Titre: Title:	Game-theoretic decentralized model predictive control of thermal appliances in discrete-event systems framework
Auteurs: Authors:	Saad A. Abobakr, Waselul H. Sadid, & Guchuan Zhu
Date:	2018
Type:	Article de revue / Article
Référence: Citation:	Abobakr, S. A., Sadid, W. H., & Zhu, G. (2018). Game-theoretic decentralized model predictive control of thermal appliances in discrete-event systems framework. IEEE Transactions on Industrial Electronics, 65(8), 6446-6456. https://doi.org/10.1109/tie.2018.2793204

URL de PolyPublie: PolyPublie URL:	https://publications.polymtl.ca/2993/
Version:	Version finale avant publication / Accepted version Révisé par les pairs / Refereed
Conditions d'utilisation: Terms of Use:	Tous droits réservés / All rights reserved

Document publié chez l'éditeur officiel
Document issued by the official publisher

Titre de la revue: Journal Title:	IEEE Transactions on Industrial Electronics (vol. 65, no. 8)
Maison d'édition: Publisher:	IEEE
URL officiel: Official URL:	https://doi.org/10.1109/tie.2018.2793204
Mention légale: Legal notice:	©2018 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

A Game-Theoretic Decentralized Model Predictive Control of Thermal Appliances in Discrete-Event Systems Framework

Saad A. Abobakr, *Student Member, IEEE*, Waselul H. Sadid, and Guchuan Zhu, *Senior Member, IEEE*

Abstract—This paper presents a decentralized model predictive control (MPC) scheme for thermal appliances coordination control in smart buildings. The general system structure consists of a set of local MPC controllers and a game-theoretic supervisory control constructed in the framework of discrete-event systems (DES). In this hierarchical control scheme, a set of local controllers work independently to maintain the thermal comfort level in different zones, and a centralized supervisory control is used to coordinate the local controllers according to the power capacity and the current performance. Global optimality is ensured by satisfying the Nash equilibrium at the coordination layer. The validity of the proposed method is assessed by a simulation experiment including two case studies. The results show that the developed control scheme can achieve a significant reduction of the peak power consumption while providing an adequate temperature regulation performance if the system is $\mathcal{P}$-observable.

Index Terms—Smart buildings; Thermal appliance control; Model predictive control; Discrete-event systems; Game theory.

Nomenclature

i, j	Indices of zones and controllers.
T_i	Interior temperature of Zone i.
$R_{i,ij}^{a,d}$	Thermal resistances of Zone i or cross Zone i, j.
C_i, Φ_i	Heat capacity of and power input to Zone i.
$\mathcal{L}, \mathcal{K}$	Regular languages.
$\mathcal{M}_\mathcal{L}$	Finite-state machine representing $\mathcal{L}$.
Σ, s	Finite set of events and sequence of events.
$Pow(\Sigma)$	Power set of Σ.
$\pi_j(s)$	Operator of partial observation indexed by j.
f^j, η^j	Cost function and utility function indexed by j.
u^j	Control action indexed by j.

I. Introduction

THE peak power load in buildings can cost as much as 200 to 400 times the regular rate [1]. Peak power

Saad A. Abobakr and Guchuan Zhu are with the Department of Electrical Engineering, Ecole Polytechnique de Montreal, P.O. Box 6079, Station Centre-Ville, Montreal, QC H3T 1J4, Canada. Waselul H. Sadid is with Computer Science and Engineering Department, East West University, Dhaka, Bangladesh, `saad.abobakr, guchuan.zhu@polymtl.ca, sadid.wh@ewubd.edu`

reduction has therefore a crucial importance for achieving the objectives of improving cost-effectiveness in building operations. Controlling thermal appliances in heating, ventilation, and air conditioning (HVAC) systems is considered to be one of the most promising and effective ways to achieve this objective. As the highest consumers of electricity with more than one-third of the energy usage in a building [2], and due to their slow dynamic property, thermal appliances have been prioritized as the equipment to be regulated for peak power load reduction [3].

There exists a rich set of conventional and modern control schemes that have been developed and implemented for the control of building systems in the context of the Smart Grid, among which Model Predictive Control (MPC) is one of the most frequently adopted techniques. This is mainly due to its ability to handle constraints, time varying processes, delays, and uncertainties, as well as disturbances. In addition, it is also easy to incorporate multiple-objective functions in MPC [4], [5]. There has been a considerable amount of research aimed at minimizing energy consumption in smart buildings, among which the technique of MPC plays an important role [4]–[12].

MPC can be formulated into centralized, decentralized, distributed, cascade, or hierarchical structures [4], [6], [7]. In a centralized MPC, the entire states and constraints have to be considered to find a global solution of the problem. While in the decentralized model predictive control (DMPC), the whole system is partitioned into a set of subsystems, each with its own local controller. As all the controllers are engaged in regulating the entire system [6], a coordination control is required for DMPC to ensure the overall optimality.

Some centralized MPC-based thermal appliance control schemes for temperature regulation and power consumption reduction were implemented in [2], [8]–[10]. In [11], a robust DMPC based on H_∞-performance measurement was proposed for HVAC control in a multi-zone building in the presence of disturbance and restrictions. In [13], centralized, decentralized, and distributed controllers based on MPC structure, as well as proportional-integral-derivative (PID) control, were applied to a three-zone building to track the temperature and to reduce the power consumption. A hierarchical MPC was used for power management of an intelligent grid in [14]. Charging electrical vehicles was integrated in the design to balance the load and production. An application of DMPC to minimize the computational load is reported in [12]. A term enabling the regulation flexibility was integrated into the cost function to tune the level of guaranteed quality of service.

Game theory is another notable tool, which has been extensively used in the context of smart buildings to assist the decision making process and to handle the interaction between energy supply and request in energy demand management. Game theory provides a powerful means for modeling the cooperation and interaction of different decision makers (players) [15]. In [16], a game-theoretic scheme based on Nash equilibrium (NE) is used to coordinate appliance operations in a residential building. A game-theoretic MPC was established in [17] for demand side energy management. The proposed approach in [18] was based on cooperative gaming to control two different linear coupled systems. A game interaction for energy consumption scheduling is proposed in [19] by taking into consideration the coupled constraints. This approach can shift the peak demand and reduce the peak to average ratio.

Inspired by the existing literature and in view of the advantages of using discrete-event systems (DES) to schedule the operation of thermal appliances in smart buildings as reported in [3], our goal is to develop a DMPC-based scheme for thermal appliance control in the framework of DES. Initially, the operation of each appliance is expressed by a set of *states* and *events*, which represent the status and the actions of the corresponding appliance. A system can then be represented by a finite-state machine (FSM) as a regular language over a finite set of events in DES [20]. Indeed, appliance control can be constructed using the MPC method if the operation of a set of appliances is schedulable. Compared to *trial and error* strategies, the application of the theory and tools of DES allow for the design of complex control systems arising in the field of the Smart Grid to be carried out in a systematic manner.

Based on the architecture developed in [21], we propose a two-layer structure for decentralized control as shown in Fig. 1. A supervisory controller at the upper layer is used to coordinate a set of MPC controllers at the lower layer. The local control actions are taken independently relying only on the local performance. The control decision at each zone will be sent to the upper layer and a game theoretic scheme will take place to distribute the power over all the appliances while considering power capacity constraints. Note that HVAC is a heterogenous system consisting of a group of subsystems that have different dynamics and natures [4]. Therefore, it might not be easy to find a single dynamic model for control design and power consumption management of the entire system. Indeed, with a layered structure, an HVAC system can be split into a set of subsystems to be controlled separately. A coordination control, as proposed in the present work, can be added to manage the operation of the whole system. The main contributions of this work are twofold:

1) We propose a new scheme for DMPC-based game-theoretic power distribution in the framework of DES for reducing the peak power consumption of a set of thermal appliances while meeting the prescribed temperature in a building. The developed method is capable of verifying *a priori* the feasibility of a schedule and allows for the design of complex control schemes to be carried out in a systematic manner.

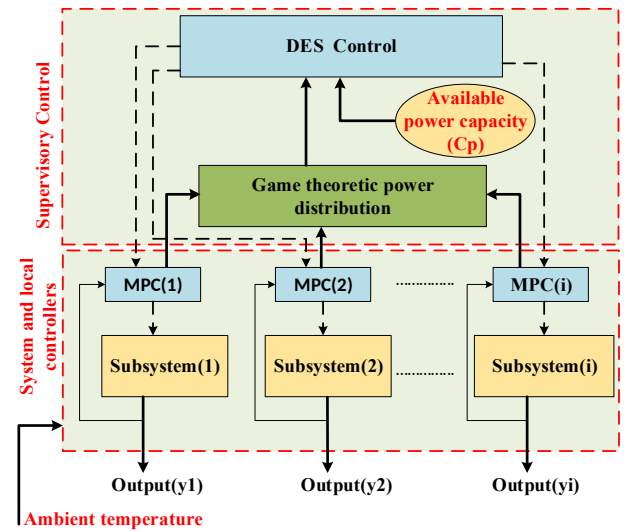2) We establish an approach to ensure the system performance by considering some observability properties of

Fig. 1. Architecture of a decentralized MPC-based thermal appliance control system.

