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**Numerical and Analytical Calculation of the Hydrodynamic Coefficients of a
Simplified Francis Turbine Radial Seal under Whirling Vibrations**

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Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise en sciences appliquées*
Génie mécanique

Août 2023

POLYTECHNIQUE MONTRÉAL

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présenté par **Emilie QUENEDEY**

en vue de l'obtention du diplôme de *Maîtrise en sciences appliquées*
a été dûment accepté par le jury d'examen constitué de :

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RÉSUMÉ

L'énergie hydraulique représente plus de 60% de la production d'énergie au Canada, et 96% des sources d'énergie au Québec. De nombreux barrages hydroélectriques au Québec sont équipés de roue de turbine de type Francis. Ces roues sont confinées radialement au sein du stator, avec des chambres latérales incluant d'étroits joints radiaux. Lors du fonctionnement de la turbine, un débit de fuite turbulent traverse ces joints radiaux, avant de s'écouler dans les chambres latérales. Une déviation radiale du rotor modifie l'écoulement dans les joints, créant ainsi un gradient de pression important. Ce nouveau champ de pression génère des forces d'excitation sur le rotor, qui est alors soumis à des vibrations selon une orbite circulaire. Ces forces sont caractérisées par des coefficients dynamiques de masse, d'amortissement et de rigidité. Ceux-ci constituent des variables d'entrée primordiales à l'analyse de la dynamique de la ligne d'arbre de la turbine, car ils permettent de prédire les vibrations de l'ensemble du système rotorique. Cependant, les caractéristiques hydrodynamiques du débit de fuite autour de la roue de turbine sont peu connues, et sont généralement négligées lors des calculs traditionnels de la dynamique de la ligne d'arbre. Ainsi, notre objectif est de déterminer ces coefficients dynamiques du rotor en prenant en compte l'influence de ces joints hydrodynamiques, afin d'améliorer la précision des analyses de la ligne d'arbre.

Les forces radiales et tangentielles exercées sur le rotor, pour une vitesse de rotation donnée, peuvent être représentées par des polynômes quadratiques et linéaires de la fréquence orbitale, dont les coefficients polynomiaux correspondent aux coefficients rotodynamiques. Ainsi, pour évaluer ces coefficients, il est nécessaire de déterminer les forces radiales et tangentielles agissant sur le rotor afin de les interpoler. Une modélisation numérique de la dynamique des fluides (CFD) réalisée avec le logiciel ANSYS CFX permet de calculer ces forces pour différentes fréquences orbitales. De plus, un modèle numérique acoustique-structurel basé sur la méthode des éléments finis (FEM) a été développé avec ANSYS Mechanical APDL : des analyses harmoniques, avec imposition du mouvement orbital au rotor, sont effectuées pour déterminer les forces résultantes, pour plusieurs fréquences orbitales également. Pour une géométrie de rotor cylindrique entouré d'un joint radial annulaire, les forces obtenues à partir de ces deux modèles numériques sont comparées à celles obtenues par un modèle analytique basé sur la théorie de l'écoulement potentiel.

Contrairement à la CFD, le modèle acoustique-structurel néglige le cisaillement du fluide, ce qui entraîne un gradient de pression en phase avec le mouvement du rotor, et une force fluide purement radiale au rotor dans son mouvement. L'absence de composante tangentielle dans la force de réaction empêche l'évaluation de l'amortissement direct et de la rigidité croisée

du rotor. En revanche, la CFD prend en compte la dynamique complète de l'écoulement, ce qui permet d'évaluer la force tangentielle.

Le modèle CFD a permis d'étudier la dépendance des coefficients dynamiques par rapport à la configuration du confinement radial. Il a ainsi été observé que la position du joint radial par rapport à la chambre radiale plus large a un impact significatif sur la stabilité du système joint-rotor. Lorsque le joint est en amont de la chambre, le rotor devient instable du fait de son amortissement négatif, tandis qu'un joint placé en aval de la chambre radiale confère une rigidité positive. Par conséquent, lorsqu'un joint se situe en amont de la chambre, l'ajout d'un second joint aval permet de réduire cette instabilité.

Par ailleurs, des études paramétriques menées sur une géométrie de confinement latéral, comportant un joint en amont de la chambre, montrent que les coefficients hydrodynamiques sont également sensibles aux dimensions radiales du joint et de la chambre, ainsi qu'au débit massique et à la fréquence angulaire du rotor. Ces différents paramètres exercent donc une influence directe sur la stabilité du rotor.

Les coefficients dynamiques du rotor qui résultent de ces divers modèles permettent d'améliorer l'analyse de la ligne d'arbre.

ABSTRACT

Turbulent leakage flow passes through the radial seals and side chambers of a Francis turbine runner. A small deviation of the rotor changes the radial seal flow, inducing a strong pressure gradient. It gives rise to fluid reaction forces acting on the rotor, which is subjected to whirling vibrations. These forces can be modelled by dynamic force coefficients in the shaft-line dynamics and are important for predicting whirling vibrations. However, the hydrodynamic characteristics of this leakage flow around a Francis turbine runner are not well known and are not included in the traditional shaft line rotor dynamic calculation. We determine the rotor dynamic coefficients to improve the accuracy of shaft line analyses.

Radial and tangential forces for a given rotating speed are written as quadratic and linear polynomials of the whirl frequency. The polynomial coefficients correspond to the rotor dynamic coefficients. A computational fluid dynamics (CFD) simulation by ANSYS CFX computes these forces for different whirl frequencies. Harmonic analyses on a vibroacoustic finite element model (FEM) are also performed to obtain these forces. Their interpolation as a function of the whirl frequency yields the rotordynamic coefficients. These numerical models are compared with an analytical potential flow theory model for a plain annular seal geometry.

Both numerical models demonstrate similar radial forces for the plain annular seal configuration. However, in opposition to CFD, the vibroacoustic model does not account for fluid shear, neglecting the phase shift between pressure gradient and rotor displacement, and consequently, the absence of tangential forces. This limitation prevents the evaluation of direct damping and cross-coupled stiffness.

Since CFD considers all flow dynamics, it allows for the study of the dependence of the rotordynamic coefficients on the radial clearance configuration. Placing the seal downstream of the side chamber promotes stability, while having an upstream seal generates the opposite effect. Therefore, installing a second downstream seal can improve rotor stability if the runner radial clearance features an upstream seal. Furthermore, parametric studies on a clearance-and-side-chamber geometry highlight that hydrodynamic coefficients are also sensitive to radial casing dimensions, mass flow rate, and rotor angular frequency, all of which have a direct influence on rotor stability. The resulting rotor dynamic coefficients help to improve the shaft line analysis.

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LIST OF SYMBOLS AND ACRONYMS

CFD	Computational Fluid Dynamics
DNS	Direct Numerical Simulation
FEM	Finite Element Method
FSI	Fluid Structure Interaction
FVM	Finite Volume Method
SST	Shear Stress Transport

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CHAPTER 1 INTRODUCTION

To limit global warming, the COP21 agreement in 2015 established a limit of 1.5°C increase in global temperature since the pre-industrial period, which could be reached by early 2030's or even by 2024 (Agence Science-Press, 2023). To prevent exceeding a 2°C increase, a drastic reduction in CO_2 emissions is crucial, and the Intergovernmental Panel on Climate Change (IPCC) has stressed the need to switch from fossil fuels to renewable energy sources. Several countries have made significant progress in this regard. In Canada, hydroelectric energy accounted for 61% of the total energy production in 2018, with Quebec alone contributing over 50% of the country's hydroelectricity (Humphrys, 2020). In fact, more than 99% of Quebec's electricity is derived from renewable sources, primarily hydroelectric power, which accounts for over 96% of the total energy mix (HydroQuébec, 2023). In general, with advancements in solar and wind energy technologies, hydropower is relied upon more and more to balance the renewable energy power input. The advantage of hydraulic energy lies in its constant availability, unaffected by variable natural factors. Therefore, regulating the flow of water through the turbine allows for the generation of the desired amount of hydropower, as in Quebec (Hydro-Québec, 2023). Hydraulic energy can also be easily stored by pumping water upstream of dams through pump-turbines (ANDRITZ, 2023).

There exists several types of hydraulic turbines, with the Francis turbine being the most prevalent variant (Hydro-Québec, 2023). As illustrated in Fig. 1.1, the configuration of a Francis turbine involves a spiral case surrounding the turbine runner. The stator encloses the runner radially. During the turbine operation, the main flow circulates through the spiral case and enters radially into the runner with a uniform velocity. It drives the runner into rotation, and is then ejected axially through the draft tube (GridPro_Blog, 2020). However, a small fraction of the main flow, called leakage flow, circulates within the radial side chambers between the runner and stator. These radial confinements are constituted of an extremely thin gap, commonly called hydrodynamic seal, and a larger side chamber (Weber et al., 2016). These seals are specifically designed to minimize leakage flow and mitigate its negative impact on the efficiency of large machines like Francis turbines, as noted by Staubli (1998).

Undesirable and potentially unacceptable self-excited vibrations can occur in hydraulic turbines due to the flow within hydrodynamic seals (Henry, 1991). Turbulent leakage flow passes axially through the seal, in addition to a circumferential shear flow generated by the rotation of the rotor. Muszynska (2005) explains that during turbine operation, strong pressure gradients arise within the hydrodynamic seals, thereby inducing radial and tangential forces

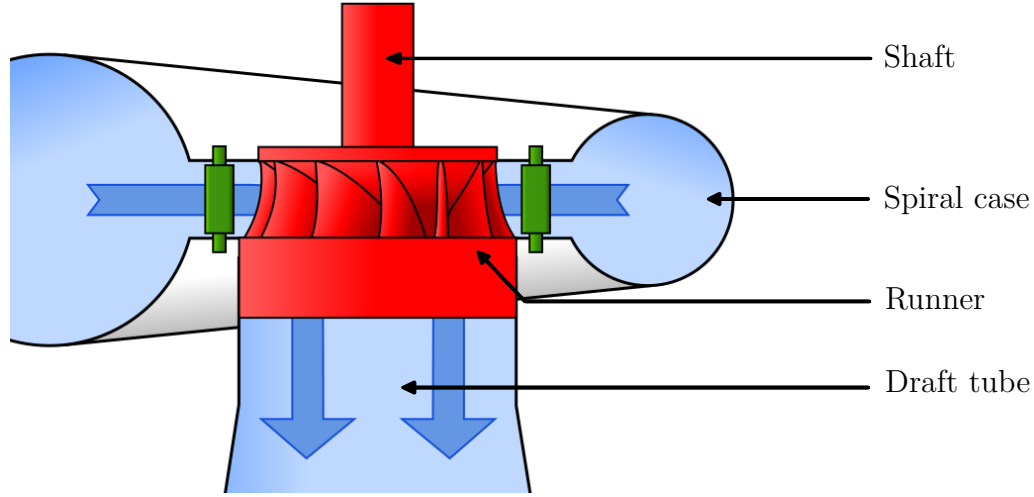


Figure 1.1 Schematic of a Francis turbine (WikipediaCommons, 2016)

on the rotor. The circular orbit of the whirl motion being increased by the fluid forces, it may contribute to a self-excited whirl vibration.

Manufacturers seek to prevent these unwanted vibrations from occurring as they disrupt the normal operation of rotating machines and pose a threat to the long-term sustainability of turbines. Therefore, it is essential to conduct dynamic shaft line analysis prior taking any action on the turbine. The behavior of rotor dynamics and the forces exerted on the rotor system depend on the rotordynamic coefficients of mass, damping and stiffness. These coefficients serve as crucial inputs for shaft line analysis.

The methodology presented by Staubli and Bissig (2001) proposes to determine the rotordynamic coefficients by analyzing the reaction forces at various whirl frequencies. While analytical models, such as those proposed by Blevins (1990) or Childs (1983a, 1983b), have been developed to calculate forces and rotordynamic coefficients for simple plain annular seals, these theories are challenging to apply to more complex radial clearance geometries. Computational fluid dynamics (CFD) emerges as a powerful tool to comprehensively study the flow dynamics within turbine radial seals. CFD enables the evaluation of reaction forces on the rotor in realistic turbine runner configurations. Additionally, it allows for the comparison of forces based on the seal's position relative to the larger side chamber, as demonstrated by Nishimura et al. (2018). Alternatively, numerical vibroacoustic models based on the finite element method (FEM) are less numerically expensive than CFD. However, these models do not account for the effect of velocity profiles, which are crucial in capturing the complete flow behavior.

In this thesis, we present a method to calculate a rotor seal dynamic coefficients, which can

be extended to real turbine runner seal systems. First, we carry out a comprehensive review of the existing literature to establish the research objectives of our project. Then, we apply the methodology of Staubli and Bissig (2001). Two numerical models, using CFD and FEM respectively, are set up to evaluate the fluid forces required to obtain the dynamic coefficients of the rotor seal. An analytical model based on potential flow theory presented by Blevins (1990) is developed for comparison with the numerical models. Finally, the results section compares the predictions of these models in terms of forces. The variation of forces and rotordynamic coefficients as a function of geometric and input parameters, and as a function of radial clearance configuration, is studied. These results lead to a discussion of the sensitivity of rotor stability to these dynamic coefficients. To conclude, we provide a comprehensive summary of our work, outlining the achieved objectives, discussing the limitations encountered, and offering prospects for future research directions.

This project was done in collaboration with Andritz Hydro Canada Inc., based in Pointe-Claire, Canada.

CHAPTER 2 LITERATURE REVIEW

The next chapter reviews the research into rotor seal dynamic coefficients and whirling vibrations that occur in these seals. The turbine hydrodynamic seals involved are presented, and the flow dynamics that arise during turbine operation are explained. These vibrations are related to the rotor seal dynamic coefficients which are then detailed. Finally, an overview of the various analytical and numerical models for determining these dynamic coefficients is presented.

2.1 Geometry of hydrodynamic seals

Most turbomachines possess hydrodynamic seals around the rotor. They consist in very narrow gaps, sometimes with the shape of a labyrinth or just as a small plain gap between the runner and the stator. These seals are designed to reduce leakage flow, to minimize its deleterious effect on efficiency in large machines such as Francis and pump-turbines (Staubli, 1998).

In Francis turbines, side chambers separate the runner from the stator. They become narrower around the runner crown and band hydrodynamic seals, with gaps of the order of 0.001 m for a runner of radius approximately 2 m, as illustrated in Fig. 2.1. During the turbine operation, a turbulent axial flow passes through those hydrodynamic seals, in addition to a circumferential shear flow driven by the rotor rotation. The cylindrical surface of the runner band and the stator in this region gives an annular shape of the narrow radial seal around the rotor band. This master focuses on the influence of fluid dynamics within these seals on the dynamics of the entire runner. Because the other fluid structure interactions between the flow and the runner are not of interest, the bladed runner can be assimilated to a solid shaft section in the context of this master.

The fluid dynamics that takes place in the annular seal is largely dependent on the rotor vibration, such as motion around a circular orbit. Such rotor motion directly modifies the seal geometry and the local pressure distribution on the rotor surface as well as the flow which can transition from laminar to turbulent. Strong gradients emerge in the flow (Staubli, 1998), leading to radial and tangential exciting forces on the rotor, as it will be depicted further.

It is necessary to strike a balance between the impact of the seals on machine efficiency and the dynamic stability of the shaft line. To properly account for the destabilising role of the seal in the design of the shaft line, a precise knowledge of the mentioned mechanisms is required.

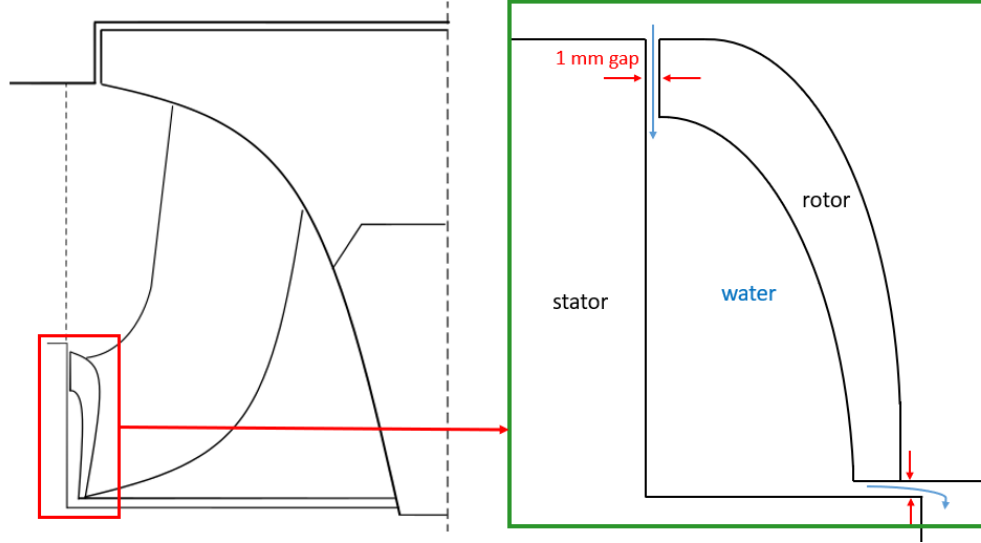


Figure 2.1 Schematic of a Francis runner side chamber and radial seal, reproduced from (Weber et al., 2016)

According to Muszynska (2005) The hydrodynamic seal present in runner-stator system is similar to a fluid film bearing. Muszynska summarises this general dynamic behaviour in a schematic reproduced in Fig. 2.2. Both hydrodynamic seals and bearings depict a rotor-stator system, the narrow gap between which is filled with fluid. The rotor is elastically supported and rotates on itself, while the stator is fixed. The confined fluid is set in motion by the rotation of the rotor under the effect of viscous friction. The resulting circumferential flow generates reaction forces on the rotor, which can produce self-excited vibrations such as "fluid whirl".

The theory of bearing fluid films is mature since they are present in most rotating machines.

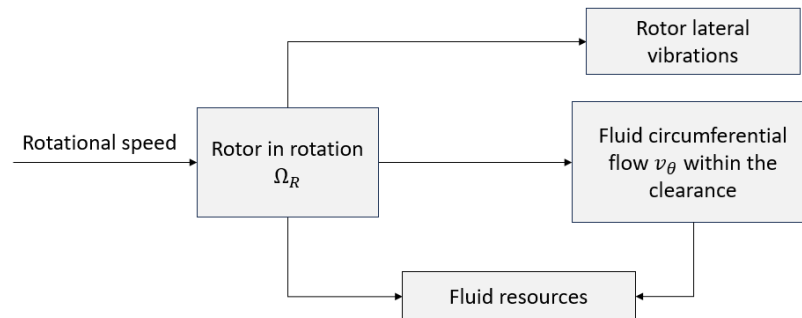


Figure 2.2 Schematic of the rotor/fluid system scenario that corresponds to hydrodynamic seals and bearings, reproduced from Muszynska (2005)

Rao (1991) explains that the thin viscous film between the rotor and the stator of the bearing supports the load from the rotor, as the squeeze film effect increases the damping and stiffness of the system. If the fluid is not highly viscous, the fluid force can induce negative stiffness instead, hence instability (Brennen, 2011).

However a comparison between the dynamical behaviour of hydrodynamic seals and fluid film bearings highlights major distinctions between both. First, The fluid contained in bearings is an oil lubricant of high viscosity (Rao, 1991), while only water is present in the radial seal around the turbine runner. Moreover, seals present an axial pressure gradient resulting from friction losses, this is why axial flow must be accounted for, which is not the case for bearings (Childs, 1983b).

Non-viscous fluid combined with pressure drops leads to fully-developed turbulent flows in the seal, in opposition to viscous often creeping flow in bearings. This difference prevents the direct applicability of fluid bearings approaches to characterise hydraulic turbines hydrodynamic seal flows. Other models are required to understand the fluid dynamics that occur in turbine hydrodynamic seals.

While fluid dynamics is easier to understand in annular geometries, it can vary for more complex configurations. In particular, the presence and shape of fluid domains upstream and downstream of radial seals can lead to recirculation and separation zones in the seals. In addition, the position of the seals in relation to the wider radial chamber can influence rotor stability. Nishimura et al. (2018); Weber et al. (2016) conclude that if hydrodynamic seals are located downstream of the side chambers, they tend to stabilize the runner, but upstream seals are responsible for the rise of radial fluid forces on the rotor, that can be destabilizing (Nishimura et al., 2016, 2018; Weber et al., 2016).

2.2 Influence of the rotor rotation

The turbine runner is attached to the shaft and put in rotation at the angular frequency Ω_R . In hydrodynamic turbines, the runner rotates at relatively small rotation rates around 0 to 10 Hz (Presas et al., 2015), hence inertial effects of the runner, including the gyroscopic effect, are typically neglected. However, the phenomenon of whirling vibration can appear within the fluid-rotor system in the hydrodynamic seal. In hydrodynamic runner chambers, whirl vibrations are characterised by a rotating solid body put into motion under fluid-structure interactions (FSI). It usually describes a circular orbit in its casing. The rotor rotation provides the required energy to feed the self-excited whirling vibrations (Muszynska, 2005). As illustrated in Fig. 2.3, whirling instability appears when the rotor angular frequency of

rotation Ω_R reaches the first threshold of instability, which is usually below the fundamental frequency of the rotor lateral bending mode Ω_{R_1} , as explained by Muszynska (2005).

When Ω_R increases from the first threshold of instability to Ω_{R_1} , the vibration magnitude, *i.e.*, the eccentricity ϵ , remains relatively constant, and the whirling vibrations dominate the runner vibrations. However when Ω_R becomes close to Ω_{R_1} , the eccentricity ϵ starts increasing, and the fluid whirl disappears temporarily. It starts again as Ω_R crosses a second threshold of instability, above Ω_{R_1} .

When $\Omega_R \approx 2\Omega_{R_1}$ fluid whirl is replaced by fluid whip, which differs from whirl as the fluid frequency is equal to Ω_R . As illustrated by Fig. 2.4, during fluid whip the rotor is deformed according to its first bending mode shape (Muszynska, 2005). The present study only focuses on fluid whirl, as the turbine runner rotation is usually too low to observe fluid whip.

