F_r and F_t were calculated from measured forces (F_x, F_y) using Eq. (5.6) under each feed rate. The results are shown in Table 5-1. The average radial forces in CW and CCW directions were generally similar. They were used in the calculation of the global radial compliance. F_t is generally dependent of feed rate variation, as it decreases with respect to the feed rate increases.

$$F_C^2 = F_x^2 + F_y^2 = F_t^2 + F_r^2 \quad (5.4)$$

$$\gamma = \arctan2(Y - Y_0, X - X_0) \approx \omega_{feed} t \quad (5.5)$$

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix}_{2 \times n} = \begin{bmatrix} \cos \gamma & \sin \gamma \\ \sin \gamma & -\cos \gamma \end{bmatrix}_{2 \times 2} \begin{bmatrix} F_x \\ F_y \end{bmatrix}_{2 \times n} \quad (5.6)$$

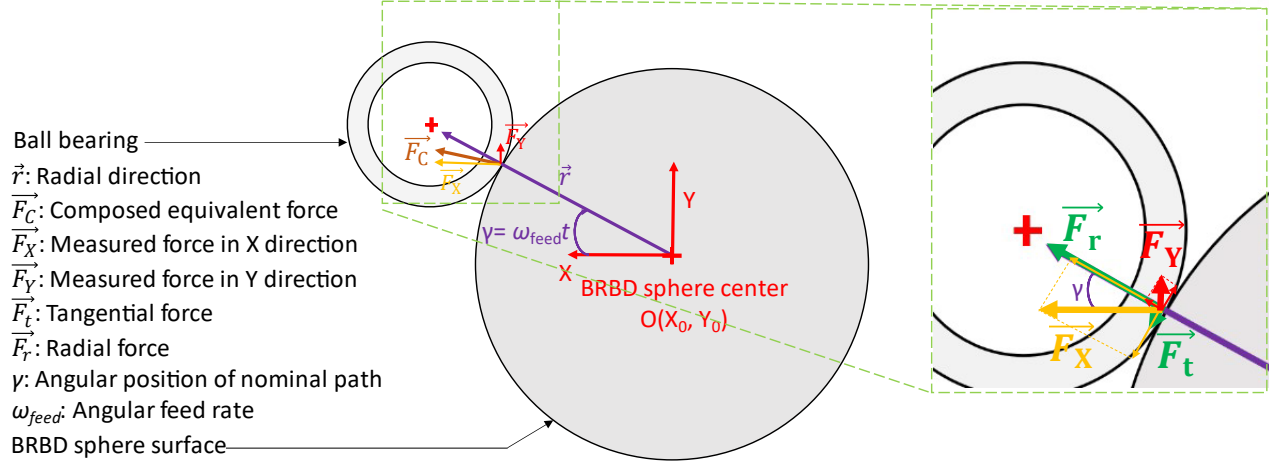


Figure 5-3: Forces at the contact point between the sphere and ball bearing. The X and Y directions were chosen as opposite to the positive direction of dynamometer.

Table 5-2: Average radial force and average tangential forces

Feed rates / [mm/min]	F_r (CW) / N	F_t (CW) / N	F_r (CCW) / N	F_t (CCW) / N
1000	1456.97	-64.11	1450.69	-134.50
2000	1459.28	-58.03	1450.33	-161.74
4000	1460.56	-101.63	1452.64	-174.22
6000	1458.89	-124.15	1450.89	-195.29
8000	1456.94	-182.47	1453.83	-207.71

The zero-engagement position, where the forces between the sphere and bearing are zero, was selected as the reference for calculating global radial compliance under 200 μm engagement. Hence, the variation of radial forces from zero to 200 μm engagement is F_r , and the engagement

variation is $\Delta\rho$. The radial compliance $C_{r,i}$ at each angular position i and the average radial compliance $\bar{C}_r$ can be calculated using Eq. (5.7) and Eq. (5.8).

$$C_{r,i} = \frac{\Delta\rho_i}{\Delta F_{r,i}} = \frac{\Delta\rho_i}{F_{r,i} - 0} = \frac{\Delta\rho_i}{F_{r,i}} \quad (5.7)$$

$$\bar{C}_r = \frac{\sum_{i=1}^n C_{r,i}}{n} \quad (5.8)$$

As in Figure 5-4, the average radial compliance $\bar{C}_r$ are generally the same (around $0.138 \mu\text{m}/\text{N}$) for the CW and CCW directions under the tested feed rates. The radial compliance plots have an oval shape, meaning that the machine tool shows different compliance behaviour in the X and Y directions: X is more compliant than Y. Besides, the compliances in Y+ and Y- directions are also asymmetric, as the oval shape is asymmetric with respect to the horizontal axis. These correspond with results in the static test, that the X direction is more compliant than the Y direction, and the compliances along the Y+ and Y- direction are asymmetric.

In Figure 5-4, at lower feed rates, the radial compliance C_r at each angular position varies with fewer oscillations around $\bar{C}_r$. But in general, they have the same magnitude and varying trends, meaning that these tests of radial compliance were still quasi-static and the feed rate does not influence too much on the compliance variation, as in this simplified test setup, no cutting forces variation caused by the rotation of the cutting tool were included.

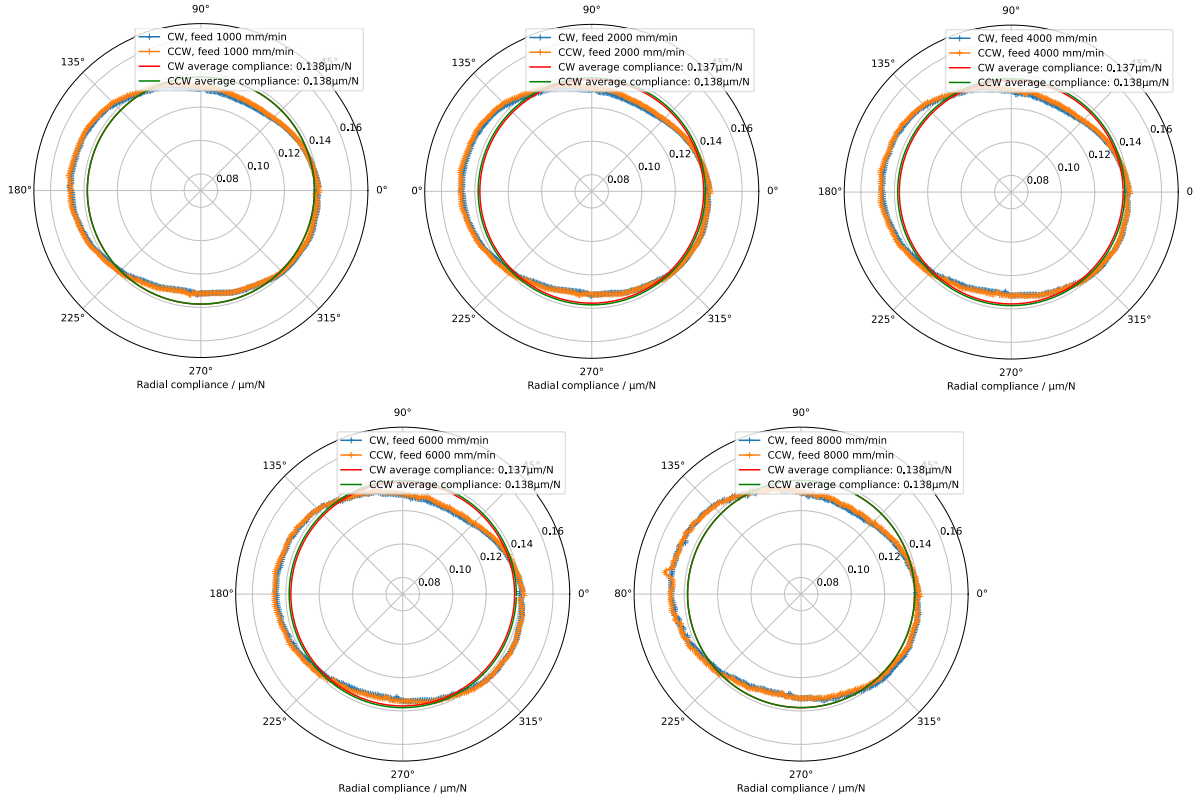


Figure 5-4: Radial compliance plots under all feed rates

The projected engagements in the X and Y directions ($E_{V,X}$, $E_{V,Y}$) can be calculated from the radial engagements by using Eq. (5.9) by using trigonometry and $\Delta\rho$. It equals also to the outer product of the compliance matrix and the force matrix, with C_{XX} for calculating $E_{V,X}$ caused by the forces in the X axis, and C_{XY} for calculating $E_{V,X}$ caused by forces in the Y direction. A similar nomenclature is applied for C_{YY} and C_{YX} . i represents tests under different feed rates, and n is the number of testing points under each feed rate. The full compliance matrix can be calculated using Eq. (5.10) by applying the pseudo-inverse using code `Numpy.linalg.pinv` in Python. The results under all feed rates for the CW and CCW directions are plotted in Figure 5-5. C_{XX} and C_{YY} decrease within $0.003 \mu\text{m/N}$, while the absolute of C_{XY} and C_{YX} increase significantly as the feed rate increases.

$$\begin{bmatrix} E_{V,X} \\ E_{V,Y} \end{bmatrix}_{2 \times n,i} = \begin{bmatrix} \Delta\rho \cos\gamma \\ \Delta\rho \sin\gamma \end{bmatrix}_{2 \times n,i} = \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix}_{2 \times 2,i} \times \begin{bmatrix} F_X \\ F_Y \end{bmatrix}_{2 \times n,i} \quad (5.9)$$

$$\begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix}_{2 \times 2,i} = \begin{bmatrix} E_{V,X} \\ E_{V,Y} \end{bmatrix}_{2 \times n,i} \times \begin{bmatrix} F_X \\ F_Y \end{bmatrix}_{n \times 2,i}^+ \quad (5.10)$$

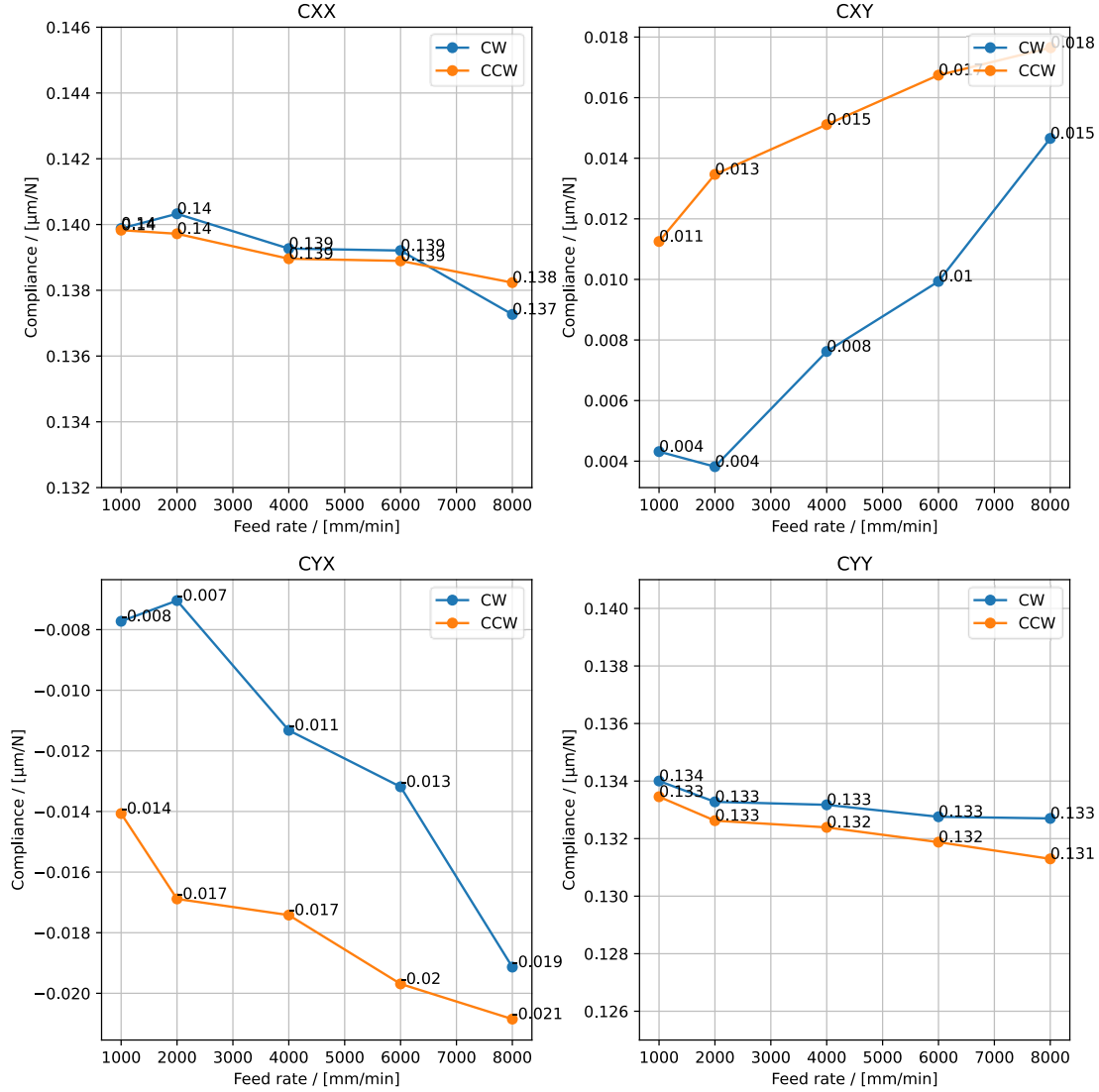


Figure 5-5: Compliance matrix plots under all feed rates for CW and CCW direction

5.2 Tool tip force prediction by a physic-based model from feed motor data

The motor dynamic equilibrium equation using the Coulomb friction model is firstly written as Eq (5.11), where the motor feed torque T_m overcomes the static friction torque $T_{f,stat}$, tool tip disturbance torque T_d , dynamic load torque due to reflected inertia on the motor J_e and viscous friction $T_{f,visc}$. ω is the motor's actual speed, and B is the viscous friction coefficient [23].

$$T_m = J_e \frac{d\omega}{dt} + T_{f,visc} + T_{f,stat} + T_d \quad (5.11)$$

$$T_{f,visc} = B\omega \quad (5.12)$$

$$T_{f,stat} = K_f \text{sign}(\omega) \quad (5.13)$$

$$T_d = K_d F_d \quad (5.14)$$

By rearranging Eq. (5.11-5.14), the tool tip disturbance force can be related to the feed motor data as in Eq. (5.15), where ω is the rotary speed, K_f is coulomb friction torque coefficient, K_d is a disturbance torque gain, and F_d as the tool tip disturbance force. A constant term ζ was added here to keep the residual distribution normal. As some coefficients were missing, the coefficients were calculated by using pseudo-inverse on the training dataset under all feed rates, as Eq. (5.16). The results are shown in Table 5-3. The force predictions along the X and Y direction on the testing dataset can be calculated with the coefficients obtained in the training dataset using Eq (5.17). The training and testing dataset were separated from overall dataset constructed from the exported data from the dynamometer and machine controller with the ratio of 8:2.

