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## Research Article

# Type-2 Fuzzy Modeling for Acoustic Emission Signal in Precision Manufacturing

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This paper presents an application of type-2 fuzzy logic on acoustic emission (AE) signal modeling in precision manufacturing. Type-2 fuzzy modeling is used to identify the AE signal in precision machining. It provides a simple way to arrive at a definite conclusion without understanding the exact physics of the machining process. Moreover, the interval set of the output from the type-2 fuzzy approach assesses the information about the uncertainty in the AE signal, which can be of great value for investigation of tool wear conditions. Experiments show that the development of the AE signal uncertainty trend corresponds to that of the tool wear. Information from the AE uncertainty scheme can be used to make decisions or investigate the tool condition so as to enhance the reliability of tool wear.

## 1. Introduction

Related to advances in machine tools, manufacturing systems and material technology, machining practice is changing from conventional machining to precision machining, even high-precision machining. The scale of precision machining becomes finer and closer to the dimensional scale of material properties. As a result, the acoustic emission (AE) from microscopic sources becomes significant [1].

AE is the class of phenomena whereby transient elastic waves are generated by the rapid release of energy by a localized source or sources within a material, or transient elastic wave(s) so generated (ANSI/ASTM E 610-89). Emissions from process changes, like tool wear, chip formation, can be directly related to the mechanics of the process. AE-based sensing methodologies for tool condition and cutting process monitoring have been studied since 1977 [2]. Signal processing schemes were used to treat AE signal to extract the most useful information, for example, time series analysis [3, 4], Fourier transform [5, 6], Gabor transform [7–9], and wavelet transform [10–13], and so forth. Because the information obtained during the machining process is vague, incomplete or imprecise, these conventional methods need a large number of cutting experiments and additional assumptions in

many circumstances for effective uncertainty handling. These requirements reduce the reliability of the models and increase money and time consumption. Moreover, the general mathematical relation cannot be used to map the nonlinear relationship between the AE signal and tool wear condition [14]. Artificial intelligence methods have played an important role in modern tool condition monitoring (TCM) to observe the relation between tool wear and AE signal such as neural networks [15, 16], fuzzy logic [17], and fuzzy neural network [18–22]. The increased use of artificial intelligence within TCM has enabled the development of more robust and comprehensive strategies.

It is believed that a relatively uncontaminated AE signal can be obtained because AE frequency range is much higher than that of machine vibrations and environmental noises and does not interfere with the cutting operation. AE can be effectively used for TCM applications at the precision scale. In fact, it is impossible to get an accurate AE signal. It is because that machining process varies considerably depending on the part material, temperature, cutting fluids, chip formation, the tool material, temperature, chatter and vibration, and so forth. Additionally, AE sensors are very sensitive to environmental changes such as changes in temperature,



humidity, circuit noise, and even the locating error of the sensors. Moreover, changes of cutting conditions also affect the behaviour of acoustic emission signals. None of previous studies considered the uncertainty in AE signal.

The aim of this paper is to present an innovative type-2 Takagi-Sugeno-Kang (TSK) fuzzy modeling to capture the uncertainties in the AE signal in machining process in order to overcome the challenges in TCM. Type-2 TSK fuzzy modeling method is not only a powerful tool to model high complex nonlinear physical processes, but also a great estimator for the ambiguities and uncertainties associated with the system. It is capable to arrive at a definite conclusion without understanding the exact physics of the machining process. In this paper, type-2 TSK fuzzy modeling is implemented to filter the raw AE signal directly from the AE sensor during turning process. Furthermore, its output interval set assesses the uncertainty information in AE, which is of great value to a decision maker and can be used to investigate the complicated tool wear condition during machining process.

This paper is divided into four sections. Section 1 gives a brief overview of previous studies on AE-based TCM. Section 2 introduces TSK fuzzy logic and the type-2 TSK fuzzy modeling algorithm. Section 3 presents a case study where type-2 TSK fuzzy approach is used to filter the raw AE signal directly from the AE sensor and identify the uncertainty interval of AE. The experimental results show the effectiveness and advantages of type-2 TSK fuzzy modeling. Conclusion is given in Section 4.

## 2. Type-2 TSK Fuzzy Uncertainty Modeling

**2.1. TSK Fuzzy Logic.** Fuzzy logic has been originally proposed by Zadeh in his famous paper “Fuzzy Sets” in 1965 [23]. Fuzzy logic provides a simple way to obtain a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. TSK fuzzy logic system (FLS) [24, 25] was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and a mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviour of the system in those regions. TSK FLS has a powerful capability of explaining complex relations among variables using rule consequents which are functions of the input variables. This is due to the model’s properties: on one hand being a general non-linear approximator that can approximate every continuous mapping and on the other hand being a piecewise linear model that is relatively easy to interpret [26] and whose linear sub-models can be exploited for control and fault detection [27].

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of the system. For a MISO first-order type-1 TSK model, its  $k$ th rule can be expressed as

$$\text{IF } x_1 \text{ is } Q_{1k}, x_2 \text{ is } Q_{2k}, \dots, x_n \text{ is } Q_{nk}, \quad (1)$$

$$\text{THEN } Z \text{ is } w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n,$$

where  $x_1, x_2, \dots, x_n$  and  $Z$  are linguistic variables;  $Q_{1k}, Q_{2k}, \dots$ , and  $Q_{nk}$  are the fuzzy sets on universe of discourses  $U, V, \dots$ , and  $W$ , and  $p_0^k, p_1^k, \dots$ , and  $p_n^k$  are regression parameters.

A Gaussian MF can be expressed by the following formula for the  $v$ th variable:

$$Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right], \quad (2)$$

where  $x_v^{k*}$  is the mean of the  $v$ th input feature in the  $k$ th rule for  $v \in [0, n]$  ·  $\sigma$  is the standard deviation of Gaussian MF.

Based on Zadeh’s concept of type-2 fuzzy sets and extension principle [28], practical algorithms for conjunction, disjunction, and complement operations of type-2 fuzzy sets are obtained by extending previous studies [29]. Prior work [30] also introduced embedded interval-valued type-2 fuzzy sets and developed a general formula for the extended composition of type-2 relations, which is considered as an extension of the type-1 composition. The characterization in the definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first-order approximation of uncertainty and, therefore, type-2 fuzzy sets provide a second-order approximation. They play an important role in modeling uncertainties that exist in fuzzy logic systems [31] and are becoming increasingly important in the goal of “Computing with Words” [32] and “Computational Theory of Perceptions” [33]. A complete type-2 fuzzy logic theory with the handling of uncertainties was also established [34]. Because of its larger number of design parameters for each rule, it was believed that type-2 FLS have the potential to be used in control [35] and other areas where a type-1 model may be unable to perform well [36]. Type-2 TSK FLS and its structures were presented in 1999 [37].

