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Prompt uncertainty estimation with GUM framework for on-machine tool coordinate metrology

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Abstract

This paper validates the uncertainty evaluated following the Guide to the Expression of Uncertainty in Measurement (GUM) for on-machine probing with a five-axis machine tool. A partly synthetic input covariance matrix is assembled for Monte Carlo and GUM frameworks, which separately estimate the uncertainty of on-machine probed point sets and obtained geometric features. The differences between the GUM and Monte Carlo results lie within the stipulated tolerances with comparable coverage regions and marginal distributions. This validates the GUM framework, which is on average 24 and 249 times faster for on-machine measurement of a gauge block and a precision sphere, respectively.

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Keywords: Uncertainty; On-machine measurement; GUM; Monte Carlo; Five-axis machine tool

1. Introduction

When machine tools come to operation as measuring systems, they boost production efficiency because of the synergy between measurement and manufacture. Like in any other measurement procedure, the estimates for measurand obtained by these apparatus are complete only when they come with uncertainty metrics [1]. This is essential to establish the traceability of the measurement process.

On the other hand, generating an accurate uncertainty assessment is usually a time-consuming process. This stems from the computation cost of the propagation of uncertainty, which provides the basis of uncertainty estimators. The complexities associated with the analytical method for the propagation of uncertainty, known as the Markov formula [2], have brought about the development of numerical methods. The Monte Carlo method (MCM) is an iterative numerical approach to propagate distributions and is favourable for complex measurement functions or those with no closed-form mathematical expression. Moreover, according to the central limit theorem [2], this method has promising convergence properties. Despite that, calling the measurement function once in every MCM trial highly increases the computation time, especially when it comes to an adaptive MCM where the convergence criteria require conducting an adequately large number of trials. Supplement 2 to GUM [3] specifies an alternative analytical solution for the propagation of uncertainty, referred to as the GUM uncertainty framework (GUF). Validated with an adaptive MCM, GUF is an efficient replacement for the inefficient MCM.

The GUF application in the metrology of machine tools has remained limited probably because of the convolutions of covariance analysis. These difficulties confine the GUF implementation to its single-output form, specified by GUM [1] as the law of propagation of uncertainty [4, 5]. In this study, however, a full covariance estimator allows for conducting GUF. Adhering to Supplement 2 to GUM [3], we then assess the validity of the GUF results by comparing them with the MCM estimates, considering predefined numerical tolerances. The conformity of GUF to the MCM results is also examined through ellipsoidal coverage regions and marginal probability distribution functions (PDFs).

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2. Uncertainty evaluation

2.1. Measurement function, input, and output quantities

The forward kinematic model of the machine serves as the on-machine measurement function. This model receives the probed axis positions and the machine parameters to calculate the compensated Cartesian coordinates in the workpiece frame using a chain of homogeneous transformation matrices that gives the relative position of the tool frame, as the stylus tip centre, with respect to the workpiece frame. When probing a point set of size n, like in estimating a ring's diameter by probing n points around its surface, the recorded $5n$ axis positions (five joint positions per probed point) together with 13 machine parameters, describing the geometric status of the machine, form the input data of uncertainty estimators. Therefore, the number of input quantities is $N = 5n + 13$. The 5×5 repeatability matrices of probed axis positions [6] and the 13×13 covariance matrix associated with the machine parameters [7] separately encode information on the correlations involved in on-machine measurement since the data on the machine parameters and the probed axis positions is acquired independently. As a result, the full covariance matrix of the input quantities is not directly obtainable from observations. To assemble these effects and obtain a single full covariance matrix $\boldsymbol{U_x}$ of dimension $(5n+13)\times(5n+13)$, a covariance estimator simulates pseudo-repeatability tests as though the replicated tests of on-machine probing and those of geometric error indication (with SAMBA [8]) occur at the same time, which approximates the full covariance matrix of the input quantities. Details on the covariance simulator can be found in [9].

The $3n$ compensated Cartesian coordinates of the probed points in the workpiece frame are the output quantities of the measurement function, thus the number of output quantities is $m = 3n$ and covariance matrix $\boldsymbol{U_y}$ is of dimension $3n\times 3n$. When a probed point set is further processed to estimate the actual value of geometric feature attributes, the final covariance matrix diagonals are the scalar values (variance) associated with the estimates.

