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<td><strong>Auteurs:</strong></td>
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<td><strong>Date:</strong></td>
<td>2016</td>
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<tr>
<td><strong>Type:</strong></td>
<td>Article de revue / Journal article</td>
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**Document publié chez l’éditeur officiel**
Document issued by the official publisher

**Titre de la revue:** Sensors

**Maison d’édition:** MDPI

**URL officiel:** [https://doi.org/10.3390/s16010087](https://doi.org/10.3390/s16010087)

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Article

Silica Bottle Resonator Sensor for Refractive Index and Temperature Measurements

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Received: 26 November 2015; Accepted: 6 January 2016; Published: 9 January 2016

Academic Editor: Vittorio M. N. Passaro

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Abstract: We propose and theoretically demonstrate a bottle resonator sensor with a nanoscale altitude and with a length of several hundreds of microns made on the top of the fiber with a radius of tens of microns for refractive index and temperature sensor applications. The whispering gallery modes (WGMs) in the resonators can be excited with a taper fiber placed on the top of the resonator. These sensors can be considered as an alternative to fiber Bragg grating (FBG) sensors. The sensitivity of TM-polarized modes is higher than the sensitivity of the TE-polarized modes, but these values are comparable and both polarizations are suitable for sensor applications. The sensitivity ~150 (nm/RIU) can be reached with a bottle resonator on the fiber with the radius 10 µm. It can be improved with the use of a fiber with a smaller radius. The temperature sensitivity is found to be ~10 pm/K. The temperature sensitivity can decrease ~10% for a fiber with a radius \( r_{co} = 10 \) µm instead of a fiber with a radius \( r_{co} = 100 \) µm. These sensors have sensitivities comparable to FBG sensors. A bottle resonator sensor with a nanoscale altitude made on the top of the fiber can be easily integrated in any fiber scheme.

Keywords: refractive index sensor; temperature sensor; bottle resonator

1. Introduction

A bottle resonator made on the surface of the optical fiber is a smooth parabolic perturbation of the fiber radius with a nanoscale altitude, which looks like a bottle. Operation of the bottle resonator is based on whispering gallery modes (WGMs) circulating on the surface of the resonator perpendicular to the fiber axis. The parabolic thickness profile of the bottle resonator, like a linear harmonic oscillator, provides light confinement along the fiber axis (Figure 1). Similarly to the electromagnetic field of surface plasmon-polaritons (SPPs) the electromagnetic field of WGMs is localized near the surface of the resonator. This field distribution makes WGMs useful for sensor applications [1–3]. Contrary to SPP devices [4–8], WGM devices are completely dielectric, that is free from metal components which exhibit loss such as in metal films or particles.

In this paper we consider a silica fiber bottle resonator with a nanoscale altitude for refractive index and temperature sensing applications. WGMs of a bottle resonator can be excited with the evanescent field of biconically tapered fiber (Figure 1). The excited WGMs appear as transmission dips in the output spectrum of a tapered fiber. The shift of these dips with the change in the refractive index or temperature can be used for sensing applications. In order to position our sensors amongst others let us consider the sensitivity of several widely used sensors, for example, fiber Bragg grating (FBG), WGM, and surface plasmon resonance (SPR) sensors. The temperature resolution of a FBG...
example is closely connected with small thermo-optical and thermal expansion coefficients of the fiber material, which increases the sensitivity of WGM sensors. The sensitivity is approximately 10 pm/K at near room temperatures [13]. In [14], WGM temperature sensors with an associated detectable resonance wavelength shift of 1.56 × 10^{-6} μm/K have been shown theoretically that the minimum resolvable temperature can be as small as 1.11 × 10^{-8} K [14]. The thermal responsivity of WGM sensors is a crucial factor in their design. As an example of recent WGM sensor achievements it is worth mentioning the crystalline MgF₂ disc resonator with a sensitivity of 1.09 nm/RIU. Refractive index sensitivities of 30, 570, and 700 nm/RIU have been reported in a microsphere resonator, a capillary-based optofluidic ring resonator, and a nanowire loop resonator, respectively [13]. The thermal responsivity of WGM sensors is significantly higher. As an example for prism-coupled and grating-coupled SPR sensors it is ~7000 nm/RIU and ~3000 nm/RIU, respectively [11]. In [12] it has been shown theoretically that the temperature sensitivity of SPR sensors can be as high as 4 nm/K. A comprehensive review of the current state of the art of physical and biological WGM sensors can be found in Ref. [13].

