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# Distributed Product Flow Control in a Network of Inventories With Stochastic Production and Demand

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**ABSTRACT** This paper presents a cooperative distributed model for the control of product flow in a network of cooperative inventory systems (ISs). In each IS, a linear input/output equation describes the balance between product demand vs production, as well as the product flows among ISs. The peculiarity of the problem is that both the production and demand are stochastic and cannot be controlled. In addition, each IS has just deterministic information on its inventory level (i.e., the quantity of stored product) which is not shared in the network. Other information is available in each IS regarding the forecast of both its related production and demand. The goal of the proposed model, on the overall network, is to keep both the product stored in each IS and the product exchanged among ISs, around given values as planned a priori. The only available control is related to the product flow between neighboring ISs, which has to be locally computed. Section II describes the proposed approach, which is based on the dual decomposition of the problem, which enables reaching the optimal control. This problem can represent the abstraction of a series of problems, which are not just related to logistics. As examples, section III presents two case studies: a network of virtual power utilities; and the control of risk in the transport of dangerous goods by road.

**INDEX TERMS** Inventory systems, distributed control, dual decomposition, logistics, virtual power utilities, dangerous goods transport.

## I. INTRODUCTION

Different large-scale complex systems, as logistics networks [1]–[3], water resources [4], and power grids [5], [6] can be modelled as a network of interacting inventory systems (IS) subject to local dynamics. Specifically, each IS can produce, stock, and locally distribute the same kind of product. Some of such large-scale complex systems may have the peculiarity that their production, as their customer demand, cannot be controlled. So, the control is limited to the exchange of product either among themselves or with external vendor/customers. The control objective is to track a desired level in each local inventory, satisfying local demand. In this paper, the IS network is taken into account as cooperative, balancing the divergences between demand and available

resources in each IS, in order to reduce inventory costs as well as improving service levels for the whole network [7]. Controlling the product and information flows among such subsystems represents a critical problem [8].

While reference values, either for product flow or for inventory levels, may be planned a priori, at the operational level, it is important to control product flows in the network and inventory levels such that they are close to the agreed reference planned values. As an example, a microgrid power system can be modelled as an IS, whose product is the power coming by renewable energy sources, that in case of sources as wind and sun depend on the uncontrolled weather conditions [9]. Power overproduction (or shortage) can be solved both by the use of local energy storage (e.g., batteries) and by exchanging power either with other cooperating microgrids or with an external power grid. The operational problem is to control power exchanges with other grids, with the aim

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of staying close to the agreed power exchange value (i.e., the control reference value) while satisfying local power demand, and leaving the local energy storage at the required level (i.e., the state reference value).

Hereinafter, these systems are referred to as a cooperative network of inventory systems (CNISs) composed of a set of input-output inventory systems (ISs) and a set of connecting links. Each IS is represented by a local linear model and characterized by a specific cost function to be minimized. In the general case, the IS model may include the inventory level (i.e., the state variable), a local production/demand modelled as an uncontrolled stochastic process, and the product input/output flows (i.e., the control variables), which are used to achieve each local objective. The links represent the infrastructure used both for the product flow and for the information communication between two neighbouring ISs. Some studies [10]–[13] have successfully shown that large-scale complex systems in different domains can be described by a CNIS, specifically, modeling their dynamics as linear systems and considering quadratic cost functions for each IS. Standard centralized control techniques may entail severe limitations mainly due to the lack of adaptability to a variable scenario where some ISs and/or related links may be either available or not (e.g. due to communication failure) in the evolution of processes. Therefore, for large-scale networked systems, a centralized control scheme may be inappropriate.

Various distributed model predictive control [14] strategies have been proposed where the computation of the optimal control is supported by the prediction of the stochastic variables. A potential way to classify them is to distinguish between cooperative and non-cooperative approaches. In a cooperative distributed model predictive control strategy, each agent optimizes a central objective function while considering the effect of all control actions on all subsystems of the network. In a non-cooperative distributed control strategy, each agent makes decisions on its own subsystem while considering the effect of network interactions only locally [15]. The performance of the network in this case converges to a Nash equilibrium [16]. Fundamental literature describing distributed model predictive control methods can be found in [17]–[24].

A relevant case study of distributed control applications is the power distribution field. Mehleri *et al.* [25] presented a mathematical programming approach for the optimal design of distributed energy systems. The objective is the optimal selection of distributed energy resource technologies and heat exchange connections among buildings, which minimizes the investment and operational cost of the overall distributed energy system. Guo *et al.* [26] developed a Lyapunov-based cost minimization algorithm that considers energy and demand management decisions. The authors performed a collaborative decentralised energy consumption scheduling algorithm in multiple households. Mudumbai *et al.* [27] proposed a distributed control algorithm for the frequency control and the optimal economic dispatch of power generators, where each generator independently adjusts its

power-frequency set points only using the aggregate power imbalance in the network. In [28], the problem of sending information packets from a source node to a destination node using a network of cooperative wireless relays to minimize the total energy consumption is considered.

Distributed control of cooperative systems can optimize the operations of a group of decision makers (DMs) that share a single global objective. The difficulty in optimally solving such problems arises when the global state of the system is either fully or partially nested in the different systems [29]. Dual decomposition is a method for splitting a large-scale optimization problem into multiple small-scale optimization subproblems. Although information exchange among the subsystems is required, the decomposition method is useful for the improvement of computational efficiency in distributed computing environments. Moreover, the method received renewed attention for distributed control [30]. In [31], dual decomposition has been used as a distributed iterative procedure that allows the agents to be aware of the impact of their actions on the global objective. Other approaches based on dual decomposition have been proposed for application in dangerous goods (DG) transport optimization [11], utility maximization in energy systems [12], [32] and flight transport [33].

The main contribution of this paper is to extend the current state-of-the-art of the distributed control of cooperative inventory systems through the application of the dual decomposition approach, defining optimal consensus between subsystems when their production and demand rate are stochastic variables. The distributed optimal consensus control about the desired values of product flows within the network depends on the minimization cost of each ISs under a cooperative strategy. An important aspect of the paper is that the proposed approach is described with the required abstraction to allow its application in different domains.

The rest of the paper is structured as follows. Section 2 describes the mathematical formulation of the CNIS model. Section 3 shows the application of the proposed strategy to two different case studies associated with transport networks and virtual power utilities. Section 4 concludes the paper, discussing the performance of the proposed approach, and illustrating interesting challenges for future research.

