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# GEOMETRIC OPTIMIZATION OF A SELF-ADAPTIVE ROBOTIC LEG

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## ABSTRACT

Inspired by underactuated mechanical fingers, this paper demonstrates and optimizes the self-adaptive capabilities of a 2-DOF Hoecken's-Pantograph robotic leg allowing it to overcome unexpected obstacles encountered during its swing phase. A multi-objective optimization of the mechanism's geometric parameters is performed using a genetic algorithm to highlight the trade-off between two conflicting objectives and select an appropriate compromise. The first of those objective functions measures the leg's passive adaptation capability through a calculation of the input torque required to initiate the desired sliding motion along an obstacle. The second objective function evaluates the free-space trajectory followed by the leg endpoint using three criteria: linearity, stance ratio, and height-to-width. In comparison with the initial geometry based on the Hoecken's linkage, the selected final mechanism chosen from the Pareto front shows an important improvement of the adaptation capabilities, at the cost of a slight decrease in the stance phase duration. This paper expands on mechanical self-adaptive design philosophy, which recently attracted a lot of attention in the field of grasping, to legged locomotion and paves the way for subsequent experimental validation of this approach.

**Keywords:** optimization; robotic leg; underactuation; linkage; kinetostatic analysis.

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## RÉSUMÉ

En utilisant une approche similaire aux mécanismes de doigts sous-actionnés, les capacités d'adaptation d'une architecture de jambe robotique à deux DDL de type Hoecken's-Pantographe sont optimisées dans cet article afin de lui permettre de surmonter des obstacles imprévus lors de sa phase de vol. Une optimisation multiobjectif des paramètres géométriques du mécanisme a été effectuée afin de mettre en évidence l'opposition existant entre deux objectifs contradictoires et choisir un compromis. Le premier de ces objectifs

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29 mesure la capacité d'adaptation passive de la jambe en calculant le couple d'entrée requis pour amorcer le  
30 glissement désiré le long d'un obstacle. La deuxième fonction objectif évalue la trajectoire de base suivie  
31 par l'extrémité de la jambe en se basant sur trois critères : linéarité, ratio de la phase de support, et rapport  
32 hauteur/largeur. En comparaison avec la géométrie initiale pasée sur le mécanisme de Hoecken, le méca-  
33 nisme final trouvé sur le front de Pareto présente une amélioration marquée des capacités d'adaptation, au  
34 coût d'une légère réduction de la durée de la phase de support. Cet article étend la philosophie de l'auto-  
35 adaptation mécanique, qui a récemment beaucoup attiré l'attention dans le domaine de la préhension, à celui  
36 de la marche, et ouvre la voie à une validation expérimentale de cette approche.

37 **Mots-clés :** optimisation ; jambe robotique ; sous-actionnement ; mécanisme à membrures ; analyse cinéto-  
38 statique.

## 39 1. INTRODUCTION

40 While the simplicity, energy efficiency, and speed of wheeled locomotion are hard to match, walking  
41 robots are often a preferred alternative when navigating uneven terrains. Successful robotic leg designs are  
42 often serial mechanisms comprised of several actuated joints, such as in the Star $LETH$  (Hutter et al., 2012)  
43 or the HyQ (Semini et al., 2011) robots, or consist of much simpler compliant links, which, despite their  
44 simplicity, generate a very efficient dynamical gait for the robot and help successful navigation in rough  
45 terrains, as exemplified by the RHex (Saranli et al., 2001). On the other hand, mechanical linkages can also  
46 guide the endpoint of a robotic leg using as few as a single actuator and are largely unaffected by the payload  
47 while in stance phase conversely to designs using compliant links. However, while the limited number of  
48 actuators required with the designs based on linkages is a significant advantage, it is impossible for the  
49 leg to depart from the generated trajectory if the presence of an obstacle requires an adaptation. Active  
50 reconfiguration with the use of additional actuators has however been previously proposed, notably for the  
51 Theo Jansen linkage (Nansai et al., 2015).

52 Among linkages able to generate a suitable leg trajectory for a walking application, the combination of  
53 the Hoecken's linkage, also sometimes referred to as Chebyshev's lambda mechanism, and a pantograph  
54 for amplification purposes has been studied by many (Ottaviano et al., 2005; Tao and Ceccarelli, 2011;  
55 Liang et al., 2012), including the authors (Fedorov and Birglen, 2017). In the present paper, the geometric  
56 parameters of a two degree-of-freedom (DOF) variant of this architecture are, for the first time, optimized to  
57 take advantage of its self-adaptive capabilities. The desired objective is to allow the leg to "give in" without  
58 any sensing or control and slide along an obstacle following an unexpected collision, rather than trying to  
59 pursue an unfeasible trajectory. While a similar behaviour has been previously obtained by making use of  
60 electronic reflex generation (Park et al., 2013; Focchi et al., 2013), this effect is here intended to be obtained  
61 purely mechanically. It should be emphasized that, in our case, the leg does not avoid the collision with the  
62 obstacle, and that it remains in contact with the colliding object during the whole adaptation sequence.

63 The proposed approach is directly inspired by the design of self-adaptive mechanisms in the field of  
64 underactuated robotic hands (Birglen et al., 2008; Birglen, 2009): a single input force is distributed to  
65 several output phalanges, the motions of which are triggered by contacts between the finger and the grasped  
66 object. With the use of preloaded springs and mechanical limits, the closing sequence of the phalanges can

67 be achieved without any control or sensors. Similarly, a contact during the swing phase of the proposed  
68 leg mechanism passively triggers the secondary DOF which allows the leg to depart from its trajectory to  
69 accommodate the obstacle.

70 Ultimately, using a purely mechanical solution to generate a complex behaviour, such as obstacle over-  
71 coming, can be more affordable than relying on the complex software control of multiple actuators, and  
72 could prove to be useful for applications where environmental factors such as extreme temperatures or ra-  
73 diation impact the use of electronic controllers. Similar considerations have guided the recent emergence  
74 of underactuated grippers as affordable solutions for the grasping of complex objects. The expansion of  
75 this design philosophy to other applications might prove to be fruitful, as initially investigated in (Khakpour  
76 et al., 2014) for cable robots and is further investigated here.

## 77 **2. MECHANISM DESCRIPTION**

### 78 **2.1. Geometry**

79 The leg mechanism described in this paper can be separated in two basic linkages: a four-bar linkage  
80 acting as a path generator, and a pantograph. More specifically, the geometric parameters of the four-bar are  
81 initially matching the ones of the Hoecken's linkage, although they will be altered following the optimization  
82 described in Section 4. When driven by the rotation  $\theta_1$  of the input crank, this linkage generates a trajectory  
83  $M_1$  suitable for a walking application due to the existence of a linear portion and the proportion of the cycle  
84 (close to 70%) that is spent in this phase, as illustrated in Fig. 1a. The second element of the mechanism  
85 is the pantograph, *i.e.*, a linkage characterized by a constant ratio of the distance between its two *guiding*  
86 *points* to the distance between either of its *guiding points* and its *following point*, see Fig. 1a. The pantograph  
87 performs three functions in the mechanism, listed here by increasing order of importance for our application:

- 88 1. Amplify the trajectory  $M_1$ , which is inputted to one of the pantograph's guiding points.
- 89 2. Ensure that the leg endpoint (*i.e.*, the following point) is the lowest point of the mechanism.
- 90 3. Add a second degree of freedom,  $\theta_2$ , to the mechanism. The associated motion  $M_2$  is applied to the  
91 pantograph's second guiding point, and therefore, also affects the position of the following point.