DES, namely co-observability and $\mathcal{P}$-observability. This approach provides a means for deciding whether a local controller requires more power to satisfy the desired specifications by enabling events through a sequence based on the observation.

In the remainder of the paper, Section II presents the model of building thermal dynamics. The settings of centralized and decentralized MPC are addressed in Section III. Section IV introduces the basic notions of DES and presents a heuristic algorithm for searching the NE employed in this work. The concept of $\mathcal{P}$-Observability and the design of the supervisory control based on decentralized DES are presented in Section V. Simulation studies are carried out in Section VI, followed by some concluding remarks provided in Section VII.

II. MODELING OF BUILDING THERMAL DYNAMICS

In this section, the continuous time differential equation is used to present the thermal system model as proposed in [10]. The model will be discretized later for the predictive control design. The dynamic model of the thermal system is given by:

$$\frac{\mathrm{d}T_i}{\mathrm{d}t} = \frac{1}{C_i R_i^a}(T_a - T_i) + \frac{1}{C_i}\sum_{j=1,j\neq i}^{M}\frac{1}{R_{ji}^d}(T_j - T_i) + \frac{1}{C_i}\Phi_i,$$

(1)

where M is the number of zones, T_i, $i \in \{1,\ldots,M\}$, is the interior temperature of Zone i (the indoor temperature), T_j is the interior temperature of a neighboring Zone j, $j \in \{1,\ldots,M\}\backslash i$, T_a is the ambient temperature (the outdoor temperature), R_i^a is the thermal resistance between Zone i and the ambient temperature, R_{ji}^d is the thermal resistance between Zone i and Zone j, C_i is the heat capacity of Zone i, and Φ_i is the power input to the thermal appliance located in Zone i. Note that the first term on the right hand side of (1) represents the indoor temperature variation rate of a zone due to the impact of the outdoor temperature, and the second term captures the interior temperature of a zone due to the effect of

thermal coupling of all of the neighboring zones. Therefore, it is a generic model, widely used in the literature.

The system (1) can be expressed by a continuous time state-space model as:

$$\dot{x} = Ax + Bu + Ed$$
$$y = Cx, \tag{2}$$

where $x = [T_1, T_2, \ldots, T_M]^T$ is the state vector and $u = [u_1, u_2, \ldots, u_M]^T$ is the control input vector. The output vector is $y = [y_1, y_2, \ldots, y_M]^T$ where the controlled variable in each zone is the indoor temperature, and $d = [T_a^1, T_a^2, \ldots, T_a^M]^T$ represents the disturbance in Zone i. The system matrices $A \in \mathbb{R}^{M \times M}$, $B \in \mathbb{R}^M$, and $E \in \mathbb{R}^M$ in (2) are given by:

$$A = \begin{bmatrix} A_1 & \frac{1}{R_{21}^d C_1} & \cdots & \frac{1}{R_{M1}^d C_1} \\ \frac{1}{R_{12}^d C_2} & A_2 & \cdots & \frac{1}{R_{M2}^d C_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{R_{1M}^d C_N} & \frac{1}{R_{2M}^d C_N} & \cdots & A_M \end{bmatrix},$$

$$B = \mathrm{diag}\begin{bmatrix} B_1 & \cdots & B_M \end{bmatrix}, E = \mathrm{diag}\begin{bmatrix} E_1 & \cdots & E_M \end{bmatrix},$$

with

$$A_i = -\frac{1}{R_i^a C_i} - \frac{1}{C_i} \sum_{j=1, j \neq i}^{M} \frac{1}{R_{ji}^d}, B_i = \frac{1}{C_i}, E_i = \frac{1}{R_i^a C_i}.$$

In (2), C is an identity matrix of dimension M. Again, the system matrix A captures the dynamics of the indoor temperature and the effect of thermal coupling between the neighboring zones.

III. PROBLEM FORMULATION

The control objective is to reduce the peak power while respecting comfort level constraints, which can be formalized as an MPC problem with a quadratic cost function which will penalize the tracking error and the control effort. We begin with the centralized formulation and then we find the decentralized setting by using the technique developed in [22].

A. Centralized MPC setup

For the centralized setting, a linear discrete time model of the thermal system can be derived from discretizing the continuous time model (2) by using the standard zero-order hold with a sampling period T_s, which can be expressed as:

$$x(k+1) = A_d x(k) + B_d u(k) + E_d d(k),$$
$$y(k) = C_d x(k), \tag{3}$$

where $x(k) \in \mathbb{R}^M$ is the state vector, $u(k) \in \mathbb{R}^M$ is the control vector, and $y(k) \in \mathbb{R}^M$ is the output vector. The materices in the discrete-time model can be computed from the continous-time model in (2) and are given by $A_d = e^{AT_s}$, $B_d = \int_0^{T_s} e^{As} B \mathrm{d}s$, and $E_d = \int_0^{T_s} e^{As} D \mathrm{d}s$; $C_d = C$ is an identity matrix of dimension M.

Let $x_d(k) \in \mathbb{R}^M$ be the desired state and $e(k) = x(k) - x_d(k)$ be the vector of regulation error. The control to be fed

into the plant is resulted by solving the following optimization problem at each time instance t:

$$f = \min_{u_i(0)} e(N)^T P e(N) + \sum_{k=0}^{N-1} e(k)^T Q e(k) + u^T(k) R u(k) \tag{4a}$$

s.t $x(k+1) = A_d x(k) + B_d u(k) + E_d d(k),$
$$x_0 = x(t), \tag{4b}$$
$$x_{\min} \leq x(k) \leq x_{\max}, \tag{4c}$$
$$0 \leq u(k) \leq u_{\max}, \tag{4d}$$

for $k = 0, \ldots, N$, where N is the prediction horizon. In the cost function in (4), $Q = Q^T \geq 0$ is a square weighting matrix to penalize the tracking error, $R = R^T > 0$ is square weighting matrix to penalize the control input, and $P = P^T \geq 0$ is a square matrix that satisfies the Lyapunov equation

$$A_d^T P A_d - P = -Q \tag{5}$$

for which the existence of matrix P is ensured if A in (2) is a strictly Hurwitz matrix. Note that in the specification of state and control constraints, the symbol "$\leq$" denotes componentwise inequalities, i.e., $x_{i\min} \leq x_i(k) \leq x_{i\max}$, $0 \leq u_i(k) \leq u_{i\max}$, for $i = 1, \ldots, M$.

The solution of the problem (4) provides a sequence of controls $U^*(x(t)) = \{u_0^*, \ldots, u_N^*\}$, among which only the first element $u(t) = u_0^*$ will be applied to the plant.

B. Decentralized MPC Setup

For the DMPC setting, the thermal model of the building will be divided into a set of subsystems. In the case where the thermal system is stable in open loop, i.e., the matrix A in (2) is strictly Hurwitz, we can use the approach developed in [22] for decentralized MPC design. Specifically, for the considered problem, let $x^j \in \mathbb{R}^{n_j}$, $u^j \in \mathbb{R}^{n_j}$, and $d^j \in \mathbb{R}^{n_j}$ be the state, control, and disturbance vectors of the jth subsystem with $n_1 + n_2 + \cdots + n_m = M$. Then for $j = 1, \cdots, m$, x^j, u^j, and d^j of the subsystem can be represented as:

$$x^j = W_j^T x = \begin{bmatrix} x_1^j & \cdots & x_{n_j}^j \end{bmatrix}^T \in \mathbb{R}^{n_j}, \tag{6a}$$

$$u^j = Z_j^T u = \begin{bmatrix} u_1^j & \cdots & u_{m_j}^j \end{bmatrix}^T \in \mathbb{R}^{n_j}, \tag{6b}$$

$$d^j = H_j^T d = \begin{bmatrix} d_1^j & \cdots & d_{l_j}^j \end{bmatrix}^T \in \mathbb{R}^{n_j}. \tag{6c}$$

where $W_j \in \mathbb{R}^{M \times n_j}$ collects the n_j columns of identity matrix of order n, $Z_j \in \mathbb{R}^{M \times n_j}$ collects the m_j columns of identity matrix of order m, and $H_j \in \mathbb{R}^{M \times n_j}$ collects the l_j columns of identity matrix of order l. Note that the generic setting of the decomposition can be found in [22]. The dynamic model of the jth subsystems is given by:

$$x^j(k+1) = A_d^j x^j(k) + B_d^j u^j(k) + E_d^j d^j(k),$$
$$y^j(k) = x^j(k), \tag{7}$$

where $A_d^j = W_j^T A_d W_j$, $B_d^j = W_j^T B_d Z_j$, and $E_d^j = W_j^T E_d H_j$ are sub-matrices of A_d, B_d and E_d, respectively, which are in general dependent on the chosen decoupling

matrices W_j, Z_j and H_j. As in the centralized setting, the open-loop stability of the DMPC are guaranteed if A_d^j in (7) is strictly Hurwitz for all $j = 1, \ldots, m$.