2.2. Monte Carlo method

MCM evaluates with the measurement function (the machine's forward kinematic model) an adequately large sample of input quantities $\boldsymbol{X}$, drawn at random from a joint distribution, resulting in a corresponding sample of output quantities $\boldsymbol{Y}$ [3, 10]. In an adaptive MCM, this procedure completes in h sequences, each including M=10^4 trials. During each sequence, matrix $\boldsymbol{G_X}$ accumulates M drawn input vectors $\boldsymbol{x}$ and matrix $\boldsymbol{G_Y}$ stores the corresponding evaluated output vectors $\boldsymbol{y}$ (Fig. 1a). The latter provides essential statistical information to evaluate the uncertainty of the measurement function's outputs. As a result, regardless of the number of input and output quantities, an estimate for the joint PDF of the measurand is obtainable by MCM. Given a desirable coverage probability of p, the MCM simulator initially conducts $h_0 =$ 10 sequences and then calculates output estimates $\boldsymbol{y}$, associated standard uncertainty $\boldsymbol{u_y}$, maximum eigenvalue $\lambda_{\max}$ of the output correlation matrix, and coverage factor k_p (Fig. 1a). Comparing with predefined numerical tolerances, if convergence does not hold, it then conducts one more sequence and continues until the results converge. More details on the MCM procedure can be found in [7].

2.3. GUM uncertainty framework

The Taylor series of a function evaluates it with an infinite sum of the terms formed by the function's derivatives at a single point. Considering the first two terms of the Taylor series of a measurement function at the expectation values of the input quantities, GUM estimates the measurement function in a small neighbourhood of these expected values. Drawing an analogy between this neighbourhood and the standard uncertainty intervals of the input values, the law of propagation of uncertainty evaluates the standard uncertainty of the output quantities.

The GUF outcome is an estimate for the covariance matrix of the output variables. Then, compared to MCM, GUF reveals limited information about the measurand. Particularly, GUF does not provide any joint PDF for the output quantities. Nonetheless, fitting certain distributions, such as normal, to the obtained covariance matrix might approximate the true joint PDF of the output quantities.

Supplement 2 to GUM [3] develops this concept for multi-output measurement function $\boldsymbol{Y} = f(\boldsymbol{X})$, where $\boldsymbol{X} = [X_1, \dots, X_N]^{\mathrm{T}}$ is a vector of N input quantities and $\boldsymbol{Y} = [Y_1, \dots, Y_m]^{\mathrm{T}}$ is a vector of m output quantities. Covariance matrix $\boldsymbol{U_y}$ associated with output estimates $\boldsymbol{y}$ is [3]

$$\boldsymbol{U_y} = \boldsymbol{C_x}\,\boldsymbol{U_x}\,\boldsymbol{C_x^T} \tag{1}$$

where $\boldsymbol{U_x}$ is the covariance matrix associated with best estimates $\boldsymbol{x}$ of input quantities $\boldsymbol{X}$. In this equation, $\boldsymbol{C_x}$ is the sensitivity matrix at $\boldsymbol{X} = \boldsymbol{x}$ of dimension $m \times N$:

$$\boldsymbol{C_x} = \begin{bmatrix} \dfrac{\partial Y_1}{\partial X_1} & \cdots & \dfrac{\partial Y_1}{\partial X_N} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial Y_m}{\partial X_1} & \cdots & \dfrac{\partial Y_m}{\partial X_N} \end{bmatrix} \tag{2}$$

Given the complexity of the on-machine measurement function, numerical differentiation is a suitable means to estimate the partial derivatives stored in the sensitivity matrix $\boldsymbol{C_x}$ given in Eq. (2). According to GUM [1], the step size for differentiating the measurement function with respect to the j^{th} input quantity X_j equals the corresponding standard uncertainty $u(x_j)$. Fig. 2 illustrates the numerical differentiation procedure using the symmetric derivative.

This is a resupply of March 2023 as the template used in the publication of the original article contained errors. The content of the article has remained unaffected.