92 **2.2. Self-adaptive behaviour**

93 As is often the case with self-adaptive fingers where the motion of a phalanx may not be triggered until  
 94 contact is established with the grasped object, the secondary DOF  $\theta_2$  is here constrained using a preloaded  
 95 spring and a mechanical limit. In normal operation (*i.e.*, if no obstacles are encountered), this DOF is  
 96 therefore not triggered and the leg endpoint follows the *free-space trajectory*, illustrated in Fig. 1b, solely  
 97 generated by the rotation of  $\theta_1$ .

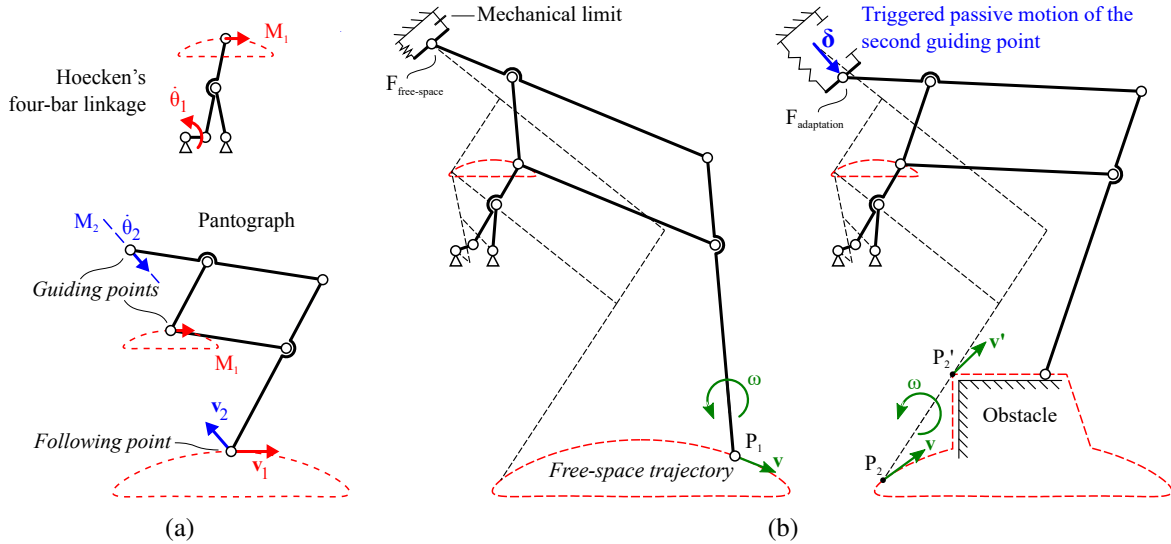


Fig. 1. Description of the mechanism's initial geometry: (a) constituent linkages, (b) simulation of the leg encountering an obstacle during its swing phase and subsequent passive adaptation

98 If a collision occurs during the swing phase of the leg motion, the continued actuation of  $\theta_1$  coupled with  
 99 the obstacle resistance at the point of contact cause an increase of the mechanism's internal forces, which  
 100 is used to overcome the preloaded spring. Then, the resultant motion of  $\theta_2$  combined to the rotation of  
 101  $\theta_1$  allows the leg endpoint to depart from the free-space trajectory and slide along the obstacle. Similarly  
 102 to self-adaptive fingers, the spring acting on the  $\theta_2$  does not store any useful energy to the walk but only  
 103 prevents incoherent motion of the leg endpoint, and resists the adaptation movement. In order to reduce the  
 104 required actuation effort, the spring stiffness and preloading should therefore be kept as low as possible, *i.e.*,  
 105 of the minimal magnitude required to balance the inertial effects to which point F is subjected. Since these  
 106 effects are not apparent in the kinetostatic analysis performed in this paper, the selection of the spring is, for  
 107 now, done during the initial testing of the physical linkage (for the first 3d-printed prototype, of total linkage  
 108 mass of  $\sim 100$  g described in (Fedorov and Birglen, 2017), a spring with a stiffness of  $k = 0.05$  N/mm and

109 an initial preload of 0.36 N was experimentally deemed suitable).

110 In Fig. 1b, the passive adaptation induces a translation (denoted by the vector  $\delta$ ) of the pantograph's  
 111 guiding point,  $F$ , from its initial position ( $F_{free-space}$ ) to a new location ( $F_{adaptation}$ ). The displacement  $M_2$   
 112 between these two points can be generated by any motion generator, the simplest choices being a prismatic  
 113 joint (for which  $M_2$  is a straight line) or a revolute one (for which  $M_2$  is an arc), as in (Fedorov and Birglen,  
 114 2017). The nature of this generator is however left arbitrary.

115 Due to the geometry of the pantograph linkage, the vertical component of  $\delta$  must be negative to generate  
 116 a rising motion of the leg endpoint. A purely vertical translation of  $F$  would therefore seem advantageous,  
 117 but such a design would render the passive adaptation much more difficult, as will be shown in Section  
 118 3.1. At the other extreme, a completely horizontal  $\delta$  makes adaptation very easy, but could not result in the  
 119 desired vertical motion of the leg endpoint along an obstacle. An intermediate orientation as illustrated in  
 120 Fig. 1b must therefore be selected for the allowed range of motion of point  $F$ .

### 121 2.3. Initial geometric parameters

122 Although several straight line linkages can be suitable for the generation of the free-space trajectory, the  
 123 Hoecken's linkage, with only three links, was selected here for its simplicity. Its geometric parameters are  
 124 presented in Fig. 2 ( $h_1$  and  $w_1$  refer respectively to the trajectory's height and width) and Table 1, where  $l_{ij}$   
 125 refers to the distance between points  $i$  and  $j$ .

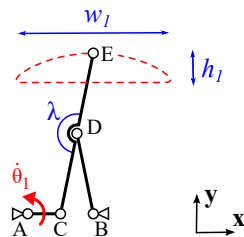


Fig. 2. Hoecken's linkage

Geometric parameter	Value
Coordinates of A	(0.00 0.00)
Coordinates of B	(2.00 0.00)
$l_{AC}$	1.00
$l_{BD}=l_{CD}=l_{DE}$	2.50
$\lambda$	180°

Table 1. Initial geometry (four-bar)

126 Next comes the dimensioning and positioning of the pantograph. The geometry of this linkage can be  
 127 described using only two ratios:

- 128 •  $\rho$  is the amplification ratio which relates the sizes of similar triangles FGE and FHJ (see Fig. 3).
- 129 •  $\alpha$  is the shape factor of triangles FGE and FHJ, defined as the ratio of  $l_{EG}$  to  $l_{FG}$  (see Fig. 3 again).

