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Pre-posterior optimization of sequence of measurement and intervention actions under structural reliability constraint

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Abstract

It is common to assess the condition of an existing infrastructure using reliability analysis. When, based on the available information, an existing structure has an estimated failure probability above the admissible level, the default solution often is to either strengthen or replace it. Even if this practice is safe, it may not be the most economical. In order to economically restore and improve our existing infrastructure, the engineering community needs to be able to assess the potential gains associated with reducing epistemic uncertainties using measurements, before opting for costly intervention actions, if they become necessary. This paper provides a pre-posterior analysis framework to (1) optimize sequences of actions minimizing the expected costs and satisfying reliability constraints, and (2) quantify the potential gain of making measurements in existing structures. Illustrative examples show that when the failure probability estimated based on the present state of knowledge does not satisfy an admissible threshold, strengthening or replacement interventions can be sub-optimal first actions. The examples also show that significant savings can be achieved by reducing epistemic uncertainties.

Keywords: Pre-posterior, reliability, measurement, optimization, uncertainty, infrastructure management

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1. Introduction

With increased awareness about the extent of deficiencies of existing infrastructures, the US National Academy of Engineering has identified restoration and improvement of urban infrastructure as one of the grand engineering challenges of the 21st century [1]. It is common to assess the condition of an existing infrastructure by reliability analysis using prior knowledge about capacities and demands. When an existing structure has

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an estimated failure probability above an admissible level, $p_{\mathcal{F}} > p_{\mathcal{F}}^{\{\text{adm.}\}}$, the default solution often is to perform a *structural intervention action*, such as strengthening or replacement. However, it is known that the prior information about capacities and demands of an existing structure is characterized by epistemic uncertainties. By gathering additional information, it is often possible to reduce these uncertainties and alter the failure probability estimate. Therefore, in order to assess the true condition of an existing infrastructure and economically restore and improve it, the engineering community needs to be able to estimate the potential gains associated with reducing epistemic uncertainties using *information gathering actions*, instead of directly opting for costly structural interventions based on findings from prior knowledge.

Uncertainties and their classification have received much attention from the scientific community, e.g. [2–4]. Uncertainties are most often classified as either *aleatory* or *epistemic*, depending on whether they are attributed to inherent variability or to lack of knowledge. According to this classification, epistemic uncertainties are reducible and aleatory uncertainties are not. Several researchers have noted that, during the design phase, the uncertainties in structural properties are inherently random and, therefore, aleatory in nature [4, 5]. However, once the structure is constructed, the uncertainties in structural properties become epistemic in nature. In a sense, the constructed structure is viewed as a realization from a population of structures having the same design. Naturally, if we were able to precisely measure the properties (e.g., as-built dimensions, material constants, member capacities) of an existing structure, no uncertainties in these quantities would remain. Of course, it is not possible to accurately measure all structural properties. Nevertheless, any direct or indirect observations about these quantities can serve to reduce the corresponding epistemic uncertainties. Note that measuring a structural property may either increase or decrease the estimated failure probability, depending on the measurement outcome [5, 6]. Section 3.1.1 presents considerations that this aspect requires during the planning of measurement actions.

Maintenance planning for structures has been addressed in previous research related to structural health monitoring, decision theory and reliability theory. For instance, Faber [5] proposed a general framework for assessment of existing structures based on reliability theory considering evidences obtained during inspection. Pozzi and Der Kiureghian [7] used the concept of value of information (VoI) [8] to quantify the value of measuring the evolution of structural performance as a support to maintenance interventions. In a similar way, Glisic et al. [9] used VoI to quantify, in economic terms, the impact of monitoring on decision making. Straub and Faber [10] used decision and VoI theory to build an adaptive decision framework for identifying inspection planning strategies that minimize maintenance costs. In their framework, inspections are performed in a sequence, and the decision to perform an inspection is based on the outcome of the previous inspection.

Engineering decision analysis can be made in three stages [11–13]: prior decision analysis, posterior decision analysis and pre-posterior decision analysis. This paper deals with *pre-posterior* decision analysis, where the planning of information gathering actions is made based on the prior probabilistic model of uncertainties. In this scheme, the consequences (e.g. costs) of the possible outcomes of measurement or other information gathering actions are weighed with their probabilities of occurrence. This approach to measurement actions planning is similar to what was proposed by Artstein and Wets as the *theory of sensors* [6]. Interested readers may also consult other relevant work performed in the field of maintenance-action optimization [14–17]. In the field of reliability-based optimization, Royset et al. [18–20] studied several aspects related to the design of new structures, notably optimal design under constraints. Der Kiureghian et al. [21] were among the firsts to study inverse reliability problems, where parameter values satisfying a reliability constraint are sought. More recently, Lehkỳ and Novák [22] also approach this problem using a method based on Artificial neural network. Despite all these related aspects previously addressed in the literature, solving the problem posed in this paper requires further investigations related to the optimization of sequences of information gathering and intervention actions.

This paper presents a pre-posterior framework for optimizing sequences of actions minimizing the expected costs and satisfying reliability constraints for an existing structure. This framework is intended to: (1) provide optimized sequences of information gathering and intervention actions, and (2) quantify the potential gains of measuring structures instead of directly opting for costly strengthening and replacement interventions. The paper is organized in the following order: Section 2 presents the formulation for assessing the reliability of an existing structure, Section 3 presents the mathematical framework for the pre-posterior decision analysis for sequences of actions, and Section 4 presents illustrative applications of the proposed methodology.

2. Assessing the reliability of an existing structure

The safety and serviceability of an existing structure is usually assured by verifying that, given the available knowledge, the structure has a failure probability (complement of reliability) lower or equal to an admissible value, i.e. $p_{\mathcal{F}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$. Let $\mathbf{V} = [V_1, V_2, \cdots, V_n]^{\mathrm{T}}$ denote the set of random variables defining the state of the structure and $f_{\mathbf{V}}(\mathbf{v})$ represent its joint probability density function (PDF). The failure probability is defined as

$$p_{\mathcal{F}} = \int_{\Omega} f_{\mathbf{V}}(\mathbf{v}) d\mathbf{v} \tag{1}$$

where

$$\Omega \equiv \{\mathbf{v} | \cup_k \cap_{i \in C_k} G_i(\mathbf{v}) \leq 0\} \tag{2}$$

is the failure domain. This formulation is written in terms of unions of intersections of componental failure events. The i^{th} component is defined in terms of a limit state function $G_i(\mathbf{V})$ with $\{G_i(\mathbf{V}) \leq 0\}$ indicating its failure. The union operation is over min cut sets C_k, $k = \{1, 2, \cdots\}$, where each min cut set represents a minimal set of components whose joint failure constitutes failure of the structure. The intersection operations are over components within each min cut set. Special cases of this formulation are series structural systems, when each min cut set has a single component, parallel structural systems, when there is only one cut set, and structural component, when there is only one min cut set with a single component. See Der Kiureghian [23] for more details about this formulation.

The limit-state functions $G_i(\mathbf{V})$ defining the component states are usually made up of sub-models representing component capacity and demand values. Such a sub-model typically has the form

$$R(\mathbf{X}, \epsilon) = \hat{R}(\mathbf{X}) + \epsilon \tag{3}$$

where $\hat{R}(\mathbf{X})$ represents an idealized mathematical model and ϵ is the model error, which is usually considered to have the Normal distribution. The additive error model is based on an assumption of normality, which is usually satisfied by an appropriate transformation of the model, see [24]. Physics-based models of structural components are generally biased so that the mean of ϵ, μ_ϵ, can be nonzero. The standard deviation, σ_ϵ, represents a measure of quality of the model. The vector $\mathbf{V}$ collects random variables $\mathbf{X}$ and ϵ for all sub-models. In addition, it may include any uncertain parameters $\mathbf{\Theta}$ involved in the definition of the distributions of $\mathbf{X}$ and ϵ for the various sub-models.

