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Relationship between fracture spacing and bed thickness in sedimentary rocks: Approach by means of Michaelis–Menten equation

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ABSTRACT

Fractures occur in nearly all rocks at the Earth's surface and exert essential control on the mechanical strengths of rock masses and permeability. The fractures strongly impact the stability of geological or man-made structures and flow of water and hydrocarbons, CO_2 and storing waste. For this, the dependence of opening mode fracture spacing (s) on bed thickness (t) in sedimentary basins (reservoirs) is studied in this context. This paper shows that the Michaelis–Menten equation can provide an algebraic expression for the nonlinear s-t relationship. The two parameters have clear geological meanings: a is the maximum fracture spacing which can no longer increase with increasing t, and b is the characteristic bed thickness when $s = 0.5a$. The tensile fracture strength (C) of the brittle beds during the formation of tensile fractures can be estimated from the two parameters. For sandstones of 16 areas reported in the literature, C ranges from 2.7 MPa to 15.7 MPa with a mean value of 8 MPa, which lies reasonably within the range of tensile strengths determined experimentally. This field-based approach by means of Michaelis–Menten equation provides a new method for estimating the tensile fracture strength of rock layers under natural conditions.

1. Introduction

No rock mass of larger than few cubic meters is uninfluenced by fractures that are resulted from brittle deformation at relatively shallow depths. Most of them are opening mode fractures with little or no displacement parallel to the fracture plane. In sedimentary basins (reservoirs) composed of interlayered beds with contrasted lithology and texture and thus distinct mechanical properties (Fig. 1), the opening mode fractures are ubiquitous in competent beds (e.g. limestone and sandstone) whereas incompetent layers (e.g. mudstone and shale) generally deform in a macroscopically ductile manner. Fracture spacing, which is the perpendicular distance between adjacent, mutually parallel fractures of the same set, and its relationship with bed thickness, has been extensively studied. For example, the spacing of fractures affects the mechanical properties, stability and permeability of rock masses (e.g. Bai and Pollard, 2000; Tan et al., 2014; Ji et al., 2021). The related knowledge is essential for management of the geotechnical projects (e.g. excavation of subsurface tunnels, shale gas fracking and quarrying operations).

Two categories of fracture saturation have been identified. (1) The median fracture spacing (s) in a single layer first decreases progressively with increasing applied extensional strain and then reaches a constant value even if strain continuously increases (Type 1, see Fig. 2a). (2) The median fracture spacing in a given sedimentary terrain with a wide range of bed thickness (a couple of millimeters to several dozens of meters) increases monotonically first quasi-linearly and then non-linearly, and eventually reaches a constant value with increasing bed thickness t (Type 2, see Fig. 2b). The first type (Fig. 2a) has been considerably studied by field investigations (e.g. Narr and Suppe, 1991; Ji and Saruwatari, 1998; Ji et al., 1998; Tan et al., 2014; Jiang et al., 2016; Bao et al., 2019; Ji et al., 2021), laboratory experiments (Rives et al., 1992; Wu and Pollard, 1995) and numerical modeling (Bai and Pollard, 2000; Li and Yang, 2007; Chemenda et al., 2021). However, the second one (Fig. 2b) has received little attention although its implications are significant (McQuillan, 1973; Ladeira and Price, 1981; Angelier et al., 1989; Souffaché and Angelier, 1989; Chemenda et al., 2021). For example, what can be the biggest median size of a monolith, which is a single massive stone without persistent fractures, in a region? The large monoliths (e.g. mass of about 9×10^6 kg, Yangshan,

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Fig. 1. Regularly spaced opening-mode fractures (joints) in a flat-lying, middle Ordovician bed at Cap Ferré, Quebec, Canada.

