

**Titre:** Development of a model to predict the water retention curve using basic geotechnical properties  
Title:

**Auteurs:** Michel Aubertin, Mamert Mbonimpa, Bruno Bussière, & Robert P. Chapuis  
Authors:

**Date:** 2003

**Type:** Rapport / Report

**Référence:** Aubertin, M., Mbonimpa, M., Bussière, B., & Chapuis, R. P. (2003). Development of a model to predict the water retention curve using basic geotechnical properties. (Rapport technique n° EPM-RT-2003-01). <https://publications.polymtl.ca/2604/>  
Citation:

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**Institution:** École Polytechnique de Montréal

**Numéro de rapport:** EPM-RT-2003-01  
Report number:

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**EPM-RT-2003-01**

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WATER RETENTION CURVE USING BASIC  
GEOTECHNICAL PROPERTIES**

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**January 2003**

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**EPM-RT-2003-01**

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**JANUARY 2003**

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Bibliothèque nationale du Canada, 2003

EPM-RT-2003-01

*Oxygen diffusion and consumption in unsaturated cover materials*

by: Michel Aubertin<sup>A,1</sup>, Mamert Mbonimpa<sup>A,1</sup>, Bruno Bussière<sup>B,1</sup> et Robert P. Chapuis<sup>A</sup>.

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## ABSTRACT

The water retention curve (WRC) has become a key material function to define the unsaturated behavior of soils and of other particulate media. In many instances, it can be very useful to have an estimate of the WRC early in a project, when little or no test results are available. Predictive models, based on easy to obtain geotechnical properties, can also be employed to evaluate how changing parameters (e.g. porosity or grain size) affect the WRC. In this paper, the authors present a general set of equations developed for predicting the relationship between volumetric water content  $\theta$  (and the corresponding degree of saturation  $S_r$ ) and suction  $\psi$ . The proposed WRC model is a modified version of the Kovács (1981) model, which makes a distinction between water retention due to capillary forces and retention by adhesion. The complete set of equations is given together with complementary relationships developed for specific applications on granular materials and on plastic/cohesive (clayey) soils. It is shown that the model provides a simple and practical means to estimate the water retention curve from basic properties. A discussion follows on the capabilities and limitations of the model.

**Keywords:** water retention curve, unsaturated soils, prediction, porosity, grain size, liquid limit.

## RÉSUMÉ

La courbe de rétention d'eau (CRE) est devenue une fonction clé pour définir le comportement non saturé des sols et d'autres matériaux meubles. Dans beaucoup de cas, il peut être très utile d'avoir une évaluation de la CRE dans les premières phases d'un projet, lorsque peu ou pas de résultats d'essais sont disponibles. Des modèles prédictifs, basés sur les propriétés géotechniques de base, peuvent aussi être utilisés pour évaluer comment le changement des paramètres (en termes de porosité ou de granulométrie) affecte la CRE. Dans cet article, les auteurs présentent un ensemble d'équations développées pour prédire la relation entre la teneur en eau volumique  $\theta$  (et le degré de saturation  $S_r$ , correspondant) et la succion  $\psi$ . Le modèle proposé pour la prédiction de la CRE est une version modifiée du modèle de Kovács (1981), qui fait une distinction entre la rétention d'eau due aux forces capillaires et celle par adhésion. Ce jeu d'équations est donné avec des relations complémentaires développées pour des applications spécifiques sur des matériaux granulaires et sur des sols (argileux) plastiques/cohérents. Il est montré que le modèle constitue un moyen simple et pratique pour estimer la courbe de rétention d'eau à partir des propriétés géotechniques de base. Une discussion suit sur les capacités et les limitations du modèle.

**Mots clés:** courbe de rétention d'eau, sols non saturés, prédiction, porosité, granulométrie, limite de liquidité.

## **ACKNOWLEDGEMENTS**

The post-doctoral grant provided to the first author (Mamert Mbonimpa) by the Institut de Recherche en Santé et Sécurité du Travail du Québec (IRSST) is thankfully acknowledged. Special thanks also go to Antonio Gatien and to the graduate students who performed the diffusion tests over the years. The authors also received financial support from NSERC and from a number of industrial participants to the industrial NSERC Polytechnique-UQAT Chair on Environment and Mine Wastes Management.



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## LIST OF NOTATIONS

$a_c$	adhesion coefficient (-)
$A_v$	surface of voids (cm <sup>2</sup> )
$b$	coarse grained material parameter to calculate $h_{co,G}$ (cm <sup>2</sup> )
$b_1$	coarse grained material parameter to estimate $\psi_a$ (cm <sup>x</sup> l <sup>+1</sup> )
$b_2$	coarse grained material parameter to correlate $h_{co,G}$ and $\psi_a$ (-)
$C_\psi$	corrector factor (-)
$C_U$	uniformity coefficient (-) ( $C_U=D_{60}/D_{10}$ )
$d$	diameter of a tube (cm)
$D_{10}$	diameter corresponding to 10% passing on the cumulative grain-size distribution curve (cm)
$D_{60}$	diameter corresponding to 60% passing on the cumulative grain-size distribution curve (cm)
$d_{eq}$	equivalent pore size diameter (cm)
$D_H$	equivalent grain size diameter (cm)
$e$	void ratio (-)
$D_r$	specific gravity of the solid particles (-)
$h_c$	capillary rise in a tube (cm)
$h_{cmax}$	maximum capillary rise [L]
$h_{cmin}$	minimum capillary rise [L]
$h_{co}$	equivalent capillary rise in a porous material (cm)
$h_{co,G}$	equivalent capillary rise in a coarse grained material (cm)
$h_{co,P}$	equivalent capillary rise in a plastic/cohesive material (cm)
$m$	pore size distribution parameter in the MK model (-)
$n$	porosity (-)
$S_a$	adhesion component of the degree of saturation (-)
$S_a^*$	truncated adhesion component of the degree of saturation (-)
$S_c$	capillary component of the degree of saturation (-)
$S_m$	specific surface area per unit mass of solid (m <sup>2</sup> /g)

$S_r$	degree of saturation (-)
$\sigma_w$	surface tension of water (N/m)
$u_a$	air pressure (cm)
$u_w$	water pressure (cm)
$V_v$	volume of the voids (cm <sup>3</sup> )
$w$	gravimetric water content (-)
$w_L$	liquid limit (%)
$w_P$	plastic limit (%)
$x_1$	coarse grained material parameter to estimate $\psi_a$ (-)
$x_2$	coarse grained material parameter to correlate $h_{co,G}$ and $\psi_a$ (-)
$\psi$	suction (cm)
$\xi$	plastic/cohesive material parameter required to calculate $h_{co,P}$ (cm)
$\rho_s$	solid grain density (kg/m <sup>3</sup> )
$\alpha$	shape factor (-)
$\chi$	material parameter used to estimate $S_m$ (-)
$\lambda$	material parameter used to estimate $S_m$ (m <sup>2</sup> /g)
$\theta$	volumetric water content (-)
$\psi_0$	suction at complete dryness ( $S_r = 0$ ) (cm)
$\psi_{90}$	suction corresponding to a degree of saturation of 90% (cm)
$\psi_{95}$	suction corresponding to a degree of saturation of 95% (cm)
$\psi_a$	air entry value or AEV (cm)
$\psi_{a,exp}$	air entry value determined from the experimental data (cm)
$\psi_{a,MK}$	air entry value determined from the WRC predicted with the MK model (cm)
$\psi_r$	residual suction (corresponding to the residual water content) (cm)
$\psi_{r,exp}$	residual suction determined from the experimental data (cm)
$\beta_w$	contact angle (-)
$\gamma_w$	unit weight of water (kN/m <sup>3</sup> )

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## 1. INTRODUCTION

In geotechnical engineering, subsurface water is often divided into free water in the saturated zone and moisture retained in the unsaturated (vadose) zone (e.g. Bowles 1984; Smith 1990; McCarthy 1998). Even if the behavior of water and soils under saturated condition can be treated as a special case of the more general partially saturated situation (e.g. Fredlund and Rahardjo 1993), such distinction is justified by the physical differences that characterize the two zones. Owing to the particularities that exist in unsaturated soils, special attention should be paid to adequately define the response of water above the phreatic surface.

Geotechnique for unsaturated media is a rapidly expanding field, which is related to a wide variety of applications in soil mechanics and soil physics including: estimation of field capacity (Meyer and Gee 1999); efficiency analysis of covers with capillary barrier effects (Nicholson et al. 1989; Aubertin et al. 1993, 1995a, 1996, 1997; Khire et al. 1995; O’Kane et al. 1998; Morris and Stormont 1999; Bussière 1999); bearing capacity of unsaturated foundation materials (Rassam and Williams 1999); seepage through dams (Chapuis et al. 1996, 2001; Fredlund 2000a); compressibility and swelling soil response (Fredlund, 1987; Rampino et al. 2000; Hung and Fredlund 2000); shear strength and constitutive behavior (e.g. Alonso et al. 1990; Fredlund et al. 1996, Vanapalli et al. 1996; Cui and Delage 1996); contaminant transport (Badv and Rowe 1996; Lim et al. 1998; Barbour 1998, Esposito et al. 1999); land subsidence (Thu and Fredlund 2000); freeze-thaw response of road structures (Konrad and Roy 2000). Fredlund (2000a) presents an interesting overview of various applications of unsaturated soil mechanics.

The distribution and motion of water in the unsaturated zone are closely related to forces caused by molecular attraction responsible for water adhering to solid surfaces (i.e. hygroscopic or adsorbed water on soil particles) and for surface tension causing capillary retention due to cohesion between water molecules at the interface with air (e.g. Bear 1972; Marshall et al. 1996). In saturated media, the adsorption forces tend to reduce the space available for water flow, hence reducing the effective porosity and hydraulic conductivity, while capillary forces disappear in the

absence of a water-air interface. In partly saturated conditions, these two distinct but complementary types of force affect the hydrogeological and mechanical behavior of soils. The corresponding properties, including hydraulic conductivity and shear strength parameters, are no longer material constants but rather depend on the relative amount of water and gas in the pore space.

The quantity of water retained in a soil by capillary and adhesion forces depends on many factors, namely: shape, size and distribution of pore space; mineralogy and surface activity of solid grain particles; chemical composition of interstitial water. Richards (1931) equation, which is a combination of Darcy's law and conservation equation, is commonly used to solve groundwater flow problems for variably saturated conditions. In this equation, the hydraulic conductivity is a function of the state variables, which are typically expressed in terms of volumetric water content  $\theta$  or of suction  $\psi$  (e.g. Leong and Rahardjo 1997a; Leij et al. 1997). For a given porous media, these two parameters,  $\theta$  and  $\psi$ , are related to each other and form what is being considered as a fundamental material characteristic identified here as the water retention curve WRC. Many other names are used for the  $\theta$ - $\psi$  relationship, including the soil-water characteristic curve (SWCC) which however becomes less appropriate when dealing with other types of porous material (such as ceramic plates or geotextiles) and with man made particulate media (such as mined rock wastes and tailings).

In a porous system, increasing the value of suction  $\psi$  (defined by  $u_a - u_w$ , where  $u_a$  is the air (gas) pressure and  $u_w$  is the water pressure) tends to reduce  $\theta$ . The desaturation is typically more pronounced in coarse grained materials (such as sand and gravel) than in fine grained materials (such as silt and clay). The value of  $\theta$  at a given  $\psi$  also depends on the path, whether it occurs during a wetting or drying phase. Different paths induce somewhat different curves (e.g. Parlange 1976; Mualem 1984), but such hysteresis phenomena will not be addressed directly here, as only the drainage path is considered (to simplify the presentation).



When  $\theta$  is diminished, the flow of water is more difficult because of the reduced area available in the water phase. The unsaturated hydraulic conductivity  $k_u$  thus depends on  $\theta$  (or  $\psi$ ). At a sufficiently low water content, the water phase becomes discontinuous and  $k_u$  is reduced to a very small (near zero) value.

To perform groundwater flow analyses as required in a number of applications in geotechnique and soil physics, engineers and scientists must be able to quantify the  $k_u$  function. Because direct measurement of  $k_u$  can be difficult, time consuming and costly, it is customary to use, as a starting point, some relationship between  $\theta$  and  $\psi$  (e.g. Mualem 1976, 1986; Fredlund et al. 1994) because the WRC can be evaluated more easily than  $k_u$  in the laboratory or in the field. The resulting  $\theta$ - $\psi$  data, obtained from test measurements, are plotted and used to derive specific mathematical functions with curves that run through the data points. Such fitting equations have been proposed by Gardner (1956), Brooks and Corey (1964), Brutsaert (1966), Haverkamp et al. (1977), Vauclin et al. (1979), van Genuchten (1980), Bumb et al. (1992), Fredlund and Xing (1994), and Kosugi (1994), to name a few. These mathematical expressions with the corresponding parameter values deduced experimentally or otherwise, can be very useful for particular applications when the necessary data is available. Many of these equations have been compared to each other over the years (e.g. Khire et al. 1995; Leij et al. 1997; Leong and Rahardjo 1997b; Burger and Shackelford, 2001), showing that they each have some advantages and limitations, depending on the technique and data bank used for comparison.

In some cases, it can be very useful to obtain *a priori* an estimate of the WRC from easily and rapidly obtained properties. Unfortunately, most predictive models, sometimes referred to as pedotransfer functions (e.g. Verecken et al. 1992; Schaap and Leij 1998; Elsenbeer 2001), are based on empirically derived expressions that bear little physical meanings, and this limits their significance when trying to extend their use outside the range of properties considered for their development and calibration.

In the following, the authors propose a set of equations ensuing from the physical model originally developed by Kovács (1981). This model, which includes a capillary and an adhesion component, was previously used to predict the WRC of tailings from hard-rock mines (Aubertin

et al. 1998). In this paper, the model is generalized and all its components are explicitly defined so that it can be applied to a variety of homogeneous media, including coarse and fine grained soils, tailings, and bentonite mixtures.

