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# Scan-to-CAD alignment of damaged airfoil blade point clouds through geometric dissimilarity assessment

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**Abstract**

This paper presents a method for accurate alignment of the scanned point clouds of damaged blades with their nominal CAD model, which is an essential task in automated inspection for remanufacturing. The geometric dissimilarity of the underlying surface of the local neighborhoods of each measured data point and its nearest corresponding point on the CAD model is evaluated using a metric combining the average curvature Hausdorff distance and average Euclidean Hausdorff distance. The algorithm eliminates unreliable pairs with high geometric dissimilarity values in damaged regions from the matching process. The effectiveness of the proposed method is verified by experimental tests.

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*Keywords:* Damaged blade inspection; Scan-to-CAD Registration; Averaging-out error; Geometric dissimilarity.

## 1. Introduction

Due to working in a harsh environment, aero-engine blades are likely to be damaged or deformed from their original geometric shape over time. Considering the high material and tooling cost of manufacturing aero-engine blades, remanufacturing of damaged blades is a promising and economical choice for blade restoration to a desirable working condition [1-3].

In general, the remanufacturing process of damaged blades involves a pre-inspection to detect the material-missing regions on the blade's surface as well as checking for the conformance of the blade to the specified tolerances at the undamaged regions, including the surface profile tolerance and section-specific tolerances [4-6]. The inspection starts with 3D scanning of the damaged blade to capture the geometry of the part surface in the form of a point cloud. Optical 3D scanners, such as laser scanners or structured-light scanners, are preferred for data acquisition since they can quickly capture high-density point clouds with good accuracy [7, 8].

For any section-specific or surface profile inspection of the blades, the scanned point cloud data must be compared to the nominal CAD model, while these two initially lie in two different coordinate systems. The scanned point cloud data lies in the measurement coordinate system (MCS) and the CAD model is located in the design coordinate system (DCS) [9]. As can be seen in Fig. 1, the measured point cloud data has an arbitrary relative position and orientation with respect to DCS. A scan-to-CAD alignment (aka registration) is thus required to bring the measured point cloud to a common coordinate system with the CAD model. Due to the fact that there is a considerable geometric nonconformity between scanned point cloud data of a damaged blade and its CAD model in defective regions, registration of the damaged blades is a challenge. If the effects of the geometric nonconformities are not considered during aligning two geometric representations, an incorrect matching result will be obtained between two datasets.

Traditionally, the iterative closest point (ICP) algorithm [10] and its variants [11] are applied to best match the scanned point cloud data and CAD model of freeform surfaces [12, 13]. The main idea behind applying the ICP algorithm is to iteratively minimize the sum of the squared distances between measured data points and their corresponding points on the CAD model.

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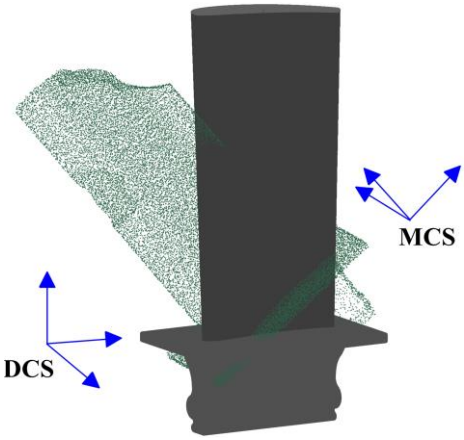


Fig. 1. Initial position and orientation of the scan data relative to the CAD model.

A rigid body transformation (i.e., a translation vector $\boldsymbol{T}$ and a rotation matrix $\boldsymbol{R}$) is applied to match every measured point $p_i$ with its corresponding point $q_i$ on the CAD model by minimizing the least-squares objective function $E$ of Eq. (1) [13]. $N_P$ is the number of points in the scanned point cloud.

$$E = \sum_{i=1}^{N_P} \left\| \boldsymbol{R}.p_i + \boldsymbol{T} - q_i \right\|^2 \quad (1)$$

The performance of the ICP method highly depends on the initial relative position and orientation of the measurement point cloud and CAD model. When two datasets are significantly far from each other, it is easy for the ICP algorithm to fall into a local optimum [14]. Therefore, it is necessary to find a proper initial estimation of the rigid transformation between two datasets using a coarse alignment and bring both sets close to each other before ICP fine alignment. Principal component analysis (PCA) is an effective technique to statistically estimate the principal axes of datasets and roughly bring two datasets close to each other [15]. Employing the PCA-based technique, the scanned point cloud is translated to share the same centroid with its nominal model, and then the point cloud is rotated to align its principal axes with those of the CAD model.