3. Validation of GUF with an adaptive Monte Carlo method

Supplement 2 to GUM [3] specifies the validation procedure of GUF using an adaptive MCM. After indicating the numerical tolerances for convergence criteria of the adaptive MCM, these metrics also define the required accuracy of the GUF results. This standard defines:

$$\begin{aligned} \boldsymbol{d}_{\boldsymbol{y}} &= |\boldsymbol{y}^{GUF} - \boldsymbol{y}^{MCM}| \\ \boldsymbol{d}_{\boldsymbol{u}(\boldsymbol{y})} &= |\boldsymbol{u}(\boldsymbol{y})^{\mathrm{GUF}} - \boldsymbol{u}(\boldsymbol{y})^{\mathrm{MCM}}| \\ d_{\lambda_{\max}} &= |\lambda_{\max}{}^{\mathrm{GUF}} - \lambda_{\max}{}^{\mathrm{MCM}}| \\ d_{k_p} &= |k_p{}^{GUF} - k_p{}^{MCM}| \end{aligned} \tag{3}$$

where $\boldsymbol{d}_{\boldsymbol{y}}$, $\boldsymbol{d}_{\boldsymbol{u}(\boldsymbol{y})}$, $d_{\lambda_{\max}}$, and d_{k_p} are the absolute differences between the MCM and GUF results (denoted by the respective superscripts) respectively for best estimates $\boldsymbol{y}$, associated standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$, the largest eigenvalue $\lambda_{\max}$ of the output correlation matrix, and coverage factor k_p. If numerical tolerance for $\boldsymbol{y}$ and $\boldsymbol{u}(\boldsymbol{y})$ is $\boldsymbol{\delta}$ and that for $\lambda_{\max}$ and k_p is ρ and κ_p, respectively, the adaptive MCM validates the GUF results if all the absolute differences given by Eq. (3) are smaller than the corresponding numerical tolerances:

$$\begin{aligned} \boldsymbol{d}_{\boldsymbol{y}} &\leq \boldsymbol{\delta} \\ \boldsymbol{d}_{\boldsymbol{u}(\boldsymbol{y})} &\leq \boldsymbol{\delta} \\ d_{\lambda_{\max}} &\leq \rho \\ d_{k_p} &\leq \kappa_p \end{aligned} \tag{4}$$

4. Results and discussion

Experimental on-machine measurements on a gauge block (Fig. 1b) and a precision sphere (Fig. 1c) with a Mitsui Seiki HU40-T five-axis horizontal machining centre have already validated the MCM results by examining whether the uncertainty intervals provide the desired coverage probability [9]. Each compensated coordinate of a probed point is regarded as having four significant decimal digits when expressed in mm. Then, the numerical tolerance for best estimates $\boldsymbol{y}$ and associated standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$ is $\boldsymbol{\delta}$=0.05 $\mathbf{1}_{3n\times1}$ µm, where $\mathbf{1}_{3n\times1}$ is a column vector of ones of length $3n$. The numerical tolerance for the largest eigenvalue $\lambda_{\max}$ of the output correlation matrix and coverage factor k_p is also ρ=κ_p=0.05. Table 1 includes the absolute differences for best estimates $\boldsymbol{y}$ and associated standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$, determined for two points probed on the gauge block with a calibrated length of 500.0095 mm [9]. For point 1, the probed positions for linear axes X, Y, and Z are -138.5650, 94.6560, and 196.2320 mm, and those for the rotary axes B and C are -42° and -293°, respectively. These values for point 2 are -61.2880, 48.3440, 246.8170 mm, -64°, and -124°, respectively.

This table also includes the estimate for the gauge's length and its standard uncertainty along with the absolute differences between these quantities obtained with the two evaluation methods. The largest eigenvalue $\lambda_{\max}$ and coverage factor k_p given by GUF and their absolute difference are also presented in

Table 2. All the absolute differences listed in these two tables are smaller than the stipulated numerical tolerances, which validates the GUF results. For best estimates $\boldsymbol{y}$, the maximum absolute difference is 72% of the specified numerical tolerance. This metric is 38% for standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$. For maximum eigenvalue $\lambda_{\max}$ and coverage factor k_p, these differences are even smaller, that is, 1% and 9% of the numerical tolerance, respectively. Verifying these criteria for different points probed at various positions on the machine tool holds the credibility of the GUF results.

