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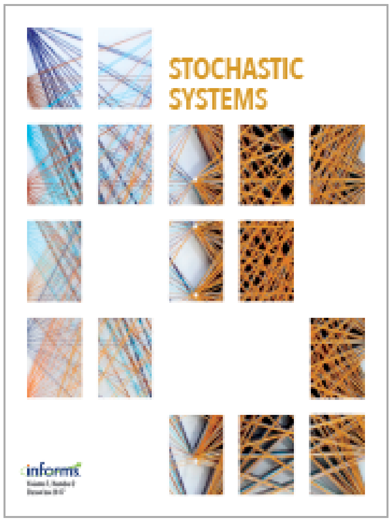

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Optimally Scheduling Public Safety Power Shutoffs

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Abstract. In an effort to reduce power system-caused wildfires, utilities carry out public safety power shutoffs (PSPSs), in which portions of the grid are deenergized to mitigate the risk of ignition. The decision to call a PSPS must balance reducing ignition risks and the negative impact of service interruptions. In this work, we consider three PSPS scheduling scenarios, which we model as dynamic programs. In the first two scenarios, we assume that N PSPSs are budgeted as part of the investment strategy. In the first scenario, a penalty is incurred for each PSPS declared past the Nth event. In the second, we assume that some costs can be recovered if the number of PSPSs is below N while still being subject to a penalty if above N. In the third, the system operator wants to minimize the number of PSPSs such that the total expected cost is below a threshold. We provide optimal or asymptotically optimal policies for each case, the first two of which have closed-form expressions. Lastly, we establish the applicability of the first PSPS model's policy to critical peak pricing and obtain an optimal scheduling policy to reduce the peak demand based on weather observations.

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Keywords: dynamic programming • public safety power shutoffs • optimal policy • wildfires

1. Introduction

Electric power systems have caused a number of recent wildfires: for example, in California, United States (2017 and 2018); in Texas, United States (2011); and in Victoria, Australia (2009) (Teague et al. 2010, Russell et al. 2012, Jazebi et al. 2019, Jeffery et al. 2019, Abatzoglou et al. 2020, Rhodes et al. 2020). The consequences of these events are exacerbated by the extreme weather conditions during which they often occur (Miller et al. 2017, Keeley and Syphard 2018). To mitigate fires, utilities like Northern California's Pacific Gas & Electricity (PG&E) have implemented precautionary grid deenergization events (i.e., intentional load shedding) (Abatzoglou et al. 2020, Pacific Gas & Electricity 2020b, Rhodes et al. 2020). These events are referred to as public safety power shutoffs (PSPSs) by PG&E and in the literature. However, deenergizing part of the grid has important consequences on the affected areas (e.g., temporary business and school closures, medical baseline customers, and loss of revenue) (Abatzoglou et al. 2020, Rhodes et al. 2020, Pacific Gas & Electricity 2022).

In this work, we formulate dynamic programs for PSPS scheduling in a specific geographic area of a power grid. The models use observations of natural phenomena to balance estimated wildfire risks with the cost of deenergization. Our models assume that initial mitigation investments are made (for example, tree trimming, better line insulation, and undergrounding of lines) (Pacific Gas & Electricity 2021). We study the following three scenarios.

1. N PSPSs are budgeted. We assume that the system operator is allowed to exceed the budget but is penalized for each extra PSPS (e.g., because of contractual agreements regarding equipment rental and associated extension costs and because of fixed on-the-ground staff expenditures) (Section 2.1).

2. The system operators can recover part of their incurred costs if the number of PSPSs is below the budget. For example, community resource centers, which are used to support the population during outages, do not need to be deployed; costs of personnel salary and food are avoided; and portable backup generators or hotel ticket vouchers do not need to be provided to medical baseline consumers (Pacific Gas & Electricity 2020a) (Section 2.2).

3. The system operator minimizes the number of PSPSs such that the total expected cost is below a threshold that depends on the wildfire mitigation strategy investment. This scenario applies to, for example, jurisdictions where the system operator is not able to forecast needs in term of PSPSs (e.g., because of limited experience in regions having not faced power line-caused wildfires in the recent past) (Section 2.3).

For each case, we provide the optimal scheduling policy or an asymptotically equivalent form of it. Next, we leverage an intermediary result from Scenario 1's treatment to optimally solve the problem of critical peak pricing (CPP) (Siano 2014). Under some of Scenario 1's conditions (viz., a high penalty for extra PSPSs and an asymptotically infinite scheduling period with a fixed ratio of budget to time horizon), both problems exhibit the same structure. For this reason, both are covered here. In Section 4, we adapt the PSPS policy to CPP, and in Section 5.2, we implement the CPP policy in a numerical example.

In the CPP paradigm, loads enrolled see very high electricity prices when the utility needs to reduce demand (e.g., because of extreme hot or cold weather) and pay a discounted rate at other times (Herter 2007, Chen et al. 2013). CPP events are called a day ahead by the utility (Vardakas et al. 2014). The total number of CPP events must not exceed a contracted limit, which we show is equivalent to the first PSPS scenario's relaxed model. We use its optimal policy for scheduling CPP events based on natural phenomenon observations.

Scheduling PSPSs is a topic of relatively recent interest. For example, PG&E only started using large-scale PSPSs in 2019 (Abatzoglou et al. 2020). In their work, Rhodes et al. (2020) formulated an optimal PSPS scheduling problem, in which the wildfire risks are balanced with the costs of power outages. In Astudillo et al. (2022), a multiperiod optimal PSPS scheduling problem was formulated as a deterministic mixed-integer linear program. Energy storage was considered in their formulation to increase grid reliability when subject to PSPSs. In Rhodes and Roald (2022), the authors used a rolling horizon formulation to include new information about wildfires. The cost of line reenergization was also considered in Rhodes and Roald (2022). Fairness considerations were added in the Kody et al. (2022b) PSPS scheduling formulation, which is expressed as a mixed-integer linear model predictive control problem. Learning-based approaches were used in Umunnakwe et al. (2022) to estimate wildfire risks, the outputs of which were then used to plan preventive line outages or load shedding events. Hong et al. (2022) developed a data-driven decision-making method for efficiently planning load shedding and PSPS events based on a data set generated from optimal power flow computations. In a related work, Kody et al. (2022a) formulated an optimal infrastructure investment strategy to reduce both wildfire risks and PSPS impacts. Our work differs from that in that the decision making is sequential and natural phenomena are modeled as stochastic processes.

We now review the literature on CPP scheduling. Joo et al. (2007) combines market price prediction and swing options to call CPPs in a way that maximizes the profit of the system operator. In Zhang et al. (2009), load price elasticity is used to balance utility profits and load costs when calling CPP. CPP scheduling was framed as a dynamic program in Chen et al. (2013), and threshold policies for different types of CPPs were proposed. Our work differs from Chen et al. (2013) in that multiple natural phenomena can be considered in the decision-making process. We also prove that the threshold policy is optimal for fixed-period CPP, which was not done in Chen et al. (2013). However, we remark that Chen et al. (2013) can accommodate a number of variants that our approach cannot (e.g., variable peak pricing, variable-period CPP, and multiple-group CPP).

We obtain our optimal policies by adapting dynamic programming-based techniques for the multiple secretary problem (Kleinberg 2005, Babaioff et al. 2007, Arlotto and Gurvich 2019). This problem is an extension of the secretary problem first published in a scientific journal by Lindley (1961), in which the $N \geq 2$ best candidates with random abilities must be selected. Freeman (1983) and Ferguson (1989) provide detailed surveys of the secretary problem.

The remainder of the paper is as follows. The PSPS scheduling problem is introduced in Section 2. Our three models and their respective policies are presented in Sections 2.1–2.3. We provide detailed and original derivations of Scenarios 1 and 2 policies in Sections 3.1 and 3.2 and leverage the results of Chen and Blankenship (2004) and Chen and Feinberg (2007) to provide an optimal policy for Scenario 3 in Section 3.3. In Section 4, we apply the results of Section 2.1 to CPP. Section 5 presents numerical simulations for the first two PSPSs and the CPP models. We conclude in Section 6.

2. PSPS Scenarios

We now present our novel PSPS scheduling models and their respective optimal or asymptotically optimal policies. We consider a sequence of days indexed by $t = 1, 2, \ldots, T$. Let $C_I > 0$ be the initial investment in infrastructure upgrades made to reduce wildfire risks. We consider a single geographic area prone to power line-caused wildfires (e.g., supplied by aging or hardly accessible transmission lines). Let $u_t = 1$ be the decision taken on day t to call for a PSPS in this region for the next day (i.e., at $t+1$). Let $u_t = 0$ be the complementary decision to operate the grid normally for the next day. Let $\mathbf{u} = (u_1, u_2, \ldots, u_T)^\top \in \{0,1\}^T$. For the first two scenarios, we let $0 \le k < N$ be the remaining PSPS budget at round t:

$$k = N - \sum_{i=0}^{t-1} u_i.$$

Let $\mathbf{X}_t \in \mathbb{R}^n$ be a random vector made of observations and measurements from $n \in \mathbb{N}$ different natural phenomena and other factors impacting wildfire ignitions. The variable $\mathbf{X}_t$ is observed at the end of day t. The entries of $\mathbf{X}_t$ can represent factors such as ambient temperature or wind speed (Pacific Gas & Electricity 2020a). We assume that $\mathbf{X}_t$ is a discrete-time, finite-state Markov process with known transition probabilities. Let $\mathcal{X} \subset \mathbb{R}^n$ be the state space of $\mathbf{X}_t$. Let $\mathbf{P} \in \mathbb{R}^{n \times n}$ be the transition matrix. We denote the probability of moving from state $\mathbf{y} \in \mathcal{X}$ to $\mathbf{x} \in \mathcal{X}$ by $\Pr[\mathbf{X}_{t+1} = \mathbf{x} | \mathbf{X}_t = \mathbf{y}]$.

For $t = 1, 2, \ldots, T$, let $f_t : \mathbb{R}^n \longmapsto \{0,1\}$ be a function mapping $\mathbf{X}_t$ to a binary output indicating the high risk ($f_t(\mathbf{X}_t) = 1$) or low risk ($f_t(\mathbf{X}_t) = 0$) of ignition given that the lines are energized during day t. The function f_t is assumed to be deterministic, known, and set according to the system operator needs and knowledge of its geographical area and infrastructure. For example, f_t could take the form

$$f_t(\mathbf{X}_t) = \begin{cases} 1, & \text{if } \mathbf{X}_t(i) \ge \mathbf{\Delta}_t(i),\ i = 1, 2, \ldots, n \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $\mathbf{\Delta}_t(i)$ is the risk threshold for observation i. This function is inspired by PG&E's "minimum fire conditions" (Pacific Gas & Electricity 2020b). For example, PG&E uses thresholds like 40 km/h for sustained wind speed, 72 km/h for wind gust, and an air humidity below 20% (Pacific Gas & Electricity 2020a). Hence, if the entries of $\mathbf{X}_t$ are greater than or equal to these values for the wind speed and gust and less than or equal to this value for the humidity, then $f_t(\mathbf{X}_t) = 1$.

