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ARC ROUTING PROBLEMS WITH TIME DURATION CONSTRAINTS AND UNCERTAINTY

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ARC ROUTING PROBLEMS WITH TIME DURATION CONSTRAINTS AND UNCERTAINTY

présenté par : CHEN Boxiao

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DEDICATION

To my great parents,

who are always standing by me, my mother Hongqu and father Guangming

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RÉSUMÉ

Le problème résolu dans cette thèse est divisé en deux parties: 1) la localisation-affectation pour les tournées sur les arcs en tenant compte des caractéristiques des secteurs, c'est-à-dire le problème de conception des secteurs (SDP), 2) le routage robuste sur les arcs avec contraintes de durée (RARPTD).

Les objectifs de cette recherche sont les suivants: 1) développer une formulation mathématique pour la conception des secteurs en considérant le temps de passage sur les arcs et le niveau de service requis. Le résultat du modèle mathématique donne une solution optimale pour le problème de localisation-affectation pour les tournées sur les arcs, 2) proposer un algorithme heuristique efficace, qui assure à la fois des coûts acceptables et de bonnes caractéristiques des secteurs. L'algorithme heuristique donne une solution applicable pour le problème de conception des secteurs, 3) développer une formulation mathématique déterministe pour le problème de tournées sur les arcs avec contraintes de durée et concevoir un jeu de données d'incertitude des temps de parcours et de service, 4) proposer une formulation résoluble pour le problème de tournées robustes sur les arcs avec contraintes de durée.

Les essais sont conduits avec des données générées aléatoirement et avec un cas réel de réseau. L'analyse des résultats démontre que l'algorithme heuristique en trois étapes est plus facile à utiliser que l'algorithme branch-and-cut. De plus, l'algorithme heuristique en trois étapes peut générer une bonne solution avec des secteurs concis et bien conçus. En ce qui concerne le

RARPTD, les essais montrent que les réseaux de petite taille peuvent être résolus rapidement. L’analyse de sensibilité indique que: 1) il existe toujours deux façons d'améliorer la robustesse de la solution optimale: payer le prix de la robustesse ou ajuster l'allocation des arcs aux secteurs, 2) lorsque le nombre de véhicules augmente, la solution optimale sous faible niveau d'incertitude peut être plus robuste, mais le coût des solutions optimales sous le même niveau d'incertitude augmente.

ABSTRACT

The problem solved in this thesis is divided into two parts: 1) location-allocation arc routing with considering the characteristics of sectors, namely, the sector design problem (SDP), 2) robust arc routing with time duration based on the sectoring result of sector design (RARPTD).

The objectives of this research are: 1) to develop a location-allocation arc routing mathematical formulation with considering the deadheading time and required service level. The result of the mathematical model provides an optimal solution for the location-allocation arc routing problem, 2) to design an effective and efficient heuristic algorithm, which ensures both acceptable cost and good sector characteristics. The heuristic algorithm provides an applicable solution for the sector design problem, 3) to develop the deterministic mathematical formulation for the arc routing problem with time duration and deadheading time and design the uncertainty support set of the service time and deadheading time, 4) to propose a solvable formulation for the robust arc routing problem. The result of the robust formulation provides the robust optimal solution for the robust arc routing problem with time duration.

Experiments are conducted with randomly generated instances and a real network case. The results analysis demonstrates that the three-stage heuristic algorithm is computationally more tractable than the branch-and-cut algorithm and could yield high quality solution with compact and good shaped sectors. As for the part of RARPTD, experiments demonstrate that small-sized networks can be solved to optimality quickly and sensitivity analysis indicates: 1) there are

always two ways to improve the robustness of the optimal solution: pay the price of robustness or adjust the allocation of required edges, 2) when the number of vehicles increases, the optimal solution under low uncertainty level can be more robust but the cost of the optimal solutions under the same uncertainty level increases.

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LIST OF ABBREVIATIONS

ARP	Arc routing problem
ARPTD	Arc routing problem with time duration
CARP	Capacitated arc routing problem
CARPDD	Capacitated arc routing problem with deadheading demand
LARP	Location-allocation routing problem
L-AARP	Location-allocation arc routing problem
RARPTD	Robust arc routing problem with time duration
RPP	Rural postman problem
SARP	Sectoring arc routing problem
SDP	Sector design problem
TCARP	Time constrained arc routing problem
TSA	Three-stage algorithm
VRP	Vehicle routing problem

CHAPTER 1 INTRODUCTION

1.1 Problem definition

Road maintenance operations are conducted out of a set of depots spatially distributed on a transportation network. Each depot is responsible for providing maintenance service to a sector of the network. In this thesis, I present several algorithms for the design of service sectors and robust arc routing problem in a road network that support daily maintenance operations, which is one of the most expensive maintenance operations in both China and Canada. Service demands in each sector are met by a fleet of service vehicles. Typical service demands include visually checking the operational status of each road segment, evaluating the function of the auxiliary facilities, reporting the defects of the road and so on. Service vehicles operate on routes that start and end at the depot assigned to each sector.

The sector design and schedule of the maintenance routes are considerably important. For instance, every year, Quebec`s winter lasts about 5 months. Coming along with the winter is the thick layer of snow on the road network. One of the most troublesome problems is that roads covered with snow make the transportation difficult. One way to deal with this problem is to remove snow on roads. Daily snow removal operations include visually checking the operational status of each road segment, reporting the defects, shoveling snow and so on. Unreasonable routes of the snowplows will lead to significantly high operational cost. Therefore, the scientific schedule of removal routes is quite important. vc

The problem solved in this thesis is divided into two parts: 1) location-allocation arc routing with considering the characteristics of sectors, called sector design for simplicity. The contents of this part have been published in the International Journal of Production Research by Chen et al. (2017); 2) robust arc routing based on the sectoring result of sector design. In the next sub-sections these two sub-problems are defined in more details.

1.1.1 Sector design

The sector design problem (SDP) addressed in this thesis consists of determining depots' locations and their respective service sectors on a road network in order to serve the whole set of road segments under given operational constraints. The number of vehicles required to provide maintenance service to a given sector depends on the workload of each sector. In this thesis, the workload is defined as the total working time, including service time and deadheading time. In order to precisely assess the workload within each sector, the routing operation must be taken into consideration. The objective is to minimize the overall cost with an acceptable service level. Moreover, the characteristics of the resulting sectors need to be taken into consideration.

In conclusion, with considering the characteristics of sectors, the SDP is to achieve that (1) each arc is assigned to a depot; (2) each arc is serviced on a route that starts and ends at its assigned depot; (3) each depot is assigned with a number of vehicles depending on its workload; (4) the working duration of each vehicle does not exceed a given threshold; and (5) the total cost is minimized.

1.1.2 Robust arc routing with time duration

The arc routing problem with time duration constraints (ARPTD) is an extension of the arc routing problem (ARP), which is different from the classical capacitated arc routing problem (CARP). The classical capacitated arc routing problem was introduced by Golden and Wong (1981), which is usually defined on a connected undirected graph, consisting of determining a set of vehicle routes such that (1) each vehicle starts at the depot, services a subset of the edges it traverses, then returns to the depot; (2) the total demand serviced by a vehicle does not exceed capacity; (3) each required edge is serviced exactly once; and (4) the sum of deadheading costs is minimized.

However, when we study the problem of road network daily maintenance routing problem, we find that the work time duration rather than capacity of each maintenance vehicle is limited compared with CARP. The work time included both the service time and deadheading time. In practice, the service time and deadheading time are often subject to significant uncertainty. In this thesis, it is assumed that both the service times on required arcs and deadheading times are uncertain and belong to a polyhedral uncertainty set Q. Then, the robust arc routing problem with time duration constraints (RARPTD) is studied.

1.2 Research objectives

In this study, the general objective is to make the scientific planning of daily maintenance operations on road maintenance networks. However, when we study the problem of road network daily maintenance planning problem, if depot location, arcs allocation and robust arc routing are considered at the same time in building a mathematical formulation, the formulation becomes too complex and contains too many variables to be solved to optimality. Therefore, the problem studied in this thesis is divided into two parts:

1) Location-allocation arc routing with considering the characteristics of sectors, called sector design for simplicity, which concerns locating the maintenance depots, partitioning the large network into sectors and deterministic arc routing. The specific objectives in this part can be summarized as:

- Develop a location-allocation arc routing mathematical formulation with considering the deadheading time and required service level without the sector design component. The objective is to minimize the overall cost. The formulation will be solved with a branch-and-cut algorithm. The result of the mathematical formulation provides an optimal solution for the location-allocation arc routing problem.
- Define the sectoring evaluation and design an effective and efficient heuristic algorithm, which ensures both acceptable cost and good sector characteristics. Apply a post-algorithm to optimize the maintenance route within each sector. The heuristic algorithm provides an applicable solution for the sector design problem.
- Compare the results of the branch-and-cut algorithm and the heuristic algorithm with both some randomly generalized instances and a real network case.

2) Robust arc routing based on the sectoring result of sector design, which aims at realizing the robust and cost-saving arc routing under service time and deadheading time uncertainty. The specific objectives in this part can be summarized as:

- Develop the deterministic mathematical formulation for the arc routing problem with time duration and deadheading time and design the uncertainty support set of the service time and deadheading time, which is a more general polyhedral set and less conservative.
- Find the robust counterpart of the deterministic formulation to describe the robust arc routing problem and transform it into a solvable integer linear programming with duality techniques. Solve it with the branch-and-cut algorithm. The result of the robust integer linear programming provides the robust optimal solution for the arc routing problem with uncertain service time and deadheading time.
- Conduct experiments with randomly generated instances to verify the performance of the robust formulation and also apply sensitivity analysis to a sector obtained from the sectoring result of the real network. The uncertainty level and the number of vehicles are varied to conduct the sensitivity analysis and draw some conclusions.