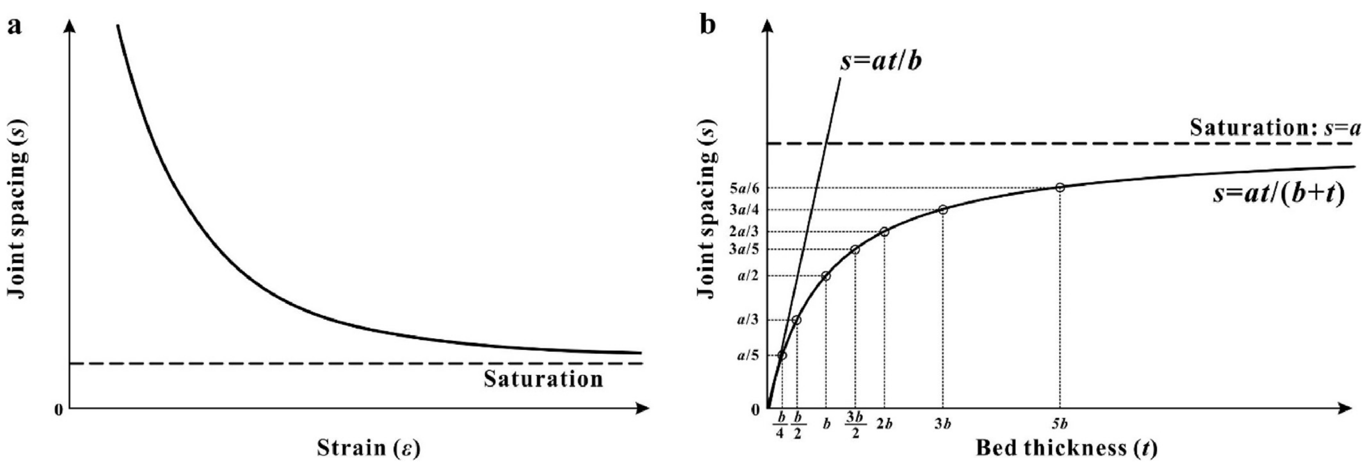

Fig. 2. Two categories of fracture saturation: (a) Type 1: fracture spacing no longer decreases even with increasing strain; and (b) Type 2: fracture spacing creases rising with increasing bed thickness.

Nanjing, China; mass of about 1.65×10^6 kg, Baalbek, Lebanon) form famous landmarks which receive a lot of attention from tourists. Can the maximum median joint spacing (a) be assessed for a terrain of layered sedimentary rocks? The value of a represents a state of joint saturation over which fracture spacing ceases increasing with increasing bed thickness. Thus, a is the largest median dimension of an intact rock bed without development of the joints (Angelier et al., 1989; Souffaché and Angelier, 1989). This question is of paramount importance for providing constraints on the lower limit for fluid permeability and the upper limit for mechanical stability of sedimentary strata in a given region. Furthermore, an algebraic expression of s-t relationship, if exists, should lead to better understanding its mechanical origin and physical meaning of each parameter.

2. Approach by means of Michaelis–Menten equation

Typical s-t relations (Table 1 and Fig. 3) have been obtained by best fitting of the data from 16 sandstone terrains reported in the literature (Novikova, 1947; Ladeira and Price, 1981; Angelier et al., 1989; Aydan and Kawamoto, 1990; Gross, 1993; Ruf et al., 1998; Ji et al., 1998, 2021; Cilona et al., 2016; Saein and Riahi, 2019). The geological settings, lithological descriptions and structural aspects have been given in these original papers and cited therein. The relations between the median fracture spacing (s) and bed thickness (t) can be well described by a non-linear equation:

$$s = \frac{at}{b + t} \tag{1}$$

where a is the maximum median fracture spacing which remains unchanged even if bed thickness (t) increases (Angelier et al., 1989; Souffaché and Angelier, 1989), and b is a constant which is numerically equal to the bed thickness when $s = 0.5a$ (Fig. 2b). This simple form of mathematic equation, named after German biochemist Leonor Michaelis and Canadian physician Maud Menten (Michaelis and Menten, 1913), is one of the well-known models in the field of steady-state enzyme kinetics (Johnson and Goody, 2011; Cornish-Bowden, 2015; Srinivasan, 2022). Michaelis and Menten (1913) took this formula describing the rate of enzymatic reactions (i.e. rate of formation of product) as a function of the concentration of a substrate. Their paper has achieved enormous influence on the study of biochemistry since 1913. The Michaelis–Menten equation possesses the following important characteristics (Fig. 2b):

Table 1
Parameters of the Michaelis-Menten equation for sandstones measured from 16 localities.

