

Supplementary material: Folded metaporous material for sub-wavelength and broadband perfect sound absorption

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I. MICRO-LATTICE: EQUIVALENT FLUID

A. Equivalent fluid model

The analysis is performed in the linear harmonic regime at the circular frequency ω with the implicit time dependence $e^{i\omega t}$.

The equivalent fluid of the micro-lattice is quasi-isotropic. Its equivalent bulk modulus K is scalar while its equivalent density ρ is a diagonal 3×3 matrix in its principal directions $R_1 = (O, \mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$

$$\rho_{R_1} = \begin{bmatrix} \rho^{//} & 0 & 0 \\ 0 & \rho^{//} & 0 \\ 0 & 0 & \rho^\perp \end{bmatrix}, \quad (S1)$$

where $\rho^{//}$ is the in-plane density and $\rho^\perp$ is the out-of-plane density.

According to the Johnson-Champoux-Allard-Lafarge (JCAL) model¹, the expression of the equivalent fluid properties can be approximated by means of nine parameters in the case of the considered quasi-isotropic medium. The equivalent density can be written as

$$\rho(\omega) = \frac{\rho^a}{\phi} \alpha(\omega), \quad (S2)$$

where ρ^a is the density of the saturating fluid, i.e. the air medium in the present case, ϕ the open porosity, and $\alpha(\omega)$ the dynamic tortuosity matrix. The equivalent bulk modulus can be written as

$$K(\omega) = \frac{\gamma P_0}{\phi} \left(\gamma - \frac{\gamma - 1}{\alpha'(\omega)} \right)^{-1}, \quad (S3)$$

where P_0 is the static pressure, γ the specific heat ratio and $\alpha'(\omega)$ the thermal tortuosity.

The Johnson *et al.* model², initially derived for isotropic

fluid and straightforwardly adapted to the considered quasi-isotropic fluid, defines the dynamic tortuosities as

$$\begin{cases} \alpha^{//}(\omega) = \alpha_\infty^{//} - \frac{i\nu}{\omega} \frac{\phi}{q_0^{//}} \sqrt{1 + \frac{i\omega}{\nu} \left(\frac{2\alpha_\infty^{//} q_0^{//}}{\phi \Lambda^{//}} \right)^2}, \\ \alpha^\perp(\omega) = \alpha_\infty^\perp - \frac{i\nu}{\omega} \frac{\phi}{q_0^\perp} \sqrt{1 + \frac{i\omega}{\nu} \left(\frac{2\alpha_\infty^\perp q_0^\perp}{\phi \Lambda^\perp} \right)^2}, \end{cases} \quad (S4)$$

where $\nu = \eta/\rho^a$ is the kinematic viscosity of the saturating fluid, η is the dynamic viscosity, and α_∞ , Λ and q_0 are the high frequency limit of the tortuosity, the viscous characteristic length and the viscous static permeability of the porous medium, respectively in the in-plane ($//$ superscript) and out-of-plane ($\perp$ subscript) directions.

The Champoux-Allard-Lafarge model further defines the thermal tortuosity as^{1,3}

$$\alpha'(\omega) = 1 - \frac{i\nu'}{\omega} \frac{\phi}{q_0'} \sqrt{1 + \frac{i\omega}{\nu'} \left(\frac{2q_0'}{\phi \Lambda'} \right)^2}, \quad (S5)$$

where $\nu' = \nu/Pr$, Pr is the Prandtl number, and Λ' and q_0' are the thermal characteristic length and the static thermal permeability respectively.

The JCAL parameters of the considered micro-lattice depend on the lattice constant governed by the infill-factor (IF) in practice. The procedure to obtain the JCAL parameters from numerical simulations and inverse characterization is described in Ref. [4]. The JCAL parameters of the considered micro-lattice obtained from inverse characterization are presented in Fig. S1.

B. Frame transformation

The coordinate system $\mathbf{R}_0 = (O, \mathbf{x}, \mathbf{y}, \mathbf{z})$ is the rotation of $\mathbf{R}_1 = (O, \mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$ about $\mathbf{y} = \mathbf{y}_1$ by an angle β . The transformation matrix from $\mathbf{R}_0$ to $\mathbf{R}_1$ is $\mathbf{P}_0^1$ where

$$\mathbf{P}_0^1 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}. \quad (S6)$$

