

POLYTECHNIQUE MONTRÉAL

affiliée à l'Université de Montréal

**Modèles de fiabilité et de maintenance prédictive de systèmes sujets à des
défaillances interactives**

Toualith Jean-Marc MEANGO

Département de mathématiques et de génie industriel

Thèse présentée en vue de l'obtention du diplôme de *Philosophiæ Doctor*

Génie industriel

Août 2020

POLYTECHNIQUE MONTRÉAL

affiliée à l'Université de Montréal

Cette thèse intitulée:

Modèles de fiabilité et de maintenance prédictive de systèmes sujets à des défaillances interactives

Présenté par **Toualith Jean-Marc MEANGO**

en vue de l'obtention du diplôme de *Philosophiæ Doctor*

a été dûment acceptée par le jury d'examen constitué de :

Luc ADJENGUE, Ph. D., président

Mohamed-Salah OUALI, Doctorat, membre et directeur de recherche

Jonathan JALBERT, Ph. D., membre

Claver DIALLO, Ph. D., membre externe

DÉDICACE

A mes grands-parents, mes parents, mes sœurs et mes frères dont Emmanuel,

REMERCIEMENTS

Il y a un grand nombre de personnes que j'aimerais remercier qui ont contribué directement et indirectement à la complétion de cette thèse. Cela dit, je me permets de remercier en premier Celui qui est et qui m'a conduit sur le chemin de chacune de ces personnes. Pour moi qui effectue des recherches en génie industriel sur des modèles de fiabilité, je vois des liens entre des phénomènes qui semblent à priori aléatoires. Pour moi, il y a dans le concept de cette Personne que l'on ne maîtrise pas l'explication de ces phénomènes que nous vivons et appréhendons parfois sous des formes stochastiques, et dont les interactions produisent la beauté du monde et des vies qui nous entourent, qu'elles soient pour nous cohérentes ou non. Ces aspects motivent ma vision du monde, ma manière d'être et ont inspirés mes recherches, qui, je l'espère auront apporté, un brin de significativité à ma vie et celles de toutes les personnes que j'ai rencontré durant mon cursus.

Je remercie en second ma famille. Mes grands-parents, ceux qui sont là et ceux qui ne sont plus, ont posé les fondements d'une famille unie dans l'amour et dans le souci du travail bien accompli. Mes parents en sont l'exemple parfait. Mon père est pour moi un icône et j'aspire à être aussi sage que lui. Le cœur de ma mère m'apprend jusqu'aujourd'hui qu'il n'y a rien de plus beau que de sacrifier sa vie pour son prochain. Avec mes frères et sœurs, je m'attèle à les imiter mais chacun à leur façon, ils y arrivent de bien plus belle manière que moi.

Mon Directeur de recherche, M. Ouali est un homme sage et au grand cœur, passionné par ses recherches, et qui traite ses étudiants comme des amis. N'eut été sa direction essentielle, sa créativité, sa diligence dans la recherche de nouvelles opportunités et son support moral constant, aucun de ces modèles qui sont présentés n'auraient vu le jour. Et, par-dessus tout, je n'aurais pas eu le courage d'aller aussi loin. Je le remercie du fond du cœur de m'avoir accordé sa confiance au moment où j'en avais le plus besoin.

Je remercie tous les professeurs à Polytechnique que j'ai eu à rencontrer. Je suis sans cesse inspiré et impressionné par leur niveau de compétence. C'est pour moi une grande fierté d'avoir appris et de continuer d'apprendre de leur expérience dans une institution d'une telle qualité. Je suis, d'ailleurs reconnaissant d'avoir partagé des idées avec plusieurs étudiants et collègues particulièrement doués de cette école. Je veux, en outre, remercier le Docteur John Santiago, ancien professeur à *Colorado Technical University*, qui est l'une des personnes clés qui m'a aidé à me lancer sur le chemin du doctorat.

Je veux remercier enfin tous mes amis de l'IFES. Nos interactions ont permis ce travail. Les personnes qui sont douces et ont un cœur tendre, sont un cadeau du ciel dans ce monde.

RÉSUMÉ

L'interaction des défaillances est une thématique qui prend une ampleur considérable dans le monde de la recherche industrielle moderne. Les systèmes sont de plus en plus complexes et leurs fonctionnements et défaillances sur le long terme sont sujets à diverses sources d'influence internes et externes. Les actifs physiques en particulier sont soumis à l'impact du temps, de l'environnement et du rythme de leur utilisation. Connaître ces sources d'influence n'est pas suffisant car il importe de comprendre quelles sont les relations qui les lient afin de planifier de façon efficiente la maintenance des actifs. En effet, cette dernière peut s'avérer très coûteuse et sa mauvaise planification peut conduire à l'utilisation de systèmes dangereux pouvant engendrer des événements catastrophiques.

La fiabilité est un vaste domaine. Elle propose une large panoplie de modèles mathématiques qui permettent de prédire le fonctionnement et les défaillances des actifs physiques. Ceci dit, les concepts des modèles les plus appliqués à ce jour se basent sur des hypothèses parfois simplistes et occultent bien souvent certaines relations de dépendances qui régissent un système. L'interaction des défaillances dans le cadre des dépendances stochastiques est abordée par de nombreux travaux de recherches. Par contre, la compréhension et l'implémentation de ces travaux demeurent un défi pour les spécialistes en maintenance qui ont besoin de modèles réalistes pour une maintenance préventive efficace.

Cette thèse traite de la fiabilité et la maintenance prédictive des actifs physiques en exploitation et sujets à divers modes de défaillance interactifs. Elle établit avant tout l'importance d'accorder une attention particulière à l'interaction des défaillances dans le domaine de la fiabilité et de la maintenance. Dans une revue de littérature, les concepts et les méthodes de modélisation et d'optimisation en fiabilité et en maintenance préventive sont présentés. Les divers types de dépendances dans un système sont discutés. Un cas d'application, à savoir celui des ponts en béton, est proposé. Les travaux entrepris par la suite fournissent avant tout un cadre pour la modélisation de la fiabilité incluant l'interaction des défaillances. A cette fin, une étude comparative des modèles existants les plus pertinents est effectuée de points de vue conceptuel, méthodologique et applicatif. Le cadre étant défini, un modèle basé sur les chocs extrêmes et les chaînes de Markov est construit afin de valoriser le caractère séquentiel des défaillances interactives. Cette proposition est améliorée pour prendre en compte la dégradation du système.

Une stratégie de maintenance prédictive est conséquemment développée. Toutes ces approches sont appliquées à un ensemble de ponts en béton observés sur plusieurs années. Cela permet d'expliquer les dépendances entre l'occurrence de déplacements et l'occurrence de fissures dans une structure. Tous ces concepts et résultats sont finalement discutés afin de déterminer des perspectives réalistes pour une étude approfondie de l'interactivité d'un point de vue fiabiliste et dans un but stratégique pour la planification de la maintenance.

ABSTRACT

Failure interaction is a subject gaining growing attention in the world of modern industrial research. Systems are becoming increasingly complex. Their life cycles are subject to various internal and external influences. Physical assets in particular are impacted by time, environment and usage. Knowing these sources of influence is not enough. Indeed, it is important to understand the relationships between them in order to plan effectively for the maintenance of assets. Maintenance can be quite expensive. Thus, poor planning can lead to dangerous systems that could cause catastrophic events.

Reliability engineering offers a wide range of mathematical models to predict failures. That being said, the concepts of the most widely applied models in the industry are often based on simplistic assumptions and tend to overlook certain dependencies within a system. Failure interaction in the context of stochastic dependencies is largely addressed in the literature. However, understanding and implementing the proposed approaches remains a challenge for maintenance specialists that need realistic models for efficient maintenance planning.

This thesis focuses on the reliability and predictive maintenance of physical assets subject to interactive failure modes. First of all, it emphasizes the importance of paying particular attention to failure interaction. In a literature review, the concepts and methods for modeling and optimizing reliability and preventive maintenance are presented. The diverse dependencies in a system are discussed. A case study is proposed, namely concrete culverts. Subsequently, the research provides a framework for modeling reliability that integrates the interaction of failures. To this end, the most relevant models in the literature are comparatively studied from a conceptual, methodological and applicative point of view. In the defined framework, a model based on extreme shocks and Markov processes is built in order to represent the sequential nature of interactive failures. This approach is extended to take into account the natural degradation of a system. A predictive maintenance strategy is consequently developed. All these models are applied to a set of concrete culverts observed over several years. The dependences between the occurrence of displacements and the occurrence of cracks in a structure are explained through these approaches. Finally, these concepts and results are discussed in order to determine realistic perspectives for in-depth studies of the impact of failure interaction on reliability and for strategic maintenance planning.

TABLE DES MATIÈRES

DÉDICACE.....	III
REMERCIEMENTS	IV
RÉSUMÉ.....	VI
ABSTRACT	VIII
TABLE DES MATIÈRES	IX
LISTE DES TABLEAUX.....	XIII
LISTE DES FIGURES.....	XIV
LISTE DES SIGLES ET ABRÉVIATIONS	XV
LISTE DES ANNEXES.....	XVI
CHAPITRE 1 INTRODUCTION.....	1
1.1 Contexte	1
1.2 Problématique.....	2
1.3 Question de recherche	3
1.4 Hypothèses de recherche	4
1.5 Objectifs	4
1.6 Méthodologie	5
1.7 Originalité et contributions.....	5
1.8 Structure du document	7
CHAPITRE 2 REVUE DE LITTÉRATURE.....	8
2.1 Concepts classiques de fiabilité.....	9
2.1.1 Expressions de fiabilité	10
2.1.2 Évaluation de la fiabilité	11
2.2 Concepts classiques de maintenance.....	13

2.2.1	Stratégies de maintenance planifiée	14
2.2.2	Stratégies de maintenance prédictive	15
2.3	Intérêt des dépendances structurelles, économiques et stochastiques	16
2.3.1	Dépendances structurelles	16
2.3.2	Dépendances économiques	17
2.3.3	Dépendances stochastiques	18
2.4	Cas d'étude des ponts en béton	19
2.4.1	Modes de défaillance	20
2.4.2	Opérations de maintenance	21
2.4.3	Données de maintenance	22
2.5	Conclusion	24
CHAPITRE 3 ARTICLE 1: FAILURE INTERACTION MODELS FOR MULTICOMPONENT SYSTEMS: A COMPARATIVE STUDY		25
3.1	Introduction	25
3.2	Concepts of interactive failures	26
3.2.1	Reliability indexes' interaction	28
3.2.2	State-based interaction	43
3.2.3	Copula-based interaction	50
3.3	Comparative study of failure interaction models	57
3.3.1	Concepts	57
3.3.2	Methods	59
3.3.3	Applications	61
3.4	Conclusion	63
CHAPITRE 4 ARTICLE 2: FAILURE INTERACTION MODEL BASED ON EXTREME SHOCK AND MARKOV PROCESSES		64

4.1	Introduction	64
4.2	Failure interaction model	67
4.2.1	Random draw interaction mechanism	67
4.2.2	Failure interaction probabilities	69
4.2.3	Time-to-failure with interaction failure modes	70
4.3	Probability estimation through a Markov process method.....	72
4.4	Evaluation of the reliability indexes.....	75
4.4.1	Expected time-to-failure.....	75
4.4.2	Survival model	76
4.5	Example of concrete culverts	77
4.6	Conclusion.....	81
CHAPITRE 5 ARTICLE 3: OPTIMAL PREDICTIVE MAINTENANCE STRATEGY BASED ON AN INTERACTIVE DEGRADATION MODEL OF DEPENDENT FAILURE MODES		83
5.1	Introduction	83
5.2	Interactive degradation model	86
5.3	Predictive maintenance strategy using interactive degradation model	91
5.4	Illustrative example	93
5.5	Conclusion.....	101
CHAPITRE 6 DISCUSSION GÉNÉRALE		104
6.1	Récapitulatif	104
6.2	Confirmation des hypothèses de recherche	107
6.3	Limites d'application des propositions	109
6.4	Défis technologiques	110
CHAPITRE 7 CONCLUSION ET RECOMMANDATIONS		112

7.1	Conclusion générale	112
7.2	De nouvelles perspectives de recherche	113
RÉFÉRENCES.....		115

LISTE DES TABLEAUX

Tableau 2.1	Expressions de fiabilité et relations analytiques entre celles-ci	12
Tableau 3.1	Classification criteria of the failure interaction models	28
Tableau 3.2	Parameters of Reliability Indexes' Interaction models	40
Tableau 3.3	Parameters of state-based interaction models	49
Tableau 3.4	Parameters of Copula-based interaction models	56
Tableau 3.5	Advantages and limits of interaction models concepts	58
Tableau 3.6	Advantages and limits of interaction models methodologies.....	60
Tableau 3.7	Advantages and limits of interaction models applications.....	62
Tableau 4.1	Probabilities $\hat{p}_k^{(S1)}$ and effect of S2 on S1	78
Tableau 4.2	Probabilities $\hat{p}_k^{(S2)}$ and effect of S1 on S2	78
Tableau 4.3	Effect of the intensity λ on the coefficients.....	80
Tableau 5.1	Degradation parameters of concrete culverts for both displacement (S1) and cracking (S2) failure modes	95
Tableau 5.2	Means and variances of the Gamma distributions of shocks S1 and S2	96
Tableau 5.3	Probabilities $\hat{p}_k^{(S1)}$ and effects of S2 on S1.....	96
Tableau 5.4	Probabilities $\hat{p}_k^{(S2)}$ and effects of S1 on S2.....	96
Tableau 5.5	Cost ratios to the corrective maintenance (overhaul) cost per unit of length.....	98
Tableau 5.6	Yearly cost-ratio of the PdM strategy for diverse values of $L_r^{(S1)}, L_r^{(S2)}$ and Q	100
Tableau 5.7	Optimum values of the PdM strategy	100
Tableau 5.8	Optimum values of the PdM strategy for different cost ratios: $\pi_r^{(S1)}$ and $\pi_r^{(S2)}$	102

LISTE DES FIGURES

Figure 4.1	Example of random draws in a pool subject to shocks and interaction	69
Figure 4.2	System subject to two competitive extreme shock processes	71
Figure 4.3	Random drawing as Markov chains considering states $i = (l_i, k_i)$	72
Figure 4.4	Random drawing as continuous Markov process	73
Figure 4.5	Distribution functions of S1, S2 and S (both failure modes) at the n -th shock	79
Figure 4.6	Variation of coefficients in function of the “drawing speed” λ	80
Figure 4.7	Expected number of displacements and cracking shocks that will provoke an interaction, considering Intensity λ	81
Figure 5.1	System subject to two competitive and interactive degradation processes	89
Figure 5.2	Flowchart of the proposed PdM strategy	94
Figure 5.3	Reliability and failure functions of S1, S2 and both failure modes	97

LISTE DES SIGLES ET ABRÉVIATIONS

cdf cumulative distribution function

MTBF Mean Time Between Failures

MTTR Mean Time To Repair

PdM Predictive Maintenance

PM Preventive Maintenance

PPM Planned Preventive Maintenance

LISTE DES ANNEXES

Annexe A Démonstrations relatives au chapitre 4	133
Annexe B Macros et captures d'écran des simulations de la stratégie de maintenance prédictive du chapitre 5	139

CHAPITRE 1 INTRODUCTION

La compétitivité et l'évolution rapide des marchés technologiques imposent à toute entreprise une sélection informée de méthodes de gestion de ses actifs physiques. L'un des défis principaux de la gestion des actifs physiques est l'optimisation intelligente des ressources assignées à leur maintenance. En Janvier 2013, après une défaillance de batteries lithium-ion sur deux avions, tous les *Boeing 787 Dreamliners* étaient indéfiniment interdits de décoller. Cet évènement, en plus de révéler des enjeux de sécurité, a eu de lourdes conséquences économiques (Ross [2013], Williard et al. [2013]). Il est primordial d'avoir une certaine capacité d'anticipation de tels évènements, des éventuelles défaillances et des coûts qu'elles pourraient engendrer. Les domaines de la fiabilité et de la maintenance sont définis autour de cette problématique. Si la fiabilité étudie le comportement d'un système face aux potentielles défaillances, la maintenance est une réponse pratique à cette potentialité. Cette maintenance se doit d'être préventive pour réduire les coûts et les conséquences de tout éventuel dysfonctionnement du système. Elle peut intégrer un modèle prédictif qui permet de représenter la dégradation du système lorsque cette dernière est régulièrement ou continuellement surveillée.

1.1 Contexte

Les méthodes utilisées aux fins de la prédiction des défaillances se basent sur des données observables, des modèles physiques ou un historique de maintenance. Les modèles physiques peuvent représenter de façon déterministe le comportement d'un système (Melchers & Beck [2018]). Ils sont en général difficiles à établir à cause de la complexité des phénomènes concernés et requièrent donc de longues périodes d'expérimentation. Cela est d'autant plus le cas s'ils concernent des systèmes introduisant de nouvelles technologies. Une alternative est de construire des modèles de prédiction qui s'approchent de l'évolution des données observables caractérisant l'état d'un système dans le temps (An et al. [2015]). C'est dans cette capacité qu'intervient le forage de données. Cette approche est moins précise que l'usage de modèles physiques. En amont, il est nécessaire d'avoir à disposition des ressources telles que des capteurs, des senseurs, des techniciens et opérateurs qualifiés (Meeker & Hong [2014]). Ce dernier aspect suscite un problème de faisabilité économique. En outre, il est évident que l'abondance et la qualité des données conditionnent la qualité des modèles. Il existe des modèles moins onéreux qui prédisent les

défaillances d'un système en se basant sur l'historique de sa maintenance. Principalement probabilistes, ils sont moins précis que les modèles physiques et les modèles basés sur la collecte de données mais peuvent être combinés à ces derniers, offrant ainsi une large variété de possibilités de modélisation (Modarres et al. [2016]). Ceci dit, ces modèles s'appuient sur des hypothèses de départ qui ne capturent, bien souvent, que des aspects limités du fonctionnement réel d'un système.

1.2 Problématique

Le fonctionnement d'un système peut être ralenti ou arrêté par différents facteurs ou de diverses manières. En d'autres termes, il peut être sujet à divers modes de défaillances; ce qui est d'autant plus le cas si le système a de multiples composants. La plupart des approches classiques considèrent ces modes comme étant indépendants. En fait, cette hypothèse d'indépendance existe principalement pour simplifier le poids analytique et diminuer la complexité des modèles de fiabilité. En réalité, les modes de défaillances d'un système interagissent à bien des niveaux.

Par exemple, la maintenance des pneus d'une flotte de véhicules met en évidence des cas d'interactivité. Selon Ammar (2009), un pneu peut être sujet à une déchirure du talon, une crevaison, une usure par bosse, une usure de la bande de roulement, une usure du flanc, etc. Par exemple, l'usure de la bande de roulement ou du flanc favorise la crevaison et l'usure par bosse. Inversement, un pneu réparé ou rénové pour cause de l'un de ces modes de défaillances s'usera en général plus rapidement qu'un pneu neuf. Un autre exemple d'interactivité est la membrane d'osmose inversée utilisée dans le cas du dessalage des eaux de mer (Greenlee et al. [2009]). Elle peut ne plus remplir sa fonction à cause d'une contamination par résidus organiques, une contamination ou une usure due à son prétraitement chimique, une obstruction due à l'accumulation de sel, des dégâts mécaniques dus à des surpressions, etc. Ces modes sont considérés comme distincts. Par contre, selon Fritzmann et al. (2007), une contamination chimique facilite l'usure de la membrane et accentue sa sensibilité aux surpressions. Un cas intéressant présenté dans cette thèse est celui des ponceaux en béton utilisés dans les réseaux routiers. Ils servent à faciliter la circulation des cours d'eaux et parfois des espèces animales tout en assurant la circulation sécuritaire des véhicules. Ces structures peuvent se fissurer, se perforer, subir des déplacements, s'affaisser, s'affouiller, être encombrées par une accumulation de masses diverses, etc. (Schall [2012]). Il est évident que si la structure s'affaisse ou s'affouille, le risque d'encombrement augmente. Par ailleurs, plus les déplacements sont importants plus les fissures de la structure seront

proéminentes. Conséquemment, des perforations apparaîtront dans les zones fortement fissurées. Plus de détails sont abordés à ce sujet dans le prochain chapitre.

La présence de l'interactivité est indéniable pour tout système sujet à divers modes de défaillances compétitifs. Il est pourtant rare que ce phénomène soit directement mesurable. Dans les exemples susmentionnés, on ne peut en général que remarquer l'accélération de la fréquence d'un mode de défaillance en relation avec l'évolution rapide d'une variable d'état surveillant un autre mode potentiellement interactif. En effet, les liens entre divers modes sont complexes et généralement représentés par des fonctions conjointes toutes aussi complexes intégrées aux modèles de fiabilité. Ces fonctions peuvent être difficiles à interpréter physiquement et limitatives dans leur applicabilité suivant les hypothèses de départ choisies. Il serait d'un grand intérêt de développer des modèles de fiabilité qui capturent et expliquent de façon réaliste l'interactivité.

1.3 Question de recherche

La principale question sur laquelle se base cette thèse est la suivante :

Comment peut-on représenter l'interactivité des modes de défaillance d'un système de manière interprétable, l'intégrer à des modèles de fiabilité réalistes et optimiser la stratégie de maintenance appropriée ?

Pour répondre à cette question, certaines interrogations secondaires doivent être abordées :

- Quels sont les concepts pertinents d'interactivité des défaillances proposés par la littérature? Quels sont les avantages, les limites et les critères de différenciation de ceux-ci?
- Comment peut-on représenter dans des modèles de fiabilité le caractère aléatoire d'un phénomène d'interaction dont on a une connaissance limitée? Est-il possible de développer de tels modèles de fiabilité sur la base de données d'un historique de maintenance et de données observables?
- Quel impact auraient ces modèles de fiabilité dans une stratégie de maintenance prédictive visant à appréhender les conséquences de l'interactivité des défaillances et permettant de minimiser le coût de maintenance ?

1.4 Hypothèses de recherche

Telles que formulées, les interrogations relatives à ces travaux limitent le champ de recherche à des systèmes dont on soupçonne à priori un phénomène interactif et dont l'interprétation du phénomène demeure ambiguë. De ce fait, les hypothèses suivantes sont appropriées :

- Les modes de défaillances d'un système interagissent de manière à altérer la fiabilité du système considéré.
- L'interaction des modes de défaillances peut intervenir sous la forme d'une série de chocs au système, aléatoires et interdépendants.
- Les probabilités d'occurrence d'une interaction peuvent être quantifiées sous la forme de variables aléatoires.
- Les données de maintenance et de dégradation permettent de modéliser l'effet de l'interactivité.
- Une stratégie de maintenance prédictive intégrant l'interaction au sein d'un système permet d'optimiser les coûts de maintenance.

1.5 Objectifs

Afin de répondre aux questions mentionnées plus haut, les objectifs suivants ont été formulés :

- Classer les modèles de fiabilité intégrant l'interactivité des modes de défaillances.
- Représenter l'interactivité des modes de défaillances d'un système à l'aide des données de défaillance et de dégradation du système.
- Évaluer l'effet des modes interactifs de défaillance sur le comportement fiabiliste du système par comparaison à l'hypothèse d'indépendance des modes de défaillance.
- Développer un modèle de fiabilité applicable dans le cadre de la maintenance prédictive.
- Optimiser une stratégie de maintenance prédictive afin de minimiser le coût encouru lorsque l'influence des défaillances interactives est prise en compte.

1.6 Méthodologie

Le terme « réaliste » étant quelque peu suggestif, il est important que tout apport technique soit précédé d'une discussion conceptuelle sur la valeur d'un modèle de fiabilité intégrant l'interaction des défaillances. Sur la base des hypothèses susmentionnées, la méthodologie suivante est le vecteur choisi afin d'atteindre les objectifs de recherche :

- Une revue de la littérature et une classification des modèles de fiabilité intégrant l'interactivité des défaillances. Cette classification est appuyée par une étude comparative des concepts, des méthodes et des applications en maintenance rattachés à ces modèles.
- Le développement de modèles de fiabilité basés sur la représentation de l'interactivité des défaillances comme des processus de chocs aléatoires. Ces modèles sont construits à partir de données d'historiques de maintenance et de données observables.
- L'utilisation des processus de Markov capturant le caractère séquentiel des interactions.
- La validation des modèles proposés en comparant leurs courbes de fiabilité avec celles de modèles n'intégrant pas l'interactivité de défaillances.
- Le développement et l'optimisation d'une stratégie de maintenance prédictive permettant de minimiser le coût. L'optimisation d'une telle stratégie requiert des méthodes de simulation stochastique.
- L'application des méthodes développées au cas des ponts.

1.7 Originalité et contributions

Les systèmes industriels contemporains évoluent rapidement d'un point de vue technologique. Conséquemment, la modélisation des modes de défaillances s'avère être de plus en plus complexe. L'originalité de cette thèse est avant tout de rendre accessible aux gestionnaires de maintenance le concept d'interactivité des modes de défaillance d'un système et d'affiner la prévision de la fiabilité en comparaison avec des modèles plus classiques. Les travaux proposés dans cette thèse permettent de :

- Développer une approche générique de modélisation et caractériser l'interactivité sous forme de série de chocs aléatoires successifs et interdépendants.

- Quantifier et interpréter l'interactivité et son influence sur la fiabilité afin de mieux comprendre et mesurer un phénomène dont on ne peut observer que les effets indirects.
- Valoriser les données historiques et les données de dégradation dans une dynamique de maintenance prédictive.
- Minimiser le coût de la maintenance d'un système sujet à des modes interactifs de défaillance.

Trois contributions scientifiques ont été développées pour contribuer à la recherche dans le domaine de l'ingénierie de la fiabilité et la maintenance de systèmes sujets à des modes interactifs de défaillance :

- **Contribution 1: Failure interaction models for multicomponent systems: a comparative study. *SN Applied Sciences*, 1(1), 66.**

Cette contribution comprend une revue de la littérature détaillée de modèles de fiabilité, pertinents et récents, intégrant l'interaction des modes de défaillances. Elle comprend aussi une proposition de critères de classification de tels modèles. Les concepts, les méthodes et les applications en maintenance de ces approches sont comparées afin de conseiller et encadrer toute démarche de modélisation de l'interactivité.

- **Contribution 2: Failure interaction model based on extreme shock and Markov processes. *Reliability Engineering & System Safety*, 197, 106827.**

L'interaction des modes de défaillances d'un système y est représentée sous forme de chocs aléatoires interdépendants. Le phénomène est quantifié au moyen d'une série de probabilités d'occurrence de ces chocs qui permet de construire une distribution distinctive du caractère séquentiel des chocs. Les paramètres des fonctions de fiabilité sont estimés via des processus de Markov. Comme application, les phénomènes de déplacement et de fissuration dans les ponceaux en béton sont concurremment étudiés.

- **Contribution 3: Optimal repair strategy based on interactive degradation model of dependent failure modes. Soumis en Juin 2020 à: *Reliability Engineering & System Safety*. JRESS-D-20-00273.**

Le modèle de la précédente contribution est amélioré pour associer un processus de dégradation à des chocs compétitifs. Une stratégie de maintenance prédictive est définie avec pour objectif la valorisation de la séquence des défaillances interactives. Les ponts en béton font l'objet d'une étude suivant cette approche et l'impact sur le coût est évalué et interprété.

1.8 Structure du document

Le document est organisé en 7 chapitres. A la suite de cette introduction, le chapitre 2 est une revue de littérature présentant différents aspects liés à la fiabilité, la maintenance, l'interactivité des défaillances et au cas d'étude ainsi que la description des données disponibles. Les chapitres 3, 4 et 5 présentent respectivement les trois contributions susmentionnées. Une discussion générale suit au chapitre 6 afin de comparer les différents concepts et résultats présentés. Le chapitre 7 offre une conclusion générale à la thèse et des perspectives réalistes.

CHAPITRE 2 REVUE DE LITTÉRATURE

Blanchard et al. (1990) définissent un système comme un ensemble logique de composants, d'attributs et de relations qui opère dans un environnement avec lequel il interagit. Les composants sont les parties opérantes du système. Les attributs sont les propriétés des composants du système. Ils sont généralement observables au travers d'une variété de valeurs physiques et/ou chimiques que traversent les composants. Ces valeurs définissent l'état du système. Les relations sont les liens entre les attributs et les composants. En effet, les composants peuvent avoir la capacité au travers de diverses combinaisons de transmettre une information ou une énergie, d'accomplir un mouvement ou une combinaison de mouvements, de favoriser ou provoquer une réaction chimique, et/ou de garantir l'intégrité physique. Un composant est lui-même un système en relation avec d'autres systèmes. Ces relations sont au service d'une ou plusieurs fonctions.

Ces fonctions sont définies suivant des critères quantitatifs et qualitatifs en lien avec la capacité et le rendement attendus du système. Lorsqu'une fonction est significativement altérée et ne peut respecter ces critères prédéfinis, on emploie le terme de défaillance. Divers facteurs internes et externes peuvent entraîner une défaillance. De manière générale, ses effets sont physiquement mesurables, descriptibles ou perceptibles sur l'état du système. Un seuil est de ce fait défini par l'utilisateur du système au-delà duquel la fonction principale est considérée comme n'étant plus assurée. Une défaillance peut être due au dysfonctionnement d'un ou plusieurs composants, à l'altération d'un ou plusieurs attributs des composants et/ou à l'évolution des relations les reliant. Dans le cas des actifs physiques, ces modifications peuvent être causées par l'utilisateur, l'environnement ou la détérioration physique naturelle de leurs composants. Selon Crow (1975) on peut distinguer deux principaux types de systèmes : les systèmes réparables et les systèmes non-réparables. Une ampoule électrique est un exemple de système non-réparable. Si une défaillance intervient, le système en entier doit être remplacé. Une automobile est un système réparable. Ses composants sont régulièrement inspectés et réparés ou remplacés au besoin lorsqu'advient une défaillance. Dans un cas comme dans l'autre, une défaillance a des conséquences économiques et impacte la sécurité de l'utilisateur et de l'environnement. Si les conséquences économiques sont parfois évidentes, le volet sécuritaire soulève des questions éthiques que nous ne développons dans cette thèse. Ceci dit, il convient de les mentionner car la sécurité de l'utilisateur ne touche pas seulement son bien-être physique, mais tout l'ensemble de l'individu. Il est primordial que le

système industriel ne détériore pas sa santé physique mais aussi son état psychologique, sa situation sociale, ses valeurs éthiques et morales.

Nous entendons par usager dans le cadre industriel tout opérateur ou utilisateur. De manière générale, la société, l'entreprise possédant le système et son lieu géographique constituent son environnement immédiat. Des problèmes ergonomiques, opérationnels, logistiques, organisationnels, de compétence ou même éthiques peuvent influencer le bon fonctionnement des actifs. L'erreur humaine est une variable que prennent en compte bien des experts à cet effet selon Swain (1990), Griffith & Mahadevan (2011), Groth & Swiler (2013). La formation et la mise en place de méthodes de travail appropriées sont des solutions dans ces cas de figure. Lorsque les causes des défaillances sont toutes autres, il devient important de comprendre les mécanismes qui régissent le fonctionnement du système pour en prédire le comportement et prévenir tout dysfonctionnement. L'étude de la fiabilité fournit un cadre pour la conception et la maintenance d'actifs physiques sécuritaires et viables économiquement. Dhillon (1999) suggère que le concept de la fiabilité date des années 30-40 dans le domaine militaire. La fiabilité est devenue quelques décennies plus tard dans les années 50-60, une discipline scientifique et académique à part entière (Barlow & Proschan [1975]). L'évaluer est un exercice comportant de nombreux défis aussi bien conceptuels que pratiques. Nous présentons ces défis non-exhaustivement dans cette revue de littérature centrée sur des aspects que nous estimons être primordiaux pour le monde industriel.

Avant tout, quelques concepts classiques relatifs à la fiabilité et la maintenance sont présentés. Ensuite, les concepts de dépendances structurelles, économiques et stochastiques sont abordés d'un point de vue fiabiliste. En effet, ses diverses relations peuvent considérablement complexifier toute représentation de la fiabilité et son application à la maintenance. Par la suite, diverses méthodes mathématiques et probabilistes de modélisation de la fiabilité et d'optimisation de la maintenance sont étudiées. Enfin, la maintenance des pontons est décrite dans le but de bien comprendre la problématique, les modes de défaillance et les données disponibles. En effet, le ponton en béton est le système utilisé comme exemple d'application des modèles développés dans cette thèse.

2.1 Concepts classiques de fiabilité

La fiabilité s'exprime sous forme de probabilité et représente l'aptitude d'un système d'accomplir la fonction pour laquelle il a été conçu dans des conditions définies pour une durée de mission donnée (Diallo [2006]). Ces conditions peuvent être diverses en termes de performance et

relativement à l'environnement influençant et influencé par le système. Reprenant l'exemple d'un véhicule, sa fiabilité par rapport à sa capacité de transport de biens et d'usagers d'un point à un autre n'est effective que si des conditions d'efficacité énergétique, de sécurité des passagers et des biens, et de limitation des effets polluants sont respectées entre autres critères. La fiabilité a une définition plus large que simplement la notion de succès ou échec d'une tâche. Les conditions de fonctionnement non satisfaites et l'état du système à l'instant de la défaillance concourent à une définition plus adéquate de cette défaillance. Les relations d'un système avec son environnement sont à prendre en compte dans la représentation de la fiabilité.

De manière pratique, l'évaluation de la fiabilité intervient principalement à deux étapes du cycle de vie d'un système : durant sa conception et durant son utilisation. En phase de conception, Misra (1975) présente diverses problématiques qui sont abordées dans le but de maximiser la fiabilité d'un système. Entre autres, il s'agit d'allouer des redondances pour les composants critiques, de simplifier autant que faire se peut l'architecture du système, de planifier la maintenance avant la mise en service, d'établir un programme de mise à jour et d'amélioration des composants. La fiabilité dite prévisionnelle peut être appréhendée au travers de diverses méthodes : les blocs-diagramme de fiabilité, l'Analyse des Modes de Défaillance et de leurs Effets ou, une extension de cette dernière, l'Analyse des Modes de Défaillance, de leurs Effets et de leur Criticité (AMDEC), etc. (Borgovini et al. [1993], Čepin [2011], Lipol & Haq [2011], Spreafico et al. [2017], Jia et al. [2019]). Quoiqu'efficaces, toutes ces méthodes ne peuvent parfaitement représenter la réalité en situation d'exploitation. Il est en effet possible que certains cas de figure soient omis par les concepteurs, d'où l'importance d'un suivi assidu des actifs physiques durant leur cycle de vie. Ceci dit; dans cette thèse, nous nous intéressons principalement aux actifs physiques en exploitation.

2.1.1 Expressions de fiabilité

Soient S et $\bar{S}$ deux événements représentant respectivement la survie et la défaillance d'un système. Ils sont complémentaires pour un système en exploitation car ne pouvant advenir simultanément. La fiabilité ou probabilité de survie ($P[S]$) et la probabilité de défaillance ($P[\bar{S}]$) sont reliées comme suit :

$$P[S] = 1 - P[\bar{S}] \quad (2.1)$$

Dans le domaine continu, la fiabilité est en général représentée par une fonction dite de survie, généralement notée $R(t)$, qui permet de l'évaluer en fonction de l'âge du système. La probabilité de défaillance du système est évidemment aussi dépendante de l'âge. En considérant une variable continue aléatoire $T \geq 0$ de durée de vie, Finkelstein et al. (2013) s'appuient sur une définition classique de cette probabilité de défaillance notée $F(t)$ afin de traiter des questions de modélisation stochastique de la fiabilité :

$$F(t) = 1 - R(t) = P(T \leq t), \quad t \in [0, +\infty[\quad (2.2)$$

où $P(T \leq t)$ est la probabilité que la durée de vie ne dépasse pas t . Cette notation conventionnelle de probabilités est fréquemment utilisée dans toute la thèse. Cette probabilité peut suivre une loi de probabilité déterminée ou s'en rapprocher. Lorsque cela est applicable, cette densité est définie comme suit :

$$f(t) = \frac{d}{dt} F(t) \quad (2.3)$$

Il est possible d'en déduire ce que l'on désigne comme le taux de défaillance du système, usuellement noté $h(t)$ qui n'est autre que la fréquence à laquelle adviennent les défaillances. Il est exprimé en nombre de défaillances par unité de temps :

$$h(t) = \frac{f(t)}{R(t)} \quad (2.4)$$

Le tableau 2.1 qui est un extrait des travaux de Hamada et al (2008) montre comment ces diverses fonctions sont liées les unes aux autres.

2.1.2 Évaluation de la fiabilité

La fonction de survie d'un système peut être obtenue de diverses manières. Les outils qui permettent d'observer ou de tester ce système et les types de données qui sont recueillies sont des éléments déterminants dans la sélection du modèle de cette fonction. Ces données sont principalement des durées de vie obtenues d'un historique de maintenance, des variables d'état (température, vibration, etc.) discrètes ou continues, ou des résultats de tests expérimentaux (Meeker [2014]).

Tableau 2.1 Expressions de fiabilité et relations analytiques entre celles-ci

	En fonction de...			
	$f(t)$	$F(t)$	$R(t)$	$h(t)$
$f(t)$	$f(t)$	$\frac{d}{dt}F(t)$	$-\frac{d}{dt}R(t)$	$h(t)e^{-\int_0^t h(s)ds}$
$F(t)$	$\int_0^t f(s)ds$	$F(t)$	$1 - R(t)$	$1 - e^{-\int_0^t h(s)ds}$
$R(t)$	$\int_t^\infty f(s)ds$	$1 - F(t)$	$R(t)$	$e^{-\int_0^t h(s)ds}$
$h(t)$	$\frac{f(t)}{\int_t^\infty f(s)ds}$	$\frac{\frac{d}{dt}F(t)}{1 - F(t)}$	$-\frac{d}{dt}\{\ln R(t)\}$	$h(t)$

Le comportement défaillant peut, comme il a été dit précédemment, être décrit par une loi de probabilité. Cette loi permet de calculer la probabilité de défaillance en fonction du temps. Cette approche se base en général sur les durées de vie répertoriées dans l'historique de maintenance. Parmi les lois les plus fréquemment utilisées figurent les lois de Weibull, exponentielle, normale, de Poisson, etc. (Crowder et al. [1994]). Il en existe beaucoup et c'est en général par le moyen du maximum de vraisemblance que les paramètres de la loi adéquate sont déterminés. Lorsque des données sur la dégradation du système sont disponibles, des modèles connexes ayant des variables explicatives sont utilisées (Gorjian et al. [2010]). Par ailleurs, il existe des méthodes non-paramétriques pour évaluer la fiabilité à partir des durées de vie ou temps de défaillance. Elles s'avèrent très utiles pour avoir une première approximation de la fonction de survie ou si trop peu d'informations est disponible pour ajuster une loi au comportement du système. L'estimateur non paramétrique de Kaplan-Meier est probablement le plus utilisé (Kaplan & Meier [1958]).

Les récentes avancées dans le domaine de la prospection de données ont permis une floraison de modèles basés sur les données. Ils sont prisés car ils décèlent des relations complexes en les traduisant sous forme de formules mathématiques. Ils présentent dans certains cas l'avantage de pouvoir combiner dans un modèle mathématique les durées de vie et les données sur la dégradation du système quand elles sont disponibles (Nikulin, et al. [2010]). Par contre, ils requièrent une abondance de données et leur précision dépend énormément de la qualité des données. En outre,

malgré leur pouvoir de prédiction, l'interprétation des phénomènes qu'ils décrivent pourrait être ambiguë.

Dans le domaine de la fiabilité structurelle, il y a un intérêt particulier pour la conception de modèles physiques. On utilise une variété de méthodes déterministes et/ou de méthodes stochastiques (Rackwitz, R. [2001]). La méthode contrainte-résistance, par exemple, est un outil pertinent à cet effet (Bererich, Y. [2010]). Elle relie statistiquement la durée de vie ou résistance du système aux contraintes auxquelles il est soumis. Cette approche requiert une expérimentation rigoureuse afin de déterminer la fiabilité en fonction de chaque mode de défaillance. Ceci dit, son objectif principal est de fournir au concepteur du système des idées d'optimisation de sa conception. Les modèles physiques sont supposés avoir la meilleure précision si le processus d'expérimentation s'avère suffisamment rigoureux.

2.2 Concepts classiques de maintenance

Il existe principalement deux types de maintenance : la maintenance corrective et la maintenance préventive. Le terme « corrective » suggère une réaction à un événement dont il faut rectifier les causes premières. Lorsqu'une politique de maintenance corrective est employée, les actifs sont exploités jusqu'à leur défaillance. Quand cela advient, ils sont alors remplacés ou réparés. Ce genre de politique est en général utilisé quand l'arrêt non programmé ou tout mauvais fonctionnement des composants ou du système concerné ne représentent que des risques sécuritaires ou économiques minimes. La peinture, les essuie-glaces, les couvertures de sièges d'une voiture en sont des exemples. Par contre, pour une entreprise, cette politique suggère une approche intelligente des problèmes d'inventaire et de chaînes d'approvisionnement (Sheut & Krajewski [1994]).

Pour un composant tel que le moteur d'une voiture qui est crucial au fonctionnement, complexe dans son architecture et coûteux, on ne peut attendre une défaillance qui pourrait d'ailleurs rendre irréparable cet organe du véhicule. Des activités de prévention et d'entretien sont nécessaires afin de maximiser sa durée de vie. Un autre composant tel qu'un pneu est moins coûteux mais il sera remplacé s'il présente des signes avancés d'usure. S'il ne l'est pas, il peut se perforer durant l'usage du véhicule, ou se bosseler. Ces événements engendrent des risques pour la sécurité de l'utilisateur du véhicule. La maintenance préventive intervient dans ce cadre. Elle est un sujet vastement étudié dans la littérature. Selon Wang (2002), il existerait des milliers de modèles aux fins de son implémentation et son optimisation. Ils répondent au besoin de réagir à l'occurrence d'une

défaillance, mais surtout si cela est possible, de prévenir cette défaillance. Ce second aspect est dans l'optique de l'amélioration de la fiabilité. Poursuivant cet objectif, le concept de maintenance centrée sur la fiabilité a vu le jour il y a quelques décennies (Nowlan & Heap [1978], Tang et al. [2017]). Des quantités, qui ont une valeur déterminante pour la planification de la maintenance, sont alors dérivées de la fiabilité. La plus récurrente dans la littérature est la durée moyenne entre pannes notée MTBF (*Mean Time Between Failures*) :

$$MTBF = \int_0^{\infty} R(t)dt \quad (2.5)$$

De manière pratique, une politique de maintenance optimale doit conduire soit à minimiser le coût théorique des actions de maintenance, soit à maximiser la disponibilité du système (Olsen [2017]). La disponibilité est une mesure de performance du système ; c'est la proportion de temps durant lequel le système reste fonctionnel. Sa valeur asymptotique se définit comme suit :

$$A = \frac{MTBF}{MTTR+MTBF} \quad (2.6)$$

où MTTR (*Mean Time To Repair*) est le temps moyen jusqu'à réparation. Le MTTR qui est basé sur les temps de réparation peut se comprendre de manière équivalente au MTBF basé sur les temps de bon fonctionnement.

Cela dit, il existe une grande variété de stratégies préventives. Les plus pertinentes sont présentées par la suite en deux grandes catégories que sont la maintenance planifiée et la maintenance prédictive.

2.2.1 Stratégies de maintenance planifiée

En situation d'exploitation, la maintenance planifiée se base sur un historique de maintenance ou sur les recommandations des documents constructeurs. Ces documents fournissent en général une espérance de vie approximative et des directives quant aux actions à mener régulièrement pour prolonger la vie du produit. Ceci dit, même en présence de documents constructeurs, l'exploitation des actifs physiques présente certains défis spécifiques sous-évalués ou difficiles à prévoir par les concepteurs. En effet, l'erreur humaine, les aspects logistiques et organisationnels spécifiques à une compagnie, la variabilité des environnements de travail et des situations géographiques, sont entre autres des éléments qu'un modèle théorique ne peut parfaitement prévoir de manière déterministe. Ces aspects influencent pourtant directement ou indirectement un système. Cela rend

indispensable la tenue d'un historique de maintenance. Il répertorie les temps de défaillances et les temps alloués aux réparations mais aussi les ressources humaines et financières allouées à ces événements. Il permet donc de réévaluer la fiabilité du système tel qu'il fonctionne réellement dans son environnement.