## 2. BASIC CONCEPTS

The water retention curve (WRC) represents the relationship between the volumetric water content  $\theta$  and suction  $\psi$ . The former parameter is related to porosity  $n$  and degree of saturation  $S_r$  ( $\theta = nS_r$ ). It can also be expressed as a function of the more common massic (gravimetric) water content  $w$  (i.e.  $\theta = w(1-n)D_r$ , where  $D_r$  is the relative density of the solid particles). The suction value  $\psi$ , on the other hand, is typically expressed as a potential using either pressure units (ex. kPa), pressure head (ex. m of water), or centimetric logarithmic head (known as pF); at 20°C, 9.81kPa  $\equiv$  100cm of water  $\equiv$  pF of 2. The total suction  $\psi$  is the sum of the matric potential  $\psi_m$  and of the osmotic potential  $\pi$ , but the latter is often neglect in geotechnical applications ( $\psi = \psi_m$  is assumed here). For convenience,  $\psi$  takes a positive value under negative water pressure  $u_w$  above the phreatic water surface (where  $u_w = u_a$  or  $\psi = u_a - u_w = 0$ , with  $u_a$  taken as the reference air pressure).

A large number of experimental investigations have been devoted to developing and applying techniques to obtain the  $\theta$ - $\psi$  relationship of soils. Accordingly, various direct and indirect measurement methods have been proposed for the WRC, and many of these have been reviewed in recent textbooks and monographs (e.g. Klute 1986; Carter, 1993; Fredlund and Rahardjo 1993; Marshall et al. 1996). The evolution and understanding of the WRC and of the corresponding measurement techniques have been recently presented by Barbour (1998), who provides an interesting historical perspective.

Measurement of the WRC in the laboratory and in the field, although less demanding than that of  $k_u$ , can be relatively time consuming and expensive. In some situations, it may be very useful to have estimates of the WRC, especially during the preliminary stages of a project when the available information is limited. Predictive models for the WRC, which are usually based on simple geotechnical (pedologic) properties, such as grain size and porosity, can also be of help when analyzing and validating test results, and also for evaluating the variations that can be expected in the field within a non homogenous deposit.

Over the last two decades or so, quite a few predictive models have been proposed (see Elsenbeer 2001, and the corresponding Monograph Issue on the topic). These include a number of functional regression methods using some of the above mentioned empirical WRC equations. The fitting parameters are then related to particular textural parameters (e.g. Cosby et al. 1984; Schaap et al. 1998; Fredlund 2000b). Other discrete regression methods make no *a priori* assumption regarding the shape of the WRC, and construct empirical models from the regression equations linking the  $\theta$ - $\psi$  values and basic properties (e.g. Gupta and Larson 1979; Rawls and Brackensiek 1982; Rawls et al. 1991; Vereecken et al. 1992; Tietje and Tapkenhinrichs 1993). It appears preferable however to use predictive models for which the WRC equation is based on physical characteristics of the media. This latter type of model includes those of Arya and Paris (1981), Haverkamp and Parlange (1986), Tyler and Wheatcraft (1989, 1990), Haverkamp et al. (1999), and Arya et al. (1999).

Such a physically-based model was also proposed by Kovács (1981). Still relatively unknown and seldom used in the geotechnical field, the Kovács (1981) model recognizes the need to distinguish between capillary and adhesive forces. This approach potentially provides a more realistic view, as it can be related to the actual processes involved (e.g. Celia et al. 1995; Nitao and Bear 1996). However, the Kovács (1981) model, in its original version, did not easily lend itself to practical engineering applications because of the relatively poor definition of key parameters needed for its use. The authors have nevertheless been able to apply it, after some modifications (hence the name Modified Kovács or MK model) to the WRC of artificial silts or tailings (Aubertin et al. 1998). In the following, the MK model is extended for general applications to various types of soils, from coarse sand to clayey materials. The basic

assumptions and theoretical considerations behind these extensions are briefly presented. The descriptive and predictive capabilities of the MK model are shown using tests results taken from the literature and obtained in the author's own facilities.

### **3. THE MK MODEL**

The Kovács (1981) model is used here as the basis to develop a practical set of equations for predicting the WRC of various types of soils from commonly used geotechnical properties. The original model has been described in details in Kovács' (1981) textbook, and was also reviewed by the authors in a paper where the model has been adapted for tailings from hard rock mines (Aubertin et al., 1998). Hence, only its basic components will be recalled.

The proposed MK model presented in the following retains the sound physical concepts from which the original model was constructed. The non-trivial modifications introduced serve to generalize the statistical function used to describe the pore-size distribution of the media appearing in the capillary component. Some parameters, ill defined originally, are expressed more specifically as a function of basic soil properties. The ensuing set of equations allows the user to obtain an evaluation of the WRC, which can be employed for practical calculations involving problems with unsaturated conditions. As will be shown in the next sections, the MK model is easy to use and generally provides a good estimate of the WRC for various types of granular as well as plastic/cohesive materials. The MK model can also be used in a more classical (descriptive) manner to draw the WRC from a few relevant testing results. Because of their close relationship with material response, the model equations can be employed to evaluate the expected influence of changing properties on the WRC, such as varying grain size, porosity, or liquid limit.

### 3.1 The equivalent capillary rise

The MK model makes use of a parameter defined as the equivalent capillary rise  $h_{co}$  [L] in the porous medium. The role of this parameter is the same as the so-called average capillary rise in the original Kovács (1981) model. The adopted denomination is justified below. This parameter can be derived from the well-known expression used for the capillary rise  $h_c$  [L] of water in a tube having a diameter  $d$ . The value of  $h_c$  is given by (Smith 1990; Chin 2000):

$$[1] \quad h_c = \frac{4\sigma_w \cos \beta_w}{\gamma_w d}$$

where  $\sigma_w$  [MT<sup>-2</sup>] is the surface tension of water ( $\sigma_w = 0.073$  N/m at 20°C),  $\beta_w$  [-] is the contact angle between water and the tube surface ( $\beta_w = 0^\circ$  for quartz and glass),  $\gamma_w$  [ML<sup>-2</sup>T<sup>-2</sup>] is the unit weight of water ( $\gamma_w = 9.8$  kN/m<sup>3</sup> at 20°C). This well-known equation indicates that capillary rise is inversely proportional to the diameter of the tube. When applied to pore space in soils above the phreatic surface, this equation helps to understand why there can be a much larger capillary rise in fine grained soils, where the voids (somewhat similar to capillary tubes) are small, than in coarse grained soils where the voids are typically larger.

In soils however, the pore size is not uniform so  $h_c$  is not easily defined. This pore system can be substituted by a system of regular channels with a diameter expressed as the equivalent hydraulic pore diameter  $d_{eq}$  [L] defined as (e.g. Bear 1972; Kovács 1981):

$$[2] \quad d_{eq} = 4 \frac{V_v}{A_v}$$

where  $V_v$  [L<sup>3</sup>] and  $A_v$  [L<sup>2</sup>] are respectively the volume and surface of the voids. In practice,  $A_v$  approximately corresponds to the surface area  $A_G$  [L<sup>2</sup>] of the solid grains. By relating  $A_G$  to the massic specific surface area  $S_m$  [L<sup>2</sup>M<sup>-1</sup>], equation (2) can be transformed as (Scheidegger 1974):

$$[3] \quad d_{eq} = 4 \frac{e}{\rho_s S_m}$$

In this equation,  $e$  [-] is the void ratio ( $V_v/V_s$ , where  $V_s$  is the volume of the solid grains) and  $\rho_s$  [ $ML^{-3}$ ] is the solid grain density.

The capillary rise  $h_{co}$  of a soil above the water table, obtained by replacing diameter  $d$  (in equation 1) by the equivalent hydraulic pore diameter  $d_{eq}$ , can therefore be expressed as:

$$[4] \quad h_{co} = \frac{\sigma_w \cos \beta_w}{\gamma_w} \frac{\rho_s S_m}{e}$$

Parameter  $h_{co}$  defined by equation 4 is the equivalent capillary rise of the soil.

Although  $S_m$  can be directly measured by various techniques (e.g. Lowell and Shields 1984; Igwe 1991), in most practical cases, the value of  $S_m$  is not readily available. For coarse grained soils, the specific surface area can nevertheless be estimated from the grain size distribution using the following expression (Kovács 1981):

$$[5] \quad S_m = \frac{\alpha}{\rho_s D_H}$$

where  $\alpha$  [-] is a shape factor ( $6 \leq \alpha \leq 18$ ;  $\alpha = 10$  is used here - see discussion below), and  $D_H$  [L] is an equivalent particle diameter for a heterogeneous mixture. The equivalent diameter  $D_H$  for a heterogeneous mix of particles theoretically represents the diameter of a homogeneous mix (with a single size) that has the same specific surface area as the heterogeneous one.

Equation 4 can thus be rewritten to calculate the equivalent capillary rise in a soil above the water table:

$$[6] \quad h_{co,G} = \frac{\sigma_w \cos \beta_w}{\gamma_w} \frac{\alpha}{eD_H}$$

where subscript  $G$  stands for granular (low plasticity, low cohesion) materials, as opposed to clayey (plastic/cohesive) materials (which will be discussed below). In this equation, the contact angle  $\beta_w$  will be taken as zero (e.g. Marshall et al. 1996).

In granular soils,  $D_H$  can be evaluated by subdividing the grain size curve (e.g. Chapuis and Légaré 1992). For practical geotechnical applications, the value of  $D_H$  can also be approximated using a function of a representative size, like the effective diameter  $D_{10}$  [L] (diameter corresponding to 10 % passing on the cumulative grain-size distribution curve), and of the uniformity coefficient  $C_U$  (i.e.  $D_H = [1 + 1.17 \log(C_U)] D_{10}$ ; Aubertin et al. 1998; Mbonimpa et al. 2000, 2002). For the capillary rise of water in granular soils, equation (6) is expressed as follow:

$$[7] \quad h_{co,G} = \frac{b}{eD_{10}}$$

with

$$[8] \quad b = \frac{10\sigma_w}{[1.17 \log(C_U) + 1]\gamma_w}$$

Parameter  $b$  [L<sup>2</sup>] is expressed in cm<sup>2</sup>, while  $h_{co}$  and  $D_{10}$  are expressed in cm. For the values of  $\sigma_w$  and  $\gamma_w$  given above, equation 8 becomes:

$$[9] \quad b = \frac{0.75}{1.17 \log(C_U) + 1}$$

These equations mean that, for a uniform sand with  $D_{10} = 0.01$  cm,  $C_U = 5$  and  $e = 0.6$ ,  $b = 0.412$  cm<sup>2</sup>, the equivalent capillary rise  $h_{co,G}$  is about 69 cm, while for a silt with  $D_{10} = 0.0005$  cm,  $C_U = 20$  and  $e = 0.6$ ,  $b = 0.297$  cm<sup>2</sup>, and  $h_{co,G}$  would be 990 cm approximately. These values of

$h_{co,G}$  are much larger than the height of the capillary fringe (as defined in most textbooks) which rather represents more or less the air entry value (AEV) of the material. The physical meaning of these parameters will be further discussed below.

For clayey (plastic/cohesive) soils, the above equations can provide inadequate estimates of  $S_m$  and  $h_{co}$ , particularly when the liquid limit  $w_L$  (%) is above about 30 to 40%. For such fine grained soils, other factors influence their water retention capacity. In this case,  $S_m$  (in equation 4) is better estimated using the relationship that exists between the specific surface area and the liquid limit  $w_L$  (%). The following expression, recently proposed by Mbonimpa et al. (2002), is used here:

$$[10] \quad S_m = \lambda w_L^\chi$$

where  $\lambda$  [ $L^2M^{-1}$ ] and  $\chi$  (unitless) are material parameters. Using a fairly large number of tests results from various sources, it has been established that  $\lambda \approx 0.2 \text{ m}^2/\text{g}$  and  $\chi = 1.45$  for materials with  $22 \text{ m}^2/\text{g} \leq S_m \leq 433 \text{ m}^2/\text{g}$  and  $18 \% \leq w_L \leq 127 \%$ .

Combining equations 4 and 10 gives:

$$[11] \quad h_{co,P} = \frac{\xi}{e} w_L^{1.45}$$

where subscript  $P$  stands for plastic/cohesive materials. From the previous developments, parameter  $\xi$  [L] can be expressed as:

$$[12] \quad \xi = \frac{\sigma_w \cos \beta_w}{\gamma_w} \lambda \rho_s$$



The units for  $\xi$  and  $h_{co,P}$  are cm. For the above values of  $\sigma_w$  and  $\gamma_w$ ,  $\beta_w = 0^\circ$ ,  $\lambda \approx 0.2 \text{ m}^2/\text{g}$ , and  $\rho_s$  in  $\text{kg}/\text{m}^3$ ,  $\xi$  becomes:

$$[13] \quad \xi \approx 0.15 \rho_s$$

From equation 11 and 13, one can calculate that for a clayey soil with  $w_L = 40\%$ ,  $\rho_s = 2700 \text{ kg}/\text{m}^3$  and  $e = 0.8$ ,  $\xi = 405 \text{ cm}^2$ , and  $h_{co,P}$  is equal to  $106500 \text{ cm}$ ; for  $w_L = 80\%$ ,  $\rho_s = 2700 \text{ kg}/\text{m}^3$  and  $e = 0.8$ , one obtains  $\xi = 405 \text{ cm}^2$  and  $h_{co,P} = 290968 \text{ cm}$ . Again, these values are much larger than the water rise corresponding to the capillary fringe. It will be seen below that  $h_{co}$  should rather be compared to the potential elevation (or suction  $\psi_r$ ) for the residual water content.

### 3.2 The WRC equations

The MK model uses  $h_{co}$  as a reference parameter to define the relationship between volumetric water content  $\theta$  and matric suction  $\psi$ . As mentioned above, both the original Kovács (1981) model and the MK model consider that water is held by two types of forces, i.e. capillary forces responsible for capillary saturation  $S_c$  and adhesive forces causing saturation by adhesion  $S_a$ . The  $S_c$  component equation is obtained from a cumulative pore size distribution function, while the equation of  $S_a$  is given by an interaction law with van der Waals type attraction between grain surface and water dipoles. The  $S_c$  component is more important at relatively low suction values, while the  $S_a$  component becomes dominant at higher suction when capillary water has been withdrawn.