2.3 Description of the flow dynamics

2.3.1 Circumferential flow

Nishimura et al. (2018) explain that the rotor rotation at frequency Ω_R coupled with the whirling motion generates a perturbation in the circumferential flow through viscous frictions. The fluid layer close to the rotor rotates at the same frequency Ω_R as the rotor while the fluid has no velocity close to the stator walls. If the preswirl is neglected, the average circumferential angular velocity in the clearance is usually approximated by $\Omega_R/2$ (Muszynska, 2005).

H. Black et al. (1981) also highlighted the influence of the inlet swirl on the circumferential velocity, through the study of a seal undergoing a turbulent axial flow. In this case, the average circumferential flow velocity reaches $R\Omega_R/2$ along the seal, R being the rotor radius, and Ω_R its rotational frequency.

As most studies consider the average fluid circumferential velocity as a constant equal to $0.5\Omega_R$ (Black & Jenssen, 1971), other theories (Muszynska, 2005) take into consideration the non-linearities in the circumferential flow, especially by introducing the fluid circumferential velocity ratio λ , as a decreasing function of the rotor eccentricity ϵ , hence the whirling frequency could be written as $\lambda\Omega_R$.

In any case, the whirling frequency does not only depend on the structural stiffness and mass, it is also affected by the viscous fluid damping too. The total damping in the radial direction is usually large because of the prevalent circumferential flow (Muszynska, 2005).

The rotor eccentricity directly affects the circumferential flow. If a unidirectional vibration occurs to the rotor, the area decreases in the upper sub-channel, and increases in the lower

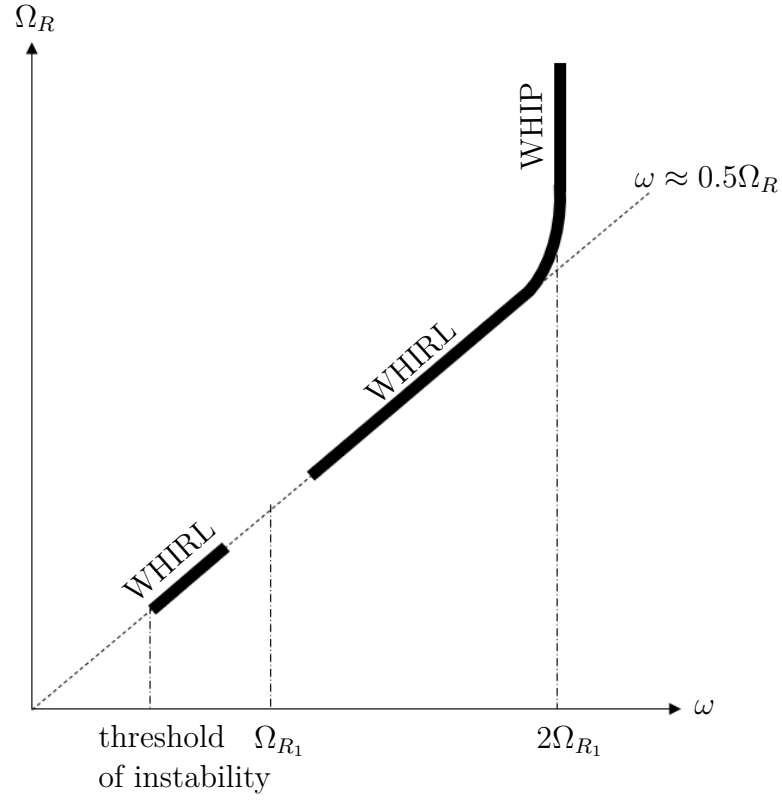


Figure 2.3 Zones of fluid whirl and fluid whip according to the rotor angular frequency Ω_R with respect to the shaft natural bending frequency Ω_{R1} , inspired by Muszynska (2005)

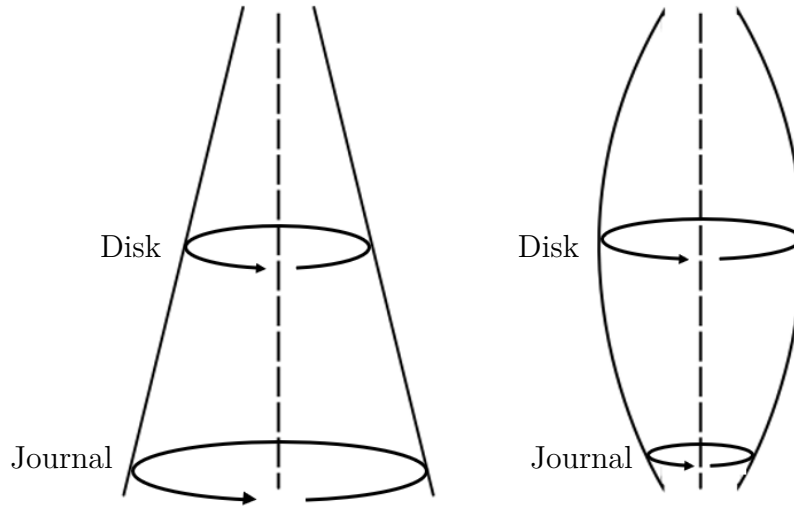


Figure 2.4 Whirl and whip modes of a rotor, reproduced from Muszynska (2005)

sub-channel. As explained by Brennen (2011) and illustrated in Fig. 2.5, conservation of the flow rate induces a flow velocity increase in the smaller sub-channel and decrease in the larger one. The static pressure distribution evolves in the opposite way, following Bernoulli's law. The underpressure in the upper sub-channel combined with the pressure rise on the lower sub-channel generates an acting force in the direction of motion, that enhances the rotor eccentricity, hence it is a destabilizing effect. This phenomenon is described as the "Bernoulli effect", which is dominant for high Reynolds numbers, according to Brennen (2011).

Overall, when the rotor is off-centred the circumferential flow induces reaction forces which act back on the rotor and generate self-excited vibrations. The rotor tends to vibrate in the lateral directions, around a circular orbit. This orbit of the whirl motion being increased by the fluid forces, it may contribute to a self-excited whirl vibration (Muszynska, 2005).

Some techniques intend to weaken this circumferential mean flow. According to Muszynska (2005), modifications of the cylindrical geometry, such as the introduction of grooves or lobes, break the circumferential flow regularity. Actually, a fraction of the circumferential flow energy is dissipated through vortices or axial backwards flows which are induced by those geometry variations. The regular circumferential velocity is thus reduced. Because plain cylindrical clearances do not present geometrical variation, they favour a strong circumferential flow.

2.3.2 Axial Flow in the annular gap

During the turbomachine operation an axial pressure gradient arises between the inlet and the outlet of the radial clearance. It induces an axial flow around the confined rotor (Brennen, 2011). In some theories like that of Childs (1983b) or Muszynska (2005), the axial flow influences the fluid reaction forces in the XY plane perpendicular to the rotor axis. Effectively, Muszynska explains that variations in the axial flow modify the amount of fluid in the circumferential flow.

Zerarka (2006); Brennen (2011); Robrecht and Pelz (2022) develop the changes in the axial flow called "Lomakin effect", which is schematized in Fig. 2.6 in the cross-section and axial plane of the seal. For short seals ($L/R \ll 1$) at high Reynolds numbers, circumferential variations are predominant on the axial velocity instead of the circumferential component, as fluid motion is mostly axial (Brennen, 2011). An eccentricity of the rotor perturbs the entrance flow circumferentially around it, as shown in Fig. 2.6.a. At the seal inlet, the flow separates due to its inertia (Robrecht & Pelz, 2022). These separation vortices combined with the sudden narrowing of the cross-section, result in a pressure loss at the seal inlet. This inlet underpressure alters the circumferential distribution of axial flow through the seal, which is also depicted in Fig. 2.6.b: more flow passes through the larger space, resulting in

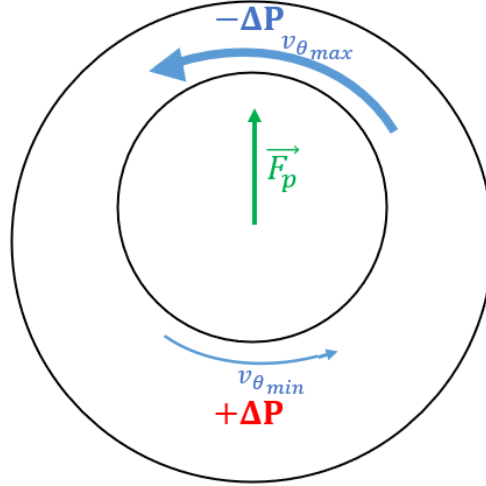


Figure 2.5 Schematic the Bernoulli effect on the circumferential flow, based on data from Brennen (2011)

a smaller pressure field, and vice versa in the smaller space. Following the Bernoulli law, a circumferential pressure gradient appears in the seal, which tends to alter the circumferential flow. The reaction force emerging from the circumferential pressure gradient is stabilizing as it is in opposition with the direction of eccentricity, and the added stiffness is positive.

According to Nishimura et al. (2018), perturbation of the circumferential flow and the inertia of the axial leakage flow both contribute to the resulting pressure gradient over the rotor in hydrodynamic seals. The flow is squeezed in the narrow passage, generating a strong pressure gradient that causes radial and tangential excitation forces on the rotor. The circular orbit of the whirl motion being increased by these fluid forces, it may contribute to a self-excited whirl vibration. Designing turbomachines requires to evaluate those vibrations which are directly affected by the runner seal direct and cross-coupled dynamic mass, damping and stiffness. Those dynamic coefficients are thus required as input data for predicting the shaft line dynamics.

2.4 Dynamic coefficients

Staubli and Bissig (2001)'s reminder holds that when considering a linear relationship between force and displacement and for small displacements around a central position, the following

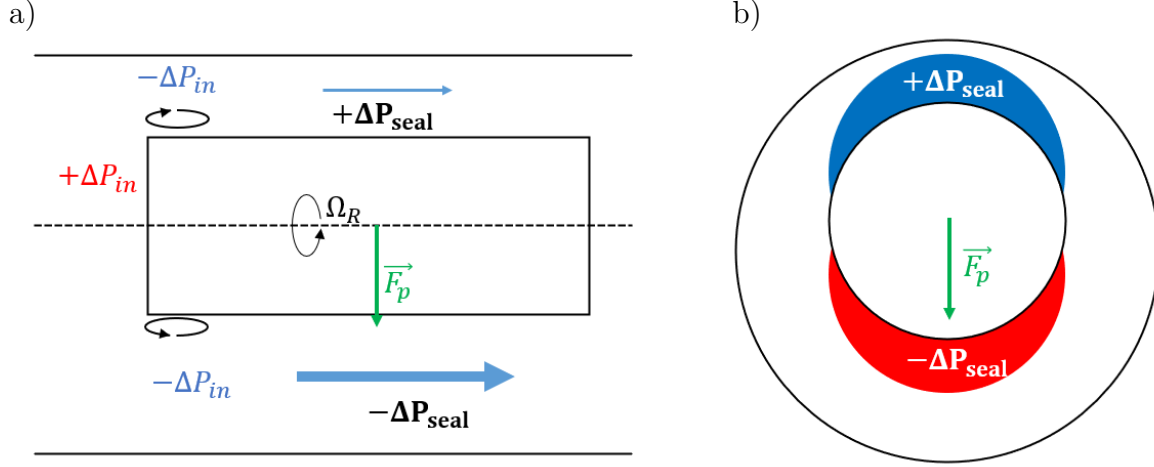


Figure 2.6 Schematic of the Lomakin effect: (a) in the axial plane; (b) in the cross-section plane, inspired by Zerarka (2006)

rotordynamic equation describes the force-displacement model of the system.

$$\begin{Bmatrix} -F_x(t) \\ -F_y(t) \end{Bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix}, \quad (2.1)$$

The stiffness matrix involves direct and cross-coupled stiffness (K and k). The damping matrix incorporates direct and cross-coupled damping (C and c), and the mass matrix includes direct added mass M . These matrices describe the linear relationship between the fluid reaction forces applied to a solid, and the solid displacement, velocity and acceleration. Based on experimental results, Staubli and Bissig (2001) also consider the cross-coupled mass terms null.

Hydrodynamic direct stiffness, denoted K , is related to the reaction force in phase with the rotor displacement. When the rotor is off-centred, the pressure distribution on its surface varies with θ , mainly through the Bernoulli and Lomakin effects. This results in circumferential variations in the pressure field around the seal. This increases the hydrodynamic stiffness on the rotor, modifying the total rotor stiffness and influencing its stability. As Lomakin and Bernoulli effects depend on the seal geometry that varies with time, the sign of the direct stiffness K can switch, when the effect overcomes the former dominant one (Brennen, 2011).

Hydrodynamic direct damping, denoted C , is related to the reaction force in phase with the rotor velocity. A positive damping dissipates the vibration energy which reduces the magnitude of vibration (Adams, 2001; Blevins, 1990), and the damping force restores the rotor to its stable position. However a negative damping transfers this vibration energy to the system, which increases the rotor vibration magnitude.

Although most models consider a stationary damping, for small rotor-stator clearances, Muszynska (2005) assumes that the damping force rotates at the whirling frequency. Its rotation induces another angular velocity-related force tangential to the rotor and superposed to the main radial damping. Muszynska considers this additional damping force as one of the main triggering forces of fluid whirl.

The added mass, denoted M , represents the inertia of the fluid put in motion by the moving solid. It coincides with the mass of fluid displaced by the rotor motion. It is positive, hence the resulting force is in phase with the fluid acceleration (Blevins, 1990). The added mass does not vary with the mean flow applied on the rotor, but is only a function of the rotor and clearance geometries.

As the natural frequency of the rotor is k_{tot}/m_{tot} , a higher added mass decreases the rotor natural frequency, which promotes instability, as lower frequencies are more frequently reached.

In addition to direct dynamic coefficients, the rotor hydrodynamic seal is also characterised by cross-coupled stiffness, denoted k , and cross-coupled damping, denoted c . They are usually neglected in dynamic analyses because of the dominant direct coefficients. Nevertheless they can be of the same order of magnitude as direct stiffness and damping in vertical axis machines according to Synnegård et al. (2016).

Andres (1993) and Hofstad (2004) state that cross-coupled stiffness stems from the swirl in the circumferential flow, and it increases with the circumferential velocity. The resulting force can be in the direction of the whirl motion, which increases the vibration magnitude (Sadiq et al., 2022). Von Pragenau (1982) assumes that a rough stator surface reduces this velocity hence leads a lower cross-coupled stiffness.

As for direct damping, a positive cross-damping coefficient increases stability of the system. However, whereas positive direct damping creates a restoring force opposite to the relative motion, the positive cross-coupled damping force acts perpendicular to the direction of motion.

2.5 Influence of the position of the seal in respect with the side chamber on the rotor stability

As explained in section 2.1 and shown in Fig. 2.1, The radial clearance of turbine runners includes a large side chamber and a narrower seal. The radial seal's placement in respect with the larger side chambers can affect rotor dynamics. It depends on whether the flow passes through the smaller clearance before or after the larger chamber, as developed by Nishimura et al. (2018); Weber et al. (2016).

Nishimura et al. (2018) evaluated numerically and analytically the force acting on a confined rotor, where the confinement is comprised of a seal upstream of the larger side chamber, and the flow can move either downwards or upwards, as presented in Fig. 2.7. For a flow from top to bottom, *i.e.*, passing through the seal before the side chamber, Nishimura et al. (2018) deduced that the fluid forces acting on the rotor are exciting as they exhibit negative damping. However if the flow passes first through the side chamber before the hydrodynamic seal, the forces are restoring, bringing positive damping to the system.

These results correlate with those of Weber et al. (2022), who observed that adding an additionnal seal at the outlet of the side chamber improved the rotor stability. Without consideration of the viscous effects, Weber et al. expressed a damping parameter c_{RSC} to characterise the stability of the seal placed either at the inlet or the outlet of the side chamber. As a seal at the inlet of the chamber brings instability, c_{RSC} would be negative, while it would be positive for a seal placed at the outlet:

$$\begin{aligned} c_{RSC} &= - \frac{\rho l^2}{2C_{ch}} \frac{Q}{2C_0} \quad \text{for a seal at the inlet,} \\ c_{RSC} &= \frac{\rho l^2}{2C_{ch}} \frac{Q}{2C_0} \quad \text{for a seal at the outlet (Weber et al., 2016).} \end{aligned} \tag{2.2}$$

with ρ the fluid density, Q the volumetric leakage flow rate, C_{ch} the side chamber radial width, and C_0 the seal radial width. Finally, these first observations imply that having a seal downstream of the side chamber tends to stabilise the rotor by adding a positive damping. The expression of C_{RSC} in Eq. 2.2 also implies that the hydrodynamic damping depends on the geometric dimensions of the seal and side chambers, as well as the flow rate.

2.6 Past models

Different analytical and numerical models were developed to calculate the dynamic coefficients of a rotor seal system. The dynamic damping, stiffness and the added mass are related to the reaction forces from the fluid to the rotor, hence it is possible to evaluate those

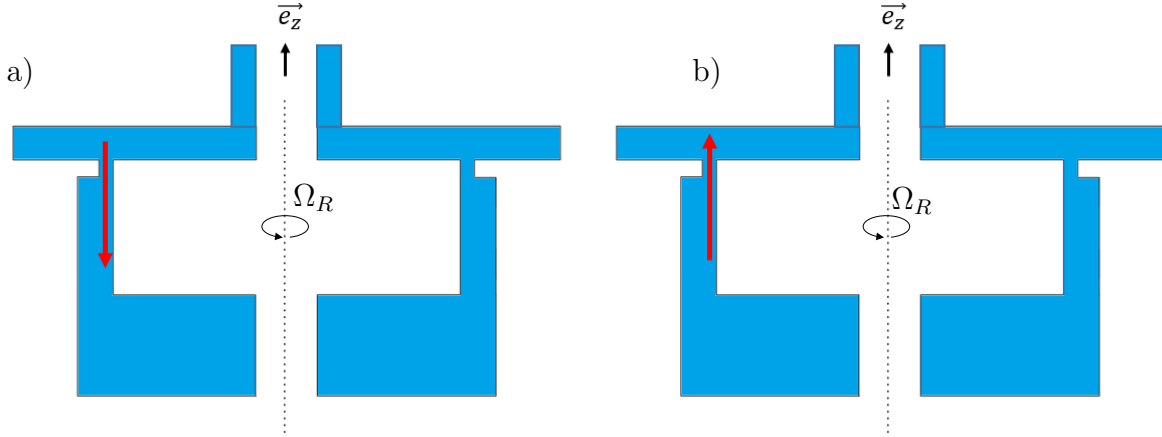


Figure 2.7 Schematic the rotor confined with a radial seal and a side chamber: (a) with a downwards flow that passes first through the seal upstream of the side chamber; (b) with an upwards flow that passes last through the seal downstream of the side chamber

coefficients from calculation of those forces.

2.6.1 Analytical models

Most of the past models study a plain annular seal structure, as shown in Fig. 2.8. The seal is of length l , the rotor of radius R_R is rotating at frequency Ω_R , named ω in Fig. 2.8.

H. Black and Jenssen (1970); Jenssen (1970) developed one of the first bulk-flow analyses to calculate the dynamic coefficients in high pressure annular seals. They assumed the circumferential flow to be a fully developed shear flow with uniform velocity magnitude $R\omega/2$. This model was valid only for hydrodynamic seals with small eccentricity. Allaire (1976) numerically calculated the dynamic coefficients for large eccentricity using Black and Jenssen's model.

Later, Childs (1983a, 1983b, 1989) developed a bulk-flow model for turbulent flow of plain annular seals with a strong pressure gradient in the axial direction. It relies on the lubrication theory of Hirs (1973), which models a fully turbulent flow along the circumferential and axial directions. Childs (1983a, 1983b) considers a fluid annulus around an axisymmetric impeller, as depicted in Fig. 2.8. He introduces perturbation terms in the flow of the order of the eccentricity ϵ and develops the dynamic coefficients that take into account the swirl in the circumferential flow. Childs (1983b) determines the dynamic coefficients for high-pressure annular finite-length seals with the circumferential flow assumed to be a function of the axial coordinate through the preswirl component. The same model is developed by Childs (1983a) for a short-seal by neglecting the circumferential velocity perturbation term, with simpler

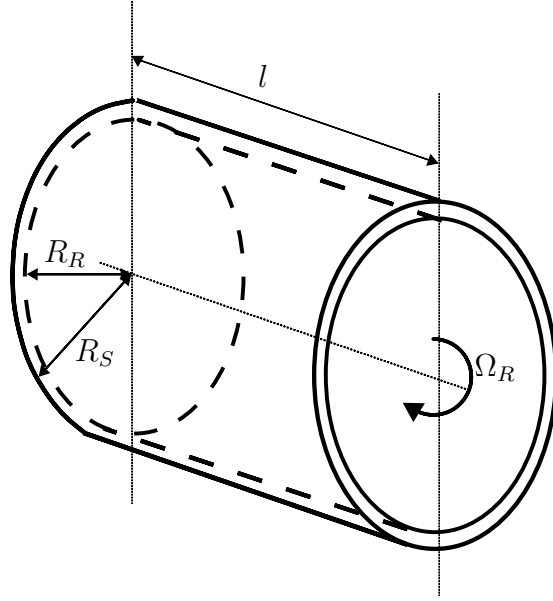


Figure 2.8 Schematic of a plain annular seal, inspired by Childs (1983a)

expression of the dynamic coefficients.

The bulk flow model of Childs (1983a, 1983b) is not applicable to more complex geometries. First, it relies on empirical friction coefficients n and m from Hirs (1973)' turbulence equations. To express the wall shear stress

$$\frac{\tau}{\frac{1}{2}\rho u^2} = m \left(\frac{\rho u h}{\eta} \right)^m, \quad (2.3)$$

the coefficients m and n are adjusted to individual experiments and depend on various parameters, such as the roughness of the surface, its curvature, the Reynolds number, the flow type and the influence of inertia effect except turbulence, as Hirs (1973) highlighted through several different experiments. This dependence prevents the application of the bulk-flow model to a particular leakage geometry different from a simple annular seal without running experiments, otherwise the flow path curvature would not be taken into account in the dynamic coefficients.