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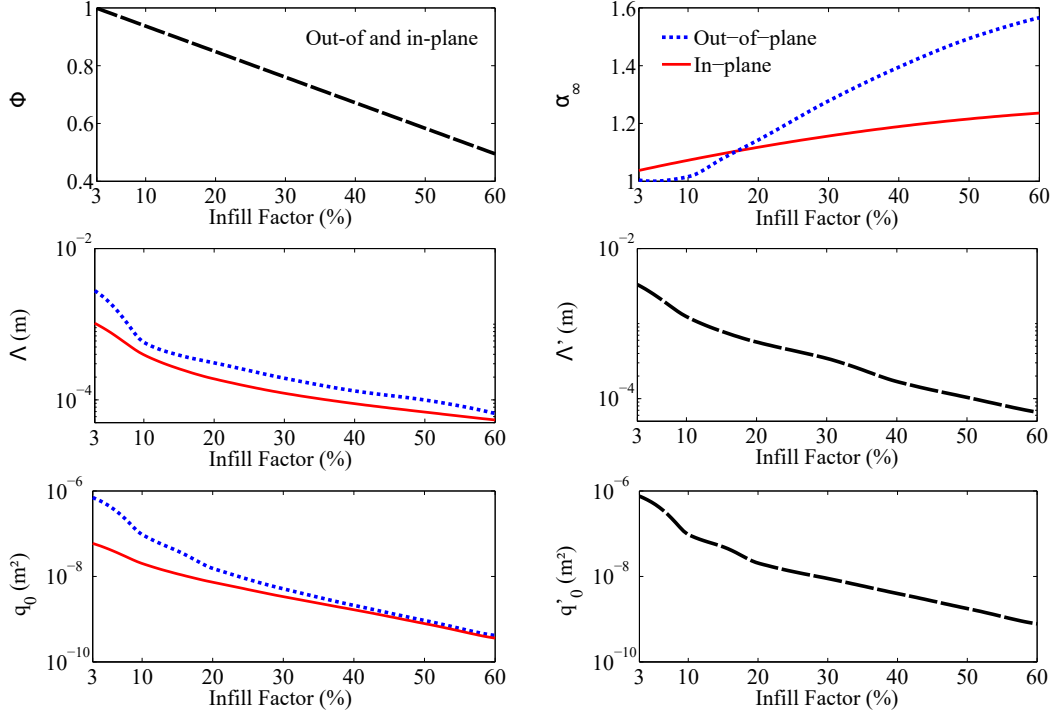


FIG. S1. JCAL parametric model, obtained by inverse characterization of homogeneous samples.

Noting ^{T} the transpose of a matrix, the matrix $\boldsymbol{\rho}$ is derived in $\mathbf{R}_0$ by

$$\boldsymbol{\rho}_{R_0} = \mathbf{P}_0^{1T} \boldsymbol{\rho}_{R_1} \mathbf{P}_0^1 = \begin{bmatrix} \rho_x & 0 & \rho_{xz} \\ 0 & \rho_y & 0 \\ \rho_{xz} & 0 & \rho_z \end{bmatrix}. \quad (\text{S7})$$

where

$$\begin{cases} \rho_x = \rho^{//} \cos^2(\beta) + \rho^\perp \sin^2(\beta), \\ \rho_y = \rho^{//}, \\ \rho_z = \rho^{//} \sin^2(\beta) + \rho^\perp \cos^2(\beta), \\ \rho_{xz} = \cos(\beta) \sin(\beta) (\rho^{//} - \rho^\perp). \end{cases} \quad (\text{S8})$$

C. Choice of the micro-lattice

The considered micro-lattice is composed of a superposition of parallel rods orthogonally alternating in the $(O, \mathbf{y}_1, \mathbf{z}_1)$ plane. Each rod is a combination of two filaments of diameter $D = 200 \mu\text{m}$ merged with one another in the $(O, \mathbf{z}_1)$ direction. A simpler micro-lattice composed of rods being single filaments of diameter $D = 200 \mu\text{m}$ is more resistive in its out-of-plane direction and does not permit to attain the desired losses and thus leads to MpS with non perfect absorption.

D. Manufacturing parameters

The FPM are made by Fused Deposition Modeling (FDM). The used slicer software and printer are *Simplify3D* and

RAISE3D Pro2, respectively. The extruded material is polylactic acid (PLA). Two nozzles are used alternatively: a $400 \mu\text{m}$ diameter nozzle to manufacture the impervious structure and a $200 \mu\text{m}$ diameter for the micro-lattice. The normals of the micro-lattices and of the FPM are aligned to simplify their manufacturing. The extrusion multiplier (EM) is a parameter that increases (>1) or decreases (<1) the linear flux of extruded filament. Usually for PLA, the EM is set to 0.9. That of the $400 \mu\text{m}$ diameter nozzle is set to 1.1 in order to slightly extrude too much PLA and thus improve the impermeability of the impervious structure. The EM of the $200 \mu\text{m}$ diameter nozzle is set to 0.9. The temperatures of the nozzles and of the bed are set to 210°C and 55°C , respectively. The printing speed of the micro-lattice and of the impervious structure are set to $3200 \text{ mm} \cdot \text{min}^{-1}$ and $1500 \text{ mm} \cdot \text{min}^{-1}$, respectively.

II. GOVERNING EQUATIONS IN A RIGID FRAME POROUS MEDIUM

As the micro-lattice is quasi-isotropic, i.e. not isotropic, the propagation equations are recalled. Using the alternative Biot's formulation⁵, De Ryck *et al.* derived the equations of motion in a macroscopically inhomogeneous porous material under the rigid frame approximation⁶. They take the usual form in the frequency domain

$$i\omega \boldsymbol{\rho} \mathbf{V} = -\text{grad} p, \quad (\text{S9})$$

$$\frac{-i\omega}{K} p = \text{div} \mathbf{V}, \quad (\text{S10})$$

where p is the fluid pressure in the material pores and $\mathbf{V}$ the equivalent velocity for the oscillatory fluid flow in the interconnected pores.

III. FEM MODEL GOVERNING EQUATIONS

Combining Eqs. (S9) and (S10), the equations of motion can be written in a form that only involves p

$$\frac{\omega^2}{K} p + \text{div}(\boldsymbol{\rho}^{-1} \text{grad} p) = 0, \quad (\text{S11})$$

where $\boldsymbol{\rho}^{-1}$ is the inverse of the density matrix. Equation (S11) is obviously also valid in air, where $K = \gamma P_0$ and $\boldsymbol{\rho}^{-1}$ is a scalar equal to $1/\rho_0$. Impervious walls are accounted by zero-flux boundary condition

$$\text{grad} p \cdot \mathbf{n} = 0, \quad (\text{S12})$$

where $\mathbf{n}$ is the normal vector to the walls.

