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UMA ABORDAGEM DE DECOMPOSIÇÃO PARA RESOLUÇÃO DE UM PROBLEMA DE PRODUÇÃO, INVENTÁRIO, DISTRIBUIÇÃO E ROTEIRIZAÇÃO

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Abstract: The aim of this study is to develop a solution to the problem of distribution of goods proposed by the Mathematical Competitive Game 2017-2018, jointly organized by the French Federation of Mathematical Games and Mathematical Modelling Company. Referred to as a production-inventory-distribution-routing problem (PIDRP), it is an NP-hard combinatorial optimization problem, which received the least attention in the literature. The research is quantitative model-based and combines exact and heuristic methods to propose a multiple-phase resolution approach to PIDRP. The results show that the use of clusters ensures practical operational aspects and provides good feasible solutions for the PIDRP in short and long-term planning. The theoretical contribution of this study lies in the PIDRP modeling strategy, and the practical contribution consists in solving a real-life PIDRP-based using optimization techniques.

Keywords: PIDRP. VRP. IRP. MIP. Supply Chain.

Resumo: O objetivo deste estudo é desenvolver uma solução para o problema de distribuição proposto pelos jogos matemáticos de 2017-2018, organizado conjuntamente pela Federação Francesa de Jogos Matemáticos e Empresa de Modelagem Matemática. Conhecido na literatura como um problema de produção-estoque-distribuição-roteamento (PIDRP), este é um problema de otimização combinatória NP-difícil, ainda pouco explorado na literatura. Esta pesquisa é baseada em modelos quantitativos e combina métodos exatos e heurísticos para propor uma abordagem de resolução de múltiplas fases para o PIDRP. Os resultados mostram que o uso de clusters respeita aspectos operacionais práticos e oferece boas soluções para o PIDRP no planejamento de curto e longo prazo. A contribuição teórica deste estudo está na estratégia de modelagem do PIDRP, e a contribuição prática consiste na solução de um PIDRP real baseado em técnicas de otimização.

Palavras-chave: PIDRP. VRP. IRP. PLIM. Cadeia de suprimentos.

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1 Introduction

Supply chain management (SCM) is a multidisciplinary and cross-functional approach to manage the supply chain activities in sourcing, procurement, conversion, and logistics. According to the Council of Supply Chain Management Professionals (CSCMP), essentially SCM "integrates supply and demand management within and across companies" (CSCMP, 2019). The purpose of SCM is to help companies achieve the objective of maximizing customer value and gain sustainable competitive advantage in a dynamic external environment (Frankel et al., 2008).

Optimize the SCM is a major issue for many organizations. This research work aims to develop a solution to the problem of distribution of goods proposed by the Mathematical Competitive Game 2017-2018, jointly organized by the French Federation of Mathematical Games and Mathematical Modelling Company (Beauzamy, 2018b). The overall proposal, which is based on a real-life problem, is to define the logistics concerning an industry of heaters and air conditioners for the current year, based on sales data obtained during the year 2015, that maximize its expected profit, answering the following questions, as presented in Beauzamy (2018b):

1. How many items must be produced daily in each plant?
2. What should be the daily warehouse's inventory levels per product?
3. When and how many items should be delivered every day to each warehouse?
4. When and how many items should be delivered daily from warehouses to shops?
5. What are the routes and schedules for the delivery trucks?

This problem is referred in the literature as Production-Inventory-Distribution- Routing Problem (PIDRP), an NP-hard combinatorial optimization problem which integrates different decisions levels (i.e., tactical and operations) on production lot-sizing, inventory management, distribution planning, and vehicle routing (Miranda et al., 2017; Mostafa & Eltawil, 2015). The main goal of this paper is to solve the proposed PIDRP through a multiple-phase solution strategy using a combination of exact and heuristic methods.

The main contributions of this research are:

1. It brings a solution to a real-life problem using optimization techniques;
2. It presents a new modeling and resolution approach for the PIDRP;
3. It disseminates the Mathematical Game, a valuable source of relevant research problems in the field of operational research;
4. It provides future research directions.

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The remainder of this paper is organized as follows: Section 2 describes the problem in detail. Section 3 presents the literature review. Section 4 introduces the methodology and assumptions considered to develop the solution. Section 5 explains the solution approach and Section 6 presents the results. Finally, in Section 6, the conclusions and some research directions are drawn.

2 Literature review

The characteristics of the problem of distribution of goods proposed in Beauzamy (2018b) take us to consider the problems related to vehicle routing, where the objective is to find the best arrangement of routes to be followed by a fleet of vehicles to serve a set of customers while minimizing the costs of transportation (Labadie et al., 2016). Likewise, it describes the necessity to meet customers' expectations, which are closely linked to the inventory capacity of the stores and the number of items available in it. The inclusion of these components leads us to analyze this context as a vehicle routing problem (VRP) that takes into account inventory issues. This type of problem is known in the literature as the Inventory Routing Problem (IRP) (Moin & Salhi, 2007), but yet, our problem has more variables to be interpreted. In a more integrated view, customer demands directly interfere with the number of items to be produced at the factories and then made available for the sales locations straight from the warehouses. At this point, it makes sense to think about the integration of lot sizing and distribution problems, leaving us in the domain of Production-Distribution Problems (Belfiore et al., 2006; Cordeiro et al., 2019; Park, 2005; Telo et al., 2017).