Reference	Locality	N	a (m)	b (m)	R^2	C* (MPa)
Angelier et al. (1989)	Taiwan, China	7	0.904	0.529	0.97	5.5
Angelier et al. (1989)	Moenkopi Fm., Colorado Plateau, USA	15	4.51	6.76	0.56	12.2
Aydan and Kawamoto (1990)	Unknown	7	0.977	0.54	0.97	5.7
Cilona et al. (2016)	Santa Susana, Simi Hills, California, USA	5	0.773	1.935	0.86	5.1
Gross (1993)	Monterey Fm., Gaviota (C), Santa Barbara, California, USA	4	0.985	0.713	0.92	5.7
Gross (1993)	Monterey Fm., Alegria (A), Santa Barbara, California, USA	8	1.296	1.3	1	6.5
Gross (1993)	Monterey Fm., Alegria (B), Santa Barbara, California, USA	7	3.13	3.28	1	10.2
Gross (1993)	Monterey Fm., Gaviota (B), Santa Barbara, California, USA	9	7.5	6.9	0.97	15.7
Ji et al. (1998)	St-Jean-Port-Joli, Quebec, Canada	97	2.798	1.932	0.67	9.6
Ji et al. (2021)	Ste-Anne-des-Monts, Quebec, Canada	109	1.656	1.7	0.91	7.4
Ladeira and Price (1981)	Flysch, UK (Interlayers <5 cm)	15	0.222	1.286	0.68	2.7
Ladeira and Price (1981)	Flysch, UK (Interlayers >5 cm)	19	0.367	1.005	0.86	3.5
Ladeira and Price (1981)	Alentejo, Portugal (Interlayers >5 cm)	39	0.779	1.262	0.95	5.1
Novikova (1947)	Russia	12	6.63	3.65	0.98	14.8
Ruf et al. (1998)	Huntingdon, Pennsylvania, USA	30	1.96	1.54	0.84	8
Saein and Riahi (2019)	Isfahan, Iran	8	3.3	3.49	0.98	10.4

N: number of measurements; R^2: coefficient of determination; C: tensile fracture strength; *: Calculated using Eq. (12).

(1) When $t << b$, s varies linearly with t and $s = at/b$. This yields a linear s-t relationship which has been observed particularly in the sedimentary rock layers with thicknesses of less than 50 cm (e.g. Huang and Angelier, 1989; Narr and Suppe, 1991; Gross, 1993; Gross et al., 1995; Ji et al., 1998; Ji and Saruwatari, 1998; Bao et al., 2019).
(2) When t becomes infinitely large, s stabilizes at a certain value a. This indicates that the fracture spacing will reach a steady-state maximum value a rather than increase infinitely with increasing bed thickness.
(3) For moderate values of bed thickness, the s-t relation is strongly nonlinear (McQuillan, 1973; Ladeira and Price, 1981; Ji et al., 2021). When $t = nb$, $s = an/(n+1)$, where n is a number. When $t = 0.5b$, b, $2b$, $3b$, $4b$ …, $s = a/3$, $a/2$, $2a/3$, $3a/4$, $4a/5$ …, respectively (Fig. 2b). The above features make the Michaelis–Menten equation be a more realistic description of the s-t relations than the linear (Narr and Suppe, 1991; Gross, 1993; Gross et al., 1995; Ji and Saruwatari, 1998) and power-law (Ji et al., 2021) equations that predict infinite increases of fracture spacing with increasing bed thickness.