The proposed set of equations for the MK model is written as follows for the degree of saturation  $S_r$ :

$$[14] \quad S_r = \frac{\theta}{n} = S_c + S_a^* (1 - S_c)$$

$$[15] \quad S_c = 1 - \left[ \left( h_{co}/\psi \right)^2 + 1 \right]^m \exp \left[ -m \left( h_{co}/\psi \right)^2 \right]$$

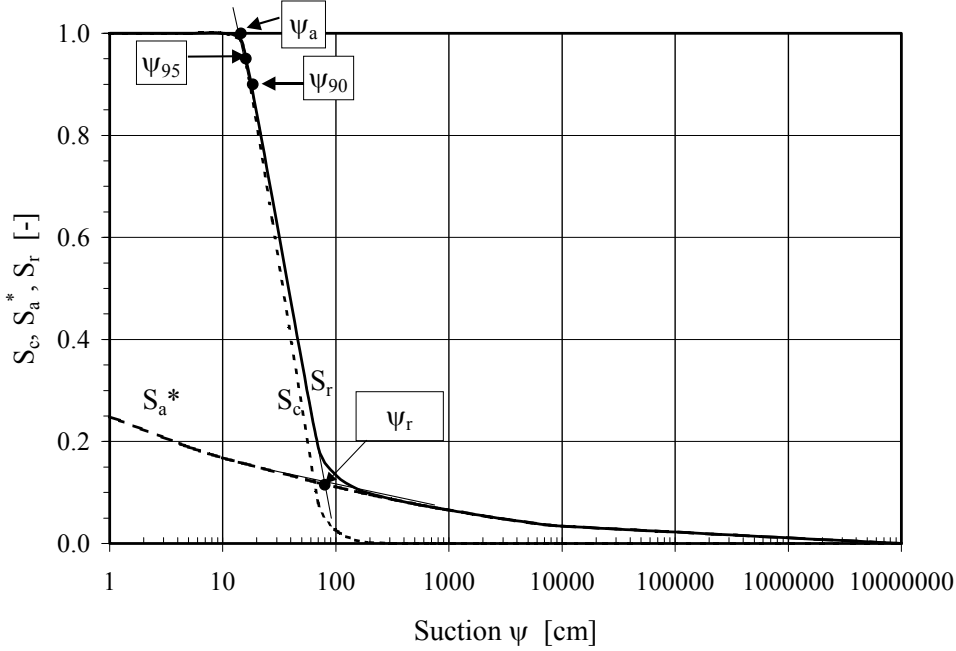
$$[16] \quad S_a^* = 1 - \langle 1 - S_a \rangle$$

$$[17] \quad S_a = a_c C_\psi \frac{\left( h_{co}/\psi_n \right)^{2/3}}{e^{1/3} \left( \psi/\psi_n \right)^{1/6}}$$

with

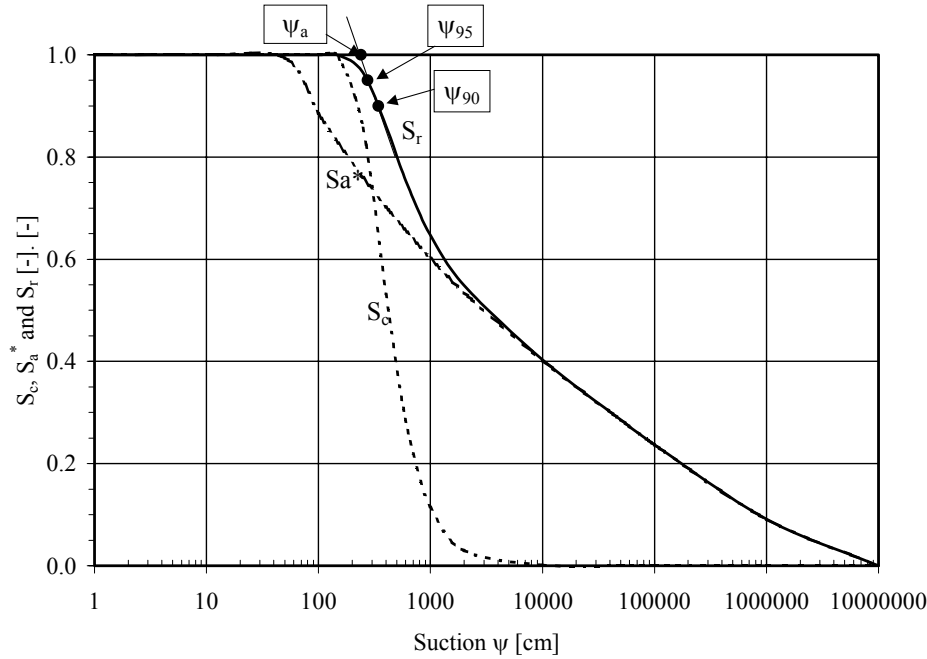
$$[18] \quad C_\psi = 1 - \frac{\ln(1 + \psi/\psi_r)}{\ln(1 + \psi_o/\psi_r)}$$

Equation 14 expresses the total degree of saturation by combining the capillary and adhesion components with contributions of  $S_c$  and  $S_a^*$  given by equations 15 to 18. In these equations  $m$  and  $a_c$  are material parameters, and  $\psi_r$  is the pressure (suction) corresponding to the residual water content ( $\psi_r$  is also equal to the water entry value - WEV - when no distinction is made between drainage and wetting path). Figures 1 and 2 show typical curves drawn from the MK model in a semi-log  $S_r$ - $\psi$  plane, illustrating the contributions of  $S_c$  and  $S_a^*$  for granular (Fig. 1a) and plastic/cohesive (Fig 1b) materials. Hypothetical (but representative) values for  $D_{10}$ ,  $C_U$ ,  $e$ ,  $w_L$ ,  $G_s$ ,  $h_{co}$ ,  $m$ ,  $a_c$ , and  $\psi_r$  have been used for these plots. The value  $\psi_r$  and the pressure (suction) corresponding to the air entry value  $\psi_a$  are also shown on these Figures together with two other parameters  $\psi_{90}$  and  $\psi_{95}$  defining suctions for preset degrees of saturation (of 90% and 95% respectively).



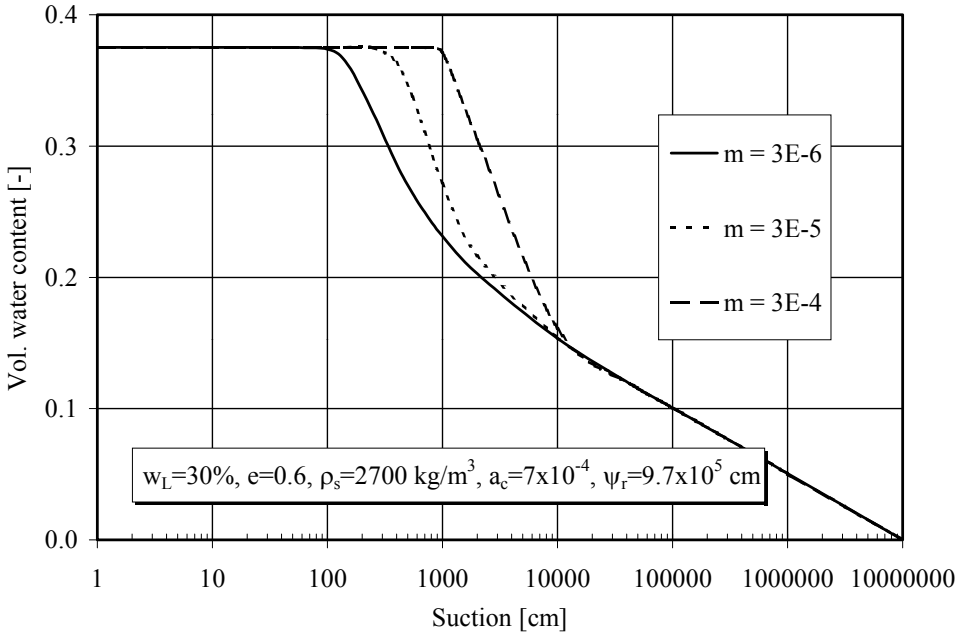
**Figure 1.** Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a non cohesive (low plasticity) soil in the MK model (for  $D_{10} = 0.006$  cm,  $C_U = 10$ ,  $e = 0.6$ ;  $\psi_r = 190$  cm,  $m = 0.1$  and  $a_c = 0.01$ );  $\psi_a$  is the pressure (suction) corresponding to the air entry value (AEV),  $\psi_r$  is the pressure corresponding to the residual water content (also called water entry value-WEV), and  $\psi_{90}$  and  $\psi_{95}$  are suctions corresponding to a degree of saturation  $S_r$  of 0.90 and 0.95 respectively.

Equation 15 provides the expression to evaluate  $S_c$  ( $0 \leq S_c \leq 1$ ) as a function of the equivalent capillary rise  $h_{co}$  (given by the expressions proposed above) and suction  $\psi$ . This equation is a generalization of the one developed by Kovács (1981), in which the statistical exponential function has been expanded to better reflect the influence of pore-size distribution through the distribution parameter  $m$ . The statistical distribution expression used for  $S_c$  is one that can also be applied for grain size curves, as there is a well-known similarity between the latter and the WRC (e.g. Aubertin et al. 1998; Fredlund et al. 2000).



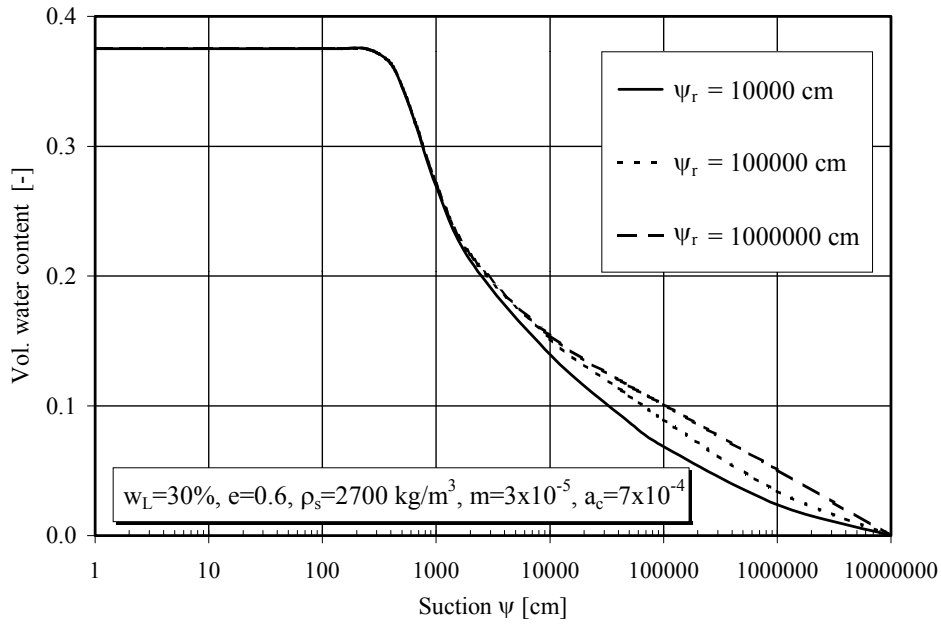
**Figure 2.** Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a cohesive (plastic) soil (for  $w_L=30\%$ ,  $e=0.6$ ,  $\rho_s =2700 \text{ kg/m}^3$ ,  $\psi_r =9.7 \times 10^5 \text{ cm}$ ,  $m=3 \times 10^{-5}$ , and  $a_c=7 \times 10^{-4}$ );  $\psi_a$  is the pressure (suction) corresponding to the air entry value (AEV),  $\psi_{90}$  and  $\psi_{95}$  are suctions corresponding to a degree of saturation  $S_r$  of 0.90 and 0.95 respectively.

Figure 3 illustrates how the  $\theta$ - $\psi$  relationship is affected by the value of parameter  $m$ . As can be seen in this figure, both the position and the slope of the curve are related to  $m$ . On the WRC, parameter  $m$  influences the air entry value  $\psi_a$  (or AEV), which theoretically correspond to the suction when the largest pores start to drain, and also to the rate of decline beyond  $\psi_a$  in the capillary range. For practical applications, the value of  $m$  will be expressed as a function of some very basic geotechnical properties.

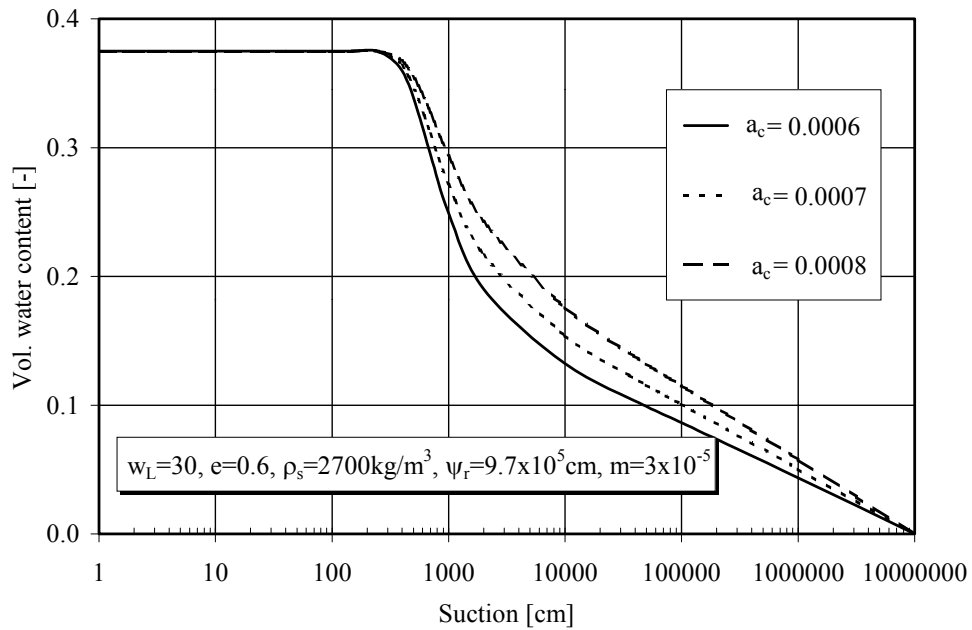


**Figure 3.** Effect of the pore size distribution parameter  $m$  on the position and shape of the WRC according to the MK model.