The next section presents the overall structure of this thesis.

1.3 Thesis structure

In this chapter, the definition of the problem studied and the research objectives are presented.

Chapter 2 presents a literature review over common methodologies and approaches used in capacitated arc routing problem (CARP), sector design problem (SDP) and robust arc routing problem with time duration (RARPTD), then, the methodology and contribution of this thesis are presented.

The SDP formulation and the three-stage algorithm are described in chapter 3, then, the computational results of some randomly generated instances and a real network case are presented to verify the efficiency and effectiveness of the three-stage algorithm.

In chapter 4, the deterministic ARPTD formulation is first built, then, the RARPTD formulation is obtained after the definition of uncertainty set and the application of duality techniques. The formulation is applied to a sector from the sectored real network. The sensitivity analysis is conducted considering the uncertainty level and number of vehicles.

Finally, chapter 5 concludes the thesis and proposes some future research directions.

CHAPTER 2 LITERATURE REVIEW

This chapter presents the review of studies about the capacitated arc routing problem (CARP), the location-allocation arc routing problem, the sectoring arc routing problem, the CARP with uncertainty and robust optimization, divided into three main sections:

- Capacitated arc routing problem
- Sector design
- Robust arc routing with time duration

Besides, Section 4 presents the methodology of this research and Section 5 situates the contribution of this research compared to existing related approaches and methodologies.

2.1 Capacitated arc routing problem

There are many relevant literature and relatively mature research about CARP. The exact algorithms for the CARP can be divided into three main categories: 1) based on cutting planes (Belenguer and Benavent 1998, 2003); 2) based on transformation of the CARP into a capacitated vehicle routing problem (Baldacci and Maniezzo 2006, Longo et al. 2006; 3) based on column generation and additional cutting planes approaches (Keenan 2001, Gómez-Cabrero et al. 2005, Letchford and Oukil 2009, Martinelli et al. 2011, Bartolini et al. 2011, Bode and Irnich 2011).

Compared with the ARPTD, there are some similar problems derived from CARP studied before for example the time constrained arc routing problem (TCARP) and the capacitated arc routing problem with deadheading demand (CARPDD).

TCARP is motivated by a postal delivery application in rural Ireland by Keenan (2005), which is a generalization of the CARP where both a service time and a traversal time are associated with each edge, while servicing the required edges, satisfying the duration limit and minimizing the total route duration. However, in the ARPTD, the objective is to minimize the deadheading cost, which is more general than the objective of minimizing the total route duration of the TCARP.

CARPDD extends the classical capacitated arc routing problem by introducing an additional capacity consumption incurred by a vehicle deadheading an edge, which was formally introduced by Kirlik and Sipahioglu (2011). In 2013, Bartolini et al. (2013) proposed an exact algorithm for the deterministic CARPDD. Compared to the ARPTD, CARPDD is more general because of the limited vehicle capacity rather than the time duration of each route and can apply to several practical situations; however, for each specific situation, there are some distinguishing characteristics that the generalized CARPDD cannot describe.

2.2 Sector design

In the following subsections the works on the location-allocation arc routing problem, and the sectoring arc routing problem are reviewed respectively.

2.2.1 Location-allocation arc routing problem

Location-allocation routing problems (LARP) deal with the combination of three types of decisions: the location of facilities, the allocation of customers, and the design of distribution routes. While most LARP papers address node routing (see the survey papers Prodhon and Prins, 2014, Drexl and Schneider, 2015), the location-allocation arc routing problem (L-AARP) is quite overlooked in the literature.

Levy and Bodin (1989) address the problem of designing districts for postal delivery. A heuristic approach is developed to firstly assign all edges to a depot, and then improve the balance of the districts.

Ghiani and Laporte (1999) address the problem of locating a set of depots in an arc routing context. It is shown that the problem can be transformed into a Rural Postman Problem (RPP) if there is a single depot to open or no bounds on the number of depots. The problem is then solved to optimality using a branch-and-cut algorithm.

Amaya et al. (2007) address the capacitated ARP with refill points. The vehicle servicing arcs must be refilled on the spot by using a second vehicle. The problem can be viewed as a L-AARP because in addition to traversing edges or arcs, the refill points must also be located on the graph. The problem is solved by a cutting plane approach.

Doulabi and Seifi (2013) address multi-depot location arc routing problems with vehicle capacity constraints. A simulated annealing algorithm is proposed to solve the relaxed formulation in order to obtain a lower bound for the problem.

Lopes et al. (2014) present some new constructive and improvement methods used within different meta-heuristic frameworks.

Compared with the SDP addressed in this paper, the L-AARP focuses only on the minimization of cost, without considering the characteristics of sectors (workload balance, compactness, etc.).

2.2.2 Sectoring arc routing problem

Sectoring (or districting) arc routing problem (SARP) is to sector a large region into sectors (or districts), to facilitate the organization, planning or control of activities performed within the network owing to the easier planning of operations in smaller sub-networks. Compared with location and vehicle routing problem, Districting Problem is a distinct stage in the organization of arc routing application. Thus, it hasn't received a lot of attention in the literature.

Hanafi et al. (1999) consider a sectoring problem in the context of municipal waste collection. The objective is to achieve workload balance among all the sectors, and to minimize the number of connected components within a sector.

Labelle et al. (2002) consider a sector design for snow removal operations. The problem is to partition the street network into sectors, and to allocate sectors to disposal site so as to minimize the total operational cost. An “assign-first-partition-second” heuristic procedure was proposed to solve the problem.

Muyldermans et al. (2002) consider the problem of district design for salt spreading operations. The problem involves partitioning a road network into non-overlapping, connected sub-networks, each with a depot. A heuristic procedure was developed in which the road network was firstly partitioned into small cycles. Then the small cycles were aggregated into districts.

Mourão et al. (2009) consider the sectoring arc routing problem, which involves the partition of a road network into sectors and the design of routes for each sector, so as to minimize the total duration of the trips. The number of sectors is pre-specified. One vehicle is associated with each sector, but it may perform several trips so as to satisfy capacity and time limit constraints. Three heuristic approaches are evaluated based on different criteria, such as routing distance, workload imbalance, and compactness, etc.

Bozkaya et al. (2011) use a tabu search heuristic embedded within a geographic information system-based decision support system to tackle the districting problem. The resulting district meets districting criteria.

Silva de Assis et al. (2014) discuss a districting problem in the context of meter reading in power distribution networks. A bi-criteria mathematical programming formulation that tries to maximize balance and compactness is proposed to optimize the current districting plan.

Butsch et al. (2014) proposes a heuristic to design districts in an arc routing context. Solutions must satisfy both hard criteria (complete and exclusive assignment as well as connectedness) and several soft criteria (balance, small deadheading, local compactness, and global compactness). The heuristic applies a construction procedure followed by a tabu search improvement phase.

Kandula and Wright (1995, 1997) also consider a combined location, sectoring and fleet sizing problem for the application of snow removal on roads. The problem was formulated as a mixed integer linear program with the location allocation components associated with district design. Their experimental results demonstrate the effectiveness of the methodology in designing districts. However, the authors have not proposed any algorithm that could be applied to solve problems other than the case described in their papers. Besides this work, no other work has been found to deal with combined location, allocation, and sectoring problem.

2.3 Robust arc routing with time duration

In this section, we review the works on the CARP with uncertainty and the robust optimization respectively.

2.3.1 CARP with uncertainty

In practice, the service time and deadheading time are often subject to significant uncertainty. The CARP with stochastic demand were dealt with in some papers. For some applications, the demand on each edge is described by a random variable with a specific probability distribution.

Fleury et al. (2004) study the robustness of deterministic CARP solutions when demands are randomized. A memetic algorithm is adapted to handle the randomness of the demands. Fleury et al. (2005) study the CARP with stochastic demand with normally distributed edge demands.

Christiansen et al. (2009) consider the CARP with stochastic demand with the demands on required edges described by Poisson distribution. The problem is formulated as a set-partitioning problem and solved by a branch-and-price algorithm with pricing carried out by dynamic programming.

Laporte et al. (2010) study the CARP with stochastic demand described by Poisson distribution in the context of garbage collection. An adaptive large-scale neighbourhood search heuristic is developed for the problem.

However, considering insufficient historical data, unclear statistical information, the difficulty to obtain the probability distribution of uncertain parameters and so on, robust optimization techniques are better used to deal with the uncertainty. Chen et al. (2017) study the robust optimization approach for the directed arc routing problem encountered in daily maintenance operations of a road network under the uncertainty of service time. Except that, there is not much relevant research on robust arc routing problem.

2.3.2 Robust optimization

The robust optimization approach was introduced for convex optimization problems by Ben-Tal and Nemirovski (1998,1999). Although there is not much relevant work on robust optimization techniques on the arc routing problem, the methodology has been applied in other different settings, a closely related one of which is the vehicle routing problem.

List et al. (2003) solve the fleet planning under uncertainty with robust optimization techniques and explore the tradeoff between the expected cost and the risk of extreme outcomes of uncertain variables.