The scheme in Figure 1 illustrates how the proposed problem can be classified according to its characteristics. A quick check in the diagram suggests that the environment can be viewed as a Production-Inventory-Distribution- Routing Problem (PIDRP) (Bard & Nananukul, 2008; Mostafa & Eltawil, 2015). The PIDRP is a NP-hard combinatorial optimization problem considered an extension of the Inventory Routing Problem (IRP), which integrates production-distribution problem (Coelho et al., 2014; Mostafa & Eltawil, 2015).

The SCM domain is very broad and covers all the planning and management activities in logistic (Frankel et al., 2008). The PIDRP specifically addresses decisions on production lot-sizing, inventory management, distribution planning, and vehicle routing. The objective of lot-sizing, which is also referred as Lot-sizing problem (LSP), is determining the economic production quantity or economic order quantity lot sizes by adjusting inventory and setup or order costs (Glock et al., 2014). The distribution planning ensures that the delivery of goods is done properly, that is, the right goods are delivered at the right amount, to the right location, at the desired time. The vehicle routing aims to find optimal routes for vehicles to deliver the goods to a set of locations (Bertazzi & Speranza, 2016; Coelho et al., 2014). Inventory management deals with different types of inventory (e.g., raw materials, working

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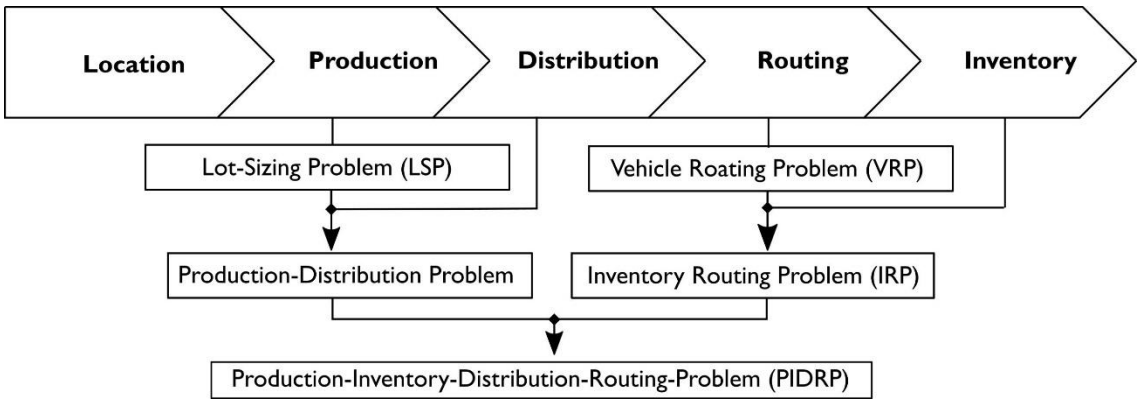
process, components, finished goods) to allow a company to meet its demands, aiming to ensure high customer service level while minimizing order and holding costs (Williams & Tokar, 2008).

Figure 1 - Problem classification



Source: Adapted from Mostafa and Eltawill (2015).

A seminal paper that addresses the issue of coordinate the different SCM functions is found in Chandra and Fisher (1994). Another key reference is Lei et al. (2006), which proposed a solution approach for the PIDRP, solving a real-life supply network coordination problem. To understand how publication on IRP and PIDRP has evolved over the years, both key terms we searched at the electronic data source Web of Science (WoS) Core Collection, broadly covering the Operational Research (OR) literature. The following search strings were applied: 1 - (TS=("Inventory Routing Problem"); 2 – (TS=(Production AND Inventory AND Distribution AND Routing AND Problem)). Papers containing the terms in either the titles, abstracts, or keywords were retrieved. The title and abstract of the papers were analyzed for consistency, and duplicates removed. As a result, 362 papers about IRP were found, which together received 4561 citations, and 124 papers about PIDRP, which collectively received 2878 citations. The results are summarized in Figure 2, where an increasing number of publications over the years on both research topics can be observed. Moreover, publications on both research topics have been well cited. From another perspective, dividing the total number of citations by the number of publications, which allow a better comparison, the papers on IRP received in average 12.6 citations, while the papers about PIDRP received 23.2 citations, almost the double. An explanation for the higher relevance of PIDRP may be attributed to the fact that it deals with more supply chain management functions, comprising more real-world scenarios.