In addition, Eq. (1) implies that

$$t/s = (b+t)/a \tag{2}$$

The ratio of bed thickness to median fracture spacing (t/s) for each single bed has been used as fracture spacing ratio (FSR, Gross, 1993) to justify the degree of Type 1 fracture saturation (Bai and Pollard, 2000). Eq. (2) indicates that FSR, which is not a constant, also depends on bed thickness. FSR increases linearly with increasing bed thickness with a slope of $1/a$ (Chemenda et al., 2021).

Table 1 lists the a and b values obtained from best-fitting of s-t data for totally 391 sandstone layers (Table S1 in Appendix A) reported in the literature (Novikova, 1947; Ladeira and Price, 1981; Angelier et al., 1989; Aydan and Kawamoto, 1990; Gross, 1993; Ji et al., 1998, 2021; Ruf et al., 1998; Cilona et al., 2016; Saein and Riahi, 2019). The ratios of a/b, obtained from these 16 localities, vary between 0.17 and 1.82, with a mean of 1.04, a median of 0.99, and a standard deviation of 0.51. Although the data are somehow scattered, a general trend can be audibly observed for these sandstones: $a \approx b$ (more precisely $a = 0.98b$, $R^2 = 0.88$, Fig. 4), indicating that the sandstones possess roughly similar features.

3. Discussion

The s-t relation has been documented to be nearly linear in relatively thin sedimentary beds ($t \le 0.5$ m, Ladeira and Price, 1981; Narr and Suppe, 1991; Gross, 1993; Ji and Saruwatari, 1998; Ji et al., 1998; Jiang et al., 2016; Bao et al., 2019; Chemenda et al., 2021):

$$s = \beta t \tag{3}$$

where β is the slope of the best-fitting line on plots of median fracture spacing versus bed thickness. The β value depends on the mechanical properties of both competent (jointed) and incompetent (unjointed) layers. According to Hobbs (1967), it yields

$$\beta = \sqrt{\frac{E_f}{G_m}}\cosh^{-1}\left(\frac{E_f\varepsilon}{E_f\varepsilon - C}\right) \tag{4}$$

where E_f is the Young's modulus of the jointing layer, G_m is the shear modulus of the incompetent layer, and ε is the far-field extensional strain of the layer-matrix system. Eqs. (3) and (4) cannot account for Type 1 fracture saturation as they show that joint spacing decreases continuously with increasing extensional strain ε. In order to obtain such a linear s-t relation, Hobbs (1967) implicitly imposed a condition that the thickness of incompetent layers (d) adjacent to the competent bed is exactly equal to $2t$. This imposed condition is obviously unrealistic because the incompetent layers (mudstone and shale) are very often thinner than the competent, jointed bed (sandstone or limestone) in sedimentary rocks (Ladeira and Price, 1981; Ji and Saruwatari, 1998; Ji et al., 2021).

Following Price and Cosgrove (1990), Ji et al. (1998), Li and Ji (2020), and Li et al. (2020), we have

$$\beta = \frac{C}{2\tau} \tag{5}$$

where τ is the interfacial shear stress between the competent and incompetent layers, and cannot exceed the shear strength of the incompetent material.

The derivations of Eqs. (3)–(5) were based on an assumption that the rock is homogeneous and isotropic, and has uniform mechanical properties such as tensile strength C. However, natural rocks display pronounced size effects on their tensile strengths:

$$C = C_0 t^{-1/k} \tag{6}$$

where C_0 is the tensile fracture strength of the rock sample with a reference length L_0, and k is the Weibull modulus of the competent material. Incorporating Eq. (6) into Eq. (5), Ji et al. (2021) obtained:

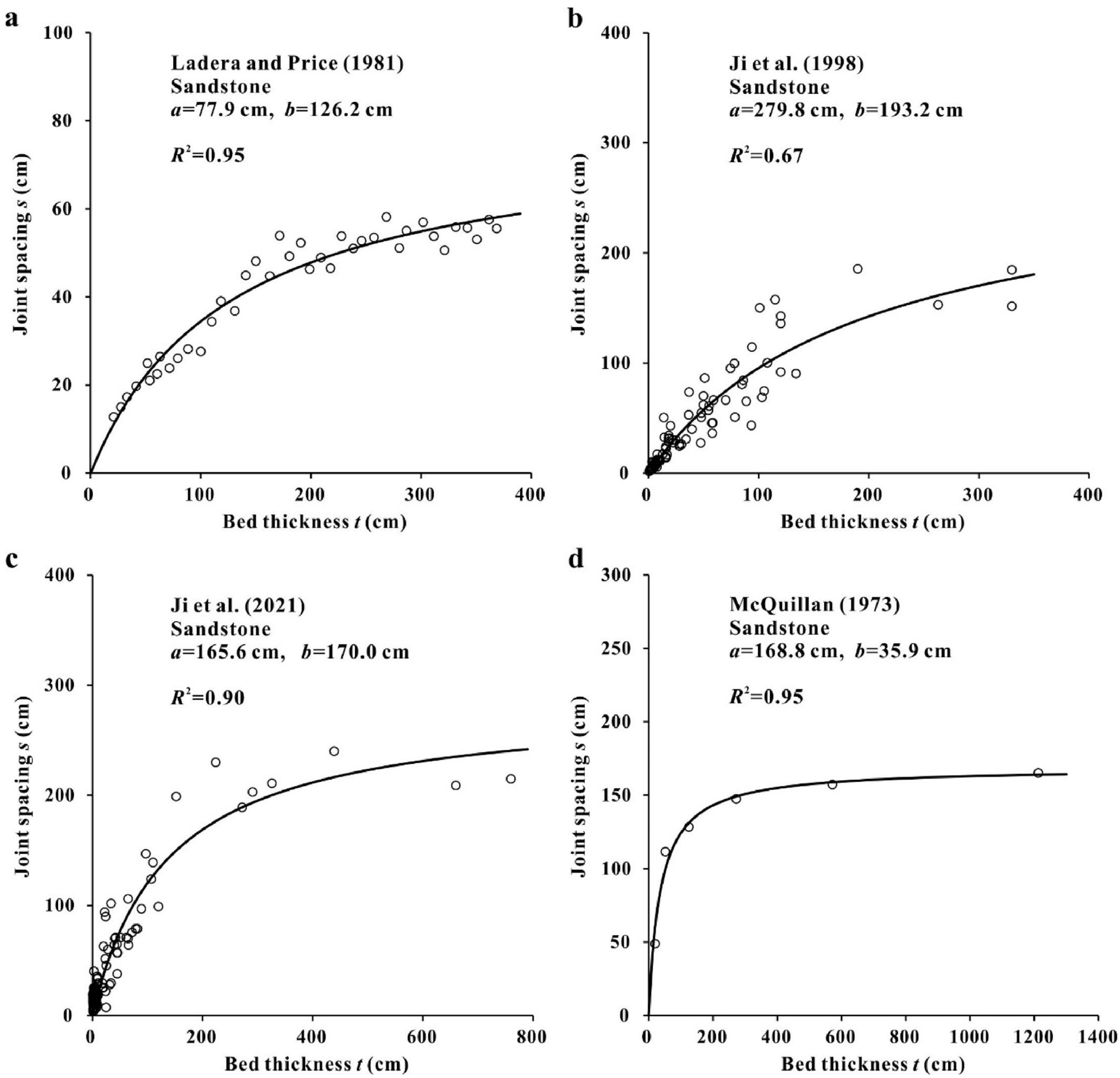

Fig. 3. Plots of joint spacing versus bed thickness for four sandstone terrains (a)–(d). The *s-t* data are fitted using the Michaelis–Menten equation with two parameters *a* and *b*. Data in (a), (b), (c) and (d) were measured by Ladeira and Price (1981), Ji et al. (1998), Ji et al. (2021) and McQuillan (1973), respectively.

$$\beta = \frac{C_0}{2\tau} t^{-1/k} \tag{7}$$

and thus,

$$s = \frac{C_0}{2\tau} t^{1-1/k} \tag{8}$$

Eq. (8) reveals a power-law relationship between s and t, and it has been applied to thick beds (0–8 m, Ji et al., 2021). Clearly, ds/dt can be given by

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{C_0}{2\tau}\left(1 - \frac{1}{k}\right) t^{-1/k} \tag{9}$$

Eq. (9) cannot be zero at any t value because ds/dt monotonically decreases with increasing t. Thus, Eq. (8) does not predict a steady state of Type 2 fracture saturation although it offers generally a good description for the variation of median fracture spacings in sedimentary beds with a wide range of bed thicknesses (Ji et al., 2021).