Une politique de maintenance planifiée s'optimise dans la plupart des cas en fonction de l'âge du système étudié. L'objectif est alors de déterminer un âge optimal du système et/ou une fréquence auxquels les actions préventives doivent être exécutées. Kumar & Westberg (1997), Jhang & Sheu (2000), Jiang & Ji (2002) développent des politiques de remplacement en fonction de l'âge. Yeh et al. (2007), Ullah & Jiang (2019) traitent des cas de remplacements à périodicité fixe. Pour un système avec de nombreux composants, ces opérations deviennent des remplacements par blocs tels que décrites par Sheu & Griffith (2002), Ke & Yao (2016). Certains modèles envisagent les actions de maintenance à des intervalles inégaux; ce sont des politiques dites séquentielles. Les politiques de Nakagawa (1988) et Lin et al. (2001) en sont des exemples.

Par ailleurs, dans une approche plus directe, il est possible d'optimiser une politique en fonction du nombre optimal de réparations que l'on peut s'autoriser. C'est le cas dans les travaux de Bai & Pham (2005). Ces divers modèles emploient des approches variées tels que les réparations minimales et la maintenance imparfaite. Ces approches interviennent afin de prendre en compte les limites opérationnelles d'une compagnie mais aussi pour optimiser le coût et le temps des activités de maintenance (Pham & Wang [1996]).

2.2.2 Stratégies de maintenance prédictive

Selon la norme NF EN 13306 X 60-319, la maintenance prédictive ou prévisionnelle est une maintenance conditionnelle (basée sur la condition). Elle est exécutée suivant les prévisions extrapolées de paramètres significatifs de la dégradation d'un système (AFNOR [2018]). La maintenance conditionnelle se base sur des moyens de surveillance de l'état d'un système (Jardine et al. [2006], Peng et al. [2010], Shin & Jun [2015]). En effet, son objectif étant de surveiller l'évolution de l'état d'un système, des moyens sont mis en place afin de répertorier soit de façon continue ou de façon discrète les variables caractérisant cette évolution. Il peut s'agir de températures, de profondeurs d'usure, de vibrations, de corrosion, etc. Ces facteurs sont observables et mesurables au moyen de capteurs et détecteurs appropriés et/ou par des inspecteurs entraînés. C'est d'ailleurs au moyen de ces facteurs que se définissent les modes de défaillances.

S'ils sont altérés au-delà d'un certain seuil, le fonctionnement du système n'est plus optimal. Reprenant l'exemple d'une automobile, cette dernière comprend sur son tableau de bord de nombreux moyens de surveiller son état dont un indicateur de la température du moteur et des témoins d'alerte de couleur rouge qui signalent une panne du moteur et/ou un danger. Bien évidemment, il ne s'agit pas que d'observer les variables et de comprendre le « comportement » du système. Il est important d'en appréhender l'impact économique et les risques de sécurité mais surtout d'intervenir quand les variables d'état signalent une défaillance (Ahmad & Kamaruddin [2012], Alaswad & Xiang [2017], Ejaz et al. [2019], Liu et al. [2017, 2020]). La maintenance prévisionnelle intervient à un niveau supérieur. Elle permet d'intervenir avant que la défaillance ne soit avérée.

De manière générale, une stratégie de maintenance prévisionnelle déterminera des seuils pour les variables d'états à partir desquelles il faudra intervenir avant que la dégradation ne devienne trop prononcée et que la défaillance ne soit déclarée (Zhou et al. [2007], Van Horenbeek & Pintelon [2013], Jiang et al. [2015]). Ces interventions peuvent être humaines ou automatisées, périodiques ou suivant un modèle dynamique dépendamment de l'abondance et la qualité des données, des modèles de fiabilité et des moyens technologiques utilisés (Yan et al. [2017], Lughofer & Sayed-Mouchaweh [2019]). Les réparations minimales et la maintenance imparfaite sont aussi des politiques de maintenance utilisées pour ce type de stratégie.

2.3 Intérêt des dépendances structurelles, économiques et stochastiques

Un système interagit avec son environnement et comprend des composants qui sont en relation les uns avec les autres. Si le fonctionnement optimal du système dépend de la qualité de ces relations, l'évaluation de sa fiabilité et la planification de la maintenance dépendent d'autant plus de la compréhension de ces mêmes relations. Cho & Parlar (1991) décrivent ces importants mécanismes en trois groupes de dépendances que nous discutons ci-dessous : dépendances structurelles, économiques et stochastiques.

2.3.1 Dépendances structurelles

Un système est conçu suivant une certaine architecture. Ses composants sont physiquement liés (i.e. par un joint, un encastrement, etc.) et partagent des fonctions particulières dans l'architecture du système. En effet, les fonctions intrinsèques aux composants se combinent pour participer à une

ou plusieurs fonctions globales relatives au système. Ces liens physiques sont largement étudiés dans le domaine de la fiabilité structurelle (Royset et al. [2001], Thoft-Cristensen & Baker [2012], Melchers & Beck [2018]). Les relations structurelles suggèrent que la défaillance d'un composant affecte le fonctionnement des composants auxquels il se retrouve physiquement lié. Pour aller plus loin, en termes de maintenance, des composants dépendants structurellement devront être arrêtés ou concomitamment démontés si l'un d'eux est défaillant (Dao & Zuo [2017], Dinh et al. [2019]).

La plupart des modèles de fiabilité prennent en compte ce type de dépendances qui est directement observable. Par exemple, l'impact sur la fiabilité d'une configuration en séries par rapport à une configuration en parallèle est aisément différenciable. Dans une configuration en série, tout composant est crucial, ce qui n'est pas le cas en redondance (Feizabadi & Jahromi [2017], Kim & Kim [2017], Wang et al. [2018], Peiravi et al. [2019]). Parmi les concepts mettant ces dépendances en lumière, on retrouve les défaillances en cascades. Il s'agit de l'effet domino. C'est un principe suivant lequel une défaillance se propage ou des défaillances sont entraînées de manière successive les unes par les autres à cause des connexions structurelles internes au système. Rahnamay-Naeini & Hayat (2016), Guo et al. (2017), Cadini, et al. (2017), Baqaee (2018) étudient les défaillances en cascades dans les infrastructures et les réseaux de production.

2.3.2 Dépendances économiques

Des composants peuvent aussi avoir des liens significatifs d'un point de vue économique. Les relations structurelles mentionnées plus haut peuvent engendrer une nécessité qu'à l'avènement d'une défaillance des composants soient conjointement démontés. Leur inspection et leur maintenance conjointes engendrent des coûts conjoints qui peuvent être exploités et peuvent représenter un avantage économique (Keizer et al. [2017]). Pour aller plus loin, les composants dépendants économiquement ne sont pas forcément structurellement liés (Dekker et al. [1997]). Si les interventions sur des sous-systèmes non dépendants structurellement demandent des compétences, des moyens logistiques ou des outils spécifiques similaires ou identiques, afin de réduire le coût de ces interventions, il conviendra de prendre en considération simultanément ces sous-systèmes. Zhou et al. (2013), Dao et al. (2014), Atashgar & Abdollahzadeh (2017) proposent des approches à ce sujet. Cela a un impact sur l'organisation de la chaîne d'approvisionnement de la compagnie gérant ces actifs physiques.

Le concept de maintenance opportuniste exploite les dépendances économiques. Thomas et al. (2008) affirment qu'il n'y a pas de normes ou de standards consensuels quant à sa définition. Par contre, son principe est généralement compris comme suit. À l'avènement d'une défaillance ou d'une action de maintenance programmée sur un composant, pour les composants liés économiquement à ce dernier, un choix optimal d'actions préventives est effectué afin de diminuer le coût sur le long terme de la maintenance du système. Lin et al. (2008), Okogbaa et al. (2008), Zequeira et al. (2008), Huynh et al. (2013) proposent des modèles de maintenance préventive opportuniste.

2.3.3 Dépendances stochastiques

Les relations qui régissent l'évolution d'un système ne sont pas que structurelles ou économiques. Il existe des liens entre les composants et leurs défaillances que l'on constate sans en avoir l'explication physique définitive (Lai & Xie [2003], Filus & Filus [2008]). Ce genre de relation est souvent représenté dans le cadre de la dépendance stochastique. Si les méthodes classiques telles que les arbres de défaillance et les blocs-diagramme de fiabilité fournissent des approches aux dépendances structurelles et économiques, elles sont en revanche inefficaces en ce qui concerne les dépendances stochastiques. Ces dernières mettent en relation les états des composants. Ces états interagissent suivant des mécanismes plus ou moins complexes car ils partagent le même environnement. Un environnement de fonctionnement commun rend plausible le fait que l'état défaillant d'un sous-système ou d'un composant en affecte un autre. Caractériser ce type de phénomène peut s'avérer particulièrement complexe si l'on n'en maîtrise pas tous les tenants et les aboutissants. En effet, ce qui est directement observable c'est une évolution simultanée des variables d'état, des indices de fiabilité, ou, un arrêt simultané ou différé d'entités supposément liés. C'est suivant ce raisonnement que Murthy & Nguyen (1985a, 1985b) définissent cette dépendance en termes d'interaction des défaillances. Vu le caractère observable et mesurable d'une défaillance et de ces effets, c'est une manière pragmatique de représenter les dépendances stochastiques qui s'accorde parfaitement avec l'étude de la fiabilité. Ils considèrent deux types d'interactions:

- Les défaillances induites ou type I : un composant cause instantanément la défaillance d'un autre avec une probabilité fixe;

- L'interaction des taux de défaillances ou l'interaction par accumulation de chocs, type II : la défaillance d'un composant augmente le taux de défaillance d'un autre composant. Le type II se comprend aussi comme une accumulation de chocs internes qui changent la vitesse à laquelle les composants tombent en panne.

Ces définitions sont utilisées à ce jour par de nombreux modèles dont certains très récents. Par exemple, Sun et al. (2006) mettent ce type d'interaction en évidence entre un arbre et un roulement mécanique. L'usure du roulement accélère le désalignement de l'arbre. Rezaei (2017) considère des phénomènes similaires entre les composants d'une turbine à vapeur. Shen et al. (2017) se servent d'une copule mathématique pour représenter ses liens stochastiques pour une machine-outil à commande numérique.

La plupart des modèles proposés par la littérature sont appliqués dans le domaine de la maintenance planifiée (Castro [2009], Fan et al. [2011], Golmakani & Moakedi [2012], Sung et al. [2013], Lai & Yan [2014]). Cela dit, on trouve d'autres modèles développés pour la maintenance conditionnelle et/ou prédictive qui établissent des relations entre les variables d'états. Plus précisément, elles théorisent comment un processus de dégradation en affecte un autre jusqu'à la défaillance. Les vitesses de dégradation et les chocs accumulés s'influencent et provoquent des augmentations, mutuelles ou non (Rasmekomen & Parlikad [2016], Hao et al. [2017], Yang et al. [2018]). Cependant, vu que la définition des dépendances stochastiques est très large, les approches de modélisation des dépendances stochastiques sont très diverses et variées. Elles bénéficieraient d'une classification afin de, premièrement définir un cadre d'application de ces approches, ensuite étudier leurs avantages et limites, et enfin de suggérer les bonnes pratiques aidant à la conception de modèles pertinents de défaillances interactives.

2.4 Cas d'étude des ponceaux en béton

Dans les réseaux de transports, les ponceaux sont en général des ouvrages aménagés sur un cours d'eau en vue d'en permettre le franchissement. Ils permettent le passage de l'eau et/ou des animaux sous une route, des chemins de fer ou d'autres structures similaires (Schall [2012]). Parmi les types les plus utilisés, on retrouve les ponceaux en béton, les ponceaux en acier et les ponceaux en Polyéthylène haute densité. Les ponceaux en béton sont des ouvrages rigides dont le cycle de vie s'étend sur plusieurs décennies. Leur construction demande que soient mobilisées une firme de conception, une main d'œuvre qualifiée de réalisation et un organisme de vérification ainsi que des

ressources matérielles dont le béton, armature d'acier, puisards, graviers, etc. Le béton est en général renforcé en raison des importantes contraintes structurelles qu'il devra subir durant plusieurs années. En effet, les ponceaux en béton sont sujets à divers facteurs qui favorisent leur dégradation au fil du temps et peuvent réduire considérablement leur durée de vie (Kim & Yoo [2005], Chen & Sun [2013]). Si une telle structure vient à s'effondrer, une partie du réseau routier peut en souffrir.

Selon Shrivastava, G. (2016), ce fut le cas dans les Antilles, à Pont Cassé, le 19 Avril 2013. Suite à des pluies torrentielles, un véhicule transportant deux personnes tomba dans un ravin de 13 mètres qui s'était rapidement formé après l'effondrement d'un ponceau. Giresini et al. (2016) investiguent un évènement similaire qui eut lieu en Italie, en Sardaigne du Nord, le 18 Novembre 2013, occasionnant des pertes en vies humaines. Perrin & Dwivedi (2006) documentent plusieurs cas d'effondrements de chaussées dans diverses villes des Etats-Unis pour démontrer le besoin d'une gestion efficace de ces actifs. Le 6 Mai 2017, l'affaissement d'un ponceau au Québec obligea les autorités compétentes à fermer la route 117 pour plusieurs jours (Rivard-Boudreau [2017]). Un tel incident représente un danger pour l'usager de la route mais aussi un problème économique en raison des coûts des réparations et à cause de la circulation des biens et personnes retardée ou redirigée. Pour l'exemple de la route 117, une enquête a montré que le ponceau montrait des signes de dégradation avancés mais que les mesures préventives ont trop tardées. Afin de mieux comprendre ce qui sera le cas d'application principal de cette thèse, les modes de défaillances et les méthodes de maintenance des ponceaux en béton sont présentés ci-dessous.

2.4.1 Modes de défaillance

Le béton subit un traitement particulier durant la construction des ponceaux pour garantir qu'il résiste aux changements de température et aux contraintes structurelles qui lui seront imposés (Dvorkin & Dvorkin [2006]). Si ce traitement, pour une raison ou une autre n'est pas correctement effectué, des défauts de matériaux apparaîtront dans la structure durant le cycle de vie du ponceau. Par contre, même avec un traitement adéquat, le ponceau est sujet à diverses contraintes dont la qualité et l'épaisseur de la structure routière qu'il supporte (Kamenchukov & Svetenok [2020]). Le ponceau peut être sujet à des fissurations transversales et/ou longitudinales qui sur le long terme peuvent conduire à des ouvertures. Les fissurations transversales sont les plus dangereuses car elles favorisent un affaissement. En outre, des infiltrations à travers la paroi ou le long du pourtour du

ponceau peuvent advenir. Hormis ces problèmes, les conditions géotechniques et la circulation routière en amont de la structure peuvent causer des mouvements et des déplacements. L'affouillement qui est un type d'érosion sous la base du ponceau est un autre mode de défaillance qui met en danger ce type de structure. Le minage, l'érosion de la base du ponceau, l'apparition d'une fosse à l'entrée ou à la sortie du ponceau et la sédimentation affectent le rendement hydraulique de la structure. Tous ces phénomènes apparaissent avec l'âge de la structure et en fonction des conditions climatiques et environnementales (Yalin [2018]). De manière additionnelle, des débris peuvent s'accumuler (animaux morts, plantes, déchets polluants, etc.) et obstruer le ponceau. Par ailleurs, des structures connexes peuvent subir des défauts similaires qui viennent perturber le fonctionnement des ponceaux. Il n'est pas rare que des murs de tête, des remblais ou des puisards soient construits autour du ponceau pour le protéger ou en optimiser le rendement. Ces autres structures peuvent aussi avoir des fissurations, des défauts de matériaux, s'affaisser et obstruer ou causer des bris aux ponceaux.

Tels que définis, il semble plausible que ces modes de défaillance soient corrélés et que de ce fait les ponceaux en béton soient sujets à des modes de défaillance interactifs. Par exemple, comme les ponceaux en béton ne se déforment pas, les mouvements et déplacements entraîneront certainement des fissurations du béton. De manière générale, un problème structurel débutant par un mouvement, une fissuration, une infiltration, ou un défaut de matériaux possède le potentiel d'entraîner une dégradation suivant l'un de ces modes et sur le long terme tous ces modes en même temps. Par exemple, une infiltration déstabiliserait l'équilibre des forces du remblai, causant du mouvement. Ce mouvement entraînerait une ouverture des joints d'assemblage, créant ainsi plus d'infiltration. De plus, les problèmes de sédimentation qui seraient une conséquence des infiltrations et seraient amplifiés par toute accumulation de débris. La dégradation du ponceau fragilise aussi les structures connexes telles que les remblais.

2.4.2 Opérations de maintenance

La maintenance des ponceaux en béton représente un défi particulier surtout au Québec où les conditions climatiques en hiver peuvent être rudes et nuire à l'intégrité de ces ponceaux. Des inspections régulières sont requises pour évaluer et répertorier l'état des ponceaux (Mitchell et al. [2005]). Des critères suivant les modes de défaillances énoncés plus haut sont établis et mesurés à des intervalles prédéterminés. Masada et al. (2007) proposent une étude de cas sur les mesures

préventives relatives aux pontons en Ohio. Par exemple, on peut choisir d'inspecter chaque 5 ans un ponton en très bon état tandis qu'un ponton montrant des signes de dégradation avancée sera inspecté chaque année. En fonction des diagnostics des inspecteurs, une combinaison des critères, en général dans une formule mathématique optimisée par des experts, détermine l'état général du ponton et désigne la priorité à lui accorder en termes d'opérations de maintenance. Ces opérations varient du nettoyage pour dégager les débris à la complète reconstruction du ponton et peuvent être très coûteuses (Meegoda [2013]). Beaver & McGrath (2005) proposent une étude sur des dizaines de milliers de pontons en Utah qui démontrent le poids économique et le défi stratégique relativement à la planification de leur maintenance.

Ceci dit, dans les travaux susmentionnés, l'approche classique d'évaluation de l'état du ponton traite indépendamment les modes de défaillance les uns des autres. La potentielle accélération mutuelle des phénomènes causant les défaillances n'est pas représentée. Comme conséquence, les actions de maintenance ciblent indépendamment les défauts et sont planifiées sans considérer cette potentielle accélération. Pourtant, l'interactivité est un phénomène dont les effets s'observent indéniablement. Il y a une perte d'information dans l'évaluation de la fiabilité. Cela peut avoir pour conséquence une certaine proportion de défaillances non anticipées malgré le caractère régulier des inspections. On peut se retrouver à faire beaucoup plus d'opérations correctives que prévues malgré les mesures préventives mises en place. Ces dernières offrent pourtant le potentiel pour l'application de méthodes de fiabilité variées dans les domaines de la maintenance planifiée et de la maintenance prédictive.

2.4.3 Données de maintenance

Beaver & McGrath (2005), Masada et al. (2007) donnent un aperçu du type de données auxquelles il faut s'attendre dans l'étude de la fiabilité des pontons. Il s'agit principalement de données d'inspection suivant plusieurs critères qui déterminent l'état de la structure, de son rendement hydrologique et des structures qui lui sont associées dont la chaussée. Tous les inspecteurs sont en général formés pour la détection des modes de défaillance, la mesure de leur amplitude et l'enregistrement adéquat de ces informations. Afin d'uniformiser les résultats des inspections, plutôt que de répertorier uniquement les valeurs réelles de toute dégradation, ces dernières sont associées à un indice de gravité sur une échelle standardisée. Cet indice de gravité permet d'identifier suivant l'avis d'ingénieurs et autres experts les besoins de maintenance et de les

planifiés. Les réseaux urbains et interurbains peuvent avoir des milliers de kilomètres de routes sous lesquels se retrouvent des milliers de ponceaux. La tenue de données sur une telle quantité de systèmes peut s'avérer ardue. Il est pourtant nécessaire de s'assurer de la fiabilité de ces données (Chu et al. [2016]) sinon toute décision ou analyse subséquente pourrait être faussée. Nous avons travaillé sur une telle base de données recueillies sur des milliers de ponceaux au cours de plusieurs années.

Le premier défi évident est la quantité abondante d'informations à trier. En effet, il s'agit de déterminer ce qui est essentiel parmi quelques millions d'informations sur la localisation, les codifications qui permettent l'identification, les commentaires qualitatifs, l'état inspecté des ponceaux, l'historique des maintenances, etc. A cet effet, les avis des experts sont primordiaux. Une variable en particulier qui a semblé incontournable était l'âge du ponceau, c'est-à-dire le nombre d'années d'utilisation depuis sa construction ou sa dernière complète rénovation. Le second défi est de nettoyer cette massive base de données qui comporte inévitablement des erreurs de mesures, de taxonomie, de logiciel, etc. Un troisième défi est de traiter les données manquantes. Même s'il existe une panoplie de méthodes pour compléter des données manquantes, plutôt que de juste les appliquer aveuglement, il faut se demander si cette information n'est pas récupérable de manière alternative, par exemple au travers des commentaires qualitatifs des inspecteurs ou des avis des experts. Un dernier défi est de transformer l'information utile sous une forme exploitable. Par exemple, si l'âge est une donnée essentielle, il n'est peut-être pas répertorié en tant que tel dans la base de données et doit être retracer en fonction des informations sur la date de construction et l'historique de maintenance. Aussi, l'historique de maintenance peut se retrouver en grande partie détaillé qualitativement dans les commentaires des inspecteurs.

Afin de faire face à ces défis, nous avons développé des macros Excel de prétraitement des données enregistrées sur les ponceaux. Elles sélectionnent parmi plusieurs centaines de variables, celles qui sont essentielles telles que suggérées par les experts. Elles permettent aussi d'indexer des informations parmi des centaines de milliers de commentaires clés faisant référence aux opérations de maintenance afin de retracer l'historique de maintenance. Elles excluent certaines erreurs de taxonomie et de mesure suivant des règles booléennes inspirées des avis des experts. Ces macros s'exécutent au bout de plusieurs heures. À la fin de ce prétraitement, les données sont validées par les experts et classées en fonction de divers critères tels que les dimensions structurelles, le matériau et la tendance de dégradation ou les activités de maintenance afin de faciliter les analyses

subséquentes. À partir de ces données nettoyées et approuvées des experts, nous avons testés les modèles proposés dans cette thèse. Roman, S. (2002) présente les meilleures pratiques dans la construction de macros Excel et Osborne (2013) fournit plus d'informations sur l'exploration et le nettoyage des données.

2.5 Conclusion

Les méthodes classiques présentées dans cette revue de littérature permettent de représenter les dépendances structurelles d'un système et d'en appréhender les dépendances économiques afin de choisir une stratégie de maintenance, qu'elle soit planifiée ou prédictive. Cependant, les dépendances stochastiques représentent un défi que les modèles classiques ne peuvent expliquer. Les avis des experts concernant les différentes relations entre les modes de défaillance des ponts en sont des exemples. Si l'expérience et l'intuition de ces spécialistes permettent de remarquer ces phénomènes, il demeure beaucoup plus difficile de proposer des modèles caractérisant ces dépendances. Le point de vue fiabiliste autour de l'interaction des défaillances est particulièrement intéressant car il offre des phénomènes physiques (défaillances avérées) à étudier. Ceci dit, la floraison de nouveaux modèles prenant en compte ces dépendances stochastiques soulève des questions quant aux concepts, méthodes et applications qu'ils offrent. Dans quel cadre peuvent-ils être implémentés? Quels en sont les avantages et les limites? Au-delà de juste chercher à les implémenter, comment peut-on les classifier et quelles leçons en tirer afin de concevoir des modèles de fiabilité et de maintenance intégrant efficacement l'interaction des défaillances?

CHAPITRE 3 ARTICLE 1: FAILURE INTERACTION MODELS FOR MULTICOMPONENT SYSTEMS: A COMPARATIVE STUDY

Toualith Jean-Marc MEANGO and Mohamed-Salah OUALI

Published by “*SN Applied Sciences*”

<https://doi.org/10.1007/s42452-018-0063-2>

Abstract—Classical reliability models consider the failure modes of a multicomponent system as independent phenomena, even if these modes concern only one component or neighbouring components. In fact, when a failure mode occurs on a given system, causes may be related to external events or to the physical degradation of any component in the system. In both cases, failure modes can partially or totally share some root causes, which may compromise the principle of independence of failure modes. This paper reviews the failure interaction models that characterise the effect of any failure mode of a component on the failure modes of its neighbors and on the entire system. Three relevant groups of interaction models are reviewed: reliability indexes’ interaction, state-based interaction and copula-based interaction. All these models share the hypothesis of stochastic dependence between failure modes, also referred to as failure interaction. They differ in their fundamental modeling concepts. Advantages and limits of each of them are emphasized in a comparative study dealing with dependency concepts, modeling methods and application domains.

Index Terms—Failure interaction, stochastic dependence, reliability analysis, multicomponent system.

3.1 Introduction

Risk management of industrial systems involves both an intelligent optimization of maintenance resources and an ability to anticipate the potential failures of its components (Blanchard & al. [1990]). Failure analysis based on reliability engineering methods is the core of this particular subject matter. Common methods used in this area, such as Failure Mode and Effects Analysis (FMEA), reliability block diagrams and Stress-Strength analysis, assume that the failure modes occur independently even if they concern only one component or neighbour components (Johnson [1988], Čepin [2011], Liu & al. [2013]). When multiple failure modes may be observed on a given

system within a relative short operation time, they remain stochastically independent. Although this fundamental assumption allows some level of simplicity when addressing the failure analysis and risk assessment, it may be quite restrictive and irrelevant due to the mutual influence or interaction of multiple failure modes within the system. This is especially true considering that most current systems (Reverse osmosis membranes, semi-conductors, etc.) have become more and more complex. Their functional decomposition shows multiple components that interact. Understanding the interactivity of failures within a multicomponent system has become a critical and complex challenge for reliability analysis.

This paper reviews failure interaction modeling in the context of multiple failure modes and provides a framework allowing to identify and select the proper models pertaining to relevant technical matters and applications. The reviewed models share the principle of stochastic dependence between failure modes also called failure interaction. While stochastic models are diverse and numerous, we chose to limit our review to failure interaction models. They seek to characterise the effect of any failure mode of a component on the failure modes of its neighbors and the entire system. This is critical in the understanding and prediction of how and why a system fails. Practical implications like maintenance methods and strategies can then be addressed through these models. They are classified into three approaches: reliability indexes' interaction, state-based interaction and copula-based interaction. Based on their fundamental modeling hypothesis, advantages and limits of each approach are emphasized in a comparative study dealing with dependency concepts, modeling methods and application domains.

The remainder of the paper is organized as follows. Section 3.2 presents the concepts of failure interaction as covered in the literature on failure analysis. Interactive failure models are discussed according to each proposed approach. Modeling equations are described in detail to show the main differences between the models reviewed. Section 3.3 presents a comparative study of the reviewed interactive failure models based on their concepts, methods and applications related to the subject matter. Section 3.4 contains some concluding remarks.

3.2 Concepts of interactive failures

Cho & Parlar (1991) present the interdependences of a system's components in three categories: economic, structural and stochastic. Economic dependence exists whenever the combination of diverse maintenance actions, if performed simultaneously, can have a significant impact on the

overall cost of the system's maintenance. Structural dependence relates to the architecture of the system. Some components may indissociably share a specific function. In this case, the maintenance of a component requires that the other components be stopped or dismantled. For example, the principle of cascading failures considers a system within which components fail successively in a specific pattern, subject to the architecture. Concepts of interactive failures or failure interaction presented in this review are beyond economic and structural considerations. They are defined as stochastic dependences. Stochastic dependence appears when the failure or the degradation of a component influences the state of another component. This category of failure dependence establishes a relation between the states of a system's components, regardless of its architecture. The inherent principle suggests that the propagation of a component's failure alters the reliability of other components, and consequently, of the overall system. In this case, relationships are determined between the components' ages, failure rates and failure times. This stochastic dependence implies that the states of components interact, in which case, the terms interactive failures and failure interaction become relevant.

Murthy & Nguyen (1985a, 1985b) present two types of failure interaction:

- Type I, induced failures: a component's failure causes the instantaneous failure of another component with a probability p or has no effect with a probability $1 - p$;
- Type II, failure rate or shock damage interaction: a component's failure changes the failure rate of another component; in other words, a component's failure acts as a shock on another component and the accumulation of shocks accelerates the failure rate of this other component.

The aforementioned types of failure interactions are largely reused, updated and extended in most models presented by this review. For example, Nakagawa and Murthy (1993) consider shock damage interactions to find the optimal replacement number to minimize the expected cost of maintenance. Failure interaction can be defined as a gradual, immediate, unidirectional or bidirectional phenomenon. The most relevant works pertaining to the subject of interactive failures or failure interaction can be classified into 3 groups of models:

- Reliability indexes' interaction: Failure rates of diverse components and/or failure modes are linked by an analytical function, which in general has an incremental effect on the system's overall failure rate. Structural and economic dependencies within the system

influence greatly the choice of the function. As a consequence, these models address in general simple architectures (series, parallel, etc.). Such models are built from failure times in maintenance logs and don't explicit physical degradation processes;

- State-based interaction: A relationship is established between the diverse degradation processes within the system. The monitoring process of state variables (Temperatures, wear, etc.) is rather important in this case since the interaction will be defined considering the variations in the degradation rates of a system's components;
- Copula-based interaction: These models are created regardless of any assumptions one might have on the evolution of the degradation processes and/or the failure times. A function (copula) that links components' reliabilities is selected based solely on the likelihood of the data.

Table 3.1 summarizes the general criteria that led to the classification of the failure interaction models.

Table 3.1 Classification criteria of the failure interaction models

Models	<i>Reliability indexes' interaction</i>	<i>State-based interaction</i>	<i>Copula-based interaction</i>
Input data	Failure times	Failure times and state variables	Failure times and state variables
Starting hypothesis of interaction	Incidental variations in the probability of failure	Incidental variations in the degradation process	None
Application field	Scheduled Maintenance	Conditional Maintenance	Scheduled and Conditional Maintenance
Focus of the models	Indexes such as survival probability, failure rate or distribution parameters	Modeling of degradation processes	Application of mathematical functions

3.2.1 Reliability indexes' interaction

Reliability indexes' interaction models are the most common in the literature. They establish an analytical relationship of dependence between the reliability indexes or parameters of a system's

components. Practically, they establish a mathematical dependence between failure rates or failure times regardless of any physical degradation parameter. They rely solely on the accuracy of the maintenance logs. They combine the concepts proposed by Murthy & Nguyen (1985a, 1985b) for systems with two components or more.

3.2.1.1 Failure interaction of two components

Satow & Osaki (2003) base their study on the hypothesis defined by Murthy & Nguyen (1985a, 1985b) and extended by Nakagawa & Murthy (1993). In their works, component 1 causes the instant failure of component 2 (type I interaction) or causes the failure of component 2 due to the accumulation of multiple shocks (type II interaction). The idea is that within physical systems components' failures can be precipitated by the failure of neighbour components through random phenomena like vibrations, frictions, etc. The failures of component 1 are assumed to follow a non-homogeneous Poisson process of intensity $h(t)$. The system has to be replaced at the N -th failure of component 1 and whenever component 2 fails. In the case of induced failures (type I interaction), p_j denotes the probability that component 2 fails instantaneously at the j -th failure of component 1. The expected time to replacement ETR is:

$$ETR = \sum_{j=0}^{N-1} \left\{ \prod_{i=0}^j (1 - p_i) \right\} \left\{ \int_0^\infty \frac{[H(t)]^j}{j!} e^{-H(t)} dt \right\}, p_0 = 0, N > 0 \quad (3.1)$$

where $H(t) = \int_0^t h(u) du$. In the case of shock damage interaction (type II interaction), component 1 causes damage with a distribution $G(x)$ to component 2 that fails when the total damages reach a threshold level Z .

$$ETR = \sum_{j=1}^{N-1} G^j(Z) \int_0^\infty \frac{[H(t)]^j}{j!} e^{-H(t)} dt, N > 1 \quad (3.2)$$

where $G^{(k)}(x)$ is the k -fold convolution function of $H(t)$ with itself, $G^{(0)}(x) = 0$ and $k = 1, 2, \dots$ For shock interaction, the case $N = 1$ is not considered since it is equivalent to induced failure. These models are used to determine a replacement policy that minimizes the cost in regard to the system age (t) and a threshold (Z) of shocks accumulated by component 2. The main difficulty of that method is the definition of the threshold and the damage distribution. Since the primarily available data are failure times from maintenance logs, one might need to rely on experimental data or a physical model in addition.

Wang & Zhang (2009) associate a maintenance technician to a system with 2 dissimilar components. Considering the case of a type II interaction, a component 1 causes random shocks on a component 2. These internal shocks are accumulated until the system fails. The failure of component 2 causes the instant failure of component 1. Component 1 is replaced upon failure and component 2 can be repaired. Like the previous model, a component is assumed to be critical to the system's primary function. Moreover, it requires extensive repairs upon its failure while some other part can be easily replaced but, still, can unbalance the stability of the system as a whole through chain reactions or random phenomenon in the shared environment. Diverse quantities are then studied for a renewal process: X_i the time between failures of the i -th component 1, Y_i the damage caused to component 2 by shocks due to the failure of the i -th component 1, Z_n the repair time of component 2 in the n -th cycle of a replacement policy. A geometric model is chosen to characterize the threshold of shocks accumulated by component 2. Another geometric model represents the evolution of the repair time Δ_n in the n -th cycle of a replacement policy. The distribution functions of X_i , Y_i and Z_n are denoted respectively by $F(t)$, $H(t)$ et $G_n(t)$ so that:

$$\begin{aligned} G_n(t) &= G(b^{n-1}t) \\ \Delta_n &= a\Delta_{n-1} = a^{n-1}\Delta \end{aligned} \quad (3.3)$$

where a and b ($0 < b \leq 1$, $0 < a \leq 1$) are the geometric ratios. Wang & Zhang (2009) define T_i and v_i as respectively the time between failures and the number of component 1 failures in the i -th operation cycle. $Z(i)$ denotes the total amplitude of accumulated shocks due to component 2 failures. Equation (3.4) illustrates the model.

$$\begin{aligned} P(v_i = k) &= P(Z_1 + Z_2 + \dots + Z_{k-1} < a^{i-1}\Delta < Z_1 + Z_2 + \dots + Z_{k-1} + Z_k), \\ P(v_i = k) &= H^{(k-1)}(a^{i-1}\Delta) - H^{(k)}(a^{i-1}\Delta) \end{aligned} \quad (3.4)$$

where $H^{(k)}(\Delta)$ is the k -fold convolution function of $H(t)$ and $k = 1, 2, \dots$. In reality, the length of repair time cannot be described as strictly geometric. There is in fact high randomness due to the human factor in the repair process. But, the model (3.4) succeeds in determining a relation between the age of a system and the repair time while including shock damage interaction.

Lai & Yan (2014) base their work on type II interactions that are also extensively studied in related early articles by Lai & Chen (2006, 2008) and Lai (2009). The model (3.5) considers a repairable component 2 and a non-repairable component 1. The main point of this model is to introduce a certain proportionality in failure rate interaction that is intuitively assumed. The potential for chain

reactions and the shared environment contribute to an acceleration of the failure rate. The failure rate of component 2 follows a non-homogeneous Poisson process of intensity $h_2(t)$. Every failure of component 2 increases the failure rate of component 1. Inversely, if component 1 fails, component 2 instantly fails. This hypothesis can be restrictive since no retroactive effect is accounted for. The failure rate of component 1 is:

$$h_1(t) = \sum_{j=0}^{\infty} h_1(t|N_2(t) = j) \times P(N_2(t) = j) \quad (3.5)$$

where $N_2(t)$ is the number of failures of component 2. Lai & Yan (2014) apply the concept of minimal repairs to component 2. They take into account the economic dependences within a system and determine a replacement policy with an optimal number of minimal repairs for a maintenance cycle of optimal duration T .

Golmakani & Moakedi (2012) use the same model (3.5) but consider 2 types of failures. A component 1 is subject to soft failures following a non-homogeneous Poisson process. A component 2 is subject to hard failures following a homogeneous Poisson process. Hard failures have an instant detectable effect and require immediate intervention when they occur. Soft failures can only be detected by a scheduled inspection because they don't stop the system but decrease its performance. Generally, hard failures pertain to the inability to perform a primary function while soft failures are related to secondary functions. A component's hard failure can be the root cause of another component's soft failure if this component serves a secondary function (e.g. protective apparel) for a more critical component. The model (3.6) is based on the assumption that a hard failure by component 2 increases the failure rate of component 1. The latter has inversely no effect on component 2 if it stops working. A coefficient p represents the percentage of increase of the failure rate of component 1 due to hard failures.

$$h_1(t) = \sum_{j=0}^{\infty} \left(1 + \frac{p}{100}\right)^j h_1^0(t) \times P(N_2(t) = j) \quad (3.6)$$

where $h_1^0(t)$ is the initial failure rate of component 1. The estimation of the coefficient p would rely on experimental data or the availability of a physical model. Yet, considering a constant incremental effect (p), suggests that the interaction have little to no variability. Though, defining the failures in consideration of the time of detection and inspection allows the authors to characterize the effects of interaction in a maintenance strategy based on the optimal number of inspections in a cycle.

Sung et al. (2013) combine the concepts of failure rate interaction (type II) and external shocks. The effect of external shocks is particularly important for mechanical systems that often have a protective external component subject to the state of the surrounding environment. Whether a system is well-designed or not, it is merely impossible to perfectly prevent an interaction with external factors. This interaction has an internal consequence as well. The external shocks occur following a non-homogeneous Poisson process of intensity $r(t)$ which suggests randomness. They cause minor failures with a probability p and catastrophic with a probability $1 - p$, exclusively to a component 2. The failures of component 2 act as internal shocks as well and increase the failure rate of component 1. The entire system fails if component 1 fails. The number of component 2 failures $N_2(t)$ follows a non-homogeneous Poisson process of intensity $\delta(t) = h_2(t) + pr(t)$ where $h_2(t)$ is the failure rate independently of the external shocks. Thus the failure rate of the system depends on the failure rate of the dominant component 1 and the number of external shocks endured by the other component. Equation (3.7) gives the failure rate of the dominant component:

$$h_1(t) = \sum_{j=0}^{\infty} h_1(t|N_2(t) = j) \times P(N_2(t) = j) \quad (3.7)$$

The survival function of the system is given by:

$$R(t) = \exp \left\{ - \int_0^t [h_1(x) + (1 - p)r(x)] dx \right\} \quad (3.8)$$

This method allows the inclusion in an analytical model of the effect of the environment on the system but the starting hypothesis remains restrictive since only systems with a single dominant component are addressed by this model. A long-term replacement policy is derived by the authors in function of an optimal number of minimal repairs, which minimizes the overall cost of maintenance.

3.2.1.2 Failure interaction in a multicomponent system

The models are relatively abundant in the case of systems with two components. They become less representative for multicomponent systems. In fact, models for multicomponent systems are less abundant in the literature since the understanding of the interaction becomes more difficult when the studied system is more complex. Jhang & Sheu (2000) compare maintenance strategies for a system with N components. Each component i can be subject to a minor failure with a probability $1 - p_i$ or cause a major failure to all other components stopping the system with a probability p_i (type I interaction). This is similar to the reasoning behind hard/soft failures but, here, a retroactive

effect considering the set of components affected is accounted for. Minimal repairs are used in the case of minor failures. All failures of component i are assumed to follow a non-homogeneous process of intensity $h_i(t)$. The failure times are simulated by random draws by the authors. $Y = \min\{Y_{1,i}, 1 \leq i \leq N\}$ is defined as the time until the first system replacement; where $Y_{1,i}$ is the time until a failure from component i stops the system. The reliability function of $Y_{1,i}$ is:

$$R_{p_i,i}(t) = \exp\left\{-\int_0^t [p_i(x)h_i(x)]dx\right\} \quad (3.9)$$

The survival function of Y is:

$$R(t) = \exp\left\{-\sum_{i=1}^N \int_0^t [p_i(x)h_i(x)]dx\right\} \quad (3.10)$$

The interactive model (3.10) is considered with both - but separately - age and block replacement policies that are common maintenance strategies. This shows the flexibility of the approach. Yet, building this model from failure times in maintenance logs would require substantive data on all components; while in reality, only critical components are properly observed for a limited period. The main results show that accounting for the interactivity of failures in the developed models helps to better anticipate the maintenance costs.

Lai (2007) broadens the scope of earlier work on type II interactions in systems with two components and studies type II interactions for multicomponent systems. He considers a system consisting of N components. One of them is assumed to be dominant and non-repairable, while the others are secondary components and can be repaired. They are mutually independent and follow non-homogeneous Poisson processes. The hypothesis of dominance is plausible if the primary function in a system is guaranteed by a single component. Though, as aforementioned, industrial systems tend to be more and more complex and multiple components can share a critical function or have more than one critical function. The failure of a secondary component increases the failure rate of the primary component. Lai (2007) proposes a replacement policy following the system's age, considering that secondary failures are corrected by minimal repairs. Li et al. (2013) use the same model with $n + 1$ components. One of them is dominant and non-repairable and the others are secondary and repairable. The secondary components are mutually independent and follow exponential laws of parameters $h_n(t) < h_{n-1}(t) < \dots < h_2(t) < h_1(t)$. A voting system of m components out of the secondary n causes the failure of the system. Such an approach adds more randomness to the model since unlike the model of Lai (2007) the set of components

interacting and failing can vary. The failures of secondary components increase the failure rate of the primary component denoted by $h_N(t)$:

$$h_N(t) = h(t|k_1, k_2, \dots, k_i, \dots, k_n) \quad (3.11)$$

where k_i is the number of failures $N_i(t)$ of component i . S_{ik} ($k = 0, 1, 2, \dots$) is defined as the random failure time of component i . The probability that k or more secondary components fail is:

$$P(S_{ik} \leq t) = \sum_{j=k}^{\infty} \frac{[R_i(t)]^j e^{-R_i(t)}}{j!} = \sum_{j=k}^{\infty} P_{ij}(t) \quad (3.12)$$

where $R_i(t) = \int_0^t h_i(x) dx$ is the survival function of a single component. $h_N(t)$ depends on the conditional probability $h_{Nk_i}(t) = P(t|N_1(t) = k_1, \dots, N_n(t) = k_n)$ and :

$$h_N(t) = \sum_{k_1} \sum_{k_2} \dots \sum_{k_n} h_{Nk_i}(t) P_{nk_n}(t) \dots P_{1k_1}(t) \quad (3.13)$$

Estimating the probability of failure of the dominant component is complex and the authors propose the use of Markov processes. A transition matrix helps to determine the performance and reliability parameters of the system. However, the use of Markov processes adds a certain complexity to the model because the number of states of the dominant component will be directly correlated to the number of secondary components.

As above-mentioned, there are cases that respect the ideas of the dominance of a single component and the unilaterality of interactions. Yet, these hypotheses are restrictive for systems with a more complex structure.

Zhang et al. (2014, 2015) also use Markov processes to characterize the states of a system with n components. They integrate the concept of interactivity in a more general manner. There is no predetermined dominant component since in most systems the primary function is performed by a set of components with the same level of criticality. This shared level of criticality influences the maintenance actions. This is why the authors include opportunistic maintenance concepts to their study. This adds to the model dynamics between failure effects and maintenance actions. A component is assumed to have a finite set of states: $\Psi = \{0, \dots, m\}$ where state 0 is the initial state, and states $1, \dots, m - 1$ reflect the deteriorating conditions. The component is subject to corrective maintenance when it reaches m . Markov chains are used to model the state transition of a component with a transition rate h . The deterioration process of a component speeds up when other components fail. Maintenance opportunity from other components is assumed to follow a Poisson

process of intensity $\mu = h_f + \mu_p$ where h_f is the failure rate of other components and μ_p is the rate of preventive maintenance on other components. A variable a identifies the maintenance actions by:

- $a = 0$: no maintenance
- $a = 1$: conducting maintenance.