## II. MODEL DEFINITION

### A. THE GENERAL PROBLEM

A CNIS is modelled as a directed graph  $G(V, L)$ .  $V$  is the set of nodes modeling the IS, whose cardinality  $N = |V|$ .  $L$  is the set of links whose cardinality is  $M = |L|$ , connecting each couple of adjacent nodes in  $V$ . The link direction has the role of defining positive values, that, as a convention, are from  $i$  to  $j$  where  $j > i$  (negative in the opposite direction).

The following minimization problem  $P$ , in a discrete time horizon  $t = 0, \dots, T$ , can be defined:

$$H^* = \min_{\underline{u}(t)} \sum_{t=0}^T \left[ H \left( \underline{x}(t), \underline{u}(t), \underline{\xi}(t) \right) \right]$$

$$\begin{aligned}
 &= \min_{\underline{u}(t)} \sum_{t=0}^T \sum_{i \in V} H_i(x_i(t), \underline{u}(t), \xi_i(t)) \quad (1) \\
 &\text{s.t. } \underline{x}(t+1) = \underline{x}(t) + B\underline{u}(t) + \underline{\xi}(t) \\
 &\quad t = 0, \dots, T-1 \\
 &\quad \underline{x}(0) = \underline{x}^0 \quad (2)
 \end{aligned}$$

In (1)  $H^*$  is optimal values of the cost function  $H(\cdot)$ . where:

- $\underline{x}(t) \in \mathbb{R}^N$  is the system state vector, whose  $i$ -th element  $x_i(t)$  represents the  $i$ -th IS state at time  $t$ . In this approach the state variables are referred to the inventory values at the nodes.
- $\underline{u}(t) \in \mathbb{R}^M$  is the control vector, representing the constant generic flow exchanged between a pair of adjacent ISs during the time interval  $(t, t + 1]$ ; the control vector is solved for each time interval over a finite time horizon  $T$ .
- $\underline{\xi}(t) \in \mathbb{R}^N$  is a noise vector defined for the time interval  $(t, t + 1]$ , whose  $i$ -th element  $\xi_i(t)$  represents the random fluctuation between production and demand present in the  $i$ -th IS.

Besides,  $H(\cdot)$  is separable in terms of functions  $H_i(\cdot)$  which are related to each  $i$ -th subsystem, as a function of each subsystem state  $x_i(t)$ , and of the shared control  $\underline{u}(t)$ .

Equation (2) represents the discrete time state equation that describes the evolution over time of the inventory stored in each IS of the network and where

- $A \in \mathbb{R}^{N \times N}$  is a diagonal matrix whose diagonal elements weights the efficiency of the ISs.
- $B \in \mathbb{R}^{N \times M}$  incidence matrix, representing the network topology.
- $\underline{x}^0$  is the vector of the initial conditions associated to the state variables at the instant 0.
- $(t, t + 1]$  is the time discretization interval.

### B. DUAL DECOMPOSITION APPROACH

In the proposed distributed approach, the problem P, has been decomposed in several subproblems solved by each node sharing local control decisions with the neighbour nodes. In the following subsection, the problem is so reformulated, as shown in the following theorem.

Problem P has some important peculiarities:

- The function  $H(\cdot)$  can be separated into different functions  $H_i(\cdot)$ , one for each IS, with independent states  $x_i(t)$  but shared control  $\underline{u}(t)$ , affecting two adjacent ISs in each component.
- The matrix  $A$  is diagonal, as it is supposed that the state of one IS does not affect that of the others.
- In addition, let the function  $H(\cdot)$  and  $H_i(\cdot)$  be strictly convex.<sup>1</sup>

These peculiarities allow solving the problem in a decentralised approach, summarized in the following theorem.

**Theorem:** Let P be the problem described by (1) and (2).

<sup>1</sup>This assumption can be relaxed into a convex requirement using specific methods, such as the alternating direction method of multipliers which can also provide reliable and accurate solutions with more efficiency [34]

The optimization problem can be separated into  $N + M$  problems by the following formulation. Specifically,  $N$  problems are defined, one for each IS:

$$\begin{aligned}
 H_i^* &= \min_{\underline{w}_i(t), \underline{v}_i(t)} \sum_{t=0}^T \left\{ \begin{aligned} &H_i(x_i(t), \underline{w}_i(t), \underline{v}_i(t), \xi_i(t)) \\ &+ \sum_{j \in S(i)} c_{i,j}(t) w_{i,j}(t) + \\ &- \sum_{p \in P(i)} c_{p,i}(t) v_{i,p}(t) \end{aligned} \right\} \\
 \text{s.t. } &x_i(t+1) = A_i x_i(t) + \sum_{p \in P(i)} v_{i,p}(t) \\
 &- \sum_{j \in S(i)} w_{i,j}(t) + \xi_i(t) \\
 &x_i(0) = x_i^0 \quad i = 1 \dots N \quad (3) \\
 &x_i(0) = x_i^0 \quad i = 1 \dots N \quad (4)
 \end{aligned}$$

where:

- $H_i^*$  are the optimal values of the minimization problem solved by each IS;
- $\underline{w}_i(t) \in \mathbb{R}^M$  and  $\underline{v}_i(t) \in \mathbb{R}^M$ , are the control variables representing the outgoing and incoming product flows from the  $i$ -th IS, respectively. Specifically, the generic component  $j$  of  $\underline{w}_i(t)$ ,  $w_{i,j}(t)$ , represents the desired outgoing flow from the  $i$ -th towards the  $j$ -th IS. Similarly, the generic component of the vector  $\underline{v}_i(t)$ ,  $v_{j,i}(t)$ , represents the desired incoming flow from the  $j$ -th towards the  $i$ -th IS.
- $\underline{c}(t) \in \mathbb{R}^N$  is a cost vector, where the component  $c_{i,j}(t)$  represents the price to create a consensus on the control shared by the  $i$ -th and  $j$ -th IS.
- $P(i) \subset V$  and  $S(i) \subset V$  are the set of predecessor and successor nodes of the  $i$ -th IS, respectively.

In addition,  $M$  problems are defined one for each link  $(i, j)$ :

$$\begin{aligned}
 K_{(i,j)}^* &= \max_{c_{i,j}} [c_{i,j}(t)(w_{i,j}(t) - v_{j,i}(t))] \\
 &\forall (i, j) \in L, \quad t = 0, \dots, T-1 \quad (5)
 \end{aligned}$$

where

- $K_{(i,j)}^*$  are the optimal values of the maximization problem.