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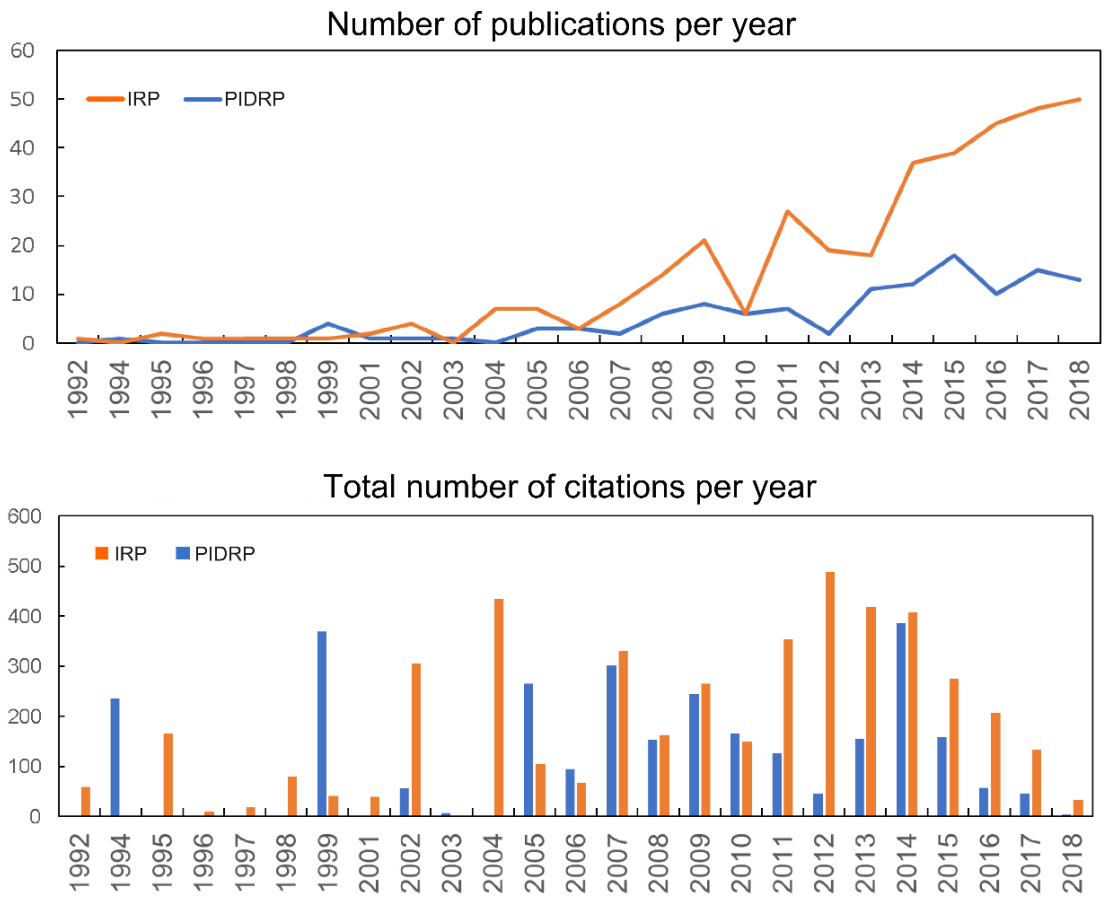
Figure 2 - Publications and citations evolution over the years



Source: the authors.

According to Mostafa and Eltawil (2015), PIDRP is a relatively novel area, which has received the least attention in the literature. The authors conducted an exhaustive review on the PIDRP and classified the literature in terms of decision level, problem characteristics, modeling approach, decision variables, objectives, contributions, type of application, and software tools. A short review is also presented in Miranda, Morabito, and Ferreira (2017) and Miranda et al. (2018). These papers identify different solution strategies for PIDRP available in the literature, mostly based on different Metaheuristics. However, none of them explored fixing routes strategies, as discussed in Beasley (1984), Campbell et al. (1998), and Campbell and Savelsbergh (2004).

3 Methodology

This research is quantitative model-based and lies within the field of Operations Research (OR), which is considered as part of Operations Management (Bertrand & Fransoo, 2002). It can be further classified as Axiomatic Normative. Axiomatic, since it aims to produce knowledge by how it manipulates certain variables and improves the quality of the mathematical solutions, and Normative, because it is

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mainly interested in developing a strategy to solve a PIDRP (Bertrand & Fransoo, 2002). Regarding the methodological choices, the scope of this research is limited to mathematical optimization-based approaches, mainly mixed integer programming (MIP) formulations.

The PIDRP is an NP-hard combinatorial optimization problem (Miranda et al., 2017). A multiple-phase solution strategy using a combination of exact and heuristic methods will be developed in this paper to solve the PIDRP proposed by the Mathematical Competitive Game (Beauzamy, 2018b). The idea is to adapt the solution method for IRP proposed by Campbell et al. (1998) to solve the multi-period, multi-product PIDRP, dividing the main problem into two phases, and decomposing the problems of each phase in easier subproblems solvable with exact methods in a reasonable time.

The PIDRP instance data for the Mathematical Games, which is based on a manufacturing company located in Europe, that makes and sells heaters and air-conditioners can be retrieved in Beauzamy (2018b). The instance includes a detailed description of the company's operations and a complete sales history data of the year 2015, all provided by the new owner of the company. It is important to note that the Mathematical Game "deals with the resolution of a real-life problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work" (Beauzamy, 2018b, p. 2).

4 Problem description

The problems proposed by Mathematical Competitive Games try to deal with practical circumstances, and the proposed challenge resembles real situations in the Industry. The conditions are delineated as the production and distribution of two products (heaters and air-conditioners) over a year, with some assumptions introduced to simplify the mathematical modeling. Each heater has a volume of 0.4 m^3, and each air-conditioner has a volume of 0.8m^3. The delivery cost is 1 Euro per Km, and each truck may carry up to 20 m^3. The supply chain is composed of 2 plants, 2 warehouses, and 20 shops. The two factories responsible for assembling those items do not have restrictions related to the production capacity per day neither to switch the manufacturing from one product to another. The goods should be delivered at two warehouses and then daily distributed to twenty stores, with both types of locations being suitable to accommodate the two types of products. The coordinates of the facilities, as well as their storage capacity, are known, as described in Beauzamy (2018b).