$S = \{(i, \omega) | 0 \leq i \leq m; 0 \leq \omega \leq 2\}$ is defined as a space where i is the state of a component and ω is the occurrence of maintenance opportunity given by:

- $\omega = 0$: no maintenance opportunity;
- $\omega = 1$: maintenance opportunity due to failures of other components;
- $\omega = 2$: maintenance opportunity due to other components' preventive maintenance.

The transition probability from state $s \in S$ to another $s' \in S$ is defined by $P_{ss'}(a)$ in the following manner:

$$\begin{aligned}
 P_{(i,\omega)(j,0)}(a) &= \begin{cases} h(\mu + h)^{-1}p_{ij} & \text{if } a = 0 \\ h(\mu + h)^{-1}p_{0j} & \text{if } a = 1 \end{cases}, \quad 0 \leq i, j \leq m \\
 P_{(i,\omega)(j,1)}(a) &= \begin{cases} h_f(\mu + h)^{-1}q_{ij} & \text{if } a = 0 \\ h_f(\mu + h)^{-1}q_{0j} & \text{if } a = 1 \end{cases}, \quad 0 \leq i, j \leq m \\
 P_{(i,\omega)(j,2)}(a) &= \begin{cases} \mu_p(\mu + h)^{-1} & \text{if } a = 0, \quad 0 \leq i = j \leq m \\ \mu_p(\mu + h)^{-1} & \text{if } a = 1, \quad 0 \leq i \leq m, j = 0 \\ 0 & \text{else} \end{cases}
 \end{aligned} \tag{3.14}$$

where p_{ij} is a transition probability that a component deteriorates from state i to state j and q_{ij} is defined as another transition probability induced by other components' failures. The authors propose the method of Hidden Markov Chains to determine the transition matrix according to observed data. This suggests a certain abundance of reliable observational data on each component's failure. By applying the concept of opportunistic maintenance to their model, they take into account the economic dependences.

Liu et al. (2015) represent the interactivity by a probability matrix $P = (p_{ij})$ considering type I interactions as defined by Murthy & Nguyen (1985a). Thus, a component i causes an instant failure to a component j with a probability p_{ij} and has no effect with a probability $1 - p_{ij}$. Failure interaction undoubtedly decreases the reliability of a system. By accounting for the

interactivity, Liu et al. (2015) identify some issues in the early stages of a system's design that could affect the constructor's warranty. This shows that the concept of failure interaction has on the long run, in addition to the physical implications of a failure, an extensive and accelerating effect on the decrease of a system's economic value. They define w as a system's period of warranty, $p_i(w)$ the probability of failure for a component i during w , $\alpha_i(w)$ the probability that a component i failure will cause the system failure, and $F_S(w)$ the distribution function of the system:

$$\begin{aligned} F_S(w) &= 1 - R_S(w) \\ \alpha_i(w) &= p_i(w)/F_S(w) \\ p_i(w) &= \Pr[T_i \leq Y_i, T_i \leq w] \end{aligned} \quad (3.15)$$

where $Y_i = \min(T_j, \forall j \in \Omega, j \neq i)$, $\Omega = \{1, 2, \dots, n\}$. The number of failures N_{ij} of a component j caused by a component i :

$$E[N_{ij}] = \frac{p_{ij} \cdot p_i(w)}{R_S(w)} \quad (3.16)$$

The authors apply their model to series and parallel architectures to adjust the warranty cost considering the number of failures due to type I interactions. This approach could be improved by including type II interactions. Zhang et al. (2018) tackle the issue of product warranty as well by considering type III interactions that we classified in this paper as type II interactions. They base their model on Satow & Osaki (2003) with two components in series. Moreover, the methods aforementioned, perform limitedly when it comes to complex architectures. Their core concepts revolve around key assumptions on some dependencies.

3.2.1.3 Interaction of failure modes from a system point-of-view

A system can be subject to diverse failure modes. These modes can influence each other, mutually or not. The models considered thus far are based on assumptions of failure interaction between the components of a system. They do not explicitly identify the dependences between diverse failure modes of the system as a whole. They could be applied in such cases by considering the modes as components in series, but the interactivity of failure modes can have a more complex form. Some recent papers intend to model this dependence.

Zequeira & Bérenguer (2006) distinguish two types of failure modes: maintainable and non-maintainable. The modes are distinguished considering the reparability in the occurrence of a failure. Preventive maintenance corrects the deterioration due to maintainable modes. Non-

maintainable modes can only be corrected by a complete overhaul of the system. Moreover, minimal repairs are considered in the case of failures. The model is applied to components in series. It puts the stochastic dependence on display by considering that the failure rate of maintainable modes $h_m(t)$ depends on the failure rate of non-maintainable modes $h_{nm}(t)$. There is a certain similarity to the hard/soft failures reasoning but, here, only maintainability distinguishes the two types of failures. A term $p(t)$ is defined as the probability that a non-maintainable mode will automatically cause a maintainable mode. This probability is estimated by considering the physical and structural characteristics of the system. In other words, there needs to be a supportive physical model or conclusive experimental model to determine how the modes are linked. Preventive maintenance is performed at periods kT where $k = 0, 1, 2, \dots$ and $T > 0$. During a period $[kT, (k+1)T[$ the maintainable failure rate is $h_m[t - (k-1)t] + p(t)h_{nm}(t)$. Then, the system's failure rate during period $[kT, (k+1)T[$ is given by (3.17):

$$h_{k,T}(t) = h_{nm}(t) + h_m[t - (k-1)t] + p(t)h_{nm}(t) \quad (3.17)$$

The authors use a mechanical coupling as an illustrative example, which shows applicability of the model on real-life systems. However, not all maintainable failure modes have the same impact or repair-time. Considering them as a single category alters the comprehension of a system's deterioration process. They also propose an imperfect preventive maintenance policy to estimate an optimal complete revision period T and an optimal number of preventive replacements N that minimize the overall cost.

Castro (2009) proposes an improvement on the model by Zequeira & Bérenguer (2006). He suggests that the occurrence of maintainable failures is correlated with the number of non-maintainable failures denoted by $N_2(t)$ aggregated following the installation of a system. This choice is motivated by the fact that the number of non-maintainable failures in a specific period is easier to comprehend than a failure rate to estimate. The failure rate of maintainable modes $h_{1,k}(t)$ during period $[kT, (k+1)T[$ is then defined as a doubly stochastic Poisson process or a Cox process. An adjustment factor $a > 1$ quantifies the effect of cumulative degradation due to non-maintainable failures in such a model:

$$h_{1,k}(t) = h_{1,0}(t - kT)a^{N_2(kT)}, \quad kT \leq t < (k+1)T, \quad a > 1 \quad (3.18)$$

where $h_{1,0}(t)$ is the failure rate of maintainable modes before the first operation of preventive maintenance. The number of failures considering the distinctive modes is integrated with a periodic preventive maintenance strategy. With minimal repairs used as troubleshooting actions, the strategy estimates the optimal number of minimal repairs N before a complete revision at period T that minimizes the overall cost. Both (3.17) and (3.18) models share the same principle of a unidirectional interactivity. It is not taken into account that two maintainable modes could be significantly distinctive and interact in a bidirectional manner.

Fan et al. (2011) have, interestingly, proposed a model with two failure modes that have bidirectional stochastic dependence. Their model is based on the works of Murthy & Nguyen (1985a, 1985b), Zequeira & Bérenguer (2006) and Castro (2009). This is important since the assumption of failure interaction regardless of architecture suggests the possibility of a retroactive effect. Most of the previous models consider this effect as immediate, instant or total (on the whole system) while it could be gradual. The failure rates are defined as doubly stochastic Poisson processes or Cox processes. The authors consider that the failure rate of mode i depends on the number of the failures in the other mode $\bar{i}$ and vice-versa. A performance variable is associated with each failure mode. In this model, t is defined in function of the number of passed preventive maintenance cycles. In fact $t = t_k + t_L$ where t_k is the time from the system's installation to the k -th maintenance operation and t_L is the operating time since the last preventive action. $h_{0,i}(t)$ is the failure rate before the first preventive maintenance. At the l -th preventive maintenance cycle, the failure rate for the mode i is:

$$h_i(t_l + \tau) = \begin{cases} h_{0,i}(y_{i,l}^+ + \tau) a_i^{\bar{N}_{i,l}^Z}, & si \ l \leq k \\ h_{0,i}(y_{i,l}^+(t) + \tau) a_i^{N_{i,l}^Z(t)}, & si \ l > k \end{cases} \quad (3.19)$$

where $a_i^{\bar{N}_{i,l}^Z}$ and $a_i^{N_{i,l}^Z(t)}$ are adjustment factors representing the effect of mode $\bar{i}$. $\bar{N}_{i,l}^Z$ is the number of failures by mode $\bar{i}$ before t_l . $N_{i,l}^Z(t)$ is the number of failures by mode $\bar{i}$ estimated at t . $y_{i,l}^+$ is the effective age of the mode i right after the l -th preventive maintenance. $y_{i,l}^+(t)$ is the predicted effective age of the mode i right after the $(l - k)$ -th preventive maintenance at t . This approach is suitable for repairable systems. As an application, Fan et al. (2011) develop a Cooperative Predictive Maintenance Model that relies on minimal repairs and imperfect maintenance with

restricted resources. A subsequent cooperative maintenance strategy for both modes is proposed and takes into account the system's effective age and the bidirectional interactions.

3.2.1.4 General interactive failure models

All the above-mentioned models share some restrictiveness due to the numerous hypotheses formulated on the structure of the system. They define a dominant component or consider a specific architecture for the overall system. Therefore, they cannot be generalized for all cases.

Sun et al. (2006, 2010) identify the interactions with an immediate effect and the interactions with a gradual effect. They assume the hypothesis of a gradual effect accurately represents physical systems. In a system with N components, they distinguish an independent failure rate denoted by $h_{Ii}(t)$ inherent to each component i ($i = 1, 2, \dots, N$) and a dependent or interactive failure rate denoted by $h_i(t)$. The interactive failure rate of one component integrates the influence of other components:

$$h_i(t) = \varphi_i[h_{Ii}(t), \vec{h}_{j_i}(t)_B, t], \quad i = 1, 2, \dots, N \quad (3.20)$$

$\vec{h}_{j_i}(t)_B$ is the vector of failure rates before any interaction influencing component i . Sun et al. (2006) use the Taylor expansion to establish a parametric expression of the dependent failure rate of one component in function of the independent failure rates of all components. The obtained analytical expression provides this idea of an update in a failure rate from independent to dependent by a linear combination with coefficients denoted by θ_{ij_i} :

$$h_i(t) = h_{Ii}(t) + \sum_{j_i} \theta_{ij_i}(t) h_{j_i}(t)_B, \quad i = 1, 2, \dots, N \quad (3.21)$$

The authors call the θ_{ij_i} interactive coefficients since they represent the weighted effect of a failure rate from a component or a failure mode on another. θ_{ij_i} is a parameter comprised between 0 (no interaction) and 1 (perfect interaction and immediate/simultaneous failure). A matrix of the coefficients can consequently be built. Such an approach characterizes the interactivity independently from any assumption on the architecture of the system. Yet, building the matrix is not straightforward and relies on substantive experimentation and the opinions of experts. The estimation of the interactive coefficients requires a partition of the failure times in order to fit independent failure rates for each component or failure mode. This requires abundant observational

data for each scenario of the $\frac{N^2}{2}$ possibilities of interaction. Zhao et al (2013) reprise the model by Sun & al. (2006).

Table 3.2 Parameters of Reliability Indexes' Interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
Satow & Osaki (2003)	2 components: One is critical, repairable and dependent; the other is secondary and non-repairable - Unidirectional interaction - Random shocks	p_j: the probability that the critical component fails instantaneously at the j-th failure of the secondary component $G(x)$: the distribution of the damage caused by the secondary component to the critical component Z: failure threshold level	ETR: the expected time to replacement of the overall system	Numerical examples
Wang & Zhang (2009)	2 components: One is critical, repairable and dependent; the other is secondary and non-repairable - Unidirectional interaction - Random shocks - Geometric functions for repair times	X_i: the time between failures of the i-th secondary component Y_i: the damage caused to the critical component by shocks due to the failure of the i-th secondary component Z_n: the repair time of the critical component in the n-th cycle of a replacement policy.	$P(v_i = k)$: the probability to have k failures in the i-th operation cycle	Numerical examples
Lai & Yan (2014), Lai & Chen (2006, 2008)	2 components: One is critical, repairable and dependent; the other is secondary and non-repairable - Unidirectional interaction - Proportionality in failure rate interaction	$N_2(t)$: the number of failures of the secondary non-repairable component	$h_1(t)$: the failure rate of the critical component	- Numerical examples - Electrical network system
Golmakani & Moakedi (2012)	2 components: One is critical, repairable and dependent; the other is secondary and non-repairable - soft failures and hard failures - Unidirectional interaction - Proportionality in failure rate interaction	$N_2(t)$: the number of failures of the secondary non-repairable component $h_1^0(t)$: the initial failure rate of the critical component p: the percentage of increase of the failure rate of the critical component due to hard failures of the secondary component	$h_1(t)$: the failure rate of the critical component	- Numerical examples - Electrical distribution system (a coupled capacitor bank and a high power transformer)

Table 3.2 (continued) Parameters of reliability indexes' interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
<i>Sung et al. (2013)</i>	2 components: One is critical, repairable and dependent; the other is secondary and non-repairable - Unidirectional interaction - Minor and catastrophic failures - External shocks	$N_2(t)$: the number of failures of the secondary non-repairable component $r(t)$: the rate of external shocks p: probability of minor failures of the critical component caused by the secondary component	$R(t)$: the survival function of the system	Numerical examples
<i>Jhang & Sheu (2000)</i>	- Multi-component - All components influence each other - Minor and catastrophic failures	p_i: probability of minor failures of a component i caused by the other components $h_i(t)$: the failure rate of each component i	$R(t)$: the survival function of the system	Numerical examples
<i>Lai (2007)</i>	- Multi-component: one of them is dominant and non-repairable, the others are secondary and mutually independent - Unidirectional interaction - Markov processes	$h_i(t)$: the failure rate of each secondary component i k_i: the number of failures of component i S_{ik} ($k = 0, 1, 2, \dots$): the random failure time of component i $P(S_{ik} \leq t)$: the probability that k or more secondary components fail	$h_N(t)$: the failure rate of the dominant component	Numerical examples
<i>Zhang et al. (2014, 2015)</i>	- Multi-component - All components influence each other - Opportunistic maintenance - Markov processes	μ_p is the rate of preventive maintenance ω: the occurrence of maintenance opportunity p_{ij}: the transition probability of a component from state i to j q_{ij}: the transition probability of a component from state i to j induced by other components' failures	$P_{ss'}$: the systems's transition probability from state s to s'	Numerical examples
<i>Liu et al. (2015)</i>	- Multi-component - All components influence each other - Period of warranty	p_{ij}: the probability that a component i causes an instant failure to a component j α_i the probability that a component i causes the system failure w: the period of warranty	N_{ij}: the number of failures of a component j caused by i during w	Numerical examples

Table 3.2 (continued) Parameters of reliability indexes' interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
<i>Zequeira & Bérengruer (2006)</i>	- Any architecture - Unidirectional interaction - Non-maintainable failure modes influence maintainable failure modes	$p(t)$: the probability that a non-maintainable failure mode will cause a maintainable failure mode $h_m(t)$: the failure rate of maintainable modes $h_{nm}(t)$: the failure rate of non-maintainable modes kT: period of preventive maintenance ($k = 0, 1, 2, \dots$)	$h_{k,T}(t)$: the system's failure rate during period $[kT, (k+1)T[$	Numerical examples
<i>Castro (2009)</i>	- Any architecture - Unidirectional interaction - Non-maintainable failure modes influence maintainable failure modes	$N_2(t)$: number of non-maintainable failures $h_{1,0}(t)$: the failure rate of maintainable modes before any preventive maintenance α: An adjustment factor that quantifies the effect of cumulative degradation due to non-maintainable failures kT: period of preventive maintenance ($k = 0, 1, 2, \dots$)	$h_{1,k}(t)$: the failure rate of maintainable modes during period $[kT, (k+1)T[$	Numerical examples
<i>Fan et al. (2011)</i>	- Any architecture - All failure modes influence each other	$\bar{N}_{i,l}^z$ and $N_{i,l}^z(t)$: resp. the number of failures due to mode $\bar{i}$ before t_l (time of l-th preventive maintenance) and the number of failures due to mode $\bar{i}$ $h_{0,i}(t)$: the failure rate due to mode i before the first preventive maintenance $a_i^{\bar{N}_{i,l}^z}$ and $a_i^{N_{i,l}^z(t)}$: adjustment factors representing the cumulative effect of failure mode $\bar{i}$ on mode i.	$h_i(t_l + \tau)$: the failure rate for mode i at the l-th preventive maintenance cycle	- Numerical examples - Simulated data
<i>Sun et al. (2006, 2010)</i>	- Any architecture - All components influence each other gradually	$\vec{h}_{ji}(t)_B$: the vector of failure rates before any interaction $h_{ji}(t)$: the independent failure rate inherent to each component i θ_{ij_i}: interactive coefficients representing the weighted effect of a failure rate from component j on i	$h_i(t)$: the interactive failure rate of component i	- Experimental data - Misalignment of a shaft interacting with the fatigue of ball bearings

They present an alternative to the classical method of Failure Modes and Effects Analysis (FMEA) that would account for the interactivity in a complex system. They apply their model to a gyroscope and demonstrate that the concept of interactive failures are indeed applicable and have a significant

influence on the overall reliability of a system subject to multiple failure modes. Wang & Li (2015) reprise the analytic model of Sun et al. (2006) as well and propose an allocation method of redundancies in the design of a system. They demonstrate that the effect of interactive failures can be minimized if they are properly modeled in the system design process. The parameters pertaining to reliability indexes' interaction are summed up in table 3.2.

3.2.2 State-based interaction

In the context of conditional maintenance, the interactivity can be represented with a system's state variables. With sensors and captors, data on the system's degradation is collected and it helps to provide a physical interpretation of interactive failures. Keizer et al. (2017) review diverse condition-based maintenance policies for systems with multiple dependent components. They consider structural, stochastic and resource dependences. Among models with stochastic dependences, they distinguish failure interaction models as follows:

- Failure induced damage: the failure of one component causes immediate damages to other components by causing an immediate failure or increasing the deterioration level;
- Load sharing: a component fails but the system keeps operating as the other components need to work harder;
- Common-mode deterioration: many components fail simultaneously.

We consider a slightly different classification since some of the reviewed models by Keizer et al. (2017) are copula-based. The following approaches are not. Chen et al. (2011) distinguish the natural failure of a system caused by its degradation process from traumatic failures due to external shocks caused by the environment. The degradation follows a stochastic process $D(t)$ and a failure threshold is fixed D_f . The shocks occur independently from the degradation process and follow a Poisson process of intensity λ . The traumatic failures are assumed to occur with a probability p that depends on the degradation level. The system is more prone to failure modes if the degradation level increases significantly. This is particularly true for mechanical systems subject to degradation. The overall system becomes more vulnerable to its environment as it ages (e.g. proneness to corrosion, vibration, physical shocks etc.). The internal components and their mechanical links are in reverse subject to the propagation of external shocks.

$$p(t) = \exp[\alpha(D(t) - D_f)] \quad (3.22)$$

where α is a parameter of the dependence.

As a matter of fact, the traumatic failure rate $h_s(t)$ depends on the probability p :

$$h_s(t) = \lambda \cdot p(t) \quad (3.23)$$

The moment of a natural failure is denoted by T_d and the moment of a traumatic failure by T_s . The system's failure times are defined as $T = \min\{T_d, T_s\}$. The authors consider an example with linear degradation $D(t) = Bt$ where B is a random variable so that $B = \beta$ for traumatic failures. Thus, the survival function of the system is:

$$R_T(t) = P(T > t) = P(T_d > t) \cdot P(T_s > t | T_d > t) \\ R_T(t) = F_B\left(\frac{D_f}{t}\right) \cdot \int_{-\infty}^{D_f/t} \exp\left[-\int_0^t \lambda \exp[\alpha(\beta u - D_f)] du\right] f_B(\beta) d\beta \quad (3.24)$$

where $f_B(\cdot)$ and $F_B(\cdot)$ are respectively the density and the cumulative distribution of B . The authors apply the renewal theory to the proposed model and derive an inspection policy that minimizes the long-term cost of maintenance. In this approach, the cumulative effect of shocks and the variability of their amplitude are not accounted for because shocks can cause traumatic failure or have no effect.

Huynh et al. (2011, 2012) aim to combine the competitive effects of degradation, internal and external shocks. They reprise the concept of natural and traumatic failures and define the degradation as a continuous stochastic process $X(t)$. A Gamma distribution is selected to model the degradation since it characterizes in a satisfying manner diverse phenomenon like erosion, corrosion, etc. The shocks follow a non-homogeneous Poisson process. The system's failure rate depends on the degradation level. Above a fixed degradation level threshold M , the system's failure rate $h(t)$ is altered. It means that the system becomes more vulnerable to internal and external shocks. It also suggests that there is a specific moment when the interaction becomes more significant. As an example, a component with a regulation or protective function could fail in an electronic system, and cause heating that would accelerate the system's failure.

$$h(X(t)) = h_1(t) \cdot \mathbf{1}_{X(t) \leq M} + h_2(t) \cdot \mathbf{1}_{X(t) > M} \quad (3.25)$$

where $\mathbf{1}_{(\cdot)}$ is an indicator function; $h_1(t)$ and $h_2(t)$ are two continuous and non-decreasing failure rates following the relation $h_1(t) \leq h_2(t)$, $\forall t \geq 0$. The number of failures due to shocks $N_s(t)$ is

a Cox process. The authors define the following variable: $\sigma_s = \inf\{t \geq 0, N_s(t) = 1\}$. The survival function of the system is then:

$$R_s(t) = P(\sigma_s > t) = P(X(t) \leq M) \cdot P(\sigma_s > t | X(t) \leq M) + P(\sigma_s > t, X(t) > M) \quad (3.26)$$

F_{σ_M} (resp. f_{σ_M}) is the distribution function (resp. p.d.f.) of the time when M is reached. $\bar{F}_1(t)$ is the survival function associated to $h_1(t)$. Huynh et al. (2011, 2012) suggest that the parameters of this model, like the threshold M , $h_1(t)$ and $h_2(t)$, can be estimated by classical statistical methods. They apply the principle of minimal repairs and the renewal theory as well and obtain an inspection and maintenance policy that minimizes the long-term cost in function of the age and the degradation level.

Do et al. (2015) consider a system consisting of two dependent components that are connected in series. This means that the two components are critical to the functioning of the system. Their dependence is expressed by the relationship of their degradation processes. The accumulation of wear is described by a scalar random variable X_t^i and a component i is assumed to have failed when a threshold L_i is reached. The accumulation of wear at $t + 1$ becomes:

$$\begin{aligned} X_{t+1}^i &= X_t^i + f(X_t^j) + \Delta X^i, j \neq i \\ f(X_t^j) &= \mu_*^j (X_t^j)^{\sigma^j} \end{aligned} \quad (3.27)$$

where ΔX^i is the independent random increment of the deterioration level, $f(X_t^j)$ represents the impact of a component j on i , and σ^j, μ^j are positive parameters quantifying the influence of a component j on i . The parameters of the acceleration of the degradation processes σ^j, μ^j would have to be estimated through experimental methods or abundant condition-based maintenance historical data. Assaf et al. (2016, 2017) present a model based on Do et al (2015) involving similar issues for multicomponent systems in the form of degradation rate-state interactions. The interaction is defined as an acceleration of the wear indicators caused to a component by neighbor components.

Song et al. (2012, 2014) work on multicomponent systems with the hypothesis that each component has 2 failure modes that are stochastically dependent. Each component is subject to degradation and shocks. This approach is motivated by the fact that a component's degradation can often be evaluated experimentally easier than the interactive effects of other components. It would be for example hard to evaluate with precision how the corrosion of a component could have caused the

deformation of another. Components are then assumed to cause random shocks on each other. A resistance limit to shocks D_i specific to the component i ($i = 1, 2, \dots, n$) is fixed. A component degrades following a process $X_i(t)$ with a failure threshold H_i . Contrary to the two aforementioned models, the cumulative effects of shocks are calculated and this increases the degradation level. W_{ij} , Y_{ij} and $S_i(t)$ are respectively defined as the amplitude of the j -th shock on component i , the proportion of the j -th shock impact on component i and the cumulative of shocks Y_{ij} . The survival function of the systems depends on $F_{W_i}(\omega)$ and $F_{X_i}(x_i, t)$ that are respectively the cumulative density functions of W_{ij} and $X_{S_i}(t) = X_i(t) + S_i(t)$. The reliability of a single component depends on the survival probability in case of a single shock and the probability that the failure threshold is not reached by the accumulated shocks:

$$F_{X_i}(x_i, t) = P(X_{S_i}(t) < x_i) = \sum_{m=0}^{\infty} \left(\int_0^{x_i} G_i(x_i - u, t) f_{Y_i}^{(m)}(u) du \right) P(N(t) = m) \quad (3.28)$$

where $G_i(t)$ is the cumulative density function of $X_i(t)$ and $f_{Y_i}^{(k)}(u)$ is the distribution function of the sum of the independent and identically distributed k variables Y_{ij} . The survival function of the system is then determined for standard architectures (series, parallel, series-parallel). For example, the reliability of a system with components in series is:

$$R(t) = \sum_{m=0}^{\infty} \prod_{i=1}^n \left[P(W_i < D_i)^m P\left(X_i(t) + \sum_{j=1}^{N(t)} Y_{ij} < H_i \mid N(t) = m\right) \right] \times P(N(t) = m) \quad (3.29)$$

A maintenance strategy that minimizes the cost is derived and an optimal inspection period is estimated. It is important to note, however, that the proposed model considers a number of shocks common to all components. The failure threshold is fixed and has no variability. The amplitude of shocks is not considered a random process when in reality it could be.

One of the more elaborate approaches is in an article by Rafie et al. (2014) that links the concept of interactive failures with a changing degradation rate. This work results from some earlier collaborative research on multiple competitive failure modes, failure limits change and shock damage interaction: Feng & Coit (2009), Peng et al. (2010), Jiang et al. (2012), Arab et al. (2013). The proposed approach considers two conditionally independent events: NHF_t meaning No Hard Failure occurs by time t and NSF_t meaning No Soft Failure occurs by time t . Hard failures are due to random shocks exceeding a threshold D_1 . These shocks occur following a Poisson process of intensity λ . The k -th random shock is denoted by W_k with a c.d.f $F_W(w)$ and the number of shocks

is denoted by $N(t)$. Soft failures are caused by the degradation. A threshold H is defined and can be reached by the combined effect of random shocks and natural degradation. The natural degradation is defined as $X(t) = \varphi + \beta t + \varepsilon$ where the initial degradation φ and the degradation rate β are random variables, and the random error ε follows a normal distribution. The degradation rate β can change from β_1 to β_2 due to a trigger J -th shock and become:

$$X(t) = \begin{cases} \varphi + \beta_1 T_J + \beta_2 (t - T_J) + \varepsilon, & J \leq N(t) \\ \varphi + \beta_1 t + \varepsilon, & J > N(t) \end{cases} \quad (3.30)$$

where T_J is the transition time, the time when the rate β changes. Y_k and $S(t)$ denote respectively the k -th shock's damage size and the total damage size due to all random shocks. The overall degradation is represented by $X_S(t) = X(t) + S(t)$ where $S(t) = \sum_{k=1}^{N(t)} Y_k$ if $N(t) > 0$ and $S(t) = 0$ otherwise. The survival function of a device subject to two dependent competing failure processes is denoted by $R(t)$. Equations (3.31) sum up the proposed model:

$$\begin{aligned} P(NHF_t) &= [F_W(D_1)]^{N(t)} \\ P(NSF_t) &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P(X_S(t) < H | J = j, N(t) = i) \times P(J = j | N(t) = i) \times P(N(t) = i) \\ P(X_S(t) < H | J = j, N(t) = i) &= \begin{cases} \int_0^t P(\varphi + \beta_1 t_j + \beta_2 (t - t_j) + \varepsilon + \sum_{k=1}^i Y_k < H) f_{T_j}(t_j | j) dt_j, & j \leq i \\ P(\varphi + \beta_1 t + \varepsilon < H), & j > i \end{cases} \quad (3.31) \\ R(t) &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P(NSF_t | J = j, N(t) = i) \times P(NHF_t | J = j, N(t) = i) \times P(N(t) = i) \end{aligned}$$

where $f_{T_j}(t_j | j)$ is the p.d.f. of T_j for $J = j$.

This approach is of even more interesting since it puts into perspective 4 distinct models of shock interaction and proposes an application on micro-electromechanical systems (MEMS). This study also contains a sensitivity analysis to provide a physical interpretation of the interactivity. The shock processes considered are:

- The generalized extreme shock model: a single shock above a critical threshold value changes the degradation rate of a system.
- The generalized δ -shock model: the interval between two sequential shocks above a critical threshold value is inferior to a limit δ so that the degradation rate changes.
- The generalized m -shocks model: An accumulation of m shocks above a critical threshold value changes the degradation rate.

- The generalized consecutive shocks model: a succession of n shocks above a critical threshold value changes the degradation rate.

Liu et al. (2016) propose an extension to this model by including the damage due to self-regulation in self-healing systems.

These diverse models of shock processes represent the internal failure interactions as well as the external effects of a system's environment. Other models considering a changing degradation rate are available in the literature such as Bian & Gebraeel. (2014), Rasmekomen & Parlikad (2016), Hao et al. (2017, Fan et al. (2017), Shen et al (2018). Yet, for all of the models, the idea that the amplitude of the shocks, internal and external, can vary or be random, needs to be studied further. In fact, the changing degradation rate is in most cases subject to a constant factor.

Liang et al (2017) build on the previous work of Liang & Parlikad (2015) and Rasmekomen & Parlikad (2016) to deal with the issue of fault propagation. They consider that the phenomena of dependences, induced or inherent to a system, can interact and lead to a cooperative acceleration of dependent failures and cause new dependences. They model the deterioration of a multicomponent system as a multi-layered vector-valued continuous-time Markov chain. They distinguish non-critical component from critical components. For a system with ν critical components, the normal deterioration is $\{X_0(t)\} = \{X_{1,0}(t), \dots, X_{\nu,0}(t)\}$. They establish the inherent deterioration rate of a critical component l :

$$\omega_{l,0}(t) = r_{l,0} + \sum_{j \in \{1,2,\dots,\nu\}/l} g(X_{j,0}(t)) \quad (3.32)$$

where $r_{l,0}$ is the intrinsic deterioration rate and $g(.)$ is the affected deterioration rate, which is a linear function of other critical components' conditions $X_{j,0}(t)$. Non-critical components also influence the deterioration of the critical components by affecting their state. After the h th malfunction of a non-critical component, the inherent deterioration of a critical component l becomes:

$$\omega_{l,h}(t) = r_{l,h} + \sum_{j \in \{1,2,\dots,\nu\}/l} g(X_{j,h}(t)) \quad (3.33)$$

The transition rate between those two states is:

$$\lambda_{i,h}(t) = \sum_{l=1}^{\nu} r_{l,h} + g(i(\nu - 1)) \quad (3.34)$$

Liang et al (2017) remark that the current knowledge of fault propagation relies heavily on the expert opinion. This suggests a potential bias which remains to be addressed.

The parameters pertaining to state-based interaction are summed up in table 3.3.

Table 3.3 Parameters of state-based interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
<i>Chen et al. (2011)</i>	- Any architecture - Natural failures and traumatic failures - Degradation and external shocks - External shocks influence the degradation process	$D(t)$: the degradation as a stochastic process D_f: a fixed failure threshold p: the probability of occurrence of traumatic failures depending on the degradation level	$R_T(t)$: the survival function of the system where T is the minimum between the moment of a natural failure and the moment of a traumatic failure	- Numerical examples - Degradation-threshold-shock of a laser device
<i>Huynh et al. (2011, 2012)</i>	- Any architecture - Degradation, internal and external shocks - The failure rate changes above a degradation level threshold	M: degradation level threshold $X(t)$: degradation as a continuous stochastic process	$R_s(t)$: the survival function of the system	Numerical examples
<i>Do et al. (2015)</i> <i>Assaf et al (2015, 2017)</i>	- Multi-component - Components influence each other through accumulation of wear - degradation rate-state interactions	ΔX^i: the independent random increment of the deterioration level of each component L_i: failure threshold σ^j, μ^j: positive parameters quantifying the influence of a component j on i	X_t^i: accumulation of wear on each component i $R(t)$: the reliability of a system	- Numerical examples - Experimental data - gearbox accelerated life testing platform
<i>Song et al. (2012, 2014)</i>	- Multi-component degradation and shocks - Cumulative shocks influence the degradation process above a failure threshold	D_i: resistance limit to shocks of a component i H_i: failure threshold of a component i W_{ij} and Y_{ij}: resp. the amplitude of the j-th shock on component i, and the proportion of the j-th shock impact on component i	$F_{X_i}(x_i, t)$: the failure function of a single component i (X_i is the degradation process) $R(t)$: the reliability of a system in series or parallel	- Numerical examples - Experimental data - Micro-electromechanical systems (MEMS)

Table 3.3 (continued) Parameters of state-based interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
<i>Rafie et al. (2014)</i> <i>Feng & Coit (2009)</i> <i>Peng et al. (2010)</i> <i>Jiang et al. (2012)</i> <i>Arab et al. (2013)</i>	- Any architecture - Soft failures by degradation - Hard failures by random shocks - Shocks change the degradation rate - Diverse shock models	H: the system's failure threshold φ: the initial degradation β: the degradation rate Y_k: the k-th shock's damage size T_j: the transition time when β changes.	$R(t)$: the survival function of the system	- Numerical examples - Experimental data - Micro-electro-mechanical systems - Multi-stents systems
<i>Liang et al (2017)</i> <i>Liang & Parlikad (2015)</i> <i>Rasmekomen & Parlikad (2016)</i>	- Multi-component - Differentiation between critical and non-critical components - Components influence each other's state and degradation rate - Fault propagation	$r_{l,0}$: the intrinsic deterioration rate of each component l $\{X_0(t)\}$: the system's normal deterioration process $\lambda_{i,h}(t)$: the transition rate of a critical component from a state where no non-critical component have failed to a state with h failures of non-critical components.	$\omega_{l,h}(t)$: the inherent deterioration of a critical component l after the hth malfunction of a non-critical component	- Case study - Heat exchanging system in oil refinery process and distillation tower - Fleet of oil immersed power transformers - Cold box unit in a petrochemical plant

3.2.3 Copula-based interaction

Copula-based interaction models differ from the above models in the sense that they rely on little to no assumption on interaction mechanisms. They define the reliability indexes and/or the state variables of components as random variables and model the dependence analytically. Copulas are used to build multivariate probability distributions which, in most cases, consider that the marginal probability distribution of each variable is uniform. A copula C can be defined as the joint distribution of p random variables $U_1, U_2, \dots, U_p$, each of which is marginally uniformly distributed, so that the joint cumulative distribution function of such a distribution is:

$$C(u_1, u_2, \dots, u_p) = Pr(U_1 \leq u_1, U_2 \leq u_2, \dots, U_p \leq u_p) \quad (3.35)$$

The copulas are parametric functions. Their parameters govern the intensity of the dependence. There is a large number of copulas. The most commonly used are Archimedean and Gaussian copulas. Archimedean copulas have a closed form. A unique parameter represents the intensity of the dependence. They have the advantage of being applicable to cases of high dimensionality. Some examples of widely known Archimedean copulas are Ali-Mikhail-Haq, Clayton, Frank and Gumbel copulas. Gaussian copulas are determined following the matrix of correlation of the considered random variables. Kolev et al. (2006) provide useful information regarding copulas in their review.

While it could be argued that some of the following models have hybrid traits resembling reliability-index and state-based interaction models, we classify them as copula-based since the interactivity between components and/or failure modes is modelled through candidate copulas which are the focus of these approaches.

Limbourg et al. (2007) explore the influence of spatial dependencies on a multi-component system's reliability. They study diverse architecture, especially voting systems. Mostly, they bring emphasis on the fact that when components are neighbor, they show dependent failures. A Gaussian Copula C is used to model the dependent failures probabilities among neighbor components. Given a state variable x , the dependent failure probability $F(x)$ of a system with n components is:

$$F(x) = C(F_1(x), \dots, F_n(x)) \quad (3.36)$$

where $F_i(x)$ ($i = 1, \dots, n$) is the failure probability distribution of each component.

A copula can directly link reliability indexes. For example, Jiayin et al. (2009) use a copula C to model the dependence of n components in a system. They propose a characterization with a copula following the type of architecture: series or parallel. The reliability of the system is written:

- For a system with n components in parallel:

$$R_{parel}(t) = 1 - C_n(1 - R_1(t), \dots, 1 - R_n(t)) \quad (3.37a.)$$

- For a series system:

$$R_S(t) = \Delta_{1-R_1(t)}^1 \dots \Delta_{1-R_n(t)}^1 C_n(u_1, u_2, \dots, u_n) \quad (3.37b.)$$

where $\Delta_{x_1}^{x_2} f(x) = f(x_2) - f(x_1)$ and $u_i = 1 - R_i$ is the failure probability of a single component i . The authors choose the Gumbel copula. The latter is Archimedean and has a unique parameter θ

that represents the degree of correlation of the random variables. This parameter can be estimated by the maximum likelihood method.

Similarly, Jia et al. (2014) demonstrate how a copula can be used to evaluate reliability indexes for multicomponent systems with failure interactions among the components. They provide an illustration through the Clayton copula. They especially address the survival function, the failure rate and the meantime to failure for series, parallel, and k-out-of-n systems. Eryilmaz (2014) studies k-out-of-n systems as well considering n dependent components with different weights and diverse failure distributions. Examples with Clayton and Gumbel copulas are included in the paper. This shows the large applicability of copula-based models. Yet, even though the methods proposed could be applied; usability would be another issue. The mathematical complexity suggests a certain expertise or supporting algorithms. There is also a certain limitation into the assumption that diverse components would all interact between each other following the same mechanism and a single interaction copula. In response to that matter, Navarro & Durante (2017) take a step further in the determination of a joint reliability function of residual lifetimes for a multicomponent system. Their study of coherent systems leads them to define special cases of dependence mechanisms:

- All the components of the system are working;
- Some components have failed at given times;
- Some components have failed at some unknown failure times.

A distortion function allows the distinction of such cases and is the copula linking the failure functions of all the components. Yet, even though the parameters of the chosen function are tailored to specific cases, it is supposed that all the components involved follow the same failure interaction mechanism.

Xu et al. (2017) study a multistate manufacturing system and bring an emphasis on the fact that in most cases not all components in a system are stochastically dependent. They define the failure interaction function as follows:

$$P(x) = \prod_{i=1}^m C_i(x; \partial_i) \prod_{j=1}^n P_j(x) \quad (3.38)$$

where $C_i(x)$ is the general failure rate based on a copula of components with failure interaction, $P_j(x)$ is the failure rate of a dependent component j , and ∂_i is the interaction coefficient of

component i . For high accuracy and to deal with problems of small data, Xu et al. (2017) suggest that the Grey system theory be used for the estimation of the parameters rather than classical methods like the Moment Estimation Method or Maximum Likelihood Estimation Method. Yet, the issue of the mathematical complexity of the overall model remains and is combined with the empirical nature of the Grey system theory. It limits the physical interpretation of the interaction mechanisms.

Copulas can also link degradation processes in diverse ways. Guo et al. (2013) intend to determine the joint reliability R of two degradation processes that cause dependent competitive risks. They propose a reliability model based on a copula C . The latter models the interdependence of the two phenomena and integrates the reliability associated to each degradation process R_1 and R_2 . The authors suggest two distinct definitions of the copula.

$$\begin{aligned} R(t) &= R_1(t) + R_2(t) - 1 + C(1 - R_1(t), 1 - R_2(t), \theta) \\ &\quad \text{or} \\ R(t) &= C(R_1(t), R_2(t), \theta) \end{aligned} \quad (3.39)$$

θ is a vector of the copula parameters. It governs the amplitude of the dependence's strength. Simulated data are used in this study. θ is estimated by the maximum likelihood method. Gumbel, Clayton, t- and Gaussian copulas are comparatively applied in the model. The goodness of fit is evaluated by the following criteria: Log-likelihood (LL), Akaike information criterion (AIC), Bayesian information criterion (BIC). Moreover, the models obtained are compared among each other following the average relative error (ARE). Guo et al. (2013) identify the Gaussian copula as the best candidate function for both copula definitions since it gives relatively more precise results.

Wang & Pham (2012) study competitive dependent risks related to degradation and random shocks. The survival function $R(t)$ of a system with n components is determined in function of the number of probable fatal shocks $N(t)$ through a copula C .

$$R(t) = C(R_1(t), \dots, R_n(t)) \cdot P(N(t) = 0) \quad (3.40)$$

Constant copulas (Normal, Plackett, Gumbel, Clayton, t-, Gaussian, etc.) and time-varying copulas (Normal, Rotated-Gumbel, Symmetrized Joe-Clayton) are comparatively considered as candidates. The strength of the fit is evaluated according to diverse criteria: Log-likelihood (LL), Akaike information criterion (AIC), Bayesian information criterion (BIC). Wang & Pham. (2012) demonstrate that time-varying copulas produce better results than constant copulas because they

better fit the simulated data. This shows that there is a possible variability in the phenomenon of failure interaction. An & Sun (2016) follow the same train of thoughts as Wang & Pham (2012) and develop a similar model for dependent degradation processes and shock loads above a certain level. This model is tested on a MEMS application with diverse candidate copulas.

Xi et al. (2014) use a distinctive approach that accounts for the interactivity within a complex system. They develop a sampling method involving a copula in the context of Residual-Useful-Life (RUL) prediction. Their method consists of two steps: first, statistical learning of the historical data and second real-time RUL prediction. A copula C is used to model the dependences between failure times T_i and degradation levels T_N of the system considered:

$$C(u_i, u_N) = C(F_i(T_i), F_N(T_N)), \quad T_i \leq T_N \text{ for } i < N \quad (3.41)$$

where F is an N -dimensional distribution function with marginal functions $F_1, \dots, F_N$. The copula is selected following the Bayesian approach of Huard et al. (2006). While this method does not explicitly model the dependency between failure modes, it builds a relation between degradation levels and any other type of failure. This represents how the degradation of the system makes it more prone to its diverse failure modes.

Mercier & Pham (2012) take a different direction. They consider a system with two units and model the failure interaction by a deterioration process following a bivariate non-decreasing Lévy process:

$\left(X_t = \left(X_t^{(1)}, X_t^{(2)}\right)\right)_{t>0}$, a process with range $\mathbb{R}_+^2$ starting from $(0, 0)$. In Tankov (2003), Peter Tankov introduced Lévy copulas to model the dependencies between components of a multidimensional spectrally positive Lévy process. The system is considered as failed when it reaches a failure zone $\mathcal{L} \subset \mathbb{R}_+^2$. The failure time is:

$$\sigma_{\mathcal{L}} = \inf\{t \geq 0 | X_t \in \mathcal{L}\} \quad (3.42)$$

Considering $L_1 > 0$ and $L_2 > 0$ as the respective failure thresholds of units 1 and 2, three situations are studied:

- Units set in series: $\mathcal{L} = \mathbb{R}_+^2 \setminus [0, L_1[\times [0, L_2[;$
- Units set in parallel: $\mathcal{L} = [L_1, \infty[\times [L_2, \infty[;$

both components of $(X_t)_{t>0}$, standing for different wear indicators of a single system.