In this part of the paper we give a short overview of the theory used to simulate the operation of proposed sensors. A bottle resonator can be described with a truncated harmonic-oscillator profile [15]:

\[ R(z) = R_b \left[ 1 + (\Delta k z)^2 \right]^{-1/2} \]

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where \( R_b \) is the fundamental resonance radius and \( \Delta k \) is the modal index sensitivity.

Structure under investigation: a fiber with bottle resonator is excited with a tapered fiber. The dips in the output spectrum correspond to the WGMs circulating in the resonator.

Figure 1. Structure under investigation: a fiber with bottle resonator excited with a tapered fiber. The dips in the output spectrum correspond to the WGMs circulating in the resonator.

2. Theoretical Analysis

In this part of the paper we give a short overview of the theory used to simulate the operation of proposed sensors. A bottle resonator can be described with a truncated harmonic-oscillator profile [15]:

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where \( R_b = r_{co} + \Delta r_{co}, r_{co} \) is the radius of the fiber without of a resonator, \( \Delta r_{co} \) is the maximum altitude of the resonator. \( \Delta k \) is a parameter, which can be obtained, for example, from an experiment. The electric field of a bottle resonator mode in the scalar approximation in adiabatical approximation in cylindrical coordinates \((r,q,z)\) can be presented as [16]:

\[
E(r, q, z) = \Psi_{m,p,q}(z) \Phi_m(r, z) \exp(i\mu q)
\]  

(2)

where an integer \( m \) \((m = 0,1,2, \ldots)\) is an azimuthal number. It gives the number of field nodes around the circumference. An integer \( p \) \((p = 1,2, \ldots)\) is a radial quantum number. It gives the number of power maxima along the radius, and \( q \) \((q = 0,1,2, \ldots)\) is the discrete or continuous axial quantum number. Here:

\[
\Phi_m(r, z) = \text{Ai}\left(\frac{2^{1/3}m^{2/3}}{r_{co}} (r_{co} - r) - \alpha_p \right)
\]

(3)

where \( \alpha_p \) is \( p \)-th root of the Airy function [17]. The amplitude \( \Psi_{m,p,q}(z) \) in the case of a harmonic oscillator profile can be estimated using the one-dimensional Schrödinger equation [15,16,18] and described by the relation:

\[
\Psi_{m,p,q}(z) = \frac{1}{\sqrt{\pi 2^{m+1} (q!)^2}} \Delta E_m^{1/4} H_q\left(\frac{\Delta E_m}{2} z\right) \exp\left(-\frac{\Delta E_m}{4} z^2\right)
\]

(4)

where \( H_q(x) \) is the Hermite polynomial. \( \Delta E_m = 2U_{m,p} \Delta k/R_b \). \( U_{m,p} \) can be estimated with the relation [19,20]:

\[
U_{m,p} \approx m \left[1 + \frac{\alpha_p}{2^{1/3}m^{2/3}} - \frac{n_{cl}}{m} \left(\frac{n_{co}}{n_{cl}}\right)^{\frac{1}{2}} \left(\frac{n_{co}}{n_{cl}}\right)^{\frac{1}{2}} + \frac{3}{10} \frac{\alpha_p^2}{2^{2/3}m^{4/3}} \right]
\]

(5)