Moreover Childs (1983b) reduced the 3-dimensional problem to a bidimensional one, by only keeping the axial and circumferential velocity components, and neglecting the radial variations. As underlined by Hsu and Brennen (2002), Childs assumes the velocity profiles in the seal to be radially uniform. Therefore recirculation and separation of the flow are not considered. Although they barely happen in a simple annular seal, it becomes necessary to include these flow dynamics in the case of a more complex seal geometry.

The above bulk-flow models mostly rely on empirical equations, hence their complexity. Simpler models exist in the literature such as those based on the potential flow theory. Blevins (1990) revisits the potential flow model for an ideal fluid between two coaxial cylinders coupled with fluid, as depicted in Fig. 2.9. Simplicity of the model lies in the assumption of an inviscid and irrotationnal flow. Both cylinders remain rigid, but they can move relative to each other. The potential flow theory yields reaction forces on each cylinders as functions of the added mass of the cylinder. The resulting rotor added mass is:

$$M = \rho\pi R_R^2 \left(\frac{1 + (R_R/R_S)^2}{1 - (R_R/R_S)^2} \right), \quad (2.4)$$

where R_R and R_S are the rotor and stator radii. For a narrower radial gap $R_S - R_R$, the added mass of both cylinders drastically increase along with fluid forces that tend to dampen the motion of one cylinder relative to the other.

Potential flow theory yields expressions of the reaction forces on the rotor as well as the added mass, but it does not account for the axial flow. It simply relates the flow dynamics that occur in the circumferential and radial direction along the annular seal.

2.6.2 Numerical models

Analytical models cannot consider all the fluid dynamics. If the errors from those simplifications are negligible for plain annular seals, they are too significant for real hydraulic turbines hydrodynamic seal prototypes. Numerical models become necessary to answer that problem, especially Computational Fluid Dynamics (CFD) models.

Nishimura et al. (2016, 2018) performed CFD simulations on ANSYS CFX 15.0 to investigate the forces due to the pressure gradient on a columnar rotor. CFD allows them to consider a complex geometry of side chambers and radial seals.

Staubli and Bissig (2001) express the fluid forces as functions of the whirl frequency and the rotor dynamic coefficients, and use a CFD model to calculate those forces, whose expression yields the dynamic coefficients. This numerical method is applicable on a real turbomachine complex seal geometry. This method was applied by several authors on more complex seal geometries, such as Hofstad (2004); Staubli and Bissig (2001); Weber et al. (2022).

CFD simulations can either be performed for steady or transient conditions. Transient simulations are run in the absolute co-ordinate system, in which the rotor motion induces variations of the fluid domain. Hence the mesh must be adapted at each increment, which has a significant computational complexity. Instead, for seal geometries, steady-state simulations

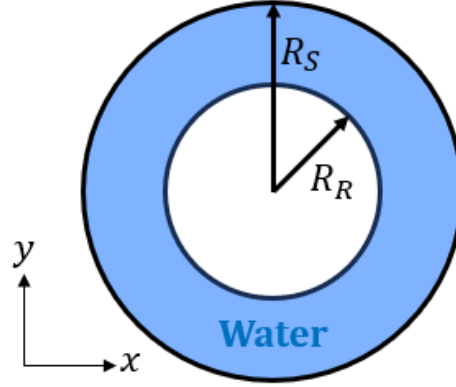


Figure 2.9 Incompressible inviscid fluid between two coaxial cylinders, inspired by Blevins (1990)

are performed. The geometry is axisymmetric and coupled with a circular rotor orbit about the concentric position, thus calculations can be performed in a relative coordinate system attached to the whirling rotor center. In this frame of reference, the fluid domain is steady-state. This is why the mesh remains identical as each increment, which drastically reduces the computational complexity.

Although CFD includes most of the fluid dynamics, it is numerically expensive. Vibroacoustic models based on the Finite Element Method (FEM) are less complex in calculation. For an acoustic fluid domain, FSI coupling problems can be solved with a Finite Element Analysis (FEA) applied to the solid and fluid domains, as did Berthet (2021). Simplicity of the model lies in the fluid acoustic property, which is considered ideal, hence inviscid. However it does not consider the velocity profile and shear stresses.

Vibroacoustic FSI problems based on FEM were performed by Edessa et al. (2012), who studied the bending vibration modes of immersed cylinders. Mohd Azman et al. (2021) determined the whirling vibrations of a shaft with the same method. Although vibroacoustic studies usually yield the natural frequencies, mode shapes and added mass via modal analyses, no publication could be found on the calculation of dynamic damping and stiffness using acoustic structural models.

2.7 The moment whirl

Although the influence of rotor translation under circular orbits on the dynamic coefficients is the core of the previous literature review, a misalignment of the shaft along its axis under angular displacement can also alter the rotor dynamics. This is why some studies interpolate

the bending moment in addition to the radial and tangential forces, in order to derive the bending coefficient, that is further included in the shaft line analysis, as did Weber et al. (2022). The bending moment is of great influence mostly for long annular seals (Andres, 1993). Childs (1982) demonstrated on finite-length annular seals that the force and moment dynamic coefficients are coupled: force coefficients depend on and angular displacement of the seal, and moment coefficients depend on the seal displacement. However given the dimensions of the considered annular seals that are characteristic of short-seals, the bending moment is of no interest here.

CHAPTER 3 LITERATURE REVIEW SYNTHESIS AND OBJECTIVES

In turbomachine design, it is crucial to evaluate vibrations that are influenced by the dynamic coefficients of the runner seal, known as its direct and cross-coupled dynamic mass M , damping C , and stiffness K , and its cross-coupled damping c and stiffness k . Under external excitations the rotor is off-centered, leading to a deformed seal with a smaller and a larger gap. The flow through the narrow passage experiences a significant pressure gradient, resulting in radial and tangential excitation forces on the rotor. These fluid forces can enhance the circular orbit of the whirl motion and potentially contribute to self-excited whirl vibrations. The rotor stability depends on its hydrodynamic mass, damping and stiffness, and these parameters can help or hinder the rotor from reaching a stable state. Hence, to accurately predict the dynamics of the shaft line, it is necessary to consider these coefficients as input data.

Most analytical models like that of Blevins (1990) and Childs (1983a, 1983b) consider the simple case of a plain annular seal without considering the side chambers and the position of the seal against those chambers. However these different cavities modify the flow dynamics, thus the resulting dynamic coefficients are significantly altered. Numerical CFD models were developed to study more realistic seal prototypes. For instance Nishimura et al. (2016, 2018) investigated the reaction forces on a rotor with consideration of the upstream and downstream seal cavities and the side chambers further down the seals. Although they did not evaluate the rotor dynamic coefficients, Weber et al. (2022) performed transient CFD simulations of a simplified configuration of a Kaplan turbine radial seal to determine the runner dynamic coefficients. It can be appropriate to simplify the seal geometry as it can significantly reduce the complexity in calculation. However uncertainties resulting from geometrical simplifications must be evaluated. Moreover, these studies did not examine the sensitivity of the dynamic coefficients and resulting fluid forces to the geometric parameters of the rotor seal.

Furthermore, CFD has a high numerical cost. Vibroacoustic models using FEM are less complex in calculation since the acoustic elements assume an inviscid and irrotational flow.

Based on the literature review, five main objectives were identified, to which this research project aims to respond:

- develop a numerical vibroacoustic FEM model under ANSYS Mechanical APDL to

obtain the rotor dynamic coefficients, and compare the obtained forces with CFD calculated forces in order to evaluate the limits of the vibroacoustic model compared to CFD;

- compare the CFD and FEM forces with results from an analytical model based on potential flow theory on a plain annular seal geometry in order to verify the calculations of the forces;
- investigate the influence of the seal geometry on the flow dynamics and the resulting forces;
- investigate the influence of the position of the seal with respect to the side chamber on the flow dynamics;
- evaluate the sensitivity of the rotor stability and its dynamic coefficients to radial gap geometrical parameters and input parameters.

CHAPTER 4 METHODOLOGY

4.1 Theoretical concepts

For a given rotor seal configuration, the following methodology based on the work of Staubli and Bissig (2001) is applied. As shown in Fig. 4.1, inside the radial seals we consider a cylindrical rotor rotating at rate Ω_R and drawing a circular orbit of eccentricity ϵ , at whirl frequency ω . The rotor eccentricity changes the seal geometry, which thus has a smaller and a larger gap of width $C_0 - \epsilon$ and $C_0 + \epsilon$ respectively. In the relative coordinate system of reference (x, y) attached to the rotor center motion and whirling at rate ω , the force components along these axes, F_x and F_y , fit the radial and tangential reaction forces, respectively. The viscous forces are assumed to be negligible compared to the pressure reaction forces, thus the total reaction forces are supposed to be only generated by the pressure field exerted on the rotor surface. In the whirling frame of reference, they correspond to:

$$\begin{aligned} F_x = F_r &= \int_0^L \int_0^{2\pi} p_{rotor} \cos \varphi R d\varphi dz, \\ F_y = F_t &= \int_0^L \int_0^{2\pi} p_{rotor} \sin \varphi R d\varphi dz. \end{aligned} \tag{4.1}$$

4.1.1 Dynamic equation of the rotor

To relate the fluid forces to the rotor dynamic coefficients, like Staubli and Bissig (2001) we assume a linear relationship between the fluid forces and the rotor displacement (x_R, y_R) , velocity and acceleration in the reference frame (x_0, y_0) , which gives:

$$-\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} = \begin{bmatrix} K_{xx} & K_{yx} \\ -K_{xy} & K_{yy} \end{bmatrix} \begin{Bmatrix} x_R \\ y_R \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{yx} \\ -C_{xy} & C_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x}_R \\ \dot{y}_R \end{Bmatrix} + \begin{bmatrix} M_{xx} & M_{yx} \\ -M_{xy} & M_{yy} \end{bmatrix} \begin{Bmatrix} \ddot{x}_R \\ \ddot{y}_R \end{Bmatrix}. \tag{4.2}$$

In the assumption of small deformation and for a rotational symmetry, the cross-coupled dampings C_{xy} and C_{yx} , as well as the cross-coupled stiffnesses K_{xy} and K_{yx} from Eq. 4.2 are equal, of magnitude c and k respectively. The cross-coupled mass terms M_{xy} and M_{yx} are neglected based on former experimental results, as mentioned by Staubli and Bissig. The

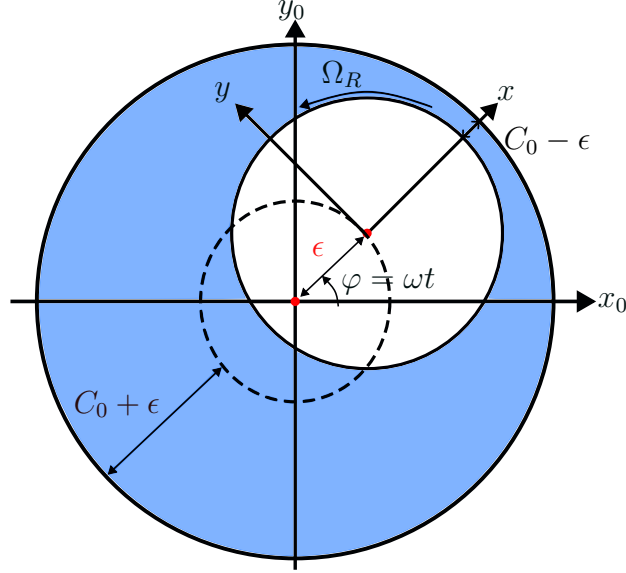


Figure 4.1 Schematic of the whirling rotor; reference frame (x_0, y_0) and whirling frame (x, y) of reference

resulting dynamic equation of the rotor-seal structure is:

$$\begin{Bmatrix} -F_x(t) \\ -F_y(t) \end{Bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{Bmatrix} x_R \\ y_R \end{Bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{Bmatrix} \dot{x}_R \\ \dot{y}_R \end{Bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x}_R \\ \ddot{y}_R \end{Bmatrix}, \quad (4.3)$$

where the radial and tangential forces, F_x and F_y , depend on the rotordynamic direct mass M , direct damping C , direct stiffness K , cross-coupled damping c and cross-coupled stiffness k . Those dynamical coefficients determine the rotor stability.

4.1.2 Stability of the rotor

Damping and stiffness both influence the stability of the rotor, while the mass only influences its frequency. Without cross-coupled damping and stiffness, the rotor is stable when both direct damping C and stiffness K are positive. Effectively, a null stiffness generates static instability, as it decreases the resistance of the rotor while increasing its displacement in response to an external force. A dynamic instability is due to a negative damping: more energy is given to the system as it oscillates, leading to increasing vibrations.

However, in the presence of cross-coupled stiffness and damping, the assessment of rotor stability becomes more complex. A comprehensive criterion that takes into account all relevant coefficients is required to evaluate the system's stability.

The characteristic equation of the system corresponds to the determinant of Eq. 4.3 with

$F_x = F_y = 0$. With :

$$\begin{Bmatrix} x_R \\ y_R \end{Bmatrix} = \begin{Bmatrix} X_0 \\ Y_0 \end{Bmatrix} e^{\alpha t}, \quad (4.4)$$

this characteristic equation is :

$$\alpha^4 + \frac{2C}{m}\alpha^3 + \left[\frac{C^2 + c^2}{m^2} + \frac{2K}{m} \right] \alpha^2 + \frac{2}{m^2}(KC + kc)\alpha + \frac{K^2 + k^2}{m^2} = 0 \quad (4.5)$$

If all α -roots have negative real parts, the system is stable and non-oscillating. On the contrary, the system is unstable if at least one root has a positive real part, as recalled by Lity (1966). Lity uses the Routh-Hurwitz stability criterion to determine the sign of the roots' real parts. It yields three stability criteria that the hydrodynamic coefficients must fulfill at the same time to be in a stable state:

$$\begin{aligned} C &> 0, \\ \frac{C}{M} + \frac{KC - kc}{C^2 + c^2} &> 0, \\ \frac{C}{M} - \frac{k^2}{KC + kc} &> 0. \end{aligned} \quad (4.6)$$

4.1.3 Evaluation of the dynamic coefficients

The circular displacement of the rotor in the reference frame (x_0, y_0) is:

$$\begin{Bmatrix} x_R(t) \\ y_R(t) \end{Bmatrix} = \begin{Bmatrix} \epsilon \cos(\omega t) \\ \epsilon \sin(\omega t) \end{Bmatrix}. \quad (4.7)$$

Introducing Eq. 4.7 into Eq. 4.3 and initializing the time to $t = 0$, F_x/ϵ and F_y/ϵ are equal to:

$$\begin{aligned} \frac{F_x}{\epsilon} &= M\omega^2 - c\omega - K, \\ \frac{F_y}{\epsilon} &= -C\omega + k. \end{aligned} \quad (4.8)$$

Linear and quadratic regressions of F_x/ϵ and F_y/ϵ over the whirl frequency ω yield the polynomial coefficients which are the direct and cross-coupled dynamical coefficients. For a fixed rotor eccentricity ϵ , the radial and tangential forces F_x and F_y must be calculated, for several whirl frequencies ω . Two numerical models aim to calculate those force components. They are compared to an analytical model for the plain annular seal.

4.2 Different configurations of seals

The methodology involves studying two seal geometries with increasing complexity in their shape. There are pictured in Fig. 4.2, with representation of the flow by the red arrows, and the dashed red lines indicate the permeable frontiers. The first geometry, shown in Fig. 4.2.a, consists of a simple plain annular seal of width 1.6 mm, which surrounds a rotor of radius $R_R = 2012$ mm, length $l = 183$ mm, off-centred by $\epsilon = 0.6$ mm. These dimensions are inspired from seal configurations obtained from the industrial partner. The fluid enters axially at the inlet and exits at the outlet.

For a more comprehensive examination of the flow dynamics in rotordynamic seals of Francis turbines, further studies are conducted on a geometry reproduced from Nishimura et al. (2018), depicted in Fig. 4.2.b. In this second configuration, the fluid enters from an upper cavity, passes through a radial seal of width C_0 , and a side chamber of width C_{ch} and length l . The fluid exits through a cavity located below the radial gaps. The geometric dimensions correspond to those presented by Nishimura et al. (2018).

Compared to the plain annular seal configuration, this second studied rotor seal geometry is a more realistic representation of Francis turbine radial confinements. Such radial clearance is schematized in Fig. 4.2.c, based on prototypes from ANDRITZ Hydro Canada Inc. In this existing configuration, the flow enters the fluid domain radially, with the majority of the flow passing through the runner. However, a small portion of leakage flow drops axially within the narrow radial gaps, comprised of first narrow seal, followed by a larger side chamber and a second radial seal.

4.3 Analytical flow-potential model

A bi-dimensional analytical model based on potential flow theory is formulated for the plain annular seal geometry represented in Fig. 4.2.a. It stems from the potential flow theory for two coaxial cylinders coupled by fluid by Blevins (1990). Blevins assumes that the flow is inviscid, incompressible and irrotationnal. Under these assumptions, the flow can be described by the flow potential ϕ , a scalar defined as:

$$\vec{v}_f = \vec{\nabla} \phi, \quad (4.9)$$

with $\vec{v}$ the fluid velocity vector.

The flow must verify the equations of continuity and Euler:

$$\begin{aligned} \rho \vec{\nabla} \cdot \vec{v}_f &= 0, \\ \frac{\partial \vec{v}_f}{\partial t} + (\vec{v}_f \cdot \vec{\nabla}) \vec{v}_f &= -\frac{1}{\rho} \vec{\nabla} p, \end{aligned} \quad (4.10)$$

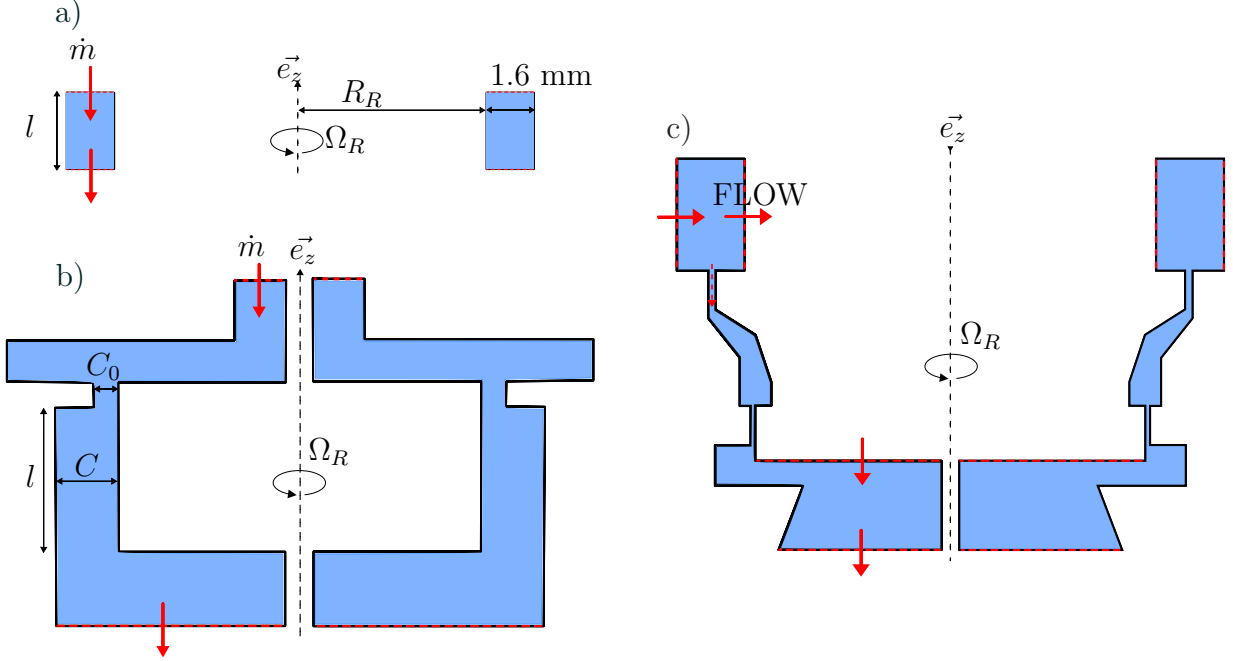


Figure 4.2 Schematic of the studied seal geometries: (a) a plain annular seal, and (b) radial gaps reproduced from (Nishimura et al., 2018). (c) Schematic of side chambers of a real Francis runner prototype

ρ and p being the fluid density and the pressure. For an incompressible flow, substituting $\vec{v}_f$ with $\vec{\nabla}\phi$, these equations become:

$$\begin{aligned} \Delta\phi &= 0, \\ \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 &= -\frac{p}{\rho}. \end{aligned} \quad (4.11)$$

These two equations are the Laplace and Euler equation that ϕ must verify. The writing of the Euler equation as a function ϕ is described in Appendix A.

The outer cylinder is fixed, and the rotor horizontal perturbation in displacement is :

$$\vec{\epsilon} = \begin{Bmatrix} \epsilon \cos(\omega t) \\ \epsilon \sin(\omega t) \end{Bmatrix}, \quad (4.12)$$

with ω the whirl frequency and ϵ the amplitude of displacement, which are represented in Fig. 4.1. The rotor also rotates around its axial axis at frequency Ω_R . It induces a circumferential flow mean angular frequency $\Omega_f \approx 0.5\Omega_R$. Thus the base flow is considered steady-state and purely circumferential, rotating at frequency $\Omega_f = 0.5\Omega_R$. The fluid radial velocity v_{f_r} is assumed to be null because $R_S - R_R \ll l$, with R_R , R_S the rotor and stator radii, and l the rotor axial length. We suppose that there is a fluid axial velocity, v_{f_z} .