In this work, Eqs. (S11) and (S12) are implemented in the Mathematical module of *Comsol Multiphysics* for the simulations by FEM.

IV. ANALYTICAL MODEL

A. Introduction

The aim of the analytical model is to predict the behavior of the MpS composed of four FPM $i = 1, 2, 3, 4$ of different $\{\xi_i; IF_i\}$ mounted in an impedance tube in the rigid backing configuration. Three symmetric images of the MpS are created by the lateral impervious walls of the impedance tube acting as mirrors for the pressure field. Thus, an infinite periodical arrangement is formed. Its motif is composed of the MpS and its three images. The FPM P is simplified by the quartet of identical straight tubes numbered $J_h(P) = 4(P - 1) + \{1, 2, 3, 4\}$ and their three images numbered $J_i(P) = J_h(P) + 16\{1, 2, 3\}$. Finally four quartets of quartets, i.e. $4^3 = 64$ straight tubes, must be taken into account analytically as depicted in Fig. S2 where the red squares delimit the impedance tube lateral walls. These tubes have a square cross section of width $w_x^{[J]} = w_y^{[J]} = Q(\xi)$ and a height $b^{[J]} = L_a(\xi)$. The quasi-isotropic equivalent fluid filling these tubes is not in its principal direction but rotated of an angle $\beta(\xi)$ around $\mathbf{y} = \mathbf{y}_1$. The justification of this postulate is explained in section V.

The analytical model can consider more complex configurations than that of the MpS mounted in an impedance tube. It can consider M rectangular straight tubes and any incident plane wave direction. The tubes are closed at the lower end and open at the upper end. They are separated by impervious walls and filled with quasi-isotropic equivalent fluids out of

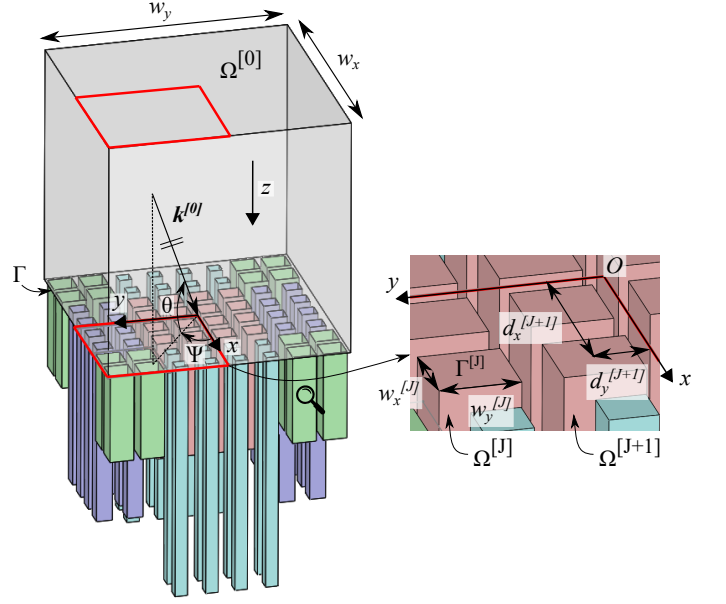


FIG. S2. Diagram of the analytical model. The represented geometry corresponds to the computation domain when simulating a MpS mounted in an impedance tube. The red square delimits a MpS composed of four quadruplets of straight tubes filled with different micro-lattices. The grey cube represents the air domain $\Omega^{[0]}$. Each of the other color symbolizes a micro-lattice defined by a IF and a β .

their principal directions. The Cartesian coordinate system $\mathbf{R}_0 = (O, \mathbf{x}, \mathbf{y}, \mathbf{z})$ is used. The position vector is $\mathbf{x} = \{x, y, z\}$, the $(O, \mathbf{z})$ axis is normal to the straight tubes and points towards their rigid backing. The azimuth and elevation angles are noted ψ and θ , respectively. The domains are noted Ω . The air domain and J^{th} straight tube are indicated by the superscript $[0]$ and $[J]$ with $J \in [1; M]$, respectively. The straight tube J is centered around $(x, y) = (d_x^{[J]}, d_y^{[J]})$, of width $w_x^{[J]}$ and $w_y^{[J]}$, and of height $b^{[J]}$. The computation domain is of lateral dimensions w_x and w_y . The position $z = 0$ is noted Γ . The interface between $\Omega^{[0]}$ and $\Omega^{[J]}$ is noted $\Gamma^{[J]}$.

The model uses a mode matching technique inspired from [7]. In the porous straight tubes, the propagation is put in equation by the transfer matrix method (TMM) only accounting for normal propagation. In air, Bloch waves are considered to account for the finite size of the computation domain and the engendered periodic pattern. Note that the analytical model does not require any length correction at the opening of the tubes because it uses a mode matching technique accounting for the high order modes.