Let $A_t > 0$ be the estimated cost of power line-caused wildfires at time t. This represents loss of life, material damage, reconstruction, repair, and service interruption. The power line-induced wildfire damage cost at t is then given by $A_t f_t(\mathbf{X}_t)$ (i.e., the estimated cost weighted by the risk indicator function). Let $a_t > 0$ be the revenue loss of the utility when deenergizing the grid at time t. Let $s_{1,t} > 0$ and $s_{2,t} > 0$ be the costs of deenergizing and reenergizing the grid, respectively. The scalar coefficient $s_{1,t}$ includes, for example, the costs of crews and special equipment dispatched to deenergize the grid. The scalar coefficient $s_{2,t}$ also includes the costs related to the personnel on the field in addition to the cost of specialized equipment required to inspect the network before reenergization (Pacific Gas & Electricity 2020b). The deenergization costs can be modeled as $\max\{s_{1,t+1}(u_t - u_{t-1}), 0\}$ (i.e., $s_{1,t+1}$ is incurred only if a PSPS is called ($u_t = 1$) when the grid was operating nominally ($u_{t-1} = 0$) before). Similarly, reenergization costs can be expressed as $\max\{s_{2,t+1}(u_{t-1} - u_t), 0\}$ (i.e., are nonzero only if the ongoing PSPS is ended ($u_t = 0$ and $u_{t-1} = 1$)). Observing that we can either deenergize or reenergize the grid but not both at same time, the total de-reenergization costs can be written as a single term:

$$\max\{s_{1,t+1}(u_t - u_{t-1}), 0\} + \max\{s_{2,t+1}(u_{t-1} - u_t), 0\} \equiv \max\{s_{1,t+1}(u_t - u_{t-1}), s_{2,t+1}(u_{t-1} - u_t)\}.$$

We assume that the grid is energized before and after the time range (i.e., $u_0 = u_{T+1} = 0$).

2.1. Scenario 1: Cost Penalty for Additional PSPSs

We assume that the wildfire mitigation strategy does not strictly rely on infrastructure upgrades and that PSPSs can be scheduled to reduce the strategy's total costs, C_I. In the first two scenarios, we assume that C_I is decomposed into two components: (i) a reduced budget for infrastructure upgrades and PSPS-induced financial losses when N PSPSs have been planned for, which we denote $\overline{C}_I(N) < C_I$, and (ii) the operational and/or damage costs associated with calling a PSPS or not at each day. We can use, for example, the value of lost load (VoLL) (Kariuki and Allan 1996, Willis and Garrod 1997, Ratha et al. 2013) for a day to model the financial losses from the former component.

In Scenario 1, we assume that additional costs are incurred for each PSPS after the budget, N, has been reached. These costs represent, for example, the value of lost load per extra day without power and the cost of personnel and equipment contract extensions. These costs are not recovered if the number of PSPSs is below N because of contracted obligations (e.g., surveillance personnel and hired equipment, like helicopters, for post-PSPS inspections) (Pacific Gas & Electricity 2021). Let $\gamma > 0$ be the cost penalty per extra power shutoff. We model the additional costs as $\gamma \max\left\{0, \sum_{t=1}^{T} u_t - N\right\}$.

The total cost with penalty for Scenario 1 is:

$$c_{\mathrm{p}}(N, \mathbf{u}) = \sum_{t=1}^{T} \left(a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t - u_{t-1}), s_{2,t+1}(u_{t-1} - u_t)\}\right) + \gamma \max\left\{0, \sum_{t=1}^{T} u_t - N\right\} + \overline{C}_I(N). \tag{2}$$

Our objective is to minimize the expectation of $c_{\mathrm{p}}(N, \mathbf{u})$ given $\overline{C}_I(N)$. We herein omit $\overline{C}_I(N)$ in both Scenarios 1 and 2 objective functions because it does not influence their optima. The PSPS scheduling problem is expressed as the following problem, which we later rewrite as a dynamic program:

$$\begin{aligned} \min_{\substack{u_t \\ t=1,2,\ldots,T}} \quad & \mathbb{E}[c_{\mathrm{p}}(N, \mathbf{u})] \\ \text{subject to} \quad & u_t \in \{0,1\} \\ & u_0 = u_{T+1} = 0, \end{aligned} \tag{3}$$

and note that the expectation is taken with respect to the random variables $\mathbf{X}_t$, $t = 1, 2, \ldots, T$.

Problem (3) is approximately solved by the following closed-form, easily implementable policy. The Proposition 1 policy is optimal for an asymptotically equivalent form of Problem (3). This will be shown in Section 3.1.2.

Proposition 1. *Consider Problem* (3). *Let $d = T - t$ and the current day's observations be $\mathbf{X}_d = \mathbf{x}$. Then, $u_d = 1$ if*

$$\mathbb{E}[f_{d-1}(\mathbf{X}_{d-1}) | \mathbf{X}_d = \mathbf{x}] \geq \frac{1}{A_{d-1}}(g_{d-1}(k-1|1,\mathbf{x}) - g_{d-1}(k|0,\mathbf{x}) + a_{d-1} + (1-u_{d+1})s_{1,d-1} - u_{d+1}s_{2,d-1}),$$

where

$$\begin{aligned} g_d(k|u_{d+1} = u, \mathbf{X}_{d+1} = \mathbf{x}) = \sum_{\boldsymbol{\xi} \in \mathcal{X}} \min\{& g_{d-1}(k|0,\boldsymbol{\xi}) + \mathbb{E}[A_{d-1}f_{d-1}(\mathbf{X}_{d-1})|\mathbf{X}_d = \boldsymbol{\xi}] + us_{2,d-1}, \\ & g_{d-1}(k-1|1,\boldsymbol{\xi}) + a_{d-1} + (1-u)s_{1,d-1}\}\Pr[\mathbf{X}_d = \boldsymbol{\xi}|\mathbf{X}_{d+1} = \mathbf{x}], \end{aligned} \tag{4}$$

with

$$g_0(k|u,\mathbf{x}) = us_{2,T+1} + \sum_{\boldsymbol{\xi} \in \mathcal{X}} \mathbb{E}[A_{T+1}f_{T+1}(\mathbf{X}_{T+1})|\mathbf{X}_T = \boldsymbol{\xi}]\Pr[\mathbf{X}_T = \boldsymbol{\xi}|\mathbf{X}_{T-1} = \mathbf{x}] \tag{5}$$

$$g_d(0|u,\mathbf{x}) = us_{2,d-1} + \sum_{i=0}^{d} \sum_{\boldsymbol{\xi} \in \mathcal{X}} \mathbb{E}[A_{T-i+1}f_{T-i+1}(\mathbf{X}_{T-i+1})|\mathbf{X}_{T-i} = \boldsymbol{\xi}]\mathbf{P}^{(i+1)}_{\mathbf{x},\boldsymbol{\xi}}, \tag{6}$$

for all $\mathbf{x} \in \mathcal{X}, u \in \{0,1\}, k \geq 1$, and $d \geq 1$ and where $\mathbf{P}^{(i+1)}_{\mathbf{x},\boldsymbol{\xi}} = \Pr[\mathbf{X}_{n+i+1} = \boldsymbol{\xi}|\mathbf{X}_n = \mathbf{x}]$ for any n.

2.2. Scenario 2: Cost Adjustment

In the second scenario, the decision maker is subject to a cost adjustment if the number of shutdowns differs from N. As in Scenario 1, there is a penalty if the number of shutoffs is above N. Conversely, if this number is below N, part of the anticipated value of load lost is recovered because of reduced outages (e.g., because of on-call personnel or avoided need for the extra fuel). We model the adjustment as proportional to the value of lost load for a day, $\lambda > 0$, and express it as $\lambda\left(\sum_{t=1}^{T} u_t - N\right)$.

In Scenario 2, the total cost with adjustment is defined as

$$c_{\mathrm{a}}(N, \mathbf{u}) = \sum_{t=1}^{T} \left(a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t - u_{t-1}), s_{2,t+1}(u_{t-1} - u_t)\}\right) + \lambda\left(\sum_{t=1}^{T} u_t - N\right) + \overline{C}_I(N), \tag{7}$$

where the cost penalty in (2) has been replaced by the cost adjustment. The PSPS scheduling problem with cost adjustment is given by

$$\begin{array}{ll}\min\limits_{\substack{u_t\\ t=1,2,\ldots,T}} & \mathbb{E}[c_{\mathrm{a}}(N,\mathbf{u})]\\ \text{subject to} & u_t \in \{0,1\}\\ & u_0 = u_{T+1} = 0.\end{array} \tag{8}$$

The solution to Problem (8) is given by the following proposition. We later establish Proposition 2's optimality in Section 3.2.

Proposition 2. *Consider Problem* (8). *On day $d = T - t$, after receiving the current observations, $\mathbf{X}_t = \mathbf{x}$, and then, $u_d = 1$ if*

$$\mathbb{E}[f_{d-1}(\mathbf{X}_{d-1})\,|\,\mathbf{X}_d = \mathbf{x}] \geq \frac{1}{A_{d-1}}\left(h_{d-1}(1,\mathbf{x}) - h_{d-1}(0,\mathbf{x}) + a_{d-1} + \lambda + (1-u_{d+1})s_{1,d-1} - u_{d+1}s_{2,d-1}\right), \tag{9}$$

where

$$\begin{aligned}h_d(u_{d+1}=u,\mathbf{X}_{d+1}=\mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \max\{&h_{d-1}(1,\boldsymbol{\xi}) + \mathbb{E}[A_{d-1}f_{d-1}(\mathbf{X}_{d-1})\,|\,\mathbf{X}_d=\boldsymbol{\xi}] + us_{2,d-1},\\ &h_{d-1}(0,\boldsymbol{\xi}) + a_{d-1} + \lambda + (1-u)s_{1,d-1}\}\Pr[\mathbf{X}_d=\boldsymbol{\xi}\,|\,\mathbf{X}_{d+1}=\mathbf{x}],\end{aligned}$$

with $h_0(u,\mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathbf{X}} \mathbb{E}[A_{T+1}f_{T+1}(\mathbf{X}_{T+1})\,|\,\mathbf{X}_T=\boldsymbol{\xi}]\Pr[\mathbf{X}_T=\boldsymbol{\xi}\,|\,\mathbf{X}_{T-1}=\mathbf{x}] + us_{2,T+1}$.

Similarly to the first scenario, Proposition 2 provides a closed-form optimal policy for the PSPS scheduling with cost adjustment problem that can be readily implemented by system operators.

2.3. Scenario 3: Minimum Number of PSPSs

In the third scenario, no budget for PSPSs has been set. Instead, we suppose that the system operator wishes to minimize the expected number of PSPS events such that the expected cost of the whole wildfire prevention strategy is below a threshold, which is itself less than or equal to C_I. Similarly to previous sections, the strategy models both the initial reduced investment and the costs of PSPSs. We suppose that a reduced investment $\tilde{C}_I < C_I$ was made to upgrade the power infrastructure and passively reduce wildfire risks. We let $\overline{\alpha} > 0$ be the difference between (i) the nominal investment C_I and (ii) the initial infrastructure investment $\tilde{C}_I$. The system operator must then schedule PSPSs such that the additional costs incurred are less than or equal to $\overline{\alpha} = C_I - \tilde{C}_I$ and power line-caused wildfire risks are mitigated in a cost-effective manner. We include Scenario 3 for completeness, although it does not admit a readily implementable, closed-form policy. We consider Scenario 3 because it may be of interest when the number of PSPSs is not constrained and difficult to forecast. Although we have not identified a readily implementable, closed-form policy at this time, we provide a value function-based analysis to optimally schedule PSPSs when its number is minimized.