Signs + and – correspond to TE and TM polarization, respectively. \( c \) is the speed of light in vacuum. \( n_{co} \) and \( n_{cl} \) are refractive index of the fiber and surrounding medium, respectively. In the first approximation \( r_{co}, k_r n_{co} \approx m \), where \( k_r = \omega/c = 2\pi/\lambda_r \), and the WGM frequency, \( \omega_r \), can be estimated using the geometry of a sample. This frequency corresponds to the condition for constructive interference of the wave upon a round trip of the resonator. The resonant wavelength of the WGM is

\[
\lambda_{m,p,q} = 2\pi n_{co} \left[ \left(\frac{U_{m,p}}{R_b}\right)^2 + \left(q + \frac{1}{2}\right) \Delta E_m \right]^{-1/2}
\]

(6)

In the case of the bottle resonator a smooth (nm) parabolic perturbation of the fiber radius can be described as

\[
R(z) = r_{co} + \Delta r(z) = r_{co} + \Delta r_{co} = \frac{z^2}{2R}, \text{for } 0 < z < L
\]

(7)

where \( L = (2\Delta r_{co})^{1/2} \) is the length of resonator. \( R \) is the radius of the curvature of the bottle resonator. As one can see in Equations (1) and (7) \( (\Delta k)^2 = \frac{2\Delta r_{co}}{R_b L^2} \). Following [13] the WGM excitation process can be simulated with the \( \delta \)-function \( C \delta(z - z_c) \), where \( C \) is the coupling parameter. \( z_c \) is the point near the top of the resonator on the z-axis, which is directed along the fiber axis, where the tapered fiber touches the resonator. In this case [18],

\[
\Psi_{m,p,q}(z) = CG(\lambda, z_c, z)
\]

(8)
The resonator Green’s function can be presented as

$$G(NJ, \lambda, z, z') = \frac{\cos[\psi(\lambda, z, z') + \pi/4] \cos[\psi(\lambda, z, z') + \pi/4]}{2\beta(\lambda, z, z') \cos[\psi(\lambda, z, z')]$$

where

$$\psi(\lambda, z, z') = \int_b^{z'} \beta(\lambda, z) dz$$

Here, $$\beta(\lambda, z, z')$$ is the propagation constant and the $$z$$ and $$z'$$ are turning points, where $$\beta(\lambda, z, z') = 0$$ [18]. The WGM does not propagate beyond these points along the length of the fiber. We want to emphasize that the semiclassical theory fails near the turning points, since the axial wavelength, which is proportional to $$\beta(\lambda, z, z')$$, reaches infinity at the turning points [19].

3. Results and Discussion

3.1. WGMs of the Bottle Resonator

Let us consider a silica fiber with the radius $$r_c = 30 \mu m$$. Following Equation (4), one can simulate the field distribution along the radius of the fiber for different modes (Figure 2). All calculations have been performed in Matlab with double precision. As one can see in Figure 2, the maximum of the field moves closer to the fiber axis as the radial quantum number increases.

As we already mentioned, the resonator is like a linear harmonic oscillator provides light confinement along the fiber axis. Using relation Equation (4) we have simulated the electric field intensity distribution along WGM along the length of the resonator (z-axis) with $$\Delta r_o = 3.8 \text{ nm}$$. We have also considered the resonator with three different lengths $$L = 500, 1000, \text{ and } 1500 \mu m$$ have been considered. We have also simulated the electric field intensity distribution in the WGM along the length of the (Figure 3a). We have also simulated the electric field intensity distribution in the WGM along the resonator with $$L = 500 \mu m$$ and three different lengths $$\Delta r_o = 1.8, 3.8, \text{ and } 4.8 \text{ nm}$$ (Figure 3b). As one can see in Equation (4) the WGM field becomes more concentrated near the top of the resonator with increasing length of the resonator $$L$$. As an example, if the length $$L = 500 \mu m$$ the WGM field is concentrated in the vicinity of 0.4 of the length of the resonator that is $$200 \mu m$$ near the top of the resonators (Figure 3a). If the length of the resonator is increased up to $$L = 1500 \mu m$$ and the altitude is the same $$\Delta r_o = 3.8 \text{ nm}$$ the WGM field is concentrated in the vicinity of 0.23 of the length of the resonator that is $$345 \mu m$$ near the top of the resonators (Figure 3a).