B. Field description

1. Air

In $\Omega^{[0]}$, the components of the incident wave vector $\mathbf{k}^{[0]}$ of norm $k^{[0]}$ are

$$\begin{cases} k_x^i = -k^{[0]} \cos(\theta) \cos(\psi), \\ k_y^i = -k^{[0]} \cos(\theta) \sin(\psi), \\ k_z^i = k^{[0]} \sin(\theta). \end{cases} \quad (\text{S13})$$

The incident pressure field is then

$$p^i(\mathbf{x}) = e^{-i(k_z^i z + k_x^i x + k_y^i y)}. \quad (\text{S14})$$

In $\Omega^{[0]}$, according to the Bloch Floquet theorem, the pressure field takes the form

$$p^{[0]}(\mathbf{x}) = p^i(\mathbf{x}) + \sum_{m=-N}^N \sum_{n=-N}^N R_{mn} e^{i(k_{z,mn} z - k_{x,m} x - k_{y,n} y)}, \quad (\text{S15})$$

where considering N Bloch modes in x and in y directions, $m \in [-N; N]$ and $n \in [-N; N]$ are the number of the x and y Bloch mode, respectively. The components of the reflected wave vectors are

$$\begin{cases} k_{x,m} = k_x^i + \frac{2m\pi}{w_x}, \\ k_{y,n} = k_y^i + \frac{2n\pi}{w_y}, \\ k_{z,mn} = \sqrt{(k^{[0]})^2 - (k_{x,m})^2 - (k_{y,n})^2}, \\ \text{Re}(k_{z,mn}) \geq 0, \text{Im}(k_{z,mn}) \leq 0. \end{cases} \quad (\text{S16})$$

2. Porous filled straight tubes

Equations (S9) and (S10) can be cast in a first-order differential matrix system that only involves the propagation along z in $\Omega^{[J]}$

$$\frac{\partial}{\partial z} \begin{pmatrix} p^{[J]} \\ V_z^{[J]} \end{pmatrix} = \mathbf{A} \begin{pmatrix} p^{[J]} \\ V_z^{[J]} \end{pmatrix}, \quad (\text{S17})$$

where, in $\mathbf{R}_0$, the state matrix is

$$\mathbf{A}_{R_0} = \begin{bmatrix} -i \frac{\rho_{xz}^{[J]}}{\rho_x^{[J]}} k_x^i & -i \omega \bar{\rho}_z^{[J]} \\ \frac{-i \omega}{K^{[J]}} & -i \frac{\rho_{xz}^{[J]}}{\rho_x^{[J]}} k_x^i \end{bmatrix}, \quad (\text{S18})$$

with

$$\begin{cases} \bar{\rho}_z^{[J]} = \rho_z^{[J]} - \frac{(\rho_{xz}^{[J]})^2}{\rho_x^{[J]}}, \\ \frac{1}{K^{[J]}} = \frac{1}{K^{[0]}} - \frac{(k_x^i)^2}{\rho_x^{[J]} \omega^2} - \frac{(k_y^i)^2}{\rho_y^{[J]} \omega^2}. \end{cases} \quad (\text{S19})$$

The transverse propagation is neglected in $\Omega^{[J]}$. As the straight tubes are of small width, the impact of the transverse

modes is negligible in the frequency range of interest, i.e., the porous media are considered locally reacting. In $\mathbf{R}_0$, the state matrix reduces to

$$\mathbf{A}_{R_0} = \begin{bmatrix} 0 & -i \omega \rho_z^{[J]} \\ \frac{-i \omega}{K^{[J]}} & 0 \end{bmatrix}. \quad (\text{S20})$$

The link between the state vector $(p^{[J]}, V_z^{[J]})^\top$ at coordinates z and $z = b^{[J]}$ is given by means of a matrix exponential

$$\begin{pmatrix} p^{[J]} \\ V_z^{[J]} \end{pmatrix}_z = \exp_m(\mathbf{A}(z - b^{[J]})) \begin{pmatrix} p^{[J]} \\ V_z^{[J]} \end{pmatrix}_{b^{[J]}}. \quad (\text{S21})$$

The matrix exponential is the transfer matrix of the transfer matrix method. The position $z = b^{[J]}$ is that of the rigid backing of the straight tube J . Then, $V_z^{[J]}(b^{[J]}) = 0$ and the pressure amplitude at any $\mathbf{x} \in \Omega^{[J]}$ is

$$p^{[J]}(z) = \cos \left(\sqrt{\frac{\rho_z^{[J]}}{K^{[J]}}} \omega (z - b^{[J]}) \right) p^{[J]}(b^{[J]}). \quad (\text{S22})$$

The pressure field then takes the form

$$p^{[J]}(\mathbf{x}) = B^{[J]} \cos \left(k_z^{[J]} (z - b^{[J]}) \right), \quad (\text{S23})$$

where

$$k_z^{[J]} = \omega \sqrt{\frac{\rho_z^{[J]}}{K^{[J]}}}, \quad \text{Re}(k_z^{[J]}) \geq 0, \text{Im}(k_z^{[J]}) \leq 0. \quad (\text{S24})$$

C. Continuity Conditions

The continuity conditions of the pressure and normal component of the velocity fields are written at the interface Γ .