An example of a type-2 MF, whose vertices have been assumed to vary over some interval of value, is depicted in Figure 1. The *footprint of uncertainty* (FOU) associated with this type-2 MF is a bounded shaded region. FOU represents the entire interval type-2 fuzzy set  $\tilde{Q}$ . Upper MF and Lower MF are two type-1 MFs that are bounds for the FOU of a type-2 set  $\tilde{Q}$ . The intersections of crisp input  $x^0$  show that there are lower MF degree  $\underline{u}$  and upper MF degree  $\bar{u}$  with respective lower and upper MFs. Detailed type-2 fuzzy sets and interval type-2 FLS background material can be found in [38].

A generalized  $k$ th rule in the first-order type-2 TSK fuzzy MISO system can be expressed as

$$\begin{aligned} &\text{IF } x_1 \text{ is } \tilde{Q}_{1k}, x_2 \text{ is } \tilde{Q}_{2k}, \dots, x_n \text{ is } \tilde{Q}_{nk}, \\ &\text{THEN } Z \text{ is } \tilde{w}^k = \tilde{p}_0^k + \tilde{p}_1^k x_1 + \tilde{p}_2^k x_2 + \dots + \tilde{p}_n^k x_n, \end{aligned} \quad (3)$$

where  $\tilde{p}_0^k, \tilde{p}_1^k, \dots, \tilde{p}_n^k$  are consequent parameters,  $\tilde{w}^k$  is the output from the  $k$ th IF-THEN rule in an  $M$ -rule FLS,  $\tilde{Q}_{1k}, \tilde{Q}_{2k}, \dots, \tilde{Q}_{nk}$  are fuzzy sets on the universe of discourse.

One way to obtain a type-2 model directly from a type-1 model is by extending the cluster center,  $x_{jk}^*$ , from a point to



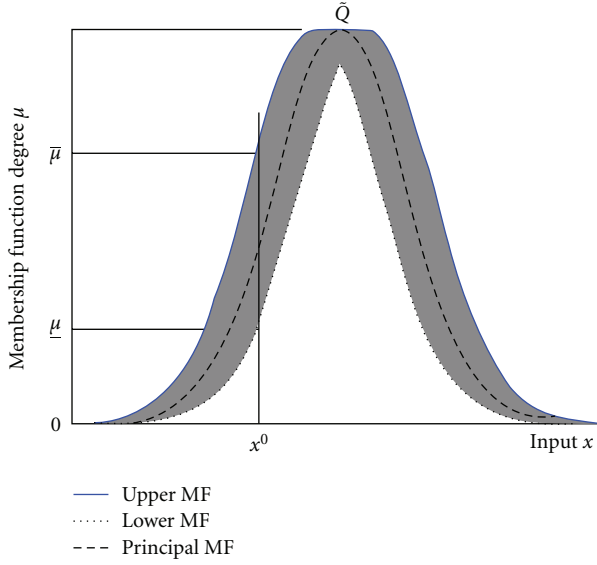


FIGURE 1: Type-2 Gaussian MF.

a constant-width interval-valued fuzzy set,  $\tilde{x}_{jk}^*$  as shown in Figure 2. The size of the interval is  $2a_j$ :

$$\tilde{x}_{jk}^* = [x_{jk}^*(1 - a_j^k), x_{jk}^*(1 + a_j^k)]. \quad (4)$$

Consequent parameter  $\tilde{p}_j^k$  is obtained by extending the consequent parameter  $p_j^k$  from its type-1 counterpart using the following expression:

$$\tilde{p}_j^k = [p_j^k - s_j^k, p_j^k + s_j^k], \quad (5)$$

where  $j \in [0, n]$ , and  $s_j^k$  denotes the spread of fuzzy numbers  $\tilde{p}_j^k$ .

Hence, the premise MF is changed from type-1 fuzzy set into type-2 fuzzy set, that is,

$$\tilde{Q}_{jk} = \exp \left[ -\frac{1}{2} \left( \frac{x_j - x_{jk}^*(1 \pm a_j^k)}{\sigma_j^k} \right)^2 \right], \quad (6)$$

where  $\sigma_j^k$  is the standard deviation of Gaussian MF.

Type-2 FLSs are very useful in circumstances in which it is difficult to determine an exact membership function for a fuzzy set. They can be used to handle rule uncertainties and even measurement uncertainties. Type-2 FLSs moves the world of FLSs into a fundamentally new and important direction. To date, type-2 FL moves in progressive ways where type-1 FL is eventually replaced or supplemented by type-2 FL [39].

**2.2. Type-2 Fuzzy Modelling Algorithm.** The diagram of type-2 TSK fuzzy modelling algorithm is shown in Figure 3. This algorithm is initially presented in [40]. Type-2 TSK fuzzy approach includes two steps: the first step is type-1 fuzzy modelling to eliminate noise components in the AE signal and the second consists of expanding the type-1 fuzzy

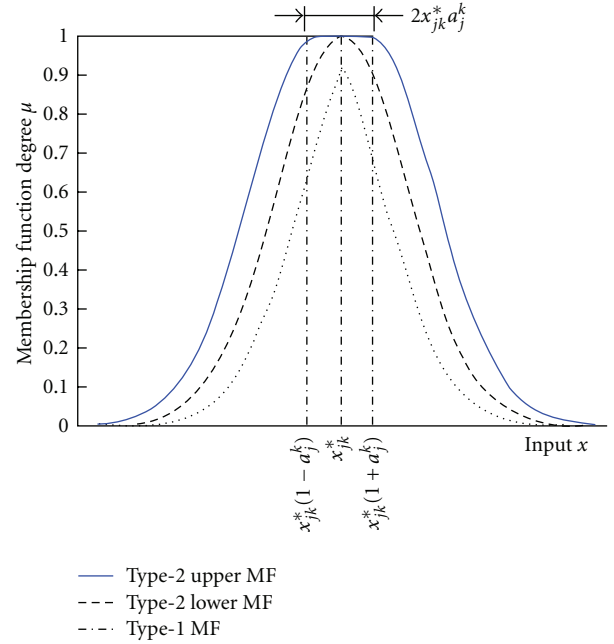


FIGURE 2: Spread of cluster center.