Let $e^j = W_j^T e$. The jth sub-problem of the DMPC is then given by:

$$f^j = \min_{u^j(0)} \sum_{k=0}^{\infty} e^{jT}(k) Q_j e^j(k) + u^{jT}(k) R_j u^j(k)$$
$$= \min_{u^j(0)} e^{jT}(k) P_j e^j(k) + e^{jT}(k) Q_j + u^{jT}(k) R_j u^j(k)$$
(8a)

s.t. $\quad x^j(t+1) = A_d^j x^j(t) + B_d^j u^j(0) + E_d^j d^j,$

$$x^j(0) = W_j^T x(t) = x^j(t), \tag{8b}$$
$$x_{\min}^j \le x^j(k) \le x_{\max}^j, \tag{8c}$$
$$0 \le u^j(0) \le u_{\max}^j, \tag{8d}$$

where the weighting matrices are $Q_j = W_j^T Q W_j$, $R_j = Z_j^T R Z_j$, and the square matrix P_j is the solution of the following Lyapunov equation

$$A_d^{jT} P_j A_d^j - P_j = -Q_j. \tag{9}$$

At each sampling time, every local MPC provides a local control sequence by solving the problem (8). Finally, the closed-loop stability of the system with this DMPC scheme can be assessed by using the procedure proposed in [22].

IV. Game Theoretical Power Distribution

A *normal-form* game is developed as a part of the supervisory control to distribute the available power based on the total capacity and the current temperature of the zones. The supervisory control is developed in the framework of DES [20], [23] to coordinate the operation of the local MPCs.

A. Fundamentals of DES

DES is a dynamic system which can be represented by transitions among a set of finite states. The behavior of a DES requiring control and the specifications are usually characterized by regular languages. These can be denoted by $\mathcal{L}$ and $\mathcal{K}$ respectively. A language $\mathcal{L}$ can be recognized by a finite-state machine (FSM), which is a 5-tuple:

$$\mathcal{M}_{\mathcal{L}} = (Q, \Sigma, \delta, q_0, Q_{\mathsf{m}}),$$

where Q is a finite set of states, Σ is a finite set of events, $\delta : Q \times \Sigma \to Q$ is the transition relation, $q_0 \in Q$ is the initial state, and Q_{m} is the set of marked states. Note that the event set Σ includes the control components u^j of the MPC setup. The specification is a subset of the system behavior to be controlled, i.e., $\mathcal{K} \subseteq \mathcal{L}$. The controllers issue control decisions to prevent the system from performing behavior in $\mathcal{L} \setminus \mathcal{K}$, where $\mathcal{L} \setminus \mathcal{K}$ stands for the set of behaviors of $\mathcal{L}$ that are not in $\mathcal{K}$. Let s be a sequence of events and denote by $\overline{\mathcal{L}} := \{s \in \Sigma^* \mid (\exists s' \in \Sigma^*) \text{ such that } ss' \in \mathcal{L}\}$ the *prefix closure* of a language $\mathcal{L}$.

The closed behavior of a system, denoted by $\mathcal{L}$, contains all the possible event sequences the system may generate. The marked behavior of the system is $\mathcal{L}_{\mathsf{m}}$, which is a subset of the closed behavior, representing completed tasks (behaviors), and is defined as $\mathcal{L}_{\mathsf{m}} := \{s \in \mathcal{L} \mid \delta(q_0, s) = q' \wedge q' \in Q_{\mathsf{m}}\}$. A language $\mathcal{K}$ is said to be $\mathcal{L}_{\mathsf{m}}$-closed if $\mathcal{K} = \overline{\mathcal{K}} \cap \mathcal{L}_{\mathsf{m}}$.

B. Decentralized DES

The decentralized supervisory control problem considers the synthesis of $m \ge 2$ controllers that cooperatively intend to keep the system in $\overline{\mathcal{K}}$ by issuing control decisions to prevent the system from performing behavior in $\mathcal{L} \setminus \overline{\mathcal{K}}$ [23], [24]. Here we use $\mathcal{I} = \{1, \ldots, m\}$ as an index set for the decentralized controllers. The ability to achieve a correct control policy relies on the existence of at least one controller that can make the correct control decision to keep the system within $\overline{\mathcal{K}}$.

In the context of the decentralized supervisory control problem, Σ is partitioned into two sets for each controller $j \in \mathcal{I}$: controllable events $\Sigma_{c,j}$ and uncontrollable events $\Sigma_{uc,j} := \Sigma \setminus \Sigma_{c,j}$. The overall set of controllable events is $\Sigma_c := \bigcup_{j=\mathcal{I}} \Sigma_{c,j}$. Let $\mathcal{I}_c(\sigma) = \{j \in \mathcal{I} | \sigma \in \Sigma_{c,j}\}$ be the set of controllers that control event σ.

Each controller $j \in \mathcal{I}$ also has a set of observable events, denoted by $\Sigma_{o,j}$, and unobservable events $\Sigma_{uo,j} = \Sigma \setminus \Sigma_{o,j}$. To formally capture the notion of partial observation in decentralized supervisory control problems, the natural projection is defined for each controller $j \in \mathcal{I}$ as $\pi_j : \Sigma^* \to \Sigma_{o,j}^*$. Thus for $s = \sigma_1 \sigma_2 \ldots \sigma_m \in \Sigma^*$, the partial observation $\pi_j(s)$ will contain only those events $\sigma \in \Sigma_{o,j}$:

$$\pi_j(\sigma) = \begin{cases} \sigma, & \text{if } \sigma \in \Sigma_{o,j}; \\ \varepsilon, & \text{otherwise}, \end{cases}$$

which is extended to sequences as follows: $\pi_j(\varepsilon) = \varepsilon$, and $\forall s \in \Sigma^*$, $\forall \sigma \in \Sigma$, $\pi_j(s\sigma) = \pi_j(s)\pi_j(\sigma)$. The operator π_j eliminates those events from a sequence that are not observable to controller j. The inverse projection of π_j is a mapping $\pi_j^{-1} : \Sigma_{o,j}^* \to Pow(\Sigma^*)$ such that for $s' \in \Sigma_{o,j}^*$, $\pi_j^{-1}(s') = \{u \in \Sigma^* \mid \pi_j(u) = s'\}$, where $Pow(\Sigma)$ represents the power set of Σ.

C. Co-Observability and Control Law

When a global control decision is made, at least one controller can make a correct decision by disabling a controllable event through which the sequence leaves the specification $\mathcal{K}$. In that case, $\mathcal{K}$ is called *co-observable*. Specifically, a language $\mathcal{K}$ is *co-observable* w.r.t. $\mathcal{L}$, $\Sigma_{o,j}$, and $\Sigma_{c,j}$ $(j \in \mathcal{I})$ if [23]

$$(\forall s \in \overline{\mathcal{K}})(\forall \sigma \in \Sigma_c) \; s\sigma \in \mathcal{L} \setminus \overline{\mathcal{K}} \Rightarrow$$
$$(\exists j \in \mathcal{I}) \; \pi_j^{-1}[\pi_j(s)]\sigma \cap \overline{\mathcal{K}} = \emptyset.$$

In other words, there exists at least one controller $j \in \mathcal{I}$ that can make the correct control decision (i.e., determine that $s\sigma \in \mathcal{L} \setminus \overline{\mathcal{K}}$) based only on its partial observation of a sequence. Note that an MPC has no feasible solution when the system is not co-observable. However, the system can still work without assuring the performance.