Once two datasets are initially matched through an appropriate coarse registration, the ICP algorithm iteratively matches the whole point cloud to the CAD model. Original ICP algorithm and other purely Euclidean distance-based least-squares minimization approaches are precise when aligning two identical geometric shapes. Any geometric nonconformity between two datasets to be aligned affects the registration results of the original ICP. When the scanned point cloud data of the damaged blade is aligned to its CAD model, the original ICP algorithm makes effort to minimize the squared distance between corresponding pairs in both material-missing (damaged) and undamaged regions. Consequently, the algorithm averages out the individual distances between corresponding pairs in order to globally minimize the least-squares objective function. As a result, the estimated errors between measured data points and CAD model at the damaged regions become smaller than the actual errors, and the estimated errors at the undamaged regions become larger than the actual errors [16].

Some studies have tried to propose new methods to weaken the effects of averaging-out errors on the registration of damaged blades. Zhang et al. [17] and Liu et al. [18] extracted the airfoil sections in the non-defective regions of the polygonal mesh model of the scanned damaged blade to align the point cloud data and nominal model. They employed the geometric features of extracted cross-sectional data, i.e., convex hull centroid and the centroid of minimal area bounding-box, for best-matching the two datasets. Calculation of the centroid location from the polygonal model is likely to be subject to area bias and centroid miscalculation [19, 20]. Li et al. [21] proposed a modified ICP algorithm to align the scan data with the nominal model based on the curvature and distance of each measured data point from its corresponding closest point on the nominal model. The point-to-point evaluation of geometric features of corresponding pairs makes the algorithm sensitive to noise. The performance of the methods presented in Refs. [17, 18, 21] depends on the user-defined thresholds. In addition, these methods require manual settings for the alignment of two geometric representations.

We have recently proposed a fine-tuned alignment method for the registration of damaged blades to remove the averaging-out errors of the original ICP [16]. In [16], we evaluated the effectiveness of our method using various numerical case studies. The present paper is an extension to [16] to examine the performance of the proposed method in an experimental case study with a real scanned point cloud. As the flowchart of Fig. 2 presents, the proposed scan-to-CAD registration approach includes three main steps: coarse (rough) alignment to bring two datasets sufficiently close to each other using the PCA method, fine alignment to iteratively best-match the whole point cloud to CAD model, and fine-tuned alignment to best-match only the reliable data points of undamaged regions with the CAD model in an iterative way to remove the averaging-out errors resulted from the original ICP. A correspondence search method is utilized to automatically assess the geometric dissimilarity of each corresponding pair and remove the unreliable pairs of damaged regions from the registration process.

## 2. Fine-tuned alignment algorithm

The fine-tuned alignment algorithm aims to automatically detect and remove data points of damaged regions and align the rest of the data points with the CAD model. This algorithm mainly contains two steps: correspondence search and transformation calculation (see Fig. 2). The appropriate correspondence search is the key to the correct alignment. A uniform point-sampled dataset of CAD model with the average point spacing equal to the scanned point cloud data is generated. The local neighborhood and Gaussian curvature at each data point of measured point cloud and CAD datasets are computed and inputted to the fine-tuned alignment algorithm. The Territory claiming (TC) algorithm [22] is used for establishing the local neighborhood of points, and the Gaussian curvature at each point is calculated through local quadric surface fitting as discussed in detail in [23].