As we already mentioned, the resonator is like a linear harmonic oscillator provides light confinement along the fiber axis.
length of the resonator that is ~200 μm near the top of the resonators (Figure 3a). If the length of the resonator is increased up to \( L = 1500 \) μm and the altitude is the same Δr = 3.8 nm the WGM field is concentrated in the vicinity 0.23 of the length of the resonator that is ~345 μm near the top of the resonators (Figure 3a). If the altitude of the resonator is increased keeping a constant length \( L = 500 \) μm, the field of the WGM will be concentrated closer to the top of the resonator. For example if \( \Delta r = 1.8 \) nm the field is concentrated in the vicinity of 0.5 of the length of the resonator that is ~250 μm. The sensitivity of a bottle resonator sensor is different for TE and TM modes. It can be estimated from the corresponding change in the refractive index, \( \Delta n = \frac{\Delta \lambda}{\lambda} \), of the WGMs are functions of the refractive index of the surrounding medium. WGMs circulate on the surface of the resonator. They have to be sensitive to any changes in the refractive index of the surrounding medium like SPPs. Each excited WGM appears as a transmission dip in the output spectrum of the tapered fiber (Figure 1). This dip will shift along the wavelength axis as the refractive index of the surrounding medium changes. This shift, \( \Delta \lambda \), is divided by the corresponding change in the refractive index and characterizes the sensor’s sensitivity to the refractive index of the surrounding medium

\[
\frac{\Delta \lambda}{\lambda} = \frac{\Delta n}{n}
\]

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For TM modes, respectively.

Figure 4 illustrates the sensitivity of the bottle resonator to the refractive index as a function of the fiber radius for TE and TM-polarizations. In our simulations the length $L = 500 \, \mu m$ and the altitude $z = 10 \, cm$ of the fiber were constant. From the Figure 4, it can be seen that the sensitivity of the WGMs with TM-polarization is better than the sensitivity of the WGMs with TE-polarization although the values are comparable (Figure 4). The sensitivities of all modes become almost equal to each other for fibers with radius $rco = 100 \, \mu m$ and $ncl = 1$. The shift in the resonant wavelength with the temperature can be estimated in the first approximation as

$$\frac{d\lambda}{dn}_{TM} \approx \frac{\lambda^2}{2\pi c_0 n_{cl}^3\left(n_{cl}^2 - n_{cl}^2\right)}$$  

(14)

For TM modes, respectively.

3.3. Temperature Sensing

The WGM wavelength is a function of the refractive index and the radius of the fiber (see Equations (5) and (6)), which are functions of the temperature, i.e., a bottle resonator sensor can be used as a temperature sensor. Let us investigate its sensitivity to temperature. We assume that the sensor is placed in air or vacuum that is $n_{cl} = 1$. The shift in the resonant wavelength with the temperature can be estimated in the first approximation as

$$\Delta \lambda = \frac{\lambda^2}{2\pi c_0 n_{cl}^3\left(n_{cl}^2 - n_{cl}^2\right)} \Delta T$$  

(15)

where $\Delta T$ is the change in the temperature. $\alpha = dr/(dT)$ is the coefficient of thermal expansion, which is the fractional increase in radius per unit rise in temperature. It changes slightly with temperature in the range between $-0.2 \times 10^{-6} \, K^{-1}$ at $-50 \, ^\circ C$ and $-0.7 \times 10^{-6} \, K^{-1}$ at $250 \, ^\circ C$ [21]. $dn/dT$ is the thermo-optical coefficient. The thermo-optical coefficient of silica at room temperature is $dn/dT \approx 9.2 \times 10^{-6} \, K^{-1}$. It decreases more or less linearly down to $-3 \times 10^{-6} \, K^{-1}$ at liquid nitrogen temperature [22]. This dependence of the thermo-optical coefficient on the temperature has been taken
into account in our simulations. As one can see in Equation (15) the influence of thermal expansion on the sensor sensitivity is less than the influence of the thermo-optic effect by a factor of approximately ten. As we see from our simulations the influence of the thermal expansion on the sensor’s sensitivity, which can be described as the relation:

\[ S_T = \Delta \lambda / \Delta T \]  