Equations 16 and 17 define the adhesion component, again as a function of  $h_{co}$  and  $\psi$ . A truncated value of  $S_a$  (i.e.  $S_a^*$ ) is used to make sure that the adhesion component does not exceed unity at low suction ( $0 \leq S_a^* \leq 1$ ). In equation 16,  $\langle y \rangle = 0.5(y + |y|)$ ; for  $S_a \geq 1$ ,  $S_a^* = 1$ , and for  $S_a < 1$ ,  $S_a^* = S_a$ . Equation 17 is based on Kovács (1981) expression in which a sixth order hyperbola is used to relate the adhesion saturation (due to the film of water adsorbed on grain surface) to suction. In this equation,  $a_c$  is the adhesion coefficient (dimensionless) and  $\psi_n$  is a normalization parameter introduced for unit consistencies ( $\psi_n = 1$  cm when  $\psi$  is given in cm, corresponding to  $\psi_n = 10^{-3}$  atmosphere). Parameter  $C_\psi$  (equation 18), taken from Fredlund and Xing (1994), forces the water content to zero when  $\psi$  reaches a limit imposed by thermodynamic equilibrium ( $\theta = 0$  at  $\psi = \psi_0 = 10^7$  cm of water, corresponding approximately to complete dryness). In equation 18,  $\psi_r$  represents the suction at residual water content, which will be shown below to depend on basic soil properties (as is the case with  $\psi_a$  and  $h_{co}$ ). The effect of varying the value  $\psi_r$  in the MK model can be seen on Figure 4. Figure 5 shows the influence of the adhesion coefficient  $a_c$  on the WRC (for typical clayey soils).



**Figure 4.** Effect of the residual suction  $\psi_r$  on the WRC according to the MK model.



**Figure 5.** Effect of the adhesion coefficient  $a_c$  on the WRC according to the MK model.

## 4. APPLICATIONS

The MK model presented above includes a set of equations that provides an estimate of the WRC from full saturation ( $S_r = 1, \theta = n$ ) to complete dryness ( $S_r = 0 = \theta$ ). To apply the model however, three open parameters in the constitutive equations have to be defined explicitly. These are parameter  $m$  in equation 15,  $a_c$  in equation 17, and  $\psi_r$  in equation 18. Based on investigations carried by the authors on a diversity of soils and particulate media, it has been found that the values of  $m$ ,  $a_c$  and  $\psi_r$  can be related to some very basic properties.

The experimental data used here for granular materials have been taken from various investigations performed on the properties of sands and low plasticity silts and tailings. The authors results have been obtained with either plate extractor or Tempe cell according to procedure described in Aubertin et al. (1995b, 1998), while some others have been taken from the literature (see Table 1). All the experimental results on plastic/cohesive materials have been taken from the literature (see Table 2). However, to limit the possibility of excessive shrinkage during testing (see Discussion), the clayey soils retained cover a relatively small range of liquid limit and porosity values.

### 4.1 Residual suction

To apply the model, the residual suction  $\psi_r$  introduced in the expression of factor  $C_\psi$  was evaluated first. For granular materials, values of  $\psi_r$  were determined using the tangent method described by Fredlund and Xing (1994) (see Figure 1). The curves used to this end were obtained by fitting the experimental data with a descriptive equation (i.e. the van Genuchten (1980) model implemented in the code RETC; van Genuchten et al. 1991); the tangent method was then applied on the corresponding WRC best fit (N.B. the best fit curve could also have been obtained with the MK model).

Table 1. Nature, origin, and basic geotechnical properties of the granular materials.

Source	Material	$D_{10}$ [cm]	$C_u$ [-]	$e$ [-]
Sydor (1992)	Coarse Sand	0.05800	1.3	0.750
	Borden Sand	0.00910	1.7	0.590
	Modified Borden Sand	0.00800	1.8	0.640
Bruch (1993)	B.Creek sand consolidated at 5 kPa	0.00930	2.6	0.269
	B. Creek sand consolidated 10 kPa	0.00930	2.6	0.267
Ricard (1994)	Tailings Bevcon	0.00035	11.4	0.674
		0.00035	11.4	0.710
		0.00035	11.4	0.795
	Tailings Senator	0.00031	11.9	0.798
		0.00031	11.9	0.929
	Tailings Sigma	0.00033	14.6	0.695
		0.00033	14.6	0.746
		0.00033	14.6	0.802
	Tailings Sigma +10% bentonite	0.00010	35.0	0.698
0.00010		35.0	0.944	
Kissiova (1996)	Tailings Bevcon	0.00038	11.1	0.570
		0.00038	11.1	0.680
		0.00038	11.1	0.920
	Tailings Sigma	0.00034	14.7	0.660
		0.00034	14.7	0.720
	Sacrete sand	0.01450	3.5	0.570
0.01450		3.5	0.630	
MacKay (1997)	London Silt	0.00060	5.5	0.634
	Ottawa Sand	0.00937	1.7	0.587
Lim et al. (1998)	B. Creek sand consolidated at 5 kPa	0.00930	2.6	0.618
Rassam and Williams (1999)	Tailings at 50m	0.00600	5.0	0.637
	Tailings at 150m	0.00184	9.5	0.637
Authors own results	Tailings Sigma (fine)	0.00040	9.8	0.720
		0.00040	9.8	0.740
	Tailings Sigma (coarse)	0.00040	8.6	0.640
		0.00040	8.6	0.710
		0.00040	8.6	0.780
	Till	0.00035	4.5	0.620
		0.00035	4.5	0.700
		0.00035	4.5	0.720
0.00035		4.5	0.790	

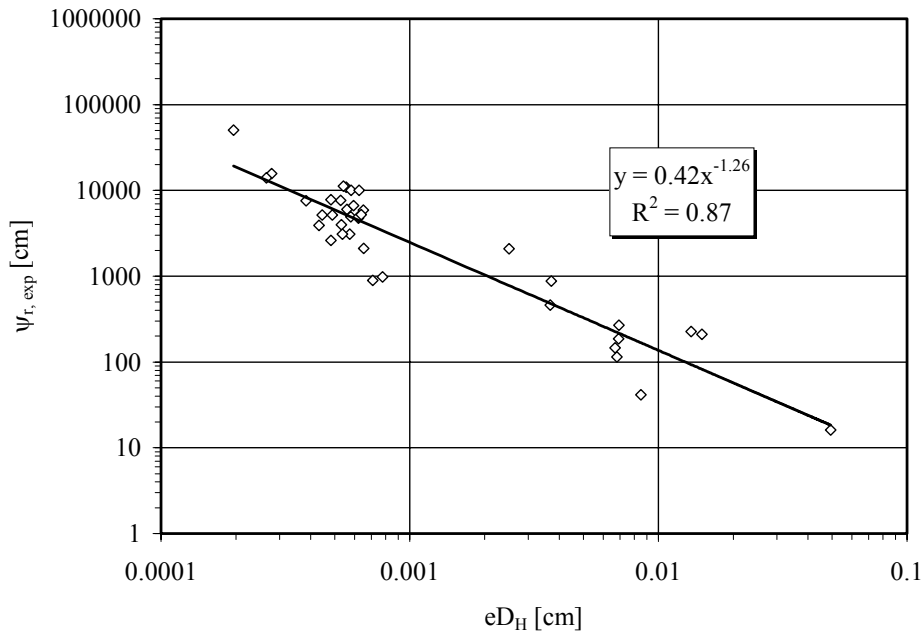


Table 2. Nature, origin, and basic geotechnical properties of the plastic/cohesive materials.

Source	Material	$w_L$ [%]	$G_s$ [-]	$e$ [-]
Alimi-Ichola and Bentoumi (1995)	Gault clay	40	2.650	0.942
Vanapalli et al. (1996)	Sandy clay till (200 kPa prec.)	35.5	2.730	0.430
	Sandy clay till (25 kPa prec.)	35.5	2.730	0.540
	Sandy clay till (25 kPa prec.)	35.5	2.730	0.545
Huang et al. (1998)	Silty sand PPCT11	22.2	2.680	0.536
	Silty sand PPCT21	22.2	2.680	0.502
	Silty sand PPCT16	22.2	2.680	0.463
	Silty sand PPCT26	22.2	2.680	0.425
O'Kane et al. (1998)	Till Cover	40	2.770	0.493
Vanapalli et al. (1998)	Guadalix Red silty clay	33	2.660	0.480
Fredlund (1999) (in Soilvision)	Record 3713	35.5	2.65	0.545
	Record 3714	35.5	2.65	0.545
	Record 3715	35.5	2.65	0.449
	Record 3716	35.5	2.65	0.474
	Record 3717	35.5	2.65	0.474
	Record 3718	35.5	2.65	0.518
	Record 3720	35.5	2.65	0.546
	Record 3728	35.5	2.65	0.438
	Record 55	35.5	2.65	0.475
	Record 65	35.5	2.73	0.546
	Record 66	35.5	2.73	0.438
	Record 70	35.5	2.73	0.444
	Record 71	35.5	2.73	0.518
	Record 72	35.5	2.73	0.472
	Record 73	35.5	2.73	0.545
	Record 75	35.5	2.73	0.430
Record 76	35.5	2.73	0.372	

After considering various geotechnical parameters (alone or combined) with a physical significance, it was established that the following relationship provides an adequate estimate of the residual suction (see Figure 6).

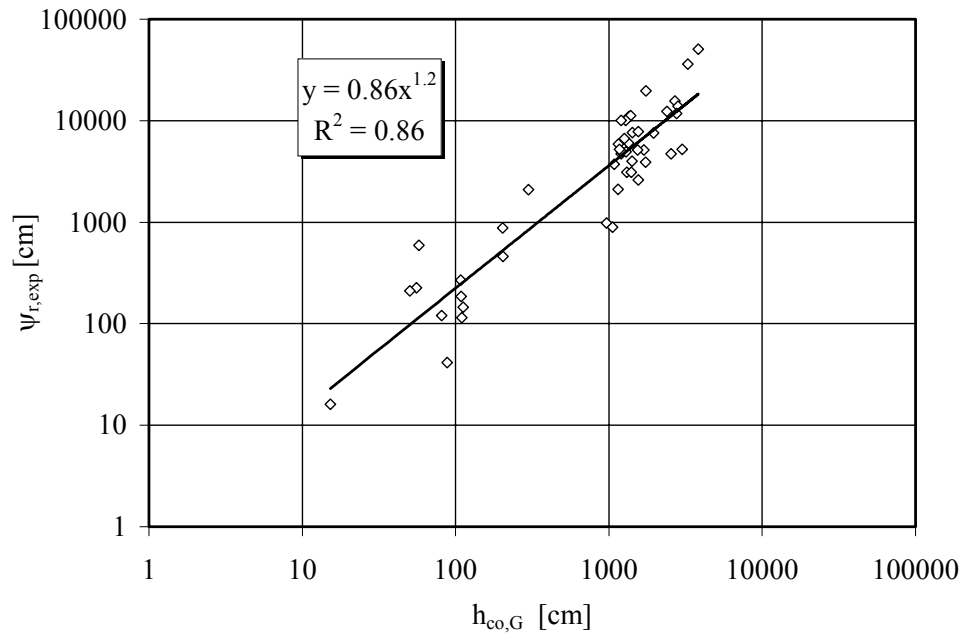
$$[19] \quad \psi_r = \frac{0.42}{(eD_H)^{1.26}}$$



**Figure 6.** Relationship between the residual suction  $\psi_{r, \text{exp}}$  determined from the experimental data (with best fit WRC), the void ratio  $e$ , and the equivalent grain size diameter  $D_H$  for granular materials identified in Table 1 ( $D_H$  is defined by equations 5 and 6).

Based on this expression and on equation 7, the authors also developed a simple relationship between  $\psi_r$  and the equivalent capillary rise  $h_{co, G}$  for granular materials, which is expressed as follow (see Figure 7):

$$[20] \quad \psi_r = 0.86 h_{co, G}^{1.2}$$



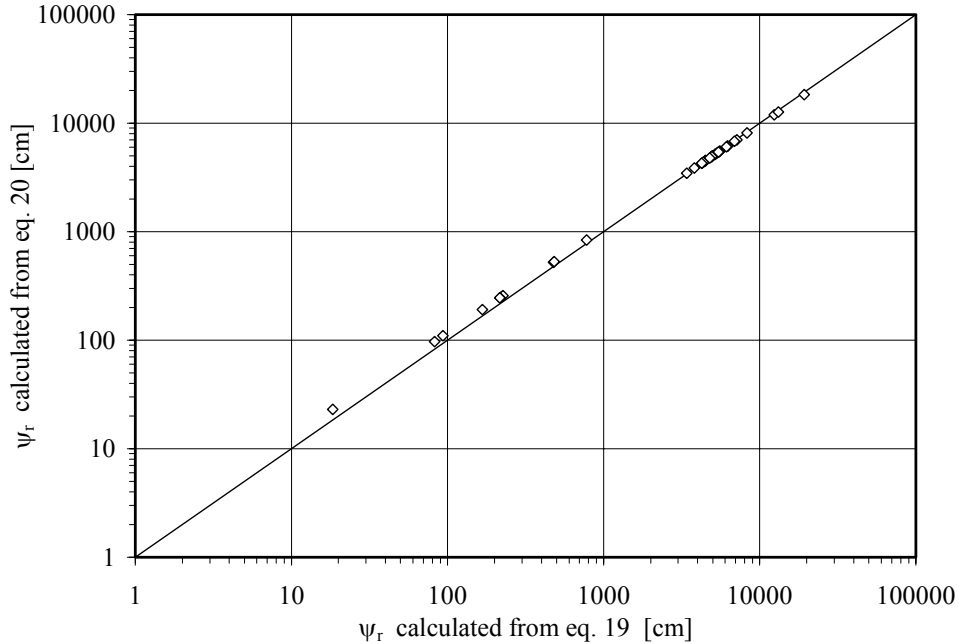
**Figure 7.** Relationship between the residual suction  $\psi_{r,exp}$  determined from the measured data (and best fit WRC), and the equivalent capillary rise  $h_{co,G}$  (eq. 7) for granular materials.