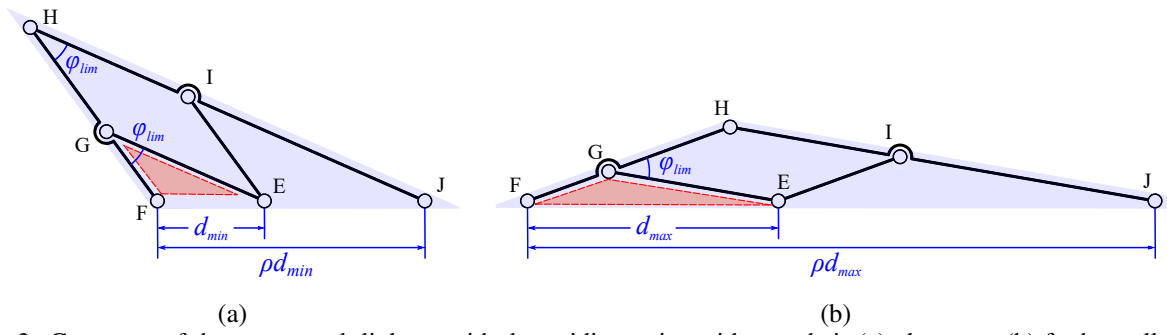


Fig. 3. Geometry of the pantograph linkage with the guiding points either at their (a) closest or (b) farthest allowed positions

130 An important consideration when designing the mechanical leg is to prevent the pantograph from reaching  
 131 a singular configuration, of which an indicator is the shape of the parallelogram EGHJ: neither of its angles  
 132 should be allowed to become smaller than a threshold value defined by  $\phi_{lim}$ . This condition allows to set a  
 133 design constraint on the permissible values of  $d$ , the distance between the guiding points E and F. First, the  
 134 law of cosines is used with triangle EFG in both limit configurations illustrated in Fig. 3, *i.e.*:

$$d_{min}^2 = l_{FG}^2 + l_{EG}^2 - 2 \cdot l_{FG} \cdot l_{EG} \cdot \cos(\phi_{lim}) \quad (1a)$$

$$d_{max}^2 = l_{FG}^2 + l_{EG}^2 - 2 \cdot l_{FG} \cdot l_{EG} \cdot \cos(\pi - \phi_{lim}). \quad (1b)$$

135 Eqs. (1a) and (1b) can be combined using the previously defined ratio  $\alpha$  to establish yet another ratio,  
 136  $d_{max}/d_{min}$ , which is maximal when  $\alpha = 1$ , *i.e.*, FGE and FGH are isosceles triangles:

$$137 \quad \frac{d_{max}}{d_{min}} = \sqrt{\frac{1 + \alpha^2 + 2\alpha \cdot \cos(\phi_{lim})}{1 + \alpha^2 - 2\alpha \cdot \cos(\phi_{lim})}}. \quad (2)$$

138 The position of point E, one of the pantograph's guiding points, is always the location of the four-bar's  
 139 coupler point. Therefore, only six parameters remain unknown for the design of the pantograph:

- 140 • The  $x$  and  $y$  coordinates of  $F_{free-space}$ , the location of guiding point F during the free-space trajectory.
- 141 • The  $x$  and  $y$  coordinates of  $F_{adaptation(max)}$ , the location guiding point F at the maximal position in  
 142 the chosen range of  $\theta_2$ .
- 143 • The pantograph's  $\rho$  and  $\alpha$  ratios.

144 As illustrated in Fig. 4, a translation  $\delta_{max}$  of F from  $F_{free-space}$  to  $F_{adaptation(max)}$  results in a shift of all the  
 145 points of the free-space trajectory by vector  $(\rho - 1)\delta_{max}$ , thereby defining the maximal adaptation trajectory.



146 Both the free-space and maximal adaptation trajectories have an identical shape to the Hoecken's linkage  
 147 output curve, but scaled up to a height  $h_2 = \rho h_1$  and a width  $w_2 = \rho w_1$ .

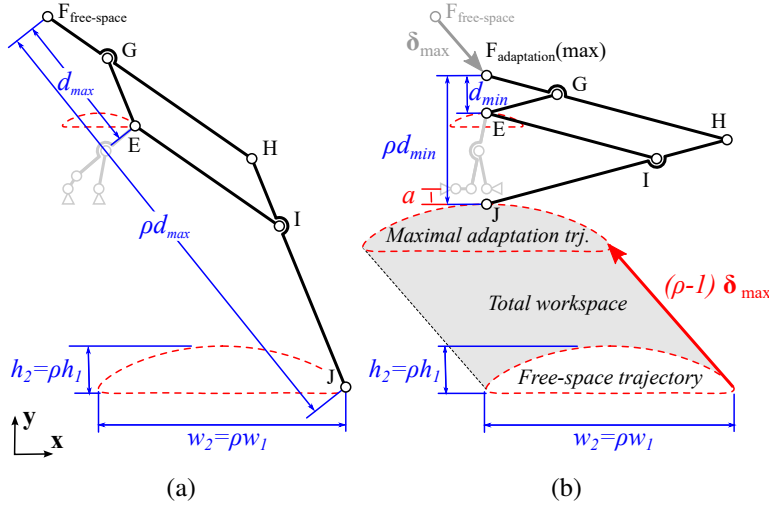


Fig. 4. Dimensioning of the pantograph, (a)  $F = F_{free-space}$ ,  
 (b)  $F = F_{maximal\ adaptation}$

Geometric parameter	Value
Coordinates of $F_{free-space}$	(-1.33 11.17)
Coordinates of $F_{max.adaptation}$	(2.00 7.35)
$l_{EG} = l_{FG} = l_{HI}$	4.73
$l_{EI} = l_{GH} = l_{IJ}$	11.40

Table 2. Initial geometry (pantograph)

148 To ensure a unique solution for the dimensioning of the pantograph, which simplifies the subsequent  
 149 optimization, six constraints have been selected. Using Fig. 4 as a reference, these constraints are listed  
 150 below:

- 151 • Ensure a sufficiently large *total workspace* (i.e., the area theoretically reachable by the leg's endpoint  
 152 following motions of  $\theta_1$  and  $\theta_2$ ).  
 153  $\Rightarrow$  *Constraint C<sub>1</sub>*: the vertical distance  $(\rho - 1)\delta_{max}^T \mathbf{y}$  between the free-space and maximal adaptation  
 154 trajectories is arbitrary chosen to be three times the height  $h_2$  of the free-space trajectory:

$$(\rho - 1)\delta_{max}^T \mathbf{y} = 3h_2. \quad (3)$$

- 156 • Allow for easy passive adaptation of the leg to collisions. As will be detailed in Section 3.1, the ori-  
 157 entation of  $\delta$  has a critical effect on the torque required for the leg to slide along the surface of an  
 158 obstacle. A compromise must be selected between the ease of adaptation and the proportion of the  
 159 workspace located directly above the free-space trajectory.

- 160  $\Rightarrow$  *Constraint C<sub>2</sub>*: the horizontal distance  $(\rho - 1)\delta_{max}^T \mathbf{x}$  between the free-space and maximal adap-

161 tation trajectories is arbitrarily chosen at half the width  $w_2$  of the free-space trajectory:

$$162 \quad (\rho - 1)\boldsymbol{\delta}_{max}^T \mathbf{x} = w_2/2. \quad (4)$$

163 • Prevent interference between the leg's endpoint and Hoecken's linkage.