Sungur et al. (2008) study a robust VRP with demand uncertainty aiming at minimizing transportation costs while satisfying all demands in a given bounded uncertainty set. Computational results showed that the robust solution incurs only a small additional cost over deterministic solution.

Erera et al. (2010) consider the VRP with stochastic demands and duration constraints. The sum of expected route durations over all possible realizations is minimized with robust optimization techniques by solving the optimization problem of an adversary. A set of vehicle routes is first selected before the uncertain demands realized. Additional travel time is considered due to recourse action after demands realizing.

Souyris et al. (2012) study the problem of dispatching technicians to service distributed equipment with service time uncertain. The problem is formulated as a vehicle routing problem with soft time windows and a robust optimization model is proposed and solved by branch-and-price.

Agra et al. (2013) solve the robust vehicle routing problem with time windows by two different robust formulations and compare the two robust formulations on a test bed composed of maritime transportation instances.

Gounaris et al. (2013) derive robust counterparts of different deterministic Capacitated VRP formulation, use the robust capacity rounded inequalities as the cuts to expedite solving and then compare the performance of all the robust formulations.

The methodology adopted in this thesis is presented in the next section.

2.4 Methodology

As it can be seen from the literature review, the proposed approaches and models have some limitations and shortcomings to design sectors and (robust) routes efficiently, three of which are listed below and addressed in later chapters of this thesis.

- The deadheading time is ignored in most cases.

- The location-allocation problem and sectoring problem are considered separately.
- Stochastic routing is done without considering insufficient historical data or computational intractability.

In this thesis, the deadheading time is considered both in the algorithm design, to assess to workload within each sector more precisely, and in routing formulations. Location-allocation arc routing problem and sectoring arc routing design problem are combined to obtain both good overall cost and good characteristics of the partitioned sectors. The robust optimization is applied with a general polyhedral uncertainty set to deal with uncertain demands, overcoming the difficulties of insufficient historical data and computational intractability of statistics method. Specific methodology used in this thesis is summarized as follows.

- The location-allocation arc routing problem and deterministic arc routing problem with time duration are both formulated as mixed integer linear programming models, aiming at minimizing the overall cost and the deadheading cost respectively.
- The sectoring evaluation is designed according to the literature. Based on the sectoring evaluation, combining the sweeping method and greedy method applied to the VRP, a three-stage heuristic algorithm is proposed to solve the sector design problem. In order to precisely assess the workload within each sector, the routing operation is taken into consideration in the first stage.

- Define the uncertain service time and deadheading time with a general polyhedral support set, then, derive the robust counterpart of the deterministic formulation of the arc routing problem with time duration. Duality techniques are applied to deal with the infinite constraints caused by the uncertainty set.

- Conduct experiments with randomly generated instances and a real network case to verify the effectiveness and efficiency of the three-stage heuristic algorithm. Do sensitivity analysis of robust solutions under different levels of uncertainty and different numbers of vehicles.

The next section presents the contribution of this thesis.

2.5 Contribution of this thesis

The contribution of this thesis can be summarized as follows,

- As far as the sector design problem is concerned, existing work always consider location-allocation arc routing and sectoring arc routing separately. When considering location-allocation arc routing problem without sectoring component, the formulation will only focus on the minimization of overall cost, thus, ignoring the characteristics of sectors, resulting in overlapped and imbalanced sectors. We combined location-allocation and sectoring in the three-stage algorithm in this thesis to overcome above shortcomings and also improve the solving efficiency compared with the exact algorithm.

- In practical, when maintenance vehicles service roads, there will be both service time and deadheading time consumed, however, classical capacitated arc routing problem ignores the deadheading time. In this thesis, the deadheading time is considered for both the design of three-stage algorithm and the building of the mathematical formulations.

- In this thesis, the deterministic mathematical formulation for the ARPTD is first proposed. Then, the robust counterpart of the deterministic formulation under a general polyhedral uncertainty set is developed. The existing approach to deal with the robust problem (Sungur et al. 2008) assumes service and deadheading times could attain their worst-case realizations simultaneously. Compared with that, the general polyhedral uncertainty set can avoid overly conservative solutions as some statistical information can be injected into it such as the correlation between variables. This is the first time of the general polyhedral uncertainty set to be used in solving the robust arc routing problem.

CHAPTER 3 SECTOR DESIGN

In this chapter, the sector design problem (SDP) is addressed, which is defined as a location-allocation arc routing problem with considering some sought-after characteristics of sectors.

The sector design problem (SDP) is addressed within the following structure. Section 1 first presents the formulation of the location-allocation arc routing problem (L-AARP) without considering any characteristics of sectors, aiming at minimizing the overall cost. Combining the sweeping method and the greedy method applied to the VRP, Section 3 proposed a three-stage heuristic algorithm to solve the sector design problem. The sectoring evaluation is designed according to the literature in Section 4. Experiments are conducted with randomly generated instances and a real network case in Section 5 to verify the effectiveness and efficiency of the three-stage heuristic algorithm. Finally, Section 6 summarizes this chapter and draws some conclusions and remarks.

The contents of this chapter have been published in the International Journal of Production Research by Chen et al. (2016).

3.1 Formulation

The sector design problem (SDP) addressed in this thesis consists of determining depots' locations, their respective service sectors and routes within each sector. The number of service vehicles allotted to each sector must be appropriate to the assigned workload and in this thesis, the workload is defined as the total working time, including service time and deadheading time. In order to precisely assess the workload within each sector, the routing operation is taken into consideration both when building the formulation of the location-allocation arc routing problem (L-AARP) and designing the three-stage heuristic algorithm. The objective is to minimize the overall cost with an acceptable service level. In this section, the formulation of the location-allocation arc routing problem (L-AARP) is given, where the characteristics of sectors are not considered compared with the sector design problem (SDP).

The mathematical formulation for the L-AARP is defined on a directed graph $G = (V, A)$, with a set of vertices V, and an arc set A. The potential locations of the depots are selected from the set V. The set of potential locations is denoted as S. Thus, S is a subset of V. Each arc $a \in A$ has a service time. The problem is to find a set of maintenance depots, such that: (1) each arc in A is assigned to a depot; (2) each arc is serviced on a route that starts and ends at its assigned depot; (3) each depot is assigned with a number of vehicles depending on its workload; (4) the working duration of each vehicle does not exceed a given threshold; (5) the total cost is minimized.

3.1.1 Notation

The following notation is used in the proposed formulation.

Parameters:

V set of vertices, $v \in V$

A set of arcs, $a \in A$

S set of potential locations of depots. S is a subset of V

K set of vehicles, $k \in K$

TV number of vehicles available for all sectors

$O(v)$ set of arcs leaving vertex, $v \in V$

$I(v)$ set of arcs entering vertex, $v \in V$

Y subset of vertices, $Y \subseteq V$

$A(Y)$ the set of arcs whose two end nodes are both in Y

F fixed cost of locating a depot at any site s, $s \in S$

f fixed cost of using a vehicle

TD the time duration limit of a route

L_{as} distance from depot s to arc a, $s \in S$, $a \in A$. L_{as} is the distance of the shortest path between s and the starting node of arc a.

d_a length of arc a, $a \in A$

r maximal service distance of an open depot. Penalty cost will occur on arcs located outside of the maximum service distance of the assigned depot.

s_a service time on arc a, $a \in A$

t_a deadheading time on arc a, $a \in A$

γ required service level. It is defined as the fraction of the arcs that are located within the maximal service distance of the maintenance depot. $\gamma \leq 1$.

P unit penalty cost of the arcs located outside of the maximum service distance

Q unit deadheading travel cost

i_{as} is defined as a binary parameter that takes the value of 1 if the arc a is outside the coverage of the maximal service distance of a potential depot.

$$i_{as} = \begin{cases} 1, if\ L_{as} \geq r \\ 0, otherwise \end{cases}$$

Then, matrix I is defined, $I = \{i_{as} | s \in S, a \in A\}$.

Decision variables:

b_s = 1, if a depot is built at site s; 0, otherwise, $s \in S$.

x_{as}^k =1, if arc a is serviced by vehicle k based at depot s; 0, otherwise, $s \in S$, $k \in K$, $a \in A$.

y_{as}^k number of times arc a is traversed (without servicing) by vehicle k at depot s, $s \in S$, $k \in K$, $a \in A$.

z_{ks} = 1, if vehicle k is used at depot s; 0, otherwise, $s \in S$, $k \in K$.

3.1.2 Objective function

The objective is to minimize the overall service cost D, which includes the fixed cost of using the depots D_1, the fixed cost of vehicles D_2, the deadheading travel cost D_3, and the total penalty cost D_4. D_1 is obtained by multiplying the fixed cost of locating a depot by the total number of depots built. D_2 is calculated by multiplying the fixed cost of a vehicle by the total number of vehicles used. Unit deadheading cost is multiplied by the total length of arcs deadheaded to obtain the deadheading travel cost D_3. As for the total penalty cost D_4, it is obtained by multiplying the unit penalty cost by the total length of the arcs located outside of the maximum service distance.