At the outset of our analysis, the PDF of $\mathbf{V}$ represents our prior state of knowledge about the structure and its future loads. We designate this by using the notation $f_{\mathbf{V}}^{\{0\}}(\mathbf{v})$. The corresponding estimate of the failure probability is denoted $p_{\mathcal{F}}^{\{0\}}$. If $p_{\mathcal{F}}^{\{0\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$, the reliability constraint ($p_{\mathcal{F}}^{\{\text{adm.}\}}$) is satisfied and no further action is necessary. When $p_{\mathcal{F}}^{\{0\}} > p_{\mathcal{F}}^{\{\text{adm.}\}}$, actions are necessary to reduce the failure probability estimate.

As we take actions to modify the structure, learn about the random variables, or improve the models, the distribution of $\mathbf{V}$ changes. We show this by changing the superscript $\{0\}$. Specifically, $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ denotes the distribution of $\mathbf{V}$ after an ordered set of actions $\{a_{1:i}\} = \{a_1, \cdots, a_i\}$. The corresponding failure probability estimate is denoted $p_{\mathcal{F}}^{\{a_{1:i}\}}$. Our aim is to find an optimal sequence of future actions $\mathcal{A}_{\text{opt}} = \{a_1, \cdots, a_n\}$ that minimizes the expected costs, while assuring that $p_{\mathcal{F}}^{\{a_{1:i}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$.

3. Optimization framework

This section presents the formulation of the optimization framework for identifying the sequence of future actions that minimizes the expected costs and satisfies the failure probability constraint. Sub-section 3.1 presents the mathematical formulation of the optimization problem, Sub-section 3.2 discusses computational issues, and Sub-section 3.3 describes the effects of structural intervention and information gathering actions on the random variables involved in the estimation of the failure probability.

3.1. Formulation of the optimization framework

As mentioned in Section 2, when $p_{\mathcal{F}}^{\{0\}} > p_{\mathcal{F}}^{\{\text{adm.}\}}$, actions are necessary to reduce the failure probability estimate. Let $\mathcal{A} = \{a_1, \cdots, a_i\}$ denote an ordered set of candidate actions so that action a_i can take place only after actions $\{a_1, \cdots, a_{i-1}\}$ have been completed. Example actions include replacement or strengthening of the structure, measurement of component capacities, measurement of variables involved in the capacity or demand models, proof testing of the structure, etc. (Sub-section 3.3 provides a more exhaustive description of these actions). Each action a_i will alter our state of knowledge about one or more of the random variables so that $f_{\mathbf{V}}^{\{a_{1:i-1}\}}(\mathbf{v})$ will change to $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ after action a_i is taken. If action a_i is a structural intervention, e.g., replacement or strengthening, the new distribution $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ is that of the new or strengthened structural design. If the action is one of information gathering, $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ is derived from $f_{\mathbf{V}}^{\{a_{1:i-1}\}}(\mathbf{v})$ by conditioning on the observations, while accounting for possible measurement errors. However, since the analysis is performed before observations are made, one needs to consider all possible realizations of the observations with their corresponding prior probabilities. This aspect requires pre-posterior analysis. The corresponding probability estimate $p_{\mathcal{F}}^{\{a_{1:i}\}}$ should be regarded as the conditional probability of failure, given the observations. Thus, it is a function of the future observations. In the outcome space of these observations, the domain where $p_{\mathcal{F}}^{\{a_{1:i}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$ constitutes the event that actions $\{a_1, \cdots, a_i\}$ will lead to satisfaction of the reliability constraint. The next sub-section elaborates on the characterization of this domain and specification of the probability of success in satisfying the reliability constraint.

Our task is to identify an optimized sequence of future actions $\mathcal{A}_{\text{opt}} = \{a_1, \cdots, a_n\}$ so that $\mathcal{A}_{\text{opt}}$ minimizes the expected costs subject to $p_{\mathcal{F}}^{\{\mathcal{A}_{\text{opt}}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$. For intervention actions (e.g., strengthening or replacement), the probability of satisfying the reliability constraint based on the prior state of knowledge is either zero or one. This is because for any strengthening or replacement design, the estimated failure probability is either greater than, or less than the admissible value. The only reason for contemplating intervention actions with zero probability of satisfying the reliability constraint is to consider them subsequent to other actions that may improve our state of knowledge. For example, a partial retrofit may become a viable option after measurements have shown that the capacity is likely to be greater than initially estimated.

3.1.1. Actions involving measurements

This sub-section describes the method for computing the probability of success for actions involving measurements. Assume that the first action a_1 of a sequence $\mathcal{A}$ consists in measuring a structural property or structural response. Given a measurement outcome $m^{\{a_1\}} \in \mathbb{R}$ and the conditioned PDF $f_{\mathbf{V}}^{\{a_1\}}(\mathbf{v})$, the failure probability conditional on the measurement outcome is $p_{\mathcal{F}}^{\{a_1\}}$. Because the outcome of the measurement is unknown a-priori, it is treated as a random variable $M^{\{a_1\}}$. The subset of outcomes of $M^{\{a_1\}}$ leading to satisfaction of the reliability constraint is

$$\mathcal{M}^{\{a_1\}} = \{m^{\{a_1\}} : p_{\mathcal{F}}^{\{a_1\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}\} \tag{4}$$

Thus, the probability that after taking action a_1 the reliability constraint will be satisfied is

$$p_{\text{succeed}}^{\{a_1\}} = \int_{\mathcal{M}^{\{a_1\}}} f_{M^{\{a_1\}}}(m^{\{a_1\}}) dm^{\{a_1\}} \tag{5}$$

in which $f_{M^{\{a_1\}}}(m^{\{a_1\}})$ is the PDF of the measurement outcome $M^{\{a_1\}}$. The conditional probability of failure as a function of the measurement $m^{\{a_1\}}$, $p_{\mathcal{F}}^{\{a_1\}}$, the subset of measurement outcomes $\mathcal{M}^{\{a_1\}}$, its complement $\overline{\mathcal{M}}^{\{a_1\}}$, and the probability of meeting the reliability constraint, $p_{\text{succeed}}^{\{a_1\}}$ are illustrated in Figure 1. For a_1 there is a probabil-

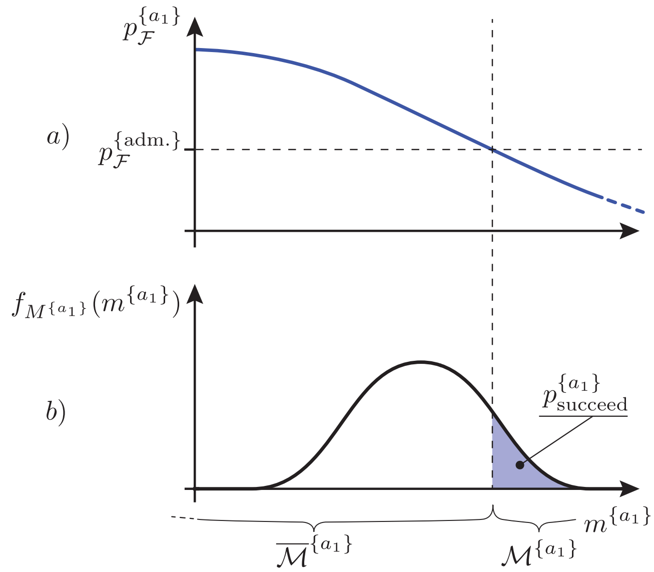

Figure 1: Outcome space of a measurement: a) failure probability conditioned on the measurement outcome $p_{\mathcal{F}}^{\{a_1\}}$, b) PDF of the measurement outcome $M^{\{a_1\}}$ with the shaded area showing probability $p_{\text{succeed}}^{\{a_1\}}$ of meeting the reliability constraint.

ity $1 - p_{\text{succeed}}^{\{a_1\}}$ that the measurement action will not satisfy the reliability constraint. Therefore, for all measurement outcomes $m^{\{a_1\}} \in \overline{\mathcal{M}}^{\{a_1\}}$, it is necessary to plan for at least one additional measurement or intervention action. Figure 2 shows the outcome space of two successive measurements. Figure 2(a) depicts the subset of successful outcomes of the first measurement $\mathcal{M}^{\{a_1\}}$ (shaded area). Figure 2(b) depicts the subset of successful outcomes of the second measurement, conditional on a first unsuccessful measurement, $\mathcal{M}^{\{a_{1:2}\}}$ (darkly shaded area). The boundary of $\mathcal{M}^{\{a_{1:2}\}}$ is nonlinear because of interaction between the previous unsuccessful measurement outcome and the new measurement. (A previous unsuccessful measurement far from the boundary of success requires a more favorable outcome of the second measurement to assure success.)