$$F_d = \frac{1}{K_d} T_m - \zeta - \frac{J_e}{K_d} \frac{d\omega}{dt} - \frac{B}{K_d} \omega - \frac{K_f}{K_d} \text{sign}(\omega) \quad (5.15)$$

$$\left[\frac{1}{K_d}, \zeta, \frac{J_e}{K_d}, \frac{B}{K_d}, \frac{K_f}{K_d} \right]_{x,y} = F_{x,y} \begin{bmatrix} T_{m,train} \\ -1 \\ -\frac{d\omega}{dt}_{train} \\ -\omega_{train} \\ -\text{sign}(\omega_{train}) \end{bmatrix}_{x,y}^+ \quad (5.16)$$

$$F_{x,y,pred} = \left[\frac{1}{K_d}, \zeta, \frac{J_e}{K_d}, \frac{B}{K_d}, \frac{K_f}{K_d} \right]_{x,y} \begin{bmatrix} T_{m,test} \\ -1 \\ -\frac{d\omega}{dt}_{test} \\ -\omega_{test} \\ -\text{sign}(\omega_{test}) \end{bmatrix}_{x,y} \quad (5.17)$$

Table 5-3: Physic-based model coefficients obtained from the training dataset

	$\frac{1}{K_d}$	ζ	$\frac{J_e}{K_d}$	$\frac{B}{K_d}$	$\frac{K_f}{K_d}$
X	-343.00	-45.15	46.21	254.00	570.32
Y	-385.31	-42.56	56.71	204.55	656.37

The model was examined on the testing dataset. The predicted composed equivalent force $F_{c,pred}$ and the predicted radial forces $F_{r,pred}$ can be calculated using Eq. (5.18-5.19). The predicted radial volumetric errors $E_{v,pred}$ can be calculated by multiplying $F_{r,pred}$ with the radial compliance C_r validated in section 5.1. The results are shown in Figure 5-6 and Figure 5-7.

$$F_{r,pred} = F_{x,pred}\cos\gamma + F_{y,pred}\sin\gamma \quad (5.18)$$

$$E_{v,pred} = F_{r,pred}C_r \quad (5.19)$$

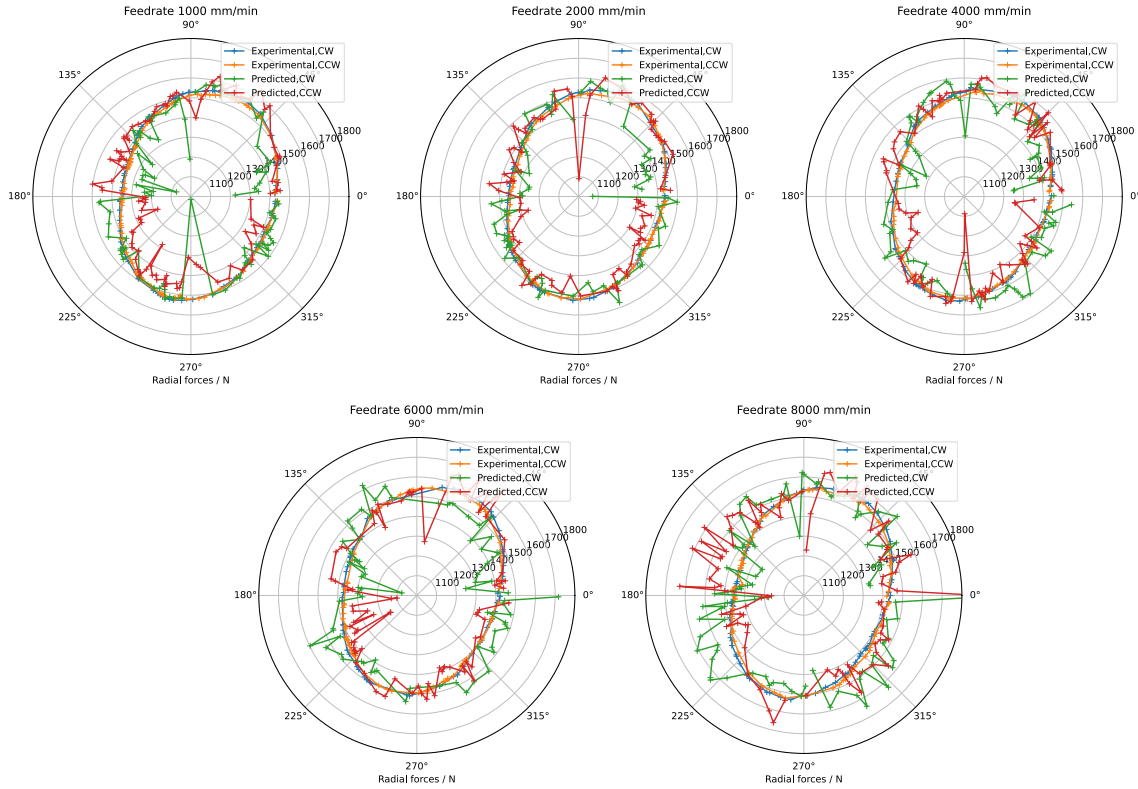


Figure 5-6: Predicted and experimental radial forces plots using a physical-based model on the testing dataset.

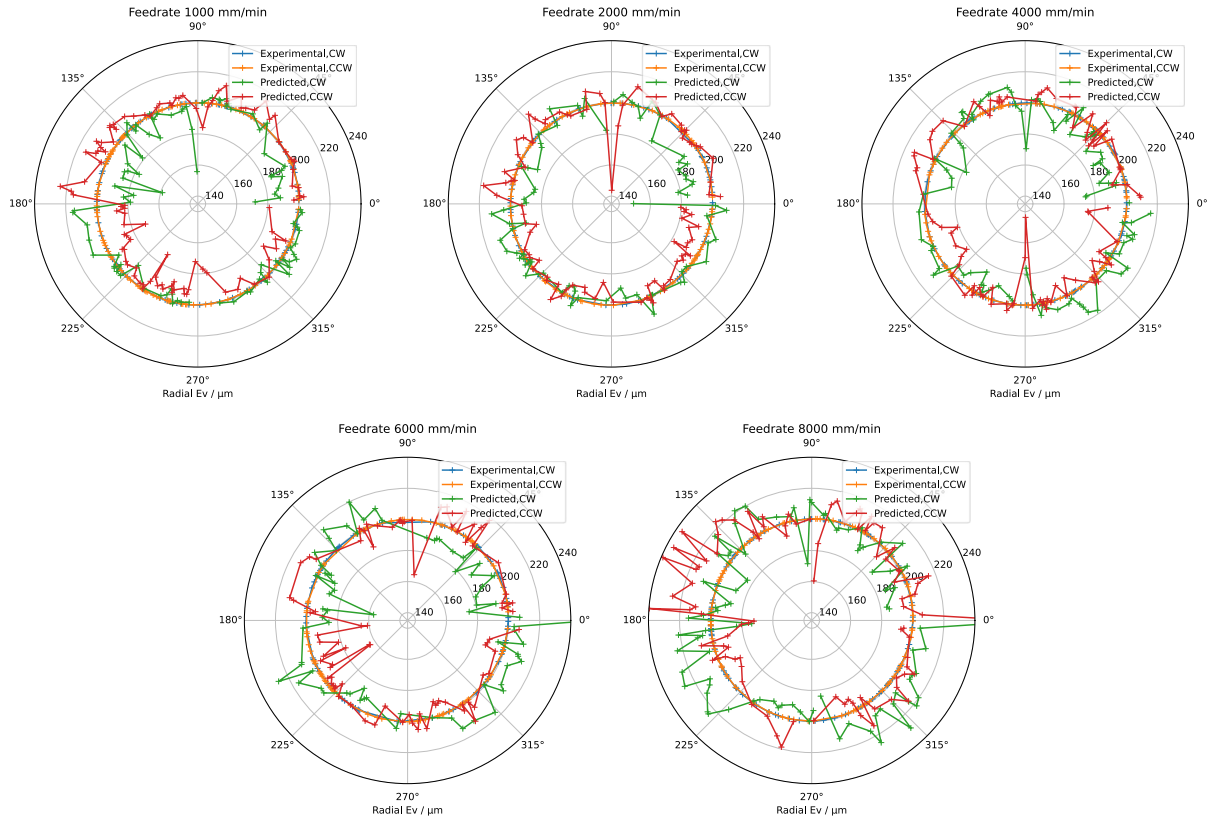


Figure 5-7: Predicted and experimental radial volumetric error plots using a physical-based model on the testing dataset.

The feed rates in the test influence the predicted data: at higher feed rates, the predicted forces depart more from the experimental forces than at lower feed rates, meaning that more prediction errors appear. A similar trend appears in the prediction of volumetric errors since the radial volumetric errors were calculated as the product of radial compliance and predicted forces. The $E_{v,pred}$ is generally at least $20 \mu\text{m}$ away from the experimental data, and even further compared to the experimental data at higher feed rate. At the movement reversal points (90° , 180° , 270° and 360° angular position), the radial force prediction peaks far from the experimental data. The Coulomb friction part in the model does not capture the friction variation, as it is modelled by $\text{sign}(\omega)$ and results in a sudden change at those points, which is not the actual friction variation status. Hence, the forces predicted also have a sudden change. Besides, the current exported from the controller is not cleaned. It has the noises, especially at higher feed rate. Using the uncleaned signal from the controller for calculating the predicted forces can highly influence the prediction

results. Hence, the prediction errors in the predicted forces could be even more at higher feed rate.

Clear evidence is that this physical-based equilibrium model cannot be used directly to predict the radial forces and volumetric errors. Firstly, with only the Coulomb and viscous friction model, it cannot capture the friction variation at certain angular position. A more precise model is needed. Secondly, there is no compensation on the feed motor torque signal. More compensations, such as the servo motor's signal compensation and machine tool's dynamic errors, should be added to the physical-based model to improve the model's performance. New models should be used to predict tool tip forces and volumetric errors from the feed motor data. The neural network model may deal with these problems without using manually created friction models and signal compensation equations.

5.3 Tool tip force prediction by neural network models

An alternative way to predict the forces in the X and Y directions and radial forces by torques with higher precision is to use neural network models. Two different neural network models are presented in the following parts: Neural Network Model 1 (NNs_1) and Neural Network Model 2 (NNs_2). Both of them are trained on the 80% of data acquired based on all feed rates and predicted on the 20% of data acquired based on all feed rates.

5.3.1 Tool tip force prediction separately in two directions (model NNs_1).

Firstly, forces in the X and Y directions were treated separately by two models. The inputs of each model were selected based on the correlation coefficients matrix in Figure 5-8. The term $d\omega/dt$ was kept in the model's inputs as it has a correlation coefficient of higher than 0.5 with the force. ω or $\text{sign}(\omega)$ need to be kept to include the friction's influence even though they are not highly correlated with the forces. As ω and $\text{sign}(\omega)$ are also correlated, the term $\text{sign}(\omega)$ was discarded. A HalvingGrid search method in the SciPy package in Python was used for the hyperparameter tuning. The predicted radial volumetric errors were calculated based on Eq. (5.18-5.19) from the predicted forces. The final neural network model (NNs_1) is illustrated in Figure 5-9, and the final prediction results under all feed rates on the testing data set are

illustrated in Figure 5-10. The results under different feed rates are shown in Figure 5-11 to Figure 5-15.

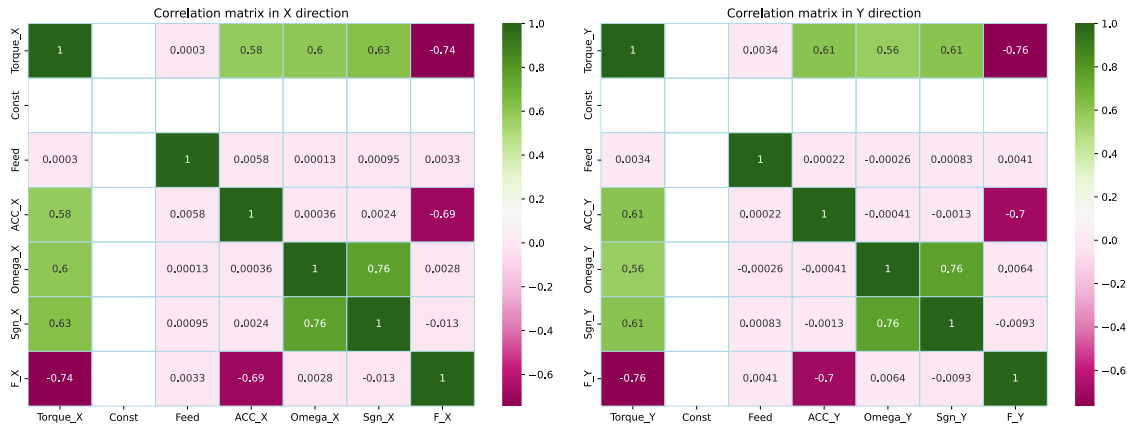


Figure 5-8: Correlation matrix of each term in X and Y direction separately. In each direction, “Torque” is the feed motor torque T_m , “Const” is the const ζ , “Feed” is the feed rate, “ACC” is $d\omega/dt$, “Omega” is ω , “Sgn” is the term $\text{sign}(\omega)$ and “F” is the tool tip forces F_d in each direction.

