

Titre: Decentralized Game Theoretical Investigation for the Residential
Title: Microgrid Power Network Problem

Auteur: Mohamad Aziz
Author:

Date: 2023

Type: Mémoire ou thèse / Dissertation or Thesis

Référence: Aziz, M. (2023). Decentralized Game Theoretical Investigation for the Residential
Citation: Microgrid Power Network Problem [Thèse de doctorat, Polytechnique Montréal].
PolyPublie. <https://publications.polymtl.ca/55115/>

 **Document en libre accès dans PolyPublie**
Open Access document in PolyPublie

URL de PolyPublie: <https://publications.polymtl.ca/55115/>
PolyPublie URL:

**Directeurs de
recherche:** Hanane Dagdougui, Issmaïl El Hallaoui, & Georges Zaccour
Advisors:

Programme: Doctorat en génie industriel
Program:

POLYTECHNIQUE MONTRÉAL

affiliée à l'Université de Montréal

**Decentralized Game Theoretical Investigation for the Residential Microgrid
Power Network Problem**

MOHAMAD AZIZ

Département de mathématiques et de génie industriel

Thèse présentée en vue de l'obtention du diplôme de *Philosophiæ Doctor*

Génie industriel

Août 2023

POLYTECHNIQUE MONTRÉAL

affiliée à l'Université de Montréal

Cette thèse intitulée :

**Decentralized Game Theoretical Investigation for the Residential Microgrid
Power Network Problem**

présentée par **Mohamad AZIZ**

en vue de l'obtention du diplôme de *Philosophiæ Doctor*

a été dûment acceptée par le jury d'examen constitué de :

Michel GENDREAU, président

Hanane DAGDOUGUI, membre et directrice de recherche

Issmail EL HALLAOUI, membre et codirecteur de recherche

Georges ZACCOUR, membre et codirecteur de recherche

Miguel F. ANJOS, membre

Mads ALMASSALKHI, membre externe

DEDICATION

To My Family and Loved Ones

ACKNOWLEDGEMENTS

First and foremost, I would like to express my heartfelt gratitude to my PhD supervisors, Prof. Hanane Dagdougui, Prof. Issmail El Hallaoui, and Prof. George Zaccour for their unwavering understanding, guidance, and support throughout the years. Their expertise and encouragement have been invaluable, both professionally and personally.

I would also like to extend my sincere appreciation to my colleagues at Polytechnique and Gerad for their valuable advice and technical assistance. Their collaboration and support have greatly contributed to the success of my research.

I am forever indebted to Jad Kabbara, Moataz El Naghi, Ahmad Taha, Kenan Rijleh, and all my friends in Montreal and abroad, who have been pillars of strength during the most challenging and emotionally demanding times. Your friendship and support have been truly invaluable.

To my Canadian family, Glenn Wight and Susie Henderson, I cannot thank you enough for your constant encouragement throughout the years. Your belief in me has been a source of inspiration.

Lastly, there are no words to express my immense gratitude to my parents, Ismail Aziz and Fatme Doghman, my life partner Cassie Wight, my brothers and sisters-in-law Ahmad Aziz, Mahmoud Aziz, Mira Jammoul, and Maha Diab, and my nieces and nephews Rayan, Adam, and Maya. Your unconditional love, unwavering support, and belief in my abilities have been the driving force behind my accomplishments. I am truly fortunate to have such an incredible family.

Thank you all from the bottom of my heart.

RÉSUMÉ

Les recherches actuelles dans le domaine de l'énergie et des systèmes énergétique portent sur la conversion, la distribution et le stockage de l'énergie. Une récente étude de l'agence internationale de l'énergie indique que "pour répondre aux besoins énergétiques mondiaux d'ici 2035, il faudra 40 000 milliards de dollars d'investissements en raison de l'augmentation de la demande et du remplacement des installations de production et de traitement". Afin de préserver l'environnement et réduire les émissions GES, il est essentiel de résoudre les problèmes liés au réseau électrique et d'accélérer le remplacement des installations conventionnelles existantes. Dans cette recherche proposée, nous étudierons le problème du réseau électrique composé de plusieurs acteurs, tels que les décideurs politiques, les sociétés de production d'électricité et les prosommateurs (c'est-à-dire les ménages capables de produire leur propre énergie, par exemple grâce à des systèmes solaires photovoltaïques), en prenant en compte les facteurs environnementaux et socio-économiques.

Dans le problème étudié, les acteurs interagissent pour mettre en œuvre une politique dans laquelle chacun a ses propres objectifs et sa propre dynamique du système. Il y a une interdépendance dans les fonctions de coût, car les décisions d'un acteur affectent les bénéfices des autres. En raison de la prolifération croissante des ressources énergétiques résidentielles distribuées, un grand nombre de prosommateurs sont apparus sur le marché de l'électricité. La communauté des micro-réseaux (MG), un groupe de prosommateurs avec une entité de contrôle commune, est une composante prometteuse du réseau intelligent qui pourrait potentiellement créer un marché de l'électricité libre. Dans le réseau électrique étudié, les acteurs sont les suivants : (1) les décideurs politiques, qui contrôlent la taxe sur les technologies polluantes (dites "noires") et les subventions accordées aux technologies vertes, (2) les sociétés de production d'électricité (représentant le réseau principal), qui contrôlent l'équipement, le stockage de l'énergie et le type de technologie de production, et dont les actions sont basées sur le prix de production et les taxes/subventions liées à la technologie utilisée, et (3) les prosommateurs, qui ont le contrôle sur la demande, sont influencés par le prix du marché et ont le choix entre la consommation, le stockage pour une utilisation ultérieure ou la revente

de leur génération locale.

Dans cette thèse, le problème du réseau électrique régissant l'interaction entre le MG, qui représente la coalition des prosommateurs, et le réseau principal, qui représente les sociétés de production d'électricité, a été formulé. L'objectif est de trouver la décision optimale pour chaque acteur, en considérant l'optimalité dans le sens de l'équilibre du jeu formulé. Le réseau principal vise à minimiser les coûts associés à l'écetage de la demande pendant les heures de pointe, tandis que l'objectif du MG est de minimiser les coûts globaux de la coalition formée par ses prosommateurs. Premièrement, une nouvelle formation d'un MG résidentiel comprenant une coalition de prosommateurs avec des systèmes photovoltaïques est développée. Dans le MG proposé, les ménages sont connectés via une banque d'alimentation virtuelle composée de batteries de stockage domestiques, qui assure la communication entre les ménages et le réseau principal. En appliquant la théorie des jeux en champ moyen (MFG), nous trouvons des stratégies d'équilibre de Nash qui permettent à une telle formation de minimiser une combinaison linéaire des coûts de production d'énergie des ménages, des coûts de consommation d'énergie et des revenus de l'énergie vendue. L'approche résultante est testée à travers une étude de cas d'un MG à Montréal, au Québec, Canada.

Deuxièmement, une formulation théorique du jeu pour un MG résidentiel basé sur la gestion de la demande à base des incitations est proposée, dans laquelle un mécanisme de tarification incitative est adopté pour encourager la réduction de la demande pendant les heures de pointe. L'objectif est de maximiser le confort des ménages en identifiant le prix d'équilibre optimal de l'électricité. Les ménages communiquent entre eux et avec le gestionnaire du réseau de distribution (DSO) et cherchent à minimiser leurs coûts d'électricité. Troisièmement, une étude numérique approfondie du modèle mathématique est présentée. Des stratégies d'équilibre de Nash, qui minimisent les coûts énergétiques totaux, sont développées en appliquant la théorie des jeux en champ moyen.

À notre connaissance, cette recherche proposée est la première à étudier le problème des réseaux électriques en utilisant et en appliquant la théorie des jeux en champ moyen. Les résultats de ces études sont les suivants : (i) une nouvelle méthodologie générale de tarification pour les micro-réseaux, (ii) des stratégies d'équilibre de Nash pour les décideurs politiques,

les sociétés de production et les prosommateurs, (iii) une optimisation en temps réel conduisant à une réduction des coûts globaux pour tous les acteurs du jeu du réseau électrique. Enfin, le réseau électrique formulé permet une interaction optimale entre les fournisseurs (sociétés de production) et les prosommateurs, contribuant ainsi à la réduction des impacts environnementaux, sociaux et des coûts de production.

ABSTRACT

As the demand for energy continues to grow and environmental concerns become increasingly important, power network issues arise, necessitating the replacement of facilities, implementation of demand response strategies, and greater integration of renewable energy resources. This trend is expected to lead to the emergence of a substantial number of prosumers (households that both consume and produce energy) in the electricity market. These prosumers will play a crucial role in the development of smart grids and could potentially contribute to the establishment of a free electricity market. By free electricity market, we mean a platform where buyers and sellers can freely exchange electric energy in accordance with the sector's regulations. This thesis focuses on addressing the power network problem that arises from the interaction between a residential community microgrid, representing a coalition of prosumers, and the main grid, which represents power generation companies. The problem is tackled using decentralized game theoretical techniques, with optimality defined as the equilibrium of the formulated game.

Firstly, a novel configuration for a residential community microgrid is developed, which includes a coalition of prosumers equipped with solar panel and energy storage systems. The constructed microgrid is composed of households that are interconnected through a virtual power bank that facilitates communication between them and the main grid. By employing mean field game theory, Nash Equilibrium strategies are found, under which such sharing minimizes a linear combination of the households' energy costs.

Secondly, a game theoretical formulation is presented for an incentive-based residential community microgrid, where an incentive-based pricing mechanism is adopted to encourage peak demand reduction. The constructed microgrid for this particular framework consists of a coalition of prosumers households and an aggregator. The main grid incentive demand curve is shared with the residential community through the aggregator.

Finally, a follow-up computational investigation is conducted for the constructed incentive-based residential community microgrid optimization problem, in which Nash equilibrium

strategies are found via an application of mean-field game theory. This computational analysis provides valuable insights into the real-time optimal decision-making processes within the residential community microgrid.

Overall, this thesis delves into the power network problem arising from the interaction between residential prosumers and power generation companies, employing decentralized game theoretical techniques to find equilibrium solutions. The research explores different scenarios, including the incorporation of solar panel systems, incentive-based pricing mechanisms, and the utilization of mean-field game theory, all aimed at achieving optimal energy management within the residential community microgrid.

TABLE OF CONTENTS

DEDICATION	iii
ACKNOWLEDGEMENTS	iv
RÉSUMÉ	v
ABSTRACT	viii
TABLE OF CONTENTS	x
LIST OF TABLES	xiv
LIST OF FIGURES	xv
CHAPTER 1 INTRODUCTION	1
1.1 Thesis Objective	2
1.2 Thesis Structure	4
1.3 Previously Published Material	5
CHAPTER 2 LITERATURE REVIEW	6
2.1 Motivation Behind the Microgrid Network	6
2.1.1 Economical Aspect of the Network	8
2.2 Game Theory Literature Review	9
2.2.1 Strategic Games	10
2.2.2 Mixed Games	11
2.2.3 Bidding in the Standard Auctions	12
2.2.4 Mean Field Game Theory	12
2.3 Dynamical Games Formulation for the Power Network Problems Literature Survey	14
2.3.1 Demand Response: Summary and Objectives	16

2.3.2	Renewable Energy Resources (RERs) and Battery Storage system Summary and Objectives	19
2.3.3	Computational Models for the Microgrid Optimization Method	20

CHAPTER 3 A DECENTRALIZED GAME THEORETIC APPROACH FOR VIRTUAL STORAGE SYSTEM AGGREGATION IN A RESIDENTIAL COMMUNITY MICROGRID

	MICROGRID	25
3.1	Abstract	25
3.2	Introduction	25
3.2.1	Relevant Work	26
3.2.2	Mean Field Game Theory	28
3.2.3	Contributions	29
3.2.4	Paper Structure	30
3.3	Description of the game	31
3.3.1	System structure	31
3.3.2	Virtual Power Bank	31
3.4	MGO: System Dynamics	33
3.4.1	Household Generation Dynamics	33
3.4.2	Household Operation Cost Model	34
3.4.3	Energy Demand Management	34
3.4.4	Household Power Bank Account	35
3.5	MGO: Dynamical Game Formulation	36
3.5.1	MG Equilibrium Price: P_{MG}	37
3.5.2	Decentralized Mean Field Optimal Control	38
3.5.3	System Performance and Cost Functions	40
3.5.4	Existence and Uniqueness of the HJB Equation [1]	43
3.6	Algorithmic Implementation of Decentralized Community MGO Problem . .	44
3.6.1	Boundary Condition and Approximation Methods	44
3.6.2	Numerical Algorithm for Uniform Agents	46
3.6.3	Case Study: Montreal, Quebec	47

3.6.4	Computational Performance	52
3.6.5	Decentralized Approach Versus Centralized Approach	53
3.7	Concluding Remarks	55

CHAPTER 4 COOPERATIVE GAME THEORETICAL APPROACH FOR RESIDENTIAL USERS THROUGH LOAD AGGREGATOR IN SMART GRID

4.1	Abstract	56
4.2	Introduction	56
4.3	Conceptual design of the proposed game framework	59
4.3.1	Conceptual design	59
4.3.2	MFG Framework	60
4.4	MGO: System Dynamics	62
4.4.1	Main grid Pricing Mechanism	62
4.4.2	Aggregator System Dynamics	63
4.4.3	MG Equilibrium Price: P_{MG}	64
4.4.4	Household Generation Dynamics	66
4.4.5	Household Operation Cost Model	66
4.4.6	Energy Demand Management	67
4.4.7	Household Energy Profile	69
4.5	MGO: Dynamical Game Formulation	70
4.5.1	Mean Field Fokker Planck Equation	70
4.5.2	Mean Field Hamilton Jacobean Bellman Equation	71
4.5.3	Mean Field Loop	75
4.6	Conclusion	76

CHAPTER 5 COMPUTATIONAL INVESTIGATION OF RESIDENTIAL USERS THROUGH LOAD AGGREGATOR SMART GRID POWER OPTIMIZATION PROBLEM

5.1	Abstract	78
5.2	Introduction	78
5.2.1	MG Architecture [2] and Paper Objectives	79
5.2.2	Computational Investigation for the MGO problem in Literature	80

5.2.3	Paper Objective	81
5.2.4	Paper Outline	82
5.3	MGO System Dynamics	82
5.3.1	Main Grid Pricing Mechanism	82
5.3.2	MG Equilibrium Price: P_{MG}	84
5.3.3	Household Generation and Operation Cost Dynamics	85
5.3.4	Energy Load Management	86
5.3.5	Household Energy Profile [2]	87
5.4	MGO: Dynamical Game Formulation	88
5.4.1	Mean Field Fokker Planck Equation	89
5.4.2	Mean Field Hamilton Jacobean Bellman Equation	89
5.4.3	Mean Field Loop	92
5.5	Algorithmic Implementation For the MGO via MFG	93
5.5.1	Boundary Specifications and Discretization Techniques	93
5.5.2	Algorithmic procedure for the Uniform Households	95
5.6	Numerical Results and Robustness Analysis: Montreal MG Case Study . . .	96
5.6.1	Numerical Results	97
5.6.2	Algorithms Computational Performances	101
5.7	Conclusion	102
CHAPTER 6 GENERAL DISCUSSION		104
CHAPTER 7 CONCLUSION AND RECOMMENDATIONS		106
7.1	Summary of Contributions	106
7.2	Limitations	107
7.3	Future Work	108
REFERENCES		109

LIST OF TABLES

Table 3.1	Optimal Solution considering SC2	43
Table 3.2	Algorithms Performance for Montreal, Quebec Case Study . . .	52
Table 3.3	Numerical Performance Versus Population	54
Table 4.1	Optimal Solution considering SC2	75
Table 5.1	Optimal Solution considering SC2	92
Table 5.2	Algorithms Performance for the Uniform Household Case . . .	102

LIST OF FIGURES

Figure 2.1	Conceptual framework of a Smartgrid	15
Figure 2.2	Microgrid Architecture	16
Figure 3.1	Community Microgrid Blockdiagram	32
Figure 3.2	Load Probability Density Function	48
Figure 3.3	Energy Generation Density Function	49
Figure 3.4	Generic Household i Energy Profile and MG Equilibrium Price	50
Figure 3.5	Power Bank Energy Profile	51
Figure 3.6	Household i Decision Variables and Demand Origin Fractions	52
Figure 3.7	Centralized Versus Decentralized Performance	54
Figure 4.1	Major-Minor Agent Community MG Block Diagram	60
Figure 4.2	Winter Credit Option (WCO)–Pricing Scheme	63
Figure 5.1	Major-Minor Agent Community Microgrid Block Diagram . .	80
Figure 5.2	Winter Credit Option (WCO)–Pricing Scheme	83
Figure 5.3	Initial Probability Density Function at $t = 0$ for $y_c \theta$	97
Figure 5.4	Propagation of the $\mu(\theta, y_c, t)$ versus time	98
Figure 5.5	Propagation of the cost function $v(\theta, y_c, t)$ versus time	99
Figure 5.6	Generic Agent Energy Profile	100
Figure 5.7	Generic Agent Energy Profile During Peak Event	101

Nomenclature

δ	Net Energy i.e. $\delta = b + \theta - y$.
γ	Energy Withdrawn Through the VB in kWh .
ι	Household Credit/Incentive During PE.
λ	MG Aggregated Credit Saving.
$\mathcal{T}$	Maingrid Energy Threshold During PE.
μ_θ	Probability Density Function for θ .
μ_y	Probability Density Function for y .
Ω_θ	Range of all Generated Energy.
Ω_y	Range of all Demand.
τ	Household Individual Threshold During PE.
θ	Generated Energy in kWh .
b	Energy profile in kWh .
C	Total Generation and Storage Cost in $\$$
C_u	MG Pricing Mechanism Function.
$C_{B,M}$	Unit Cost for Battery Operation and Maintenance in $\text{¢} / kWh$.
C_{max}	Maximum Battery Capacity in kWh .
G_{INC}	Irradiance in W/m^2 .
G_{MG}	MG Aggregate Load.

G_{STC}	Irradiance at STC.
i	Indices of Households.
k_s	Maximum Power Temperature Coefficient.
$K_{O\&M}$	Unit Cost for PV Operation and Maintenance in $\text{¢}/kWh$.
N	Number of Households.
P_{MA}	Maingrid Energy Price in $\text{¢}/kWh$.
P_{MG}	MG Energy Price.
P_{PV}	Generated PV Power in kW .
P_{STC}	Power at STC.
r	Energy Sold in MG in kWh .
T	Final Time.
T_c	PV cell Temperature in $^{\circ}C$.
T_r	PV room Temperature $^{\circ}C$.
u_{γ}	Decision Variable for Withdrawn Energy.
u_c	Decision Variable for the Controllable Load.
u_r	Decision Variable for Sold Energy.
w_1	Fraction of Energy Met from the MG VB.
w_2	Fraction of Energy Met from the maingrid.
W_{ϵ}	Standard Wiener Process.
W_{Θ}	Standard Wiener Process for the PV generation.
y	Household Load in kWh .

- AG Microgrid Aggregator.
- DER Distributed Energy Resources.
- DR Demand Response.
- DSO Distribution System Operator.
- FPK Fokker-Plank-Kolmogorov Equation.
- HJB Hamilton-Jacobi-Bellman Equation.
- i Indices of Households.
- MF Mean Field.
- MFG Mean Field Game Theory.
- MG Microgrid.
- MGO Microgrid Optimization Problem.
- N Set of all Households.
- PDE Partial Differential Equation.
- PE Peak Event.
- PV Photovoltaic Panel.
- RMG Residential Microgrid.
- STC Standard Temperature Condition.
- VPB Virtual Power Bank

CHAPTER 1 INTRODUCTION

Current research in the field of energy and power systems focuses on energy conversion, distribution, and storage. According to a recent study conducted by the International Energy Agency, meeting the world's energy supply needs by 2035 will require a substantial investment of \$40 trillion. This investment is necessary due to increasing demand and the need to replace production and processing facilities. As energy demand continues to rise and environmental awareness grows, power network issues become prominent, leading to the replacement of facilities and a heightened focus on environmental welfare.

This proposed research aims to examine the power network problem, which involves policymakers, power generation companies, and households (known as prosumers). In this dissertation, I investigate this optimization problem by considering various environmental and socioeconomic factors.

In the power network problem, multiple decision-makers interact to implement a policy, each with their own objective function and system dynamics. The interaction between these decision-makers is manifested through the coupling of their cost functions, as the actions of one agent impact the payoffs of others. With the growing adoption of residential distributed energy resources, a significant number of prosumers have emerged in the electricity market. Among them, the microgrid community, which may comprise a group of individually controllable prosumers, holds great potential as a key component of the smart grid, ultimately leading to the possibility of a free electricity market.

Within the constructed energy network, three distinct sets of players are involved. Firstly, the policymaker wields control over the taxation imposed on conventional technology and the subsidies provided for green technology. Secondly, the power generation companies, representing the main grid within this framework, exercise authority over energy supply, storage, and the choice of technology for production. Their actions are guided by the generation price and the tax/subsidy policy applied to the utilized technology. Lastly, the consumers possess control over their own energy demand and are influenced by market prices. They have the

flexibility to decide between immediate consumption or storing energy for future use.

1.1 Thesis Objective

In this thesis, the focus lies on addressing the power network problem that governs the interaction between the microgrid, representing a coalition of prosumers, and the main grid, which represents the power generation companies. The primary objective is to determine the optimal decisions for each player within this framework, with optimality defined as achieving equilibrium in the formulated game. The main grid aims to minimize the cost associated with peak demand offset, while the microgrid seeks to minimize the overall cost incurred by its prosumers coalition, comprising households.

Firstly, a novel configuration is developed for a residential community microgrid, featuring a coalition of prosumer households equipped with solar photovoltaic systems. Within this constructed microgrid, households are interconnected via a virtual power bank, which consists of their storage batteries and facilitates communication between households and the main grid. By applying mean field game theory, Nash Equilibrium strategies are determined. These strategies enable efficient sharing that minimizes a linear combination of the households' energy generation cost, energy consumption cost, and revenue from energy sales. The approach is validated through a case study involving a community microgrid in Montreal, Quebec, Canada.

Secondly, a game theoretical formulation is presented for an incentive-based residential community microgrid. This formulation involves the adoption of an incentive-based pricing mechanism, aiming to encourage peak demand reduction. The incentive demand curve is shared with the residential community through an aggregator, whose objective is to maximize the welfare of the community by determining the optimal equilibrium electricity price. The aggregator in this framework represents the mediator of the microgrid which relays the main grid incentive demand curve, and peak demand credit and finds the microgrid equilibrium price. Each household communicates with one another and with the distributed system operator (DSO) through the aggregator, while aiming to minimize their local electricity costs.

Thirdly, a subsequent computational investigation is conducted for an incentive-based resi-

dential community. Similar to the previous scenario, an incentive-based pricing mechanism is employed to encourage peak demand reduction and share the incentive demand curve through the aggregator. Nash Equilibrium strategies, minimizing the total cost functions associated with the residential community's energy consumption, are determined using mean field game theory. The decentralized residential community microgrid (RCM) electric energy problem is computationally analyzed within the context of a microgrid with controllable load households connected to the main grid, featuring an incentive-based pricing strategy.

To the best of our knowledge, the work in this dissertation represents the pioneering effort in exploring the power network problem using mean field game theory. The significant contributions of this thesis include:

- Introducing a novel and comprehensive pricing methodology for the microgrid, which provides a robust framework for optimizing economic interactions within the network.
- Establishing Nash Equilibrium strategies for key stakeholders such as the policy maker, generation companies, and consumers. These strategies enable efficient decision-making processes and foster a balanced equilibrium within the power network.
- Developing real-time optimization techniques that effectively reduce the overall costs for all players involved in the power network game. By leveraging the principles of mean field game theory, optimal solutions are obtained, leading to enhanced economic efficiency.
- The formulated power network facilitates optimal interactions between suppliers and consumers, resulting in tangible benefits such as reduced environmental impact, improved social outcomes, and minimized production expenses.

In summary, this research contributes to novel insights into the residential microgrid power optimization problem by employing mean field game theory. The outcomes encompass an innovative pricing methodology, Nash Equilibrium strategies, real-time optimization, and improved economic, environmental, and social outcomes for all participants within the power network.

1.2 Thesis Structure

The remaining chapters of this thesis are organized as follows, Chapter 2 provides a comprehensive literature review, which is essential for understanding the contributions made by this thesis. It covers key concepts from dynamical game theory and delves into the microgrid power network problem.

In Chapter 3, we present the first published journal paper, where we introduce an innovative architecture for the residential community featuring a virtual power bank. Within this chapter, we propose a novel formation for a residential community microgrid comprising prosumer households equipped with solar photovoltaic systems. By employing mean field game theory, Nash Equilibrium strategies are identified, enabling efficient sharing that minimizes a linear combination of the households' energy generation cost, energy consumption cost, and revenue from sold energy. To validate this approach, a case study is conducted on a community microgrid situated in Montreal, Quebec, Canada.

Chapter 4 encompasses the second journal paper, which introduces two significant elements to the previously constructed microgrid: (i) the load aggregator and (ii) households with smart houses and demand response capabilities. Within this chapter, a game theoretical formulation is presented for an incentive-based residential community microgrid. The formulation includes the development of an incentive-based pricing mechanism aimed at encouraging peak demand reduction and sharing the incentive demand curve with the residential community through the aggregator.

Chapter 5 focuses on the third journal paper, which serves as a follow-up to the work presented in Chapter 4. This chapter presents a computational investigation of the decentralized residential community microgrid electric energy problem using mean field game theory. Specifically, the analysis is conducted on a microgrid featuring controllable load households connected to a main grid with an incentive-based pricing strategy.

Lastly, Chapter 6 serves as the conclusion, where we discuss the limitations of the presented work and outline potential avenues for future research exploration.

1.3 Previously Published Material

The main chapters in this thesis are based on papers published (Chapters 3, and 4) or under review (Chapters 5) at peer-reviewed venues.

- Chapter 3 has been published as [3]:

Mohamad Aziz, Hanane Dagdougui, and Issmail El-hallaoui. 2022. A Decentralized Game Theoretic Approach for Virtual Storage System Aggregation in a Residential Community. In: IEEE Access 10 (2022), pp. 34846–34857.

Contributions of Authors: Mohamad Aziz led the project, writing algorithmic implementation, analyzing the collected data, running the simulations, and writing the paper. Hanane Dagdougui and Issmail El-hallaoui contributed to the editing of the paper and provided general guidance and technical direction.

- Chapter 4 under-revision as [2]:

Mohamad Aziz, Hanane Dagdougui, and Issmail El-hallaoui. 2023. Cooperative Game Theoretical Approach for Residential Users Through Load Aggregator in Smart Grid: Formulation and Proof of Concept. Submitted to: European Journal of Control.

Contributions of Authors: Mohamad Aziz led the project, writing the problem formulation and presenting the arguments sufficient for implementing the mean field game theory on the constructed microgrid optimization problem. Hanane Dagdougui and Issmail El-hallaoui contributed to the editing of the paper and provided general guidance and technical direction.

- Chapter 5 is based on a submitted manuscript, [4]:

Mohamad Aziz, Hanane Dagdougui, and Issmail El-hallaoui. Computational Investigation for Residential Users Through Load Aggregator Smart Grid Power Optimization Problem via Mean Field Game Theory. In: Cahiers du GERAD, Montreal (2023).

Contributions of Authors: Mohamad Aziz led the project, writing the algorithmic implementation and running the simulation result for the constructed case study. Hanane Dagdougui and Issmail El-hallaoui contributed to the editing of the paper and provided general guidance and technical direction.

CHAPTER 2 LITERATURE REVIEW

2.1 Motivation Behind the Microgrid Network

The global population has been steadily expanding and has just reached eight billion according to the United Nations' recent release. Many future projections estimate that over the coming decades, it will stabilize probably at a level just below ten billion. Over the same period, the standard of living as measured by per capita gross economic product, especially in areas currently less well off, is projected to significantly improve largely because of technology. The result of both of these factors is that the global gross economic product is expected to increase by a factor of six-seven over the next five decades. Based on past trends, such an increase would also result in an increased demand for material commodities including the products made by the chemical industry by perhaps a factor of five and an increase in overall energy demand by more than a factor of three but an electrical energy demand by perhaps a factor of seven according to World Energy. A legitimate concern is raised whether such a magnitude of materials and energy demand increase, even if only approximately accurate, can be sustainably satisfied. That is, can such a demand be satisfied without compromising the ability of future generations to meet their quality of life aspirations [5]?

Meeting current and future demands for energy is often quoted as the most pressing problem facing humanity in the 21st century. This problem has to be addressed in a global environment where resources are constrained and diminishing, while at the same time raising standards of living for a growing population and preventing catastrophic climate change. Recent experiences has shown that energy production is increasingly competing with other demands on the same resources, such as commodity chemicals production and food production, and all compete for water as a primary resource. In moving to a sustainable future, radical changes will have to take place in how we produce, transport, and utilize energy. The solutions will be many and diverse, and highly dependent on geographic location and patterns of consumption [6].

Today's electrical grid is composed of multiple networks and multiple power suppliers with several operators employing many coordination and operation strategies. The "smart grid" refers to the incorporation of recent advances in information and communication technologies into the grid; this is in order to increase the connectivity, automation, and coordination among various suppliers, consumers, and networks that perform transmission or distribution tasks. The long-term goal is to conceive a power system where highly intermittent suppliers can be accommodated, peak demand can be reduced, and dependency on pollutant fossil fuels with their volatile price can be decreased [7].

Sustainability is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs". The United Nations recognizes major components of sustainability to be social development, economic development, and environmental protection. These are described as "three interdependent and mutually reinforcing pillars" [8]. The sustainable development of energy systems is becoming increasingly important for policy and decision-makers worldwide. The sustainable development of energy systems requires consideration of all three sustainability dimensions: environmental, economic, and social. The framework comprises scenario analysis, life cycle assessment, life cycle costing, social sustainability assessment, and multicriteria decision analysis, which are used to assess and identify the most sustainable energy options [9].

The current interest in sustainability focuses on a number of areas including environmental protection, health and safety, energy efficiency, product stewardship, industrial ecology, renewable resources, water management, green chemistry, and national security. But perhaps the greatest challenges to sustainability are the issues of climate change and carbon management, especially those associated with energy production and consumption [5].

Renewable energy sources are considered a cleaner way to produce energy as they emit a reduced amount of greenhouse gases compared to traditional sources of energy. However, there are negative impacts associated with their use that have to be considered to ensure a more sustainable energy system as a whole [8].

2.1.1 Economical Aspect of the Network

Electricity is the prime mover of modern society. Per capita consumption of electricity is an indicator of a country's growth. This indicator depends on the availability, quality, and reliability of power and most particularly its overall cost. The cost of electricity governs its usage [10].

The advent of new generation, storage, and demand response technologies for improving energy efficiency will dramatically change the characteristics of supply and demand for electricity at both micro and macro levels. Consequently, econometric models of consumer behavior are extremely important for understanding the qualitative and quantitative characteristics of demand response and efficient integration of the related technologies [11]. The closely related measure of responsiveness and sensitivity of demand to small variations in price, i.e., the price elasticity of demand is also examined within the same quantitative framework. The cost of energy production is divided into three subcategories below. Generation companies use these data along with the decisions taken by the policymakers to find their production strategy.

- Capital cost comprises the cost of construction and installation of a power plant.
- Total annualized cost is the sum of annual capital cost plus annual fixed cost plus annual variable cost excluding the fuel cost plus the annual fuel cost.
- The levelised costs or unit energy costs represent the cost of energy generated over the lifetime of a power plant, expressed per unit of energy. It is calculated by dividing the total annualized cost by the total annual energy generation in the same year [9].

As for the consumer-supplier relation, electricity pricing is a major issue to consumers for its liberal use. But due to the demand of social and technological advancement, the change in power networks and their mode of operation are inevitable. Revolutionizing electric power industries encompasses technical and also non-technical economical issues. For maintaining reliable power available in deregulated and smart grid environments, recurring and non-recurring expenditures in different head components have increased a lot compared to regulated systems, resulting in a high cost of electricity. Limiting this cost within the reach

of consumers is thus the chief motivation of research in this area, i.e., optimizing electricity pricing in modern power networks [10].

2.2 Game Theory Literature Review

Game theory provides a formal analytical framework with a set of mathematical tools to study the complex interactions among rational players. Throughout the past decades, game theory has made a revolutionary impact on a large number of disciplines ranging from engineering, economics, political science, philosophy, and even psychology [12]. In recent years, there has been significant growth in research activities that use game theory for analyzing communication and power networks.

Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another welfare [12].

In the language of game theory, a game refers to any social situation involving two or more individuals [12]. The individuals involved in that game may be called players. A player may be interpreted as an individual or as a group of individuals making a decision. The models of game theory are highly abstract representations of classes of real-life situations. Their abstractness allows them to be used to study a wide range of phenomena. For example, the theory of Nash Equilibrium has been used to study oligopolistic and political competition. The theory of mixed strategy equilibrium has been used to explain the distributions of tongue length in bees and tube-length flowers. The theory of repeated games has been used to illuminate social phenomena like threats and promises. The theory of the core reveals a sense in which the outcome of trading under a price system is stable in an economy that contains many agents [13].

As stated in the definition above, there are two basic assumptions that game theorists generally make about the players: (i) they are rational and (ii) intelligent:

1. A decision maker is *rational* if he makes decisions consistently in pursuit of his own

objective. In a game theory, building on the fundamental results of decision theory, we assume that each player's objective is to maximize the expected value of his own *utility* scale.

2. A decision maker is said to be *intelligent* if he knows everything that we know about the game and he can make any inferences about the situation that we can make [12].