Since, the increments in a Levy Process are supposed to be independent, there is no specific closed-form that represents the interactivity. The wear indicators contribute collaboratively to the assessment of the system's overall degradation. The studied system is assumed to be continuously monitored so that, upon failure, a signal is sent to trigger an instantaneous perfect repair. Based on this model, the authors propose a preventive maintenance policy assessed through a cost function on an infinite horizon time. The determined policy performs better than a simple periodic replacement policy. Likewise, Li et al. (2016) model the stochastic dependence between the degradation of two components due to common environment with the Clayton-Lévy copula. Yet, the lack of traceability and complexity in the Levy process is a limitation in terms of comprehending failure mechanisms and propose pragmatic maintenance actions.

Furthermore, determining the relationship between maintenance and failure interaction is a work in progress. Yang et al. (2013) establish a joint survival function for a repairable multicomponent system. The system is subject to partially perfect repair. It means that since only one component at a time fails and causes the system to fail, only the failed component is fully repaired, The latent age to failure of each component i denoted by d_i is then critical in the calculation of the system's survival function. The latter is defined as a multivariate Weibull distribution constructed via Archimedean (Gumbel-Hougaard copula) or Gaussian copulas. For a system with K components, this joint distribution is denoted by $S(d_1, \dots, d_K, \theta)$ where θ is the vector of parameters of the model estimated by Maximum Likelihood. Furthermore, the authors use hypothesis testing to test the statistical dependency of component failures. Yet, even though this paper offers some insight into a relationship between maintenance actions (partially perfect repair) and failure dependencies, it remains limited since maintenance operations can be quite diverse (minimal repair, perfect repair, etc.). Zhang & Yang (2015) and Yang et al. (2017) rely on this previously proposed model to develop a maintenance approach that involves renewal theory. They specifically use Clayton and Gaussian copulas. But, the complexity of the obtained model requires the use of a simulation based optimization approach with stochastic approximation.

Zhang et al. (2014) focus their interest on Accelerated Life Tests. Considering k different stress levels S_i ($i = 1, \dots, k$), they aim to define a joint survival function for a system with p failure modes by using a copula $\hat{C}_p$. They do so by making the assumptions that the distribution families, the mechanism of each competing failure mode and the copula used to construct the joint survival

function will not change under different stress levels. In reality, the interaction mechanisms might be affected by the difference of stress levels. The survival copula $\hat{C}_p(R_{i1}(t), \dots, R_{ip}(t)|\theta_c)$, where $R_{ij}(t)$ is the reliability considering failure mode j under S_i and θ_c is the parameter vector, can be Archimedean as suggested by the authors. In fact, they use the Gumbel-Hougaard copula due to its relative simplicity. The parameters pertaining to copula-based interaction are summed up in table 3.4.

Table 3.4 Parameters of Copula-based interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
<i>Limbouurg et al. (2007)</i>	- Multi-component - Gaussian copula - Voting system	$F_i(x)$: the failure probability of each component i	$F(x)$: the failure probability of the system	Numerical examples
<i>Jiayin et al. (2009)</i>	- Multi-component - Gumbel copula - Series or parallel architecture	$R_i(t)$: the survival function of each component i	$R(t)$: the survival function of the system	- Case study - Drive shaft of a machine only subject to bending moment
<i>Jia et al. (2014)</i> <i>Eryilmaz (2014)</i>	- Multi-component - Gumbel-Hougaard, Clayton copulas - k-out-of-n systems	$R_i(t)$: the survival function of each component i	$R(t)$: the survival function of the system	Numerical examples
<i>Xu et al. (2017)</i>	- Multi-component - Grey system theory - Coherent systems	$P_j(x)$: the failure rate of each dependent component j	$P(x)$: the failure rate of the system	- Numerical examples - Cylinder engine manufacturing system
<i>Guo et al. (2013)</i>	- Multi-component 2 degradation processes - Gaussian copulas - Series or parallel architecture	$R_i(t)$: the survival function of each component i	$R(t)$: the survival function of the system	Numerical examples
<i>Wang & Pham (2012)</i>	- Multi-component - degradation and random shocks - time-varying copulas - Series or parallel architecture	$R_i(t)$: the survival function of each component i $N(t)$: the number of probable fatal shocks	$R(t)$: the survival function of the system	Numerical examples
<i>Xi et al. (2014)</i>	- Multi-component - Bayesian approach	T_i: the failure times T_N: the degradation levels	RUL: the remaining-useful-life of the system	- Case study - Electric cooling fan

Table 3.4 (continued) Parameters of Copula-based interaction models

Models	Assumptions	Crucial parameters	Output	Illustrative examples
Mercier & Pham (2012) Li et al. (2016)	2 components - Deterioration following a Levy process - Components in series or in parallel, with different wear indicators collaboratively accelerating the failure times	$\left(X_t = \left(X_t^{(1)}, X_t^{(2)}\right)\right)_{t>0}$: degradation of the system with component 1 and 2 as a Levy process $\mathcal{L} \subset \mathbb{R}_+^2$: Failure zone	$\sigma_{\mathcal{L}}$: the failure time of the system	Numerical examples
Yang et al. (2013) Zhang & Yang (2014) Yang et al. (2017)	- Multi-component - Gumbel-Hougaard, Clayton and Gaussian copulas	d_i: the latent age to failure of each component i	$R(t)$: the survival function of the system	- Case studies - Cylinder head production system - Car body assembly process - Forklift vehicle system
Zhang et al. (2014)	- Any architecture - Gumbel-Hougaard copula - Accelerated Life Tests	$R_{ij}(t)$: the reliability considering failure mode j under S_i	$R(t)$: the survival function of the system	- Case study - Rolling ball bearings

3.3 Comparative study of failure interaction models

The above-mentioned approaches have the general advantage of presenting models that account for the phenomenon of failure interaction. Thus, they are more realistic than all classical techniques that consider failures as independent. There are other advantages and also limitations. They are divided into three different aspects: concepts, methods and applications.

3.3.1 Concepts

The conceptual differences between the models relate to the starting assumptions and the concept's meaning established to validate the dependence failure hypothesis. These aspects are summarized up in table 3.5.

Table 3.5 Advantages and limits of interaction models concepts

Groups/ References	Concepts	Advantages	Limits
<i>Reliability indexes' interaction:</i> Satow & Osaki (2003) Wang & Zhang (2009) Lai & Yan (2014) Lai & Chen (2006, 2008) Golmakani & Moakedi (2012) Sung et al. (2013) Jhang & Sheu (2000) Lai (2007) Zhang et al. (2014, 2015) Liu et al. (2015) Zequeira & Béranguer (2006) Castro (2009) Fan et al. (2011) Sun et al. (2006, 2010)	The failure rate of a mode is accelerated or updated in function of the number of failures of some other mode. The dependence is unidirectional in most cases; bidirectional in rare cases; Coefficients represent the interactivity as acceleration factors. Random shocks model the influence of the environment.	Accounting for interactive failures gives a better estimation of reliability. Models are more realistic than classical methods and help to significantly optimize the system's performance Coefficients representing the interactivity give a measure of the dependence.	The coefficients modeling the interactivity are constant while the phenomenon of interactivity could be variable. The dependence is unidirectional in most cases. Models are limited to simple architectures. Complex architecture not taken into account. Starting Hypotheses are too restrictive.
<i>State-based interaction:</i> Chen et al. (2011) Huynh et al. (2011, 2012) Do et al. (2015) Assaf et al (2015, 2017) Song et al. (2012, 2014) Rafie et al. (2014) Feng & Coit (2009) Peng et al. (2010) Jiang et al. (2012) Arab et al. (2013) Liang et al (2017) Liang & Parlikad (2015) Rasmekomen & Parlikad (2016)	Degradation and shocks are represented competitively. Random shocks model the influence of the environment. Accumulated shocks alter the degradation rate or speed. Degradation makes the system more vulnerable to random shocks.	The state and degradation of the system is represented more realistically. The models give a physical interpretation of the interactivity. The models' interpretability suggests specific maintenance procedures.	The variability of the shocks' amplitude and number is neglected. The failure threshold is considered to be fixed when in reality it is variable. State variables only capture partial aspects of the degradation.
<i>Copula-based interaction:</i> Limbourg et al. (2007) Jiayin et al. (2009) Jia et al. (2014) Eryilmaz (2014) Xu et al. (2017) Guo et al. (2013) Wang & Pham (2012) Xi et al. (2014) Mercier & Pham (2012) Li et al. (2016) Yang et al. (2013) Zhang & Yang (2014) Yang et al. (2017) Zhang et al. (2014)	A function characterizes the interaction of multiple random variables: reliability functions, failure times, degradation levels.	The interactivity can be represented even though there is limited prior information on its process.	The analytical models are rigid and complex. The copula has to be chosen from a limited set of candidate functions.

One can notice that reliability indexes' interaction models rely on elaborate assumptions about the system's structure. The dependence is one-sided and some entities are assumed to be dominant beforehand in most cases. Moreover, they are only applied to simple architectures (series, parallel, etc.). The main issue with state-based interaction models is that the interactivity within complex systems is rarely measurable by a finite set of state variables. The interactivity itself is rarely observable since only components critical to the system's primary function will be monitored. In fact, the interactions could affect more than just the components with sensors or detectors. Then, there is a risk of significant bias in the results interpretation. Copula-based interaction models are quite useful in cases when there is little prior information about the system. But, they rely on rigid and complex analytical functions selected from a finite set. The failure interaction might not respect the form of the preselected functions. Moreover, some specificities in the architecture can be distinguished through qualitative and experimental information. This information would be left out if an approach that is solely analytical is used.

3.3.2 Methods

The differences in terms of methodology are related to the diverse techniques and the selected study process to develop the starting hypotheses. Table 3.6 summarizes the issues related to methodology.

The methods used for copula-based interaction models have significant mathematical complexity in comparison to other methods. It is also difficult to associate a physical explanation to the parameters even though they use a large array of well-known statistical learning methods and tests. Reliability indexes' interaction models are applicable to diverse preventive maintenance policies. State-based interaction models are limited to the domain of conditional maintenance but have the best interpretability. Most models in the literature rely on experimental designs and simulated data when it is known that there could be higher variability in reality.

Table 3.6 Advantages and limits of interaction models methodologies

Groups/ References	Methods	Advantages	Limits
<i>Reliability indexes' interaction:</i> Satow & Osaki (2003) Wang & Zhang (2009) Lai & Yan (2014) Lai & Chen (2006, 2008) Golmakani & Moakedi (2012) Sung et al. (2013) Jhang & Sheu (2000) Lai (2007) Zhang et al. (2014, 2015) Liu et al. (2015) Zequeira & Bérenguer (2006) Castro (2009) Fan et al. (2011) Sun et al. (2006, 2010)	Scheduled maintenance; Experts' opinions to estimate the parameters of interactivity; Evaluation of conditional probabilities; Markov processes; Experimental design; Numerical examples; Simulated data.	Better system model in comparison to classical methods; Well-known statistical learning methods; Good traceability.	Possibility of significant differences between experimental conditions and reality; Limited to simple architectures; Mathematical complexity.
<i>State-based interaction:</i> Chen et al. (2011) Huynh et al. (2011, 2012) Do et al. (2015) Assaf et al (2015, 2017) Song et al. (2012, 2014) Rafie et al. (2014) Feng & Coit (2009) Peng et al. (2010) Jiang et al. (2012) Arab et al. (2013) Liang et al (2017) Liang & Parlikad (2015) Rasmekomen & Parlikad (2016)	Conditional maintenance; Hidden Markov Models to estimate degradation levels; Analytical models of degradation and random shocks; Experimental design; Numerical examples; Simulated data.	Automatic evaluation of degradation levels; Relationship between system and environment; Evaluation of the physical consequences of interactive failures;	Possibility of significant differences between experimental conditions and reality; Necessity of large data sets for precision; Experts' opinions not accounted for.
<i>Copula-based interaction:</i> Limbourg et al. (2007) Jiayin et al. (2009) Jia et al. (2014) Eryilmaz (2014) Xu et al. (2017) Guo et al. (2013) Wang & Pham (2012) Xi et al. (2014) Mercier & Pham (2012) Li et al. (2016) Yang et al. (2013) Zhang & Yang (2014) Yang et al. (2017) Zhang et al. (2014)	Predefined analytical functions; Parameters selected following the maximum likelihood; Goodness of fit and model selection by LL, AIC, BIC, ARE.	Classical statistical learning methods and tests; Diverse candidate functions.	Ambiguous interpretation of the interactivity factors; Mathematical complexity.

The methods used for copula-based interaction models have significant mathematical complexity in comparison to other methods. It is also difficult to associate a physical explanation to the parameters even though they use a large array of well-known statistical learning methods and tests. Reliability indexes' interaction models are applicable to diverse preventive maintenance policies. State-based interaction models are limited to the domain of conditional maintenance but have the best interpretability. Most models in the literature rely on experimental designs and simulated data when it is known that there could be higher variability in reality.

3.3.3 Applications

The applications are numerous when it comes to interactive failures. But how efficient are they? The criteria to consider include the performance of the methods employed in terms of financial savings or availability improvement, and the level of expertise and complexity required for the application. Research is often subject to less practical constraints than in the field of application. Usually, industrial companies are limited in terms of their capabilities for experimentation, expertise levels, and time constraints. It is therefore important to see which approaches can be applied in a general set with basic or generic tools. Table 3.7 emphasizes the issue of applicability. Copula-based interaction models are the least practical as standalone applications due to their analytical complexity. Reliability indexes' interaction models are less complex but still difficult to apply due to high computational complexity. This issue is amplified by the lack of dedicated software for reliability models integrating the interactivity. State-based interaction models would provide the best results in the industry. Yet, they require the highest economic investment in order to be applied.

Table 3.7 Advantages and limits of interaction models applications

Groups/ References	Applications	Advantages	Limits
<i>Reliability indexes' interaction:</i> Satow & Osaki (2003) Wang & Zhang (2009) Lai & Yan (2014) Lai & Chen (2006, 2008) Golmakani & Moakedi (2012) Sung et al. (2013) Jhang & Sheu (2000) Lai (2007) Zhang et al. (2014, 2015) Liu et al. (2015) Zequeira & Béranguer (2006) Castro (2009) Fan et al. (2011) Sun et al. (2006, 2010)	Age or Bloc replacement policy; Periodic preventive maintenance with minimal repairs; Warranty cost adjustment; Redundancy allocation; Opportunistic maintenance.	Straight-forward application to scheduled maintenance; Optimization of resources more efficient if the interactivity is accounted for; Rethinking the architecture of industrial systems.	Analytical complexity of models; High computational complexity (simulations); Lack of dedicated software; The results interpretation is more complex than with state variables.
<i>State-based interaction:</i> Chen et al. (2011) Huynh et al. (2011, 2012) Do et al. (2015) Assaf et al (2015, 2017) Song et al. (2012, 2014) Rafie et al. (2014) Feng & Coit (2009) Peng et al. (2010) Jiang et al. (2012) Arab et al. (2013) Liang et al (2017) Liang & Parlikad (2015) Rasmekomen & Parlikad (2016)	Failure limit maintenance policy; Inspection policy in function of the degradation level; Reliability models for real systems like Micro-electro-mechanical systems.	Identification of key parameters of complex systems following the degree of interactivity; Better estimation of the system's reliability;	Economical toll of conditional maintenance (captors, sensors, etc.); Abstract measure of the economic consequences of interactivity; Risk if biased interpretation of the interactivity.
<i>Copula-based interaction:</i> Limbourg et al. (2007) Jiayin et al. (2009) Jia et al. (2014) Eryilmaz (2014) Xu et al. (2017) Guo et al. (2013) Wang & Pham (2012) Xi et al. (2014) Mercier & Pham (2012) Li et al. (2016) Yang et al. (2013) Zhang & Yang (2014) Yang et al. (2017) Zhang et al. (2014)	Subset of more complex models; Data-driven reliability models; RUL prediction.	Versatility in the usability; Representation of a complex phenomenon by an analytical closed-form.	Not practical for the industry due to the analytical orientation; Prior information not integrated.

3.4 Conclusion

The interactivity of failure modes is a concept presented by the literature in the context of stochastic dependence. This failure interaction can be defined as a gradual, immediate, unidirectional or bidirectional phenomenon. Diverse models are reviewed and classified into three distinctive categories: Reliability indexes' interaction, state-based interaction, and copula-based interaction.

All of the models presented are more realistic than classical methods that overlook the interactivity or assume it to be negligible. Yet, they have a few limitations. Copula-based interaction models are data-driven and based on a limited set of analytical functions. Their complexity makes them impractical for the industry, especially since interpretation of these models can be ambiguous. Reliability indexes' interaction models can be paired with numerous classical statistical learning and preventive maintenance methods. But, they rely on restrictive hypotheses about a system's structure. State-based interaction models have the best interpretability. But, conditional maintenance can be costly and the state variables might capture only partial aspects of the phenomenon studied.

Another general observation about all models is how they lack representation of the variability in the interactivity phenomenon. In most cases, the parameters, factors or coefficients used to account for the interactivity are defined as constant numbers or effects rather than random variables. In future work, it would be useful to put forth an interaction model that could integrate this variability more comprehensively to the chosen coefficients or parameters of interaction.

CHAPITRE 4 ARTICLE 2: FAILURE INTERACTION MODEL BASED ON EXTREME SHOCK AND MARKOV PROCESSES

Toualith Jean-Marc MEANGO and Mohamed-Salah OUALI

Published by “*Reliability Engineering & System Safety*”

<https://doi.org/10.1016/j.res.2020.106827>

Abstract—Reliability models have overtaken the stochastic dependence that may exist between multiple failure modes of the same system. A few relevant models involve the calculation of interaction coefficients, which are difficult to interpret and quantify from maintenance logs. This paper develops a model that integrates failure interactions as successive random events. It uses an interrelated draw concept to model the dependent failure modes as consequences of extreme shocks. The model quantifies the probabilities of both sides’ failure mode effects on the reliability indexes’ system. A Markov process method is used to estimate the mean time-to-failure of a culvert system subject to two cracking and displacement dependent failure modes. The model developed reproduces the interaction phenomena within the culvert beside the interpretability of its results. A sensitivity analysis demonstrates that the intensity of the shock process strengthens its interactivity in most cases.

Index Terms—Failure interaction, stochastic dependence, reliability analysis, Markov process.

4.1 Introduction

Reliability and maintenance engineering play a key role in the sustainability sustainment of industrial assets. Common modeling methods are not able to deal with certain new challenges due to the complexity of failure modes, which have become more difficult to apprehend. Specific attention to failure interaction within a complex system is growing (Che et al. [2018], Yousefi et al. [2019], Qiu et al. [2019]). The classical methods, such as Failure Mode and Effects Analysis (FMEA), reliability block diagrams and Stress-Strength analysis, are often based on the hypothesis of stochastic independence between failure modes. Such an assumption holds as long as a system remains simple with components that have been studied and tested for a long time. Independence between failure modes becomes a shakable hypothesis with new complex technologies involved. Keizer et al. (2017) put an emphasis on maintenance policies for systems with multiple dependent

components. They show how more relevancy should be attributed to dependencies within a system in order to reduce the risk of failure. These dependencies could be structural, economic and stochastic. Failure interaction is a stochastic dependence that could help understand intricate phenomena within a multicomponent system. Such phenomena certainly impact reliability and maintenance performance as a result of failure interaction (Wang & Chen [2016]). Yet, capturing and quantifying failure interaction require a new modeling method.

Murthy & Nguyen (1985a, 1985b) present two types of failure interactions. In Type I, called induced failures, a component causes the instantaneous failure of another component with a probability p or has no effect with a probability $1 - p$. In Type II, called failure rate or shock damage interaction, a component's failure changes the failure rate of another component. In this type, the component's failure acts as a shock on another component and the accumulation of shocks accelerates the failure rate. This distinction is reproduced in most of the interaction models (Meango & Ouali, 2019).

Shock models seem like an appropriate method for representing failure dependence. Shanthikumar & Sumita (1983), followed by Gut (1999), Cha & Finkelstein (2011), and Cirillo & Hüsler (2011) propose variant extreme shock models to address failure interaction. In those models, the failure happens when numeral shocks of random magnitudes occurring at random times exceed a given critical level. Therefore, this may be an opportunity to identify the moment when an interaction becomes critical, even if its evolution cannot be continuously observed.

Meango & Ouali (2019) review and classify failure interaction models into three categories. These include the Copula-based interaction models, which are based on joint functions that link degradation processes and/or failure times. Wang & Pham (2012) develop a model associating degradation processes and random shocks using different time-varying copulas. Likewise, Xu et al. (2017), Yang et al. (2017) test a large array of Archimedean and Gaussian copulas. The interaction is usually quantified by a single parameter. The obtained models are mathematically complex and somewhat difficult to be physically interpreted.

State-based interaction models are those in which a relationship is established between the degradation processes. Models that are most related to these are Huynh et al. (2011, 2012), Song et al. (2012, 2014), Rafiee et al. (2014), which present relevant reliability models that integrate failure interactions in degradation processes with random shocks. An & Sun (2017) discuss a model

with dependent competing failure processes with shock loads above a certain level. Rafiee et al. (2017) consider a generalized mixed shock model that is only relevant to a condition-based maintenance policy. The model requires, in addition to maintenance logs, a fair amount of experimental data to estimate the probability of interaction, which is a limitation to its applicability.

The interaction models of reliability indexes are those in which the indexes of diverse failure modes are linked with each other by an analytical function that represents a specific interaction mechanism. Certain specific coefficients are often used to quantify the interaction. However, they lack flexibility in terms of their variability and interpretability. Satow & Osaki (2003) and Wang & Zhang (2009) work on a two-component system subject to random shocks. The interaction phenomenon between failures is primarily defined by the reparability of the components. Likewise, Sun et al. (2006, 2010), Zequeira & Béranger (2006), Castro (2009), Sung et al. (2013) and Zhang et al. (2014, 2015) offer some alternative models, but the failure interaction is usually reflected by a coefficient of interaction without interpretability, except for a presumed proportionality of failure rates.

This paper proposes a new method that could be classified as Reliability Indexes' Interaction, in which the interaction between multiple failure modes is explained through random draws. The intent is to provide quantification, flexibility and interpretability in the representation of an interaction, even with a limited data sample. A repairable system subject to two failure modes that occurs at random shocks is studied, regardless of its structural architecture. The shocks are associated with random draws, in which a draw symbolizes a single shock. In the proposed model, the cumulative number of overall shocks increases until one fatal shock causes one of two failure modes. A shock has an internal effect on the system and increases the probability that one of the failures will occur. We assume that a shock somehow alters the state of the system by changing the ratios of entities in the drawing pool. The probabilities of interaction are estimated through the interdependent instant probabilities of the extreme shock process. Then, the distribution function and the mean time-to-failure are derived using a Markov process.

The remainder of the paper is organized as follows. Section 4.2 presents our failure interaction model. Section 4.3 explains how the parameters of our model are estimated with Markov processes. Section 4.4 describes the details about key reliability indexes and how to evaluate them. Section

4.5 presents a case study, which is about an application of culverts in a region of Quebec. Section 4.6 contains concluding remarks.

4.2 Failure interaction model

The proposed failure interaction model is built from a combination of shock processes. We consider a repairable system, represented by a drawing pool, subject to two failure modes. The pool contains a finite number of balls that represent the normal operation (Type 0), and the failure modes 1 (Type 1) and 2 (Type 2). A draw depicts a shock that may conserve the same type (Type 0) or transform certain balls from Type 0 to Type 1 or Type 2, depending on which Type 1 or Type 2 ball was previously picked. The sequence of successive random draws denotes an interaction mechanism within the system that eventually changes the number of type 1 and 2 balls representing the two failure modes. The probability of having a certain type of shock at a given time depends on how the previous shocks have changed the system. The probability of interaction between the two failure modes characterizes how each failure mode was influenced by the sequence of previous shocks.

4.2.1 Random draw interaction mechanism

There are random draws starting with a fixed n_{pool} balls labelled 0, 1 and 2. A ball is picked up and released back into the drawing pool at periodic intervals. When ball 1 is picked, shock 1, representing failure mode 1, is recorded. In the same way, shock 2 is recorded when ball 2 is drawn. The draw of any ball 0 has no effect on the system behavior. At the k th draw, if a ball 1 is picked, a number y_k of balls 0 is replaced with y_k balls 2 in the drawing pool. If a ball 2 is picked, a number x_k of balls 0 is replaced with x_k balls 1 in the drawing pool. The numbers y_k and x_k characterize the influence of failure mode 1 on failure mode 2 and vice versa. Therefore, this drawing mechanism changes the probability of drawing any ball, considering that the overall number of balls in the pool always remains the same, n_{pool} .

Assume that at the first (1st) draw, the probability of picking a ball 0 is $p_1^{(0)}$, a ball 1 is $p_1^{(1)}$ and a ball 2 is $p_1^{(2)}$. At the second (2nd) draw, a ball 1 can be drawn either from one of balls 1 previously present in the pool or from the x_1 balls 1 eventually added if a ball 2 was previously drawn. This

results in the equality $p_2^{(1)} = p_1^{(1)} + \frac{x_1}{n_{pool}} p_1^{(2)}$. This principle also applies to the potential drawing of a ball 2 so that $p_2^{(2)} = p_1^{(2)} + \frac{y_1}{n_{pool}} p_1^{(1)}$.

Since at the time of drawing, the only possibilities are 0, 1 and 2 and n_{pool} remains the same after each draw, it is evident that $p_k^{(0)} + p_k^{(1)} + p_k^{(2)} = 1, \forall k$. One can trivially deduce from this equality the instant probability to draw a ball 0 at the 2nd draw as $p_2^{(0)} = p_1^{(0)} - \frac{y_1}{n_{pool}} p_1^{(1)} - \frac{x_1}{n_{pool}} p_1^{(2)}$.

Iteratively, at the k -th draw, the probability of picking any of 0, 1 and 2 balls is, respectively, given by:

$$\begin{cases} p_k^{(0)} = p_{k-1}^{(0)} - \gamma_{k-1} p_{k-1}^{(1)} - \theta_{k-1} p_{k-1}^{(2)} \\ p_k^{(1)} = p_{k-1}^{(1)} + \theta_{k-1} p_{k-1}^{(2)} \\ p_k^{(2)} = p_{k-1}^{(2)} + \gamma_{k-1} p_{k-1}^{(1)} \end{cases} \quad (4.1)$$

where $\theta_{k-1} = \frac{x_{k-1}}{n_{pool}}$ and $\gamma_{k-1} = \frac{y_{k-1}}{n_{pool}}$. The probability of an interaction between the two failure modes is defined by coefficients θ_{k-1} and γ_{k-1} . θ_{k-1} is the influence of the previous $(k-1)$ -th drawing of ball 2 on the probability of picking ball 1. Likewise, γ_{k-1} represents the influence of the previous $(k-1)$ -th drawing ball 1 on the probability of picking ball 2. These coefficients contribute to increasing the subsequent probabilities and having a shock related to both failure modes. There is a significant advantage of describing the interaction of failures through a series of probabilities. As above-mentioned and explained by Meango & Ouali (2019), most models will rely on a single coefficient when in reality such phenomena evolve with how failure and shock occur sequentially. The series θ_h and γ_h ($h = 1, 2, \dots$) capture that aspect but also can help evaluate if the interactivity will strengthen, weaken or be completely random considering the sequence of shocks over time.

Figure 4.1 depicts the proposed random draw mechanism in which $n_k^{(0)}$, $n_k^{(1)}$ and $n_k^{(2)}$ denote the cumulative number of balls 0, 1 and 2 drawn at the k th iteration, respectively. Drawing ball 0 does not change the state of the system. However, when ball 1 or 2 is picked, the state of the system is changed by decreasing the numbers of balls 0 and increasing those of 1 or 2.

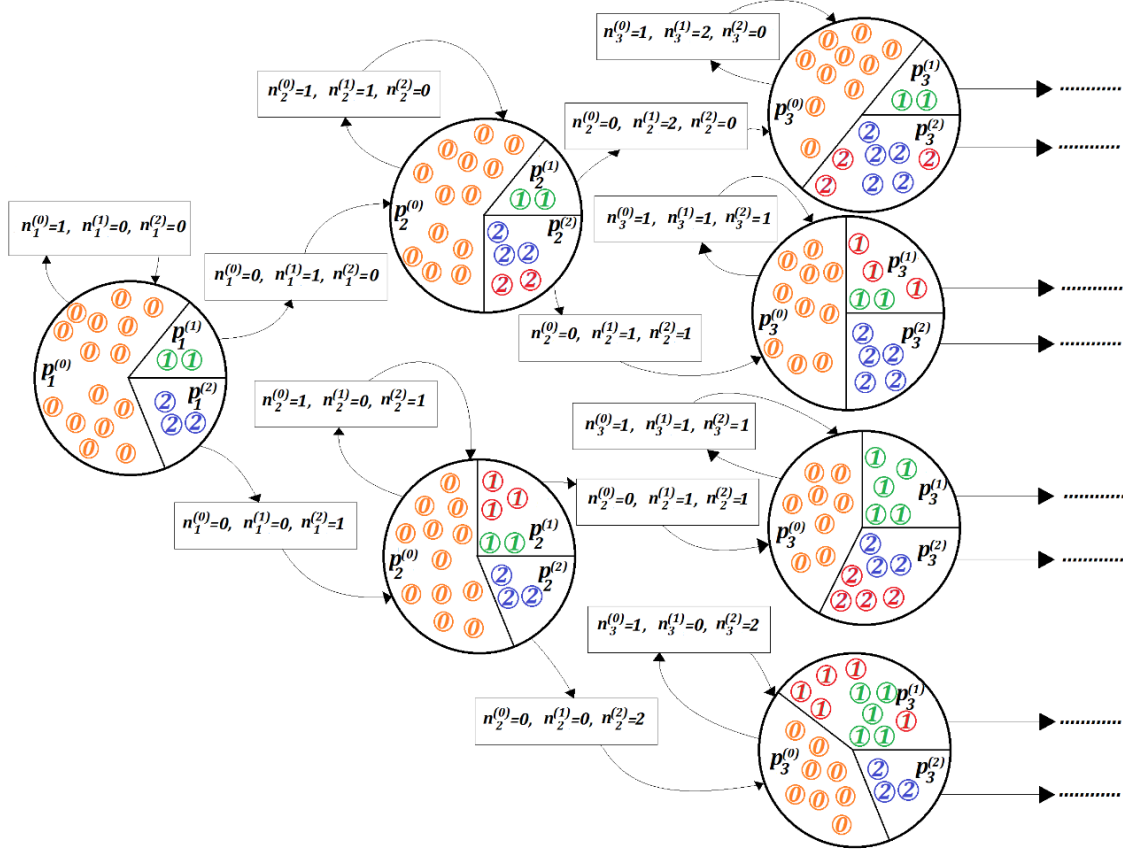


Figure 4.1 Example of random draws in a pool subject to shocks and interaction

4.2.2 Failure interaction probabilities

Understanding how the probabilities of having a certain number of shocks evolve from one draw to another is crucial in evaluating the failure probability of the system. Let $N^{(0)}(t)$, $N^{(1)}(t)$ and $N^{(2)}(t)$ be the random quantities of balls 0, 1, 2, drawn at t , respectively. The cumulative number of shocks at t is denoted by $N(t) = N^{(0)}(t) + N^{(1)}(t) + N^{(2)}(t)$. The probability of having a certain number $N^{(a)} = l$ of shocks $a \in \{0, 1, 2\}$ knowing that the overall number of shocks $N = k$ at t , corresponds to the conditional probability $P\{N^{(a)}(t) = l | N(t) = k\}$. Initially, for any type of ball, $a \in \{1, 2\}$, $P\{N^{(a)}(t) = 0 | N(t) = 0\} = 1$, which is obvious since there is only one possible case. After the 1st draw, there are two possible cases for each $a \in \{1, 2\}$: $P\{N^{(a)}(t) = 0 | N(t) = 1\} = 1 - p_1^{(a)}$ and $P\{N^{(a)}(t) = 1 | N(t) = 1\} = p_1^{(a)}$. If more balls are drawn, the number of possibilities increase multiplicatively. Then, the probability of $n_k^{(a)} = l$ is

given by $P\{N^{(a)}(t) = l | N(t) = k\}$ for any $a \in \{1, 2\}$ and $0 < l \leq k$. Using the total probability law, $P(n_k^{(a)} = l)$ is evaluated in the following manner:

$$P(n_k^{(a)} = l) = p_k^{(a)} \cdot P(n_{k-1}^{(a)} = l - 1) + (1 - p_k^{(a)}) \cdot P(n_{k-1}^{(a)} = l) \quad (4.2)$$

Appendix A.1 shows that for $1 < l \leq k$:

$$P(n_k^{(a)} = l) = \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{v_r}^{(a)}}{1 - p_{v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1 - p_h^{(a)}) \right) \quad (4.3)$$

where $v_0 \in (l, \dots, k)$, $v_1 \in (l-1, \dots, k-1)$, $\dots$, $v_{l-1} \in (1, \dots, k+1-l)$

Note that for any $l > k$, $P(n_k^{(a)} = l) = 0$, because it is impossible to have $N_a(t) > N(t)$. Note that this distribution differs from a classical one like the Binomial distribution since for the latter, an experience is repeated with constant probabilities of success and failure. In this model, the probability of success is equivalent to $p_h^{(a)}$. It evolves from one experience of drawing a ball to another since the state of the system evolves as well.

4.2.3 Time-to-failure with interaction failure modes

The changes occurring due to the shock mechanism act as two extreme shock processes on the system behavior. Considering a certain failure mode $a \in \{1, 2\}$, we denote by $Y_s^{(a)}$ the interarrival times of the shocks that change the structure of the drawing pool and could cause the system's failure. The shocks 1 and 2 arrive at times $T_j^{(a)} = \sum_{s=1}^j Y_s^{(a)}$ with a magnitude $W^{(a)}[T_j^{(a)}]$, $j \geq 1$. The system fails when a shock of type (a) has a magnitude that exceeds the threshold $l^{(a)}$. It is implied that the effects of the shocks are not cumulative regarding their magnitudes but rather affect the stability of the system and its propiency to certain type of shocks. We define $T^{(a)}$, the time-to-failure due to the $N^{(a)}$ -th shock (a) as follows:

$$T^{(a)} = \sum_{j=1}^{N^{(a)}(t)} Y_j^{(a)} \quad (4.4)$$

The lifetime of the system is then $T = \min(T^{(1)}, T^{(2)})$. For example, as shown in Figure 4.2, $T_7^{(2)}$ is the time at which a shock 2 above the threshold $l^{(2)}$ occurs and stops the system. The succession

of shocks 1 and 2 suggests that the occurrence of shock 1 could have significantly increased the probability of having shocks 2 and failure 2.

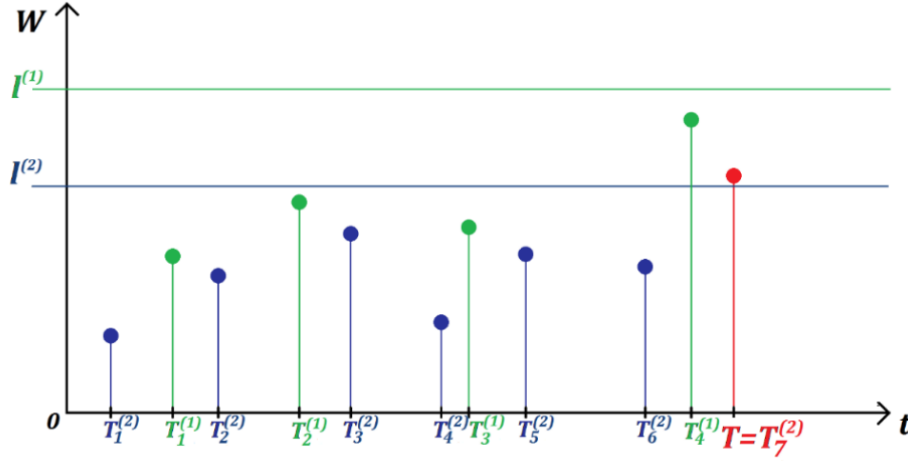


Figure 4.2 System subject to two competitive extreme shock processes

For successive random draws, the probability $P(T^{(a)} > t)$ that a fatal shock will occur past a certain time t is equal to the probability $P\{N^{(a)}(t) < n\}$ that at time t the n -th fatal failure is not reached yet:

$$P(T^{(a)} > t) = P\{N^{(a)}(t) < n\} = \sum_{l=0}^{n-1} P\{N^{(a)}(t) = l\} \quad (4.5)$$

According to the condition probability law:

$$P\{N^{(a)}(t) = l\} = \sum_{k=0}^{\infty} P\{N(t) = k\} \cdot P\{N^{(a)}(t) = l | N(t) = k\} \quad (4.6)$$

Considering a homogeneous Poisson process of intensity λ , $P\{N(t) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$ and Equation (4.3), then

$$P\{N^{(a)}(t) = l\} = \sum_{k=l}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{v_r}^{(a)}}{1-p_{v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1-p_h^{(a)}) \right) \quad (4.7)$$

The choice of representing diverse shocks all together in a Poisson process represents a slight departure from the Reliability Indexes' Interaction models mentioned in the introduction. The common approach would be the representation of each process separately and the choice of a

copula or joint function to bind the diverse processes. In this model, the interaction is represented in the sequence of shocks but also encapsulated in the shared randomness of their occurrence in time.

4.3 Probability estimation through a Markov process method

Figure 4.3 depicts the system behavior according to the two competitive Markov processes that characterize the occurrence of the two failure modes. At any time t , a system state is defined by the vector $X^{(a)}(t): (N^{(a)}(t), N(t))$. The system has a countable and finite number of states $i: (l_i, k_i)$ where $Pr_{i,j}^{(a)}$ denotes the probability of transitioning from state i to state j considering the Markov process associated to type (a) shocks. The joint probability of being at a certain state is given by:

$$P\{X^{(a)}(t) = i\} = P\{N(t) = k_i\} \cdot P(n_{k_i}^{(a)} = l_i) \quad (4.8)$$

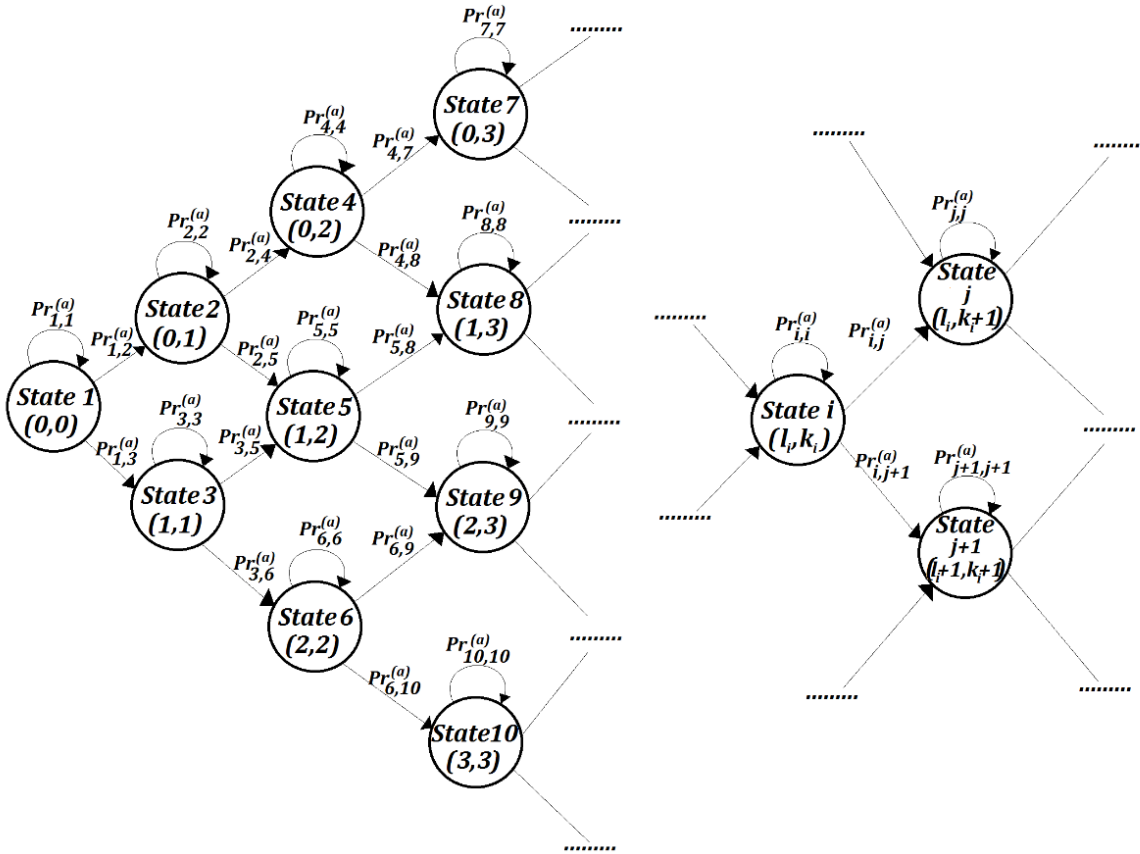


Figure 4.3 Random drawing as Markov chains considering states $i = (l_i, k_i)$

Figure 4.4 provides a more suitable representation of the Markov process.

Considering a very short interval of time s , the transition probability from state i to j has the following form:

$$\begin{aligned} Pr_{i,j}^{(a)}(t, t+s) &= P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} \\ Pr_{i,j}^{(a)}(t, t+s) &= P\{(N^{(a)}(t+s) = l_j, N(t+s) = k_j) | (N^{(a)}(t) = l_i, N(t) = k_i)\} \end{aligned} \quad (4.9)$$

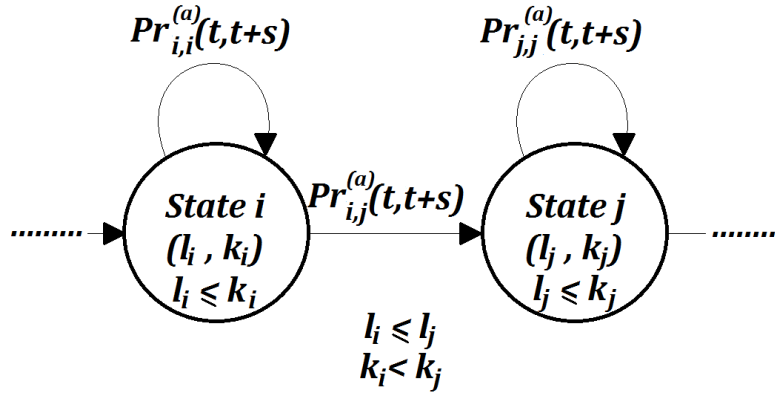


Figure 4.4 Random drawing as continuous Markov process

Under a homogenous Poisson process, appendix A.2 demonstrates that:

$$Pr_{i,j}^{(a)}(t, t+s) = \delta_{ij}^{(a)} + q_{ij}^{(a)}s + o(s) \quad (4.10)$$

$$\text{where } q_{ij}^{(a)} = \begin{cases} -\lambda_i & , i = j \\ (1 - p_{k_j}^{(a)}) \lambda_i, k_j - k_i = 1, l_j - l_i = 0 \\ p_{k_j}^{(a)} \cdot \lambda_i & , k_j - k_i = 1, l_j - l_i = 1 \\ 0 & \text{otherwise} \end{cases} \text{ and } \delta_{ij}^{(a)} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

$\delta_{ij}^{(a)}$ is Kronecker's delta. Because the drawing process is homogeneous, $q_{ij}^{(a)}$ is independent of time and $p_{k_j}^{(a)}$ can be easily estimated. If the drawing process was inhomogeneous λ_i would be replaced with $\lambda_i(t)$. According to Inamura (2006), the $q_{ij}^{(a)}$ can be estimated from continuous data observations:

$$\begin{cases} \hat{q}_{ij}^{(a)} = \frac{A_{ij}^{(a)}(T_{overall})}{G_i^{(a)}(T_{overall})} \\ \hat{q}_{ii}^{(a)} = -\sum_{j \neq i} \hat{q}_{ij}^{(a)} = -\hat{\lambda}_i \end{cases} \quad (4.11)$$

where $A_{ij}^{(a)}(T_{overall})$ is the number of times the system transitions from i to j during the overall time $T_{overall}$ of observations, $G_i^{(a)}(T_{overall})$ is the sojourn time in state i during $T_{overall}$. This implies that:

$$\hat{p}_{k_j}^{(a)} = \frac{A_{ij}^{(a)}(T_{overall})}{\hat{\lambda}_i G_i^{(a)}(T_{overall})}, \quad k_j - k_i = 1, \quad l_j - l_i = 1 \quad (4.12)$$

However, it is common that the available data will be discrete. The availability of continuous data is often reliant on the implementation of Condition-Based Maintenance tools. Inamura (2006) describes a method to estimate $\hat{q}_{ij}^{(a)}$ if only discrete data observations are available. This requires the use of the Expectation-Maximization algorithm. Given discrete time observations x^{obs} , in the M-step, the likelihood estimator can be obtained as:

$$\tilde{q}_{ij}^{(a)} = \frac{E(A_{ij}^{(a)}(T_{overall})|x^{obs})}{E(G_i^{(a)}(T_{overall})|x^{obs})} \quad (4.13)$$

As an immediate consequence, the probabilities would be:

$$\tilde{p}_{k_j}^{(a)} = \frac{E(A_{ij}^{(a)}(T_{overall})|x^{obs})}{\tilde{\lambda}_i E(G_i^{(a)}(T_{overall})|x^{obs})} \quad (4.14)$$

where $\tilde{\lambda}_i = \sum_{j \neq i} \tilde{q}_{ij}^{(a)}$.