A decentralized control law for Controller j, $j \in \mathcal{I}$, is a mapping $U^j : \pi_j(\mathcal{L}) \to Pow(\Sigma)$ that defines the set of events that Controller j should *enable* based on its partial observation of the system behavior. While Controller j can

choose to enable or disable events in $\Sigma_{c,j}$, all events in $\Sigma_{uc,j}$ must be enabled, i.e.,

$$(\forall j \in \mathcal{I})(\forall s \in \mathcal{L})\; U^j(\pi_j(s)) = \{u \in Pow(\Sigma) \mid u \supseteq \Sigma_{uc,j}\}.$$

Such a controller exists if the specification $\mathcal{K}$ is co-observable, controllable, and $\mathcal{L}_\mathsf{m}$-closed [23].

D. Normal-Form Game and Nash Equilibrium

At the lower layer each subsystem requires a certain amount of power to run the appliances according to the desired performance. Hence, there is a competition among the controllers if there is any shortage of power when the system is not co-observable. Consequently, a normal-form game is implemented in this work to distribute the power among the MPC controllers. The decentralized power distribution problem can be formulated as a normal form game as below:

A (finite, n-player) *normal-form* game is a tuple $(\mathcal{N}, \mathcal{A}, \eta)$ [25], where:

- $\mathcal{N}$ is a finite set of n players, indexed by j;
- $\mathcal{A} = \mathcal{A}^1 \times \ldots \times \mathcal{A}^n$, where $\mathcal{A}^j$ is a finite set of actions available to Player j. Each vector $a = \langle a^1, ..., a^n \rangle \in \mathcal{A}$ is called an action profile;
- $\eta = (\eta^1, ..., \eta^n)$ where $\eta^j : \mathcal{A} \to \mathbb{R}$ is a real-valued utility function for Player j.

At this point, we consider a decentralized power distribution problem with

- a finite index set $\mathcal{M}$ representing m subsystems;
- a set of cost functions F^j for subsystem $j \in \mathcal{M}$ for corresponding control action $U^j = \{u^j | u^j = \langle u_1^j, ..., u_{m_j}^j \rangle\}$, where $F = F^1 \times \ldots \times F^m$ is a finite set of cost functions for m subsystems and each subsystem $j \in \mathcal{M}$ consists of m_j appliances;
- and a utility function $\eta_k^j : F \to x_k^j$ for Appliance k of each subsystem $j \in \mathcal{M}$ consisting m_j appliances. Hence, $\eta^j = \langle \eta_1^j, ..., \eta_{m_j}^j \rangle$, with $\eta = (\eta^1, ..., \eta^m)$.

The utility function defines the comfort level of the kth appliance of the subsystem corresponding to Controller j, which is a real value $\eta_k^{j,\mathsf{min}} \leq \eta_k^j \leq \eta_k^{j,\mathsf{max}}, \forall j \in \mathcal{I}$, where $\eta_k^{j,\mathsf{min}}$ and $\eta_k^{j,\mathsf{max}}$ are the corresponding lower and upper bounds of the comfort level.

There are two ways in which a controller can choose its action: (i) select a single action and execute it; (ii) randomize over a set of available actions based on some probability distribution. The former case is called a *pure* strategy, and the latter is called a *mixed* strategy. A mixed strategy for a controller specifies the probability distribution used to select a particular control action $u^j \in U^j$. The probability distribution for Controller j is denoted by $p^j : u^j \to [0,1]$, such that $\sum_{u^j \in U^j} p^j(u^j) = 1$. The subset of control actions corresponding to the mixed strategy u^j is called the *support* of U^j.

Theorem 1: ([26, Proposition 116.1]) Every game with a finite number of players and action profiles has at least one mixed strategy Nash equilibrium.

It should be noticed that the control objective in the considered problem is to retain the temperature in each zone inside a range around a set-point rather than to keep tracking the set-point. This objective can be achieved by using a sequence of discretized power levels taken from a finite set of distinct values. Hence, we have a finite number of strategies depending on the requested power. In this context, an NE represents the control actions for each local controller based on the received power that defines the corresponding comfort level.

In the problem of decentralized power distribution, the NE can now be defined as follows. Given a capacity C for m subsystems, distribute C among the subsystems $(pw^1, \ldots, pw^m)$ $\wedge \left(\sum_{j=1}^m pw^j \leq C\right)$ in such a way that the control action $U^* = \langle u^1, \ldots, u^m \rangle$ is an NE if and only if

- $\eta^j(f^j, f_{-j}) \geq \eta^j(\tilde{f}^j, f_{-j})$ for all $\tilde{f}^j \in F^j$;
- f^* and $\langle \tilde{f}^j, f_{-j} \rangle$ satisfy (8),

where f^j is the solution of the cost function corresponds to the control action u^j for j^{th} subsystem, and f_{-j} denote the set $\{f^k \mid k \in \mathcal{M} \wedge k \neq j\}$, and $f^* = (f^j, f_{-j})$.

The above formulation seeks a set of control decisions for m subsystems that provides the best comfort level to the subsystems based on the available capacity.

Theorem 2: The decentralized power distribution problem with a finite number of subsystems and action profiles has at least one NE point.

Proof: In the decentralized power distribution problem, there are a finite number of subsystems $\mathcal{M}$. In addition, each subsystem $m \in \mathcal{M}$ conforms a finite set of control actions $U^j = \{u^1, \ldots, u^j, \ldots\}$ depending on the sequence of discretized power levels, with the probability distribution $\sum_{u^j \in U^j} p^j(u^j) = 1$. That means that the DMPC problem has a finite set of strategies for each subsystem including both pure and mixed strategies. Hence, the claim of this theorem follows from Theorem 1. ∎

E. Algorithms for Searching the Nash Equilibrium

An approach for finding a sample NE for normal-form games is proposed in [25], as presented by Algorithm 1. This algorithm is referred to as the SEM (Support-Enumeration Method), which is a heuristic-based procedure based on the space of supports of DMPC controllers and a notion of *dominated* actions that are diminished from the search space. The following algorithms show how the DMPC-based game theoretic power distribution problem is formulated to find the NE. Note that the complexity to find an exact NE point is exponential. Hence, it is preferable to consider heuristics-based approaches that can provide a solution very close to the exact equilibrium point with a much lower number of iterations.

It is assumed that a DMPC controller assigned for a subsystem $j \in \mathcal{M}$ acts as an agent in the normal-form game. Finally, all the individual DMPC controllers are supervised by a centralized controller in the upper layer. In Algorithm 1, x^j defines the support size of the control action available to subsystem $j \in \mathcal{M}$. It is ensured in the SEM that balanced supports are examined first, so that the lexicographic ordering is performed on the basis of the increasing order of the difference between the support sizes. In the case of a tie, this is followed by the balance of the support sizes.

An important feature of the SEM is the elimination of solutions that will never be NE points. Since we look for the best performance in each subsystem based on the solution

---

Algorithm 1 NE in DES

1: **for all** $x = (x^1, \ldots, x^n)$ sorted in increasing order of first $\sum_{j \in \mathcal{M}} x^j$ followed by $max_{j,k \in \mathcal{M}}(|x^j - x^k|)$ **do**

2:      $\forall j \; \tilde{U}^j \leftarrow \emptyset$ // *uninstantiated supports*

3:      $\forall j \; D^{x_j} \leftarrow \{u^j \in U^j \mid \sum_{k \in \mathcal{M}} |u^{j,k}| = x^j\}$ // *domain of supports*

4:      **if** RecursiveBacktracking($\tilde{U}, D^x, 1$) returns NE U^* **then**

5:          **return** U^*

6:      **end if**

7: **end for**

---

of the corresponding MPC problem, we want to eliminate the solutions that result always in a lower performance than the other ones. An exchange of a control action $u^j \in U^j$ (corresponding to the solution of the cost function $f^j \in F^j$) is *conditionally dominated* given the sets of available control actions U_{-j} for the remaining controllers, if $\exists \tilde{u}^j \in U^j$ such that $\forall u_{-j} \in U_{-j}, \eta^j(f^j, f_{-j}) < \eta^j(\tilde{f}^j, f_{-j})$.