Figure 3 depicts the general view of the environment.

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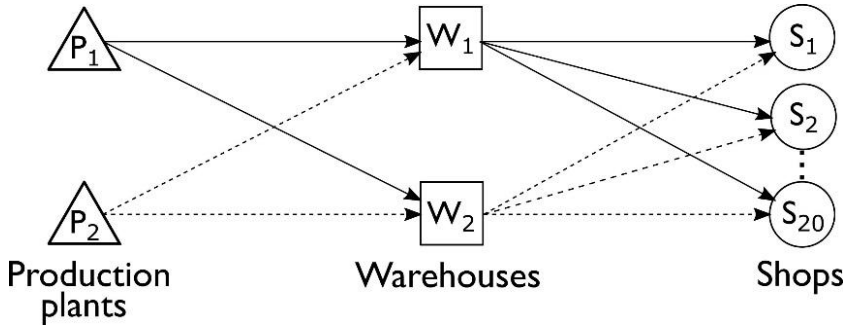
Figure 3 - Overview of the supply chain



Source: The authors.

The consumption of products is seasonal, with air-conditioners and heaters trade being complementary to each other, meaning that the overall work and sales around the year should be reasonably equally, accepting a variation of ±10%. Accordingly, the shops are considered to be open six days a week with no exemption, and consequently, it is expected that consumers can spot and purchase at least one of the products. Otherwise, a sale is lost, decreasing the potential profit.

It is assumed that on December 31st, all inventories from warehouses or shops are empty, and new planning should be put in place to attend the current year demand. The delivery plan is assumed to be served by a third-party company, through a contract comprising all the necessary trucks and drivers previewed by the new strategy. As the sales can vary by about 10%, if more trucks are required, a 20% increase in price is charged. Both considerations are important issues in the problem because the primary goal is to maximize the profit for the current year.

As the objective is to achieve maximum profit, the logistics should help to answer some questions related to that goal. Firstly, it is necessary to know the number of air-conditioners and heaters that must be produced in each plant each day, and then once the products are available to be distributed, how many items have to be kept in the warehouses. Secondly, these quantities may vary daily because of the consumption expected from the customers, so it is necessary to establish when and the number of products that may be delivered to each warehouse. These dynamics follow down until the customers, implying that it is required to determine the number of each item will be dispatched to the stores each day.

Finally, the flow of the products from the warehouses to the shops depends on the creation of routes and schedules for the trucks. The fleet of vehicles is to be considered homogeneous, that is, with the same volume capacity. This aspect is essential as the two types of products have different volumes and will be shipped together according to demand. The delivery cost is 1 EUR per km, excluding maintenance and acquisition costs, which are disregarded, and the garages of the trucks are inside the warehouses.

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4.1 Assumptions

The following assumptions are considered for the development of the model:

1. Each truck can depart and return to a warehouse once a day;
2. Using the past data, provided in Beauzamy (2018b), the sales of the coming year can be defined. A uniform law on the interval [-10%; 10%] gives the sales. Thus, using a random number generator (RNG), the upcoming sales can be calculated as follows:

$$Sales2018_{jsp} = Sales2015_{jsp} \times (0.9 + 0.2 \times RNG) \qquad \forall j \in J, \forall s \in S, \forall p \in P$$

Where J = set of days, S = set of shops, P = set of products, and RNG is a random number in [0;1] given by a uniform law, randomly round up or down this value (50% chance for each) to obtain integer data.

3. After analyzing the data presented in Beauzamy (2018b), it assumes that product 1 represents the air-conditioners and product 2 the heaters.
4. Production plants and warehouses can operate 7 days a week.

5 Solution approach

The solution approach starts by dividing the PIDRP problem into two phases to reduce the complexity of the problem and prevent a combinatorial explosion. The first phase, the leading problem, consists of a Multi-Depot Inventory Routing Problem (MDIRP) with 2 warehouses and 20 customers. The solution of the first phase is then used as a parameter to solve the second phase, which can also be modeled as an MDIRP with 2 production plants and 2 warehouses. Then, each MDIRP is converted into two Inventory Routing Problems (IRP) by solving an Assignment Problem (AP). Next, the approach proposed by Campbell et al. (1998) and Campbell and Savelsbergth (2004) to fix the routes iteratively was used. After, a MIP formulation to solve each IRP, based on a small set of fixed routes was implemented. Finally, by combining the solutions of the two-phases, the original PIDRP problem is addressed. The problem tree with all the decompositions and the order in which the problems are solved can be seen in Figure 4.