Once we define the set of players, we may distinguish two types of models: those in which the set of possible actions of individual players are primitive and those in which the set of possible actions of a group of players are primitives. Sometimes, models of the first type are referred to as non-cooperative while those of the second type are referred to as cooperative. Game theory can be divided into two branches: non-cooperative [14] and cooperative game theory [12], [15]. Non-cooperative game theory studies the strategic choices resulting from the interactions among competing players, where each player chooses its strategy independently for improving its own performance (utility) or reducing its losses (costs). For solving non-cooperative games, several concepts exist such as the celebrated Nash equilibrium [14]. While non-cooperative game theory studies competitive scenarios, cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions [12], that can strengthen the players' positions in a game.

Game theory has a very general scope, encompassing questions that are basic to all of the social sciences. It can offer insights into any economics, political, or social situations that involves individuals who have different goals or preferences [12].

2.2.1 Strategic Games

A strategic game is a model of interactive decision-making in which each decision-maker chooses his plan of action simultaneously and independently assuming all players are rational. When referring to the actions of the players in a strategic game as "simultaneous" we do not necessarily mean that these actions are taken at the same point in time. For situations to be modeled as a strategic game, it is important only that players make decisions independently, or no player is informed of the choice of any other player prior to making his own decision.

Extensive game is a detailed description of the sequential structures of the decision problems encountered by the player in strategic solution. Two scenarios for extensive games exist: (i) the perfect information and (ii) imperfect information. The *perfect information* scenario is such a game if each player, when making a decision, is perfectly informed of all the events previously occurred [13]. The game with *imperfect information or incomplete information* is a game in which, at first point in time when the players can begin to plan their moves in the game, some player already have private information about the game that other players do not know. The initial private information that a player has at this point is called the *type* of the player [12].

2.2.2 Mixed Games

Mixed games are games with on one hand an “ocean” of insignificant players (formalized by a continuum of players) and on the other hand some significant players (atoms) [16]. In cooperative game theory, games including small and large agents are denoted as mixed games. Such games are useful in modeling markets with a dominant big corporation and several small entities [17].

The class of games considered here involves a major player and many minor players. The major player has a significant role in affecting others, but each minor player possesses only a weak influence. This kind of interaction modeling is motivated by many socio-economic problems. A typical situation is an interaction between one or more large corporations and many, much smaller, competitors. Traditionally, models differentiating the strength of players have been well studied in cooperative game theory, and they are customarily called mixed games, with the players accordingly called mixed players. For non-cooperative games with large players and many small players, the work [17] examined the notion of the negligibility of small players. In this dissertation, we will adopt a population of mixed players for the development of non-cooperative stochastic dynamic games [18].

2.2.3 Bidding in the Standard Auctions

In a sealed bid auction, each bidder independently and privately picks a price and offers to buy the goods at that price. The one who bids the highest price wins. The Dutch auction is seemingly quite different: The auctioneer calls the prices, beginning with a high price and proceeding to successively lower ones. The bidder listens to the price called, notices whether any other bidder has accepted a price, and finally accepts some price if no other bidder has done so first. In English auction houses, the auctioneer begins with the lowest accepted price, i.e. *the reverse price*, and proceeds to solicit a successively higher bid from the customers until no one will increase the bid [19].

Consider first the descending auction. Each bidder must choose a price at which she will call out, conditional on no other bidder having yet called out; and the bidder who chooses the highest price wins the object at the price she calls out. Thus this game is strategically equivalent to the first-price sealed-bid auction and players' bidding functions are therefore exactly the same.

With private values, in the ascending auction, it is clear that a dominant strategy is to stay in the bidding until the price reaches your value, that is until you are just indifferent between winning or not winning. For the second-price sealed-bid private-values auction, it is optimal for the player to bid her true value, whatever other players do. Hence, in that case, 'truth telling' is a dominant strategy equilibrium. A key feature of bidding in auctions with common-values components is the winner's curse: each bidder must recognize that she wins the object only when she has the highest signal (i.e., in symmetric equilibrium). In equilibrium, bidders must adjust their bids downwards accordingly.

2.2.4 Mean Field Game Theory

Mean Field Game theory (MFG) studies the existence and generation of strategies of Nash equilibrium for stochastic dynamic games between a large population of agents [20–22]. Basically, the theory exploits the relationship between the finite and corresponding infinite limit population problems to find control solutions with negligible error. The heart of this approach is the solution to the Hamilton-Jacobi-Bellman (HJB) partial differential equation(PDE) and

the Fokker-Planck-Kolmogorov (FPK) PDE (equivalently McKean-Vlasov stochastic differential equation) of the generic agent, which is linked by the state distribution of that generic agent.

The original motivation for the development of Mean Field Game (MFG) theory lies in the code-division multiple access CDMA network optimization analysis of Huang et al. [23–25], where the problem is tackled as a centralized stochastic control problem for a finite number of users. It was the computational intractability of the centralized CDMA power control optimization problem [24] that motivated the development of MFG theory which was then called Nash Certainty Equivalence (NCE) theory. This generated the work in [26–28], where the problem is in general analyzed in a Linear Quadratic Gaussian (LQG) framework, and in [22, 28], where the nonlinear theory is developed. In particular, cost functions varying in space (called locality-dependent cost functions) were investigated in an MFG LQG framework in [29]. Independent of that sequence of papers, MFG was developed in [30–32], where it was given its current name.

Recently in [33, 34], Ferrari et al. solved an analogous decentralized optimal problem using an approach called the generalized reduced gradient (GRG) method. The approach is to find the micro-agent control strategies that optimize a macro-agent cost function using the GRG method. In essence, the GRG method is to optimize a macroscopic behavior of the system subject to coupled microscopic behavior; it relies on the existence of an accurate macroscopic evolution equation [34].

Computational investigations for MFG numerical implementations are presented in the work of Achdou et al. [35, 36], where numerical methods are given for the approximation of the stationary and evolutionary versions of the MFG models of Lasry et al. [30–32]. In particular, the existence and uniqueness properties of the various discretization schemes are established together with bounds for the solutions and convergence theorems for finite difference method. In their work, the cost function is assumed to be quadratic and Newton-Iterations techniques are used after linearization of the MFG discretized equations. In contrast, in the present work, a general framework for solving the coupled Hamilton-Jacobi-Bellman equation and Fokker-Planck-Kolmogorov equation is presented, where the cost function is evidently not

quadratic.

2.3 Dynamical Games Formulation for the Power Network Problems Literature Survey

In light of the limited availability of current energy sources and environmental concerns, the introduction of renewable energy resources (RERs) aims to meet the increasing global electricity demand. Demand response (DR) emerges as a potential approach to harness RERs and address the fluctuating electricity requirements of consumers. To implement DR within distribution networks (DN), various control techniques, decision variables, and incentives are employed. Intelligent algorithms are utilized to achieve this optimization; however, they encounter computational constraints when dealing with the uncertainties associated with RERs and the power system. Consequently, this chapter focuses on examining the limitations of intelligent algorithms for DR and presents a comparative analysis of different algorithmic approaches to the approach presented in this dissertation [37].

The introduction of Microgrid (MG) signifies notable progress in maximizing the efficient utilization of Distributed Energy Resources (DERs). Within MG, DERs are effectively controlled through advanced communication protocols and controllers, with the goal of enhancing the overall functionality of the conventional power system. Essentially, an MG can be viewed as a preliminary model of a system of systems (SOS). The central component of an MG revolves around the synchronized cooperation among interconnected local generation units within a network. Figure 2.1 depicts a visual summary of the conceptual framework of a smart grid (SG) and Figure 2.2 illustrates the comprehensive structure of a Microgrid (MG) within distribution networks (DN), showcasing its detailed architecture [38].

In recent years, there has been a growing interest in direct current (DC) microgrids among researchers and industry professionals. These MGs have been applied in various projects, including renewable energy source parks, DC-powered homes, energy storage systems (ESSs), electric vehicles (EVs), and charging stations. Several countries, such as the United Kingdom, China, Canada, United States, Kuwait, and Turkey, have already adopted demand-side management (DSM) techniques to mitigate losses in the traditional power grid. These devel-

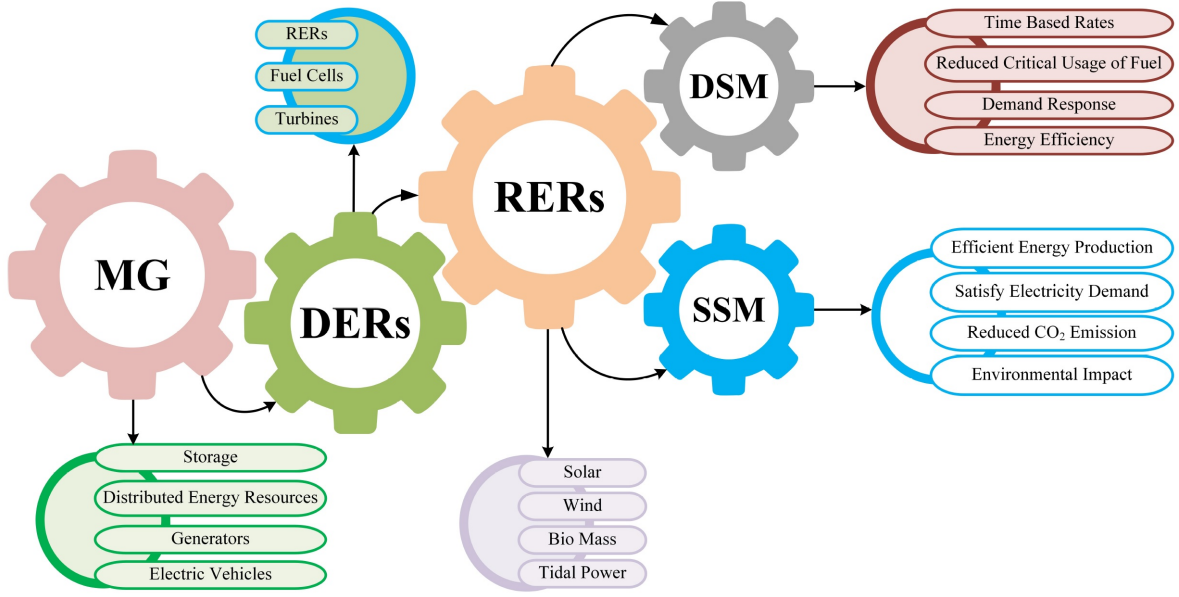


Figure 2.1 Conceptual framework of a Smartgrid

opments indicate a promising future for MGs in the market. In addition, demand response (DR) encompasses control techniques (centralized and decentralized control), decision variables (energy management and task scheduling), and motivations (incentive and price-based) to manage electricity demand. However, existing DR schemes require ongoing optimization. To address this, intelligent algorithms are employed to optimize DR techniques. The DN utilizes RERs to balance supply and demand through optimized DR methods. Nevertheless, advanced intelligent algorithms face challenges due to the stochasticity nature of the parameters causing computational burden. In this context, machine learning is often the default approach. However, these methods fall short and perform poorly as the number of households in the microgrid increases. Mean Field Game Theory algorithms offer a solution to overcome these limitations. Thus, in this thesis, we recommend the use of MFG applications to accurately quantify the uncertain nature of RERs in future DN systems, enabling optimized DR.

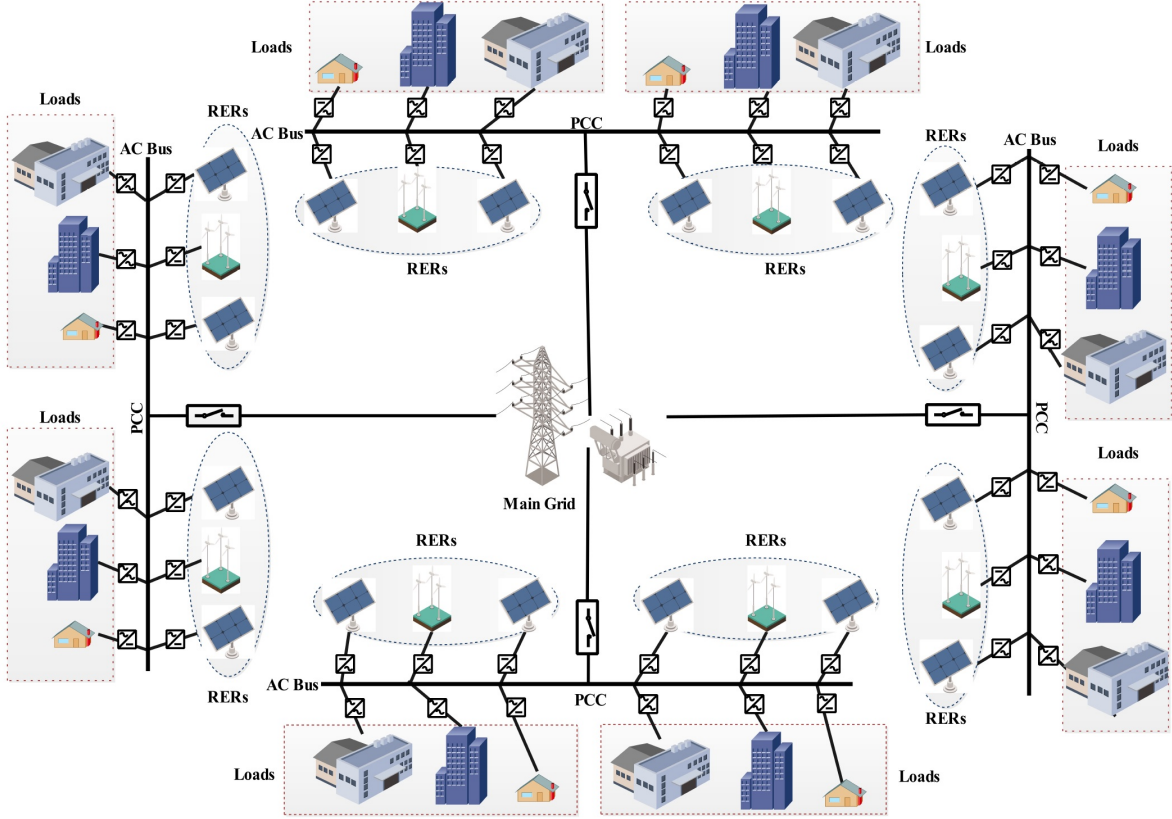


Figure 2.2 Microgrid Architecture

2.3.1 Demand Response: Summary and Objectives

Demand response (DR) involves adjusting the consumption patterns of consumers to align with times when electricity supply is abundant or when there's reduced demand, often through financial incentives or pricing strategies. Coupled with smart grid technology and energy storage solutions, demand response plays a pivotal role in ensuring the stability and dependability of power networks, especially given the fluctuations of renewable energy sources and the increasing demand for electricity. Specifically, DR refers to the modification of a user's energy portfolio beyond their usual consumption. This modification depends on two factors:

- The dynamic price rate (main grid pricing strategy), which varies in real-time.
- Incentives provided to encourage reduced electricity usage during peak hours.

DR programs play a vital role in establishing a connection between MGs and the market. These programs operate through two main streams: system reliability-oriented programs and market-oriented programs. Reliability-oriented DR programs encourage consumers to adopt controlled appliance usage by providing them with economic incentives. Participation in these programs can be voluntary or involuntary, depending on the user's preference. On the other hand, market-oriented DR programs enable consumers to receive flexible dynamic electricity prices in real-time, empowering them to adjust their load consumption based on the current electricity rates.

Market-oriented DR programs can be further categorized based on user characteristics. The first category is the demand bidding program, which benefits large energy consumers capable of flexibly adjusting their load as needed. Typically, these users face fixed electricity prices under normal conditions but receive appealing incentives as market prices increase. Another option available to consumers, particularly in industrial and commercial projects, is the real-time pricing program. In this program, the electricity generation price for consumers varies over time, while the costs associated with transmission and distribution infrastructure remain constant.

The time-of-use pricing program, which is voluntary, involves charging consumers based on varying rates that reflect basic production costs. This pricing structure encourages reduced consumption during peak hours. To provide dynamic pricing to users and ensure system reliability, the critical peak pricing program is introduced. This program focuses on supplying electricity at rates determined by real market conditions, ensuring that users pay according to the prevailing rates. To avoid any confusion, the pricing schedule is provided either a day in advance or on an hourly basis.

In this dissertation, the considered DR program is the reliability-oriented DR programs where households get incentive during peak event which is communicated a day ahead.

2.3.1.1 Advantages of Demand Response

Demand response (DR) programs offer numerous benefits to the power system, one of which is the reduction in electricity supply prices. These programs allow for swift adjustments to

market conditions, ensuring ample reserve margins and stable pricing [39]. Additionally, a network-driven DR program plays a crucial role in maintaining system reliability by balancing the supply and generation ratio through the segmentation of demand into shorter time intervals.

DR programs yield several advantages in terms of environmental impact. By curbing energy generation, these programs contribute to the avoidance of environmental harm [40, 41]. Through effective scheduling facilitated by DR, excessive fuel consumption during activities like generator startup and ramping can be minimized, resulting in decreased emissions and reduced electricity generation [42, 43]. Environment-focused DR programs prioritize addressing environmental concerns such as mitigating greenhouse gas effects, improving energy efficiency, and promoting the adoption of environmentally friendly generation methods. DR also enhances the real-time efficiency of the power system. By dividing time into short, medium, and long terms, DR ensures that electricity flows within the technical limits of the system during shorter and medium timeframes. This helps alleviate network peaks and averts system failures. In the long term, DR reduces overall generation, leading to lower network maintenance costs and improved system efficiency [44].

Furthermore, DR enables the synchronization of technical systems with dynamic market pricing. Time-based pricing, facilitated by DR, optimizes various aspects such as scheduling in advance (day-ahead and hour-ahead), incorporating distributed generation (DG) and renewable energy resources (RERs), reducing peak loads, achieving a balance between demand and supply, and implementing real-time control. This alignment not only results in reduced carbon dioxide (CO₂) emissions but also lowers network expenses [45].

2.3.1.2 Limitations of Demand Response

Barriers related to power system characteristics and energy sector liberalization hinder the full implementation of demand response (DR). These barriers include hardware requirements, coordination issues, and tax considerations.

Hardware setup, such as smart meters, presents a challenge as the cost needs to be allocated among consumers, aggregators, retailers, and distribution system operators (DSOs).

Splitting the responsibility between aggregators and consumers can help address this split incentive problem. Proper coordination among all actors involved in DR is essential, especially when adjusting demand in real-time and considering generation and transmission separately. Clear pricing rules and tax regulations in the liberalized electricity sector can help minimize coordination issues. Policymakers should prioritize these objectives to facilitate the successful deployment of DR at the local level.

Demand response with a centralized controller also known as the Demand Load Control (DLC) method is a basic form of Demand Response (DR) that involves a central operator exerting control over consumer loads. This approach becomes particularly important during emergency situations or contingencies. However, it restricts the consumer's freedom to manage their own load. The limitations of such an approach can be summarized by the complexity of security required to guarantee a safe transfer of data between the centralized unit and the households, in addition to the computational overhead of such an approach in real-time especially with a large number of households.

2.3.2 Renewable Energy Resources (RERs) and Battery Storage system Summary and Objectives

Integrating renewable energy resources (RERs) like solar and wind power provides a viable solution for reducing the environmental impact of fossil fuel-based energy generation. However, effectively incorporating these RERs into MG planning poses challenges due to their unpredictable and intermittent nature. The main technologies of distributed energy resources RERs are:

- Wind Turbine Systems: Modeling the power generated by the wind turbine relies on factors such as tower height, velocity speed exponent, wind shear, and the characteristics of the turbine in relation to the wind.
- Solar Panel PV systems: Photovoltaic (PV) technology consists of a collection of PV cells interconnected either in parallel or in series to convert solar energy into electrical energy. Modeling the power generated by the PV relies on solar radiation, module efficiency, and the surface area of the PV panel.

- Hydroelectric turbines (HT): HT utilizes the kinetic energy present in flowing water to generate electricity, employing a dam, penstock, and forebay in the process. These turbines are designed to operate without causing any adverse environmental impacts, adhering to the specifications provided by the manufacturers. The choice of an appropriate HT for a particular application relies on factors such as the net head, fluctuations in flow discharge, rotational speed, cost, and other relevant considerations.

The RER considered in this dissertation is the solar panel. In the constructed microgrid, households deploy solar panels and it is the only source of energy generation.

2.3.2.1 Battery Storage System BSS

A battery storage system (BSS) allows for the storage of electricity generated from renewable energy resources (RERs) during periods of low power consumption, typically off-peak times. This stored energy can then be fed back into the power system during peak load periods when there is a high demand for power.

In this thesis MG, households have the capacity to sell energy from their storage system during peak-event or when it is optimal. In addition, in this dissertation, we deliberately omitted an extensive analysis of the battery storage system available for households. It was assumed that households would adopt uniform battery storage systems with maximum capacity. To streamline my research, we did not consider the charging and discharging losses when modeling the generation and storage functions of households. Nonetheless, we firmly believe that the approach outlined in this thesis, along with the published papers, can be readily implemented to address the broader challenge of the microgrid (MG) problem, encompassing charging and discharging as well as transmission costs.

2.3.3 Computational Models for the Microgrid Optimization Method

This subsection presents a survey of the mathematical techniques used in literature for modeling the stochasticity in the MGO problems. In addition, we will highlight the limitations of each approach and summarize the pros and cons of the MFG approach compared to what is in the literature.

2.3.3.1 Classical Mathematical Algorithms

Different mathematical solvers are available to improve the management capabilities of an MG. In one study referenced as [46], the General Algebraic Modeling System (GAMS) package was utilized to evaluate the linear optimization problem within the MG and solve for uncertainties associated with the generation process [47]. Another work cited in [48, 49] implemented a multi-scenario Mixed-Integer Linear Programming (MILP) model for energy management. The outcome from [49] was employed to address the scheduling problem of the MG using Mixed-Integer Programming (MIP) [50]. Additionally, [51] demonstrated the effectiveness of the previous approach presented in [50] in handling large-scale MILP problems. Another technique mentioned is the Sparse Nonlinear Optimizer (SNOPT), which is based on GAMS and can solve nonlinear optimization problems [52]. The energy management system formulated as a nonlinear programming problem can be solved using SNOPT [53]. Furthermore, the Gurobi optimizer is used to optimize MILP problems [53, 54], as also seen in [55] for computational purposes.

2.3.3.2 MG Optimization Via Machine Learning

Using stochastic methods, it is possible to approximate the probability distribution function (PDF) of random variables [56]. Dantzig was a pioneer in using a stochastic approach to model uncertainty [57]. However, the stochastic approach requires knowledge of the input factors [58]. When models involve uncertainty and scenario-based generation, knowing the PDF is necessary. This involves breaking down the PDF into multiple parts, each representing a scenario-based outcome, and focusing on the preferred probability area of each part [59]. A scenario-based generation has been implemented in the context of photovoltaic (PV) and wind turbines (WT) to handle uncertainties [60]. Discrete distribution sets are used to represent scenarios for solar, wind, and load uncertainties, and Monte Carlo Simulation (MCS) and a roulette-wheel mechanism are employed to generate random scenarios [48]. To reduce computational complexity, a scenario reduction algorithm is utilized [47]. MCS is also used in modeling PV and WT, with the PDF used to supervise input parameters [50]. Analysis of results involves using histograms and other stochastic criteria indicators [61]. Fuzzy theory is

applied to renewable energy resources (RERs) [62], and fuzzy methods are used to analyze the effects of uncertain power output in distribution networks (DNs) [63]. Robust optimization is employed to address parametric uncertainty issues when PDF parameters are absent [64]. Additionally, optimization algorithms, such as robust optimization, are utilized to optimize solar and wind power systems [65].

In the domain of demand response (DR), different optimization strategies are available to achieve optimal DR strategies, including swarm-based algorithms, quasi-static techniques, and mixed-integer approaches [66,67]. Mutual operation of available distributed generators (DGs) in MGs is optimized to control load [68,69]. Integration of optimization techniques with DR enhances system stability, maximizes economic benefits, and provides control over frequent disconnections [70,71]. Automation plays a crucial role in DR, such as automated real-time pricing schemes for electric vehicle charging stations [72], network optimization protocols to balance demand and improve communication among DR actors [73,74], and hierarchy-based power network structures in combination with DR to improve distribution and sustain minimum system voltage [75]. Dynamic demand management algorithms optimize energy consumption and real-time prices, integrating with automated control systems to empower consumers in managing their energy consumption [76]. Virtual power plants (VPPs) supported by network communication manage conventional and distributed energy resources (DERs) [77]. Smart networks facilitate system sustainability through dynamic profiling of user load consumption and optimized reconfiguration of network frameworks for DR [78,79].

Smart grids (SGs) encompass the renovation of transmission and distribution networks, reduce losses, optimize grid operations, and facilitate the market for DGs with the integration of energy storage systems (ESSs) [80–83]. Integration of DR in SGs enables energy reduction during peak hours while ensuring consumer comfort [84]. MGs play a vital role in DR management by organizing users in the network, utilizing ESSs, and renewable and non-renewable energy resources through DG, and enabling bi-directional energy transactions with distribution companies [85]. MGs can operate in isolated mode by adjusting their topology and incorporating interconnecting devices, with a multi-agent management system facilitat-

ing energy sales and purchases within the MG [86, 87]. Asymmetric failures in MGs, such as short-circuits, grounding failures, and double failures, can be analyzed and addressed through techniques like asymmetric failure analysis and current and voltage matrices [88, 89]. Incentive-based DR programs contribute to MG management by reducing peak-hour consumption and optimizing energy costs [90, 91].

2.3.3.3 MFG Computational Approach

A significant aspect of the efficiency and readability of mean field games theory stems from its ability to utilize the power of differential calculus. This sets it apart from previous developments in games with an infinite number of players, which primarily relied on measure theory to represent the continuum of players. Mean field games theory takes a new mathematical direction by extensively incorporating differential calculus, a powerful tool that has seen major advancements in various applied fields.

The integration of differential calculus in mean field games theory opens up new possibilities and advantages. It allows for easier handling of player entries and exits compared to N -player game theory. Representing players as a continuum enables smooth modeling of the renewal of player generations. States, time, and player distributions in space are treated as continuous variables, and entries and exits are straightforward flows that can be technically addressed without difficulty. Overlapping generation models can be implemented seamlessly.

Another notable development in mean field games theory is the introduction of a social dimension. Statistical data on other players become fundamental components of individual strategies in these models. By approximating N -player games with the mean field games limit, incorporating corrective terms such as $1/N$ if necessary, this approach introduces a "social" aspect even in limited groups of a few hundred agents.

Considering the positive effects compared to N -player games, it is natural to view mean field games as potential solutions to N -player game problems. For instance, when dealing with relatively small N -player games, where N is on the order of a few dozen and involves player entries and exits, the mean field games limit ($N \rightarrow \infty$) often provides a good initial approximation to the N -player solution. The first term of the $1/N$ development is typically

sufficient to accurately describe the effects caused by granularity arising from the finite and relatively small value of N .

In simpler terms, the complexity of the Mean Field Game (MFG) remains unchanged as the total number of agents in the game increases. This makes MFG a promising tool for finding optimal solutions in real-time optimization problems. One of the primary advantages of MFG is its decentralized nature, which requires minimal communication between agents compared to other techniques. The key information needed for MFG algorithms is the initial probability density function of the state variables. The MFG algorithm solves the propagation of statistical information, allowing agents to determine the mean at each time step and consequently find their optimal solutions. The main advantage of the MFG approach can be summarized as follows: the MFG loop, which is a crucial feature of the approach, can be solved offline. This means that when applying the MFG approach in real-time optimization, its performance can be likened to an open-loop control problem. Therefore, the MFG approach holds great potential as an algorithm for solving the real-time microgrid power network optimization problem.

CHAPTER 3 A DECENTRALIZED GAME THEORETIC APPROACH FOR VIRTUAL STORAGE SYSTEM AGGREGATION IN A RESIDENTIAL COMMUNITY MICROGRID

3.1 Abstract

The current electricity market is better described as an oligopoly than a market of perfect competition from which, in fact, it may be rather far. The increasing penetration of residential distributed energy resources has led to a significant number of prosumers in the electricity market. Microgrid community, a group of single controllable entity prosumers, is a promising component of the smart grid which will potentially yield a free electricity market. In this paper, we present a novel formation for a residential community microgrid that includes a coalition of prosumer households with solar photovoltaic systems. These households are connected through a virtual power bank that consists of households' storage batteries and that mediates the communications between the households and the main grid. Using an application of mean field game theory, we find Nash Equilibrium strategies under which such sharing could minimize a linear combination of the households' energy generation cost, energy consumption cost and revenue of sold energy. The resulting approach is tested on a case study of a constructed community micro-grid in Montreal, Quebec, Canada. The proposed mean field game approach can help decrease the aggregated cost and the individual energy cost. A comparative analytical study on the benefit of sharing was also performed, demonstrating that each prosumer is expected to have at least a 40 percent reduction on their individual cost if belonging to a microgrid community of 100 prosumers located in Montreal city.

3.2 Introduction

Electricity, an essential commodity, is unlike other commodity markets in that it is traded through networks with sets of strict constraints both physical and operational. When modeling the competitive electricity market, one must consider the following: the electricity quantities, the market clearing price, and the strategic interactions among the participants

which determine the market efficiency. By construction, the electricity networks provide opportunities and constraints on the market participants to adopt strategic behaviors. Thus, game theory enables a potential framework and analytical tool to model the electricity network problems [92]. Furthermore, in the smart grid paradigm, several decision makers will interact to implement a policy where each has their own objective function and system dynamics. The coupling occurs in the cost functions as the decision of an agent affects the payoff of the other agents.

The concept of residential microgrid (MG) has emerged as a promising platform that coordinates and integrates a potentially large number of distributed generations, energy storage systems and local loads. MG consists of a network of distributed energy resources connected locally within a specified geographical area. Residential microgrid can operate not only in a grid-connected mode, but also in stand-alone mode. The objective of MG is to attain a steady and effective energy supply via a simplified implementation of smart grid functionality [22]. MG provides a collaborative environment wherein various participants cooperate to achieve energy savings, peak load shifting and increased social welfare by deploying flexibility assets such as energy storage systems. Coordinating these energy storage systems enables efficient use of renewable energy sources inside the residential microgrid, allowing end-users to cooperate and share the stored energy and achieve higher economic benefits. To this end, storage system aggregator can play an active role in coordinating users' charge and discharge decisions by setting the price of energy sharing; thus, maximising the social welfare. Formulating the MG network problem as a collaborative economic operation, however, is a major challenge. In fact, the problem of operational planning of residential community microgrid for end-users in an energy sharing environment is an active field of research.

3.2.1 Relevant Work

The existing research on residential microgrid mainly focuses on: (i) demand response management using various optimization techniques, (ii) interactions with the main grid through the aggregator, and (iii) the coordination of local flexibilities such as photovoltaic systems, electric vehicles and storage systems. For instance, Celik et al. in [93] proposed a central-

ized energy management system to investigate the impact of distributed residential PV and aggregator energy storage assets on the economical performance of the aggregated network. However, in [93], the energy storage system belongs to the aggregator which is optimizing the network in a centralized fashion. In [94], a novel privacy-preserving distributed parallel optimization framework that allows the participation of a large-scale aggregation of prosumers with residential PV-battery systems in the market for ancillary services is proposed. Correa-Florez et al. in [95] presented an optimization model for aggregated home energy management systems, where the aggregator manages a set of electric water heaters. Although robust optimization is used to include uncertainties, the management of resources for day-ahead energy and local markets takes place in a centralized fashion. In [96], Paudyal et al. proposed a new integrated hierarchical control framework for residential energy optimization that coordinates and controls large electric appliances taking into account residents' comfort and suppliers' rewards. However, in [96], active distributed energy resources within the residential community was not considered.

Most recent research presents models based on offline schemes where the objective is to predict the day-ahead actions. Chen et al. in [97] modeled the Monte Carlo price-based demand response management for residential appliances via real-time stochastic and robust optimization. Huang et al. in [98] adopted a Lyapunov optimization technique that minimizes the MG operation cost. Their approach uses an adaptive electricity scheduling algorithm taking into account the quality-of-service in electricity virtual queue and energy storage virtual queue. In addition, Garg et al. in [99] minimized the production cost of a MG by proposing an online convex optimization program for the MG with a single turbine-boiler generator. In [100], authors proposed a hierarchical smart grid interactive architecture to optimize the quality of service and the stability of the MG. Chen et al. in [101] proposed a smart energy management system to optimize the MG operation; the system consisted of an energy storage system and a power forecasting module. In [102, 103], Zheng et al. solved a fully distributed reactive power optimization problem for active distribution network. In essence, the approach there is to offer the potential of finding the globally optimal solution inspired by augmented Lagrangian method and the semidefinite programming convex optimization technique presented in [104].

3.2.2 Mean Field Game Theory

Game theory enables an analytical framework consisting of mathematical tools to formulate the coupled and complex interactions between the strategic rational players. Recently, game theory and its applications have been used significantly to analyze and model power networks and communication networks problems. In particular, the contributions behind these novel models and algorithms are capturing and analyzing the following characteristics of the emerging smart grid: (i) the heterogeneous aspect of the smart grid, (ii) the necessity for a low-complexity distributed algorithms that model the collaborative or competitive scenarios between the different participants of the smart grid [105], and (iii) the various participants interacting in the smart grid. Chen et al. in [106] proposed a hierarchical structure of users providing operating reserves to the power system through load aggregators. The bidding problem of load aggregators is formulated as Nash game, where both the existence and uniqueness of the market equilibrium is demonstrated. In [107], a monopolistic game-based approach for the management of energy flexibility through end-users, aggregators, and the distribution system operator is proposed. Mayorga in [108] proposed a data-based Stackelberg market strategy for a distribution market operator for the coordination of power dispatch among various virtual power plants.