The validity of this equation is confirmed by Figure 8, which shows that equations 19 and 20 give identical residual suction values.

The basic expression obtained for granular soils (eq. 19) is frequently impractical for clayey soils, because  $D_{10}$  and  $C_U$  are often unknown. Furthermore, the WRC of plastic/cohesive materials is rarely bilinear (around the residual water content) in a semi-log plot, for suction above the air entry value. Considering also that the experimental results are not customarily available at high suctions, it can be argued that the change in the slope on the WRC that indicates the residual suction is generally not well defined (see also discussion by Corey 1994). The graphical determination of the residual suction  $\psi_r$  thus becomes very difficult for these soils.

In the following applications of the MK model, it is assumed that the residual suction values  $\psi_r$  for plastic/cohesive soils can be related to the equivalent capillary rise  $h_{co,P}$  (equation 11) using the same dependency as for granular soils. Hence, the expression used for  $\psi_r$  of clayey soils becomes:

$$[21] \quad \psi_r = 0.86 \left( \frac{\xi}{e} \right)^{1.2} w_L^{1.74}$$



**Figure 8.** Comparison of the  $\psi_r$  values obtained from equations 19 (see Figure 6) and 20 (see Figure 7) for the granular materials identified in Table 1.

#### 4.2 Parameter $a_c$ and $m$

Once  $\psi_r$  was determined for each material in the data base (see Table 1 and 2), the remaining open parameters ( $a_c$  and  $m$ ) value was evaluated from a fitting procedure, so that calculated WRC match experimental data as closely as possible (with the MK model). After repeating the regression calculation for the various results on different materials, further investigations have been conducted to relate the obtained parameter values to basic geotechnical properties. For granular soils, where  $h_{co,G}$  is given by equation 7, the analysis of the available results has indicated that the value of the pore-size distribution parameter  $m$  can be closely approximated by the inverse of the uniformity coefficient (i.e.  $m=1/C_U$ ). When  $C_U = 1$ ,  $m = 1$  and Kovács (1981) original expression for  $S_c$  is then recovered from the MK model. This confirms that this newly

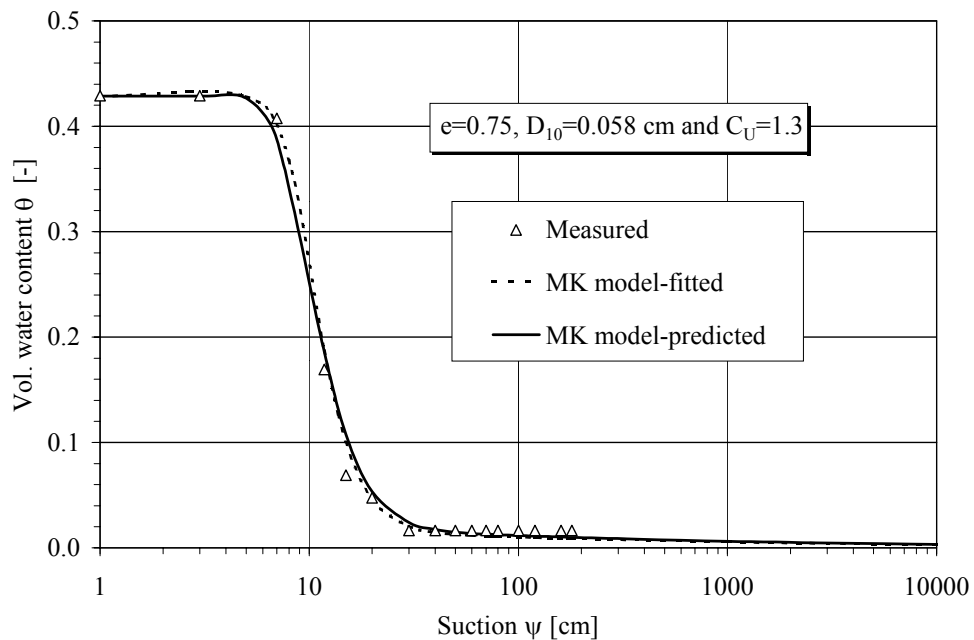
introduced parameter is closely related to the grain size distribution. For these same granular materials, the analysis indicated that the adhesion coefficient  $a_c$  is approximately constant, with  $a_c = 0.01$ .

For the plastic/cohesive soils considered here, with  $h_{co,P}$  given by equation 11, both  $m$  and  $a_c$  values can be taken as constants (with  $m = 3 \times 10^{-5}$  and  $a_c = 7 \times 10^{-4}$ ) in the predictive applications. In this case, the influence of grain-size distribution is somewhat superseded by the dominant effect of the surface activity (defined here through the  $w_L$  function).

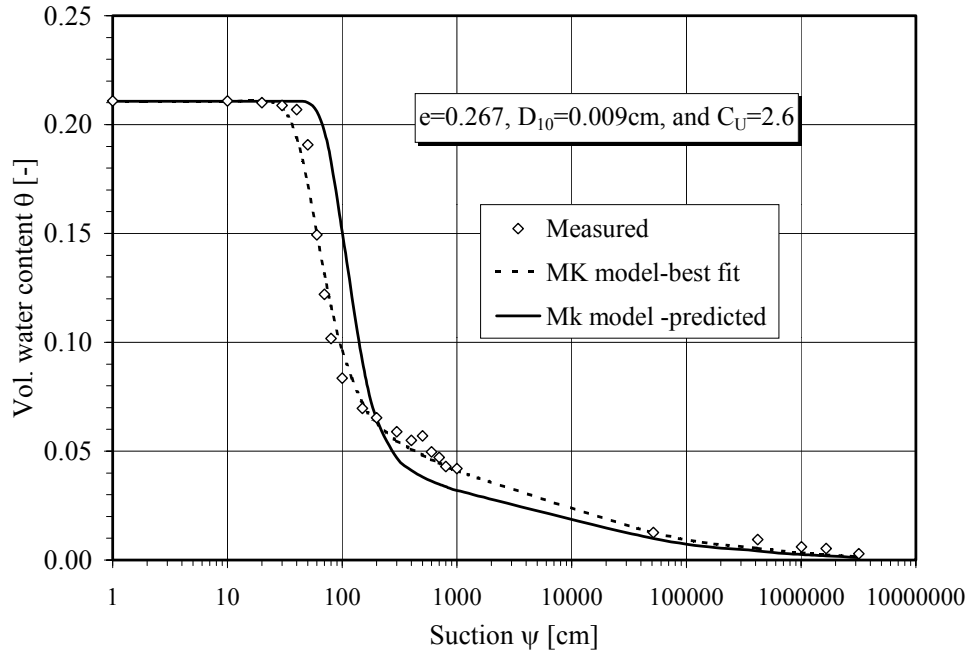
The relationships and parameter values defined for the MK model can now be used to evaluate the WRC of various materials. Tables A-1 and A-2, in Appendix, give sample results to show explicitly how the presented set of equations are used to obtain the WRC. Figures 9 to 15 compare typical fitted (with adjusted  $a_c$  and  $m$  values) and predicted (with preset  $a_c$  and  $m$  values) curves using the experimental data obtained for different granular and clayey materials. Identification and basic properties ( $D_{10}$ ,  $C_u$  and  $e$  for granular materials;  $w_L$ ,  $e$  and  $\rho_s$  for plastic/cohesive soils) are given in each figure. The value of  $a_c$  and  $m$  that leads to the best fit curves shown in Figure 9 to 15 are given in Table 3. As can be seen on these figures, there is generally a good agreement between the predicted and the measured WRC, despite the differences sometimes observed between the best fit and predicted parameter values (especially with  $m$  for loose or clayey materials). Such good agreement is obtained with most of the results identified in Tables 1 and 2. In fact, the predictions made with the MK model compare favorably to those obtained with most other models, which often include more input parameters (and thus require more information for their application), as depicted in recent review publications (e.g. Wösten et al. 2001; Schaap et al. 2001; Rawls et al. 2001; Zhuang et al. 2001). For instance, predicted volumetric water content values, at a given suction, generally approach the measured values within a range of 0.02 to 0.05, which is comparable to the best available pedotransfer functions, that often relies on particular measured points on the WRC (i.e. water content at suctions of 10, 33, or 1500 kPa for instance). It must be said however that the authors have concentrated most of their efforts on the practical development of the model equations for geotechnical applications, than on statistical analyses of the MK model capabilities (and on systematic comparison with other available functions). More work on this aspect will have to be

performed in the future. In the meantime, the proposed set of equations has been successfully used by the authors on actual field projects (e.g. Aubertin et al. 1999; Nastev and Aubertin 2000), and also by others on different types of particulate media (outside the authors' data bank). The success obtained so far are due, in part, to the sound physical description of the WRC provided by the MK model.

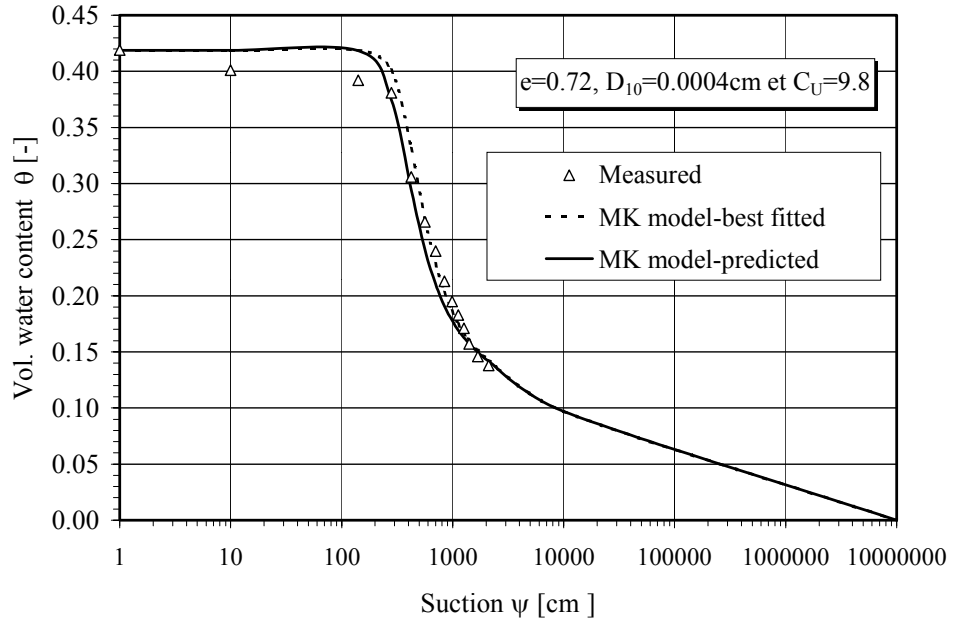
There are nevertheless some discrepancies between the predicted curves and actual data. Some aspects related to these differences are discussed below, together with other features, capabilities, and limitations of the MK model.



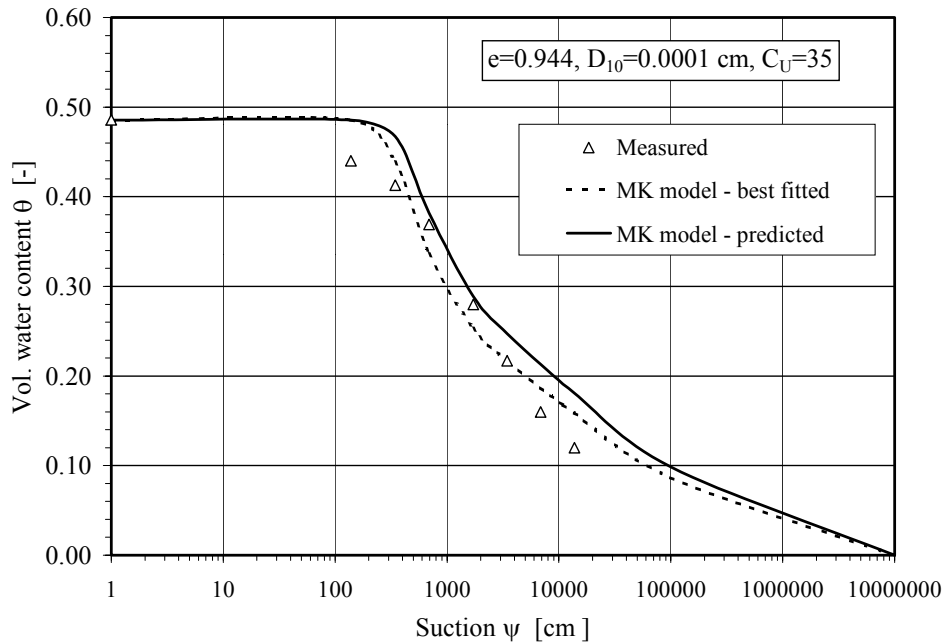
**Figure 9** . Application of the MK model to a coarse, uniform, and relatively loose sand (data from Sydor 1992).