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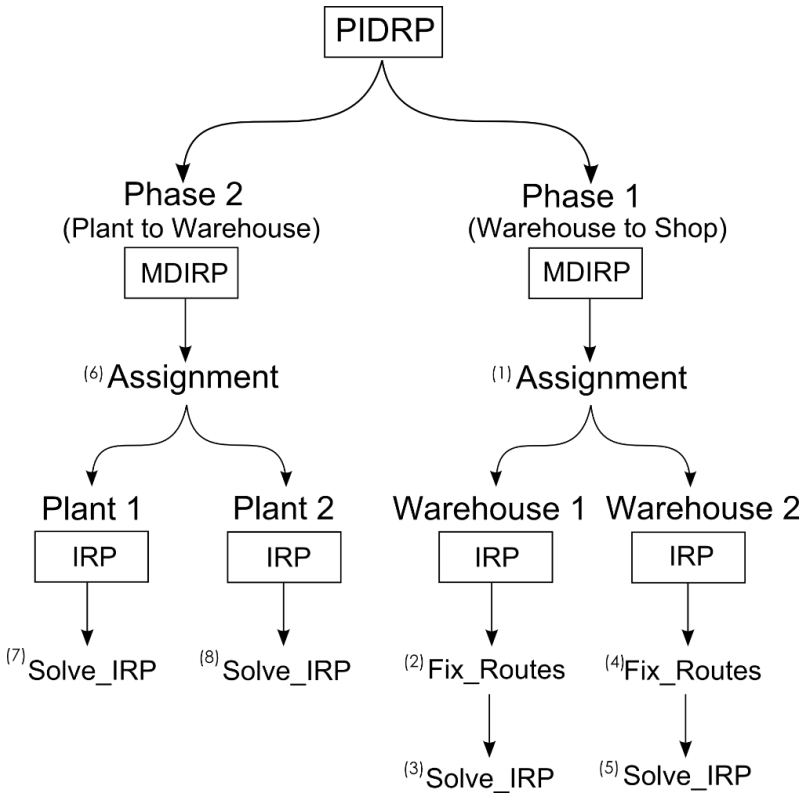
Figure 4 - Problem decomposition tree



Source: the authors.

5.1 Assignment problem

Our solution approach emphasizes practical aspects. Thus, the first step was to assign a set of shops to each warehouse, solving an Assignment Problem (AP), to reduce the complexity of companies' operations, as well as the complexity of the model. The AP formulation is described next:

Sets

J – shops;

I – warehouses;

P – product;

Parameters

d_{ij} – the rectilinear distance between $i \in I$ and $j \in J$

b_{ip} – the storage capacity of $i \in I$ for each $p \in P$

a_{jp} – the demand of the shop $j \in J$ for product $p \in P$

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Decision variables

$x_{ij} = 1$ if the shop j is served by the warehouse i, 0 otherwise.

The formulation is as follow:

$$\text{Minimize} \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \tag{1}$$

s.t.

$$\sum_{i \in I} x_{ij} = 1, \qquad \forall j \in J \tag{2}$$

$$\sum_{j \in J} a_{jp} x_{ij} \leq b_i, \quad \forall i \in I, p \in P \tag{3}$$

$$x_{ij} \in \{0,1\}, \qquad \forall i \in I, j \in J \tag{4}$$

The Objective function (1) minimizes the total cost of assigning the shop j to the warehouse i. Constraints (2) ensures that each shop j is assigned to only one warehouse i. Constraints (3) ensures that each warehouse i can supply the set of shops assigned to it. Constraints (4) defines the domain of the variables.

5.2 Fixing routes

To make the resolution approach computationally tractable, a small, but a good set of delivery routes need to be explicitly defined. In this work, the approach proposed by Campbell et al. (1998) and Campbell and Savelsbergth (2004) to select a small group of delivery routes was adopted, which is based on the concept of clusters, a group of customers that can be served with a reduced cost by a single-vehicle for extended period of time. After determining a set of disjoint clusters covering all customers, only routes visiting customers in the same cluster are considered.

The approach mentioned above consists of four steps, described in the next subsections. First, a set of clusters is generated using heuristic rules. Then, the cost of serving each cluster is estimated. Next, a collection of disjoint clusters at a minimum cost is selected. Finally, the routes within each disjoint cluster are defined. The overall procedure is summarized in Figure 6, at the end of section 3.2.3.

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5.2.1 Generating a set of clusters

Considering the enormous amount of clusters that can be generated, heuristic procedures were used (de Souza Leite, M., Santos, S. C., da Silva, A. M., & de Paula Ferreira, 2017) to limit and select a small set of clusters covering all shops. The procedure, adapted from Campbel and Savelsbergth (2004), is detailed in Figure 5. The algorithm starts by generating all possible clusters (2^{n-1}), where n is the number of shops. Then, the clusters are screened based on truck' capacity, demand, and geographic location of shops. Finally, for feasibility reason, clusters with one of each shop are also included. The parameters used in the heuristic were defined through multiple experiments, by varying the level of truckload and rectilinear distance between shops in the cluster.

Figure 5 – Generate a small set of clusters

```
H = Ø; /* small set of clusters */
S = {1,...,n};  /* set of shops */
F ← all possible (2^(n-1)) Subset of Cluster;
foreach Cluster ϵ F do
        Truck capacity ≥ Cluster's Demand;
        Cluster Demand ≥ 80% x truck capacity;
        The rectilinear distance between shops in the Cluster ≤ 5 km;
        H ← Cluster;
end
foreach Shop ϵ S do
        H ← Shop;
end
```

Source: the authors.

5.2.2 Estimating the cost of each cluster

The cost of serving each cluster is estimated by the MIP formulation presented next, taken from Campbell et al. (1998). A period of a year was used to evaluate the distribution cost of serving customers in the cluster.