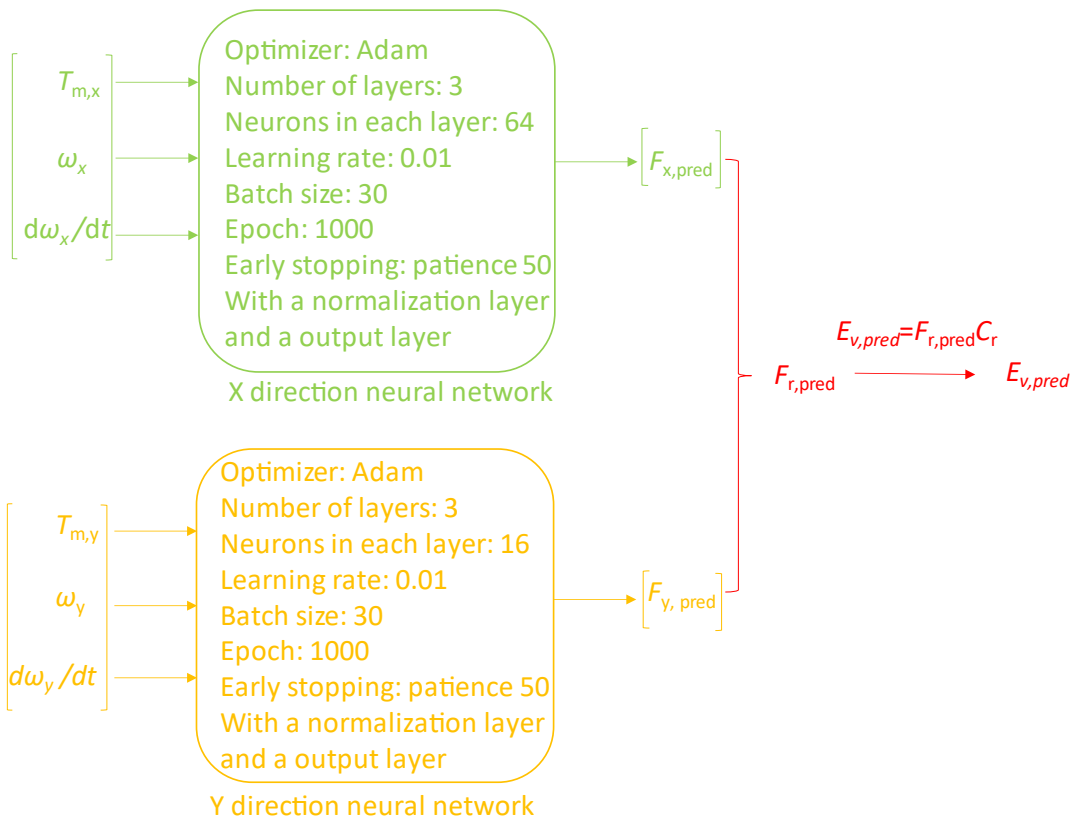


Figure 5-9: Structure of tool tip force and volumetric error prediction with model NNs_1

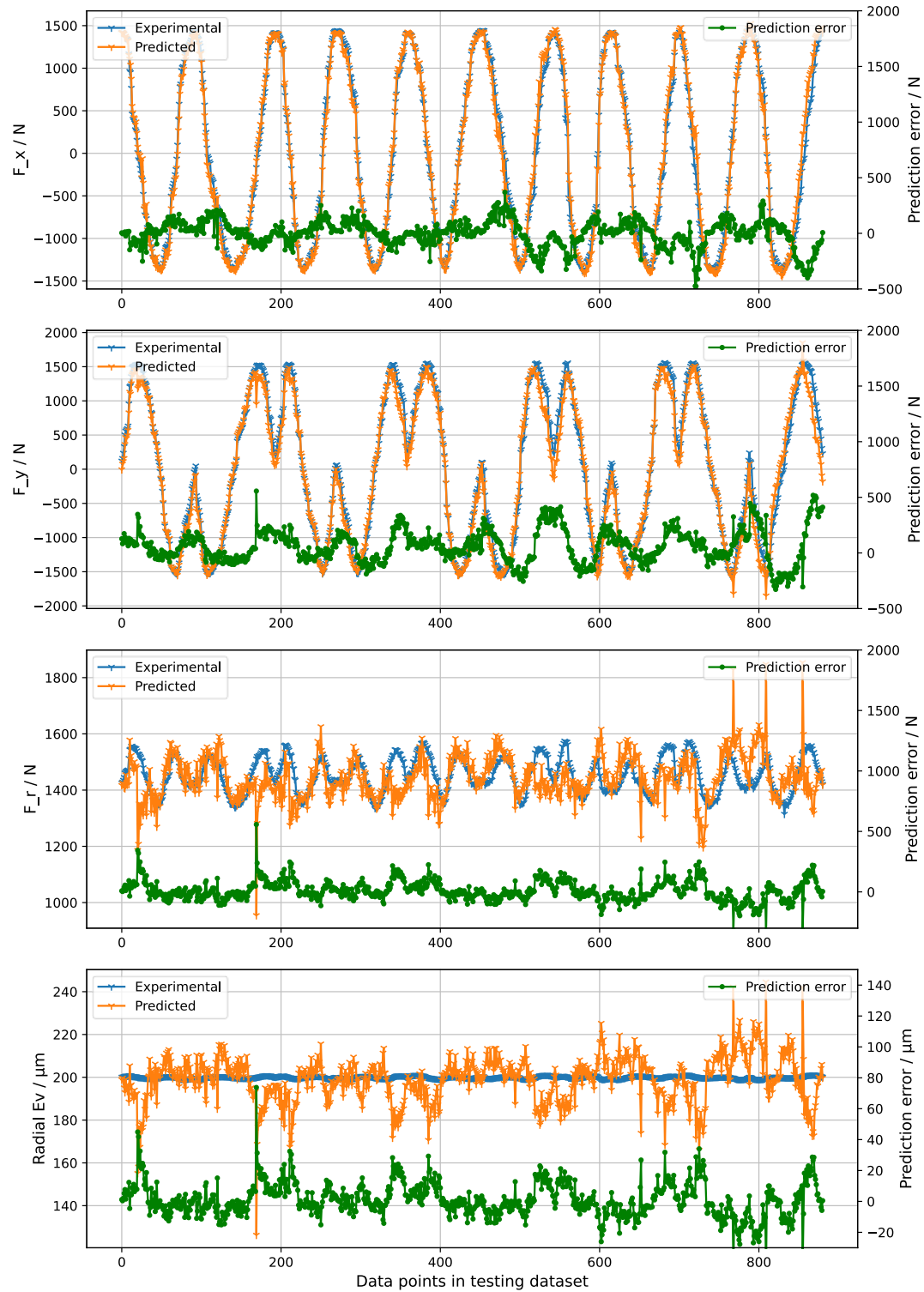


Figure 5-10: Prediction results using neural network model NNs_1. The prediction is based on the data of all feed rates altogether. F_x , F_y , and F_r are the forces in the X, Y and radial direction.

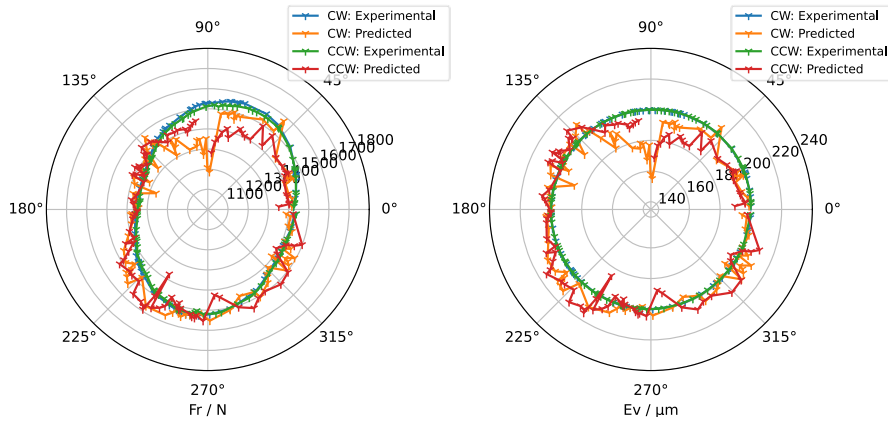


Figure 5-11: F_r (left) and E_v (right) predictions of NNs_1 under 1000 mm/min feed rate

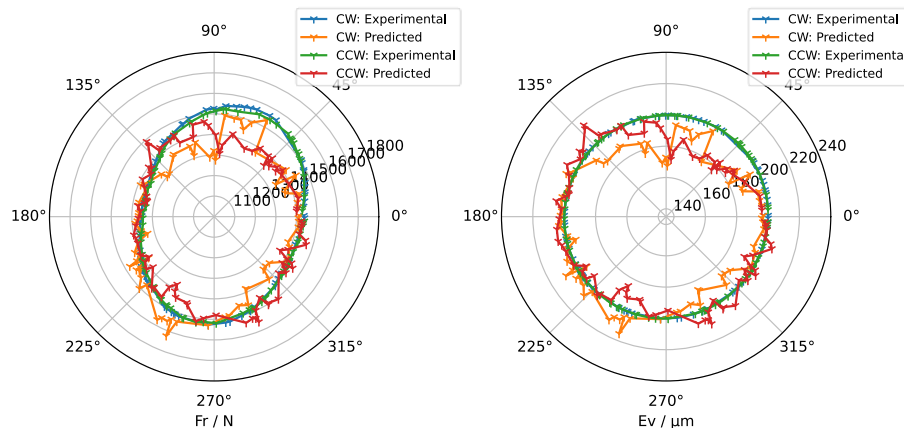


Figure 5-12: F_r (left) and E_v (right) predictions of NNs_1 under 2000 mm/min feed rate

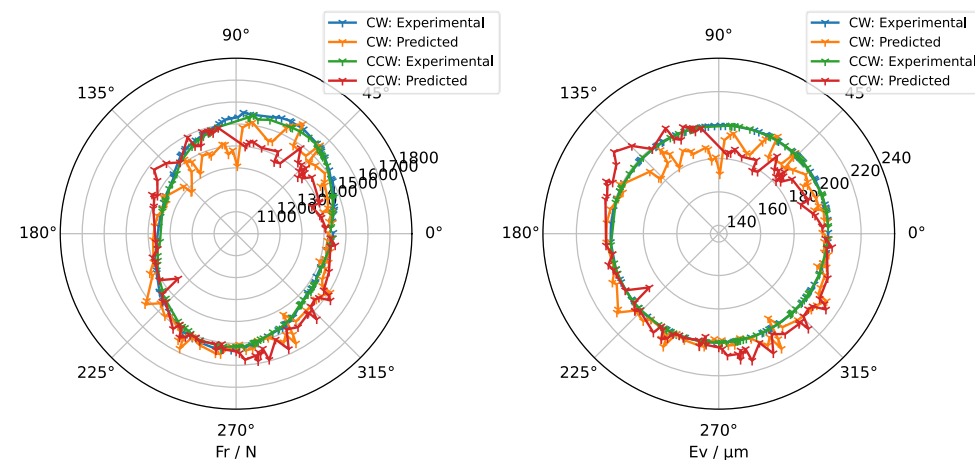


Figure 5-13: F_r (left) and E_v (right) predictions of NNs_1 under 4000 mm/min feed rate

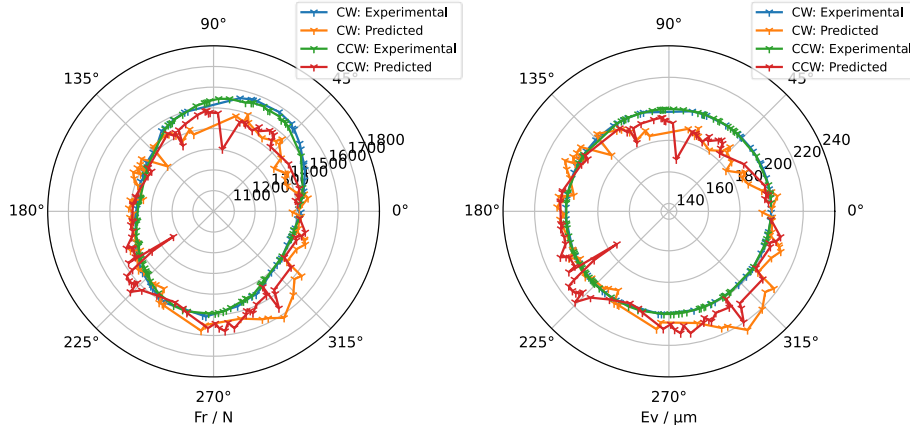


Figure 5-14: F_r (left) and E_v (right) predictions of NNs_1 under 6000 mm/min feed rate

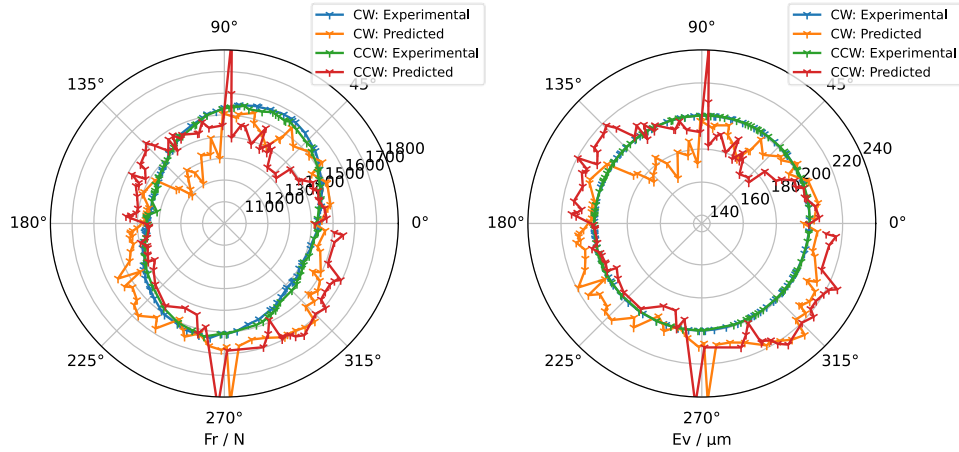


Figure 5-15: F_r (left) and E_v (right) predictions of NNs_1 under 8000 mm/min feed rate

5.3.2 Tool tip force prediction simultaneous in two directions (model NNs_2).

A second neural network model (NNs_2) combining the simultaneous prediction of tool tip forces in the X and Y direction was proposed. This model treats simultaneously the X and Y direction data in one neural network model. The model's inputs were selected based on the correlation matrix concerning all the indicators in X and Y directions at once, as shown in Figure 5-16. The final inputs and outputs of NNs_2 are shown in Figure 5-17. The prediction results under all feed rates are shown in Figure 5-18. Results under all feed rates are in Figure 5-19 to Figure 5-23.



Figure 5-16: Correlation matrix of each term in X and Y direction separately. In each direction, “Torque” is the feed motor torque T_m , “Const” is the const ζ , “Feed” is the feed rate, “ACC” is the term of $d\omega/dt$, “Omega” is the ω , “Sgn” is the term $\text{sign}(\omega)$ and “F” is the tool tip forces F_d in each direction.

