Titre: Title:	Experimental Study on the Reliability of Scaling Down Techniques Used in Direct Shear Tests to Determine the Shear Strength of Rockfill and Waste Rocks
Auteurs: Authors:	Akram Deiminiat, & Li Li
Date:	2022
Type:	Article de revue / Article
Référence: Citation:	Deiminiat, A., & Li, L. (2022). Experimental Study on the Reliability of Scaling Down Techniques Used in Direct Shear Tests to Determine the Shear Strength of Rockfill and Waste Rocks. CivilEng, 3(1), 35-50. https://doi.org/10.3390/civileng3010003

Titre de la revue: Journal Title:	CivilEng (vol. 3, no. 1)
Maison d'édition: Publisher:	Multidisciplinary Digital Publishing Institute
URL officiel: Official URL:	https://doi.org/10.3390/civileng3010003
Mention légale: Legal notice:	© 2022 Deiminiat, A., & Li, L. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/).

CivilEng

Article

Experimental Study on the Reliability of Scaling Down Techniques Used in Direct Shear Tests to Determine the Shear Strength of Rockfill and Waste Rocks

Akram Deiminiat and Li Li *

Department of Civil, Geological and Mining Engineering, Research Institute on Mining and Environment, Polytechnique Montreal, Montreal, QC H3T 1J4, Canada; akram.deiminiat@polymtl.ca
* Correspondence: li.li@polymtl.ca

Abstract: The determination of shear strength parameters for coarse granular materials such as rockfill and waste rocks is challenging due to their oversized particles and the minimum required ratio of 10 between the specimen width (W) and the maximum particle size (d_{max}) of tested samples for direct shear tests. To overcome this problem, a common practice is to prepare test samples by excluding the oversized particles. This method is called the scalping scaling down technique. Making further modifications on scalped samples to achieve a specific particle size distribution curve (PSDC) leads to other scaling down techniques. Until now, the parallel scaling down technique has been the most popular and most commonly applied, generally because it produces a PSDC parallel and similar to that of field material. Recently, a critical literature review performed by the authors revealed that the methodology used by previous researchers to validate or invalidate the scaling down techniques in estimating the shear strength of field materials is inappropriate. The validity of scaling down techniques remains unknown. In addition, the minimum required W/d_{max} ratio of 10, stipulated in ASTM D3080/D3080M-11 for direct shear tests, is not large enough to eliminate the specimen size effect (SSE). The authors' recent experimental study showed that a minimum W/d_{max} ratio of 60 is necessary to avoid any SSE in direct shear tests. In this study, a series of direct shear tests were performed on samples with different d_{max} values, prepared by applying scalping and parallel scaling down techniques. All tested specimens had a W/d_{max} ratio equal to or larger than 60. The test results of the scaled down samples with d_{max} values smaller than those of field samples were used to establish a predictive equation between the effective internal friction angle (hereafter named "friction angle") and d_{max}, which was then used to predict the friction angles of the field samples. Comparisons between the measured and predicted friction angles of field samples demonstrated that the equations based on scalping scaling down technique correctly predicted the friction angles of field samples, whereas the equations based on parallel scaling down technique failed to correctly predict the friction angles of field samples. The scalping down technique has been validated, whereas the parallel scaling down technique has been invalidated by the experimental results presented in this study.

Keywords: direct shear tests; scaling down technique; shear strength; maximum particle size; scalping technique; parallel technique

check for updates

Citation: Deiminiat, A.; Li, L. Experimental Study on the Reliability of Scaling Down Techniques Used in Direct Shear Tests to Determine the Shear Strength of Rockfill and Waste Rocks. *CivilEng* **2022**, *3*, 35–50. https://doi.org/10.3390/civileng3010003

Academic Editors: João Castro-Gomes, Cristina Fael and Miguel Nepomuceno

Received: 16 November 2021
Accepted: 5 January 2022
Published: 8 January 2022

1. Introduction

The determination of shear strength parameters is challenging for coarse granular materials such as rockfill and waste rocks due to their oversized particles and the minimum required ratio of 10 between specimen width (W) and the maximum particle size (d_{max}) of tested samples for direct shear tests [1]. For the convenience of laboratory tests, it is always preferable to use specimens as small as possible. However, when the specimens are too small, the measured shear strength can be significantly different from that of the tested material in field conditions. Thus, the tested specimens must be large enough to eliminate any specimen size effect (SSE) [2–11]. The minimum required specimen volume to avoid any SSE is called the representative element volume [12–14].

For direct shear tests, several standards have been proposed and used in practice. Among them, the ASTM D3080/D3080M-11, hereafter called ASTM, is the most popular and the most used worldwide [2,11,15–23]. It was published as ASTM D3080 in 1972 and updated every eight years by the ASTM technical committees. Recently, it was temporarily withdrawn due to over eight years passing since the last update [1]. The withdrawn rationale has nothing to do with d_{max}; therefore, it can be expected that the updated ASTM standard for direct shear tests will remain unchanged with respect to the minimum required specimen sizes. The width (W) and thickness (T) of the tested square specimen should be: $W \geq 50$ mm; $T \geq 13$ mm; $W/d_{max} \geq 10$; $T/d_{max} \geq 6$; $W/T \geq 2$. Similar requirements can be found in other standards [24–26].

For fine particle materials such as clay, silt and sand with a d_{max} smaller than or equal to 1 mm, applying ASTM in specimen preparation is not a problem because the standard direct shear test system is usually equipped with a square shear box 60 mm wide (i.e., W = 60 mm). The specimens prepared with this standard shear box automatically have a W/d_{max} ratio up to 60, a value largely exceeding the ASTM's minimum required ratio of 10. For coarse materials such as gravel, rockfill and waste rocks, applying ASTM in specimen preparation can become problematic. The problem is particularly prominent with rockfill and waste rocks, which usually contain fine particles as small as silts and coarse particles as large as boulders. Conducting laboratory tests with original field material and respecting the ASTM's requirements are economically impossible if technically not impossible.

To overcome this problem, a common practice is to prepare test samples by excluding the oversized particles [10,27–40]. The method is called the scalping scaling down technique.

Scalping technique is probably the simplest and earliest method to obtain laboratory samples having an admissible d_{max} from field materials [10,29,33,37,40–47]. By applying this technique, the particle size distribution curve (PSDC) of the obtained samples can differ from that of the field material due to the reduction in coarse particles. Some researchers made use of this method when the excluded oversized particles represented 10% to 30% [37,42,48,49].

Further modifications on scalped samples to achieve a specific PSDC have led to other scaling down techniques. When the PSDC of the scaled down sample is modified to be parallel to that of field material, the method is called the parallel scaling down technique [50–52]. The PSDC of the obtained sample thus looks like a horizontal shift of the PSDC of the field material towards the finer side in the semi-log plane of the PSDC.

The third scaling down technique, called the replacement method, consists of replacing the oversized particles by the same mass of particles having size between 4.75 mm (No. 4 sieve) and the admissible d_{max} [46,53–55]. The obtained samples can have a PSDC very different from that of the field material [2,56,57].

The fourth scaling down method, called the quadratic grain-size technique, produces a PSDC by following an equation that has nothing to do with the PSDC of field materials [10]. The physical meaning of the proposed modification is unclear, and it will not be discussed further in this study.

Until now, the parallel scaling down technique has been the most popular and the most widely used [10,15,17,30–32,38,39,57–65]. This is mainly because the parallel scaled down samples are considered to be the most faithful to the field material, due to the similarity between the PSDC of scaled down samples and that of the field material [10,32,39,40,50–52,58–61,65,66]. This is, however, a not valid justification. Recently, a critical review given by Deiminiat et al. [10] has shown that it is impossible to reproduce a PSDC strictly parallel to that of field material without adding fine particles smaller than the minimum particle size of the field material, as shown by Sukkarak et al. [64]. Adding finer particle material, either by grinding material or from a different source, results in an entirely different material from the field material. Changes in particle size and shape associated with particle breaking during sample preparation are other aspects that do not guarantee a faithful scaled down sample to the field material [10,58,61,63,65–70]. Therefore, none of the four scaling down techniques can be used to produce a scaled down sample faithful to the field material. In addition, the critical analysis of Deiminiat et al. [10] revealed that the methodology used in previous studies [33,39,46] to validate or invalidate scaling down techniques through direct comparisons between the effective internal friction angles (hereafter named "friction angle" for the sake of simplicity) of field materials and those of scaled down samples is inappropriate. The subsequent conclusion is invalid.