The objective function of the L-AARP formulation is:

$$\min D = \min(D_1 + D_2 + D_3 + D_4)$$

$$= \min(F\sum_{s\in S} b_s + f\sum_{s\in S}\sum_{k\in K} z_{ks} + Q\sum_{s\in S}\sum_{k\in K}\sum_{a\in A} d_a y_{as}^k + P\sum_{s\in S}\sum_{k\in K}\sum_{a\in A} i_{as} d_a x_{as}^k \tag{3.1}$$

3.1.3 3.1.3 Constraints

Number of service time

One of the given operational constraints is that each required arc must be serviced exactly once, therefore (3.2) is used to ensure this operational constraint.

$$\sum_{k\in K}\sum_{s\in S} x_{as}^k = 1, \forall a \in A \tag{3.2}$$

Flow constraints

For any complete vehicle route, we know that at each node, the number of arcs leaving and entering must be equal, which is called the network flow conservation constraints and described with constraints (3.3).

$$\sum_{a\in O(v)} (x_{as}^k + y_{as}^k) = \sum_{a\in I(v)} (x_{as}^k + y_{as}^k), \forall k \in K, s \in S, v \in V \tag{3.3}$$

$$\sum_{a\in O(s)} (x_{as}^k + y_{as}^k) \geq \frac{\sum_{a\in A} x_{as}^k}{|A|}, \forall k \in K, s \in S \tag{3.4}$$

Constraints (3.4) combined with (3.3) ensure that each vehicle k must leave and return to the depot if it is used at that depot to serve any arc.

Connectivity constraints

A classical type of constraints for routing problem is the connectivity constraints. A vehicle route of routing problem without connectivity constraints may break down into 2 or more disconnected tours, all of which satisfy the flow conservation constraints however some of which do not pass through the depot, as illustrated in figure 3-1. The vehicle route breaks down into 2 disconnected subtours, depot-1-2-depot and 3-4-5-6-3.

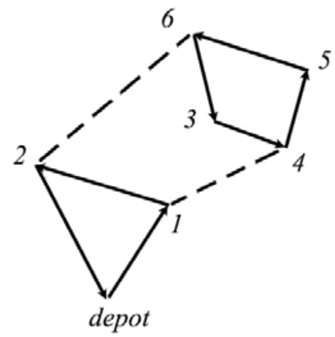


Figure 3-1 Disconnected subtours

Constraints (3.5) are the connectivity constraints, which ensure that if there is any arc in $A(Y)$ served by vehicle k of depot s, then there must be an arc in $O(Y)$ traversed or served by vehicle k of depot s (vehicle k crosses the border of Y).

$$\sum_{a \in O(Y)} (x_{as}^k + y_{as}^k) \geq x_{rs}^k, \forall Y \subseteq V, s \notin Y, r \in A(Y), k \in K, s \in S \tag{3.5}$$

Relationship between decision variables

x_{as}^k is related to both z_{ks} and b_s as we know that a vehicle is used only if it serves at least one arc and a depot is built only if it provides service to at least one arc, which are ensured by constraints (3.6) and (3.7) respectively.

$$z_{ks} \geq \frac{\sum_{a \in A} x_{as}^k}{|A|}, \forall k \in K, s \in S \tag{3.6}$$

$$b_s \geq \frac{\sum_{a \in A} \sum_{k \in K} x_{as}^k}{|A|}, \forall s \in S \tag{3.7}$$

Time duration constraints

The working duration for each maintenance vehicle is limited. Constraints (3.8) ensure that the total work duration (travel time plus service time) of each vehicle is less than the maximum allowed working duration during each shift.

$$\sum_{a\in A}(s_a x_{as}^k + t_a y_{as}^k) \le TD, \forall k \in K, s \in S \tag{3.8}$$

Service level constraints

The maximum number of arcs located out of the coverage of the maximal service distance of their assigned depot is limited by the required service level. Multiply $(1-\gamma)$ by the total number of arcs serviced by the depot to obtain the maximum number of arcs located outside of the maximal service area of the depot.

$$\sum_{k\in K}\sum_{a\in A} i_{as} x_{as}^k \le (1-\gamma)\sum_{k\in K}\sum_{a\in A} x_{as}^k\,, \forall s \in S \tag{3.9}$$

Total number of vehicles

The total number of vehicles used must be less than the number of available vehicles *TV*, which are valid constraints to tight the formulation.

$$\sum_{k\in K}\sum_{s\in S} z_{ks} \le TV \tag{3.10}$$

Integer and non-negativity constraints

These constraints guarantee the integrality and non-negativity of the decision variables in the formulation.

$$x_{as}^k, b_s, z_{ks} \in \{0,1\}, \forall a \in A, k \in K, s \in S \quad (3.11)$$

$$y_{as}^k \geq 0, integer, \forall a \in A, k \in K, s \in S \quad (3.12)$$

Complete formulation

Now the complete formulation can be presented in the following, which includes the objective function, service time of required arcs, flow constraints, connectivity constraints, relationship between decision variables, time duration constraints, service level constraints, total number of vehicles and integer and non-negativity constraints.

$$\min D = \min(D_1 + D_2 + D_3 + D_4)$$

$$= \min(F \sum_{s \in S} b_s + f \sum_{s \in S} \sum_{k \in K} z_{ks} + Q \sum_{s \in S} \sum_{k \in K} \sum_{a \in A} d_a y_{as}^k + P \sum_{s \in S} \sum_{k \in K} \sum_{a \in A} i_{as} d_a x_{as}^k)$$

(3.1)

$$\sum_{k \in K} \sum_{s \in S} x_{as}^k = 1, \forall a \in A \quad (3.2)$$

$$\sum_{a \in O(v)} (x_{as}^k + y_{as}^k) = \sum_{a \in I(v)} (x_{as}^k + y_{as}^k), \forall k \in K, s \in S, v \in V \quad (3.3)$$

$$\sum_{a \in O(s)} (x_{as}^k + y_{as}^k) \geq \frac{\sum_{a \in A} x_{as}^k}{|A|}, \forall k \in K, s \in S \quad (3.4)$$

$$\sum_{a \in O(Y)} (x_{as}^k + y_{as}^k) \geq x_{rs}^k, \forall Y \subseteq V, s \notin Y, r \in A(Y), k \in K, s \in S \quad (3.5)$$

$$z_{ks} \geq \frac{\sum_{a \in A} x_{as}^k}{|A|}, \forall k \in K, s \in S \quad (3.6)$$

$$b_s \geq \frac{\sum_{a\in A}\sum_{k\in K} x_{as}^k}{|A|}, \forall s \in S \tag{3.7}$$

$$\sum_{a\in A}(s_a x_{as}^k + t_a y_{as}^k) \leq TD, \forall k \in K, s \in S \tag{3.8}$$

$$\sum_{k\in K}\sum_{a\in A} i_{as} x_{as}^k \leq (1-\gamma)\sum_{k\in K}\sum_{a\in A} x_{as}^k, \forall s \in S \tag{3.9}$$

$$\sum_{k\in K}\sum_{s\in S} z_{ks} \leq TV \tag{3.10}$$

$$x_{as}^k, b_s, z_{ks} \in \{0,1\}, \forall a \in A, k \in K, s \in S \tag{3.11}$$

$$y_{as}^k \geq 0, integer, \forall a \in A, k \in K, s \in S \tag{3.12}$$

This section presents the formulation of the location-allocation arc routing problem (L-AARP) without considering the characteristics of sectors, with the objective of minimizing the overall cost, which will be solved by the branch-and-cut algorithm. In order to solve the location-allocation arc routing problem with considering the characteristics of sectors, based on the sectoring evaluation, a three-stage heuristic algorithm is designed in the next sections.

3.2 Branch-and-cut algorithm

This section proposes a branch-and-cut algorithm based on the mathematical formulation introduced in Section 3.1. Because the connectivity constraints (3.5) have exponential size, the connectivity constraints are separated during the search. The following three different separation techniques have been developed.

i) The exact method that detects violated constraints via solving a sequence of min-cut problems by the well-known push-relabel algorithm (Ahuja et al. 1993).

ii) A heuristic method that detects the violation via identifying isolated components by using a union-find data structure (Corman et al. 2001). For each isolated component, all violated inequalities are added.

iii) The same method with ii), except that only the most-violated inequality for each isolated component is added.

If a valid inequality is denoted as $ax \geq b$, where x is a vector of variables, a is a vector of coefficients and b is scalar, then the violation of this inequality can be defined as $b - ax$. The most-violated inequality means the one with the largest value of $b - ax$.

In addition, the following symmetry breaking constraints are introduced to speed up the process of the branch-and-cut algorithm. They ensure that at each depot, the number of arcs serviced by vehicle k must be greater than or equal to the number of arcs serviced by vehicle $k + 1$. Thus, the algorithm can avoid those solutions with the same objective value but being different only by exchanging routes of two vehicles.

$$\sum_{\square \in \square} x_{as}^{k} \geq \sum_{\square \in \square} x_{as}^{k+1}, \forall s \in S, k \in \{1, \dots, |K| - 1\} \tag{3.13}$$

We now describe the details of the algorithm. The connectivity constraints (3.5) are first relaxed, as well as the integer requirement of the variables of the original model (P), and then include the symmetry-breaking constraints into model (P). The original model (P) has turned into a new linear program (P').