Mean Field Game (MFG) theory is an emerging platform and a methodology that enables the analysis of a multi-agent coordination problems in which the dynamic of each individual agent is impacted by the statistical behaviour of the population, and in which the individual agent effect on population distribution diminishes as the number of agents increases [109]. MFG theory studies the existence and generation of Nash equilibrium strategies for sophisticated dynamical games with large population of agents [21, 22, 110]. Basically, the theory finds control solutions with negligible error to the finite population problem compared to the infinite limit population problems. The core of the MFG approach is embedded in the solutions to the the Fokker-Planck-Kolmogorov (FPK) and Hamilton-Jacobi-Bellman (HJB) partial differential equations of the generic agent, which are coupled by the state distribution of the generic agent. Independent of that sequence of papers, MFG was developed in [32]. Numerically applying the MFG approach is only concerned with estimating the aggregated

effect (i.e. mass), thus, the numerical complexity of the MFG approach is indifferent to the size of the population.

In a smart grid paradigm, MFG has been applied to many types of populations such as thermostatically controlled loads in [111], [112] and electric vehicles in [113]. However, a concrete construction of mean-field games in a community microgrid that addresses all factors of energy exchanged has not yet been designed. In particular, the design of a decentralized autonomous MG where households, locally connected to the same MG, are independently able to find their best responses has not been fully addressed in the current framework of smart grids. In [114], Eduardo et al. addressed the dynamic population games and a droop control via hierarchical microgrid management system. The notion of community energy storage has been addressed in literature before where the game formed was non-cooperative [115] and where equilibrium prices for MGs and the main grid are only set by a centralized unit. Contrary to [115], this paper models the MGO as a cooperative game where households' aim to find their best response that minimizes their cost function and in turn minimizes the aggregate cost of the MG.

3.2.3 Contributions

The objective of this paper is to bridge the gap between the aforementioned research directions on residential microgrid and the application of MFG theory. We intend to optimize the energy exchange within a residential microgrid community operating in grid-connected mode focusing on the formulation and the optimization of the microgrid problem. In this respect, we present a novel formation for a residential community microgrid that includes a coalition of prosumer households connected through a virtual power bank which mediates the communications within the MG and with the main grid.

To the best of the authors' knowledge, no prior studies have modeled the community microgrid optimization (MGO) problem as a dynamical game without a centralized unit. The proposed framework for microgrid management can be seamlessly integrated within recent grid technologies that eliminate the need for a central operator. Specifically, blockchain can be integrated with the presented mean field game theoretic optimization whereby energy

crowd-sourcing can be performed in the operation time-scale; see the recent studies [116,117] and the references therein for how blockchain can be utilized in grid management. The main contributions of this paper are as follows:

- We present the concept of virtual power bank (VPB) which depicts a virtual network of the households' storage systems. In addition, the VPB is connected to the main grid and it plays the role of local electricity market, enabling sharing among a large number of prosumers in the microgrid.
- We develop the first application of non-linear decentralized mean field game theory to the residential community microgrid along with the associated numerical algorithms that solve the constructed decentralized non-linear MGO problem. We demonstrate that the finite problem converges to the infinite limit population problem with negligible error.
- We formulate the MGO problem for the constructed MG via means of dynamical game theory, where the connected households cooperate to minimize the total energy cost of the formulated microgrid while simultaneously maximizing their net profit individually via means of decentralized mean field game theory.
- We prove the concept of the proposed approach through a test case simulations conducted on a constructed community microgrid in Montreal, Canada. We find that every prosumer can achieve a minimum of 40 percent reduction of their individual cost by joining a microgrid community composed of at least 100 prosumers.

3.2.4 Paper Structure

The rest of the paper is structured as follows: Section II presents the microgrid architecture, its relevant elements, and the assumptions of the proposed approach. Section III discusses the formulation of the system dynamics including the models of solar PV, prosumers' total operational cost, and the dynamics of the virtual power bank. Section IV presents the game formulation problem via means of dynamical MFG where we specify the mean field Hamilton-Jacobi-Bellman equation (MF-HJB) and the mean field Fokker-Planck-Kolmogorov

equation (MF-FPK) for each household. Subsequently, Section V describes the algorithmic implementation of the decentralized community MGO problem including the numerical results for a community MG. Finally, in section VII we conclude the paper.

3.3 Description of the game

3.3.1 System structure

Figure 3.1 illustrates the structure of the proposed residential community MG, which is operating in grid-connected mode. Each household has solar panels, representing their energy generation source, and has energy storage systems (i.e. batteries). Households' batteries are connected to each other through the virtual power bank. In the case of energy shortage in a household, the household will buy the required energy deficit from the MG through the virtual bank. If there is a shortage in the MG power bank, households buy their required energy from the main grid through the power bank. The proposed architecture presented in Figure 3.1 consists of a large number of prosumer households connected locally to each other through the virtual power bank, which connects the MG to the main grid. In this framework, the households' represented in Figure 3.1 are uniform houses, in the sense that all households are identical in terms of loads (i.e. appliances installed), generation potential (i.e. solar panel technology) and the internal structures of these houses. This network of uniform entities can be seen in any new construction of chalet complexes or mass built townhouses; thus the microgrid constructed in this paper can be used to model these projects and the presented algorithm and framework can be generalised in future research to include non-uniform houses. For the scope of this paper uniform houses microgrid is considered.

3.3.2 Virtual Power Bank

In this framework, the network of households' batteries and households' energy profiles constructs the virtual power bank (VPB). Hence, the VPB can be defined as a virtual coalition of households' batteries (i.e. a virtual aggregator) available on the microgrid scale (community level), and connected to the main grid and to all prosumers within the microgrid.

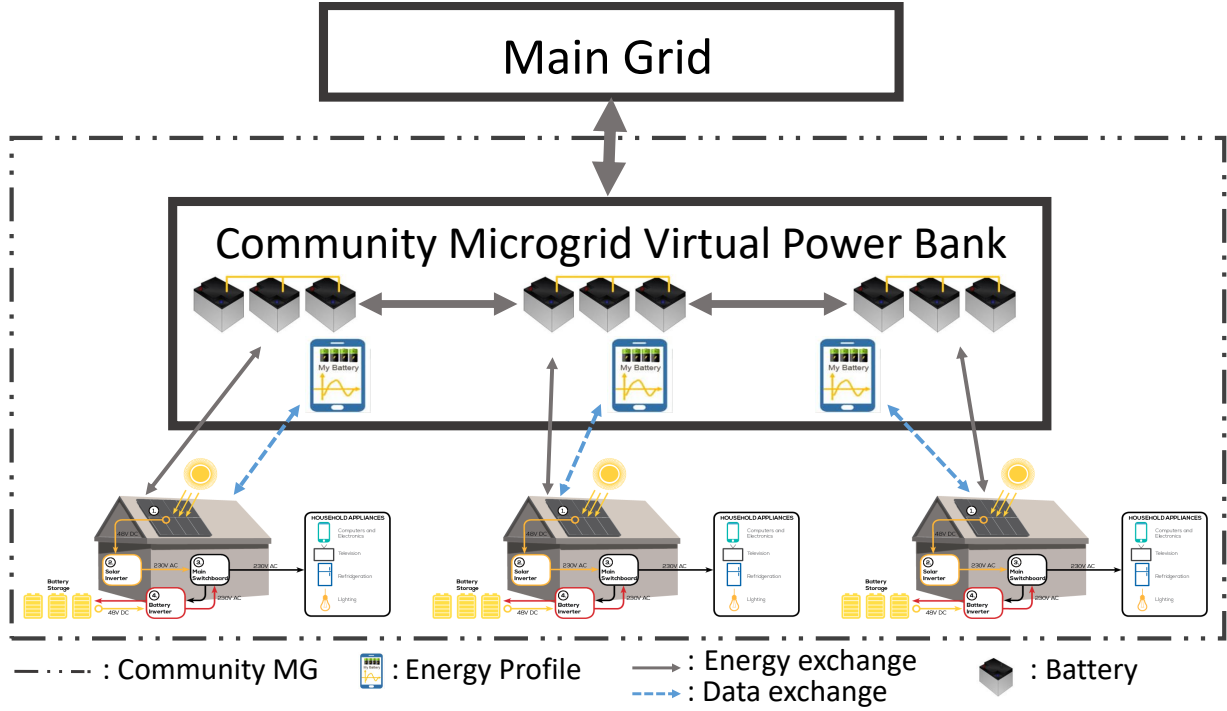


Figure 3.1 Community Microgrid Blockdiagram

By construction, the aggregated capacity of the power bank will become high enough to trade energy with the main grid and to provide energy sharing among a large number of prosumers. In this context, MG is composed of a sufficiently large number of households and a virtual power bank wherein prosumers have the possibilities to exchange energy with each other and the main grid. Similar concepts related to virtualizing storage system have been discussed in [118] and recently in [119], however, both papers assume a centralised energy storage system.

Recall Figure 3.1, each prosumer in the MG has an account in the virtual power bank and their energy balance represents the net amount of energy available in their related batteries. Households can store into their batteries, withdraw from their batteries, or buy from the power bank. Households with an energy surplus can make profit by selling their excess energy to other households through the virtual power bank.

3.4 MGO: System Dynamics

Each household has a set of time dependent variables: load and generation, which depend on their location, the deployed technology, and net storage or in other word the balance in the virtual power bank. Their decisions are influenced by the equilibrium market price for electricity in the MG and main grid. Each household coordinates their energy consumption in order to minimize their individual costs so they collectively minimize the aggregated MG operation cost.

3.4.1 Household Generation Dynamics

In this paper, we assume the MG is a two dimensional area. In addition, the community MG is composed of N households, where N is a sufficiently large number for the MFG theory to be applicable. Aziz et al. in [1, 120] have shown that the infinite control problem can be approximated by a decentralized MFG finite number problem for $N \geq 200$. Later in subsection 3.6.5 we will show that a MG of 100 prosumers is sufficiently large enough for the MFG approach to be implemented. All the variables are time dependent but the time subscript is removed to simplify the notations.

For simplicity, we consider the scenario where households are homogeneous i.e. they deploy the same technology for generation and storage. Using [121, 122] the power generated by each solar photovoltaic pannels (PV) at household i is given by:

$$P_{PV}^i = P_{STC}^i \times \frac{G_{ING}^i}{G_{STC}} \times (1 + k_s(T_c^i - T_r^i)) \quad (3.1)$$

Correspondingly, the generated energy θ^i by household i at time t is governed by the following stochastic differential equation:

$$d\theta^i = P_{PV}^i dt + \epsilon_\theta^i dW_\theta^i, \quad (3.2)$$

where W_θ^i , $1 \leq i \leq N$ are N independent Wiener process (i.e. Brownian Motion) and ϵ_θ is a positive constant.

Assuming that households are homogeneous, the stochasticity in PV power generation will be the result of $(G_{ING}^i, T_c^i, T_r^i)$ i.e. the effects of the technology deployed and the sun position along with air temperature. Denote by h^i the set of generation variables i.e. $h^i := \{G_{ING}^i, T_c^i, T_r^i\}$. Also, we assume that each household has adequate technology to determine T_c and T_r .

3.4.2 Household Operation Cost Model

Dealing with PV, solar radiation incurs zero fuel cost. Hence, the cost is mainly due to the operation and maintenance costs (O&M) of the PVs and to the cost of storage. In this paper, we assume the cost of operation and maintenance O&M is $K_{O\&M} = 0.1095(\$/kWh)$ for generation and $C_{B,M} = 0.001(\$/kWh)$ for storage. Thus, the cost of generation for household i is given to be:

$$dC^i(t) = K_{O\&M} \times \theta^i(t) + C_{B,M} \times C_{max} \times dt, \quad (3.3)$$

where C_{max} is assumed uniform for all households.

3.4.3 Energy Demand Management

The energy demand is met by withdrawing from the household's storage, from the MG through the power bank (i.e. from other households) or from the main grid through the power bank. For simplicity, we assume that household i , $1 \leq i \leq N$ has a time varying energy demand profile, $y^i(t)$, which has to be met at every time t , $0 \leq t \leq T$. Also, for the scope of this paper we assumed $y^i(t)$ a sufficiently smooth function for all i , $1 \leq i \leq N$ ([123], [124]), and the probability density function for the MG community household energy demand, $\mu_y(t, y)$, is known by each household. Hence, household i at time t observes their profile $y^i(t)$ and by using $\mu_y(t, y)$, household i can find the expected aggregated demand in the MG.

3.4.4 Household Power Bank Account

Each household has an account at the power bank. By analogy to financial banks, each household has a chequing account, an investment account, and a credit account, denoted by $b^i(t)$, $r^i(t)$, and $\gamma^i(t)$ respectively. Contrary to classical financial banks, the MG virtual power banks aims to maximize social welfare instead of maximizing its own profit.

The VPB plays the role of a local electricity market, in which prosumers of the microgrid community decide on the resource type, resource amount and price of electricity. The MG community is composed of two layers: (1) virtual layer represented by the VPB and (2) physical energy layer represented by distribution grid or the physical microgrid distribution grid. In the virtual layer, the estimated generation, consumption and energy price are calculated, while in the physical layer, represented by the distribution grid or microgrid control, the physical distribution of electricity is accomplished. In the proposed framework, we assume that grid constraints are not stringent; the focus of this paper is the proof of concept that MFG framework presents a potential approach for the MGO problem.

Recall $\theta^i(t)$ and $y^i(t)$ are the amount of energy generated by household i and the load of household i at time t respectively. The set of decision variables, $u_r^i(t)$ and $u_\gamma^i(t)$, represent the controls for the amount of energy sold through or withdrawn from the MG power bank, respectively. Define

$$\delta^i(t) := \theta^i(t) + b^i(t) - y^i(t) \quad (3.4)$$

as the net energy after meeting current demand. The dynamics of $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ are as follows :

$$b^i(t + dt) = \delta^i(t) - r^i(t) + \gamma^i(t) \quad (3.5)$$

$$r^i(t) = u_r^i(t) \delta^i(t) \quad (3.6)$$

where $0 < u_r^i(t) \leq 1$ when $\delta^i(t) > 0$ and $u_r^i(t) = 0$ when $\delta^i(t) \leq 0$ and,

$$\gamma^i(t) = \begin{cases} u_\gamma^i(t) C_{max} dt - \delta^i(t) & \text{if } \delta^i(t) \leq 0 \\ u_\gamma^i(t) (C_{max} dt - \delta^i(t)) & \text{if } \delta^i(t) > 0 \end{cases} \quad (3.7)$$

where $0 \leq b^i(t) \leq C_{max}$ for all t and where C_{max} is the storage battery capacity for household i and $0 \leq u_\gamma^i(t) \leq 1$ for all t . The dynamics are formulated such that household i will not contribute to the MG power bank at time t when household i has a shortage (i.e. $\delta^i(t) \leq 0$) and that $y^i(t)$ is always met for all time t and for all i , $1 \leq i \leq N$. Using (3.5), (3.6) and (3.7) the dynamics of $b(t)$ can be derived to the following set of differential equations. For simplicity we will drop the household superscript i . The observations can be categorized in two scenarios; (i) scenario one (SC1) where $\delta(t) \leq 0$ and (ii) scenario two (SC2) where $\delta(t) \geq 0$. In SC1, we have $\delta(t) \leq 0$ then $u_r^* = 0$, $0 \leq u_\gamma^*$ and:

$$db = C_{max} u_\gamma^* dt \quad (3.8)$$

SC2 where $\delta(t) > 0$ will be divided into two cases:

- Case 1: It is optimal to sell, then $0 < u_r^*$, $u_\gamma^* = 0$ and

$$db = \frac{1 - u_r^*}{1 + u_r^*} (P_{PV} - dy) dt \quad (3.9)$$

- Case 2: It is optimal to recharge, then $u_r^* = 0$, $u_\gamma^* > 0$ and

$$db = \frac{1 - u_\gamma^*}{1 + u_\gamma^*} (P_{PV} - dy) dt + \frac{u_\gamma^*}{1 + u_\gamma^*} C_{max} dt \quad (3.10)$$

Assuming P_{PV} and y are piece-wise continuous and differentiable and using (3.8), (3.9) and (3.10) the readers can prove that db is continuous and differentiable and that u_γ and u_r are continuous.

3.5 MGO: Dynamical Game Formulation

The objective of each household is to minimize their cost function over the time period $0 \leq t \leq T_f$. Treating households as prosumers, household i at time t can either withdraw from or sell through the power bank. Thus, in the case of energy surplus at time t (i.e. $\delta^i(t) > 0$), household i can either sell their surplus to the MG through the power bank (i.e. $u_r^i(t) > 0$) or buy from the power bank to fill its battery (i.e. $u_\gamma^i(t) > 0$). On the other hand,

in the case of energy shortage at time t (i.e. $\delta^i(t) \leq 0$), household i will withdraw energy from the power bank to meet its current load and optimally fill its battery; thus, $u_r^i(t) = 0$, $u_\gamma^i(t) \geq 0$ and $\gamma^i(t) \geq |\delta^i(t)|$. The state variables of each household at time t are: energy generated $\theta(t)$, energy balance in the power bank $b(t)$, demand $y(t)$, $\gamma(t)$ and $r(t)$. Denote by $\mu_y(y, t)$ and $\mu_\theta(\theta, t)$ the probability density function for y and θ at time t , respectively. In this paper, $\mu_y(y, t)$ is expected to follow a normal distribution while $\mu_\theta(\theta, t)$ is expected to follow a beta distribution function [125].

3.5.1 MG Equilibrium Price: P_{MG}

The overall objective of the MGO is to minimize the aggregated cost. Thus, regarding the energy equilibrium price in the MG, we will adopt a pricing mechanism that maximizes the social welfare (i.e. households will minimize their individual costs which in turn minimizes the aggregated cost and maximizes the social welfare in the MG).

Walrasian Equilibrium is an economic concept in decentralized economic systems that aims to ensure the equilibrium between the demand and the supply. The primary goal is to find the optimal pricing mechanism based on the demand and the supply in order to maximize the social welfare in the community microgrid. The Walrasian Equilibrium can be defined based on three main components: (i) maximizing the service of the prosumers, (ii) market clearing, and (iii) maximizing the social welfare [126]. In essence the equilibrium price is found such that: (i) every household maximize his/her utility given prices, and (ii) markets clear (i.e. the total demand for each commodity just equals the aggregate endowment at every time t). For those reasons we will use Walrasian equilibrium (also known as the competitive equilibrium) to find the MG equilibrium price [127, 128]. In short, the key result of the Walrasian Equilibrium theory is the fundamental first welfare theorem.

Theorem 1 *First Welfare Theorem [128]. If the tuple (price, $\{\text{load}\}_1^N$) where N is the total number of households in the MG forms a Walrasian Equilibrium then the load matrices $\{\text{load}\}_1^N$ are pareto-optimal.*

In other word if the tuple y and P_{MG} are pareto-optimal (i.e. households minimize their

objective function and the VB has a clear market) then the energy exchange economy through the VB follow a Walrasian equilibrium.

The aggregated demand in the MG at time t is the sum of the households' demands at time t i.e. $\sum_{i=1}^N y^i(t)$. Assuming that the retail price of energy is proportional to the first order derivative of the time-dependent generation cost and using the fundamental first welfare theorem, the pareto-optimal pricing mechanism denoted by P_{MG}^* in the MG is proportional to the aggregate cost of meeting the total load in the MG. Thus, $P_{MG}^*(t)$ is proportional to $\nabla C_u(G_{MG}(t))$ where C_u is the price function which is concave with respect to $G_{MG}(t)$ where $G_{MG}(t)$ is the aggregated load in the MG i.e.

$$G_{MG}(t) = \sum_{i=1}^N y^i(t) = N \times E\{y(t)\} = N \int_{\Omega_y} y \mu_y(y, s) dy \quad (3.11)$$

where Ω_y is the range of $y(t)$ for all $0 \leq t \leq T$.

The main grid energy pricing mechanisms considered here is mainly based on a flat rate (i.e. P_{MA} is constant over time). Households in the MG know which mechanism is applied and have full access to P_{MA} at any t , $0 \leq t \leq T$.

3.5.2 Decentralized Mean Field Optimal Control

The MFG approach is a decentralized control theory where each agent finds the expected value of the mean population and accordingly finds its best response. Compared to finite games, in mean-field games, the players (households) do not react to actions from individual players but rather to the aggregate behavior of all players. The result of such approach is proven to be the Nash Equilibrium i.e. households have no incentive to deviate. Each household action has a negligible influence on the aggregated value of the energy produced or required in the microgrid. Hence, household individual action has negligible effect on P_{MG} i.e .

$$\frac{P_{PV}^i}{\sum_{1 \leq i \leq N} P_{PV}^i} \sim \frac{y^i}{\sum_{1 \leq i \leq N} y^i} \sim \epsilon.$$

The state variables are $\{\theta^i(t), y^i(t), \gamma^i(t), r^i(t), b^i(t)\}$. The decision variables are: $u_r^i(t)$ and $u_\gamma^i(t)$. Each agent knows the probability density functions $\mu_y(t)$ and $\mu_\theta(t)$ for $y(t)$ and $\theta(t)$

respectively for all t , $0 \leq t \leq T$.

The MFG framework permits an analysis in terms of large population decentralized control problems where each user employs its local state information and an estimation of the MG aggregated behavior. The central equilibrium theorem of MFG theory provides a scenario in which the impact of all other users on a given user constitutes a deterministic component in its evolution when all the other agents use the MFG Nash Equilibrium strategy. More specifically, subject to technical conditions given in [21], which include the below five conditions:

1. The control value space U is a compact subset of $\mathbb{R}^m$ and the final time T satisfies $0 < T < \infty$.
2. The dynamics governing the states are smooth and together with their derivatives are uniformly Lipschitz in (θ, y, b) and u .
3. For any given smooth mean field density μ , the loss function $L(\cdot, \cdot)$ is smooth and together with its derivatives is uniformly Lipschitz in (θ, y, b) .
4. The value function $V(\cdot, \cdot)$ lies in $C^{1,2}[0, T]$.
5. The infimization operation in (3.17) and (3.19) yields a unique solution continuous and uniformly Lipschitz in (θ, y, b) .

The main infinite population Mean Field Game Nash Equilibrium Theorem (see [21, 110]) states that for an infinite family of individual agent nonlinear stochastic dynamical systems, here of the form (3.2), and individual expected cost to go performance functions, (3.15), where here L is of the form (3.12), there exist best response feedback control laws as in (3.18) and in Table 3.1, where any agent's control depends upon the information set which consists solely of the probability density function of the generated energy and load (i.e. $\mu_\theta \mu_y$) and the individual agent's state. Furthermore, the best response control laws, the resulting generic agent's state distribution and its optimal cost to go (i.e its value function) are given by the infinite population MFG equations MF-HJB, given respectively in (3.17) and (3.19)

below, and which together give the Nash equilibrium for the infinite population system for both scenarios.

Theorem 2 *MFG ϵ -Nash Theorem.*

Assume that the conditions of the MFG Nash Equilibrium Theorem hold and hence that the best response control laws $\mathcal{U}^\infty = \{u_0^i = u_0^i(t, x^i | \mu_t), 1 \leq i < \infty\}$ generating a Nash equilibrium for an infinite agent population system and associated performance functions exist. Then $\mathcal{U}^N = \{u_0^i = u_0^i(t, x^i | \mu_t), 1 \leq i \leq N\}$ yields a (strong) ϵ -Nash equilibrium for all ϵ , i.e. $\forall \epsilon > 0 \exists N(\epsilon)$ s.t. $\forall N \geq N(\epsilon)$

$$J^i(u_0^i, u_0^{-i}) - \epsilon \leq \inf_{u^i \in \mathcal{U}} J^i(u^i, u_0^{-i}) \leq J^i(u_0^i, u_0^{-i}),$$

where $u^i \in \mathcal{U}$, the set of all past dependent controls of the form $u^i = u^i(t, x^i | \mu_t)$. $\square$

3.5.3 System Performance and Cost Functions

The cost function $L(b, y, \theta, \gamma, r, t)$ is given by

$$\begin{aligned} L(\bullet) := & E \int_0^{T_f} (\gamma(t) (w_1(t) P_{MG}(t) + w_2(t) P_{MA}(t)) \\ & - r(t) P_{MG}(t) + C_{O\&M}(t)) dt \end{aligned} \quad (3.12)$$

where w_1 and w_2 denote the fraction of demand consumed from the MG and the main grid respectively.

Recall that energy exchange happens through the VPB. household i meets his/her load from both MG surplus or maingrid. In our framework there is no centralized unit, the VPB is the medium where energy is exchange. The aggregate energy sold by households and aggregated load is calculated. Recall the assumption that household i buys from himself/herself when withdrawing from his/her battery. Fairness in the community MGO problem is interpreted as follows: all households in the MG incur the same share from both energy sold in the MG and from energy withdrawn from the maingrid. Thus fairness is guaranteed when households

incur the same share from both the MG and the maingrid (i.e. w_1 and w_2 are the same for all households).

Adopting the fairness assumption, w_1 and w_2 are calculated as follows:

$$\begin{aligned} w_1(t) &= \min \left\{ 1, \frac{\sum_i \theta^i(t)}{\sum_i y^i(t)} = \frac{E\{\theta(t)\}}{E\{y(t)\}} = \frac{\int_{\Omega_\theta} \theta \mu_\theta d\theta}{\int_{\Omega_y} y \mu_y dy} \right\} \\ w_2(t) &= 1 - w_1(t) \end{aligned} \quad (3.13)$$

where Ω_θ is the range of $\theta(t)$ for all t , $0 \leq t \leq T$. Recall the P_{MG} in (3.11), the expected cost function can be written as:

$$l(\bullet) := C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) + \gamma(t) w_2 P_{MA}$$

Hence the cost function in (3.12) becomes:

$$\begin{aligned} L(\bullet) &= \int_0^{T_f} (C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) \\ &\quad + \gamma(t) w_2 P_{MA}) dt \end{aligned} \quad (3.14)$$

In (3.14), coupling occurs in the cost function where the aggregated demand and the aggregated generated energy affect the decision of the household, particularly in P_{MG} . Following the cost function $L(\bullet)$ in (3.14) the cost to go $J(b, y, \theta, \gamma, r, s)$ is given by:

$$\begin{aligned} J(\bullet) &= E \left[\int_s^{T_f} L(b_t, y_t, \theta_t, \gamma_t, r_t) dt \right] \quad s.t. \\ b_s &= b, y_s = y, \theta_s = \theta, \gamma_s = \gamma, r_s = r \end{aligned} \quad (3.15)$$

and accordingly the value function $v(\bullet)$ is given by:

$$v(b, y, \theta, \gamma, r, s) = \inf_{u_\gamma, u_r} J(b, y, \theta, \gamma, r, s) \quad (3.16)$$

Assuming that all functions are sufficiently smooth then the mean field Hamilton-Jacobi-Bellman equations (MF-HJBs) for each scenario are:

- SC1: $\delta(t) \leq 0$, $u_r^* = 0$ and the MF-HJB-SC1 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= -C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma} \left\{ \gamma (w_1 P_{MG} + w_2 P_{MA}) - r P_{MG} + C_{max} u_\gamma \frac{\partial v}{\partial b} \right\}
\end{aligned} \tag{3.17}$$

and thus:

$$u_\gamma^* = \inf_{u_\gamma} \left\{ C_{max} u_\gamma \frac{\partial v}{\partial b} \right\} \Rightarrow u_\gamma^* = \begin{cases} 0 & \text{if } \frac{\partial v}{\partial b} \geq 0 \\ 1 & \text{if } \frac{\partial v}{\partial b} < 0 \end{cases} \tag{3.18}$$

- SC2: $\delta > 0$ i.e. household can either sell the surplus, fill the battery or do nothing. The MF-HJB-SC2 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma, u_r} \{ \gamma (w_1 P_{MG} + w_2 P_{MA}) - r P_{MG} \} \\
&+ \inf_{u_\gamma, u_r} \left\{ \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \right\} \\
&+ \inf_{u_\gamma} \left\{ \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b} \right\}
\end{aligned} \tag{3.19}$$

thus:

$$\begin{aligned}
\{u_\gamma^*, u_r^*\} &= \inf_{u_\gamma, u_r} \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \\
&+ \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b}
\end{aligned} \tag{3.20}$$

Using (3.20) and taking into account that (i) P_{PV} represents the rate of change in energy production θ (i.e. supply), (ii) dy represents rate of change in load y (i.e. consumption) and (iii) the term $\frac{\partial v}{\partial b}$ represents the rate of change in cost to go function v with respect to household balance b i.e.

$$\frac{\partial v}{\partial b} = \frac{v(t + dt) - v(t)}{b(t + dt) - b(t)}$$

The optimal solutions for the MF-HJB-SC2 equation in (3.19) are derived. The optimal solutions are presented in Table 3.1.

Table 3.1: Optimal Solution considering SC2

Scenario	$P_{PV} - dy$	$\frac{\partial v}{\partial b}$
$u^*r = 1$ $u^\gamma = 0$	≥ 0	≥ 0
$u^*r = 0$ $u^\gamma = 0$	> 0	< 0
$u^*r = 0$ $u^\gamma = 0$	< 0	≥ 0
$u^*r = 0$ $u^\gamma = 1$	≤ 0	< 0

3.5.4 Existence and Uniqueness of the HJB Equation [1]

Consider the following Hamilton Jacobi Bellman equation scheme:

$$-\frac{\partial v}{\partial t} + \sup_{\{u^i \in U\}} G(t, x, u, -\frac{\partial v}{\partial x}) = 0 \quad (3.21)$$

$$v|_{t=T} = h(x) \text{ and } (t, x) \in [0, T] \times \mathbb{R}^n,$$

where G is the generalized Hamiltonian defined by:

$$G(t, x, u, p, P) \triangleq \frac{1}{2} \text{tr} \left(P \sigma(t, x, u) \sigma(t, x, u)^\top \right) + \langle p, b(t, x, u) \rangle - f(t, x, u). \quad (3.22)$$

for all $(t, x, u, p, P) \in [0, T] \times \mathbb{R}^n \times U \times \mathbb{R}^n \times \mathbb{S}^n$ and consider the following assumptions:

1. The control value space U is a compact subset of $\mathbb{R}^m$ and the final time T satisfies $0 < T < \infty$.
2. The maps $b : [0, T] \times \mathbb{R}^n \times U \rightarrow \mathbb{R}^n$, $\sigma : [0, T] \times \mathbb{R}^n \times U \rightarrow \mathbb{R}^{n \times m}$, $f : [0, T] \times \mathbb{R}^n \times U \rightarrow \mathbb{R}$ and $h : \mathbb{R}^n \rightarrow \mathbb{R}$ are uniformly continuous and uniformly Lipschitz in x , i.e. there exists a uniform Lipschitz constant $L > 0$ such that for $\varphi(t, x, u) = b(t, x, u)$, $\sigma(t, x, u)$,

$f(t, x, u)$ and $h(x)$ such that $\forall t \in [0, T], x, \hat{x} \in \mathbb{R}^n, u \in U$

$$\begin{aligned} |\varphi(t, x, u) - \varphi(t, \hat{x}, u)| &< L(|x - \hat{x}|), \\ |\varphi(t, 0, u)| &\leq L. \end{aligned}$$

Theorem 3 ([129], Chapter 4, Theorem 5.2, p. 190, Theorem 6.1, p.198)

Let T1 and T2 hold. Then the value function $V(x, t)$ of the corresponding stochastic optimal control problem on $D \times [0, T]$ is linearly bounded and uniformly Lipschitz in $x \in D$, and $V(\cdot, \cdot) = v(\cdot, \cdot)$, where $v(\cdot, \cdot)$ is the unique viscosity solution of the HJB (3.21). Furthermore, $v \in C^{1,2}[0, T]$ if and only if it is a classical solution to (3.21). $\square$

3.6 Algorithmic Implementation of Decentralized Community MGO Problem

Here, we will provide the algorithms that solve the MF-HJB in both cases: MF-HJB-SC1 and MF-HJB-SC2.

3.6.1 Boundary Condition and Approximation Methods

In order to numerically solve the MFG equations (HJB and FPK), we need to specify the following: 1) boundary conditions and 2) the approximation methods used for discretizing the partial differential equations.

3.6.1.1 Boundary Conditions

Below are the boundary conditions for the MGO problem:

- $0 \leq t \leq T$, where T is to be specified.
- $0 \leq u_r, u_\gamma \leq 1$ and the admissible set of control is $U_{admissible} := [0, 1]$.
- Power is assumed to be positive and bounded i.e.
 $0 \leq P_{PV} \leq P_{PV}^{max}$, where P_{PV}^{max} is to be specified.
- $\mu_\theta(\theta, t)$ and $\mu_y(y, t)$ are specified and known to the households.

- Final cost is zero i.e. $v(\theta, b, y, T) = 0$.
- Final household i account balance $b^i(T) = 0$ for all $1 \leq i \leq N$.
- C_{max} is uniform for all household and is to be specified.

3.6.1.2 Approximation Method

To solve the partial equations numerically, these conditions must be satisfied and we have used these approximations methods.

Time, energy, household balance, and demand step sizes are denoted by Δt , $\Delta \theta$, Δb and Δy respectively. For the convergence of the numerical algorithms, the step sizes should satisfy the necessary conditions in [130] regarding the stability of the Fixed Point Argument Method. In particular, in this case the conditions are as follows:

$$\frac{|P_{PV}^{max}|\Delta t}{(\Delta \theta)^2} \leq \frac{1}{2}; \quad \frac{\Delta t}{(\Delta y)^2} \leq \frac{1}{2}; \quad \frac{|\Upsilon|\Delta t}{(\Delta b)^2} \leq \frac{1}{2} \quad (3.23)$$

where $\Upsilon := \max [C_{max}, \max (P_{PV} - \delta y)]$.