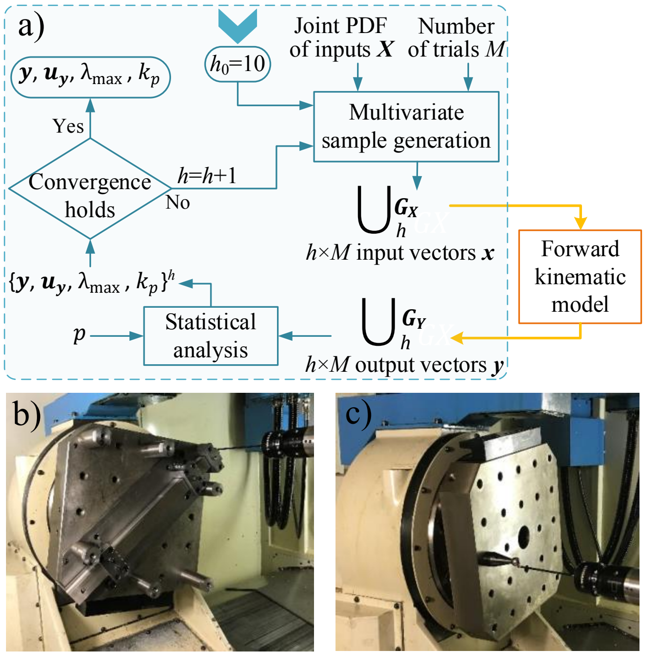

Fig. 1. a) Data flow for uncertainty assessment in on-machine measurement using an adaptive Monte Carlo method, and on-machine measurement of b) gauge block and c) precision sphere.

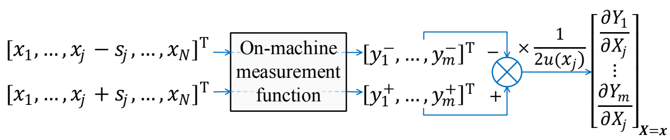

Fig. 2. Numerical differentiation of the output quantities $\boldsymbol{Y}$ of on-machine measurement function at $\boldsymbol{X} = \boldsymbol{x}$ with respect to X_j, the j^{th} input quantity.

For the three compensated coordinates of point 1, Fig. 3a-c compare the ellipsoidal and rectangular coverage regions for coverage probability p=95% obtained with GUF and MCM, which encompass 2000 random output points. The smallest coverage region for each pair of output quantities is also estimated with a finite element method specified in Supplement

2 to GUM [3]. The difference between the coverage areas for coordinates X and Y does not exceed 1% for both the