After estimating the probabilities, the interaction coefficients can be derived and their evolution can be interpreted. It is safe to assume for $k \geq 1$ that:

$$\begin{cases} \hat{p}_{k+1}^{(0)} = \hat{p}_k^{(0)} - \hat{\gamma}_k \hat{p}_k^{(1)} - \hat{\theta}_k \hat{p}_k^{(2)} \\ \hat{p}_{k+1}^{(1)} = \hat{p}_k^{(1)} + \hat{\theta}_k \hat{p}_k^{(2)} \\ \hat{p}_{k+1}^{(2)} = \hat{p}_k^{(2)} + \hat{\gamma}_k \hat{p}_k^{(1)} \end{cases} \quad (4.15)$$

$\hat{\gamma}_k$ and $\hat{\theta}_k$ can be estimated by solving Equation (4.15):

$$\begin{cases} \hat{\theta}_k = \frac{\hat{p}_{k+1}^{(1)} - \hat{p}_k^{(1)}}{\hat{p}_k^{(2)}}, \hat{p}_{2,k} > 0 \\ \hat{\gamma}_k = \frac{\hat{p}_{k+1}^{(2)} - \hat{p}_k^{(2)}}{\hat{p}_k^{(1)}}, \hat{p}_{1,k} > 0 \end{cases} \quad (4.16)$$

4.4 Evaluation of the reliability indexes

This section develops the expected time-to-failure and the survival model under the failure interaction mechanism.

4.4.1 Expected time-to-failure

The expected time-to-failure provides a general idea of the system lifespan, which may help in designing appropriate preventive maintenance plans. The system fails at $T = \min(T^{(1)}, T^{(2)})$ which means that the system fails when a fatal shock of type 1 or 2 occurs after a certain sequence of shocks. According to Wald's equation (Wald, 1944):

$$E[T^{(a)}] = E[N^{(a)}(t)] \cdot E[Y_j^{(a)}] \quad (4.17)$$

It is demonstrated in appendix A.3 that:

$$E\{N^{(a)}(t) | N(t) = k\} = \sum_{l=1}^k p_l^{(a)} \quad (4.18)$$

Using the law of total expectation, the expected number of shocks (a) during t is:

$$E[N^{(a)}(t)] = \sum_{k=1}^{\infty} P\{N(t) = k\} \cdot \sum_{l=1}^k p_l^{(a)} \quad (4.19)$$

Considering that m is the maximum number of shocks ever recorded on the type of system studied throughout its lifespan, and assuming that $\sum_{l=m+1}^k p_l^{(a)} \approx p_m^{(a)}(k - m)$, then the following approximation can be made:

$$\begin{aligned} E[N^{(a)}(t)] \approx & \sum_{k=1}^m P\{N(t) = k\} \sum_{l=1}^k p_l^{(a)} + \left[\sum_{l=1}^m p_l^{(a)} - m p_m^{(a)} \right] [1 - \sum_{k=0}^m P\{N(t) = k\}] \\ & + p_m^{(a)} [\lambda t - \sum_{k=1}^m k \cdot P\{N(t) = k\}] \end{aligned} \quad (4.20)$$

We denote by X_i ($i = 0, 1, \dots$) the interarrival times in between successive draws, regardless of the types of balls drawn. Considering that drawing a ball 1 or 2 happens in between successive draws, regardless of the balls drawn, $Y_j^{(a)}$ can be expressed in function of random variables X_i .

where $\forall a \in \{1, \dots, M\}, x_{k-1}^{(a)} = \sum_{j=1}^M x_{j,k-1}^{(a)}$ and $\theta_{k-1}^{(a)} = \sum_{j=1}^M \theta_{j,k-1}^{(a)}$.

The system would experience failure at $T = \min(T^{(0)}, \dots, T^{(M)})$. The survival probability would then be:

$$R_S(t) = \prod_{a=1}^M P(N^{(a)}(t) < n) = \prod_{a=1}^M \left\{ \sum_{j=0}^{n-1} P\{N^{(a)}(t) = j\} \right\} \quad (4.25)$$

In order to estimate the interaction probabilities of such a model, M competitive Markov processes would have to be studied. Availability and separation of data could in that case become a bigger challenge, since shocks would need to be identifiable distinctly and sequentially. In the following section, an example based for only two competitive failure modes is provided.

4.5 Example of concrete culverts

This example deals with the maintenance of identical concrete culverts. The condition of each culvert is inspected yearly and different degradation indices associated with certain dominant failure modes are measured using a five-level scale. Once a critical level of degradation is reached, it prompts a maintenance intervention. However, according to the experts of those culverts, two particular failure modes have a mutual effect on each other when one of them occurs. The cracking in a concrete culvert may increase the risk of unwanted infiltration and, in the long run, deformation and even slumping of the entire structure. It is also plausible that intermittent hydrological problems, such as floods and clutter, would weaken the culvert's structure. In our case, two failure modes are considered: severe displacement (S1) and severe cracking (S2). The severe cracking is the major cause of degradation of a culvert. Based on a dataset giving the age of the culverts and their degradation indices at the time of inspection, the following probabilities are estimated using Equations (4.11) to (4.16) (Tables 4.1 and 4.2).

While the influence of S2 on S1 seems to be random, the interaction due to the effect of S1 on S2 is gradual. S2 remains the biggest risk that could lead to severe cracking. Considering the probability of failure $F(t) = 1 - R(t)$, one can see how the system is affected by the interaction of S1 and S2 in Figure 4.5.

Table 4.1 Probabilities $\hat{p}_k^{(S1)}$ and effect of S2 on S1

Probabilities $\hat{p}_k^{(S1)}$		$\hat{\theta}_k$ (effect of S2 on S1)	
$k = 1$	0.2414	$k = 1$	0.1116
$k = 2$	0.3041	$k = 2$	0.0242
$k = 3$	0.3180	$k = 3$	0.0060
$k = 4$	0.3215	$k = 4$	0.0110
$k = 5$	0.3282		

Table 4.2 Probabilities $\hat{p}_k^{(S2)}$ and effect of S1 on S2

Probabilities $\hat{p}_k^{(S2)}$		$\hat{\gamma}_k$ (effect of S1 on S2)	
$k = 1$	0.5624	$k = 1$	0.0332
$k = 2$	0.5704	$k = 2$	0.0518
$k = 3$	0.5862	$k = 3$	0.0634
$k = 4$	0.6063	$k = 4$	0.1032
$k = 5$	0.6395		

A global sensitivity analysis is performed regarding the effect of the intensity of the drawing Poisson process on the model. As presented in Table 4.3, the uncertainty interval [0.0800, 0.1000] is considered for a nominal value of 0.0879 shocks/year. Interestingly, the coefficients $\hat{\theta}_k$ and $\hat{\gamma}_k$ still capture the phenomenon of interaction with respect to the “speed” of the drawing process. The displacement failure mode appears to be influenced randomly by the cracking one; while the cracking failure mode is gradually evolving due to the displacement one. Furthermore, as the “speed” of the drawing process increases, the influence of displacements on cracking is stronger, as shown in Figure 4.6.

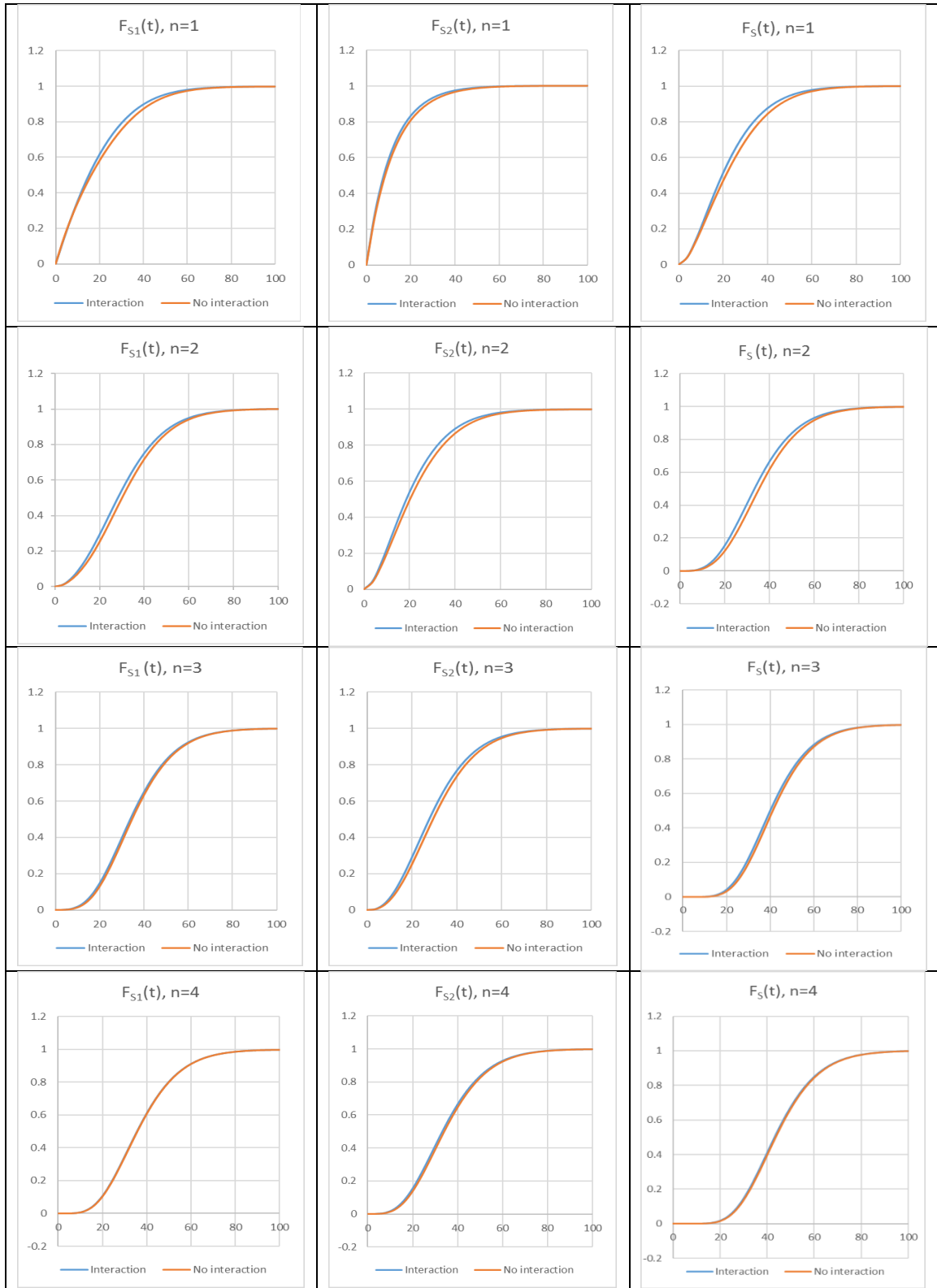


Figure 4.5 Distribution functions of S_1 , S_2 and S (both failure modes) at the n -th shock

Table 4.3 Effect of the intensity λ on the coefficients

Coefficients	Intensity λ									
	0.0800	0.0825	0.0850	0.0875	0.0879	0.0900	0.0925	0.0950	0.0975	0.1000
$\hat{\theta}_1$	0.1115	0.1116	0.1116	0.1116	0.1116	0.1115	0.1115	0.1114	0.1114	0.1113
$\hat{\theta}_2$	0.0236	0.0238	0.0240	0.0242	0.0242	0.0244	0.0245	0.0247	0.0248	0.0249
$\hat{\theta}_3$	0.0060	0.0060	0.0060	0.0060	0.0060	0.0060	0.0061	0.0061	0.0061	0.0061
$\hat{\theta}_4$	0.0104	0.0106	0.0108	0.0110	0.0110	0.0112	0.0113	0.0115	0.0116	0.0118
$\hat{\gamma}_1$	0.0271	0.0290	0.0309	0.0329	0.0332	0.0348	0.0368	0.0388	0.0408	0.0429
$\hat{\gamma}_2$	0.0459	0.0478	0.0497	0.0515	0.0518	0.0533	0.0550	0.0568	0.0584	0.0602
$\hat{\gamma}_3$	0.0588	0.0602	0.0617	0.0631	0.0634	0.0646	0.0660	0.0675	0.0689	0.0704
$\hat{\gamma}_4$	0.0679	0.0789	0.0902	0.1014	0.1032	0.1129	0.1242	0.1359	0.1474	0.1594

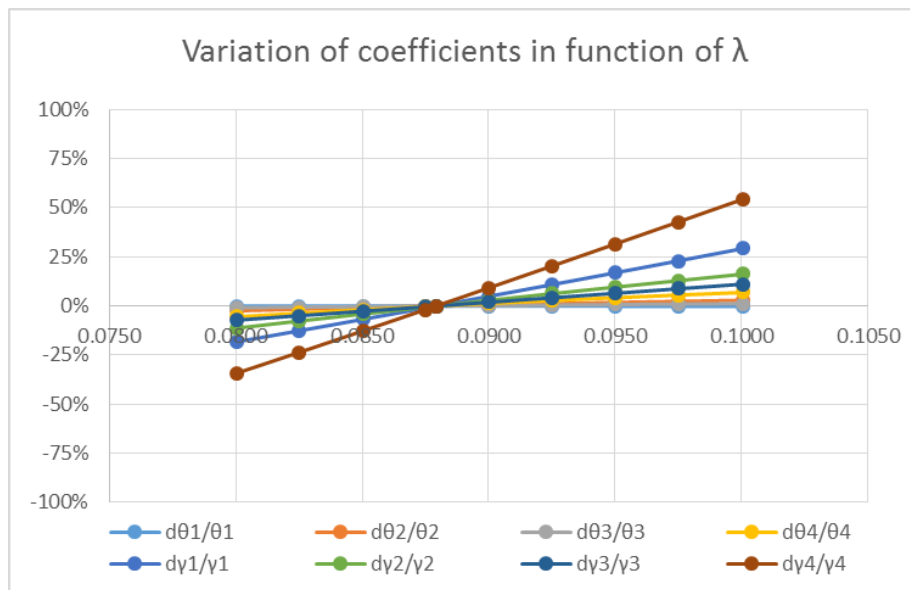
**Figure 4.6 Variation of coefficients in function of the “drawing speed” λ**

Figure 4.7 shows how the drawing “speed” increases the expected number of shocks. The effect is more prominent on the cracking shocks (S2).

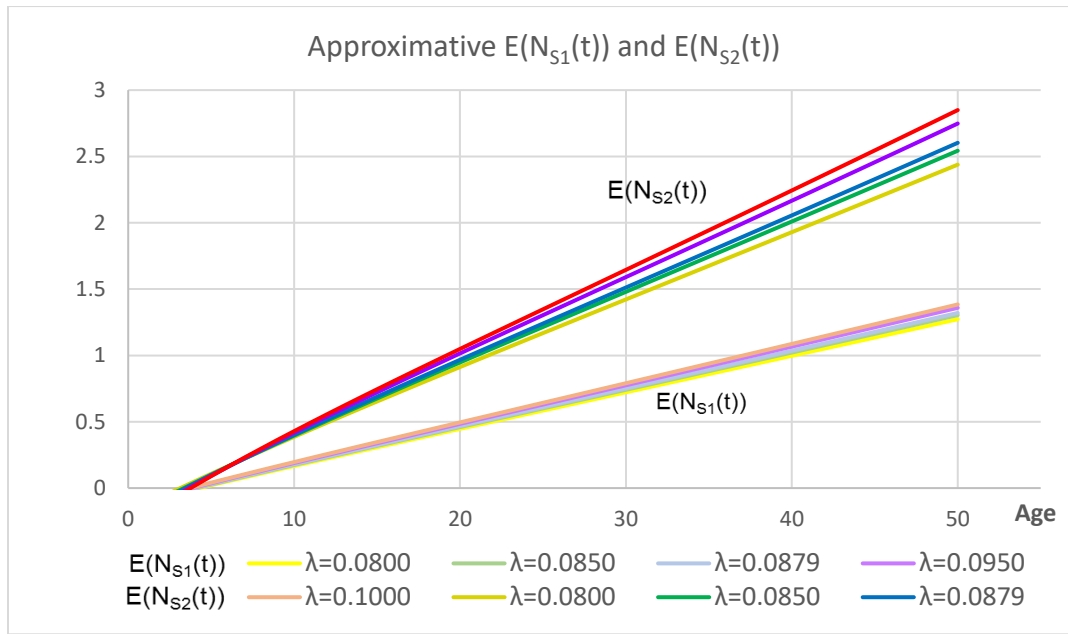


Figure 4.7 Expected number of displacements and cracking shocks that will provoke an interaction, considering Intensity λ

4.6 Conclusion

Failure interaction is a prominent concept, since failure mechanisms are intertwined in complex systems. A reliability index interaction model is proposed in this paper. It characterizes interactions through extreme shocks and random draws. The reasoning behind the presence of interactivity is stochastic and not defined by structural or economic considerations. In contrast to other interaction models, the occurrence in time of diverse shocks is represented by a single Poisson process as a measure of their indistinctive shared randomness. The coefficients representing interactivity are series that capture the sequential aspect of the physical phenomena. The probabilities of interaction are quantifiable, random and variable, as they should be. These probabilities are integrated into the evaluation of reliability functions, mean-time-to-failure and the expected number of shocks. The parameters of the proposed model are estimated using a Markov process. This method captures the sequential aspect in the phenomenon of interaction. The number of possible states, while relatively high, does not affect the computation of the method very much, since there is a straightforward link between the instant probabilities of shock and the generating matrices. A case study on concrete culverts is used to evaluate the probability that cracking will provoke displacement and

vice-versa. The results show that the cracking seems to be significantly accelerated by displacements. In practice, such findings would be critical in designing appropriate maintenance strategies to jointly treat displacements and cracking, whenever there are signs of cracking.

It is important to point out that the proposed model considers a Poisson process to represent the rate at which all shocks will arrive, independent of their nature. Common practice would be to consider shock rates distinctly in function of their origin and search for a correlation factor. Having distinct shock rates leaves a gray area as to whether they are independent or if they already encompass some aspect of the dependence. The common Poisson process for all shocks provides a simpler model, but is also free from all preconceived assumptions of failure interactions. Although in this article a homogeneous Poisson process is chosen, it would make the model broader if inhomogeneous models were explored.

Furthermore, as previously stated, interpretability is a big factor in the construction of useful reliability models. The objective of such tools is to provide engineers an understanding of phenomena that are hard to observe and provide insight on possible solutions. The variability in the evaluation of interaction probabilities captures whether the evolution is progressive, which in general is rather problematic. Maintenance practitioners could have a preventive approach that is comprehensive of this evolution. However, it should be noted that the model would not accurately represent self-healing systems, since the fundamental assumptions do not take into account a response to a shock from the system. In fact, the reaction to a shock could be delayed or a system could have built-in counter measures. Still, one could go a step further by broadening the principles of the random drawings to satisfy this aforementioned situation.

In future work, a condition-based approach will be developed to take into account the degradation of the system. By doing so, its reaction to shocks will be more explicit and provide more interpretability. Moreover, since interactivity indicates the need to address failure modes conjointly, an opportunistic maintenance approach based on similar principles of random drawing and a failure interaction model will be explored.

CHAPITRE 5 ARTICLE 3: OPTIMAL PREDICTIVE MAINTENANCE STRATEGY BASED ON AN INTERACTIVE DEGRADATION MODEL OF DEPENDENT FAILURE MODES

Toualith Jean-Marc MEANGO and Mohamed-Salah OUALI

Submitted on June 2020 to “*Reliability Engineering & System Safety*”. JRESS-D-20-00273.

Abstract—This paper proposes an optimal predictive maintenance with a state-based interaction model that integrates failure interactions as successive random events and combines them with the effect of natural degradation. Interaction mechanisms are represented through specific sequences, increased occurrence and size of shocks. Competitive shock processes, their mutual effects and their collective contribution to a system’s degradation are modelled by a combination of Gamma distributions. To exploit knowledge on this state-based interactivity scheme, a predictive maintenance strategy is proposed. It considers the direct impact of repairs on interaction probabilities and the degradation processes. Stochastic simulation is performed to determine optimal inspection frequency and predictive maintenance thresholds of the interacting failure modes. A case study on concrete culverts that are subject to two dependent failure modes, cracking, and displacement demonstrates the efficiency of the proposed predictive maintenance strategy that integrates failure interaction.

Index Terms— Failure interaction; Interactive degradation; Reliability model; Stochastic dependence; Predictive maintenance.

5.1 Introduction

The viability of a business depends largely on the management of its physical assets. With regards to this, maintenance represents a significant percentage of operational costs. The choice of maintenance equipment, personnel and methods determines the extent of such costs. The latter are unavoidable and need to be optimized and properly planned. This suggests a discussion surrounding scheduled maintenance and condition-based tools, considering the type of equipment to be preserved and the financial capacity of the company (Wang [2002]). Scheduled maintenance or Planned Preventive Maintenance (PPM) relies mainly on failure times and maintenance logs. It is largely used because it encompasses a large array of simple models. Predictive Maintenance (PdM) is a condition-driven preventive maintenance (PM) strategy (Mobley [2002]). It requires more means than just documenting failure times to justify

maintenance interventions. For example, vibration monitoring, thermography, tribology, are some techniques can be used to monitor the system's condition. Thus, this strategy is generally considered to be more precise, since physical indicators of the state of a system are monitored. Moreover, not only are they monitored, but previsions are extrapolated from their observed behavior as suggested by the norm NF EN 13306 X 60-319 (AFNOR [2018]). In terms of interpretability, PdM models have the advantage of providing a direct physical interpretation to the evolution of the system, and therefore, they offer a better understanding of the failure phenomenon and point towards the choice of an appropriate preventive action to take, and specific technicians, and practices, before the failure occurs

Moreover, as modern systems evolve in terms of their complexity, concern is growing regarding the ability of maintenance models to respond to the intricacies of failure mechanisms (Zio [2009]). Systems, as described by Cho & Parlar (1991), can have structural, economical and stochastic dependencies. Structural and economical dependencies can, in many cases, be addressed in a straightforward way, unlike stochastic dependencies, which are relationships with no deterministic physical model. While most models in the literature tend to consider independence among failures as a simplifying hypothesis, the reality is that the dependencies in a system will influence how failures occur and interact. If it is not comprehensively accounted for in a maintenance model, the interaction between failures might accelerate the speed at which a piece of equipment fails and produces hidden costs. In fact, the real costs might be slightly larger than planned costs, due to an underestimation of the reliability indexes of a system.

Murthy & Nguyen (1985a, 1985b) define two concepts of failure interactions: induced failures, when a component causes the instantaneous failure of another with a probability p , and failure rate or shock damage interaction when one component's failure changes the failure rate of another. Models that are comprehensive of failure interaction are mostly based on these concepts. Meango & Ouali (2019) review and classify these models into three categories: copula-based, state-based and reliability indexes' interaction.

Copula-based interaction models use joint functions of degradation processes and/or failure times. Limbourg et al. (2007), Jia et al. (2014), Eryilmaz (2014), Xu et al. (2017) model the interaction between reliability functions and failure rates by using Gaussian and/or Archimedean copulas. Guo et al. (2013), Wang & Pham (2011) link degradation processes and random shocks with constant and time-varying copulas. Some maintenance models are built based on this approach. Mercier & Pham (2012) represent correlated wear indicators by a Lévy process and optimize a preventive maintenance cost function. Li et

al. (2016) propose a condition-based decision rule for maintenance grouping, considering a stochastic dependence modelled by Lévy copulas. Copula-based interaction models use diverse analytical closed forms with a constant parameter representing the extent of the interaction. Yet, with this approach, the effect of maintenance on an interaction is not explicitly addressed. Alongside the mathematical intricacies of a copula, this brings high complexity to the choice of maintenance procedures and interpretation of the physical effects of failure interaction.

Reliability indexes' interaction models study how failure rates from diverse components or failure mechanisms in a system accelerate with each other. They are based on maintenance logs and failure times. Most of the associated maintenance models analyze the cost of a replacement, and how it is affected by failure interaction (Jhang & Sheu [2000], Satow & Osaki [2003], Lai [2007], Lai [2009], Lai & Chen [2006, 2008], Sung et al. [2013], Lai & Yan [2016]). Diverse concepts of preventive maintenance are addressed through these models. Wang & Zhang (2009) consider a geometric process repair and renewal reward theorem. Zequeira & Bérenguer (2006), Castro (2009) propose periodic imperfect preventive maintenance models that minimize the cost rate for an infinite time span. Golmakani & Moakedi (2012) determine an optimal periodic inspection interval on a finite time horizon. While diverse, these approaches do not correspond with maintenance measures that directly mitigate failure interaction. Fan et al. (2011) and Zhang et al. (2014, 2015) study better alternatives that cooperatively allocate resources like opportunistic maintenance. However, since they are built for scheduled maintenance, these models inherit the same limitations in terms of the interpretability of physical phenomena. Furthermore, in most reliability indexes' interaction models, failure interaction between two entities is represented by constant coefficients, which brings rigidity to the models.

State-based interaction models link state variables and provide an interpretation to physical degradation, external and internal shocks. They are used in the development of condition-based strategies. Peng et al. (2010), Chen et al. (2011), Huynh et al. (2011, 2012), Song et al. (2012, 2014) work on dependent degradation and shock processes and adopt periodic inspection, preventive maintenance and replacement to minimize the long-run cost rate. Liang & Parlikad (2015) focus on optimizing asset availability and operational costs by integrating them to their work fault propagation. Yousefi et al. (2019) study optimal on-condition thresholds and inspection intervals for systems where corrective maintenance must be avoided. The choice of representation for the degradation process is a central matter in condition-based models. Jiang et al. (2012) present shifting failure thresholds as a consequence of interaction and a basis for preventive maintenance policies. They propose a model that represents shock sizes as normal random

variables and natural degradation as following a linear path. Do et al. (2015) use gamma distributions instead in their condition-based policy since they represent physical degradation well. For most PdM strategies in the literature, the cost functions are similar. They are based on the renewal theory (Zhou et al., 2007; Nguyen et al., 2014, 2015, 2017) and have slight differences depending on whether they consider perfect and/or imperfect maintenance and inspections with fixed or updated periods. Stochastic simulation is used to optimize the proposed approaches. Although all of these models carry the economic toll of condition monitoring, they have the advantage of providing more precise and comprehensive results. Yet, they share a similar rigidity as reliability indexes' interaction models with regards to coefficients and parameters.

Meango & Ouali (2020) propose a flexible approach. In a reliability indexes' interaction model based on extreme shocks and Markov processes, interaction between two failure modes is represented as a series of probabilities. In this paper, this model is extended to integrate degradation processes and cumulative shocks regarding two distinct failure modes. This new state-based interaction model is then used as the basis of a PdM strategy. Simulation is used to find the optimal inspection period and repair thresholds of the interactive failure modes in order to minimize the proposed cost function.

The remainder of the paper is organized as follows. Section 5.2 introduces the interactive degradation model that combines the interaction processes of failure modes. Section 5.3 develops the optimal predictive maintenance model that minimizes the total average cost over time. Section 5.4 presents a real case concerning the maintenance of concrete culverts. Section 5.5 discusses and concludes the proposed approach and the obtained results.

5.2 Interactive degradation model

The proposed model considers a system subject to two dependent failure modes. These failure modes are themselves caused by distinct natural degradation mechanisms and random shocks. On one hand, we assume that the natural degradation mechanisms, denoted by (a) (i.e. 1 and 2), are independent from each other. Meaning that, the degradation mechanism that could lead to failure mode 1 does not affect the degradation mechanism that could lead to failure mode 2, and vice versa. A time-dependent Gamma distribution is adopted to represent each natural degradation mechanism as follows: $X_0^{(a)}(t) \sim \text{Gamma}(\alpha_0^{(a)}t, \beta_0^{(a)})$ where $\alpha_0^{(a)}t$ and $\beta_0^{(a)}$ denote the shape and inverse scale parameters of the Gamma distribution, respectively. The choice of gamma distributions is motivated by their flexibility and

suitability in the representation of physical deterioration as emphasized by Do et al. (2015). The advantage of the assumption of independence between natural degradation paths is that the direct effects of interaction are observable through the occurrence and sequence of the competitive shocks.

On the other hand, shocks occur following a homogeneous Poisson process of intensity λ . The model distinguishes shocks in terms of their effect on the degradation level of the system as related to failure modes. A shock is denoted by a (1 or 2) if it has physical effects that could lead to the failure mode a (1 or 2). Each shock could either have no effect or cause instant jumps in the overall degradation. The system fails when the cumulative degradation exceeds a given threshold. The interaction partially resides in the fact that a shock 1 increases the probability that the next shock will be a shock 2, and vice versa. Equations (5.1) formulate the basis of the interaction model at the k -th shock ($k > 1$).

$$\begin{cases} p_k^{(1)} = p_{k-1}^{(1)} + \theta_{k-1} p_{k-1}^{(2)} \\ p_k^{(2)} = p_{k-1}^{(2)} + \gamma_{k-1} p_{k-1}^{(1)} \\ p_k^{(0)} = p_{k-1}^{(0)} - \gamma_{k-1} p_{k-1}^{(1)} - \theta_{k-1} p_{k-1}^{(2)} \end{cases} \quad (5.1)$$

where $p_k^{(1)}$, $p_k^{(2)}$, $p_k^{(0)}$ denote the probabilities that the k -th shock is a shock 1, a shock 2 and a shock that has no effect on the system's degradation, respectively. θ_{k-1} and γ_{k-1} denote the interaction probabilities that represent the influence of a previous shock 2 on the occurrence of shock 1, and the influence of a previous shock 1 on the occurrence of shock 2, respectively.

The number of distinct types of shocks is important in the construction of the interactive degradation model. $N(t)$ denotes the overall number of shocks at time t , while $N^{(a)}(t)$ is the number of shocks $a = 1, 2$ at time t . Considering that all shocks follow a homogeneous Poisson process of intensity λ , $P\{N(t) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$, Equations (5.2) express the probability of having l shocks $a = 1, 2$ at t .

$$\begin{cases} P\{N^{(a)}(t) = 1\} = \sum_{k=1}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \sum_{v=1}^k \frac{p_v^{(a)}}{1-p_v^{(a)}} \prod_{h=1}^k (1-p_h^{(a)}) \\ P\{N^{(a)}(t) = l\} = \sum_{k=l}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{v_r}^{(a)}}{1-p_{v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1-p_h^{(a)}) \right), l > 1 \end{cases} \quad (5.2)$$

Instant shock probabilities $p_k^{(a)}$ are estimated using Markov processes, which lead to the estimation of interaction probabilities θ_k , and γ_k . In order to reduce the computational toll of estimating $P\{N^{(a)}(t) = l\}$, Equations (5.2) can be approximated. Considering the maximum number, denoted by m , of shocks ever

recorded on the type of system studied throughout its lifespan, a finite sum can be estimated (see Meango & Ouali (2020) for further explanation). After the estimation of parameters $\hat{p}_k^{(a)}$, the following applies:

$$\begin{cases} P\{N^{(a)}(t) = 1\} \approx \sum_{k=1}^m \frac{(\lambda t)^k e^{-\lambda t}}{k!} \sum_{v=1}^k \frac{\hat{p}_v^{(a)}}{1 - \hat{p}_v^{(a)}} \prod_{h=1}^k (1 - \hat{p}_h^{(a)}) \\ P\{N^{(a)}(t) = l\} \approx \sum_{k=l}^m \frac{(\lambda t)^k e^{-\lambda t}}{k!} \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{\hat{p}_{v_r}^{(a)}}{1 - \hat{p}_{v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1 - \hat{p}_h^{(a)}) \right), 1 < l \leq m \\ P\{N^{(a)}(t) = l\} \approx 0, l > m \end{cases} \quad (5.3)$$

Now, assume that $T_j^{(a)}$ is the arrival time of a single j -th shock and $W_j^{(a)}$ is the degrading effect of the j -th shock a on the system. Thus, the cumulative effect of $N^{(a)}$ shocks a denoted by $S^{(a)}(t)$ is given simply with the following equations:

$$S^{(a)}(t) = \begin{cases} \sum_{j=1}^{N^{(a)}(t)} W_j^{(a)} & \text{if } N^{(a)}(t) > 0 \\ 0 & \text{if } N^{(a)}(t) = 0 \end{cases} \quad (5.4)$$

To express the dependence of the damage sizes between a given shock and a previous one, we introduce $\bar{a} = 1$ when $a = 2$, $\bar{a} = 2$ when $a = 1$, and the counting variable $M^{(\bar{a})}(j)$ that is the number of shocks $\bar{a}$ prior to the j -th shock a . $W_j^{(a)}$ is the sum of the shock size denoted by $Z_j^{(a)}$ intrinsic to the failure mode a , and an additional shock size resulting from prior shocks $\bar{a}$ and denoted by $\varepsilon_j^{(a)}$. This interactive effect is assumed to impact only the next shock, rather than create a domino effect. The cumulative effect of $N^{(a)}$ shocks a can be rewritten as:

$$S^{(a)}(t) = \begin{cases} \sum_{j=1}^{N^{(a)}(t)} Z_j^{(a)} + \sum_{i=1}^{M^{(\bar{a})}\{N^{(a)}(t)\}} \varepsilon_i^{(a)} & \text{if } N^{(a)}(t) > 0 \\ 0 & \text{if } N^{(a)}(t) = 0 \end{cases} \quad (5.5)$$

Overall, the degradation process as related to failure mode a , denoted by $X_S^{(a)}(t)$, is expressed in the following manner and presented in figure 5.1:

$$X_S^{(a)}(t) = X_0^{(a)}(t) + S^{(a)}(t)$$

$$X_S^{(a)}(t) = \begin{cases} X_0^{(a)}(t) + \sum_{j=1}^{N^{(a)}(t)} Z_j^{(a)} + \sum_{i=1}^{M^{(\bar{a})}\{N^{(a)}(t)\}} \varepsilon_i^{(a)} & \text{if } N^{(a)}(t) > 0 \\ X_0^{(a)}(t) & \text{if } N^{(a)}(t) = 0 \end{cases} \quad (5.6)$$

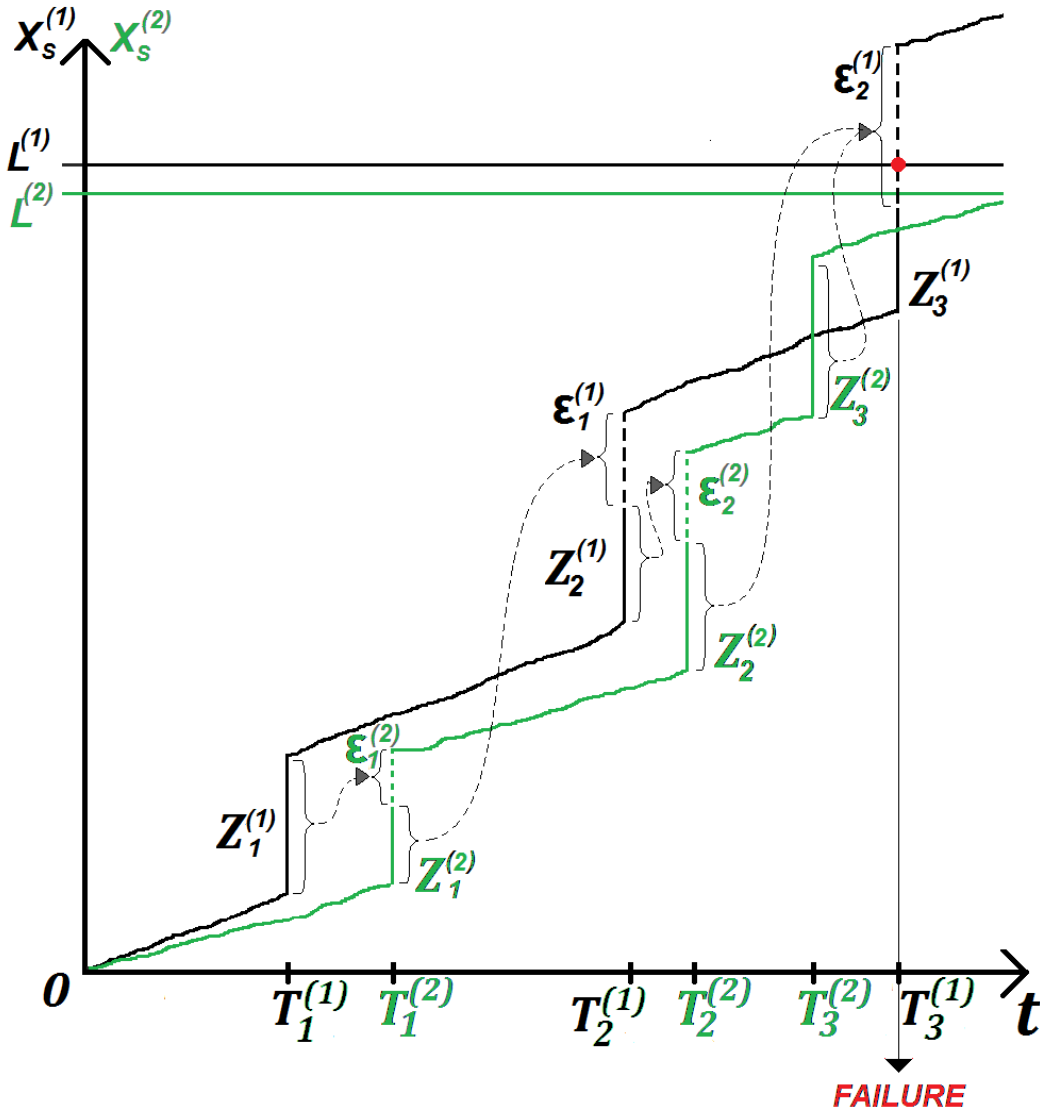


Figure 5.1 System subject to two competitive and interactive degradation processes

The system will fail if $X_s^{(a)}(t)$ passes a certain degradation level threshold $L^{(a)}$. The probability of that occurrence ought to be studied to evaluate the reliability function of the system. The cumulative distribution function (cdf) of $X_s^{(a)}(t)$ can be derived as:

$$\begin{aligned}
P(X_S^{(a)}(t) < x) &= P(X_S^{(a)}(t) < x | N^{(a)}(t) = 0) \cdot P(N^{(a)}(t) = 0) \\
&+ \sum_{l=1}^{\infty} \left\{ \sum_{v=0}^{\infty} P(X_S^{(a)}(t) < x | N^{(a)}(t) = l, M^{(\bar{a})}(l) = v) \cdot P(M^{(\bar{a})}(l) = v) \right\} \\
&\cdot P(N^{(a)}(t) = l)
\end{aligned} \tag{5.7}$$

Considering that k draws occurred, the probability $P(M^{(\bar{a})}(l) = v)$ depends on specific configurations where the last shock is of type a :

$$P(M^{(\bar{a})}(l) = v) = \sum_{k=\max(v+1, l)}^{\infty} P(n_{k-1}^{(\bar{a})} = v, n_{k-1}^{(a)} = l-1) \cdot p_k^{(a)}, \tag{5.8}$$

From Equation (5.2) it can be deduced for positive integers c and d that:

$$\begin{cases} P(n_{c+d}^{(\bar{a})} = c, n_{c+d}^{(a)} = d) = \sum_{s_0=d}^{c+d} \sum_{s_1=d-1}^{s_0-1} \dots \sum_{s_{d-1}=1}^{s_{d-2}-1} \left(\prod_{r=0}^{d-1} \frac{p_{s_r}^{(a)}}{p_{s_r}^{(\bar{a})}} \right) \left(\prod_{h=1}^{c+d} p_h^{(\bar{a})} \right) \\ P(n_c^{(\bar{a})} = c, n_c^{(a)} = 0) = \prod_{s=1}^c p_s^{(\bar{a})} \\ P(n_0^{(\bar{a})} = 0, n_0^{(a)} = 0) = 1 \end{cases} \tag{5.9}$$

Knowing k , the above equation can help determine $P(n_{k-1}^{(\bar{a})} = v, n_{k-1}^{(a)} = l-1)$ by recurrence, since, where it applies:

$$\begin{aligned}
P(n_s^{(\bar{a})} = c, n_s^{(a)} = d) &= P(n_{s-1}^{(\bar{a})} = c, n_{s-1}^{(a)} = d) \cdot p_s^{(0)} + P(n_{s-1}^{(\bar{a})} = c-1, n_{s-1}^{(a)} = d) \cdot p_s^{(\bar{a})} \\
&+ P(n_{s-1}^{(\bar{a})} = c, n_{s-1}^{(a)} = d-1) \cdot p_s^{(a)}
\end{aligned} \tag{5.10}$$

Furthermore, it is assumed that $Z_j^{(a)} \sim \text{Gamma}(\alpha_Z^{(a)}, \beta_Z^{(a)})$ and $\varepsilon_i^{(a)} \sim \text{Gamma}(\alpha_\varepsilon^{(a)}, \beta_\varepsilon^{(a)})$. $Z_j^{(a)}$ are amplitudes solely due to shocks a and $\varepsilon_i^{(a)}$ are additional amplitudes of shocks a due to previous shocks $\bar{a}$. In practice, to determine $(\alpha_Z^{(a)}, \beta_Z^{(a)})$, a gamma distribution is fitted to a sample of observed single shocks a with no prior shocks $\bar{a}$. To determine $(\alpha_\varepsilon^{(a)}, \beta_\varepsilon^{(a)})$, a gamma distribution is fitted to a sample of

observed single shocks a preceded by single shocks $\bar{a}$. $\alpha_Z^{(a)}$ is subtracted from the shape parameter of that distribution to obtain $\alpha_\varepsilon^{(a)}$ with the assumption that $\beta_Z^{(a)} = \beta_\varepsilon^{(a)}$. This suggests that additional shocks $\varepsilon_i^{(a)}$ have the potential to be as damageable as shocks $Z_j^{(a)}$ to the system. The choice of gamma distributions is not only suitable for physical degradation, but also offers alternatives in the combination of the processes. As a direct consequence, $\sum_{j=1}^l Z_j^{(a)} + \sum_{i=1}^v \varepsilon_i^{(a)} \sim \text{Gamma}(l\alpha_Z^{(a)} + v\alpha_\varepsilon^{(a)}, \beta_Z^{(a)})$.