Let $c_{t+1} : \{0,1\} \longmapsto \mathbb{R}$ be the operating cost function (i.e., the cost function without adjustment or penalty terms) for a single day $t+1$:

$$c_{t+1}(u_t) = a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t - u_{t-1}), s_{2,t+1}(u_{t-1} - u_t)\}.$$

The minimum number of PSPS scheduling problems is

$$\begin{array}{ll}\min\limits_{\substack{u_t\\ t=1,2,\ldots,T}} & \mathbb{E}\left[\sum_{t=1}^{T} u_t\right]\\ \text{subject to} & u_t \in \{0,1\}\\ & u_0 = u_{T+1} = 0\\ & \mathbb{E}\left[\sum_{t=1}^{T} c_{t+1}(u_t)\right] \leq \overline{\alpha},\end{array} \tag{10}$$

and the optimal scheduling policy for Problem (10) is presented in Proposition 3.

Proposition 3. *Consider the minimum number of PSPS scheduling Problem* (10), *and assume that it is feasible for $\mathbf{X}_1 = \mathbf{x}$. Let*

$$\alpha_t = \begin{cases}\overline{\alpha}, & \text{if } t = 1\\ \phi_t^{\star}(\mathbf{X}_t = \mathbf{x}), & \text{if } t \geq 2,\end{cases} \tag{11}$$

where $\phi_t^\star(\mathbf{X}_t=\mathbf{x})$ is defined by the following recursion:

$$(u_t^\star,\phi_{t+1}^\star) = \begin{cases} \left(1, \underset{\phi'(\mathbf{x}')\in\Phi_{T-t}(\mathbf{x}')}{\arg\min} \sum_{\mathbf{x}'\in\mathcal{X}} \Pr[\mathbf{X}'_{t+1}=\mathbf{x}'\,|\,\mathbf{X}_t=\mathbf{x}]V_{T-t-1}\left(\mathbf{x}',\phi'(\mathbf{x}')\right)\right), \\ \qquad \textit{if}\;\; \mathbb{E}[c_{t+1}(0)\,|\,\mathbf{X}_t=\mathbf{x}] + \sum_{\mathbf{x}'\in\mathcal{X}} b_{T-t-1}(\mathbf{x}')\Pr[\mathbf{X}'_{t+1}=\mathbf{x}'\,|\,\mathbf{X}_t=\mathbf{x}] > \alpha_t \\ \qquad \textit{and}\; \mathbb{E}[c_{t+1}(1)\,|\,\mathbf{X}_t=\mathbf{x}] + \sum_{\mathbf{x}'\in\mathcal{X}} b_{T-t-1}(\mathbf{x}')\Pr[\mathbf{X}'_{t+1}=\mathbf{x}'\,|\,\mathbf{X}_t=\mathbf{x}] \le \alpha_t \\ \left(0, \underset{\phi'(\mathbf{x}')\in\Phi_{T-t}(\mathbf{x}')}{\arg\min} \sum_{\mathbf{x}'\in\mathcal{X}} \Pr[\mathbf{X}'_{t+1}=\mathbf{x}'\,|\,\mathbf{X}_t=\mathbf{x}]V_{T-t-1}\left(\mathbf{x}',\phi'(\mathbf{x}')\right)\right), \;\; \textit{otherwise}, \end{cases} \tag{12}$$

for $t=1,2,\ldots,T$ and where $\Phi_{T-t}(\mathbf{x},\alpha)$, $b_{T-t}(\mathbf{x},\alpha)$, and V_T are, respectively, defined in (40), (42), *and* (41) *from Section* 3.3.1.

Then, at day t, after receiving the current day's observations, $\mathbf{X}_t=\mathbf{x}$, it is optimal to declare a PSPS for the next day if $u_t^\star=1$.

The derivation of the Proposition 3 policy is given in Section 3.3. We remark that although the problem formulation is of interest, to the authors' best knowledge, it does not admit a closed-form policy. This is a topic for future work.

3. Analysis of Policies

In this section, we provide a comprehensive derivation for each optimal or asymptotically optimal policy.

3.1. Asymptotically Optimal Policy for Scenario 1

We now show that (3) is a relaxation of a modified multiple secretary problem (Arlotto and Gurvich 2019), which has a closed-form optimal policy. Then, we establish the asymptotic exactness of the relaxation under certain conditions in Section 3.1.2.

3.1.1. Problem Reformulation. In this section, we state the problem to which (3) is a relaxation.

We observe that for any $\gamma>0$, (3) can be shown via the Lagrangian to be a relaxation of the following constrained problem:

$$\begin{aligned} \min_{\substack{u_t\\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t-u_{t-1}), s_{2,t+1}(u_{t-1}-u_t)\}\right] \\ \text{subject to} \quad & u_t\in\{0,1\} \\ & u_0=u_{T+1}=0 \\ & \mathbb{E}\left[\sum_{t=1}^{T}u_t\right]\le N. \end{aligned} \tag{13}$$

In (13), the cost penalty for additional PSPSs is now modeled as an expected budget constraint. We also consider (14), in which the expected constraint has been replaced by a deterministic constraint:

$$\begin{aligned} \min_{\substack{u_t\\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t-u_{t-1}), s_{2,t+1}(u_{t-1}-u_t)\}\right] \\ \text{subject to} \quad & u_t\in\{0,1\} \\ & u_0=u_{T+1}=0 \\ & \sum_{t=1}^{T}u_t\le N. \end{aligned} \tag{14}$$

We remark that (13) is a relaxation of (14) because the deterministic constraint implies that the expected one is satisfied. This follows from the fact that (13) entails that the budget constraint holds for all random variable sequence $\{\mathbf{X}_t\}_{t=1}^T$ outcomes. This differs from (14), which can allow a PSPS budget above or below N for some sequences as long as the budget constraint holds in the expected sense over all random variable sequences. We use the law of iterated expectations (Ross 2014, proposition 5.1) to rewrite the objective of (14) as

$$\begin{aligned}\min_{\substack{u_t\\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T}\mathbb{E}[a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \max\{s_{1,t+1}(u_t-u_{t-1}), s_{2,t+1}(u_{t-1}-u_t)\}\,|\,\mathbf{X}_t]\right]\\ \text{subject to} \quad & u_t \in \{0,1\}\\ & u_0 = u_{T+1} = 0\\ & \sum_{t=1}^{T} u_t \le N.\end{aligned} \tag{15}$$

We rewrite (15) as an equivalent dynamic program as in Arlotto and Gurvich (2019). Let $w \in \mathbb{R}$ be the cumulative cost incurred as of round t; that is,

$$w = \sum_{i=1}^{t-1} a_{i+1}u_i + A_{i+1}f_{i+1}(\mathbf{x}_{i+1})(1-u_i) + \max\{s_{2,i+1}(u_{i-1}-u_i), s_{1,i+1}(u_i-u_{i-1})\},$$

where $\mathbf{x}_i$ is the realization of $\mathbf{X}_i$ at round i. Let $d = T-t$ be the number of remaining days before the end of the PSPS program. To facilitate the analysis of the problem in its dynamic programing form, for the remainder of this subsection, all subscripts refer to the remaining number of decisions d (e.g., $d=0$ or $d=-1$ is equivalent to $t=T$ and $t=T+1$, respectively). Then, given decision u_{d+1} and observations $\mathbf{x}_{d+1}$ (i.e., the decision and observations from the previous round), the value function is

$$\begin{aligned}v_d(w,k\,|\,u,\mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{&v_{d-1}\big(w + \mathbb{E}[A_{d-1}f(\mathbf{X}_{d-1})\,|\,\mathbf{X}_d=\boldsymbol{\xi}] + us_{2,d-1}, k\,|\,0,\boldsymbol{\xi}\big),\\ &v_{d-1}\big(w + a_{d-1} + (1-u)s_{1,d-1}, k-1\,|\,1,\boldsymbol{\xi}\big)\}\Pr[\mathbf{X}_d=\boldsymbol{\xi}\,|\,\mathbf{X}_{d+1}=\mathbf{x}],\end{aligned} \tag{16}$$

with the boundary conditions at $d=0$ and $d-1=-1$:

$$v_0(w,k\,|\,u,\mathbf{x}) = w + \mathbb{E}[A_{-1}f(\mathbf{X}_{-1})\,|\,\mathbf{X}_0=\boldsymbol{\xi}_j] + us_{2,-1}, \tag{17}$$

for all $w\in\mathbb{R}, k=1,2,\ldots,N, \mathbf{x}\in\mathbf{X}, u\in\{0,1\}$, and $d=1,2,\ldots,T$, and

$$v_d(w,0\,|\,u,\mathbf{x}) = w + \sum_{i=0}^{d}\sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[A_{i-1}f_{i-1}(\mathbf{X}_{i-1})\,|\,\mathbf{X}_i=\boldsymbol{\xi}]\mathbf{P}^{(i+1)}_{\mathbf{x},\boldsymbol{\xi}} + us_{2,d-1}, \tag{18}$$

for all $w\in\mathbb{R}, \mathbf{x}\in\mathbf{X}, u\in\{0,1\}$, and $d=1,2,\ldots,T$.

The dynamic program (16)–(18) admits a closed-form, easily implementable optimal policy. We then show that Problem (14), which is equivalent to (16)–(18), is asymptotically equivalent to the original Problem (3) under certain conditions. Problems (16)–(18) can be interpreted as an instance of the multiple secretary problem (Arlotto and Gurvich 2019) with the addition of a switching cost and a regularizer. The switching cost further adds a time-dependent constraint to the problem. This will lead to more conservative decision making in general but to a more sensitive policy immediately following a PSPS. The regularizer takes the form of an offset, which penalizes the worst decision that can be taken (i.e., selecting a day with low expected wildfire risks). It can be interpreted as a sparsity regularizer. Consequently, it may lead to decision sequences for which the budget may not be fully used, which was not considered in prior multiple secretary formulations. We adapt the results (Arlotto and Gurvich 2019, appendix C) to include switching costs and the regularizer and obtain the optimal policy for (16)–(18), as shown in Proposition 1.