In the rotor frame of reference, the radial displacement of the rotor is:

$$\tilde{r} = \epsilon \cos(\gamma - \omega t) = \epsilon \cos \theta, \quad (4.13)$$

with $\theta \approx \gamma - \omega t$ for small amplitude of displacements ϵ . As shown in Fig. 4.3, the angles γ and θ are the angular coordinates in the reference and whirling frames of reference respectively. The angle ωt separates the reference frame (x_0, y_0) from the whirling frame of reference. The structural radial displacement yields the structural velocity using the material derivative:

$$v_s = \frac{D\tilde{r}}{Dt} = \frac{\partial \tilde{r}}{\partial t} + v_{f_r} \frac{\partial \tilde{r}}{\partial r} + \frac{v_{f_\theta}}{r} \frac{\partial \tilde{r}}{\partial \theta} + v_{f_z} \frac{\partial \tilde{r}}{\partial z}, \quad (4.14)$$

with $v_{f_r} = 0$, $v_{f_\theta} = r\Omega_r = 0.5r\Omega_R$. It yields:

$$v_s = \dot{\epsilon} \cos \theta - \frac{1}{2} \epsilon \Omega_R \sin \theta. \quad (4.15)$$

The structural radial displacement is independent of v_{f_z} as we assume that $\tilde{r}$ is not a function of z and neglect axis tilt and flex.

Non penetration boundary conditions constrain the fluid radial velocity on the rotor surface at $r = R_R$, and on the stator surface at $r = R_S$.

$$\begin{aligned} v_{f_r}(R_R) &= v_s(R_R) = \dot{\epsilon} \cos \theta - \frac{1}{2} \epsilon \Omega_R \sin \theta, \\ v_{f_r}(R_S) &= 0. \end{aligned} \quad (4.16)$$

The flow potential must verify the Laplace equation of Eq. 4.11 and these boundary conditions. Appendix B presents the derivation of the flow potential which leads to:

$$\phi = \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(-\dot{\epsilon} \cos \theta + \frac{1}{2} \epsilon \Omega_R \sin \theta \right). \quad (4.17)$$

The Euler equation of Eq. 4.11 expresses the pressure distribution p as a function of the flow potential ϕ . Appendix C calculates p as a function of ϕ which gives:

$$\begin{aligned} p(R_R) &= -\rho \frac{\partial \phi}{\partial t} + [...], \\ &= \rho \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(\ddot{\epsilon} \cos \theta - \frac{1}{2} \Omega_R \dot{\epsilon} \sin \theta + \omega \dot{\epsilon} \sin \theta + \frac{1}{2} \epsilon \omega \Omega_R \cos \theta \right) + [...]. \end{aligned} \quad (4.18)$$

Integration along the x - and y -axis of the terms included in "[...]" is null, thus there are not detailed.

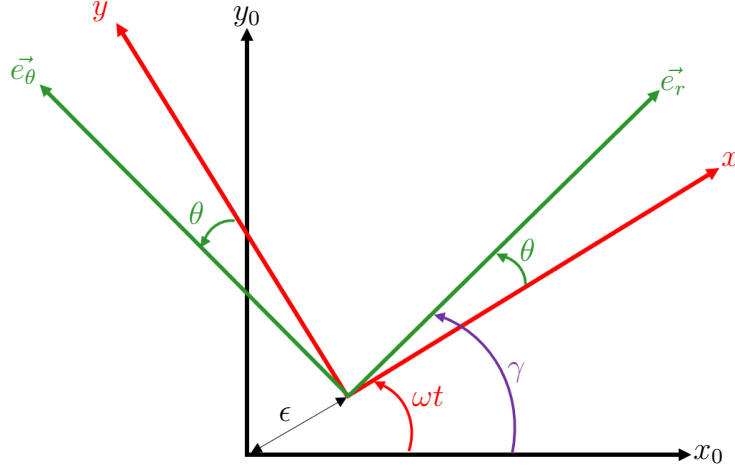


Figure 4.3 Reference frame (x_0, y_0) , whirling frame of reference (x, y) and cylindrical frame of reference attached to the rotor center $(\vec{e}_r, \vec{e}_\theta)$

Integration of p over the rotor surface along the x - and y -axis yields the fluid forces acting on the cylinder due to fluid-structure interactions:

$$\begin{aligned} F_x &= \int_0^{2\pi} p(R_R) R_R \cos \theta d\theta L, \\ F_y &= \int_0^{2\pi} p(R_R) R_R \sin \theta d\theta L. \end{aligned} \quad (4.19)$$

The resulting forces are:

$$\begin{aligned} F_x &= \rho\pi R_R^2 \left(\frac{R_R^2 + R_S^2}{R_R^2 - R_S^2} \right) \left(\ddot{\epsilon} + \frac{1}{2}\epsilon\omega\Omega_R \right) L, \\ F_y &= 0. \end{aligned} \quad (4.20)$$

Actually, projecting the force over the y -axis yields a purely imaginary force component $F_y = -jF_x$, which means it is shifted of 90° , *i.e.*, it is along the x -axis. Thus there is no tangential force, but only of radial force of magnitude F_x .

As developed in the methodology, the rotor dynamic coefficients can be calculated from interpolation of the forces against the whirl frequency ω . Because $\vec{\epsilon} = \epsilon \cos(\omega t) \vec{e}_x + \epsilon \sin(\omega t) \vec{e}_y$, F_x/ϵ can be written as a polynomial of ω :

$$\frac{F_x}{\epsilon} = \rho\pi R_R^2 \left(\frac{R_R^2 + R_S^2}{R_R^2 - R_S^2} \right) (-\omega^2 + \frac{1}{2}\omega\Omega_R) L. \quad (4.21)$$

Therefore, for a given rotor frequency Ω_R and eccentricity ϵ , the radial force is calculated for every whirl frequency ω . Furthermore, Eq. 4.21 expresses the ratio F_x/ϵ as a polynomial of

ω , similarly to Eq. 4.8.

4.4 CFD model

Analytical models are easily applied to plain annular seal, as represented in Fig. 4.2.a. However, the geometry of Francis turbine radial confinements is more complex, and resolving the flow dynamics occurring in these configurations thus requires adapted tools such as numerical models.

Therefore, a CFD model of the rotor-seal system is set up in ANSYS CFX 2022 R1. Based on the work by Staubli and Bissig (2001), the axisymmetric geometry is coupled with a circular rotor orbit about the concentric position, hence steady-state calculations run in the whirling frame of reference (x, y) . More precisely, the mesh is initially deformed with an off-centred rotor, and the fluid domain is put in rotation at the whirl frequency ω . The following no-slip conditions are applied in the relative frame of reference attached to the rotor center motion:

$$\begin{aligned}\Omega_{rotor} &= \Omega_R - \omega \quad \text{on the rotor surface,} \\ \Omega_{stator} &= -\omega \quad \text{on the stator surface.}\end{aligned}\tag{4.22}$$

Running steady-state simulations in the rotating reference frame allows doing without a moving mesh during simulations, which significantly reduces the numerical cost. Moreover this method is applicable to any seal geometry.

Regarding the inlet and outlet boundary conditions, an axial mass flow rate $\dot{m}$ is imposed at the top opening, and the static pressure is set to 0 at the bottom opening.

A shear-stress transport (SST) model of turbulence closes the turbulence equations.

4.5 Vibroacoustic FEM model

A second numerical model, which is an acoustic-structural FEM model, is implemented for the simple annular seal of Fig. 4.2.a, in ANSYS Mechanical APDL 2022 R1. As shown in Fig. 4.4, the infinitely rigid cylindrical rotor is modeled with a centered point mass m_s linked with RBE2 connections to solid elements SOLID186 of an annular surface of outer radius $R_R = 2012$ mm and of width 10 mm, which interact with the acoustic FLUID220 elements. As did Blais (2021), ANSYS Mechanical APDL reads the fluid outer surface as a rigid wall if there is no specific condition imposed. Hence the stator wall is automatically simulated at the outer boundary of the fluid domain.

The rotor point mass is attached by perpendicular springs to the stator wall to simulate the rotor structural rigidity. This approach allows for the simulation of fluid structure interac-

tions, as the two springs constitute elastic elements which enable the motion of the rotor submitted to the fluid forces. In order to maintain the conditioning of the stiffness matrix, the disc should not be too rigid with respect to the spring. Therefore the point mass is set to $M_s = 200$ kg and the structural stiffness is $k_s = 4000$ N/m.

In terms of degrees of freedom, the bending moment of the rotor system is not considered, thus only the rotor translation in the (x, y) plane is of interest. Therefore, translation against the z -axis and rotations around the x - and y -axis are blocked.

The fluid domain is modeled by acoustic elements which solve the Helmholtz equation, hence the fluid is necessarily inviscid and irrotational, as emphasized by Blais (2021). The absence of viscosity implies that the fluid cannot be set in rotation by the rotor. Therefore, to account for the rotor's rotation, the fluid domain is treated as a solid body rotating as a whole, disregarding boundary layers and velocity profiles. Meanwhile, the rotor itself is considered fixed without any velocity. More precisely, simulations are run in the rotor frame of reference, where the fluid rotates relative to the rotor, at the angular velocity $\Omega_{fluid/rotor} = \Omega_F - \Omega_R = (1/2 - 1)\Omega_R = -1/2\Omega_R$. This fluid circular motion is applied through a mean flow effect, supported in the version 2022 R1 of ANSYS Mechanical APDL, which consists in applying a mean flow velocity in all fluid nodes. It corresponds to an acoustic body load applied with the VMEN option in the BF command. ANSYS Mechanical solves the convective wave equation using the defined mean flow velocity.

Under a displacement of circular motion $(x_R, y_R) = (\epsilon \cos(\omega t), \epsilon \sin(\omega t))$ imposed to the rotor point mass, harmonic analyses are run for different whirl frequencies of excitation ω . The integration of the pressure gradient along the x - and y -axes yields the radial and tangential forces.

CHAPTER 5 RESULTS AND DISCUSSION

In this section, we present the verification of the numerical models by comparing them with the developed analytical model on the plain annular seal geometry. Then we describe the flow dynamics in the seal, and variations of this flow from one geometry to another, in function of the direction of the flow, and when additional seals are added to the geometry. Finally, we show the variation of radial and tangential forces in function of geometrical and input parameters. We present the deduced influence of those parameters on the stability of the rotor.

5.1 Studies of convergence

The vibroacoustic model uses the FEM, while the CFD model employs the Finite Volume Method (FVM). Both of these methods rely on the use of a mesh. Increasing the level of mesh refinement leads to more accurate results; however, a finer mesh also results in longer computation times. Prior to analyzing any numerical data, it is necessary to sufficiently refine the mesh to ensure mesh independence, meaning that the results should not significantly vary upon further mesh refinement.

To determine the point of mesh independence, a convergence study is conducted. Simulations are performed on progressively refined meshes. Fig. 5.1 shows the logarithmic representation of the absolute error against the rotor radial force F_x , using the plain annular seal geometry on ANSYS APDL 2022 R1, as presented in section 4.5. Different mesh element size ratios, denoted as $h = \Delta x / \Delta x_{ref}$, are considered. Here, Δx_{ref} represents the element size of the mesh of reference, which is the most refined mesh available. In this figure, the absolute error of F_x decreases with the mesh aspect ratio h . A constant slope represents an order of convergence of approximately 5 N, for $h \leq 4$, hence the numerical precision is significant. More precisely, the absolute error of F_x remains below 1 N for $h \leq 2$, while F_x is of the order of 1000 N. Since calculations are relatively fast for $h = 2$, and the mesh is refined enough to achieve mesh independence, the subsequent results are obtained using the mesh corresponding to $h = 2$.

A similar convergence analysis is conducted for the CFD model. Fig. 5.2 depicts the convergence study for the mesh associated with the geometry of Fig. 4.2.b. The absolute error of the radial force F_x and the tangential force F_y is calculated using the same methodology as described above for the vibroacoustic model. The finest mesh ($h = 1$), which forces are used as a reference, corresponds to a refinement of 60 elements in the radial direction within the 1

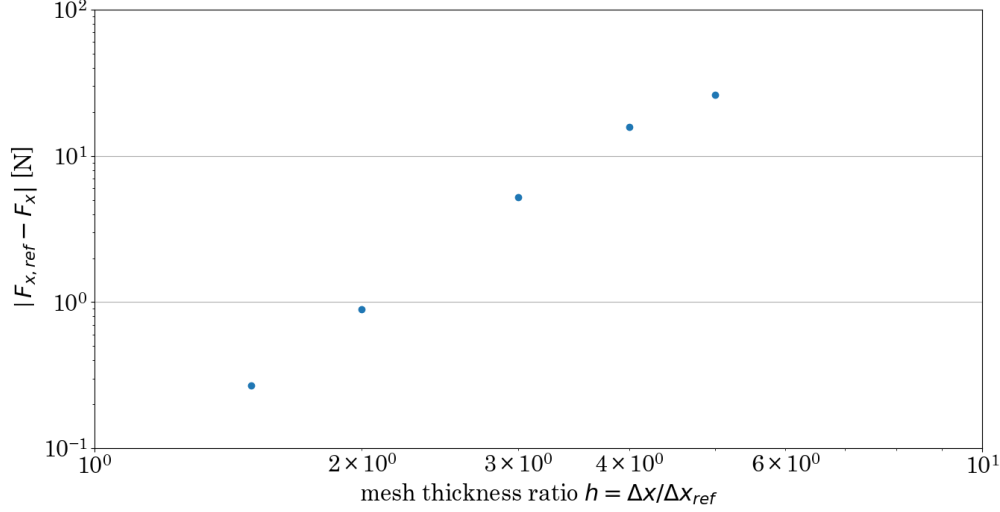


Figure 5.1 Absolute error of the vibroacoustic model radial force $|F_{x_{ref}} - F_x|$ against the mesh element size ratio h

mm-wide seal. For $h \leq 4$, the absolute error of F_x and F_y is approximately divided by 10 when the ratio h is divided by 2, which again characterises a high precision of the mesh. For subsequent studies, the mesh corresponding to $h = 2$ is utilized, in order to have the most precise results, as the time of calculation remained reasonable.

5.2 Description of the flow dynamics in the seal

The flow is characterised by its velocity and the pressure field, which are related by the Bernoulli equation. The fluid reaction forces acting on the rotor are a result of the pressure distribution over its surface.

5.2.1 Comparison of the flow from the two numerical models

Flow in the vibroacoustic model

In the vibroacoustic model, the velocity profiles of the flow are disregarded. The imposed displacement is $\{\epsilon\} = \epsilon_x + i\epsilon_y = \epsilon \cos(\omega t) + i\epsilon \sin(\omega t)$, with $\epsilon = 0.6$ mm, and no axial flow is applied at first. Fig. 5.3 shows the real and imaginary responses of the rotor at $\omega \approx 6.3$ rad/s, corresponding to an excitation frequency $f = 1$ Hz. The real and imaginary components represent the forced displacements along the x -axis ($\epsilon_x = \epsilon \cos(\omega t)$), and along the y -axis ($\epsilon_y = \epsilon \sin(\omega t)$), respectively. In Fig. 5.3.a, the pressure gradient is exclusively along the x -axis, while the pressure gradient of the imaginary response is along the y -axis. The colorbars

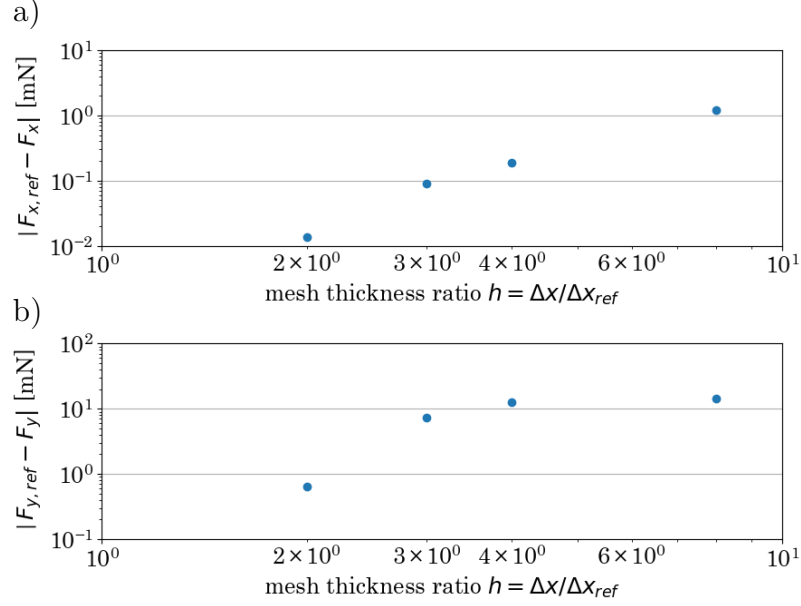


Figure 5.2 Absolute error against the mesh element size ratio h of (a) the radial force $|F_{x_{ref}} - F_x|$, and (b) the tangential force $|F_{y_{ref}} - F_y|$

indicate the same pressure gradient for both graphs. According to these observations, the pressure gradient is in phase with the rotor displacement. This behavior persists across all frequencies ω . Furthermore, these results are not affected by the introduction of an axial mean flow in the acoustic domain.

The absence of a phase shift between the pressure gradient and structural displacement was expected due the absence of inertial effects from the axial flow, and the inability of the acoustic fluid elements to account for dissipative effects and shear stress. Consequently, this absence of phase shift results in a null tangential force F_y .

Other simulations were performed with the application of an axial flow, however it did not alter the results.

Flow in the CFD model

CFD enables comprehensive simulation of fluid dynamics, albeit at the expense of high computational costs. To compare with the vibroacoustic flow, several CFD simulations were performed on the plain annular seal, with an axial mass flow rate of $\dot{m} = 5$ kg/s. The axial mass flow rate is defined as $\dot{m} = \rho S_z v_{f_{z,inlet}}$, with ρ the fluid density, $S_z = 2\pi R_R C_0$ the cross-section surface in the plane normal to the z -axis, $C_0 = R_S - R_R$ the mean radial clearance, and $v_{f_{z,inlet}}$ the inlet axial velocity. For $\dot{m} = 5$ kg/s, $v_{f_{z,inlet}} \approx 0.25$ m/s. The input

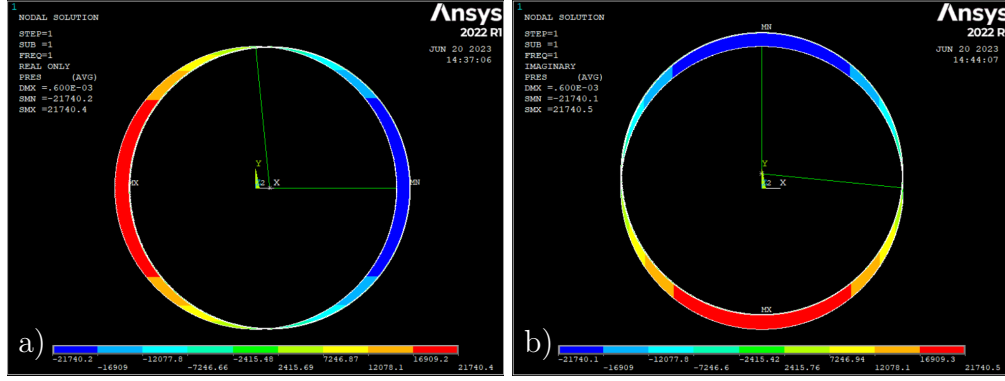


Figure 5.3 Pressure field on the plain annular seal from ANSYS Mechanical APDL 2022. a) pressure field for the solution real part ; b) pressure field for the solution imaginary part

rotor angular frequency is $\Omega_R = 5$ rad/s, which fixes the average circumferential velocity $\bar{v}_\theta = \bar{R}\Omega_R \approx 10$ rad/s, with a mean radius $\bar{R} = 1/2(R_R + R_S) \approx 2.01$ m.

Several variables are used to describe the output flow. The average axial velocity in the middle cross-section ($z = 0$) of the seal is also defined as:

$$\bar{v}_{f_z}(z = 0) = \int_0^{2\pi} \int_{R_R}^{R_S} v_{f_z} r \cdot dr \cdot d\theta(z = 0). \quad (5.1)$$

The Rossby number $R_o = \bar{v}_{f_z} / \bar{R}\Omega_R$ represents the ratio between the average axial and circumferential velocities in the seal. Finally, the rotational, axial and whirl Reynolds numbers are defined as:

$$Re_{\Omega_R} = \Omega_R R_R C_0 / \nu; \quad Re_z = v_{f_z} C_0 / \nu; \quad Re_\omega = \omega R_R C_0 / \nu \quad (5.2)$$

As outlined in chapter 2, the rotor angular frequency Ω_R is usually between 0 and 10 Hz, and the observed whirl frequency ω approximates $0.5\Omega_R$, resulting in a range of 0 to 5 Hz. For two whirl frequencies $\omega = 10$ rad/s ≈ 1.6 Hz and $\omega = 20$ rad/s ≈ 3.2 Hz within this range, the above different parameters are calculated and summarized in Table 5.1. These two frequencies are with the range of whirl frequencies. In the two configurations, the Rossby number is equal to $R_o \approx 0.02$, thus the circumferential flow is dominant in the seal for the considered configuration. The different Reynolds numbers also testify of a turbulent circumferential flow.

The flow in the geometry of Fig. 4.2.a is presented in Fig. 5.4, for the two whirl frequencies $\omega = 10$ rad/s and $\omega = 20$ rad/s. Fig. 5.4.a-c show discrepancies of the flow in the XY -plane when ω varies: at $\omega = 10$ rad/s the velocities are maximal close to the rotor, and null near

Table 5.1 Variation of the rotordynamic coefficients with $\dot{m}$, evaluated with the CFD model

ω (rad/s)	$v_{f_{z,inlet}}$ (m/s)	$\bar{v}_{f_\theta}$ (m/s)	$\bar{v}_{f_z}(z=0)$	R_o	Re_{Ω_R}	Re_z	Re_ω
10	0.247	10.064	0.22	0.022	1.61×10^4	355	3.22×10^4
20	0.247	10.064	0.23	0.023	1.61×10^4	355	6.44×10^4

the stator wall, which correlates with the no slip boundary conditions at the rotor and stator surfaces. However, although the same conditions are imposed at $\omega = 20$ rad/s, the resulting fluid circumferential velocity $v_{f_\theta} = r\omega$ is maximal at the rotor surface, while it was expected to be null because of the no slip boundary conditions. This unexpected result can be due to the mesh resolution: for high rates ω , the boundary layer is so thin that it fits entirely inside a single cell. The axial flow shown in Fig. 5.4.b-d does not vary significantly with ω . It is present at the inlet but it is rapidly dominated by the circumferential flow through the seal, which induces an axial flow almost null. However this phenomenon may undoubtedly vary if the mass flow rate $\dot{m}$ is increased at the inlet.