*Proof*

The shared control  $\underline{u}(t)$  is redefined with the new variables  $\underline{w}_i(t)$  and  $\underline{v}_i(t)$ , with additional constraints:

$$w_{i,j}(t) - v_{j,i}(t) = 0 \quad \forall (i, j) \in L, t = 0, \dots, T-1 \quad (6)$$

Under standard assumptions for strong duality, the problem is rewritten as:

$$H^* = \max_{\underline{c}(t)} \min_{\substack{\underline{w}_i(t) \\ \underline{v}_i(t)}} \sum_{t=0}^K \left\{ \begin{aligned} &\sum_{i \in V} [H_i(x_i(t), \underline{w}_i(t), \underline{v}_i(t), \xi_i(t)) \\ &+ \sum_{j \in S(i)} c_{i,j}(t) (w_{i,j}(t) - v_{i,j}(t)) \\ &+ \sum_{p \in P(i)} c_{p,i}(t) (w_{p,i}(t) - v_{i,p}(t))] \end{aligned} \right\} \quad (7)$$

For fixed  $c_{i,j}(t)$  and  $c_{p,i}(t)$ , the inner maximization decomposes into  $N$  separate optimization problems as

described in (3) subject to (4). According to the saddle point algorithm or Usawa's algorithm [35], the prices  $c_{i,j}(t)$  and  $c_{p,i}(t)$  can be locally traded and updated in a distributed manner using the gradient method, resulting in the  $M$  different problems as described in (5). The interested reader may refer to [36] and [37] for more details.

#### Optimization Procedure:

The possibility of solving the problem in a distributed way strongly depends on the possibility of efficiently minimizing function (3) subject to (4), while also taking into account the characteristics of the noise  $\xi_i(t)$ . Under the hypothesis that an efficient way to minimize (3) is available, the following iterative algorithm can be applied to solve problems (3), (4), and (5):

1. each IS solves its corresponding problem (3) and (4), considering an initial fixed vector  $\underline{c}(t)$ ;
2. the components  $\underline{w}_i(t)$ ,  $\underline{v}_i(t)$  that are solutions of problems (3), (4) are sent to the respective shared links, where a link trader agent (LTA) aims to improve the solution of each maximization problem (5) by varying the related component in  $\underline{c}(t)$ ;
3. each LTA sends the updated component  $\underline{c}(t)$  to the adjacent ISs for a new computation of  $H_i^*$ , optimized in  $\underline{w}_i(t)$ ,  $\underline{v}_i(t)$ ;
4. the algorithm terminates when a satisfactory convergence is reached, e.g., when  $|w_{i,j}(t) - v_{j,i}(t)| < \varepsilon_{i,j}$ ,  $\forall (i,j) \in L$  and  $\varepsilon_{i,j}$  is a given very small value. This stopping criterion can also be achieved in a distributed way, freezing a stable solution obtained by each LTA, that is, when the above condition is satisfied for a given number of instants.

The maximization in (5) can be approached with well-known control techniques (e.g., a gradient search method as in [38]). To reduce the number of iterations, more recent specific methodologies [39] can be applied.

#### Note 1:

A particular case for (1) is the cost function of a linear quadratic stochastic tracking problem:

$$\begin{aligned}
 J^* &= \min_{\underline{u}(t)} J(\underline{x}(t), \underline{u}(t), \underline{\xi}(t)) \\
 &= \min_{\underline{u}(t)} \sum_{t=1}^K \left[ (\underline{x}(t) - \underline{x}^*(t))^T Q_t (\underline{x}(t) - \underline{x}^*(t)) \right] \\
 &\quad + \sum_{t=0}^{K-1} (\underline{u}(t) - \underline{u}^*(t))^T R_t (\underline{u}(t) - \underline{u}^*(t)) \quad (8) \\
 \text{s.t. } &\underline{x}(t+1) = A\underline{x}(t) + B\underline{u}(t) + \underline{\xi}(t) \\
 &t = 0 \dots T-1 \\
 &\underline{x}(0) = \underline{x}^0 \quad (9)
 \end{aligned}$$

where:

- $\underline{x}^*(t)$  and  $\underline{u}^*(t)$  are the reference values for the inventory and flow, respectively;
- $Q_t \in \mathbb{R}^{N \times N}$ ,  $Q_t > 0$ ; hereinafter  $Q_t$  are supposed to be diagonal;

- $R_t \in \mathbb{R}^{M \times M}$ ,  $R_t > 0$ ; hereinafter  $R_t$  are supposed to be diagonal;
- $A \in \mathbb{R}^{N \times N}$  is a diagonal matrix;
- $B \in \mathbb{R}^{N \times M}$  is the CNIS incidence matrix.
- $\underline{\xi}(t)$  is the process noise or disturbance at time  $t$ .

While this problem can be solved in a centralized way [40], this approach may be impractical for very large and time varying networks, where communication, infrastructural links or the same IS may fail in time.

It is worthwhile to underline that the presented model can represent the abstraction of several problems, which may be also apparently distant from distributed inventory management problems. Specifically, in the following section, two different applications of the proposed dual decomposition approach are presented.

The first case study is related to a network of cooperating virtual power utilities. In this case, the product is power that can be generated by renewable energy sources, and that is consumed locally by customers. Such power can be either locally stored or exchanged with other virtual power utilities.

The second case study concerns the deliveries of DG transport by a road network. The road infrastructure is divided in regions (which are the ISs) where a certain number of DG trucks (whose risk is refereed in the abstraction of the model as the product) are expected to travel in the road infrastructure according to a prescheduled plan.

In both cases the aim is to find a control minimizing the divergence of the current IS state and flow with respect to the planned one. The two instances also reflect the peculiarity of the proposed problem where the DMs cannot control the production and demand rate. Meanwhile, the aim of the problem is to achieve a global objective on the network balancing only the flow values between adjacent subsystems through local control strategies.

### III. APPLICATION EXAMPLES

#### A. CASE STUDY 1: NETWORK OF COOPERATING VIRTUAL POWER UTILITIES (NCVPU)

In a network of cooperating virtual power utilities (NCVPU), virtual power utilities (VPUs) are distributed over geographically extensive areas. It is assumed that each VPU represents a cluster of distributed generators (e.g., wind turbines and photovoltaic plants) whose production cannot be controlled but depends on the availability of renewable energy sources. In each VPU a set of customers is also present, whose power demand cannot be controlled, as depending on real-time needs. Each VPU represents a node of the network. A VPU has an energy storage system (ESS) and can control the exchange of power in real time with other VPUs and with the energy market (EM). An LTA operates over each link of the network. The LTA has the task of finding an agreement between the two VPUs that are adjacent to that link. The objective is to cooperate to make the overall VPUs have the state and the control working around a reference level. The control is agreed on by information exchange with other adjacent VPUs. Each VPU is numbered from 1 to  $N-1$ , and

the EM, which is connected to at least one VPU, is assigned the index  $N$ . Time is discretized in time intervals of 1 h.