The techniques used for discretizing the 1st and 2nd order derivative for any function g over variable ρ are as follows:

- 1st order derivative:

$$\begin{aligned} \text{Forward in } \rho : \frac{\partial g(., \rho)}{\partial \rho} &= \frac{g(., \rho + \Delta \rho) - g(., \rho)}{\Delta \rho} \\ \text{Backward in } \rho : \frac{\partial g(., \rho)}{\partial \rho} &= \frac{g(., \rho) - g(., \rho - \Delta \rho)}{\Delta \rho} \end{aligned} \quad (3.24)$$

- 2nd order derivative:

$$\frac{\partial^2 g}{\partial \rho^2} = \frac{g(., \rho + \Delta \rho) - 2g(., \rho) + g(., \rho - \Delta \rho)}{(\Delta \rho)^2} \quad (3.25)$$

3.6.2 Numerical Algorithm for Uniform Agents

The MGO presented in this paper focuses on a community MG in which the statistical information is found based on prior historical data. In addition, the statistical information, $\mu_\theta(t, \theta)$ and $\mu_y(t, y)$, are known to the households. The algorithmic implementation of the MGO presents the numerical solution for the MGO's MF-HJB equations. Algorithm 1 generates a best response strategy for a generic household in the community MG. Note that household i , $1 \leq i \leq N$, observes his state variables and has complete information of his system dynamics. This is revealed in the input section of Algorithm 1. In addition, recall that household i , $1 \leq i \leq N$, observes $\gamma^i(t)$ and $r^i(t)$, and controls $u_\gamma^i(t)$ and $u_r^i(t)$. The preliminary steps for Algorithm 1 are:

1. Discretize the MF-HJB equations in (3.17), (3.19), and (3.20) using (3.24) and (3.25),
2. Discretize the state differential equations in (3.2), (3.8), (3.9) and (3.10) using (3.24) and (3.25),
3. Find Δt , $\Delta \theta$, Δy and Δb satisfying (3.23),
4. Using households previous historical data find $\mu_\theta(t, \theta)$ and $\mu_y(t, y)$ for all $0 \leq t \leq T$.

Algorithm 1 Computation of a Generic Agents' Best Response Control Along a Sample Path

For a generic household of the community MG $1 \leq i \leq N$:

Input: $\mu_\theta(t, \theta)$, $\mu_y(t, y)$, $P_{MA}(t)$ for all $0 \leq t \leq T$, C_{max} , $\Omega_\theta := [\theta_{min}, \theta_{max}]$, $\Omega_y := [y_{min}, y_{max}]$, G_{ING} , G_{STC} , T_c , T_r

Initialization:

Generate a Brownian Process W_θ^i

Set $k = 0$, $t = T$, $v(T) = b(T) = 0$, & $u_\gamma^i(T) = u_r^i(T) = 0$

Loop: At iteration k , execute the following:

Substitute G_{ING} , G_{STC} , T_c , T_r in (3.2) and calculate $\theta^i(t)$

Using (3.4) find $\delta^i(t)$ and using (3.6) and (3.7) find $\gamma^i(t)$ and $r^i(t)$ respectively.

Calculate: $\Lambda_\theta = \int_{\Omega_\theta} \theta \mu_\theta(t, \theta) d\theta$, $\Lambda_y = \int_{\Omega_y} y \mu_y(t, y) dy$,

Using (3.13) find $w_1(t)$ and following that $w_2(t) = 1 - w_1(t)$ and using (3.11) find $P_{MG}(t)$

If $\delta^i(t) \leq 0$ then consider Mf-HJB-SC1 in (3.17) and thus $u_r^{*,i}(t - \Delta t) = 0$ and $u_\gamma^{*,i}(t - \Delta t)$ is found using (3.18)

Else, calculate $dy = \frac{y(t+\Delta t) - y(t)}{\Delta t}$ and using Mf-HJB-SC2 and the results in Table I find $u_r^{*,i}(t - \Delta t)$ and $u_\gamma^{*,i}(t - \Delta t)$

Increment k

If $T - \Delta t \times k \leq 0$ exit the loop.

Output: $v^i(t)$, $\theta^i(t)$, $P_{MG}(t)$, $u_r^i(t)$ & $u_\gamma^i(t) = 0$

3.6.3 Case Study: Montreal, Quebec

In this paper, we consider a community microgrid in Montreal, Quebec as our case study. The residential pricing in Montreal's main grid is a flat rate, thus regardless of the time of use or current load and aggregate demand, the energy price is flat and known to households and $P_{MA} = 7.13 \text{cent/kWh}$. The case study is done over the scope of a summer day where the starting time $t = 0$ is 12:00 a.m. and T is $T = 11 : 59$ p.m. and households' PV area is assumed uniform and equals about $6m^2$. Previous data has been collected and studied for both households' loads and PV generation. In this framework, it is assumed that the initial statistical information of the load and energy are known to all households and Figure 3.2 and Figure 3.3 depict the load and PV generation statistical information respectively in the constructed microgrid. The case study presented here can be implemented in any season and weather conditions and main grid dynamical pricing scheme has been introduced in a future publication.

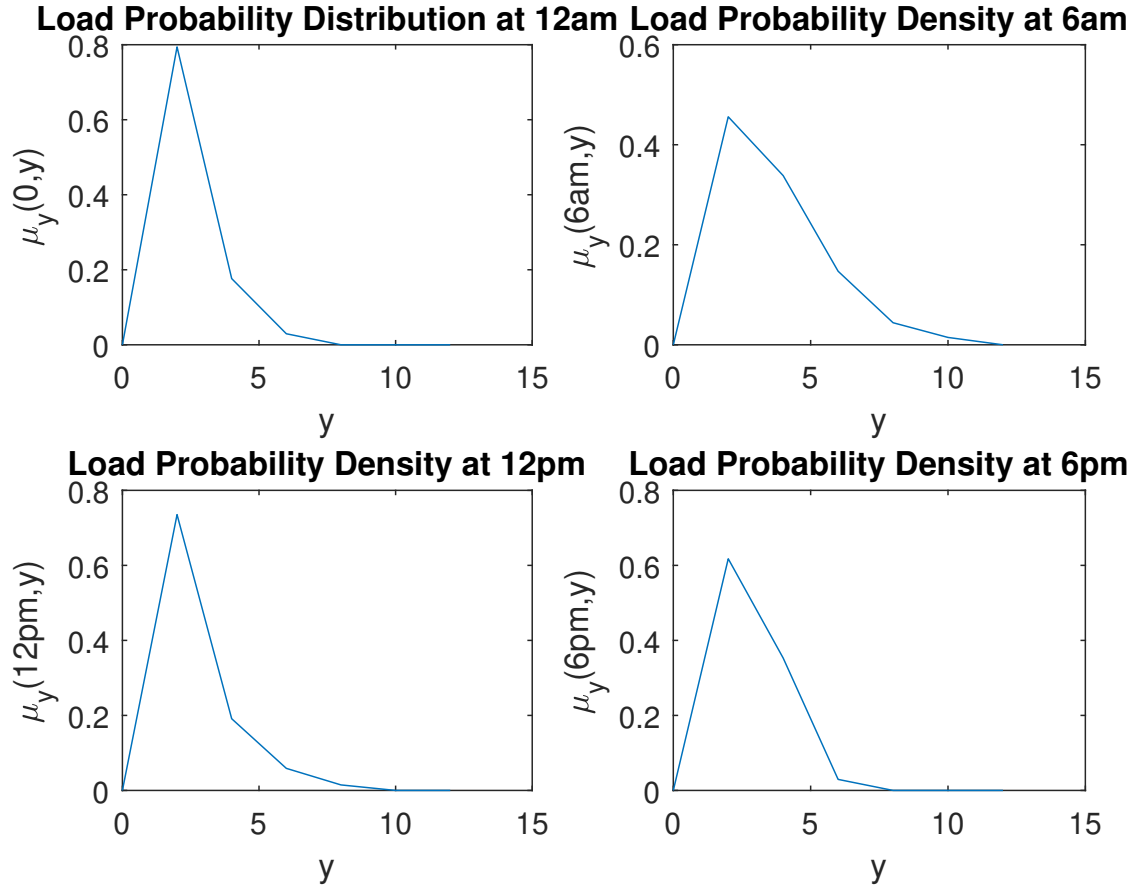


Figure 3.2 Load Probability Density Function

The numerical results showing a generic household's i profile where their state variables $y^i(t)$ (i.e. load) and $\theta^i(t)$ (i.e. generated energy) are shown in Figure 3.4. In addition, Figure 3.4 depicts the expected cost to go along with the microgrid Walrasian equilibrium price.

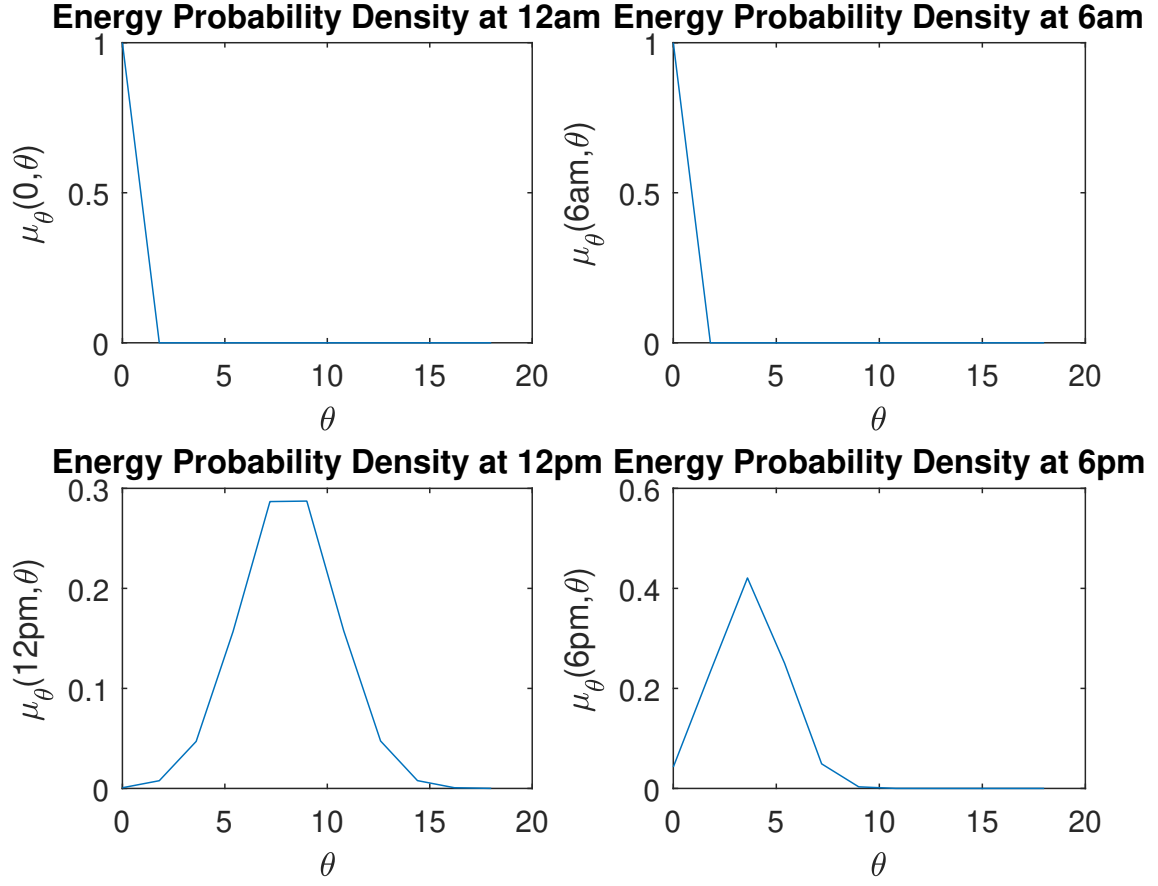


Figure 3.3 Energy Generation Density Function

Figure 3.5 presents the energy profile of household i in the virtual power bank i.e $b^i(t)$, $\delta^i(t)$, $\gamma^i(t)$ and $r^i(t)$.

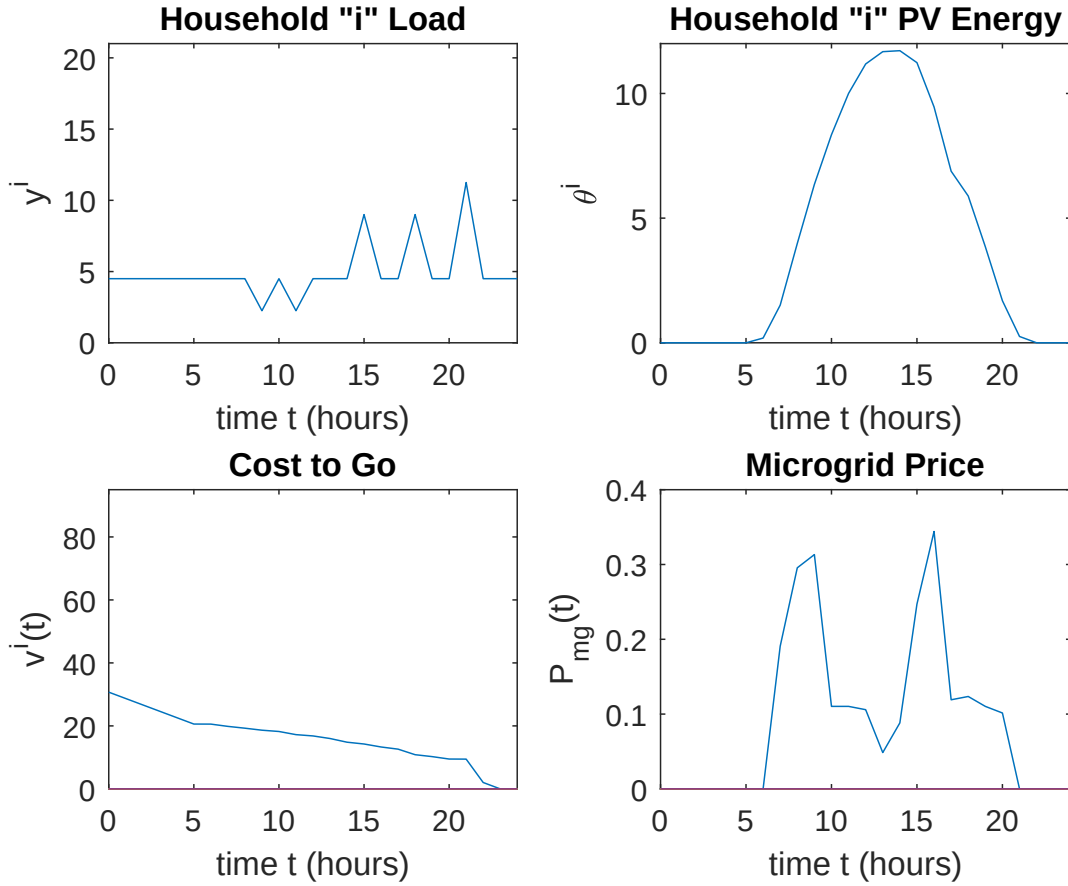


Figure 3.4 Generic Household i Energy Profile and MG Equilibrium Price

Figure 3.6 depicts household i decision variables i.e. $u_r^i(t)$ and $u_\gamma^i(t)$ along with $w_1(t)$ and w_2 representing the portion of energy consumed from the MG and the main grid, respectively.

From the numerical results, the reader can infer the following: (i) household i sells their stored energy when their expected short term demand in the near future is met by their expected generated energy and (ii) household i charges their battery from the main grid when their expected short-term demand exceeds their expected short-term generation.

In addition, we studied the scenario where MG does not exist and households only interact with the main grid. In this scenario, the generic household presented in Figure 3.4 will incur 55.65\$ whereas the same generic agent incurred only 30.71\$ when joining the MG. Running the comparison of the stand-alone cost incurred by a generic household i and the cost incurred by joining the community MG for all i , $1 \leq i \leq N$, household i cost was reduced on average

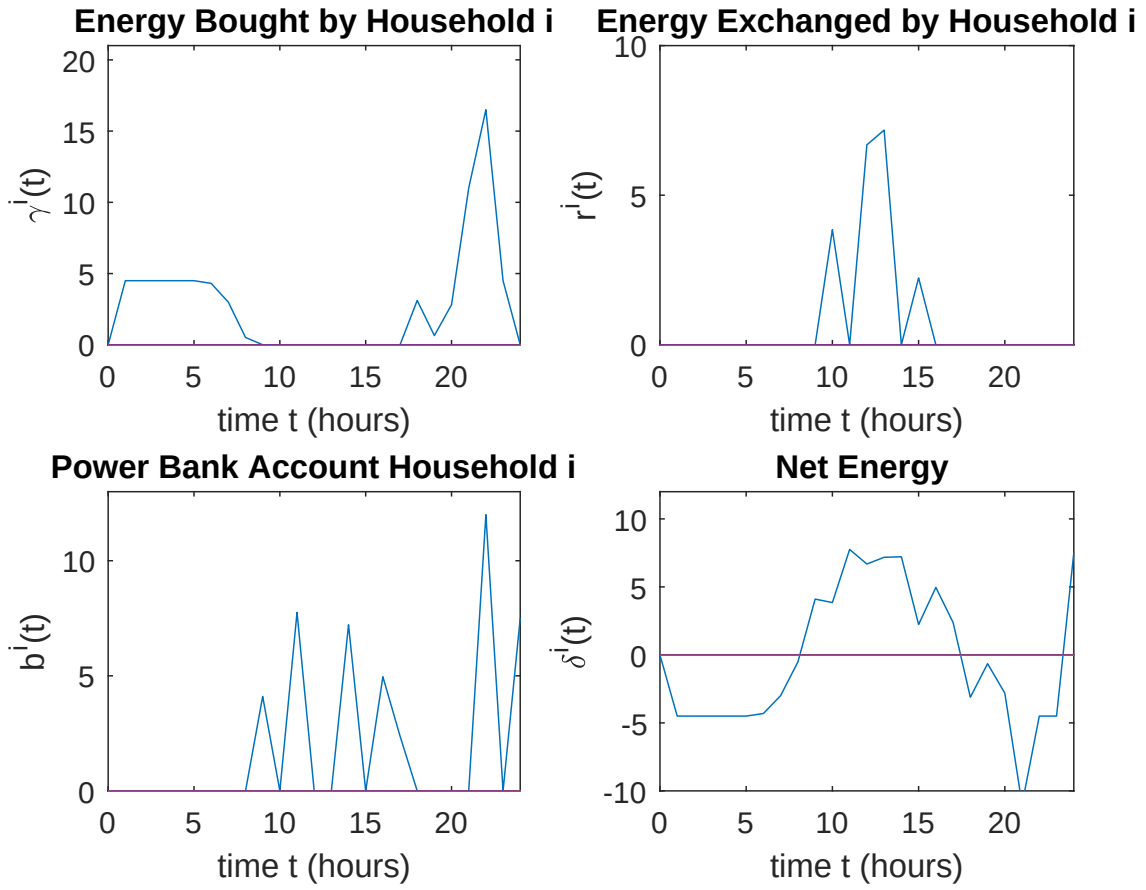


Figure 3.5 Power Bank Energy Profile

by 23.78\$ by joining the MG which accounts for 42% of their total cost. In addition to the significant 40% cost reduction, the proposed approach will reduce the peak energy in the main grid.

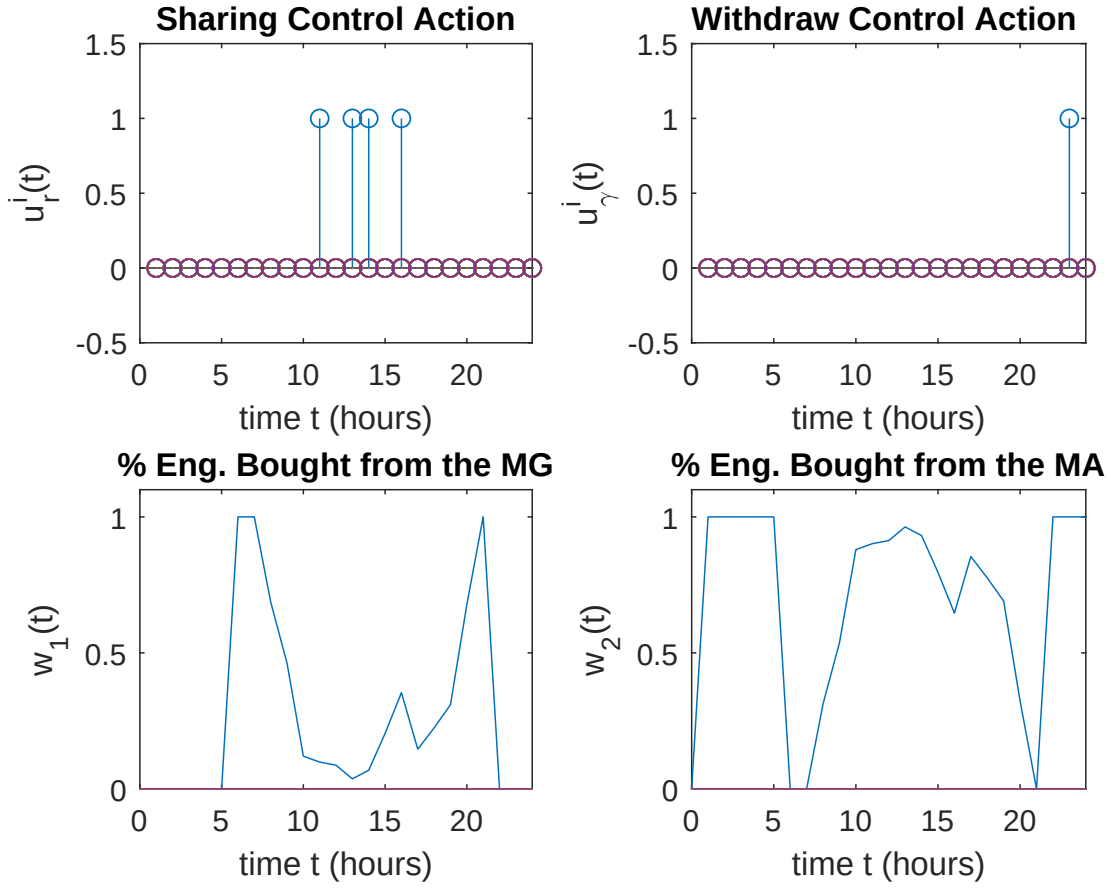


Figure 3.6 Household i Decision Variables and Demand Origin Fractions

3.6.4 Computational Performance

Table 3.2: Algorithms Performance for Montreal, Quebec Case Study

Scenario	Number of Households	Run.-Time sec	Memory KB	Dim.
Montreal, Quebec	100	8.98	8,192	7

Table 3.2 presents the algorithm's computational performance for the Montreal, Quebec case. The specifications of the computer on which the simulations were generated are as follows: 1) processor: Intel(R) Core(TM) i7-4600U CPU @ 2.10GHZ , 2) RAM: 8GB and 3) system type: 64-operating system, x64-based processor.

3.6.5 Decentralized Approach Versus Centralized Approach

The loss function for each household in the infinite population decentralized case is given by:

$$L(\bullet) := E \int_0^{T_f} (\gamma(t) (w_1(t)P_{MG}(t) + w_2(t)P_{MA}(t)) - r(t)P_{MG}(t) + C_{O\&M}(t))dt \quad (3.26)$$

where P_{MG} is dependent on the estimated aggregated generated power and load. We recall that this loss function is utilized to calculate the MFG-equilibrium control laws that are employed for a finite population according to the MGF methodology.

On the other hand, in the centralized scenario all the information is known to the centralized unit; thus, the aggregated demand and P_{MG} the mass effect is calculated using the collected real data. Figure 3.7 presents the numerical comparison between the centralized and decentralized control performances in the 50 and 100 MG prosumers' cases. In addition, table 3.3 documents the numerical performance of the proposed algorithm as the number of microgrid households increases.

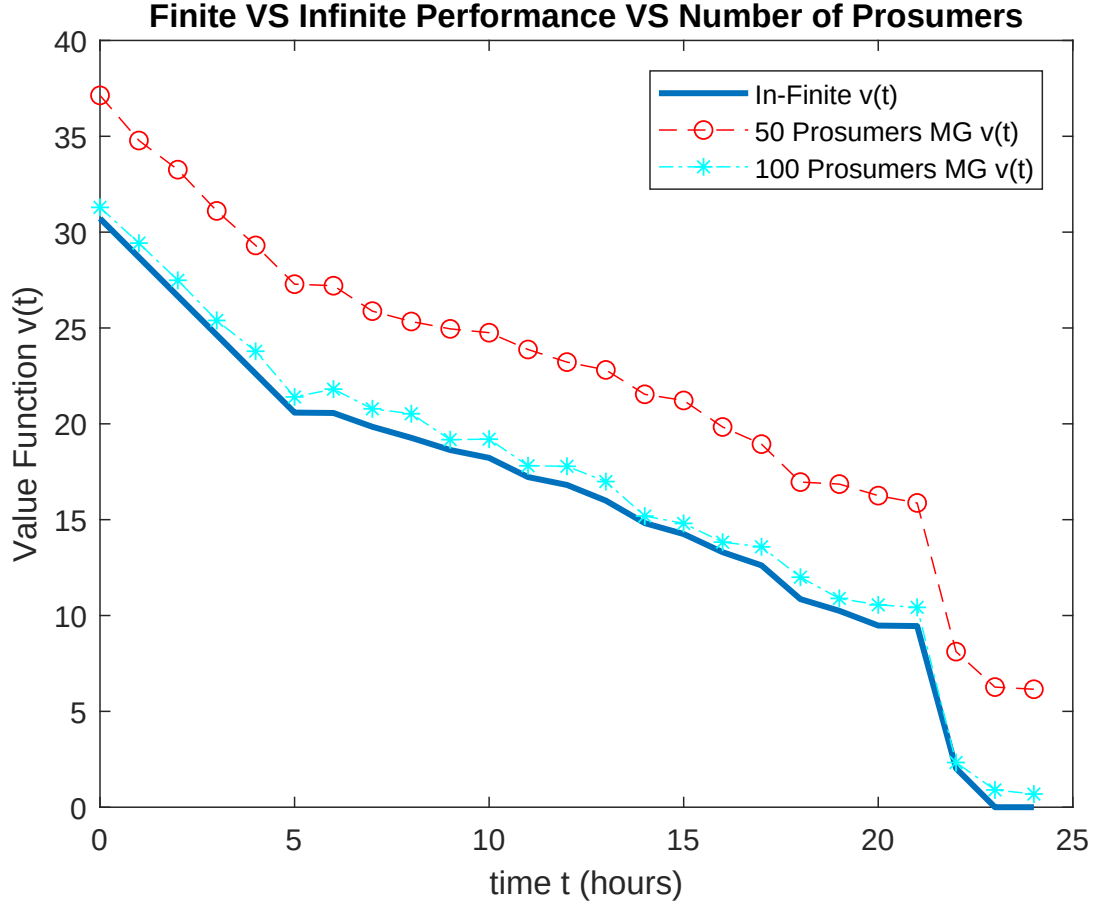


Figure 3.7 Centralized Versus Decentralized Performance

Table 3.3: Numerical Performance Versus Population

Scenario	Number of Households	Run.-Time sec	Memory KB
Cent.	50	6.55	12,762
MFG	50	8.98	8,192
Cent.	100	13.75	22,647
MFG	100	8.98	8,192

Analyzing the data in Table 3.3 and the two results in Figure 3.7, one can infer that the MFG methodology equilibrium tends to approximate the performance of the centralized control as the number of agents is sufficiently large enough, in this case ($N \geq 100$).

3.7 Concluding Remarks

The work in this paper presents a game theoretic analysis for a community microgrid. The novel architecture of such MG is constructed by the deployment of a virtual power bank which (i) contains the households' batteries and (ii) mediates the communication between the MG households and the communication between the MG and the main grid. This paper presents a formulation for the decentralized MGO problem for the constructed community MG via mean field game theory. We propose a framework that solves both the MF-HJB-SC1 and the MF-HJB-SC2 equations in (3.17) and (3.19), respectively. The solutions for the MGO are proven to be Nash Equilibrium strategies that minimize household individual cost functions and in turn the aggregate cost in the MG by at least 40%.

Extending the framework presented in this paper to solve the general MGO problem where the main grid electricity market is free (i.e. P_{MA} is based on the demand and supply solely) and households possess controllable loads will be further investigated in future work.

CHAPTER 4 COOPERATIVE GAME THEORETICAL APPROACH FOR RESIDENTIAL USERS THROUGH LOAD AGGREGATOR IN SMART GRID

4.1 Abstract

Incentive-based demand response aggregators are widely recognized as a powerful strategy to increase the flexibility of residential community MG (RCM) while allowing consumers' assets to participate in the operation of the power system in critical peak times. RCM implementing demand response approaches are of high interest as collectively, they have a high impact on shaping the demand curve during peak time while providing a wide range of economic and technical benefits to consumers and utilities. The penetration of distributed energy resources such as battery energy storage and photovoltaic systems introduces additional flexibility to manage the community loads and increase revenue. This paper proposes a game theoretical formulation for an incentive-based residential community microgrid, where an incentive-based pricing mechanism is developed to encourage peak demand reduction and share the incentive demand curve with the residential community through the aggregator. The aggregator's objective is to maximize the welfare of the residential community by finding the optimal community equilibrium electricity price. Each household communicates with each other and with the distributed system operator (DSO) through the aggregator and aims to minimize the local electricity cost.

4.2 Introduction

With the development of demand-side management technologies, residential consumers possess a great potential to provide flexible services to power systems [106]. An individual household possessing distributed energy resources (DER) and demand response technologies has no impact on the power system demand curve in his/her region and hence cannot contribute significantly to power grid flexibility. To overcome this issue, multiple households integrated with DERs can be clustered by an aggregator to better trade the available flex-

ibility and shape the demand curve. Incentive-based DR is a promising direction for peak load reduction/shaving, electricity bill reduction, and profit maximization for households, aggregators and electricity providers. In this context, several decision-makers may interact to implement a policy where each has its own objective function and system dynamics. The coupling occurs in the cost function as a local decision of a household affects the payoff of other houses. Three sets of players are involved: (1) the distribution system operator (DSO) who has control over the supplied power, incentive-based DR program, and electricity pricing mechanism, (2) the aggregator of the community MG who has control over the community's pricing mechanism and who mediates the communication between the households themselves and the DSO, and (3) the residential households who have the control over their demands and their distributed energy resources. The resulting problem will then include all sets of players interacting such that there is a coupling in the cost and/or dynamics.

The concept of microgrids (MG) has emerged as a promising platform that integrates a cluster of distributed energy generation and storage systems, local demand, grouped together within a limited geographical area to achieve reliable and adequate energy supply [98, 131]. A new feature of the concept of MG is an RCM with load aggregators (LA). Different approaches in the literature have been developed to solve related problems via the application of game theory.

Among the game theory approaches and in this paper, Mean Field Game Theory (MFG) will be conducted to formulate and solve the optimization problem. MFG was first developed in [30, 32] to explore Nash equilibrium when the number of agents tends to infinity for differential games. MFG studies the existence and generation of strategies of Nash equilibria for stochastic dynamic games between a large population of agents [20–22]. Numerically, applying the MFG approach is only concerned with estimating the aggregated effect (i.e. the mean), thus, the numerical complexity of the MFG approach is indifferent to the size of the population. In a smart grid paradigm, MFG has been applied to many populations, such as thermostatically controlled loads in [111] and electric vehicles in [113]. However, a concrete construction of mean-field games in a community of smart homes has not yet been designed. This work thus builds on these developments to bridge the gap between MFG theory and

relevant applications in residential community microgrid.

In recent research, several game theoretical approaches have been used to solve the smart grid optimization problem. For a recent summary, Jasim *et al.* in [132] presented the smart grid problem in a centralized fashion and solved the optimization problem via an application of the Virulence Optimization Algorithm and Earth Worm Optimization Algorithm. In [133] Dong *et al.* formulated the energy blockchain problem via a decentralized peer-to-peer Stackelberg game theoretical approach. The drawback of such an approach is the extensive communication between the peers which will result in a less efficient price converging time and more communication compared to the mean-field game theory approach implemented in this paper. In [134] Mishra *et al.* used a non-cooperative decentralized game theoretical formulation and found the optimal solution via an application of the proximal decomposition algorithm. This is similar to the approach presented in our paper except in our framework the converging price is determined via mean field game theory and real-time optimal pricing and control are determined after solving the MFG game making our approach a faster and more efficient in terms of real-time applications. Another Stackelberg game theoretical formulation has been presented in [135] where Tang *et al.* developed a game-theoretic optimization scheme based on the intermediary aggregator to unlock the considerable flexibility potential of buildings. The authors presented the smart grid where the aggregator owns the renewable energy and storage systems, limiting the physical architecture of the smart grid to a centralized one.

The contributions of this paper are given as follows. We propose a game theoretical formulation for the incentive-based residential community MG (RCM) with load aggregator using the decentralized mean field game theory. We present a high-level formulation of the load aggregated energy problem and a proof of concept that major-minor agent mean field game theory is a plausible way to find the optimal pricing mechanism deployed in the RMC possessing a load aggregator and operating in grid-connected mode. The incentive-based pricing mechanism is adopted by the electric utility to reduce peak demand while sharing the incentive demand curve with the residential MG through a load aggregator. The MG aggregator's objective is to maximize the welfare of the MG by finding the optimal MG equilibrium electricity price. In the constructed MG, household demand is divided into two types: (i)

fixed, and (ii) elastic (i.e. controllable load)). Households communicate with each other and the main grid through the aggregator and they aim to minimize their electricity cost. This proposed research will lead to: (i) A new general pricing methodology for the community MG and for the main grid, (ii) the Nash-Equilibrium strategies for the households (i.e. minor agents) and community MG aggregator (i.e. major agent), and (iii) real-time optimization resulting in households' energy cost and reduction of peak demand cost.