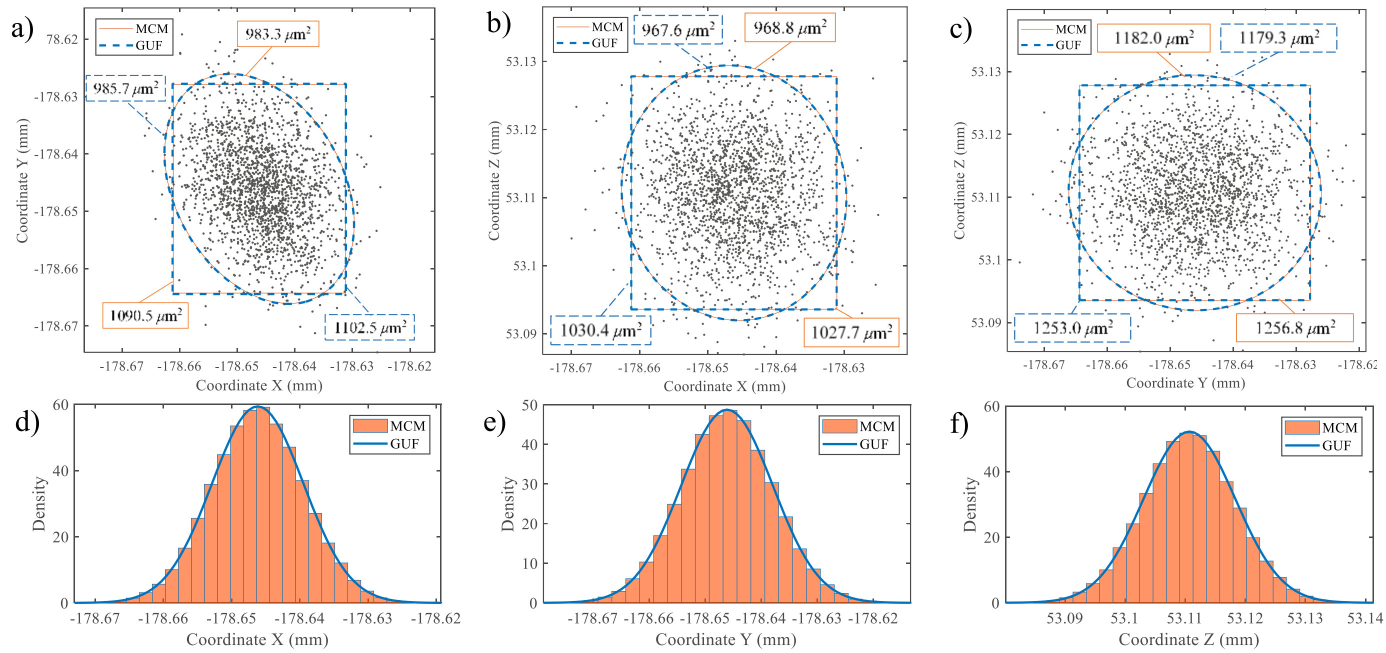

Fig. 3. a-c) Ellipsoidal and rectangular coverage regions (for coverage probability p=0.95) obtained by the adaptive MCM and GUF for the compensated coordinates of a point probed on the gauge block. Also shown are 2000 random output points. d-f) Comparison between the marginal histograms given by the adaptive MCM and GUF.

Table 1. The adaptive MCM and GUF results for best estimates $\boldsymbol{y}$ and associated standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$ together with their absolute differences obtained for the compensated coordinates of two points probed on a gauge block and its estimated length.

	Best estimate $\boldsymbol{y}$ (mm)				Standard uncertainty $\boldsymbol{u}(\boldsymbol{y})$ (μm)				MCM validates GUF?
	MCM	GUF	$\boldsymbol{d_y}$	$\boldsymbol{d_y}/\boldsymbol{\delta}$ (%)	MCM	GUF	$\boldsymbol{d_{u(y)}}$	$\boldsymbol{d_{u(y)}}/\boldsymbol{\delta}$ (%)	
Coordinate X of point 1	-178.6461	-178.6462	2.89E-05	58	6.7	6.7	4.38E-03	9	Yes
Coordinate Y of point 1	-178.6461	-178.6461	3.61E-05	72	8.2	8.2	1.23E-02	25	Yes
Coordinate Z of point 1	53.1107	53.1107	2.54E-05	51	7.6	7.6	4.25E-03	9	Yes
Coordinate X of point 2	179.1009	179.1009	1.38E-05	28	6.8	6.8	1.71E-03	3	Yes
Coordinate Y of point 2	179.0881	179.0881	1.18E-05	24	8.4	8.4	1.20E-02	24	Yes
Coordinate Z of point 2	53.1067	53.1068	1.79E-05	36	7.7	7.7	1.91E-02	38	Yes
Gauge's length	499.9966	499.9966	1.21E-05	24	9.0	9.0	6.76E-03	14	Yes

Table 2. The adaptive MCM and GUF results for the largest eigenvalue $\lambda_{\max}$ of the output correlation matrix and coverage factor k_p along with their absolute differences obtained for the compensated coordinates of two points and the estimated length listed in Table 1.