More generally, for any subsequent measurement action $a_i \in \mathcal{A}$, $i = 2, \cdots, n$, the subset of successful measurement outcomes $\mathcal{M}^{\{a_{1:i}\}} \subseteq \mathbb{R}^i$ is

$$\mathcal{M}^{\{a_{1:i}\}} = \left\{ m^{\{a_{1:i}\}} : p_{\mathcal{F}}^{\{a_{1:i}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}} \wedge m^{\{a_{1:i-1}\}} \notin \mathcal{M}^{\{a_{1:i-1}\}} \right\} \tag{6}$$

In Eq.(6), the subset of successful measurement outcomes $\mathcal{M}^{\{a_{1:i}\}}$ is obtained while excluding the previous subset of successful measurement outcomes $\mathcal{M}^{\{a_{1:i-1}\}}$. Mea-

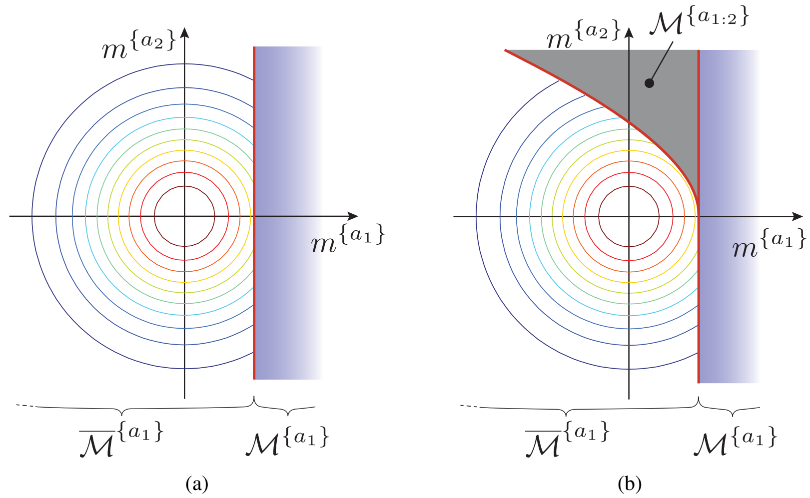

Figure 2: Outcome space of two successive measurement outcomes: a) subset of successful first measurement outcomes, b) subset of successful second measurement outcomes conditioned on first unsuccessful measurement outcome

surement outcomes $m^{\{a_{1:i-1}\}} \in \mathcal{M}^{\{a_{1:i-1}\}}$ are excluded because a_i would only be taken if all previous measurement actions were unsuccessful. The conditional probability of success of the i^{th} measurement action given no success up to the $(i-1)^{\text{th}}$ action is,

$$p_{\text{succeed}}^{\{a_i\}} = \frac{1}{c^{\{a_{1:i}\}}} \cdot \int_{\mathcal{M}^{\{a_{1:i}\}}} f_{M^{\{a_{1:i}\}}}(m^{\{a_{1:i}\}}) dm^{\{a_{1:i}\}} \tag{7}$$

where $f_{M^{\{a_{1:i}\}}}(m^{\{a_{1:i}\}})$ is the joint PDF of the i measurements and

$$c^{\{a_{1:i}\}} = \int_{\overline{\mathcal{M}}^{\{a_{1:i-1}\}}} f_{M^{\{a_{1:i}\}}}(m^{\{a_{1:i}\}}) dm^{\{a_{1:i}\}} \tag{8}$$

is a normalization constant. Equation 7 is obtained by dividing the probability of intersection of no success in the first $i-1$ measurements and success in the i^{th} measurement by the probability of no success in the first $i-1$ measurements.

3.1.2. *Expected costs for sequences of actions*

In order to compute the expected costs, actions must be added to the set $\mathcal{A}$ until the sequence of n actions has a cumulative probability $p_{c,\text{succeed}}^{\{\mathcal{A}\}} = p_{c,\text{succeed}}^{\{a_{1:n}\}} = 1$. When $p_{c,\text{succeed}}^{\{\mathcal{A}\}} = 1$, it is certain that the sequence of actions planned are sufficient to satisfy the reliability constraint. The cumulative probability that a sequence of actions $\{a_1, \cdots, a_i\}$, $i \in \{2, \cdots, n\}$ will result in meeting the reliability constraint is given by

$$p_{c,\text{succeed}}^{\{a_{1:i}\}} = p_{c,\text{succeed}}^{\{a_{1:i-1}\}} + p_{\text{succeed}}^{\{a_{1:i}\}} \tag{9}$$

where the probability of satisfying the reliability constraint using a sequence of actions $\{a_1, \cdots, a_i\}$, $i \in \{2, \cdots, n\}$ is,

$$p_{\text{succeed}}^{\{a_{1:i}\}} = p_{\text{succeed}}^{\{a_i\}} \times (1 - p_{c,\text{succeed}}^{\{a_{1:i-1}\}}) \tag{10}$$

Note that $c^{\{a_{1:i}\}}$ presented in Eq. 8 is identical to $1 - p_{c,\text{succeed}}^{\{a_{i-1}\}}$. Figure 3 presents an example of the probability mass function $p_{\text{succeed}}^{\{a_{1:i}\}}$ and the corresponding cumulative probability mass function $p_{c,\text{succeed}}^{\{a_{1:i}\}}$, plotted against the cumulative cost of actions $C(\{a_{1:i}\})$. As illustrated in Figure 3, it is likely that a subset of $\mathcal{A}$ will reach a $p_{c,\text{succeed}}^{\{a_{1:i}\}}$ close to one, so that in most cases, performing only the first few actions in $\mathcal{A}$ will be sufficient to satisfy the reliability constraint.

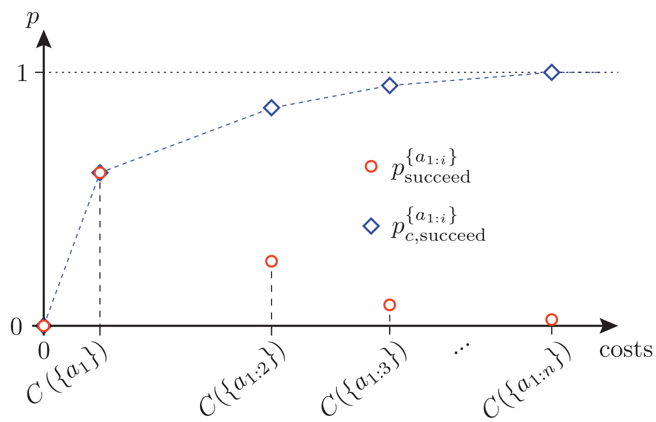

Figure 3: Probability mass function $p_{\text{succeed}}^{\{a_{1:i}\}}$ and corresponding cumulative probability mass function $p_{c,\text{succeed}}^{\{a_{1:i}\}}$ against the cumulative cost of actions $C(\{a_{1:i}\})$.