The rest of the paper is structured as follows: Section II presents the conceptual design and the proposed game framework, and its relevant elements. Section III discusses the formulation of the system dynamics including the models of solar PV, prosumers' total operational cost, and the dynamics of the aggregator. Section IV presents the dynamical game formulation problem via means of major-minor agent mean field game theory. Finally, in section V we conclude the paper.

4.3 Conceptual design of the proposed game framework

This section highlights the conceptual design and the MG architecture considered in our framework and the motivation behind formulating the problem via the MFG framework.

4.3.1 Conceptual design

Figure 4.1 depicts the block diagram for the constructed residential community MG. The community MG is composed of an aggregator (i.e. the major agent) and N households that will form N minor agents in the formulated game where N is a sufficiently large number.

By construction, the aggregator mediates the energy exchange between the MG and the main grid along with the energy exchange within the MG (i.e. between the households themselves and the households and the main grid). Households exchange energy with the MG and with the main grid. Households are located within the geographical area of the MG, and they communicate and exchange energy with each other and with the main grid through the aggregator of the community MG. The aggregator is the major agent in the community MG, and its objective is to maximize the social welfare of the MG households. The aggregator's role is to set the equilibrium price in the MG and sends the aggregated demand to the main grid. The aggregator collects the price of electricity from the main grid and then forwards the

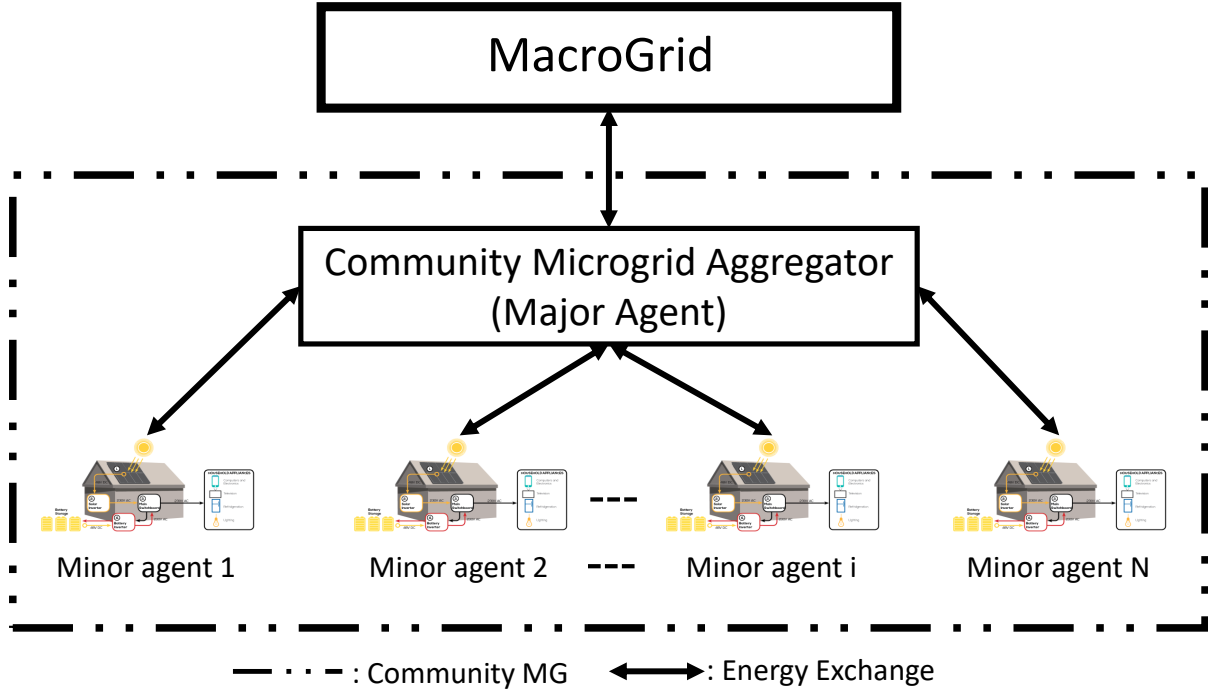


Figure 4.1 Major-Minor Agent Community MG Block Diagram

MG equilibrium price and the main grid electricity price to households. Households in the MG react to the message and the prices provided by the aggregator. Households' objective function is to minimize their energy cost and their demand is assumed to be divided into two types:

- Fixed demand that represents the demand of all the non-controllable appliances.
- Elastic demand that represents the demand of all controllable appliances.

4.3.2 MFG Framework

The framework focus on a large population game involving stochastic dynamical systems with mixed typed agents: (i) a major agent (i.e. the MG AG), and (ii) a population of N minor agents (N households where N is very large). The major and minor agents are coupled via both: (i) their individual nonlinear stochastic dynamics, and (ii) their individual cost functions. In particular, the aggregated demand and generation of the prosumer affect the dynamics and cost function of the LA, whereas the MG equilibrium price which is the

state dynamics of AG affects the individual prosumer cost function and dynamics. A distinct feature of the mixed agent MFG problem is that even asymptotically, (as the population size N approaches infinity) the noise process of the major agent causes random fluctuation of the mean-field behavior of the minor agents. The major player has a significant influence on others. The minor players individually have negligible impact, but they collectively contribute in the mean field coupling terms in the individual dynamics and costs.

For large population dynamic games, it is unrealistic for a player to collect detailed state information about all other players, and a central issue is the development of low-complexity solutions so that each player may implement a strategy based on local information. Since minor agents are responding to the same major player, the behavior of the minor players will have an inherent correlation. Subsequently, their aggregate effect as a random process is directly impacted by the major player, and from the major player's perspective, when determining its equilibrium strategy, it cannot "freeze" the aggregate effect by treating it as an exogenous signal. So, for the decision-making of an individual player (especially the major player), it is crucial to identify how the aggregate effect statistically evolves.

The objective of this framework is to develop a decentralized control synthesis under large population conditions such that each player's strategy uses only limited information. In particular, we assume that the state variable of the major player is available to all players, while the state of each minor player is always known to itself but can be estimated by the aggregator using the mean field framework.

For this reason, we need to introduce additional system states based on available information for the control synthesis of the players. To tackle the model with mixed players, our fundamental approach is to determine the aggregate effect, represented by a set of aggregate quantities, of the minor players such that within the population limit the major player optimally responds to that aggregate effect while each minor player optimally responds to the major player and that aggregate effect combined, and such that all the minor players also collectively produce the same aggregate effect initially presumed. The key feature of the MFG approach is finding the closed-loop solutions formed by the Mean Field Hamilton Jacobean Bellman equation (MF-HJB) and the Mean Field Fokker Planck Kolomogrov

Equation (MF-FPK). The later equations will be presented in section 4.5.

4.4 MGO: System Dynamics

Recall Figure 4.1, each household in the MG has a generation potential and a storage system. Each household possesses a smart house (i.e. household i at any time t is able to monitor and control his/her appliances). Hence, the households, i.e. the minor agents, possess demand response technologies to minimize their individual costs. Household i at any time t can store into his/her battery, withdraw from their battery, or demand energy from the MG aggregator. Households with an energy surplus can make a profit by selling their excess energy to other households or the main grid through the aggregator. Hence, each household has a set of time-dependent variables: load (divided into two types fixed, and elastic), generation, which depends on the location and technology deployed, and net storage (i.e. energy stored in his/her battery). Households' decisions are influenced by the aggregator (the major agent) dynamics (i.e. the equilibrium market price for electricity in the MG and main grid). On the other hand, the aggregator system dynamics are the electricity price in the MG and its decision variables are affected by the aggregated demand responses of the households and the main grid pricing mechanism.

4.4.1 Main grid Pricing Mechanism

In this framework, the adopted power network scheme consists of only one main energy company generator. The current electricity market price in this paper is denoted by p_{MA} dollars per kWh. The MA proposed an incentive-based pricing mechanism if consumers are able to reduce power demand during peak demand events. Households will receive a credit if they reduce their power demand during peak demand events. For the scope of this paper, the pricing portfolio will be studied which works as follows: the day before a peak demand event, households will receive a notification about the peak event, households are entitled to a MA credit\$/kWh for every kilowatt-hour (kWh) curtailed (i.e. that is, not consumed compared with your usual energy use).

Figure 4.2 depicts the main grid pricing scheme, the readers can infer that the peak threshold

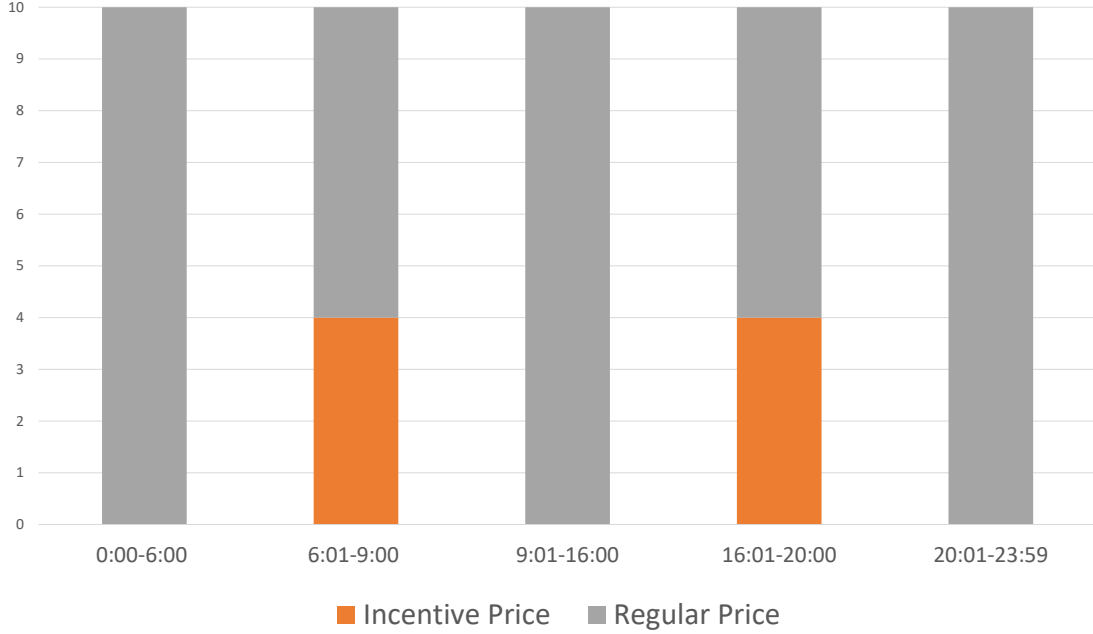


Figure 4.2 Winter Credit Option (WCO)–Pricing Scheme

credit line is assumed to be 4 kWh; and thus, during a peak event, if the household's demand is less than 4KWh, the household will be credited MA credit\$/kWh.

4.4.2 Aggregator System Dynamics

The aggregator gathers the load information from the households (i.e. the minor agents), and accordingly, sets the equilibrium price in the MG. The aggregator objective is to maximize the social welfare of the MG and to guarantee fairness in the community MG (i.e. both internally and externally; internally by setting the MG energy price and externally by setting the penalty and incentive incurred when dealing with the main grid among the households).

The system variables of the aggregator are: (i) the total demand of households, (ii) the electricity price of the MG, and (iii) the assigned MG penalty and incentive price increment and subsidy. In [3], the equilibrium market price was based on Walrasian Equilibrium where the objective was to find the price that maximizes the social welfare assuming a flat rate main grid pricing mechanism. In this framework, the main grid is deploying an incentive-based pricing mechanism to reduce peak cost and encourage participating MGs to keep their

aggregate demand less than a penalty threshold. Thus, the major agent (i.e. the aggregator) objective is to find the MG equilibrium price that maximizes the social welfare taking into account the following: (i) household objective is to minimize his/her costs (i.e. households are assumed to make decision-based on their best response) (ii) main grid peak pricing mechanism and the assigned demand curve to be attained and (iii) aggregator objective is to maintain and control the operation of the MG in the efficient and normal modes (i.e. to keep the aggregated demand curve of the MG in the incentive zone during peak hours). Hence, the MG pricing mechanism will adopt an incentive and penalty-based approach. Denote by p_{MG}^b the base price in the MG, u_A the aggregator price control coefficient such that $-1 \leq u_A \leq 1$, and p_A the price value increment such that $p_{min}^A \leq p_A \leq p_{max}^A$ where p_{max}^A and p_{min}^A are predefined constants. To insure social welfare in the MG, we will use Walrasian equilibrium theory to find the MG equilibrium base price denoted by p_{MG}^b which will be the optimal price during off-peak time as well [127, 128].

4.4.3 MG Equilibrium Price: P_{MG}

The MG price will be divided into two components: (i) the base price [3], the Walrasian equilibrium price that maximizes the social welfare, and (ii) the incentive/penalty during peak time. The Walrasian equilibrium price is found such that: (i) every household maximizes his/her utility given prices, and (ii) market clearing (i.e. the total demand for each commodity just equals the aggregate endowment at every time t), and (iii) it maximizes the social welfare. For more details about the Walrasian equilibrium, we refer the readers to [3].

The aggregated demand in the MG at time t is the sum of the households' demands at time t . Assuming that the retail price of energy is proportional to the first order derivative of the time-dependent generation cost and using the fundamental first welfare theorem, the Pareto-optimal pricing mechanism denoted by p_{MG}^* in the MG is proportional to the aggregate cost of meeting the total load in the MG. Thus, $p_{MG}^*(t)$ is proportional to $\nabla C_u(G_{MG}(t))$ where C_u is the price function which is concave with respect to $G_{MG}(t)$, where $G_{MG}(t)$ is the aggregated

load in the MG i.e.

$$G_{MG}(t) = \sum_{i=1}^{i=N} y^i(t) = N \times E\{y(t)\} = N \int_{\Omega_y} y \mu_y(y, s) dy \quad (4.1)$$

where Ω_y is the range of $y(t)$ for all $0 \leq t \leq T$ and where μ_y represents the load probability density function. Thus, the aggregator will adopt the Walrasian Equilibrium during the off-peak-event time and will use it as the base price during the peak event. The LA will set an incentive and penalty credit during peak events to guarantee the grid functioning in the winter credit incentive threshold. The electricity provider will announce the peak event a day ahead; thus the aggregator and households are aware of the market energy price a day-ahead. Accordingly, the objective of the aggregator is to minimize the day-ahead cost in the MG by setting the clearing equilibrium price for the MG based on off-peak pricing, peak pricing, and the aggregated households' generation and load.

The framework presented in this paper treats the households connected to the MG as one entity; (i.e. the credit collected based on the aggregated demand withdrawn from the main grid). During peak-event, households will be encouraged to reduce their demand $\gamma^i(t)$ in case of shortage and to increase their $r^i(t)$ contribution in case of a surplus. The credit will be calculated based on the aggregated load withdrawn from the main grid, households will be divided into two categories: penalized and incentivized ones. Denote by τ the threshold set by the main grid during peak-event. Households who withdraw from the main grid by an amount less than τ will get credited and who withdraw more than τ will be penalized. The MG will be considered as a whole entity and the overall MG threshold will be $\mathcal{T} = \tau \times N$. The aggregated credited saving of the MG at time t is $\lambda(t)$ where

$$\lambda(t) = \left(\mathcal{T} - \sum_{i=1}^N (\gamma^i(t) - r^i(t)) \right) \quad (4.2)$$

The LA incentive guarantees fairness and to set p_{MG} is Nash-Equilibrium during peak-event. Fairness is guaranteed by setting incentive credit and penalty credit: (i) households will be credited if they are contributing energy during PEs or if the energy withdrawn is less than τ , (ii) household will be penalized if the withdrawn load at time t is greater than τ .

- Incentive credit:

$$\iota^i(t) = \begin{cases} (\tau + r^i(t)) \times \text{MA credit} & r^i(t) \geq 0 \& \gamma^i(t) = 0 \\ (\tau - \gamma^i(t)) \times \text{MA credit} & r^i(t) = 0 \& \gamma^i(t) \leq \tau \end{cases} \quad (4.3)$$

- Penalty credit for household i when $\gamma^i(t) \geq \tau$ & $r^i(t) = 0$

$$\iota^i(t) = (\tau - \gamma^i(t)) \times \text{MA credit} \quad (4.4)$$

Equations (4.3) and (4.4) guarantee that households with a surplus to better off selling during peak-event and force households with a shortage to remain under the preset threshold.

4.4.4 Household Generation Dynamics

The community MG is composed of N households, where N is a sufficiently large number for the MFG theory to be applicable. All the variables are time-dependent, and the time subscript is removed to simplify the notations.

For simplicity, we consider the scenario where households are homogeneous i.e. they deploy the same technology for generation and storage. The generated energy Θ^i by household i at time t is governed by the following stochastic differential equation [3]:

$$d\Theta^i = P_{PV}^i dt + \epsilon_{\Theta}^i dW_{\Theta}^i, \quad (4.5)$$

where W_{Θ}^i , $1 \leq i \leq N$ are N independent Wiener process (i.e. Brownian Motion).

Denote the probability density function governing the energy generation by $\mu_{\theta}(\theta, t)$; thus, the probability density will be calculated and will be assumed known to the aggregator.

4.4.5 Household Operation Cost Model

The cost is mainly due to the operation and maintenance costs (O&M) of the PVs and to the cost of storage.

$$C^i(t) = K_{O\&M} \times \Theta^i(t) + C_{B,M} \times C_{max} \times dt, \quad (4.6)$$

where C_{max} is the battery maximum capacity which is assumed uniform for all households i , $1 \leq i \leq N$ and dt is the time increment.

4.4.6 Energy Demand Management

The energy demand is met by withdrawing from the household's storage, from the MG through the power bank (i.e. from other households), or from the main grid or the microgrid. For simplicity, we assume that household i , $1 \leq i \leq N$ has a time-varying energy demand profile denoted by $y^i(t)$. Households in this framework are assumed to be smart houses with two types of demand: (i) fixed demand denoted by $y_f^i(t)$, and (ii) controllable demand denoted by $y_c^i(t)$. Household i sets the mode of operation for the controllable appliances. His/her decision is influenced by the aggregator message i.e. (p_{MG} and p_{MA}).

$$y^i(t) = y_f^i(t) + y_c^i(t) \quad (4.7)$$

Each appliance is scheduled to operate or remain idle in each time interval (e.g., a quarter) during the day. A whole cycle of the time interval is considered as $\mathbb{T} = t, t + dt, \dots, T$.

4.4.6.1 Fixed-Load i.e. Must-Run Appliances

The first type of load represents all the must-run appliances. These appliances consume a fixed amount of energy during a fixed period per day without any flexibility for their mode of operation. The Fixed load does not change regardless of the microgrid or main grid electricity prices. Examples of such appliances include lighting appliances, a television, and a refrigerator. Recall $y_f^i(t)$ the fixed load of household i and denote by $\mu_{y_f}(y_f, t)$ the probability density function representing the fixed load appliances. Similarly to the generation density function, the probability density function governing the fixed demand μ_{y_f} will be calculated from previously collected data.

4.4.6.2 Controllable Load

The second type of load represents the elastic controllable appliances. The operation of these appliances is quite elastic, such as water heaters and air conditioners, and their mode of operation affects the overall controllable load. This mode of operation will be directly affected by electricity prices (maingrid and microgrid prices). Hence, the electricity market price directly influences the mode of operation. The elastic appliances possess a minimum and maximum energy consumption and depending on the mode of operation the energy consumption level is determined. The operation mode depends on the household preference; for instance, the preferred house set temperature (20-24 °C). Households monitor the electricity price and accordingly, they set the mode of operation for the elastic appliances, for instance, if the electricity price is high they can reduce the heater by a degree or two, and during summer increase the air conditioner by a degree or two.

Using such modes of transition between modes of operation in the elastic appliances, the nature of controllable appliances, and the fact that a load of each mode of operation depends on external conditions (such as external temperature, appliance insulation, etc.), the increment stochastic differential equation will be deployed to model the propagation of the controllable load at time t for each appliance. Thus the total controllable load $y_c(t)$ will be represented by the following equation:

$$dy_c = u_c(t)dt + \epsilon_c\psi(t); \quad (4.8)$$

where $u_c(t)$ represents the increment control vector action for each elastic appliance and $\psi(t)$ is a Brownian motion covering the stochastic nature of the elastic appliances. The Brownian motion represents the stochastic elements governing the external temperature along with the installation elements for the elastic appliances. In essence, households with control the mode of operation via $u_c(t)$ taking into account the preferred mode of operation (i.e. for the heating system to keep the house in the adequate temperature range). We need to note that elastic appliances' modes of operation are independent; thus the control actions are also independent.

4.4.7 Household Energy Profile

Each household has an energy profile consisting of load, generation, storage, energy withdrawn, and energy sold. $\Theta^i(t)$ and $y^i(t)$ are the amount of energy generated by household i and the load of household i at time t respectively. The set of decision variables, $u_c^i(t)$, $u_r^i(t)$, and $u_\gamma^i(t)$, represent the controllable load, the discharging control action (i.e. the amount of energy sold by household i , and the charging control (i.e. the amount of energy withdrawn from the MG to charge the battery of household i) respectively. Define

$$\delta^i(t) := \Theta^i(t) + b^i(t) - y^i(t) \quad (4.9)$$

as the net energy after meeting the current demand where $b^i(t)$ represents the energy balance of the household i . The dynamics of $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ are as follows:

$$b^i(t + dt) = b^i(t) + P_{pv}^i(t)dt - u_c^i(t)dt - dy_f(t)dt \quad (4.10)$$

$$+ \epsilon_\theta dW_\theta - \epsilon_c dW_c - r^i(t) + \gamma^i(t)$$

$$r^i(t) = \begin{cases} u_r^i(t)\delta^i(t) \\ \text{s.t. } u_r^i(t) = 0 \text{ when } \delta^i(t) \leq 0 \text{ \& } 0 \leq u_r^i(t) \leq 1 \end{cases} \quad (4.11)$$

$$\gamma^i(t) = \begin{cases} u_\gamma^i(t)C_{max}dt & \text{if } \delta^i(t) \leq 0 \\ u_\gamma^i(t)(C_{max}dt - \delta^i(t)) & \text{if } \delta^i(t) > 0 \\ \text{s.t. } 0 \leq u_\gamma(t) \leq 1 & \text{for all } 0 \leq t \leq T \end{cases} \quad (4.12)$$

where $0 \leq b^i(t) \leq C_{max}$ for all t and where C_{max} is the storage battery capacity for household i . Two scenarios to be considered; (i) scenario one (**SC1**) where $\delta^i(t) \leq 0$ i.e. there is a shortage at household i and household i is a pure consumer, and (ii) scenario two (**SC2**) where $\delta(t) \geq 0$ where household i is a prosumer i.e. household i has the potential to sell his overflow energy or charge his battery for future use.

In SC1, we have $\delta(t) \leq 0$ then $u_r^* = 0$, $0 \leq u_\gamma^*$ and:

$$db = C_{max}u_\gamma dt + P_{PV}dt - u_c dt - dy_f dt + \epsilon_b dW_b \quad (4.13)$$

denote by function $f := P_{Pv} - u_c - dy_f$ then db is can be written as follows:

$$db = (C_{max}u_\gamma + f)dt + \epsilon_b dW_b \quad (4.14)$$

On the other hand scenario SC2 where $\delta(t) > 0$ will be divided into two sub-cases:

- Case 1: It is optimal to sell (i.e. $u_\gamma^* = 0$):

$$db = \frac{1 - u_r^*}{1 + u_r^*} f dt + \epsilon_b dW_b \quad (4.15)$$

- Case 2: It is optimal to recharge (i.e. $u_r^* = 0$):

$$db = \frac{1 - u_\gamma^*}{1 + u_\gamma^*} f dt + \frac{u_\gamma^*}{1 + u_\gamma^*} C_{max} dt + \epsilon_b dW_b \quad (4.16)$$

Assuming P_{PV} and y are piece-wise continuous and differentiable and using (4.13), (4.15) and (4.16) the readers can prove that db is continuous and differentiable and that u_γ and u_r are continuous.

4.5 MGO: Dynamical Game Formulation

4.5.1 Mean Field Fokker Planck Equation

The MF-FPK enables the aggregator to estimate the propagation in the load and generation. The Markov process used to model the elastic demand can be written in a stochastic differential equation via the following process.

Each appliance is independent of the rest of the appliances thus the overall probability density function for the aggregated elastic demand will be the product of the appliance's Fokker Planck Kolmogorov solution. The appliances possessing elastic demand in a household can be summarized by the water heater, HVAC, and dimmer-enabled appliances (light and any controlled appliance). Thus for an HVAC, we will assume the set temperature can take one of these values $[21^\circ\text{C}, 22^\circ\text{C}, 23^\circ\text{C}]$. As for the water heater the set temperature is $[70^\circ\text{C}, 80^\circ\text{C}, 90^\circ\text{C}]$. By considering the above assumption and deploying the increment partial differential

equation the propagation of the density function for the elastic load can be solved by the following Fokker Plank Kolmogorov equation.

Hence, the aggregator in the constructed framework will be solving the Fokker Planck Kolmogorov equation affiliated with the controllable load and the generated energy and will find the propagation of $\mu(y_c, \theta, t)$ with respect to t . The fixed demand $y_f(t)$ probability density function $\mu_{y_f}(y_f, t)$ will be found by analyzing the collected data over the span of 5 years, the aggregator at any time t is able to find the probability density function $\mu(y_c, y_f, \theta, t) := \mu(y_c, \theta, t) \times \mu_{y_f}(y_f, t)$. Recall equations (4.5) and (4.8) representing the partial differential equation for the generation θ and the controllable load y_c the mean field Fokker Plank Kolmogorov (MF-FPK) equation will be presented in the following equation:

$$\begin{aligned} \frac{\partial \mu(y_c, \theta, t)}{\partial t} = & -\frac{\partial}{\partial \theta} P_{PV} \mu(y_c, \theta, t) + \frac{1}{2} \frac{\partial^2}{\partial \theta^2} (\epsilon_\theta^2 \mu(y_c, \theta, t)) \\ & - \frac{\partial}{\partial y_c} [u_c \mu(y_c, \theta, t)] + \frac{1}{2} \frac{\partial^2}{\partial y_c^2} (\epsilon_c^2 \mu(y_c, \theta, t)) \end{aligned} \quad (4.17)$$

4.5.2 Mean Field Hamilton Jacobean Bellman Equation

The MF-HJB equation can be derived from the cost-to-go function. The MF-HJB solves for the optimal solutions to minimize the cost function of the households. Recall that the objective of the aggregator is to maximize the social welfare in the MG which is achieved by minimizing the aggregated energy bill of the main grid and setting the penalty and incentive fairly among the households. Thus, the cost-to-go function is the aggregated cost of energy in the MG and the coupling occurs in the cost function between the major agent and the minor agent. The aggregator control actions are the MG optimal energy price along with the penalty and incentive granted to the households. Households who are consuming beyond the incentivized threshold will be penalized and households who are providing energy to MG during that peak demand period will be provided with incentives.

The MG connection to the main grid has two modes of operation: (i) PE, and (ii) off-peak demand. For formulating the off-peak scenario, we will use the MFG formulation in [3] with one modification where we include the controllable load. The equilibrium price in this scenario is the Walrasian equilibrium, we need to note that the incentive and penalty control actions

are inactive during the off-PE. Hence, the HJB equation of the off-PE scenario is formulated as follows; we find the optimal control actions that minimize the cost-to-go function. The aggregator is minimizing the cumulative cost of energy in the MG. The MG consists of N households, and the aggregator cost function represents the summation of households' cost functions. Assuming N to be large enough, the aggregated cost of the N households can be written by $N \times$ (the generic household i cost function). This assumption is guaranteed due to the fact that N is sufficiently large enough for the MFG approach to be implemented, and the fact that households are assumed to be homogeneous and their statistical information (generation and load probability density functions) is known to the aggregator. For the off-peak scenario, we refer the reader to [3].

Regarding the second scenario, the PE, the cost function $L(b, y, \theta, \gamma, r, t)$ for a generic house i in the MG is represented as follows:

$$L(\bullet) := E \int_0^{T_f} (\gamma(t) (w_1(t)P_{MG}(t) + w_2(t)P_{MA}(t)) - r(t)P_{MG}(t) + \iota^i(t) + C_{O\&M}(t))dt \quad (4.18)$$

where w_1 and w_2 denote the fraction of demand consumed from the MG and the main grid respectively. P_{MG} in equation (4.18) represents aggregator action and the equilibrium price which is found via the Walrasian equilibrium. Recall equations (4.3), and (4.4) representing the incentive and penalty during the PE duration. The generic household i cost function will be subject to two cases: (i) energy shortage case (i.e. $\delta^i \leq 0$) and (ii) energy surplus case (i.e. $\delta^i(t) \geq 0$). During the first case, household i at time t is a pure energy consumer in the sense that $u_r^i(t) = 0$; thus $r^i(t) = 0$. The generic agent control action at time t will be the elastic appliances mode of operation control $u_c^i(t)$ and the battery charging control $u_\gamma^i(t)$.

4.5.2.1 Energy Fairness coefficients (w_1 and w_2)

In the presented framework, energy fairness is interpreted as follows: all households in the MG incur the same share from both the MG and the main grid (i.e. w_1 and w_2 are the same

for all households).

Thus, w_1 and w_2 in equation (4.18) are calculated as follows:

$$\begin{aligned} w_1(t) &= \min \left\{ 1, \frac{\sum_{i=1}^N r^i(t)}{\sum_{i=1}^N \gamma^i(t)} \right\} \\ w_2(t) &= 1 - w_1(t) \end{aligned} \quad (4.19)$$

Recall the P_{MG} in (4.1), the expected cost function can be written as:

$$l(\bullet) = C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) + \gamma(t) w_2 P_{MA} + \iota$$

Hence the cost function in (4.18) becomes:

$$\begin{aligned} L(\bullet) &= \int_0^{T_f} (C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) \\ &\quad + \gamma(t) w_2 P_{MA} + \iota) dt \end{aligned} \quad (4.20)$$

In (4.20), coupling occurs in the cost function where the aggregated demand and the aggregated generated energy affect the decision of the household, particularly in P_{MG} . Following the cost function $L(\bullet)$ in (4.20) the cost to go $J(b, y, \theta, \gamma, r, s)$ is given by:

$$\begin{aligned} J(\bullet) &= E \left[\int_s^{T_f} L(b_t, y_t, \theta_t, \gamma_t, r_t) dt \right] \quad s.t. \\ b_s &= b, y_s = y, \theta_s = \theta, \gamma_s = \gamma, r_s = r \end{aligned} \quad (4.21)$$

and accordingly the value function $v(\bullet)$ is given by:

$$v(b, y, \theta, \gamma, r, s) = \inf_{u_\gamma, u_r} J(b, y, \theta, \gamma, r, s) \quad (4.22)$$

Assuming that all functions are sufficiently smooth then the mean field Hamilton-Jacobi-Bellman equations (MF-HJBs) for each scenario are:

- SC1: $\delta(t) \leq 0$, $u_r^* = 0$ and the MF-HJB-SC1 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= -C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma} \left\{ \gamma (w_1 P_{MG} + w_2 P_{MA}) + C_{max} u_\gamma \frac{\partial v}{\partial b} - \iota \right\} \\
&+ \inf_{u_c} \left\{ u_c \frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2} \frac{\partial^2 v}{\partial y_c^2} \right\}
\end{aligned} \tag{4.23}$$

where $\iota(t) = (\tau - \gamma(t)) \times \text{MA credit}$ and thus u_γ^* and u_c^* are given by the following equations:

$$\begin{aligned}
u_\gamma^* &= \inf_{u_\gamma} \left\{ C_{max} u_\gamma \left(\frac{\partial v}{\partial b} + \text{MA credit} \right) \right\} \\
\Rightarrow u_\gamma^* &= \begin{cases} 0 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} \geq 0 \\ 1 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} < 0 \end{cases}
\end{aligned} \tag{4.24}$$

$$u_c^* = -\text{sgn} \left(\frac{\partial v}{\partial y_c} \right) \tag{4.25}$$

- SC2: $\delta(t) > 0$ i.e. household can either sell the surplus, fill the battery or do nothing. The MF-HJB-SC2 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma, u_r, u_c} \{ \gamma (w_1 P_{MG} + w_2 P_{MA}) - r P_{MG} - \iota \} \\
&+ \inf_{u_\gamma, u_r, u_c} \left\{ \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \right\} \\
&+ \inf_{u_\gamma, u_c} \left\{ \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b} \right\} + \inf_{u_c} \left\{ u_c \frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2} \frac{\partial^2 v}{\partial y_c^2} \right\}
\end{aligned} \tag{4.26}$$

thus:

$$\begin{aligned} \{u_\gamma^*, u_r^*\} = \inf_{u_\gamma, u_r} & \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \\ & + \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b} \end{aligned} \quad (4.27)$$

$$u_c^* = - \operatorname{sgn} \left(\frac{\partial v}{\partial y_c} \right) \quad (4.28)$$

Using (4.27) and taking into account that (i) P_{PV} represents the rate of change in energy production θ (i.e. supply), (ii) dy represents rate of change in load y (i.e. consumption) and (iii) the term $\frac{\partial v}{\partial b}$ represents the rate of change in cost to go function v with respect to household balance b i.e.

$$\frac{\partial v}{\partial b} = \frac{v(t + dt) - v(t)}{b(t + dt) - b(t)}$$

The optimal solutions for the MF-HJB-SC2 equation in (4.26) are derived. The optimal solutions are presented in Table 4.1.