To correctly evaluate the reliability of a scaling down technique, a series of shear tests should be performed on several scaled down specimens having different d_{max} values. A relationship between the shear strength and d_{max} can then be established and used to predict the shear strength of the field material by applying the extrapolation technique [15,16,19,58,59,62,63,71–73]. This methodology was followed by several researchers [74,75]. However, their direct shear tests were performed by using a W/d_{max} ratio equal to or even smaller than the minimum required value of 10 stipulated by the ASTM standard, exactly the procedure carried out by other researchers [9,16,19–23,31,58,59,62,63,73,76]. Recently, Deiminiat et al. [10] have shown that the minimum required W/d_{max} ratio of 10, stipulated by the ASTM standard, is too small to eliminate SSEs. Deiminiat et al. [11]) further showed that the minimum required W/d_{max} ratio should be around 60 to avoid any SSE. The published experimental results obtained by using the minimum required W/d_{max} ratio of 10 and the subsequent conclusions are not reliable. The validation or invalidation of scaling down techniques shown in previous studies is questionable. Further validation or invalidation of the scaling down techniques is necessary against reliable experimental results. To this end, a series of direct shear tests were performed by using specimens having W/d_{max} ratios equal to or larger than 60, prepared by applying the scalping and parallel scaling down techniques; the replacement scaling down technique could not be applied because the d_{max} value of the "field" material is too close to the critical value of 4.75 mm. It is important to note that the shear strength of coarse granular materials is not only controlled by d_{max}, but also by particle shapes, content of fine or gravel particles, compact or relative density, water content, normal stress, specimen shape, etc. One methodology is to simultaneously consider all the influencing parameters together. This is good for a specific project of design and construction, but it is not suitable in research because the test results would be a consequence of the combined effects of several influencing factors. The results do not allow a good and accurate understanding of the effect of each individual influencing parameter. The unique scope of this paper is to verify the validity/invalidity of scaling down techniques associated with variation in the d_{max} value; thus, the only allowed changing parameter is the d_{max} value. For one given material, all other influencing parameters must be kept constant.

In this paper, some of the experimental results are presented. The test results are then used to test the validity of the scalping and parallel scaling down techniques through the processes of curve-fitting and prediction by extrapolation.

2. Laboratory Tests

2.1. Testing Materials

In this study, two types of waste rocks, called WR1 and WR2, were tested. Figure 1 shows a photograph of WR1 (Figure 1a) and a photograph of WR2 (Figure 1b). The two waste rocks contained a wide range of sub-angular and sub-rounded particles. They were used to prepare three testing materials, called M1, M2 and M3. M1 and M2 were made of WR1 and WR2, whereas M3 was made of WR2 based on the PSDC of WR1.

The largest shear box had a square section of 300 mm by 300 mm; therefore, the largest admissible d_{max} was 5 mm in order to have W/d_{max} ratios not smaller than 60 [11].

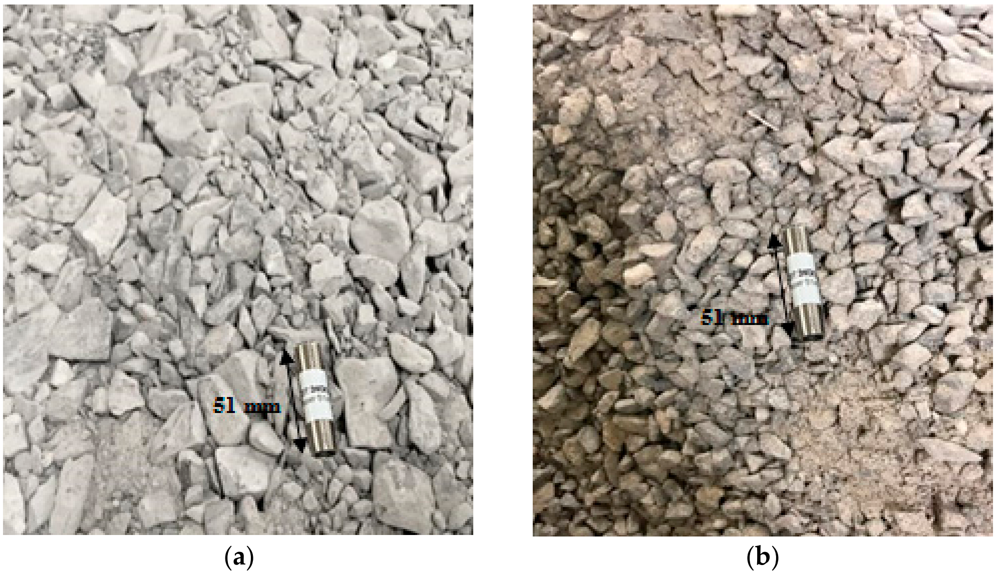

Figure 1. Photos of (**a**) WR1 and (**b**) WR2.

To prepare the testing samples with different d_{max} values, a portion of waste rocks was first sorted with sieves of opening sizes of 5, 3.36, 2.36, 1.4, 1.19, 0.85, 0.63, 0.315, 0.16 and 0.08 mm. Thus, all the particles larger than 5 mm were excluded. The obtained samples with d_{max} = 5 mm were considered as "field" materials. To avoid any confusion with the in situ field materials, the laboratory "field" materials are hereafter called field samples. Figure 2 shows the PSDCs of field samples of M1, M2 and M3.

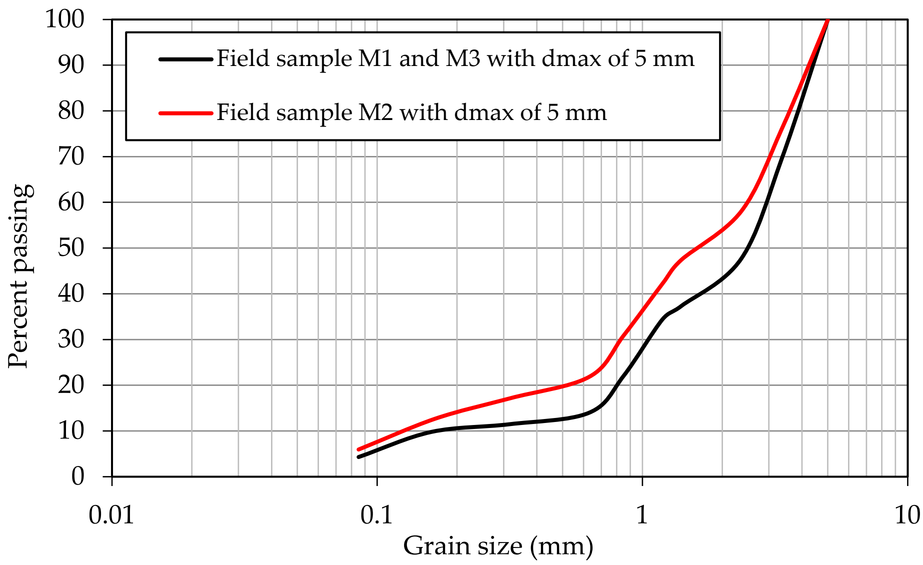

Figure 2. PSDCs of field samples M1, M2 and M3.

Table 1 shows the different portions of field samples M1, M2 and M3. They were used as the base materials for making scaled-down samples with d_{max} values of 1.19, 1.4, 2.36, and 3.36 mm by applying the scalping and parallel scaling down techniques.

To apply the scalping scaling down technique, one can either calculate the required mass of each range of particle size based on Table 1 to obtain a sample by controlled mixture (hereafter called the controlled scalped sample), or directly pour field sample through a sieve with the target d_{max} to obtain a sample without any control (hereafter called the uncontrolled scalped sample). For a given admissible d_{max}, the scalped samples obtained by applying the two methods should be identical. In reality, difference can appear as the source materials are not entirely homogeneous. In this study, controlled scalped samples were obtained from field sample M1, whereas uncontrolled scalped samples of field sample M2 were obtained directly from WR2. Controlled scalped samples made of field sample M3 were obtained by again considering the PSDC of field sample M1. Figure 3 shows the PSDC of scalped samples of M1 and M3 (Figure 3a) and those of M2 (Figure 3b); the PSDCs of field samples M1 to M3 are also plotted on the figure.

Table 1. Portion distributions of field samples M1, M2 and M3.