Procedure 1 Recursive Backtracking

Input: $\tilde{U} = \tilde{U}^1 \times \ldots \times \tilde{U}^m$; $D^x = (D^{x_1}, \ldots, D^{x_m})$; j
Output: NE U^* or failure

1: **if** $j = m + 1$ **then**

2:      $\tilde{U} \leftarrow \{(\gamma^1, \ldots, \gamma^m) \mid (\gamma^1, \ldots \gamma^m) \leftarrow feasible(u^1, \ldots, u^m), \forall u^j \in \prod_{j \in \mathcal{M}} \tilde{U}^j\}$

3:      $\tilde{U} \leftarrow \tilde{U} \setminus \{(\gamma^1, \ldots, \gamma^m) \mid (\gamma^1, \ldots \gamma^m)$ does not solve the control problem$\}$

4:      **if** Program 1 is feasible for $\tilde{U}$ **then**

5:          **return** found NE U^*

6:      **else**

7:          **return** failure

8:      **end if**

9: **else**

10:      $\tilde{U}^j \leftarrow D^{x_j}$

11:      $D^{x_j} \leftarrow \emptyset$

12:      **if** IRDCA($\tilde{U}^1, \ldots, \tilde{U}^j, D^{x_{j+1}}, \ldots, D^{x_m}$) succeeds **then**

13:          **if** RecursiveBacktracking($\tilde{U}, D^x, j + 1$) returns NE U^* **then**

14:              **return** found NE U^*

15:          **end if**

16:      **end if**

17: **end if**

18: **return** failure

In addition, the algorithm for searching NE points relies on recursive backtracking (Procedure 1) to instantiate the search space for each player. We assume that in determining conditional domination, all the control actions are feasible, or are made feasible for the purposes of testing conditional domination. In adapting SEM for the decentralized MPC problem in DES, Procedure 1 includes two additional steps: (i) if u^j is not feasible, then we must make the prospective control action

feasible (where feasible versions of u^j are denoted by γ^j) (Line 2); and (ii) if the control action solves the decentralized MPC problem (Line 3). The input to Procedure 2 (Line 12

Procedure 2 Iterated Removal of Dominated Control Actions (IRDCA)

Input: $D^x = (D^{x_1}, \ldots, D^{x_m})$
Output: Updated domains or failure

1: **repeat**

2:      *dominated* $\leftarrow$ *false*

3:      **for all** $j \in \mathcal{M}$ **do**

4:          **for all** $u^j \in D^{x_j}$ **do**

5:              **for all** $\tilde{u}^j \in \{U^j\}$ **do**

6:                  **if** u^j is conditionally dominated by $\tilde{u}^j$ given $D\_^{x_j}$ **then**

7:                      $D^{x_j} \leftarrow D^{x_j} \setminus \{u^j\}$

8:                      *dominated* $\leftarrow$ *true*

9:                      **if** $D^{x_j} = \emptyset$ **then**

10:                          **return** failure

11:                      **end if**

12:                  **end if**

13:              **end for**

14:          **end for**

15:      **end for**

16: **until** *dominated = false*

17: **return** D^x

---

in Procedure 1) is the set of domains for the support of each MPC. When the support for an MPC controller is instantiated, the domain contains only the instantiated supports. The domain of other individual MPC controllers contains the supports of x^j that were not removed previously by earlier calls to this procedure.

Remark 1: In general, the NE point may not be unique and the first one founded by the algorithm may not necessarily be the global optimum. However, in the considered problem, every NE represents a feasible solution that guarantees that the temperature in all the zones can be kept within the predefined range, as far as there is enough power, while meeting the global capacity constraint. Thus, it is not necessary to compare different power distribution schemes as long as the local and global requirements are assured. Moreover, and most importantly, using the first NE will drastically reduce the computational complexity.

Program 1 Feasibility Program TGS (Test Given Supports)

Input: $U = U^1 \times \ldots \times U^m$
Output: u is an NE if there exist both $u = (u^1, \ldots, u^m)$ and $v = (v^1, \ldots, v^m)$ such that:

1: $\forall j \in \mathcal{M}, u^j \in U^j : \sum_{u_{-j} \in U_{-j}} p^{-j}(u_{-j}) \eta^j(u^j, u_{-j}) = v^j$

2: $\forall j \in \mathcal{M}, u^j \notin U^j : \sum_{u_{-j} \in U_{-j}} p^{-j}(u_{-j}) \eta^j(u^j, u_{-j}) \leq v^j$

3: $\forall j \in \mathcal{M}, u^j \in U^j : p^j(u^j) \geq 0$

4: $\forall j \in \mathcal{M}, u^j \notin U^j : p^j(u^j) = 0$

5: $\forall j \in \mathcal{M} : \sum_{u^j \in U^j} p^j(u^j) = 1$

We also adopted a feasibility program from [25], as shown in Program 1, to determine whether or not a potential solution is an NE. The input is a set of feasible control actions corresponding to the solution to the problem (8), and the output is a control action that satisfies NE. The first two constraints ensure that the MPC has no preference for one control action over another within the input set and it must not prefer an action that does not belong to the input set. The third and the fourth constraints check that the control actions in the input set are chosen with a non-zero probability. The last constraint simply assesses that there is a valid probability distribution over the control actions.

Remark 2: It is pointed out in [25] that Program 1 will prevent any player from deviating to a pure strategy aimed at improving the expected utility, which is indeed the condition for assuring the existence of NE in the considered problem.

V. SUPERVISORY CONTROL

A. Decentralized DES in the Upper Layer

In the framework of decentralized DES, a set of m controllers will cooperatively decide the control actions. In order for the supervisory control to accept or reject a request issued by an appliance, a controller decides which events are enabled through a sequence based on its own observations. The schedulability of appliances operation depends on two basic properties of DES: controllability and co-observability. We will examine a schedulability problem in decentralized DES, where the given specification $\mathcal{K}$ is controllable but not co-observable.

When $\mathcal{K}$ is not co-observable, it is possible to synthesize the extra power, so that all the MPC controllers guarantee their performance. To that end, we resort to the property of $\mathcal{P}$-*observability* and denote the content of additional power for each subsystem by $\Sigma_j^p = \{\textsf{extp}^j\}$. A language $\mathcal{K}$ is called $\mathcal{P}$-observable w.r.t. $\mathcal{L}$, $\Sigma_{o,j} \cup (\cup_{j \in \mathcal{I}} \Sigma_j^p)$, and $\Sigma_{c,j}$ $(j \in \mathcal{I})$ if

$$(\forall s \in \overline{\mathcal{K}})(\forall \sigma \in \Sigma_c) \ s\sigma \in \mathcal{L} \backslash \overline{\mathcal{K}} \Rightarrow$$
$$(\exists j \in \mathcal{I}) \ \pi_j^{-1}[\pi_j(s)]\sigma \cap \overline{\mathcal{K}} = \emptyset.$$

B. Control Design

DES is used as a part of the supervisory control in the upper layer to decide whether any subsystem needs more power to accept a request. In the control design, a controller's view $\mathcal{C}_j$ is first developed for each subsystem $j \in \mathcal{M}$. Figure 2 illustrates the process for accepting or rejecting a request issued by an appliance of subsystem $j \in \mathcal{M}$.
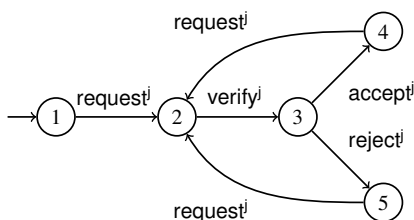

Fig. 2. Accepting or rejecting a request of an appliance.

If the distributed power is not sufficient for a subsystem $j \in \mathcal{M}$, this subsystem will request for extra power ($\textsf{extp}^j$) from the supervisory controller, as shown in Fig. 3. The controller of the corresponding subsystem will accept the request of its appliances after getting the required power.
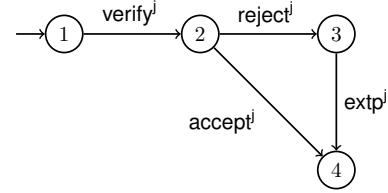

Fig. 3. Extra power provided to subsystem $j \in \mathcal{M}$.

Finally, the system behavior $\mathcal{C}$ is formulated by taking the synchronous product [20] of $\mathcal{C}_j, \forall j \in \mathcal{M}$, and $\textsf{extp}^j, \forall j \in \mathcal{M}$. Let $\mathcal{L}_{\mathcal{C}}$ be the language generated from $\mathcal{C}$ and $\mathcal{K}_{\mathcal{C}}$ be the specification. A portion of $\mathcal{L}_{\mathcal{C}}$ is shown in Fig. 4. The states to avoid are denoted by double circle. Note that, the supervisory controller ensures this by providing additional power.
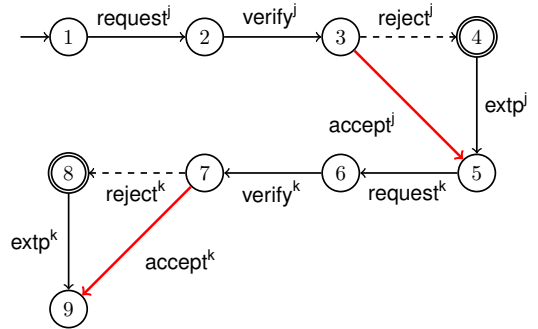

Fig. 4. A portion of $\mathcal{L}_{\mathcal{C}}$.