$P(X_S^{(a)}(t) < x | N^{(a)}(t) = l, M^{(\bar{a})}(l) = v)$ can be estimated with approaches from Mathai (1982) or Moschopoulos (1985). It can more simply be approximated with the Moment-Matching-Method:

$$\left\{ \begin{array}{l} P(X_S^{(a)}(t) < x | N^{(a)}(t) = l, M^{(\bar{a})}(l) = v) \approx \frac{\gamma(\alpha_{l,v}^{(a)}, \beta_{l,v}^{(a)} x)}{\Gamma(\beta_{l,v}^{(a)})} \\ \alpha_{l,v}^{(a)} = \frac{\{\alpha_0^{(a)} \beta_0^{(a)} t + [l\alpha_Z^{(a)} + v\alpha_\varepsilon^{(a)}] \beta_Z^{(a)}\}^2}{\alpha_0^{(a)} (\beta_0^{(a)})^2 t + [l\alpha_Z^{(a)} + v\alpha_\varepsilon^{(a)}] (\beta_Z^{(a)})^2} \\ \beta_{l,v}^{(a)} = \frac{\alpha_0^{(a)} (\beta_0^{(a)})^2 t + [l\alpha_Z^{(a)} + v\alpha_\varepsilon^{(a)}] (\beta_Z^{(a)})^2}{\alpha_0^{(a)} \beta_0^{(a)} t + [l\alpha_Z^{(a)} + v\alpha_\varepsilon^{(a)}] \beta_Z^{(a)}} \end{array} \right. \quad (5.11)$$

The proposed combination of Gamma processes narrows the definition of the interaction mechanisms to the occurrence, sequence and size of shocks. Indeed, the probability that the system will be affected by the next shock increases as a result of precedent shocks. Moreover, the size of the next shock will depend on precedent shocks.

The reliability function is evaluated by assuming that both failure modes have the same importance. Either one can independently stop the system. Though, their interdependencies are encapsulated in the distributions of the two competitive shock processes. Considering degradation thresholds $L^{(a)}$, the system's reliability is:

$$R(t) = \prod_{a=1}^2 P(X_S^{(a)}(t) < L^{(a)}) \quad (5.12)$$

In the next section, a predictive maintenance strategy is developed based on the proposed interactive degradation model.

5.3 Predictive maintenance strategy using interactive degradation model

We consider a repairable system subject to two failure modes 1 and 2. The system is inspected at the constant period Q . At each inspection moment, a repair action is performed on each failure mode (a) when

the threshold $L_r^{(a)}$ is reached. This repair action is considered as preventive maintenance (PM) since it occurs before failure. It restores the degradation level to its initial value. The system undergoes an overhaul maintenance (i.e. corrective maintenance) if the cumulative degradation of the more expeditious failure mode a exceeds its threshold $L^{(a)}$. We assume that the overhaul maintenance makes the whole system in “as good as new” condition. The repair actions eliminate the potential effect of prior shocks on the next shock. If the system fails in between inspections, it stops functioning and no action is performed until the next inspection. The time to repair is not considered in this strategy, but rather, repairs are assumed to be instantaneous and their impact is represented within the costs.

Considering the inspection cost C_i , the repair costs $C_r^{(1)}$ and $C_r^{(2)}$, the overhaul cost C_{oh} and the downtime cost per unit of time C_{dt} , the cumulative maintenance cost of the system at time t is given by:

$$C^t(L_r^{(1)}, L_r^{(2)}, Q) = C_i N_i(t) + C_r^{(1)} N_r^{(1)}(t) + C_r^{(2)} N_r^{(2)}(t) + C_{oh} + C_{dt} \rho(t) \quad (5.13)$$

where $N_i(t)$, $N_r^{(1)}(t)$ and $N_r^{(2)}(t)$ are respectively the number of inspections and repairs (PM) related to failure mode 1 and 2. $\rho(t)$ is the downtime due to an undetected failure.

Using the renewal theory as proposed by Do et al. (2015), the long run expected cost per unit of time is:

$$C(L_r^{(1)}, L_r^{(2)}, Q) = \frac{E[C^H(L_r^{(1)}, L_r^{(2)}, Q)]}{E[H]} \quad (5.14)$$

where H represents the life cycle of the system and $E[.]$ is the expectation.

Through stochastic simulation, the cost criterion and subsequently the minimum cost rate are evaluated to obtain the optimal values of the decision parameters:

$$C(L_r^{(1)*}, L_r^{(2)*}, Q^*) = \min_{L_r^{(1)}, L_r^{(2)}, Q} \left\{ C(L_r^{(1)}, L_r^{(2)}, Q), 0 \leq L_r^{(1)} < L^{(1)}, 0 \leq L_r^{(2)} < L^{(2)}, 0 < Q < \infty \right\} \quad (5.15)$$

A large number of iterations is recommended for such simulation schemes. It is also necessary to understand all mechanisms intervening in the maintenance decision process. Figure 5.2 describes this process, which takes inspiration from Do et al. (2015). It can be outlined in the following manner.

The state of the system is updated after each maintenance action. $X_{s,j}^{(a)}(t)$ is defined as the state after j inspections. The total number of inspections at t is then:

$$N_i(t) = \lfloor t/Q \rfloor \quad (5.16)$$

We define variable $\tau(j) = t - jQ$ and probabilities $p_{j,k}^{(a)}$ ($k = 1, 2, \dots$) as instant probabilities of shocks a between the j -th and $(j + 1)$ -th inspection ($j \geq 0$). The probability of having a certain number of shocks a in the interval $]jQ, (j + 1)Q]$ is:

$$P\{N_j^{(a)}(\tau) = l\} = \sum_{k=l}^{\infty} \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!} \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{j,v_r}^{(a)}}{1-p_{j,v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1-p_{j,h}^{(a)}) \right) \quad (5.17)$$

In the first period $]0, Q]$, $p_{0,k}^{(a)}$ ($k = 1, 2, \dots$) can be determined through Markov processes. In subsequent intervals $]jQ, (j + 1)Q]$ where $j > 0$, previous states become important and $p_{j,k}^{(a)}$ ($k = 1, 2, \dots$) are updated at jQ in the following manner:

$$p_{j,k}^{(a)} = \begin{cases} p_{j-1,k+l_{j-1}}^{(a)} & \text{if } X_{S,j-1}^{(a)}(Q) < L_r^{(a)} \quad (PM) \\ p_{0,k}^{(a)} & \text{if } X_{S,j-1}^{(a)}(Q) \geq L_r^{(a)} \quad (PM \text{ occurred}) \end{cases}, \quad j > 0 \quad (5.18)$$

where $l_{j-1} = N_{j-1}(Q)$. This updating method alters the state and the probability of failure in the interval $]jQ, (j + 1)Q]$. The degradation level is updated as well upon maintenance. In the first period $]0, Q]$, the degradation level is similar as in section 5.2. In subsequent intervals $]jQ, (j + 1)Q]$ where $j > 0$, previous states determine the maintenance actions:

$$X_{S,j}^{(a)}(\tau) = \begin{cases} X_{S,j-1}^{(a)}(Q) + X_0^{(a)}(\tau) + \sum_{k=1}^{N_j^{(a)}(\tau)} Z_k^{(a)} + \sum_{v=1}^{M_j^{(\bar{a})}\{N_j^{(a)}(\tau)\}} \varepsilon_v^{(a)} & \text{if } X_{S,j-1}^{(a)}(Q) < L_r^{(a)} \quad (PM) \\ X_0^{(a)}(\tau) + \sum_{k=1}^{N_j^{(a)}(\tau)} Z_k^{(a)} + \sum_{v=1}^{M_j^{(\bar{a})}\{N_j^{(a)}(\tau)\}} \varepsilon_v^{(a)} & \text{if } X_{S,j-1}^{(a)}(Q) \geq L_r^{(a)} \quad (PM \text{ occurred}) \end{cases} \quad (5.19)$$

where $M_j^{(\bar{a})}$ retraces all possible configurations of the process in this interval.

5.4 Illustrative example

The illustrative example deals with concrete culverts that are used in road systems to allow water or even wildlife to circulate under a road. The failure of such structures can sometimes lead to the collapse of roads and can leave communities stranded until an overhaul happens. Since this is an event to avoid at all costs, the failure of a culvert is defined at a certain level of physical deterioration and/or congestion of the structure. Diverse failure modes are anticipated, and several state variables are recorded throughout the lifespan of culverts. Generally, the condition of culverts regarding each variable is inspected regularly using a five-level scale. When reached, the two lowest levels are critical and considered as failures of the systems. They prompt corrective maintenance actions.

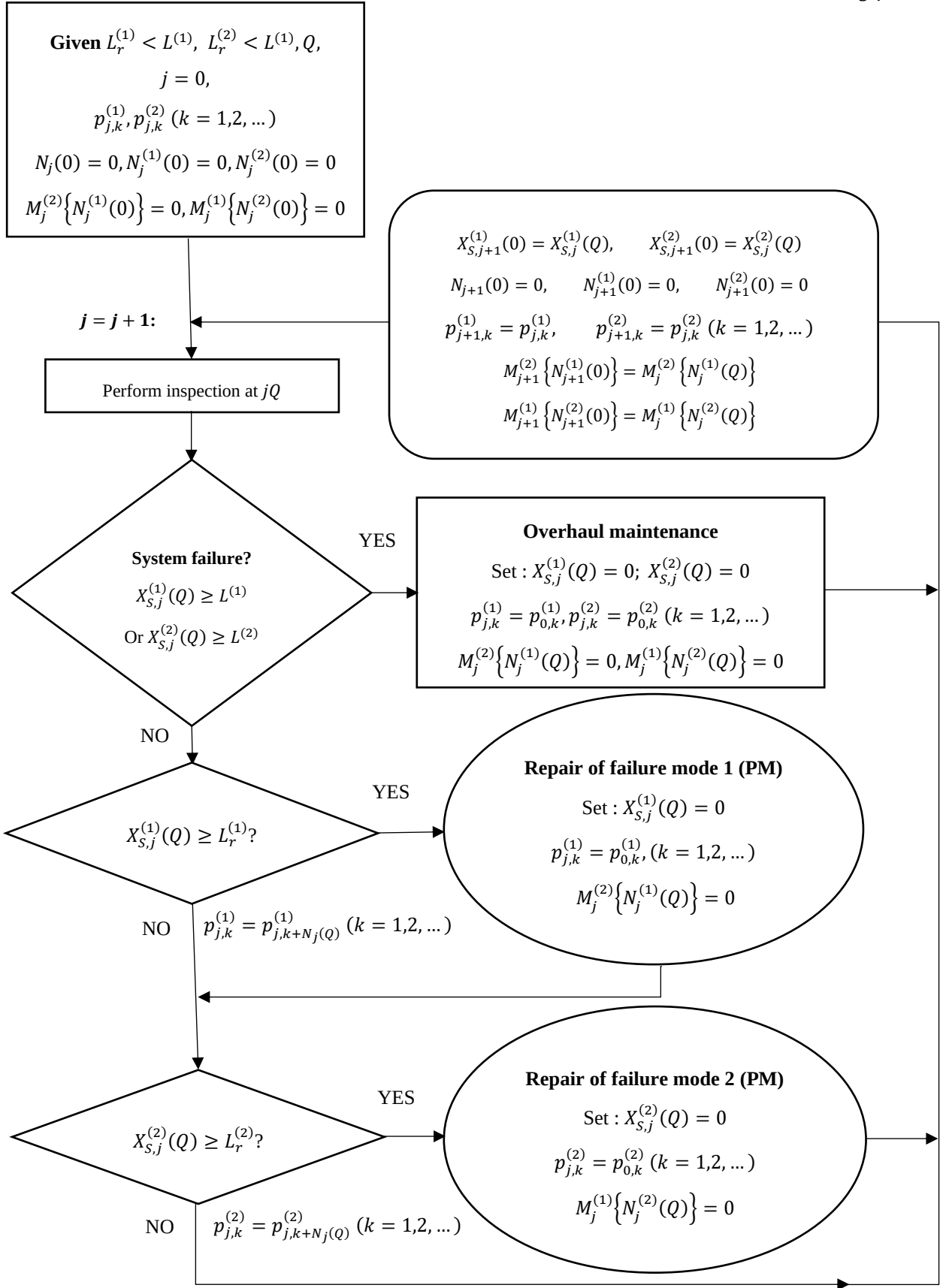


Figure 5.2 Flowchart of the proposed PdM strategy

According to experts, there are two crucial degradation mechanisms to monitor that have a mutual effect on each other: cracking and displacement. Cracking increases the risk of infiltration, causes deformation and even slumping. The inverse effect, displacement threatening the integrity of the structure, is also plausible. Thus, two failure modes are studied in this case: severe displacement and severe cracking, denoted by S1 and S2, respectively.

The dataset used is comprised of the age of the culverts and the degradation levels at the time of inspection. Degradation levels and intensities of shocks are evaluated on the a five-level scale. The equivalent physical measures of displacement and cracking and their damageability are evaluated by trained inspectors. In the article by Meango & Ouali (2020), all changes in degradation levels are considered to be the result of shocks. Here, a differentiation is made between sudden shocks and continuous deterioration. Since sequences are key to this study, it should be noted that the data is not raw. Indeed, issues regarding missing inspection data have been addressed prior to the implementation of the models. Data have been duly cleaned and transformed to observe the proper indicators of sequences. Expert opinions, quality control methods like Shewhart charts (Deng et al. [2012]) and even data mining methods such as pattern recognition helped to that extent. Table 5.1 provides the different parameters of the model for S1 and S2. Table 5.2 provides the means and variances of $Z_j^{(S1)}$, $Z_j^{(S2)}$, $\varepsilon_i^{(S1)}$ and $\varepsilon_i^{(S2)}$.

Table 5.1 Degradation parameters of concrete culverts for both displacement (S1) and cracking (S2) failure modes

Parameters	$a = S1$	$a = S2$
$\alpha_0^{(a)}$	0.0043	0.0160
$\beta_0^{(a)}$	0.8422	1.1790
$\alpha_Z^{(a)}$	11.3475	3.7742
$\beta_Z^{(a)}$	9.8023	2.2834
$\alpha_\varepsilon^{(a)}$	1.0966	0.0638
$\beta_\varepsilon^{(a)}$	9.8023	2.2834
$L^{(a)}$	3	3

Table 5.2 Means and variances of the Gamma distributions of shocks S1 and S2

Shocks	$Z_j^{(S1)}$	$Z_j^{(S2)}$	$\epsilon_i^{(S1)}$	$\epsilon_i^{(S2)}$
Mean	1.1576	1.6529	0.1119	0.0279
Variance	0.1181	0.7239	0.0114	0.0122

In terms of interactive impact through damage sizes, the mean of $\epsilon_i^{(S1)}$ as a result of prior S2 shocks represents a nearly 10% addition to S1 shocks. The mean of $\epsilon_i^{(S2)}$ as a result of prior S1 shocks represents an approximately 2% addition to S2 shocks. Failure interaction increases the sizes of shocks and those cumulated shocks accelerate the failure of the system.

With a rate λ of 0.0995 shocks/year, the interaction probabilities are estimated in both tables 5.3 and 5.4. These results emphasize that cracking represents a bigger risk than displacement in producing damaging shocks to the culverts.

Table 5.3 Probabilities $\hat{p}_k^{(S1)}$ and effects of S2 on S1

Probabilities $\hat{p}_k^{(S1)}$	$\hat{\theta}_k$ (effect of S2 on S1)
$k = 1$ 0.2728	$k = 1$ 0.1164
$k = 2$ 0.3154	$k = 2$ 0.0619
$k = 3$ 0.3404	$k = 3$ 0.0400
$k = 4$ 0.3581	$k = 4$ 0.0282
$k = 5$ 0.3718	

Table 5.4 Probabilities $\hat{p}_k^{(S2)}$ and effects of S1 on S2

Probabilities $\hat{p}_k^{(S2)}$	$\hat{\gamma}_k$ (effect of S1 on S2)
$k = 1$ 0.3663	$k = 1$ 0.1337
$k = 2$ 0.4027	$k = 2$ 0.1271
$k = 3$ 0.4428	$k = 3$ 0.1295
$k = 4$ 0.4869	$k = 4$ 0.1354
$k = 5$ 0.5354	

Figure 5.3 shows how the reliability is affected by interactive failures and the contribution of each failure mode to the system's failure function. The probability of failures due to S1 and S2 are undervalued when interaction is not accounted for. The overall effect is that the system is deemed more reliable than it really is if failure interaction is not integrated in the model.

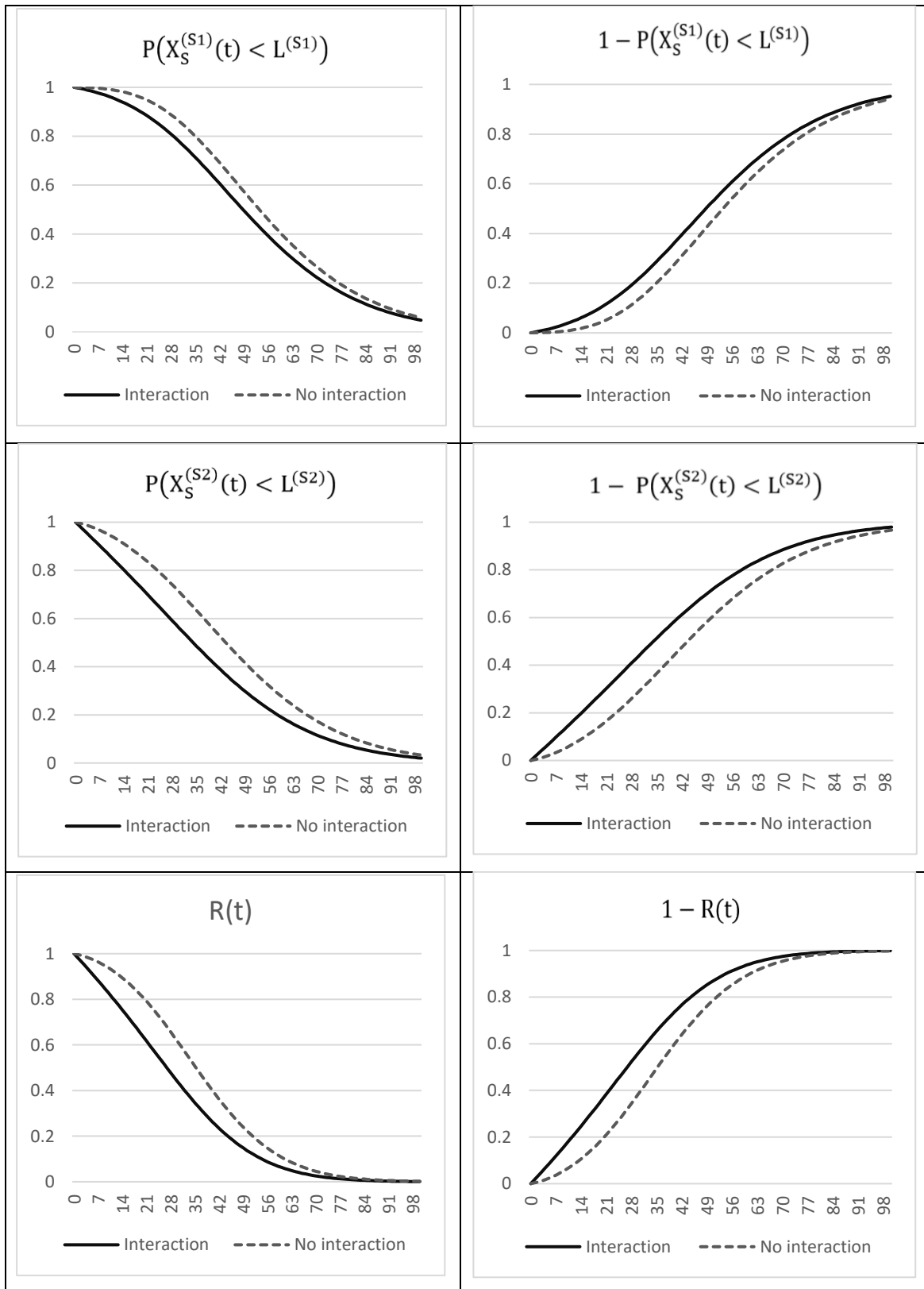


Figure 5.3 Reliability and failure functions of S1, S2 and both failure modes

S2 has a slightly bigger impact and brings down the reliability curve of the whole system slightly faster than S1. Instant probabilities $\hat{p}_k^{(S2)}$ produce this effect.

Overall, instant probabilities $\hat{p}_k^{(S1)}$, $\hat{p}_k^{(S2)}$ and additional shocks $\varepsilon_i^{(S1)}$, $\varepsilon_i^{(S2)}$ that are a direct consequence of the interaction phenomenon decrease the reliability of the system. Maintenance resources might be misallocated or undervalued if such variables were overlooked. Once the reliability parameters are known, a maintenance policy can be tailored to the behavior of the system, considering all known costs. The unplanned and planned repair costs are, in general, aggregate values of diverse maintenance actions on culverts such as cleaning, filling, realignment, insertion, etc.

Further information on practices regarding culverts can be found in research by Watanabe & Koga (2012), Regier (2015) and Meegoda & Zou (2015). In this paper, we use annual cost ratios to the overhaul corrective maintenance cost as outlined in Table 5.5. Therefore, the cost function provides a ratio to C_{oh} . Doing so is equivalent to studying the real costs and provides a sense of the proportions of the maintenance expenses compared to one another and to the overall corrective maintenance cost.

Table 5.5 Cost ratios to the corrective maintenance (overhaul) cost per unit of length

Cost ratios	$\pi_i = \frac{C_i}{C_{oh}}$	$\pi_r^{(1)} = \frac{C_r^{(1)}}{C_{oh}}$	$\pi_r^{(2)} = \frac{C_r^{(2)}}{C_{oh}}$	$\pi_{dt} = \frac{C_{dt}}{C_{oh}}$
Annual cost ratio per unit of length (%)	1.25	7.14	7.50	12.50

Two cases are systematically considered and compared: interaction and no interaction. As outlined in the previous section, a simulation has to be performed. Many iterations are required. One way to decide on a maximum value for the inspection period Q and thus reduce the number of computations is to search for the minimum of an age-based preventive maintenance policy. An evident consequence of the differences in reliability functions, as shown in Figure 5.3, would be that the age-based policy integrating interaction would provide an optimum at an earlier age than the alternative without an interaction.

The proposed repair strategy is applied, and several values of the decisional parameters are tested. Results from one simulation compared to another might slightly differ depending on the number of iterations and the chosen input values. Indeed, the candidate values of the parameters $L_r^{(S1)}, L_r^{(S2)}, Q$ are limited in number and condition the choice of the optimum. The principle is to approximate ideal values with sufficient runs and candidate values while keeping a balanced understanding of the computational toll. Since only 5 degradation levels are evaluated by inspectors, we define intermediary levels to have more options to choose from and to compare alternatives with and without an interaction. Table 5.6 shows samples from different simulations. The minimum prices per inspection period Q are in bold.

The first observation from the simulation is that, generally, long inspection periods will increase the total average cost per year. This is due to the overhaul maintenance cost, which could also be understood as a set-up, installation or replacement cost. On one hand, if inspections are not performed often enough, that cost occurs early, as the life cycle of the system is shortened. On the other hand, if inspection periods are too short, the overall cost increases as well, meaning useless inspections are performed. Another observation is that the highest costs occur when the preventive maintenance thresholds are the highest.

Table 5.7 shows the optimum values from the simulations, with a policy that considers interaction and with a policy that does not consider interaction. A policy without interaction basically considers that interaction probabilities θ_k , and γ_k are null. The probability of the occurrence of shocks does not grow as a result of previous shocks. Also, such a policy does not represent additional shock sizes $\varepsilon_i^{(S1)}$ and $\varepsilon_i^{(S2)}$, which in related simulations are fixed to value 0. A low threshold means that action should be taken upon early signs of degradation. As previously established, S2, which has a bigger impact on reliability, requires the most caution and indeed has a low threshold. Yet, acting too early may increase the overall cost.

As a general observation, involving failure interaction in the policy affects the overall cost, since without considering this, unplanned failures are more likely to happen. The 0.11% difference between both costs seems insignificant at first, but in the region studied, concrete culverts are in the order of thousands, are used over several decades, and are used on kilometers of roads.

Table 5.6 Yearly cost-ratio per unit of length of the PdM strategy for diverse values of $L_r^{(S1)}, L_r^{(S2)}$ and Q

Candidate thresholds		$\pi(L_r^{(S1)}, L_r^{(S2)}, Q) = \frac{c(L_r^{(S1)}, L_r^{(S2)}, Q)}{c_{oh}} (\%)$				
		Candidate inspection periods Q				
$L_r^{(S1)}$	$L_r^{(S2)}$	1	2	3	4	5
1	1	5.75	5.18	5.14	5.40	5.56
	1.5	5.51	5.15	5.27	5.40	5.59
	2	5.62	5.30	5.55	5.72	5.85
	2.5	5.85	5.65	5.81	6.08	6.33
1.5	1	5.63	5.00	5.10	5.32	5.51
	1.5	5.52	5.11	5.15	5.32	5.55
	2	5.61	5.23	5.28	5.63	5.80
	2.5	5.75	5.52	5.71	6.05	6.22
2	1	5.59	5.07	5.14	5.32	5.53
	1.5	5.42	5.08	5.33	5.41	5.84
	2	5.56	5.27	5.59	5.82	5.89
	2.5	5.78	5.61	5.81	6.20	6.37
2.5	1	5.70	5.25	5.35	5.48	5.68
	1.5	5.59	5.35	5.38	5.60	5.83
	2	5.59	5.29	5.62	5.78	5.95
	2.5	5.90	5.75	5.74	6.20	6.53

Table 5.7 Optimum values of the PdM strategy

Simulations	Optimal values for 1000 runs (with a 99% confidence interval)			
	Q^*	$L_r^{(S1)*}$	$L_r^{(S2)*}$	$\pi(L_r^{(S1)*}, L_r^{(S2)*}, Q^*) = \frac{c(L_r^{(S1)*}, L_r^{(S2)*}, Q^*)}{c_{oh}} (\%)$
With interaction	2	1.5	1.0	$5.00 \pm (7.76 \times 10^{-3})$
Without interaction	2	1.5	1.5	$5.11 \pm (6.41 \times 10^{-3})$

Moreover, the corrective maintenance or replacement cost of a single concrete culvert could easily be in the order of thousands of dollars. These additional costs could be avoided if S2 or the cracking of concrete culverts is addressed in very early stages, before it accelerates and causes more displacements.

Furthermore, we consider diverse preventive maintenance costs to see whether, regardless of these, our observations hold. Table 5.8 shows the observed optimum values from the simulations, with interaction and a policy of not considering interaction. The overall cost with interaction always remains inferior or equal to the cost without interaction. An interesting observation is that, with higher preventive maintenance costs, there is a tendency to have higher thresholds $L_r^{(S1)*}$ and $L_r^{(S2)*}$. Even though the threshold for S2 remains the lowest, there is a tendency to rely less on preventive maintenance as its cost grows. This is logical, since preventive maintenance is only a viable choice if the related overall expenses are more advantageous than just letting the system fail and relying on corrective maintenance. However, in the case of culverts, the cost of corrective maintenance will always be very high in comparison to preventive interventions, and frequent preventive actions will extend the life span of the systems.

5.5 Conclusion

The stochastic dependence of failures is a subject that has gained a lot of traction over the past few years in the research community. If relatively simple systems like culverts can be subject to complex relationships affecting their reliability, the eventuality that more complex systems would be significantly affected by failure interaction is plausible. In this paper, the interactive degradation model offers an approach that integrates natural degradation and distinguishes it from competitive shock processes. Instant probabilities of shocks grow according to the sequences of these shocks, which characterize the interaction phenomenon. Moreover, additional damages represented with Gamma distributions occur, depending on those sequences. Going one step further, a repair strategy is proposed to emphasize the important effect of failure interaction on maintenance policies. It considers the above-mentioned sequences to evaluate the cost of maintenance for a system subject to two failure modes. The optimal degradation thresholds and inspection period to perform repair actions are estimated through stochastic simulation.

Furthermore, our intuition on the effect of failure interaction on maintenance policies and the related costs are confirmed with an application of the proposed models on concrete culverts. It demonstrates that acknowledgment and integration of stochastic dependence in the maintenance model provides the right choice of repair action thresholds. The concrete culverts should ideally be inspected at least once every 2 years. Since cracking is gradually influenced by displacement and facilitates displacements, the threshold for preventive maintenance actions should be chosen in such a way that cracking is addressed at an early stage. Yet, if it is addressed too early, it increases the overall cost of applying the strategy.

However, it is possible that relationships between the failure modes could be more complex. The current model is based on the hypothesis that the mechanism of the natural degradation remains unchanged regardless of the number of shocks that occur. Moreover, the shocks are altogether supposed to follow a homogenous Poisson process. While this might be accurate for a simple and steady structure like a concrete culvert, in other cases, the shock processes might be more intricate. Additional shock damage sizes due to interaction could accumulate and grow exponentially in a domino effect.

In future work, accelerations of the natural degradation speed and a nonhomogeneous Poisson process could be integrated to the current reliability model. Exploring the idea of a domino effect on the accumulation of additional shock damage sizes would also be beneficial. Furthermore, it would be of great interest to consider a maintenance model involving variable inspection periods for the benefit of having a more dynamic strategy that would capitalize better on the knowledge of failure interaction mechanisms.

CHAPITRE 6 DISCUSSION GÉNÉRALE

6.1 Récapitulatif

Quelles sont les bonnes pratiques dans la conception de modèles caractérisant l'interaction des défaillances? La première contribution de la thèse répond à cette question d'un point de vue conceptuel, méthodologique et applicatif. Avant tout, divers modèles d'interaction des défaillances sont revus et classés en trois catégories :

- Modèles d'interaction basés sur les indices de fiabilité : ils ont pour but de mettre en relation les indices de divers modes de défaillance tels que la probabilité de défaillance et le taux de défaillance. Ces relations expriment l'interactivité sous forme de coefficients augmentant ces indices. Ces coefficients sont parfois estimés expérimentalement.
- Modèles d'interaction basés sur l'état : ils ont un lien direct avec la maintenance prédictive. Ils sont construits sur la base des liens stochastiques entre les variables d'état d'un système. Vu qu'il s'agit de systèmes physiques, la dégradation et les chocs sont des facteurs prééminents pour déterminer l'état. L'interaction des défaillances s'exprime là encore en termes de coefficient d'accélération de la dégradation ou de la probabilité de chocs précipitant l'atteinte d'un seuil de dégradation critique.
- Modèles d'interaction basés sur une copule : une copule est un outil mathématique qui représente la dépendance analytiquement. La plupart des modèles utilise des copules mais tout en se basant sur des hypothèses préalables sur les mécanismes de dépendances. Ce type de modèle interactif se distingue par l'absence de ces hypothèses. La copule a pour paramètres des coefficients qui mesurent la corrélation des temps de défaillances, des indices de fiabilités et/ou des variables d'états du système.

D'un point de vu conceptuel, les hypothèses de départ d'un modèle d'interactivité sont clés. Elles déterminent le cadre d'application de ce dernier et en définissent les limites. Il est important que ces hypothèses ne soient pas trop larges comme dans le cas des modèles basés sur les copules. En effet, la difficulté de n'émettre aucune supposition à valider ou infirmer est que la corrélation qui découle des copules est difficile à interpréter. Les mécanismes des phénomènes étudiés demeurent masqués par des valeurs statistiques, généralement sans dimension physique. Cependant, définir

des hypothèses trop restrictives limite grandement l'utilisation. Plusieurs modèles faisant interagir les indices de fiabilité souffrent de cet inconvénient. L'interaction est parfois considérée comme unilatéral ou est représenté en fonction des dépendances structurelles qui identifient des composants plus importants que d'autres. En réalité, tel que plusieurs fois mentionnées dans cette thèse, les dépendances dites stochastiques ont une incidence qui peut être indépendante des dépendances structurelles ou en dépasser la compréhension. L'interaction des défaillances peut aussi être bilatérale ou réciproque dans un système. Une bonne approche est de s'inspirer de phénomènes physiques, ce que font de façon intéressante les modèles basés sur l'état. Un aspect qui ressort certainement de la panoplie de concepts utilisés, c'est que l'étude de la séquence d'arrivée des défaillances doit motiver les hypothèses initiales.

Les méthodes, pour la plupart techniquement avancées, s'inscrivent dans le cadre défini par les concepts de départ. Elles peuvent être statistiques et/ou probabilistes, basées sur des données. Les données peuvent être des temps de défaillances ou des données issues des outils de maintenance conditionnelle. Si seuls les temps de défaillances sont disponibles, le phénomène interactif se décrit uniquement par son influence sur l'âge du système. On a pu remarquer à partir de la première contribution que dans ces cas-ci les chercheurs s'appuient souvent sur des données expérimentales pour estimer des coefficients interactifs. La disponibilité de données sur l'état du système à divers stades de son cycle de vie, informe grandement sur la présence de phénomènes interactifs, un autre avantage que présente les modèles basés sur l'état du système. Par contre, vu l'importance des séquences des mécanismes qui engendre une défaillance, plus que de juste observer des variables d'état, il faut être capable de remarquer les variations que subissent ces facteurs, les classer et/ou les ordonner et les relier stochastiquement les unes aux autres. Les méthodes probabilistes Markoviennes se prêtent bien à ce genre de besoin. Les méthodes Bayésiennes sont une alternative qui gagnerait à être étudiée.

Les champs d'application des dépendances stochastiques sont variés. Elles peuvent servir dans la phase de conception du système, que ne couvre pas cette thèse. Des références à ce sujet sont mentionnées dans la revue de littérature. Les modèles basés sur les indices de fiabilité servent généralement à mettre en place des stratégies de maintenance planifiée tandis que ceux basés sur l'état sont en relation avec la maintenance prédictive. Le coût économique de la maintenance prédictive peut représenter un frein pour l'application de cette dernière et une étude budgétaire au niveau de l'entreprise est toujours nécessaire avant de la sélectionner. Quoi qu'il en soit, elle

demeure un avantage économique et stratégique sur le long terme. En effet, la plupart des modèles proposés peuvent être mis à jour au fur et à mesure que les caractéristiques physiques et les phénomènes interactifs se dévoilent dans les données récupérées. Cependant, la première contribution avait révélé que bien des données proposées étaient simulées ou provenaient de travaux expérimentaux. La rareté d'applications en situation d'exploitation levait une ombre sur la réelle applicabilité des modèles proposés. C'est en cela qu'étaient nécessaires la proposition d'un modèle respectant les besoins conceptuels et méthodologiques mentionnés plus haut et son application à une situation d'exploitation.

Les seconde et troisième contributions (resp. Chapitres 4 et 5) répondent à ce besoin. Le chapitre 4 présente l'interaction via une série de probabilités d'occurrence de chocs aléatoires. Nous considérons ces probabilités comme des indices de fiabilité. Elles sont combinées pour définir la probabilité de défaillance du système due à un choc extrême. Le modèle est de ce fait un modèle d'interaction basé sur les indices de fiabilité. Un principe de tirage dans une urne et de transformation de l'urne inspire cette approche. En effet, en situation d'exploitation, c'est ainsi que l'interaction est perçue. On entrevoit les effets croissants à divers stades de son évolution. Le modèle physique qui explique la croissance de la probabilité d'occurrence d'un choc demeure caché et est équivalent à l'ajout d'un nombre aléatoire de boules à cette urne qui représente le système. Les distributions qui en découlent contiennent un grand nombre de paramètres que l'on peut estimer avec des processus de Markov. Cette approche permet de mettre en lumière l'état du système et les séquences des défaillances par des processus de comptage des chocs compétitifs.

Le chapitre 5 permet de faire un pas supplémentaire en ajustant le modèle pour qu'il soit basé sur l'état. Cela permet de bénéficier des avantages discutés plus haut. Une représentation physique de l'effet direct de l'interaction est possible sous la forme de distributions Gamma. Les paramètres de ces distributions sont estimés de manière classique via des méthodes d'apprentissage statistique. Cette estimation généralement basée sur le maximum de vraisemblance se fait suite à un exercice de séparation des données n'incluant aucun choc que l'on pourrait attribuer aux interactions et à l'alternative incluant des chocs dus aux interactions. Ces distributions informent sur l'effet d'un choc relatif à un mode de défaillance sur un choc subséquent relatif à un autre mode de défaillance. Cette quantification influe sur l'analyse de la fiabilité et la gestion de la maintenance. En effet, la stratégie de maintenance prédictive proposée intègre la notion de chocs qui produisent l'interaction mais aussi la notion de réparations qui modifient les mécanismes d'interactions.

L'état du système est simulé et les variables significatives représentant sa condition sont extrapolées sur plusieurs décennies. En effet, les probabilités d'occurrence des chocs et les niveaux de dégradation sont mis à jour en fonction des actions entreprises. La prise en compte de cet aspect permet d'optimiser efficacement une politique de maintenance prédictive et de prévoir les temps d'inspection ainsi que des seuils d'intervention en ce qui concerne les niveaux de dégradation.

Le cas d'étude des ponceaux traité par les chapitres 4 et 5 s'avère idéal car les données répertoriées sont en situation d'exploitation, comprennent des temps de défaillances, mais aussi des mesures des niveaux de dégradation. Mais, elles n'ont pas été utilisées sans transformation. L'exercice à effectuer face à une telle base de données est de séparer les chocs externes du processus de dégradation inhérent au système. Les avis d'expert et la littérature ont permis de comprendre comment un ponceau se dégrade lorsqu'il est dans des conditions idéales. Une fois cette connaissance acquise, il a fallu séparer les données d'inspection suivant des méthodes statistiques telles que les cartes de contrôle. Une fois les données séparées, il est possible de déterminer le taux de chocs par unité de temps λ . Ce taux peut varier d'une base de données à une autre ou d'une entreprise à une autre. Il dépend des méthodes de travail, de l'environnement et de l'exploitation faite du système. De plus, les experts ont une intuition des modes de défaillance critiques et des influences mutuelles qu'ils ont. C'est suivant leur conseil que l'étude s'est accentuée autour des problématiques de fissurations et de mouvements. Il s'avère que ces phénomènes ont des influences mutuelles qui rendent la structure moins fiable que ce que l'on prévoit sans les interactions. Ces influences produisent une augmentation de 0.11% sur le budget de maintenance prédictive lorsqu'elles ne sont pas prises en compte.

Les chapitres 4 et 5 s'inscrivent dans le cadre des conclusions du chapitre 3. Mais, il est important de noter que l'accent est principalement mis sur un système sujet à deux modes de défaillance interactifs.

6.2 Confirmation des hypothèses de recherche

Les modes de défaillances d'un système interagissent-ils de manière à altérer la fiabilité du système considéré? Il semble qu'au regard de la littérature et de nos propres recherches ce soit le cas. Une différence notable est remarquée dans les chapitres 4 et 5. Elle est plus proéminente et précipite la défaillance du système étudié. Le décalage des courbes des figures 4.5 et 5.3 peut s'interpréter comme le potentiel d'un ponceau de s'effondrer quelques années avant la prédiction de défaillance

d'une courbe ne représentant pas l'interactivité. De plus, l'hypothèse selon laquelle l'interaction interviendrait sous la forme d'une série de chocs aléatoires est validée. Le modèle inspiré de tirages aléatoires représente bien ce phénomène et les liens entre fissuration et mouvement en sont un exemple pratique. En effet, la fissuration rend un ponton plus sensible aux chocs. Inversement, les mouvements soumettent le ponton à des charges additionnelles non prévues durant sa conception et son installation. Cela facilite l'élargissement des fissures.

La quantification de l'interaction et ses effets est une problématique qui motive l'hypothèse sur les probabilités d'occurrence de l'interaction. Ces dernières sont dénotées par θ_k et γ_k et sont déduites des probabilités instantanées de chocs. Ces séries de coefficients donnent une valeur stochastique à l'interaction et dépendent des séquences de chocs observées dans la base de données. Additionnellement, l'amplitude de choc additionnel $\varepsilon_i^{(a)}$ informe sur la contribution physique de l'interaction à la dégradation. Cette information est importante pour le responsable de maintenance dans le diagnostic des défaillances. Elle pourrait même orienter la construction d'un modèle physique représentant l'interaction ou inspirer la conception de système s'auto-ajustant à ce type de choc additionnel. Par ailleurs, les données de maintenance et de dégradation permettent de représenter l'effet de l'interactivité. Il n'a pas été nécessaire de faire des expériences supplémentaires. Cependant, les avis des experts ont donné des indications importantes sur les données, à savoir l'identification des indices de dégradation qui pourraient être affectés par les effets des dépendances et les éventuelles sources de bruit dans la base de données nettoyée tel que suggéré dans le chapitre 2.

Un modèle de fiabilité est plus performant s'il prend en compte une représentation des phénomènes interactifs. Il va de soi qu'une stratégie de maintenance prédictive obéisse au même paradigme. Son objectif est de prévoir les ressources à allouer à la maintenance préventive d'un système dont on suit l'état. Si l'on prédit cet état plus précisément on évite de gaspiller des ressources. Dans le chapitre 5, les paramètres de cette stratégie sont la période d'inspection et les seuils de dégradation suggérant des actions préventives. Il ne s'agit pas juste d'intervenir avec la période et les seuils minimums. D'ailleurs pour ce cas de figure, le tableau 5.6 montre que l'on dépense plus que dans certains cas où l'on interviendrait moins. Il en est de même pour le cas inverse. Le tableau 5.7 montre en plus que même si les valeurs optimales varient quelque peu avec les coûts des opérations, une politique de maintenance intégrant l'interactivité est plus profitable que son alternative. Nous

pouvons donc affirmer qu'une stratégie de maintenance prédictive intégrant l'interaction au sein d'un système permet d'optimiser les coûts de maintenance.

6.3 Limites d'application des propositions

Les modèles proposés pourraient bénéficier de quelques améliorations. Une première remarque que l'on pourrait faire concerne le taux de chocs par unité de temps λ . Ce taux est le paramètre d'une loi de Poisson que l'on utilise pour tous les chocs indistinctement les uns des autres. Le choix d'une distribution homogène simplifie l'estimation des probabilités d'interaction et se prête bien au cas d'application des pontons. En effet, à l'image du tirage aléatoire du chapitre 4, un opérateur constatera ou non un choc pour des inspections prévues à des périodes prédéterminées et peu nombreuses sur l'espace de plusieurs années. En revanche, pour un système complexe sujets à diverses sources de chocs et avec des situations d'exploitation variées, une distribution homogène pourrait être inappropriée.

Une seconde remarque importante concerne les amplitudes des chocs. Elles sont représentées pour chaque mode de défaillance par des distributions gamma ayant des paramètres d'échelle identiques. Les cas où ces paramètres seraient différents ne sont pas traités par la thèse. Par ailleurs, les chocs additionnels dus à l'interaction sont considérés comme une conséquence ponctuelle qui ne se reproduit pas sur les chocs suivants. Ceci implique que les chocs additionnels n'en occasionnent pas d'autres. Il n'y a pas d'effet cumulatif de l'interaction où un choc augmenterait l'amplitude de chaque choc additionnel suivant.

Un troisième aspect qui limite les modèles de fiabilité de cette thèse est la charge computationnelle attachée à l'estimation des paramètres. Il n'existe pas encore de logiciel dédié qui permette de représenter l'interaction des défaillances, même pour les modèles présentés dans le chapitre 3. De plus, les modèles que nous proposons sont appliqués à un cas avec deux modes de défaillances. Ils sont extensibles à des situations avec plus de deux modes de défaillance comme le montre l'équation 4.24. La charge computationnelle deviendrait en revanche beaucoup trop grande. Une abondance de données permettant de retracer tous les phénomènes interactifs serait aussi nécessaire. La fonction de défaillance du cas d'étude des pontons estime à partir de deux processus de Markov concurrents dix paramètres qui sont les termes de deux séries. Plus le nombre de modes de défaillances augmente, plus il y a de paramètres à estimer et de processus de Markov à construire. Dépendamment de l'information disponible, si cette dernière est incomplète, ces

processus peuvent requérir des simulations. En outre, avec l'estimation de tous ces divers paramètres, survient le besoin de mesures statistiques de la qualité du modèle globale. Notre approche a été la validation par rapport à des fonctions excluant l'interactivité. Ceci dit, la qualité de l'ajustement aux données doit être étudiée plus en profondeur pour les modèles de fiabilité globaux.