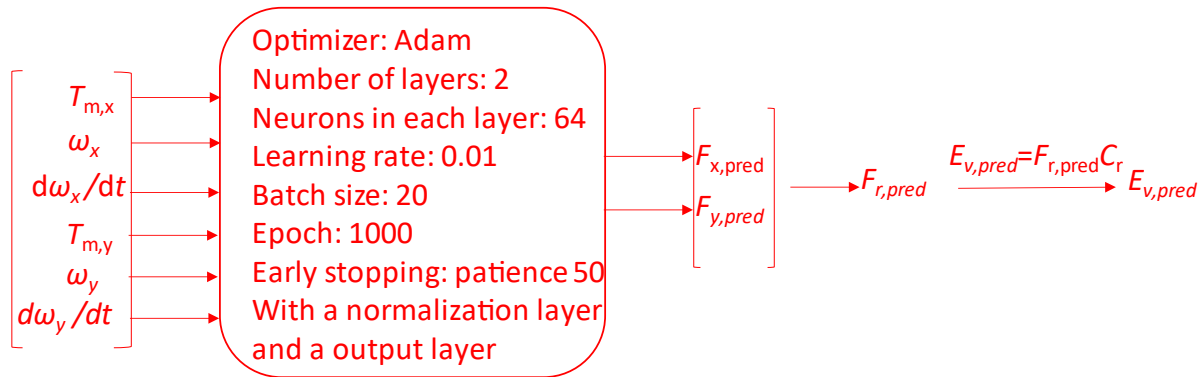


Figure 5-17: Structure of tool tip force and volumetric error prediction with model NNs_2

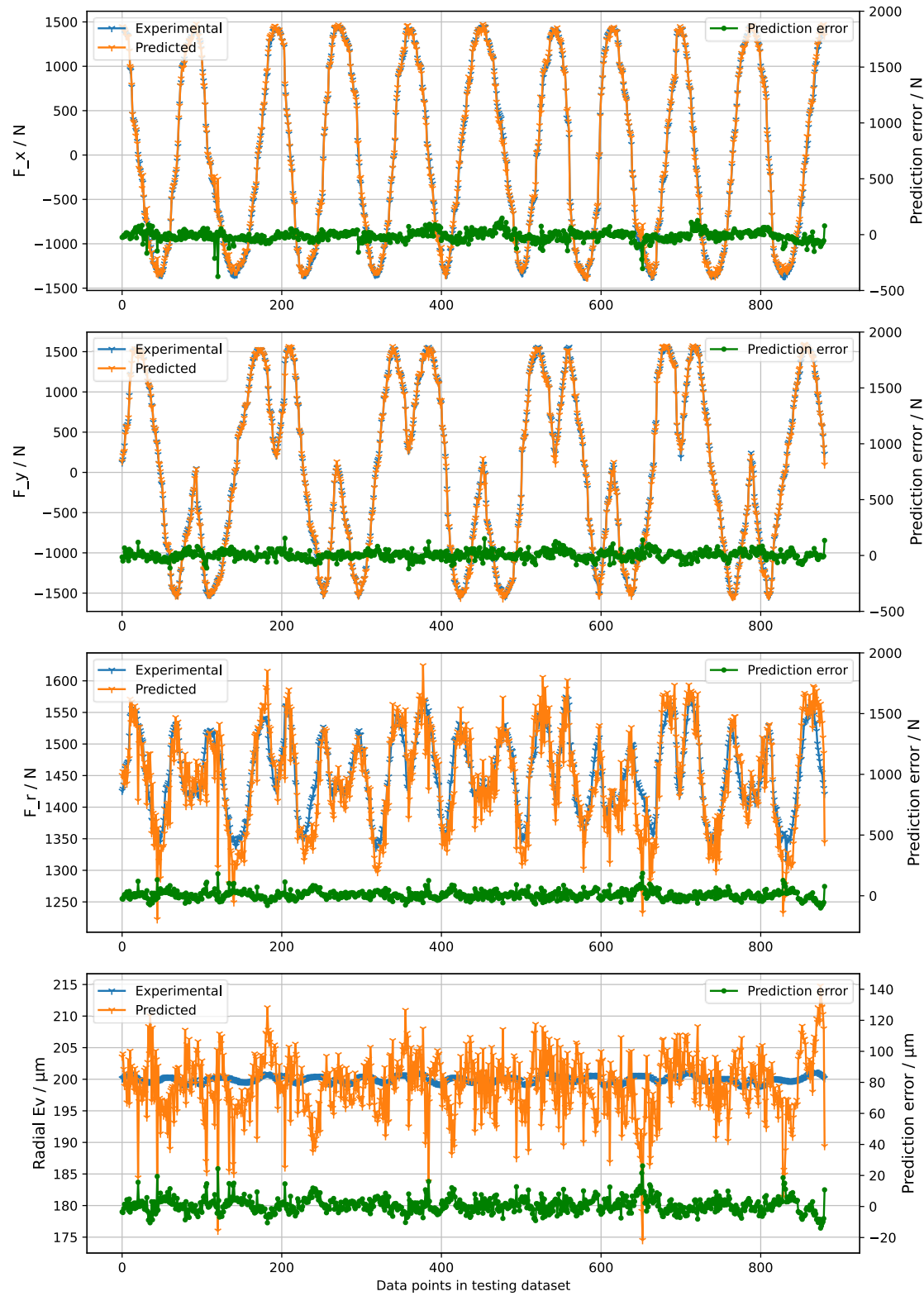


Figure 5-18: Prediction results using neural network model NNs_2. The prediction is based on the data of all feed rates. F_x , F_y , and F_r are the forces in the X, Y and radial direction.

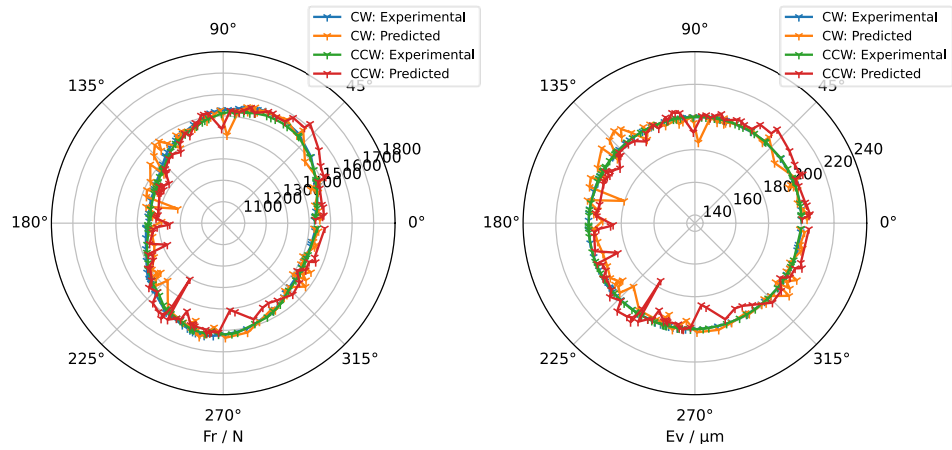


Figure 5-19: F_r (left) and E_v (right) predictions of NNs_2 under 1000 mm/min feed rate

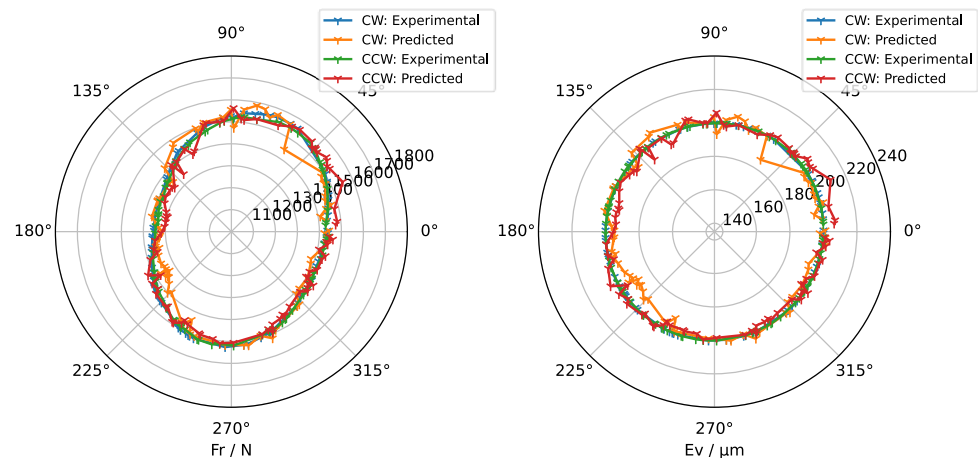


Figure 5-20: F_r (left) and E_v (right) predictions of NNs_2 under 2000 mm/min feed rate

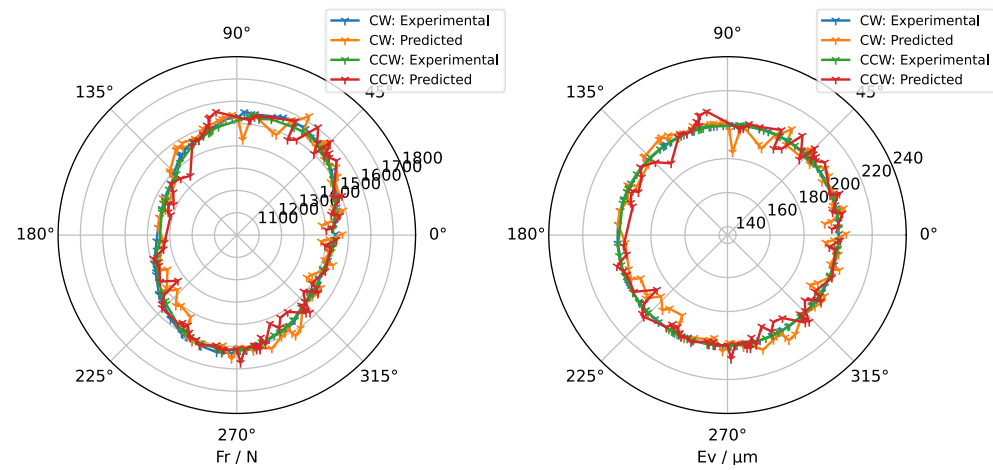


Figure 5-21: F_r (left) and E_v (right) predictions of NNs_2 under 4000 mm/min feed rate

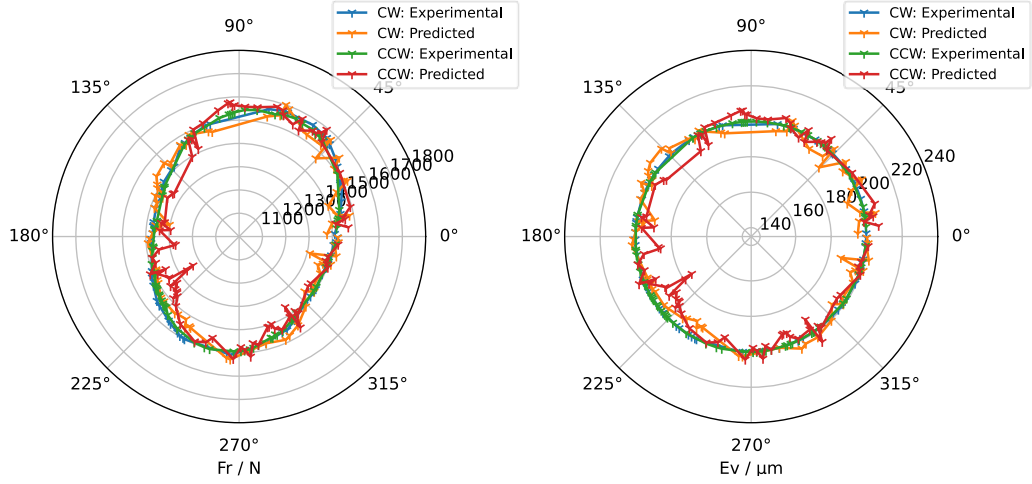


Figure 5-22: F_r (left) and E_v (right) predictions of NNs_2 under 6000 mm/min feed rate

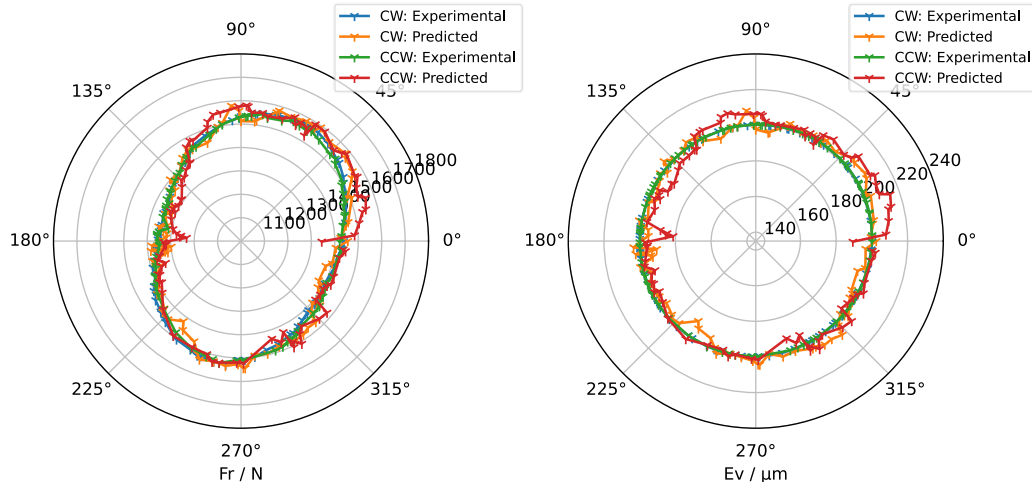


Figure 5-23: F_r (left) and E_v (right) predictions of NNs_2 under 8000 mm/min feed rate

5.3.3 Summary

By comparing the prediction results between the physical model and two neural network models, both neural network models have better performance than the physical model. A commonly used physical model with Coulomb and viscous friction may not be adequate for predicting the tool tip forces in dynamic working conditions. Compensations such as the friction variation at the movement reversal points and feed torque signal compensation need to be added in.

Based on the results from Figure 5-10 to Figure 5-23, NNs_2 predicts F_r mostly within the range of 1450 ± 150 N, and E_v of mostly 200 ± 10 μm ; however, NNs_1 predicts F_r mostly more than the range of 1500 ± 200 N, and E_v of more than 200 ± 20 μm . Especially at higher feed rates,

NNs_2 performs better than NNs_1 on the F_r prediction. Furthermore, the RMSEs between experimental and predicted indicators (F_x , F_y , F_r and E_v) on the testing dataset are shown in Table 5-4. NNs_2 generally performs better than NNs_1 on predicting F_x , F_y , F_r and E_v . NNs_2 treated the features in X and Y directions simultaneously. It may learn the dynamic errors between the X and Y directions, such as the mismatch errors in the X and Y directions. A better neural network model may be needed to further improve the precision of tool tip force and volumetric errors prediction based on machine compliance behaviour.

Table 5-4: RMSEs of experimental and predicted indicators using two different neural network models under all feed rates on the testing dataset.

Neural network model	RMSE of F_x / N	RMSE of F_y / N	RMSE of F_r / N	RMSE of E_v / μm
NNs_1	117.09	160.70	82.38	11.10
NNs_2	47.25	37.12	35.44	4.91

CHAPTER 6 GENERAL DISCUSSION

The static and dynamic tests give a general methodology to model the tool tip forces and volumetric errors based on machine compliance behaviour from the feed motor data. Nevertheless, the work has some limitations as listed in part 6.1. Discussions on the static and dynamic test are presented in part 6.2 and 6.3.

6.1 Limitations of the work

- a) The influence of tool compliance, bearing compliance, spindle compliance and influence on tool orientation were not decoupled from the volumetric errors defined in the work. Hence, the results are subjected to the specific setup.
- b) The static test only identified the three translational compliances. The cross-compliances in the tridimensional space should be added for more accurate results in the real working condition.
- c) The spindle was blocked up and was not rotating. Hence, the cutting force varying frequency's influence on compliance was not tested. The frequency's influence should also be included in the real cutting condition.
- d) Compliances of the rotational axis should also be considered in the real working condition.

6.2 Discussion on the static test

In the static test, the static translational volumetric compliance was tested separately along the X, Y and Z directions. The static translational volumetric compliance was calculated using the linear interpolation method and modelled with the linear and tanh function terms. Both methods prove that, with the tested tool, the static translational volumetric compliance has nonlinear behaviour under smaller commands (or smaller E_{XYZ} as they are numerically opposite) and smaller tool tip forces where the friction status changes. The behaviour is almost linear behaviour under larger tool tip forces and where the friction status does not change. The nonlinear behaviour was captured by the tanh function.