1. Pressure continuity

The pressure continuity condition reads

$$\forall \mathbf{x} \in \Gamma^{[J]}, p^{[0]}(\mathbf{x}) = p^{[J]}(\mathbf{x}). \quad (\text{S25})$$

2. Normal component of the velocity

The mass conservation Eq. (S9) implies that the normal component of the velocity $V_z^{[J]}$ is

$$V_z^{[J]} = \frac{-1}{i \omega} \left(\frac{1}{\rho_{cx}^{[J]}} \frac{\partial p^{[J]}}{\partial x} + \frac{1}{\bar{\rho}_z^{[J]}} \frac{\partial p^{[J]}}{\partial z} \right), \quad (\text{S26})$$

where $\rho_{cx}^{[J]} = \rho_{xz}^{[J]} - \rho_x^{[J]} \rho_z^{[J]} / \rho_{xz}^{[J]}$. In the general case of anisotropic fluids out of their principal directions, the normal

component of the velocity is coupled to the transverse variations of pressure. As the transverse variations of pressure are neglected in $\Omega^{[J]}$, $V_z^{[J]}$ reduces to

$$V_z^{[J]} = \frac{-1}{i\omega\rho_z^{[J]}} \frac{\partial p^{[J]}}{\partial z}. \quad (\text{S27})$$

Finally, the normal component of velocity continuity over Γ is

$$\frac{1}{\rho^{[0]}} \frac{\partial p^{[0]}}{\partial z} \Big|_{z=0} = \sum_J \frac{1}{\rho_z^{[J]}} \frac{\partial p^{[J]}}{\partial z} \Big|_{z=0} \Pi, \quad \Pi = 1 \ \forall (x, y) \in \Gamma^{[J]}, \ \Pi = 0 \ \forall (x, y) \in (\Gamma \setminus \Gamma^{[J]}). \quad (\text{S28})$$

3. Projection of the continuity conditions over the domains modes

The projection of Eq. (S25) over the $\Omega^{[J]}$ modes reads

$$\int_{d_x^{[J]} - w_x^{[J]}/2}^{d_x^{[J]} + w_x^{[J]}/2} \int_{d_y^{[J]} - w_y^{[J]}/2}^{d_y^{[J]} + w_y^{[J]}/2} \left[p^{[0]}(z=0) - p^{[J]}(z=0) \right] dx dy = 0. \quad (\text{S29})$$

This equation can be rewritten

$$B^{[J]} \cos(k_z^{[J]} b^{[J]}) e^{-Cb^{[J]}} w_x^{[J]} w_y^{[J]} = I_{x,0}^{[J]} I_{y,0}^{[J]} + \sum_{m,n} R_{mn} I_{x,m}^{[J]} I_{y,m}^{[J]}, \quad (\text{S30})$$

where

$$I_{\alpha,b}^{\pm[J]} = \int_{d_\alpha^{[J]} - w_\alpha^{[J]}/2}^{d_\alpha^{[J]} + w_\alpha^{[J]}/2} e^{-ik_{\alpha,b}\alpha} d\alpha = w_\alpha^{[J]} e^{\pm ik_{\alpha,b} d_\alpha^{[J]}} \text{sinc}(k_{\alpha,b} w_\alpha^{[J]}/2). \quad (\text{S31})$$

The projection of Eq. (S28) over the $\Omega^{[0]}$ modes reads

$$\int_{-d_x/2}^{d_x/2} \int_{-d_y/2}^{d_y/2} \left[\frac{1}{\rho^{[0]}} \frac{\partial p^{[0]}}{\partial z} \Big|_{z=0} - \sum_J \frac{1}{\rho_z^{[J]}} \frac{\partial p^{[J]}}{\partial z} \Big|_{z=0} \Pi \right] e^{i(k_{x,q}x + k_{y,s}y)} dx dy = 0. \quad (\text{S32})$$

This equation can be rewritten

$$\frac{id_x d_y}{\rho^{[0]}} (-k_z^i \delta_{0q} \delta_{0s} + k_{z,q} R_{qs}) - \sum_J \frac{k_z^{[J]}}{\rho_z^{[J]}} B^{[J]} \sin(k_z^{[J]} b^{[J]}) I_{x,q}^{+[J]} I_{y,s}^{+[J]} = 0. \quad (\text{S33})$$

D. Linear system

The combination of Eqs. (S30), (S33) allows to eliminate the unknown amplitudes $B^{[J]}$

$$R_{qs} - \sum_{m,n} R_{mn} \frac{\rho^{[0]}}{id_x d_y k_{z,q} s} \sum_J \frac{I_{x,m}^{[J]} I_{y,n}^{[J]}}{w_x^{[J]} w_y^{[J]} \rho_z^{[J]}} k_z^{[J]} \tan(k_z^{[J]} b^{[J]}) I_{x,q}^{+[J]} I_{y,s}^{+[J]} = \frac{k_z^i}{k_{z,q} s} \delta_{0q} \delta_{0s} + \frac{\rho^{[0]}}{id_x d_y k_{z,q} s} \sum_J \frac{I_{x,0}^{[J]} I_{y,0}^{[J]}}{w_x^{[J]} w_y^{[J]} \rho_z^{[J]}} k_z^{[J]} \tan(k_z^{[J]} b^{[J]}) I_{x,q}^{+[J]} I_{y,s}^{+[J]}. \quad (\text{S34})$$

