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## Spatial Correlation and MIMO Capacity at 2.4GHz

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### Abstract

Multiple-Input Multiple-Output (MIMO) systems are commonly used methods in order meet the increasing capacity need in wireless communication systems. However, the channel capacity in the multi-antenna architecture depends not only on signal-to-noise ratio (SNR) but also the spatial correlation between antenna elements. Therefore, detailed information is required about the transmission medium during the system design. We used the ray tracing method for the radio propagation channel modelling during this study. In multi-antenna systems, measurements were made for  $0.02\lambda$  receiving antenna spacing in a laboratory consisting of a full anechoic room, and the spatial correlation between far-field signals was observed. We also examined the impact of correlation on the channel capacity in a  $2 \times 2$  MIMO system. The test results showed high level of correlation between received signals in cases where one and two beams were effective as reflections even though the distance between the antennas was bigger than the wavelength. Furthermore, we observed that the channel capacity increased as the correlation between sub-channels increased.

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*Keywords:* MIMO systems, ray tracing model, spatial correlation, channel capacity

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### 1. Introduction

In line with the developments in the wireless and mobile system technology and the resulting increase in the user demands, faster, reliable and high capacity communication systems are needed. The desired high performance can be achieved in fixed transmission power and bandwidth with the Multiple-Input-Multiple-

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Output (MIMO) technology based on the use of multiple antennas in the receiving and transmitting ends of the system [1]. MIMO systems enables more efficient use of the limited source of frequency as the transmission is carried out at the same time and in the same frequency over the parallel sub-channels formed by multiple antenna elements [2].

MIMO systems are currently used both at base stations and wireless mobile devices. The distance between the elements of the antenna array is crucial in achieving the desired efficiency in multiple antenna systems. It is recommended that this distance be higher than  $11 - 13\lambda$  for vertically distributed dipole antennae at base stations and higher than  $0.5\lambda$  and  $0.2\lambda$  for mobile units [2]. As the antenna range of multiple antenna wireless mobile systems has diminished, the features of the transmission channel used and the characteristics of the antenna array and the spatial correlation between them have become essential parameters that need to be taken into account during system planning [3].

While designing MIMO systems, it was assumed that the sub-channels formed by the multiple antenna elements at both ends were ideally separated from one another [4]. Therefore, it is commented that the total channel capacity linearly increases with the number of antenna used [1, 4, 5, 6]. However, when physical propagation media are taken into account, channel capacity is affected by many factors, the most important one of these factors is the spatial correlation between sub-channels [5, 6, 7]. The correlation increase between received signals can cause a significant decrease in MIMO architecture performance [9].

As system designs have got more complicated in recent years, more detailed information is needed on transmission media. The ray tracing method, which is one of the deterministic channel models, provides very detailed information on transmission medium [10]. Therefore, it is widely used for characterizing urban transmission channels [11]. The ray tracing model is based on the calculation of all transmission paths formed by directly (line of sight-LOS), reflection, diffraction and scattering between receiving and transmitting antenna [10, 12].

Many algorithms can be used in MIMO systems to increase capacity and to model the transmission channel. The spatial correlation value among the sub-channels forming the system is the critical point and stands out during system design. In this study, spatial correlation value between signals in receiver antenna arrays are analyzed with measurements based on the distance between antenna elements. Multi-antenna system measurements were made in the Electromagnetic Diagnostic and Measurement Laboratory of ITU Electronics and Communication Department, which consists of a full anechoic room. The transmission channel modeling was performed using the optical ray tracing method. Also the necessary measurements in two antenna MIMO systems were made and the relation between correlation and channel capacity was examined.

The organization of the paper is as follows: Brief information about the spatial correlation and ray tracing which is used to characterize propagation channel and MIMO capacity is given in Section II. In Section III, we give our analysis results and finally in Section IV, we conclude the paper.