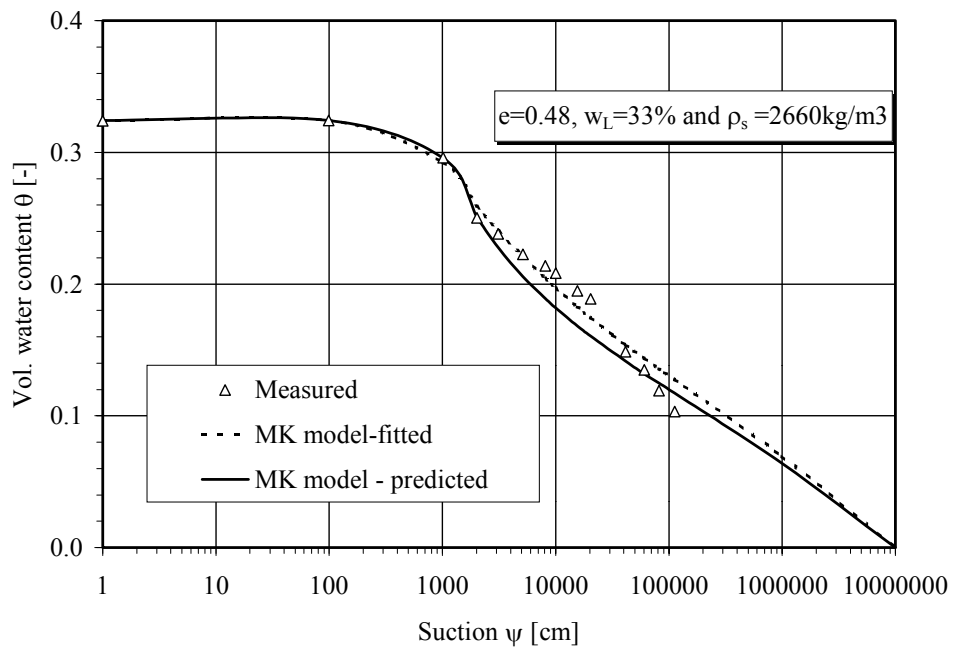
**Figure 10.** Application of the MK model to a fine, uniform, and dense sand (data from Bruch 1993).



**Figure 11.** Application of the MK model to tailings Sigma (silty material, well-graded, and loose) (data from authors).

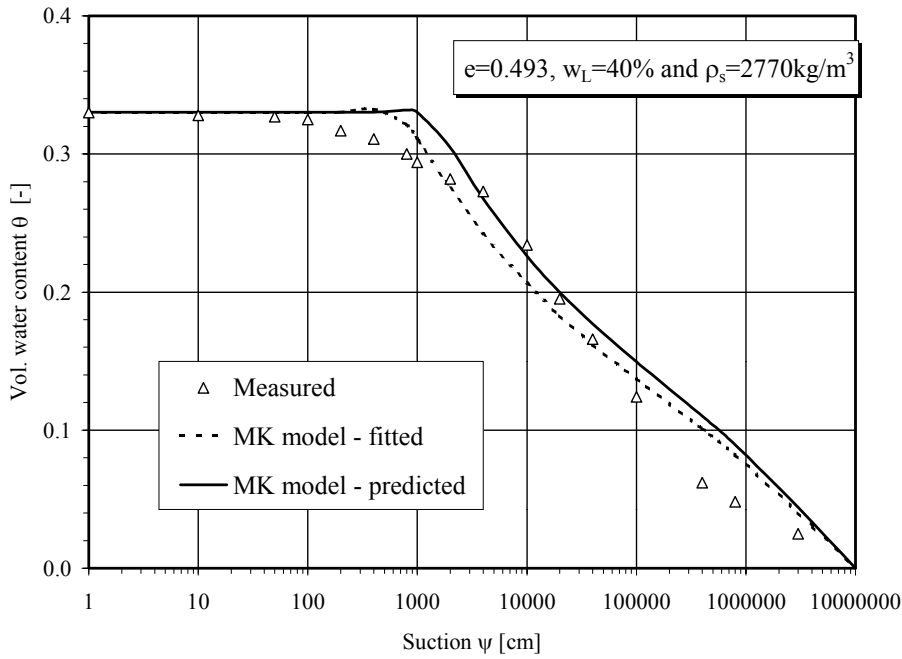


**Figure 12.** Application of the MK model to tailings Sigma mixed with 10 % bentonite (data from authors).

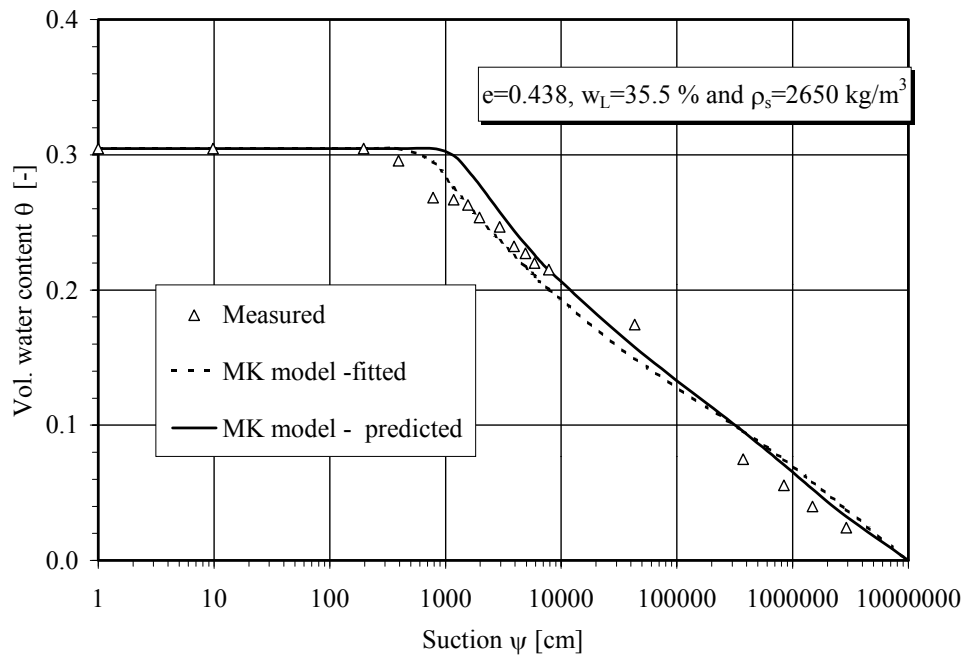


**Figure 13.** Application of the MK model to Guadalix red silty clay (data from Vanapalli et al. 1998)





**Figure 14.** Application of the MK model to a till (data from O’Kane et al. 1998)



**Figure 15.** Application of the MK model to Indian Head Till (data from Fredlund 1999)

Table 3. Parameters  $m$  and  $a_c$  leading to the best fit WRC shown in Figures 9 to 15, and the empirically determined values.

Material	Fitted WRC		Predicted WRC	
	$m$	$a_c$	$m$	$a_c$
Coarse, uniform and relatively dense sand (data from Sydor 1992), see Figure 9	0.827	0.007	0.769	0.01
Fine and dense sand (data from Bruch 1993), see Figure 10	0.091	0.013	0.388	0.01
Tailings Sigma (silty material, coarse and loose) (Authors data), see Figure 11	0.161	0.010	0.102	0.01
Tailings Sigma mixed with 10% bentonite (Ricard 1994), see Figure 12	0.019	0.009	0.029	0.01
Guadalix Red silty clay (data from Vanapalli et al. 1998), see Figure 13	$3.6 \times 10^{-6}$	$7.6 \times 10^{-4}$	$3.0 \times 10^{-5}$	$7.0 \times 10^{-4}$
Till (data from O’Kane et al. 1998), see Figure 14	$8.1 \times 10^{-6}$	$6.5 \times 10^{-4}$	$3.0 \times 10^{-5}$	$7.0 \times 10^{-4}$
Indian Head Till (Record 728; data from Fredlund 1999), see Figure 15	$1.0 \times 10^{-9}$	$7.0 \times 10^{-4}$	$3.0 \times 10^{-5}$	$7.0 \times 10^{-4}$

## 5. DISCUSSION

The proposed functions presented above provide a simple means to estimate the WRC for granular and plastic/cohesive soils, as well as for other particulate media. The MK model equations have been developed as an extension of the Kovács’ (1981) model, which was selected initially because of its sound physical bases regarding water retention phenomena (Ricard, 1994; Aubertin et al. 1995b, 1998). It is considered important that the model distinguish capillary and adhesion forces as being responsible for moisture retention. The evaluation of these two

components for predictive purposes requires only basic geotechnical properties such as the grain size (through  $D_{10}$ ,  $C_U$ ), porosity  $n$  (or void ratio  $e$ ), solid grain density ( $\rho_s$ ), and liquid limit ( $w_L$ ).

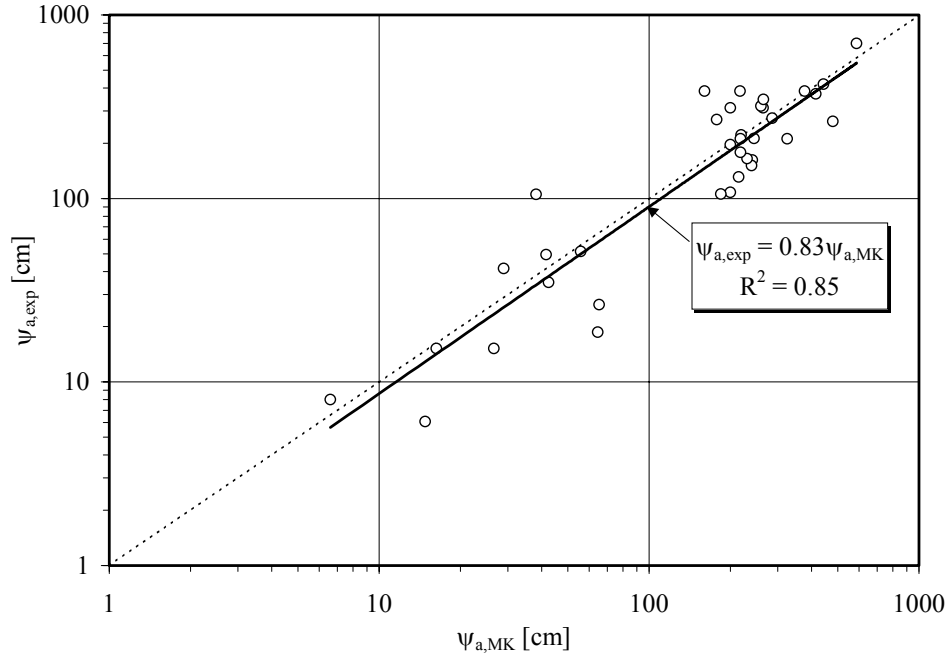
When analyzing the quality of predictions made with the MK model and the discrepancies sometimes observed, one should take into account the variety of the WRC measurement techniques and the associated uncertainties and limitations. These may have a significant effect on the curves and on the estimates for parameters  $\psi_r$ ,  $m$ , and  $a_c$  in the MK model. In that regard, it can be expected that the quality of the predictions is influenced by the data themselves. Despite this situation, the accordance between predicted and measured WRC is generally good, and the results can be used for preliminary analysis. The proposed model however shows some limitations in its predictive capabilities. Some of these are discussed in the following.

### **5.1 Prediction of the residual suction and air entry value**

The residual suction  $\psi_r$  and the air entry value  $\psi_a$  (or AEV) are familiar points defined on the WRC, that are often used for practical applications. The residual suction  $\psi_r$  has already been defined above with the MK model. However, the actual value of this suction (and of the corresponding volumetric water content) is difficult to determine precisely, and even its physical meaning has been questioned (e.g. Corey 1994). For the applications shown here, there is more than one approach that can be used to define  $\psi_r$ . It can be measured on the fitted curves for the available experimental data (with MK, or with other models such as the van Genuchten 1980 equation used in the preliminary phase of the model development); it corresponds to the intersection point between two tangents (see Fig. 1a). It can also be evaluated on the predicted WRC. When the best fit and predicted curves are close to each other (like in Figure 9), the residual suctions obtained from the two approaches are then almost identical, and correspond well to the value predicted by equation 19. However, when the best fit and predicted curves differ (like in Figure 10 and 12), the residual suction can then take fairly different values depending on the way it is determined.

For the MK model application, equation 19 is used to define suction  $\psi_r$ , which is then used in the WRC construction from the set of equations (eq. 14 to 18). This means that the predicted  $\psi_r$  (from eq. 19) may be somewhat different than the one deduced from the complete WRC obtained from the model. Furthermore, strictly speaking according to the MK model (for the best fit or predicted WRC), residual suction is reached only when the capillary component  $S_c$  goes to (almost) zero, but this condition is not exactly equal to the intersection point between the two tangents. Hence, at this point, one must infer that this specific state can only be approximated with the predictive tools at hand.

A somewhat similar situation can be described for the air entry value. The AEV can also be estimated with the set of equations proposed above. However, more than one method exists to determine the AEV (e.g. Corey, 1994; Fredlund and Xing, 1994; Aubertin et al. 1998), so different values may be obtained for a given information. Furthermore, because the MK model aims at describing the entire WRC, it is not particularly efficient in defining precisely specific points on the curve, such as the AEV. Hence, in this case, one may need to make some adjustments to better reflect the observed  $\psi_a$ ; the adjustments to the predicted curves may be especially significant for loose (compressible) materials. Fortunately, the  $\psi_a$  determined from actual experimental data ( $\psi_{a,exp}$ ), can be related to values obtained from the WRC predicted by the MK model ( $\psi_{a,MK}$ ). A relationship for that purpose is shown in Figure 16. Values  $\psi_{a,exp}$  and  $\psi_{a,MK}$  were determined using the tangent method (see Figure 1). The curves used to determine  $\psi_{a,exp}$  were obtained by fitting the experimental data with a descriptive equation; again the van Genuchten (1980) model was used (N.B. fitted curves with MK would provide essentially the same AEV). The relationship presented in Figure 16 indicates that the MK model tends to overpredict  $\psi_a$ . A correction factor of 0.83 nevertheless allows a fairly good representation of the AEV deduced from experimental data (see best fit line on the figure).