Sets

J – shops

R – delivery routes

Parameters

c_r – the cost of executing the route $r \in R$

Q – vehicle capacity

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C_j – storage capacity of the shop $j \in J$

T – planning horizon

U_j – demand rate of shop $j \in J$

Decision variables

y_{jr} – volume delivered to shop $j \in J$ on route $r \in R$

z_r – number of times the route $r \in R$ is executed

Formulation

$$\text{Minimize} \sum_{r \in R} c_r z_r \tag{5}$$

s.t.

$$\sum_{j:j \in r} y_{jr} \leq \min\left(Q, \sum_{j:j \in r} C_j\right) z_r \qquad \forall r \in R \tag{6}$$

$$y_{jr} \leq \min\left(Q, C_j\right) z_r \qquad \forall r \in R, \quad \forall j \in r \tag{7}$$

$$\sum_{r:j \in r}^{R} y_{jr} = TU_j \qquad \forall j \in J \tag{8}$$

$$z_r \text{ integer}, \quad y_{jr} \geq 0 \quad \forall r \in R, j \in J \tag{9}$$

The Objective function (5) minimizes the distribution cost. Constraints (6) defines the maximum volume delivered on the route *r* in the planning period. Constraints (7) ensures that the total volume delivered to the customer does not exceed the capacity of the vehicle and the storage capacity of the customer, multiplied by the number of times the route is performed. Constraints (8) ensures that the total volume delivered is equal to the shops' demand rate. Constraints (9) define the domain of variables.

In order to obtain the cost of an optimal route (cr) for a subset of customers in a cluster and, later one, to define the visit order of the shops within each cluster, the Travelling Salesman Problem (TSP) is solved employing the MIP formulation below, adapted from (Miller et al., 1960).

Sets

I – Shops

J – Shops

R –delivery routes

I_r –shops in each route $r \in R$

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Parameters

d_{ij} – the rectilinear distance between $i \in I$ and $j \in J$

u_i, u_j – are arbitrary real numbers with $i \in I$ and $j \in J$

Decision variables

x_{ij} = 1 if shop *j* is served after shop *i*, 0 otherwise.

The formulation is as follow:

$$\text{Minimize} \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \tag{10}$$

s.t.

$$\sum_{j \in I_r} x_{ij} = 1, \qquad \forall i \in I_r \tag{11}$$

$$\sum_{i \in I_r} x_{ij} = 1, \qquad \forall j \in I_r \tag{12}$$

$$u_i - u_j + n x_{ij} \leq n - 1 \qquad \begin{array}{c} \forall i \neq j \neq 1 \\ i \in I, j \in J \\ u_i \geq 0 \end{array} \tag{13}$$

$$x_{ij} \in \{0,1\}, \qquad \forall i \in I, j \in J \tag{14}$$

The Objective function (10) minimizes the total distance traveled by the trucks. Constraints (11) ensure that each truck arrives at the shop i only once. Constraints (12) ensure that each truck j departs from each shop j only one time. Constraints (13) eliminate the sub tours, ensuring that all shops are served. Constraints (14) define the domain of variables.

5.2.3 Selecting the clusters

Finally, the set partitioning problem is solved to select a collection of disjoint clusters at a minimum cost.

Sets

S – clusters

J – shops

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Parameters

c_j – estimated cost of serving the cluster $j \in J$

a_{ij} = 1 if $j \in S_i$, 0 otherwise

Decision variables

x_j =1 if cluster $j \in J$ is selected, 0 otherwise

The formulation is as follow:

$$\text{Minimize} \sum_{j \in J} c_j x_j \tag{15}$$

s.t.

$$\sum_{j \in J} a_{ij} x_j = 1, \qquad \forall i \in S \tag{16}$$

$$x_j \in \{0,1\}, \qquad \forall j \in J \tag{17}$$

The Objective function (15) minimizes the total cost of the selected clusters. Constraints (16) indicate that each shop j must be covered by only one cluster. Constraints (17) represent the domain of the variables

The overall procedure for fixing the routes is summarized in Figure 6. First, a set of clusters (H) is generated using heuristic rules, as described in Figure 5. Then, all possible subset of clusters (G) based on each cluster in H is generated. Next, for each subset of each cluster, a TSP is solved to calculate the cost of executing the routes. Then, the cost of serving each cluster is estimated using a MIP formulation. Afterward, the partitioning problem is solved to select a collection of disjoint clusters at a minimum cost. In the end, the TSP is solved to define the routes within each subset of the disjoint clusters.

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Figure 6 – The algorithm to fix the routes

```
W = Ø; /* set of disjoint clusters */
R = Ø; /* set of routes */
H; /* small set of clusters */
foreach Cluster ϵ H and i ϵ 1,...,sizeof(H) do
    G = Ø;
    Int m = |Cluster| ≤ 4;
    float Cost[m];
    G ←all possible (2^(m-1)) Subset of Cluster;
    float distancia[sizeof(G)];
    foreach g ϵ G and j ϵ 1,...,sizeof(G) do
        Subset.distancia[j] ← execute TSP
    end
    Cluster.cost[i] ← execute cluster_cost_estimate
end
W ← execute Set_Partitioning
foreach w ϵ W do
    R ← execute TSP
end
```

Source: the authors.