(P')

$$\min D = \min(D_1 + D_2 + D_3 + D_4)$$

$$= \min(F \sum_{s \in S} b_s + f \sum_{s \in S} \sum_{k \in K} z_{ks} + Q \sum_{s \in S} \sum_{k \in K} \sum_{a \in A} d_a y_{as}^k + P \sum_{s \in S} \sum_{k \in K} \sum_{a \in A} i_{as} d_a x_{as}^k$$

subject to:

(3.2) – (3.4), (3.6) – (3.10), (3.13) and

$$0 \leq x_{as}^k, b_s, z_{ks} \leq 1, \forall a \in A, k \in K, s \in S \tag{3.14}$$

$$y_{as}^k \geq 0, \forall a \in A, k \in K, s \in S \tag{3.15}$$

After model (P') is solved, the constraint set is updated by adding the violated connectivity constraints (5) and try to solve (P') again. This process continues iteratively until the optimum found for (P') is feasible for constraints (3.5). Then, if there are fractional variables, we branch on one fractional variable to generate two new sub-problems. If all the variables are integer, we explore another sub-problem, i.e., for each sub-problem, the previous procedure is repeated.

The branch-and-cut algorithm is implemented in C++, using IBM ILOG CPLEX Concert Technology, version 12.6. The standard CPLEX cuts are automatically added. All the CPLEX parameters are set to their default values.

3.3 A three-stage heuristic algorithm

Solving a sector design problem involves not only partitioning the entire road network into sectors but also determining the maintenance routes within each sector with considering good characteristics of sectors. One way to overcome this complexity is a three-stage procedure in which the first stage is to decide the clusters using the adapted sweeping method, and the second stage is to assign the clusters to the potential sites of depots based on minimum cost criteria. Finally, in the third stage, the locations of depots and their respective service sector are determined through a number of iterations. A post-algorithm procedure is applied to optimize the maintenance routes within each sector and determine a set of minimum cost routes in a service sector. Details of the proposed algorithm are described in the following subsections.

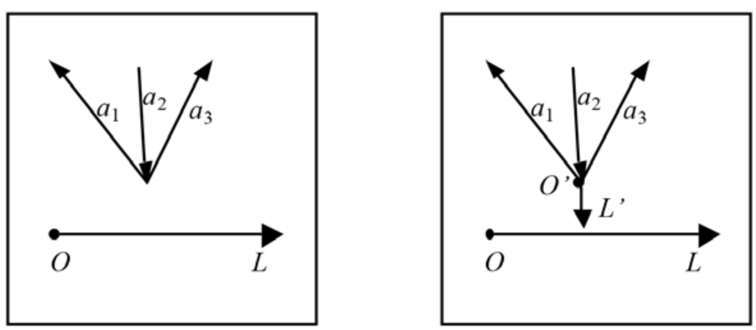
3.3.1 Clustering the arcs (road segments)

A cluster is a set of arcs that will be served by a single vehicle. And the workload of a vehicle should not exceed its working duration *TD*. The sweeping method for solving the vehicle routing problem has been adapted to build clusters. The adapted sweeping method (ASM) is proposed to assign nearby arcs to each cluster without exceeding the time duration constraint. The rationale implied by this method is to mimic the behavior of road maintenance service in order to achieve sector compactness.

Firstly, all the arcs in *A* are ordered by the following procedure.

Select a vertex O centrally located on graph G as the pole, and the polar axis, L, is set to be from O to the right direction. Put G in the polar coordinates system. For each arc a, $a \in A$, the starting vertex and the ending vertex can be represented respectively as (ρ_a, θ_a) and (ρ_a', θ_a'). Represent arc a by the polar coordinates of the vertex with smaller angular coordinate.

Order all the arcs in A in non-decreasing order of their angular coordinates. For those arcs whose angular coordinates are equal, order them in non-decreasing order of their radial coordinate. There exist some arcs for which both their radial and angular coordinates are equal. That is, these arcs have an intersection. In this case, a new polar coordinate system is built with the intersection as the new pole O'. L' is set to be from O' to the down direction. Order these arcs in non-decreasing order of their new angular coordinates.



(a) three arcs have equal polar coordinates (b) new polar coordinates

Figure 3-2 An instance having arcs with equal polar coordinates

For example, in the polar coordinate system shown in Figure 3-2(a), the three arcs, a_1, a_2, a_3, have the same polar coordinates. In Figure 3-2(b), a new polar coordinates system is built. Thus these three arcs are ordered in non-decreasing order of their angular coordinates in the new system as a_3, a_2, a_1.

Denote the resulting sequence of arcs as $\bar{A}$. The steps of the ASM are as follows:

Step 1: Set $n = 1$.

Step 2: Let A_n be the set of the arcs that belong to cluster n. Let C_n denote the workload of servicing all the arcs in A_n. Set A_n to be Ø. Set C_n to be 0.

Step 3: Get the first arc from $\bar{A}$, and denote it as $\bar{a}$.

Step 4: Include $\bar{a}$ into A_n, and update the workload C_n. C_n can be obtained by solving a Directed Rural Postman Problem (DRPP).

Step 5: If $C_n < TD$, then go to Step 6; otherwise remove $\bar{a}$ from A_n, close cluster n, set $n = n + 1$ and go to Step 2.

Step 6: Remove $\bar{a}$ from $\bar{A}$. If $\bar{A}$ is empty, then stop; otherwise go to Step 3.

In Step 4, we used the method introduced by Bartolini et al. (2011) to transform the DRPP into a Vehicle Routing Problem (VRP). Then, an integer linear programming formulation with Miller-Tucker-Zemlin (Kulkarni & Bhave, 1985) constraints is developed for the VRP and solved by a branch-and-bound algorithm. In Step 5, when a specific arc cannot be inserted into a cluster without violating the capacity constraint, this cluster is closed. When the ASM method terminates, the value of n is the total number of vehicles needed, denoted as N. Thus, $A = \{A_n\}_{n=1}^{N}$.

3.3.2 Assigning the clusters

The objective of this stage is to get an assignment of clusters to potential sites satisfying the service level requirement. Firstly, we introduce how to calculate the service level and the incurred penalty cost associated with each assignment.

Upon the assignment of cluster A_n to a potential site s ($A_n \rightarrow s$), denote the subset of arcs in A_n whose L_{as} is less than or equal to r as A_n^*. The service level associated with this assignment ($A_n \rightarrow s$) is defined as:

$$\gamma(A_n \rightarrow s) = \frac{|A_n^*|}{|A_n|} \tag{3.16}$$

The associated penalty cost is defined as:

$$D_4(A_n \rightarrow s) = \sum_{a \in A_n - A_n^*} P \cdot d_a \tag{3.17}$$

In order to assign a cluster A_n $(n \in N)$ to a potential site, the following procedure is proposed:

Step 1: For each $s \in S$, do:

i) calculate the associated service level γ ($A_n \rightarrow s$);

ii) calculate the associated penalty cost $D_4(A_n \rightarrow s)$.

Step 2: Let S^* be the set of possible sites that satisfy the service level. If $S^* = \emptyset$, then assign cluster A_n to the site with the highest service level and stop; otherwise go to Step 3.

Step 3: Assign cluster A_n to the site (in S^*) with the minimum penalty cost. If there are more than one site in S^* with the minimum penalty cost, choose one randomly.

The above procedure is repeated until all the clusters have been assigned to one site.

3.3.3 Determining the location of depots

The principle of this stage is to locate a certain number of depots to provide maintenance service with a given service level, so that the total service cost is minimized. The objective is achieved by merging any two sites to see if the total service cost could be reduced. A merge operation is defined as follows.

Merge operation: Denote s_i and s_j as two potential sites (s_i, $s_j \in S$). Note that a potential site can be an empty site without assigned clusters. Thus, to merge s_i and s_j is to re-assign those clusters, if there are any, that were assigned to site s_i to site s_j, then to shut down s_i (if s_i was an open depot).

The service level of site s_j is updated using equation (3.16). If the required service level γ is satisfied, the merge is acceptable; otherwise, it is declined. Therefore, given a set of the potential sites S, the procedure of this stage can be described as follows:

Step 1: Order the potential sites in S in a random sequence. Denote the sequence of sites as RS. Set m=1.

Step 2: Denote the m-th site of RS as s_0. Remove the m-th site from RS, updating RS to RS^*. Denote the n-th site of RS^* as s_n.

Step 3: **for** n = 1 to $|RS^*|$ **do**

i) merge s_0 and s_n;

ii) if the merge is acceptable, calculate the saving cost sc_{0n};

Step 4: Denote n^* as the site that incurs the largest positive saving, and merge s_0 and s_{n^*}.

Step 5: Update the corresponding clusters that are assigned to s_0 and s_{n^*}. if $m<|S|$, set $m=m+1$, then to step 2; otherwise go to step 6.

Step 6: Calculate the objective value D of the solution as defined in equation (3.1).

In Step 3, a merge operation will incur the decrease of fixed cost and the increase of penalty cost. The saving cost is thus the summation of the two items.

The above procedure is embedded into an iterative structure that facilitates the search for a good solution. At each iteration, the sequence of sites RS is updated by randomly swapping a number of sites. The best D^* found and the associated solution are updated.

When the algorithm terminates, the sectoring of the network, including the location of each depot, as well as its corresponding service sector, is determined.

A post-algorithm procedure is applied to optimize the maintenance routes within each sector and determine a set of minimum cost routes in a service sector, which comes from the previous work (Chen et al. 2014)). The branch-and-cut algorithm is used in the post-algorithm procedure. The total cost is then updated.

3.4 Sectoring evaluation

Besides the total service cost of a sectoring result, a number of complementary criteria has been proposed in literature as we described in Section 2.2.2 to evaluate a solution. Three criteria adopted in this thesis are described as follows:

(1) Imbalance

The imbalance of a sectoring result is defined as the difference between the maximum average sector workload and the minimum average sector workload. The average sector workload is defined as the total workload of the sector divided by the number of vehicles assigned to this sector.