In decision theory, optimal decisions are those that maximize the expected value of a utility function [25]. Accordingly, the optimization problem at hand consists in finding a sequence of actions $\mathcal{A}_{\text{opt}}$ so that

$$\mathcal{A}_{\text{opt}} = \arg\min_{\mathcal{A}}\{E[C(\mathcal{A})]|p_{c,\text{succeed}}^{\{\mathcal{A}\}} = 1\} \tag{11}$$

in which $E[C(\mathcal{A})]$ is the expected cost for a sequence of measurement and intervention actions $\mathcal{A}$ obtained as

$$E[C(\mathcal{A})] = \sum_{i=1}^{n} \left(p_{\text{succeed}}^{\{a_{1:i}\}} \times C\left(\{a_{1:i}\}\right)\right) \tag{12}$$

where $p_{\text{succeed}}^{\{a_{1:i}\}}$ is the probability of occurrence of a sequence of i actions leading to success.

Decision makers may adopt optimized management policies by planning to perform actions sequentially as defined in $\mathcal{A}_{\text{opt}}$ until $p_{\mathcal{F}}^{\{a_{1:i}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$, $i \in \{1, \cdots, \#\mathcal{A}_{\text{opt}}\}$. By following this procedure, the cost of taking actions will, on average, be equal to $E[C(\mathcal{A}_{\text{opt}})]$. In implementation, each time an action is taken, the subsequent sequence of future actions can be re-optimized. Doing this, the expected cost is likely to be smaller than $E[C(\mathcal{A}_{\text{opt}})]$.

3.2. Computational issues

Implementation of the proposed framework requires addressing four computational issues: (a) Computation of the conditional failure probability $p_{\mathcal{F}}^{\{a_{1:i}\}}$ according to the distribution $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ for each realization of the measurements; (b) computation of the probability of success $p_{\text{succeed}}^{\{a_i\}}$ after action a_i, conditional on lack of success in all previous actions; and (d) solution of the optimization problem in Eq. (11). For (a), existing reliability methods for component and systems, such as FORM, SORM [23, 26] or various sampling methods [27–29] can be used. In cases where the limit-state functions depend on complex FEM analyses, advanced meta-modeling techniques can be used to speed-up calculations [30–33]. Remarks for computational issues (b) and (c) are presented below.

In most cases, a closed-form solution to the integral in Eq. (7) for $p_{\text{succeed}}^{\{a_i\}}$ is not available. Therefore, an approximation must be used. It is noted that for viable candidate actions, the probability $p_{\text{succeed}}^{\{a_i\}}$ should not be too small. If the probability of success is indeed small, the action is useless (unless its cost is negligible, in which case it can be taken without further analysis). We assume that, from the context of the problem, the analyst will be able to identify and exclude non-viable actions from consideration. Thus, given that $p_{\text{succeed}}^{\{a_i\}}$ is not small, say it is of order 0.1 or greater, a simple Monte Carlo solution approach can be used. The algorithm essentially requires repeated simulations of the measurement outcomes $m^{\{a_i\}} = \{m^{\{a_1\}}, \cdots, m^{\{a_i\}}\}$ according to the distribution $f_M^{\{a_{1:i}\}}(m^{\{a_{1:i}\}})$, constructing the corresponding conditional distribution $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$, computing the conditional failure probability $p_{\mathcal{F}}^{\{a_{1:i}\}}$ (by any of the methods mentioned above), and counting the fraction of simulations for which $p_{\mathcal{F}}^{\{a_{1:i}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$. The fraction asymptotically approaches $p_{\text{succeed}}^{\{a_i\}}$ as the number of simulations grow. We employ this approach in the example presented in Sub-section 4.

Given that there are n distinct possible actions, $n!$ is an upper bound for the number of possible sequences of actions. Because the complexity of the problem is $\mathcal{O}(n!)$, optimization algorithms should be used in order to find efficient sequences of actions in a reasonable time. A number of algorithms are available that can solve this problem. One example algorithm is presented in Sub-section 4.1.2.

3.3. Structural intervention and measurement actions

The main categories of actions considered in this paper are: capacity interventions, demand limitation, measurements, model refinement, and increased risk acceptance. Each category and its cost are described below. Note that each category may contain subcategories of actions each having its specific effect and cost.

When assessing the capacity of an existing structure, prior knowledge for $\mathbf{V}$ is available from construction data, code provisions, previous measurements or the literature. The prior knowledge is used to assign the probability distribution $f_{\mathbf{V}}^{\{0\}}(\mathbf{v})$. For example, using code provisions, the compressive strength $f_c^{'}$ of concrete may be characterized by a Lognormal distribution having parameters $\lambda^{\{0\}}$ and $\zeta^{\{0\}}$, $\ln\mathcal{N}(\lambda^{\{0\}},\zeta^{\{0\}})$. As described in the introduction, for an existing structure, this PDF describes the lack of knowledge regarding the actual value of $f_c^{'}$ rather than an inherent variability.

Capacity interventions (a_{CI}) - Capacity interventions increase the capacity of the structure with respect to safety or serviceability limit states. Two types of capacity interventions are considered: replacement (a_{CI1}) and strengthening (a_{CI2}). The replacement of a structure is usually done so that the distribution $f_{\mathbf{V}}^{\{a_{\text{CI1}}\}}(\mathbf{v})$ of the random variables $\mathbf{V}$ of the new structure guarantees that $p_{\mathcal{F}}^{\{a_{\text{CI1}}\}} \leq p_{\mathcal{F}}^{\{\text{adm.}\}}$, i.e., $p_{\text{succeed}}^{\{\text{CI1}\}} = 1$. The corresponding cost is denoted $C^{\{\text{CI1}\}}$.

In the case of a strengthening intervention, the amount of strengthening needed to satisfy the reliability constraint depends on the capacity of the structure. Hence, prior measurement actions that inform on the capacity of the structure can influence the probability of success of a strengthening action. For this reason, it may be desirable to consider candidate-strengthening actions that, when taken alone, do not satisfy the admissible failure probability but may do so subsequent to measurement actions. If such strengthening candidate actions are considered, then $p_{\text{succeed}}^{\{\text{CI2}\}} \in \{0,1\}$. When $p_{\text{succeed}}^{\{\text{CI2}\}} = 1$, the action by itself is sufficient to satisfy the admissible reliability threshold. When $p_{\text{succeed}}^{\{\text{CI2}\}} = 0$, the capacity strengthening intervention is not sufficient to satisfy the admissible threshold without also taking other actions. The cost of strengthening is denoted $C^{\{\text{CI2}\}}$.

Demand limitation (a_{DL}) - A demand limitation action decreases the demand on the structure. In the case of a bridge, limiting the demand may consist in limiting the weight of trucks allowed to travel over the bridge or reducing the number of lanes. This action essentially modifies the distribution of the demand variables so that it shifts towards smaller demand values. For this action, $p_{\text{succeed}}^{\{\text{DL}\}} \in \{0, 1\}$. The case $p_{\text{succeed}}^{\{\text{DL}\}} = 0$ corresponds to the situation where limiting the demand alone is not sufficient to satisfy the reliability constraint. However, this option may become viable subsequent to an information gathering action. Limiting the demand on a structure has an indirect cost for the owner or the society, which we denote as $C^{\{\text{DL}\}}$.