system to its type-2 counterpart to obtain the information of uncertainty in AE signal. The first step is type-1 TSK fuzzy approach. Subtractive clustering method [41] (see Appendix A) combined with a least-square estimation algorithm is used to cope with the nonlinearity of the AE and the uncertainty of imprecise data from measurement. A detailed description for this type-1 fuzzy modelling can be found in [42, 43]. The second step is type-2 TSK fuzzy approach. The type-1 MFs are considered as the principal MFs of a type-2 FLS, the antecedent MFs are extended as interval type-2 fuzzy memberships by assigning uncertainty to cluster centers using (4), and the consequent parameters are extended as fuzzy numbers by assigning uncertainty to consequent parameter values using (5). The type-2 TSK fuzzy inference engine is presented in Appendix B. Through enumerative search of optimum values of spreading percentage of cluster centers and consequent parameters, the best approach for analysing AE signal is obtained. The Detailed description for this modelling algorithm can be found in [44]. Examples of application of this algorithm can be found in [45–47].

Compared with traditional methods and its type-1 counterpart, type-2 fuzzy modeling can not only obtain a modeling result directly from the input-output data sets, but it can also capture the uncertainty interval of the result [34, 48]. The information about uncertainties in the type-2 interval output is very helpful for decision making.

### 3. Experimental Study

**3.1. Experimental Setup.** The experiment described in this paper was taken on the BOEHRINGER CNC lathe. The workpiece material was Titanium Metal matrix Composite (Ti MMC) 10% wt. TiC/Ti-6Al-4V where the microstructural response of cast Ti-6Al-4V-based composite contains



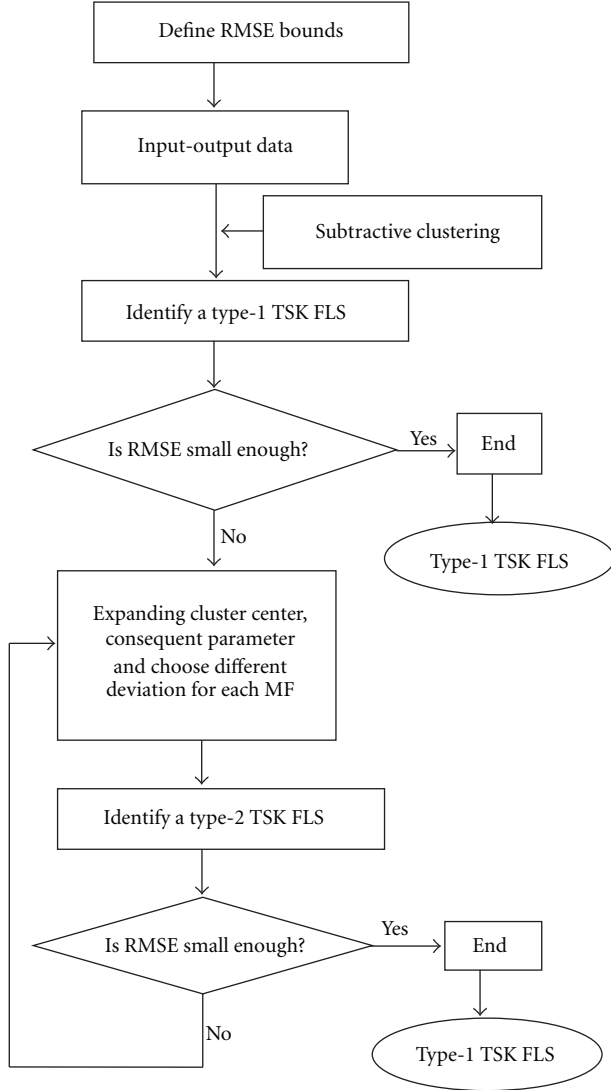


FIGURE 3: Diagram of subtractive clustering-based type-2 TSK fuzzy approach.

10 vol.-% TiC reinforcement. This kind of material is widely used in aerospace and military applications for its high hardness, light weight, high bending strength, fracture toughness, higher modulus, and elevated temperature resistance and high wear resistance. Consequently, its machining is very difficult.

The cutting tool insert was carbide from SECO tools (CNMG 120408 MF1 CP200). Turning test was done on a cylinder of Ti MMC 2.5" diameter in dry machining conditions. The machining speed was chosen to exceed the manufacturer's recommendation in order to see and check the tool cutting speed limits (80 m/min). The cutting depth was kept at 0.15 mm and the cutting feed was 0.1 mm.

The aim of this study was to find out the relation between AE and tool wear. During the test, every time when cutting length reached 10 mm, the machine was stopped to manually measure the tool wear parameter ( $VB_B$ ). Figure 4 shows one example of a raw AE signal  $AE_{raw}$  directly from AE sensor. During the first 5~8 s, the cutting tool is approaching

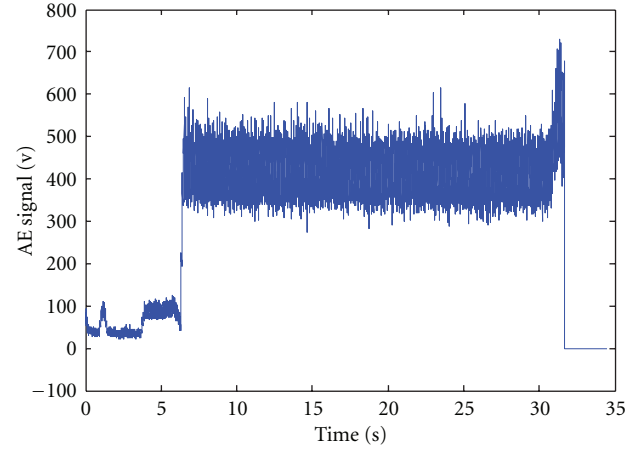


FIGURE 4: AE signal from cutting process.

TABLE 1: Number of rules and parameters of type-1 TSK modeling.