Range of Particle Size	Portion (%)		Sieve Opening Size (mm)	Passing (%)	
	M1 and M3	M2		M1 and M3	M2
3.36–5 mm	30.3	23.8	5	100.0	100.0
2.36–3.36 mm	22.0	18.1	3.36	69.7	76.2
1.40–2.36 mm	10.4	10.7	2.36	47.7	58.1
1.19–1.40 mm	2.9	5.2	1.4	37.3	47.4
0.85–1.19 mm	12.3	11.3	1.19	34.4	42.2
0.63–0.85 mm	8.0	9.1	0.85	22.1	30.9
0.315–0.63 mm	2.5	4.7	0.63	14.0	21.8
0.16–0.315 mm	1.8	4.5	0.315	11.5	17.1
0.08–0.16 mm	5.4	6.7	0.16	9.8	12.7
<0.08 mm	4.3	5.9	0.08	4.3	5.9

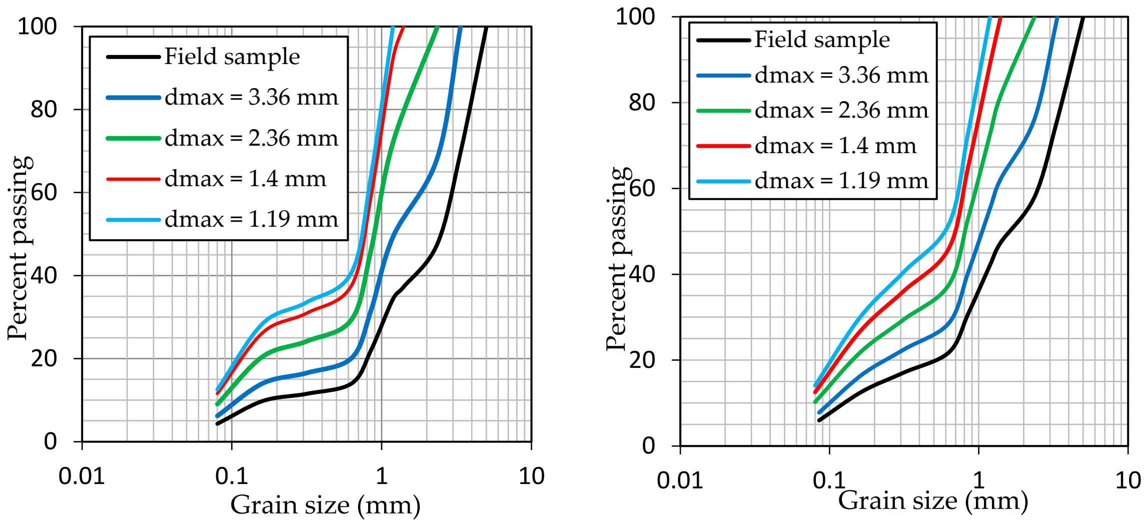

Figure 3. PSDCs of scalped samples of field sample M1 and M3 (**a**) and M2 (**b**).

To apply the parallel scaling down technique, one has to determine the required mass of each portion by considering the PSDC of the field sample and applying the following equation [50]:

$$d_{p.s} = d_{p.f}/N \tag{1}$$

where $d_{p.s}$ and $d_{p.f}$ are the particle sizes of the scaled down sample and field sample having a percentage passing p, respectively; N is the scaled down ratio between the d_{max} values of field and scaled down samples. The scaled down sample is thus a material obtained from the controlled mixture, not a fully natural material.

As an example, one explains how to prepare the parallel scaled down sample having d_{max} = 3.36 mm from the field sample with d_{max} = 5 mm. One first obtains the scaled down ratio N = 1.488 (=5 mm/3.36 mm). Afterwards, applying this scaled down ratio to all the ranges of particle sizes of the field sample results in new ranges of particle sizes for the scaled down sample, as shown in Table 2. For the calculated range of particle sizes, which do not have matched sieves, approximation has to be made, as shown in Table 2. In addition, fine particles in the range from 0.053 to 0.1 mm and fine particles smaller than 0.053 mm have to be added. The parallelism between the PSDCs of scaled down samples and field samples is impossible for these fine particle parts. The same method has been followed by several researchers [33,40,50,65,66,74]. An alternative method is addressed in Section 4.

Table 2. Calculation and selection of particle sizes for making parallel scaled samples with d_{max} = 3.36 mm for field samples M1, M2 and M3.

Range of Particle Size of Field Material	Range of Particle Sizes of Parallel Scaled Down Samples		Portion (%)	
	Calculated	Chosen	M1 and M3	M2
3.36–5 mm	2.26–3.36 mm	2.36–3.36 mm	30.3	23.8
2.36–3.36 mm	1.60–2.26 mm	1.60–2.36 mm	22.0	18.1
1.40–2.36 mm	0.95–1.60 mm	1.0–1.60 mm	10.4	10.7
1.19–1.40 mm	0.80–0.95 mm	0.80–1.0 mm	2.9	5.2
0.85–1.19 mm	0.56–0.80 mm	0.56–0.80 mm	12.3	11.3
0.63–0.85 mm	0.42–0.56 mm	0.42–0.56 mm	8.0	9.1
0.315–0.63 mm	0.21–0.42 mm	0.21–0.42 mm	2.5	4.7
0.16–0.315 mm	0.10–0.21 mm	0.10–0.21 mm	1.8	4.5
0.08–0.16 mm	0.053–0.10 mm	0.053–0.10 mm	5.4	6.7
<0.08 mm	<0.053 mm	<0.053 mm	4.3	5.9

Figure 4 shows the PSDC of parallel scaled down samples made of field samples M1 and M3 (Figure 4a) and M2 (Figure 4b); the PSDCs of the field samples are also plotted on the figure.

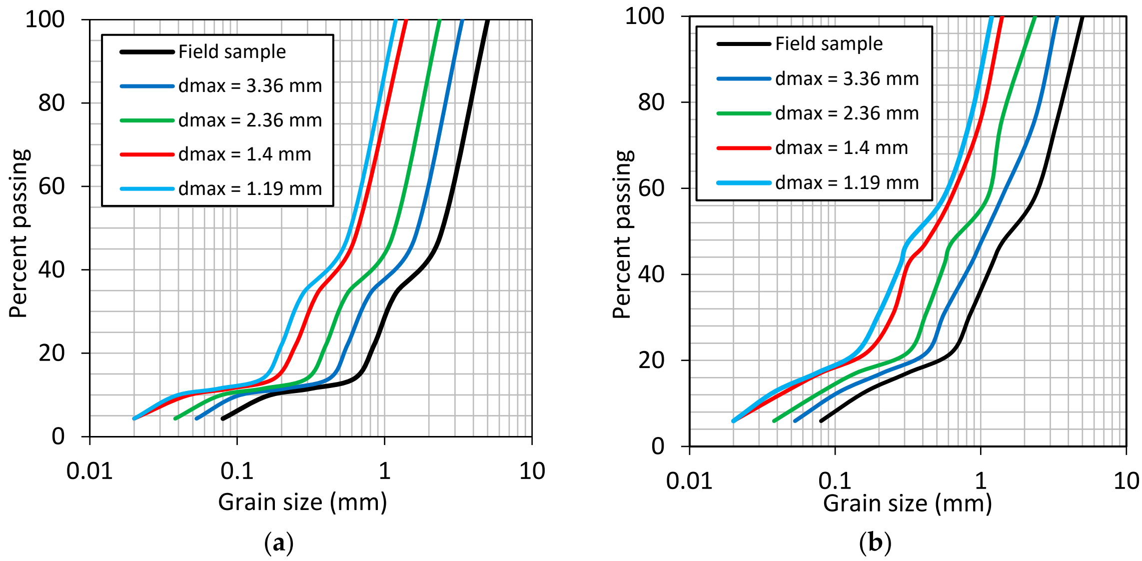

Figure 4. PSDCs of the field sample and parallel samples with different d_{max} values for: (**a**) M1 and M3; (**b**) M2.

2.2. Direct Shear Tests

In the geotechnical laboratory of Polytechnique Montreal, several square shear boxes are available. Only two (the large one, with dimensions of 300 mm × 300 mm × 180 mm, and the small one, 100 mm × 100 mm × 45 mm) have been used to have W/d_{max} ratios not smaller than 60.

As previously outlined, the scope of this study is to analyze the reliability of scaling down techniques. It is thus very important to ensure that variations in the measured friction angle of one given material prepared by following one scaling down technique are only due to the variations in d_{max}, instead of a result due to the combined effects of several influencing factors. The scaled down samples and the field samples should have the same compactness (void ratio) and the same moisture content, under the same normal stresses. All the samples were thus prepared with dry waste rocks. Another advantage associated with dry materials is the removal of any possible influence of loading rate on the shear test results [33,77]. The tested specimens were prepared by slowly placing the materials in the shear boxes to determine the loosest state. The density of the loosest field sample was first obtained by considering the volume of the large shear box of 300 mm × 300 mm × 180 mm and the mass of the filled material at the loosest state. The specific gravity (G_s) of the sample was measured to be equal to 2.65 by following ASTM C128 [78]. The maximum void ratio (e_{max}) of the loosest field sample can then be obtained. To obtain the same void ratio and density for scaling down specimens, the required masses were calculated by using the volume of the large shear box and the values of G_s and e_{max} of the field sample.