Measurements (a_{ME}) - Measurements can reduce the epistemic uncertainty associated with some of the variables in $\mathbf{V}$. These uncertainties generally have two components: (1) statistical uncertainty and (2) lack of knowledge. The first kind is present in the parameters of distribution models, which are estimated from limited data. This type of uncertainty can be reduced by increasing the data size through additional measurements. For example, the uncertainty in the estimates of the mean and variance of the yield strength of reinforcing bars in a reinforced concrete (RC) structure can be reduced by performing additional sample tests. Uncertainty due to lack of knowledge is associated with the properties of an existing structure. Note that a single, error-free measurement, if physically possible, can eliminate this type of uncertainty for a property value. The cost of a measurement action is denoted $C^{\{\text{ME}\}}$.

One measurement action is that of measuring the real value of an uncertain quantity of the existing structure represented by a random variable V. Before measuring, the outcome of the measurement is unknown. Therefore, the measurement outcome is also a random variable. We denote it as $\hat{V} = V + e_V$, where e_V denotes the measurement error. Thus, the distribution of the future measurement outcome is defined by the prior distribution of V and the distribution of the measurement error. If random variables $\mathbf{V}$ are dependent, the conditional distribution $f_{\mathbf{V}}^{\{\text{ME}\}}(\mathbf{v})$ must be derived conditional on the measurement outcome $\hat{V}$.

Load tests are measurements providing lower-bound information about the capacity R of a structure. A load test may be conducted up to a demand level acceptable with respect to the prior knowledge of the capacity. The admissible proof load $S^{\{\text{adm.}\}}$ is back-calculated from inverse reliability analysis, where the admissible probability of failure is $p_{\mathcal{F}}^{\{\text{adm.}\}}/\gamma$. γ is a safety factor with a value greater than or equal to one in order to avoid a failure during the test [34, 35]. After a successful load test, the evidence that the structure has not failed, i.e. $\{R > S^{\{\text{adm.}\}}\}$, is used to obtain the conditional distribution $f_{\mathbf{V}}^{\{\text{ME}\}}(\mathbf{v})$.

Another type of measurement action aims at calibrating the error in a sub-model, such as a structural capacity model. Suppose $R(\mathbf{x}, \epsilon) = \hat{R}(\mathbf{x}) + \epsilon$ is a capacity model where the model error ϵ has the normal distribution with mean μ_ϵ and standard deviation σ_ϵ. These parameters in general are uncertain with a prior distribution $f^{\{0\}}_{\mu_\epsilon \sigma_\epsilon}$. Using experiments reproducing the failure of the system or component studied, it is possible to generate samples of ϵ and, thereby, update the distribution of the parameters. Specifically, the discrepancies between predicted and observed capacity values $(R - \hat{R})$ during these tests (with known $\mathbf{x}$ values) constitute realizations of the random variable $\epsilon + e$, where e is the measurement error in the experiment. Using the Bayesian updating rule [36], this data can be used to update the distribution of the mean and standard deviation to $f^{\{ME\}}_{\mu_\epsilon \sigma_\epsilon}$.

Model refinement (a_{MR}) - An alternative to calibrating the error in a model is to refine the model and reduce its bias and variability. Model refinement in general changes the formulation of the limit-state functions $G_i(\mathbf{V})$ and may introduce a new set of random variables $\mathbf{V}$. As a result, the prior distribution $f^{\{0\}}_{\mathbf{V}}$ is revised to $f^{\{\mathrm{MR}\}}_{\mathbf{V}}$. The cost of model refinement is denoted $C^{\{\mathrm{MR}\}}$.

Increase risk acceptance (a_{IR}) - When a system or a component has a failure probability greater than the prescribed threshold, it might be desirable to accept a higher risk by increasing the admissible failure probability. This decision may have an impact on insurance premiums and on financial provisions necessary to cover the cost of a potential failure. The cost of increasing risk acceptance is denoted $C^{\{\mathrm{IR}\}}$.

4. Illustrative examples

This section presents illustrative applications of the proposed methodology to two example structures. The chief aim of these examples is to illustrate the formulation and development of the optimal sequence of actions. For this reason a simple structure is considered so as not to burden the reader with unnecessary details.

The first example investigates the reliability of the central column supporting a two-span bridge against buckling (component reliability) and the second example investigates a similar structure supported by two columns (system reliability). The required level of reliability is set at $p_{\mathcal{F}}^{\{\text{adm.}\}} = 0.0013$, which is equivalent to reliability index $\beta = 3$. For each case, we first determine if the column(s) meets this requirement based on the available information. Since the requirement is not met, we develop an optimal plan for a sequence of actions to undertake to assure satisfaction of the reliability constraint, while minimizing expected costs.

4.1. Example 1 - component reliability problem

Figure 4 shows the layout of the considered structure. It is known a-priori that the columns height is $H = 9\,\text{m}$ and that its rectangular section has a depth of $d = 3\,\text{m}$ and a width of $w = 0.25\,\text{m}$. The column is made of reinforced concrete with its elastic modulus E having a lognormal distribution with prior mean $\mu_E^{\{0\}} = 33.5\,\text{GPa}$ and standard deviation $\sigma_E^{\{0\}} = 3\,\text{GPa}$ (corresponding to distribution parameters $\lambda^{\{0\}} = 3.51$ and $\zeta^{\{0\}} = 0.0894$). The contribution of the reinforcement to the flexural stiffness is

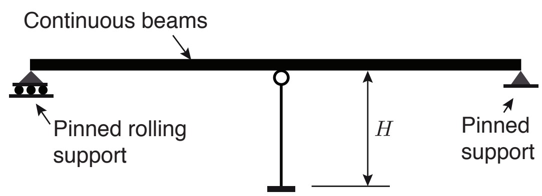

Figure 4: Example two span bridge where the component studied is the central column.

neglected. The top end of the column is pinned and the bottom end is partially fixed by a concrete slab lying on the ground. The effective length coefficient K is represented by a uniform distribution within the interval $(0.7, 1.0)$. The buckling capacity model for this slender column is given by

$$\hat{R} = \frac{\pi^2 EI}{(KH)^2} \tag{13}$$

where $I = dw^3/12$ is the moment of inertial in the weak direction of the column. The true log-capacity is defined by $\ln R = \ln \hat{R} + \epsilon$, where the model error ϵ is a Gaussian random variable having mean μ_ϵ and standard deviation σ_ϵ. It is known that the standard deviation is $\sigma_\epsilon = 0.05$. However, the model bias μ_ϵ is unknown and our prior information is that it is normally distributed with prior mean $\mu_\mu^{\{0\}} = 0.05$ and standard deviation $\sigma_\mu^{\{0\}} = 0.05$. Here, the prior mean of the mean error is greater than zero (the model is conservatively biased), representing the conservative nature of the design model. This could be due to, e.g., the effect of neglecting the contribution of the reinforcement to the section moment of inertia. The total dead load supported by the column is known to be $D = 4000\,\text{kN}$. The column weight is neglected. The maximal live load, L, applied on the column is described by a lognormal distribution with prior mean $\mu_L^{\{0\}} = 600\,\text{kN}$ and standard deviation $\sigma_L^{\{0\}} = 50\,\text{kN}$. The set of random variables defining this problem is $\mathbf{V} = \{E, K, L, \epsilon, \mu_\epsilon\}$. The first four random variables are assumed to be statistically independent, ϵ depends on μ_ϵ.

The column failure is represented by the limit state function

$$G(\mathbf{V}) = R(E, K, \epsilon) - D - L \tag{14}$$

Reliability analysis with the prior information yields the estimated failure probability $p_{\mathcal{F}}^{\{0\}} = 0.0088$ ($\beta \approxeq 2.37$). Since this is greater than the admissible failure probability $p_{\mathcal{F}}^{\{\text{adm.}\}} = 0.0013$, actions must be undertaken to satisfy the reliability constraint. The subsequent sections define the candidate actions considered and determine the optimal sequence of actions that will reduce the estimated failure probability below the admissible threshold, while minimizing the expected costs.