Cutting section (mm)	0~10	10~20	20~30	30~40	40~50
Number of data sets	5500	8200	5700	4500	4000
Number of rules	24	23	29	34	22
Standard deviation	1.1209	0.9016	1.034	1.1932	1.0606

the workpiece and gradually reaching the cutting depth. After 30 s, the cutting tool leaves the surface of workpiece. The middle period is the steady cutting period, which contains the most useful information for tool wear condition investigation. In the experiment, five AE signal sets were recorded according to different cutting sections: 0~10 mm, 10~20 mm, 20~30 mm, 30~40 mm, and 40~50 mm. This paper focuses on filtering and capturing the uncertainty on AE signals during the five continuous cutting periods using type-2 TSK fuzzy modeling.

**3.2. Data Processing.** First, type-1 TSK fuzzy filtering (top part of Figure 3) was used to eliminate noise components in the AE signal. Demirli's extended subtractive clustering identification algorithm [49] was used to estimate both the number and initial location of cluster centers and extract the TSK fuzzy rules from input/output data. The clustering parameters are preinitialized. The cluster radius is confined to the range [0.15; 1.0] with a step size of 0.15. The accept ratio and the reject ratio are both considered in the range [0; 1.0] with a step size of 0.1. The squash factor is considered in the range [0.05; 2] with a step size of 0.05. Combined with a least-square estimation algorithm, the fuzzy systems for each cutting length were identified. Table 1 lists the number of rules identified and the standard deviation used for the five AE signal sets. List of cluster centres can be found in [17].

Traditionally, the AE signal is characterized using AE root-mean-square (RMS) measurement in well-controlled tensile tests. To compare AE signal obtained by fuzzy filtering with the one by traditional filter, AE RMS values (illustrated



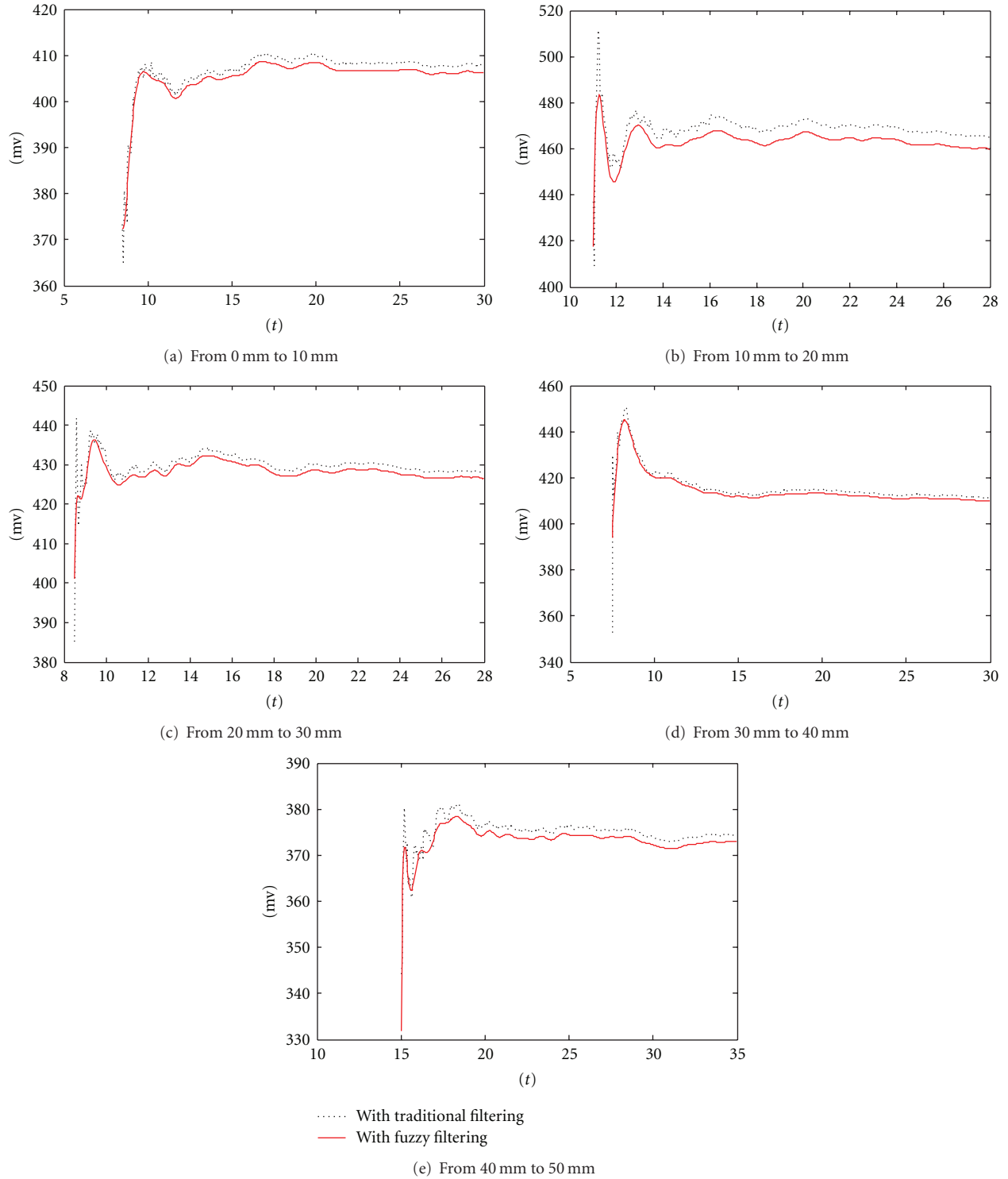


FIGURE 5: AE RMS value for different cutting section.

in Figure 5) and AE mean values (depicted in Figure 6) are calculated for both cases. The dotted curves represent the values obtained by traditional filtering, and the solid ones represent the values obtained by fuzzy filtering. In Figure 5, the dotted curves are above the solid curves. This means

that the AE RMS values obtained using the traditional filter are larger than those obtained with fuzzy filtering. The difference could be caused by computation of different algorithm, where fuzzy filtering generates fuzzy rules directly from the input-output data acquired from sensor, without