As illustrated in Fig. 3 and Table 1, Eq. (1) is of the form of Michaelis–Menten equation, and it successfully depicts the *s-t* variation with two eigenvalues: a is the median fracture spacing when Type 2 saturation has been reached, and b is the characteristic bed thickness when $s = 0.5a$ (Fig. 2b). The empirical equation proposed by Chemenda et al. (2021) (in their Eq. (17)) is actually a variant of the Michaelis–Menten equation.

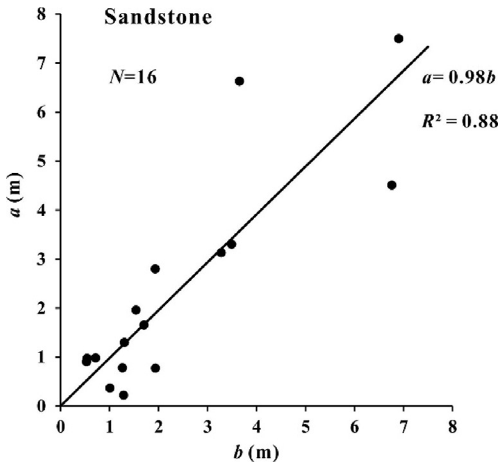

Fig. 4. Plots of a versus b for 16 sandstone terrains.

Ladeira and Price (1981) proposed that the nonlinear s-t curve (Fig. 3a and d) can be divided into two straight lines: the first with a large slope while the second with a significantly small or even zero slope. The two straight lines are assumed to represent two distinct mechanisms of fracturing. Ladeira and Price (1981) attributed the independence of fracture spacing on bed thickness to permeability-controlled hydraulic fracturing. Based on two dimensional (2D) finite-difference modeling of a three-layer composite (a competent layer embedded in between two incompetent layers), Chemenda et al. (2021) reached a conclusion different from that of Ladeira and Price (1981). Chemenda et al. (2021) suggested that the transition from the first mechanism to the second one occurs when the ratio t/s reaches a critical value of about 2. According to Eq. (2), the critical bed thickness (t_c) at which the transition occurs is given by

$$t_c \approx 2a - b \tag{10}$$

When $t > t_c$, the layer-parallel normal stress in the middle of the competent bed is changed from tensile to slightly compressive or zero, and therefore no new opening mode fracture can be initiated in the middle of the competent bed. Furthermore, the fractures initiated by strain-mismatch-induced tensile stresses from the interfaces between the competent and incompetent layers cannot cut through the whole competent bed. As a result, the median fracture spacing s remains unchanged with increasing bed thickness. However, their explanation is not supported by the measured data (Table S1) that fracture spacing keeps changing with increasing t even when $t > t_c$. As illustrated in Fig. 2b, the fracture spacing displays an initial rapid nonlinear increase at $t < 2b$, followed by a more graduate nonlinear increase at $t \geq 2b$. Furthermore, Chemenda et al. (2021) cannot interpret the cases where $t_c = 2a - b \leq 0$ (e.g. Ladeira and Price, 1981; Cilona et al., 2016).

Based on the balance determination of elastic energy and rupture energy, Souffaché and Angelier (1989) obtained the following relation between a and C (the tensile fracture strength of the rock):

$$a = \frac{2\pi C^2}{\nu \rho g E} \tag{11}$$

or

$$C = \left(\frac{\nu \rho g E a}{2\pi}\right)^{1/2} \tag{12}$$

where ρ, ν, and E are the density, Poisson's ratio and Young's modulus of the competent layer, respectively, and g is the gravitational acceleration.