Table 3 shows the tested specimens along with their specimen sizes to d_{max} ratios and e_{max}. For each sample, direct shear tests were repeated three times to obtain three values of friction angle; each value was obtained by performing three direct shear tests with normal stresses of 50, 100 and 150 kPa, respectively. These values are relatively small, due to the limited capacity of air compressor on the large size specimens of 300 mm × 300 mm. The void ratios of the tested specimens after the application of the normal stresses before applying shear strains were estimated and are presented in Table 4. It can be seen that the void ratios of the tested specimens decrease slightly as the applied normal stress increases from 0 to 50 kPa. The decrease degree becomes smaller when the normal stress is further increased from 50 to 150 kPa. When the system became stable, shear loads were applied by using a strain rate of 0.025 mm/s (1.5 mm/min). A total number of 243 direct shear tests were performed to complete the test program in this study.

Figure 5 shows typical shear stress–shear displacement curves obtained with the large shear box on the field and scaled down samples of M1 (graphs on left column), M2 (graphs in the center column) and M3 (graphs on right column). One sees that the shear stress and displacement curves of different specimens of the same material under a given normal stress (σ_n) have the same variation trend. For example, for M1 under a normal stress of 50 kPa, all the shear stress and displacement curves of field, scalped and parallel scaled down samples exhibit a loose sand-like mechanical behavior. This indicates that the tested samples are all very loose and their compactness states are close to each other.

Table 3. Tested specimens along with sizes to d_{max} ratios and e_{max} for M1, M2 and M3.

Samples	d_{max} (mm)	e_{max} [79]			Large Shear Box		Small Shear Box	
		M1	M2	M3	W/d_{max}	T/d_{max}	W/d_{max}	T/d_{max}
Field sample	5	0.59	0.70	0.68	60	36	–	–
Scalping down technique samples	3.36	0.58	0.69	0.66	89	54	–	–
	2.36	0.57	0.68	0.68	127	76	–	–
	1.4	0.60	0.67	0.65	214	129	71	32
	1.19	0.60	0.66	0.67	252	151	84	38
Parallel scaling down technique samples	3.36	0.58	0.68	0.66	89	54	–	–
	2.36	0.61	0.67	0.65	127	76	–	–
	1.4	0.60	0.68	0.68	214	129	71	32
	1.19	0.62	0.67	0.66	252	151	84	38

Table 4. Void ratio (e) of the tested specimens after the application of normal stresses (σ_n) before applying shear strains.

Samples	d_{max} (mm)	e of M1 under σ_n of			e of M2 under σ_n of			e of M3 under σ_n of		
		50 kPa	100 kPa	150 kPa	50 kPa	100 kPa	150 kPa	50 kPa	100 kPa	150 kPa
Field sample	5	0.52	0.51	0.49	0.64	0.63	0.63	0.62	0.62	0.61
Scalping down technique samples	3.36	0.50	0.49	0.48	0.65	0.62	0.62	0.60	0.60	0.60
	2.36	0.49	0.48	0.48	0.64	0.61	0.61	0.63	0.63	0.62
	1.4	0.52	0.51	0.50	0.63	0.60	0.61	0.60	0.60	0.59
	1.19	0.52	0.50	0.50	0.63	0.60	0.60	0.62	0.61	0.61
Parallel down technique samples	3.36	0.51	0.50	0.49	0.62	0.61	0.60	0.60	0.60	0.59
	2.36	0.52	0.51	0.50	0.63	0.61	0.61	0.61	0.60	0.59
	1.4	0.53	0.52	0.50	0.64	0.62	0.61	0.62	0.62	0.61
	1.19	0.53	0.52	0.51	0.63	0.61	0.61	0.60	0.60	0.60

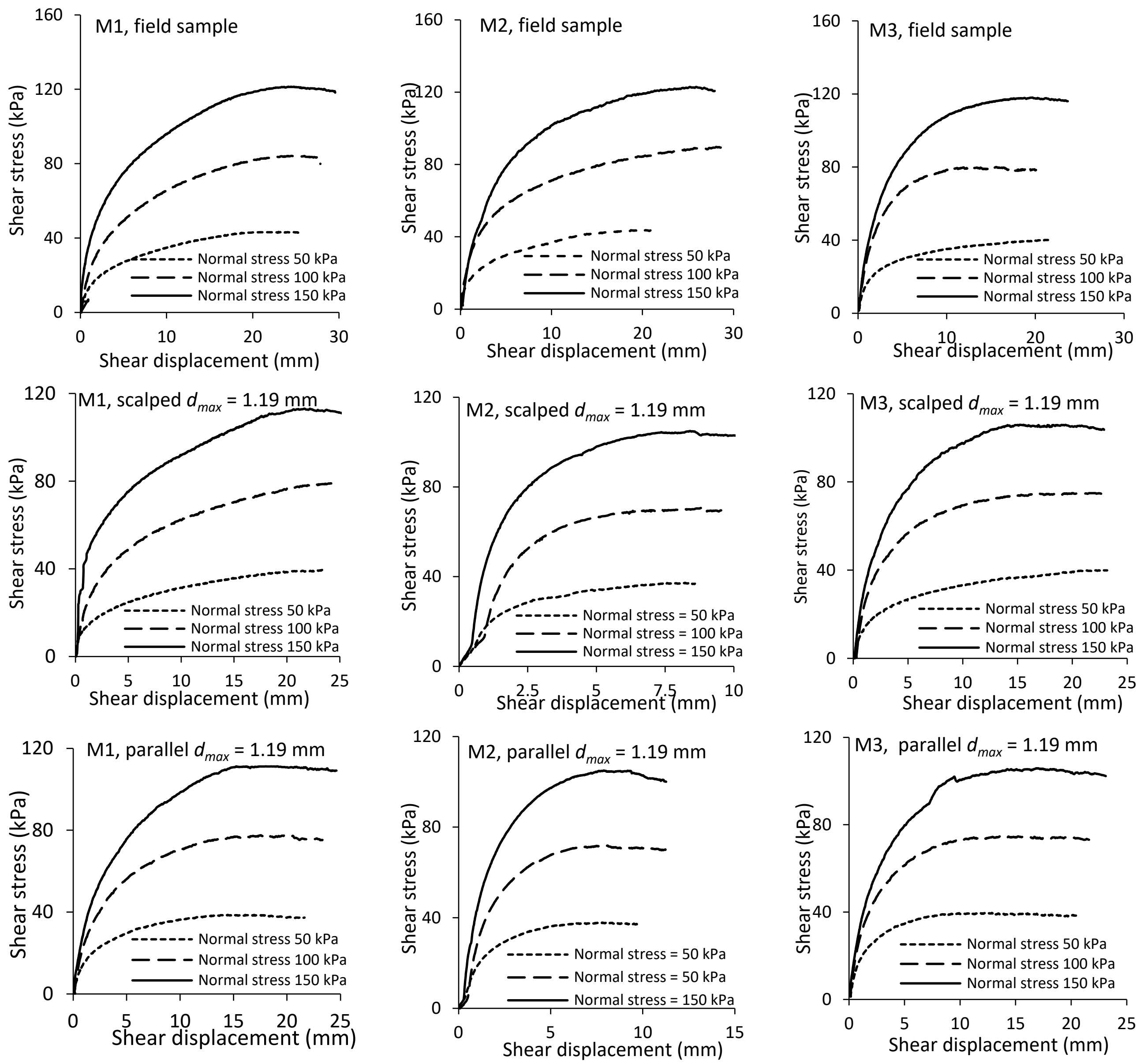

Figure 5. Shear stress vs. shear displacement curves obtained with the large shear box on field samples and scaled down samples with d_{max} value of 1.19 mm for M1 (graphs on left column), M2 (graphs in the center column) and M3 (graphs on right column).

2.3. Experimental Results

For each sample with three shear stress–shear displacement curves obtained by direct shear tests under three normal stresses, three peak shear strength values can be obtained. A friction angle can then be determined by linear fitting on the three points.

Table 5 presents the friction angles of the field, scalping and parallel scaled down samples with different d_{max} values for the three materials. The average friction angles were then calculated for each sample. Notably, all the friction angles increased as d_{max} increased, even though the tested waste rocks had sub-angular and sub-rounded shapes (see Figure 1). This trend shows a typical behavior of rounded particle materials, not angular or sub-angular materials. This aspect is further addressed in Section 4.

Table 5. Measured friction angles (φ) of the field and scaled down samples made of M1, M2 and M3.