## **2. System Description**

### *2.1. Spatial Correlation and Ray Tracing Model*

As a result of many years of research, the level of the effect of classical channel features such as delay spread and Doppler spread on the system performance was found [13]. As distinct from the classical channel model, the effect of angle of arrival (AOA) on system performance in the spatial channel model characterized by angle spread has been studied in recent years. Angle spread has a significant effect on the spatial correlation between the element pairs of the antenna array.

Spatial correlation gives the measure of the relation between signals received by two antenna fed by the same source and situated on different points in space [14]. Correlation is a function of signal conditions such as the angle of arrival, the power azimuth spectrum (PAS), antenna spacing and the system bandwidth [15, 16].

The signals received by two sequent elements of the receiving antenna array fed by a transmitting antenna are  $V_n$  and  $V_{n+1}$ , the coefficient of the spatial correlation between them is mathematically defined as follows [2, 5].

$$r(d) = \frac{E\{V_n^*(d, \theta)V_{n+1}(d, \theta)\} - E\{V_n^*(d, \theta)\}E\{V_{n+1}(d, \theta)\}}{\prod_{i=n}^{n+1} \sqrt{\langle V_i^* V_i \rangle - \langle V_i^* \rangle \langle V_i \rangle}}, \quad (n=1, 2, \dots, N) \quad (1)$$

In (1),  $N$  represents the number of receiving antennas,  $d$  represents distance between antennas,  $\theta$  represents angle of arrival and  $E\{\cdot\}$  represents average value function and  $*$  denotes the complex conjugate.

The relation between *narrowband signals* coming from a single source to two sequent antenna in an antenna array with  $d$  distance in a *far-field* is as follows [14, 15]:

$$V_n = \frac{1}{M} \sum_{m=1}^M \alpha_m \exp\left\{j2\pi n \frac{d}{\lambda} \cos(\theta_m)\right\}. \quad (2)$$

where  $M$  represents the number of sub-paths,  $\lambda$  represents the wavelength in meters as determined by the carrier frequency,  $\alpha_m$  represents the power of signal from  $m^{\text{th}}$  sub-path and  $\theta_m$  represents the AoA of the  $m^{\text{th}}$  sub-path.

Wireless transmission channels contain many objects that can affect the signal. Besides, the physical and electrical features of these objects affect the ray [10]. Transmission channels can be modeled statistically or as a deterministic. Nowadays, the deterministic channel model called ray tracing model is widely used. The ray tracing model can give detailed information on radio propagation channel as it calculates all possible paths between the receiver and the transmitter [12, 17]. However, according to statistical models, its biggest disadvantage is that its calculating load is too much.

In far-field, the ray sent from a source to a receiving antenna is affected by the objects in the transmission channel and forms signals in the receiver whose amplitudes and angles of arrival are different. However, some of these signals received by the receiving antenna have amplitudes that are small enough to omit. The omissions reduce the functional complexity of the ray tracing model.

The transmission models in which predominantly line of sight (LoS) beam is received by a receiving antenna can be defined as single ray models. In the case that the multiple antenna elements with  $d$  distance receive single ray with LoS beam ( $M=1$ ), the spatial correlation connection can be expressed as follows [3, 8, 14, 16]:

$$r_v(d) = \int_{-\pi}^{\pi} \exp\left(-j2\pi \frac{d}{\lambda} \cos(\theta)\right) p_{\theta}(\theta) d(\theta). \quad (3)$$

where  $\theta$  represents the incident angle relative to the normal,  $p_{\theta}(\theta)$  represents AoA's probability density function.

There are different probability density functions (PDFs) for AoA in the literature but most of the studies involved fixed, uniform and Gaussian distribution cases. The probability density function terms in cases where the AoA displayed uniform and Gaussian distribution were defined in studies conducted by [2, 3, 8].

As a result of analyses, the spatial correlation result was achieved using the Bessel function [3, 8] when the AoA was uniformly distributed and the deviation from the average was  $2\pi$ .

Fig. 1 shows the curves for various probability density functions of AoA and different standard deviation values. When the curves were analyzed, it was realized that as the average deviation value for AoA with Gaussian distribution increased, the correlation between the received signals decreased.