The Poisson's ratio ν and Young's modulus E of a rock can be calculated from the data of its P- and S-wave velocities (V_p and V_s) and densities by

$$V_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \tag{13}$$

$$V_s = \sqrt{\frac{G}{\rho}} \tag{14}$$

$$E = \frac{9KG}{G + 3K} \tag{15}$$

$$\nu = \frac{E}{2G} - 1 \tag{16}$$

where G and K are the shear and bulk moduli, respectively. A survey of literature shows that P- and S-wave velocities and densities of 67 sandstone samples (Table S2 in Appendix A) have been measured at confining pressures ranging from 10 MPa to 300 MPa (Hughes and Cross, 1951; Freund, 1992; Johnston and Christensen, 1992; Ji et al., 2002). These sandstones consist of silicate clasts and clay, and have their porosities varying from 0.1% to 14.9%, with an average value of 6% (standard deviation: 3.7%). The data of elastic wave velocities and densities allow us to calculate the average Young's modulus, Poisson's ratio and density for sandstones at various confining pressures. As shown in Table 2, either E or ν increases with increasing confining pressure due to progressive closure of microcracks and pores (Ji et al., 2007). At 100 MPa, which corresponds to a lithostatic pressure at a depth of about 4 km where joints are formed, we have $\nu = 0.143 \pm 0.06$ (standard deviation), $E = 58.51 \pm 11.29$ GPa, and $\rho = 2.53 \pm 0.114$ g/cm^3. This experimentally measured E value is very close to those used in the mechanical model of Schopfer et al. (2011) ($E = 56.11$–57.3 GPa), but

Table 2
Statistical data of elastic properties for 67 sandstone samples (density of 2.53 ± 0.114 g/cm^3, and porosity of 6.035% ± 3.735%)*.

Confining pressure (MPa)	Shear modulus (GPa)		Young's modulus (GPa)		Bulk modulus (GPa)		Poisson's ratio	
P	G	St Dev	E	St Dev	K	St Dev	ν	St Dev
20	19.55	4.15	41.84	9.83	16.72	6.51	0.069	0.079
50	23.77	4.67	53.28	10.76	24.59	7.67	0.123	0.072
80	24.62	4.41	55.91	9.87	26.14	7.58	0.138	0.063
100	25.64	4.96	58.51	11.29	28.2	7.68	0.143	0.060
150	26.32	5.07	60.43	11.51	29.69	7.91	0.150	0.058
200	26.87	5.16	61.78	11.7	30.55	8.26	0.152	0.059
250	27.12	5.16	62.42	11.74	30.92	8.29	0.153	0.058
300	27.4	5.16	63.1	11.76	31.31	8.38	0.154	0.058

St Dev: standard deviation; *: Data compiled in Handbook of Seismic Properties of Minerals, Rocks and Ores (Ji et al., 2002).

significantly higher than those in the other models (e.g. 1 GPa, Angelier et al., 1989; Souffaché and Angelier, 1989; 30 GPa, Jain et al., 2007; and 40 GPa, Bai and Pollard, 2000). It is noted that the Poisson's ratio of sandstone increases with increasing density and clay content, but decreases with increasing porosity and silicate clast content (see Ji et al. (2018) for more details). A highly compacted monocrystalline aggregate of quartz (zero porosity) has a very low Poisson's ratio ($\nu = 0.08$), and quartz arenites containing microcracks, micropores and secondary minerals may even display negative Poisson's ratios at low confining pressures (Ji et al., 2018).

The tensile fracture strengths C of 16 sandstones, computed from Eq. (11), are listed in Table 1. The value of C ranges from 2.7 MPa to 15.7 MPa with a mean value of 8 MPa (standard deviation: 3.8 MPa). For example, the sandstone from the Sainte-Anne-des-Monts (Quebec, Canada) has tensile strength of about 7.4 MPa. These values derived from the field measured s-t data are consistently within the range of tensile strengths of sandstones measured experimentally in laboratory (e.g. Kubota et al., 2008; Gong and Zhao, 2014; Lu et al., 2017; Lan et al., 2019).