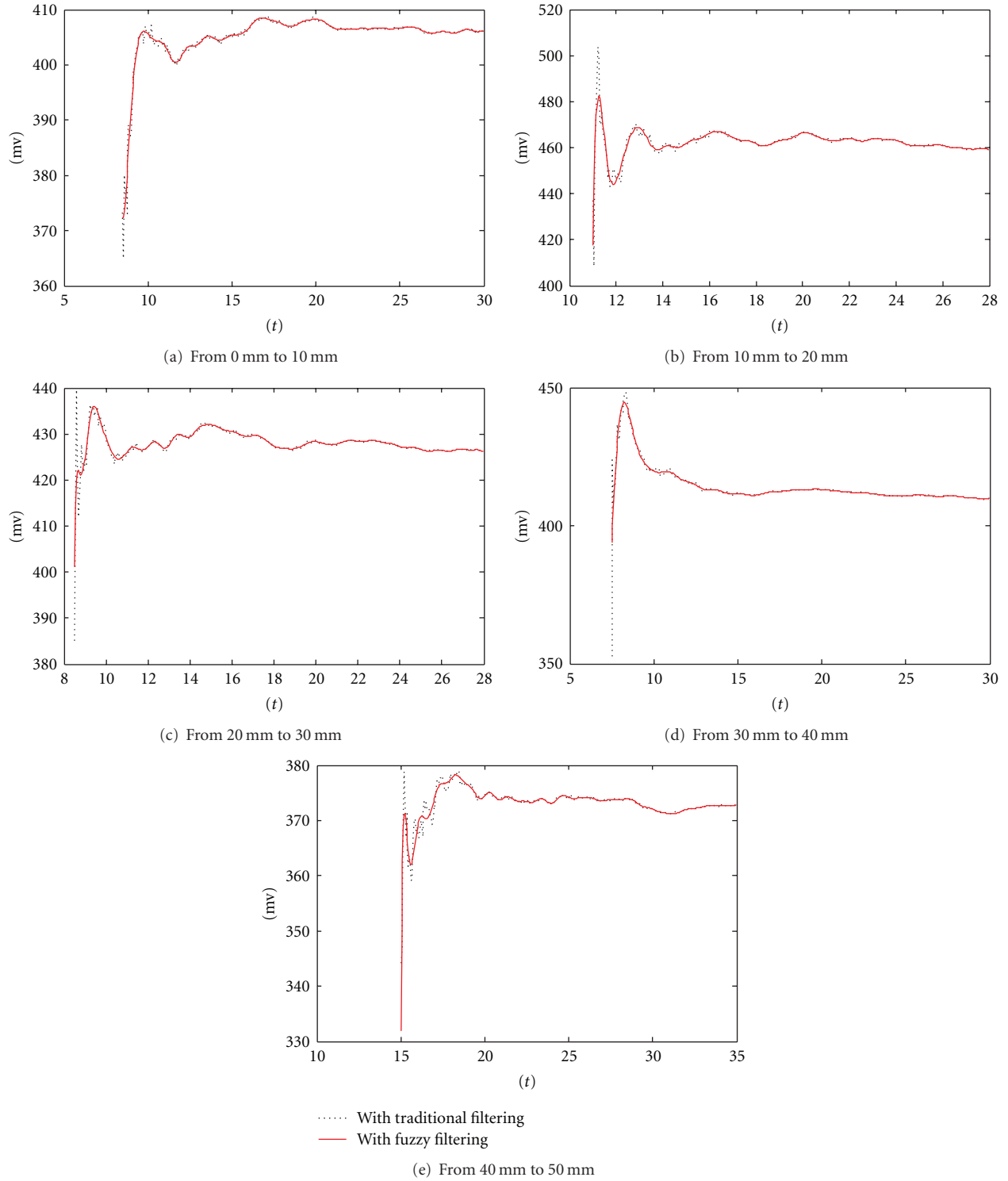


FIGURE 6: AE mean value for different cutting section.

traditional user-defined high-pass and low-pass filters. The mean values obtained by the two methods are almost same as shown in Figure 6.

The second step consists of expanding the type-1 fuzzy system to a type-2 system. Because the AE signals used are relatively uncontaminated, uncertainty in the AE signal is

much smaller than the raw AE signal value. The spreading percentage for clusters is confined to the range  $[0.0\%; 0.01\%]$  with a step size of  $0.0001\%$ . The spreading percentage for the consequent parameters is considered as  $2\%$ . Spreading percentage for clusters and consequent parameters can be found in [17]. The information on uncertainty in the five



TABLE 2: The variations in modeling results from the five AE signal sets.

Cutting section (mm)	Variation (mv)								$VB_B$ (mm)
	$V_1$		$V_2$		$V_3$		$V_4$		
	MAX	MIN	MAX	MIN	MAX	MIN	MAX	MIN	
0~10	1.1092	0.0262	1.0257	0.0003	2.1350	0.0108	13.6804	0.0670	0.050
10~20	4.7649	0.0214	4.5361	0.0006	9.3011	0.0018	28.7809	0.1597	0.100
20~30	5.9340	0.0604	5.660	0.0155	11.5940	0.0759	92.8960	0.1784	0.146
30~40	4.2204	0.0270	3.9954	0.0007	19.3474	0.0296	117.362	0.1980	0.196
40~50	12.630	0.0210	12.097	0.0001	24.7279	0.0040	128.294	0.0464	0.373

AE signal sets is shown in Figure 7 between the type-2 fuzzy output upper boundary  $\overline{AE}$  (dotted curve) and the lower boundary  $\underline{AE}$  (dashed curve). The overall identified AE signal value  $AE_{fuzzy}$  is shown as solid curve.

As indicated on Figure 7, there is the uncertainty interval for each cutting section. The most significant is shown on Figure 7(e). Table 2 summarizes the value of maximum and minimum variations between  $\overline{AE}$  and  $E_{fuzzy}$ ,  $\underline{AE}$  and  $AE_{fuzzy}$ ,  $\overline{AE}$  and  $\underline{AE}$ , also  $AE_{fuzzy}$  and the raw AE signal value  $AE_{raw}$ , where, representatively,

$$\begin{aligned}
 V_1 &= |\overline{AE} - AE_{fuzzy}|, \\
 V_2 &= |\underline{AE} - AE_{fuzzy}|, \\
 V_3 &= |\overline{AE} - \underline{AE}|, \\
 V_4 &= |AE_{raw} - AE_{fuzzy}|.
 \end{aligned} \tag{7}$$

The greatest variation of each cutting instant is  $V_3$  between the upper boundary and lower boundary of type-2 interval output. The biggest value is 24.7279 mv in the final cutting section, as same as that on Figure 7(e). The last colon on Table 2 lists  $VB_B$  measured in the end of each cutting section.