Proof of Proposition 1. Let $e_{d-1}(\boldsymbol{\xi}) = \mathbb{E}[A_{d-1}f_{d-1}(\mathbf{X}_{d-1})\,|\,\mathbf{X}_d=\boldsymbol{\xi}]$ and $p_{\mathbf{x}}(\boldsymbol{\xi}) = \Pr[\mathbf{X}_d=\boldsymbol{\xi}\,|\,\mathbf{X}_{d+1}=\mathbf{x}]$ to simplify notation. We first show that

$$v_d(w,k\,|\,u,\mathbf{x}) = w + g_d(k\,|\,u,\mathbf{x}), \tag{19}$$

for $d=0,1,\ldots,T$. We proceed by induction. In the base case $d=0$, (19) holds trivially because of the boundary conditions, and for $d=1$, we have two instances: $k=0$ and $k\ge 1$. For $k=0$,

$$v_1(w,0\,|\,u,\mathbf{x}) = w + \sum_{\boldsymbol{\xi}\in\mathbf{X}} \mathbb{E}[A_0f_0(\mathbf{X}_0)\,|\,\mathbf{X}_1=\boldsymbol{\xi}]\Pr[\mathbf{X}_1=\boldsymbol{\xi}\,|\,\mathbf{X}_2=\mathbf{x}] + us_{2,0}. \tag{20}$$

For $k \geq 1$,

$$\begin{aligned} v_1(w,k|u,\mathbf{x}) &= \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{v_0\big(w+e_0(\boldsymbol{\xi})+us_{2,0},k|0,\boldsymbol{\xi}\big), v_0\big(w+a_0+(1-u)s_{1,0},k-1|1,\boldsymbol{\xi}\big)\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{w+e_0(\boldsymbol{\xi})+us_{2,0}, w+a_0+(1-u)s_{1,0}+s_{2,-1}\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= w + \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{e_0(\boldsymbol{\xi})+us_{2,0}, a_0+(1-u)s_{1,0}+s_{1,-1}\}p_{\mathbf{x}}(\boldsymbol{\xi}), \end{aligned} \tag{21}$$

where the second equality follows from the boundary condition (17). Similarly, for $g_1(k|u,\mathbf{x})$, we have for $k = 0$ that

$$g_1(0|u,\mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathbf{X}} \mathbb{E}\,[A_0 f_0(\mathbf{X}_0)|\mathbf{X}_1 = \boldsymbol{\xi}]\Pr[\mathbf{X}_1 = \boldsymbol{\xi}|\mathbf{X}_2 = \mathbf{x}] + us_{2,0}. \tag{22}$$

For $k \geq 1$, we have

$$\begin{aligned} g_1(k|u,\mathbf{x}) &= \sum_{\boldsymbol{\xi}\in\mathcal{X}} \max\{g_0(k|0,\boldsymbol{\xi})+e_0(\boldsymbol{\xi})+us_{2,0}, g_0(k-1|1,\boldsymbol{\xi})+a_0+(1-u)s_{1,0}\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= \sum_{\boldsymbol{\xi}\in\mathcal{X}} \max\{e_0(\boldsymbol{\xi})+us_{2,0}, a_0+(1-u)s_{1,0}+s_{2,-1}\}p_{\mathbf{x}}(\boldsymbol{\xi}), \end{aligned} \tag{23}$$

where we last used the boundary conditions (5). Using (22) in (20) and (21) in (23), we obtain

$$v_1(w,k|u,\mathbf{x}) = w + g_1(k|u,\mathbf{x}),$$

which establishes the base case. Now, let $d \longrightarrow d+1$. Then, for $k = 0$, by definition we have

$$v_{d+1}(w,0|u,\mathbf{x}) = w + \sum_{i=0}^{d+1}\sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[A_{i-1}f_{i-1}(\mathbf{X}_{i-1})|\mathbf{X}_i = \boldsymbol{\xi}]\mathbf{P}^{(i+1)}_{\mathbf{x},\boldsymbol{\xi}} + us_{2,d} \tag{24}$$

$$g_{d+1}(0|u,\mathbf{x}) = \sum_{i=0}^{d}\sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[A_{i-1}f_{i-1}(\mathbf{X}_{i-1})|\mathbf{X}_i = \boldsymbol{\xi}]\mathbf{P}^{(i+1)}_{\mathbf{x},\boldsymbol{\xi}} + us_{2,d}. \tag{25}$$

For $k \geq 1$, we obtain

$$\begin{aligned} v_{d+1}(w,k|u,\mathbf{x}) &= \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{v_d\big(w+e_d(\boldsymbol{\xi})+us_{2,d},k|0,\boldsymbol{\xi}\big), v_d\big(w+a_d+(1-u)s_{1,d},k-1|1,\boldsymbol{\xi}\big)\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= w + \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{v_d\big(w+e_d(j)+us_{2,d},k|0,\boldsymbol{\xi}\big)+e_d(\boldsymbol{\xi})+us_{2,d}-w-e_d(\boldsymbol{\xi})-us_{2,d}, \\ &\qquad v_d\big(w+a_d+(1-u)s_{1,d},k-1|1,\boldsymbol{\xi}\big)+a_d+(1-u)s_{1,d}-w-a_d-(1-u)s_{1,d}\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= w + \sum_{j\in\mathcal{X}} \min\{g_d(k|0,\boldsymbol{\xi})+e_d(\boldsymbol{\xi})+us_{2,d}, g_d(k-1|1,\boldsymbol{\xi})+a_d+(1-u)s_{1,d}\}p_{\mathbf{x}}(\boldsymbol{\xi}) \\ &= w + g_{d+1}(k|u,\mathbf{x}), \end{aligned}$$

where we used the induction hypothesis (19) and the definition of g given in (4) to obtain the third and final equalities, respectively. This completes the induction proof, and we have established (19) for $d = 0,1,\ldots,T$.

We now derive an optimal policy using (4). The structure of $v_d(w,k|u_{d+1},\mathbf{X}_{d+1})$ implies that $u_d = 1$ if and only if

$$\begin{aligned} &v_{d-1}\big(w+\mathbb{E}[A_{d-1}f(\mathbf{X}_{d-1})|\mathbf{X}_d = \mathbf{x}]+u_{d+1}s_{2,d-1},k|0,\mathbf{x}\big) \\ &\quad \geq v_{d-1}\big(w+a_{d-1}+(1-u_{d+1})s_{1,d-1},k-1|1,\mathbf{x}\big). \end{aligned}$$

Substituting (19) yields

$$g_{d-1}(k|0,\mathbf{x}) + w + \mathbb{E}[A_{d-1}f(\mathbf{X}_{d-1})|\mathbf{X}_d = \mathbf{x}] + u_{d+1}s_{2,d-1} \geq g_{d-1}(k-1|1,\mathbf{x}) + w + a_{d-1} + (1-u_{d+1})s_{1,d-1}.$$

Thus, we have $u_d = 1$ if and only if

$$\mathbb{E}[f(\mathbf{X}_{d-1})|\mathbf{X}_d = \mathbf{x}] \geq \frac{1}{A_{d-1}}\left(g_{d-1}(k-1|1,\mathbf{x}) - g_{d-1}(k|0,\mathbf{x}) + a_{d-1} + (1-u_{d+1})s_{1,d-1} - u_{d+1}s_{2,d-1}\right),$$

which completes the proof. □

In this policy, the difference $g_{d-1}(k|0,\mathbf{x}) - g_{d-1}(k-1|1,\mathbf{x})$ ensures that a PSPS is scheduled only if the cost of the wildfire at the next round weighted by the expected risk is greater than the expected cost reduction of a PSPS when k have been previously called. This is done to account for the budget constraint. The other terms modify the threshold to account for fixed costs of calling a PSPS. In particular, the switching costs promote consecutive over isolated deenergization events. The regularizer integrates the fact that the optimal strategy may be to call less than N PSPSs. This differs from Arlotto and Gurvich (2019), in which the optimal solution always uses the full budget.

3.1.2. Asymptotically Exact Relaxation. We now show that under certain conditions, the average cost or per-round form of (14), in which the total expected cost is scaled by the time horizon, is asymptotically equivalent to the original Problem (3) per-round form (i.e., the latter is an asymptotically exact relaxation of the former).

We first establish sufficient conditions for (13) to be an exact relaxation of (3) in Lemma 1. To this end, let $r : \{0,1\}^T \longmapsto \mathbb{R}$ be the objective function of (14). Let $P : \{0,1\}^T \longmapsto \mathbb{R}^+$, where $P(\mathbf{u}) = \mathbb{E}\left[\max\left\{0, \sum_{t=1}^T u_t - N\right\}\right]$ is the penalty for additional PSPSs. Recall that γ is the coefficient of the penalty term in (3).

Lemma 1. *Suppose $n < T$. Let*

$$\alpha = \min\left\{ r(\mathbf{u}) \,\middle|\, \mathbf{u} \in \{0,1\}^T, u_0 = u_{T+1} = 0, \mathbb{E}\left[\sum_{t=1}^T u_t\right] \leq N \right\} - \min\{r(\mathbf{u}) | \mathbf{u} \in \{0,1\}^T, u_0 = u_{T+1} = 0\}.$$

Then, for all $\gamma > \alpha$, (3) is an exact relaxation of (13).

Proof of Lemma 1. We first observe that

$$\min\left\{ P(\mathbf{u}) \,\middle|\, \mathbf{u} \in \{0,1\}^T, u_0 = u_{T+1} = 0, \mathbb{E}\left[\sum_{t=1}^T u_t\right] > N \right\} = 1.$$

Then, by Sinclair (1986, theorem 1), for $\gamma > \alpha$, the optima of (3) are also optima of (13).

Second, we show the converse, which was omitted in Sinclair (1986) (i.e., the optima of (13) are optimal for (3) when $\gamma > \alpha$). Let $o_{(3)}$ and $o_{(13)}$ be the optimal values of (3) and (13), respectively. Let $\mathbf{u}^\star$ be an optimum of (13). For all $\gamma > 0$, we have

$$\begin{aligned} o_{(13)} &= r(\mathbf{u}^\star) \\ &= r(\mathbf{u}^\star) + \gamma P(\mathbf{u}^\star) \end{aligned}$$

because $\mathbf{u}^\star$ is feasible for (13) (i.e., satisfies $\mathbb{E}\left[\sum_{t=1}^T u_t\right] \leq N$ and thus, $P(\mathbf{u}^\star) = 0$). By Sinclair (1986, theorem 1), $o_{(3)} = o_{(13)}$ for $\gamma > \alpha$. Hence,

$$r(\mathbf{u}^\star) + \gamma P(\mathbf{u}^\star) = o_{(3)},$$

and $\mathbf{u}^\star$ is also an optimum of (3). Therefore, if $\gamma > \alpha$, then (3) and (13) have the same optima. It follows that (3) is an exact relaxation of (13) under this condition. □

Lemma 1 states that if the marginal cost of being over the PSPS budget is large enough, then the relaxation is exact.

We now show that the (14) and (13) per-round forms are equivalent, almost surely, when the number of days and the budget constraint threshold N grow to infinity with a fixed ratio $\rho \in (0,1)$.

Consider the Markov decision processes (MDPs) $(\mathcal{X}, \{0,1\}, \mathbf{P}, r_t)$ used in (13) and (14). Then, the Markov chain induced by any policy is the same for both MDPs because $\mathbf{X}_t$, $t = 1, 2, \ldots, T$, models natural phenomena, and its evolution is independent of the decisions $\mathbf{u}$. Let $\mathcal{M}_{\mathbf{X}}$ denote this Markov chain.

Recall that $\mathcal{M}_{\mathbf{X}}$ is ergodic if it is recurrent and aperiodic. An ergodic Markov chain $\mathcal{M}_{\mathbf{X}}$ possesses a unique stationary distribution $\mathbf{s} \in \mathbb{R}_+^n$ such that $\mathbf{s} = \mathbf{sP}$ and $\mathbf{1}^\top\mathbf{s} = 1$. Finally, $\mathcal{M}_{\mathbf{X}}$ is stationary if the initial state of the Markov chain is distributed according to $\mathbf{s}$ (i.e., $\mathbf{X}_0 \sim \mathbf{s}$). For $\mathcal{M}_{\mathbf{X}}$ stationary, we therefore have $\mathbf{X}_t \overset{\mathrm{d}}{=} \mathbf{X} \overset{\mathrm{d}}{=} \mathbf{X}_0 \sim \mathbf{s}$ for all $t \geq 0$, where $\overset{\mathrm{d}}{=}$ means that they are equal in distribution.