Souffaché and Angelier (1989) also obtained an analytical solution for b:

$$b = \frac{2\left[\sigma_0 - (1-2\nu)P_f + \nu\rho' gz\right]}{\nu\rho g} \tag{17}$$

where σ_0 is the residual tensile stress within the competent layer after breakage; P_f is the pore fluid pressure, ρ' is the mean density of the overlying rocks from the surface to depth z. $P_f = 0$ if the rock is dry, and $P_f = \rho_f gz$ under the hydrostatic conditions (i.e. the fluid velocity is zero, the fluid pressure variation occurs in the vertical rather than the horizontal direction due to the weight of the fluid), where ρ_f is the density of fluid. Assuming that $\sigma_0 = 0$ (no stress transfer occurs across an opening mode fracture) and $\rho \approx \rho'$, Eq. (17) can be simplified as

$$P_f = \frac{\nu\rho g}{1-2\nu}\left(z - \frac{b}{2}\right) \tag{18}$$

For the sandstone, it is taken that $\rho = 2530$ kg/m^3, $g = 9.8$ m/s^2, $\nu = 0.143$, and $z = 4000$ m. Under the conditions, Eq. (18) implies that P_f, which depends essentially on the depth z, varies slightly with b because $z \gg 0.5b$. At a depth of 4 km where fractures were presumably formed, the fluid pressure calculated from Eq. (18) is about 19 MPa, which is higher than the tensile fracture strengths computed from Eq. (12). $P_f > C$, indicating that the effective stress ($\sigma_3{}^* = \sigma_3$ - P_f) is indeed negative and thus tensile at the fracture-forming depth. This further supports the proposition of Ladeira and Price (1981) that hydraulic fracture mechanism operates and plays an important role in the development of opening mode fractures in sandstone beds.

Further work is needed to improve our understanding of the mechanical processes to produce the s-t relation following the Michaelis–Menten equation. Bed thickness-dependent flaws (shape and distribution) and three dimensional (3D) mechanical interaction between the competent bed and its adjacent incompetent layers are believed to be two critical factors to control the nonlinear variation of fracture spacing with bed thickness (Chemenda et al., 2021; Ji et al., 2021). The study has an importance in practical applications such as hydrology, oil and gas industry and mining engineering.

4. Conclusions

Two types of fracture saturation have been identified: fracture spacing s ceases lessening with increasing extensional strain (Type 1) and halts rising with augmenting bed thickness (Type 2). The Michaelis–Menten equation can provide a good description for the nonlinear dependence of median fracture spacing s on bed thickness t in sedimentary beds such as sandstones and limestones. The two eigenvalues of this equation: a (the maximum median fracture spacing when Type 2 saturation has been reached) and b (the characteristic bed thickness at which $s = 0.5a$). The value of a corresponds to the largest average size of a monolith that can occur in a given terrain of sedimentary rocks. Based on original derivations of Souffaché and Angelier (1989), the tensile fracture strength C is estimated during the formation of tensile fractures in sandstone beds from the parameters of the Michaelis–Menten equation. The value of C ranges from 2.7 MPa to 15.7 MPa with a mean value of 8 MPa, which lies reasonably in the range of tensile strengths of sandstones determined by experimental measurements. The obtained parameters of a and b enable the researchers to reproduce the nonlinear s-t curves with adequate precision, and to facilitate the comparison between different regions and statistical analysis of the rock mechanical properties. The Michaelis–Menten equation with determined eigenvalues a and b, although more rigorous theoretical analyses and quantitative experimental data are needed to improve our understanding, is particularly useful to more precisely estimate the fracture spacing at depth from bed thickness measured from well cores. Thus, the equation has potential applications in the hydrology, petroleum industry and geotechnical engineering.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jrmge.2022.11.003.

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