Table 4.1: Optimal Solution considering SC2

Scenario	$P_{PV} - dy$	$\frac{\partial v}{\partial b} + \text{MA credit}$
$u^* r = 1$ $u^\gamma = 0$	≥ 0	≥ 0
$u^* r = 0$ $u^\gamma = 0$	> 0	< 0
$u^* r = 0$ $u^\gamma = 0$	< 0	≥ 0
$u^* r = 0$ $u^\gamma = 1$	≤ 0	< 0

4.5.3 Mean Field Loop

The MFG loop is solved solely by the major agent which relays the optimal pricing decision to the minor agents. In this approach the aggregator does not have control over the appliances and full autonomy is assumed at the minor agents. The MFG approach with the assumptions that households are rational players and guaranteed fairness provided the best response

approach in the MG. The MFG loop represents the coupling in the mean-field game theory where the dynamics and control actions are both coupled and time-variant. The initial statistical information is used in the Hamilton Jacobian Bellman (HJB) equation to find the optimal energy price for the MG and household energy profile controls. The optimal decisions are then used to update the statistical information by solving the Fokker Plank Kolmogorov (FPK) equation when the mean field controls are implemented. The mean field loop aims to find the fixed point convergence i.e. the pair of the probability density function and its corresponding set of optimal control actions.

Recall the equations representing the MF-FPK and MF-HJB in (4.17), (4.23), and (4.26), the MFG loop can be summarized by the following diagram:

$$\begin{array}{ccc}
 \mu(y_c, \theta, t) & \xrightarrow{(4.1, 4.3, 4.4)} & p_{MG}(t) \xrightarrow{(4.23, 4.26)} v(b, y, \theta, \gamma, r, t) \\
 \nwarrow & & \swarrow \\
 & & u_c^*(t), u_\gamma^*(t), u_r^*(t)
 \end{array} \tag{4.29}$$

The loop in equation (4.29) summarizes the MFG approach and the coupling in the MF-HJB and MF-FPK along with the coupling in the cost functions and system dynamics of the generic agents and aggregators. The uniqueness of the MFG approach is represented by the converging solution of MF-FPK and control actions represented by $\mu(y_c, \theta, t)$ and $[u_c^*, u_\gamma^*, u_r^*]$ of the loop (4.29).

4.6 Conclusion

In summary, this paper presents a theoretical formulation for residential MG users through a load aggregator via an application of mean-field game theory. The paper addresses the system dynamics governing the energy profile of the households and the load aggregator. Here, we propose a game theoretical formulation for an incentive-based residential community, where an incentive-based pricing mechanism is adopted to encourage peak demand reduction and share the incentive demand curve with the residential community through the aggregator.

The aggregator's objective is to maximize the welfare of the residential community by finding the optimal community equilibrium electricity price. Each household communicates with each others and with the main grid through the aggregator and aims to minimize the local electricity cost. We have derived the essential equations for the approach along with the MFG-loop governing the coupling between the MF-FPK and MF-HJB equations. Consequently, to extend the work in this paper, the mathematical and computational investigations for the MG optimization problem with incentive-based pricing will be implemented using the problem architecture and formulation presented in this paper.

CHAPTER 5 COMPUTATIONAL INVESTIGATION OF RESIDENTIAL USERS THROUGH LOAD AGGREGATOR SMART GRID POWER OPTIMIZATION PROBLEM

5.1 Abstract

Incentive-based demand response aggregators are widely recognized as a powerful strategy to increase the flexibility of residential community microgrid (RCM) while allowing consumers' assets to participate in the power system operation in critical peak times. This paper presents the computational investigation for an incentive-based residential community [2], where an incentive-based pricing mechanism is adopted to encourage peak demand reduction and share the incentive demand curve with the residential community through the aggregator. Nash Equilibrium strategies, which minimize the total RCM energy cost functions, are found by applying mean field game theory. Computational investigations of the decentralized RCM electric energy problem via mean field game theory are presented for the MG with controllable load household and connected to a main grid with an incentive-based pricing strategy.

Keywords: RCM, aggregator, demand response, Mean Field Game Theory, Dynamical Game, Pricing Mechanism, Dynamical Pricing, Stochastic Control.

5.2 Introduction

The smart grid is much automated and can be integrated into the main supply with distributed generation resources (DER) sources (primarily renewable), energy storage systems (ESSs), and advanced metering infrastructure. Due to using high technology devices in generation and distribution systems, the smart grid represents an advanced digital system [136]. In addition, the smart grid functionalities, such as real-time monitoring, load balancing, and accurate billing, require the granular collection of smart metering data at frequent time intervals [137].

In this paper, we propose a computational game-theoretical investigation for the incentive-based residential community microgrid with a load aggregator (RCMLG) via an application of Mean Field Game theory (MFG). Mean field game theory is a mathematical framework that studies the behavior of large populations of interacting agents. It combines concepts from game theory and statistical physics to analyze situations where each agent aims to optimize its own objective function while accounting for the aggregate behavior of the entire population. In mean-field games, the interactions between agents are approximated by a mean field, representing the average effect of all other agents on each individual's decision-making process. This approach allows for the analysis of complex systems with a large number of agents, making it applicable to various fields such as economics, biology, and engineering. Mean field game theory has been extensively studied and developed since its introduction by Lasry and Lions [30–32], and separately by Caines, Huang, and Malhame [21, 22] in 2006, leading to valuable insights into the dynamics and equilibria of large-scale interactive systems. This paper represents a continuation of the work in [2]. The energy network considered in this framework consists of a main grid generation company (MA) that adopts an incentive-based pricing mechanism to reduce peak demand and it shares the incentive demand curve with the residential microgrid (MG) through the aggregator. The aggregator's objective is to maximize the welfare of the microgrid by finding the optimal equilibrium electricity price in the microgrid. In the constructed MG, household demand is divided into two types: (i) fixed, and (ii) controllable. Households communicate with each other and the main grid through the aggregator and they aim to minimize their individual electricity bill. The numerical outcomes of this research will lead to (i) A new general pricing methodology for the MG connected to an incentive-based pricing MA, (ii) the Nash-Equilibrium strategies for the households and community MG aggregator, and (iii) real-time optimization resulting in a reduction of both households' energy bill and MA peak demand cost.

5.2.1 MG Architecture [2] and Paper Objectives

Considering the block diagram in [2] represented in Figure 5.1, the community MG is composed of an aggregator and N households, in the formulated game N is a sufficiently large

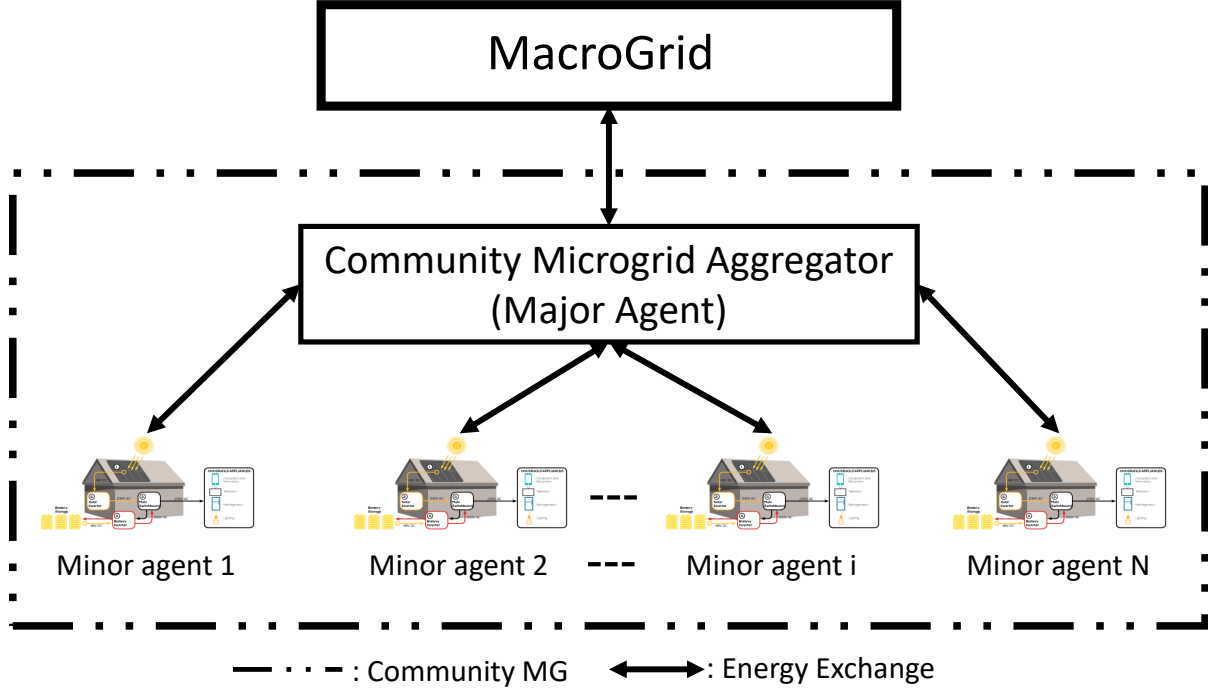


Figure 5.1 Major-Minor Agent Community Microgrid Block Diagram

number. By construction, the aggregator mediates the energy exchange between the microgrid and the main grid along with the energy exchange within the microgrid (i.e. between the households themselves and the households and the main grid).

5.2.2 Computational Investigation for the MGO problem in Literature

This subsection surveys mathematical techniques used in literature to model stochasticity in MGO problems. It discusses classical mathematical algorithms, machine learning algorithms, and the MFG computational approach.

- **Classical Mathematical Algorithms:** Various mathematical solvers improve Microgrid (MG) management capabilities. A general algebraic modeling system (GAMS) is used to evaluate linear equations and handle uncertainties in generation [138]. Multi-scenario mixed-integer linear programming MILP models were used in [139]. In addition, sparse nonlinear optimizer (SNOPT), which is based on GAMS, was used in [140] to address scheduling and optimization problems. These algorithms demonstrate effectiveness in

handling large-scale MILP problems.

- Machine Learning Algorithms: Noyan et al. introduced stochastic methods to approximate the probability distribution function (PDF) of random scenarios in [141]. Scenario-based generation, using discrete distribution sets and Monte Carlo Simulation (MCS), handles uncertainties in photovoltaic (PV) and wind turbines (WT) were introduced in [142]. Fuzzy theory and robust optimization are applied to renewable energy resources (RERs) and address parametric uncertainty was introduced in [62].
- MFG computational approach: Mean field games theory utilizes differential calculus to represent players as a continuum. It simplifies the handling of player entries/exits and enables seamless modeling of player generations. The introduction of a social dimension incorporates statistical data on other players. Mean field games offer advantages over N-player games and provide initial approximations to N-player solutions. MFG is a decentralized approach that requires minimal communication between agents. Its offline-solving capability makes it promising for real-time microgrid power network optimization.

5.2.3 Paper Objective

In this paper, the objective is to optimize the cost of the energy exchanged in community MG which is operating in a grid-connected mode where the MA adopts an incentive-based pricing mechanism. The focus is on numerically optimizing the MG optimization problem (MGO). In this context and as seen in Figure 5.1, the community MG is composed of a sufficiently large number of households and an aggregator. Households that are located within the geographical area of the MG communicate and exchange energy with each other and with the main grid through the aggregator of the community MG. The objective of the aggregator is to maximize the social welfare of the MG households. The aggregator sets the equilibrium price in the MG and sends the aggregated demand to the main grid provider. The aggregator collects the price of electricity from the main grid and then forwards the MG equilibrium price and the main grid electricity price to households. Households in the MG react to the message and the prices provided by the aggregator. Households' objective

function is to minimize their energy cost and their demand is assumed to be divided into two types: (i) fixed demand, and (ii) controllable demand. Fixed demand represents all demand of all the appliances that cannot be controlled. Controllable demand represents all appliances that can be monitored and controlled as they are running.

The scope of this paper is to numerically solve the MGO via an application means of dynamical game theory. The objective of the connected households is to minimize their total cost and the aggregator to maximize the social welfare of the MG via means of major agent minor agents mean field game theory. This paper presents a high-level formulation of the load aggregated energy problem and a proof of concept that major-minor agent mean field game theory is a plausible way to find the optimal pricing mechanism deployed in the microgrid and the optimal load management strategies for the micro-grid households in an incentive main grid formulation.

5.2.4 Paper Outline

The rest of the paper is structured as follows: Section II presents a summary of the MGO system dynamics presented in [2]. Section III focuses on the formulation MGO via an application of mean-field game theory. Section IV discusses the algorithmic implementation of the MGO. Section V presents the numerical simulation results for the MGO along with the numerical robust analysis for the proposed algorithms. Finally, in section VI we conclude the paper.

5.3 MGO System Dynamics

This section presents a high-level summary of the MGO problem system dynamics from [2]. For the detailed formulation, we refer the readers to [2].

5.3.1 Main Grid Pricing Mechanism

In this framework, the power network scheme studies a case MG in Montreal, Quebec where there is only one main energy company generator, Hydro-Quebec (HQ). The current electricity market price in this paper is denoted by p_{MA} dollars per kWh. HQ is motivated in

adopting an incentive-based pricing mechanism. Currently, HQ is offering two portfolios for households to choose from:

1. Winter Credit Option (WCO),
2. Rate Flex Dynamic Pricing (RFD)

The aim of these portfolios is to reduce peak costs. For the scope of this paper, Portfolio (1) will be studied. Portfolio (1) works as follows: the day before a peak demand event, households will receive a notification about the peak event, and during an event, households are entitled to a 0.5\$/kWh for every kilowatt-hour (kWh) curtailed (i.e., that is, not consumed compared with your usual energy use).

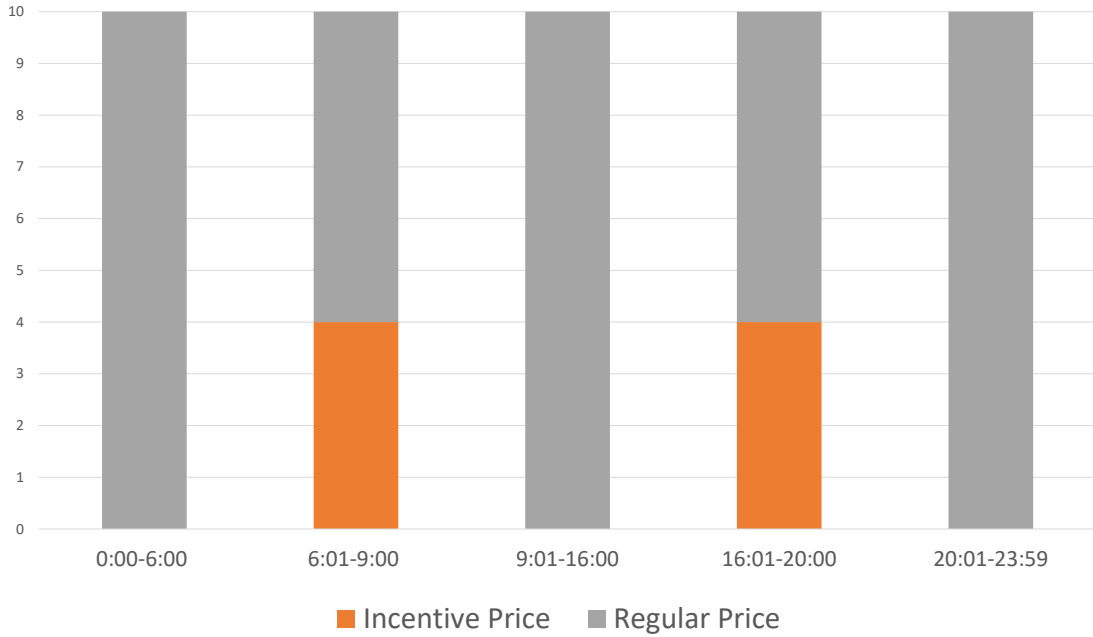


Figure 5.2 Winter Credit Option (WCO)–Pricing Scheme

Figure 5.2 depicts the main grid pricing scheme for the WCO portfolios, the readers can infer that the peak threshold credit line is denoted by τ and here $\tau = 4Kwh$; and thus, during a peak event, if the household's demand is less than 4KWh, the household will be credited MA credit = 0.5\$/Kwh.

5.3.2 MG Equilibrium Price: P_{MG}

The microgrid price will be divided into two components: (i) the base price [3], the Walrasian equilibrium price that maximizes the social welfare, and (ii) the incentive/penalty during peak time. For more details about the Walrasian equilibrium, we refer the readers to [3] and for the incentive/penalty component, we refer the reader to [2].

Denote by $p_{MG}^w(t)$ the Walrasian equilibrium MG price, $p_{MG}^w(t)$ is proportional to $\nabla C_u(G_{MG}(t))$ where C_u is the price function which is concave with respect to $G_{MG}(t)$ where $G_{MG}(t)$ is the aggregated load in the MG i.e.,

$$G_{MG}(t) = \sum_{i=1}^{i=N} y^i(t) = N \times E\{y(t)\} = N \int_{\Omega_y} y \mu_y(y, s) dy \quad (5.1)$$

where Ω_y is the range of $y(t)$ for all $0 \leq t \leq T$.

The AG will adopt the Walrasian Equilibrium during the off-peak-event time, and as its base price during the peak event. The AG will set an incentive and penalty credit during peak events to guarantee the grid functioning in the winter credit incentive threshold. During peak-event, the MG will be considered as one entity and the aggregated threshold is $\mathcal{T} = \tau \times N$. The aggregated credited saving of the MG at time t is $\lambda(t)$ where

$$\lambda(t) = \left(\mathcal{T} - \sum_{i=1}^N (\gamma^i(t) - r^i(t)) \right) \quad (5.2)$$

The AG aims to guarantee fairness and to set p_{MG} as Nash-Equilibrium during the peak-event.

- Incentive credit:

$$\iota^i(t) = \begin{cases} (\tau + r^i(t)) \times \text{MA credit} & r^i(t) \geq 0 \& \gamma^i(t) = 0 \\ (\tau - \gamma^i(t)) \times \text{MA credit} & r^i(t) = 0 \& \gamma^i(t) \leq \tau \end{cases} \quad (5.3)$$

- Penalty credit for household i when $\gamma^i(t) \geq \tau$ & $r^i(t) = 0$

$$\iota^i(t) = (\tau - \gamma^i(t)) \times \text{MA credit}, \quad (5.4)$$

where $r^i(t)$ and $\gamma^i(t)$ represents the amount of energy sold or bought by household i at time t .

5.3.3 Household Generation and Operation Cost Dynamics

In this framework, we assume a uniform setting where all households are homogeneous, using identical generation and storage technologies. This assumption streamlines the modeling process and numerical simulations. However, it's worth noting that the algorithms we've presented can also accommodate non-uniform scenarios. The straightforward way to account for heterogeneity is to adjust the probability density function. By making this singular update in the algorithms' input, one can derive the Mean Field Game (MFG) equilibrium for non-uniform households. However, if we were to incorporate heterogeneity as a system variable, this would increase both the complexity and computational performance of the algorithms due to adding extra dimensions to model such a scenario. The MFG algorithm will certainly generate optimal solutions for the nonuniform MGO case.

The generated energy Θ^i by household i at time t is governed by the following stochastic differential equation [3]:

$$d\Theta^i = P_{PV}^i dt + \epsilon_{\Theta}^i dW_{\Theta}^i, \quad (5.5)$$

where W_{Θ}^i , $1 \leq i \leq N$ are N independent Wiener process (i.e. Brownian Motion). Denote the probability density function governing the energy generation by $\mu_{\theta}(\theta, t)$. For the scope of this paper, the propagation of the probability density function for the generation function will be found by analyzing the data collected over the span of 5 years for an MG consisting of 300 households.

The generation cost is mainly due to the operation and maintenance costs (O&M) of the PVs and the cost of storage. Assuming the cost of operation and maintenance O&M is $K_{O\&M} = 0.0535(\$/kWh)$ for generation and $C_{B,M} = 0.001(\$/kWh)$ for storage, the cost of

generation for household i is given to be:

$$C^i(t) = K_{O\&M} \times \Theta^i(t) + C_{B,M} \times C_{max} \times dt, \quad (5.6)$$

where C_{max} is the battery maximum capacity which is assumed uniform for all households i , $1 \leq i \leq N$ and dt is the time increment.

5.3.4 Energy Load Management

For simplicity, we assume that household i , $1 \leq i \leq N$ has a time-varying energy demand profile denoted by $y^i(t)$. Households in this framework are assumed to possess a smart house technology with two types of demand: (i) fixed demand denoted by $y_f^i(t)$, and (ii) controllable demand denoted by $y_c^i(t)$. Household i demand $y^i(t)$ is given by:

$$y^i(t) = y_f^i(t) + y_c^i(t). \quad (5.7)$$

Denote by $\mu_{y_f}(y_f, t)$ the probability density function representing the fixed load appliances. Similarly to the generation density function, the probability density function governing the fixed demand will be calculated from collected data over the span of 4 years for 300 households.

For the controllable load, households monitor the electricity price and accordingly, they set the mode of operation for the controllable appliances. Using such modes of transition between modes of operation in the controllable appliances, the nature of controllable appliances, and the fact that a load of each mode of operation depends on external conditions (such as external temperature, appliance insulation, etc.), the increment stochastic differential equation will be deployed to model the propagation of the controllable load at time t for each appliance. Thus the total controllable load $y_c(t)$ will be represented by the following equation:

$$dy_c = u_c(t)dt + \epsilon_c W_c(t); \quad (5.8)$$

where $u_c(t)$ represents the increment control vector action for each controllable appliance and

$W_c(t)$ is a Brownian motion covering the stochastic nature of the controllable appliances, and $y_c^i(t)$ is bounded by $Y_{min}^c \leq y_c^i(t) \leq Y_{max}^c$ for all $1 \leq i \leq N$. In this framework, households are assumed to be identical, hence Y_{min}^c and Y_{max}^c are considered uniform for all households and represent the boundary conditions for the controllable load to be in the household's comfort mode of operation. In other words, Y_{min}^c and Y_{max}^c represent the minimum load and maximum load respectively for the controllable appliances to function within the minimum mode and maximum mode of their respective comfortable mode of operation.

5.3.5 Household Energy Profile [2]

Each household has an energy profile consisting of load, generation, storage, energy withdrawn, and energy sold. $\Theta^i(t)$ and $y^i(t)$ are the amount of energy generated by household i and the load of household i at time t respectively. The set of decision variables, $u_c^i(t)$, $u_r^i(t)$, and $u_\gamma^i(t)$, represent the controllable load, the discharging control action (i.e., the amount of energy sold by household i , and the charging control (i.e., the amount of energy withdrawn from the MG to charge the battery of household i) respectively. Define

$$\delta^i(t) := \Theta^i(t) + b^i(t) - y^i(t) \quad (5.9)$$

as the net energy after meeting the current demand. The dynamics of $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ are as follows:

$$b^i(t + dt) = b^i(t) + P_{pv}^i(t)dt - u_c^i(t)dt - dy_f(t)dt \quad (5.10)$$

$$+ \epsilon_\theta dW_\theta - \epsilon_c dW_c - r^i(t) + \gamma^i(t)$$

$$r^i(t) = \begin{cases} u_r^i(t)\delta^i(t) \\ \text{s.t. } u_r^i(t) = 0 \text{ when } \delta^i(t) \leq 0 \text{ \& } 0 \leq u_r^i(t) \leq 1 \end{cases} \quad (5.11)$$

$$\gamma^i(t) = \begin{cases} u_\gamma^i(t)C_{max}dt & \text{if } \delta^i(t) \leq 0 \\ u_\gamma^i(t)(C_{max}dt - \delta^i(t)) & \text{if } \delta^i(t) > 0 \\ \text{s.t. } 0 \leq u_\gamma(t) \leq 1 & \text{for all } 0 \leq t \leq T \end{cases} \quad (5.12)$$

where $0 \leq b^i(t) \leq C_{max}$ for all t and where C_{max} is the storage battery capacity for household i . Two scenarios are considered: (i) scenario one (**SC1**), where $\delta^i(t) \leq 0$ i.e., there is a shortage at household i and household i is a pure consumer, and (ii) scenario two (**SC2**) where $\delta(t) \geq 0$ where household i is a prosumer i.e., household i has the potential to sell his overflow energy or charge his battery for future use.

In SC1, we have $\delta(t) \leq 0$ then $u_r^* = 0$, $0 \leq u_\gamma^*$ and:

$$db = C_{max}u_\gamma dt + P_{PV}dt - u_c dt - dy_f dt + \epsilon_b dW_b. \quad (5.13)$$

If one denotes by function $f := P_{PV} - u_c - dy_f$, then db is can be written as follows:

$$db = (C_{max}u_\gamma + f)dt + \epsilon_b dW_b. \quad (5.14)$$

On the other hand, scenario SC2, where $\delta(t) > 0$, will be divided into two sub-cases:

- Case 1: It is optimal to sell (i.e. $u_r^* = 0$):

$$db = \frac{1 - u_r^*}{1 + u_r^*} f dt + \epsilon_b dW_b \quad (5.15)$$

- Case 2: It is optimal to recharge (i.e. $u_r^* = 0$):

$$db = \frac{1 - u_\gamma^*}{1 + u_\gamma^*} f dt + \frac{u_\gamma^*}{1 + u_\gamma^*} C_{max} dt + \epsilon_b dW_b \quad (5.16)$$

Assuming P_{PV} and y are piece-wise continuous and differentiable and using (5.13), (5.15), and (5.16), the readers can prove that db is continuous and differentiable and that u_γ and u_r are continuous.

5.4 MGO: Dynamical Game Formulation

The objective of this framework is to develop a decentralized control synthesis under large population conditions such that each player's strategy uses only limited information. In particular, we assume that the aggregator's decisions are communicated to the households,

while the state of each household is always known to itself but can be estimated by the aggregator using the mean field framework. The key feature of the MFG approach is finding the closed-loop solutions formed by the Mean Field Hamilton-Jacobeian-Bellman equation (MF-HJB) and the Mean Field Fokker Planck-Kolmogrov-Equation (MF-FPK).

5.4.1 Mean Field Fokker Planck Equation

The aggregator in the constructed framework will be solving the Fokker Planck Kolmogrov equation affiliated with the controllable load and the generated energy and will find the propagation of $\mu(y_c, \theta, t)$ with respect to t . The aggregator at any time t is able to find the probability density function $\mu(y_c, y_f, \theta, t) := \mu(y_c, \theta, t) \times \mu_{y_f}(y_f, t)$. Recall equations (5.5) and (5.8) representing the partial differential equation for the generation θ and the controllable load y_c the mean field Fokker Plank Kolmogrov (MF-FPK) equation will be presented in the following equation:

$$\begin{aligned} \frac{\partial \mu(y_c, \theta, t)}{\partial t} = & -\frac{\partial}{\partial \theta} P_{PV} \mu(y_c, \theta, t) + \frac{1}{2} \frac{\partial^2}{\partial \theta^2} (\epsilon_\theta^2 \mu(y_c, \theta, t)) \\ & - \frac{\partial}{\partial y_c} [u_c \mu(y_c, \theta, t)] + \frac{1}{2} \frac{\partial^2}{\partial y_c^2} (\epsilon_c^2 \mu(y_c, \theta, t)) \end{aligned} \quad (5.17)$$

5.4.2 Mean Field Hamilton Jacobean Bellman Equation

The microgrid connection to the main grid has two modes of operation: (i) peak event, and (ii) off-peak demand. For the off-peak scenario, we will use the MFG formulation in [3] with one modification where we include the controllable load and the controllable appliances. The equilibrium price in this scenario is the Walrasian equilibrium, we need to note that the incentive and penalty control actions are inactive during the off-peak event. Regarding the second scenario, the peak-event, the cost function $L(b, y, \theta, \gamma, r, t)$ for a generic house i in the microgrid is represented as follows:

$$\begin{aligned}
L(\bullet) := & E \int_0^{T_f} (\gamma(t) (w_1(t)P_{MG}(t) + w_2(t)P_{MA}(t)) \\
& - r(t)P_{MG}(t) + \iota^i(t) + C_{O\&M}(t))dt
\end{aligned} \tag{5.18}$$

where w_1 and w_2 denote the fraction of demand consumed from the MG and the main grid respectively. From [2], w_1 and w_2 in equation (5.18) are as follows:

$$\begin{aligned}
w_1(t) = & \min \left\{ 1, \frac{\sum_{i=1}^N r^i(t)}{\sum_{i=1}^N \gamma^i(t)} \right\} \\
w_2(t) = & 1 - w_1(t)
\end{aligned} \tag{5.19}$$

The cost function in (5.18) becomes:

$$\begin{aligned}
L(\bullet) = & \int_0^{T_f} (C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) \\
& + \gamma(t)w_2P_{MA} + \iota)dt
\end{aligned} \tag{5.20}$$

Hence, the cost to go $J(b, y, \theta, \gamma, r, s)$ is given by:

$$\begin{aligned}
J(\bullet) = & E \left[\int_s^{T_f} L(b_t, y_t, \theta_t, \gamma_t, r_t) dt \right] \quad s.t. \\
& b_s = b, y_s = y, \theta_s = \theta, \gamma_s = \gamma, r_s = r
\end{aligned} \tag{5.21}$$

and accordingly the value function $v(\bullet)$ is given by:

$$v(b, y, \theta, \gamma, r, s) = \inf_{u_\gamma, u_r} J(b, y, \theta, \gamma, r, s) \tag{5.22}$$

The generic household i cost function will be subject to two cases: (i) energy shortage case (i.e. $\delta^i \leq 0$) and (ii) energy surplus case (i.e. $\delta^i(t) \geq 0$). Assuming that all functions are sufficiently smooth then the mean field Hamilton-Jacobi-Bellman equations (MF-HJBs) for each scenario are:

- SC1: $\delta(t) \leq 0$, $u_r^* = 0$ and the MF-HJB-SC1 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= -C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma} \left\{ \gamma (w_1 P_{MG} + w_2 P_{MA}) + C_{max} u_\gamma \frac{\partial v}{\partial b} - \iota \right\} \\
&+ \inf_{u_c} \left\{ u_c \frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2} \frac{\partial^2 v}{\partial y_c^2} \right\}
\end{aligned} \tag{5.23}$$

where $\iota(t) = (\tau - \gamma(t)) \times \text{MA credit}$ and thus u_γ^* and u_c^* are given by the following equations:

$$\begin{aligned}
u_\gamma^* &= \inf_{u_\gamma} \left\{ C_{max} u_\gamma \left(\frac{\partial v}{\partial b} + \text{MA credit} \right) \right\} \\
\Rightarrow u_\gamma^* &= \begin{cases} 0 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} \geq 0 \\ 1 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} < 0 \end{cases}
\end{aligned} \tag{5.24}$$

$$u_c^* = -\text{sgn} \left(\frac{\partial v}{\partial y_c} \right) \tag{5.25}$$

- SC2: $\delta(t) > 0$ i.e. household can either sell the surplus, fill the battery or do nothing. The MF-HJB-SC2 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} &= C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma, u_r, u_c} \{ \gamma (w_1 P_{MG} + w_2 P_{MA}) - r P_{MG} - \iota \} \\
&+ \inf_{u_\gamma, u_r, u_c} \left\{ \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \right\} \\
&+ \inf_{u_\gamma, u_c} \left\{ \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b} \right\} + \inf_{u_c} \left\{ u_c \frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2} \frac{\partial^2 v}{\partial y_c^2} \right\}
\end{aligned} \tag{5.26}$$

thus:

$$\begin{aligned} \{u_\gamma^*, u_r^*\} = \inf_{u_\gamma, u_r} & \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy) \frac{\partial v}{\partial b} \\ & + \frac{u_\gamma}{1 + u_\gamma} C_{max} \frac{\partial v}{\partial b} \end{aligned} \quad (5.27)$$

$$u_c^* = - \operatorname{sgn} \left(\frac{\partial v}{\partial y_c} \right) \quad (5.28)$$

The optimal solutions for the MF-HJB-SC2 equation in (5.26) are derived. The optimal solutions are presented in Table 5.1.

Table 5.1: Optimal Solution considering SC2

Scenario	$P_{PV} - dy$	$\frac{\partial v}{\partial b} + \text{MA credit}$
$u^* r = 1$ $u^\gamma = 0$	≥ 0	≥ 0
$u^* r = 0$ $u^\gamma = 0$	> 0	< 0
$u^* r = 0$ $u^\gamma = 0$	< 0	≥ 0
$u^* r = 0$ $u^\gamma = 1$	≤ 0	< 0

5.4.3 Mean Field Loop

The mean field loop aims to find the fixed point convergence i.e., the pair of the probability density function and its corresponding set of optimal control actions. Recall the equations representing the MF-FPK and MF-HJB in (5.17), (5.23), and (5.26), the MFG loop can be summarized by the following diagram:

$$\begin{array}{ccc} \mu(y_c, \theta, t) & \xrightarrow{(5.1, 5.3, 5.4)} & p_{MG}(t) \xrightarrow{(5.23, 5.26)} v(b, y, \theta, \gamma, r, t) \\ \nwarrow & & \swarrow \\ \nwarrow & & \swarrow \\ & & u_c^*(t), u_\gamma^*(t), u_r^*(t) \end{array} \quad (5.29)$$

The loop in equation (5.29) summarizes the MFG approach, the coupling in the MF-HJB and MF-FPK, and the coupling in the cost functions and system dynamics of the generic agents and aggregators. The uniqueness of the MFG approach is represented by the converging solution of MF-FPK and control actions represented by $\mu(y_c, \theta, t)$ and $[u_c^*, u_\gamma^*, u_r^*]$ of the loop (5.29).

5.5 Algorithmic Implementation For the MGO via MFG

In this section, we will present the computational procedures that address the MF-HJB equation in both scenarios: (i) a day with no peak event, and (ii) a day with peak event.

5.5.1 Boundary Specifications and Discretization Techniques

To solve the MFG equations (HJB and FPK) using numerical methods, two aspects need to be determined: 1) the boundary conditions and 2) the approximation techniques employed to discretize the partial differential equations.