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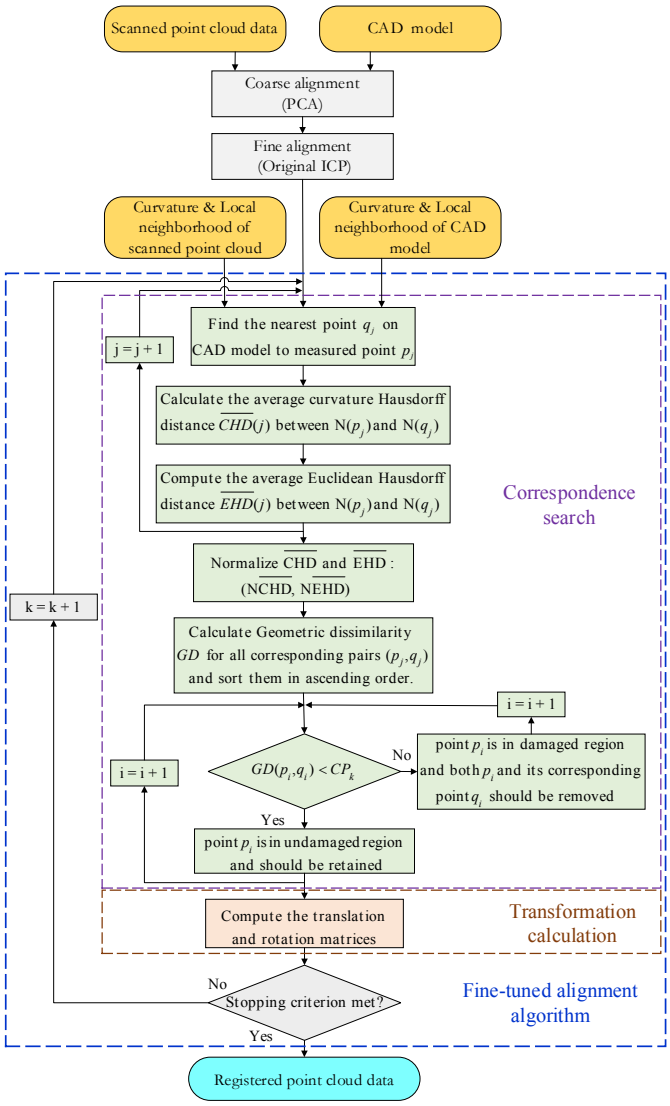


Fig. 2. Flowchart of the proposed fine-tuned registration algorithm.

For each point of the scan, the nearest point on the CAD model is found as its corresponding point first. Then the Hausdorff distance is utilized for a group-to-group evaluation of Euclidean distance and curvature of the local neighborhood of each measured point and its nearest point on the CAD model. A geometric dissimilarity function is defined by combining the curvature Hausdorff distance (*CHD*) and Euclidean Hausdorff distance (*EHD*) to assess the closeness of the local neighborhood of corresponding pairs and remove the unreliable pairs with high geometric dissimilarity value from the transformation computation process. The Hausdorff distance measures how far two datasets are from each other [24, 25]. In the present study, the average Hausdorff distance is employed to measure the closeness of curvature and Euclidean distance between the local neighborhoods of each measured point $p$ and its corresponding closest point $q$ on the CAD model. The average Hausdorff distance between two sets A and B is computed by Eq. (2).

$$\overline{HD}(A,B) = \frac{\sum_{a\in A}\min_{b\in B}\|a-b\| + \sum_{b\in B}\min_{a\in A}\|b-a\|}{|A|+|B|} \quad (2)$$

where | . | denotes the cardinality of a set, and || . || denotes the distance between elements of the sets (i.e., points), which can be determined by various distance definitions. We compute both the average Gaussian curvature Hausdorff distance ($\overline{CHD}$) and average Euclidean Hausdorff distance ($\overline{EHD}$) between point $p$ and $q$ to recognize the geometric dissimilarities in the damaged regions based on the shape changes and positional distance of local neighboring points *N(p)* and *N(q)*.

Once the $\overline{CHD}$ and $\overline{EHD}$ values between local neighborhoods *N(p)* and *N(q)* of each corresponding pair *(p, q)* are calculated, the $\overline{CHD}$ and $\overline{EHD}$ values are normalized using the min-max normalization approach of Eq. (3) to scale these two variables into the [0,1] interval and make them unitless:

$$N\overline{CHD}(N(p),N(q)) = \frac{\overline{CHD}(N(p),N(q)) - \min(\overline{CHD})}{\max(\overline{CHD}) - \min(\overline{CHD})}$$
$$N\overline{EHD}(N(p),N(q)) = \frac{\overline{EHD}(N(p),N(q)) - \min(\overline{EHD})}{\max(\overline{EHD}) - \min(\overline{EHD})} \quad (3)$$

The geometric dissimilarity *GD* (*p, q*) for each corresponding pair *(p, q)* is then defined as the combination of the normalized average curvature Hausdorff distance ($N\overline{CHD}(N(p),N(q))$) and the normalized average Euclidean Hausdorff distance ($N\overline{EHD}(N(p),N(q))$):

$$GD(p,q) = N\overline{CHD}(N(p),N(q)) + N\overline{EHD}(N(p),N(q)) \quad (4)$$

In each iteration, the *GD* value of each pair is compared to the cut-off point $CP_k$ to decide whether the measured point $p$ is in the undamaged region (i.e., $GD(p,q) < CP_k$) or belongs to the damaged region (i.e., $GD(p,q) \geq CP_k$). If the point $p$ is in damaged regions, the pair *(p, q)* is removed from the registration process. To compute the cut-off point $CP_k$ in the *k*th iteration of the fine-tuned alignment algorithm, the data points are sequenced in ascending order of the *GD* values of all corresponding pairs. The *GD* versus the data point index plot is considered as a rotated L-curve the corner of which divides the plot into data points with relatively large *GD* values and data points with relatively small *GD* values. Since the data points with relatively large *GD* values belong to the damaged regions, we calculate the corner of the L-curve and set its corresponding *GD* value as the $CP_k$ value to remove the corresponding pairs satisfying $GD \geq CP_k$. For more details on how to calculate $CP_k$, the readers are referred to [16].