164  $\Rightarrow$  *Constraint C<sub>3</sub>*: the maximal adaptation trajectory is below the lower limit of the four-bar's en-  
165 velope. For the Hoecken's linkage, the distance  $a$  between the origin and the maximal adaptation  
166 trajectory, c.f. Fig. 4, is therefore chosen equal to the length of the crank:

$$167 \quad a = l_{ac}. \quad (5)$$

168 • Keep the mechanism as compact as possible.

169  $\Rightarrow$  *Constraint C<sub>4</sub>*: the centers of the maximal adaptation trajectory and that of  $M_1$ , the trajectory  
170 generated by the Hoecken's linkage, are horizontally aligned.

171  $\Rightarrow$  *Constraint C<sub>5</sub>*: the shape factor  $\alpha$  of the pantograph is set at 1.

172 • Avoid singularities for all possible configurations of the mechanism.

173  $\Rightarrow$  *Constraint C<sub>6</sub>*: the limit angle  $\phi_{lim}$  is set at  $30^\circ$  and Eq. (2) thus becomes:

$$174 \quad \frac{d_{max}}{d_{min}} = 3.73. \quad (6)$$

175 The numerical parameters chosen for constraints  $C_1$  and  $C_2$  have here been arbitrary selected to demonstrate  
176 the subsequent geometry optimization procedure, but can be altered depending on specific requirements for  
177 the mechanism. The unique geometric parameters satisfying these six constraints for the Pantograph linkage  
178 are computed using an iterative method and are shown in Fig. 4 and Table 2 (the origin is still coincident  
179 with point A of the four-bar).

### 180 3. PERFORMANCE EVALUATION

#### 181 3.1. Ease of adaptation

182 The actuation torque  $\tau_{in}$  (acting on  $\theta_1$ ) required to perform the normal motion as well as the adaptation  
 183 described in Section 2.2 can be expressed as a function of the preloading force  $f_p$  (acting on  $\theta_2$ ), the friction  
 184 coefficient  $\mu$  at the obstacle contact location, and the mechanism's configuration at the moment of this  
 185 contact. This relationship can be found out by performing a static analysis on the mechanism. Two classes  
 186 of contacts can be defined: in a *Type I* contact, the collision occurs at the leg endpoint and the orientation of  
 187 the normal unit vector  $\mathbf{n}$  at this point depends on the obstacle, whereas, in a *Type II* contact, collision occurs  
 188 elsewhere along the terminal link of the leg and the orientation of the latter changes  $\mathbf{n}$ . In both cases, the  
 189 unit vector  $\mathbf{t}$  is defined tangent to the relative sliding motion of the leg and the obstacle (c.f. Fig. 5).

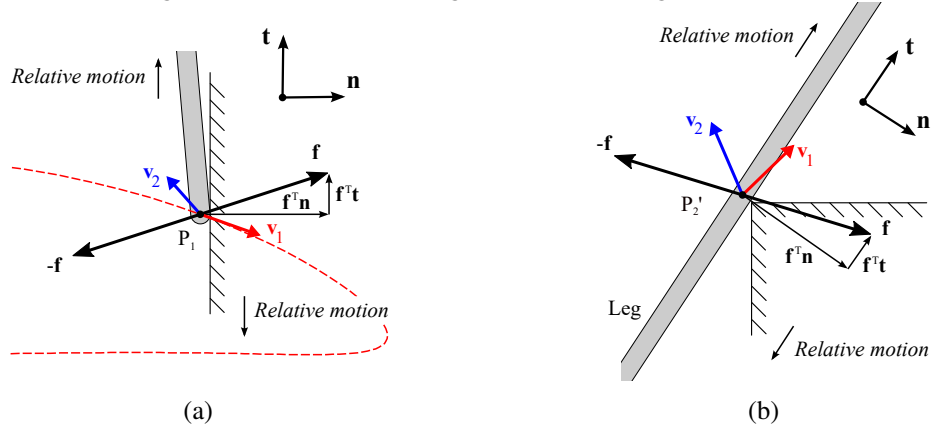


Fig. 5. Velocities and forces at the contact points, with either (a) *Type I* or (b) *Type II* contacts.

190 Using a Coulomb friction model with a coefficient  $\mu$  and considering the edge of the friction cone, the  
 191 contact force  $\mathbf{f}$  is first expressed as:

$$192 \quad \mathbf{f}^T \mathbf{t} = \mu \mathbf{f}^T \mathbf{n}. \quad (7)$$

193 For a *Type I* contact,  $\mathbf{f}$  can also be written as:

$$194 \quad \mathbf{f} = \mathbf{J}^{*-T} \boldsymbol{\tau}, \quad (8)$$

195 where  $\boldsymbol{\tau} = \begin{bmatrix} \tau_{in} & -f_p \end{bmatrix}^T$  and  $\mathbf{J}^*$  is a square submatrix of the mechanism's Jacobian, mapping the endpoint

196 linear velocity  $\mathbf{v}$  to the velocities of the DOFs, *i.e.*,

$$197 \quad \mathbf{v} = \mathbf{J}^* \dot{\boldsymbol{\theta}} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \dot{\boldsymbol{\theta}}, \quad (9)$$

198 with  $\dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T$ . The matrix  $\mathbf{J}^*$  can be interpreted geometrically as  $\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}$  where  $\mathbf{v}_i$  is the derivative  
 199 of the leg endpoint's position with respect to  $\dot{\theta}_i$ . As illustrated in Fig. 5a,  $\mathbf{v}_1$  is always tangent to the freespace  
 200 trajectory while the orientation of  $\mathbf{v}_2$  is tangent to  $M_2$ , the motion associated to  $\theta_2$ . More specifically, if a  
 201 prismatic joint is used for  $M_2$  its direction would be parallel to the vector  $\boldsymbol{\delta}$ . For a *Type II* contact, one  
 202 can similarly define local velocities  $\mathbf{v}_i$  by evaluating  $\mathbf{J}^*$  at point P' which can be very conveniently obtained  
 203 using screw theory (Davidson and Hunt, 2004).

204 Eqs. (7) to (9) can then be rearranged to yield the expression of the actuation torque required to overcome  
 205 the preloading of the triggered motion:

$$206 \quad \tau_{in} = \frac{-\mathbf{v}_1^T (\mathbf{n} + \mu \mathbf{t})}{\mathbf{v}_2^T (\mathbf{n} + \mu \mathbf{t})} f_p. \quad (10)$$

207 One might think that an easy way to decrease the ratio  $\tau_{in}/f_p$  would be to maximize the dot product  
 208  $\mathbf{v}_2^T (\mathbf{n} + \mu \mathbf{t})$ , which is the denominator of Eq. (10), by making  $\mathbf{v}_2$  parallel to  $\mathbf{n}$ . If a vertical obstacle and the  
 209 generation of  $M_2$  by a prismatic joint are assumed, this strategy would be equivalent to making the vector  $\boldsymbol{\delta}$   
 210 horizontal. In turn, this would result in an unacceptable reduction of the available workspace directly above  
 211 the free-space trajectory, which explains the compromise imposed by the pantograph design constraints  $C_1$   
 212 and  $C_2$ .

213 Impact mechanics are neglected for the calculation of the required torque. Indeed, the worst case scenario  
 214 is that, following a collision, the leg is forced to a standstill due to the impact forces and the inertial effects.  
 215 However, the static conditions, for which Eq. (10) is derived, thereby become valid.