(2) Overlap

Define M as the total number of sectors. The sector area m, SA_m, is defined as the area of the polygon (with all the interior angles smaller than 270 degree), which covers all the arcs within sector m. Denote TA as the area of the complete network. The overlap of a sectoring solution is defined as:

$$overlap = \frac{\left(\sum_{m=1}^{M} SA_m\right) - TA}{TA} \tag{3.18}$$

The overlap concerns the shape of sectors. Small overlap indicates clear boundaries, and is preferable.

(3) Compactness

The Compactness Ratio (*CR*) is calculated to evaluate the compactness of each sector. Denote SP_m as the perimeter of the minimum polygon that covers sector *m*. The compactness ratio of sector *m* is defined as:

$$\mathrm{CR_m} = \frac{\sqrt{\mathrm{SA_m}}}{\mathrm{SP_m}} \tag{3.19}$$

Thus the compactness ratio of a sectoring solution is given by:

$$CR = \sum_{m=1}^{M} \frac{|A_m|}{|A|} \cdot CR_m \tag{3.20}$$

where A_m is the set of arcs that are served in sector *m*.

The larger the value of *CR*, the more compact a sectoring result is. Denote the Compactness Ratio of the network without sectoring as *OCR*. Setting *OCR* as a reference, smaller difference between *OCR* and *CR* is expected, which represents better compactness of the sectoring.

3.5 Computational results

In this section, some experiments are conducted and analyzed to evaluate the quality and the efficiency of the algorithms. All the tests were performed on a personal computer with a 2.53 GHz duo processor and 8.0 GB of RAM. The algorithm was coded in MATLAB 14.0.

3.5.1 Problem settings

Different sizes of graphs were randomly generated to mimic the shape of real road networks. The graph generation procedure is similar to the method proposed by Belenguer et al. (2006).

Some important parameters of the networks are developed based on the real data we received from a road maintenance agency in Shanghai. They are shown as follows:

(1) The length (d_a) of a road segment was set as the Euclidean distance between the two end nodes of the arc (unit: *km*). The average service speed of the vehicles in service is 15km/h. Thus, the service time (s_a) on an arc was defined as $4d_a$ (min). The average travel speed of the vehicles not in service is 40 km/h.

(2) The fixed cost of using a depot F was set to be 1000, and the fixed cost of employing a vehicle f was set to be 200. The unit penalty cost P was set to be 20. The unit deadheading cost was set to be 2.

(3) The required service level was set to be 0.9. The maximum allowed working duration of each vehicle (C) was set to be 240 minutes.

3.5.2 Comparison of the branch-and-cut algorithm and the TSA

The first set of experiments compares the branch-and-cut algorithm with the three-stage algorithm (TSA). The ratio κ quantifies the difference in terms of total cost. It is given by $\kappa = (D_{TSA} - D_{B\&C})/D_{B\&C}$, where D_{TSA} is the objective value of the TSA solution, and $D_{B\&C}$ is the objective value of the branch-and-cut approach.

Table 3-1 shows the results based on the performance measure κ in percentage for different problems. The instances have 12 to 20 vertices, and 23 to 61 arcs, respectively. The column "CPU(s)" gives the CPU seconds of both algorithms to solve each instance. For the branch-and-cut algorithm, the best upper bound found within four hours of calculation is used for comparison. For each instance the gap from the lower bound is also reported in the table.

Comparing with the branch-and-cut algorithm, the total cost of the sectoring solution obtained from the TSA is slightly higher, with a ratio from 0.49% to 12.2%. The results also show the efficiency of the TSA. The TSA takes much less computation time, taking only seconds for each run regardless of the problem size. Thus we will use the TSA for solving medium and large sized problems in the following experiments.

Table 3-1 Comparison of the B&C algorithm and the TSA

No	$\|V\|$	$\|A\|$	r	B&C			TSA		κ (%)
				CPU(s)	*gap(%)*	$D_{b\&c}$	*CPU(s)*	D_{TSA}	
a-1	12	23	18	22.36	0	3167.64	38.41	3334.04	5.25
a-2	12	25	18	2.5	0	3072.69	29.24	3204.56	4.29
a-3	12	30	18	8.87	0	3275.62	32.62	3506.95	7.06
b-1	16	35	18	13467.42	0	3914.73	40.15	3975.27	1.55
b-2	16	37	18	21.22	0	3217.1	42.33	3340.17	3.83
b-3	16	43	18	14410.28	3.11	3483.87	39.78	3909.46	12.2
c-1	20	48	20	751.13	0	3942.1	78.12	4231.4	7.34
c-2	20	52	20	14400.03	2.53	4101.11	80.34	4489.67	9.47
c-3	20	61	20	14400	2.30	3947.5	75.35	3966.86	0.49

3.5.3 Experiment on a real network

In this sub-section, we evaluate the performance of the TSA for solving a real network of high-speed freeways in the city of Shanghai. The complete network consists of 31 vertices and 88 arcs as shown in Figure 3-3. The total mileage of the network is about 200 kilometers. A majority of the network is two-way roads, with each way maintained independently.

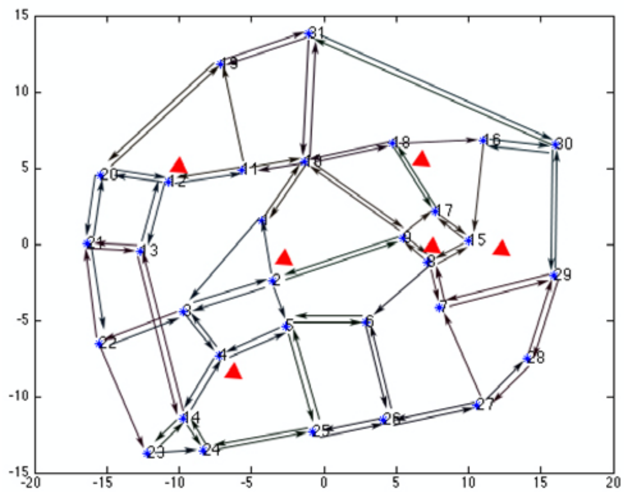
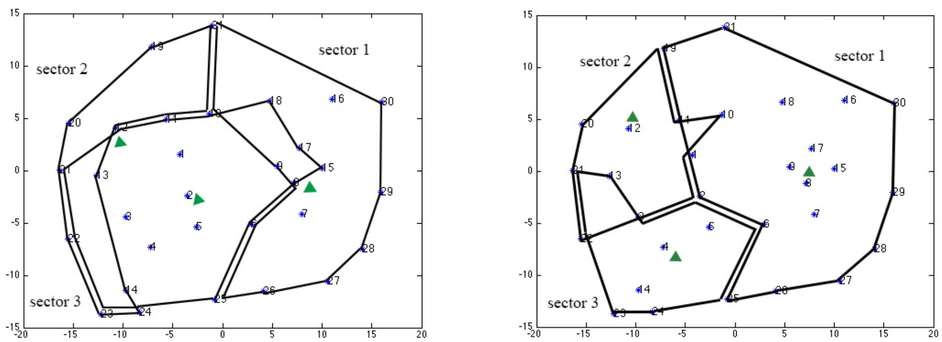


Figure 3-3 Network of the high-speed freeways in the city of Shanghai

The set of potential sites *S* is {2, 4, 8, 12, 15, 18} (indicated as triangles in Figure 3-3). The sectoring problem of the network has been solved by the branch-and-cut algorithm, as well as by the three-stage heuristic algorithm. Both sectoring solutions are illustrated in Figure 3-4. The complete network has been partitioned into three sectors in both solutions, each of which corresponds to the area serviced by a single depot.

Table 3-2 summarizes both solutions, which include the depot of each sector, as well as the number of vehicles used in each sector.



(a) Branch-and-cut solution (b) TSA solution

Figure 3-4 Sectoring solutions of the network in Figure 3-3

Table 3-2 Summarization of both sectoring solutions

Sector	Branch-and-cut solution		TSA solution	
	Location	Number (vehicles)	Location	Number (vehicles)
1	8	4	8	6
2	12	2	12	2
3	2	4	4	3

Besides the total costs, both solutions are evaluated in terms of those criteria described in Section 6. The results are shown in Table 3-3.

Table 3-3 Performance measures for the sectoring solutions shown in Figure 3-4

Solution	Total cost	Imbalance (min)	Overlap (%)	Compactness	
				CR	OCR
Branch-and-cut	5160.6	34.1	11.63	0.213	0.268
TSA	5661.2	31.0	7.86	0.227	

From Table 3-3, although the total cost of the TSA solution is 9.70% higher than the total cost of the branch-and-cut solution, the TSA outperforms the branch-and-cut algorithm in terms of both overlap and compactness. The objective of the branch-and-cut algorithm is merely to minimize the total cost regardless of the shape of the sector. Therefore, arcs of one sector may locate in the neighbor sector as long as the total cost is saved, which results in overlap of sectors. For TSA algorithm, overlap is owing to the definition of the area of the sector, all interior angles of the polygon being smaller than 270 degree. It can be almost avoided that complete arcs in one sector locate in another sector. Thus, the performance is better in terms of the compactness and overlap of each sector.

Therefore, we conclude that with the TSA a clearer sectoring result will be obtained with better shape and compactness of each sector. Even though the TSA solution is more expensive, it is more appealing and preferable in practice.