After the geometric dissimilarity assessment and retaining only the reliable pairs of undamaged regions, the algorithm computes the rigid body transformation that minimizes the sum of the squared error between these reliable pairs of the two datasets. The computed rotation and translation are then applied to align the scanned point cloud data with the CAD model. The algorithm terminates the iteration when the global root mean square error of matching points for two successive iteration falls below a threshold value.

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## 3. Results and discussion

We conducted numerical case studies in [16] to validate the proposed fine-tuned alignment algorithm. Here, we compare the registration result of the proposed alignment method and the standard ICP algorithm for the experimental case study using scanned point cloud data of a damaged blade. The damaged blade was scanned using an ATOS Core 200 (GOM, Braunschweig, Germany) structured-light 3D scanner. Figure 3 shows the damaged blade, scanned point cloud data, and the nominal CAD model. As can be seen in Fig. 3, the scanned point cloud data has an arbitrary relative position and orientation with respect to the CAD model. The scanned point cloud contains 950,202 points with an average point spacing of 0.08 mm. The damaged blade contains voids and tip damage (see Fig. 3), which are common material-missing type damages on the surface of the aero-engine blades. The point cloud of the CAD model is also obtained by the uniform sampling of the surface with the average point spacing equal to scanned point cloud data (i.e., 0.08 mm) to analyze the same surface area on both CAD surface and the underlying surface of the scan data for the subsequent curvature and distance analysis.

In the present study, the PCA method and original ICP algorithm are applied respectively for rough and fine matching [16]. The classical point-to-point minimization algorithm has been used to globally minimize the root mean square error (*RMSE*) of the measured data from the CAD model and compute the transformation parameters $\boldsymbol{T}$ and $\boldsymbol{R}$. The iteration is terminated when the change of the global registration error falls below the threshold, which is set to be $\mu=10^{-6}$.

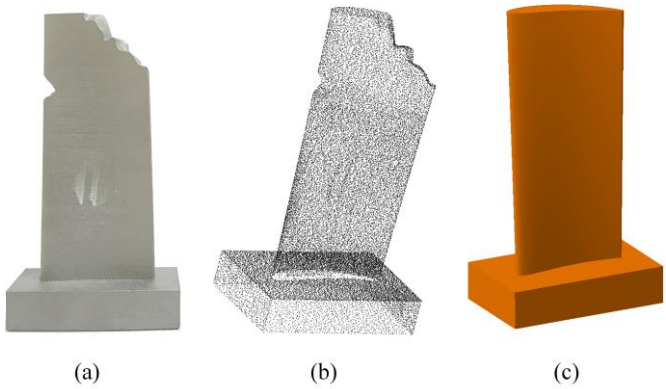


Fig. 3. (a) Damaged blade, (b) scanned point cloud of the damaged blade, and (c) the nominal CAD model.

Figure 4(a) shows the point cloud data of the damaged blade with its colormap based on the absolute deviations from the CAD model after being aligned using the fine-tuned registration procedure. Figure 4(b), illustrates the removed data points (in black) at the end of the last iteration of the fine-tuned registration. Using the proposed method, almost all data points of the damaged regions are eliminated from the registration process. It is seen in Fig. 4(b) that some data points of the undamaged regions in the trailing edge and sharp edges of the blade tail are also removed as unreliable points. Since only a tiny portion of data points in undamaged regions are removed as unreliable points, it does not affect the accuracy of the registration outcome. It should be noted that, in the present work, we have employed the raw scanned point cloud data of the blade as input, which is contaminated by outliers at the high-curvature features.