Fig. 5.5.a presents the pressure field p for different XY planes of the seal, at different axial levels $z \in [0; l/4; l/2; 3l/4; l]$, where l is the axial length of the seal. The angle $\theta = 0^\circ$ corresponds to the smallest seal gap when the rotor is eccentric, and $\theta = 180^\circ$ represents the largest gap. The pressure field exhibits a minimal value at $\theta \approx 135^\circ$ and is constant at its maximal value for $200^\circ \leq \theta \leq 360^\circ$. The pressure distribution over the rotor does not vary with z , thus the pressure field at $z = l/2$ reflects the pressure distribution across the seal. Since the displacement is only along the x -axis in the CFD simulation, the pressure gradient is out of phase with the displacement, in opposition to vibroacoustic results. Hence a tangential force F_y is accounted for in CFD.

Fig. 5.5.b illustrates the pressure field at $z = l/2$, for various frequencies ω . As ω changes, the minimum pressure still occurs at $\theta \approx 135^\circ$. However, the pressure gradient increases significantly with increasing ω . Consequently the amplitude of the reaction forces grows with ω .

Finally, CFD results for both configurations showed that the pressure forces acting on the rotor were over 1000 times greater than the viscous forces, justifying the assumption that viscous forces should be neglected, claimed in chapter 4.

Force comparison from the numerical and analytical models for different frequencies ω

The flow dynamics at a specified rate ω induce radial and tangential forces F_x and F_y on the rotor. Referring to Eq. 4.8 of the methodology, the rotor seal dynamic coefficients are

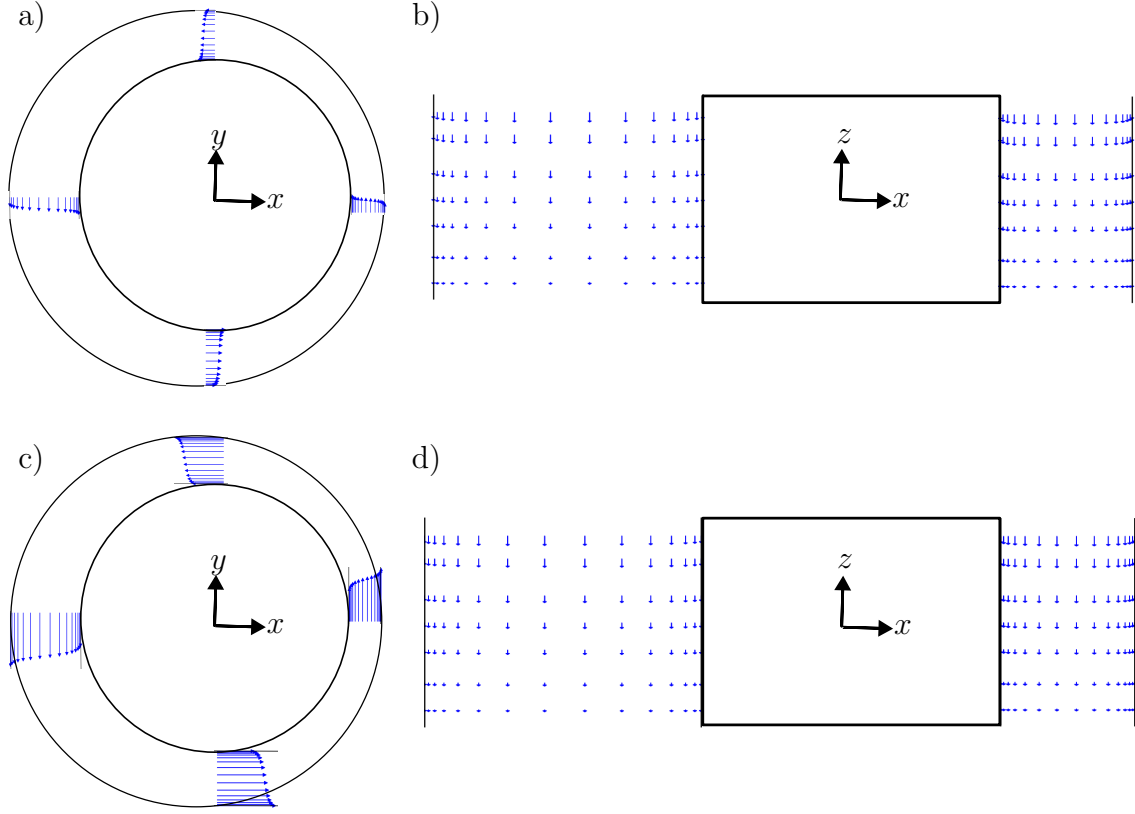


Figure 5.4 Schematic of the velocity profiles in the plain annular seal at $\Omega_R = 5$ rad/s, $\dot{m} = 5$ kg/s: (a)(b) at $\omega = 10$ rad/s, in the XY plane at $z = 0$, and in the axial ZX plane at $y = 0$; (c)(d) at $\omega = 5$ rad/s, in the XY plane at $z = 0$, and in the axial ZX plane at $y = 0$

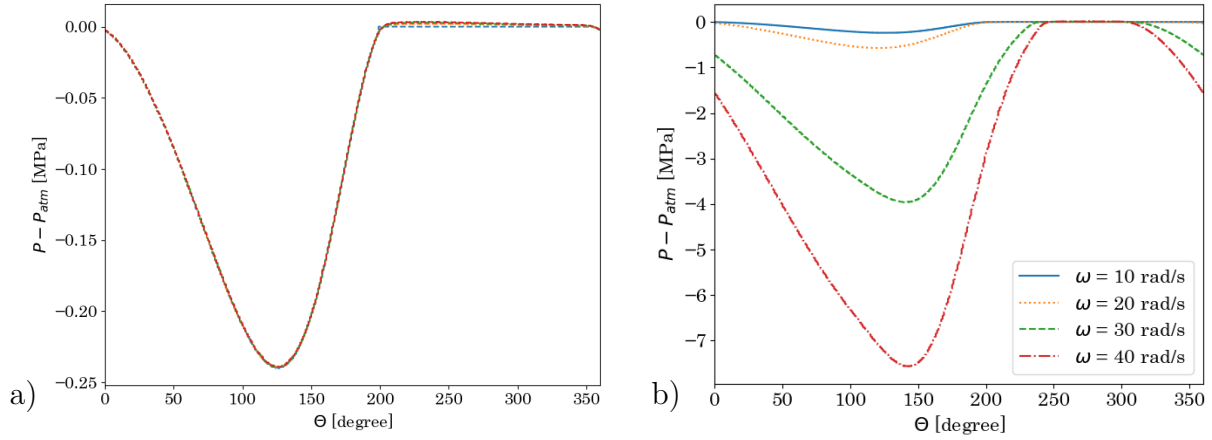


Figure 5.5 Pressure field on the rotor surface of the plain annular seal for a mass flow rate $\dot{m} = 5$ kg/s and runner rotating speed $\Omega_R = 5$ rad/s: a) at $\omega = 10$ rad/s at 5 different axial positions z in the seal; b) for different frequencies ω at $z = 0$ in the middle XY plane of the seal

deduced from the interpolation of the ratios F_x/ϵ and F_y/ϵ as functions of the whirl frequency ω . Hence variations of the forces as a function of ω from the different models are compared. Here the mass flow rate applied in CFD is set to $\dot{m} = 0$, because the analytical and vibroacoustic do not consider the axial flow.

To verify both numerical models, Fig. 5.6 shows the comparison of the radial force over the eccentricity F_x/ϵ from the analytical with the results from the numerical methods for the plain annular seal of Fig. 4.2.a. For all models F_x/ϵ follows the same quadratic trend, which is related to the added mass, according to Eq. 4.8. However, the CFD forces start to deviate from each other for approximately $\omega \geq 30$ rad/s. The velocity profile which are considered in CFD can explain the lower prediction of the radial force with this method. Effectively, for high whirl frequencies ω , the flow becomes more turbulent with an increase of fluctuations in the flow, which can impact the resulting pressure distribution. Flow fluctuations are not accounted for in the analytical and vibroacoustic models. Overall, the similar tendency of F_x/ϵ verifies the numerical models. The correlation between the CFD model and the other models was expected, because the flow in the CFD simulation of the plain annular seal did not show any disturbance, as recirculation, or separation.

Both analytical and vibroacoustic models consider an irrotationnal flow thus the circumferential shear stress is not included. Moreover, it was underlined in section 4.3 and section 5.2.1 that the analytical and vibroacoustic models do not include the effect of axial flow on the rotor. Therefore, the variation of pressure obtained from the vibroacoustic model is in phase

with the displacement, which generates a purely radial reaction force, as does the analytical model. As a consequence, only the CFD model evaluates the tangential force F_y . Since the evaluation of direct damping C and cross-coupled stiffness k relies on F_y , as expressed in Eq. 4.8, only the CFD model can determine all the dynamic coefficients of the rotor seal. As a consequence, all the following studies use CFD.

5.2.2 Flow inside the radial gaps

Although the study of the flow in plain annular seals without upstream and downstream flow domains helps to understand the dynamics in radial seals, it is not representative of real turbomachine radial gaps. These clearances include hydrodynamic seals coupled with larger side chambers. This subsection emphasizes to what extent geometrical simplifications of the radial gaps alter the flow, and the resulting forces.

The geometry of Nishimura et al. (2018) presented in Fig. 4.2.b is comprised of a side chamber, and also includes cavities upstream and downstream of the radial gaps. The seal is upstream of the side chamber as the axial flow rate $\dot{m}$ is applied at the upper entrance, as shown in Fig. 4.2.b.

Fig. 5.7 displays the flow through the seal and side chamber for this configuration, at $\omega = 5$ rad/s, at different heights z , with $z = 0$ being at the upper opening of the seal. Because of no-slip boundary conditions, the circumferential flow through the seal and side chamber is faster close to the rotor, and it has no velocity close to the stator, at all heights z . At the inlet of the seal in Fig. 5.7.a ($z = -0.1$ mm), the flow is accelerated in the smaller clearance region, like at the outlet of the seal, at $z = -5$ mm (Fig. 5.7.b). At this height, the flow varies almost linearly with the radius r . In Fig. 5.7.c, the velocity profiles in the XY plane in the side chamber at $z = -25$ mm does not separate. Fig. 5.7.d shows the axial flow in the radial gaps. The axial flow is higher in the seal than in the side chamber, this is why the flow in the seal is plotted in an extra rectangle for a better representation. Through the seal, a small recirculation appears close to the rotor wall, at the seal inlet. In the smaller region of the seal, the recirculation bubble is negligible, as it is 0.001 mm large. The recirculation is more important for larger regions of the seal: close to the rotor, the bubble is approximately 0.1 mm large at its largest width. Moreover, the larger clearance of the seal also exhibits recirculation near the stator wall, with a bubble width of 0.002 mm. Strong recirculation vortices also occur within the side chamber, reaching a width of 2 mm.

Overall, the flow is stronger within the seal than in the side chamber; the circumferential flow is accelerated in the smaller clearance region, and decelerated in the larger clearance;

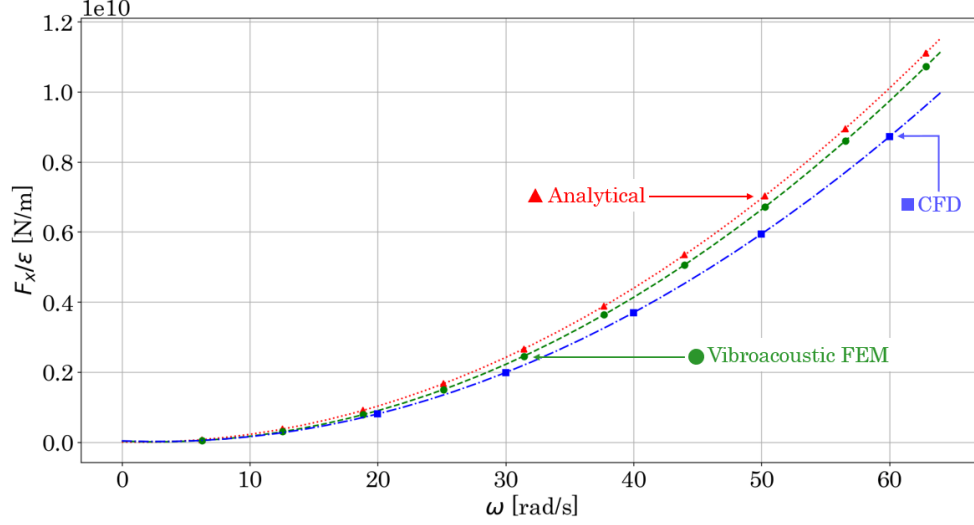


Figure 5.6 F_x/ϵ from the three models, on the plain annular seal of Fig. 4.2, at $\Omega_R = 5$ rad/s, $\dot{m} = 0$ kg/s

and the axial flow recirculates in the radial gaps, with dominant recirculation vortices in the larger radial clearance. In the seal, where the flow is stronger than in the side chamber, these vortices reduce the effective section of the seal in its larger region. According to the Bernoulli equation, the actual velocity in the seal larger clearance is higher than velocity without recirculation. Thus the flow recirculation decreases the pressure in the larger clearance. Although the presence of recirculation vortices is consistent, particularly in relation to abrupt cross-sectional changes, it should be borne in mind that these recirculation zones depend on the turbulence model chosen, in this case the SST model. A complete resolution of the turbulence using the DNS (Direct Numerical Simulation) method, for example, would eliminate this dependence on the turbulence model, but such simulations have an exorbitant computational cost, not least because of the extreme mesh refinement required (Perron et al., 2013).

In Fig. 5.8 the corresponding pressure distribution is plotted at several heights z in the seal and the side chamber. Fig. 5.8.a represents the axial positions where the pressure distribution is plotted, as well as the θ -positions in the deformed seal. Pressure distributions of Fig. 5.8.b show the largest pressure gradients at z_1 , z_2 and z_3 close to the seal entrance. The heights z_1 and z_2 are located within the recirculation bubbles, while the height z_3 is located at the end of these vortices. Thus, as explained previously, the larger underpressure at z_1 and z_2 is expected, due the effective section of the seal is reduced by the recirculation vortices. At these three levels, the underpressure appears at $\theta \approx 160^\circ$, and an overpressure at $\theta = 0^\circ$. When the flow goes through the seal at z_4 and z_5 , Δp decreases, and the underpressure shifts

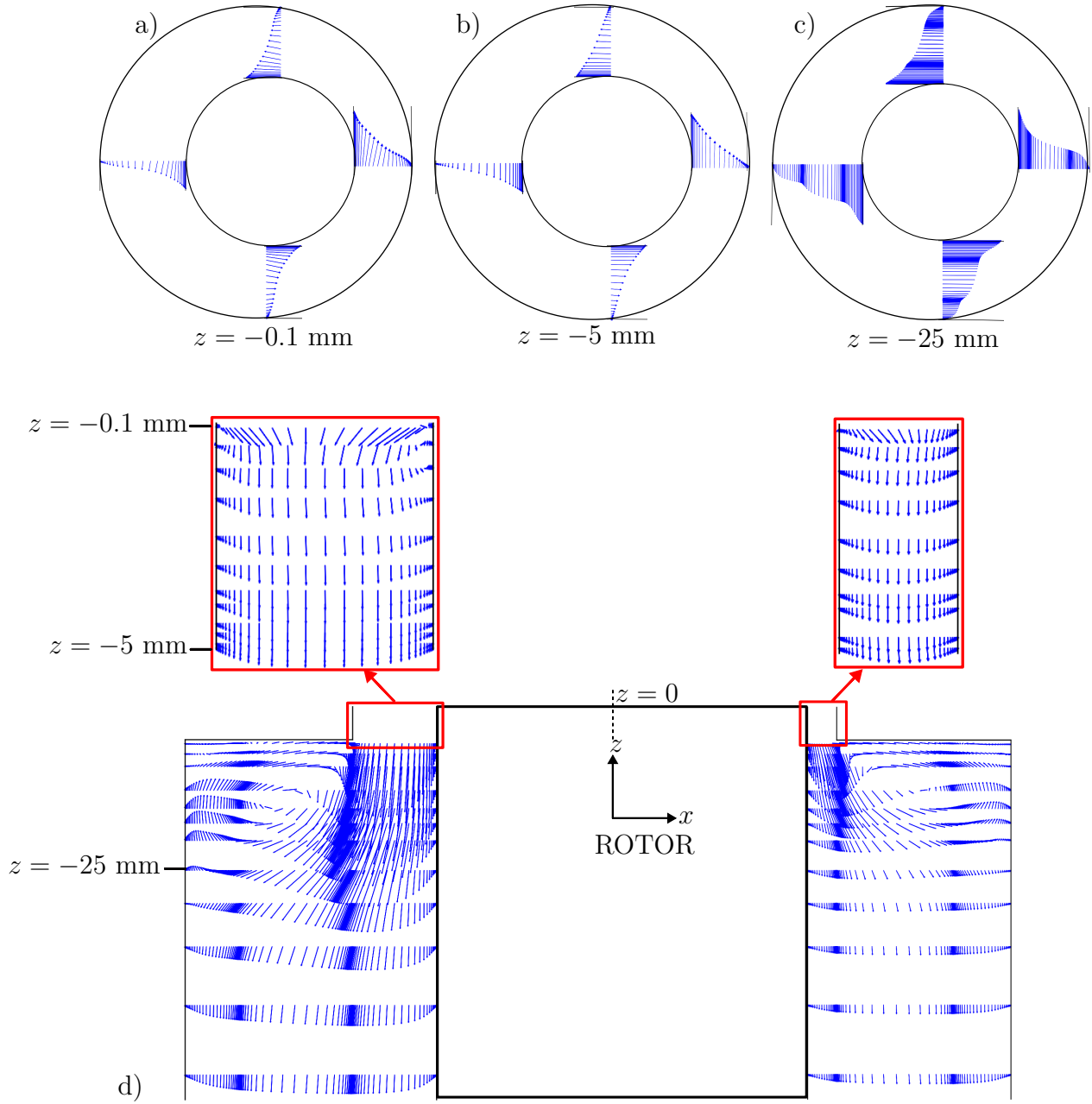


Figure 5.7 Velocity profiles for a downward flow, upstream seal in Fig. 4.2: (a) in the XY plane of the seal at $z = -0.1 \text{ mm}$; (b) in the XY plane of the seal at $z = -5 \text{ mm}$; (c) in the XY plane of the side chamber at $z = -25 \text{ mm}$; (d) in the axial ZX plane of the seal and side chamber, at $\theta = 0^\circ$. Note that proportions of the geometry are not to scale. The radial gap has been stretched for the visualization.

to $\theta \approx 130^\circ$, while the overpressure is still at $\theta = 0^\circ$. Through the side chamber at z_6 and z_7 , p draws a sinusoid with an overpressure at $\theta \approx 220^\circ$, and a underpressure at 0° .

Fig. 5.9 shows the dimensionless forces by unit height $\bar{F}_{z_i} = \Delta F / \Delta z$ obtained from the pressure distributions depicted in Fig. 5.8, integrated over θ . The total force $\bar{F}_{tot}$ is plotted in black. For a circular motion at frequency ω around the z -axis, the instantaneous rotor speed $\dot{\epsilon}$ occurs solely along the y -axis, and the instantaneous displacement ϵ along the x -axis. By projecting the forces onto these two axes, they manifest as stiffness in the x -direction and damping in the y -direction. Almost all forces by unit height $\bar{F}_{z_i}$ exhibit negative damping, as their y -component is positive, in the direction of instantaneous velocity $\dot{\epsilon}$. The forces at the seal inlet ($\bar{F}_{z_1}$, $\bar{F}_{z_2}$, $\bar{F}_{z_3}$) barely exhibit damping as their y -component is approximately null. Furthermore, most forces in the seal, as well as in the side chambers, display positive stiffness, except in the side chamber at height z_7 , where the x -terms is positive and tends to increase the eccentricity ϵ . The total reaction force $\bar{F}_{tot}$ exhibits negative damping and positive stiffness, primarily due to dominant forces within the seal. Finally, the recirculation through the seal has a significant effect on the reaction forces. The forces by unit height $\bar{F}_{z_1}$, $\bar{F}_{z_2}$ arise through the recirculation vortices at the inlet of the seal, while $\bar{F}_{z_3}$ and $\bar{F}_{z_4}$ arise downstream to these recirculation vortices. Both forces that appear through the vortices are significantly higher than the others, providing a larger positive stiffness. Therefore, the presence of recirculation vortices through the radial seals helps to improve the stability of the rotor.

The rotor-seal dynamic coefficients are found by interpolating F_x/ϵ and F_y/ϵ as polynomials of ω , referring to Eq. 4.8. Hence it is necessary to calculate F_x and F_y for different whirl frequencies ω . Fig. 5.10 presents the variation of F_x and F_y as a function of ω . Numerical and experimental results by Nishimura et al. (2018), and the CFD results are plotted. The three data show the same evolution of the forces F_x and F_y : they both increase with ω , but F_x seems to be quadratic, while F_y is linear with ω . This correlates with the analytical expression of F_x and F_y of Eq. 4.8. Determination of the dynamic coefficients is performed in section 5.3.

5.2.3 Flow inside the radial seal downstream to the side chamber

If the flow enters the seal before or after the larger side chamber, the dynamics through the seal can change. Subsection 5.2.2 considered the flow in a seal located upstream of the side chamber, as reminded in Fig. 5.11.a. For a downstream seal represented in Fig. 5.11.b, the flow represented by the red arrows enters the side chamber before the seal. This was simulated by interchanging the inlet and outlet boundary conditions.