3.5.4 Sensitivity analysis for medium and large sized problems

In this part, a sensitivity analysis is conducted by varying each of the following parameters: (1) service level γ; and (2) percentage of two-way roads q. Random medium and large sized planar networks have been generated following the same procedure in problem settings. These instances have 25 to 42 vertices, 59 to 157 arcs, respectively. They consist of two major categories: a) networks with an approximate equivalence of one-way and two-way roads (q = 0.5); and b) networks with a majority of two-way roads (q = 0.2).

Table 3-4 Instances details

Instance No.	$\|V\|$	$\|S\|$	$\|A\|$	r	q	Instance No.	$\|V\|$	$\|S\|$	$\|A\|$	r	q
a-1-1	25	5	59	20	0.5	b-1-1	25	5	78	20	0.2
a-1-2	25	5	66	20	0.5	b-1-2	25	5	81	20	0.2
a-1-3	25	5	68	20	0.5	b-1-3	25	5	82	20	0.2
a-2-1	30	5	82	20	0.5	b-2-1	30	5	99	20	0.2
a-2-2	30	5	84	20	0.5	b-2-2	30	5	103	20	0.2
a-2-3	30	5	95	20	0.5	b-2-3	30	5	105	20	0.2
a-3-1	36	5	91	22	0.5	b-3-1	36	5	114	22	0.2

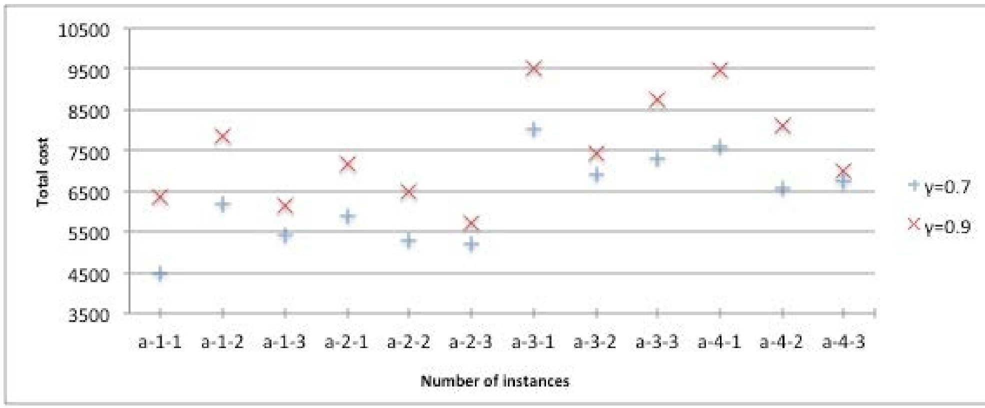
Table 3-4 Instances details (cont'd and end)

a-3-2	36	5	103	22	0.5	b-3-2	36	5	114	22	0.2
a-3-3	36	5	107	22	0.5	b-3-3	36	5	118	22	0.2
a-4-1	42	6	122	22	0.5	b-4-1	42	6	142	22	0.2
a-4-2	42	6	131	22	0.5	b-4-2	42	6	151	22	0.2
a-4-3	42	6	135	22	0.5	b-4-3	42	6	157	22	0.2

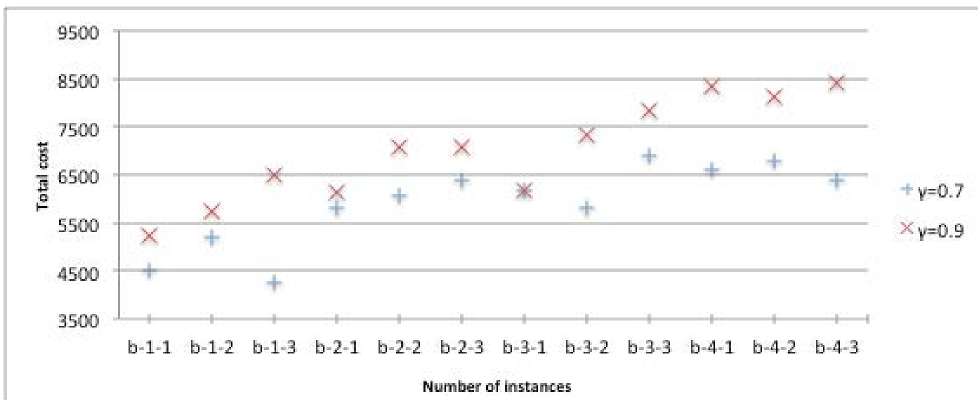
For each group, we randomly generated 3 instances, each of which has the same number of vertices, but different number of arcs. Table 3-4 summarizes the detail information about the instances.

Service level

Two service levels were used in the experiments, namely medium service level (γ = 0.7) and high service level (γ = 0.9). Figure 3-5 illustrates the performance in terms of the total cost for solving four groups of instances in each category.



(a) Instances with *q* = 0.5



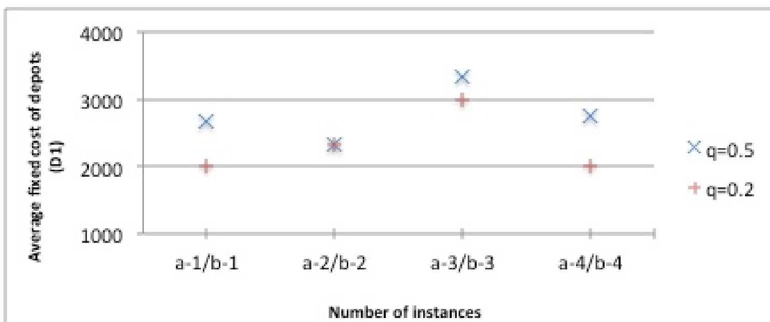
(b) Instances with *q* = 0.2

Figure 3-5 Performance comparison with different service level

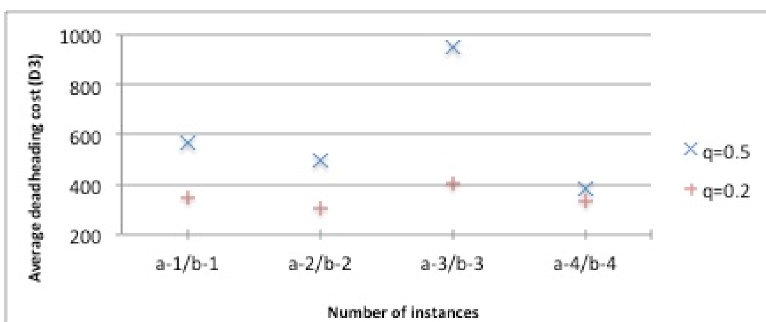
It is clearly indicated in the figure that more cost is incurred with the increase of the service level. This is mainly because the number of sectors increases with higher service level. The tendency is the same for each percentage of two-way roads (*q*). The cost increase ranges from 0 to 53.4% with more two-way roads in the network (*q*=0.2). It ranges from 4.2% to 41.7% with more one-way roads in the network (*q*=0.5).

Percentage of two-way roads

The network structure varies with different percentage of two-way roads. In order to understand the impact of the network structure on the efficiency of conducting maintenance service, we analyze some key cost components in the total cost. For each category of instances, Figure 3-6 illustrates the performance in terms of fixed cost of using the maintenance depots (D_1), and the deadheading travel cost (D_3). Note that each data point on the figure is an average of 3 instances.



a) Fixed cost of maintenance depots (D_1)



b) Deadheading travel cost (D_3)

Figure 3-6 Performance comparison with different q

It is observed from the results that:

1) With the same size of the network, the number of sectors needed for instances with q = 0.5 is usually more than that for instances with q = 0.2. This incurs a higher fixed cost of depots as shown in Figure 3-6(a). The cost increase can be up to 37.5%.

2) The deadheading cost of instances with q = 0.5 is higher than that of instances with q = 0.2, as shown in Figure 3-6(b). The increase can be up to 133.4%.

3) The penalty costs of the two categories are of the same level.

The reason of the above observations is that the accessibility of those networks with more two-way roads is better.

The proposed heuristic algorithm is also evaluated in terms of those criteria described in Section 3.2. Service level is set to be 0.9 in this round of experiment. Table 3-5 shows the results of the different performance measures, namely work load imbalance, sector overlap, and compactness of the sectoring. The results indicate that the proposed algorithm yields high quality sectoring solutions for both categories of instances with compact and good shaped sectors.

It is also indicated in Table 3-5 that the algorithm performs better, in terms of workload balance and sector overlap, when solving those instances from category b)(q = 0.2). This occurs because:

(1) More deadhead travel is needed for those networks with more one-way roads (category a). Thus, higher workload imbalance occurs especially when two-way roads are not well distributed over the network.

(2) With more two-way roads, the compactness of a cluster tends to be higher since workload is more aggregated. This is beneficial to partition the network into sectors with less overlap.