This equality can be written in matrix form, with $\mathbf{R}$ and $\mathbf{S}$ being vectors of size $2N+1$, $\mathbf{D}$ being a $(2N+1) \times (2N+1)$ matrix and $\mathbf{I}$ the identity matrix

$$(\mathbf{I} - \mathbf{D})\mathbf{R} = \mathbf{S}, \quad (\text{S35})$$

where

$$\begin{cases} R_{(i)} &= R_{qs}, \\ D_{(i,j)} &= \frac{\rho^{[0]}}{id_x d_y k_{z,q} s} \sum_J \frac{I_{x,m}^{[J]} I_{y,n}^{[J]}}{w_x^{[J]} w_y^{[J]} \rho_z^{[J]}} k_z^{[J]} \tan(k_z^{[J]} b^{[J]}) I_{x,q}^{+[J]} I_{y,s}^{+[J]}, \\ S_{(i)} &= \frac{k_z^i}{k_{z,q} s} \delta_{0q} \delta_{0s} + \frac{\rho^{[0]}}{id_x d_y k_{z,q} s} \sum_J \frac{I_{x,0}^{[J]} I_{y,0}^{[J]}}{w_x^{[J]} w_y^{[J]} \rho_z^{[J]}} k_z^{[J]} \tan(k_z^{[J]} b^{[J]}) I_{x,q}^{+[J]} I_{y,s}^{+[J]}, \end{cases} \quad (\text{S36})$$

where noting $\lfloor x \rfloor$ the largest integer below the real number x ,

$$\begin{cases} m = \lfloor \frac{j-1}{2N+1} \rfloor - N, \\ n = j-1 - \lfloor \frac{j-1}{2N+1} \rfloor (2N+1) - N, \\ q = \lfloor \frac{i-1}{2N+1} \rfloor - N, \\ s = i-1 - \lfloor \frac{i-1}{2N+1} \rfloor (2N+1) - N. \end{cases} \quad (\text{S37})$$

The absorption coefficient A is finally expressed as

$$A = 1 - \sum_{m,n=-N}^N \frac{\text{Re}(k_{z,mn})}{k_z^i} ||R_{mn}||^2. \quad (\text{S38})$$

E. Numerical validation

In order to illustrate the ability of the analytical model to predict the behavior of straight tubes filled with anisotropic equivalent fluid, a realistic configuration is presented. It mimics a sample mounted in an impedance tube of width $W = 41.9$ mm equal to that of the tube used for experimental characterization and composed of three quartets of square section tubes and a single rectangular section tube (to underline that the model is also valid for rectangular section tubes) filled with quasi-isotropic micro-lattices out of their principal directions. Thus, the sample is made of 13 tubes. The FEM model considers these 13 tubes as depicted in Fig. S3 (a) and rigid boundary conditions are applied on the walls of the impedance tube. The analytical model must then consider 4×13 tubes to account for the images of the sample due to the rigid walls of the impedance tube as depicted in Fig. S3 (b). The dimensions of the sample's tubes and the properties of their micro-lattices are presented in Tab. S1.

The absorption coefficient of the numerical sample is computed by means of FEM and by the analytical model. The corresponding absorption curves are presented in Fig. S3 by black crosses and red dashed line respectively. The correlation is excellent when considering 4 Bloch modes analytically. To consider no Bloch mode is to compute the absorption of the sample by means of the sum of the admittance of each porous-filled tube. The correlation with the FEM computation is then weak (solid blue line) which highlights the impact of the Bloch modes and the necessity to use the presented analytical model instead of an admittance sum.

V. SIMPLIFICATION OF THE FPM GEOMETRY FOR ANALYTICAL MODEL USE

The method used to find the geometrical parameters $Q(\xi)$, $L_a(\xi)$ and $\beta(\xi)$ of a FPM simplified by a quarter of straight tubes is given hereafter.

The simplification of a FPM by a quartet of straight tubes and their dimensions (Q and L_a) do not depend on the filling fluid. For simplicity, an isotropic fill-

TABLE S1. Parameters of the numerical sample composed of straight tubes of square or rectangular cross section, for validation test inside a $W = 41.9$ mm width tube.

J	{1,2,3,4}	{5,6,7,8}	{9,10,11,12}	13
b/W	1,91	1,43	1,19	0,716
w_x/W	0,138	0,160	0,174	0,446
w_y/W	0,138	0,160	0,174	0,224
$TR(\%)$	5	10	15	20
$\beta(^{\circ})$	68	60	53	0

ing fluid is considered and its JCAL parameters are $\phi = 0.78$, $\alpha_{\infty} = 1.13$, $\Lambda = 207 \mu\text{m}$, $\Lambda' = 216 \mu\text{m}$, $q_0 = 10.8 \cdot 10^{-9} \text{m}^2$, $q'_0 = 15.9 \cdot 10^{-9} \text{m}^2$.

First, the absorption coefficient of a FPM filled by the homogeneous isotropic fluid and mounted in an impedance tube is computed by FEM (see Fig. S4, (a)).