Denote by $U/\mathcal{L}_{\mathcal{C}}$ the controlled system under the supervision of $U = \wedge_{j=1}^{\mathcal{M}} U^j$. The closed behavior of $U/\mathcal{L}_{\mathcal{C}}$ is defined as a language $\mathcal{L}(U/\mathcal{L}_{\mathcal{C}}) \subseteq \mathcal{L}_{\mathcal{C}}$, such that

(i) $\varepsilon \in \mathcal{L}(U/\mathcal{L}_{\mathcal{C}})$, and
(ii) $\forall s \in \mathcal{L}(U/\mathcal{L}_{\mathcal{C}})$ and $\forall \sigma \in U(s)$, $s\sigma \in \mathcal{L}_{\mathcal{C}} \Rightarrow s\sigma \in \mathcal{L}(U/\mathcal{L}_{\mathcal{C}})$.

The marked behavior of $U/\mathcal{L}_{\mathcal{C}}$ is $\mathcal{L}_{\mathsf{m}}(U/\mathcal{L}_{\mathcal{C}}) = \mathcal{L}(U/\mathcal{L}_{\mathcal{C}}) \cap \mathcal{L}_{\mathsf{m}}$.

When the system is not co-observable, the comfort level cannot be achieved. Consequently, we have to make the system $\mathcal{P}$-observable to meet the requirements. The states followed by $\textsf{reject}^j$ for a subsystem $j \in \mathcal{M}$ must be avoided. It is assumed that when a request for subsystem j is rejected ($\textsf{reject}^j$), additional power ($\textsf{extp}^j$) will be provided in the consecutive transition. As a result, the system becomes controllable and $\mathcal{P}$-observable and the controller can make the correct decision.

Algorithm 2 shows the implemented DES mechanism for accepting or rejecting a request based on the game-theoretic power distribution scheme. When a request is generated for subsystem $j \in \mathcal{M}$, its acceptance is verified through the event $\textsf{verify}^j$. If there is an enough power to accept the $\textsf{request}^j$, the event $\textsf{accept}^j$ is enabled and $\textsf{reject}^j$ is disabled. When

there is a lack of power, a subsystem $j \in \mathcal{M}$ requests for extra power expt^j to enable the event accept^j and disable reject^j.

A unified modeling language (UML) activity diagram is depicted in Fig. 5 to show the execution flow of the whole control scheme. Based on the analysis from [27], the following theorem can be established.

Theorem 3: There exists a set of control actions $\{U^1, \ldots, U^m\}$ such that the closed behavior of $\wedge_{j=1}^m U^j / \mathcal{L}_\mathcal{C}$ is restricted to $\mathcal{K}_\mathcal{C}$ (i.e., $\mathcal{L}(\wedge_{j=1}^m U^j / \mathcal{L}_\mathcal{C}) \subseteq \mathcal{K}_\mathcal{C}$) if and only if

  (i) $\mathcal{K}_\mathcal{C}$ is controllable w.r.t. $\mathcal{L}_\mathcal{C}$ and Σ_{uc},

  (ii) $\mathcal{K}_\mathcal{C}$ is $\mathcal{P}$-observable w.r.t. $\mathcal{L}_\mathcal{C}$, π_j and $\Sigma_{c,j}$, and

  (iii) $\mathcal{K}_\mathcal{C}$ is $\mathcal{L}_m$-closed.

Algorithm 2 DES-based Admission Control

Input:
- $\mathcal{M}$: set of subsystems
- m: number of subsystems
- j: subsystem $\in \mathcal{M}$
- pw^j: power for subsystem $j \in \mathcal{M}$ from the NE
- $cons$: total power consumption of the accepted requests
- C: available capacity

1: $cons = 0$
2: **if** $\sum_{j \in \mathcal{M}} pw^j <= C$ **then**
3:     $j = 1$
4:     **repeat**
5:         **if** there is a request from $j \in \mathcal{M}$ **then**
6:             enable accept^j and disable reject^j
7:             $cons = cons + pw^j$
8:         **end if**
9:         $j \leftarrow j + 1$
10:     **until** $j <= m$
11: **else**
12:     $j = 1$
13:     **repeat**
14:         **if** there is a request from $j \in \mathcal{M}$ **then**
15:             request for extp^j
16:             enable accept^j and disable reject^j
17:             $cons = cons + pw^j$
18:         **end if**
19:         $j \leftarrow j + 1$
20:     **until** $j <= m$
21: **end if**

VI. SIMULATION STUDIES

A. Simulation Setup

The proposed DMPC has been implemented on a Matlab-Simulink platform. In the experiment, the YALMIP toolbox [28] is used to implement the MPC in each subsystem, and the DES centralized controllers view $\mathcal{C}$ is generated using the Matlab Toolbox DECK [29]. As each subsystem represents a scalar problem, there is no concern regarding the computational effort. A four-zone building, equipped with one heater in each zone, is considered in the simulation. The building layout is represented in Fig. 6. Note that the thermal coupling occurs
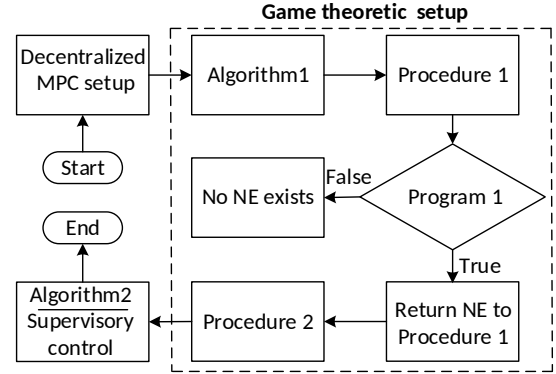

Fig. 5. UML activity diagram of the proposed control scheme.

through the doors between the neighboring zones and the isolation of the walls is supposed to be very high ($R_{\text{wall}}^d = \infty$). Note also that experimental implementations or the use of more accurate simulation software, e.g., EnergyPlus [30], may provide a more reliable assessment of the proposed work.
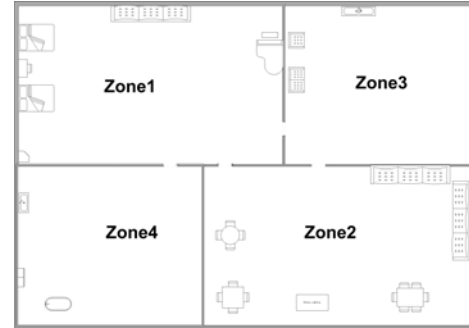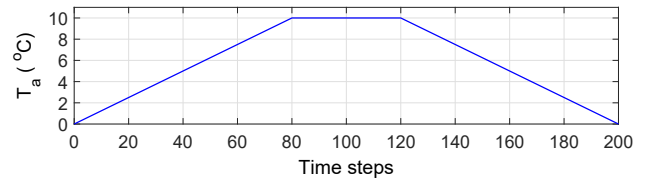

Fig. 6. Building layout.

Fig. 7. Ambient temperature

In this simulation experiment, the thermal comfort zone is chosen as $(22 \pm 0.5)^\circ C$ for all the zones. The prediction horizon is chosen to be $N = 10$ and $T_s = 1.5$ time steps which is about 3 minutes in the corresponding real time-scale. Thus, the simulation is equivalent to about 10 hours operation in a real time scale, which is implementable with the currently available computing technology. The system is decoupled into four subsystems, corresponding to a setting with $m = M$. Furthermore, it has been verified that the system and the decomposed subsystems are all stable in open loop. The variation of the outdoor temperature is presented in Fig. 7.

The co-observability and $\mathcal{P}$-observability properties are tested with high and low constant power capacity constraints, respectively. It is supposed that the heaters in Zone 1 and 2

need 800 W each as the initial power while the heaters in Zone 3 and 4 require 600 W each. The requested power is discretized with a step of 10 W. The initial indoor temperatures are set to 15 °C inside all the zones. The parameters of the thermal model are listed in Table I and Table II based on the configuration of [3], and the system decomposition is based on the approach presented in [22]. The submatrices A_i, B_i, and E_i can be computed as presented in Section III-B with the decoupling matrices given by $W_i = Z_i = H_i = e_i$, where e_i is the ith standard basic vector of $\mathbb{R}^4$.