5.3 Inventory Routing Problem

Here the MIP formulation for a multi-product and multi-period IRP, adapted from Campbell et al. (1998) is presented. It aims to determine when and how many items should be delivered every day from warehouses to the shops and from plants to the warehouses by changing a set of shops to the set of warehouses.

Sets

J – shops;

R – routes;

P – products;

T – periods

Parameters

C_v – truck capacity

V_p – *the* volume of each product $p \in P$

D_{pj} – *the* demand of each shop $j \in J$ for each product $p \in P$ in period $t \in T$

E_{jp} – *the* storage capacity of shop $j \in J$ for each product $p \in P$

C_p – cost of the product $p \in P$

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c_r – *the* cost of executing the route $r \in R$

c_{lp} – *the* penalty for sales lost of product $p \in P$

Decision variables

Q_{jr}^{tp} – *the* volume of $p \in P$ delivered to $j \in J$ on $t \in T$ using $r \in R$

S_{pj}^{t} – sales of $p \in P$ for shop $j \in J$ in period $t \in T$

L_{pj}^{t} – sales lost of product $p \in P$ by the shop $j \in J$ in $t \in T$

X_{tr} – number of times the route $r \in R$ is executed during $t \in T$

I_{pj}^{t} – the stock of product $p \in P$ in shop $j \in J$ in period $t \in T$

u_{pj}^{t} – lower bound of *the* volume of $p \in P$ delivered to $j \in J$ in period $t \in T$

U_{pj}^{t} – upper bound of *the* volume of $p \in P$ delivered to $j \in J$ in period $t \in T$

The formulation is as follow:

$$\text{Max.} \sum_{t\in T}\sum_{j\in J}\sum_{p\in P}\left(c_p S_{pj}^{t} - cl_p L_{pj}^{t}\right) - \sum_{t\in T}\sum_{r\in R} c_r X_{tr} \tag{18}$$

s.t.

$$I_{pj}^{t} = I_{j}^{t-1} + \sum_{r\in R} Q_{jr}^{tp} - S_{pj}^{t} \qquad \forall t\in T, j\in J, p\in P \tag{19}$$

$$I_{pj}^{0} = 0 \qquad \forall j\in J, p\in P \tag{20}$$

$$I_{pj}^{t} \le E_{pj} \qquad \forall t\in T, j\in J, p\in P \tag{21}$$

$$u_{pj}^{t} = \max\left(0, D_{pj}^{t} - I_{pj}^{t-1} - L_{pj}^{t}\right) \qquad \forall t\in T, j\in J \; p\in P \tag{22}$$

$$U_{pj}^{t} = D_{pj}^{t} + E_{pj} - I_{pj}^{t-1} \qquad \forall t\in T, j\in J, p\in P \tag{23}$$

$$u_{pj}^{t} \le Q_{jr}^{tp} \le D_{pj}^{t} \qquad \forall t\in T, p\in P, r\in R \tag{24}$$

$$\sum_{i\in I_r} Q_{jr}^{tp} v_p \le C_v X_{tr} \qquad \forall t\in T, p\in P, r\in R \tag{25}$$

$$L_{pj}^{t} \ge D_{pj}^{t} - I_{pj}^{t-1} - \sum_{r\in R_j} Q_{jr}^{tp} \qquad \forall t\in T, j\in J, p\in P \tag{26}$$

$$S_{pj}^{t} = D_{pj}^{t} - L_{pj}^{t} \qquad \forall t\in T, j\in J, p\in P \tag{27}$$

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$$S_{pj}^{t} \leq D_{pj}^{t} \qquad \forall t \in T, j \in J, p \in P \tag{28}$$

$$\begin{aligned} &X_{tr} \text{ integer}, S_{pj}^{t} \geq 0, L_{pj}^{t} \geq 0 && \forall t \in T, j \in J \\ &I_{pj}^{t} \geq 0, Q_{jr}^{p} \geq 0, D_{pj}^{t} \geq 0 && \forall p \in P, r \in R \end{aligned} \tag{29}$$

The Objective function (18) maximizes the total profit, maximizing sales, minimizing sales lost and distribution costs. Constraints (19) establish the inventory level at the end of each period t. Constraints (20) define the initial stock of each shop i equal to zero. Constraints (21) ensure that the inventory level at the end of each period is less or equal to the shop's storage capacity. Constraints (22-24) define a lower and upper bound for the quantity delivered from the warehouse to shops, and Constraints (25) ensure that the quantity delivered to shops is less or equal the truck capacity times the number of times the route is executed. The Constraints (22-25) were taken from Campbell et al. (1998). Constraints (26-28) link the sale lost with sales and demand. Constraints (29) define the domain of variables.

6 Results

The computational experiments were carried out on an Intel(R) Core(TM) i7-5500U CPU 2.40 GHz with 6 GB RAM using the IBM ILOG CPLEX Optimization Studio V12.7.0. and JavaScript. The problem decomposition tree exhibited in Figure 4 will be used as a reference to present the main results.

The first step was to assign each shop to a single warehouse, solving problem 1. As a result, warehouse 1 will serve the shops in the set S1, and warehouse 2 will serve the shops on the set S2, as illustrated in Figure 7.