Samples	d_{max} (mm)	M1		M2		M3	
		φ (°)	Avg. φ (°)	φ (°)	Avg. φ (°)	φ (°)	Avg. φ (°)
Field sample	5	39.5 40.1 39.8	39.8	38.7 38.6 38.9	38.7	38.4 38.3 38.7	38.5
Scalping down technique samples	3.36	38.7 39.1 39.2	39.0	37.6 37.5 37.3	37.5	37.4 38.1 37.8	37.8
	2.36	37.9 38.4 38.2	38.2	37.3 37.2 36.6	37.0	37.1 37.5 36.9	37.2
	1.40	37.6 37.2 37.1	37.3	35.4 35.8 35.6	35.6	36.2 35.9 35.6	35.9
	1.19	37.2 37.0 37.0	37.1	35.2 35.5 35.7	35.5	35.9 35.7 35.3	35.6
Parallel down technique samples	3.36	39.0 38.7 38.0	38.6	37.1 37.9 38.5	37.8	37.5 36.8 37.2	37.2
	2.36	38.7 38.1 37.2	38.0	36.2 36.6 37.2	36.6	37.3 36.8 36.1	36.7
	1.4	37.4 36.9 37.1	37.1	35.5 35.8 36.8	36.0	36.4 35.7 36.0	36.0
	1.19	37.0 36.1 35.7	36.3	35.4 35.9 36.5	35.9	35.9 35.1 35.6	35.5

3. Validation of Scaling Down Techniques

The friction angles obtained by direct shear tests on scaled down samples prepared by applying scalping and parallel scaling down techniques are first used to establish relationships between friction angle φ and d_{max} values.

Figure 6 shows the variation of the average friction angle as function of d_{max} for samples M1 (Figure 6a), M2 (Figure 6b) and M3 (Figure 6c). The relationships established by applying curve-fitting technique on the test results of scaled down samples are presented in Table 6. The measured friction angles of field samples are also plotted on Figure 6, whereas the friction angles of the field samples predicted by applying the curve-fitting equations are presented in Table 6. From the figure, one sees that the friction angles of the field samples can be correctly predicted by the curve-fitting equations of scalped samples, but fail to be predicted by the curve-fitting equations of parallel scaled down samples. These results thus tend to indicate that the scalping scaling down technique can be used for sample preparation in direct shear tests, whereas the parallel scaling down technique is not appropriate for sample preparation in direct shear tests.

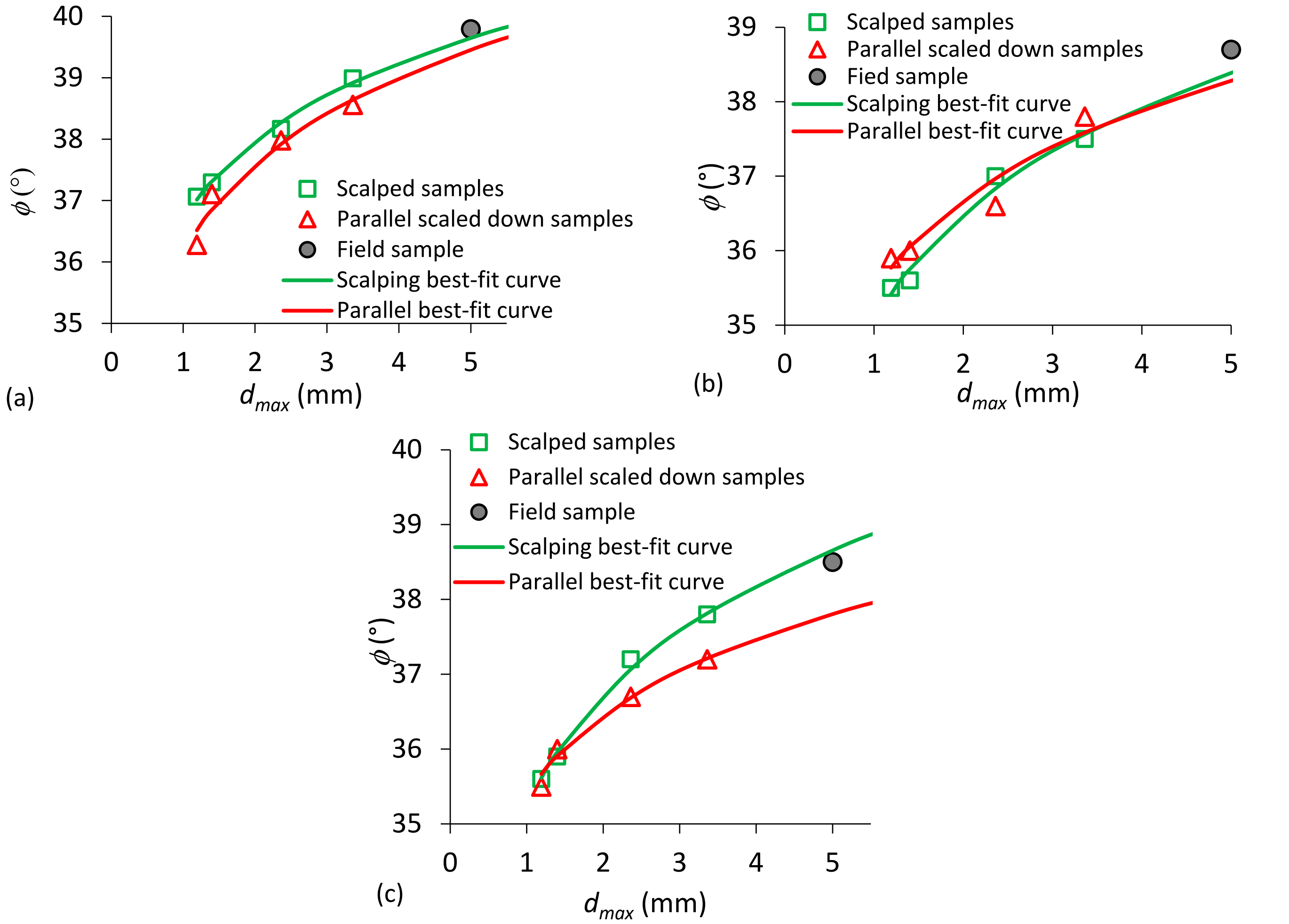

Figure 6. Variations of average φ values as a function of d_{max}, obtained by direct shear tests on scaled down and field samples (**a**) M1, (**b**) M2 and (**c**) M3.

Table 6. The φ values of field samples measured and predicted by applying the scalping and parallel prediction equations for field samples of M1, M2 and M3.

Material	Scaling down Technique	Curve Fitting Equations Based on the Test Results of Scaled Down Samples	R^2	Friction Angle φ (°) of Field Samples (d_{max} = 5 mm)	
				Predicted	Measured
M1	Scalping	$\varphi = 1.834718\ \mathrm{Ln}(d_{max}) + 36.6953$	0.97	39.6	39.8
	Parallel	$\varphi = 2.045707\ \mathrm{Ln}(d_{max}) + 36.16165$	0.86	39.4	
M2	Scalping	$\varphi = 2.070025\ \mathrm{Ln}(d_{max}) + 35.05597$	0.96	38.4	38.7
	Parallel	$\varphi = 1.749663\ \mathrm{Ln}(d_{max}) + 35.46517$	0.80	38.3	
M3	Scalping	$\varphi = 2.118069\ \mathrm{Ln}(d_{max}) + 35.24663$	0.96	38.7	38.5
	Parallel	$\varphi = 1.488585\ \mathrm{Ln}(d_{max}) + 35.4078$	0.86	37.8	

4. Discussion

In this paper, the reliability of scalping and parallel scaling down techniques used to prepare samples for direct shear tests has been evaluated through experimental studies. All the direct tests have been performed by using W/d_{max} ratios not smaller than 60, a value recently established by Deiminiat et al. [11] to avoid any SSE. Equations were established by applying a curve-fitting technique on the test results of the scaled down sample. They were then used to predict the friction angles for field samples. The comparisons between the measured and predicted friction angles of field samples tended to show that the scalping technique can be used to predict the friction angle of field samples, whereas the application of a parallel scaling down technique cannot guarantee a reliable prediction of the friction angle of field materials. Despite these interesting results, however, the test program was realized with several limitations. For instance, the three samples (M1, M2 and M3) were made of two types of dry waste rocks. The direct shear tests were realized by delicately placing the materials in shear boxes to reach the loosest state. This was to ensure that the variations in the test results are only due to the variation in d_{max} value for one given material with one chosen scaling down technique. More tests are needed where tested samples are prepared with more materials of different source origins having different particle shapes, initial fine and gravel contents, compactness, and moisture contents under different ranges of normal stresses to determine whether the conclusions are generally valid or only specifically valid for the tested (specific) materials under the tested (specific) conditions. In addition, the differences between the d_{max} values of scaled down and field samples are not very large. More experimental work is thus necessary, using larger shear boxes with field samples having larger d_{max} values. The reliability of the replacement scaling down technique can also be studied. In all cases, it is important to note that any new tests should be performed by following the methodology presented in this paper.