Let Π be the policy space. Let $\Pi^{(14)}$ be the set of all feasible policies for Problem (14). We now present a lemma stating an equivalent average per-round constraint for policies in $\Pi^{(14)}$.

Lemma 2. *Suppose $\mathcal{M}_X$ is ergodic and stationary, and consider the policy $\pi^{(14)} \in \Pi$. Let $T, N \to \infty$, where T is an integer and $N = \rho T$. Then, $\pi^{(14)} \in \Pi^{(14)}$ if and only if $\mathbb{E}[\pi^{(14)}(\mathbf{X})] \leq \rho$ almost surely.*

Proof of Lemma 2. Let $\pi^{(14)} \in \Pi^{(14)}$. By definition, we have for all $T, N \geq 1$,

$$\sum_{t=1}^{T} u_t^{\pi^{(14)}} \leq N,$$

and therefore,

$$\frac{1}{T}\sum_{t=1}^{T} \pi^{(14)}(\mathbf{X}_t) \leq \frac{N}{T},$$

where we now consider the per-round average constraint and used the fact that $u_t^{\pi^{(14)}} = \pi^{(14)}(\mathbf{X}_t)$.

We take the limit as $T, N \to \infty$, with $T \in \mathbb{N}$ and N/T kept constant at ρ. We have

$$\begin{aligned} \lim_{\substack{T,N \to +\infty \\ \text{subject to } T \in \mathbb{N} \\ N = \rho T}} \frac{1}{T}\sum_{t=1}^{T} \pi^{(14)}(\mathbf{X}_t) &\leq \lim_{\substack{T,N \to +\infty \\ \text{subject to } T \in \mathbb{N} \\ N = \rho T}} \frac{N}{T}, \\ &= \rho. \end{aligned} \tag{26}$$

Using a strong law of large numbers for ergodic Markov chains (Serfozo 2009, chapter 1, theorem 74), we obtain for the left-hand side:

$$\lim_{\substack{T,N \to +\infty \\ \text{subject to } N = \rho T}} \frac{1}{T}\sum_{t=1}^{T} \pi^{(14)}(\mathbf{X}_t) = \sum_{\mathbf{x} \in \mathcal{X}} \pi^{(14)}(\mathbf{x})\mathbf{s}(\mathbf{x}) \quad \text{almost surely,} \tag{27}$$

if the right-hand term is absolutely convergent. This condition is met in (14) because $\pi^{(14)}(\mathbf{X}) \in \{0, 1\}$ for all $\mathbf{x} \in \mathcal{X}$ and $\operatorname{card}\mathcal{X}$ is finite. We rewrite (26) as

$$\mathbb{E}_{\mathbf{X} \sim \mathbf{s}}[\pi^{(14)}(\mathbf{X})] \leq \rho \quad \text{almost surely.} \tag{27}$$

Hence, if $T, N \to \infty$ such that $T \in \mathbb{N}$ and $\rho = N/T$, then the constraint defining $\Pi^{(14)}$ is equivalent to (27). It follows that $\pi^{(14)} \in \Pi^{(14)}$ if and only if $\mathbb{E}[\pi^{(14)}(\mathbf{X})] \leq \rho$. □

Similarly, we now present the per-round constraint that defines all policies in $\Pi^{(13)}$.

Lemma 3. *Suppose $\mathcal{M}_X$ is ergodic and stationary, and let $\pi^{(13)} \in \Pi$. As $T, N \to +\infty$, where T is an integer and $N = \rho T$, $\pi^{(13)} \in \Pi^{(13)}$ if and only if $\mathbb{E}[\pi^{(13)}(\mathbf{X})] \leq \rho$.*

Proof of Lemma 3. The first steps are similar to the Lemma 2 proof. Let $\Pi^{(13)}$ be the set of all feasible policies for (13). Let $\pi^{(13)} \in \Pi^{(13)}$. By definition,

$$\begin{aligned} &\mathbb{E}\left[\sum_{t=1}^{T} u_t^{\pi^{(13)}}\right] \leq N \\ \iff &\sum_{t=1}^{T} \mathbb{E}\left[u_t^{\pi^{(13)}}\right] \leq N \\ \iff &\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[u_t^{\pi^{(13)}}\right] \leq \frac{N}{T} \\ \iff &\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[\pi^{(13)}(\mathbf{X}_t)] \leq \frac{N}{T}. \end{aligned} \tag{28}$$

We evaluate the limit on both sides as $T, N \to +\infty$ when $T \in \mathbb{N}$ and the ratio $\rho = N/T$ is kept constant. We have

$$\begin{aligned} \lim_{\substack{T,N\to+\infty \\ \text{subject to } N=\rho T}} \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\pi^{(13)}(\mathbf{X}_t)] &\le \lim_{\substack{T,N\to+\infty \\ \text{subject to } N=\rho T}} \frac{N}{T} \\ &= \rho. \end{aligned} \tag{29}$$

Because $\mathcal{M}_{\mathbf{X}}$ is stationary, we have $\mathbf{X}_t \overset{d}{=} \mathbf{X} \sim \mathbf{s}$ for all $t \ge 0$, and thus,

$$\begin{aligned} \lim_{\substack{T,N\to+\infty \\ \text{subject to } N=\rho T}} \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\pi^{(13)}(\mathbf{X}_t)] &= \lim_{\substack{T,N\to+\infty \\ \text{subject to } N=\rho T}} \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\pi^{(13)}(\mathbf{X})] \\ &= \mathbb{E}[\pi^{(13)}(\mathbf{X})]. \end{aligned} \tag{30}$$

Using (29) and (30) leads to

$$\mathbb{E}[\pi^{(13)}(\mathbf{X})] \le \rho, \tag{31}$$

and we have shown that (28) is equivalent to (31). It follows that $\pi^{(13)} \in \Pi^{(13)}$ if and only if $\mathbb{E}[\pi^{(13)}(\mathbf{X})] \le \rho$ as $T \to \infty$ and $N = \rho T$. □

Next, we establish the asymptotic relationship between round-averaged forms of (13) and (14).

Theorem 1. *Suppose $\mathcal{M}_{\mathbf{X}}$ is ergodic and stationary. The per-round average form of (13) is an asymptotically exact relaxation of the (14) round-averaged formulation, almost surely, as $T, N \to \infty$ with $\rho = N/T$, $T \in \mathbb{N}$.*

Proof of Theorem 1. From Lemmas 2 and 3, we have $\pi^{(14)} \in \Pi^{(14)}$ if and only if $\mathbb{E}[\pi^{(14)}(\mathbf{X})] \le \rho$ almost surely and $\pi^{(13)} \in \Pi^{(13)}$ if and only if $\mathbb{E}[\pi^{(13)}(\mathbf{X})] \le \rho$ when $T, N \to \infty$ with $\rho = N/T$. Therefore, $\Pi^{(13)} = \Pi^{(14)}$ almost surely as $T, N \to \infty$ with $\rho = N/T$. This implies that the per-round average forms of Problems (13) and (14) are asymptotically equivalent almost surely because their objective functions are the same. In other words, the per-round form of (13) is an asymptotically exact relaxation of the (14) per-round form. □

Lastly, we relate the original Problem (3) to the final form (14) for which we have an optimal PSPS scheduling policy.

Theorem 2. *If $\gamma > \alpha$ and $\mathcal{M}_{\mathbf{X}}$ is ergodic and stationary, then the per-round form of (3) is an exact relaxation of (14), almost surely, as $T, N \to \infty$, where T is an integer and $N = \rho T$. Otherwise, the expected minimum cumulative loss of (14) is greater than or equal to the minimum of (3).*

Proof of Theorem 2. We remark that the averaged problem (i.e., dividing the objective function by T) only scales down the objective, which remains finite as $T \to \infty$. First, we prove the asymptotic exactness of the relaxation when $\gamma > \alpha$ and $\mathcal{M}_{\mathbf{X}}$ is stationary and as $T, N \to \infty$ such that $N = \rho T$. Problem (3) is a relaxation of (13). Because we have assumed $\gamma > \alpha$, Lemma 1 ensures that the relaxation is exact. Thus, (13) optima are also optimal for (3). Problem (13) is in turn a relaxation of (14). Under the theorem's assumption, we can invoke Theorem 1, which establishes that the per-round form of (13) is an asymptotically exact relaxation of the per-round Problem (14). Similarly, (14) optima are asymptotically optimal for (13) almost surely as $T, N \to \infty$ such that $N = \rho T$. The optima of (14) are, therefore, asymptotically optimal for (3) almost surely as well when $\gamma > \alpha$. The relaxation is, therefore, exact asymptotically.

Finally, we discuss the case for which the assumptions are not satisfied. From this justification, we have that regardless of the values of γ, N, and T and the stationarity of $\mathcal{M}_{\mathbf{X}}$, (3) is a relaxation of (14). Problem (14) optimal costs are, therefore, either greater than or equal to the minimum of (3). □

We note that under any conditions, the decisions provided by Proposition 1's policy are always feasible for PSPS scheduling and lead to a minimum that is equal to or greater than (3). This is because (16) is a reformulation of (3) for which Proposition 1 provides the optimal solution.

3.2. Optimal Policy for Scenario 2

We now prove Proposition 2. We remove $\overline{C}_I(N)$ and $-\lambda N$ from the problem formulation because they are constants and do not impact the minima. We write (8) as

$$\begin{aligned} \min_{\substack{u_t \\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \lambda u_t + \max\{s_{2,t+1}(u_{t-1}-u_t), s_{1,t+1}(u_t-u_{t-1})\}\right] \\ \text{subject to} \quad & u_t \in \{0,1\} \\ & u_0 = u_{T+1} = 0. \end{aligned} \tag{32}$$

Using the law of iterated expectation (Ross 2014, proposition 5.1), as in (15), gives

$$\begin{aligned}\min_{\substack{u_t\\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T}\mathbb{E}[a_{t+1}u_t + A_{t+1}f_{t+1}(\mathbf{X}_{t+1})(1-u_t) + \lambda u_t + \max\{s_{2,t+1}(u_{t-1}-u_t), s_{1,t+1}(u_t-u_{t-1})\}|\mathbf{X}_t]\right]\\ \text{subject to} \quad & u_t \in \{0,1\}\\ & u_0 = u_{T+1} = 0. \end{aligned} \tag{33}$$

Problem (33) represents a Markov decision process, in which a decision maker observes the random variable $\mathbf{X}_t$ at time t and then makes the decision, u_t, to call a PSPS for the next day. Similarly to Section 3.1, let $d = T - t$, and let all subscripts now refer to the backward time index d. Let $z_d(w|u_{d+1}, \mathbf{X}_{d+1})$ be the expected total cost given the past decision u_{d+1} and the observation vector $\mathbf{X}_{d+1}$ when d days remain and the accumulated cost is w. Problem (33) can be expressed as the following dynamic program:

$$\begin{aligned} z_d(w|u,\mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{& z_{d-1}\big(w + \mathbb{E}[A_{d-1}f(\mathbf{X}_{d-1})|\mathbf{X}_d = \boldsymbol{\xi}] + us_{2,d-1}|0,\boldsymbol{\xi}\big),\\ & z_{d-1}\big(w + a_{d-1} + \lambda + (1-u)s_{1,d-1}|1,\boldsymbol{\xi}\big)\}\Pr[\mathbf{X}_d = \boldsymbol{\xi}|\mathbf{X}_{d+1} = \mathbf{x}], \end{aligned} \tag{34}$$

with the boundary conditions

$$z_0(w|u,\mathbf{x}) = w + \sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[A_{-1}f_{-1}(\mathbf{X}_{-1})|\mathbf{X}_0 = \boldsymbol{\xi}]\Pr[\mathbf{X}_0 = \boldsymbol{\xi}|\mathbf{X}_1 = \mathbf{x}] + us_{2,-1},$$

for all $w \in \mathbb{R}, \mathbf{x} \in \mathcal{X}$, and $u \in \{0,1\}$. The optimal policy for the cost adjustment problem is given in Proposition 2 of Section 2.2. Its proof is stated next.