	$\lambda_{\max}$				k_p				MCM validates GUF?
	MCM	GUF	$d_{\lambda_{\max}}$	$d_{\lambda_{\max}}/\rho$ (%)	MCM	GUF	d_{k_p}	d_{k_p}/κ_p (%)	
Point set (Table 1)	1.313	1.312	4.80E-04	1	3.55	3.55	4.50E-03	9	Yes
Length	1.000	1.000	0	0	1.96	1.96	8.45E-04	2	Yes

ellipsoidal and rectangular coverage regions. For these coordinates, the smallest coverage area is 982.8 μm^2, which is by less than 1% different from the GUF ellipsoidal coverage area. These differences for the X-Z and Y-Z pairs also do not exceed 1%, whose smallest areas of coverage region are 962.6 and 1178.5 μm^2, respectively. Fig. 3d-f demonstrate the closeness between the MCM marginal PDFs for the compensated coordinates of point 1 and the normal PDFs fitted based on best estimates and the associated covariance matrix given by GUF.

For almost all the considered tasks, the adaptive MCM converges in h=10 sequences, including. M=10^5 trials [9]. For the estimation of the gauge's length, where the probed point set includes two points (n=2), the adaptive MCM completes in 167 s, on a computer with an Intel i7 processor running at 4.2 GHz, 32 GB of RAM, and Windows 10. This time for GUF is 7 s, 24 times faster than the adaptive MCM. In a different task, the adaptive MCM estimates the diameter of a sphere [9] and the associated uncertainty in 1967 s, whereas GUF completes in 12 s, being 164 times faster. Fig. 4 compares the computation times between the adaptive MCM and GUF for the sphere's diameter obtained from the point sets with different sizes varying from n=10 to 25. The computation time of GUF increases almost linearly proportional to the size of point set n. On average, GUF is 249 times more efficient than MCM in the sphere identification. This notable reduction in the computation cost mainly originates from the costly covariance simulator that operates based on an MCM algorithm (for both the MCM and GUF uncertainty schemes) and has to recur every MCM trial, whereas this occurs only once in GUF. This difference is more evident for larger point sets, where the number of calls for the forward kinematic model rises.

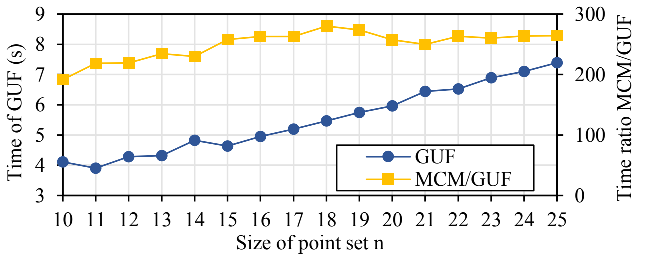

Fig. 4. The computation time of GUF for uncertainty evaluation of the sphere's diameter from the point sets with different sizes and the time ratio of the adaptive MCM to GUF.

5. Conclusions

An adaptive MCM developed for uncertainty assessment in on-machine measurement examines the feasibility of GUF. We apply these methods to obtain the best estimates and the associated standard uncertainty of the length of a gauge block and the diameter of a precision sphere. The computation cost of the uncertainty evaluation is also measured for the sphere's diameter obtained from sets of n=10 to 25 points. The summarized conclusions are as follows:

1. The adaptive MCM validates the GUF application for uncertainty assessment in on-machine probing and part verification. For the studied case of the gauge block measurement, the maximum absolute differences between the MCM and the GUF results are 72% and 38% of the specified numerical tolerance, respectively for best estimates and the associated standard uncertainty. These measures are 1% and 9% for maximum eigenvalue $\lambda_{\max}$ of the output correlation matrix and coverage factor k_p, respectively.
2. Besides complying with the criteria specified by Supplement 2 to GUM, the ellipsoidal and rectangular coverage regions as well as the marginal PDFs given by GUF closely approximate those obtained by MCM, which further validates GUF.
3. The GUF implementation dramatically decreases the uncertainty computation time. This method evaluates the uncertainty associated with a compensated bipoint probed on the gauge block and that with its length estimate in 7 s, which is 24 times faster than the adaptive MCM (167 s). For a point set of size 25 probed on the sphere, GUF gives the uncertainty associated with the compensated points and with the sphere's diameter in 12 s, whereas this time for MCM is 1967 s, i.e. 164 times longer. On average, GUF is 249 times more efficient in the sphere identification.

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