In this study, the parallel scaled down samples were prepared by considering a given percentage and reducing the ranges of particle sizes. Approximations had to be made for the calculated sizes, which did not have any match with available sieves [33,40,50,65,66,74]. In future, the following process of preparation can be considered:

- Calculate the scaled down ratio N;
- Draw the PSDC of the scaled down sample, which is parallel to the PSDC of the field sample in the semi-log plane;
- Determine the percentage of each available sieve.

Most previous studies showed a decreasing trend in the friction angle of sub-angular and angular materials as d_{max} increased [10,15,31,32,40,58,59,62,63,74,80]; however, the experimental results obtained with sub-angular and sub-rounded materials presented in Table 5 and Figure 6 show an increase in the friction angles as d_{max} increases. This difference is probably due to the fact that most previous experimental studies were realized by using large confining pressures. Large shear stresses were thus necessary to shear the tested samples. Particle crushing during the application of confining and/or shear stresses could be an associated and pronounced phenomenon [34,38,70,81,82]. The decrease in friction angle with increasing d_{max} was explained by the breakage of rock particles. The strength of rock decreases with specimen size, known as the size effect of rock strength [83–85]; therefore, the friction angle of coarse particle materials decreases with increasing d_{max} values [34]. In this study, however, the maximum value of the normal stresses was 150 kPa. The PSDCs of tested samples before and after shear tests shown in Figure 7 clearly indicate that there was no particle crushing or breakage during and after the application of normal and shear stresses. Size effects of rock strength were not involved. The trend in friction angle obtained in this study corresponded to what is usually observed in practice: at the same compact state, sand usually has a smaller friction angle than rockfill because the former usually has smaller d_{max} values than the latter.

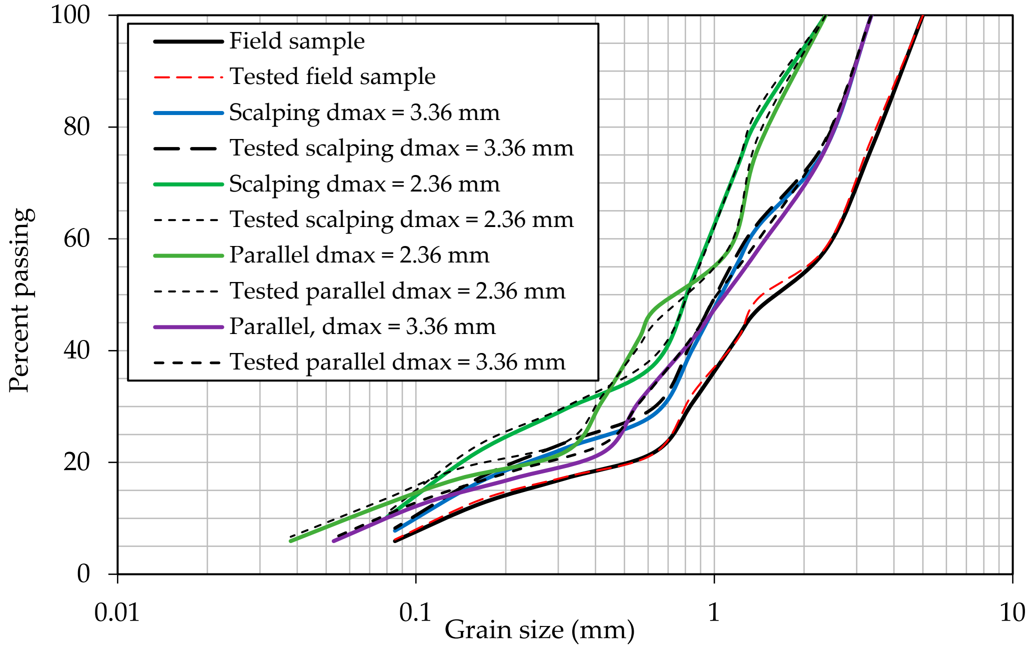

Figure 7. PSDCs of scaled down and field samples of M2 before (solid lines) and after (dashed lines) direct shear tests.

Finally, because scaling down techniques are not only used in direct shear tests, but also used in triaxial compression tests, more experimental investigation is necessary, performing triaxial compression tests with scaled down samples to test the validity of the scaling down technique. Of course, the tested specimens used in triaxial compression tests must be large enough to avoid any SSE [11].

5. Conclusions

In this study, the validity of scalping and parallel scaling down techniques used to prepare samples for direct shear tests has been for the first time evaluated through experimental work by using W/d_{max} ratios not smaller than 60. The test results are thus exempt from SSE. The experimental results show that the friction angles with scaled down samples prepared by both scalping and parallel scaling down techniques decrease as the d_{max} values increase even though the particle shapes are not rounded. This variation trend is quite different from that presented in the literature, probably due to the low normal stresses applied in this study. In addition, the comparisons between the friction angles obtained by measurements and those predicted by applying curve-fitting equations established on the friction angles of scaled down samples indicate that the scalping technique can be used to predict the friction angle of field samples, whereas the application of parallel scaling down technique cannot guarantee a reliable prediction on the friction angle of field materials.

Author Contributions: Conceptualization, L.L. and A.D.; methodology, A.D. and L.L.; formal analysis, A.D. and L.L.; investigation, A.D. and L.L.; writing—original draft preparation, A.D.; writing—review and editing, L.L. and A.D.; supervision, L.L.; project administration, L.L.; funding acquisition, L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Natural Sciences and Engineering Research Council of Canada (NSERC RGPIN-2018-06902) and Fonds de recherche du Québec—Nature et Technologies (FRQNT 2017-MI-202860).

Data Availability Statement: Not applicable.