As shown in Figures 8 and 9, the development trends of maximum and minimum variations are the same as the tool wear trend. The maximum changes in tool wear condition and AE signal both occurred during the period when the cutting length was changed from 40 mm~50 mm. It is observed that during the initial cutting period (cutting length from 0~40 mm), the variations of AE signal correspond to the initial stages of wear occurring. The period with the most significant variations (cutting section 40~50 mm) corresponds to the period of relatively rapid wear or failure of the cutting tool. Along with the increasing of uncertainty in AE signal, the development of wear is continuous and monotonically increasing. The sufficient information from AE uncertainty scheme can be used to make decision or investigate tool condition so as to enhance the reliability of tool wear estimation.

## 4. Conclusion

This paper presented a type-2 fuzzy modelling method to determine AE in precision machining. The interval output

of type-2 fuzzy modelling provides the information of uncertainty in AE estimations. The sufficient information from AE uncertainty scheme can be used to make decision or investigate tool condition so as to enhance the reliability of tool wear estimation. By applying type-2 fuzzy logic to AE-based tool conditions monitoring, it is possible to automate online diagnosis of cutting tool condition.

Type-2 FLS can model and analyse the uncertainties in machining from the vague information obtained during machining process. The estimation of uncertainties can be used for proving the conformance with specifications for products or autocontrolling of machine system. The application of type-2 fuzzy logic on uncertainty estimation in high precision machining can enable the unmanned use of flexible manufacturing systems and machine tools. It has great meaning for continuous improvement in product quality, reliability, and manufacturing efficiency in machining industry.

## Appendices

### A. Subtractive Clustering

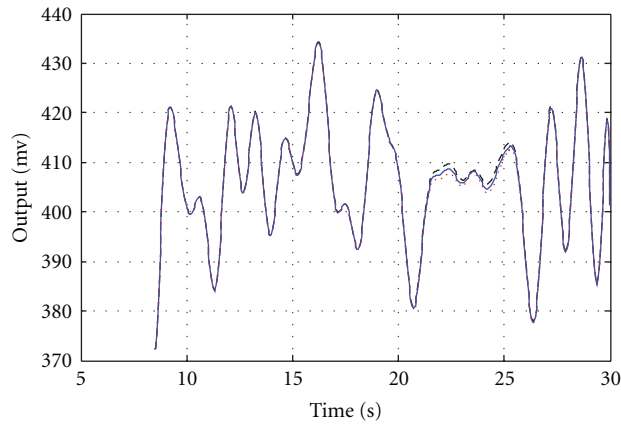
Subtractive clustering identification algorithm [41] is to estimate both the number and initial location of cluster centers and extract the TSK fuzzy rules from input/output data. Subtractive clustering operates by finding the optimal data point to define a cluster centre based on the density of surrounding data points. This method is a fast clustering method designed for high-dimension problems with a moderate number of data points. This is because its computation grows linearly with the data dimension and as the square of the number of data points. A brief description of Chiu's subtractive clustering method is as follows.

Consider a collection of  $q$  data points  $\{x_1, x_2, \dots, x_n\}$  specified by  $m$ -dimensional  $x_j$ . Without loss of generality, assume the feature space is normalized so that all data are bounded by a unit hypercube. Calculate potential for each point by using equation below:

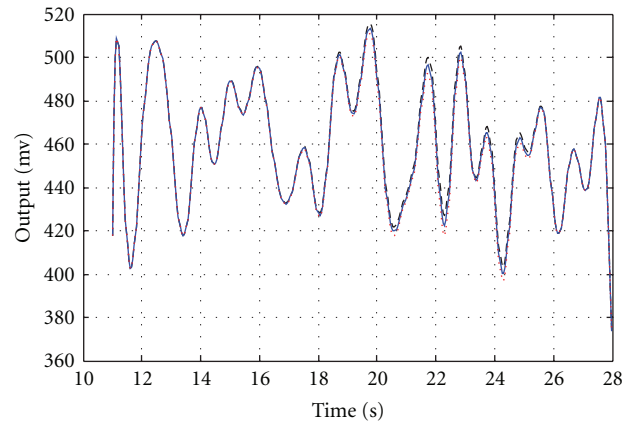
$$p_i = \sum_{j=1}^q e^{-\alpha \|x_i - x_j\|^2}, \quad \alpha = \frac{4}{r_a^2}, \tag{A.1}$$

where  $\|\cdot\|$  denotes the Euclidean distance. It is noteworthy that only the fuzzy neighbourhood within the radius  $r_a$  contributes to the measure of potential.