Referring to  $P$ , the variables and the parameters are as follows:

- $x_i(t)$ , [kWh], is the state variable of each VPU in terms of energy stored in the local ESS at instant  $t$ ;
- $x_i^*(t)$ , [kWh], is the reference value of the state with respect to an optimal EES load at instant  $t$ ;
- control vector  $\underline{u}(t)$  which consists of  $w_{i,j}(t)$  and  $v_{i,j}(t)$ , [kW], which are, respectively, the power exchange outgoing from the  $i$ -th to the  $j$ -th VPU (or to the EM) on link  $(i,j)$ , and incoming from the  $k$ -th VPU (or from the EM) to the  $i$ -th VPU on link  $(k,i)$ , during the time interval  $(t, t + 1]$ ;
- $w_{i,j}^*(t)$  and  $v_{i,j}^*(t)$  are reference values for the power exchange during the time interval  $(t, t + 1]$ ;
- $\xi_i(t)$ , [kW], represents a stochastic power balance during the time interval  $(t, t + 1]$ , i.e., the difference between the demand and the produced power in each VPU;
- $q_i$  and  $r_i$  parameters are weights which are here taken as constant in time, related to the state and the different control components;
- $0 < a_i < 1$  is an efficiency factor related to the ESS.

This problem has been applied between Italy and Morocco, assuming that two VPUs are available in each country. Wind turbines are the generators available in the four VPUs. They are connected according to the topology shown in Fig. 1, where VPU <sub>$i$</sub>  is available in Monte Settepani ( $i = 1$ ), Tanger ( $i = 2$ ), Capo Vado ( $i = 3$ ) and Essaouira ( $i = 4$ ). The wind power potential available in the four regions is based on the statistics given in the literature [41]. The computation of the optimal control is performed on a horizon of seven days, so  $K = 168$ .

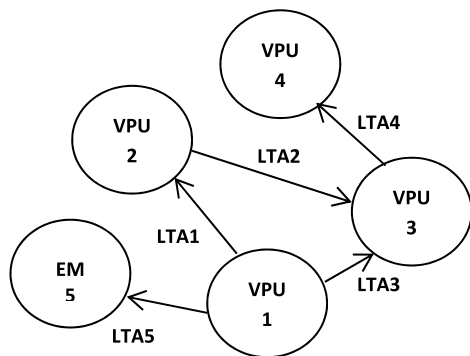


FIGURE 1. The NCVPU model including an energy market (EM).

The ESS reference values have been set to  $x_i^*(t) = 0 \forall t$ , while the control reference values have been set to  $w_{i,j}^*(t) = 0$  and  $v_{i,j}^*(t) = 0, \forall t$ . For the proposed case study, the  $\xi_i(t)$  values are shown in Fig. 2. The characteristics of  $\xi_i(t)$  can be separated in a deterministic component (for example, a prediction of power production and demand can be available

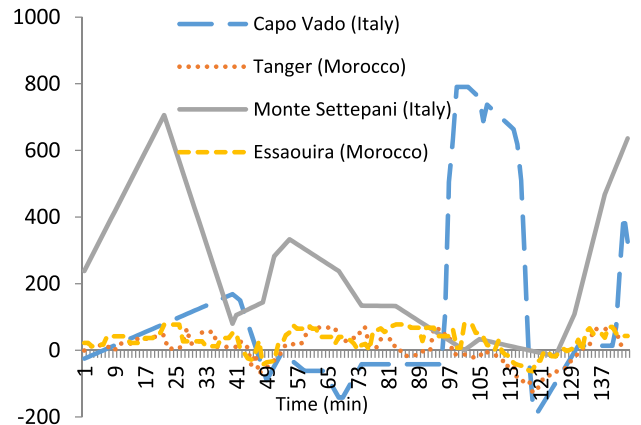


FIGURE 2. The values of  $\xi_i(t)$  for the four VPUs of Fig. 1.

by forecast models) and a stochastic component (due to the errors in forecasting).

At each time step  $t$ , an LTA communicates with its adjacent VPUs, proposing an improvement of  $c_{i,j}(t)$  parameters. Each VPU can compute its optimal desirable flows  $w_i(t)$ ,  $v_i(t)$  according to a local LQ stochastic tracking algorithm and send them back to each respective LTA. The iteration lasts until an acceptable convergence is reached.

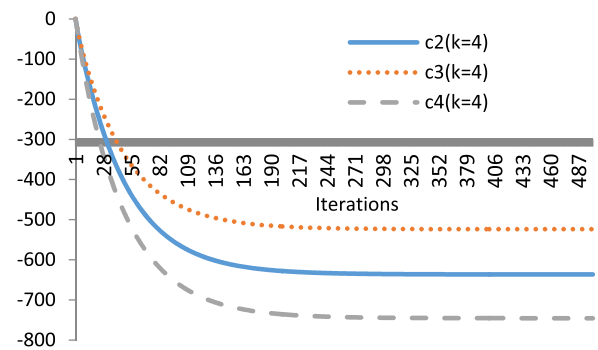


FIGURE 3. The trend of the costs  $c_{2,3}^*(t)$ ,  $c_{1,3}^*(t)$ , and  $c_{3,4}^*(t)$  during the agreement process.