Acknowledgments: The authors acknowledge the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC RGPIN-2018-06902), Fonds de recherche du Québec—Nature et Technologies (FRQNT 2017-MI-202860), and industrial partners of the Research Institute on Mines and the Environment (RIME UQAT-Polytechnique; http://rime-irme.ca/ (accessed on 7 December 2021)). Samuel Chenier, Eric Turgeon and Noura El-Harrak are gratefully acknowledged for their assistance in the laboratory work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. *ASTM D3080/D3080M-11*; Standard Test Method for Direct Shear Test of Soils under Consolidated Drained Conditions (Withdrawn 2020). ASTM International: West Conshohocken, PA, USA, 2011.
2. Rathee, R.K. Shear strength of granular soils and its prediction by modeling techniques. *J. Inst. Eng.* **1981**, *62*, 64–70.
3. Palmeira, E.M.; Milligan, G.W.E. Scale effects in direct shear tests on sand. In Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio De Janeiro, Brazil, 13–18 August 1989; Volume 1, pp. 739–742.
4. Cerato, A.; Lutenegger, A. Specimen size and scale effects of direct shear box tests of sands. *Geotech. Test. J.* **2006**, *29*, 507–516.
5. Wu, P.-K.; Matsushima, K.; Tatsuoka, F. Effects of specimen size and some other factors on the strength and deformation of granular soil in direct shear tests. *Geotech. Test. J.* **2008**, *31*, 45–64.
6. Mirzaeifar, H.; Abouzar, A.; Abdi, M.R. Effects of direct shear box dimensions on shear strength parameters of geogrid-reinforced sand. In Proceedings of the GeoMontreal 2013, Montreal, QC, Canada, 29 September–3 October 2013.
7. Omar, T.; Sadrekarimi, A. Effect of triaxial specimen size on engineering design and analysis. *Int. J. Geo-Eng.* **2015**, *6*, 1–17. [CrossRef]
8. Moayed, R.Z.; Alibolandi, M.; Alizadeh, A. Specimen size effects on direct shear test of silty sands. *Int. J. Geotech. Eng.* **2016**, *11*, 198–205. [CrossRef]
9. Zahran, K.; Naggar, H.E. Effect of sample size on TDA shear strength parameters in direct shear tests. *Transp. Res. Rec.* **2020**, *2674*, 1110–1119. [CrossRef]
10. Deiminiat, A.; Li, L.; Zeng, F.; Pabst, T.; Chiasson, P.; Chapuis, R. Determination of the shear strength of rockfill from small-scale laboratory shear tests: A critical review. *Adv. Civ. Eng.* **2020**, *2020*, 1–18. [CrossRef]
11. Deiminiat, A.; Li, L.; Zeng, F. Experimental study on the minimum required specimen width to maximum particle size ratio in direct shear tests. *CivilEng* 2021, *Submitted*.
12. Drugan, W.; Willis, J. A micromechanics-based nonlocal constitutive equation and estimates of representative volume element size for elastic composites. *J. Mech. Phys. Solids* **1996**, *44*, 497–524. [CrossRef]
13. Kanit, T.; Forest, S.; Galliet, I.; Mounoury, V.; Jeulin, D. Determination of the size of the representative volume element for random composites: Statistical and numerical approach. *Int. J. Solids Struct.* **2003**, *40*, 3647–3679. [CrossRef]
14. Wen, R.; Tan, C.; Wu, Y.; Wang, C. Grain size effect on the mechanical behavior of cohesionless coarse-grained soils with the discrete element method. *Adv. Civ. Eng.* **2018**, *2018*, 4608930. [CrossRef]
15. Gupta, A.K. Triaxial behavior of rockfill materials. *Electron. J. Geotech. Eng.* **2009**, *14*, 1–18.
16. Pankaj, S.; Mahure, N.; Gupta, S.; Sandeep, D.; Devender, S. Estimation of shear strength of prototype rockfill materials. *Int. J. Eng. Sci.* **2013**, *2*, 421–426.
17. Honkanadavar, N.P.; Kumar, N.; Ratnam, M. Modeling the behavior of alluvial and blasted quarried rockfill materials. *Geotech. Geol. Eng.* **2014**, *32*, 1001–1015. [CrossRef]
18. Amirpour Harehdasht, S.; Hussien, M.N.; Karray, M.; Roubtsova, V.; Chekired, M. Influence of particle size and gradation on shear strength–dilation relation of granular materials. *Can. Geotech. J.* **2019**, *56*, 208–227. [CrossRef]
19. Zhang, Z.; Sheng, Q.; Fu, X.; Zhou, Y.; Huang, J.; Du, Y. An approach to predicting the shear strength of soil-rock mixture based on rock block proportion. *Bull. Int. Assoc. Eng. Geol.* **2019**, *79*, 2423–2437. [CrossRef]
20. Cai, H.; Wei, R.; Xiao, J.Z.; Wang, Z.W.; Yan, J.; Wu, S.F.; Sun, L.M. Direct shear test on coarse gap-graded fill: Plate opening size and its effect on measured shear strength. *Adv. Civ. Eng.* **2020**, *2020*, 1–13. [CrossRef]
21. Nicks, J.E.; Gebrenegus, T.; Adams, M.T. Interlaboratory large-scale direct shear testing of open-graded aggregates: Round one. In Proceedings of the IFCEE 2021, Dallas, TX, USA, 10–14 May 2021; pp. 361–370. [CrossRef]
22. Rasti, A.; Adarmanabadi, H.R.; Pineda, M.; Reinikainen, J. Evaluating the effect of soil particle characterization on internal friction angle. *Am. J. Eng. Appl. Sci.* **2021**, *14*, 129–138. [CrossRef]
23. Saberian, M.; Li, J.; Perera, S.T.A.M.; Zhou, A.; Roychand, R.; Ren, G. Large-scale direct shear testing of waste crushed rock reinforced with waste rubber as pavement base/subbase materials. *Transp. Geotech.* **2021**, *28*, 100546. [CrossRef]
24. *AS 1289.6.2.2*; Soil Strength and Consolidation Tests-Determination of the Shear Strength of a Soil-Direct Shear Test Using a Shear Box. Standards Australia: Sydney, Australia, 1998.

25. *Eurocode 7*; Geotechnical Design-Part 1: General Rules. EN 1997–1. British Standards: London, UK, 2004.
26. BS 1377. *Methods of Test for Soils for Civil Engineering Purposes*; Shear Strength Tests (Total Stress); British Standard Institution: London, UK, 1990.
27. Hall, E.B. A triaxial apparatus for testing large soil specimens. In *Triaxial Testing of Soils and Bitminous Mixtures*; ASTM: West Conshohocken, PA, USA, 1951; Volume 106, p. 152. [CrossRef]
28. Holtz, W.; Gibbs, H.J. Triaxial shear tests on pervious gravelly soils. *J. Soil Mech. Found. Div.* **1956**, *82*, 1–22. [CrossRef]
29. Leslie, D. Large scale triaxial tests on gravelly soils. In Proceedings of the Second Panamerican Conference on Soil Mechanics and Foundation Engineering, Sao Paulo, Brazil, 1 July 1963; Volume 1, pp. 181–202.
30. Marachi, N.; Seed, H.; Chan, C. Strength characteristics of rockfill materials. In Proceedings of the 7th International Conference on Soil Mechanics and Foundation Engineering, Mexico City, Mexico, 25–29 August 1969; pp. 217–224.
31. Marachi, N.D.; Chan, C.K.; Seed, H.B. Evaluation of properties of rockfill materials. *J. Soil Mech. Found. Div.* **1972**, *98*, 95–114. [CrossRef]
32. Varadarajan, A.; Sharma, K.G.; Venkatachalam, K.; Gupta, A.K. Testing and modeling two rockfill materials. *J. Geotech. Geoenviron. Eng.* **2003**, *129*, 206–218. [CrossRef]
33. Hamidi, A.; Azini, E.; Masoudi, B. Impact of gradation on the shear strength-dilation behavior of well graded sand-gravel mixtures. *Sci. Iran.* **2012**, *19*, 393–402. [CrossRef]
34. Ovalle, C.; Frossard, E.; Dano, C.; Hu, W.; Maiolino, S.; Hicher, P.-Y. The effect of size on the strength of coarse rock aggregates and large rockfill samples through experimental data. *Acta Mech.* **2014**, *225*, 2199–2216. [CrossRef]
35. Chang, W.-J.; Phantachang, T. Effects of gravel content on shear resistance of gravelly soils. *Eng. Geol.* **2016**, *207*, 78–90. [CrossRef]
36. Yang, G.; Jiang, Y.; Nimbalkar, S.; Sun, Y.; Li, N. Influence of particle size distribution on the critical state of rockfill. *Adv. Civ. Eng.* **2019**, *2019*, 1–7. [CrossRef]
37. Dorador, L.; Villalobos, F.A. Scalping techniques in geomechanical characterization of coarse granular materials. *Obras Proy.* **2020**, *28*, 24–34. [CrossRef]
38. Ovalle, C.; Linero, S.; Dano, C.; Bard, E.; Hicher, P.-Y.; Osses, R. Data compilation from large drained compression triaxial tests on coarse crushable rockfill materials. *J. Geotech. Geoenviron. Eng.* **2020**, *146*, 06020013. [CrossRef]
39. Kouakou, N.; Cuisinier, O.; Masrouri, F. Estimation of the shear strength of coarse-grained soils with fine particles. *Transp. Geotech.* **2020**, *25*, 100407. [CrossRef]
40. MotahariTabari, S.; Shooshpasha, I. Evaluation of coarse-grained mechanical properties using small direct shear test. *Int. J. Geotech. Eng.* **2018**, *15*, 667–679. [CrossRef]
41. Hennes, R.G. The strength of gravel in direct shear. In *International Symposium on Direct Shear Testing of Soils*; ASTM: West Conshohocken, PA, USA, 1953; Volume 131, pp. 51–62.
42. Zeller, J.; Wullimann, R. The shear strength of the shell materials for the Go-Schenenalp Dam, Switzerland. In Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering, London, UK, 12–24 August 1957; Volume 2, pp. 399–415.
43. Morgan, C.C.; Harris, M.C. Portage mountain dam—II materials. *Can. Geotech. J.* **1967**, *4*, 142–166. [CrossRef]
44. Hall, E.B.; Smith, T. Special tests for design of high earth embankments on US-101. In *Highway Research Record*; University of Michigan: Ann Arbor, MI, USA, 1971.
45. Williams, D.J.; Walker, L.K. *Laboratory and Field Strength of Mine Waste Rock*; Research Report No. CE 48; University of Queensland: Brisbane, Australia, 1983.
46. Donaghe, R.T.; Torrey, V.H. *Strength and Deformation Properties of Earth-Rock Mixtures*; No. WES/TR/GL-85-9; Army Engineer Waterways Experiment Station Vicksburg, Geotechnical Lab, USACE: Vicksburg, MI, USA, 1985.
47. Seif El Dine, B.S.; Dupla, J.C.; Frank, R.; Canou, J.; Kazan, Y. Mechanical characterization of matrix coarse-grained soils with a large-sized triaxial device. *Can. Geotech. J.* **2010**, *47*, 425–438. [CrossRef]
48. Fragaszy, R.J.; Su, J.; Siddiqi, F.H.; Ho, C.L. Modeling strength of sandy gravel. *J. Geotech. Eng.* **1992**, *118*, 920–935. [CrossRef]
49. Bareither, C.A.; Benson, C.H.; Edil, T.B. Comparison of shear strength of sand backfills measured in small-scale and large-scale direct shear tests. *Can. Geotech. J.* **2008**, *45*, 1224–1236. [CrossRef]
50. Lowe, J. Shear strength of coarse embankment dam materials. In Proceedings of the 8th International Congress on Large Dams, Edinburgh, UK, 4–8 May 1964; Volume 3, pp. 745–761.
51. Tombs, S.G. Strength and Deformation Characteristics of Rockfill. Ph.D. Thesis, Imperial College, London, UK, 1969.
52. Charles, J.A. Correlation between Laboratory Behaviour of Rockfill and Field Performance with Particular Preference to Scammonden Dam. Ph.D. Thesis, Imperial College, London, UK, 1973.
53. Frost, R.J. Some testing experiences and characteristics of boulder-gravel fills in earth dams. In *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesion-Less Soils*; ASTM International: West Conshohocken, PA, USA, 1973; pp. 207–233.
54. Donaghe, R.T.; Townsend, F.C. *Compaction Characteristics of Earth-Rock Mixtures*; Report 1, No. WES-MP-S-73-25-1; Army Engineer Waterways Experiment Station Vicksburg: Vicksburg, MS, USA, 1973.