TABLE I
THERMAL MODEL PARAMETERS

Zone	1	2	3	4
R_i^a	69.079	69.079	105.412	105.412
C_i	0.94	0.94	0.78	0.78

TABLE II
THERMAL MODEL PARAMETERS

R_{ji}^d	$R_{12}^d=R_{21}^d$	$R_{13}^d=R_{31}^d$	$R_{14}^d=R_{41}^d$	$R_{23}^d=R_{32}^d$
Value	709.2	1063.8	1063.8	1063.8

B. Case 1: Co-observability validation

In this case, we validate the proposed scheme to test the co-observability property for various power capacity constraints. We split the whole simulation time into two intervals, $[0, 60)$ and $[60, 200]$, with $2800\ W$ and $1000\ W$ as power constraints, respectively. At the start-up, a power capacity of $2800\ W$ is required as the initial power for all the heaters.

The zone's indoor temperatures are depicted in Fig. 8. It can be seen that the DMPC has the capability to force the indoor temperatures in all zones to stay within the desired range if there is enough power. Specifically, the temperature is kept in the comfort zone until 140 time steps. After that, the temperature in all the zones attempts to go down due to the effect of the ambient temperature and the power shortage. In other words, the system is no longer co-observable and hence, the thermal comfort level cannot be guaranteed.

The individual and the total power consumption of the heaters over the specified time intervals are shown in Fig. 9 and Fig. 10, respectively. It can be seen that in the second interval, all the available power is distributed to the controllers, and the total power consumption is about 36% of the maximum peak power.

C. Case 2: $\mathcal{P}$-Observability validation

In the second test, we consider three time intervals $[0, 60)$, $[60, 140)$, and $[140, 200]$. In addition, we raise the level of power capacity to 1600 W in the third time interval. The indoor temperature of all zones is shown in Fig. 11. It can be seen that the temperature is maintained within the desired range over all the simulation time steps, despite the diminishing of the outdoor temperature. Therefore, the performance is achieved
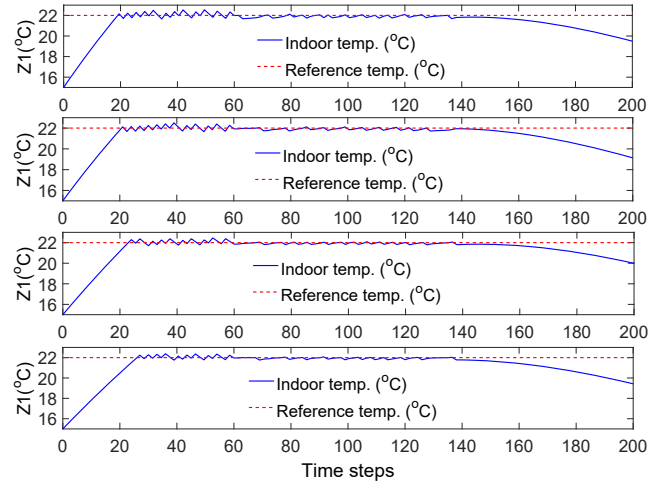

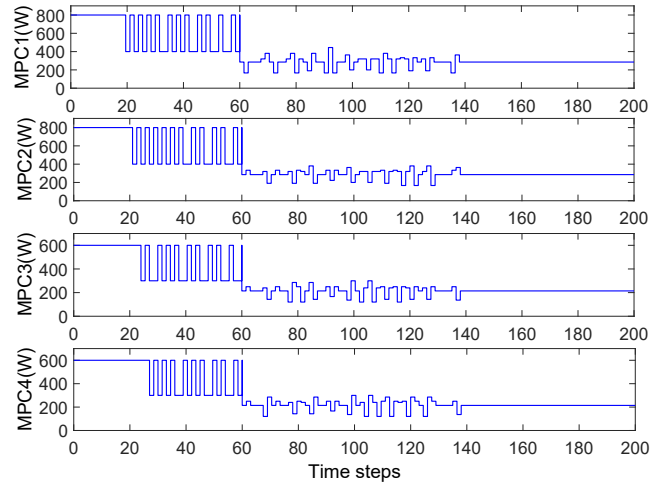Fig. 8. Temperature in each zone in Case 1.

Fig. 9. The individual power consumption in Case 1.

and the system becomes $\mathcal{P}$-observable. The individual and the total power consumption of the heaters are illustrated in Fig. 12 and Fig. 13, respectively. Note that the power capacity applied over the third interval (1600 W) is about 57% of the maximum power, which is a significant reduction of power consumption. It is worth noting that when the system fails to ensure the performance due to the lack of the supplied power, the upper layer of the proposed control scheme can compute the amount of extra power that is required to ensure the system performance. The corresponding DES will become
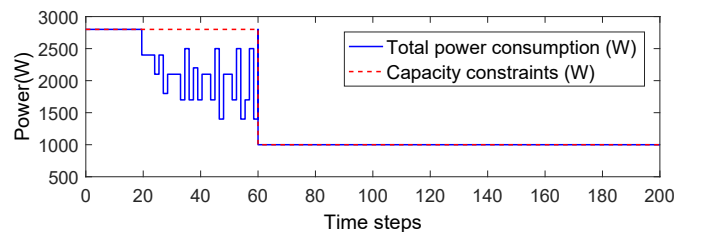

Fig. 10. Total power consumption in Case 1.

$\mathcal{P}$-observable if extra power is provided.

Finally, it is worth noting that the simulation results confirm that in both Case 1 and Case 2, power distributions generated by the game-theoretic scheme are always fair. Moreover, as the attempt of any agent to improve its performance does not degrade the performance of the others, it eventually allows avoiding the selfish behavior of the agents.
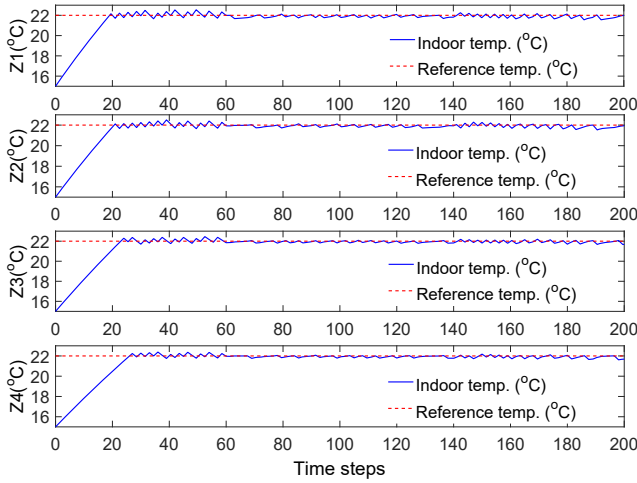

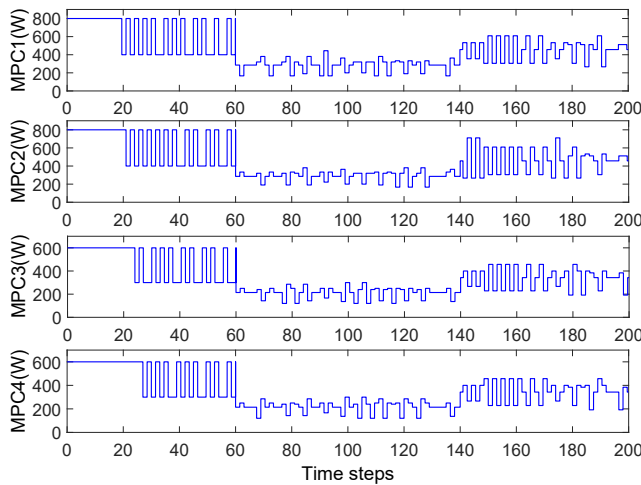Fig. 11. Temperature in each zone in Case 2.

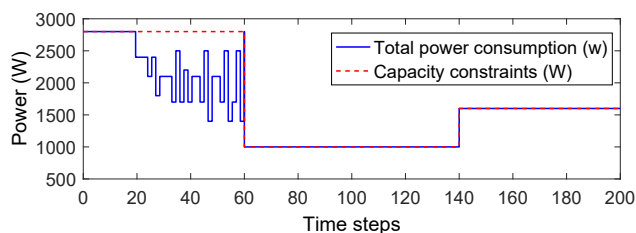Fig. 12. The individual power consumption in Case 2.

Fig. 13. Total power consumption in Case 2.

VII. Concluding Remarks

This work presented a hierarchical decentralized scheme consisting of a decentralized DES supervisory controller based on a game-theoretic power distribution mechanism and a set of local MPC controllers for thermal appliance control in smart buildings. The impact of observability properties on the behavior of the controllers and the system performance have been thoroughly analyzed, and algorithms for running the system in a numerically efficient way have been provided. Two case studies were conducted to show the effect of co-observability and $\mathcal{P}$-observability properties related to the proposed strategy. The simulation results confirmed that the developed technique can efficiently reduce the peak power while maintaining the thermal comfort within an adequate range when the system was $\mathcal{P}$-observable. In addition, the developed system architecture has a modular structure and can be extended to appliance control in a more generic context of HVAC systems. Finally, it might be interesting to address the applicability of other control techniques, such as those presented in [31], [32], to smart building control problems.