216 An example of the predicted evolution of the input torque during the sliding motion of the leg following  
 217 a *Type II* contact, calculated for the initial geometric parameters assuming  $\mu = 0$ , is shown in Fig. 6. The  
 218 maximal value of  $\tau_{in}/f_p$  is in this case 8.21 at the very beginning of the sliding motion when the leg has  
 219 not yet departed from the free-space trajectory. This maximal value quantifies the ease of adaptation to any  
 220 obstacle for which a contact would be established at this particular point.

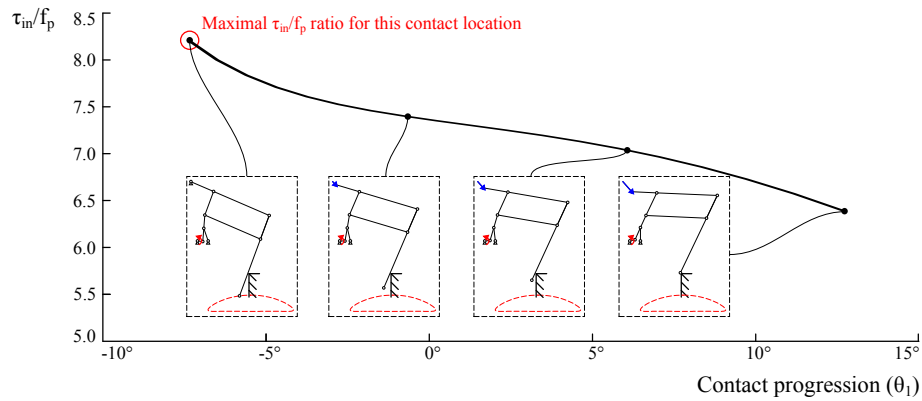


Fig. 6. Required input torque for an adaptation following a *Type II* contact with  $\mu = 0$  and a collision point at  $(9.5, -6.0)$

221 A similar contact simulation can be performed for all possible collision points as illustrated in Fig. 7a.  
 222 These points form the *adaptation workspace*, which is a smaller subset of the total workspace (points reach-  
 223 able by the endpoint following rotations of  $\theta_1$  and  $\theta_2$ ). Indeed, no collision can occur at points located above  
 224 the terminal link, such as point P in Fig. 7a, since they are not swept by the leg during normal motion along  
 225 the free-space trajectory. A variation of the initial geometry is therefore introduced to increase the area of  
 226 this adaptation workspace, referred to as the *workspace-maximizing shape*, in which the terminal link is  
 227 altered so that it is vertical at the beginning of the swing phase. This increases the range of possibly over-  
 228 comable obstacles (c.f. Fig. 7b). It is important to note that altering the shape of this link without changing  
 229 the coordinates of the joints still affects the  $\tau_{in}/f_p$  ratio since the contact location and the orientation of  $\mathbf{n}$  are  
 230 different. Indeed, all possible obstacle contacts occur at the endpoint (*Type I*) for the workspace-maximizing  
 231 shape which has the drawback of increasing the required input torques for adaptation.

232 Since it is numerically faster to evaluate the required torque for *Type I* contacts, the adaptation perfor-  
 233 mance index  $I_{adap}$  proposed here is based on the workspace-maximizing shape of the terminal link. As-  
 234 suming vertical obstacles ( $\mathbf{n} = [1 \ 0]^T$ ) and no friction ( $\mu = 0$ ), the  $\tau_{in}/f_p$  ratio is evaluated for all points  
 235 comprising the swing phase of the free-space trajectory, as plotted in Fig. 8, based on the geometry illus-  
 236 trated in Fig. 7b. The value of  $I_{adap}$  is defined as the root mean square (RMS) of the ratio  $\tau_{in}/f_p$  along this  
 237 curve, which is equal to 6.23 for the initial geometry of this particular example. A maximal ratio of 10.56 is  
 238 reached at point Q, illustrated in Fig. 7b.

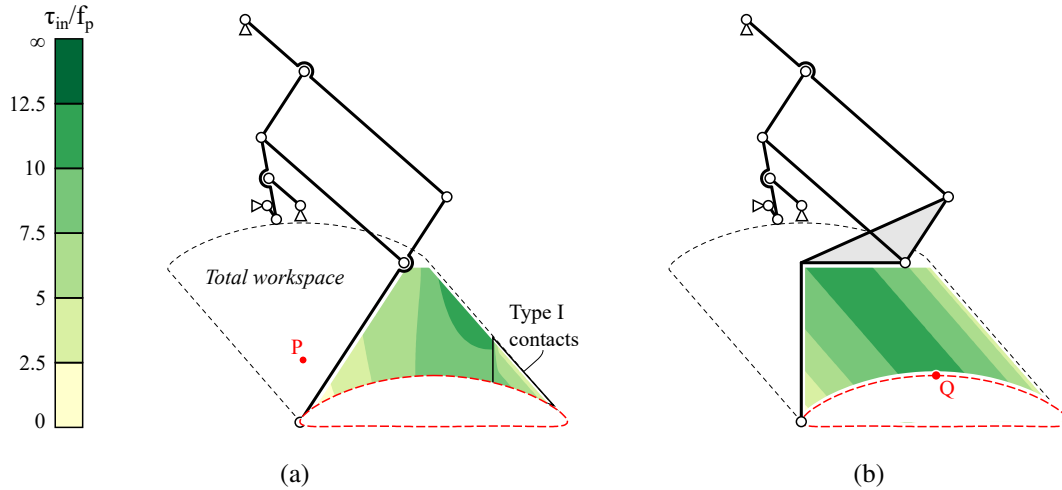


Fig. 7. Required input torque for all possible collision points with  $\mu = 0$ , and different shapes of the terminal link: (a) straight terminal link, (b) workspace-maximizing shape with  $\mathbf{n} = [1 \ 0]^T$

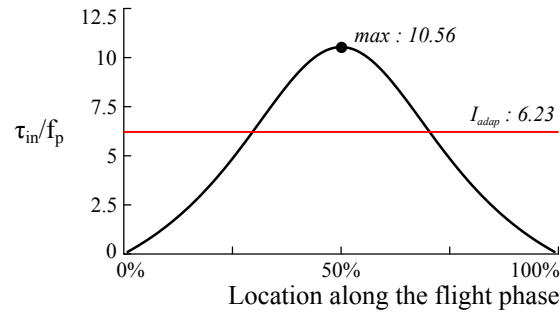


Fig. 8. Evaluation of  $I_{adap}$  for the initial geometry

239 While this index assumes, for comparison purposes, only frictionless endpoint contacts with vertical  
 240 obstacles, which might not be realistic in practical applications, there still exists a direct correlation between  
 241  $I_{adap}$  and the torque required for the terminal link to slide along an obstacle. A discussion of the mechanism's  
 242 adaption to specific obstacles follows in Section 5.

243 Moreover, other than the trade-off between the adaptation workspace area and the required torque for  
 244 adaptation, there exists an important reason to favoring link (*i.e.*, *Type II*) rather than endpoint (*i.e.*, *Type I*)  
 245 contacts with obstacles: potential interference *after* a successful adaptation. An interference is here defined  
 246 as a contact on the back of the leg which prevents it from returning to the free-space trajectory. This effect,  
 247 illustrated in Fig. 9, is even more pronounced when the pantograph is operated in what is referred to, in the  
 248 literature, as the "ostrich mode" (Ottaviano et al., 2005), *i.e.*, the pantograph's links are located aft of the  
 249 guiding points.