La résolution du modèle de maintenance prédictive du chapitre 5 requiert de la simulation. Ceci rajoute à la charge computationnelle susmentionnée. En plus, le nombre de valeurs candidates pour atteindre les valeurs optimales peut augmenter cette charge. Par ailleurs, la stratégie de maintenance est définie pour une période d'inspection et des seuils d'intervention fixes. La possibilité de réévaluer ces valeurs optimales à chaque inspection n'est pas développée. Avoir une périodicité variable pour les inspections pourrait permettre une réduction plus considérable des coûts et capitaliserait sur la connaissance de la variabilité des phénomènes interactifs. Les réparations sont en outre considérées comme étant parfaites. Les principes de maintenance imparfaite, de réparations minimales et des temps de réparations ne sont pas abordés. Ces approches auraient des effets distincts sur les phénomènes interactifs. Enfin, vu que le point focal de nos recherches est la dépendance stochastique, l'accent n'est pas mis sur le volet structurel et économique. De ce fait, ces types de dépendances ne sont pas explicitement étudiés.

6.4 Défis technologiques

Même si le risque associé à la non-détection de l'interactivité est véritable, les enjeux technologiques liés à la représentation de défaillances interactives constituent un facteur à prendre en compte. Ces enjeux interviennent à divers niveaux : entre autres l'entreprise, les techniciens, les opérateurs, les experts et les chercheurs.

En effet, une entreprise a la responsabilité de développer des systèmes sécuritaires mais elle a aussi besoin de garantir un certain profit. Même si cette entreprise n'est pas privée, la gestion des actifs physiques est un exercice qui est crucial. Alors, elle s'accorde une fine marge d'erreur qui dépend des régulations en place, de ses capacités financières, et de ses objectifs de performance et de productivité. De ce fait, la prise en compte de l'interactivité des défaillances doit représenter un avantage économique pertinent et un ajout à la sécurité des biens et des personnes. Pour des entreprises gérant des biens critiques telles que le réseau routier et les systèmes de transports, il semble être indéniable que cette problématique soit traitée.

Pour les techniciens et les opérateurs, l'enjeu prend une autre dimension car il ne s'agit pas simplement d'avoir conscience de la possibilité de l'interaction des défaillances. Il est primordial qu'une formation, adéquate et pratique, vienne soutenir cette notion. Cela dit, l'expérience des techniciens et des opérateurs est en général riche d'informations qualitatives, qui bien souvent ne sont pas valorisées à leur juste titre. Ces informations renseignent sur d'éventuelles mécanismes qui se révèlent durant la maintenance ou les longues heures d'opérations d'un système. Par ailleurs, si les modèles théoriques font floraison dans la littérature, les outils de mesures des dépendances stochastiques tels que des logiciels spécialisés de diagnostic et de pronostic semblent quasiment inexistantes. Ils sont pourtant déterminants dans l'exercice des fonctions des techniciens qui ont besoin d'une base concrète pour cibler des problèmes spécifiques.

Pour les experts et les chercheurs, la notion d'interaction est connue. C'est d'ailleurs leur effort commun qui devrait conduire à la conception de logiciels spécialisés. Leurs connaissances combinées peuvent permettre de réorienter la manière dont un système est observé, dont les données sont collectées et classées, dont les techniciens et les opérateurs sont formés. Pourtant, il semblerait que les efforts soient parfois divergents. La complexité mathématique des modèles proposés par la recherche semble dissuader les experts et les entreprises quant à leur utilisation. Les chercheurs en outre se retrouvent face à la difficulté de l'interdisciplinarité d'une telle problématique. En plus des compétences mathématiques avancées que requièrent leur recherche, l'avancée de leurs travaux dépend de la participation des experts pour les systèmes physiques étudiés mais aussi de l'apport de compétences diverses tels que le génie informatique et le génie logiciel.

Les propositions de cette thèse, quoique mathématiquement avancées, restent accessibles car utilisant comme base des principes probabilistes élémentaires. Les tirages aléatoires et les processus de chocs sont une manière intuitive de comprendre l'interaction et l'interpréter. En l'absence de logiciel dédié, des macros dans Microsoft Excel ont permis d'effectuer les simulations nécessaires comme en témoigne l'annexe B. Ces méthodes sont prêtes à l'emploi. Elles requièrent juste que l'entreprise soit déjà inscrite dans une démarche de maintenance prédictive ou choisisse de s'y orienter. Le prétraitement de données mentionné dans le chapitre 2 et le récapitulatif de ce chapitre est aussi un prérequis.

CHAPITRE 7 CONCLUSION ET RECOMMANDATIONS

7.1 Conclusion générale

En fin de compte, qu'est-ce que cela implique que de garantir la fiabilité de systèmes sujets à des défaillances interactives? Au début de cette thèse, nous présentons des méthodes classiques d'analyse de la fiabilité et de gestion de la maintenance. Nous examinons ensuite diverses approches qui viennent remettre en question l'hypothèse d'indépendance qui soutient ces modèles classiques. Cela suggère que des actifs plus fiables sont des systèmes dont on peut représenter et gérer les dépendances structurelles, économiques et stochastiques. Parmi ces dépendances, les dépendances stochastiques se démarquent en raison de la possibilité qu'elles offrent d'étudier des phénomènes difficiles à modéliser ou dont les modèles physiques n'existent pas encore. Un parcours de la littérature montre que la recherche s'accroît autour l'interactivité des défaillances dans l'optique de déterminer les causes qui accélèreraient le processus de dégradation ou la probabilité de défaillance d'un système.

Une première remarque générale est que, tout système sujet à divers modes de défaillances, a le potentiel de présenter un risque sécuritaire et/ou économique additionnel dû à cette interactivité. Le chapitre 3 le met en évidence mais aussi permet de prendre connaissance des approches pertinentes de modélisation des dépendances stochastiques, de les classer et de mettre en lumière les bonnes pratiques pour la conception de tels modèles. Les chapitres 4 et 5 ouvrent la porte sur de nouveaux modèles d'interaction basés les indices de fiabilité et l'état du système. Ils utilisent une nouvelle distribution qui représente le caractère évolutif et aléatoire de l'interactivité. Nous sommes en effet en mesure de quantifier cette dernière sous la forme d'une série de probabilités et de distributions Gamma de chocs. Ces valeurs probabilistes et statistiques donnent une idée de l'ampleur des phénomènes étudiés mais aussi permet de construire une stratégie de maintenance prédictive qui traite les dépendances stochastiques et face des prévisions en conséquence. Au travers de cette stratégie, les temps d'inspections et les seuils critiques d'intervention optimaux sont déterminés.

L'étude des ponts en béton consolide la validité de ces trouvailles car il s'agit d'une situation réelle d'exploitation mettant en jeu la sécurité de personnes et de biens. Elle présente un réel enjeu sécuritaire et économique. Les phénomènes de fissuration et de mouvement de la structure sont

étudiés et montrent de manière générale que la fissuration devrait être traitée dès les premiers signes. En effet, elle peut grandement affaiblir la structure et favorise des mouvements. Le phénomène inverse est aussi vérifié mais dans des proportions différentes. L'application de la stratégie de maintenance prédictive proposée révèle les conséquences économiques de l'interactivité de ces défaillances sur les coûts de maintenance. Vu qu'ils affectent la fiabilité des ponts et précipite une défaillance catastrophique de la structure, ils sont à l'origine de risques et coûts supplémentaires que la stratégie proposée permettrait d'éviter.

Dans une ère technologique où les efforts de modélisation déterministe trainent le pas en comparaison des alternatives, l'avenir de l'ingénierie de la fiabilité se basera sur sa capacité à concevoir de tels modèles stochastiques. Les entreprises et les entités gouvernementales ont d'ailleurs la responsabilité d'avancer vers une intégration dynamique de ses modèles qui révèlent des risques réels pour la sécurité des biens et des personnes. Cela dit, face aux propositions des chapitres 4 et 5, le chapitre 6 présente les limites de ces approches et soulèvent des problématiques d'ordre technologique. Suivent quelques recommandations aux sujets de ces potentielles pistes de recherche.

7.2 De nouvelles perspectives de recherche

Il est primordial d'approfondir l'exploration des dépendances au sein d'un système. L'objectif étant de concevoir des systèmes plus sécuritaires et économiques, il convient de rappeler qu'à ces aspects se greffent des questions technologiques. Aussi, sont recommandées les voies de recherches suivantes:

- Développement et standardisation des mesures de séquentialité dans les données de maintenance;
- Développement de mesures statistiques de la qualité d'ajustement des modèles de fiabilité intégrant les dépendances stochastiques;
- Intégration dynamique des avis d'experts à l'estimation des probabilités d'interaction par des méthodes bayésiennes;
- Représentation de l'ensemble des chocs interactifs par un processus de Poisson non-homogène dans la fonction de défaillance du chapitre 4;

- Intégration de distributions Gamma ayant des paramètres d'échelle différents à la représentation de la dégradation naturelle du chapitre 5;
- Modélisation d'un effet domino des chocs additionnels dus à l'interactivité des défaillances
- Optimisation des méthodes de calculs des approches proposées pour des systèmes sujets à plus de deux modes de défaillance.
- Mise à jour dynamique des temps d'inspection et des seuils d'intervention de la stratégie de maintenance prédictive du chapitre 5;
- Maintenance prédictive opportuniste intégrant la maintenance imparfaite et les principes d'interactivité de cette thèse;
- Conception d'applications et de logiciels spécialisés de mesure et d'exploitation des dépendances stochastiques.

Les efforts de recherche suivant ces perspectives citées en amont permettront certainement de donner un nouveau visage à l'ingénierie de la fiabilité tout en pérennisant la riche variété de connaissances acquises et développées à ce jour.

RÉFÉRENCES

- [1] AFNOR, N. (2018). EN-13306-X-60-319: Maintenance terminology. *Norm, "Association Française de Normalisation*.
- [2] Ahmad, R., & Kamaruddin, S. (2012). An overview of time-based and condition-based maintenance in industrial application. *Computers & industrial engineering*, 63(1), 135-149.
- [3] Alaswad, S., & Xiang, Y. (2017). A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering & System Safety*, 157, 54-63.
- [4] Ammar, K. (2009). *Optimisation des remplacements de pneus sous contraintes de fiabilité et de coûts: cas d'une société de transport en commun* (Doctoral dissertation, École Polytechnique de Montréal).
- [5] An, D., Kim, N. H., & Choi, J. H. (2015). Practical options for selecting data-driven or physics-based prognostics algorithms with reviews. *Reliability Engineering & System Safety*, 133, 223-236.
- [6] An, Z., & Sun, D. (2017). Reliability modeling for systems subject to multiple dependent competing failure processes with shock loads above a certain level. *Reliability Engineering & System Safety*, 157, 129-138.
- [7] Arab, A., Keedy, E., & Feng, Q. (2013). Reliability analysis for implanted multi-stent systems with stochastic dependent competing risk processes. Paper presented at the *IIE Annual Conference. Proceedings*.
- [8] Assaf, R., Do, P., Scarf, P., & Nefti-Meziani, S. (2017, June). Diagnosis for systems with multi-component wear interactions. In *Prognostics and Health Management (ICPHM), 2017 IEEE International Conference on* (pp. 96-102). IEEE.
- [9] Assaf, R., Do, P., Scarf, P., & Nefti-Meziani, S. (2016). Wear rate-state interaction modelling for a multi-component system: Models and an experimental platform. *IFAC-PapersOnLine*, 49(28), 232-237.
- [10] Atashgar, K., & Abdollahzadeh, H. (2017). A joint reliability and imperfect opportunistic maintenance optimization for a multi-state weighted k-out-of-n system considering economic

- dependence and periodic inspection. *Quality and Reliability Engineering International*, 33(8), 1685-1707.
- [11] Bai, J., & Pham, H. (2005). Repair-limit risk-free warranty policies with imperfect repair. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 35(6), 765-772.
- [12] Baqaee, D. R. (2018). Cascading failures in production networks. *Econometrica*, 86(5), 1819-1838.
- [13] Barlow, R. E., & Proschan, F. (1975). *Statistical theory of reliability and life testing: probability models*. Florida State Univ Tallahassee.
- [14] Beaver, J. L., & McGrath, T. J. (2005). Management of Utah highway culverts. *Transportation research record*, 1904(1), 113-123.
- [15] Bereriche, Y. (2010). *Contribution de l'approche contrainte/résistance à l'évaluation de la fiabilité des structures* [Mémoire de maîtrise ès sciences, Université Laval]. CorpusUL. <http://hdl.handle.net/20.500.11794/21638>.
- [16] Blanchard, B. S., Fabrycky, W. J., & Fabrycky, W. J. (1990). *Systems engineering and analysis* (Vol. 4). New Jersey;: Prentice Hall.
- [17] Bian, L., & Gebraeel, N. (2014). Stochastic modeling and real-time prognostics for multi-component systems with degradation rate interactions. *IIE Transactions*, 46(5), 470-482.
- [18] Borgovini, R., Pemberton, S., & Rossi, M. (1993). *Failure mode, effects, and criticality analysis (FMECA)*. RELIABILITY ANALYSIS CENTER GRIFFISS AFB NY.
- [19] Bozbulut, A. R., & Eryilmaz, S. (2018). Generalized extreme shock models and their applications. *Communications in Statistics-Simulation and Computation*, 1-11.
- [20] Cadini, F., Agliardi, G. L., & Zio, E. (2017). A modeling and simulation framework for the reliability/availability assessment of a power transmission grid subject to cascading failures under extreme weather conditions. *Applied energy*, 185, 267-279.
- [21] Castro, I. (2009). A model of imperfect preventive maintenance with dependent failure modes. *European Journal of Operational Research*, 196(1), 217-224.

- [22] Čepin, M. (2011). Reliability Block Diagram. In *Assessment of Power System Reliability* (pp. 119-123). Springer London.
- [23] Cha, J. H., & Finkelstein, M. (2011). On new classes of extreme shock models and some generalizations. *Journal of Applied Probability*, 48(1), 258-270.
- [24] Che, H., Zeng, S., Guo, J., & Wang, Y. (2018). Reliability modeling for dependent competing failure processes with mutually dependent degradation process and shock process. *Reliability Engineering & System Safety*, 180, 168-178.
- [25] Chen, B., & Sun, L. (2013). The impact of soil properties on the structural integrity of high-fill reinforced concrete culverts. *Computers and Geotechnics*, 52, 46-53.
- [26] Chen, L., Ye, Z., & Huang, B. (2011). *Condition-based maintenance for systems under dependent competing failures*. Paper presented at the Industrial Engineering and Engineering Management (IEEM), 2011 IEEE International Conference on.
- [27] Ching, W. K., Huang, X., Ng, M. K., & Siu, T. K. (2013). *Markov chains. Models, algorithms and applications*. Springer Science+ Business Media.
- [28] Cho, D. I., & Parlar, M. (1991). A survey of maintenance models for multi-unit systems. *European Journal of Operational Research*, 51(1), 1-23.
- [29] Chu, X., Ilyas, I. F., Krishnan, S., & Wang, J. (2016, June). Data cleaning: Overview and emerging challenges. In *Proceedings of the 2016 International Conference on Management of Data* (pp. 2201-2206).
- [30] Cirillo, P., & Hüsler, J. (2011). Extreme shock models: An alternative perspective. *Statistics & probability letters*, 81(1), 25-30.
- [31] Crow, L. H. (1975). *Reliability analysis for complex, repairable systems* (No. AMSAA-TR-138). ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY ABERDEEN PROVING GROUND MD.
- [32] Crowder, M. J., Kimber, A., Sweeting, T., & Smith, R. (1994). *Statistical analysis of reliability data* (Vol. 27). CRC Press.
- [33] Dao, C. D., & Zuo, M. J. (2017). Selective maintenance of multi-state systems with structural dependence. *Reliability engineering & system safety*, 159, 184-195.

- [34] Dao, C. D., Zuo, M. J., & Pandey, M. (2014). Selective maintenance for multi-state series–parallel systems under economic dependence. *Reliability Engineering & System Safety*, 121, 240-249.
- [35] Dekker, R., Wildeman, R. E., & Van der Duyn Schouten, F. A. (1997). A review of multi-component maintenance models with economic dependence. *Mathematical methods of operations research*, 45(3), 411-435.
- [36] Dhillon, B. S. (1999). *Design reliability: fundamentals and applications*. CRC press.
- [37] Diallo, C. (2006). *Développement d'un modèle d'identification et de gestion des pièces de rechange* [Thèse de doctorat, Université Laval]. CorpusUL. <https://corpus.ulaval.ca/jspui/bitstream/20.500.11794/18334/1/23705.pdf>.
- [38] Dinh, D. H., Do Van, P., & Iung, B. (2019, June). Modeling the impact of disassembly operations on the degradation process of multi-component systems. In *11th International Conference on Mathematical Methods in Reliability*, MMR 2019.
- [39] Do, P., Scarf, P., & Iung, B. (2015). Condition-based maintenance for a two-component system with dependencies. *IFAC-PapersOnLine*, 48(21), 946-951.
- [40] Do, P., Voisin, A., Levrat, E., & Iung, B. (2015). A proactive condition-based maintenance strategy with both perfect and imperfect maintenance actions. *Reliability Engineering & System Safety*, 133, 22-32.
- [41] Dohi, T., Ashioka, A., Osaki, S., & Kaio, N. (2001). Optimizing the repair-time limit replacement schedule with discounting and imperfect repair. *Journal of Quality in Maintenance Engineering*.
- [42] Dvorkin, L., & Dvorkin, O. (2006). *Basics of concrete science*. St-Petersburg: Stroi-Beton, 590319702-7.
- [43] Ejaz, I., Alvarado, M., Gautam, N., Gebraeel, N., & Lawley, M. (2019). Condition-Based Maintenance for Queues With Degrading Servers. *IEEE Transactions on Automation Science and Engineering*, 16(4), 1750-1762.

- [44] Eryilmaz, S. (2014). Multivariate copula based dynamic reliability modeling with application to weighted-k-out-of-n systems of dependent components. *Structural Safety*, 51, 23-28.
- [45] Fan, H., Hu, C., Chen, M., & Zhou, D. (2011). Cooperative predictive maintenance of repairable systems with dependent failure modes and resource constraint. *Reliability, IEEE Transactions on*, 60(1), 144-157.
- [46] Fan, M., Zeng, Z., Zio, E., & Kang, R. (2017). Modeling dependent competing failure processes with degradation-shock dependence. *Reliability Engineering & System Safety*, 165, 422-430.
- [47] Feizabadi, M., & Jahromi, A. E. (2017). A new model for reliability optimization of series-parallel systems with non-homogeneous components. *Reliability Engineering & System Safety*, 157, 101-112.
- [48] Feng, Q., & Coit, D. W. (2010). Reliability analysis for multiple dependent failure processes: an MEMS application. *International Journal of Performability Engineering*, 6(1), 100.
- [49] Filus, J. K., & Filus, L. Z. (2008). Some alternative approaches to system reliability modeling. In *Recent Advances in Reliability and Quality in Design* (pp. 101-135). Springer, London.
- [50] Finkelstein, M., & Cha, J. H. (2013). Stochastic modeling for reliability. Shocks, burn-in and heterogeneous populations. *Springer Series in Reliability Engineering*, London: Springer.
- [51] Fritzmann, C., Löwenberg, J., Wintgens, T., & Melin, T. (2007). State-of-the-art of reverse osmosis desalination. *Desalination*, 216(1-3), 1-76.
- [52] Giresini, L., Puppio, M. L., & Sassu, M. (2016). Collapse of corrugated metal culvert in Northern Sardinia: analysis and numerical simulations. *International Journal of Forensic Engineering*, 3(1-2), 69-85.
- [53] Golmakani, H. R., & Moakedi, H. (2012). Periodic inspection optimization model for a two-component repairable system with failure interaction. *Computers & Industrial Engineering*, 63(3), 540-545.

- [54] Gorjian, N., Ma, L., Mittinty, M., Yarlagadda, P., & Sun, Y. (2010). A review on reliability models with covariates. In *Engineering Asset Lifecycle Management* (pp. 385-397). Springer, London.
- [55] Greenlee, L. F., Lawler, D. F., Freeman, B. D., Marrot, B., & Moulin, P. (2009). Reverse osmosis desalination: water sources, technology, and today's challenges. *Water research*, 43(9), 2317-2348.
- [56] Griffith, C. D., & Mahadevan, S. (2011). Inclusion of fatigue effects in human reliability analysis. *Reliability Engineering & System Safety*, 96(11), 1437-1447.
- [57] Groth, K. M., & Swiler, L. P. (2013). Bridging the gap between HRA research and HRA practice: A Bayesian network version of SPAR-H. *Reliability Engineering & System Safety*, 115, 33-42.
- [58] Guo, C., Wang, W., Guo, B., & Peng, R. (2013). Maintenance optimization for systems with dependent competing risks using a copula function. *Eksploatacja i Niezawodność*, 15, 9-17.
- [59] Guo, H., Zheng, C., Iu, H. H. C., & Fernando, T. (2017). A critical review of cascading failure analysis and modeling of power system. *Renewable and Sustainable Energy Reviews*, 80, 9-22.
- [60] Gut, A. (1999). Extreme shock models. *Extremes*, 2(3), 295-307.
- [61] Hamada, M. S., Wilson, A., Reese, C. S., & Martz, H. (2008). *Bayesian reliability*. Springer Science & Business Media.
- [62] Hao, S., Yang, J., Ma, X., & Zhao, Y. (2017). Reliability modeling for mutually dependent competing failure processes due to degradation and random shocks. *Applied Mathematical Modelling*, 51, 232-249.
- [63] Huard, D., Evin, G., & Favre, A.-C. (2006). Bayesian copula selection. *Computational Statistics & Data Analysis*, 51(2), 809-822.
- [64] Huynh, K. T., Barros, A., & Bérenguer, C. (2013). A reliability-based opportunistic predictive maintenance model for k-out-of-n deteriorating systems. *CHEMICAL ENGINEERING*, 33.

- [65] Huynh, K. T., Barros, A., Berenguer, C., & Castro, I. T. (2011). A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events. *Reliability Engineering & System Safety*, 96(4), 497-508.
- [66] Huynh, K. T., Castro, I. T., Barros, A., & Berenguer, C. (2012). Modeling age-based maintenance strategies with minimal repairs for systems subject to competing failure modes due to degradation and shocks. *European Journal of Operational Research*, 218(1), 140-151.
- [67] Inamura, Y. (2006). Estimating continuous time transition matrices from discretely observed data. *Bank of Japan*, 06-07.
- [68] Jardine, A. K., Lin, D., & Banjevic, D. (2006). A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical systems and signal processing*, 20(7), 1483-1510.
- [69] Jhang, J.-P., & Sheu, S.-H. (2000). Optimal age and block replacement policies for a multi-component system with failure interaction. *International Journal of Systems Science*, 31(5), 593-603.
- [70] Jia, L., Ren, Y., Yang, D., Feng, Q., Sun, B., & Qian, C. (2019). Reliability analysis of dynamic reliability block diagram based on dynamic uncertain causality graph. *Journal of Loss Prevention in the Process Industries*, 62, 103947.
- [71] Jia, X., Wang, L., & Wei, C. (2014). Reliability research of dependent failure systems using copula. *Communications in Statistics-Simulation and Computation*, 43(8), 1838-1851.
- [72] Jiang, L., Feng, Q., & Coit, D. W. (2012). Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds. *Reliability, IEEE Transactions on*, 61(4), 932-948.
- [73] Jiang, R., & Ji, P. (2002). Age replacement policy: a multi-attribute value model. *Reliability Engineering & System Safety*, 76(3), 311-318.
- [74] Jiang, X., Duan, F., Tian, H., & Wei, X. (2015). Optimization of reliability centered predictive maintenance scheme for inertial navigation system. *Reliability Engineering & System Safety*, 140, 208-217.

- [75] Jiayin, T., Ping, H., & Qin, W. (2009). *Copulas model for reliability calculation involving failure correlation*. Paper presented at the Computational Intelligence and Software Engineering, 2009. CiSE 2009. International Conference on.
- [76] Johnson, R. A. (1988). 3 Stress-strength models for reliability. *Handbook of statistics*, 7, 27-54.
- [77] Kamenchukov, A. V., & Svetenok, V. S. (2020). Improving the Operational Reliability of Reinforced Concrete Culverts on Highways. *MS&E*, 753(4), 042060.
- [78] Kaplan, E. L., & Meier, P. (1958). Nonparametric estimation from incomplete observations. *Journal of the American statistical association*, 53(282), 457-481.
- [79] Ke, H., & Yao, K. (2016). Block replacement policy with uncertain lifetimes. *Reliability Engineering & System Safety*, 148, 119-124.
- [80] Keizer, M. C. O., Flapper, S. D. P., & Teunter, R. H. (2017). Condition-based maintenance policies for systems with multiple dependent components: A review. *European Journal of Operational Research*, 261(2), 405-420.
- [81] Kim, H., & Kim, P. (2017). Reliability–redundancy allocation problem considering optimal redundancy strategy using parallel genetic algorithm. *Reliability Engineering & System Safety*, 159, 153-160.
- [82] Kim, K., & Yoo, C. H. (2005). Design loading on deeply buried box culverts. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(1), 20-27.
- [83] Kolev, N., Anjos, U. D., & Mendes, B. V. D. M. (2006). Copulas: a review and recent developments. *Stochastic Models*, 22(4), 617-660.
- [84] Kumar, D., & Westberg, U. (1997). Maintenance scheduling under age replacement policy using proportional hazards model and ttt-plotting. *European journal of operational research*, 99(3), 507-515.
- [85] Lai, C. D., & Xie, M. (2003). Concepts of stochastic dependence in reliability analysis. In *Handbook of reliability engineering* (pp. 141-156). Springer, London.
- [86] Lai, M.-T. (2007). Periodical replacement model for a multi-unit system subject to failure rate interaction. *Quality & quantity*, 41(3), 401-411.

- [87] Lai, M.-T. (2009). A discrete replacement model for a two-unit parallel system subject to failure rate interaction. *Quality and Quantity*, 43(3), 471-479.
- [88] Lai, M.-T., & Chen, Y.-C. (2006). Optimal periodic replacement policy for a two-unit system with failure rate interaction. *The international journal of advanced manufacturing technology*, 29(3-4), 367-371.
- [89] Lai, M.-T., & Chen, Y.-C. (2008). Optimal replacement period of a two-unit system with failure rate interaction and external shocks. *International Journal of Systems Science*, 39(1), 71-79.
- [90] Lai, M.-T., & Yan, H. (2014). Optimal number of minimal repairs with cumulative repair cost limit for a two-unit system with failure rate interactions. *International Journal of Systems Science*(ahead-of-print), 1-8.
- [91] Li, H., Deloux, E., & Dieulle, L. (2016). A condition-based maintenance policy for multi-component systems with Lévy copulas dependence. *Reliability Engineering & System Safety*, 149, 44-55.
- [92] Li, Z., Gao, Q., Liu, S., & Liu, G. (2013). *Reliability analysis for a multi-unit system with failure rate interaction*. Paper presented at the Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE), 2013 International Conference on.
- [93] Liang, Z., Parlikad, A. K., Srinivasan, R., & Rasmekomen, N. (2017). On fault propagation in deterioration of multi-component systems. *Reliability Engineering & System Safety*, 162, 72-80.
- [94] Liang, Z., & Parlikad, A. K. (2015). A condition-based maintenance model for assets with accelerated deterioration due to fault propagation. *IEEE Transactions on Reliability*, 64(3), 972-982.
- [95] Limbourg, P., Kochs, H., Ehtle, K., & Eusgeld, I. (2007, March). Reliability prediction in systems with correlated component failures ü an approach using copulas. In *Architecture of computing systems (ARCS), 2007 20th international conference on* (pp. 1-8). VDE.
- [96] Lin, D., Zuo, M. J., & Yam, R. C. (2001). Sequential imperfect preventive maintenance models with two categories of failure modes. *Naval Research Logistics (NRL)*, 48(2), 172-183.

- [97] Lin, Z., Smith, R., & Yin, Q. S. (2008, June). A New Maintenance Strategy of Combining Preventive Maintenance and Opportunistic Maintenance. In *Safety and Reliability* (Vol. 28, No. 3, pp. 28-48). Taylor & Francis.
- [98] Lipol, L. S., & Haq, J. (2011). Risk analysis method: FMEA/FMECA in the organizations. *International Journal of Basic & Applied Sciences*, 11(5), 74-82.
- [99] Liu, B., Liang, Z., Parlikad, A. K., Xie, M., & Kuo, W. (2020). Condition-based maintenance for systems with aging and cumulative damage based on proportional hazards model. In *Value Based and Intelligent Asset Management* (pp. 211-231). Springer, Cham.
- [100] Liu, B., Wu, J., & Xie, M. (2015). Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty. *European Journal of Operational Research*, 243(3), 874-882.
- [101] Liu, B., Wu, S., Xie, M., & Kuo, W. (2017). A condition-based maintenance policy for degrading systems with age-and state-dependent operating cost. *European Journal of Operational Research*, 263(3), 879-887.
- [102] Liu, H., Yeh, R. H., & Cai, B. (2017). Reliability modeling for dependent competing failure processes of damage self-healing systems. *Computers & Industrial Engineering*, 105, 55-62.
- [103] Liu, H. C., Liu, L., & Liu, N. (2013). Risk evaluation approaches in failure mode and effects analysis: A literature review. *Expert systems with applications*, 40(2), 828-838.
- [104] Lughofer, E., & Sayed-Mouchaweh, M. (Eds.). (2019). *Predictive Maintenance in Dynamic Systems: Advanced Methods, Decision Support Tools and Real-World Applications*. Springer.
- [105] Masada, T., Sargand, S. M., Tarawneh, B., Mitchell, G. F., & Gruver, D. (2007). Inspection and risk assessment of concrete culverts under Ohio's highways. *Journal of Performance of constructed Facilities*, 21(3), 225-233.
- [106] Meango, T. J. M., & Ouali, M. S. (2019). Failure interaction models for multicomponent systems: a comparative study. *SN Applied Sciences*, 1(1), 66.
- [107] Meango, T. J. M., & Ouali, M. S. (2020). Failure interaction model based on extreme shock and Markov processes. *Reliability Engineering & System Safety*, 197, 106827.

- [108] Meegoda, J. N. (2013). Long-term maintenance of culverts. *Transportation Research Board, Washington, DC*.
- [109] Meeker, W. Q., & Escobar, L. A. (2014). *Statistical methods for reliability data*. John Wiley & Sons.
- [110] Meeker, W. Q., & Hong, Y. (2014). Reliability meets big data: opportunities and challenges. *Quality Engineering*, 26(1), 102-116.
- [111] Melchers, R. E., & Beck, A. T. (2018). *Structural reliability analysis and prediction*. John Wiley & Sons.
- [112] Mercier, S., & Pham, H. H. (2012). A preventive maintenance policy for a continuously monitored system with correlated wear indicators. *European Journal of Operational Research*, 222(2), 263-272.
- [113] Misra, K. B. (1975). *On optimal reliability design: a review*. IFAC Proceedings Volumes, 8(1), 27-36.
- [114] Mitchell, G. F., Masada, T., Sargand, S. M., & Tarawneh, B. (2005). *Risk assessment and update of inspection procedures for culverts* (No. FHWA/OH-2005/002). United States. Federal Highway Administration.
- [115] Mobley, R. K. (2002). *An introduction to predictive maintenance*. Elsevier.
- [116] Modarres, M., Kaminskiy, M. P., & Krivtsov, V. (2016). *Reliability engineering and risk analysis: a practical guide*. CRC press.
- [117] Murthy, D., & Nguyen, D. (1985a). Study of a multi-component system with failure interaction. *European Journal of Operational Research*, 21(3), 330-338.
- [118] Murthy, D., & Nguyen, D. (1985b). Study of two-component system with failure interaction. *Naval Research Logistics Quarterly*, 32(2), 239-247.
- [119] Nakagawa, T. (1988). Sequential imperfect preventive maintenance policies. *IEEE Transactions on Reliability*, 37(3), 295-298.
- [120] Nakagawa, T., & Murthy, D. (1993). Optimal replacement policies for a two-unit system with failure interactions. *Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle*, 27(4), 427-438.

- [121] Navarro, J., & Durante, F. (2017). Copula-based representations for the reliability of the residual lifetimes of coherent systems with dependent components. *Journal of Multivariate Analysis*, 158, 87-102.
- [122] Nguyen, K. A., Do, P., & Grall, A. (2014). Condition-based maintenance for multi-component systems using importance measure and predictive information. *International Journal of Systems Science: Operations & Logistics*, 1(4), 228-245.
- [123] Nguyen, K. A., Do, P., & Grall, A. (2015). Multi-level predictive maintenance for multi-component systems. *Reliability engineering & system safety*, 144, 83-94.
- [124] Nguyen, K. A., Do, P., & Grall, A. (2017). Joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance. *Reliability Engineering & System Safety*, 168, 249-261.
- [125] Nikulin, M. S., Limnios, N., Balakrishnan, N., Kahle, W., & Huber-Carol, C. (2010). *Advances in degradation modeling*. Birhäuser, Boston.
- [126] Nowlan, F. S., & Heap, H. F. (1978). *Reliability-centered maintenance*. United Air Lines Inc San Francisco Ca.
- [127] Okogbaa, O. G., Otieno, W., Peng, X., & Jain, S. (2008). Transient analysis of maintenance intervention of continuous multi-unit systems. *IIE Transactions*, 40(10), 971-983.
- [128] Olsen, F. W. (2017). *Assessment method for review of existing maintenance strategies based on reliability, availability, risk, and cost parameters* (Master's thesis, University of Stavanger, Norway).
- [129] Osborne, J. W. (2013). *Best practices in data cleaning: A complete guide to everything you need to do before and after collecting your data*. Sage.
- [130] Peiravi, A., Karbasian, M., Ardakan, M. A., & Coit, D. W. (2019). Reliability optimization of series-parallel systems with K-mixed redundancy strategy. *Reliability Engineering & System Safety*, 183, 17-28.
- [131] Peng, H., Feng, Q., & Coit, D. W. (2010). Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes. *IIE transactions*, 43(1), 12-22.

- [132] Peng, Y., Dong, M., & Zuo, M. J. (2010). Current status of machine prognostics in condition-based maintenance: a review. *The International Journal of Advanced Manufacturing Technology*, 50(1-4), 297-313.
- [133] Perrin Jr, J., & Dwivedi, R. (2006). Need for culvert asset management. *Transportation research record*, 1957(1), 8-15.
- [134] Pham, H., & Wang, H. (1996). Imperfect maintenance. *European journal of operational research*, 94(3), 425-438.
- [135] Qiu, Q., Cui, L., & Kong, D. (2019). Availability and maintenance modeling for a two-component system with dependent failures over a finite time horizon. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 233(2), 200-210.
- [136] Rackwitz, R. (2001). Reliability analysis—a review and some perspectives. *Structural safety*, 23(4), 365-395.
- [137] Rafiee, K., Feng, Q., & Coit, D. W. (2014). Reliability modeling for dependent competing failure processes with changing degradation rate. *IIE transactions*, 46(5), 483-496.
- [138] Rafiee, K., Feng, Q., & Coit, D. W. (2017). Reliability assessment of competing risks with generalized mixed shock models. *Reliability Engineering & System Safety*, 159, 1-11.
- [139] Rahnamay-Naeini, M., & Hayat, M. M. (2016). Cascading failures in interdependent infrastructures: An interdependent Markov-chain approach. *IEEE Transactions on Smart Grid*, 7(4), 1997-2006.
- [140] Rasmekomen, N., & Parlikad, A. K. (2016). Condition-based maintenance of multi-component systems with degradation state-rate interactions. *Reliability Engineering & System Safety*, 148, 1-10.
- [141] Rezaei, E. (2017). A new model for the optimization of periodic inspection intervals with failure interaction: A case study for a turbine rotor. *Case studies in engineering failure analysis*, 9, 148-156.
- [142] Rivard-Boudreau, E. (2017, July 14). Affaissement du ponceau de la route 117 : aucune réparation depuis cinq ans. Retrieved from: <https://ici.radio->

canada.ca/nouvelle/1045281/affaissement-du-ponceau-de-la-route-117-aucune-reparation-depuis-cinq-ans

- [143] Roman, S. (2002). *Writing Excel macros with VBA*. " O'Reilly Media, Inc."
- [144] Ross, P. E. (2013). Boeing's battery blues [News]. *IEEE Spectrum*, 50(3), 11-12.
- [145] Royset, J. O., Kiureghian, A. D., & Polak, E. (2001). Reliability-based optimal design of series structural systems. *Journal of Engineering Mechanics*, 127(6), 607-614.
- [146] Satow, T., & Osaki, S. (2003). Optimal replacement policies for a two-unit system with shock damage interaction. *Computers & Mathematics with Applications*, 46(7), 1129-1138.
- [147] Schall, J. D. (2012). *Hydraulic design of highway culverts* (No. FHWA-HIF-12-026). United States. Federal Highway Administration.
- [148] Shanthikumar, J. G., & Sumita, U. (1983). General shock models associated with correlated renewal sequences. *Journal of Applied Probability*, 20(3), 600-614.
- [149] Shen, G., Zeng, W., Han, C., Liu, P., & Zhang, Y. (2017). Determination of the average maintenance time of CNC machine tools based on type II failure correlation. *Eksploracja i Niezawodność*, 19(4).
- [150] Shen, J., Elwany, A., & Cui, L. (2018). Reliability analysis for multi-component systems with degradation interaction and categorized shocks. *Applied Mathematical Modelling*, 56, 487-500.
- [151] Sheu, S. H., & Griffith, W. S. (2002). Extended block replacement policy with shock models and used items. *European Journal of Operational Research*, 140(1), 50-60.
- [152] Sheut, C., & Krajewski, L. J. (1994). A decision model for corrective maintenance management. *The International Journal of Production Research*, 32(6), 1365-1382.
- [153] Shin, J. H., & Jun, H. B. (2015). On condition based maintenance policy. *Journal of Computational Design and Engineering*, 2(2), 119-127.
- [154] Shrivastava, G. (2016). A Road Collapse at Pont Cassé, Dominica: Hydrologic and Hydraulic Aspects. *WEST INDIAN*, 59.

- [155] Song, S., Coit, D. W., Feng, Q., & Peng, H. (2012). Component reliability with competing failure processes and dependent shock damage. *In IIE annual conference. Proceedings* (p. 1). Institute of Industrial and Systems Engineers (IISE).
- [156] Song, S., Coit, D. W., Feng, Q., & Peng, H. (2014). Reliability analysis for multi-component systems subject to multiple dependent competing failure processes. *Reliability, IEEE Transactions on*, 63(1), 331-345.
- [157] Spreafico, C., Russo, D., & Rizzi, C. (2017). A state-of-the-art review of FMEA/FMECA including patents. *Computer Science Review*, 25, 19-28.
- [158] Sun, Y., & Ma, L. (2010). Estimating Interactive Coefficients for Analysing Interactive Failures. *Maintenance and Reliability*, 2(46), 67-72.
- [159] Sun, Y., Ma, L., Mathew, J., & Zhang, S. (2006). An analytical model for interactive failures. *Reliability Engineering & System Safety*, 91(5), 495-504.
- [160] Sung, C.-K., Sheu, S.-H., Hsu, T.-S., & Chen, Y.-C. (2013). Extended optimal replacement policy for a two-unit system with failure rate interaction and external shocks. *International Journal of Systems Science*, 44(5), 877-888.
- [161] Swain, A. D. (1990). Human reliability analysis: Need, status, trends and limitations. *Reliability Engineering & System Safety*, 29(3), 301-313.
- [162] Tang, Y., Liu, Q., Jing, J., Yang, Y., & Zou, Z. (2017). A framework for identification of maintenance significant items in reliability centered maintenance. *Energy*, 118, 1295-1303.
- [163] Tankov, P. (2003). *Dependence structure of spectrally positive multidimensional Lévy processes*. Unpublished manuscript.
- [164] Thoft-Cristensen, P., & Baker, M. J. (2012). *Structural reliability theory and its applications*. Springer Science & Business Media.
- [165] Thomas, É., Levrat, É., & Iung, B. (2008). Overview on opportunistic maintenance. *IFAC Proceedings Volumes*, 41(3), 245-250.
- [166] Ullah, A., & Jiang, W. (2019). Optimal periodic replacement policy for a warranted product subject to multi modes failure process. *Journal of Management Analytics*, 6(2), 154-172.

- [167] Van Horenbeek, A., & Pintelon, L. (2013). A dynamic predictive maintenance policy for complex multi-component systems. *Reliability Engineering & System Safety*, 120, 39-50.
- [168] Wald, A. (1944). On cumulative sums of random variables. *The Annals of Mathematical Statistics*, 15(3), 283-296.
- [169] Wang, G., Duan, F., & Zhou, Y. (2018). Reliability evaluation of multi-state series systems with performance sharing. *Reliability Engineering & System Safety*, 173, 58-63.
- [170] Wang, G. J., & Zhang, Y. L. (2009). A geometric process repair model for a two-component system with shock damage interaction. *International Journal of Systems Science*, 40(11), 1207-1215.
- [171] Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European journal of operational research*, 139(3), 469-489.
- [172] Wang, J., & Li, M. (2015). Redundancy Allocation for Reliability Design of Engineering Systems With Failure Interactions. *Journal of Mechanical Design*, 137(3), 031403.
- [173] Wang, R., & Chen, N. (2016, December). A survey of condition-based maintenance modeling of multi-component systems. In *2016 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)* (pp. 1664-1668). IEEE.
- [174] Wang, Y., & Pham, H. (2012). Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas. *Reliability, IEEE Transactions on*, 61(1), 13-22.
- [175] Williard, N., He, W., Hendricks, C., & Pecht, M. (2013). Lessons learned from the 787 Dreamliner issue on lithium-ion battery reliability. *Energies*, 6(9), 4682-4695.
- [176] Xi, Z., Jing, R., Wang, P., & Hu, C. (2014). A copula-based sampling method for data-driven prognostics. *Reliability Engineering & System Safety*, 132, 72-82.
- [177] Xu, L., Chen, Y., Briand, F., Zhou, F., & Givanni, M. (2017). Reliability Measurement for Multistate Manufacturing Systems with Failure Interaction. *Procedia CIRP*, 63, 242-247.
- [178] Yang, Q., Hong, Y., Zhang, N., & Li, J. (2017). A copula-based trend-renewal process model for analysis of repairable systems with multitype failures. *IEEE Transactions on Reliability*, 66(3), 590-602.

- [179] Yan, J., Meng, Y., Lu, L., & Li, L. (2017). Industrial big data in an industry 4.0 environment: Challenges, schemes, and applications for predictive maintenance. *IEEE Access*, 5, 23484-23491.
- [180] Yang, Q., Zhang, N., & Hong, Y. (2013). Reliability analysis of repairable systems with dependent component failures under partially perfect repair. *IEEE Transactions on Reliability*, 62(2), 490-498.
- [181] Yang, Z., Zhao, J., Guo, C., & Li, L. (2018). Degradation and Reliability Modeling of Two-Component System with Degradation Rate Interaction. *International Journal of Performability Engineering*, 14(11).
- [182] Yeh, R. H., Chen, M. Y., & Lin, C. Y. (2007). Optimal periodic replacement policy for repairable products under free-repair warranty. *European Journal of Operational Research*, 176(3), 1678-1686.
- [183] Yousefi, N., Coit, D. W., Song, S., & Feng, Q. (2019). Optimization of On-condition Thresholds for a System of Degrading Components with Competing Dependent Failure Processes. *Reliability Engineering & System Safety*, 106547.
- [184] Zequeira, R. I., & Bérenguer, C. (2006). Periodic imperfect preventive maintenance with two categories of competing failure modes. *Reliability Engineering & System Safety*, 91(4), 460-468.
- [185] Zequeira, R. I., Valdes, J. E., & Berenguer, C. (2008). Optimal buffer inventory and opportunistic preventive maintenance under random production capacity availability. *International Journal of Production Economics*, 111(2), 686-696.
- [186] Zhang, N., Fouladirad, M., & Barros, A. (2017). Warranty analysis of a two-component system with failure interaction. Risk, Reliability and Safety: Innovating Theory and Practice: *Proceedings of ESREL 2016* (Glasgow, Scotland, 25-29 September 2016).
- [187] Zhang, N., Fouladirad, M., & Barros, A. (2018). Evaluation of the warranty cost of a product with type III stochastic dependence between components. *Applied Mathematical Modelling*, 59, 39-53.
- [188] Zhang, N., & Yang, Q. (2015). Optimal maintenance planning for repairable multi-component systems subject to dependent competing risks. *IIE Transactions*, 47(5), 521-532.