To locally investigate the averaging-out errors resulted from the original ICP algorithm, the point cloud data of both damaged blade and CAD model are sectioned by 17 equidistant sectional planes parallel to XY-plane of the CAD model from the bottom (Z=20 mm) to top (Z=100 mm) of the blade. Then, the data points in 0.1 mm distance from each sectional plane are specified as sectional data. The post-alignment errors are analyzed for each sectional data to compare the performance of the proposed method and standard ICP algorithm. Figure 5 shows the *RMSE* of sectional data points of the scanned point cloud from the CAD model after the original ICP (in red) and fine-tuned registration (in black). The sectional planes 4-6 and 12-17 are in damaged regions, and sectional planes 1-3 and 7-11 are in undamaged areas. As can be seen in Fig. 5, the post-alignment sectional *RMSE* values of the ICP method in damaged regions are smaller than the post-alignment sectional *RMSE* values of the proposed method and in undamaged regions are larger than the *RMSE* values of the proposed method. The maximum absolute deviation between the two is 21.5 µm at the tip of the blade (sectional plane #17). As discussed earlier, these averaging-out errors result from global minimization of the least-squares objective function of ICP. It should be noted that the averaging-out error values depend on the size and the geometry of damages on the scanned damaged blade. The results of aligning the scanned point cloud data of the damaged blade with the CAD model demonstrated that the proposed fine-tuned scan-to-CAD alignment method is successful in avoiding the averaging-out errors of the original ICP algorithm.

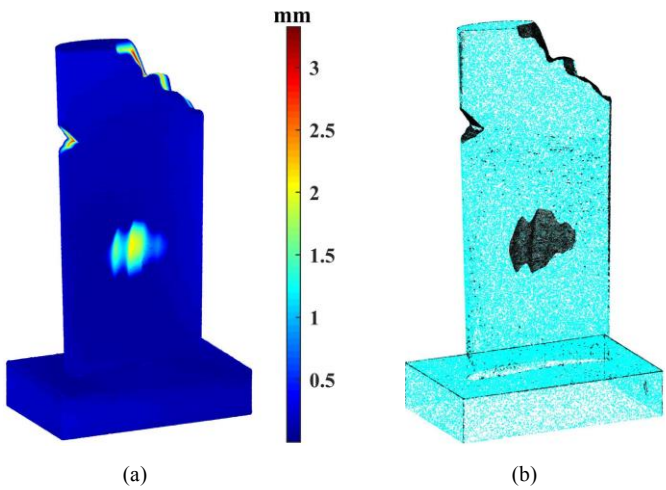


Fig. 4. (a) Error colormap of the aligned scanned point cloud data, and (b) the removed data points (in black) after the last iteration of the fine-tuned alignment.

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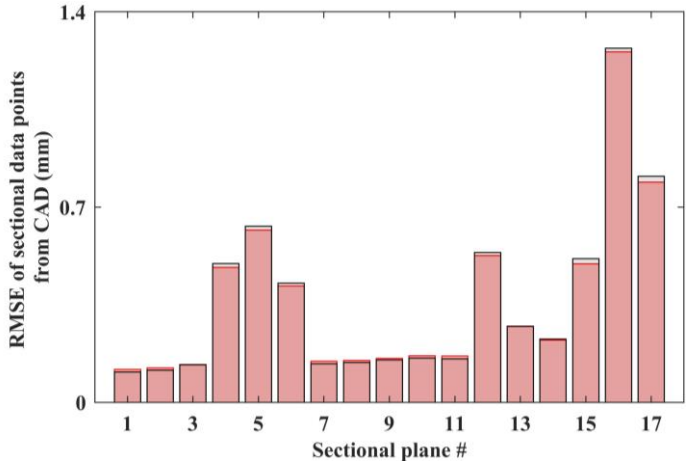


Fig. 5. Deviation of post-alignment *RMSE* of sectional data points from the CAD sectional data points after original ICP registration (in red) and the proposed method (in black).

## 4. Conclusion

This paper presents an accurate and automatic scan-to-CAD registration method for alignment of the scanned point clouds of damaged blades with their nominal CAD model. Since the least-squares minimization objective function of ICP-based algorithms globally minimizes the distance between the data points of both damaged and undamaged regions and their corresponding closest points on the CAD model, the geometric nonconformities between two datasets in defective areas lead to averaging-out errors. An effective method is developed to avoid averaging-out errors of the original ICP algorithm. The average curvature Hausdorff distance and average Euclidean Hausdorff distance are combined to measure the geometric dissimilarity between local neighborhoods of each measured data point and its closest point on the CAD model. The correspondence search step of the proposed fine-tuned alignment algorithm gradually removes the unreliable corresponding pairs with high geometric dissimilarity from the transformation calculation step and helps the algorithm avoid the averaging-out error. The registration results of the scanned point cloud data of a damaged blade have demonstrated the effectiveness of the proposed scan-to-CAD alignment method in eliminating the averaging-out errors resulted from the original ICP method.

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