References

[1] T. X. Nghiem, M. Behl, R. Mangharam, and G. J. Pappas, "Green scheduling of control systems for peak demand reduction," in *50th IEEE CDC and 2011 ECC*, pp. 5131–5136, 2011.

[2] V. Chandan and A. Alleyne, "Optimal partitioning for the decentralized thermal control of buildings," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 5, pp. 1756–1770, 2013.

[3] W. H. Sadid, S. A. Abobakr, and G. Zhu, "Discrete-event systems-based power admission control of thermal appliances in smart buildings," *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 2665–2674, 2017.

[4] A. Afram and F. Janabi-Sharifi, "Theory and applications of HVAC control systems–a review of model predictive control (MPC)," *Building and Environment*, vol. 72, pp. 343–355, 2014.

[5] T. Salsbury, P. Mhaskar, and S. J. Qin, "Predictive control methods to improve energy efficiency and reduce demand in buildings," *Computers & Chemical Engineering*, vol. 51, pp. 77–85, 2013.

[6] P.-D. Moroşan, R. Bourdais, D. Dumur, and J. Buisson, "Building temperature regulation using a distributed model predictive control," *Energy and Buildings*, vol. 42, no. 9, pp. 1445–1452, 2010.

[7] R. Scattolini, "Architectures for distributed and hierarchical model predictive control–a review," *J. Proc. Cont.*, vol. 19, pp. 723–731, 2009.

[8] I. Hazyuk, C. Ghiaus, and D. Penhouet, "Model predictive control of thermal comfort as a benchmark for controller performance," *Automation in Construction*, vol. 43, pp. 98–109, 2014.

[9] G. Mantovani and L. Ferrarini, "Temperature control of a commercial building with model predictive control techniques," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2651–2660, 2015.

[10] J. Yao, G. T. Costanzo, G. Zhu, and B. Wen, "Power admission control with predictive thermal management in smart buildings," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2642–2650, 2015.

[11] S. Al-Assadi, R. Patel, M. Zaheer-Uddin, M. Verma, and J. Breitinger, "Robust decentralized control of HVAC systems using H_{∞}-performance measures," *J. of the Franklin Inst.*, vol. 341, no. 7, pp. 543–567, 2004.

[12] M. Liu, Y. Shi, and X. Liu, "Distributed MPC of aggregated heterogeneous thermostatically controlled loads in smart grid," *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp. 1120–1129, 2016.

[13] P.-D. Morosan, R. Bourdais, D. Dumur, and J. Buisson, "Building temperature regulation using a distributed model predictive control," *Energy and Buildings*, vol. 42, no. 9, pp. 1445–1452, 2010.

[14] F. Kennel, D. Görges, and S. Liu, "Energy management for smart grids with electric vehicles based on hierarchical MPC," *IEEE Trans. Ind. Informat.*, vol. 9, no. 3, pp. 1528–1537, 2013.

[15] R. Deng, Z. Yang, M.-Y. Chow, and J. Chen, "A survey on demand response in smart grids: Mathematical models and approaches," *IEEE Trans. Ind. Informat.*, vol. 11, no. 3, pp. 570–582, 2015.

[16] Z. Zhu, S. Lambotharan, W. H. Chin, and Z. Fan, "A game theoretic optimization framework for home demand management incorporating local energy resources," *IEEE Trans. Ind. Informat.*, vol. 11, no. 2, pp. 353–362, 2015.

[17] E. R. Stephens, D. B. Smith, and A. Mahanti, "Game theoretic model predictive control for distributed energy demand-side management," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1394–1402, 2015.

[18] J. Maestre, D. Muñoz De La Peña, and E. Camacho, "Distributed model predictive control based on a cooperative game," *Optimal Control Applications and Methods*, vol. 32, no. 2, pp. 153–176, 2011.

[19] R. Deng, Z. Yang, J. Chen, N. R. Asr, and M.-Y. Chow, "Residential energy consumption scheduling: A coupled-constraint game approach," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1340–1350, 2014.

[20] P. J. Ramadge and W. M. Wonham, "Supervisory control of a class of discrete event processes," *SIAM J. Control Optim.*, vol. 25, no. 1, pp. 206–230, 1987.

[21] G. T. Costanzo, G. Zhu, M. F. Anjos, and G. Savard, "A system architecture for autonomous demand side load management in smart buildings," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 2157–2165, 2012.

[22] A. Alessio, D. Barcelli, and A. Bemporad, "Decentralized model predictive control of dynamically coupled linear systems," *Journal of Process Control*, vol. 21, no. 5, pp. 705–714, 2011.

[23] K. Rudie and W. M. Wonham, "Think globally, act locally: Decentralized supervisory control," *IEEE Trans. Automat. Contr.*, vol. 37, no. 11, pp. 1692–1708, 1992.

[24] A. F. R. Cieslak, C. Desclaux and P. Varaiya, "Supervisory control of discrete-event processes with partial observations," *IEEE Trans. Automat. Contr.*, vol. 33, no. 3, pp. 249–260, 1988.

[25] E. N. R. Porter and Y. Shoham, "Simple search methods for finding a Nash equilibrium," *Games & Eco. Beh.*, vol. 63, pp. 642–662, 2008.

[26] M. J. Osborne, *An Introduction to Game Theory*. Oxford: Oxford University Press, 2002.

[27] S. Ricker, "Asymptotic minimal communication for decentralized discrete-event control," in *2008 WODES*, pp. 486–491, 2008.

[28] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *IEEE Int. Symp. CACSD*, pp. 284–289, 2004.

[29] S. H. Zad, *Discrete Event Control Kit*. Concordia University, 2013.

[30] D. B. Crawley *et al.*, "EnergyPlus: creating a new-generation building energy simulation program," *Ene. & Buil.*, vol. 33, pp. 319–331, 2001.

[31] Y. Wu, R. Lu, P. Shi, H. Su, and Z.-G. Wu, "Adaptive output synchronization of heterogeneous network with an uncertain leader," *Automatica*, vol. 76, pp. 183–192, 2017.

[32] Y. Wu, X. Meng, L. Xie, R. Lu, H. Su, and Z.-G. Wu, "An input-based triggering approach to leader-following problems," *Automatica*, vol. 75, pp. 221–228, 2017.

Saad A. Abobakr (S'17) was born in Baniwaleed, Libya, in 1978. He received the B.Sc. degree in control engineering from the College of Electronic Technology, Baniwaleed, Libya, in 2001, and the M.Sc. degree in control and instrumentation engineering from Libyan Academy,Tripoli, Libya, in 2008. He is currently pursuing the Ph.D. degree at École Polytechnique de Montréal, Montreal, QC, Canada. His research is oriented toward the application of model predictive control to demand response management in smart buildings, demand side energy management, building energy systems modeling and simulation, and HVAC control system.

Waselul H. Sadid received the B.Sc. degree in electrical and electronic engineering from Rajshahi University of Engineering and Technology, Rajshahi, Bangladesh, the M.Sc. degree Waselul H. Sadid received the B.Sc. degree in electrical and electronic engineering from the Rajshahi University of Engineering and Technology, Rajshahi, Bangladesh, the M.Sc. degree in computer science and engineering from the Bangladesh University of Engineering and Technology, Dhaka, Bangladesh, and the Ph.D. degree in electrical and computer engineering from Concordia University, Montreal, QC, Canada, in 2000, 2005, and 2014, respectively.

He is currently serving as an assistant professor in the department of computer Science and engineering of East West University, Dhaka, Bangladesh. Before that he was an industrial postdoc intern awarded by the Mitacs-Accelerate Graduate Research Internship Program at Fusion Energy Inc., Laval, QC, Canada, and École Polytechnique de Montréal, Montreal. His research interests include decentralized control of discrete-event systems, applications of game theory, demand side energy management in smart buildings, and Internet of Things.

Guchuan Zhu (M'07-SM'12) received the M.S. degree in electrical engineering from Beijing Institute of Aeronautics and Astronautics, Beijing, China, in 1982; the Ph.D. degree in mathematics and control from École des Mines de Paris, Paris, France, in 1992; and the graduate Diploma degree in computer science from Concordia University, Montreal, QC, Canada, in 1999.

Dr. Zhu joined École Polytechnique de Montréal, Montréal, in 2004, where he is currently a Professor in the Department of Electrical Engineering. His current research interests include control of distributed parameter systems, nonlinear and robust control, and optimization with their applications to microsystems, aerospace systems, communication networks, and smart grid.