- [189] Zhang, X. P., Shang, J. Z., Chen, X., Zhang, C. H., & Wang, Y. S. (2014). Statistical inference of accelerated life testing with dependent competing failures based on copula theory. *IEEE Transactions on Reliability*, 63(3), 764-780.
- [190] Zhang, Z., Wu, S., Lee, S., & Ni, J. (2014). Modified iterative aggregation procedure for maintenance optimisation of multi-component systems with failure interaction. *International Journal of Systems Science*, 45(12), 2480-2489.
- [191] Zhang, Z., Wu, S., Li, B., & Lee, S. (2015). (n, N) type maintenance policy for multi-component systems with failure interactions. *International Journal of Systems Science*, 46(6), 1051-1064.
- [192] Zhao, T., Qi, X., & Liu, J. (2013). *Interactive failure analysis of Gyroscope based on net failure mode and effect analysis*. Paper presented at the Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE), 2013 International Conference on.
- [193] Zhou, X., Xi, L., & Lee, J. (2007). Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation. *Reliability Engineering & System Safety*, 92(4), 530-534.
- [194] Zhou, Y., Zhang, Z., Lin, T. R., & Ma, L. (2013). Maintenance optimisation of a multi-state series-parallel system considering economic dependence and state-dependent inspection intervals. *Reliability Engineering & System Safety*, 111, 248-259.
- [195] Zio, E. (2009). Reliability engineering: Old problems and new challenges. *Reliability Engineering & System Safety*, 94(2), 125-141.

ANNEXE A DEMONSTRATIONS RELATIVES AU CHAPITRE 4

A.1 Demonstration that $P(n_k^{(a)} = l) = \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{v_r}^{(a)}}{1-p_{v_r}^{(a)}} \right) (\prod_{h=1}^k (1-p_h^{(a)}))$

Assume that for $1 < l \leq k$, $P(n_k^{(a)} = l) = p_k^{(a)} \cdot P(n_{k-1}^{(a)} = l-1) + (1-p_k^{(a)}) \cdot P(n_{k-1}^{(a)} = l)$,
 $l = 1, 2, \dots$

$$\begin{aligned} P(n_k^{(a)} = l) &= p_k^{(a)} \cdot P(n_{k-1}^{(a)} = l-1) \\ &+ (1-p_k^{(a)}) \left\{ p_{k-1}^{(a)} \cdot P(n_{k-2}^{(a)} = l-1) \right. \\ &+ (1-p_{k-1}^{(a)}) \left\{ \dots \dots \dots \dots \dots \dots \dots \right. \\ &\left. \left. \left. + (1-p_{l+2}^{(a)}) \left\{ p_{l+1}^{(a)} \cdot P(n_l^{(a)} = l-1) + (1-p_{l+1}^{(a)}) \left\{ p_l^{(a)} \cdot P(n_{l-1}^{(a)} = l-1) \right\} \right\} \right\} \right\} \end{aligned}$$

Under the condition $p_{k+1}^{(a)} < 1$, it follows that:

$$P(n_k^{(a)} = l) = \frac{1}{1-p_{k+1}^{(a)}} \sum_{v=l}^k P(n_{v-1}^{(a)} = l-1) \cdot p_v^{(a)} \prod_{h=v+1}^{k+1} (1-p_h^{(a)})$$

$P(n_k^{(a)} = l)$ can be written without the quantities $n_v^{(a)}$ as follows:

$$\begin{aligned} P(n_k^{(a)} = l) &= \frac{1}{1-p_{k+1}^{(a)}} \sum_{v_0=l}^k \left\{ \frac{1}{1-p_{v_0}^{(a)}} \sum_{v_1=l-1}^{v_0-1} \left[\frac{1}{1-p_{v_1}^{(a)}} \sum_{v_2=l-2}^{v_1-1} \left(\dots \left(\frac{1}{1-p_{v_{l-2}}^{(a)}} \sum_{v_{l-1}=1}^{v_{l-2}-1} P(n_{v_{l-1}-1}^{(a)} \right. \right. \right. \right. \\ &= 0) \cdot p_{v_{l-1}}^{(a)} \prod_{h=v_{l-1}+1}^{v_{l-2}} (1-p_h^{(a)}) \dots \left. \right) \cdot p_{v_2}^{(a)} \prod_{h=v_2+1}^{v_1} (1-p_h^{(a)}) \left. \right] \\ &\cdot p_{v_1}^{(a)} \prod_{h=v_1+1}^{v_0} (1-p_h^{(a)}) \left. \right\} \cdot p_{v_0}^{(a)} \prod_{h=v_0+1}^{k+1} (1-p_h^{(a)}) \end{aligned}$$

where $v_0 \in (l, \dots, k)$, $v_1 \in (l-1, \dots, k-1)$, $\dots$, $v_{l-2} \in (1, \dots, k+1-l)$, $v_{l-1} \in (1, \dots, k-l)$.

Since $P(n_{v_{l-1}-1}^{(a)} = 0) = \prod_{h=1}^{v_{l-1}-1} (1 - p_h^{(a)})$, then $P(n_k^{(a)} = l)$ can be transformed in a combination of the probabilities $p_h^{(a)}$, thus providing the following general expression for any $a \in \{1, 2\}$ and for $1 < l \leq k$:

$$P(n_k^{(a)} = l) = \sum_{v_0=l}^k \sum_{v_1=l-1}^{v_0-1} \dots \sum_{v_{l-1}=1}^{v_{l-2}-1} \left(\prod_{r=0}^{l-1} \frac{p_{v_r}^{(a)}}{1 - p_{v_r}^{(a)}} \right) \left(\prod_{h=1}^k (1 - p_h^{(a)}) \right)$$

where $1 < l \leq k, v_0 \in (l, \dots, k), v_1 \in (l-1, \dots, k-1), \dots, v_{l-1} \in (1, \dots, k+1-l)$.

A.2 Demonstration that $Pr_{ij}^{(a)}(t, t+s) = \delta_{ij}^{(a)} + q_{ij}^{(a)}s + o(s)$

Considering a very short interval of time s ,

$$\begin{aligned} Pr_{ij}^{(a)}(t, t+s) &= P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} \\ &= P\{(N^{(a)}(t+s) = l_j, N(t+s) = k_j) | (N^{(a)}(t) = l_i, N(t) = k_i)\} \end{aligned}$$

where i, j, k_j, l_j, k_i, l_i are positive integers. It is assumed throughout this paper that:

$$\begin{aligned} &P\{(N^{(a)}(t+s) = l_j, N(t+s) = k_j) | (N^{(a)}(t) = l_i, N(t) = k_i)\} \\ &= P\{N^{(a)}(t+s) = l_j, N(t+s) = k_j | N^{(a)}(t) = l_i, N(t) = k_i\} \\ &= P\{N^{(a)}(t+s) - N^{(a)}(t) = l_j - l_i, N(t+s) - N(t) = k_j - k_i | N^{(a)}(t) = l_i, N(t) = k_i\} \end{aligned}$$

This leads to deducing the following based on the Bayesian rule:

$$\begin{aligned} &P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} \\ &= P\{N^{(a)}(t+s) - N^{(a)}(t) = l_j - l_i | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} \\ &\times P\{N(t+s) - N(t) = k_j - k_i | N^{(a)}(t) = l_i, N(t) = k_i\} \end{aligned}$$

There are limited distinct cases to consider, such as $k_j = k_i$ and $k_j > k_i$. In other cases, the transition probability is null. If $k_j - k_i = 0$, it is an absolute certainty that there will be no shock (a) if the drawing process does not produce a new shock.

Thus, $P\{N^{(a)}(t+s) - N^{(a)}(t) = 0 | N(t+s) - N(t) = 0, N^{(a)}(t) = l_i, N(t) = k_i\} = 1$ and $P\{X^{(a)}(t+s) = i | X^{(a)}(t) = i\} = P\{N(t+s) - N(t) = 0 | N^{(a)}(t) = l_i, N(t) = k_i\}$.

If the drawing process follows a homogeneous Poisson process, then,

$$P\{X^{(a)}(t+s) = i | X^{(a)}(t) = i\} = 1 - \lambda_i s + o(s)$$

where $o(s)$ denotes a general and unspecified remainder term of smaller order than s .

In the eventuality that $k_j > k_i$, the possible transition probabilities would be non-null if $l_j \geq l_i$ and $l_j - l_i \leq k_j - k_i$, in such case:

$$\begin{cases} P\{N^{(a)}(t+s) - N^{(a)}(t) = 0 | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} = 1 - p_{k_j}^{(a)} \\ P\{N^{(a)}(t+s) - N^{(a)}(t) = 1 | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} = p_{k_j}^{(a)} \end{cases}$$

If the drawing process follows a homogeneous Poisson process, then,

$$P\{N(t+s) - N(t) = 1 | N^{(a)}(t) = l_i, N(t) = k_i\} = \lambda_i s + o(s)$$

It ensues that:

$$\begin{cases} P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} = (1 - p_{k_j}^{(a)}) \lambda_i s + o(s), & l_j - l_i = 0 \\ P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} = p_{k_j}^{(a)} \lambda_i s + o(s), & l_j - l_i = 1 \end{cases}$$

If more than one shock occurs, then $k_j - k_i > 1$ and the following cases are distinguished:

$$\begin{aligned} P\{N^{(a)}(t+s) - N^{(a)}(t) = 0 | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} \\ = \prod_{h=k_{i+1}}^{k_j} (1 - p_v^{(a)}) \end{aligned}$$

$$\begin{aligned} P\{N^{(a)}(t+s) - N^{(a)}(t) = 1 | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} \\ = \sum_{v=k_{i+1}}^{k_j} \frac{p_v^{(a)}}{1 - p_v^{(a)}} \prod_{h=k_{i+1}}^{k_j} (1 - p_h^{(a)}) \end{aligned}$$

$$\begin{aligned} P\{N^{(a)}(t+s) - N^{(a)}(t) = l_j - l_i | N(t+s) - N(t) = k_j - k_i, N^{(a)}(t) = l_i, N(t) = k_i\} \\ = \sum_{v_0=k_i+l_j-l_i}^{k_j} \sum_{v_1=k_i+l_j-l_i-1}^{v_0-1} \dots \sum_{v_{l_j-l_i-1}=k_{i+1}}^{v_{l_j-l_i-2}-1} \left(\prod_{r=0}^{l_j-l_i-1} \frac{p_{v_r}^{(a)}}{1 - p_{v_r}^{(a)}} \right) \left(\prod_{h=k_{i+1}}^{k_j} (1 - p_h^{(a)}) \right), \end{aligned}$$

$$1 < l_j - l_i \leq k_j - k_i, v_0 \in (k_i + l_j - l_i, \dots, k_j), v_1 \in (k_i + l_j - l_i - 1, \dots, k_j - 1),$$

$$\dots \dots \dots, v_{l_j-l_i-1} \in (k_i + 1, \dots, k_j - (l_j - l_i - 1))$$

Under a homogenous Poisson process,

$$P\{N(t+s) - N(t) = k_j - k_i | N^{(a)}(t) = l_i, N(t) = k_i\} = o(s)$$

It can then be deduced that:

$$P\{X^{(a)}(t+s) = j | X^{(a)}(t) = i\} = o(s), \quad 0 \leq l_j - l_i \leq k_j - k_i, \quad k_j - k_i > 1$$

The above observations are summed up as follows:

$$Pr_{i,j}^{(a)}(t, t+s) = \delta_{ij}^{(a)} + q_{ij}^{(a)}s + o(s)$$

$$\text{where } q_{ij}^{(a)} = \begin{cases} -\lambda_i & , i = j \\ (1 - p_{k_j}^{(a)})\lambda_i, k_j - k_i = 1, l_j - l_i = 0 \\ p_{k_j}^{(a)} \cdot \lambda_i & , k_j - k_i = 1, l_j - l_i = 1 \\ 0 & \text{otherwise} \end{cases} \text{ and } \delta_{ij}^{(a)} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \text{ is Kronecker's delta.}$$

A.3 Demonstration that $E\{N^{(a)}(t) | N(t) = k\} = \sum_{l=1}^k p_l^{(a)}$

The expected quantities of $N^{(a)}(t)$ knowing $N(t)$ are given by:

$$E\{N^{(a)}(t) | N(t) = 0\} = 0$$

$$E\{N^{(a)}(t) | N(t) = k\} = E[n_k^{(a)}] = \sum_{l=1}^k l \cdot P(n_k^{(a)} = l), k = 1, 2, 3, \dots$$

Knowing that $P(n_k^{(a)} = l) = P(n_{k-1}^{(a)} = l-1) \cdot p_k^{(a)} + P(n_{k-1}^{(a)} = l) \cdot (1 - p_k^{(a)})$ and

$P(n_k^{(a)} = k) = \prod_{j=1}^k p_j^{(a)}$, $E\{N^{(a)}(t) | N(t) = k\}$ can be developed in the following manner:

$$\begin{aligned} E\{N^{(a)}(t) | N(t) = k\} &= \sum_{l=0}^{k-1} \{l \cdot P(n_{k-1}^{(a)} = l) \cdot p_k^{(a)}\} - (k-1) \cdot P(n_{k-1}^{(a)} = k-1) \cdot p_k^{(a)} \\ &+ \sum_{l=1}^{k-1} P(n_{k-1}^{(a)} = l-1) \cdot p_k^{(a)} + \sum_{l=1}^{k-1} l \cdot \{P(n_{k-1}^{(a)} = l) \cdot (1 - p_k^{(a)})\} + k \\ &\cdot P(n_k^{(a)} = k) \end{aligned}$$

$$E\{N^{(a)}(t)|N(t) = k\} = E[n_{k-1}^{(a)}] + p_k^{(a)} \cdot \sum_{l=1}^k P(n_{k-1}^{(a)} = l - 1)$$

Knowing that $\sum_{l=1}^k P(n_{k-1}^{(a)} = l - 1) = 1$, recurrence reasoning leads to obtain:

$$E\{N^{(a)}(t)|N(t) = k\} = \sum_{l=1}^k p_l^{(a)}$$

A.4 Approximation of $E[N^{(a)}(t)]$

$$E[N^{(a)}(t)] = \sum_{k=1}^{\infty} P\{N(t) = k\} \cdot \sum_{l=1}^k p_l^{(a)}$$

$$E[N^{(a)}(t)] = \sum_{k=1}^m P\{N(t) = k\} \cdot \sum_{l=1}^k p_l^{(a)} + \sum_{k=m+1}^{\infty} P\{N(t) = k\} \cdot \left[\sum_{l=1}^m p_l^{(a)} + \sum_{l=m+1}^k p_l^{(a)} \right]$$

where m is the maximum number of shocks ever recorded on the type of system studied throughout its lifespan. Under the assumption that all shocks a beyond m occur with instant probabilities $p_l^{(a)} \approx p_m^{(a)}$ ($l > m$) then:

$$\sum_{l=m+1}^k p_l^{(a)} \approx p_m^{(a)}(k - m), \quad k > m$$

It follows that:

$$\begin{aligned} E[N^{(a)}(t)] &\approx \sum_{k=1}^m P\{N(t) = k\} \cdot \sum_{l=1}^k p_l^{(a)} + \sum_{k=m+1}^{\infty} P\{N(t) = k\} \cdot \sum_{l=1}^m p_l^{(a)} \\ &\quad + \sum_{k=m+1}^{\infty} P\{N(t) = k\} \cdot p_m^{(a)}(k - m) \\ E[N^{(a)}(t)] &\approx \sum_{k=1}^m P\{N(t) = k\} \sum_{l=1}^k p_l^{(a)} + \left[\sum_{l=1}^m p_l^{(a)} \right] \cdot \left[\sum_{k=m+1}^{\infty} P\{N(t) = k\} \right] \\ &\quad + p_m^{(a)} \left[\sum_{k=m+1}^{\infty} k \cdot P\{N(t) = k\} - m \sum_{k=m+1}^{\infty} P\{N(t) = k\} \right] \end{aligned}$$

$$E[N^{(a)}(t)] \approx \sum_{k=1}^m P\{N(t) = k\} \sum_{l=1}^k p_l^{(a)} + \left[-m \cdot p_m^{(a)} + \sum_{l=1}^m p_l^{(a)} \right] \cdot \left[\sum_{k=m+1}^{\infty} P\{N(t) = k\} \right] \\ + p_m^{(a)} \left[\sum_{k=m+1}^{\infty} k \cdot P\{N(t) = k\} \right]$$

$$E[N^{(a)}(t)] \approx \sum_{k=1}^m P\{N(t) = k\} \sum_{l=1}^k p_l^{(a)} + \left[-m \cdot p_m^{(a)} + \sum_{l=1}^m p_l^{(a)} \right] \\ \cdot \left[\sum_{k=0}^{\infty} P\{N(t) = k\} - \sum_{k=0}^m P\{N(t) = k\} \right] \\ + p_m^{(a)} \left[\sum_{k=1}^{\infty} k \cdot P\{N(t) = k\} - \sum_{k=1}^m k \cdot P\{N(t) = k\} \right]$$

$$E[N^{(a)}(t)] \approx \sum_{k=1}^m P\{N(t) = k\} \sum_{l=1}^k p_l^{(a)} + \left[\sum_{l=1}^m p_l^{(a)} - m p_m^{(a)} \right] \left[1 - \sum_{k=0}^m P\{N(t) = k\} \right] \\ + p_m^{(a)} \left[\lambda t - \sum_{k=1}^m k \cdot P\{N(t) = k\} \right]$$

ANNEXE B MACROS ET CAPTURES D'ÉCRAN DES SIMULATIONS DE LA STRATÉGIE DE MAINTENANCE PRÉDICTIVE DU CHAPITRE 5

B.1 Macro de simulation de la stratégie de maintenance prédictive

Sub Simulation()

'Initialization

Iteration = 1000

'Clean simulation

Range(Sheets("Simulation").Cells(4, 21), Sheets("Simulation").Cells(1003, 116)).ClearContents

'Loop and Record (set new period, set new prob, record costs, record failure time)

Qins = Sheets("Simulation").Cells(2, 21).Value 'inspection parameter

For N = 1 To 16

LS1 = Sheets("Simulation").Cells(2, 22 + 6 * (N - 1)).Value 'PdM threshold S1

LS2 = Sheets("Simulation").Cells(2, 23 + 6 * (N - 1)).Value 'PdM threshold S2

For i = 1 To Iteration

Range(Sheets("IntProb").Cells(3, 2), Sheets("IntProb").Cells(7, 3)) = Range(Sheets("IntProb").Cells(11, 2), Sheets("IntProb").Cells(15, 3)).Value 'Starting probabilities

'Starting simulation Parameters

```
Sheets("Simulation").Cells(2, 5) = 0  
Sheets("Simulation").Cells(9, 5) = 0  
Sheets("Simulation").Cells(9, 6) = 0  
Sheets("Simulation").Cells(9, 7) = 0  
Sheets("Simulation").Cells(9, 9) = 0  
Sheets("Simulation").Cells(12, 3) = 0  
Sheets("Simulation").Cells(12, 4) = 0  
Sheets("Simulation").Cells(12, 5) = 0  
Sheets("Simulation").Cells(12, 7) = 0  
Sheets("Simulation").Cells(12, 12) = 0  
Sheets("Simulation").Cells(12, 13) = 0  
Sheets("Simulation").Cells(12, 14) = 0  
Sheets("Simulation").Cells(12, 15) = 0
```

'PdM Loop

```
Do Until (Sheets("Simulation").Cells(12 + Qins, 3).Value >= Sheets("Simulation").Cells(8, 2).Value Or Sheets("Simulation").Cells(12 + Qins, 4).Value >= Sheets("Simulation").Cells(8, 3).Value)
```

```
    Sheets("Simulation").Cells(9, 5) = Sheets("Simulation").Cells(9, 5) + 1 'Number of cumulated inspections (- 1)
```

```
    If (Sheets("Simulation").Cells(12 + Qins, 3).Value < Sheets("Simulation").Cells(8, 2).Value And Sheets("Simulation").Cells(12 + Qins, 4).Value < Sheets("Simulation").Cells(8, 3).Value) Then
```

```
        If Sheets("Simulation").Cells(12 + Qins, 3).Value >= LS1 Then
```

```
            Sheets("Simulation").Cells(12, 3) = 0
```

```
            Sheets("Simulation").Cells(12, 12) = 0
```

```
            Sheets("Simulation").Cells(12, 14) = 0
```

```
            Sheets("Simulation").Cells(9, 6) = Sheets("Simulation").Cells(9, 6) + 1
```

```
            Range(Sheets("IntProb").Cells(3, 2), Sheets("IntProb").Cells(7, 2)) = Range(Sheets("IntProb").Cells(11, 2), Sheets("IntProb").Cells(15, 2)).Value 'Restarting probabilities
```

```
        Else
```

```
            Sheets("Simulation").Cells(12, 3) = Sheets("Simulation").Cells(12 + Qins, 3).Value
```

```
            Sheets("Simulation").Cells(12, 12) = Sheets("Simulation").Cells(12 + Qins, 12).Value
```

```
            Sheets("Simulation").Cells(12, 14) = Sheets("Simulation").Cells(12 + Qins, 14).Value
```

```
            If Sheets("Simulation").Cells(12 + Qins, 16) = 1 Then
```

```
                Sheets("IntProb").Cells(3, 2) = Sheets("IntProb").Cells(12, 2)
```

```
                Sheets("IntProb").Cells(4, 2) = Sheets("IntProb").Cells(13, 2)
```

```
                Sheets("IntProb").Cells(5, 2) = Sheets("IntProb").Cells(14, 2)
```

```
                Sheets("IntProb").Cells(6, 2) = Sheets("IntProb").Cells(15, 2)
```

```
            End If
```

```
If Sheets("Simulation").Cells(12 + Qins, 16) = 2 Then
    Sheets("IntProb").Cells(3, 2) = Sheets("IntProb").Cells(13, 2)
    Sheets("IntProb").Cells(4, 2) = Sheets("IntProb").Cells(14, 2)
    Sheets("IntProb").Cells(5, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(6, 2) = Sheets("IntProb").Cells(15, 2)
End If

If Sheets("Simulation").Cells(12 + Qins, 16) = 3 Then
    Sheets("IntProb").Cells(3, 2) = Sheets("IntProb").Cells(14, 2)
    Sheets("IntProb").Cells(4, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(5, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(6, 2) = Sheets("IntProb").Cells(15, 2)
End If

If Sheets("Simulation").Cells(12 + Qins, 16) >= 4 Then
    Sheets("IntProb").Cells(3, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(4, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(5, 2) = Sheets("IntProb").Cells(15, 2)
    Sheets("IntProb").Cells(6, 2) = Sheets("IntProb").Cells(15, 2)
End If

End If

If Sheets("Simulation").Cells(12 + Qins, 4).Value >= LS2 Then
    Sheets("Simulation").Cells(12, 4) = 0
    Sheets("Simulation").Cells(12, 13) = 0
    Sheets("Simulation").Cells(12, 15) = 0
```

```
Sheets("Simulation").Cells(9, 7) = Sheets("Simulation").Cells(9, 7) + 1
```

```
Range(Sheets("IntProb").Cells(3, 3), Sheets("IntProb").Cells(7, 3)) = Range(Sheets("IntProb").Cells(11, 3),  
Sheets("IntProb").Cells(15, 3)).Value 'Restarting probabilities
```

```
Else
```

```
Sheets("Simulation").Cells(12, 4) = Sheets("Simulation").Cells(12 + Qins, 4).Value
```

```
Sheets("Simulation").Cells(12, 13) = Sheets("Simulation").Cells(12 + Qins, 13).Value
```

```
Sheets("Simulation").Cells(12, 15) = Sheets("Simulation").Cells(12 + Qins, 15).Value
```

```
If Sheets("Simulation").Cells(12 + Qins, 16) = 1 Then
```

```
    Sheets("IntProb").Cells(3, 3) = Sheets("IntProb").Cells(12, 3)
```

```
    Sheets("IntProb").Cells(4, 3) = Sheets("IntProb").Cells(13, 3)
```

```
    Sheets("IntProb").Cells(5, 3) = Sheets("IntProb").Cells(14, 3)
```

```
    Sheets("IntProb").Cells(6, 3) = Sheets("IntProb").Cells(15, 3)
```

```
End If
```

```
If Sheets("Simulation").Cells(12 + Qins, 16) = 2 Then
```

```
    Sheets("IntProb").Cells(3, 3) = Sheets("IntProb").Cells(13, 3)
```

```
    Sheets("IntProb").Cells(4, 3) = Sheets("IntProb").Cells(14, 3)
```

```
    Sheets("IntProb").Cells(5, 3) = Sheets("IntProb").Cells(15, 3)
```

```
    Sheets("IntProb").Cells(6, 3) = Sheets("IntProb").Cells(15, 3)
```

```
End If
```

```
If Sheets("Simulation").Cells(12 + Qins, 16) = 3 Then
```

```
    Sheets("IntProb").Cells(3, 3) = Sheets("IntProb").Cells(14, 3)
```

```
    Sheets("IntProb").Cells(4, 3) = Sheets("IntProb").Cells(15, 3)
```

```
    Sheets("IntProb").Cells(5, 3) = Sheets("IntProb").Cells(15, 3)
```

```
Sheets("IntProb").Cells(6, 3) = Sheets("IntProb").Cells(15, 3)
```

```
End If
```

```
If Sheets("Simulation").Cells(12 + Qins, 16) >= 4 Then
```

```
    Sheets("IntProb").Cells(3, 3) = Sheets("IntProb").Cells(15, 3)
```

```
    Sheets("IntProb").Cells(4, 3) = Sheets("IntProb").Cells(15, 3)
```

```
    Sheets("IntProb").Cells(5, 3) = Sheets("IntProb").Cells(15, 3)
```

```
    Sheets("IntProb").Cells(6, 3) = Sheets("IntProb").Cells(15, 3)
```

```
End If
```

```
End If
```

```
Sheets("Simulation").Cells(12, 5) = Sheets("Simulation").Cells(12 + Qins, 5).Value
```

```
Sheets("Simulation").Cells(12, 7) = Sheets("Simulation").Cells(12 + Qins, 7).Value
```

```
End If
```

```
Sheets("Simulation").Cells(2, 5) = Sheets("Simulation").Cells(2, 5).Value + 1 'Age Adjustment
```

```
Loop
```

```
Sheets("Simulation").Cells(9, 9) = Sheets("Simulation").Cells(112, 17).Value
```

```
Sheets("Simulation").Cells(i + 3, 21 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 5).Value + 1 'Number of cumulated inspections
```

```
Sheets("Simulation").Cells(i + 3, 22 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 6).Value
```

```
Sheets("Simulation").Cells(i + 3, 23 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 7).Value
```

```
Sheets("Simulation").Cells(i + 3, 24 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 8).Value
```

```
Sheets("Simulation").Cells(i + 3, 25 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 9).Value
```

```
Sheets("Simulation").Cells(i + 3, 26 + 6 * (N - 1)) = Sheets("Simulation").Cells(9, 10).Value
```

```
Next
```

```

Sheets("Summary").Cells(7 * (N - 1) + 2, 3 + Qins) = Sheets("Simulation").Cells(1007, 21 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 3, 3 + Qins) = Sheets("Simulation").Cells(1007, 22 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 4, 3 + Qins) = Sheets("Simulation").Cells(1007, 23 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 5, 3 + Qins) = Sheets("Simulation").Cells(1007, 24 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 6, 3 + Qins) = Sheets("Simulation").Cells(1007, 25 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 7, 3 + Qins) = Sheets("Simulation").Cells(1007, 26 + 6 * (N - 1)).Value
Sheets("Summary").Cells(7 * (N - 1) + 8, 3 + Qins) = Sheets("Simulation").Cells(1009, 26 + 6 * (N - 1)).Value

```

Next

'Index Minimum of PdM policy

```
MinCost = WorksheetFunction.Min(Range(Sheets("Cost Summary").Cells(3, 2 + Qins), Sheets("Cost Summary").Cells(18, 2 + Qins)))
```

For j = 1 To 16

```
If Sheets("Cost Summary").Cells(j + 2, 2 + Qins) = MinCost Then
```

```
    Sheets("Cost Summary").Cells(j + 2, 2 + Qins).Font.Color = RGB(255, 0, 0)
```

```
End If
```

Next

End Sub

B.2 Capture d'écran des fenêtres de la simulation sous Microsoft Excel

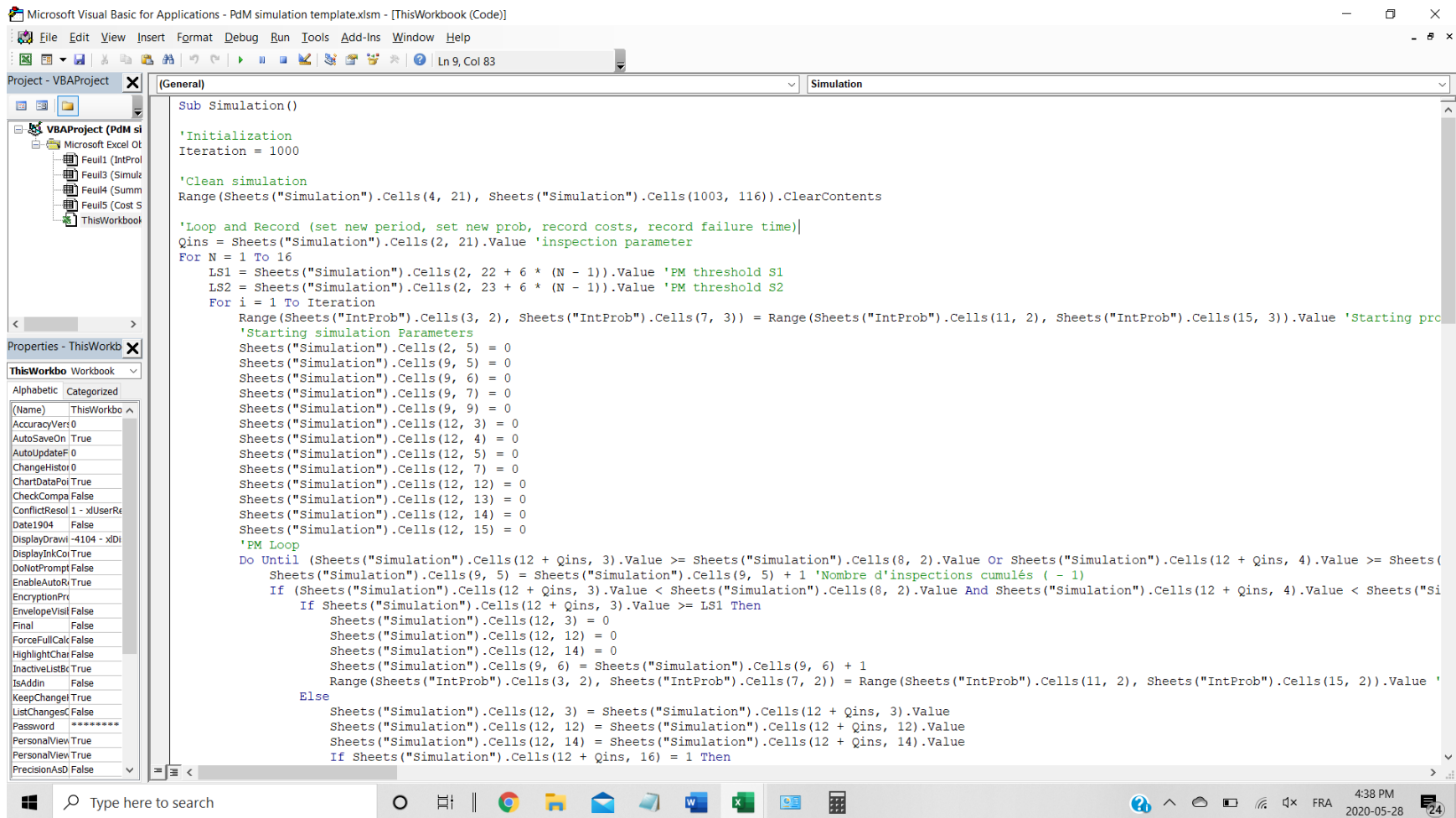


Figure B.1 Capture d'écran de la Macro de simulation de la stratégie de maintenance prédictive sous Microsoft Excel

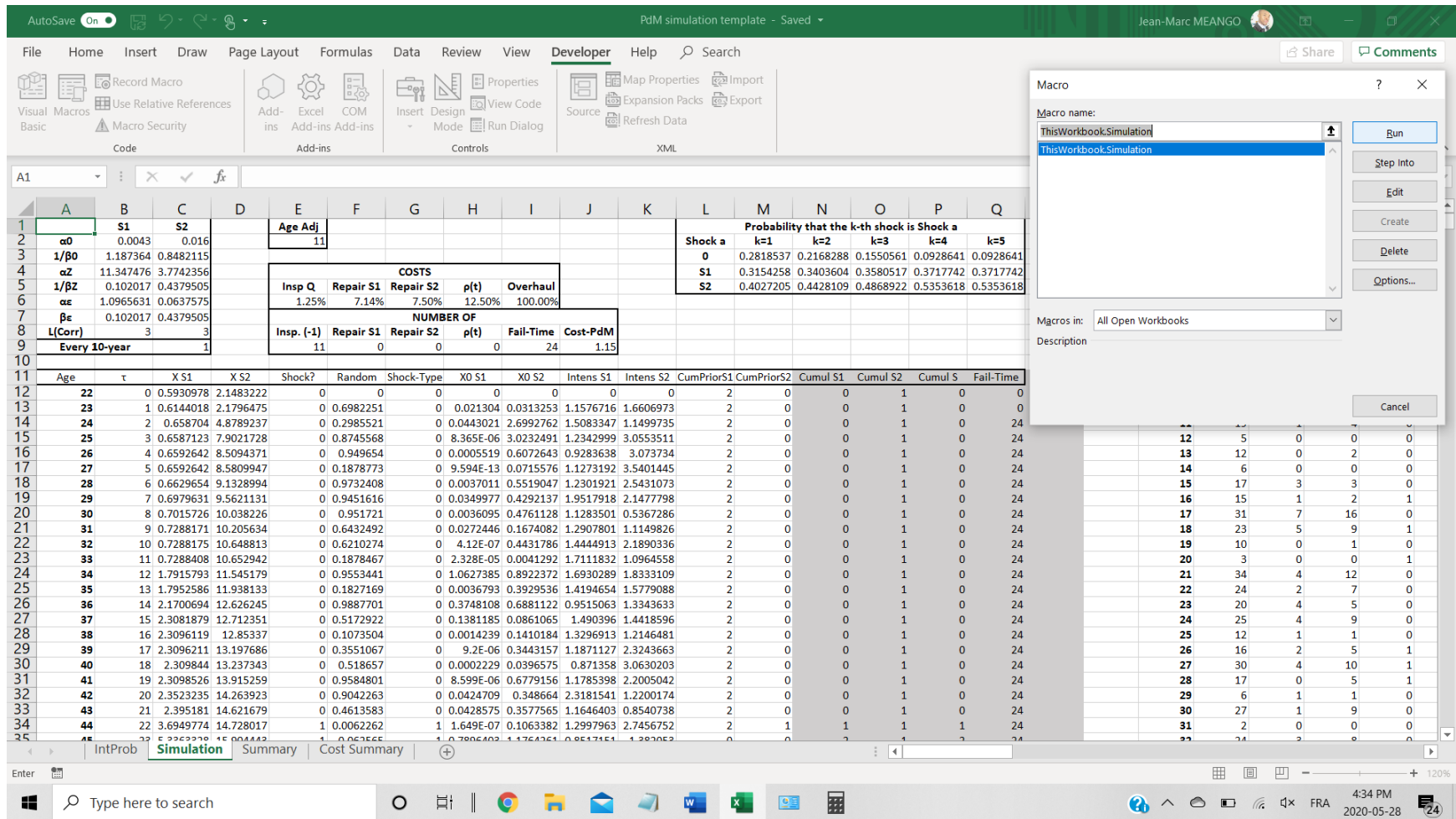


Figure B.2 Capture d'écran partielle 1 du feuillet « Simulation » sous Microsoft Excel

The screenshot displays the Microsoft Excel interface with the 'Simulation' tab selected. The worksheet contains a table with the following columns: Iteration, Insp., Repair S1, Repair S2, p(t), Fail-Time, Cost-PdM, and several other columns labeled with letters (AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO, AP, AQ, AR). The table contains numerical data for iterations 1 through 35. The Excel interface includes the ribbon with tabs like File, Home, Insert, Draw, Page Layout, Formulas, Data, Review, View, Developer, Help, and Search. The status bar at the bottom shows the time as 4:36 PM on 2020-05-28.

Iteration	Insp.	Repair S1	Repair S2	p(t)	Fail-Time	Cost-PdM	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR
1	10	2	1	1	19	1.467857	8	0	0	0	16	1.1	13	0	2	0	26	1.3125	17	3	2	0	34	1.576786
2	29	6	13	1	57	2.891071	24	2	6	0	48	1.892857	16	2	3	0	32	1.567857	7	1	0	1	13	1.283929
3	16	1	2	0	32	1.421429	18	1	4	1	35	1.721429	15	0	2	0	30	1.3375	10	1	1	1	19	1.396429
4	8	0	0	0	16	1.1	19	4	3	0	38	1.748214	12	1	0	0	24	1.221429	14	0	1	0	28	1.25
5	12	1	2	0	24	1.371429	14	2	2	0	28	1.467857	6	1	0	0	12	1.146429	16	4	2	1	31	1.760714
6	12	1	3	0	24	1.446429	3	0	0	0	6	1.0375	20	3	3	1	39	1.814286	7	0	0	0	14	1.0875
7	17	3	3	0	34	1.651786	5	0	0	0	10	1.0625	11	2	1	1	21	1.480357	10	0	0	1	19	1.25
8	16	1	2	0	32	1.421429	10	1	0	0	20	1.196429	25	5	5	1	49	2.169643	16	2	2	0	32	1.492857
9	24	3	10	0	48	2.264286	11	0	0	0	22	1.1375	6	1	0	1	11	1.271429	6	0	0	1	11	1.2
10	7	0	1	0	14	1.1625	32	7	12	0	64	2.8	22	2	5	0	44	1.792857	12	1	0	1	23	1.346429
11	19	1	4	0	38	1.608929	7	0	1	1	13	1.2875	16	2	1	1	31	1.542857	10	1	0	0	20	1.196429
12	5	0	0	0	10	1.0625	22	3	4	1	43	1.914286	13	0	1	0	26	1.2375	11	1	0	1	21	1.333929
13	12	0	2	0	24	1.3	3	0	0	1	5	1.1625	13	0	1	0	26	1.2375	11	0	1	0	22	1.2125
14	6	0	0	0	12	1.075	21	4	4	0	42	1.848214	7	0	0	1	13	1.2125	23	2	4	0	46	1.730357
15	17	3	3	0	34	1.651786	15	1	3	0	30	1.483929	8	2	0	1	15	1.367857	11	2	1	0	22	1.355357
16	15	1	2	1	29	1.533929	6	1	0	0	12	1.146429	3	0	0	1	5	1.1625	19	2	2	0	38	1.530357
17	31	7	16	0	62	3.0875	6	0	0	0	12	1.075	17	2	3	1	33	1.705357	3	0	0	0	6	1.0375
18	23	5	9	1	45	2.444643	10	1	1	0	20	1.271429	26	2	6	1	51	2.042857	10	1	0	1	19	1.321429
19	10	0	1	0	20	1.2	10	0	0	0	20	1.125	9	0	1	0	18	1.1875	11	1	1	0	22	1.283929
20	3	0	0	1	5	1.1625	5	0	0	1	9	1.1875	6	1	0	1	11	1.271429	10	0	0	0	20	1.125
21	34	4	12	0	68	2.610714	22	2	5	0	44	1.792857	19	4	3	0	38	1.748214	8	1	0	0	16	1.171429
22	24	2	7	0	48	1.967857	17	2	3	0	34	1.580357	11	1	0	0	22	1.208929	9	0	0	1	17	1.2375
23	20	4	5	0	40	1.910714	21	2	2	0	42	1.555357	23	4	4	0	46	1.873214	8	0	0	1	15	1.225
24	25	4	9	0	50	2.273214	22	3	4	0	44	1.789286	15	1	1	1	29	1.458929	7	0	0	0	14	1.0875
25	12	1	1	0	24	1.296429	18	2	5	0	36	1.742857	15	1	1	0	30	1.333929	3	0	0	0	6	1.0375
26	16	2	5	1	31	1.842857	14	4	2	0	28	1.610714	23	4	2	0	46	1.723214	14	2	0	0	28	1.317857
27	30	4	10	1	59	2.535714	22	3	7	0	44	2.014286	31	3	5	1	61	2.101786	12	1	0	0	24	1.221429
28	17	0	5	1	33	1.7125	22	3	6	0	44	1.939286	19	2	2	0	38	1.530357	7	0	0	1	13	1.2125
29	6	1	1	0	12	1.221429	15	1	3	1	29	1.608929	20	3	2	1	39	1.739286	12	0	2	0	24	1.3
30	27	1	9	0	54	2.083929	3	0	0	0	6	1.0375	7	0	0	0	14	1.0875	22	2	3	0	44	1.642857
31	2	0	0	0	4	1.025	5	0	0	1	9	1.1875	28	5	7	0	56	2.232143	30	6	7	0	60	2.328571
32	24	9	8	0	48	2.114286	16	2	2	0	22	1.402857	14	1	1	0	38	1.334286	11	1	0	0	32	1.208929

Figure B.3 Capture d'écran partielle 2 du feuillet « Simulation » sous Microsoft Excel

AutoSave On PdM simulation template - Saving... Jean-Marc MEANGO

File Home Insert Draw Page Layout Formulas Data Review View Developer Help Search

Clipboard Font Alignment Number Styles Cells Editing Ideas

K26

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	C
1	Average Cost-PdM		Q														
2	L-r S1	L-r S2	1	2	3	4	5										
3	1	1	5.75%	5.18%	5.14%	5.40%	5.56%										
4		1.5	5.51%	5.15%	5.27%	5.40%	5.59%										
5		2	5.62%	5.30%	5.55%	5.72%	5.85%										
6		2.5	5.85%	5.65%	5.81%	6.08%	6.33%										
7	1.5	1	5.63%	5.00%	5.10%	5.32%	5.51%										
8		1.5	5.52%	5.11%	5.15%	5.32%	5.55%										
9		2	5.61%	5.23%	5.28%	5.63%	5.80%										
10	2	2.5	5.75%	5.52%	5.71%	6.05%	6.22%										
11		1	5.59%	5.07%	5.14%	5.32%	5.53%										
12		1.5	5.42%	5.08%	5.33%	5.41%	5.84%										
13	2.5	2	5.56%	5.27%	5.59%	5.82%	5.89%										
14		2.5	5.78%	5.61%	5.81%	6.20%	6.37%										
15		1	5.70%	5.25%	5.35%	5.48%	5.68%										
16	2.5	1.5	5.59%	5.35%	5.38%	5.60%	5.83%										
17		2	5.59%	5.29%	5.62%	5.78%	5.95%										
18		2.5	5.90%	5.75%	5.74%	6.20%	6.53%										
19																	
20																	

IntProb Simulation Summary Cost Summary

Type here to search

5:55 PM 2020-05-28

Figure B.4 Capture d'écran du feuillet « Cost Summary » sous Microsoft Excel