The friction was modelled using adjusted Dahl's theory without considering the speed's influence. It used an exponential term to capture the friction variations at the motion start and the movement reversal points. A simple Coulomb friction model may not be adequate at the lower tool tip

forces range (lower commands, smaller E_{XYZ} range) as it cannot capture the gradual friction variations. The same applies to the movement reversal points where the friction status changes.

The final model linking the feed motor torque with the static translational volumetric errors was established. It can predict the static translational volumetric errors directly from feed motor torque.

6.3 Discussion on the dynamic test

One physical-based model and two neural network models were proposed to connect the feed motor torques with the tool tip forces. The models were combined with measured radial volumetric compliance to further predict the radial volumetric errors. The radial compliance shows an oval shape, meaning that the compliance in the X and Y directions are not the same: X is more compliant than Y. This agrees with the static test results.

There are insufficiencies in the model with Coulomb and viscous friction to predict the tool tip forces and radial volumetric errors. Firstly, the Coulomb and viscous friction model cannot capture friction variations around 90° , 180° , 270° and 360° . Hence, there is a failure of tool tip forces prediction at those angular positions. Secondly, no feed motor torque signals from the encoder are compensated at higher feed rates. It is difficult to compensate the dynamic errors using a physical-based model.

Both neural network models (NNs_1 and NNs_2) showed better performance than the physical-based model, as they may automatically learn, such as the friction variations around 90° , 180° , 270° and 360° angular positions, as well as may automatically compensate the signal. NNs_2 performed better than NNs_1 since NNs_2 simultaneously learned in the X and Y directions. NNs_2 has the potential of self-learning the dynamic error existing between X and Y directions.

CHAPTER 7 CONCLUSION AND RECOMANDATIONS

7.1 Overall conclusion

- a) The static translational volumetric compliance in the X, Y and Z directions and the radial compliances are detected. The machine tool tested shows different compliance behaviours along the X, Y and Z directions: Z is the stiffest, and X is the most compliant;
- b) The static translational volumetric compliance shows nonlinear behaviour when the tool tip forces are lower and the friction status changes. When the friction status does not change, the compliance has almost linear behaviour;
- c) In the static test, the static translational volumetric errors can be predicted directly from feed motor data;
- d) Under the dynamic working condition, the tool tip forces can be predicted using the neural network model NNs_2. Under all feed rates, it has an RMSE of radial force prediction of 35.44 N and RMSE of radial volumetric errors prediction of 4.91 μm .

7.2 Recommendations for the future work

Future works can focus on improving the model's generalizability for different working conditions, such as:

- a) Decouple the machine tool internal compliance, tool compliance, spindle compliance and bearing compliance;
- b) Improve and simplify the friction model in the machine tool;
- c) Test the cross-compliance between axes in a tridimensional space;
- d) Test the compliances of the rotary axes;
- e) Improve the neural network model for dynamic force predicting.

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APPENDIX A CALIBRATION OF TWO SCALES

Two scales were used in dynamometer calibration: Scale A, SA (OHAUS Trooper) and Scale B, SB (FG-200K AL) as in Figure A-1. They were used to obtain the uncertainties of the weights used to calibrate the dynamometer. The preliminary weights for calibrating SA are cylindrical steel weights with ISO 17025 certificates. Successive calibrations from the SA to the lower range of SB and all ranges of SB are illustrated in Figure A-1. The calibration calculation was based on the GUM [31]. The linear calibration curve, the parameters of fit and their uncertainties were obtained, which were used to predict the uncertainties of the reading error of the dynamometer.

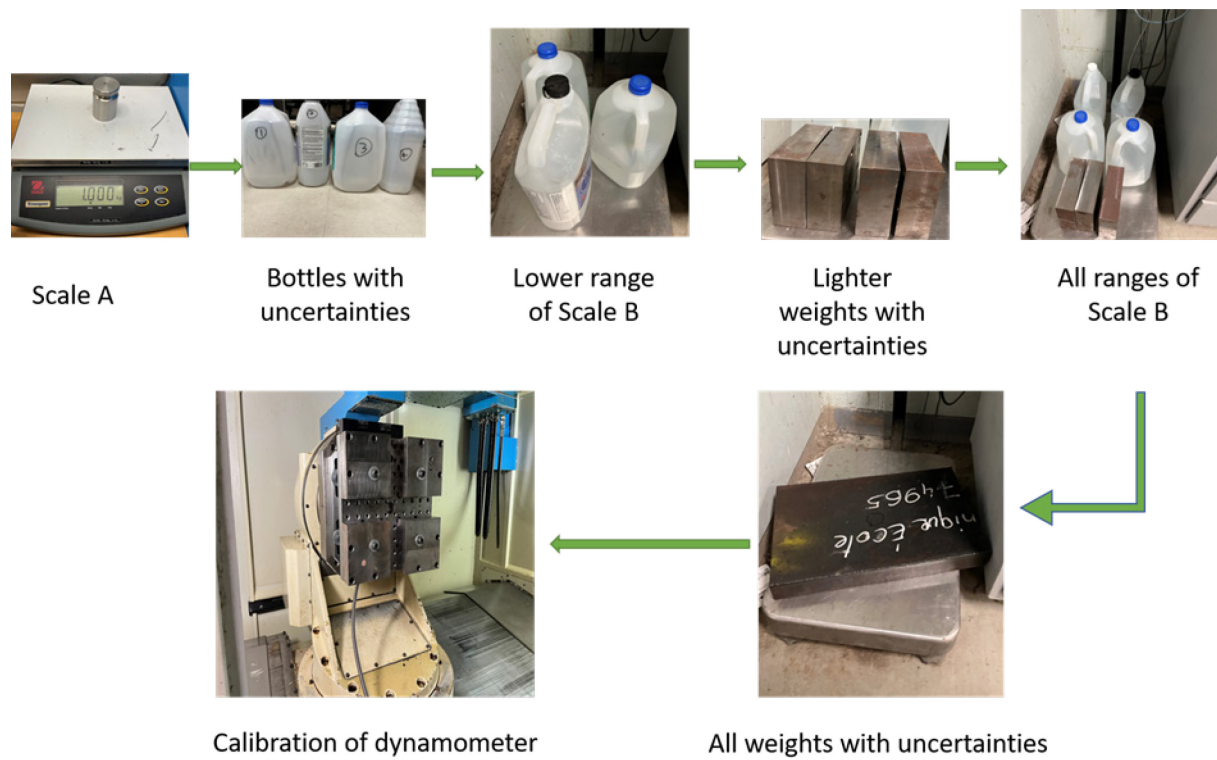


Figure A-1: SA and SB calibration procedure

The reading error $\varepsilon(r)$ at any reading r can be regressed from Eq (A.1-2), where r_0 is a fixed reference value (0 in this test), and ε_k is the difference between experimental reading and reference weights. The regression is based on the least square method based on Eq (A.2). The coefficients a and b are the regression coefficients obtained by Eq (A.3, A.4). Given any reading r_k , the predicted reading error can be written as $\varepsilon(r_k)$. $\theta_k = r_k - r_0$. The coefficient $r(a, b)$ is the estimated correlation coefficient. s^2 in Eq (A.8) also adds the term of $u_{weights}^2$ to include the

estimated uncertainties of each calibrated weight. The calculation was based on Eq (A.1-9) to obtain the regression coefficient a , b , their uncertainties and the predicted reading error uncertainties as in Eq (A. 10). In this case, the difference between the reading and the true weights can be obtained, and the its uncertainty can be obtained.

$$\varepsilon(r) = a + b(r - r_0) \quad (\text{A. 1})$$

$$S = \sum_{k=1}^n [\varepsilon_k - a - b(r - r_0)]^2 \quad (\text{A. 2})$$

$$a = \frac{(\sum \varepsilon_k)(\sum \theta_k^2) - (\sum \varepsilon_k \theta_k)(\sum \theta_k)}{D} \quad (\text{A. 3})$$

$$b = \frac{n \sum \varepsilon_k \theta_k - (\sum \varepsilon_k)(\sum \theta_k)}{D} \quad (\text{A. 4})$$

$$s^2(a) = \frac{s^2 \sum \theta_k^2}{D} \quad (\text{A. 5})$$

$$s^2(b) = \frac{ns^2}{D} \quad (\text{A. 6})$$

$$r(a, b) = -\frac{\sum \theta_k}{\sqrt{n \sum \theta_k^2}} \quad (\text{A. 7})$$

$$s^2 = \frac{\sum [\varepsilon_k - \varepsilon(r_k)]^2 + u_{weights}^2}{n - 2} \quad (\text{A. 8})$$

$$D = n \sum \theta_k^2 - \left(\sum \theta_k \right)^2 \quad (\text{A. 9})$$

$$u_c^2[\varepsilon_r] = u^2(a) + (r - r_0)^2 u^2(b) + 2(r - r_0)u(a)u(b)r(a, b) \quad (\text{A. 10})$$

To simplify the form of Eq (A.10), the term concerning the correlation coefficient $r(a, b)$ can be eliminated. The correction and uncertainty curve can be written as in Eq (A.11) and Eq (A.12), respectively. The elimination proceeded with the help of Eq (A.13-16), where $r_0 = (\sum r_k)/n$. The uncertainties of the predicted reading error will be represented as in Eq (A. 16).

$$\varepsilon(r) = a' + b(r - \bar{r}) \quad (\text{A. 11})$$

$$u_c^2[\varepsilon_r] = u^2(a') + (r - \bar{r})^2 u^2(b) \quad (\text{A. 12})$$

$$r(a', b) = 0 \quad (\text{A. 13})$$

$$a' = a + b(\bar{r} - r_0) \quad (\text{A.14})$$

$$\bar{r} = r_0 - \frac{s(a)r(a,b)}{s(b)} \quad (\text{A.15})$$

$$s^2(a') = s^2(a)[1 - r^2(a,b)] \quad (\text{A.16})$$

The uncertainties were combined with reparability to obtain the combined uncertainty u_c of the reading. It was multiplied with a coverage factor $k=2$ to get the final expanded uncertainty U . The calibration procedures' results were illustrated as follows.

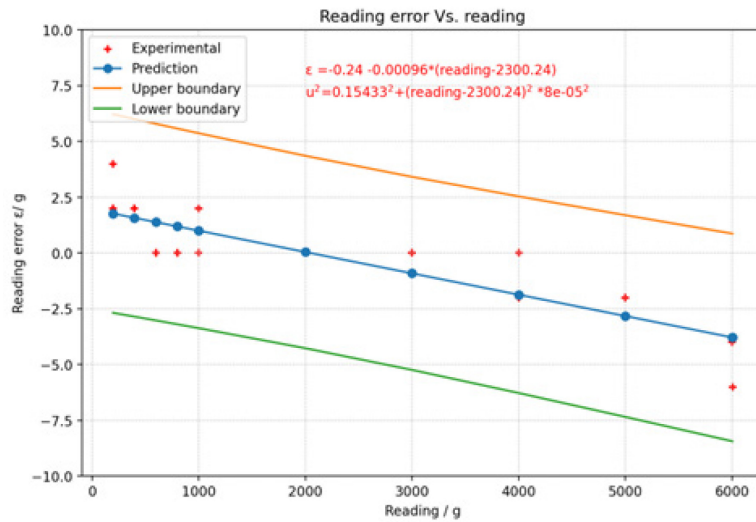


Figure A-2: SA calibration curve and results

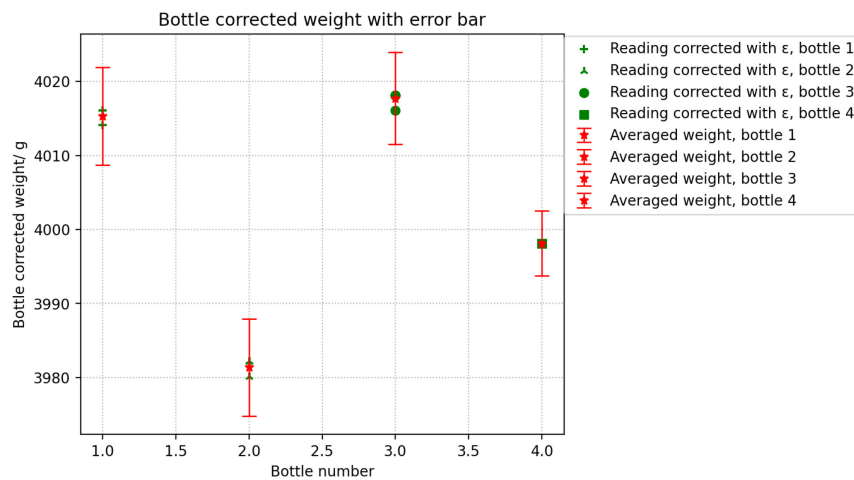


Figure A-3: Bottles of water with uncertainties for calibrating the lower range of SB