Figure 7 - The result of the assignment problem 1

$$\text{Warehouse 1} \rightarrow S_1 = \{1,3,4,6,7,11,12,14,17,18\}$$
$$\text{Warehouse 2} \rightarrow S_2 = \{2,5,8,9,10,13,15,16,19,20\}$$

Source: the authors.

Then, the procedure to fix the routes (described in section 3.2) and the IRP model was applied. The collection of disjoint clusters selected is illustrated in Figure 8. A total of 40 routes and their respective costs were also defined.

Figure 8 - Selected clusters

$$\text{Warehouse 1} \rightarrow W_1 = \{\{1,3,4\},\{6,7,11\},\{12,14\},\{17,18\}\}$$
$$\text{Warehouse 1} \rightarrow W_2 = \{\{2,5\},\{8,15,19\},\{9,10,13\},\{16,20\}\}$$

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Source: the authors.

The expected profit of Phase I, calculated by subtracting the transportation costs from the sales revenue, and the computational time are exhibited in Table 1. Fill rate is a Key Performance Indicator (KPI) that measures the ability to meet demand without sales lost, also called demand satisfaction rate (Kaganski et al., 2017). Since a high penalty for sales lost was adopted, the IRP model forced all deliveries, ensuring a 100% fill rate. The main decisions taken in this step are how many items will be delivered from the warehouse to shops and the schedule for the delivery trucks.

Table 1 - Results of phase I

Description	Revenue (€)	Profit (€)	Fill rate (%)	Computational time (min)
Warehouse 1 to S_1	8,466,600	8,452,428	100	1.2
Warehouse 2 to S_2	9,049,500	9,036,034	100	1.1
Subtotal	17,516,100	17,488,462	100	2.3

Source: the authors.

At the fourth step, each warehouse was assigned to a single production plant (problem 6), as illustrated in Figure 9, forming two different routes.

Figure 9 - The result of the assignment problem 6

$$\text{Plant 1} \rightarrow \text{Warehouse 1}$$
$$\text{Plant 2} \rightarrow \text{Warehouse 2}$$

Source: the authors.

Toward the fifth step, the IRP for each production plant, ensuring a 100% fill rate for the warehouses was solved. The main results of Phase II are presented in Table 2. The main decisions taken are: how many items should be produced each day in each plant, how many items should be kept in the warehouses, what day of the week, and how many items deliveries to each warehouse.

Table 2 - Results of phase II

Description	Transp. Cost (€)	Fill rate (%)	Computational time (min)
Plant 1 to Warehouse 1	8,080	100	3.2
Plant 2 to Warehouse 2	8,656	100	4
Subtotal	16,736	100	7.2

Source: Authors

Assuming that each truck can go through a route only once a day, the sum of the number of times a route is executed each day gives the number of required trucks, which the contract with a Truck Agency should be based. The recommendation is that the company contract 15 trucks, considering the frequency and the number of trucks required per day. Eventually, the number of trucks required will be

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higher, considering the demand and that the planning starts with empty stock in the warehouses and shops.

Finally, the overall results are shown in Table 3. Where the low of expected profit over a year is given by Equation (30).

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S} \text{Demand}^{t}_{pj} \times \text{Price}_{p} \times 99.75\% \qquad \forall t\in T, p\in P, s\in S \qquad (30)$$

Table 3 - Overall result

Expected Profit (€)	Logistic Cost (%)	Fill rate (%)	Computational time (min)
17,471,702	0.25	100	20.8

Source: Authors.

7 Conclusions

This research work solves the real-life problem- based distribution of goods of the Mathematical Game 2017-2018 (Beauzamy, 2018b) proposing a multiple-phase solution strategy, which combines exact and heuristic methods, considering its practical operational aspects, since it defines clusters and fixes a small number of routes, providing feasible solutions that can be used for different contexts, such as long or short-term planning.

The main limitation of this research work consists in not providing a comparative example. Unfortunately, the award-winning solution approach could not be retrieved from the website of the Mathematical Competitive Game 2017-2018. However, during the workshop were the different solution approaches were presented, the expected profit obtained with the solution approach proposed diverged in less than 0.5% when compared to the winner solution, despite the simplifications, and presented a lower computational time. Also, looking from a different perspective, it offers more practical aspects for companies' operations since it limits the number of routes. Our proposal is also in accordance with the final comments provided by the evaluation team, as can be seen in Beauzamy (2018a). Even though it can be considered a simple resolution approach, this research paper can stimulate other participants to disclose and compare their models, as well as serve as an example case for further development of new solutions approaches for the PIDRP.

For future works, to increase the total expected profit, the heuristic procedures to reduce the number of clusters could be improved. Other approaches to fixe the routes, such as metaheuristics and matheuristics, could also be tested with some modifications (de Assis et al., 2016; de Oliveira et al., 2017; Miranda et al., 2017). More details about the last proposition can be found in Doerner and Schmid (2010), Villegas et al. (2013), and Bertazzi and Speranza (2016). For economic reasons, the solution

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should be accessed (implemented) using an open-source solver, making it more accessible to small companies. Around 45% of the papers on PIDRP were published in the last four years, and no Systematic Literature Review (SLR) or Meta-Analysis on this research topic were identified, revealing another important avenue for future research.

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