5.5.1.1 Boundary Specifications

The following are the prescribed boundary specifications for implementing the MFG algorithm on the constructed MGO problem:

- $0 \leq t \leq T$, where T is to be quantified.
- $0 \leq u_r, u_\gamma \leq 1$ and the load control is $u_c := [-1, 1]$.
- Power is assumed to be positive and bounded i.e.,
 $0 \leq P_{PV} \leq P_{PV}^{max}$, where P_{PV}^{max} is to be quantified.
- Controllable load $y_c^i(t)$ is bounded for all $1 \leq i \leq N$, i.e. $Y_{min}^c \leq y_c^i(t) \leq Y_{max}^c$, where Y_{min}^c and Y_{max}^c are defined.
- $\mu_\theta(\theta, t)$ and $\mu_y(y, t)$ are specified and known to the households.
- Final cost is zero i.e., $v(\theta, b, y, \gamma, r, T) = 0$.

- Final household i account balance $b^i(T) = 0$ for all $1 \leq i \leq N$.
- C_{max} is uniform for all households and is to be quantified.
- During peak events, MA credit, and the demand curve threshold τ are to be quantified and communicated to households through the microgrid aggregator.

5.5.1.2 Discretization Techniques

To achieve numerical solutions for the partial equations, it is imperative to meet these requirements and employ the designated approximation techniques. The time, energy, household balance, and demand are represented by Δt , $\Delta \theta$, Δb , and Δy , respectively, as step sizes. For the numerical algorithms to converge effectively, these step sizes must fulfill the essential conditions outlined in [130], which ensure the convergence of the Fixed Point Argument Method. Specifically, in this scenario, the conditions are as follows:

$$\begin{aligned} \frac{|P_{PV}^{max}|\Delta t}{(\Delta \theta)^2} &\leq \frac{1}{2}; \quad \frac{\Delta t}{(\Delta y)^2} \leq \frac{1}{2}; \\ \frac{|\Upsilon|\Delta t}{(\Delta b)^2} &\leq \frac{1}{2}; \quad \frac{|Y_{max}^c|\Delta t}{(\Delta y_c)^2} \leq \frac{1}{2} \end{aligned} \quad (5.30)$$

where $\Upsilon := \max [C_{max}, \max (P_{PV} - \delta y)]$. The methods employed to discretize the first and second-order derivatives of a function g with respect to the variable ρ are outlined below:

- 1st order derivative:

$$\begin{aligned} \text{Forward in } \rho : \frac{\partial g(\cdot, \rho)}{\partial \rho} &= \frac{g(\cdot, \rho + \Delta \rho) - g(\cdot, \rho)}{\Delta \rho} \\ \text{Backward in } \rho : \frac{\partial g(\cdot, \rho)}{\partial \rho} &= \frac{g(\cdot, \rho) - g(\cdot, \rho - \Delta \rho)}{\Delta \rho} \end{aligned} \quad (5.31)$$

- 2nd order derivative:

$$\frac{\partial^2 g}{\partial \rho^2} = \frac{g(\cdot, \rho + \Delta \rho) - 2g(\cdot, \rho) - g(\cdot, \rho - \Delta \rho)}{(\Delta \rho)^2} \quad (5.32)$$

5.5.2 Algorithmic procedure for the Uniform Households

The presented numerical algorithm focuses on the constructed residential community micro-grid where statistical information is derived from historically collected data. In this framework, we are assuming that the households possess knowledge of the statistical information, denoted by $\mu_\theta(t, \theta)$ and $\mu_y(t, y)$. The MFG algorithmic implementation provides a numerical solution for the converging solution for mean-field equations presented in the MFG-loop in equation (5.29). Algorithm 2 constitutes an iterative method for finding a fixed point solution for the loop in equation (5.29) with a converging error ϵ_f to be defined:

Algorithm 2 MFG-FPK and MFG-HJB Numerical Solution

Input: $\mu_0(\theta, y)$ C_{max} θ_{min} θ_{max} Y_{min}^c Y_{max}^c T ϵ_f

Initialization:

Solve (5.17) by assuming $u_r^0 = 0$, $u_\gamma^0 = 0$, $u_c^0 = -1$ and denote the solution by $\mu_t^0(\theta, y)$ and assign $k = 1$

Loop: At iteration k , execute the following:

Substitute $\mu_t^{k-1}(\theta, y)$ in (5.23) and (5.26) and solve the MF-HJB in both scenarios in the backward direction and denote the solution by $v^k(\theta, y, t)$

Find the optimal controls $u_{r,\gamma,c}^k$ using the following equations (5.24), (5.25), (5.27), and (5.28) and Table 5.1

Substitute $u_{r,\gamma,c}^k$ in (5.23) and (5.26) and find $\mu_t^k(\theta, y)$ by solving the MF-FPK in (5.17) in the forward direction

If $|\mu_t^k(\theta, y) - \mu_t^{k-1}(\theta, y)| < \epsilon_f$, let $\mu_t^k(\theta, y) = \mu_t(\theta, y)$ and exit the loop.

Output: $\mu_t(\theta, y)$

Algorithm 3 is employed to generate an optimal strategy for an individual household within the MG, here optimality is defined as the best response strategy. It is important to note that each household, denoted as i (where $1 \leq i \leq N$), possesses complete information about their respective state variables and system dynamics, as indicated in the input section of Algorithm 3. Furthermore, household i also observes the values of $\gamma^i(t)$ and $r^i(t)$, and has control over $u_\gamma^i(t)$, $u_r^i(t)$, and $u_c^i(t)$. The generic household best response algorithm is as follows:

Algorithm 3 Computation of a Generic Agents' Best Response Control Along a Sample Path During both off-peak event day and peak-event days

For a generic household of the community MG $1 \leq i \leq N$:

Input: Convergent probability density functions (i.e. $\mu_\theta(t, \theta)$, $\mu_y(t, y)$ from Algorithm 2), $P_{MA}(t) \forall t$,
 C_{max} , $\Omega_\theta := [\theta_{min}, \theta_{max}]$, $\Omega_{y_f} := [y_{min}^f, y_{max}^f]$,
and $\Omega_{y_c} := [Y_{min}^c, Y_{max}^c]$

Off-Peak Event Input: MA credit is set to zero, threshold $\tau(t) = 0$, and incentive and penalty $\iota^i(t)$ is set to zero. Here P_{MG} is found via (5.1).

Peak Event Input: MA credit, and threshold $\tau(t)$ are defined and communicated to the MG households via the aggregator. $\iota^i(t)$ is found using (5.3) and (5.4) in case of incentive and in case of penalty respectively. Subsequently $P_{MG}^i(t)$ is found.

Initialization:

Generate two Brownian Process W_θ^i & W_c^i for both $\theta^i(t)$ and $y_c^i(t)$ respectively.

Set $k = 0$, $t = T$, $v(T) = b(T) = 0$, $u_\gamma^i(T) = 0$,

$u_r^i(T) = 0$, and $u_c^i(T) = 0$.

Loop: At iteration k , execute the following:

Substitute $p_{pv}(t)$ in (5.5) and calculate $\theta^i(t)$.

Using (5.9), find $\delta^i(t)$ and using (5.11), (5.12), and (5.8) find $\gamma^i(t)$, $r^i(t)$, and $y_c(t)$ respectively.

Calculate: $\Lambda_\theta = \int_{\Omega_\theta} \theta \mu_\theta(t, \theta) d\theta$, $\Lambda_y = \int_{\Omega_y} y \mu_y(t, y) dy$,

Using (5.19) find $w_1(t)$ and following that $w_2(t) = 1 - w_1(t)$ and using (5.1) find $P_{MG}(t)$

If $\delta^i(t) \leq 0$ then consider Mf-HJB-SC1 in (5.23) and thus $u_r^{*,i}(t - \Delta t) = 0$ and $u_\gamma^{*,i}(t - \Delta t)$ is found using (5.24) and $u_c^{*,i}(t - \Delta t)$ is found using (5.25).

Else, consider MF-HJB-SC2 in (5.26) and calculate $dy_f = \frac{y_f(t+\Delta t) - y_f(t)}{\Delta t}$ and using (5.27), and (5.28), and the results in Table 5.1, find $u_r^{*,i}(t - \Delta t)$, $u_\gamma^{*,i}(t - \Delta t)$, and $u_c^{*,i}(t - \Delta t)$.

Increment k

If $T - \Delta t \times k \leq 0$, exit the loop.

Output: $v^i(t)$, $\theta^i(t)$, $P_{MG}(t)$, $u_r^i(t)$, $u_\gamma^i(t)$, & $u_c^i(t) = 0$

5.6 Numerical Results and Robustness Analysis: Montreal MG Case Study

In this section, we examine the computational results of the proposed algorithms for solving the constructed MGO problem, using the Montreal Microgrid case study as the basis. The residential microgrid considered in this study assumes that all households have uniform generation and load potential. In other words, each household deploys similar solar panel

systems and utilizes similar household appliances.

5.6.1 Numerical Results

The initial probability density function $\mu_0(\theta, y)$, represented in Figure 5.3, is assumed to be known to all agents.

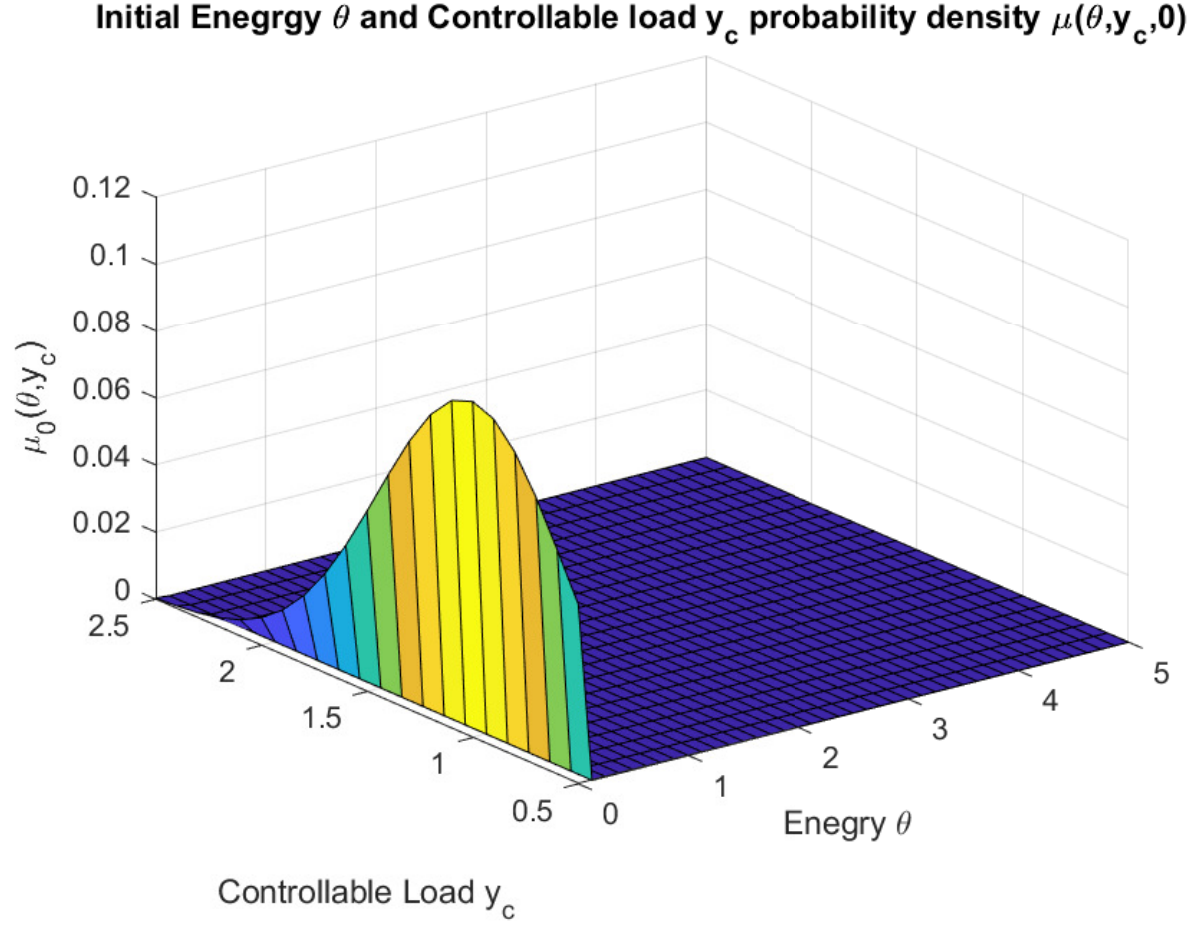


Figure 5.3 Initial Probability Density Function at $t = 0$ for y_c θ

Applying Algorithm 2, the convergent solution $\mu_t(\theta, y_c)$ to the MF-FPK equation (5.17) is shown in Figure 5.4.

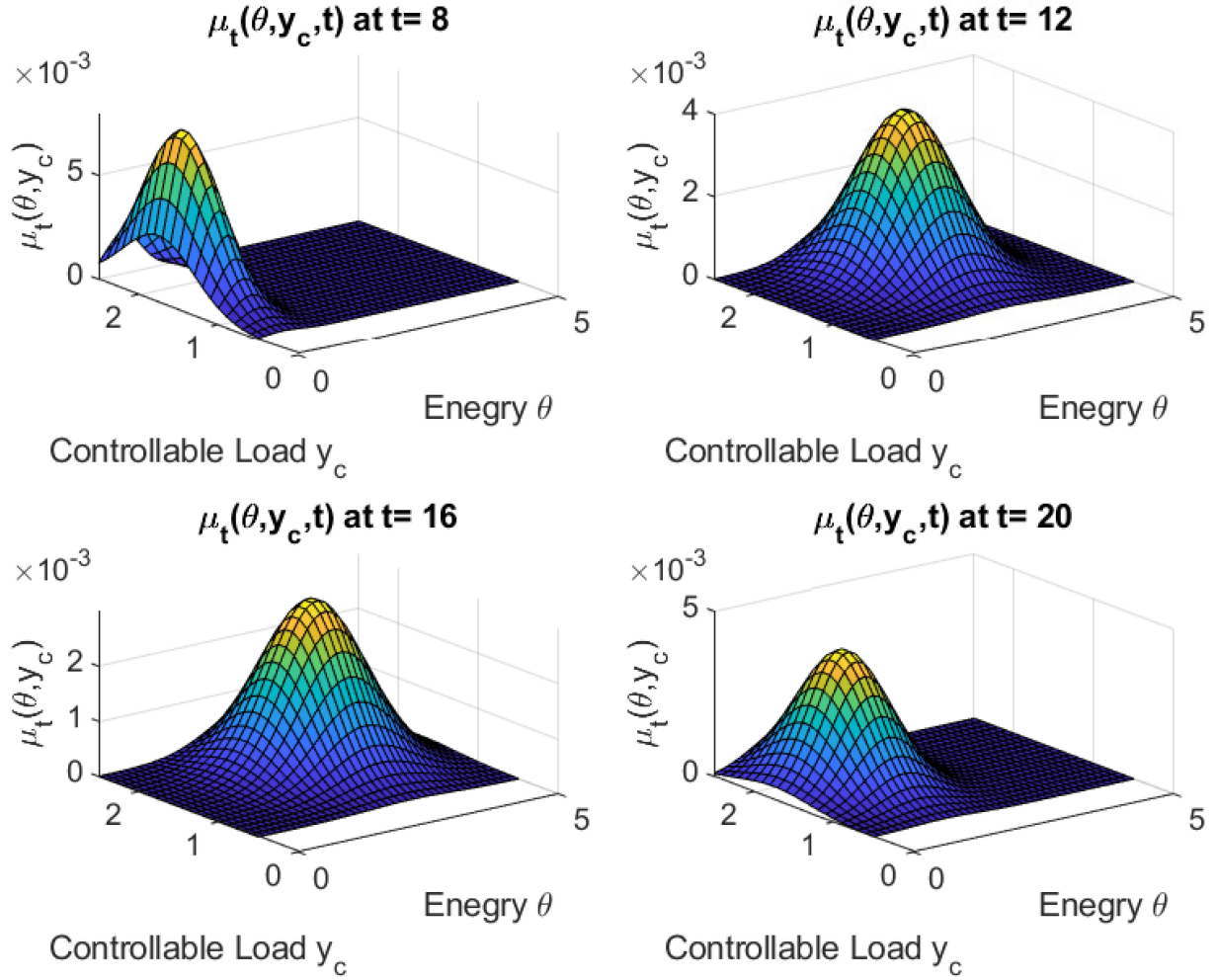


Figure 5.4 Propagation of the $\mu(\theta, y_c, t)$ versus time

Figure 5.5 depicts the solution to the MF-HJB for the overall population in the normal day event. Here, we focused on presenting the probability density function during the time of the day where $\theta(t) > 0$ i.e. from 8h:00 to 20h:00.

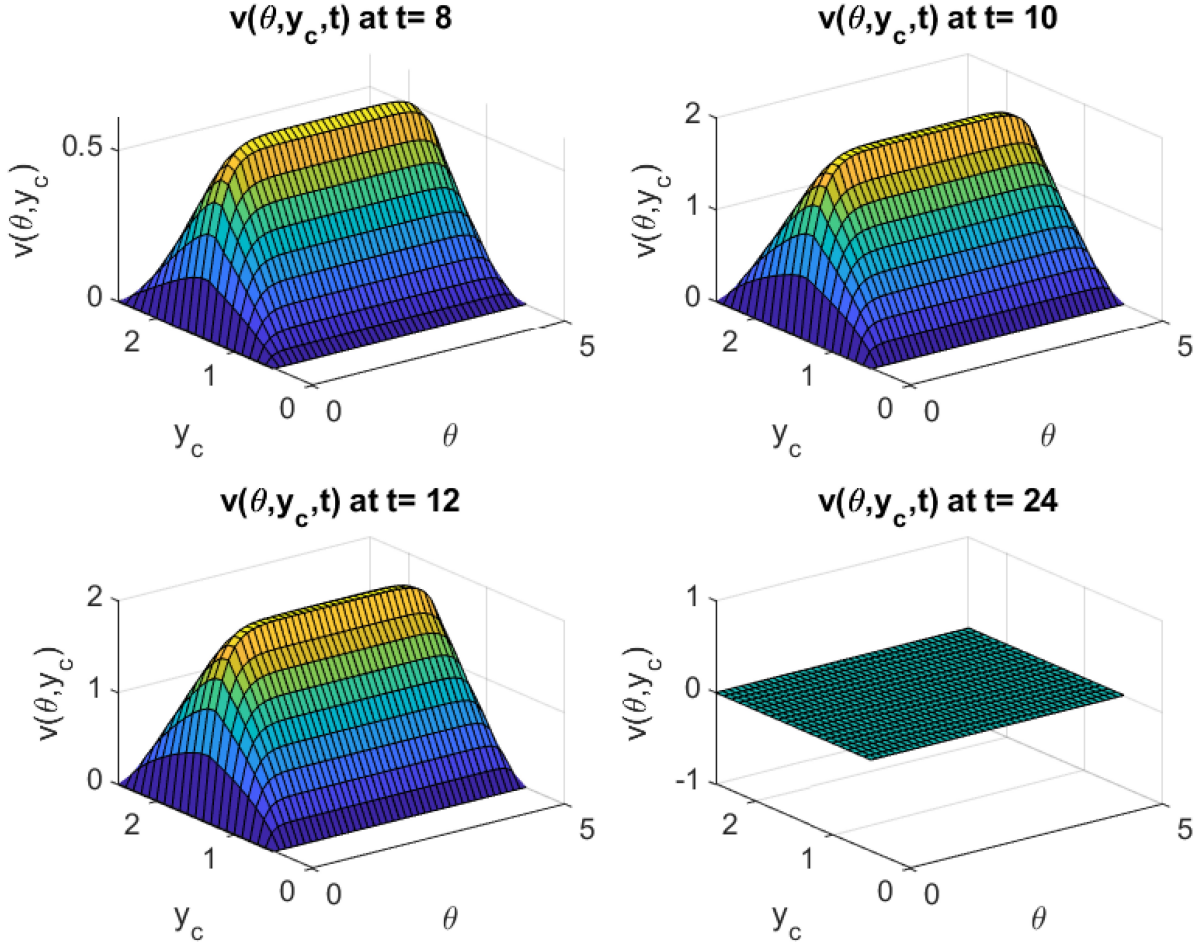


Figure 5.5 Propagation of the cost function $v(\theta, y_c, t)$ versus time

Figure 5.6 depicts the results of Algorithm 3 for the generic agent i where $b^i(0) = 2.8$ and $\theta^i(0) = 0$ and for this experiment $C_{max} = 5$, $\theta_{min} = 0$, $\theta_{max} = 3kWh$ $y_c = [0.5, 2.5]$ and $T = 24$. The generic agent i is fed the converging results of Algorithm 2 (i.e. $\mu(\theta, y_c, t)$) which allows the generic agent i to find the equilibrium solutions of $p_m g(t)$, $w_1(t)$ and $w_2(t)$ for all t where $0 \leq t \leq T$.

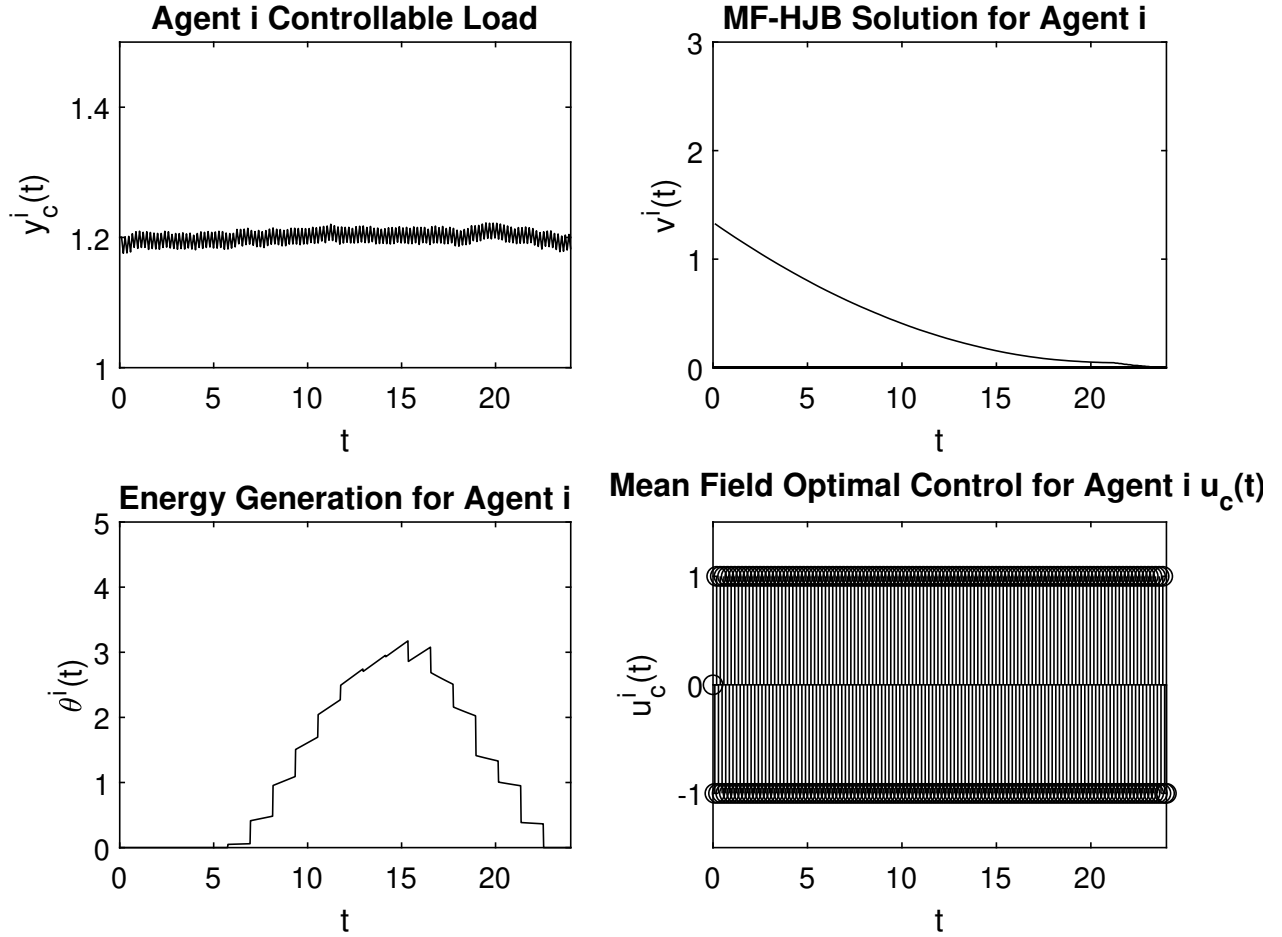


Figure 5.6 Generic Agent Energy Profile

Figure 5.7 illustrates the energy profile of the generic agent i during a peak event. It demonstrates that the optimal decision for the controllable load during a peak event is to remain within the minimum mode of operation. Furthermore, the numerical results indicate a 20.27% reduction in energy costs on the peak event day compared to a normal day. The energy cost for the generic agent on a normal day was \$1.31, whereas, during the peak event day, it was reduced to \$1.09 due to the incentive provided by the main grid. This aligns with Hy-

dro Quebec's recommendation to lower the controllable appliance to the minimum mode of operation within the household's preference set during peak events.

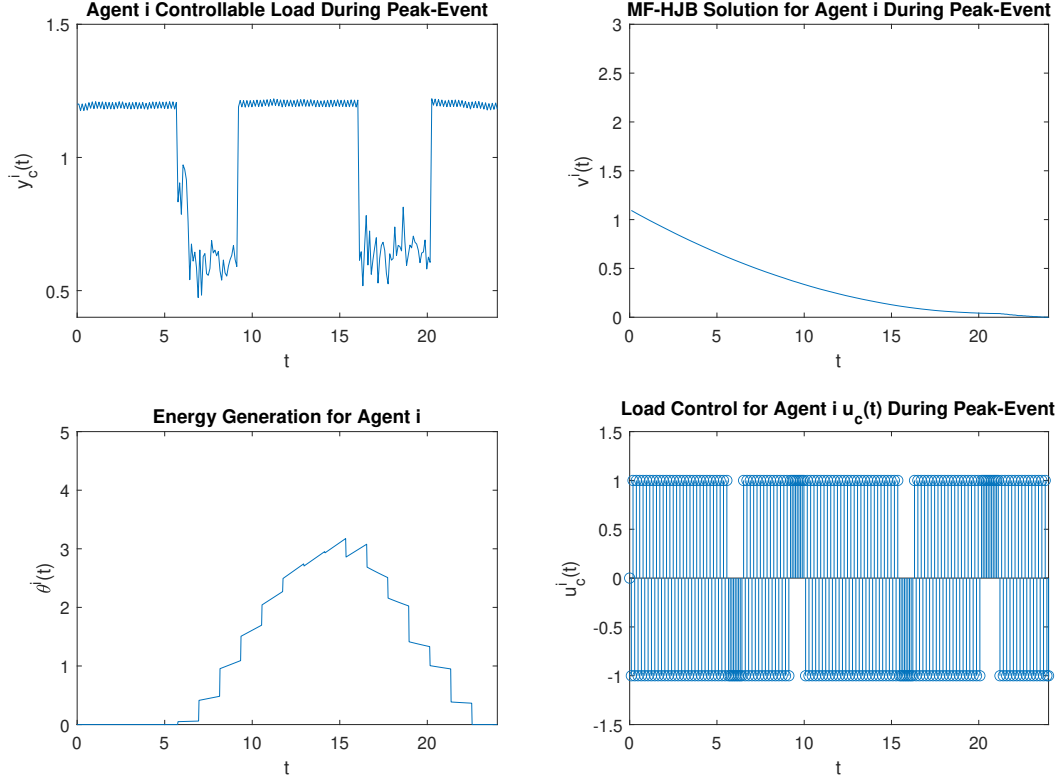


Figure 5.7 Generic Agent Energy Profile During Peak Event

5.6.2 Algorithms Computational Performances

Table 5.2 presents the algorithms' computational performance for the uniform cases, where $\epsilon_f = 10^{-4}$, where ϵ_f signifies the stopping condition for the numerical implementation of the finite difference method. The number of iterations represents the number of iterations required for executing Algorithm 2.

Table 5.2: Algorithms Performance for the Uniform Household Case

Scenario	# Iter.	Run.-Time sec	Memory KB	Dim.
Normal Day	7	263.271	32,422	4
Peak-Event Day	13	532.56	42,044	4

The specifications of the computer on which the simulation was held are 1) Processor Intel(R) Core(TM) i5-6300U CPU @ 2.40GHz, 2496 Mhz, 2 Core(s), 4 Logical Processor(s), 2) RAM: 8GB and 3) system type: 64-operating system, x64-based processor.

5.7 Conclusion

The paper introduces multiple algorithms that address the optimization problem in residential microgrid power network problems. These algorithms aim to find Nash Equilibrium strategies for all the households in the MG using the decentralized MFG methodology. The study analyzes various scenarios, including uniform households in both peak event and off-peak event scenario where the MG is in grid-connected mode and where the main grid deploys an incentive base pricing mechanism. In addition, numerically we have demonstrated the following conclusions:

- The proposed MFG algorithm yields a convergent fixed point probability density function for the MFG-loop in equation (5.29).
- Numerically, the algorithm presented in this paper produced a Nash equilibrium strategy for the constructed MGO problem resulting in reducing the energy bill for the households in the MG and simultaneously reducing the peak-event for the main grid.
- In terms of running time and complexity, the costs associated with implementing the decentralized mean field approach are lower compared to the centralized stochastic control case. Furthermore, these costs do not increase with the population size.

For future research, the algorithms introduced in this study can be expanded to address a broader problem formulation in which the residential microgrid will be consisting of non-uniform households that possess both controllable and shiftable loads, and the MG will be in a grid-connection mode where the main grid deploy a more general dynamic pricing (for example; time of use pricing mechanism).

CHAPTER 6 GENERAL DISCUSSION

In this dissertation, we investigated the residential microgrid power problem where the microgrid operates in grid-connected mode via mean field game theory. The residential community microgrid is composed of households that are integrated with renewable energy generation and battery storage systems. The residential microgrid problem was studied over the scope of three journal papers incorporated in this dissertation.

The dissertation contributions start with an introduction of an innovative approach to forming a residential community microgrid, involving a coalition of prosumer households equipped with solar photovoltaic systems. These households are interconnected through a virtual power bank that utilizes their storage batteries and facilitates communication between the households and the main grid. By employing mean field game theory, we solve Nash Equilibrium strategies that minimize a combination of energy generation costs, energy consumption costs, and revenue from sold energy. The proposed approach and algorithm are then validated over a constructed community microgrid located in Montreal, Quebec, Canada. The proposed mean field game approach reduces both aggregated and individual energy costs. Furthermore, a comparative analytical study highlights the benefits of sharing, indicating that each prosumer belonging to a microgrid community of 100 prosumers in Montreal can expect a minimum 40 percent reduction in their individual costs.

The second objective of this dissertation focuses on investigating the residential microgrid comprised of households utilizing demand response strategies. The microgrid operates in grid-connected mode, and the main grid implements an incentive-based pricing mechanism during peak events. To study this problem, we present a game-theoretical approach for an incentive-based residential community microgrid, wherein an incentive-driven pricing mechanism is developed to encourage peak demand reduction and distribute the demand curve incentive among the residential community through the aggregator. The primary goal of the aggregator is to maximize the welfare of the residential community by determining the optimal equilibrium electricity price. Each household interacts with one another and with the DN through the aggregator and seeks to minimize their individual electricity costs. Here,

we presented the high-level problem formulation and proved the mean-field game theory as a viable optimization approach to solve the problem.

The third objective of this dissertation focuses on the computational investigation of the residential microgrid consisting of households participating in demand response and distributed energy resources. Firstly, we represent the computational investigation for a residential community operating in grid-connection mode, where the main grid deploys an incentive-based pricing mechanism to encourage peak demand reduction. The incentive demand curve with the residential community through the aggregator. Secondly, we solved Nash Equilibrium strategies, which minimize the total RCM energy cost functions by applying mean field game theory. The computational investigation shows a reduction in both the energy bill for the residential microgrid energy bill and the peak event cost. The algorithmic implementation is tested on a constructed community microgrid located in Montreal, Quebec, Canada.

CHAPTER 7 CONCLUSION AND RECOMMENDATIONS

This thesis has examined the residential community microgrid optimization problem via a dynamical game-theoretical formulation. In the following section, we summarize the key contributions made in this research. Additionally, we address certain limitations of the study and outline potential avenues for future research investigation.

7.1 Summary of Contributions

The first contribution of the dissertation was bridging the gap between the aforementioned research directions on residential microgrids and the application of mean field game theory. To the best of our knowledge, no prior studies have modeled the optimization problem of community microgrids as a dynamical game without a centralized unit. To achieve this, we developed a novel framework for a residential community microgrid that includes a coalition of prosumer households connected through a virtual power bank, facilitating communication within the microgrid and with the main grid. To solve the formulated problem, we developed the first application of non-linear decentralized mean field game theory to the residential community microgrid, along with the associated numerical algorithms to solve the constructed decentralized non-linear problem. The results demonstrate that the finite problem converges to the infinite limit population problem with negligible error. Additionally, numerically we proved that a generic agent prosumer in the MG achieves a minimum of a 40 percent reduction in their individual cost by joining a microgrid community composed of at least 100 prosumers.

Another contribution of the dissertation is the extension of the previously formulated game to include the case of households with a DR. we constructed a game theoretical framework utilizing decentralized mean field game theory to address the incentive-based residential community microgrid with a load aggregator. Our approach involves a high-level description of the energy problem, specifically focused on load aggregation, and presents a proof of concept that demonstrates the applicability of mean-field game theory to determine the optimal

pricing mechanism in the residential community microgrid.