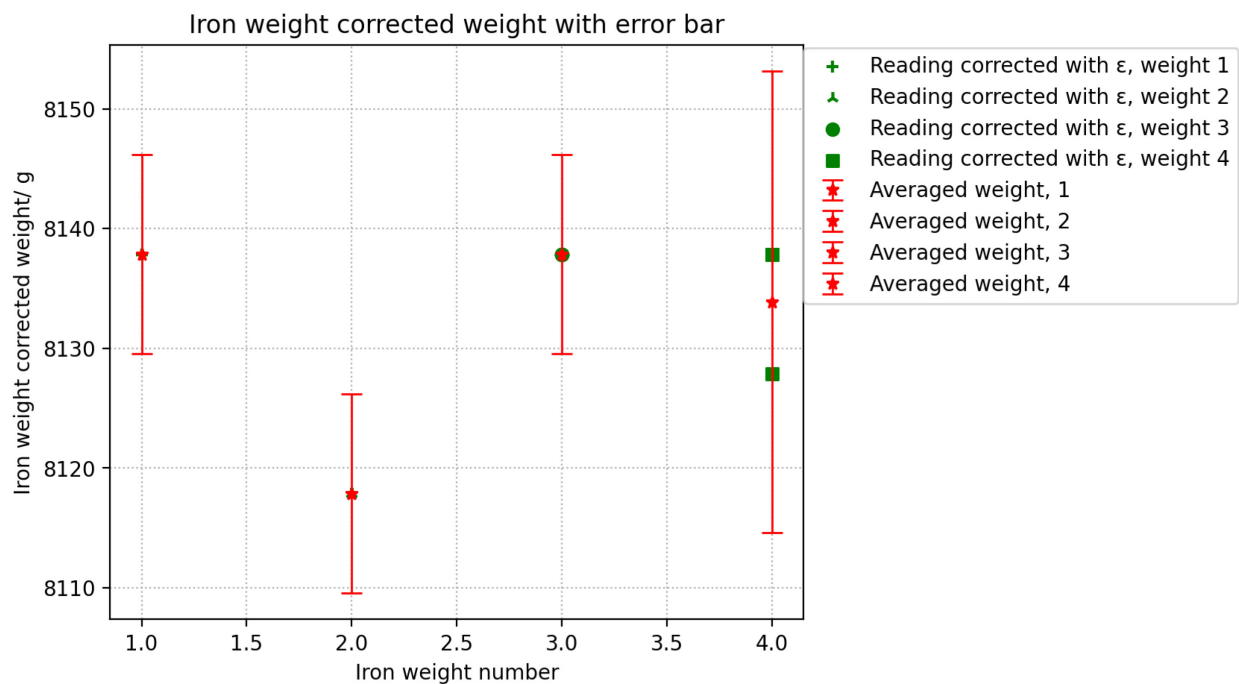


Figure A-4: Four iron blocks with uncertainties for calibration of SB

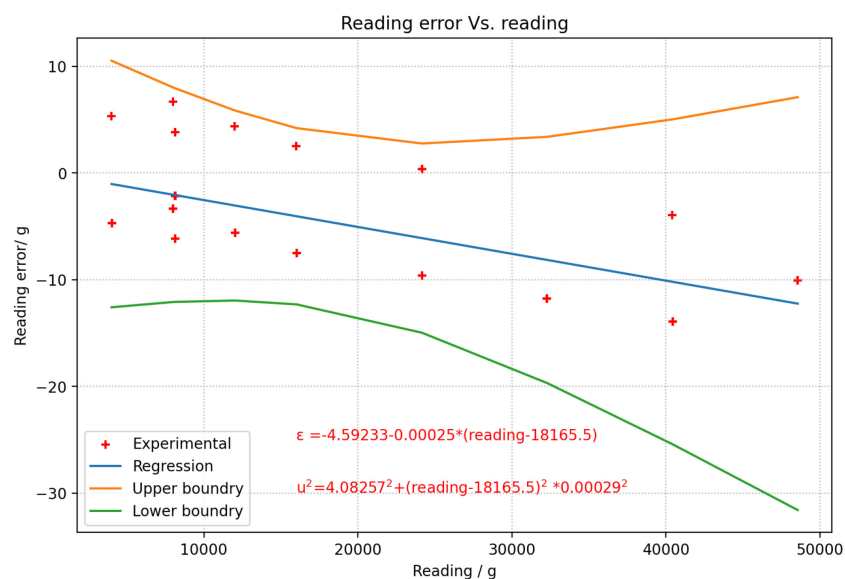


Figure A-5: SB calibration curve and results

APPENDIX B DYNAMOMETER COMPENSATION FOR STATIC TEST

The dynamometer calibration was proceeded by a succession of calibrations from two scales, SA and SB, to the dynamometer with the help of weights with estimated uncertainties. The calibration of SA and SB is in APPENDIX A. The weights, fixtures and screws to calibrate the dynamometer are listed below with uncertainties

Table B-1: Averaged weights and uncertainties of weights used in the tests

Item	Description	Measured by	Average weights / g	Repeatability / g	U (k=2) / g
1	Iron block	SB	8140.0	0.0	8.3
2	Iron block	SB	8120.0	0.0	8.3
3	Iron block	SB	8140.0	0.0	8.3
4	Iron block	SB	8136.0	5.5	19.3
5	Iron block	SB	12932.0	4.5	17.7
6	Iron block	SB	45768.0	4.5	26.9
7	Fixture and screws	SB	23570.0	0	8.7

The dynamometer is previously attached to the table of a machine tool (HU-40T). The C axes were indexed from 0 to 360 degrees with each 10-degree increment to obtain the force reading of dynamometer self-weight projected in X and Y direction (W_{Dx} , W_{Dy}) on the measurement. Next, calibrated iron blocks with total weights of (W) were fixed on the dynamometer table attached to a machine tool, and the procedure was repeated thrice to get the repeatability of the measurement. The dynamometer forces readings (F_x , F_y) were compared with the projected weights of W in the X and Y directions. In this case, the reading errors could be obtained. The Z direction reading was calibrated by setting the dynamometer table at a plane surface and adding three different weights on its surface since measuring the forces generated by the rotating weights in the Z direction on this machine tool was impossible. The setup is illustrated below in Figure B-1.

$$\varepsilon_x = (F_x - W_{Dx}) - W \sin \theta \quad (\text{B.1})$$

$$\varepsilon_y = (F_y - W_{Dy}) - W \cos \theta \quad (\text{B.2})$$

$$\varepsilon_z = F_z - W_z \quad (\text{B.3})$$

The final error curves of the dynamometer readings were illustrated in Figure B-2, Figure B-3 and Figure B-4. It is impossible to check the errors in the negative force reading in the Z direction. The calibration results were then used in the static compliance test to obtain the expanded uncertainties of the compliances tested.

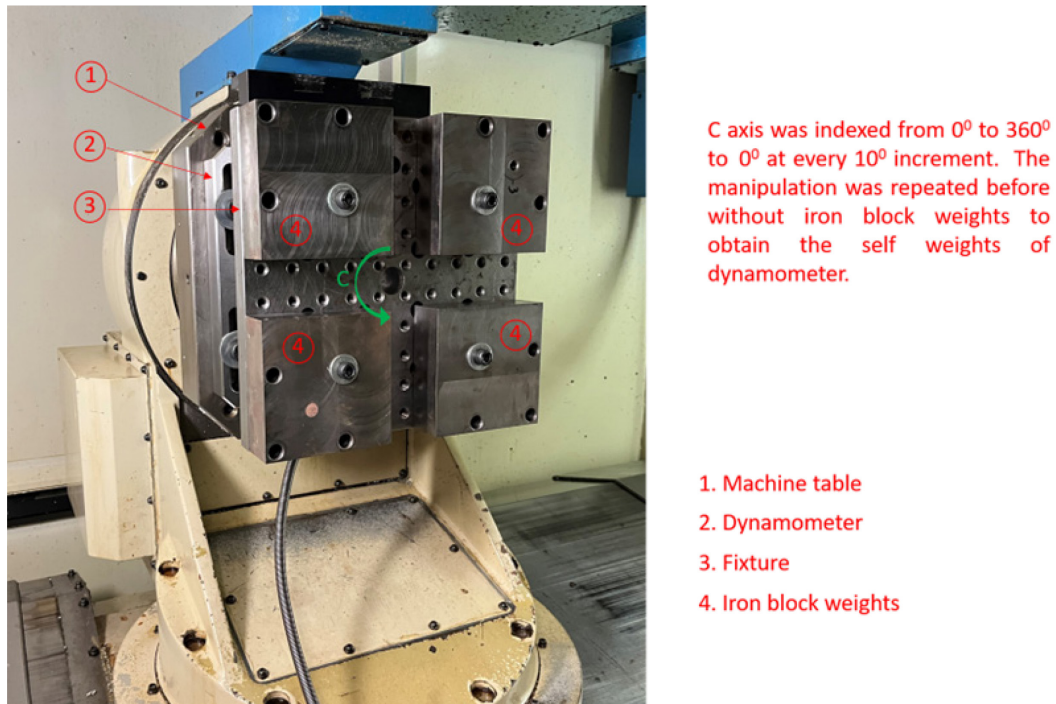


Figure B-1: Setup for dynamometer calibration in X and Y directions

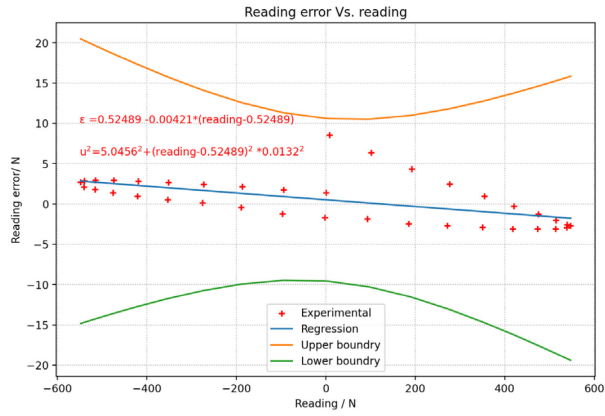


Figure B-2: Dynamometer calibration results in the X direction

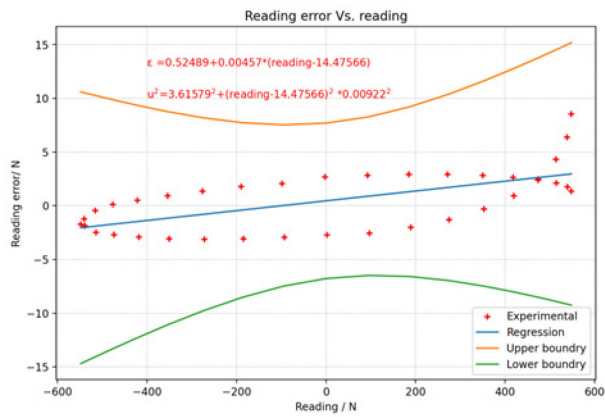


Figure B-3: Dynamometer calibration results in the Y direction

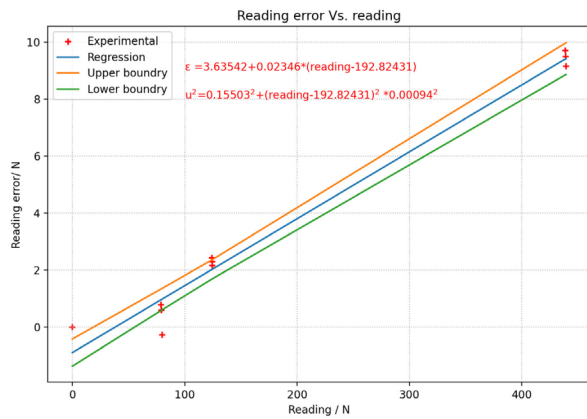


Figure B-4: Dynamometer calibration results in the Z direction

APPENDIX C PLOTS OF RAW DATA AND DATA AFTER MATCHING

Data from two machine encoders and the dynamometer were preprocessed to match the data size of dynamometer forces and encoder data. The dynamometer data was squeezed into the size of the encoder data to match the numbers of features and labels. This part aims to show the properness of the data-matching strategy by showing the raw data and the data after matching. The plots of raw data and data after matching are shown in the following figures.

1. Raw static test data plots

The static tests are repeated with different weights added to the dynamometer surfaces. All the data are from the same measurements. The raw force data of no weight is plotted below. The data of tests repeated with different weights added on the dynamometer surface follow the same trend.