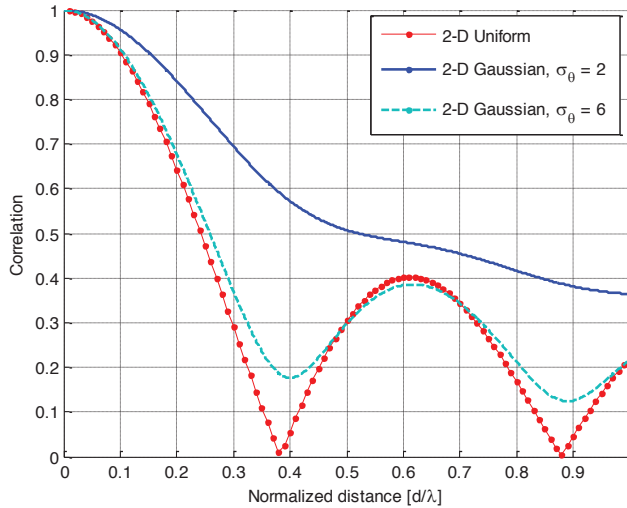


Fig. 1. Correlation for different AoA distributions.

In wireless communication channels, there is seldom a direct path between the base station and the mobile device. Therefore, models and equations based on the assumption that the transmission is conducted in empty space do not yield accurate results regarding physical propagation media. The two ray model based on geometric optics (GO) principle is considerably useful for modeling physical media as it contains both the LoS ray and the ray reflected from the surface between the receiver and the transmitter [8,18]. The equations and definitions related to the two ray model in which predominantly two rays reach the elements of the receiving antenna with  $d$  distance, are defined in detail in [8] study.

## 2.2. MIMO Capacity

Multiple-Input-Multiple-Output (MIMO) system is a technology widely used to meet the increasing demand in communication systems thanks to the high spectral efficiency it provides [19, 20]. MIMO is technique that can use at both ends of the system the multi-antenna receiving and transmitting structures that can efficiently benefit from the spatial dimension besides the time and frequency dimensions [9]. This way it provides a channel capacity beyond the traditional techniques.

For a time invariant MIMO channel with additive white Gaussian noise, when the transmission power is uniformly distributed among transmitting antenna elements and the number of  $n_T$  being transmitter antenna elements,  $n_R$  being the number of receiving antenna elements, the channel capacity can be calculated as follows [1, 6, 7]:

$$C = \log_2 \det \left( I_{n_R} + \frac{\rho}{n_T} HH^* \right) \text{bps/Hz.} \quad (4)$$

where  $H$  represents the channel matrix,  $H^*$  represents the complex conjugate transpose of  $H$ ,  $\rho$  represents the average signal-to-noise ratio (SNR),  $I$  represents the identity matrix, and  $\det(\cdot)$  is the determinant of a matrix. The expression in (4) indicates that not only the channel capacity depends on the SNR value but also the spatial correlation between the received signals impacts the capacity. Only when the correlation between the received signals is zero can the maximum channel capacity be achieved and it can be calculated using the following link ( $n_T = n_R = n$ ) [5]:

$$C_{\max} = n \log_2 (1 + \rho)_{\text{bps/Hz}} \quad (5)$$

This study deals with the effect of the correlation between far-field narrowband signals on the channel capacity when two antennas are used at receiving and transmitting ends. The following modeling was used for the analysis of the measurement results.

A MIMO system consisting of two antennas and having a correlation between channels can be defined as in (6).

$$Y = HX \quad (6)$$

where  $X = [x_1 \ x_2]^T$  is the transmitted signal vectors,  $Y = [y_1 \ y_2]^T$  is the received signal vectors.  $T$  means transpose. Channel matrix has the following form.