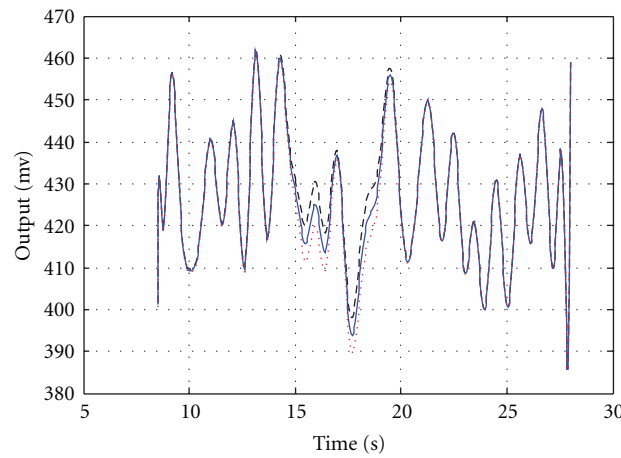




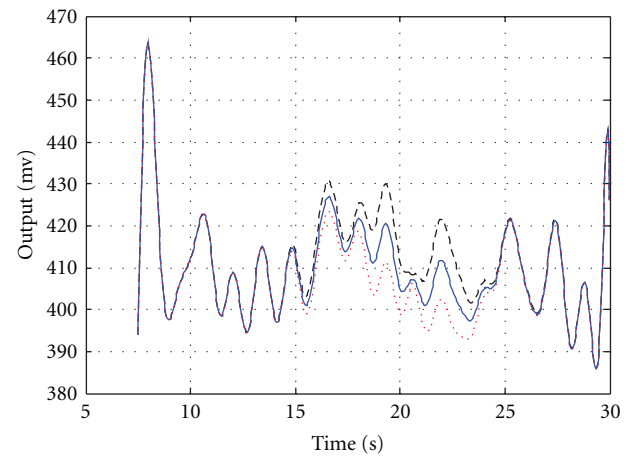
(a) From 0 mm to 10 mm



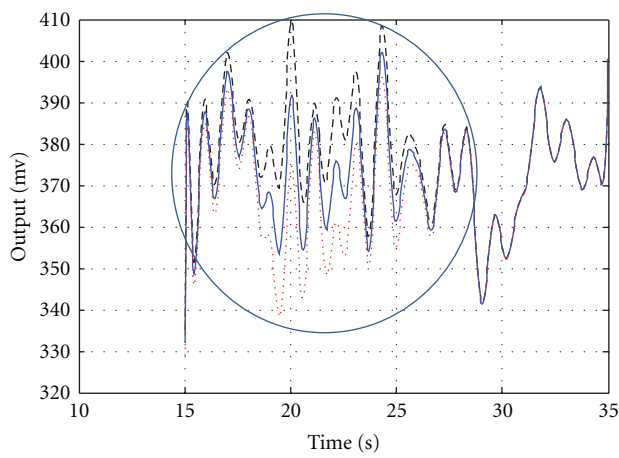
(b) From 10 mm to 20 mm



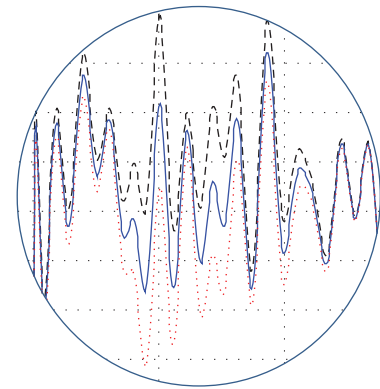
(c) From 20 mm to 30 mm



(d) From 30 mm to 40 mm



(e) From 40 mm to 50 mm



(f) Enlarged view for uncertainty interval on (e)

— Average  
 --- Upper  
 ..... Lower

— Average  
 --- Upper  
 ..... Lower

FIGURE 7: Uncertainties in AE signal in different cutting sections.



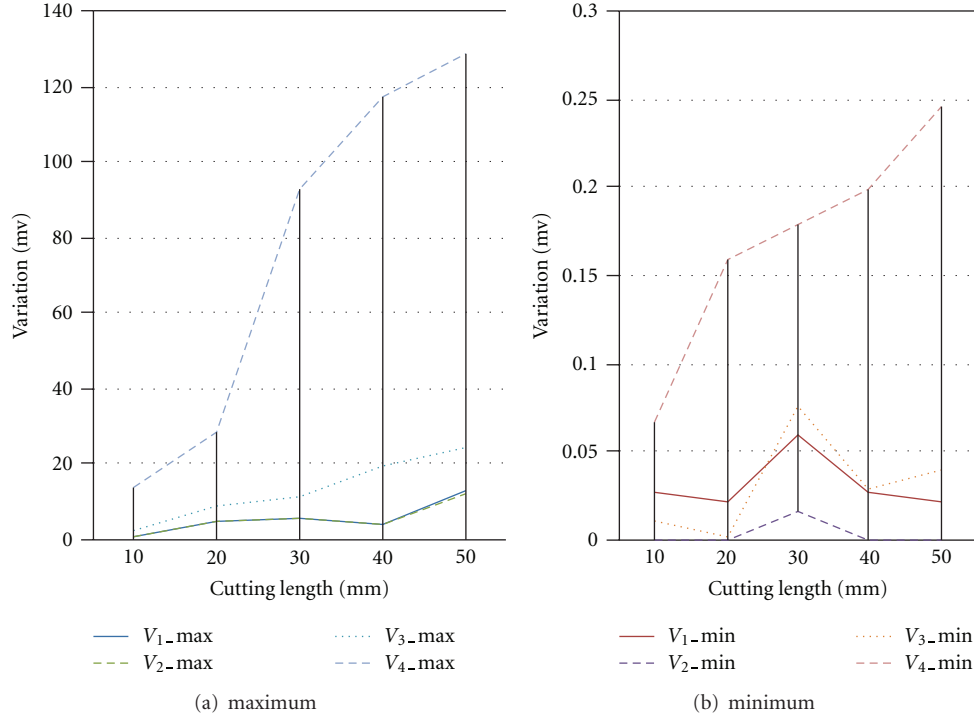


FIGURE 8: Maximum and minimum variations in different cutting sections.

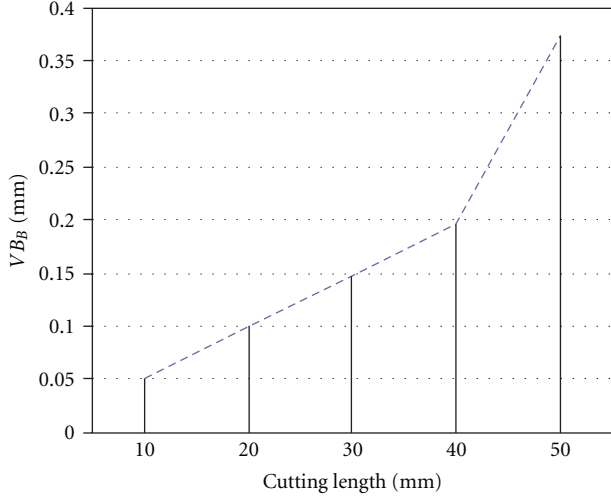


FIGURE 9: Tool wears in different cutting sections.

After the potential of every data point has been computed, the data point with the highest potential is selected as the first cluster center. Assume  $x_1^*$  is the location of the first cluster center, and  $p_1^*$  is its potential value, then revise the potential of each data point  $x_j$  by the formula

$$p_i \Leftarrow p_i - p_1^* e^{-\beta \|x_i - x_1^*\|^2}, \quad (\text{A.2})$$

where  $\beta = 4/r_b^2$  and  $r_b = \eta r_a$ .

When the potential of all data points have been reduce, by (A.2), the data point with the highest remaining potential

is selected as the second cluster center. Then further reduce the potential of each data points. Generally, after  $k$ th cluster center has been obtained, the potential of each data point is revised by formula

$$p_i \Leftarrow p_i - p_k^* e^{-\beta \|x_i - x_k^*\|^2}, \quad (\text{A.3})$$

where  $x_1^*$  is the location of the  $k$ th cluster center and  $p_1^*$  is its potential value.