Proof of Proposition 2. Proposition 2 follows from the same proof technique as for Proposition 1, in which we use the relation

$$z_d(w|u,\mathbf{x}) = w + h_d(u,\mathbf{x})$$

for all $w \in \mathbb{R}$, $u \in \{0,1\}$, $\mathbf{x} \in \mathbf{X}$ and $d = T, T-1, \ldots, 1, 0$. Then, given the structure of $z_d(w,u,\mathbf{x})$, the optimal policy is given by a threshold policy as well. Using the recursion and solving for $\mathbb{E}[f_{d-1}(\mathbf{X}_{d-1})|\mathbf{X}_d = \mathbf{x}]$ yields (9). □

3.3. Optimal Policy for Scenario 3

We discuss Scenario 3 in which the number of PSPSs is minimized. Problem (10) can be rewritten as

$$\begin{aligned}\min_{\substack{u_t\\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T}\mathbb{E}[u_t|\mathbf{X}_t]\right]\\ \text{subject to} \quad & u_t \in \{0,1\}\\ & u_0 = u_{T+1} = 0\\ & \mathbb{E}\left[\sum_{t=1}^{T}\mathbb{E}[c_{t+1}(u_t)|\mathbf{X}_t]\right] \le \overline{\alpha}, \end{aligned} \tag{35}$$

where we have used the law of iterated expectations (Ross 2014, proposition 5.1) to reexpress the objective and the constraint functions. Problem (35) can be solved using value iteration as in Chen and Blankenship (2004) and computing the optimal policy as in Chen and Feinberg (2007). Adopting the Chen and Blankenship (2004) notation, we let $\Phi_\tau(\mathbf{x})$ be the set of feasible constraint thresholds for a problem starting at day t, defined as

$$\Phi_\tau(\mathbf{X}_t = \mathbf{x}) = \left\{\alpha \in \mathbb{R} \,|\, \exists \mathbf{u} \in \{0,1\}^\tau, u_0 = u_{T+1} = 0 \text{ such that } \sum_{i=t}^{t+\tau}\mathbb{E}[c_{i+1}(u_i)|\mathbf{X}_t] \le \alpha\right\}. \tag{36}$$

Let $\phi: \mathcal{X} \longmapsto \mathbb{R}^n$ be the constraint threshold for the initial state $\mathbf{x} \in \mathcal{X}$. Let $\overline{V} \ge 0$ be a large scalar. Let $V_\tau(\mathbf{x},\alpha)$ be the value function or total expected cost for Problem (35) with the time horizon τ, initial state $\mathbf{x}$, and constraint threshold α. Based on Chen and Blankenship (2004, theorem 3.1), we reexpress (35) as the dynamic program

$$V_{\tau+1}(\mathbf{x},\alpha) = \mathcal{T}_{F_\tau(\mathbf{x},\alpha)}V_\tau(\mathbf{x},\alpha), \tag{37}$$

for $\tau \geq 1$, where the operator $\mathcal{T}_{F_\tau}$ is defined as

$$\mathcal{T}_{F_\tau(\mathbf{x},\alpha)} V(\mathbf{x},\alpha) = \begin{cases} \min_{(u,\phi') \in F_\tau(\mathbf{x},\alpha)} \left\{ u + \sum_{\mathbf{x}' \in \mathcal{X}} V(\mathbf{x}', \phi'(\mathbf{x}')) \Pr[\mathbf{X}' = \mathbf{x}' \,|\, \mathbf{X} = \mathbf{x}] \right\}, & \text{if } F_\tau(\mathbf{x},\alpha) \neq \emptyset \\ \overline{V}, & \text{if } F_\tau(\mathbf{x},\alpha) = \emptyset, \end{cases} \tag{38}$$

and the set $F_\tau(\mathbf{x},\alpha)$ is given by

$$F_\tau(\mathbf{x},\alpha) = \left\{ (u,\phi') \,\middle|\, u \in \mathcal{U}, \phi'(\mathbf{x}') \in \Phi_\tau(\mathbf{x}') \;\; \forall \mathbf{x}' \in \mathcal{X}, \right.$$
$$\left. \mathbb{E}[c_{\tau+t+1}(u_t) \,|\, \mathbf{X} = \mathbf{x}] + \sum_{\mathbf{x}' \in \mathcal{X}} \phi'(\mathbf{x}') \Pr[\mathbf{X}' = \mathbf{x}' \,|\, \mathbf{X} = \mathbf{x}] \leq \alpha \right\}. \tag{39}$$

Next, we present a detailed description of the value function and use it to compute an optimal policy for (10).

3.3.1. Value Function. We evaluate (36)–(39) for the objective and constraints of (35) to obtain the following value function. First, for a time horizon of $\tau = 1$ where the last decision is taken at day T, we have

$$V_1(\mathbf{x},\alpha) = \begin{cases} 0, & \text{if } \mathbb{E}[c_{T+1}(0) \,|\, \mathbf{X}_T = \mathbf{x}] \leq \alpha \\ 1, & \text{if } \mathbb{E}[c_{T+1}(0) \,|\, \mathbf{X}_T = \mathbf{x}] > \alpha \text{ and } \mathbb{E}[c_{T+1}(1) \,|\, \mathbf{X}_T = \mathbf{x}] \leq \alpha \\ \overline{V}, & \text{if } b_1(\mathbf{x}) \equiv \min_{u \in \{0,1\}} \mathbb{E}[c_{T+1}(u) \,|\, \mathbf{X}_T = \mathbf{x}] > \alpha \end{cases}$$

for all $\mathbf{x} \in \mathcal{X}$ and $\alpha \in \mathbb{R}$. For a time horizon $\tau \geq 2$, we let

$$b_\tau(\mathbf{x}) = \min_{u_1 \in \{0,1\}} \mathbb{E}[c_{T-\tau+2}(u_1) \,|\, \mathbf{X}_{T-\tau+1} = \mathbf{x}] + \sum_{t=2}^{\tau} \min_{u_t} \left\{ \sum_{\mathbf{x}' \in \mathbf{X}} \mathbb{E}[c_{T-t+2}(u_t) \,|\, \mathbf{X}_{T-t+1} = \mathbf{x}'] \mathbf{P}^{t-1}_{\mathbf{x},\mathbf{x}'} \right\}. \tag{40}$$

The value function is $V_\tau(\mathbf{x},\alpha)$ if $b_\tau(\mathbf{x}) > \alpha$ or otherwise, given by

$$\begin{aligned} V_\tau(\mathbf{x},\alpha) = \min_{u, \phi'(\mathbf{x}')} \quad & u + \sum_{\mathbf{x}' \in \mathcal{X}} \Pr[\mathbf{X}'_{T-\tau+2} = \mathbf{x}' \,|\, \mathbf{X}_{T-\tau+1} = \mathbf{x}] V_{\tau-1}\big(\mathbf{x}', \phi'(\mathbf{x}')\big) \\ \text{subject to} \quad & u \in \{0,1\} \\ & \phi'(\mathbf{x}') \in \Phi_{\tau-1}(\mathbf{x}') \\ & \mathbb{E}[c_{T-\tau+2}(u) \,|\, \mathbf{X}_{T-\tau+1} = \mathbf{x}] + \sum_{\mathbf{x}' \in \mathcal{X}} \phi'(\mathbf{x}') \Pr[\mathbf{X}'_{T-\tau+2} = \mathbf{x}' \,|\, \mathbf{X}_{T-\tau+1} = \mathbf{x}] \leq \alpha, \end{aligned} \tag{41}$$

where

$$\Phi_\tau(\mathbf{x}) = [b_\tau(\mathbf{x}), +\infty[. \tag{42}$$

3.3.2. Optimal Policy. We compute an optimal policy for (35) and therefore, for (10) using the value function given in the previous section and the approach of Chen and Feinberg (2007). Let $f_t : \{(\mathbf{x},\alpha) \in \mathcal{X} \times \Phi_{N-t}(\mathbf{x})\} \longmapsto \mathcal{U} \times \mathbb{R}^n$ such that $f_t(\mathbf{x},\alpha) = (f_t^u(\mathbf{x},\alpha), f_t^\phi(\mathbf{x},\alpha)) = (\overline{u}_t, \overline{\phi}_t) \in F_{T-t}(\mathbf{x},\alpha)$, where

$$(\overline{u}_t, \overline{\phi}_t) = \underset{(u,\phi') \in F_{T-t}(\mathbf{x},\alpha)}{\arg\min} \left\{ u + \sum_{\mathbf{x}' \in \mathcal{X}} V_{T-t}(\mathbf{x}', \phi'(\mathbf{x}')) \Pr[\mathbf{X}'_{t+1} = \mathbf{x}' \,|\, \mathbf{X}_t = \mathbf{x}] \right\},$$

for $t \in \{1, 2, \ldots, T\}$. By Chen and Feinberg (2007, theorem 4), if $V_T(\mathbf{x}, \overline{\alpha}) < \overline{V}$, then optimal policy at time $t \in \{1, 2, \ldots, T\}$ is

$$u_t^\star = f_t^u(\mathbf{x}_t, \alpha_t).$$

The next round's constraint threshold is given by

$$\alpha_{t+1} = f_t^\phi(\mathbf{x}_t, \alpha_t)(\mathbf{x}_{t+1}),$$

for $t = 1, 2, \ldots, T-1$, and $\alpha_1 = \overline{\alpha}$, the constraint threshold specified in the problem for the whole time horizon. Lastly, after evaluating (41) for $\tau = 1, 2, \ldots, T$, we compute $u_t^\star$ and α_{t+1} using (11) and (12) for $\mathbf{X}_t = \mathbf{x}$.

Finally, the optimal PSPS scheduling policy is presented in Proposition 3. For completeness, the proof is given here.