Fig. 5.12.a presents the velocity profiles in an XY cross-plane of the side chamber, at $z = -25$

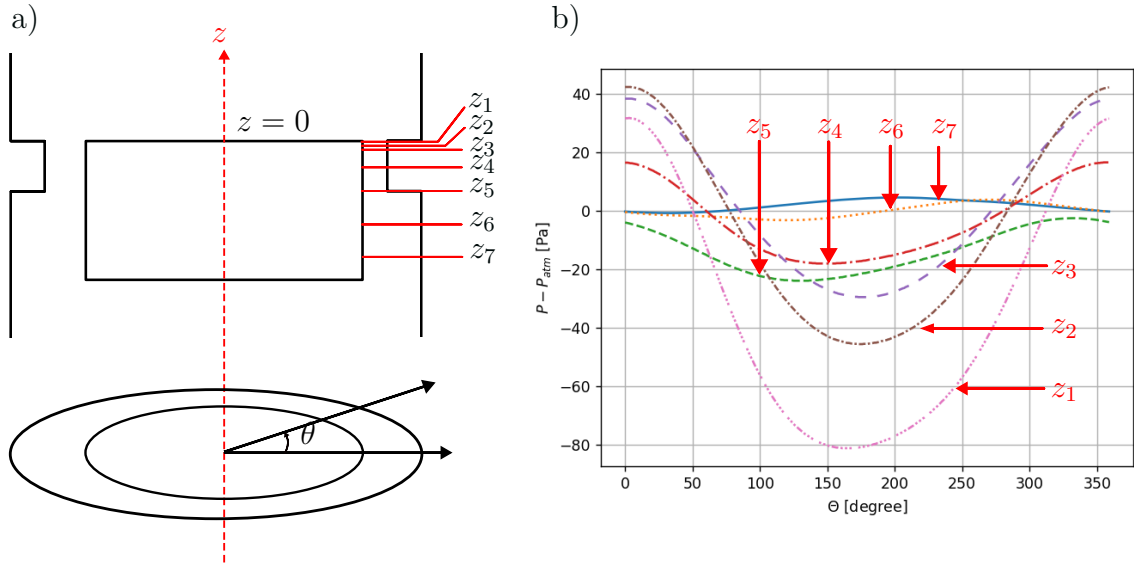


Figure 5.8 Pressure distribution in the upstream seal and side chamber of Fig. 4.2.b at $\omega = 5$ rad/s, $\Omega_R = 5$ rad/s, $\dot{m} = 0.25$ kg/s at $z_1 = -0.1$ mm; $z_2 = -0.6$ mm; $z_3 = -1$ mm; $z_4 = -3$ mm; $z_5 = -5$ mm; $z_6 = -25$ mm; $z_7 = -45$ mm

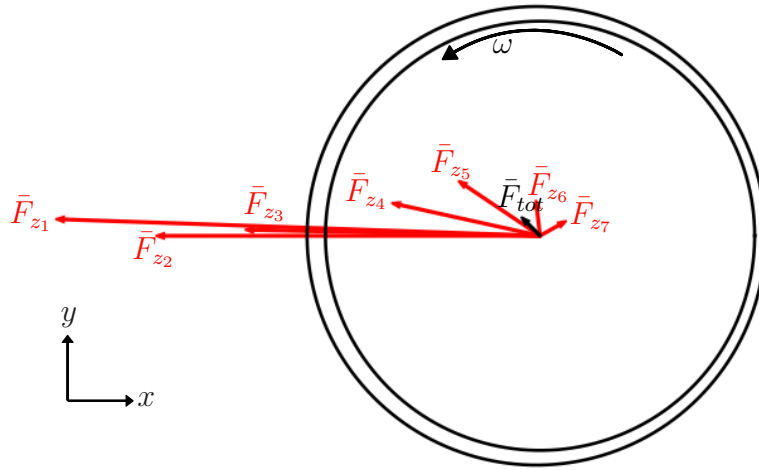


Figure 5.9 Dimensionless reaction linear forces $\bar{F}/dz$ in red, and dimensionless total force $\bar{F}_{tot}$ in black, for the upstream seal configuration of Fig. 4.2.b, at $\omega = 5$ rad/s

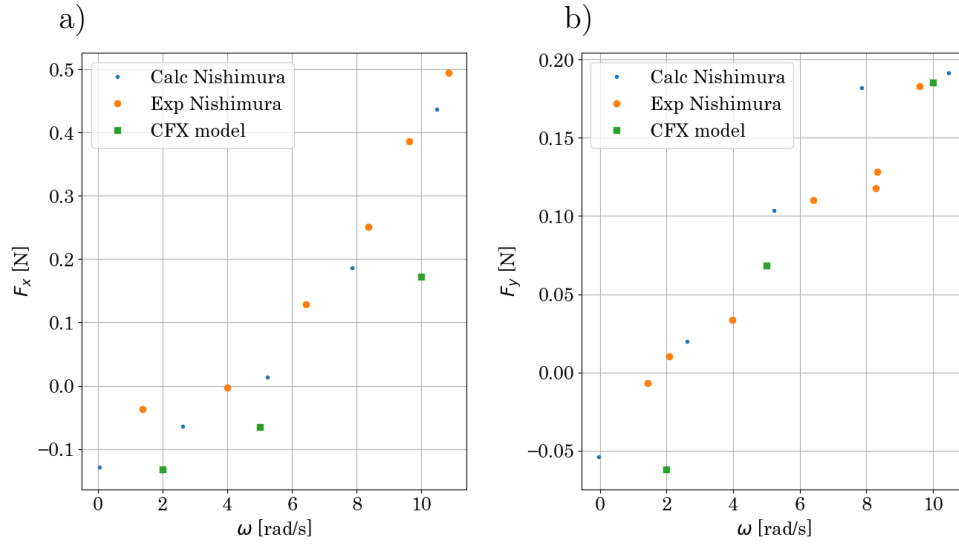


Figure 5.10 (a) Variation of the reaction force components for the upstream seal configuration, in function of the whirl frequency ω , numerically and experimentally by Nishimura et al. (2018), and from the CFD model: (a) variation of the radial force F_x ; (b) variation of the tangential force F_y

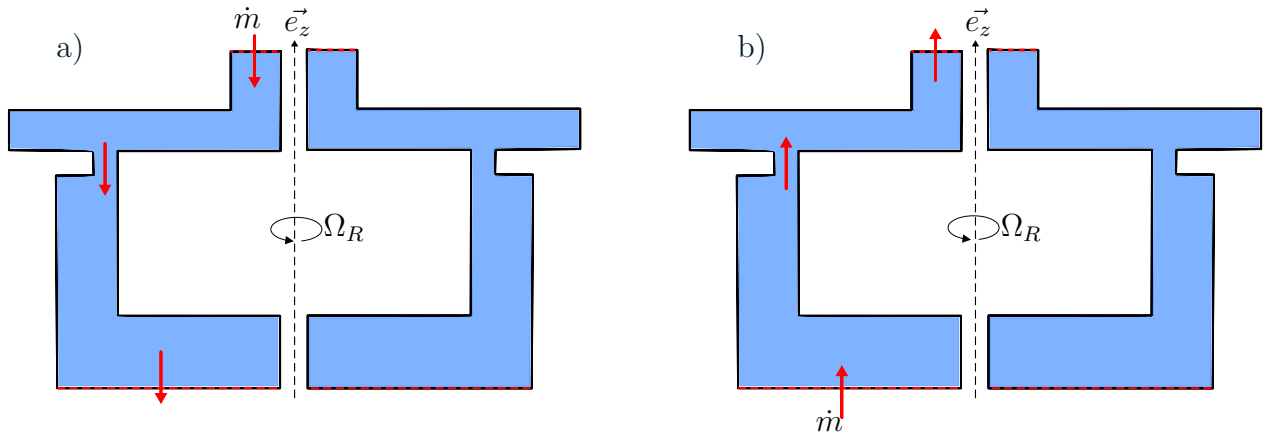


Figure 5.11 (a) Downward flow through the seal upstream of the side chamber (b) Upward flow through the seal downstream of the side chamber

mm. The flow behaves similarly to the upstream seal presented in Fig. 5.7. At the seal inlet at $z = -5$ mm in Fig. 5.12.b, separation occurs close to the stator. The smaller clearance region presents the most significant separation, as well as the highest circumferential flow. According to Fig. 5.12.c, near the seal outlet at $z = -0.1$ mm, there is almost no separation, however the smaller clearance still presents the highest fluid circumferential velocities. The flow in the radial gaps also varies in the ZX axial plane, as shown in figure Fig. 5.12.d. As in the case of a seal upstream of the side chamber, the axial flow is higher within the seal than in the side chamber. To better represent this, the flow in the seal is depicted in a separate rectangle. The side chamber flow exhibits small recirculation vortices at the chamber inlet, close to the rotor. This is due to the right angle of the rotor. These vortices are more pronounced in the larger side chamber clearance, which also corresponds to the region where the side chamber flow is the highest. Through the seal, recirculation vortices occur close to the stator walls at the seal entrance, particularly in regions with larger clearances. The recirculation vortices though the smaller clearance of the seal are close to the stator and are approximately 0.02 mm large, while in the larger clearance of the seal, large recirculation vortices occur near the stator wall, having an approximate width of 0.2 mm. This recirculation is explained by the right angle of the stator surface, between the side chamber and the seal.

Overall, when the seal is positioned downstream of the side chamber, the circumferential flow experiences compression within the smaller gap as well. However, the flow in the ZX plane within the side chamber is more uniform compared to the case of the seal upstream of the side chamber. As it was observed in subsection 5.2.2, strong recirculation vortices in the side chamber and radial seal are observed in this scenario, especially in the larger clearance regions. When the seal is downstream of the side chamber, the recirculation vortices mostly occur through the seal instead of the chamber region. They are also larger, up to 0.2 mm wide, compared with 0.1 mm wide vortices when the seal is upstream of the side chamber. Once again, it should be borne in mind that these recirculation zones depend on the turbulence model chosen.

Fig. 5.13 displays the pressure distribution at $\omega = 5$ rad/s at several axial levels z_i in the seal and the side chamber for the downstream seal configuration. Fig. 5.13.b shows that the largest pressure gradients are close to the bottom of the seal as shown by the curves at z_4 and z_5 . These two heights are located within the recirculation vortices. This coincides with the reduction of the effective section by the vortices, which induce an increase of the velocity, thus the underpressure is more important at the level of these vortices, referring to the Bernoulli equation. Nevertheless, p is higher in the side chamber than within the seal. The pressure distribution is closer to a sinusoid at all axial levels, than for an upstream seal configuration. But the maximum pressure is now for $\theta \approx 90^\circ$ in the side chamber, while the

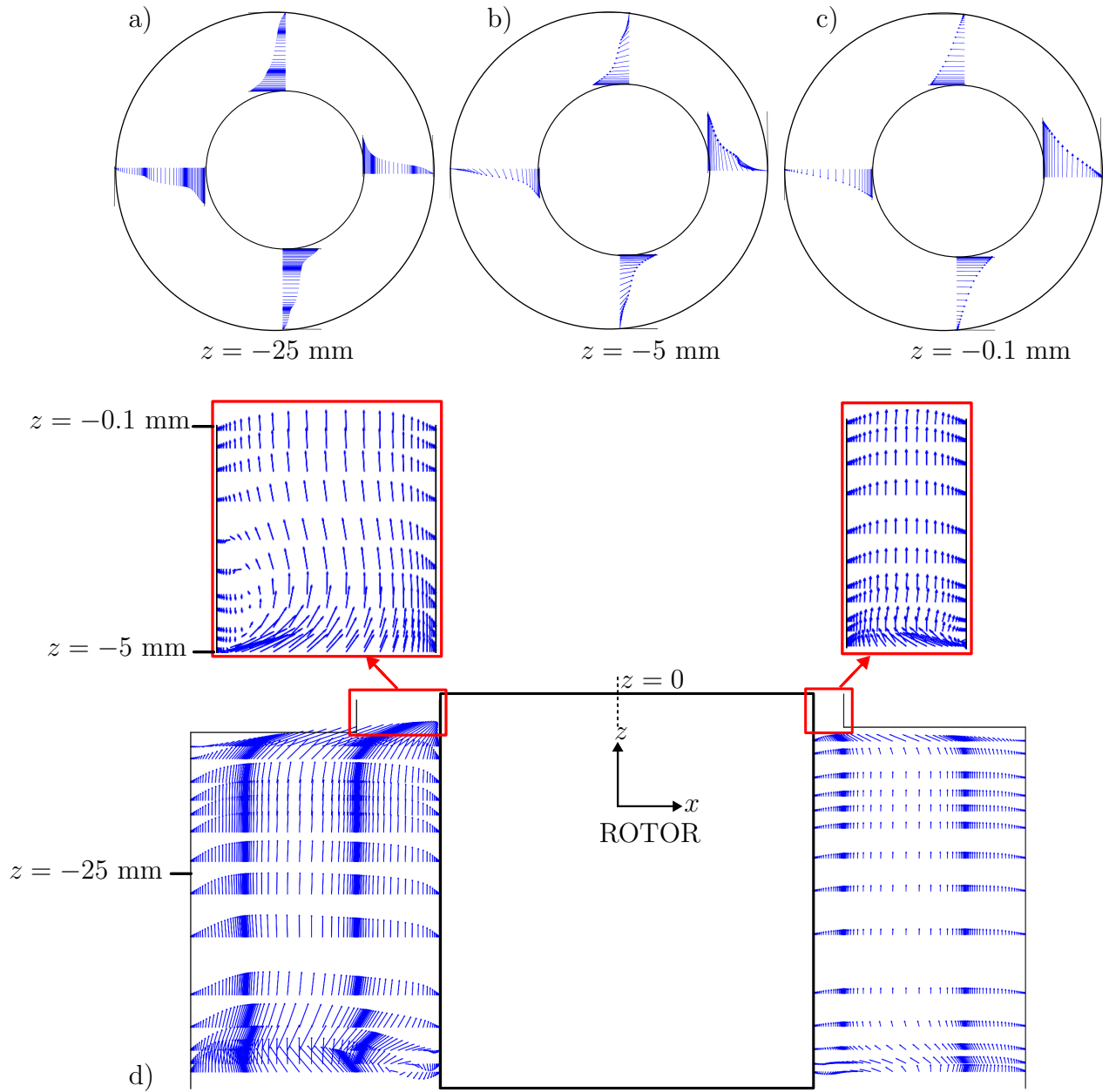


Figure 5.12 Velocity profiles for an upward flow, downstream seal in Fig. 4.2: (a) in the XY plane of the side chamber at $z = -25$ mm; (b) in the XY plane of the seal at $z = -5$ mm; (c) in the XY plane of the seal at $z = -0.1$ mm; (d) in the axial ZX plane of the seal, at $\theta = 0^\circ$. Note that proportions of the geometry are not to scale. The radial gap has been stretched for the visualization.

underpressure occurs at $\theta \approx 250^\circ$. In the seal, the direction of the pressure gradient is not significantly affected by the position of the seal, as the underpressure occurs at $\theta \approx 150^\circ$, with an overpressure in the smallest radial clearance at $\theta = 0^\circ$. On the whole, the position of the seal with respect to the side chamber influences the pressure distribution throughout the radial cavities. The larger pressure gradient remains in the seal although the pressure field is not necessarily maximal in this region. The location of the overpressures and underpressures also varies with the flow direction. It directly modifies the direction of the reaction forces, which are either restoring or exciting.

Fig. 5.14 shows the dimensionless forces by unit height $\bar{F}_{z_i} = \Delta F / \Delta z$ obtained from the pressure distributions depicted in Fig. 5.13, integrated over θ . The total force $\bar{F}_{tot}$ is plotted in black. As explained previously, the forces manifest as stiffness in the x -direction and damping in the y -direction. Here, all forces by unit height contribute to positive stiffness as they all have negative x -terms that tend to decrease the rotor displacement. As it was already observed for a seal located upstream of the side chamber in subsection 5.2.2, the forces $\bar{F}_{z_4}$ and $\bar{F}_{z_5}$ provide a larger positive stiffness, as the recirculation vortices at heights z_4 and z_5 amplify the underpressure in the larger clearance region. The forces in the side chamber also result in positive damping, because their negative y -component is opposed to $\dot{\epsilon}$. However, the forces in the seal $\bar{F}_{z_1}$ to $\bar{F}_{z_4}$ provide negative damping. The total force $\bar{F}_{tot}$ represents positive damping and stiffness, hence it is restoring. The influence of the position of the seal compared to the side chamber can drastically change to rotor stability. However in most low head hydraulic turbines configurations, the leakage flow passes through the seal before entering the side chambers. Consequently, a destabilizing effect by the seals is likely. The total force in the side chamber can be calculated for several whirl frequencies ω . Fig. 5.10 presents the variation of F_x and F_y in function of the whirl frequency. As in Fig. 5.10, numerical and experimental results by Nishimura et al. (2018), and the CFD results are plotted. F_x behaves similarly for the downstream and the upstream seal, as it increases with ω , but F_y linearly decreases ω . Referring to the expression $F_y = -C\omega + k$ of Eq. 4.8, the slope of F_y coincides with the direct damping. More specifically, a downstream seal provides positive damping, while an upstream seal exhibits negative damping. Finally, the downstream seal configuration is more stable than an seal upstream of the side chamber.

5.2.4 Flow in combined upstream and downstream seals

Since a seal downstream of the side chambers provides positive damping, it is interesting to know whether a rotor that is unstable in the presence of a seal upstream of the side chamber can be stabilized by an additional downstream seal. Therefore, based on actual modifications to a side chamber by Weber et al. (2016), the previous geometry was modified, as shown in

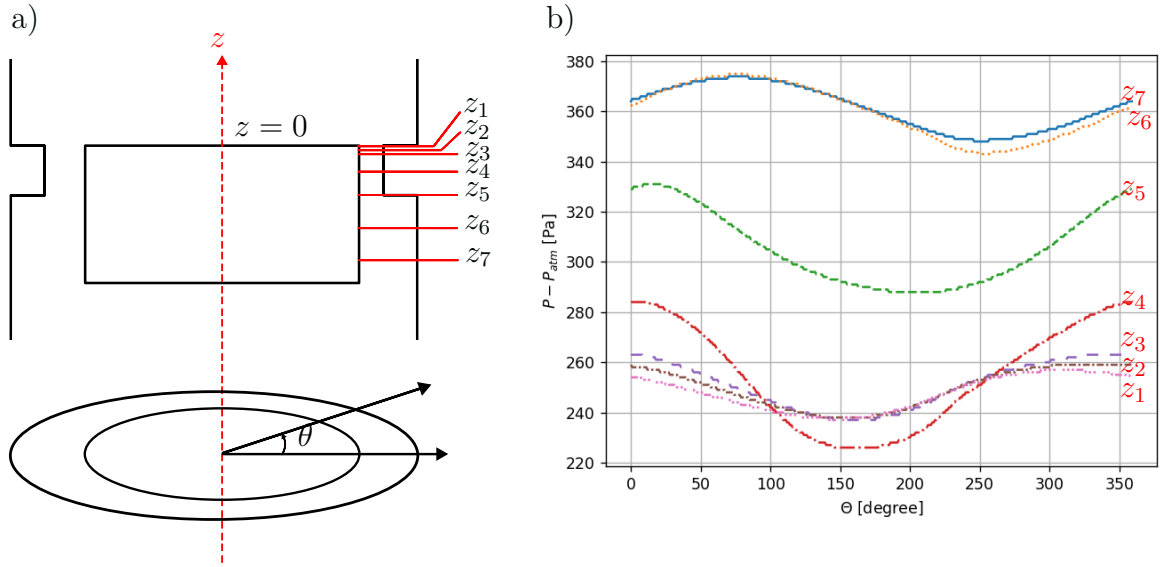


Figure 5.13 Pressure distribution in the downstream seal and side chamber of Fig. 4.2.b at $\omega = 5$ rad/s, $\Omega_R = 5$ rad/s, $\dot{m} = 0.25$ kg/s at $z_1 = -0.1$ mm; $z_2 = -0.6$ mm; $z_3 = -1$ mm; $z_4 = -3$ mm; $z_5 = -5$ mm; $z_6 = -25$ mm; $z_7 = -45$ mm

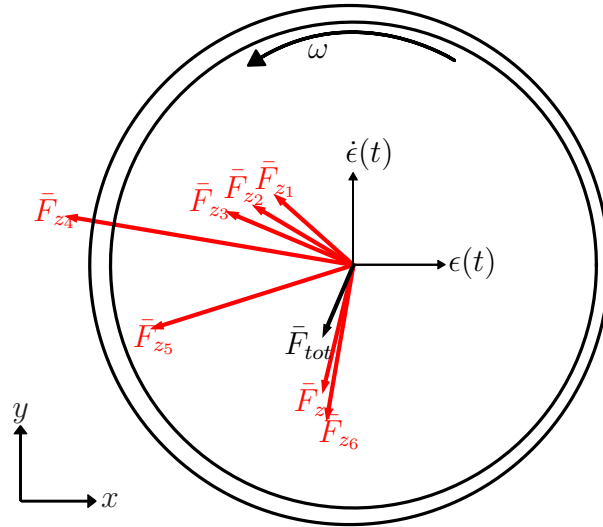


Figure 5.14 Dimensionless reaction linear forces $\bar{F}/dz$ in red, and dimensionless total force $\bar{F}_{tot}$ in black, for the downstream seal configuration of Fig. 4.2.b, at $\omega = 5$ rad/s

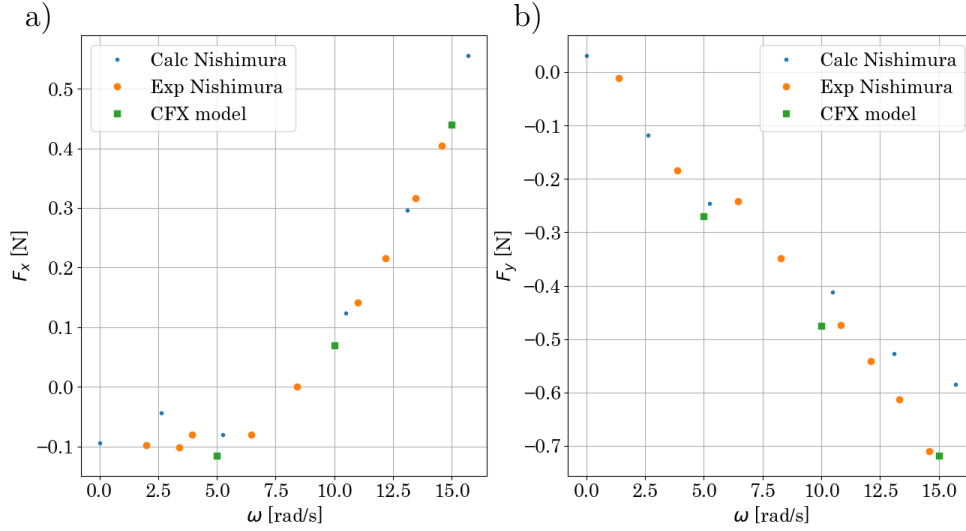


Figure 5.15 Variation of the reaction force components for the downstream seal configuration, in function of the whirl frequency ω , numerically and experimentally by Nishimura et al. (2018), and from the CFD model: (a) variation of the radial force F_x ; (b) variation of the tangential force F_y

Fig. 5.16: in addition to a seal upstream of the side chamber (seal 1), a second seal downstream of the side chamber for a flow coming from top to bottom is installed (seal 2). The axial width of this second seal is decreased from 4 to 0.5 mm. The flow conditions are similar of those in subsection 5.2.2, *i.e.*, an axial mass flow rate $\dot{m} = 0.25$ kg/s is input at the upper entrance, and a static pressure $p = 0$ is imposed at the outlet. Therefore the flow moves downwards, as shown by the red arrows.