Fig. 3 displays the convergence of  $c_{1,3}(t)$ ,  $c_{2,3}(t)$ , and  $c_{3,4}(t)$ . The agreement is reached after approximately 400 iterations. Fig. 4 shows the cost function for the four VPUs. It is worth highlighting that the EM has no state variable, and it is supposed to comply with the controls of the VPUs. It is also worthwhile to observe that the local cost function (to be minimized) increases with the search for the overall cooperative agreement. Fig. 5 shows the power agreed on between VPUs, at instant  $t = 4$ , where  $u_i$  is related to the power exchanged by LTA <sub>$i$</sub>  computed as  $v_i - w_i$ . It can be observed that negative values of  $c_{3,4}(4)$  are reflected in a positive  $u_4(4)$  value as outgoing flow with respect to 3. The same can be said for  $c_{2,3}(4)$  and  $c_{1,3}(4)$ . It is relevant to note that the disturbance  $\xi_3(t)$  for values of  $t$  around 20 (Fig. 2) is only partially sent to EM on  $u_5$ , while the rest is used to

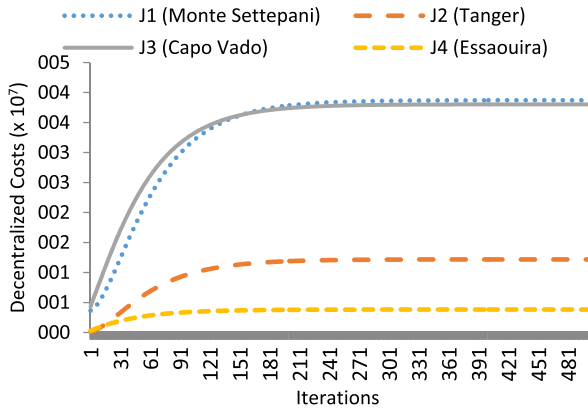


FIGURE 4. The cost functions for the VPU's using the distributed control.

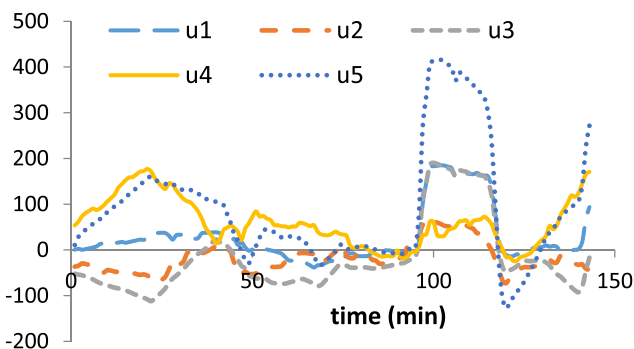


FIGURE 5. Trends of the optimal values for the  $u_i^*$ ,  $i = 1, \dots, 5$  elements of the control variables under the distributed control.

feed the internal demand and other VPUs in the network. The disturbance  $\xi_1(t)$  for values of  $t$  around 100 (Fig. 2) is reflected in a greater flow in  $u_5$ .

### B. CASE STUDY 2: DANGEROUS GOODS TRANSPORT BY ROAD

A risk-based approach to define deliveries in DG transport by road is taken into account. The original model has been introduced in [11], and a preliminary version of this work has been proposed in [42], where moderate exchange of information in the whole network among the DMs is allowed.

The model is focused on a central DM that aims to rearrange a predefined scheduled plan of DG deliveries from one depot (origin) to  $D$  service stations (destinations) to minimize both the risk due to possible accidents and the cost due to delays in deliveries. It is assumed that an initial planned schedule for deliveries is given. The motivation to modify it is twofold. First, the planning is usually computed according to an economic objective (e.g., shortest path). Second, in real time, the updated information about traffic and possible exposure of the population along the roads to the DG risk can be available with better precision. The average planned speeds of the vehicles (e.g., tank trucks) are also known (e.g., computed using a prediction model for traffic conditions). Therefore, the main objective of the DM (e.g., a national or regional authority) is to modify the scheduled route of

vehicles, taking into account the DG quantities travelling through a specific area and considering the related exposure and risk. This approach is formulated to balance two different objectives that are commonly referred to by different stakeholders involved in DG transport: the total risk on the road network, of main interest to national or regional authorities, and the compliance between the real delivery times with respect to the planned ones, of main interest to transportation companies. As a simplifying assumption, the DG flow in the network is modeled as continuous and not as discretized.

The optimization problem can be stated as follows:

$$\begin{aligned} \min_{i,q} \alpha' & \sum_{t=0}^{T-1} \sum_{d \in D} \sum_{n \in N} \left( i_n^d(t) - \hat{i}_n^d(t) \right)^2 \frac{(r_n(t))^2}{l_n^2} \\ & + \beta' \sum_{d \in D} \sum_{n \in N} \left( i_n^d(T) - \hat{i}_n^d(T) \right)^2 \frac{(r_n(T))^2}{l_n^2} \\ & + \gamma' \sum_{t=0}^{T-1} \sum_{d \in D} \sum_{n \in N} \sum_{m \in S(n)} \left( q_m^d(t) l_n - i_n^d(t) \hat{v}_m^d(t) \right)^2 \end{aligned} \quad (10)$$

$$\begin{aligned} \text{s.t. } i_n^d(t+1) &= i_n^d(t) + \sum_{m \in P(n)} q_{m,n}^d(t) - \sum_{m \in S(n)} q_{n,m}^d(t) + \xi_n^d(t), \\ i_n^d(0) &= \hat{i}_n^d, \quad t=0, \dots, T-1, \forall n \in N, \forall d \in D \end{aligned} \quad (11)$$

where:

- $i_n^d(t)$  [gasoline tonne equivalent (GTE)] is the state vector for each IS, which represents the DG mass quantity present at the  $n$ -th node at instant  $t$  directed to destination  $d$ ,  $d \in D$ , where  $D \subseteq V$  is the set of planned (IS) destinations.
- $\hat{i}_n^d(t)$  [GTE] is an input variable related to the planned quantity present at the  $n$ -th node directed to destination IS  $d$  at time  $t$ ; in this case study, they are assumed to be positive only at the origin and at the destination nodes, being zero at the transition nodes, meaning that there is a required low exposure level of the transition nodes during the DG transfer.
- $r_n(t)$  [inh] is an input variable related to the time-dependent value of risk on the  $i$ -th region at instant  $t$  (e.g., the overall number of inhabitants that are present at instant  $t$  in a buffer of 100 m of the road segment associated with IS  $n$ ).
- $l_n$  [km] is an input parameter representing the size of the  $n$ -th IS.
- $q_{n,m}^d(t)$  [GTE] is a decision variable related to the optimal DG flows in link  $(n, m)$  directed to destination IS  $d$  during the time interval  $(t, t + 1]$ .
- $\hat{v}_m^d(t)$  [km] is a decision variable related to the optimal speed (expressed as the space covered in one time interval) for DG vehicles that are in transit towards link  $(n, m)$  directed to destination IS  $d$  during the time interval  $(t, t + 1]$ .
- $\xi_n^d(t)$  [GTE] is a stochastic variable defining the planned arrivals of DG at IS  $n$  from the outside of the network directed to destination IS  $d$  during the time interval  $(t, t + 1]$ .