55. Houston, W.; Houston, S.L.; Walsh, K.D. Compacted high gravel content subgrade materials. *J. Transp. Eng.* **1994**, *120*, 193–205. [CrossRef]
56. Donaghe, R.T.; Townsend, F.C. Scalping and replacement effects on the compaction characteristics of earth-rock mixtures. In *Soil Specimen Preparation for Laboratory Testing*; ASTM: West Conshohocken, PA, USA, 1976; pp. 248–277.
57. Feng, G.; Vitton, S.J. Laboratory determination of compaction criteria for rockfill material embankments. In Proceedings of the International Conference on Soil Mechanics and Foundation Engineering, Hamburg, Germany, 6–12 September 1997; Volume 1, pp. 485–488.
58. Varadarajan, A.; Sharma, K.G.; Abbas, S.M.; Dhawan, A.K. The role of nature of particles on the behavior of rockfill material. *Soils Found.* **2006**, *46*, 569–584. [CrossRef]
59. Abbas, S.M. *Behavior of Rockfill Materials (Based on Nature of Particles)*; Lambert Academic Publishing: Saarbrücken, Germany, 2011.
60. Rao, S.V.; Bajaj, S.; Dhanote, S. Evaluations of strength parameters of rockfill material for Pakaldul hydroelectric project, Jammu and Kashmir—A case study. In Proceedings of the Indian Geotechnical Conference, Kochi, India, 15–17 December 2011; pp. 991–994.
61. Stober, J.N. Effects of Maximum Particle Size and Sample Scaling on the mechanical Behavior of Mine Waste Rock; A Critical State Approach. Master's Thesis, Colorado State University, Fort Collins, CO, USA, 2012.
62. Vasistha, Y.; Gupta, A.K.; Kanwar, V. Medium triaxial testing of some rockfill materials. *Electron. J. Geotech. Eng.* **2013**, *18*, 923–964.
63. Honkanadavar, N.P.; Dhanote, S.; Bajaj, S. Prediction of shear strength parameter for prototype alluvial rockfill material. In Proceedings of the Indian Geotechnical Conference, Chennai, India, 15–17 December 2016.
64. Sukkarak, R.; Pramthawee, P.; Jongpradist, P.; Kongkitkul, W.; Jamsawang, P. Deformation analysis of high CFRD considering the scaling effects. *Geomech. Eng.* **2018**, *14*, 211–224.
65. Ovalle, C.; Dano, C. Effects of particle size–strength and size–shape correlations on parallel grading scaling. *Geotech. Lett.* **2020**, *10*, 191–197. [CrossRef]
66. Dorador, L.; Villalobos, F.A. Analysis of the geomechanical characterization of coarse granular materials using the parallel gradation method. *Obras y Proy.* **2020**, *27*, 50–63. [CrossRef]
67. Lee, K.L.; Seed, H.B. Drained strength characteristics of sands. *J. Soil Mech. Found. Div.* **1967**, *93*, 117–141. [CrossRef]
68. Charles, J.A.; Watts, K.S. The influence of confining pressure on the shear strength of compacted rockfill. *Geotechnique* **1980**, *30*, 353–367. [CrossRef]
69. Ramamurthy, T. A geo-engineering classification for rocks and rock masses. *Int. J. Rock Mech. Min. Sci.* **2004**, *41*, 89–101. [CrossRef]
70. Wang, Y.; Shao, S.; Wang, Z. Effect of particle breakage and shape on the mechanical behaviors of granular materials. *Adv. Civ. Eng.* **2019**, *2019*, 1–15. [CrossRef]
71. Wei, H.; Frossard, E.; Hicher, P.Y.; Dano, C. Method to evaluate the shear strength of granular material with large particles. In Proceedings of the Soil Behavior and Geo-Micromechanics, GeoShanghai 2010 International Conference, Shanghai, China, 3–5 June 2010; pp. 247–254.
72. Frossard, É.; Hu, W.; Dano, C.; Hicher, P.Y. Rockfill shear strength evaluation: A rational method based on size effects. *Géotechnique* **2012**, *62*, 415–427. [CrossRef]
73. Xu, Y. Shear strength of granular materials based on fractal fragmentation of particles. *Powder Technol.* **2018**, *333*, 1–8. [CrossRef]
74. Bagherzadeh, A.; Mirghasemi, A.A. Numerical and experimental direct shear tests for coarse grained soils. *Particuology* **2009**, *7*, 83–91. [CrossRef]
75. Xu, Y.; Williams, D.J.; Serati, M.; Vangsness, T. Effects of scalping on direct shear strength of crusher run and crusher run/geogrid interface. *J. Mater. Civ. Eng.* **2018**, *30*, 04018206. [CrossRef]
76. Yaghoubi, E.; Arulrajah, A.; Yaghoubi, M.; Horpibulsuk, S. Shear strength properties and stress–strain behavior of waste foundry sand. *Constr. Build. Mater.* **2020**, *249*, 118761. [CrossRef]
77. Lambe, T.; Whitman, R. *Soil Mechanics*; John Wiley & Sons: New York, NY, USA, 1969.
78. *ASTM C128-15*; Standard Test Method for Relative Density (Specific Gravity) and Absorption of Fine Aggregate. ASTM International: West Conshohocken, PA, USA, 2015.
79. *ASTM C29/C29M-07*; Standard Test Method for Bulk Density (Unit Weight) and Voids in Aggregate. ASTM International: West Conshohocken, PA, USA, 2007.
80. Dorador, L.; Anstey, D.; Urrutia, J. Estimation of geotechnical properties on leached coarse material. In Proceedings of the 70th Canadian Geotechnical Conference, Ottawa, ON, Canada, 1–4 October 2017.
81. Matsuoka, H.; Liu, S. Simplified direct box shear test on granular materials and its application to rockfill materials. *Soils Found.* **1998**, *38*, 275–284. [CrossRef]
82. Boakye, K. Large In Situ Direct Shear Tests on Rock Piles at the Questa Min, Taos County, New Mexico. Ph.D. Thesis, Institute of Mining and Technology, Socorro, NM, USA, 2008.
83. Li, L.; Aubertin, M.; Simon, R. A multiaxial failure criterion with time and size effects for intact rock. In *Rock Mechanics for Industry, Proceedings of the 37th US Rock Mechanics Symposium, Vail, CO, USA, 6–9 June 1999*; Amadei, B., Kranz, R.L., Scott, G.A., Smeallie, P.H., Eds.; A.A. Balkema: Rotterdam, The Netherlands, 1999; Volume 2, pp. 653–659.

84. Li, L.; Aubertin, M.; Simon, R. Stability analyses of underground openings using a multiaxial failure criterion with scale effects. In *Frontiers of Rock Mechanics and Sustainable Development in the 21st Century, Proceedings of the 2nd Asian Rock Mechanics Symposium, Beijing, China, 11–14 September 2001*; Wang, S., Fu, B., Li, Z., Eds.; A.A. Balkema: Lisse, The Netherlands, 2001; pp. 251–256.
85. Li, L.; Aubertin, M.; Simon, R.; Deng, D.; Labrie, D. Influence of scale on the uniaxial compressive strength of brittle rock. In *Rock Mechanics: Meeting Society's Challenges and Demands*; Eberhardt, E., Stead, D., Morrison, T., Eds.; Taylor & Francis: Abingdon, UK, 2007; Volume 1, pp. 785–792.