Then, the absorption coefficient of a quartet of straight tubes filled by the same isotropic fluid is computed by the analytical model, thus considering four quartets of straight tubes to account for the images of the sample (see Fig. S4, (b)). The best Q and L_a are searched such that the analytical absorption curve fits the FEM one around the first absorption peak. The circles markers and the solid green line of Fig. S4 depict the absorption coefficient of $\xi = 2$ FPM computed by FEM and obtained by the analytical model with the corresponding best Q and L_a , respectively. The correlation is almost perfect for the first absorption peak. Then a small shift appears between the absorption computed by the two methods. The process was repeated for eleven values of $\xi \in [0.25; 2]$. The relations $Q(\xi)$ and $L_a(\xi)$ were then interpolated from the discrete set of values as depicted in Fig. S5 (a), (b). From a geometric point of view, ξ can be increased above 2 and until the walls of the structure merge and thus stop to create cavities. If acoustically impervious walls being infinitely thin existed, then ξ and thus L_a could take any value. Thus, the first resonance frequency of a thin FPM could be at low frequency. However, as the increase of L_a is obtained at the cost of a decrease Q^2 , the higher ξ is, the higher the quality factors of modes is creating thin absorption peaks. Other resonators such as Helmholtz resonators should then be preferred for low

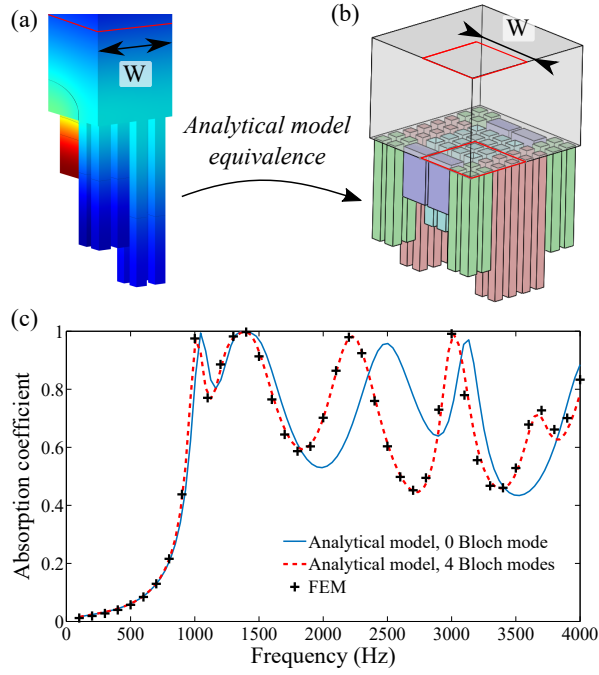


FIG. S3. (color online) Simulation of the numerical test sample (composed of straight tubes, filled by anisotropic micro-lattice and described in Tab. S1) mounted in a $W = 41.9$ mm width impedance tube. (a) Pressure field computed by FEM. (b) Geometry considered by the analytical model. The red square delimits the impedance tube. (c) Absorption coefficient computed by FEM, by the analytical model with 0 and 4 Bloch modes.

frequencies attenuation.

The pressure wave follows approximately a helix inside the cavities of the FPM. The parametric equation of a helix of radius r , height L_b and pitch L_b/ξ is

$$\begin{cases} x(s) = r \cos(2\pi\xi s) \\ y(s) = r \sin(2\pi\xi s) \\ z(s) = sL_b \\ s \in [0; 1]. \end{cases} \quad (\text{S39})$$

The arc length of the helix is $L_a = \sqrt{(2\pi\xi r)^2 + L_b^2}$. The tangent to the helix forms a constant angle β with its normal and is expressed by

$$\tan \beta = \frac{\sqrt{L_a^2 - L_b^2}}{L_b}. \quad (\text{S40})$$

Thus, $\beta(\xi)$ is derived from Eq. (S40) and the previously interpolated $L_a(\xi)$. It is depicted in Fig. S5 (c). This angle is of no importance if the FPM are filled by an isotropic material as it was the case when looking for the $L_a(\xi)$ and $Q(\xi)$ relations. However, this angle matters in practice because the FPMs are ultimately filled by quasi-isotropic MLs. Quasi-isotropy is also known as polar anisotropy to highlight the fact that the considered medium has two main

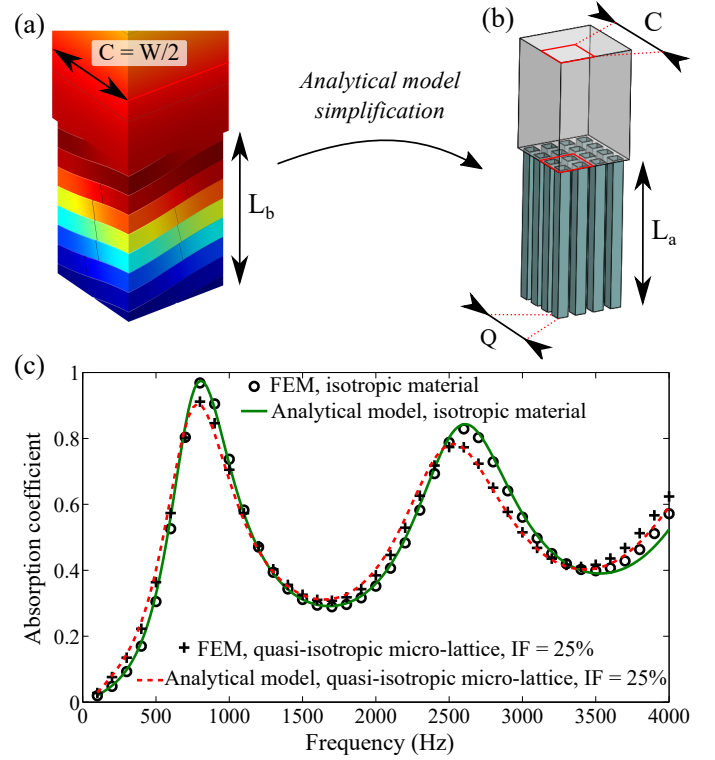


FIG. S4. (color online) Simulation of a $L_b = 30$ mm single $\xi = 2$ FPM mounted in a $C = W/2 = 20.85$ mm width impedance tube. (a) Pressure field computed by FEM. (b) Simplified geometry considered by the analytical model. The red square delimits the impedance tube. (c) Absorption coefficient computed by FEM (markers) and analytical model considering simplified geometry (lines). FPM filled with isotropic material (circles and solid green line) for retrieving the corresponding Q and L_a . FPM filled with the $IF = 25\%$ quasi-isotropic micro-lattice (crosses and red dashed line) for verification of the derived value of β .