4.1.1. Management actions

Table 1 lists a summary of the considered actions and their costs and effects. Each action is detailed below. We assume that limiting the allowable live load or increasing risk acceptance have costs higher than replacing the structure and are not considered as viable actions.

Table 1: Summary of management actions and their costs and effects.

Management action	Units of costs	Effect
Replace, (a_{CI1})	500	Replaces the column with one that satisfies the reliability constraint
Strengthen, (a_{CI2})	200	Increases capacity by increasing column moment of inertia
Load test, (a_{ME1})	5	If test passes, guarantees that $R > D + L^{\{\text{adm.}\}}$
Measure elastic modulus, (a_{ME2})	10	Reduces epistemic uncertainty in the estimate of column elastic modulus
Calibrate capacity-model error, (a_{ME3})	200	Reduces epistemic uncertainty in the estimate of model bias (mean error)
Refine capacity model, (a_{MR})	10	Reduces epistemic uncertainty in the estimate of effective length coefficient

Replacement and strengthening (CI1, CI2) - Replacement of the structure with a new one that satisfies the reliability constraint, i.e. $p_{\text{succeed}}^{\{a_{\text{CI1}}\}} = 1$, would cost $C^{\{\text{CI1}\}} = 500$. As a strengthening intervention, the inertia of the concrete column can be increased by 5%, for a cost $C^{\{\text{CI2}\}} = 200$. Reliability analysis shows that, when taken alone, this level of strengthening intervention leads to $\beta = 2.72$, which is insufficient to satisfy the reliability constraint. Thus, $p_{\text{succeed}}^{\{a_{\text{CI2}}\}} = 0$.

Load test (ME1) - A load test will be performed up to an admissible live load $L^{\{\text{adm.}\}}$, leading to a failure probability no greater than $p_{\mathcal{F}}^{\{\text{adm.}\}}$, i.e. $\gamma = 1$ (see Sub-section 3.3). It is assumed that the load can be controlled with high precision so that there is no error in the applied load value. The evidence that $\{R \geq D + L^{\{\text{adm.}\}}\}$ will be used to update the failure probability according to the rule

$$P_{\mathcal{F}}^{\{ME1\}} = \frac{Pr(R \leq D + L \cap R \geq D + L^{\{\text{adm.}\}})}{Pr(R \geq D + L^{\{\text{adm.}\}})} \tag{15}$$

The cost of performing a load test is $C^{\{\text{ME1}\}} = 5$. This amount includes the insurance costs covering a potential failure (with probability $p_{\mathcal{F}}^{\{\text{adm.}\}}$) during the test.

Measure elastic modulus (ME2) - The elastic modulus E of the concrete in the column will be measured using a non-destructive test. The logarithm of the measured value is represented by $\ln \hat{E} = \ln E + e$, where e is the measurement error having a normal distribution with zero mean (unbiased measurement) and standard deviation $\sigma_e = 0.05$. This yields a lognormal distribution for $\hat{E}$ with parameters $\lambda^{\{ME2\}} = \lambda^{\{0\}} = 3.51$ and $\zeta^{\{ME2\}} = \sqrt{(\zeta^{\{0\}})^2 + \sigma_e^2} = 0.102$. The cost of measuring E is $C^{\{\text{ME2}\}} = 10$.

Calibration of capacity model error (ME3) - We consider conducting n tests with specimens similar to the bridge column to calibrate the bias in the model error. Let $\bar{\epsilon} = 1/n \cdot (\epsilon_1 + \cdots + \epsilon_n)$ denote the sample mean of the discrepancies $\ln R - \ln \hat{R}$ to be observed. From the Bayesian theory of conjugate pair distributions [37], it is known that for the case under consideration the posterior distribution of μ_ϵ is normal with mean

$$\mu_\mu^{\{ME3\}} = \frac{\mu_\mu^{\{0\}}(\sigma_\epsilon/\sqrt{n})^2 + \bar{\epsilon}(\sigma_\mu^{\{0\}})^2}{(\sigma_\epsilon/\sqrt{n})^2 + (\sigma_\mu^{\{0\}})^2} \tag{16}$$

and variance

$$(\sigma_\mu^{\{ME3\}})^2 = \frac{(\sigma_\epsilon/\sqrt{n})^2(\sigma_\mu^{\{0\}})^2}{(\sigma_\epsilon/\sqrt{n})^2 + (\sigma_\mu^{\{0\}})^2} \tag{17}$$

However, since the experiments are yet to be performed, $\bar{\epsilon}$ remains unknown and we must use our prior information to determine its distribution. Since our present knowledge is that $\epsilon \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon)$, assuming the observations are statistically independent, we have $\bar{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon/\sqrt{n})$, where μ_ϵ has the prior distribution $\mu_\epsilon \sim \mathcal{N}(\mu_\mu^{\{0\}}, \sigma_\mu^{\{0\}})$. To generate a sample of the future observation $\bar{\epsilon}$, we first simulate μ_ϵ according to its prior distribution, then generate a sample according to $\bar{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon/\sqrt{n})$. Alternatively, one can generate a sample of $\bar{\epsilon}$ by using the distribution $\bar{\epsilon} \sim \mathcal{N}(\mu_\mu^{\{0\}}, ((\sigma_\epsilon/\sqrt{n})^2 + (\sigma_\mu^{\{0\}})^2)^{1/2})$. Equations 16 and 17 then yield the posterior mean and variance of the model bias μ_ϵ. The conditional failure probability $p_{\mathcal{F}}^{\{ME3\}}$ for each generated $\bar{\epsilon}$ is computed using the posterior distribution $\mu_\mu \sim \mathcal{N}(\mu_\mu^{\{ME3\}}, \sigma_\mu^{\{ME3\}})$. The option of conducting $n = 2$ tests at a total cost of $C^{\{\text{ME3}\}} = 200$ is considered.

Refine capacity model (MR) - Refining the capacity model would remove some of the uncertainty in the effective length coefficient K by modeling the flexibility of the foundation. If the refined model was exact, it would produce a deterministic value for K. We assume the model will have an error, ϵ_K, that is uniformly distributed within the interval $(-0.025, +0.025)$. Thus, our model of the effective length coefficient is $\hat{K} = K + \epsilon_K$. With our current state of knowledge, $\hat{K}$ is the sum of two uniformly distributed random variables. The cost of developing the refined model is $C^{\{\text{MR}\}} = 10$.

4.1.2. Optimization algorithm and numerical resolution

This section presents the choices made regarding the optimization algorithm and the reliability calculation technique. As described above, there are $n = 6$ candidate actions considered for this example. The optimized sequence of actions $\mathcal{A}_{\text{opt}}$ is obtained using a greedy optimization algorithm [38] that is adapted to this problem. Although the greedy algorithm is known for occasionally leading to sub-optimal solutions, it is chosen for its simplicity and fast convergence. It is noted that the proposed framework is independent of the specific optimization algorithm selected and that other algorithms capable of solving this problem are available [38]. The study of the performance of any particular optimization algorithm for solving this class of problems is beyond the scope of this paper.