The aim of the first two terms of the cost function are as follows:

- in case  $\hat{i}_n^d(t) = 0$  (meaning that the IS  $n$  is only a transition node towards destination  $d$ ), the ISs characterized by a higher risk are penalized;
- otherwise, if  $\hat{i}_n^d(t) > 0$  (set only if  $n = d$ , thus when  $n$  is the destination of the shipment), the optimizer will try to obtain values as close as possible to the planned inventory level.

On the other hand, the third term is a penalty term introduced to respect the desired planned speed for each specific DG shipment, considering that the classical relationship of macroscopic traffic models among traffic density, speed and flow holds [34]. The dynamic equation is in the form of a classic conservation law for each IS. Interested readers can find a more detailed description of this model in [11]. This model can be assimilated as one considering a CNIS introduced in (1), where the state vector  $\underline{x}(t)$  contains all inventories  $i_n^d(t)$  for each  $n$  and  $d$ , whereas control vector  $\underline{u}(t)$  contains DG flows  $q_{n,m}^d(t)$  for all links  $(n, m)$ . Each node, which identifies each IS, may obviously represent an origin node, a destination node, or simply a transition node. The optimization problem to be solved by each DM could be again brought back to an LQ tracking problem. Thus, the solution can be efficiently computed for each IS  $n$ .

This optimization algorithm has been applied to a small demonstrative network composed of 5 ISs and 6 links representing logistic areas. The unit time step is  $\Delta t=10$  minutes. At the initial instant, it is supposed that a known quantity of DG is stored in node 1. The destinations are represented by ISs 4 and 5.

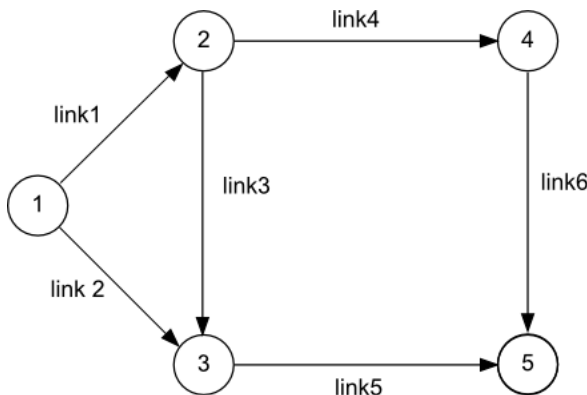


FIGURE 6. The network with 5 ISs and 6 LTA links.

Fig. 6 shows the network topology. The values  $\hat{i}_n^d(t)$  are all set equal to zero, except for:  $\hat{i}_1^4(0) = \hat{i}_1^5(0) = 100$ ;  $\hat{i}_4^4(\bar{t})=100$ , for  $\bar{t} \geq 10$ ;  $\hat{i}_5^5(\bar{t})=100$ , for  $\bar{t} \geq 14$ . The risk values  $r_i(t)$  are all set to 1, except for:  $r_2(\bar{t})=100$ ;  $r_4(\bar{t})=25$ , and for  $6 \leq \bar{t} \leq 13$ . The dimension is set to  $d_i = 10$  for each region.

A reasonable global optimum has been achieved in fewer than 200 iterations, as shown in Fig. 7, showing the convergence of the values  $c_{i,j}(t)$ .

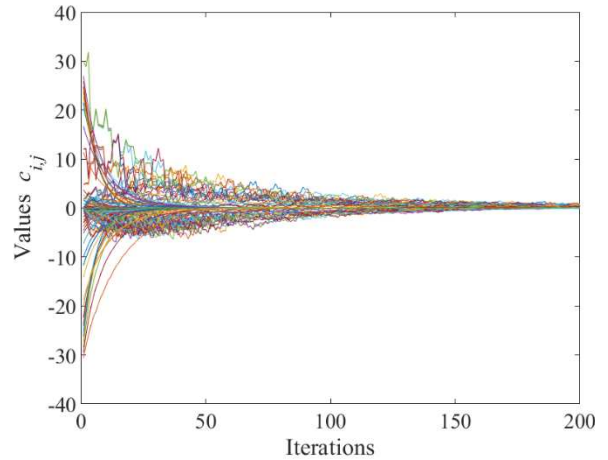


FIGURE 7. The trend of values  $c_{i,j}(t)$ .

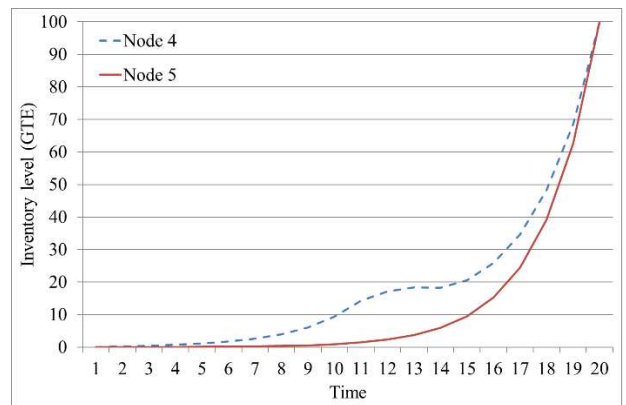


FIGURE 8. The values of inventory at the destination nodes, IS 4 and IS 5.

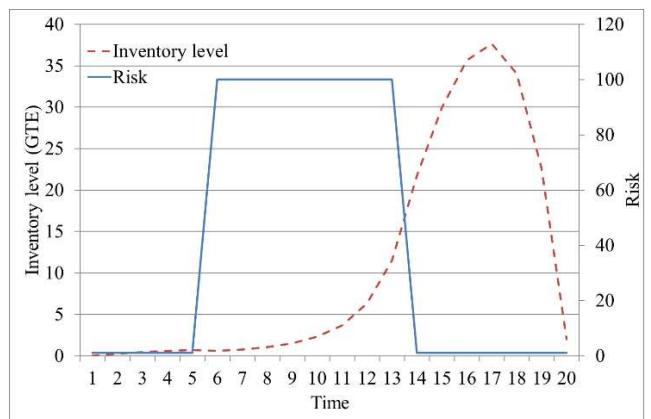


FIGURE 9. Risk value and Inventory value at IS 2.