The process of acquiring new cluster center and revising potential repeats by using the Algorithm 1.

## B. Type-2 TSK Fuzzy Inference Engine

For the most general structure of T2 TSK FLS—Model I, antecedents are T2 fuzzy sets and consequents are T1 fuzzy sets. Membership grades are interval sets, that is,

$$\mu_v^k = \left[ \underline{\mu}_v^k, \bar{\mu}_v^k \right], \quad (\text{B.1})$$

where  $\underline{\mu}_v^k$  and  $\bar{\mu}_v^k$  are the lower value and upper value of the  $v$ th input variable in the  $k$ th rule.

The explicit dependence of the total firing interval for  $k$ th rule can be computed as

$$\underline{f}^k = \underline{\mu}_1^k(x_1) \cap \underline{\mu}_2^k(x_2) \cap \cdots \cap \underline{\mu}_n^k(x_n), \quad (\text{B.2})$$

$$\bar{f}^k = \bar{\mu}_1^k(x_1) \cap \bar{\mu}_2^k(x_2) \cap \cdots \cap \bar{\mu}_n^k(x_n), \quad (\text{B.3})$$



```

if  $p_k^* > \bar{\varepsilon} p_1^*$ , accept  $x_k^*$  as a cluster center and continue.
  else if  $p_k^* < \varepsilon p_1^*$ , reject  $x_k^*$  and end the clustering process.
    else let  $d_{\min}$  = shortest of the distances between  $x_k^*$  and all previously found cluster centers.
      if  $(d_{\min}/r_a) + (p_k^*/p_1^*) \geq 1$  accept  $x_k^*$  as a cluster center and continue.
      else reject  $x_k^*$  and set the potential at  $x_k^*$  to 0. Select the data point with the next highest potential
        as the new  $x_k^*$  and reset.
      end if
    end if
end if

```

ALGORITHM 1

where variable  $\bar{f}^k$  and  $\underline{f}^k$  denote lower value and upper value of fire strength. The symbol  $\cap$  is a conjunction operator, which is a T-norm. It can be either MIN operator  $\wedge$  or product operator  $*$ .

The interval value of the consequent of the  $k$ th rule  $w^k$  is  $w^k = [w_l^k, w_r^k]$ , where

$$w_l^k = \sum_{j=1}^n c_j^k x_j + c_0^k - \sum_{j=1}^n s_j^k |x_j| - s_0^k, \quad (B.4)$$

$$w_r^k = \sum_{j=1}^n c_j^k x_j + c_0^k + \sum_{j=1}^n s_j^k |x_j| - s_0^k.$$

Here  $w_l^k$  and  $w_r^k$  denote lower and upper values of consequent output for  $k$ th rule.  $c_j^k$  and  $s_j^k$  denote the centre (mean) and the spread of fuzzy number  $\tilde{p}_j^k$ .

So, the extended output of the IT2 TSK FLS can be calculated by using following equation:

$$\begin{aligned} \tilde{w} &= [w_l, w_r] \\ &= \int_{w_l^1 \in [w_l^1, w_r^1]} \cdots \int_{w_l^n \in [w_l^n, w_r^n]} \frac{1}{\sum_{k=1}^M f^k w^k / \sum_{k=1}^M f^k}. \end{aligned} \quad (B.5)$$

Hence  $\tilde{w} = [w_l, w_r]$  is an interval type-1 set, the two endpoints  $w_l$  and  $w_r$  can be obtained by using equations below:

$$w_l = \frac{\sum_{k=1}^n f_l^k w_l^k}{\sum_{k=1}^n f_l^k}, \quad w_r = \frac{\sum_{k=1}^n f_r^k w_r^k}{\sum_{k=1}^n f_r^k}. \quad (B.6)$$

This interval set of the output has the information about the uncertainties that are associated with the crisp output, and this information can only be obtained by working with T2 TSK FLS. To compute  $\tilde{w}$ , two endpoints  $w_l$  and  $w_r$  must be computed. In order to compute  $w_l$  and  $w_r$ ,  $f_l^k$  and  $f_r^k$  have to be determined.  $w_l$  and  $w_r$  can be obtained by using the iterative procedure *KM Algorithm* [30]. Here, the computation procedure for  $w_l$  and  $w_r$  is briefly provided as follows:

Without loss of generality, assume that the precomputed  $w_r^k$  are arranged in ascending order:  $w_r^1 \leq w_r^2 \leq \cdots \leq w_r^m$ , then, we have the following.

*Step 1.* Compute  $w_r$  in (B.6) by initially setting

$$f_r^k = \frac{\left( \frac{f^k + \bar{f}^k}{2} \right)}{2} \quad (B.7)$$

for  $k = 1, \dots, R$ , where  $\bar{f}^k$  and  $\underline{f}^k$  have been previously computed using (B.2) and (B.3), respectively, and let  $w_r' \equiv w_r$ .

*Step 2.* Find  $y$  ( $1 \leq y \leq m-1$ ) such that  $w_r^y \leq w_r' \leq w_r^{y+1}$ .

*Step 3.* Compute  $w_r$  in with  $f_r^k = \bar{f}^k$  for  $k \leq y$  and  $f_r^k = \underline{f}^k$  for  $k > y$ , and let  $w_r'' \equiv w_r$ .

*Step 4.* If  $w_r'' \neq w_r'$ , then go to Step 5. If  $w_r'' = w_r'$ , then stop. And set  $w_r'' \equiv w_r$ .

*Step 5.* Set  $w_r' = w_r''$  and return to Step 2.

The procedure for computing  $w_l$  is very similar to the one just given for  $w_r$ . Replace  $w_r^k$  by  $w_l^k$ , and compute  $w_l$ . In Step 2 find  $z$  ( $1 \leq z \leq m-1$ ) such that  $w_l^z \leq w_l' \leq w_l^{z+1}$ . Additionally, in Step 3 compute  $w_l$  with  $f_r^k = \bar{f}^k$  for  $k \leq z$

and  $f_r^k = \underline{f}^k$  for  $k > z$ .

In an interval type-2 TSK FLS, output  $\tilde{w}$  is an interval type-1 fuzzy set, so the crisp output of any interval type-2 TSK FLS can be obtained by using the average value of  $w_l$  and  $w_r$ , that is,

$$w^* = \frac{w_l + w_r}{2}. \quad (B.8)$$

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