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad (7)$$

While modeling MIMO systems, it is assumed that the transmission power is uniformly distributed among transmitting antenna elements and the transmitted signals are uncorrelated. Therefore, correlation equation is defined only among the received signals. The spatial correlation terms generally defined in (1) takes the following form for two-antennas MIMO systems [6]:

$$r = \frac{h_{11}h_{21}^* + h_{12}h_{22}^*}{\sqrt{(|h_{11}|^2 + |h_{12}|^2)(|h_{22}|^2 + |h_{21}|^2)}} \quad (8)$$

Formula (9) is achieved when the capacity term in (4) for MIMO systems is expressed according to the correlation between channels.

$$C = \log_2 \left( 1 + \rho + (1 - |r|^2) \left( \frac{\rho}{2} \right)^2 \right)_{\text{bps/Hz}} \quad (9)$$

This expression clearly shows that the spatial correlation between sub-channels directly affects the channel capacity. When the channels are not correlated on one another ( $r=0$ ), this connection turns into (5) and the maximum channel capacity is reached. In completely related channels ( $r = 1$ ), the channel capacity can be calculated using the Shannon formula [6].

Fig. 2 shows the curves for channel capacities usually expected for various signal-to-noise ratio values in cases where the spatial correlation value among sub-channels varies from 0 to 1. As it is shown in the Fig.2, the channel capacity for all SNR values decreases significantly when  $|r| > 0.6$ .

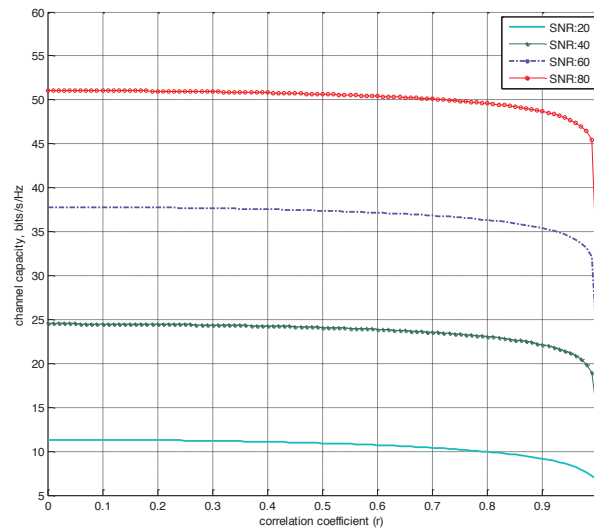


Fig. 2. Channel Capacity Against Absolute Value of Correlation Coefficient

### 3. Measurements Set-up and Results

It is assumed that the total channel capacity of ideal multi-antenna systems linearly increases with the number of antenna elements. However, it was observed while implementing the MIMO technology that the spatial correlation among antenna elements had a significant effect on the channel capacity. Therefore, correlation analyses and solution suggestions for increasing the capacity in multi antenna systems are very popular research topics.

We made measurements in the Electromagnetic Diagnostic and Measurement Laboratory of ITU Electronics and Communication Department in order to analyze the spatial correlation among the signals coming to the receiving antenna array in MIMO systems in relation to the distance between antennas. Figure 3 shows the modeling for the test set-up in the laboratory. In addition, we formed a 2 x 2 MIMO architecture and analyzed the effect of the correlation on the capacity by sending rays from two transmitting antennas to a receiving antenna array.

The laboratory mechanism is isolated from the outdoor environment with its cabin containing an external display, and the electromagnetic absorbing elements inside the laboratory make it completely anechoic room. We benefitted from this feature of this highly equipped laboratory and implemented the ray tracing method in a controlled manner for situations in which one or two rays were effective.

During the measurements, we sent uncorrelated signals of equal power from two different positions (P1 and P2) of sources to the receiving antenna array whose elements were distributed at 3.25 mm distances. The distance between the transmitting antennas is 10 cm. The antenna array used on the test equipment consists of approximately 800 elements. 2.4 GHz frequency was used during the measurements and the distance was determined as  $0.02\lambda$ . The fact that this value is much lower than  $0.5\lambda$  recommended for mobile systems

increases the importance of the results. Moreover, the noise was kept at a minimum level during the measurements. In addition, the direct connection of the Network Analyzer used in the test to the transmitting antennas resulted in a very high SNR value (70 dB).