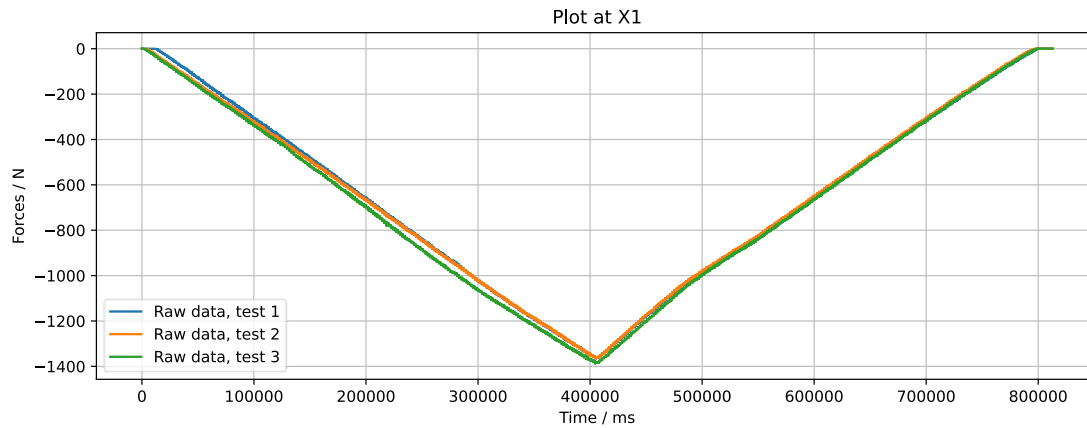


Figure C-1: Raw forces data plots at position X1

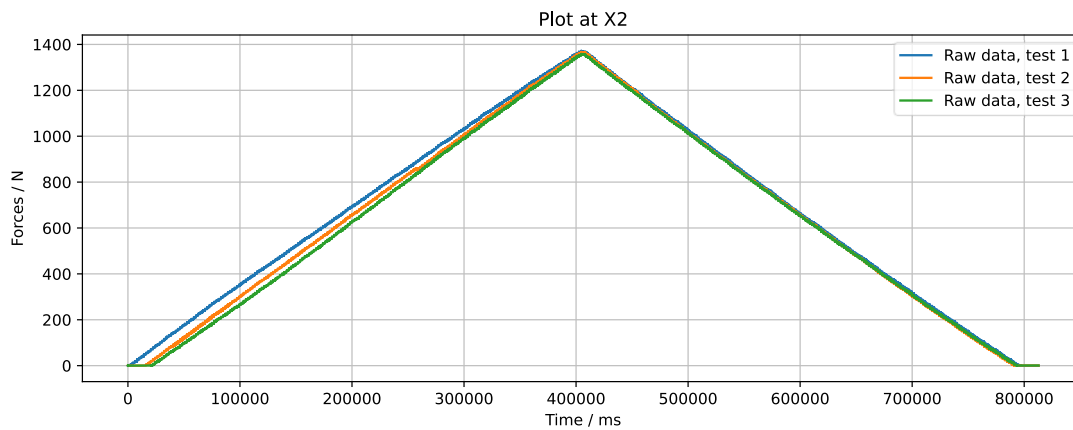


Figure C-2: Raw force data plots at position X2

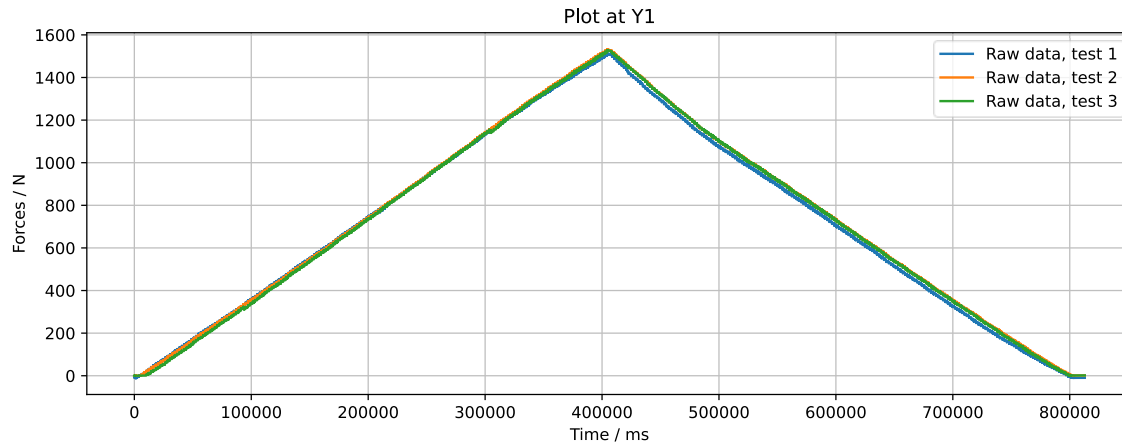


Figure C-3: Raw force data plots at position Y1

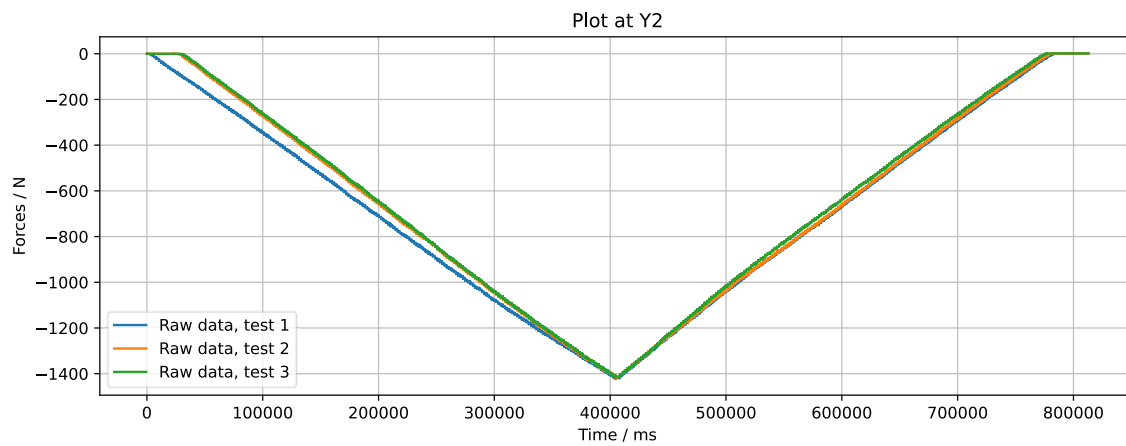


Figure C-4: Raw force data plots at position Y2

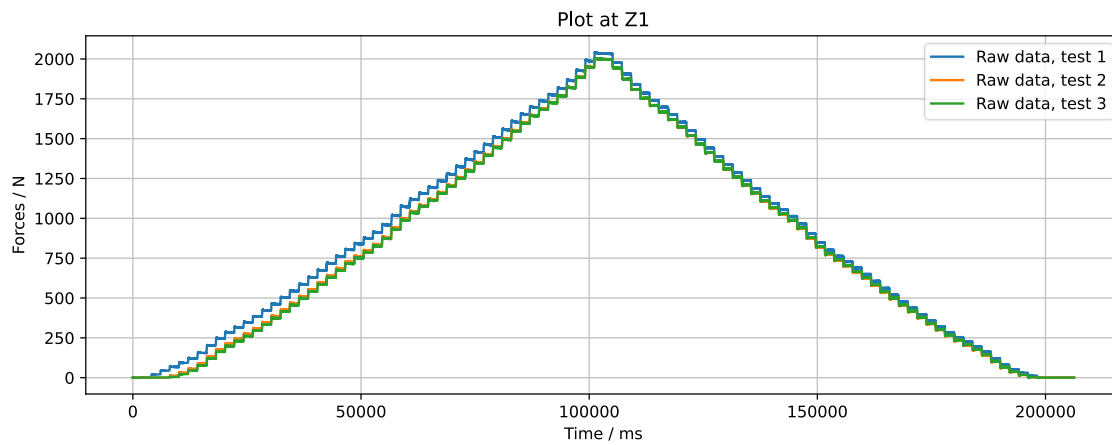


Figure C-5: Raw force data plots at position Z1

2. Static test data plots after data matching

The positional data from the encoder has negligible variation, so the data from the encoder is first averaged at each position (1 μm interval) during inward and outward motion. The forces data are averaged and matched with the processed encoder data. The data after matching is used for modelling. The data after matching with no weight added to the dynamometer is plotted below. The data plots with different weights added on the dynamometer surface follow the same trend.



Figure C-6: Force data used for modelling after data matching

The force plots before and after data matching are similar and have no significant difference. Hence, the data matching strategy used in the static test is reliable and proper.

3. Raw dynamic test data plots

The forces were measured directly with the dynamometer. The plots of raw forces exported from the dynamometer are plotted below.

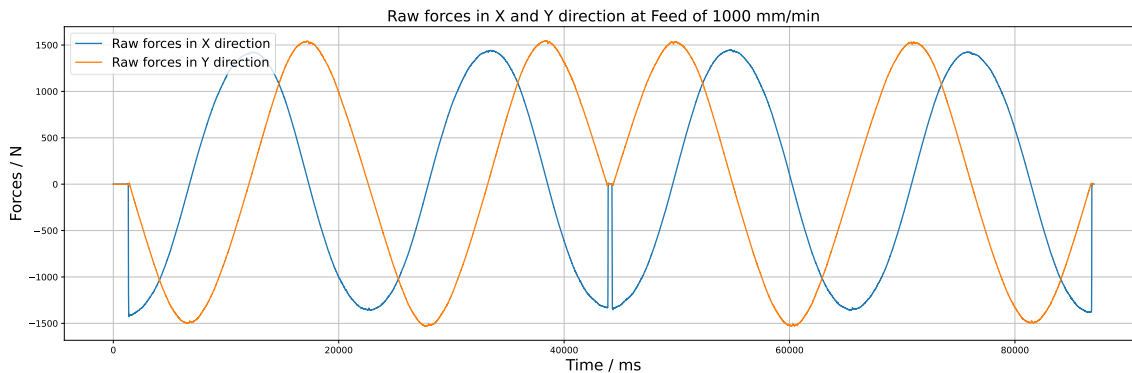


Figure C-7: Raw forces exported from dynamometer under 1000 mm/min feed rates

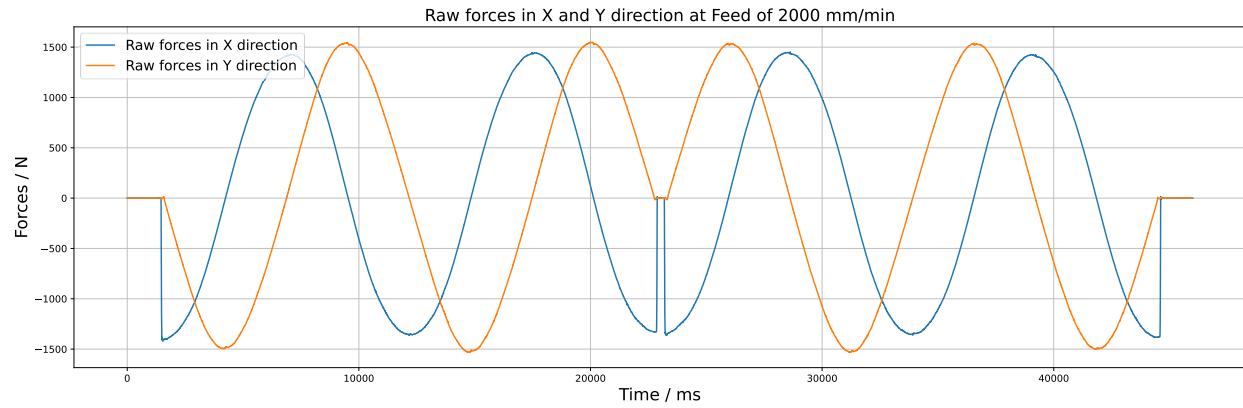


Figure C-8: Raw forces exported from dynamometer under 2000 mm/min feed rates

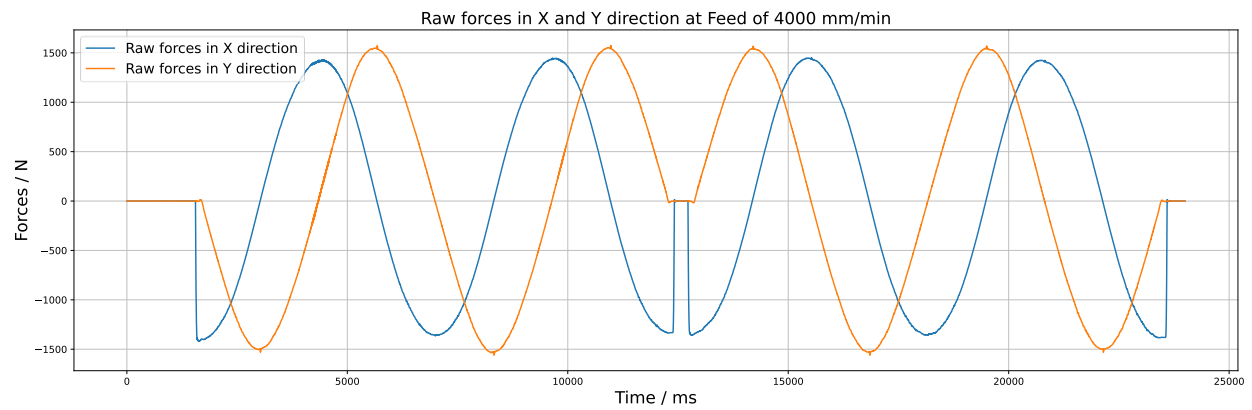


Figure C-9: Raw forces exported from dynamometer under 4000 mm/min feed rates

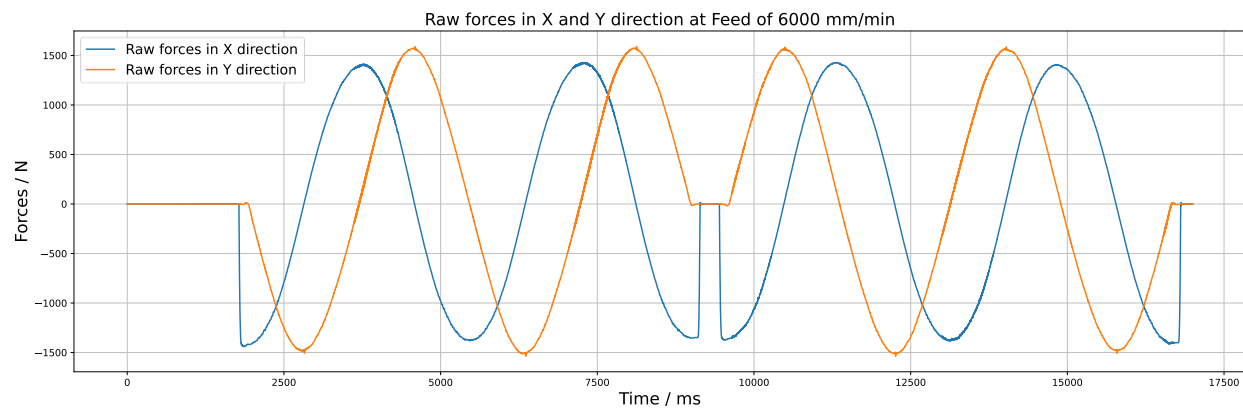


Figure C-10: Raw forces exported from dynamometer under 6000 mm/min feed rates

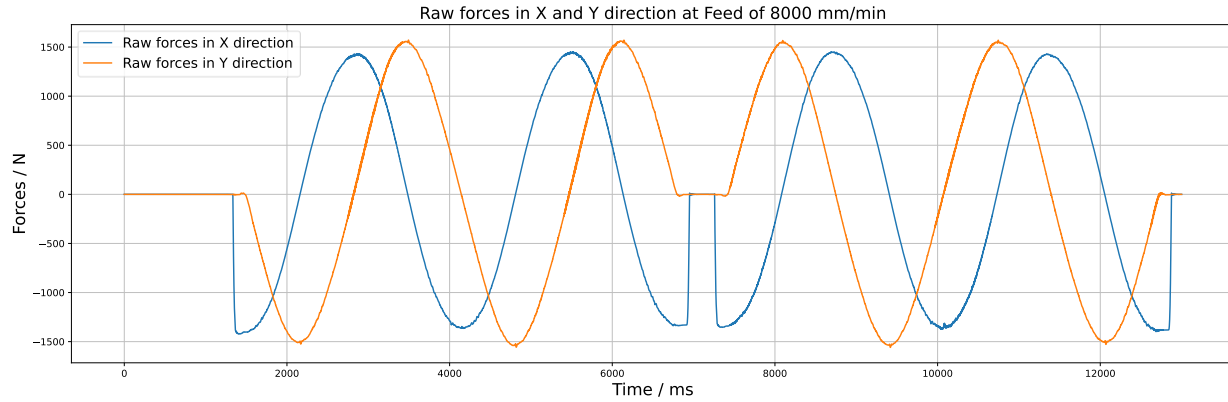


Figure C-11: Raw forces exported from dynamometer under 8000 mm/min feed rates

4. Dynamic test data plots after data matching

2 CWs and 2 CCWs circular runs were conducted in the dynamic test under different feed rates. On the machine encoder side, the data before π and after 3π angular positions in the CW and CCW motion were discarded to obtain only the data in the stable running stage of an entire circular run. The angular position at π was reselected as the new reference starting point. The relative angular positions to the reference points are 0 to 2π for CW and CCW motion in this case. The data kept in CCW motion were reversed and compared with those kept in CW motion to see the hysteresis between CW and CCW. Afterward, the dynamometer data was matched with the data from the machine encoder for the two separate runs in the CW and CCW directions. The data after matching is plotted below. The matching strategy could be considered proper since there is no significant difference between raw data and data after matching.