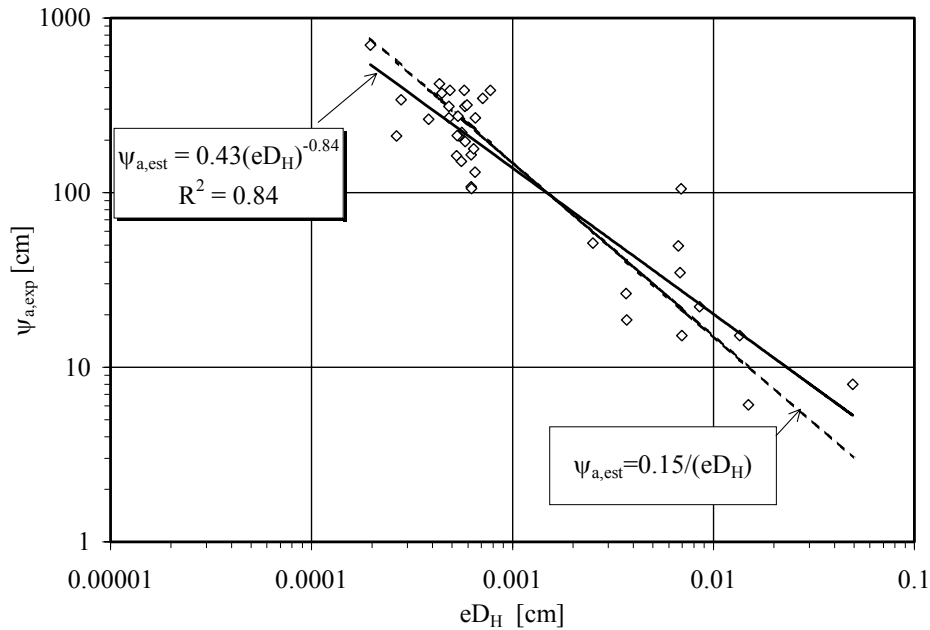
**Figure 16.** Comparison between the air entry values obtained from the experimental data  $\psi_{a,exp}$  (with best fit WRC) and obtained on the WRC predicted by the MK model ( $\psi_{a,MK}$ ) for granular materials.

One can also estimate the experimental AEV using instead some basic geotechnical properties. Various specific relationships have been proposed for that purpose (e.g. Palubarinova-Kochina 1962; Bear 1972; Yanful 1991). According to the authors' own analyses, the AEV obtained with most existing functions tend to agree with each other for coarse grained materials such as sands, but significantly diverge for fine grained materials (Aubertin et al. 1993). A recent investigation by the authors to develop a more representative relationship for granular materials has lead to the following expression (see Figure 17):

$$[22] \quad \psi_{a,est.} = \frac{b_1}{(eD_H)^{x_1}}$$

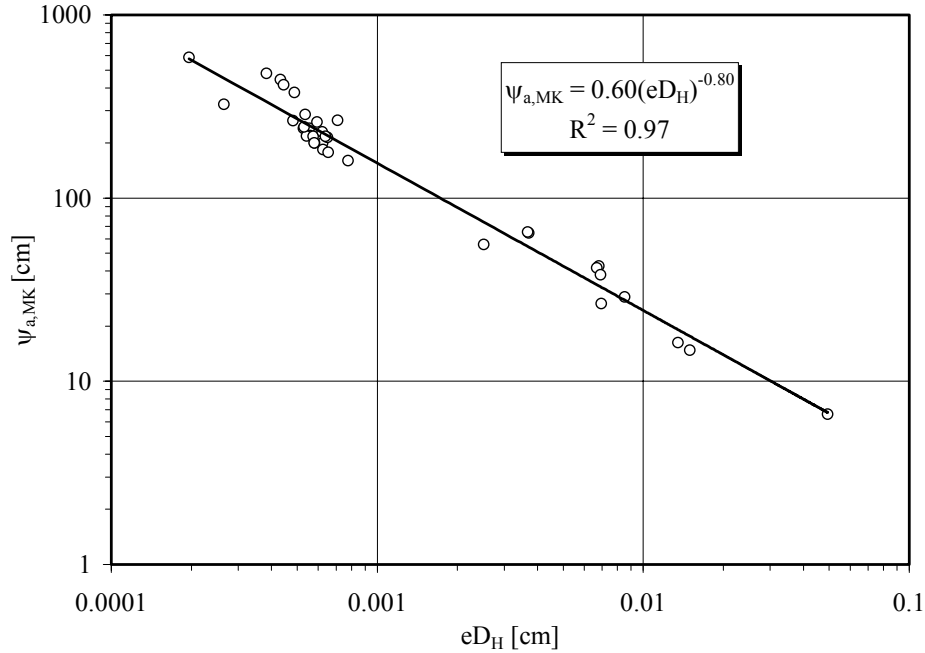
where  $b_1$  and  $x_1$  are fitting parameters and  $D_H$  has been given above in equations 6-8 (i.e.  $D_H = [1 + 1.17 \log(C_U)] D_{10}$ ). As can be seen in Figure 17, almost similar predictions (for the range of data considered here) can be obtained with  $b_1 = 0.43$ ,  $x_1 = 0.84$  and with  $b_1 = 0.15$ ,  $x_1 = 1.0$ , in

equation 22. In this equation, the estimated  $\psi_a$  is related explicitly to  $e$ ,  $D_{10}$ , and  $C_U$  as was suggested by Aubertin et al. (1998).



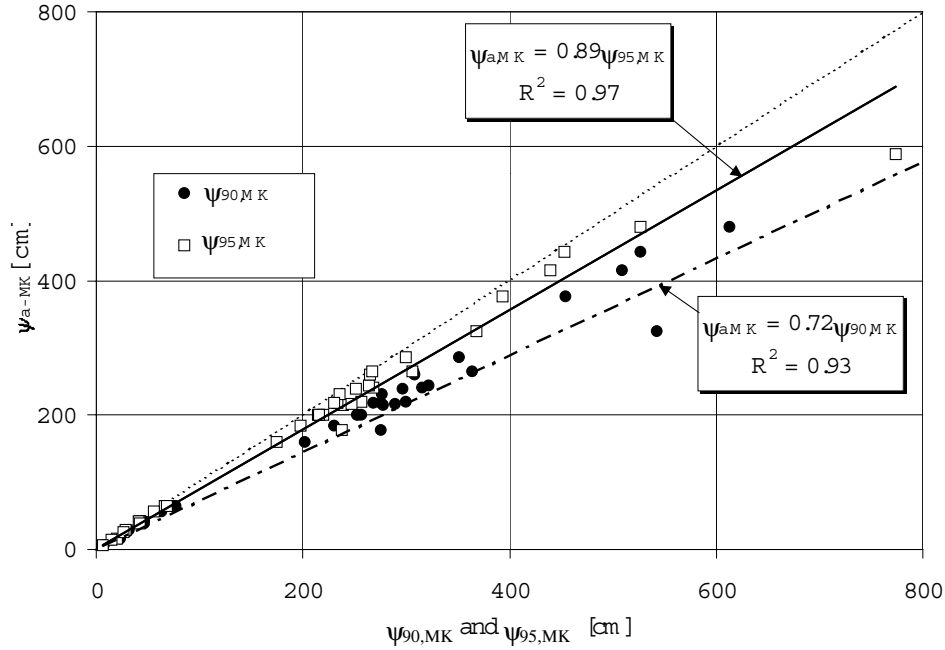
**Figure 17.** Proposed relationships for estimating the air entry value  $\psi_{a,exp}$  obtained on the experimental data for granular materials.

Equation 22 can also be used to estimate the air entry values deduced from the WRC predicted by the MK model (i.e.  $\psi_{a,MK}$ , with  $b_1 = 0.6$ ,  $x_1 = 0.8$  -see Figure 18). These simple relationships can be used to make a correction of the AEV obtained from the MK model, to better reflect the actual measured  $\psi_a$ . Hence, this allows the user to improve upon the prediction of this particular condition from the proposed equations.



**Figure 18.** Proposed relationship for estimating the air entry value  $\psi_{a,MK}$  obtained on the WRC predicted by the MK model for granular materials.

Also, it was suggested also by the authors (Aubertin et al. 1998) that the AEV could be defined from the suction corresponding to a given degree of saturation  $S_r$ . Figure 1 shows two such conditions for  $S_r$  equal to 90% and 95%. The predicted air entry value  $\psi_{a,MK}$  is compared to calculated suctions corresponding respectively these two degrees of saturation (which do not require extrapolation or any interpretation of the WRC). Figure 19 shows that there is a systematic relationship between  $\psi_{a,MK}$  and  $\psi_{95,MK}$  and  $\psi_{90,MK}$ . These results, combined with Figure 16, indicate that the experimental  $\psi_{a,exp}$  can be related to the latter two parameters (ex.  $\psi_{a,exp} = 0.83\psi_{a,MK} = 0.83 \times 0.89\psi_{95,MK} = 0.74\psi_{95,MK}$ ). This is in good agreement with previous analyses with other curve fitting equations (of Brooks and Corey 1964 and van Genuchten 1980), which have indicated that the rather steep curvature around the AEV induced by the Kovács and MK models typically tends to overestimate the actual AEV (Aubertin et al. 1998). If a rather conservative approach is required for a given project (i.e. underestimation of the AEV is favored), the authors suggest using  $\psi_a$  equal to about  $0.6 \psi_{90,MK}$ , or  $0.74 \psi_{95,MK}$  obtained on the predicted WRC from the MK model.



**Figure 19.** Relationships between the predicted air entry value  $\psi_{a,MK}$  and suctions  $\psi_{90,MK}$  and  $\psi_{95,MK}$  corresponding respectively to degrees of saturation of 90% and 95%, (obtained on the WRC predicted by the MK model) for granular materials.

Finally on this issue, the authors have not yet been successful at developing an acceptable correlation between  $\psi_a$  and basic geotechnical properties for plastic/coherent materials (mainly because of the paucity of available results). The potential user should thus be careful when using the MK model for that purpose, in the absence of an independent validation relationship. The approach presented above, associating the AEV and the suction at  $S_r=0.95$  or  $0.90$  in the MK model, may still be applicable in this case, but too few data have been analyzed at this stage to confirm their validity.

## 5.2 The equivalent capillary rise $h_{co}$

The equivalent capillary rise  $h_{co}$  constitutes a central parameter in the MK model. This parameter is defined by equations 7 and 9 for granular materials and by equations 11 and 13 for plastic/cohesive soils. This parameter theoretically represents the capillary head in a pore with a diameter equal to the equivalent hydraulic diameter  $d_{eq}$  defined according eq. 3. Considering that



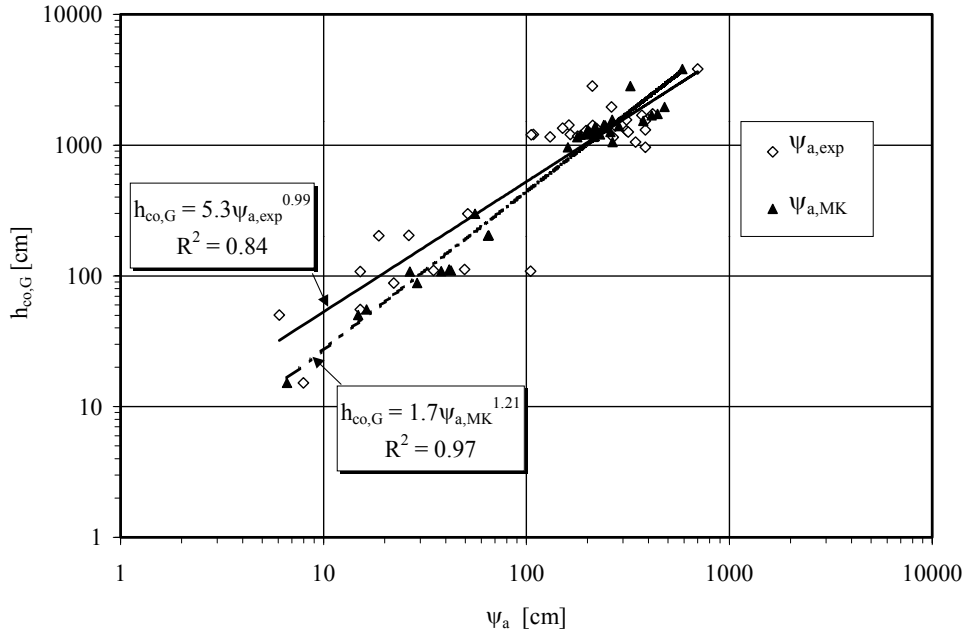
the pore diameter  $d_{eq}$  lies between the limiting values (minimal and maximal diameter) of the pore size distribution, to which a maximum ( $h_{cmax}$ ) and minimum ( $h_{cmin}$ ) capillary rise can be associated, it is expected that  $h_{co}$  should lie between  $h_{cmax}$  and  $h_{cmin}$ .

Because water can only be retained by adhesion forces above  $h_{cmax}$ , this elevation could thus be associated to the residual suction  $\psi_r$  defined on the drainage water retention curve (e.g. Lambe and Whitman 1979). Therefore, one should observe that  $h_{co} \leq \psi_r$ . This is in agreement with the relationship between  $\psi_r$  and  $h_{co}$  given by equation 20. On the other hand, the minimum capillary rise  $h_{cmin}$  can be associated the air entry value  $\psi_a$ . As shown on Figure 20 for granular materials, the equivalent capillary rise  $h_{co,G}$  and the air entry value can be correlated by the following expression:

$$[23] \quad h_{co,G} = b_2 \psi_a^{x_2}$$

From the available data, the authors have obtained  $b_2 \approx 5.3$  and  $x_2 \approx 1$  when  $\psi_a$  is determined from the experimental curve ( $\psi_{a,exp}$ ), and  $b_2 \approx 1.7$  and  $x_2 \approx 1.2$  when  $\psi_a$  is given by the MK predictive model ( $\psi_{a,MK}$ ) at full saturation (see Figure 1). As mentioned before, these relationships indicate that the equivalent capillary rise  $h_{co}$  is much larger than the air entry value, which corresponds more or less to the height of the capillary fringe in a homogeneous deposit.

With the MK model, the equivalent capillary rise  $h_{co}$  has been defined with a theoretical function of the specific surface area  $S_m$ , a property that is rarely evaluated precisely. Because  $S_m$  can vary over a wide range (from less than 10 m<sup>2</sup>/g to more than 600 m<sup>2</sup>/g for various soils; e.g. Marshall et al. 1996), the proportionality between  $h_{co}$  and  $S_m$  has a major influence on the estimation of  $h_{co}$ . In particular, for clayey soils, it has been postulated that the specific surface can be related explicitly to the liquid limit  $w_L$ , which only reflects indirectly the surface characteristics of soils. The MK model application presented here thus introduces  $S_m$  in a relatively crude (but practical) manner, and this approach can induce a relatively large uncertainty on the value of  $h_{co}$ . That is why this parameter should be viewed as a semi-empirical component introduced in an otherwise physically based model; such pragmatic simplification is very useful for engineering applications.