(16)

is negligible in comparison with the thermo-optic effect and can be neglected in simulations. As before let us consider the bottle resonator sensor with the length \( L = 500 \, \mu m \) and the altitude \( \Delta r_0 = 3.8 \, \text{nm} \), and the coupling constant \(|C|_1^2 = 2 \times 10^4 \, \text{m}^{-1}\). The transmission spectra of the tapered fiber for three different temperatures of the bottle resonator 200 K, 300 K, and 400 K have been simulated using the Green’s function Equation (9). They are presented in Figure 5. As one can see in Figure 5 the dip shifts with temperature. The bandwidths of the dips in the transmission spectrum are \( \sim 0.025 \, \text{nm} \). The sensitivity of the bottle resonator as a temperature sensor can be estimated with Equations (15) and (16). The temperature sensitivity of the sensor as a function of the fiber radius is illustrated in Figure 6 for TM and TE polarized modes. The temperature sensitivity decreases \( \sim 10\% \) as the fiber radius decreases from \( r_{co} = 100 \, \mu m \) to \( r_{co} = 10 \, \mu m \). The decrease in the sensor sensitivity is caused by the decrease in the resonant wavelength, \( \lambda_r \), with the radius of the fiber. Using Equations (5) and (6) we have obtained the rate of change of the resonant wavelength with the radius of the fiber as

\[ \frac{d\lambda_r}{dr_{co}} = \frac{\lambda_2 \alpha_p}{2^{1/3} 3\pi (n_{co} r_{co})^{5/3}} \left[ \frac{U_{m,p}}{R_b} + \Delta k \left( q + \frac{1}{2} \right) \right] \]  

(17)

Here \( r_{co} = r_{co} k_0 \) is the normalized fiber radius. For all fiber radii \( d\lambda_r/dr > 0 \), \( \lambda_r \) increases with the increase in the fiber radius. As one can see in Equation (17) and Figure 6 the rate of change of the resonant wavelength with the radius, \( d\lambda_r/dr \), increases with a decrease in the radius of the fiber, and this rate \( d\lambda_r/dr \rightarrow 0 \) as the radius of the fiber increases substantially. For our structures, where \( \Delta k \ll U_{m,p}/R_b \) Equation (17) can be simplified and presented as

\[ \frac{d\lambda_r}{dr_{co}} \approx \frac{2^{1/3} 4\pi n_{co} \alpha_p}{3 (n_{co} r_{co})^{1/3}} \left[ \alpha_p + 2^{1/3} (n_{co} r_{co})^{2/3} \right]^2 \]  

(18)

As in the case of the refractive index sensor, the sensitivity of TM polarized modes exceeds the sensitivity of TM polarized modes but these values are comparable (Figure 6). Our temperature sensor with a sensitivity of 10 pm/K can provide a temperature detection limit of 1 K if an OSA with a resolution 10 pm is used for the monitoring process. This sensitivity is comparable to the sensitivities of other WGM sensors [14].

Figure 5. The transmission spectrum of the tapered fiber as a function of the wavelength for the temperatures 300, 200, and 400 K. \( r_{co} = 30 \, \mu m, L = 500 \, \mu m, \) and \( \Delta r_0 = 3.8 \, \text{nm} \).
G.N. conducted the research work and prepared the manuscript. R.K. corrected and edited the manuscript. 

Acknowledgments: RK would like to acknowledge the Natural Sciences and Engineering Council of Canada’s Discovery Grants program and the Canada Research Chairs programs for financial support.

Conflicts of Interest: The authors declare no conflict of interest.

References


23. Sumetsky, M. Nanophotonics of optical fibers. Nanophotonics 2013, 2, 393-406. [CrossRef]

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