Proof of Proposition 3. By assumption, $V_T(\mathbf{X}_1 = \mathbf{x}, \overline{\alpha}) < \overline{V}$, and the problem is feasible. By Chen and Blankenship (2004, theorem 2) and Chen and Feinberg (2007, theorem 4), the policies (11) and (12) are an optimal policy for (37). The policy is, therefore, optimal for (10). □

4. Critical Peak Pricing

CPP is used to reduce peak demand by temporarily increasing electricity prices. Price increases must be declared a day ahead based on current observations. The maximum number of CPP events, M, is constrained by contracts between the loads and the operator. Consider, for example, Hydro-Québec's CPP program, which is in effect during the winter period from December 1st to March 31st (referred to as *rate flex D*) (Hydro-Québec 2021b). During this period, the nominal electricity price is reduced by 30% from 6.08 to 4.28 ¢/kWh for the first 40 kWh multiplied by the number of days in the month and by 22% from 9.38 to 7.36 ¢/kWh above this monthly consumption threshold. When a CPP is called, the price increases to 50 ¢/kWh from 6 a.m. to 9 a.m. and/or from 4 p.m. to 8 p.m. Finally, CPP can be called for a maximum of 100 hours (i.e., between 25 and 33 times a year). The objective of CPP scheduling is, therefore, to identify the M days that, without intervention, would have the highest demand. The load demand is correlated with several factors (e.g., weather parameters (Hor et al. 2005, Herter et al. 2007), like temperature, wind speed, precipitation, etc.; day of the week (Hahn et al. 2009); and prior demand).

In this section, we formulate a model for CPP scheduling based on weather and demand observations. The objective is to minimize peak demand costs. The model has a similar structure as PSPS Scenario 1's reformulation (14) from Section 2.1 and admits the same optimal policy. We note that (14) is not a model for PSPS scheduling. It is only used to derive an optimal closed-form policy because the per-round formulations of Problem (14) and PSPS Scenario 1 are shown to be asymptotically equivalent under certain conditions in Theorem 2.

We use the same notation as in the previous sections and for example, let $u_t = 1$ denote the decision to call for CPP during day $t+1$. We let $\mathbf{X}_t \in \mathcal{X} \subset \mathbb{R}^n$ be the vector collecting the weather and day of the week at time t. Similarly, we assume that $\mathbf{X}_t$ is a Markov process with known transition probabilities for all states. Let $q_t : \mathbb{R}^n \longmapsto \mathbb{R}$ be a function mapping the n weather readings to an estimated peak demand level (Hor et al. 2005, Herter et al. 2007, Hahn et al. 2009) at time t. We model the cost of supplying power to the grid as a quadratic function of the demand. This function denoted $c_t^{\text{power}} : \mathcal{X}, \longmapsto \mathbb{R}^+$ includes, for example, generation, import, start-up, and shutdown costs. Let $B_t \geq 0$, $C_t \geq 0$, and $D_t \geq 0$ be, respectively, the second-, first-, and zeroth-order coefficients of the cost function. Let $y > 0$ be the load curtailed during a CPP event. We assume that y is constant and known (e.g., an averaged historical value of total curtailment as estimated by the system operator) (Hydro-Québec 2021a). In future work, we will model y as uncertain as well. Lastly in this section, we let $\overline{a}_t \in \mathbb{R}$ be the revenue loss because of high prices.

We formulate the CPP scheduling problem as

$$
\begin{aligned}
\min_{\substack{u_t \\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} c_{t+1}^{\text{power}}\big(q_{t+1}(\mathbf{X}_{t+1})\big)(1-u_t) + c_{t+1}^{\text{power}}\big(q_{t+1}(\mathbf{X}_{t+1}) - y\big)u_t + \overline{a}_{t+1}u_t\right] \\
\text{subject to} \quad & u_t \in \{0,1\} \\
& \sum_{t=1}^{T} u_t \leq M.
\end{aligned}
\tag{43}
$$

Using the law of iterated expectation (Ross 2014, proposition 5.1), we obtain

$$
\begin{aligned}
\min_{\substack{u_t \\ t=1,2,\ldots,T}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} \mathbb{E}[c_{t+1}^{\text{power}}\big(q_{t+1}(\mathbf{X}_{t+1})\big)(1-u_t) + c_{t+1}^{\text{power}}\big(q_{t+1}(\mathbf{X}_{t+1}) - y\big)u_t + \overline{a}_{t+1}u_t \,|\, \mathbf{X}_t]\right] \\
\text{subject to} \quad & u_t \in \{0,1\} \\
& \sum_{t=1}^{T} u_t \leq M.
\end{aligned}
\tag{44}
$$

In other words, based on the observation vector at time t, $\mathbf{X}_t$, the system operator wishes to select up to M days for which the cost of supplying the peak demand is highest and in excess of the revenue losses minus the cost reduction induced by CPP. Recall that $d = T - t$. Let $v_d(w, k|\mathbf{X}_{d+1})$ be the expected cumulative costs at round d given the observations $\mathbf{X}_{d+1}$ when the cumulative cost is w and k of M CPPs can still be called. The associated dynamic program is

$$\begin{aligned} v_d(w,k \mid \mathbf{X}_{d+1} = \mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{&v_{d-1}(w + \mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1})) \mid \mathbf{X}_d = \boldsymbol{\xi}], k \mid \mathbf{X}_{d+1}),\\ &v_{d-1}(w + \overline{a}_{d-1} + \mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1}) - y) \mid \mathbf{X}_d = \boldsymbol{\xi}], k-1 \mid \mathbf{X}_{d+1})\}\Pr[\mathbf{X}_d = \boldsymbol{\xi} \mid \mathbf{X}_{d+1} = \mathbf{x}], \end{aligned} \tag{45}$$

with the boundary conditions

$$v_0(w,k \mid \mathbf{X}_1 = \mathbf{x}) = w + \sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[c_{-1}^{\text{power}}(q_{-1}(\mathbf{X}_{-1})) \mid \mathbf{X}_0 = \boldsymbol{\xi}]\Pr[\mathbf{X}_0 = \boldsymbol{\xi} \mid \mathbf{X}_1 = \mathbf{x}]$$

$$v_d(w,0 \mid \mathbf{X}_{d+1} = \mathbf{x}) = w + \sum_{i=0}^{d}\sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[c_{i-1}^{\text{power}}(q_{i-1}(\mathbf{X}_{i-1})) \mid \mathbf{X}_i = \boldsymbol{\xi}]\mathbf{P}_{\mathbf{x},\boldsymbol{\xi}}^{i+1},$$

for all $w \in \mathbb{R}$, $k = 0, 1, \ldots, M$ and $\mathbf{x} \in \mathcal{X}$. The optimal CPP scheduling policy is given next in Proposition 4.

Proposition 4. *Consider the CPP scheduling Problem* (44)*. At day $d = T - t$, given the observations $\mathbf{X}_d = \mathbf{x}$, a CPP event is called for the following day* (i.e., $u_d = 1$) *if*

$$\mathbb{E}[q_{d-1}(\boldsymbol{X}_{d-1}) \mid \boldsymbol{X}_d = \boldsymbol{x}] > \frac{1}{2yB_{d-1}}\left(g_{d-1}(k-1 \mid \boldsymbol{x}) - g_{d-1}(k \mid \boldsymbol{x}) + \overline{a}_{d-1} - C_{d-1}y + B_{d-1}y^2\right),$$

where

$$\begin{aligned} g_d(k \mid \mathbf{X}_{d+1} = \mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \min\{&g_{d-1}(k \mid \boldsymbol{\xi}) + \mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1})) \mid \mathbf{X}_d = \boldsymbol{\xi}],\\ &g_{d-1}(k-1 \mid \boldsymbol{\xi}) + a_{d-1} + \mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1}) - y) \mid \mathbf{X}_d = \boldsymbol{\xi}]\}\Pr[\mathbf{X}_d = \boldsymbol{\xi} \mid \mathbf{X}_{d+1} = \mathbf{x}], \end{aligned}$$

with the boundary conditions

$$g_0(k \mid \mathbf{x}) = \sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[c_{-1}^{\text{power}}(q_{-1}(\mathbf{X}_{-1})) \mid \mathbf{X}_0 = \boldsymbol{\xi}]\Pr[\mathbf{X}_0 = \boldsymbol{\xi} \mid \mathbf{X}_1 = \mathbf{x}]$$

$$g_d(0 \mid \mathbf{x}) = \sum_{i=0}^{d}\sum_{\boldsymbol{\xi}\in\mathcal{X}} \mathbb{E}[c_{i-1}^{\text{power}}(q_{i-1}(\mathbf{X}_{i-1})) \mid \mathbf{X}_i = \boldsymbol{\xi}]\mathbf{P}_{\mathbf{x},\boldsymbol{\xi}}^{i+1},$$

for all $\mathbf{x} \in \mathbb{R}, k \geq 1$, and $d \geq 1$.

Proof of Proposition 4. We use the same proof technique as in Proposition 1 with $s_{1,d} = s_{2,d} = 0$ and replace $\mathbb{E}[f(\mathbf{X}_{d-1}) \mid \mathbf{X}_{d-1} = \boldsymbol{\xi}]$ and a_{d-1} with $\mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1})) \mid \mathbf{X}_d = \boldsymbol{\xi}]$ and $\overline{a}_{d-1}$, respectively. This implies that $u_d = 1$ if

$$g_{d-1}(k \mid 0, \boldsymbol{x}) + w + \mathbb{E}[c_{d-1}^{\text{power}}(q_{d-1}(\mathbf{X}_{d-1})) \mid \mathbf{X}_d = \boldsymbol{\xi}] > g_{d-1}(k-1 \mid 1, \boldsymbol{x}) + w + \overline{a}_{d-1} + \mathbb{E}[f(\boldsymbol{X}_{d-1} - y) \mid \boldsymbol{X}_{d-1} = \boldsymbol{\xi}].$$

Letting $c_{d-1}^{\text{power}}(\mathbf{z}) = B_{d-1}\mathbf{z}^2 + C_{d-1}\mathbf{z} + D_{d-1}$ and solving for $\mathbb{E}[q_{d-1}(\mathbf{X}_{d-1}) \mid \mathbf{X}_d = \boldsymbol{\xi}]$, we obtain

$$\mathbb{E}[q_{d-1}(\mathbf{X}_{d-1}) \mid \mathbf{X}_d = \boldsymbol{\xi}] > \frac{1}{2yB_{d-1}}\left(g_{d-1}(k-1 \mid 1, \mathbf{x}) - g_{d-1}(k \mid 0, \mathbf{x}) + \overline{a}_{d-1} - C_{d-1}y + B_{d-1}y^2\right),$$

which completes the proof. □

Proposition 4 provides an optimal policy for scheduling CPPs. The policy establishes that above a peak demand provided by its right-hand term, the decision maker should call a CPP; see Figure 2. Our approach accounts for peak events that can be absorbed by the grid without raising prices (e.g., by cheap imports). Our CPP model differs from Chen et al. (2013) as it includes the quadratic cost of the demand and the revenue loss from curtailing the load. Moreover, our policy is shown to be optimal, which was not done in Chen et al. (2013).