With the greedy algorithm, the optimized sequence of actions is constructed iteratively over n loops. For each loop $k = 1, \cdots, n$, the optimized sequence of k actions is

$$\mathcal{A}_{\text{opt},k} = \mathcal{A}_{\text{opt},k-1} \cup \arg\min_{a_i} E[C(\mathcal{A}_{k,i})] \tag{18}$$

in which $\mathcal{A}_{\text{opt},0}$ is an empty set. Essentially, in each step, the algorithm looks for the next best action in the sequence. In order to compute the expected cost $E[C(\mathcal{A}_{k,i})]$, we must have $p_{c,\text{succeed}}^{\{\mathcal{A}_{k,i}\}} = 1$, i.e., the set of actions must assure satisfaction of the reliability constraint. When this condition is not satisfied, an *optimized upper bound* of the expected cost is computed for the sequence $\mathcal{A}_{k,i} = \{\mathcal{A}_{\text{opt},k-1}, a_i, a_{\text{CON}}\}$, where the concluding action a_{CON} is such that $p_{c,\text{succeed}}^{\{\mathcal{A}_{k,i}\}} = 1$. In this example, $a_{\text{CON}} = a_{\text{CI1}}$ is selected because the latter is the only action that guarantees satisfaction of the reliability constraint. The optimization procedure is repeated until $p_{c,\text{succeed}}^{\{\mathcal{A}_{\text{opt},k}\}} = 1$ and the expected cost is then computed for the optimized sequence of actions $\mathcal{A}_{\text{opt}} = \mathcal{A}_{\text{opt},k}$. Note that if

$k = n$ and $p_{c,\text{succeed}}^{\{\mathcal{A}_{\text{opt},k}\}} < 1$, additional alternative actions must be considered.

In this example, Monte Carlo simulations are used to compute the conditional probability of failure and the probability of success of for each sequence of actions. The coefficient of variation (c.o.v.) for computing a probability p by Monte Carlo simulation is

$$\delta_{\hat{p}} = \sqrt{\frac{1 - \hat{p}}{N \cdot \hat{p}}} \tag{19}$$

where $\hat{p}$ is the estimated probability. The minimum number of samples required to obtain a c.o.v. smaller than 0.05 for $\hat{p}_{\text{c,succeed}}^{\{a_{1:i}\}}$ and $\hat{p}_{\mathcal{F}}$ are about $5\,000$ and 3×10^5, respectively. The numbers of samples used in this example are greater than these minima.

4.1.3. Minimization of the expected costs of sequences of actions

Table 2 reports, for each loop and for each action a_i, the optimized upper bound of the expected costs and the probability of satisfying the reliability constraint by that action, $p_{\text{succeed}}^{\{a_i\}}$. For each loop, results corresponding to the optimal action are enclosed in a box and, previously selected actions are marked with the symbol "✓." Results presented for loop #5 are not an upper bound because no concluding action a_{CON} is required to enforce the requirement $p_{c,\text{succeed}}^{\{\mathcal{A}_{k,i}\}} = 1$.

Table 2: Optimized upper bound of the expected costs $E[C(\mathcal{A})]$ and the probability of satisfying the reliability constraint $p_{\text{succeed}}^{\{a_i\}}$ (separated by the symbol "|") computed during each optimization loop. For each loop, the upper bound of expected costs corresponding to the optimal action is enclosed in a box and actions marked with the symbol "✓" represents actions previously selected.

Action	loop #1	loop #2	loop #3	loop #4	loop#5†
Replace, (a_{CI1})	500|1	177|1	109|1	95|1	$\boxed{91\|1}$
Strengthen, (a_{CI2})	700|0	207|0.22	121|0.28	93|0.46	105|0.01
Load test, (a_{ME1})	505|0	159|0.12	98|0.12	$\boxed{91\|0.12}$	✓
Measure E, (a_{ME2})	315|0.39	$\boxed{109\|0.42}$	✓	✓	✓
Calibrate capacity model error, (a_{ME3})	597|0.21	188|0.33	$\boxed{95\|0.55}$	✓	✓
Refine capacity model, (a_{MR})	$\boxed{177\|0.67}$	✓	✓	✓	✓

†: expected costs for loop #5 are not an upper bound.

Based on the results presented in Table 2, the optimal first action is to refine the capacity model with an upper bound expected cost of 177 and $p_{\text{succeed}}^{\{a_{\text{MR}}\}} = 0.67$. After repeating the greedy optimization procedure five times, the best sequence of actions found is $\mathcal{A}_{\text{opt}} = \{a_{\text{MR}}, a_{\text{ME2}}, a_{\text{ME3}}, a_{\text{ME1}}, a_{\text{CI1}}\}$. Strengthening the structure (a_{CI2}) is found to be a suboptimal action. Figure 5 presents the probability mass function $p_{\text{succeed}}^{\{a_{1:i}\}}$ and the cumulative probability mass function $p_{c,\text{succeed}}^{\{a_{1:i}\}}$ against the costs of the optimized

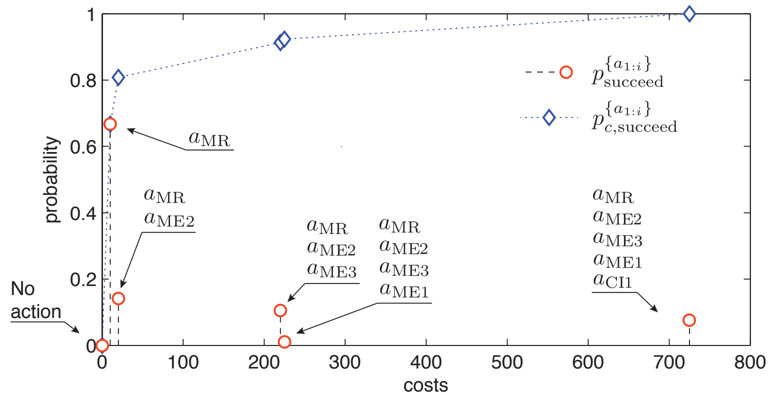

Figure 5: Probability mass function $p_{\text{succeed}}^{\{a_{1:i}\}}$ and cumulative probability $p_{c,\text{succeed}}^{\{a_{1:i}\}}$ for the costs of the optimized sequence of actions.

sequence of actions $\mathcal{A}_{\text{opt}}$. This figure shows that there is a high probability that the low-cost model refinement and measurement actions will be sufficient to reduce the estimated failure probability below the admissible threshold. The overall expected cost for the optimal sequence of actions is $E[C(\mathcal{A}_{\text{opt}})] = 91$, which is substantially lower than the cost of strengthening or replacing the structure. This is a result of the likely favorable outcomes of the low-cost candidate actions of refining the model and measuring the elastic modulus, which together have the success probability $p_{c,\text{succeed}}^{\{a_{\text{MR}},a_{\text{ME2}}\}} = 0.81$.

Results of the analysis indicate that performing a load test as a first action has a zero probability of lowering the failure probability below the admissible level. This is because the initial estimate of the failure probability of the column is large so that the admissible live load, back-calculated from the admissible failure probability, is limited to $241\,\text{kN}$, which is below the mean value. Therefore, for any non-zero cost, a load test is a sub-optimal first action because it is certain that at least one additional action will be required to satisfy the reliability constraint. When performed after having refined the capacity model, the probability of satisfying the reliability constraint with a load test increases to $p_{\text{succeed}}^{\{a_{\text{ME1}}\}} = 0.12$ (see Table 2). Despite this low probability of success, this action is expected to be a more efficient fourth action than the other alternatives because of its low cost. By adopting the optimized management strategy $\mathcal{A}_{\text{opt}}$, there is a probability 0.67 that only refining the model will be sufficient to satisfy the reliability constraint. There is a probability lower than 0.08 that replacing the structure will become necessary after having performed all information gathering actions.

4.2. Example 2 - system reliability problem

This second example presents an application of the methodology to a system reliability problem, where information gathering actions may influence the reliability of more than one component. Figure 6 shows the layout of the considered structure. The components studied are the two columns. The system is deemed to have failed if any of the two columns fails.

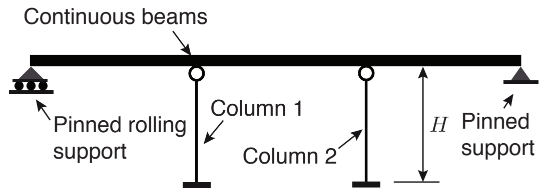

Figure 6: Example three spans bridge studied. The components studied are the two central columns. The case corresponds to a system reliability problem.