By applying the gradient method proposed in [39] to the dual problem, the convergence rate of the parameters  $c_{i,j}(t)$  is accelerated and it guarantees to reach the consensus between the decisional variables related to the optimal DG flows in the links in a minor number of iterations. This convergence has shown an important improvement with respect to both



the previous case study and to a previous work [43]. At the end of the simulation, the objective of delivering the planned product to each destination node has been achieved. As shown in Fig. 8, the IS level values for destination nodes 4 and 5 reached the optimal planned amounts in 20 time steps. Each DM also attempts to avoid the time intervals characterized by a higher value of risk, as shown in Fig. 9 for node 2. In fact, by setting a high value of  $r_i(t)$ , an agreement could be reached that the visit of a node is anticipated, postponed, or even avoided.

#### IV. CONCLUSION

In this paper, a distributed optimal control problem has been introduced for a CNIS, presenting a solution algorithm based on the dual decomposition technique. Although the technique is quite classic, it has recently received renewed attention due to its ability to decompose a problem into a team of cooperative agents [31].

Two different examples have been implemented, which refer to the special case of linear quadratic (LQ) problems, specifically applied to power exchange in an NCVPU and to DG transport.

In each instance, local sub-problems were solved at each node under a cooperative vision of the problem; supposing that each subsystem is sharing control with the adjacent subsystem without knowledge about its state.

The considered class of CNIS problems that can be found in a wide number of application areas, as resulting from the two examples, presents completely decoupled states, despite being coupled at the links connecting two ISs.

Therefore, in the authors' opinion, a CNIS seems to be an interesting class of problems that requires further investigations for at least two main reasons: first, a CNIS seems to be representative of a wide set of relevant new applications; second, the CNIS problem has a basic structure that can presumably be exploited to further design even more efficient distributed control algorithms.

Despite the proposed methodological simplicity, which has been intentionally adopted in this paper for demonstrative purposes, the CNIS model can be the starting point of several extensions, aimed at neglecting some simplifying assumptions. In addition, further improvements can be adopted to reach the algorithm convergence, which may be more suitable for a specific problem [44], [45].

In conclusion, the main contribution of the paper is twofold. First, it introduces a class of problem related to the control of complex systems composed by subsystems which share control decisions in a cooperative way when in each system the knowledge of the state of the other subsystems is unavailable.

Secondly, it presents a unifying view of different applications modeled as a CNIS, which can be efficiently implemented and optimized in a distributed way. The main peculiarity is that the limited exchange of information is related to the optimal input-output flow for each IS sent to connected agent links. This sort of privacy feature may

become relevant when the confidentiality on the state and its dynamics become a system requirement.

#### REFERENCES

- [1] M. Paolucci, R. Sacile, and A. Boccalatte, "Allocating crude oil supply to port and refinery tanks: A simulation-based decision support system," *Decis. Support Syst.*, vol. 33, pp. 39–54, May 2002. doi: [10.1016/S0167-9236\(01\)00133-6](https://doi.org/10.1016/S0167-9236(01)00133-6).
- [2] C. Bersani, R. Minciardi, R. Sacile, and E. Trasforini, "Network planning of fuelling service stations in a near-term competitive scenario of the hydrogen economy," *Socio-Econ. Planning Sci.*, vol. 43, pp. 55–71, Mar. 2009. doi: [10.1016/j.seps.2008.02.001](https://doi.org/10.1016/j.seps.2008.02.001).
- [3] X. Xu, H. Li, J. Liu, B. Ran, and L. Qin, "Passenger flow control with multi-station coordination in subway networks: Algorithm development and real-world case study," *Transportmetrica B, Transport Dyn.*, to be published. doi: [10.1080/21680566.2018.1434020](https://doi.org/10.1080/21680566.2018.1434020).
- [4] R. Minciardi, M. Robba, and R. Sacile, "Decision models for sustainable groundwater planning and control," *Control Eng. Pract.*, vol. 15, pp. 1013–1029, Aug. 2007. doi: [10.1016/j.conengprac.2006.10.017](https://doi.org/10.1016/j.conengprac.2006.10.017).
- [5] H. Dagdougui, A. Ouammi, and R. Sacile, "Optimal control of a network of power microgrids using the Pontryagin's minimum principle," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1942–1948, Sep. 2014. doi: [10.1109/TCST.2013.2293954](https://doi.org/10.1109/TCST.2013.2293954).
- [6] R. Zaccone, R. Sacile, and M. Fossa, "Energy modelling and decision support algorithm for the exploitation of biomass resources in industrial districts," *Int. J. Heat Technol.*, vol. 35, pp. S322–S329, Sep. 2017. doi: [10.18280/ijht.35Sp0144](https://doi.org/10.18280/ijht.35Sp0144).
- [7] X. Zhang and L. Wei, "A study of distributed inventory system based on VMI strategy under emergency conditions," in *Proc. 13th Int. Conf. Service Syst. Service Manage. (ICSSSM)*, Kunming, China, Jun. 2016, pp. 1–6. doi: [10.1109/ICSSSM.2016.7538474](https://doi.org/10.1109/ICSSSM.2016.7538474).
- [8] W. Zhao and D. Wang, "Simulation-based optimization on control strategies of three-echelon inventory in hybrid supply chain with order uncertainty," *IEEE Access*, vol. 6, pp. 54215–54223, 2018. doi: [10.1109/ACCESS.2018.2870856](https://doi.org/10.1109/ACCESS.2018.2870856).
- [9] Y. Xu and X. Shen, "Optimal control based energy management of multiple energy storage systems in a microgrid," *IEEE Access*, vol. 6, pp. 32925–32934, 2018. doi: [10.1109/ACCESS.2018.2845408](https://doi.org/10.1109/ACCESS.2018.2845408).
- [10] R. Minciardi and R. Sacile, "Optimal control in a cooperative network of smart power grids," *IEEE Systems J.*, vol. 6, no. 1, pp. 126–133, Mar. 2012.
- [11] C. Roncoli, C. Bersani, and R. Sacile, "A risk-based system of systems approach to control the transport flows of dangerous goods by road," *IEEE Systems J.*, vol. 7, no. 4, pp. 561–570, Dec. 2013.
- [12] H. Dagdougui and R. Sacile, "Decentralized control of the power flows in a network of smart microgrids modeled as a team of cooperative agents," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 510–519, Mar. 2014.
- [13] C. Bersani, H. Dagdougui, A. Ouammi, and R. Sacile, "Distributed robust control of the power flows in a team of cooperating microgrids," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 4, pp. 1473–1479, Jul. 2017.
- [14] E. Camponogara, D. Jia, B. H. Krogh, and S. Talukdar, "Distributed model predictive control," *IEEE Control Syst. Mag.*, vol. 22, no. 1, pp. 44–52, Feb. 2002.
- [15] W. B. Dunbar, "Distributed receding horizon control of dynamically coupled nonlinear systems," *IEEE Trans. Autom. Control*, vol. 52, no. 7, pp. 1249–1263, Jul. 2007.
- [16] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*. Philadelphia, PA, USA: SIAM, 1999.
- [17] P. D. Christofides, R. Scattolini, D. M. de la Peña, and J. Liu, "Distributed model predictive control: A tutorial review and future research directions," *Comput. Chem. Eng.*, vol. 51, pp. 21–41, Apr. 2013.
- [18] A. Bemporad, and D. Barcelli, "Decentralized model predictive control," in *Networked control systems* (Lecture Notes in Control and Information Sciences), vol. 406. London, U.K.: Springer, 2010, pp. 149–178.
- [19] A. Bemporad and M. Morari, "Robust model predictive control: A survey," in *Robustness in identification and control* (Lecture Notes in Control and Information Sciences), vol. 245. New York, NY, USA: Springer-Verlag, 1999, pp. 207–226.
- [20] R. Scattolini, "Architectures for distributed and hierarchical Model Predictive Control—A review," *J. Process Control*, vol. 19, no. 5, pp. 723–731, 2009.
- [21] N. Sandell, P. Varaiya, M. Athans, and M. Safonov, "Survey of decentralized control methods for large scale systems," *IEEE Trans. Autom. Control*, vol. AC-23, no. 2, pp. 108–128, Apr. 1978.