Fig. 5.17 illustrates the pressure distributions on the rotor surface for various second seal widths (4 mm, 2 mm, 1 mm and 0.5 mm) at the same axial levels z_i than for the former studies: $z_1 = -0.1$ mm; $z_2 = -0.6$ mm; $z_3 = -1$ mm; $z_4 = -3$ mm; $z_5 = -5$ mm; $z_6 = -25$ mm; $z_7 = -45$ mm. These distributions correspond to a rotor angular frequency $\Omega_R = 5$ rad/s, and whirl frequency $\omega = 5$ rad/s. Comparing these pressure distributions to those without the additional seal, as shown in Fig. 5.8.b, the second seal increases the pressure gradient within the side chambers, as it is shown at the axial levels z_6 and z_7 . This gradient increases as the second seal width decreases. Along the side chambers, the maximum pressure occurs at $\theta \approx 150^\circ$. While the maximum pressure through the seal (z_1 to z_5) remains at $\theta = 0^\circ$ for large second seals as in Fig. 5.17.a, reducing the second seal width progressively shifts the maximum pressure to approximately 150° at all axial levels. This behavior is particularly observable in Fig. 5.17.d for a seal width of 0.5 mm.

The variation in the pressure field modifies the reaction forces. Fig. 5.18 displays the resulting linear forces $\bar{F}/dz$ at each axial level, and the total force $\bar{F}_{tot}$ in the radial gaps. When

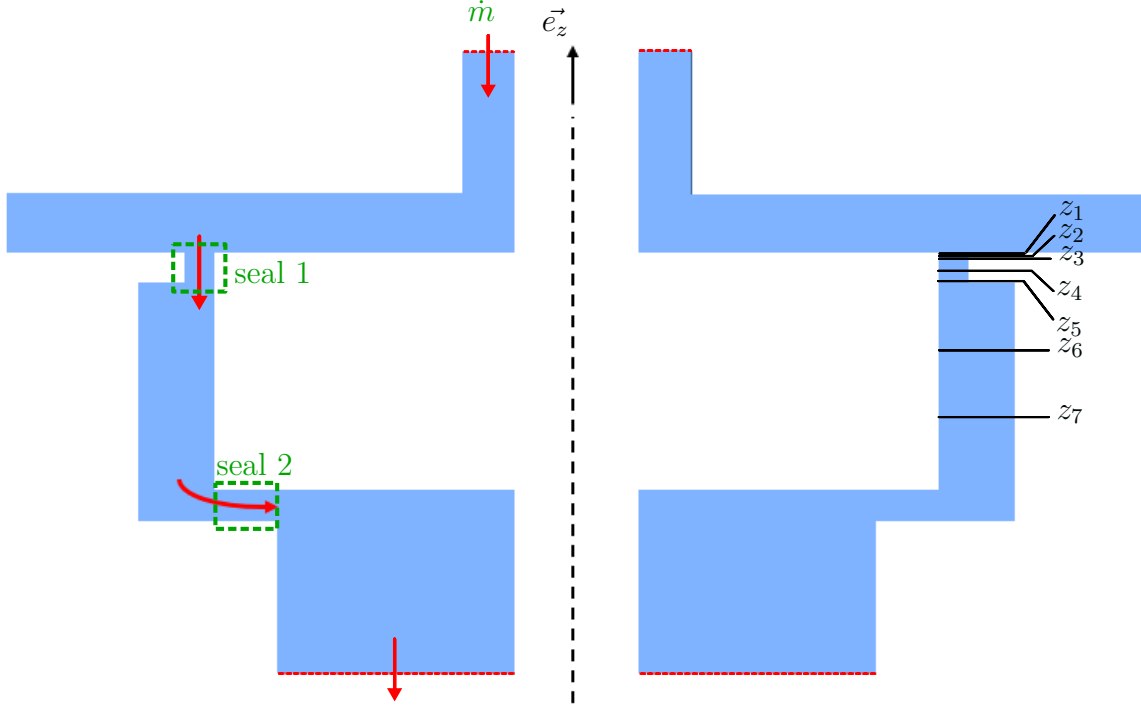


Figure 5.16 Radial gaps with the addition of a second axial seal ("seal 2") at the outlet of the side chamber, for a flow coming from top to bottom

compared to the forces without a second downstream seal shown in Fig. 5.9, the linear forces and total forces have negative y -components throughout the seal and side chamber for $\omega = 5$ rad/s, indicating positive damping. However, a narrower second seal results in a higher x -component of the total force, leading to increased negative stiffness and, consequently, an increase in rotor eccentricity ϵ . These different findings at $\omega = 5$ rad/s suggest that a second downstream seal enhances rotor damping, at the expense of an increased negative stiffness. Simulations for other whirl frequencies ω are necessary to determine the hydrodynamic coefficients specific to this new rotor seal configuration, as these coefficients are derived from regressions of F_x and F_y .

5.3 Variation of the rotor seal dynamic coefficients against geometric and input parameters

In order to better understand the impact of hydrodynamic seals on flow dynamics, and in particular the generation of self-excitation forces, parametric studies are carried out on the geometry illustrated in Fig. 4.2.b. These studies analyze how these forces, as well as the rotordynamic coefficients, vary as a function of the geometric parameters of the seal and side

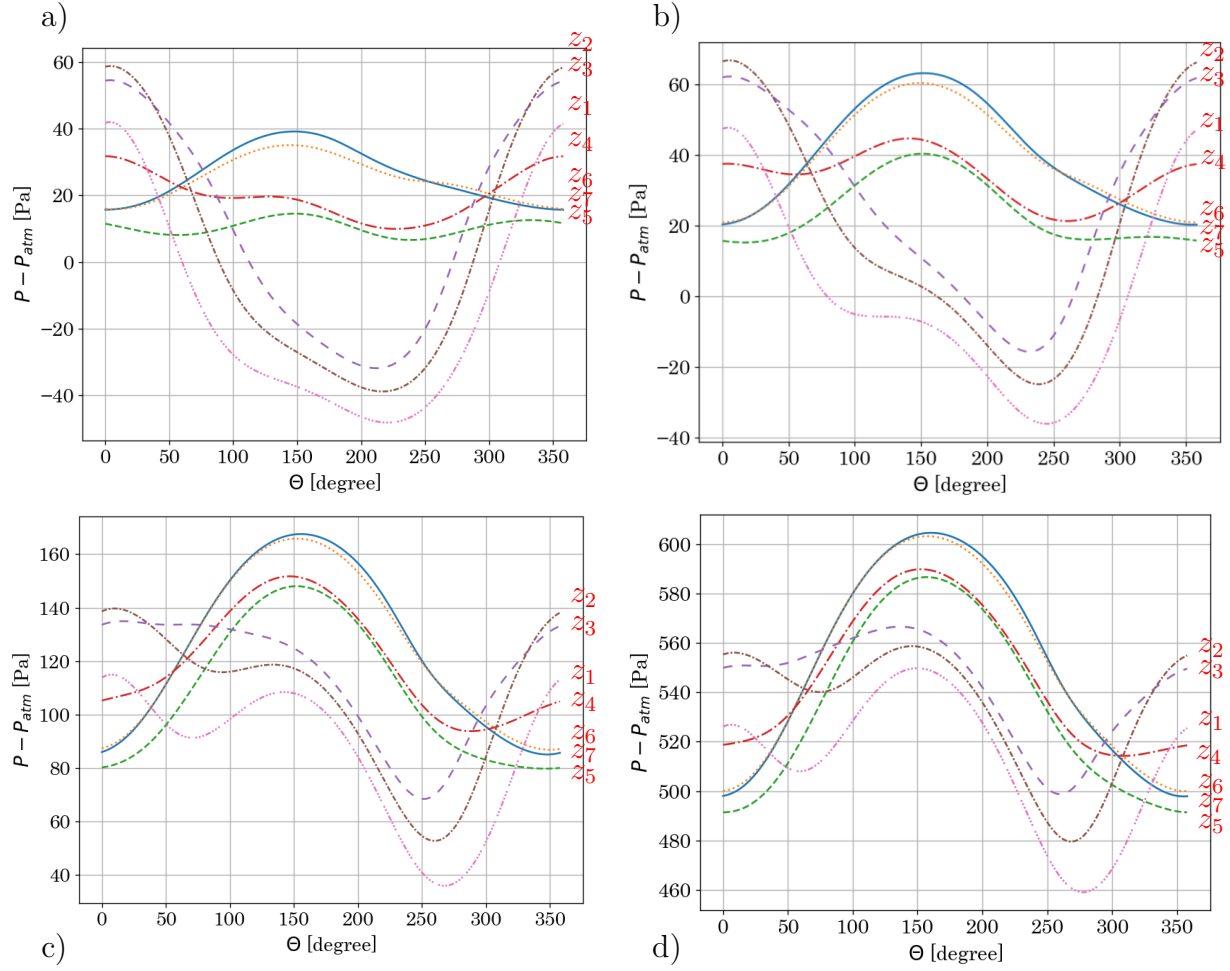


Figure 5.17 Pressure distribution in the seal and side chamber of Fig. 5.16, at the same axial levels z_1 to z_7 , for a different width of the secondary downstream seal: (a) 4 mm width; (b) 2 mm width; (c) 1 mm width; (d) 0.5 mm width

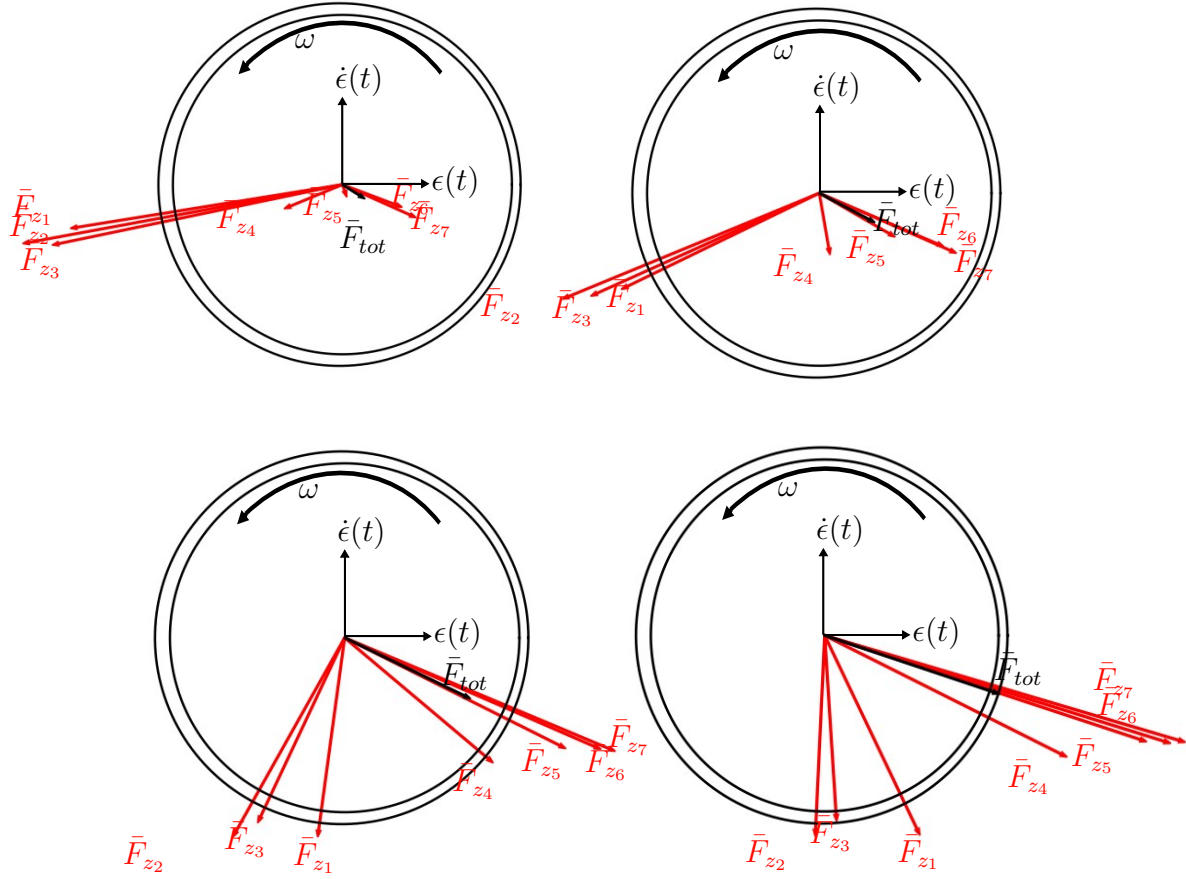


Figure 5.18 Dimensionless linear forces $\bar{F}/dz$ in red, and dimensionless total forces $\bar{F}_{tot}$ in black, within the seal and side chamber of Fig. 5.16, at the same axial levels z_1 to z_7 , for a different axial width of the secondary downstream seal: (a) 4 mm width; (b) 2 mm width; (c) 1 mm width; (d) 0.5 mm width

chamber, as well as the input mass flow rate and rotor angular frequency.

Fig. 5.19 shows the variation of F_x/ϵ and F_y/ϵ in function of ω for different mass flow rates $\dot{m}$, rotor angular frequencies Ω_R , radial seal and radial side chamber widths C_{ch} , C_0 and side chamber length l . The parameters used for the reference configuration, which is plotted on every graph, is described in Table 5.2.

As presented on Fig. 5.19.a-b, for an increasing mass flow rate $\dot{m}$ at the inlet, F_x/ϵ approximately keeps the same trend but is vertically translated downwards. For $\dot{m} = 0.10$ kg/s, the slope of F_y/ϵ is negative, but it becomes positive when $\dot{m}$ increases.

According to Fig. 5.19.c-d, when Ω_R increases, F_x/ϵ decreases and it has a more significant curvature. F_y/ϵ also decreases although its slope is almost constant.

Fig. 5.19.e-f shows that F_x/ϵ increases with C_0 . Its variations are larger at low frequency ω . Meanwhile F_y/ϵ decreases with C_0 .

According to Fig. 5.19.g-h, the side chamber length l mostly influences F_x/ϵ , which increases with l and has a more significant curvature. F_y/ϵ remains approximately constant with l .

Finally, Fig. 5.19.i-j shows that when the side chamber width C_{ch} decreases, the radial force F_x tends to vary linearly with the whirl frequency ω . Moreover the slope of F_y/ϵ increases with C_{ch} .

Referring to Eq. 4.8, these varying tendencies correspond to variations of the dynamic coefficients found by regressions of F_x/ϵ and F_y/ϵ , and summarized in Tables (5.3, 5.4, 5.5, 5.6, 5.7). Hence variation of the forces in the former parametric studies features variation of the dynamic coefficients. More precisely, some coefficients are more sensitive to a given parameter than others. As supposed by the analytical expression of the added mass expressed by Blevins (1990) in Eq. 2.4, the dynamic mass M is sensitive to the rotor parametric dimensions, as l , C_{ch} and C_0 which tend to increase M . However, Table 5.4 unexpectedly states that M also increases with Ω_R . The variation of M with Ω_R could be explained by an added stiffness, proportional to Ω_R , whose effect would be included in the mass term. Direct damping C increases negatively with $\dot{m}$, l and C_{ch} , and decreases in terms of negative values with C_0 . Direct stiffness K decreases with Ω_R , C_0 , l , C_{ch} but it increases with $\dot{m}$. The cross-coupled coefficients also vary with those parameter. The cross-coupled stiffness k diminishes towards negative values with $\dot{m}$, Ω_R , l and C_{ch} but it raises with C_0 , and the cross-coupled damping c increases with C_0 , C_{ch} and Ω_R but decreases $\dot{m}$. It does not vary linearly with l .

Table 5.2 Values of the parameters used for the reference configuration of the parametric studies

$\dot{m}$ (kg/s)	Ω_R (rad/s)	C_0 (mm)	l (mm)	C_{ch} (mm)
0.25	5	1	60	4

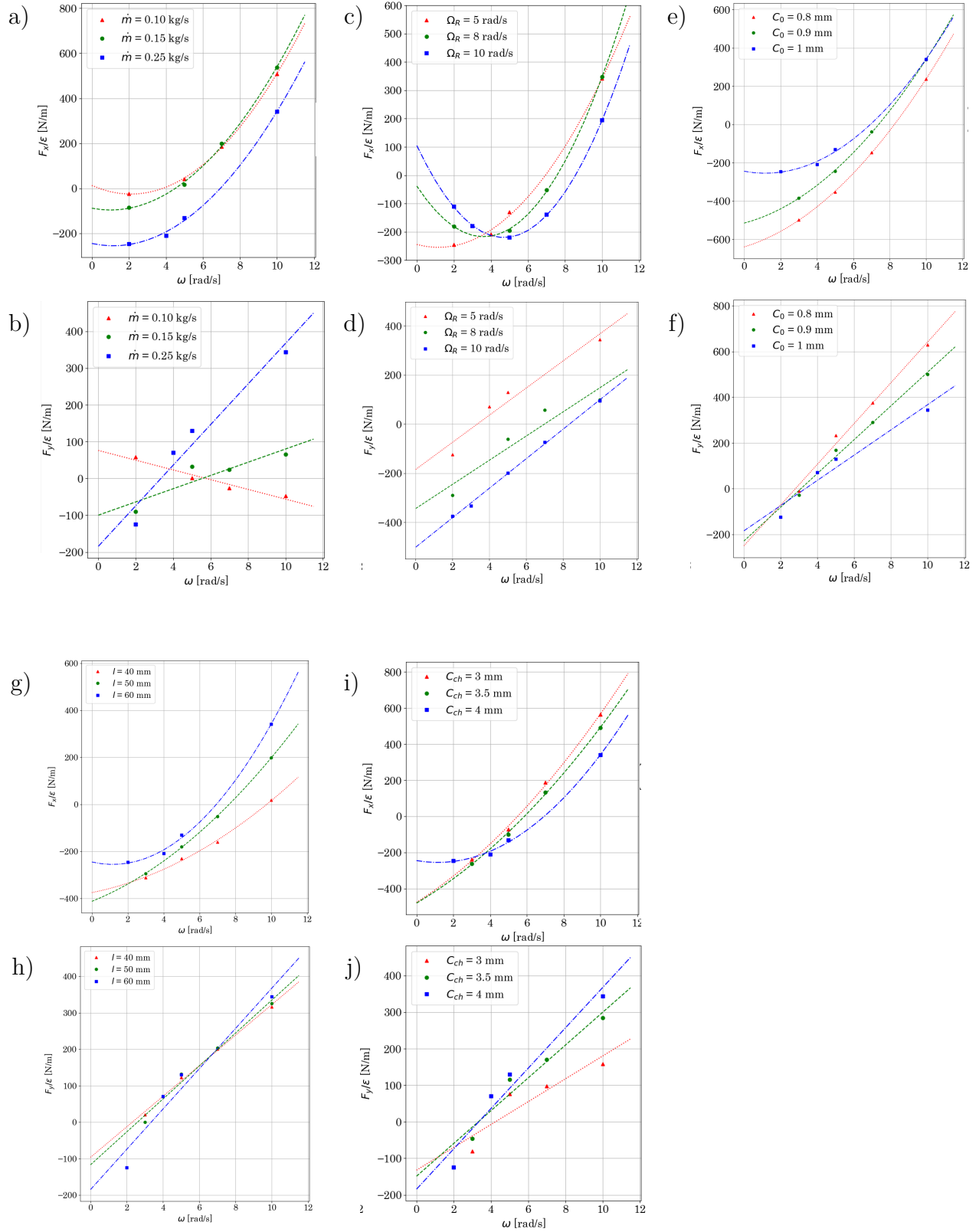


Figure 5.19 Parametric study of the dependence of F_x/ϵ and F_y/ϵ as functions of: (a,b) mass flow rate $\dot{m}$; (c,d) rotor angular frequency Ω_R ; (e,f) radial seal width C_0 ; (g,h) side chamber length l ; (i,j) and side chamber width C_{ch} , on the geometry according to Fig. 4.2.b.

Variation of the direct damping in function of those parameters can be summarised by the following expression :

$$|C| \propto \frac{\dot{m}lC_{ch}}{C_0} \quad (5.3)$$

Apart from the variation of the direct damping C of the rotor seal with the side chamber width C_{ch} , the variation of C with these parameters agrees with the analytical work of Nishimura et al. (2016) which was detailed by Weber et al. (2016), who expressed a damping parameter c_{RSC} , introduced in section 2.5:

$$\begin{aligned} c_{RSC} &= - \frac{\rho l^2}{2C_{ch}} \frac{Q}{2C_0} \quad \text{for a seal at the inlet,} \\ c_{RSC} &= \frac{\rho l^2}{2CC_{ch}} \frac{Q}{2C_0} \quad \text{for a seal at the outlet (Weber et al., 2016).} \end{aligned} \quad (5.4)$$

with Q the volumetric leakage flow rate. Nevertheless, the observed variation of the rotor seal damping with C_{ch} is inverse to the work of Weber et al..