The elastic moduli of the two columns are denoted E_1 and E_2. We assume these two random variables are jointly lognormal with prior means $\mu_1^{\{0\}} = \mu_2^{\{0\}} = 33.5\,\text{GPa}$, standard deviations $\sigma_1^{\{0\}} = \sigma_2^{\{0\}} = 3\,\text{GPa}$ and correlation coefficient $\rho^{\{0\}} = 0.9$. It follows that $\ln E_1$ and $\ln E_2$ are jointly normal with prior means $\lambda_i^{\{0\}} = \ln \mu_i^{\{0\}} - (\zeta_i^{\{0\}})^2/2 = 3.51$, standard deviations $\zeta_i^{\{0\}} = \sqrt{\ln(1 + (\delta_i^{\{0\}})^2)} = 0.0894$, and correlation coefficient $\rho_0^{\{0\}} = (\zeta_1^{\{0\}}\zeta_2^{\{0\}})^{-1} \ln(1 + \delta_1^{\{0\}}\delta_2^{\{0\}}\rho^{\{0\}}) = 0.900$, where $\delta_i^{\{0\}} = \sigma_i^{\{0\}}/\mu_i^{\{0\}} = 0.0896$ are the coefficients of variation. The description of all other random variables remains the same. The set of random variables defining this problem is $\mathbf{V} = \{E_1, E_2, K, S_l, \epsilon, \mu_\epsilon\}$.

Reliability analysis with the prior information yields the estimated system failure probability $p_{\mathcal{F}}^{\{0\}} = 0.012$ ($\beta \approxeq 2.27$). Since this is greater than the admissible failure probability $p_{\mathcal{F}}^{\{\text{adm.}\}} = 0.0013$, actions must be undertaken to satisfy the reliability constraint.

4.2.1. Management actions

The same set of actions as in the previous example is considered. Strengthening is assumed to have a cost of 300 and replacement to have a cost of 800. All other actions are assumed to have the same cost as in the previous example and, with the exception of measuring the elastic modulus, to lead to identical effects for both columns. Thus, refining the model would lead to the same change in the effective length coefficient of each column, and calibrating the capacity model by conducting experiments will improve the model for both columns. Strengthening each column by increasing its moment of inertia by 5% leads to a system reliability index of $\beta = 2.61$ based on the prior information, which is insufficient to satisfy the reliability constraint, i.e., $p_{\text{succeed}}^{\{a_{\text{CI2}}\}} = 0$. Due to the statistical dependence between E_1 and E_2, measuring the elastic modulus of one column provides information about the elastic modulus of the other column. Specifically, if we measure the logarithm of the elastic modulus of column 1 as $\ln \hat{E}_1 = \ln E_1 + e$, where $e = \mathcal{N}(0, \sigma_e)$ is the measurement error, one can show that the conditional distribution of E_2 given the observation $\hat{E}_1$ is lognormal with parameters

$$\lambda_{2|1}^{\{ME2\}} = \lambda_2^{\{0\}} + \rho_0' \zeta_2^{\{0\}} \left(\frac{\ln \hat{E}_1 - \lambda_1^{\{0\}}}{\sqrt{(\zeta_1^{\{0\}})^2 + \sigma_e^2}} \right) \tag{20}$$

$$\zeta_{2|1}^{\{ME2\}} = \zeta_2^{\{0\}} \sqrt{(1 - (\rho_0')^2)} \tag{21}$$

where

$$\rho_0' = \rho_0^{\{0\}} \frac{\zeta_1^{\{0\}}}{\sqrt{(\zeta_1^{\{0\}})^2 + \sigma_e^2}} \tag{22}$$

In the following analysis, we also explore the option of measuring the elastic moduli of both columns.

4.2.2. Minimization of action expected costs

Table 3 reports, for each loop and for each action a_i, the optimized upper bound of the expected costs and the probability of satisfying the reliability constraint by that action, $p_{\text{succeed}}^{\{a_i\}}$. For each loop, results corresponding to the optimal action are enclosed in a box and, previously selected actions are marked with the symbol "✓." Results presented for loop #5 are not an upper bound because no concluding action a_{CON} is required to enforce the requirement $p_{c,\text{succeed}}^{\{\mathcal{A}_{k,i}\}} = 1$. Results presented in Table 3 are similar to the results obtained for the first example. The main difference is that gathering information about the elastic modulus of one column provides information for the second column. As a result, there is reduced economical incentive of measuring both columns. Note that the optimal sequence identified is the same as in the previous example even if, in this case, expected costs are higher.

Note in Table 3 that the probability of satisfying the reliability constraint $p_{\text{succeed}}^{\{a'_{\text{ME2}}\}}$ computed at the third loop is small. As mentioned in Section 3.2, the Monte Carlo method is not the most suited for computing such small probabilities. However, the accuracy of $p_{\text{succeed}}^{\{a'_{\text{ME2}}\}}$ could not have changed the choice of the optimal action because action a_{ME1}, which is itself suboptimal, has a probability of success of 0.13 and a

Table 3: Optimized upper bound of the expected costs $E[C(\mathcal{A})]$ and the probability of satisfying the reliability constraint $p_{\text{succeed}}^{\{a_i\}}$ (separated by the symbol "|") computed during each optimization loop. For each loop, the action marked with the symbol "✓" represent actions previously selected and the upper bound of expected costs corresponding to the optimal action is enclosed in a box. The symbol "-" denotes an action that has been discarded.

Action	loop #1	loop #2	loop #3	loop #4	loop#5[†]
Replace, (a_{CI1})	800|1	304|1	192|1	130|1	$\boxed{121\|1}$
Strengthen, (a_{CI2})	1100|0	354|0.21	205|0.30	125|0.45	143|0.01
Load test, (a_{ME1})	805|0	275|0.10	170|0.13	$\boxed{121\|0.13}$	✓
Measure $E1$, (a_{ME2})	539|0.51	$\boxed{192\|0.39}$	✓	✓	✓
Measure $E2$, (a'_{ME2})	539|0.51	192|0.39	192|0.01	-	-
Calibrate capacity model error, (a_{ME3})	897|0.12	284|0.32	$\boxed{130\|0.60}$	✓	✓
Refine capacity model, (a_{MR})	$\boxed{304\|0.63}$	✓	✓	✓	✓

[†]: expected costs for loop #5 are not an upper bound.

cost lower than measuring E_2. For this reason, action a'_{ME2} has been discarded from consideration in the subsequent loops.

5. Discussion

The methodology presented in this paper allows having a complete picture of the expected cost of actions considered for reducing the estimated failure probability of a structure below an admissible threshold. Without minimizing the expected cost for a complete sequence of actions, decisions may be made for either cheap actions that do not necessarily satisfy the reliability constraint, or conservative actions that are not economically efficient. Furthermore, when the optimization is performed for only one action at the time, i.e., without optimizing a whole sequence, the information about the combined potential of multiple actions is missing.

The proposed pre-posterior optimization framework can be extended to the analysis of actions necessary to satisfy reliability constraints for multiple structures, while minimizing overall expected costs. Such an approach can be used for optimal maintenance management of structures within an infrastructure system.

6. Summary and conclusions

This paper provides a pre-posterior framework for optimizing sequences of information gathering and intervention actions and for quantifying the potential gain of measuring structures instead of choosing costly strengthening and replacement options. The illustrative example shows that, when a structure does not satisfy an admissible failure probability, strengthening or replacement interventions can be sub-optimal first actions. The examples also show that significant savings can be achieved by reducing the epistemic uncertainty in existing structures before costly interventions are made to assure sufficient reliability. In terms of future work, the proposed framework opens new opportunities for enhancing network-level infrastructure management.

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