- [22] L. Magni and R. Scattolini, "Stabilizing decentralized model predictive control of nonlinear systems," *Automatica*, vol. 42, no. 7, pp. 1231–1236, 2006.
- [23] A. Ferramosca, D. Limona, I. Alvaradoa, and E. F. Camacho, "Cooperative distributed MPC for tracking," *Automatica*, vol. 49, pp. 906–914, Apr. 2013.
- [24] A. N. Venkat, J. B. Rawlings, and S. J. Wright, "Stability and optimality of distributed model predictive control," in *Proc. 44th IEEE Conf. Decis. Control*, Dec. 2005, pp. 6680–6685.
- [25] E. D. Mehleri, H. Sarimveis, N. C. Markatos, and L. G. Papageorgiou, "A mathematical programming approach for optimal design of distributed energy systems at the neighbourhood level," *Energy*, vol. 44, pp. 96–104, Aug. 2012.
- [26] Y. Guo, M. Pan, Y. Fang, and P. P. Khargonekar, "Decentralized coordination of energy utilization for residential households in the smart grid," *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1341–1350, Sep. 2013.
- [27] R. Mudumbai, S. Dasgupta, and B. B. Cho, "Distributed control for optimal economic dispatch of a network of heterogeneous power generators," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1750–1760, Nov. 2012.
- [28] R. Madan, N. B. Mehta, A. F. Molisch, and J. Zhang, "Energy-efficient decentralized cooperative routing in wireless networks," *IEEE Trans. Autom. Control*, vol. 54, no. 3, pp. 512–527, Mar. 2009.
- [29] J. M. Maestre, D. M. de la Peña, and E. F. Camacho, "Distributed model predictive control based on a cooperative game," *Optim. Control Appl. Methods*, vol. 32, no. 2, pp. 153–176, 2011.
- [30] A. Rantzer, "Dynamic dual decomposition for distributed control," in *Proc. Amer. Control Conf.*, 2009, pp. 884–888.
- [31] A. Rantzer, "On prize mechanisms in linear quadratic team theory," in *Proc. 46th IEEE Conf. Decis. Control*, Dec. 2007, pp. 1112–1116.
- [32] A. D. Domínguez-García and C. N. Hadjicostis, "Distributed algorithms for control of demand response and distributed energy resources," in *Proc. 50th IEEE Conf. Decis. Control Eur. Control Conf.*, Dec. 2011, pp. 27–32.
- [33] R. L. Raffard, C. J. Tomlin, and S. P. Boyd, "Distributed optimization for cooperative agents: Application to formation flight," in *Proc. 43rd IEEE Conf. Decision Control*, Dec. 2004, pp. 2453–2459.
- [34] K. K. Sanwal, K. Petty, J. Walrand, and Y. Fawaz, "An extended macroscopic model for traffic flow," *Transp. Res. B, Methodol.*, vol. 30, no. 1, pp. 1–9, 1996.
- [35] K. J. Arrow, L. Hurwicz, and H. Uzawa, *Studies in Linear and Non-Linear Programming*. Palo Alto, CA, USA: Stanford Univ. Press, 1958.
- [36] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [37] I. Necoara and J. A. K. Suykens, "Application of a smoothing technique to decomposition in convex optimization," *IEEE Trans. Autom. Control*, vol. 53, no. 11, pp. 2674–2679, Dec. 2008.
- [38] J. Barzilai and J. M. Borwein, "Two-point step size gradient methods," *IMA J. Numer. Anal.*, vol. 8, pp. 141–148, Jan. 1988.
- [39] P. Giselsson, M. D. Doan, T. Keviczky, B. De Schutter, and A. Rantzer, "Accelerated gradient methods and dual decomposition in distributed model predictive control," *Automatica*, vol. 49, no. 3, pp. 829–833, 2013.
- [40] B. D. O. Anderson and J. B. Moore, *Optimal Control: Linear Quadratic Methods*. New York, NY, USA: Dover Publications, 2007.
- [41] A. Ouammi, H. Dagdougui, R. Sacile, and A. Mimet, "Monthly and seasonal assessment of wind energy characteristics at four monitored locations in Liguria region (Italy)," *Renew. Sustain. Energy Rev.*, vol. 14, pp. 1959–1968, Sep. 2010.
- [42] C. Roncoli, C. Bersani, and R. Sacile, "A decentralised control model for the risk-based management of dangerous good transport flows on road," in *Proc. IEEE Int. Syst. Conf.*, Vancouver, BC, Canada, Mar. 2012, pp. 1–5.
- [43] M. Raydan and B. F. Svaiter, "Relaxed steepest descent and Cauchy-Barzilai-Borwein method," *Comput. Optim. Appl.*, vol. 21, no. 2, pp. 155–167, 2002.
- [44] B. O'Donoghue, G. Stathopoulos, and S. Boyd, "A Splitting Method for Optimal Control," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 6, pp. 2432–2442, Nov. 2013.
- [45] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011.



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