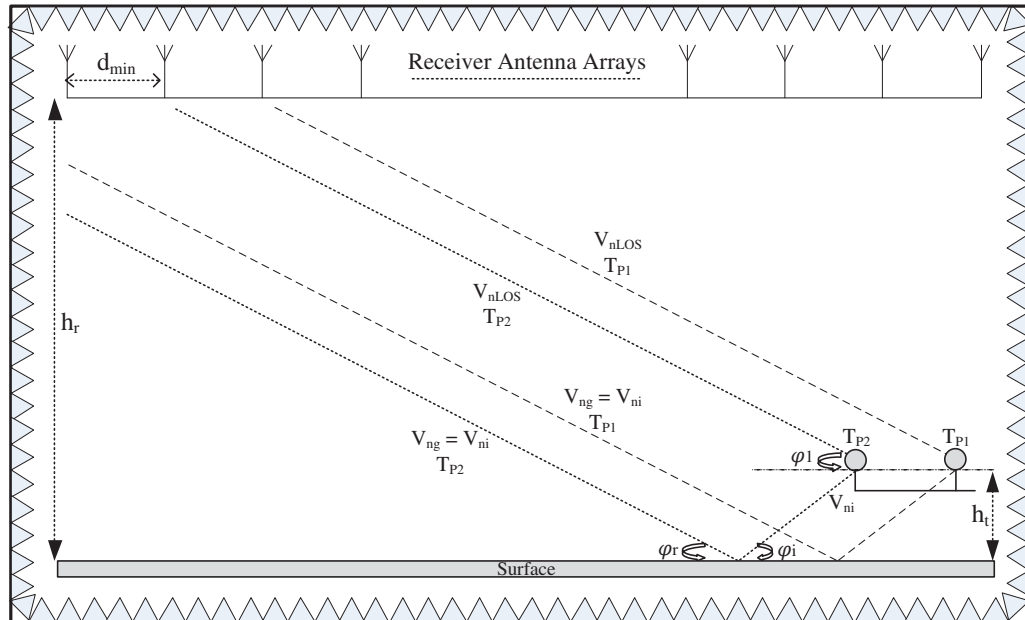


Fig. 3. Measurements Set-up

The first thing we observed while analyzing the data acquired in the tests was the angle of arrival of the signals to the receiving antennas. We gathered the antenna elements receiving the signals at the same angle of arrival at a group and analyzed the spatial correlation results in smaller data sets including about one hundred antenna elements.

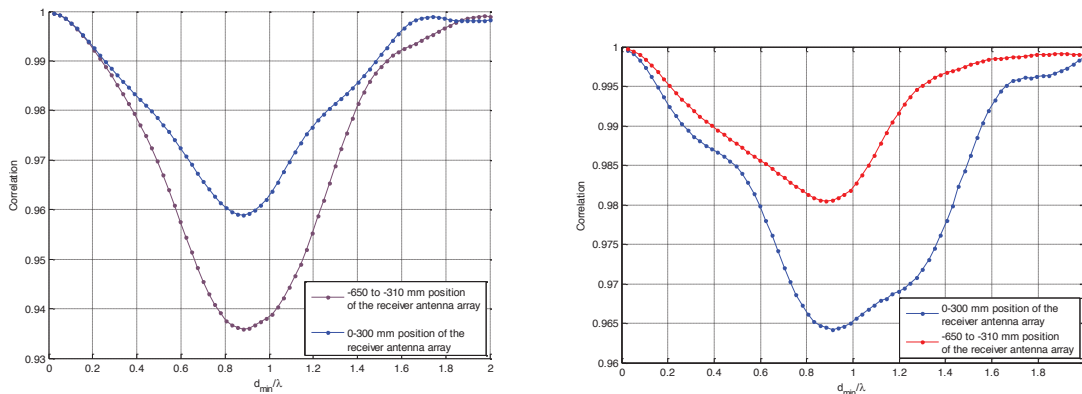


Fig. 4. (a) Far-Field Correlation Results of P1, (b) Far-Field Correlation Results of P2