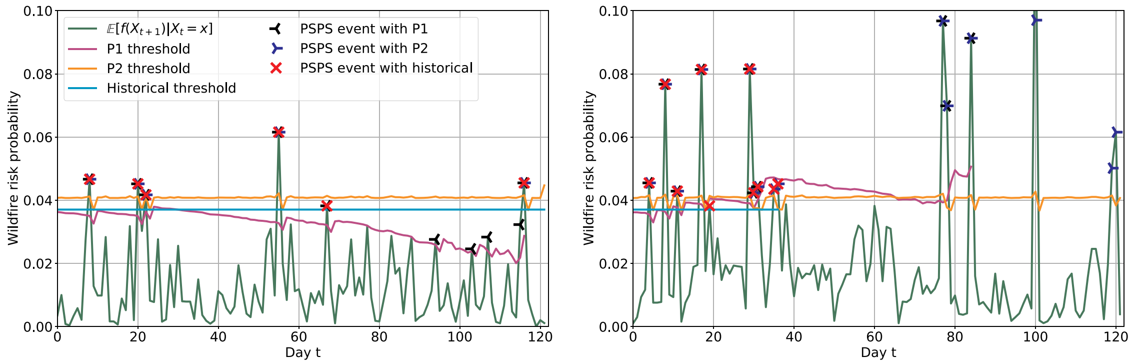
5. Numerical Examples

We now provide numerical examples for PSPS Scenarios 1 and 2 and CPP scheduling policies. We restrict ourselves to a comparison between closed-form policies. As mentioned, we do not consider PSPS Scenario 3 in this section because it does not possess a useful analytical structure, and as a result, it is significantly more computationally intensive and therefore, not readily implementable by system operators in comparison with the other scenarios. A closed-form policy may be obtainable via approximations. This is a topic for future work.

5.1. PSPS

We consider four types of weather observations to evaluate the risk of wildfire ignition: temperature, relative humidity, sustained wind, and wind gusts (Pacific Gas & Electricity 2020a). We use historical observations from

Figure 1. PSPS Scheduling Results for P1 and P2



Notes. 2019 (left panel) and 2020 (right panel). A PSPS is called at t and $u_t = 1$ if the solid green line is above the threshold. A threshold stops when its budget is depleted.

2011 to 2020 from the Sacramento International Airport weather station to model Northern California. We use data from 2011 to 2018 and from 2019 and 2020 as training and testing sets, respectively. We consider the months of June to September as the scheduling horizon and set $T = 122$. The risk thresholds of (1) are set to greater than 30°C, lower than 20%, greater than 25 km/h, and greater than 40 km/h for, respectively, the temperature, the relative humidity, and the sustained wind and wind gust speeds. These values are more risk averse compared with PG&E's values from Section 2. We note that we also consider daily maximum temperature observations in the wildfire risk function but omit the low fuel moisture, red flag warning, and on-the-ground observation thresholds (Pacific Gas & Electricity 2020b) because of the lack of available historical data. The weather phenomenon's transition matrices are calculated using the training data according to eight discretized states.

We let $A_t = \$1$B and $a_t = \$0.2$M for all t. The numerical value of A_t is set to be a fraction of PG&E's liability for recent wildfires, which was in the tens of billions (Abatzoglou et al. 2020, Rhodes et al. 2020). We set $n = 10$ and $s_1 = s_2 = \$2$M. Finally, we set the cost adjustment $\lambda = \$40.5$M (i.e., 15% of the total VoLL times the average daily demand (ADD), where VoLL = \$9,000/MWh (Sullivan et al. 2018) and ADD = 30 GWh (California Energy Commission 2021) for the state of California and Sacramento County, respectively).

The performances of P1 and P2, the policies for Scenarios 1 and 2, respectively, for the summers of 2019 and 2020 are presented in Figure 1. The policies are compared with a historical policy, which calls a PSPS on days with a wildfire risk probability (WRP) greater than the average Nth highest WRP day for every year of the training data. The expected costs for P1 and P2 and the historical policy for 2019 and 2020 are shown in Table 1. We refer to the argument of the outer expectation of, for example, Problem (15) as the expected cost (viz., the cumulative conditional expected cost of each decision given the current state). Next, we apply P1, P2, and the historical policy (average 10th highest WRP in simulated years) to 100 simulated years randomly generated using the estimated weather distribution. The average number of events and expected costs are presented in Table 2. The policies P1 and P2 successfully select the days with the highest expected WRP as shown in Figure 1. Tables 1 and 2 show that P1 and P2 outperform the historical policy in terms of expected costs, thus leading to safer operation of the power grid.

5.2. CPP

We now present numerical results for CPP scheduling. We consider four independent weather observations: the temperatures and precipitations for both Montreal, Canada and Quebec City, Canada in addition to a weekday/weekend state. We estimate q_t using a linear regression (Hor et al. 2005) on 2008–2018 data for the CPP period spanning December 1st to March 31st. We discretize $\mathbf{X}$'s components and determine the number of states

Table 1. Number of PSPS Events Called and Expected Costs for 2019 and 2020

	2019		2020	
Policy	Number of PSPS	Expected costs (billions of dollars)	Number of PSPS	Expected costs (billions of dollars)
P1	10	1.140	10	1.786
P2	5	1.072	15	1.660
Historical	6	1.240	10	1.917

Table 2. Number of PSPS Events Called and Expected Costs for 100 Simulated Years (Average (Standard Deviation))

Policy	Number of PSPS	Expected costs (billions of dollars)
P1	9.91 (0.32)	1.232 (0.231)
P2	7.78 (3.68)	1.176 (0.249)
Historical	8.23 (2.40)	1.269 (0.214)

using leave-one-out crossvalidation. We use 12 and 7 states for temperature and precipitation observations, respectively.

We set $M = 25$ and consider 10^5 clients participating to the CPP program. Each peak event is set to last 3.5 hours, and we assume that participating clients reduce their power demand by 1 kW on average (Hydro-Québec 2021a). Accordingly, we let $a_t = 3.5\,\text{h} \cdot 1\text{kW} \cdot \$0.0428/\text{kWh} \cdot 10^5 = \$15$ k and $y = 1\,\text{kW} \cdot 10^5 = 100$ MW. We set B_t, C_t, and D_t to $\$0.00245/\text{MW}^2$, $\$45.5/\text{MW}$, and $800 k for all t, respectively, so the generation cost may be higher than the price paid by customers on days with high demand.

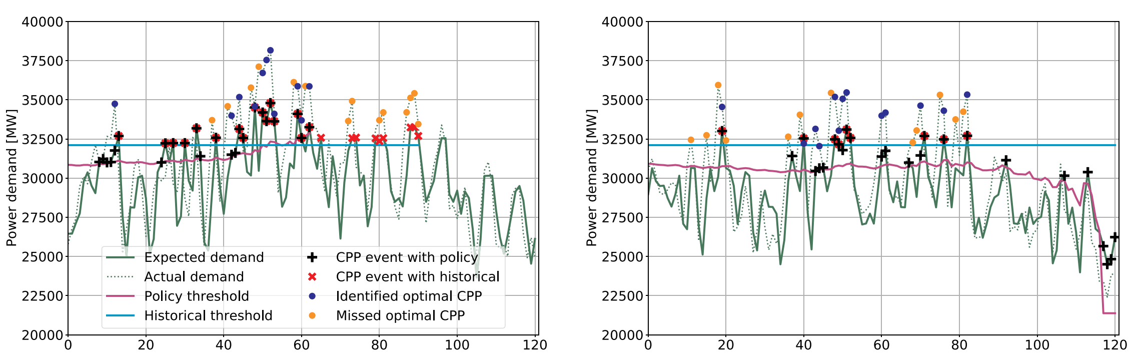
The performance of our policy for 2018–2019 and 2019–2020 winters is presented in Figure 2. In Figure 2, our policy is compared with a historical policy, which selects days with a demand greater than the average power demand for the Mth highest value for every year of the training data. The relative total cost reductions with respect to a policy selecting the days with the highest demand in hindsight are 84.1% and 88.2% in 2018–2019 and 78.6% and 35.3% in 2019–2020 for our policy and the historical policy, respectively. Figure 2, left panel shows that the historical policy can outperform our policy in particularly cold winters because they differ significantly from the historical data on which our policy is based. A larger training data set and an increased number of states could address this issue. This is a topic for future work.

Lastly, we test our policy on 100 simulated years. Figure 3 shows the constantly high relative cost reduction with respect to the hindsight policy. Our policy leads to a 98.0% average reduction with a standard deviation of 1.67% and outperforms the historical policy, which leads to an 85.9% average reduction with a standard deviation of 17.2%. The performance of our policy is thus higher and less volatile than the historical policies.

6. Conclusion

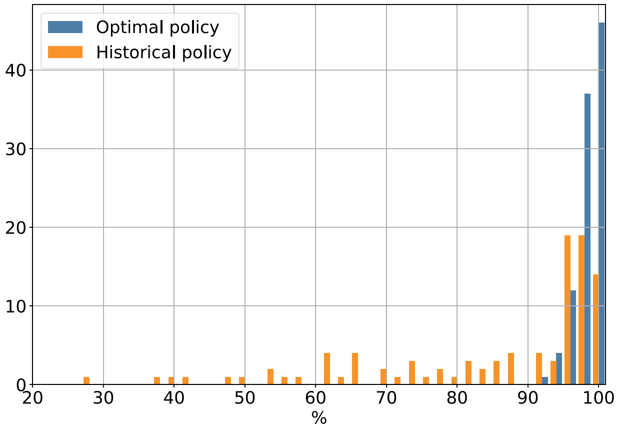
In this work, we formulate three dynamic programming models for PSPS scheduling to reduce power system-caused wildfires. We consider the trade-off between wildfire mitigation and the impacts of deenergizing communities by including the costs of wildfires, PSPS operation costs, and revenue losses for both the grid and the population. We assume that the system operator makes an initial investment to reduce wildfire risks and uses PSPSs to further decrease the risks. We consider three scenarios. First, we suppose that N PSPSs are planned to reach a desired risk level, and the operator must pay a penalty if the total number of PSPSs is above N. Second, under the same PSPS budget and penalty conditions, we assume that costs are recovered if the number of PSPS is below N. In the third model, the expected number of PSPSs is minimized subject to a total expected cost

Figure 2. CPP Scheduling Results



Notes. 2018–2019 (left panel) and 2019–2020 (right panel). A CPP is called at t and $u_t = 1$ if the solid green line is above the threshold. A threshold stops when its budget is depleted.

Figure 3. Cost Reduction Distribution for 100 Simulated Years



constraint. The first two scenarios are instances or variations of the multiple secretary problem. In each case, we adapt recent results from Arlotto and Gurvich (2019) to obtain an optimal scheduling policy for either the exact model or an asymptotically equivalent model. Lastly, we apply the first model to CPP and obtain an optimal scheduling policy. We numerically evaluate the performance of our approaches. Our simulations show that Scenarios 1 and 2 policies successfully balance wildfire risk and expected costs by selecting days with the highest expected wildfire probability. P1 and P2 outperform the historical policy in test years and in simulated years. Lastly, the CPP scheduling policy outperforms the historical policy in simulated years, on average attaining higher savings with lower variance. However, the historical policy may perform better under conditions that significantly differ from the training data (e.g., very cold winters).

Future work will focus on improving the weather model (e.g., with a larger training data set), the peak demand estimation function, and the wildfire risk probability function (e.g., with a larger number of natural phenomena, more spatial granularity, and data on actual ignitions and fire size, all of which will improve the selection of CPP and PSPS days).

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