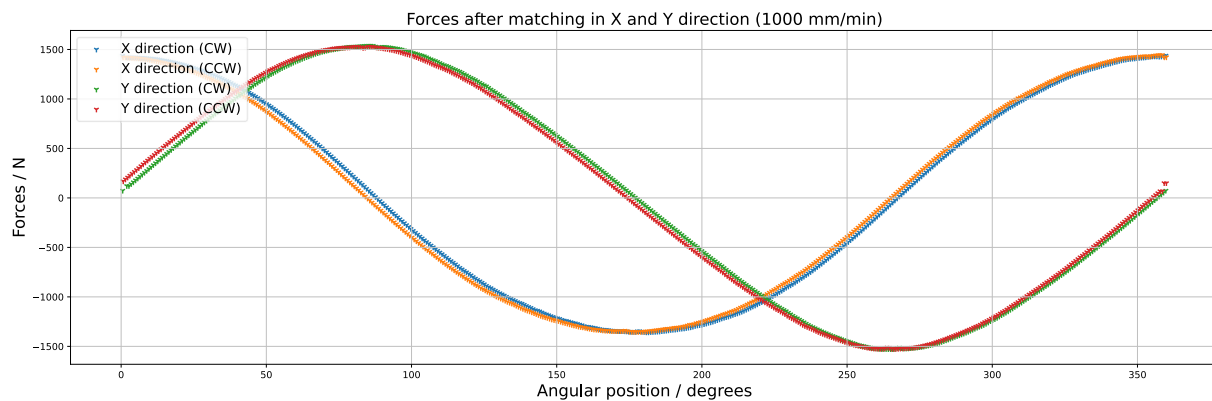


Figure C-12: Force plots after matching under 1000 mm/min feed rates

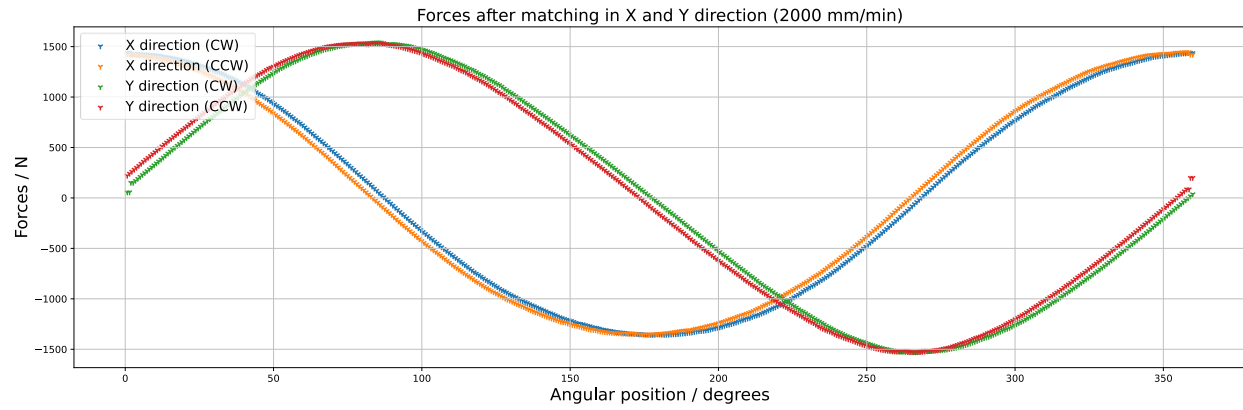


Figure C-13: Force plots after matching under 2000 mm/min feed rates

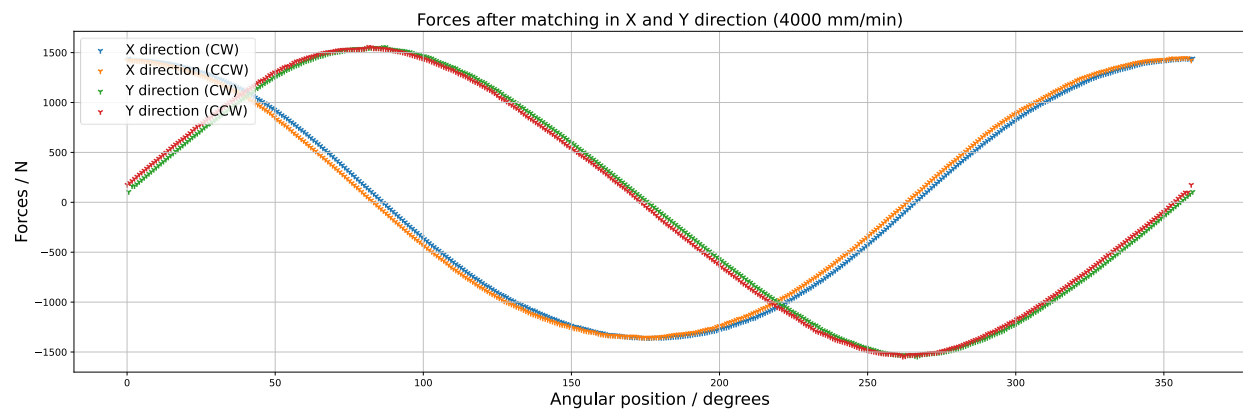


Figure C-14: Force plots after matching under 4000 mm/min feed rates

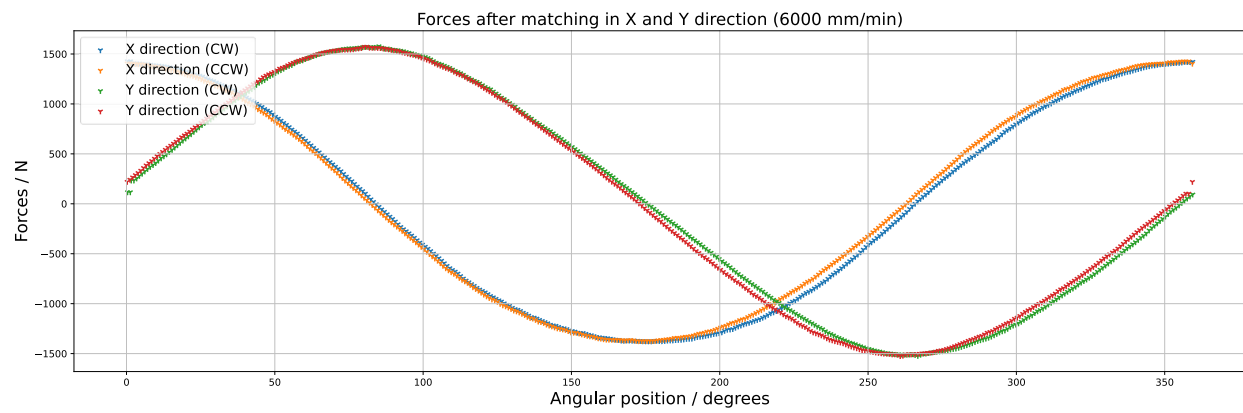


Figure C-15: Force plots after matching under 6000 mm/min feed rates

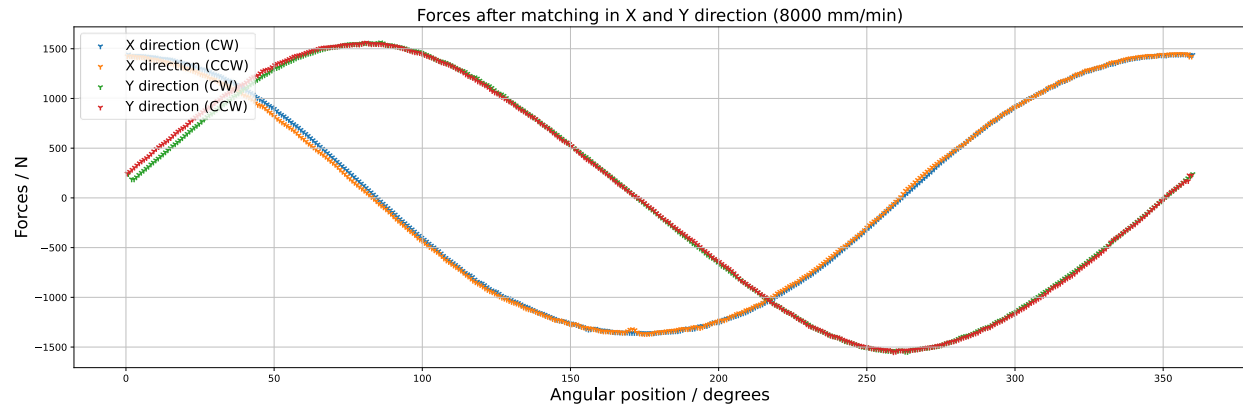


Figure C-16: Force plots after matching under 8000 mm/min feed rates

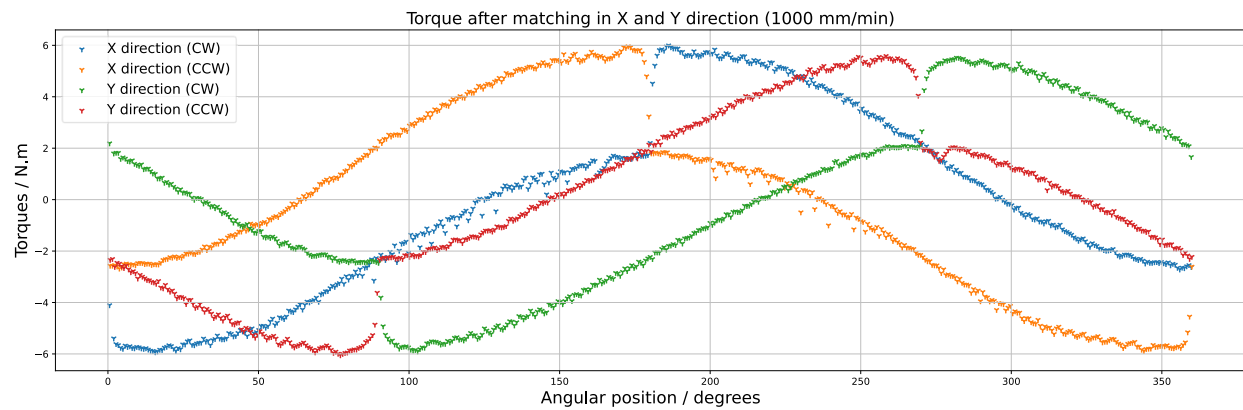


Figure C-17: Torques after matching under 1000 mm/min feed rates

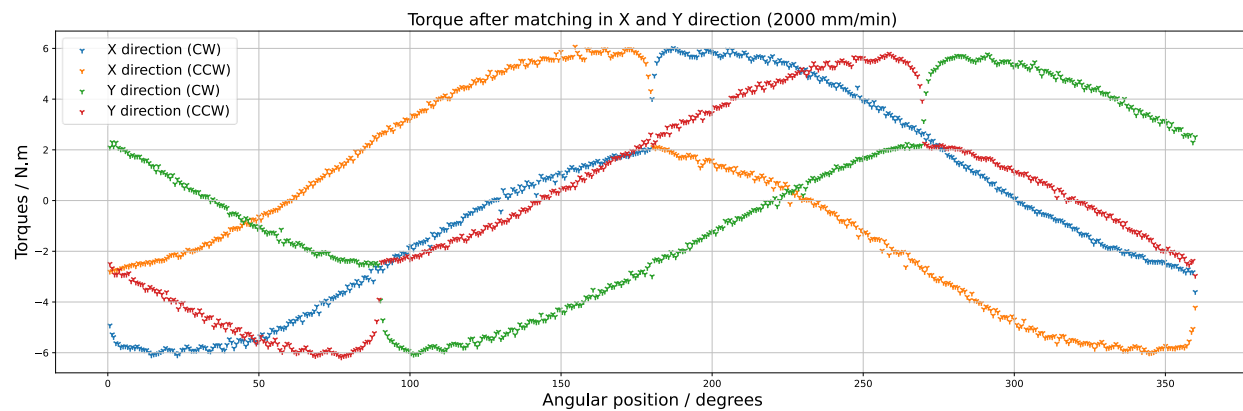


Figure C-18: Torques after matching under 2000 mm/min feed rates

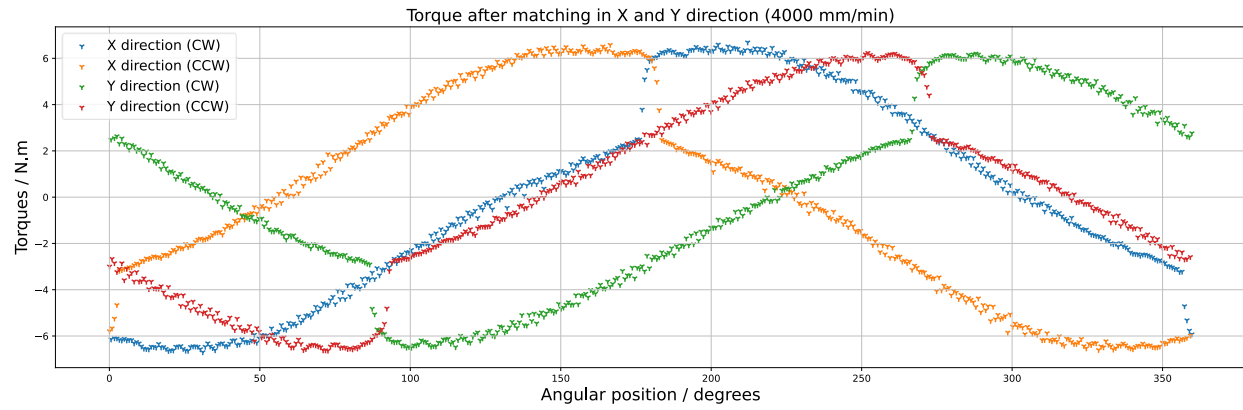


Figure C-19: Torques after matching under 4000 mm/min feed rates

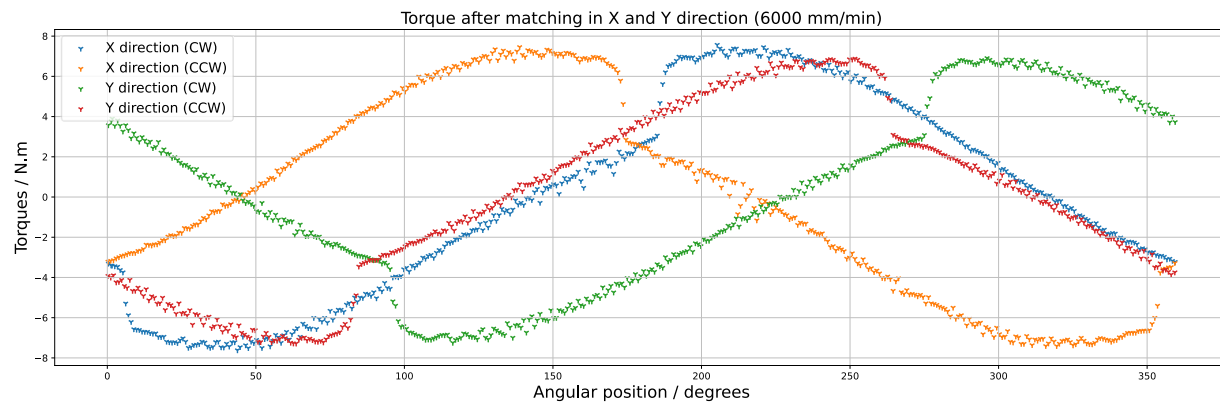


Figure C-20: Torques after matching under 6000 mm/min feed rates

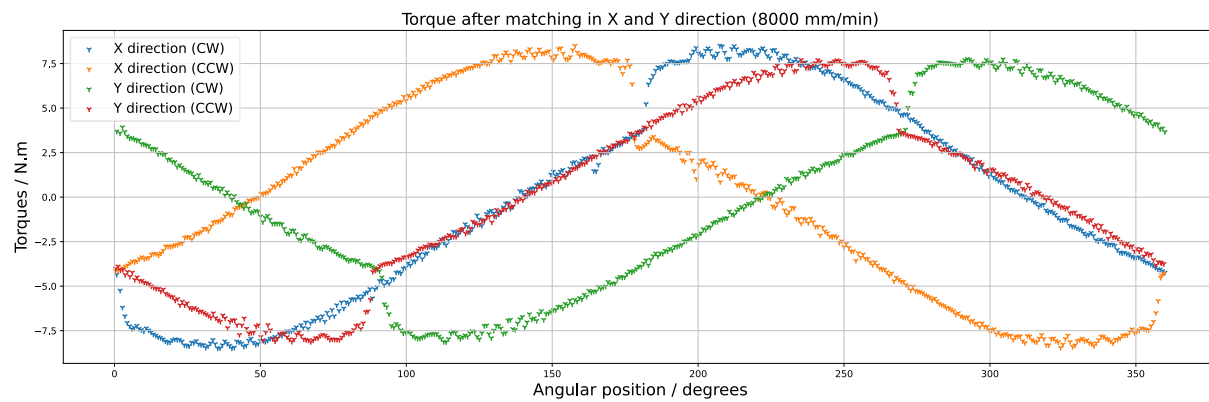


Figure C-21: Torques after matching under 8000 mm/min feed rates