5.4 Discussion of the stability of the rotor in function of the geometric and input parameters

The dynamic coefficients of mass, damping and stiffness determine the rotor stability. In order to reach a stable state, they have to fulfill the three conditions of Eq. 4.6.

The stability region based on those criteria is plotted against direct damping C and stiffness K in Fig. 5.20 for the radial gaps of Fig. 4.2.b, based on the evaluated dynamic coefficients of the former parametric studies. Every graph is dedicated to the variation of one parameter between C_0 , $\dot{m}$, Ω_R , l and C_{ch} . For every value of the varying parameter, the dynamic mass M and the cross-coupled terms k and c are fixed, and the three stability criteria of Eq. 4.6 are calculated. The line plotted for every value of the varying parameter represents the limit that separates the stable from the unstable state of the rotor. On the right of this line the three stability conditions are verified, whereas on its left, at least one of these conditions is

Table 5.3 Variation of the rotordynamic coefficients with $\dot{m}$, evaluated with the CFD model

$\dot{m}$ (kg/s)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)
0.10	8.61	13.23	-12.99	36.37	76.34
0.15	7.88	-17.99	87.96	16.01	-99.62
0.25	7.65	-55.16	244.54	17.80	-184.34

Table 5.4 Variation of the rotordynamic coefficients with Ω_R , evaluated with the CFD model

Ω_R (rad/s)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)
5	7.65	-55.16	244.54	17.80	-184.34
10	13.83	-48.67	38.03	99.56	-341.69
15	14.75	-60.00	-105.03	138.45	-501.00

Table 5.5 Variation of the rotordynamic coefficients with C_0 , evaluated with the CFD model

C_0 (mm)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)
0.8	5.92	-89.08	640.18	-28.64	-249.66
0.9	6.06	-73.84	515.67	-25.00	-228.84
1.0	7.65	-55.16	244.54	17.80	-184.34

Table 5.6 Variation of the rotordynamic coefficients with l , evaluated with the CFD model

l (mm)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)
40	2.39	-41.83	374.58	-15.15	-96.02
50	3.04	-45.10	411.81	-30.60	-116.50
60	7.65	-55.16	244.54	17.80	-184.34

Table 5.7 Variation of the rotordynamic coefficients with C_{ch} , evaluated with the CFD model

C_{ch} (mm)	M (kg)	C (N.s/m)	K (N/m)	c (N.s/m)	k (N/m)
3	4.01	-31.23	472.61	-64.00	-132.29
3.5	3.74	-44.84	479.98	-59.99	-148.86
4	7.65	-55.16	244.54	17.80	-184.34

not verified. The actual (C, K) couple of the rotor is added to the graph as a dot.

In Fig. 5.20.a, The stability region is slightly shifted towards higher direct damping and stiffness (C, K) as the radial seal width C_0 increases. Moreover, for all three C_0 widths, the rotor remains unstable.

Fig. 5.20.b shows that the stability region also narrows when the mass flow rate $\dot{m}$ increases. Moreover, although the stator never reaches stability when $\dot{m}$, it becomes less unstable when $\dot{m}$ diminishes. Actually at $\dot{m} = 0.10 \text{ kg/s}$, the rotor is close to the stability limit, as its negative damping decreases significantly, and becomes positive at lower $\dot{m}$.

In Fig. 5.20.c, the stability region significantly reduces when the rotor angular frequency Ω_R increases. However, the couple (C, K) moves further away from the stability region with higher rates Ω_R , as its stiffness sharply reduces.

It is shown in Fig. 5.20.d that the stability region decreases when the side chamber length l increases.

Finally, Fig. 5.20.e displays that when the side chamber width C_{ch} increases, the stability region also decreases, as it requires higher positive damping C and stiffness K to reach stability.

To summarize, stability region of the rotor in function of K and C decreases significantly with $\dot{m}$, Ω_R . A larger seal width C_0 decreases the stiffness but reduces the negative damping. Increasing the side chamber length l also decreases, but does not sharply modify the damping C .

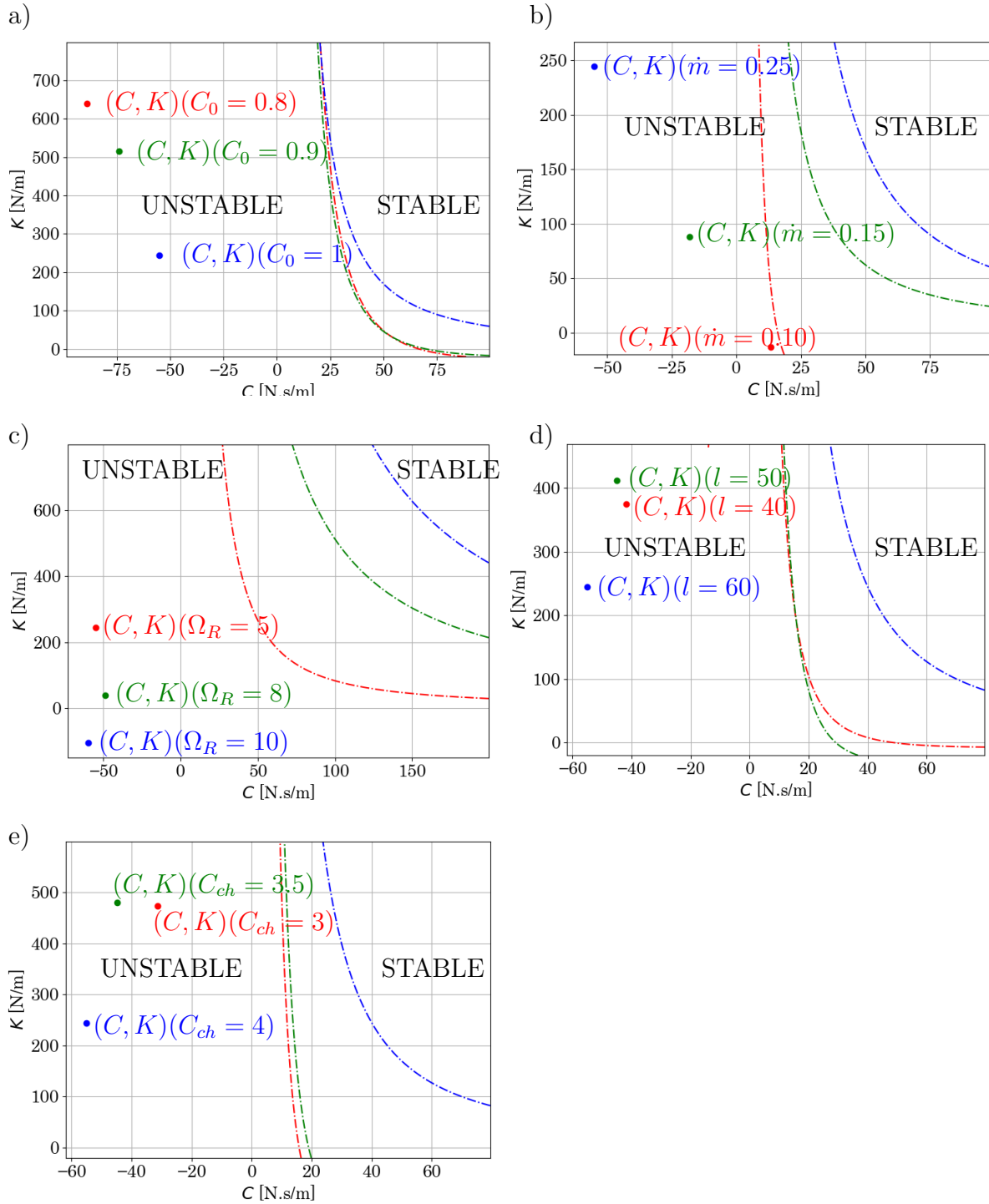


Figure 5.20 Zones of stability based on the stability criteria from Eq. 4.6, plot the C_K plane: (a) for varying seal width C_0 ; (b) for varying mass flow rate $\dot{m}$; (c) for varying rotor angular frequency Ω_R ; (d) for varying side chamber length l ; (e) for varying side chamber width C_{ch}

CHAPTER 6 CONCLUSION

6.1 Summary of Works

This master thesis sets up a methodology to determine the hydrodynamic mass, damping and stiffness of the rotor seal of a Francis turbine runner, which are required as input data for the prediction of rotor vibration. The calculation of the radial and tangential forces on the eccentricity, F_x/ϵ and F_y/ϵ , as a function of the whirl frequency ω , allows for the determination of the rotor seal dynamic coefficients by their quadratic and linear regression. Hence, knowing F_y and F_x for at least three different frequencies ω is sufficient to evaluate the dynamic coefficients.

A vibroacoustic model and a CFD model were developed to calculate the force components. They were verified by comparison with an analytical model based on potential flow theory on the geometry of a plain annular seal. The CFD model is the most efficient, as it is the only one to calculate the tangential force component F_y . Actually, both vibroacoustic and analytical models do not take the flow into account, and therefore do not take shear stress into account, which translates into a pressure gradient in phase with rotor displacement, and a purely radial force.

The developed methodology has the benefit of being applicable to any geometry of the radial gap. Application of the CFD model to a radial gap geometry composed of lateral chambers and narrower seals, with variation of key parameters and of the seal configuration led to the following observations:

- The presence of abrupt radial narrowing leads to separation and recirculation bubbles in the radial seals, which affect the pressure distribution over the rotor. The recirculation vortices are dominant in the larger clearance of the seal, and promote positive stiffness of the rotor seal.
- The position of the radial seal relative to the larger side chamber influences the flow within the radial clearance. When the seal is located downstream from the side chamber, it results in larger recirculation vortices within the larger clearance of the seal, compared to when the seal is upstream. This downstream positioning increases positive stiffness.
- The pressure gradient in the radial gaps exhibits a phase shift relative to the rotor displacement. This phase shift varies along the seal and the side chambers, resulting in reaction forces with different directions within the radial gaps. Typically, the largest

pressure gradients occur in the narrow seal, leading to the emergence of the strongest forces. In the common configuration where the seal is located upstream of the side chamber, the forces in the seal display negative damping..

- The position of the radial seal in relation to the larger side chamber has a significant impact on the pressure gradients through the radial gaps. When the seal is upstream of the chamber, These pressure gradients lead to self-exciting forces which provide negative hydrodynamic damping. If the seal is downstream, the pressure gradients inside the chamber generate stabilizing forces, resulting in positive damping.
- If the seal is located upstream of the side chamber, the damping of the rotor can be improved by the addition of a second seal downstream to the side chamber, although it can deteriorate the rotor stiffness.
- Rotordynamic coefficients are also sensitive to the geometrical dimensions of the seals and side chambers, as well as the mass flow rate and rotor angular frequency. These parameters either enhance or decrease the stability of the rotor system.

6.2 Limitations

Each numerical model used in this study has its inherent limitations. The CFD model, although capable of determining dynamic coefficients by considering the flow in its entirety, is computationally expensive. This can be particularly challenging for larger meshes with complex radial gap configurations.

On the other hand, the vibroacoustic and analytical models based on FEM address the computational complexity issue. Furthermore, their resulting radial force correlate with the one from CFD for the plain annular seal configuration. However, the vibroacoustic model does not account for the phase shift between the pressure gradient and the rotor displacement, resulting in the inability to evaluate the tangential force component. Similarly, the analytical model also lacks the ability to assess the tangential force component. Establishing a relationship between these force components would be valuable for accurately evaluating all dynamic coefficients. Additionally, both the vibroacoustic and analytical models do not capture axial flow, making it impossible to account for the Lomakin effect. Moreover, these models have not been applied to complex radial gap configurations due to these limitations. Investigating the agreement between the radial force obtained from these models and CFD for such configurations would be insightful.

6.3 Future Research

The results obtained for the geometry depicted in Fig. 4.2.b provide insights into the flow dynamics occurring in hydraulic turbine radial clearances, in the presence of a narrow radial seal. However, it is important to note that this studied geometry was reproduced from (Nishimura et al., 2018), which considers a rotor of radius 0.117 m, while Francis turbine runners can have a radius of 1 m or even larger. Consequently, the flow in radial clearances of Francis turbines tends to be more turbulent than in the studied configuration, which can influence the flow behavior. Therefore, while the current results primarily consider the influence of radial seals on the rotordynamic coefficients, increasing the precision of the model by incorporating geometric dimensions closer to realistic configurations would be beneficial. In the future, it would also be beneficial to expand the scope of the study by considering various seal geometries beyond the annular configuration analyzed in this research. Hydraulic turbines often employ different types of seals, such as labyrinth seals. Extending the current model to incorporate these new seal configurations would enable a better understanding of the flow dynamics associated with them.

Furthermore, the literature review in chapter 2 highlighted the influence of preswirl on the mean circumferential whirl frequency. While the standard assumption is $\omega = 0.5\Omega_R$, under the presence of preswirl, this value may vary slightly. It is evident that preswirl has an impact on swirl phenomena. Therefore, improving the determination of rotordynamic coefficients could be achieved by considering the effects of preswirl within the radial clearance.

Finally, subsection 5.2.4 of the Results section shed light on the fact that adding a second seal in the radial gap can improve rotor stability. However, further research on the configuration of this seal would be interesting.

By addressing these aspects in future studies, a more comprehensive understanding of the flow dynamics and improved prediction of rotordynamic coefficients can be achieved, thus enhancing the design and performance optimization of hydraulic turbines.

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APPENDIX A CALCULATION OF THE EULER EQUATION AS A FUNCTION OF THE FLOW POTENTIAL ϕ

The Euler equation as a function of the flow velocity vector $\vec{v}_f$ is:

$$\frac{\partial \vec{v}_f}{\partial t} + (\vec{v}_f \cdot \vec{\nabla}) \vec{v}_f = -\frac{1}{\rho} \vec{\nabla} p \quad (\text{A.1})$$

The second term also intervenes in the following relation:

$$\frac{1}{2} \nabla ||\vec{v}_f||^2 = \vec{v}_f \wedge (\vec{\nabla} \wedge \vec{v}_f) + (\vec{v}_f \cdot \vec{\nabla}) \vec{v}_f \quad (\text{A.2})$$

Because the flow is irrotationnal, $\vec{\nabla} \wedge \vec{v}_f = 0$, and Eq. A.2 is equivalent to:

$$\frac{1}{2} \vec{\nabla} ||\vec{v}_f||^2 = (\vec{v}_f \cdot \vec{\nabla}) \vec{v}_f \quad (\text{A.3})$$

The Euler equation of an irrotationnal flow can thus be written as:

$$\frac{\partial \vec{v}_f}{\partial t} + \frac{1}{2} \nabla v_f^2 = -\frac{1}{\rho} \vec{\nabla} p \quad (\text{A.4})$$

Potential flow theory is based on the expression of the flow potential ϕ such as $\vec{v}_f = \vec{\nabla} \phi$.
Introducing this expression into Eq. A.4 yields:

$$\begin{aligned} \frac{\partial \vec{\nabla} \phi}{\partial t} + \frac{1}{2} \vec{\nabla} (\nabla \phi)^2 &= -\frac{1}{\rho} \vec{\nabla} p, \\ \vec{\nabla} \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 + \frac{p}{\rho} \right] &= 0 \\ \Rightarrow \frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 + \frac{p}{\rho} &= F(t) \quad (\text{generalized Bernoulli equation (Blevins, 1990)}) \end{aligned} \quad (\text{A.5})$$

The term $F(t)$ is generally included in ϕ . The Euler equation as a function of ϕ is thus:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 + \frac{p}{\rho} = 0 \quad (\text{A.6})$$

APPENDIX B CALCULATION OF THE FLOW POTENTIAL ϕ

The flow potential ϕ must verify the Laplace equation:

$$\Delta\phi = 0. \quad (\text{B.1})$$

The gradient of ϕ in cylindrical coordinates is:

$$\Delta\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}. \quad (\text{B.2})$$

Therefore, the flow potential ϕ is sought in the form:

$$\phi(r, \theta) = \left(Ar + \frac{B}{r} \right) \cos \theta. \quad (\text{B.3})$$

Effectively, this form is a solution to Eq. B.1, as the derivatives of ϕ are:

$$\begin{aligned} \phi_{,r} &= \left(A - \frac{B}{r^2} \right) \cos(\theta), \\ \phi_{,rr} &= \frac{2B}{r^3} \cos \theta, \\ \phi_{,\theta\theta} &= - \left(Ar + \frac{B}{r} \right) \cos(\theta). \end{aligned} \quad (\text{B.4})$$

The flow potential must also verify the following boundary conditions:

$$\begin{cases} \phi_{,r}(R_S) = 0, \\ \phi_{,r}(R_R) = v_s(R_R) = \dot{\epsilon} \cos \theta - \frac{1}{2}\epsilon\Omega_R \sin \theta. \end{cases} \quad (\text{B.5})$$

Introducing Eq. B.3 into Eq. B.5, the constants A and B can be determined. The first boundary condition yields:

$$\begin{aligned} \phi_{,r}(R_S) &= 0 \\ \left(A - \frac{B}{R_S^2} \right) \cos \theta &= 0 \\ \Rightarrow A &= \frac{B}{R_S^2} \end{aligned} \quad (\text{B.6})$$

Using the new expression of the constant A , the second boundary condition gives:

$$\begin{aligned}
\phi_{,r}(R_R) &= v_s(R_R) = \dot{\epsilon} \cos \theta - \frac{1}{2} \epsilon \Omega_R \sin \theta, \\
\left(\frac{B}{R_S^2} - \frac{B}{R_R^2} \right) \cos \theta &= \dot{\epsilon} \cos \theta - \frac{1}{2} \epsilon \Omega_R \sin \theta, \\
B \left(-\frac{R_S^2 - R_R^2}{R_R^2 R_S^2} \right) &= \dot{\epsilon} - \frac{1}{2} \epsilon \Omega_R \tan \theta, \\
\Rightarrow B &= \frac{R_R^2 R_S^2}{R_S^2 - R_R^2} \left(-\dot{\epsilon} + \frac{1}{2} \epsilon \Omega_R \tan \theta \right).
\end{aligned} \tag{B.7}$$

The introduction of the new expression of B into Eq. B.6 gives:

$$A = \frac{R_R^2}{R_S^2 - R_R^2} \left(-\dot{\epsilon} + \frac{1}{2} \epsilon \Omega_R \tan \theta \right). \tag{B.8}$$

The expression of A and B are introduced into Eq. B.3 to obtain:

$$\phi(r, \theta) = \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(-\dot{\epsilon} \cos \theta + \frac{1}{2} \epsilon \Omega_R \sin \theta \right). \tag{B.9}$$

APPENDIX C EXPRESSION OF THE PRESSURE p AS A FUNCTION OF THE FLOW POTENTIAL ϕ

The Euler equation of Eq. 4.11 expresses the pressure distribution p as a function of ϕ :

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla \phi)^2 = -\frac{p}{\rho} \quad (\text{C.1})$$

The derivatives of ϕ are:

$$\begin{aligned} \phi_{,t} &= \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(-\ddot{\epsilon} \cos \theta + \frac{1}{2} \dot{\epsilon} \Omega_R \sin \theta \dot{\epsilon} \omega \sin \theta - \frac{1}{2} \epsilon \Omega_R \omega \cos \theta \right), \\ \phi_r &= \frac{R_R^2}{R_S^2 - R_R^2} \left(1 - \frac{R_S^2}{r^2} \right) \left(-\dot{\epsilon} \cos \theta + \frac{1}{2} \epsilon \Omega_R \sin \theta \right), \\ \phi_{,\theta} &= \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(\dot{\epsilon} \sin \theta + \frac{1}{2} \epsilon \Omega_R \cos \theta \right), \\ (\nabla \phi)^2 &= \phi_{,r}^2 + \frac{1}{r^2} \phi_{,\theta}^2. \end{aligned} \quad (\text{C.2})$$

At the rotor surface $r = R_R$, those terms become:

$$\begin{aligned} \phi_{,t} &= R_R \frac{R_R^2 + R_S^2}{R_S^2 - R_R^2} \left(-\ddot{\epsilon} \cos \theta + \frac{1}{2} \dot{\epsilon} \Omega_R \sin \theta \dot{\epsilon} \omega \sin \theta - \frac{1}{2} \epsilon \Omega_R \omega \cos \theta \right), \\ \phi_r &= \frac{R_R^2}{R_S^2 - R_R^2} \left(1 - \frac{R_S^2}{R_R^2} \right) \left(-\dot{\epsilon} \cos \theta + \frac{1}{2} \epsilon \Omega_R \sin \theta \right), \\ \phi_{,\theta} &= R_R \frac{R_R^2 + R_S^2}{R_S^2 - R_R^2} \left(\dot{\epsilon} \sin \theta + \frac{1}{2} \epsilon \Omega_R \cos \theta \right), \\ (\nabla \phi)^2 &= \phi_{,r}^2 + \frac{1}{r^2} \phi_{,\theta}^2 = \Gamma_1 \cos^2 \theta + \Gamma_2 \sin^2 \theta. \end{aligned} \quad (\text{C.3})$$

The pressure field p at the rotor surface is thus written as:

$$p = -\rho \frac{R_R^2}{R_S^2 - R_R^2} \left(r + \frac{R_S^2}{r} \right) \left(-\ddot{\epsilon} \cos \theta + \frac{1}{2} \dot{\epsilon} \Omega_R \sin \theta \dot{\epsilon} \omega \sin \theta - \frac{1}{2} \epsilon \Omega_R \omega \cos \theta \right) + \Gamma_1 \cos^2 \theta + \Gamma_2 \sin^2 \theta \quad (\text{C.4})$$

The terms proportional to $\cos^2 \theta$ and $\sin^2 \theta$ do not induce any force components, as their integration projected along the x or y -axis, is null.