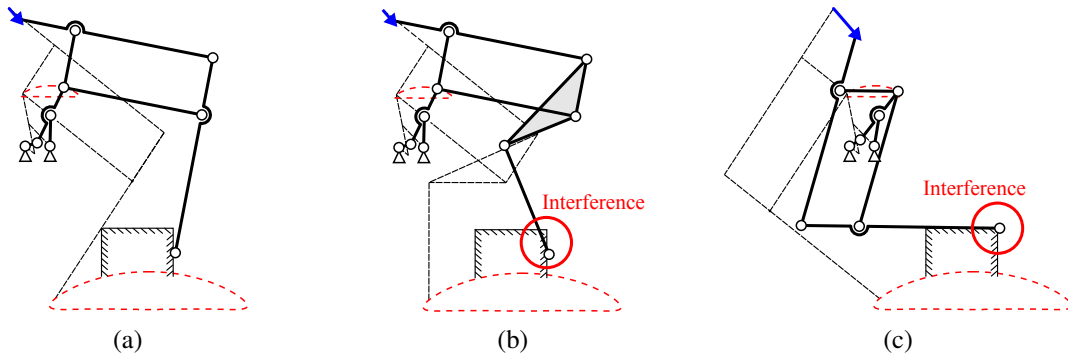


Fig. 9. Possible interference between the pantograph and the obstacle after adaptation: (a) straight terminal link, no interference (b) workspace-maximizing terminal link, slight interference, (c) "ostrich" pantograph configuration, important interference.

### 250 3.2. Free-space trajectory

251 The quality of the free-space trajectory is a another key element to consider during the design. The asso-  
 252 ciated performance index is, in addition to  $I_{adapt}$ , a second basis for comparison between various geometries.  
 253 Since this trajectory is only a scaling-up of  $M_1$ , generated by the four-bar linkage, it is easier to directly  
 254 evaluate the latter. To this aim, three criteria scored on a scale from 0 to 100% are defined:

- **Stance phase linearity:** the vertical difference  $\Delta h$  between the top and bottom points of the stance phase is compared to the total height  $h_2$  of the trajectory (in order to account for the transition between swing and stance phases, their widths are arbitrarily set at 95% of  $w_2$ , the total trajectory width, as shown in Fig. 10):

$$lin\% = \left(1 - \frac{\Delta h}{h_2}\right) \times 100\% \quad (11a)$$

- **Stance phase ratio:** the fraction of the input crank cycle that is spent in the stance phase is compared with the target fraction chosen at 0.6:

$$sta\% = \min\left(\left(\frac{\text{stance duration}}{0.6}\right), 1\right) \times 100\% \quad (11b)$$

• **Height-to-width ratio:** in order to penalize designs generating trajectories flatter than that of the Hoecken's linkage ( $h_1/w_1 = 0.19$ ), the height-to-width ratio is compared to this latter value:

$$hwr\% = \min\left(\left(\frac{h_1/w_1}{0.19}\right), 1\right) \times 100\% \quad (11c)$$

255 A free-space trajectory quality index  $I_{trj}$  combining the stance phase linearity, stance phase ratio, and  
 256 height-to-width ratio criteria into a single performance index is then defined:

$$I_{trj} = 1 - \frac{lin\%}{100\%} \cdot \frac{sta\%}{100\%} \cdot \frac{hwr\%}{100\%} \quad (12)$$

258 Table 3 details the calculation of  $I_{trj}$  for the initial geometric parameters listed in Table 1. The obtained  
 259 value, 0.03, is excellent although the mechanism fared much worse when its passive adaptation was evalu-  
 260 ated, with an  $I_{adap}$  of 6.23. It is of course impossible to minimize simultaneously both indices to satisfy the  
 two very different objectives, and a compromise between them must be investigated.

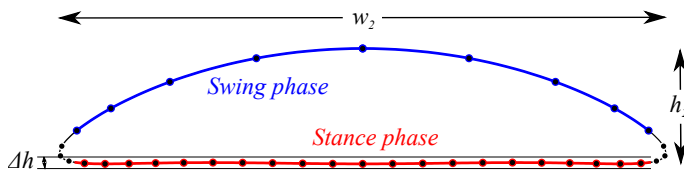


Fig. 10. Hoecken's linkage trajectory, with points drawn each 10° rotation of the input crank.

Linearity score	98.6%
Stance ratio score	98.3%
Height-to-width score	100%
<b><math>I_{trj}</math></b>	<b>0.03</b>

Table 3. Evaluation of  $I_{trj}$  for the initial geometry

261

#### 262 4. OPTIMIZATION OF THE GEOMETRY

263 Having defined two conflicting performance indices, a multi-objective optimization can be performed  
 264 on a search space comprising all possible linkages generating the trajectory  $M_1$ , in order to visualize the  
 265 associated Pareto front. For the candidate geometries comprising the Pareto set, *i.e.*, located on this front, a  
 266 decrease of one objective function can only lead to an increase of the other, so their overall merits depend  
 267 solely on the relative importance given to these two criteria.

268 For the purpose of this optimization, the search space is limited to the parameters of the crank-rocker four-  
 269 bar linkage, the geometry of which is defined by three variables:  $l_{AB}$ ,  $l_{CD}$  and  $l_{BD}$ . Two additional variables,  
 270  $l_{DE}$  and  $\lambda$ , position the coupler point E which is connected to the pantograph. Table 4 describes the allowed  
 271 ranges for these variables, which, along with the inequality constraints defined in Eqs. (13a-c), ensure that



272 the unit-length crank is able to perform a full revolution without encountering a singular configuration.

Variable	Description	Limits	Inequality Constraints
$l_{AB}$	Base link length	1.1 – 10	
$l_{CD}$	Coupler link length	1.1 – 10	$l_{AB} + l_{BD} > l_{CD} + 1$ (13a)
$l_{BD}$	Rocker link length	1.1 – 10	$l_{CD} + l_{BD} > l_{AB} + 1$ (13b)
$l_{DE}$	Additional distance to coupler point	0.1 – 10	$l_{AB} + l_{CD} > l_{BD} + 1$ (13c)
$\lambda$	Coupler link shape	0 – $2\pi$	

Table 4. Conditions for the generation of a suitable candidate geometry

273 As described by the flowchart shown in Fig. 11, both indices  $I_{trj}$  and  $I_{adap}$  are assigned to a candidate  
 274 four-bar linkage by first determining its optimal orientation, minimizing  $I_{trj}$ , and then, generating the unique  
 275 pantograph linkage satisfying constraints  $C_1$  to  $C_6$  (previously described in Section 2.3) for this particular  
 276 four-bar, which allows to compute  $I_{adap}$ .

277 Due to the highly non-linear nature of the problem, a genetic algorithm provided by MATLAB's *Opti-*  
 278 *mization Toolbox* was used with the parameters described in Table 5 to explore the search space and find  
 279 near-optimal solutions. The fitness of the best individuals obtained after 100 generations is plotted in Fig. 12.