Lastly, we investigated the computational implementation for the residential community microgrid with DR via an application of mean field game theory. The main contribution of the developed algorithms is a set of Nash equilibrium strategy for the constructed MGO problem resulting in reducing the energy bill for the households in the MG and simultaneously reducing the peak-event for the main grid. In addition, we showed that the computational costs associated with implementing the decentralized mean field approach are lower compared to the centralized stochastic control case. Furthermore, the computational cost does not increase with an increase in the population size of community microgrid households.

7.2 Limitations

When it comes to limitations, one notable aspect is the need to test and validate the proposed algorithms and frameworks using real data in real-time applications. The primary limitation of this dissertation lies in the validation of the proposed methods within real-time scenarios regardless of the fact that one of the proposed algorithm was tested on real collected data from the city of Montreal, Canada.

Another limitation pertains to the selected pricing mechanism employed by the main grid. Presently, energy providers are increasingly embracing time-of-use pricing mechanisms that are time and load dependent. In the dissertation case studies, a fixed or incentive-based pricing mechanism was considered for the main grid. However, we assert that the proposed mean field game approach can be readily adapted to address the problem with various pricing mechanisms, allowing for greater flexibility and applicability.

Lastly, the demand response considered in this dissertation consists only of the uniform households possessing controllable loads and fixed loads. In general, smart houses can be non-uniform and possess three types of loads: fixed, controllable, and shiftable load.

7.3 Future Work

Apart from the specific concepts discussed in this thesis, we find it highly intriguing to apply the presented mean field game approach in a real-time community microgrid setting. At present, only a few countries have operational community microgrids functioning in grid-connected mode. The decentralized nature of the mean field game theory adds to its appeal, as it requires minimal communication between households and the main grid, making it a viable option for prosumers to implement.

Regarding the main grid pricing mechanism, our research primarily concentrated on scenarios involving fixed and incentive-based pricing mechanisms. It would be interesting to explore the convergence of the mean-field approach and the algorithmic implementation complexity when considering a time-of-use pricing mechanism.

When it comes to demand response and smart houses, it would be interesting to explore the game-theoretical formulation for a community microgrid comprising households with shiftable and controllable loads. The inclusion of shiftable loads adds a notable level of complexity to the mean field game (MFG) approach, making it intriguing to assess how the mean field game theory compares to other optimization techniques in this context.

REFERENCES

- [1] M. Aziz and P. E. Caines, “A mean field game computational methodology for decentralized cellular network optimization,” *IEEE Transactions on Control Systems Technology*, vol. 25, no. 2, pp. 563–576, Mar 2017.
- [2] M. Aziz, H. Dagdougui, and I. Elhallaoui, “Cooperative game theoretical approach for residential users through load aggregator in smart grid: Formulation and proof of concept,” *Submitted and Under Revision: European Journal of Control*, 2023.
- [3] —, “A decentralized game theoretic approach for virtual storage system aggregation in a residential community,” *IEEE Access*, vol. 10, pp. 34 846–34 857, 2022.
- [4] —, “Computational investigation of residential users through load aggregator smart grid power optimization problem via mean field game theory,” *Cahiers du GERAD*, 2023.
- [5] J. J. Siirola, “A perspective on energy and sustainability,” in *11th International Symposium on Process Systems Engineering*, ser. Computer Aided Chemical Engineering, I. A. Karimi and R. Srinivasan, Eds. Elsevier, 2012, vol. 31, pp. 1–7.
- [6] P. I. Barton, “Energy systems engineering,” ser. Computer Aided Chemical Engineering, C. A. O. d. N. Rita Maria de Brito Alves and E. C. Biscaia, Eds. Elsevier, 2009, vol. 27, pp. 55–57.
- [7] A. Kizilkale, S. Mannor, and P. E. Caines, “Large scale real-time bidding in the smart grid: A mean field framework,” Dec 2012, pp. 3680–3687.
- [8] J. Tejeda and S. Ferreira, “Applying systems thinking to analyze wind energy sustainability,” *Conference on Systems Engineering Research*, vol. 28, pp. 213–220, 2014.
- [9] E. Santoyo-Castelazo and A. Azapagic, “Sustainability assessment of energy systems: integrating environmental, economic and social aspects,” *Journal of Cleaner Production*, vol. 80, pp. 119–138, 2014.

- [10] S. Sen, S. Sengupta, and A. Chakrabati, *Electrical Pricing: Regulated, Deregulated and Smart Grid Systems*. 6000 Broken Sound Parkway NW, Suit 300: CRC Press Taylor and Francis Group, October 2014.
- [11] M. Shekari, H. Arasteh, A. Sheikhi Fini, and V. Vahidinasab, “Demand response requirements from the cultural, social, and behavioral perspectives,” *Applied Sciences*, vol. 11, no. 23, Dec 2021. [Online]. Available: <http://dx.doi.org/10.3390/app112311456>
- [12] R. B. Myerson, *Game Theory Analysis of Conflict*. Cambridge, Massachusetts, USA: Harvard University Press, 1991.
- [13] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. Cambridge, Massachusetts, USA: The MIT Press, 1994.
- [14] T. Başar, *Dynamic noncooperative game theory*, ser. Series in Classics in Applied Mathematics. Philadelphia, PA: SIAM, 1999.
- [15] G. Owen, *Game Theory*, 3rd ed. London, U.K: Academic, 1995.
- [16] F. Fogelman and M. Quinzii, “Asymptotic value of mixed games,” *Mathematics of Operations Research*, vol. 5, no. 1, pp. 86–93, 1980.
- [17] A. Kizilkale and P. Caines, “Mean field (nce) stochastic control: Populations of major and egoist-altruist agents,” in *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*, Dec 2011, pp. 5547–5552.
- [18] M. Huang, “Large-population lqg games involving a major player: The nash certainty equivalence principle,” *SIAM Journal on Control and Optimization*, vol. 48, no. 5, pp. 3318–3353, 2010.
- [19] P. Milgrom, “Auction and bidding: A primer,” *Journal of Economic Perspectives*, vol. 13, no. 3, pp. 3–22, 1989.
- [20] P. E. Caines, *Mean Field Games*, T. Samad and J. Baillieul, Eds. Springer, 2014.

- [21] M. Huang, R. Malhame, and P. E. Caines, “Large population stochastic dynamic games: closed-loop mckean-vlasov systems and the nash certainty equivalence principle,” *Communications in Information and Systems In Communications in Information and Systems*, vol. 6, Mar. 2006.
- [22] M. Huang, R. P. Malhamé, and P. E. Caines, “Nash certainty equivalence in large population stochastic dynamic games: Connections with the physics of interacting particle systems,” in *Proceedings of the 45th IEEE Conference on Decision and Control*. IEEE, 2006, pp. 4921–4926.
- [23] M. Huang, P. E. Caines, C. Charalambous, and R. Malhame, “Power control in wireless systems: a stochastic control formulation,” in *Proc. of the 2001 American Control Conference*, vol. 2, June 2001, pp. 750–755.
- [24] M. Huang, P. E. Caines, and R. Malhame, “Uplink power adjustment in wireless communication systems: a stochastic control analysis,” *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1693–1708, oct. 2004.
- [25] M. Huang, R. Malhame, and P. E. Caines, “Stochastic power control in wireless communication systems: analysis, approximate control algorithms and state aggregation,” in *Proc. of the 42nd IEEE Conference on Decision and Control*, vol. 4, 2003, pp. 4231–4236.
- [26] M. Huang, P. E. Caines, and R. Malhame, “Individual and mass behaviour in large population stochastic wireless power control problems: centralized and nash equilibrium solutions,” vol. 1, 2003, pp. 98–103.
- [27] —, “Large-population cost-coupled lqg problems with nonuniform agents: Individual-mass behavior and decentralized nash equilibria,” *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1560–1571, 2007.
- [28] M. Huang, R. Malhame, and P. E. Caines, “Nash equilibria for large-population linear stochastic systems of weakly coupled agents.” Springer US, 2005, pp. 215–252.

- [29] M. Huang, P. E. Caines, and R. Malhame, “The nce (mean field) principle with locality dependent cost interactions,” *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2799–2805, 2010.
- [30] J. M. Lasry and P. L. Lions, “Jeux a champ moyen. i. le cas stationnaire,” *Comptes Rendus Mathematique*, vol. 343, no. 9, pp. 619–625, 2006.
- [31] —, “Jeux a champ moyen. ii. horizon fini et controle optimal,” *Comptes Rendus Mathematique*, vol. 343, no. 10, pp. 679–684, 2006.
- [32] —, “Mean field games,” *Japanese Journal of Mathematics*, vol. 2, no. 1, pp. 229–260, 2007.
- [33] G. Foderaro and S. Ferrari, “Necessary conditions for optimality for a distributed optimal control problem,” in *Proc. of the 49th IEEE Conference on Decision and Control (CDC)*, 2010, pp. 4831–4838.
- [34] G. Foderaro, S. Ferrari, and K. Rudd, “A generalized reduced gradient method for the optimal control of multiscale dynamical systems,” in *Proc. of the 52th IEEE Conference on Decision and Control (CDC)*, Dec 2013, pp. 3857–3863.
- [35] Y. Achdou, F. Camilli, and I. C. Dolcetta, “Mean field games: Convergence of a finite difference method,” *Society for Industrial and Applied Mathematics (SIAM) Journal on Numerical Analysis*, vol. 51, no. 5, pp. 2585–2612, 2013.
- [36] Y. Achdou and I. C. Dolcetta, “Mean field games: Numerical methods,” *Society for Industrial and Applied Mathematics (SIAM) Journal on Numerical Analysis*, vol. 48, no. 3, pp. 1136–1162, 2010.
- [37] S. Parhizi, H. Lotfi, A. Khodaei, and S. Bahramirad, “State of the art in research on microgrids: A review,” *IEEE Access*, vol. 3, pp. 890–925, 2015.
- [38] U. Assad, M. A. S. Hassan, U. Farooq, A. Kabir, M. Khan, S. Bukhari, Z. U. A. Jaffri, J. Oláh, and J. Popp, “Smart grid, demand response and optimization: A critical review of computational methods,” *Energies*, vol. 15, p. 2003, 03 2022.

- [39] M. A. S. Hassan, M. Chen, Q. Li, M. Mehmood, T. Cheng, and B. Li, "Microgrid control and protection state of the art: A comprehensive overview," *Journal of Electrical Systems*, vol. 14, pp. 148–164, 06 2018.
- [40] A. G. Martins, D. Coelho, C. H. Antunes, and J. Climaco, "A multiple objective linear programming approach to power generation planning with demand-side management (dsm)," *International Transactions in Operational Research*, vol. 3, no. 3, pp. 305–317, 1996. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S096960169600024X>
- [41] R. M. Shrestha and C. O. Marpaung, "Supply-and demand-side effects of carbon tax in the indonesian power sector: an integrated resource planning analysis," *Energy Policy*, vol. 27, no. 4, pp. 185–194, 1999. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0301421599000038>
- [42] J. Abaravicius and J. Pyrko, "Load management from an environmental perspective," *Energy and Environment*, vol. 17, 07 2006.
- [43] S. Holland and E. Mansur, "Is real-time pricing green? the environmental impacts of electricity demand variance," *The Review of Economics and Statistics*, vol. 90, pp. 550–561, 08 2008.
- [44] C. Batlle and P. Rodilla, "Electricity demand response tools: current status and outstanding issues," *Eur. Rev. Energy Mark.*, vol. 3, 01 2008.
- [45] E. Koliou, C. Eid, J. Chaves-Ávila, and R. Hakvoort, "Demand response in liberalized electricity markets: Analysis of aggregated load participation in the german balancing mechanism," *Energy*, vol. 71, 07 2014.
- [46] A. Khan, M. Naeem, M. Iqbal, and S. Qaisar, "A compendium of optimization objectives, constraints, tools and algorithms for energy management in microgrids," *Renewable and Sustainable Energy Reviews*, vol. 58, pp. 1664–1683, 05 2016.
- [47] N. Rezaei and M. Kalantar, "Stochastic frequency-security constrained energy and reserve management of an inverter interfaced islanded microgrid considering demand

- response programs,” *International Journal of Electrical Power and Energy Systems*, vol. 69, 07 2015.
- [48] W. Alharbi and K. Raahemifar, “Probabilistic coordination of microgrid energy resources operation considering uncertainties,” *Electric Power Systems Research*, vol. 128, 11 2015.
- [49] C. Chen, S. Duan, T. Cai, B. Liu, and G. Hu, “Smart energy management system for optimal microgrid economic operation,” *Renewable Power Generation, IET*, vol. 5, pp. 258 – 267, 06 2011.
- [50] S. Talari, M. Yazdaninejad, and M. Haghifam, “Stochastic-based scheduling of the microgrid operation including wind turbines, photovoltaic cells, energy storages and responsive loads,” *IET Generation, Transmission and Distribution*, vol. 9, 04 2015.
- [51] H. Falsafi, A. Zakariazadeh, and S. Jadid, “The role of demand response in single and multi-objective wind-thermal generation scheduling: A stochastic programming,” *Energy*, vol. 64, pp. 853–867, 01 2014.
- [52] R. Rosenthal, *GAMS—A User’s Guide*. GAMS Development Corporation: Washington, DC, USA, 2004.
- [53] I. Gurobi, *Gurobi Optimizer Reference Manual*. Optimization: Houston, TX, USA, 2015.
- [54] S. Mazzola, M. Astolfi, and E. Macchi, “A detailed model for the optimal management of a multigood microgrid,” *Applied Energy*, vol. 154, pp. 862–873, 09 2015.
- [55] D. Tenfen and E. Finardi, “A mixed integer linear programming model for the energy management problem of microgrids,” *Electric Power Systems Research*, vol. 122, 05 2015.
- [56] O. Elci and N. Noyan, “A chance-constrained two-stage stochastic programming model for humanitarian relief network design,” *Transportation Research Part B: Methodological*, vol. 108, pp. 55–83, 02 2018.

- [57] G. B. Dantzig, *Linear Programming under Uncertainty*. INFORMS, 1955, vol. 1.
- [58] S. Xie, Z. Hu, Z. Daming, Y. Li, S. Kong, W. Lin, and Y. Zheng, “Multi-objective active distribution networks expansion planning by scenario-based stochastic programming considering uncertain and random weight of network,” *Applied Energy*, vol. 219, pp. 207–225, 06 2018.
- [59] Z. Wang, C. Shen, and L. Feng, “A conditional model of wind power forecast errors and its application in scenario generation,” *Applied Energy*, vol. 212, 08 2017.
- [60] A. Rabiee, M. Sadeghi, J. Aghaei, and A. Heidari, “Optimal operation of microgrids through simultaneous scheduling of electrical vehicles and responsive loads considering wind and pv units uncertainties,” *Renewable and Sustainable Energy Reviews*, vol. 57, pp. 721–739, 05 2016.
- [61] R. Dufo-López, E. Pérez-Cebollada, J. Bernal-Agustín, and I. Martinez, “Optimisation of energy supply at off-grid healthcare facilities using monte carlo simulation,” *Energy Conversion and Management*, vol. 113, pp. 321–330, 04 2016.
- [62] L. Suganthi, S. Iniyan, and A. A. Samuel, “Applications of fuzzy logic in renewable energy systems – a review,” *Renewable and Sustainable Energy Reviews*, vol. 48, pp. 585–607, 2015.
- [63] A. Soroudi and M. Ehsan, “A possibilistic–probabilistic tool for evaluating the impact of stochastic renewable and controllable power generation on energy losses in distribution networks—a case study,” *Renewable and Sustainable Energy Reviews*, vol. 15, pp. 794–800, 01 2011.
- [64] H. Zhang, D. Yue, and X.-P. Xie, “Robust optimization for dynamic economic dispatch under wind power uncertainty with different levels of uncertainty budget,” *IEEE Access*, vol. PP, pp. 1–1, 10 2016.
- [65] E. Kuznetsova, C. Ruiz, Y.-F. Li, and E. Zio, “Analysis of robust optimization for decentralized microgrid energy management under uncertainty,” *International Journal*

- of Electrical Power and Energy Systems*, vol. 64, pp. 815–832, 2015. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0142061514004992>
- [66] A. Rahimi-Kian and H. Safamehr, “A cost-efficient and reliable energy management of a micro-grid using intelligent demand-response program,” *Energy*, vol. 91, 2015.
- [67] M. Casisi, A. Nardi, P. Pinamonti, and M. Reini, “Effect of different economic support policies on the optimal synthesis and operation of a distributed energy supply system with renewable energy sources for an industrial area,” *Energy Conversion and Management*, vol. 95, 2015.
- [68] B. Jiang and Y. Fei, “Dynamic residential demand response and distributed generation management in smart microgrid with hierarchical agents,” *Energy Procedia*, vol. 12, pp. 76–90, 2011, the Proceedings of International Conference on Smart Grid and Clean Energy Technologies (ICSGCE 2011). [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1876610211018388>
- [69] H. Chen, Z. Wang, H. Yan, H. Zou, and B. Luo, “Integrated planning of distribution systems with distributed generation and demand side response,” *Energy Procedia*, vol. 75, pp. 981–986, 2015.
- [70] R. Deng, Z. Yang, M.-Y. Chow, and J. Chen, “A survey on demand response in smart grids: Mathematical models and approaches,” *IEEE Transactions on Industrial Informatics*, vol. 11, 2015.
- [71] S. Ghosh, J. Kalagnanam, D. Katz, M. Squillante, and X. Zhang, “Integration of demand response and renewable resources for power generation management,” in *ISGT 2011*, 2011, pp. 1–7.
- [72] Q. Chen, F. Wang, B.-M. Hodge, J. Zhang, Z. Li, M. Shafie-Khah, and J. P. S. Catalão, “Dynamic price vector formation model-based automatic demand response strategy for pv-assisted ev charging stations,” *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2903–2915, 2017.

- [73] J. Koh, S. Ray, and J. Hodges, "Information mediator for demand response in electrical grids and buildings," in *2017 IEEE 11th International Conference on Semantic Computing (ICSC)*, 2017, pp. 73–76.
- [74] M. H. Yaghmaee, A. Leon-Garcia, and M. Moghaddassian, "On the performance of distributed and cloud-based demand response in smart grid," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 5403–5417, 2018.
- [75] I. D. de Cerio Mendaza, I. G. Szczesny, J. R. Pillai, and B. Bak-Jensen, "Flexible demand control to enhance the dynamic operation of low voltage networks," *IEEE Transactions on Smart Grid*, vol. 6, no. 2, pp. 705–715, 2015.
- [76] K. Ma, C. Wang, J. Yang, Q. Yang, and Y. Yuan, "Economic dispatch with demand response in smart grid: Bargaining model and solutions," *Energies*, vol. 10, 2017.
- [77] S. Mocci, N. Natale, F. Pilo, and S. Ruggeri, "Demand side integration in lv smart grids with multi-agent control system," *Electric Power Systems Research*, vol. Volume 125, pp. 23–33, 2015.
- [78] B. Wojszczyk, "Deployment of advanced smart grid solutions - global examples and lessons learned," in *2012 IEEE PES Innovative Smart Grid Technologies (ISGT)*, 2012.
- [79] A. M. Helmi, R. Carli, M. Dotoli, and H. S. Ramadan, "Efficient and sustainable reconfiguration of distribution networks via metaheuristic optimization," *IEEE Transactions on Automation Science and Engineering*, vol. 19, no. 1, pp. 82–98, 2022.
- [80] M. A. S. Hassan, M. Chen, M. A. Mehmood, M. H. Ahmed, and S. S. H. Bukhari, "A reliable approach to eliminate distributed capacitive current of transmission lines," in *2017 IEEE Conference on Energy Internet and Energy System Integration (EI2)*, 2017, pp. 1–5.
- [81] M. A. S. Hassan, G. Song, X. Kang, Z. Jiao, C. Wang, and S. Tahir, "Current differential protection for distributed transmission lines using low sampling frequency," *International Journal of Engineering Works*, vol. 2, pp. 42–47, 2015.

- [82] M. Hassan, Z. Javed, H. Armghan, N. Ali, H. A. Said, U. Farooq, A. Afzal, and M. A. S. Hassan, "Lyapunov based adaptive controller for power converters used in hybrid energy storage systems," *Sustainable Energy Technologies and Assessments*, 2020.
- [83] B. Li, M. Chen, T. Cheng, Y. Li, M. A. S. Hassan, R. XU, and T. Chen, "Distributed control of energy-storage systems for voltage regulation in distribution network with high pv penetration," 09 2018, pp. 169–173.
- [84] M. Hossain, N. Madloul, N. Abd Rahim, J. Sandeep, D. A. Pandey, and A. F. Khan, "Role of smart grid in renewable energy: An overview," *Renewable and Sustainable Energy Reviews*, vol. 60, pp. 1168–1184, 2016.
- [85] Q. Liu, R. Wang, Y. Zhang, G. Wu, and J. Shi, "An optimal and distributed demand response strategy for energy internet management," *Energies*, vol. 11, 2018.
- [86] T.-C. Ou, "Ground fault current analysis with a direct building algorithm for microgrid distribution," *International Journal of Electrical Power and Energy Systems*, vol. 53, pp. 867–875, 2013.
- [87] V.-H. Bui, A. Hussain, and H.-M. Kim, "Optimal operation of microgrids considering auto-configuration function using multiagent system," *Energies*, vol. 10, 09 2017.
- [88] R. Lasseter, "Microgrids and distributed generation," *Journal of Energy Engineering ENERG ENG-ASCE*, vol. 133, 2007.
- [89] T.-C. Ou, "A novel unsymmetrical faults analysis for microgrid distribution systems," *International Journal of Electrical Power and Energy Systems*, vol. 43, pp. 1017–1024, 2012.
- [90] P. Sheikahmadi, R. Mafakheri, S. Bahramara, M. Damavandi, and J. Catalão, "Risk-based two-stage stochastic optimization problem of micro-grid operation with renewables and incentive-based demand response programs," *Energies*, vol. 11, no. 3, Mar 2018. [Online]. Available: <http://dx.doi.org/10.3390/en11030610>

- [91] Y. Wang, M. Chen, T. Cheng, and M. A. S. Hassan, *The Research of Improved Wolf Pack Algorithm Based on Differential Evolution*. First International Conference on Intelligent Manufacturing and Internet of Things, 2018, pp. 65–76.
- [92] E. Bompard and Y. Ma, “Models of strategic bidding in electricity markets under network constraints,” *Handbook of networks in power systems I*, pp. 3–39, 2012.
- [93] B. Celik, S. Suryanarayanan, R. Roche, and T. M. Hansen, “Quantifying the impact of solar photovoltaic and energy storage assets on the performance of a residential energy aggregator,” *IEEE Transactions on Sustainable Energy*, vol. 11, no. 1, pp. 405–414, 2020.
- [94] M. Dolatabadi and P. Siano, “A scalable privacy preserving distributed parallel optimization for a large-scale aggregation of prosumers with residential pv-battery systems,” *IEEE Access*, vol. 8, pp. 210 950–210 960, 2020.
- [95] C. A. Correa-Florez, A. Michiorri, and G. Kariniotakis, “Optimal participation of residential aggregators in energy and local flexibility markets,” *IEEE Transactions on Smart Grid*, vol. 11, no. 2, pp. 1644–1656, 2020.
- [96] P. Paudyal, P. Munankarmi, Z. Ni, and T. M. Hansen, “A hierarchical control framework with a novel bidding scheme for residential community energy optimization,” *IEEE Transactions on Smart Grid*, vol. 11, no. 1, pp. 710–719, 2020.
- [97] Z. Chen, L. Wu, and Y. Fu, “Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization,” *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1822–1831, Dec 2012.
- [98] Y. Huang, S. Mao, and R. M. Nelms, “Adaptive electricity scheduling in microgrids,” *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 270–281, Jan 2014.
- [99] B. Narayanaswamy, V. K. Garg, and T. S. Jayram, “Online optimization for the smart (micro) grid,” in *2012 Third International Conference on Future Systems: Where Energy, Computing and Communication Meet (e-Energy)*, May 2012, pp. 1–10.

- [100] D. Li and S. K. Jayaweera, “Distributed smart-home decision-making in a hierarchical interactive smart grid architecture,” *IEEE Transactions on Parallel and Distributed Systems*, vol. 26, no. 1, pp. 75–84, Jan 2015.
- [101] C. Chen, S. Duan, T. Cai, B. Liu, and G. Hu, “Smart energy management system for optimal microgrid economic operation,” *IET Renewable Power Generation*, vol. 5, no. 3, pp. 258–267, May 2011.
- [102] W. Zheng, W. Wu, B. Zhang, Z. Li, and Y. Liu, “Fully distributed multi-area economic dispatch method for active distribution networks,” *IET Generation, Transmission Distribution*, vol. 9, no. 12, pp. 1341–1351, 2015.
- [103] W. Zheng, W. Wu, B. Zhang, H. Sun, and Y. Liu, “A fully distributed reactive power optimization and control method for active distribution networks,” *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1021–1033, Mar 2016.
- [104] E. Dall’Anese, H. Zhu, and G. B. Giannakis, “Distributed optimal power flow for smart microgrids,” *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1464–1475, Sep 2013.
- [105] W. Saad, Z. Han, H. V. Poor, and T. Basar, “Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications,” *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86–105, 2012.
- [106] S. Chen and R. S. Cheng, “Operating reserves provision from residential users through load aggregators in smart grid: A game theoretic approach,” *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 1588–1598, Mar 2019.
- [107] A. S. Gazafroudi, M. Shafie-Khah, F. Prieto-Castrillo, J. M. Corchado, and J. P. S. Catalão, “Monopolistic and game-based approaches to transact energy flexibility,” *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1075–1084, 2020.
- [108] T. Lu, Z. Wang, J. Wang, Q. Ai, and C. Wang, “A data-driven stackelberg market strategy for demand response-enabled distribution systems,” *IEEE Transactions on Smart Grid*, vol. 10, no. 3, pp. 2345–2357, 2019.

- [109] S. Grammatico, F. Parise, M. Colombino, and J. Lygeros, “Decentralized convergence to nash equilibria in constrained deterministic mean field control,” *IEEE Transactions on Automatic Control*, vol. 61, no. 11, pp. 3315–3329, Nov 2016.
- [110] P. E. Caines, M. Huang, and R. P. Malhamé, *Mean Field Games*. Springer International Publishing, 2018, pp. 345–372. [Online]. Available: https://doi.org/10.1007/978-3-319-44374-4_7
- [111] D. Bauso, “Dynamic demand and mean-field games,” *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6310–6323, Dec 2017.
- [112] A. De Paola, V. Trovato, D. Angeli, and G. Strbac, “A mean field game approach for distributed control of thermostatic loads acting in simultaneous energy-frequency response markets,” *IEEE Transactions on Smart Grid*, vol. 10, no. 6, pp. 5987–5999, 2019.
- [113] R. Couillet, S. M. Perlaza, H. Tembine, and M. Debbah, “Electrical vehicles in the smart grid: A mean field game analysis,” *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 6, pp. 1086–1096, Jul 2012.
- [114] E. Mojica-Nava, C. Barreto, and N. Quijano, “Population games methods for distributed control of microgrids,” *IEEE Transactions on Smart Grid*, vol. 6, no. 6, pp. 2586–2595, 2015.
- [115] C. P. Mediwaththe, E. R. Stephens, D. B. Smith, and A. Mahanti, “A dynamic game for electricity load management in neighborhood area networks,” *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1329–1336, May 2016.
- [116] S. Wang, A. F. Taha, and J. Wang, “Blockchain-assisted crowdsourced energy systems,” in *2018 IEEE Power Energy Society General Meeting (PESGM)*, 2018, pp. 1–5.
- [117] S. Wang, A. F. Taha, J. Wang, K. Kvaternik, and A. Hahn, “Energy crowdsourcing and peer-to-peer energy trading in blockchain-enabled smart grids,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 8, pp. 1612–1623, 2019.

- [118] J. Liu, N. Zhang, C. Kang, D. S. Kirschen, and Q. Xia, "Decision-making models for the participants in cloud energy storage," *IEEE Transactions on Smart Grid*, vol. 9, no. 6, pp. 5512–5521, 2018.
- [119] D. Zhao, H. Wang, J. Huang, and X. Lin, "Virtual energy storage sharing and capacity allocation," *IEEE Transactions on Smart Grid*, vol. 11, no. 2, pp. 1112–1123, 2020.
- [120] M. Aziz and P. E. Caines, "Computational investigations of decentralized cellular network optimization via mean field control," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, Dec 2014, pp. 5560–5567.
- [121] T. Ishii, K. Otani, T. Takashima, and S. Kawai, "Estimation of the maximum power temperature coefficients of pv modules at different time scales," *Solar Energy Materials and Solar Cells*, vol. 95, no. 1, pp. 386–389, 2011, 19th International Photovoltaic Science and Engineering Conference and Exhibition (PVSEC-19) Jeju, Korea, 9-13 November 2009.
- [122] N. Nikmehr and S. N. Ravadanegh, "Optimal power dispatch of multi-microgrids at future smart distribution grids," *IEEE Transactions on Smart Grid*, vol. 6, no. 4, pp. 1648–1657, Jul 2015.
- [123] U. Bessi, "Existence of solutions of the master equation in the smooth case," *SIAM Journal on Mathematical Analysis*, vol. 48, no. 1, pp. 204–228, 2016.
- [124] S. Mayorga, "Short time solution to the master equation of a first order mean field game," *Journal of Differential Equations*, vol. 268, 11 2019.
- [125] A. SoltaniNejad Farsangi, S. Hadayeghparast, M. Mehdinejad, and H. Shayanfar, "A novel stochastic energy management of a microgrid with various types of distributed energy resources in presence of demand response programs," *Energy*, vol. 160, pp. 257–274, 2018.
- [126] B. Xia, J. Yin, J. Xu, and Y. Li, "We-rec: A fairness-aware reciprocal recommendation based on walrasian equilibrium," *Knowledge-Based Systems*, vol. 182, 2019. [Online]. Available: <https://www.sciencedirect.com/science/Article/pii/S0950705119303326>

- [127] L. e on Walras, *Elements of Pure Economics, or the Theory of Social Wealth*. London, U.K.: Allen and Unwin, 1954.
- [128] P. Chavali, P. Yang, and A. Nehorai, “A distributed algorithm of appliance scheduling for home energy management system,” *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 282–290, Jan 2014.
- [129] J. Yong and X. Y. Zhou, *Stochastic controls: Hamiltonian systems and HJB equations*, ser. Applications of mathematics. New York: Springer Science and Business Media, 1999, no. 43.
- [130] L. J. and C. Y.-T, *Computational Partial Differential Equations Using MATLAB*, 2nd ed. CRC Press, 2019. [Online]. Available: <https://doi.org/10.1201/9780429266027>
- [131] D. T. Ton and M. A. Smith, “The u.s. department of energy’s microgrid initiative,” *The Electricity Journal*, vol. 25, p. 84–94, 10 2012.
- [132] A. M. Jasim, B. H. Jasim, A. Flah, V. Bolshev, and L. Mihet-Popa, “A new optimized demand management system for smart grid-based residential buildings adopting renewable and storage energies,” *Energy Reports*, vol. 9, pp. 4018–4035, 2023.
- [133] J. Dong, C. Song, S. Liu, H. Yin, H. Zheng, and Y. Li, “Decentralized peer-to-peer energy trading strategy in energy blockchain environment: A game-theoretic approach,” *Applied Energy*, vol. 325, 2022.
- [134] M. K. Mishra, K. Murari, and S. Parida, “Demand-side management and its impact on utility and consumers through a game theoretic approach,” *International Journal of Electrical Power and Energy Systems*, vol. 140, 2022.
- [135] H. Tang and S. Wang, “Game-theoretic optimization of demand-side flexibility engagement considering the perspectives of different stakeholders and multiple flexibility services,” *Applied Energy*, vol. 332, 2023.
- [136] M. Al-Kaabi, B. H. Al Igeb, and S. Y. Ali, “An overview of the smart grid attributes, architecture and components,” in *Proceedings of the 2nd International Conference on*

- Emerging Technologies and Intelligent Systems*. Springer International Publishing, 2023, pp. 461–471.
- [137] G. Wagh and S. Mishra, “A smart metering distributed privacy framework with integrity verification,” in *2023 IEEE Power and Energy Society Innovative Smart Grid Technologies Conference (ISGT)*, 2023, pp. 1–5.
- [138] A. A. Khan, M. Naeem, M. Iqbal, S. Qaisar, and A. Anpalagan, “A compendium of optimization objectives, constraints, tools and algorithms for energy management in microgrids,” *Renewable and Sustainable Energy Reviews*, vol. 58, pp. 1664–1683, 2016. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1364032115016421>
- [139] W. Alharbi and K. Raahemifar, “Probabilistic coordination of microgrid energy resources operation considering uncertainties,” *Electric Power Systems Research*, vol. 128, pp. 1–10, 2015. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0378779615001856>
- [140] H. Falsafi, A. Zakariazadeh, and S. Jadid, “The role of demand response in single and multi-objective wind-thermal generation scheduling: A stochastic programming,” *Energy*, vol. 64, pp. 853–867, 2014. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0360544213008864>
- [141] O. Elci and N. Noyan, “A chance-constrained two-stage stochastic programming model for humanitarian relief network design,” *Transportation Research Part B: Methodological*, vol. 108, pp. 55–83, 2018.
- [142] A. Rabiee, M. Sadeghi, J. Aghaei, and A. Heidari, “Optimal operation of microgrids through simultaneous scheduling of electrical vehicles and responsive loads considering wind and pv units uncertainties,” *Renewable and Sustainable Energy Reviews*, vol. 57, pp. 721–739, 2016.