**Figure 20.** Relationships between the equivalent capillary rise  $h_{co,G}$  and the air entry value for granular materials;  $\psi_{a,exp}$  and  $\psi_{a,MK}$  are obtained on the experimental and on the MK model predicted WRC respectively.

### 5.3 Parameters with preset values

In the above applications, values have been pre-assigned to the adhesion coefficient  $a_c$  (0.01 in the case of granular materials for  $w_L$  below about 30 to 40%, and  $7 \times 10^{-4}$  in the case of plastic/cohesive materials). When the MK model is adjusted to all the experimental WRC considered in this study, the actual value of  $a_c$  leading to best fit curves is not constant but rather varies from 0.001 to 0.024 for granular materials and from  $6 \times 10^{-4}$  to  $9 \times 10^{-4}$  for clayey soils. Thus, the actual deviation on  $a_c$  is fairly high, being above one order of magnitude for granular materials. Equation 17 shows that parameter  $a_c$  is linearly related to the adhesion degree of saturation  $S_a$  (see Figure 5). In practice however, this variation has an almost negligible effect on the WRC because the water content  $\theta$  is generally very small in the range of high suction, where the adhesion component  $S_a$  prevails. It is also interesting to mention that, even if parameter  $a_c$  is greater for granular materials than for plastic/cohesive soils, the adhesion degree of saturation  $S_a$

is more important for the latter than for the former materials, because  $h_{co,P}$  (see eq. 7) is much larger than  $h_{co,G}$  (see eq. 11). Hence, the value of  $a_c$  can not be linked alone to the magnitude of this component of the degree of saturation in the MK model.

A fixed value has been also assigned to the pore-size distribution parameter  $m$  ( $m=3 \times 10^{-5}$ ) in the case of plastic/cohesive materials, while  $m$  has been related to the uniformity coefficient  $C_U$  ( $m=1/C_U$ ) for granular materials. For the clayey soils considered here (see Table 2), the value of parameter  $m$  leading to best fit curves can actually vary from  $5.0 \times 10^{-6}$  to  $1.1 \times 10^{-4}$ . Figure 3 shows the effect of  $m$  on the calculated WRC, for values differing by two orders of magnitude from the preset value of  $3 \times 10^{-5}$ , i.e.  $m=3 \times 10^{-6}$  to  $3 \times 10^{-4}$ . One can see on this Figure that such large variation of  $m$  induces a much smaller variation of the capillary saturation  $S_c$  (defined with equation 15). In fact, it can be shown that the relatively large differences in  $m$  values, between the fitted and predicted situations, produce a modest variation on the  $\theta$  value on the WRC calculated with the MK model. Hence, it has been inferred that this somewhat weak dependency does not, at this point, warrant a more elaborate definition of parameter  $m$ .

In the expression (eq. 17) for the adhesion component  $S_a$ , a normalization suction  $\psi_n$  has been introduced for unit consistencies (to obtain a dimensionless adhesion coefficient  $a_c$ ). The value of  $a_c$  however depends on the selected value of  $\psi_n$ . In the presented applications, where  $\psi$  and  $h_{co}$  are given in cm,  $\psi_n$  is fixed at 1 cm (or  $10^{-3}$  atmosphere), so  $a_c=0.01$  for granular soils and  $a_c=7 \times 10^{-4}$  for plastic/cohesive materials. If  $\psi$  (and  $h_{co}$ ) is expressed in other units, as it is sometimes done, the value of  $\psi_n$  must be converted to the corresponding value. For instance, if  $\psi$  and  $h_{co}$  are given kPa, one has to use  $\psi_n \cong 0.098$  kPa.

In the model development, the specific surface was deemed to depend on the particles shape. This is taken into account here with the shape factor  $\alpha$  in equation 5. Such shape can be broadly assessed from the three typical components defined by sphericity, roundness and surface texture (Barret 1980). Sphericity is a measure of how closely the particle shape resembles a sphere, roundness refers to the curvature of the edges, and surface texture relates to local features on the grains surface. The shape factor  $\alpha$  introduced here is in fact a sphericity factor (Kovács 1981). Typical values  $\alpha$  for some regular geometrical forms are 6 for sphere, 10.4 for cube and

octahedron, and 18 for tetrahedron. Generally, the shape factor tends to depend on the grains size for many soils. Kovács (1981) has shown that the shape factor  $\alpha$  often becomes higher for finer grained soils. For simplification, a fixed value  $\alpha$  of 10 has been used in the MK model for coarse grained materials. This value is considered to be fairly representative of most granular materials, since their sphericity can generally be approximated by more or less rounded or angular cubes and octahedrons. However, most fine grained (clayey) plastic/cohesive soils are made of plate-like particles. Using a constant shape factor  $\alpha$  with a fixed value (of 10) is, in this case, not very representative. This flaw is partly overcome in the model, by expressing  $S_m$  as a function of the liquid limit  $w_L$ , which in a way reflects the interactions between water molecules and the particles. Again, this is a pragmatic choice, like many others that need to be made while constructing pedotransfer functions.

#### **5.4 Final remarks**

A fundamental characteristic of the MK model is that the degree of saturation includes a capillary and an adhesion component. The model considers that the adhesive interaction between water and grains is imputable to van der Waals forces alone. This is probably true for most granular materials. For cohesive materials however, it is expected that mineralogical and chemical composition of the grains influence the surface activity and hence, the adhesion of water on the solid particles. As mentioned previously, this aspect is treated indirectly in the MK model through the use of the liquid limit  $w_L$ . A somewhat similar approach, that takes into account the influence of surface activity, was proposed by Tinjum et al. (1997) who developed empirical equations relating the van Genuchten parameters to the plasticity index  $I_P$  ( $I_P = w_L - w_P$ , where  $w_P$  is the plastic limit) and to compaction conditions (i.e. actual and optimum water contents). The influence of compaction conditions, that may affect the microstructure (fabric) and pore-size distribution is not explicitly taken into account in the MK model and this should be kept in mind by users who may wish to apply it to compacted clay liners for instance.

Various other influence factors have not been considered in the proposed MK model, including hysteresis of the WRC (e.g. Parlange 1976; Mualem 1984), suction induced compressibility (e.g. Vanapalli et al. 1999; Al-Mukhtar et al. 1999; Ng and Pang 2000), stress history (Vanapalli et al.

1999), heterogeneous and/or multimodal pore size distribution (Burger and Shackelford 2001), and presence of very coarse particles (Yazdani et al. 2000). Work is under way to introduce some of these factors into the predictive model.

Additional ongoing work being conducted aims at using MK to evaluate the relative hydraulic conductivity  $k_r(\psi)$ , by integrating the equations presented above. Function  $k_r(\psi)$ , with a predictive model of the saturated hydraulic conductivity  $k_s$  (Mbonimpa et al. 2000, 2002) will be used to obtain the unsaturated permeability function  $k_u(\psi)$ .

## 6. CONCLUSION

The authors have presented a simple set of equations for predicting the water retention curve (WRC). These functions are derived from the Kovács (1981) model, which has been modified to better define the key input parameters, and generalized to extend its application to a variety of porous media, including granular and cohesive soils.

In the proposed model, the degree of saturation includes two components: one created by capillary forces whose contribution is more important at relatively low suction, and one associated to adhesion which mainly contributes to the WRC at large suction. The main advantage of the predictive model is that both components can be evaluated from basic (and generally available) material properties, including the effective diameter  $D_{10}$ , uniformity coefficient  $C_U$ , liquid limit  $w_L$ , void ratio  $e$  and solid grain density  $\rho_s$ . These properties are used to define the equivalent capillary rise  $h_{co}$ , which constitutes the central parameter of the MK model.

The set of equations presented to predict the WRC contains three parameters required for the model application: the residual suction  $\psi_r$ , the pore size distribution parameter  $m$ , and the adhesion coefficient  $a_c$ . In the case of granular materials, a relationship has been developed

between the value of  $\psi_r$ , determined with the experimental data (and fitted WRC), and their basic geotechnical properties; a simple relationship between  $\psi_r$  and  $h_{co}$  has also been established. Because  $\psi_r$  can be very difficult to determine for plastic/cohesive (clayey) soils, the function  $\psi_r(h_{co})$  developed for granular materials is also used for clayey soils. The two other parameters,  $m$  and  $a_c$ , have been obtained first by a curve-fit procedure so that calculated WRC match experimental data as closely as possible with the MK model. Their values were then expressed from basic geotechnical properties.

In all the cases considered here, the MK model allowed a good representation of the experimental WRC for different granular and clayey materials. It can therefore be presented as a useful tool for predicting the WRC during the preliminary phases of a project. The model is not meant to replace the required tests to obtain representative data for particular conditions, but rather provides the potential users with a simple and practical means of foreseeing the possible characteristics of the WRC. The same can be said about the complementary relationships proposed to estimate the air entry value (AEV) from material basic properties. When the experimental data becomes available, these can be used to improve upon the predictive capabilities of the proposed equations, by making the necessary adjustments for specific engineering materials like soils and tailings. The discussion presented at the end of the paper also draws attention to the inherent limitations of the proposed model, especially in regards to the way the specific surface is introduced into the equations.

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## 8. APPENDIX: SAMPLE CALCULATIONS

Table A-1 Typical calculation results to predict the WRC of granular materials: Case of Sigma tailings (see Figure 11- data from Authors).

$D_{10}$ [cm]	$e$ [-]	$C_U$ [-]	$b$ (eq. 8) [cm <sup>2</sup> ]	$h_{co,G}$ (eq. 7) [cm]
0.0004	0.72	9.8	0.347	1206

$\psi_r$ (eq. 19) [cm]	$\psi_0$ (fixed) [cm]	$\psi_n$ (fixed) [cm]	$a_c$ (fixed) [-]	$m = 1/C_U$ [-]
40603	$1 \times 10^7$	1	0.01	0.102

Suction [cm]	$S_c$ (eq. 15) [-]	$C_\psi$ (eq. 18) [-]	$S_a$ (eq. 17) [-]	$S_a^*$ (eq. 16) [-]	$S_r$ (eq. 14) [-]	Predicted $\theta$ (eq. 14) [-]	Measured $\theta$ [-]
1	1.000	1.000	1.264	1.000	1.000	0.419	0.419
10	1.000	1.000	0.861	0.861	1.000	0.419	0.401
141	0.999	0.996	0.552	0.552	1.000	0.418	0.392
281	0.793	0.992	0.490	0.490	0.895	0.374	0.381
422	0.455	0.989	0.456	0.456	0.704	0.295	0.306
563	0.254	0.985	0.433	0.433	0.577	0.242	0.266
703	0.148	0.982	0.416	0.416	0.502	0.210	0.24
844	0.090	0.978	0.402	0.402	0.456	0.191	0.213
984	0.058	0.975	0.391	0.391	0.426	0.178	0.195
1125	0.038	0.972	0.381	0.381	0.405	0.169	0.183
1266	0.026	0.968	0.372	0.372	0.389	0.163	0.171
1406	0.019	0.965	0.365	0.365	0.376	0.158	0.157
1688	0.010	0.959	0.351	0.351	0.358	0.150	0.146
2110	0.004	0.951	0.336	0.336	0.339	0.142	0.138
10000	0.000	0.850	0.231	0.231	0.231	0.097	
1000000	0.000	0.000	0.000	0.000	0.000	0.000	

Table A-2 Typical calculation results to predict the WRC of plastic/cohesive soils: Case of Indian Head Till (See Figure 15- Record 3728-data from Fredlund 1999).



$w_L$ [%]	$e$ [-]	$\rho_s$ [kg/m <sup>3</sup> ]	$\xi$ (eq. 12) [cm <sup>2</sup> ]	$h_{co,P}$ (eq. 11) [cm]
35.5	0.438	2650	394.8	159489

$\psi_r$ (eq. 21) [cm]	$\psi_0$ [cm]	$\psi_n$ (fixed) [cm]	$a_c$ (fixed) [-]	$m$ (fixed) [-]
1505828	$1 \times 10^7$	1	$7 \times 10^{-4}$	$3 \times 10^{-5}$

Suction [cm]	$S_c$ (eq. 15) [-]	$C_\psi$ (eq. 18) [-]	$S_a$ (eq. 17) [-]	$S_a^*$ (eq. 16) [-]	$S_r$ (eq. 14) [-]	Predicted $\theta$ (eq. 14) [-]	Measured $\theta$ [-]
1	1.000	1.000	2.778	1.00	1.000	0.305	0.305
9.8	1.000	1.000	1.899	1.00	1.000	0.305	0.305
196	1.000	1.000	1.152	1.00	1.000	0.305	0.305
392	1.000	1.000	1.027	1.00	1.000	0.305	0.295
784	0.916	1.000	0.914	0.91	0.993	0.302	0.268
1176	0.668	0.999	0.855	0.85	0.952	0.290	0.267
1568	0.462	0.999	0.814	0.81	0.900	0.274	0.263
1960	0.328	0.999	0.784	0.78	0.855	0.260	0.254
2940	0.161	0.999	0.733	0.73	0.776	0.236	0.247
3920	0.094	0.998	0.698	0.70	0.727	0.221	0.232
4900	0.061	0.998	0.672	0.67	0.692	0.211	0.227
5880	0.043	0.997	0.652	0.65	0.667	0.203	0.220
7840	0.024	0.996	0.621	0.62	0.630	0.192	0.215
43120	0.001	0.979	0.459	0.46	0.460	0.140	0.175
372400	0.000	0.850	0.278	0.28	0.278	0.085	0.075
838880	0.000	0.720	0.206	0.21	0.206	0.063	0.055
1486660	0.000	0.592	0.154	0.15	0.154	0.047	0.040
2916480	0.000	0.406	0.094	0.09	0.094	0.029	0.024
10000000	0.000	0.000	0.000	0.00	0.000	0.000	0.000



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