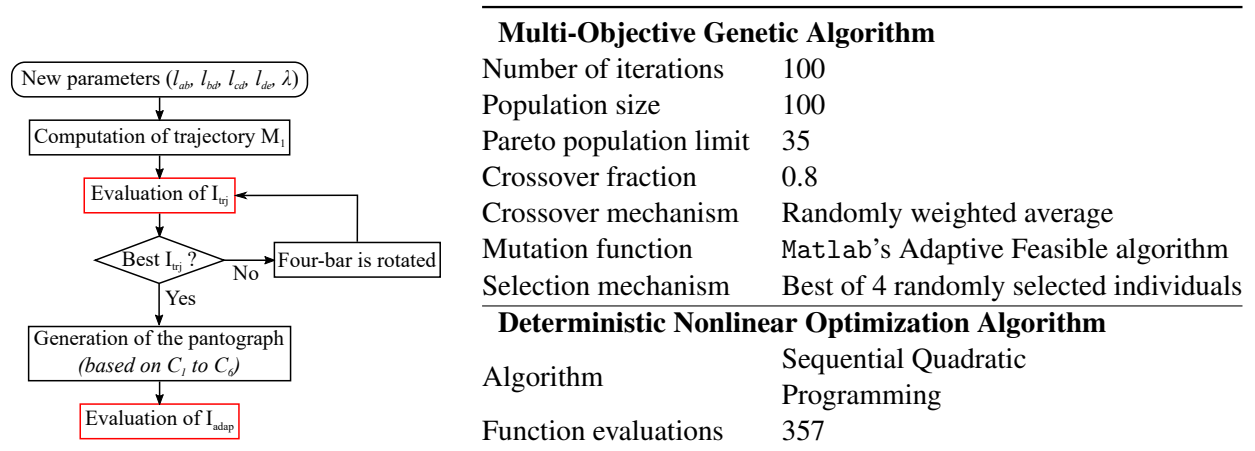


Fig. 11. Evaluation sequence of a candidate geometry

<b>Multi-Objective Genetic Algorithm</b>	
Number of iterations	100
Population size	100
Pareto population limit	35
Crossover fraction	0.8
Crossover mechanism	Randomly weighted average
Mutation function	Matlab's Adaptive Feasible algorithm
Selection mechanism	Best of 4 randomly selected individuals
<b>Deterministic Nonlinear Optimization Algorithm</b>	
Algorithm	Sequential Quadratic Programming
Function evaluations	357

Table 5. Parameters of the optimization algorithms

280 Conversely to usual optimization techniques, a genetic algorithm allows to optimize simultaneously both  
 281 objectives by keeping a large population of candidate geometries instead of a single one. Moreover, this  
 282 approach allows to avoid the pitfalls of local minima, which the function can be shown to possess. Indeed,  
 283 as an example, a specific geometry has been selected from the Pareto set for further analysis. As illustrated in  
 284 Fig. 12, this geometry, indicated by "X" in the figure, is located at (or very close to) the global minimum for

285 the particular optimization function  $I = 3.63I_{trj} + I_{adap}$ . However, a deterministic minimization algorithm  
 286 (described in Table 5) applied to the latter function with the initial geometry as starting point does not  
 287 converge towards this global minimum, but towards a local minimum, identified by "+" in the figure, see  
 288 Table 6.

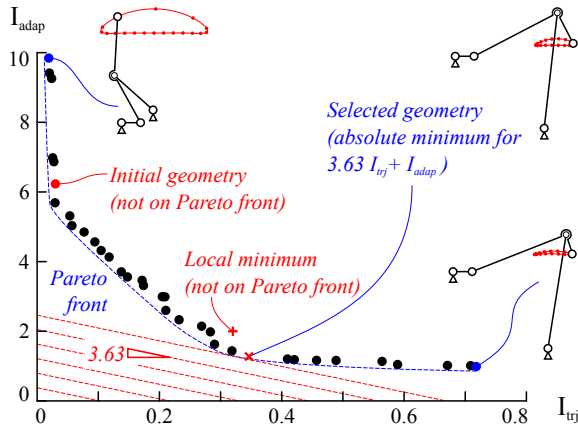


Fig. 12. Pareto set found after 100 generations

Parameter	Initial Geo.	Selected Geo.	Local Min.
$l_{AB}$	2	6.08	5.23
$l_{CD}$	2.5	4.99	8.90
$l_{BD}$	2.5 0	6.17	10.00
$l_{DE}$	2.5	1.82	3.73
$\lambda$	180°	270°	302°
$I_{trj}$	0.03	0.35	0.32
$I_{adap}$	6.23	1.27	2.01
$3.63I_{trj} + I_{adap}$	6.34	2.53	3.17

Table 6. Optimization results

## 289 5. ANALYSIS OF THE SELECTED GEOMETRY

290 The arbitrarily selected geometry (described in Fig. 13 and Table 7) features a remarkable improvement  
 291 of  $I_{adap}$  from 6.23 to 1.27 at the cost of a deterioration of  $I_{trj}$  from 0.03 to 0.35 compared to the initial  
 292 parameters. The ease of adaptation can be further improved by using a straight terminal link, which would  
 293 ensure *Type II* contacts for a large portion of the swing phase.

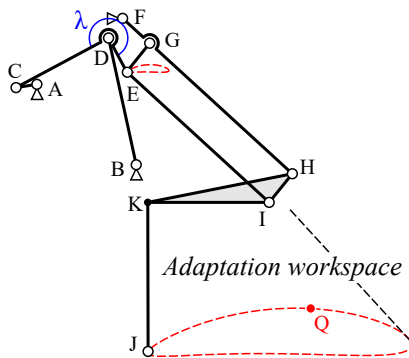


Fig. 13. Illustration of the selected geometry

Geometric parameter	Value	Geometric parameter	Value
Coord. of A	(0 0)	$l_{AC}$	1.00
Coord. of B	(4.72 -3.83)	$l_{BD}$	6.17
Coordinates of $F_{free-space}$	(4.08 3.14)	$l_{CD}$	4.99
Coordinates of $F_{max.adaptation}$	(5.27 1.84)	$l_{DE}$	1.82
		$l_{EG} = l_{FG} = l_{HI}$	1.75
		$l_{EI} = l_{GH} = l_{IJ}$	9.19
		$\lambda$	270°

Table 7. Geometric parameters of the selected design

294 As is clear from Fig. 14 and Table 8, the main drawback of this design is the reduction of the stance phase  
 295 duration from 59% to 44% of the leg cycle. A possible solution could be to increase the number of legs, or

296 to alter the angular velocity of the input crank using, for instance, non-circular gears or cams.