directions in a polar coordinate system: the out-of-plane and the radial directions. Thus, the angle between the direction of propagation and the out-of-plane direction is sufficient to describe the propagation in such medium. The angle β is that between the tangent of the helix (direction of propagation of the pressure wave) and its normal, $\mathbf{z}$, which is also the out-of-plane direction of the MLs, $\mathbf{z}_1$. Then, the pressure wave propagates with an angle β between the out-of-plane direction of the MLs and their in-plane direction. In the used Cartesian coordinate, the direction $\mathbf{y}_1$ has been chosen as the in-plane direction.

Finally, the comparison between the absorption coefficient of a single FPM filled by the quasi-isotropic ML computed by FEM considering the real geometry and by the analytical model considering the simplified geometry is very satisfactory (see Fig. S4, (c)).

In summary, the analytical model makes use of a simplified geometry for the consideration of each FPM. The geometric parameters are obtained from comparison with

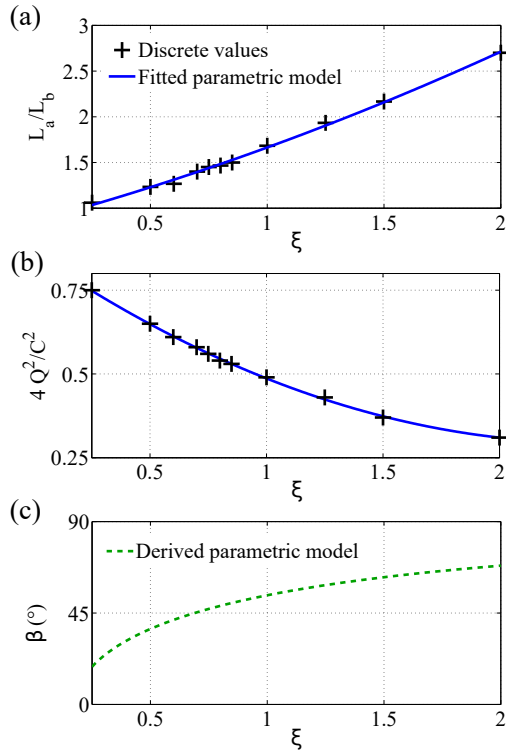


FIG. S5. Geometric parameters of the simplified FPM in function of ξ . (a) Normalized height. (b) Normalized section. (c) Frame rotation angle.

TABLE S2. Parameters of the MpS.

ξ		1.67	1.28	0.72	0
	Simulation	3	10	24	36
$IF(\%)$	Experimental	0	10	24	36
	Experimental, corrected	0	7	20	36

FEM values and geometrical considerations. There is a very good correlation between the absorption coefficient of a single FPM computed by FEM and by the analytical model. Experimentally, there is an uncertainty on the thickness of solid walls leading to an uncertainty on the experimental $Q(\xi)$, $L_a(\xi)$ and $\beta(\xi)$.

VI. COMPLEMENTARY DATA

A. Summary of the MpS parameters

The MpS was optimized numerically, printed and measured and then printed and measured anew with minor corrections. The parameters of these 3 MpS are summarized in Tab. S2. Only the IF of the filling ML varies.

B. Complementary absorption curves

Figure S6 depicts rigid backing absorption curves of 30 mm thick materials. An homogeneous ML (dashed blue line) and a single $\xi = 1$ FPM (dashed green line) reach perfect absorption at distinct frequencies. The absorption peak of the FPM is lower in frequency but narrower than that of the homogeneous ML. The solid black line represents the absorption curve of the optimized MpS and computed by means of FEM. The absorption of the corresponding sample (purple circles) follows the same trend but it is lower than expected between 1250 and 1700 Hz. Small correction of the IF suffices to adjust the measured absorption coefficient of the MpS (red crosses).

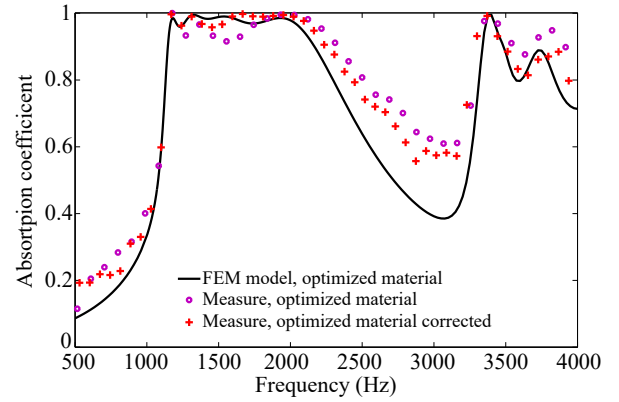


FIG. S6. (color online) Rigid-backing normal incidence absorption coefficient of MpS. Simulation of the optimized MpS by FEM model (black solid line). Measurement of optimized MpS (purple circles) and of the optimized MpS with adjusted IF (red crosses).

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