Figure 4(a) shows the curve for the correlation among the elements of the antenna array receiving the signal transmitted by the P1 point source. In this figure, correlation curves are drawn when the receiving antenna array has two different positions. The correlation result obtained when the antenna array is between -

650 mm and -310 mm has lower amplitude than the value obtained at 0-300 mm range. This occurs due to the decrease of the distance between the antennas and the source, and indicates that the correlation between signals increase as the distance between the receiver and the transmitter.

Fig.4(b) shows the far-field correlation curves in regions where the receiving antenna group consisting of one hundred elements that receive the rays spread by P2 source is between the positions defined previous case. When Figures 4(a) and 4(b) are compared, it was found that the correlation among the receiving antenna elements between the same positions was bigger for the P2 point source. The reason for this is that the distance between antennas decreased, i.e. the antenna space narrowed.

After the necessary measurements were made for two-antenna MIMO system in a full anechoic laboratory, we obtained correlation results based on the distance between the receiving antennas. Fig.5 shows the curve for the total channel capacity achieved for various antenna space values. Although the correlation value among sub-channels was about 0.9, we achieved a high channel capacity of approximately 50bps/Hz. The high SNR level achieved thanks to factors such as the highly elaborate equipment of the laboratory and the minimization of noise during the measurements enabled us to reach very high channel capacities as a result of analyses. As it is clear in Fig.5, the capacity increases when the antenna spacing increases. We analyzed Fig.2 and Fig.5 in order to compare the capacity results obtained from the measurements with ideal capacity results. The correlation among sub-channels was about 0.9 at 70 dB SNR value while the ideal and measured capacity results were practically the same.

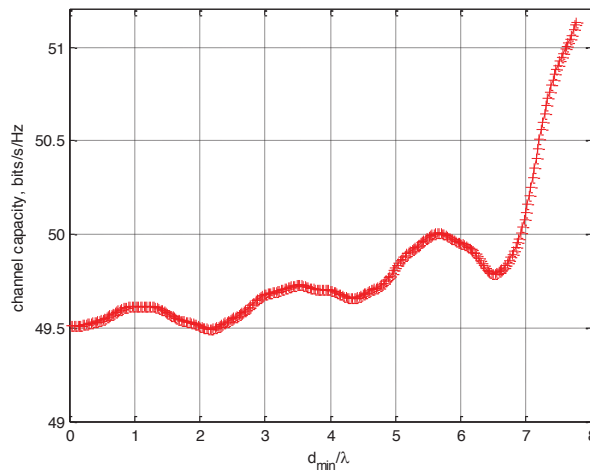


Fig.5.Total Channel Capacity

#### 4. Conclusions

In this study, we established multi-antenna system architecture in a full anechoic laboratory and conducted some tests when the receiving antenna spacing was  $0.02\lambda$ . We modeled the transmission medium when one and two rays were dominant, using the ray tracing model. The analyses results revealed that the spatial correlation among sub-channels was 0.9, which is a very high level, in both cases of the fixed point source. Furthermore, as the receiving antenna spacing increased, a certain decrease occurred in the correlation but this was too small. We studied the channel capacity at high correlation levels for the 2 x 2 MIMO systems. The analyses revealed a high value of 50bps/Hz although the correlation among sub-channels was high. The reason for this was that the SNR value was about 70 dB during the measurements. Moreover, the channel capacity increased even when the correlation level decreased slightly.

Based on our analysis, we have observed that if only one or two rays are dominant in an indoor environment, the received signals at the array elements are highly correlated even when they are further apart than a wavelength. This conclusion prevents us from reaching the maximum channel capacity of the MIMO communication channel. Hence, the channel conditions should always be checked while designing a communication system.

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