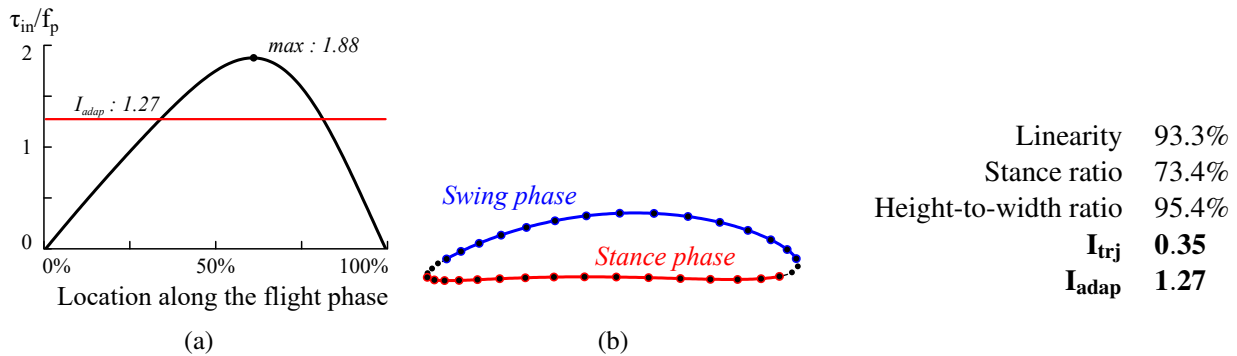


Fig. 14. Evaluation of (a)  $I_{adap}$  and (b)  $I_{trj}$  for the selected design. Points are drawn each  $10^\circ$  rotation of the input crank.

Table 8. Summary of the performance indices

297 The improved adaptation capabilities can be further demonstrated by comparing the required torque for  
 298 adaptation at the most critical point of the swing phase for the initial and selected geometries, *i.e.*, point Q  
 299 in Figs. 7b and 13. The effect of the obstacle angle and friction coefficient on *Type I* contacts, described  
 300 by Eq. (10), is plotted for both geometries in Fig. 7. Depending on the friction coefficient, adaptation  
 301 in the desired direction is shown to be possible even for overhanging obstacles, *i.e.*, whose normal has a  
 302 downwards component.

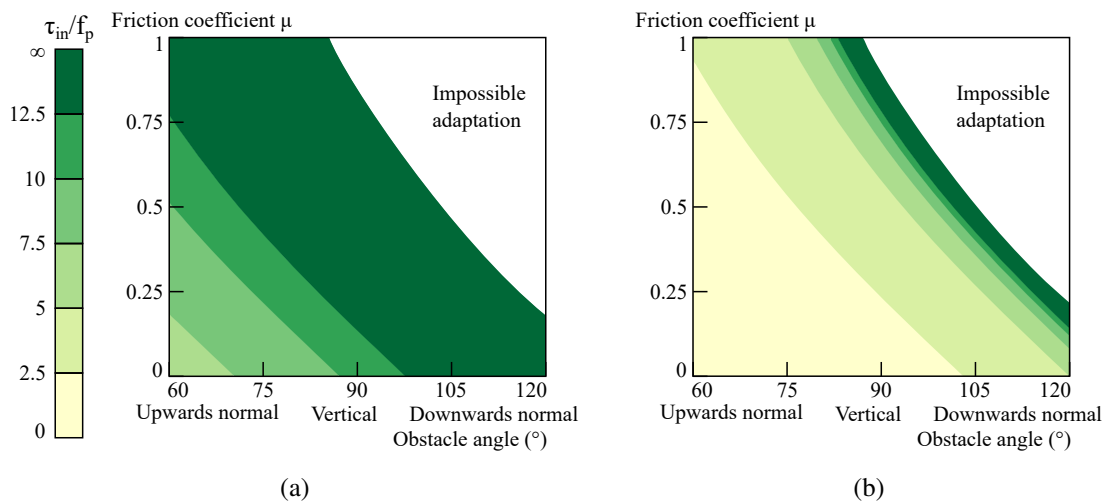


Fig. 15. Influence of obstacle angle and friction coefficient on the adaptation torque ratio at point Q: (a) initial geometry, (b) selected geometry

303 Adaptation to this latter type of obstacles has been investigated with the help of the MSC *Adams* dynamic  
 304 simulation package. Since only a single leg was simulated, the influence of the rest of mechanism was  
 305 represented by constant vertical position and only positive horizontal motion constraints for the mechanism

306 body. The latter is a conservative hypothesis, since during the swing phase of a given leg, the other legs  
307 usually actively push the body of a multi-legged robot forwards.

308 Fig. 16a shows obstacle adaptation for a straight terminal link, resulting in a *Type II* contact. After the  
309 adaptation, the leg endpoint returns to the free-space trajectory by following a direction parallel to  $\delta$ , and  
310 the next step is initiated. In Fig. 16b, the leg slides upwards along the obstacle, as predicted, even if the first  
311 contact point has a downwards normal. There is however a slight interference after the adaptation, to which  
312 the mechanism reacts by moving forwards, resulting in a sliding of point K along the obstacle. This does  
313 not cause the mechanism to get stuck during the next step, since the terminal link then undertakes a rotation  
314 around its extremity, *i.e.*, the contact point with the ground. A video of the simulations is available online at  
315 [www.youtube.com/watch?v=-ArcRu-ErvA](http://www.youtube.com/watch?v=-ArcRu-ErvA).

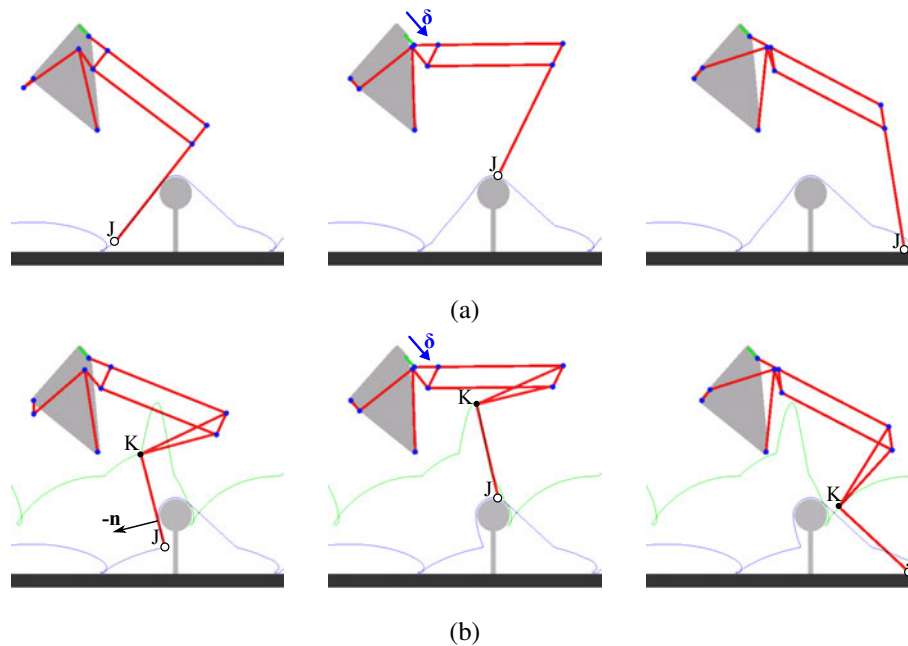


Fig. 16. Mechanism simulation using MSC Adams : Obstacle adaptation with a) a straight terminal link b) the workspace-maximizing terminal link

## 316 6. CONCLUSIONS

317 In this paper, the passive reconfiguration of a Hoecken's-Pantograph robotic leg mechanism due to the use  
318 of a second triggered DOF, in a similar manner as underactuated mechanical fingers, is investigated. The geo-  
319 metric parameters of the mechanism were optimized in order to allow it to efficiently slide along obstacles  
320 following contacts occurring during its swing phase while retaining an efficient free-space trajectory for the

321 leg endpoint, and the Pareto front representing the trade-off between these objectives was highlighted.

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