Table 3-5 Performance measures on different sectoring criteria (γ = 0.9)

instance No.	imbalance (min)	avg.	overlap (%)	avg.	compactness CR	compactness OCR	avg. CR/OCR
a-1-1	29.2		0.00		0.198	0.242	
a-1-2	45.3	42.8	0.00	2.62	0.204	0.252	0.881
a-1-3	54.0		7.86		0.252	0.248	
b-1-1	24.0		3.87		0.194	0.246	
b-1-2	1.3	11.4	0.00	1.29	0.198	0.241	0.815
b-1-3	9.0		0.00		0.205	0.246	
a-2-1	3.4		4.97		0.206	0.240	
a-2-2	26.8	17.9	0.00	2.55	0.225	0.244	0.904
a-2-3	23.4		2.67		0.234	0.251	

Table 3-5 Performance measures on different sectoring criteria (γ = 0.9) (cont'd)

b-2-1	11.1		2.72		0.224	0.239	
b-2-2	5.6	11.2	0.00	1.24	0.213	0.235	0.905
b-2-3	17.0		0.99		0.215	0.247	
a-3-1	49.4		8.89		0.224	0.253	
a-3-2	9.5	27.4	0.00	4.42	0.206	0.242	0.909
a-3-3	23.3		4.38		0.238	0.240	
b-3-1	9.5		0.50		0.194	0.242	
b-3-2	26.8	20.3	0.00	1.94	0.216	0.245	0.844
b-3-3	24.5		5.31		0.203	0.239	
a-4-1	19.3		0.00		0.209	0.243	
a-4-2	22.3	18.5	3.25	1.08	0.206	0.242	0.850
a-4-3	14.0		0.00		0.208	0.248	

Table 3-5 Performance measures on different sectoring criteria (γ = 0.9) (cont'd and end)

b-4-1	6.8		0.00		0.214	0.247	
b-4-2	15.9	10.9	0.47	0.16	0.222	0.240	0.897
b-4-3	9.9		0.00		0.214	0.238	

3.6 Summary and remarks

In this chapter, the sectors design problem was addressed, which consists of determining depots' locations, their respective service sectors and routes within each sector. The problem without considering the characteristics of sectors was formulated as a location-allocation arc routing problem and was solved by a branch-and-cut algorithm. A three-stage heuristic algorithm with sector design component and a post-algorithm procedure was developed to determine the sectors with evaluation considerations.

In the computational analysis, randomly generated planar networks with different sizes, structures, and different service levels were tested. The experiments demonstrated that the three-stage heuristic algorithm is computationally more tractable than the branch-and-cut algorithm and could yield high quality solution with compact and good shaped sectors as the scale of the network grows. Moreover, a real network was presented, solved by both algorithms. It is obvious that with the TSA a clearer sectoring result will be produced with better shape and compactness of each sector at the expense of cost.

The sensitivity analysis revealed that the three-stage algorithm performs better at solving those networks with a majority of two-way roads, since the deadheading time (shortest distance) between two arcs tends to be short.

CHAPTER 4 ROBUST ARC ROUTING WITH TIME DURATION

In this chapter, we address the robust arc routing problem with time duration (ARPTD), a variant of the capacitated arc routing problem (CARP) with uncertain demands.

This chapter is organized as follows. Section 1 first presents the deterministic formulation of the arc routing problem with time duration, aiming at minimizing the deadheading cost. Section 2 proposes a definition of the uncertainty set, which is polyhedral. We then re-express the inequalities constraints of the uncertainty set Q in matrix form. Section 3 presents the solvable formulation of robust arc routing problem with time duration (RARPTD). Experiments are conformed on randomly generated instances in section 4 to assess the performance of the robust formulation. A real network case is exhibited in section 5 and some sensitivity analyses are presented. Finally, section 6 summarizes this chapter and draws some conclusions.

4.1 Deterministic formulation

The first solved formulation for the deterministic CARP was proposed in 1990 by Belenguer (Belenguer 1990). In 1994, Welz (Welz 1994) presented another two-index formulation based on the formulation of Belenguer, in which, each edge is replaced by two opposite arcs. We extend Welz`s two-index formulation to obtain the edge assignment formulation (EA) for the ARPTD.

4.1.1 Notation

The formulation is defined on an undirected (q = 0) graph $G = (V, E)$ with vertex set $V = \{0,1, \dots, \mathrm{n}\}$ where vertex 0 is the depot. A subset $E_R \subseteq E$ of required edges must be serviced. K is the set of vehicles.

Each edge $e\{i, j\} \in E$ in the undirected connected graph is replaced by two arcs (*i*,*j*) and (*j*,*i*), denoted by $\vec{e}$ and $\overleftarrow{e}$. Thus, a new directed graph $G` = (V, A)$ is obtained. Exactly one of the two required arcs generated by the required edge must be serviced. Arcs generated by the required edges make up set A_R. Since it is easy to show that it is never necessary for any vehicle to traverse an edge more than once in a given direction, binary variables x_a^k and y_a^k are used.

$$x_a^k = \begin{cases} 1, if\ vehicle\ k\ services\ arc\ a \\ 0, otherwise \end{cases}, a \in A_R, k \in K$$

$$y_a^k = \begin{cases} 1, if\ vehicle\ k\ traverses\ arc\ a \\ 0, otherwise \end{cases}, a \in A, k \in K$$

Denote c_a as the deadheading cost of arc $a, a \in A$. Other notations are the same as defined in section 3.1.1.

4.1.2 The deterministic ARPTD formulation

Since the constraints of the deterministic ARPTD formulation are similar to those of section 3.1.3, they are not discussed in detail.

The ARPTD can be formulated as follows:

$$z(ARPTD) = min \sum_{k \in K} \sum_{a \in A} c_a y_a^k \tag{4.1}$$

$$x_a^k + y_a^k \leq 1, \forall a \in A_R, k \in K \tag{4.2}$$

$$\sum_{k \in K} (x_{\vec{e}}^k + x_{\overleftarrow{e}}^k) = 1, \forall e \in E_R \tag{4.3}$$

$$\sum_{a \in O(v)} (x_a^k + y_a^k) - \sum_{a \in I(v)} (x_a^k + y_a^k) = 0, \forall v \in V, k \in K \tag{4.4}$$

$$\sum_{a \in O(Y)} (x_a^k + y_a^k) \geq x_b^k, \forall Y \subset V, 0 \notin Y, k \in K, b \in A_R(Y) \tag{4.5}$$

$$\sum_{a \in A_R} s_a \, x_a^k + \sum_{a \in A} t_a \, y_a^k \leq TD, \forall k \in K \tag{4.6}$$

$$\sum_{a \in A} y_a^k \leq \sum_{a \in A} y_a^{k+1}, k \in \{1,2, \dots, |K| - 1\} \tag{4.7}$$

$$x_a^k \in \{0,1\}, \forall a \in A_R, k \in K \tag{4.8}$$

$$y_a^k \in \{0,1\}, \forall a \in A, k \in K \tag{4.9}$$

The objective function (4.1) minimizes the total deadheading cost. Constraints (4.2) are valid inequalities that guarantee that any vehicle will not traverse an edge more than once in a given direction. Constraints (4.3) ensure that each edge is serviced exactly once. Constraints (4.4) and (4.5) are the flow conservation and connectivity constraints respectively. Constraints (4.6) are the time duration constraints. Overtime and delay are not allowed here. Constraints (4.7) are valid inequalities for ARPTD, which are used to eliminate symmetry. Constraints (4.8) and (4.9) enforce the binary nature of all decision variables.

4.2 Definition of uncertainty set

In practice, the service time and deadheading time are often subject to significant uncertainty. Assume that both the service times on required arcs and deadheading times are uncertain and belong to a polyhedral uncertainty set Q, without known probability distributions. The nominal value of the service time and deadheading time are denoted by $\overline{s_a}$ and $\overline{t_a}$ respectively, which are the average service or deadheading time of the historical data.

First of all, the uncertainty set Q is defined with linear inequalities that the service time and deadheading time of each arc should satisfy. A polyhedral uncertainty set can be formed by choosing different families of linear inequalities according to the practical situation.

Linear inequalities for deadheading times

1) Bounds

We consider that the deadheading time of each arc belongs to a bounded set, which is constructed based on deviations around the nominal deadheading time of each arc. Let γ_a^{max} be the maximum lower deviation of the deadheading time on arc a. The maximum upper deviation is supposed to be twice the maximum lower deviation. Thus,

$$t_a \in \left[\overline{t_a} - \gamma_a^{max}, \overline{t_a} + 2 \cdot \gamma_a^{max}\right], a \in A$$

The value of γ_a^{max} is related to the level of uncertainty of the environment. A high value of γ_a^{max} indicates a high uncertainty. When $\gamma_a^{max} = 0$, the deadheading time t_a is a deterministic value, thus turning back to a deterministic problem. Besides, we should note that $t_{\vec{e}} = t_{\tilde{e}}, e \in E$.

2) Budgets

Generally, for a specific subset of arcs, the deadheading times of all its arcs will not attain its the worst realization at the same time. In other words, the deviations of the deadheading times of all arcs in that subset can be limited by a budget. A high deviation budget allows more arcs to attain their worst realization at the same time. The deviation budget constraints of arc subsets are given as follows,

$$\sum_{a \in A_l} (t_a - \overline{t_a}) \leq B_l, A_l \subseteq A, for\ l = 1,2, \dots, L$$

where A_l are specific subsets of A, in which the deviations of arcs have to satisfy the budget constraint. $\{A_l\}_{l=1}^{L}$ are disjoint and $\cup_{l=1}^{L} A_l = A$. Sets A_l can be obtained from practical experience. For example, when the underlying graph can be partitioned geographically into several districts, it is reasonable to assume that the deadheading times of all arcs in one district will not attain their worst realization at the same time and a specific level of deviation budget can be estimated from the historical data. Sets A_l can also be obtained from a statistical analysis.

We consider in this thesis the budgets are determined with respect to the lower deviation and upper deviation of t_a, according to the following formula

$$B_l = \sum_{a \in A_l} (-\gamma_a^{max}) + \mu_l [\sum_{a \in A_l} (2 \cdot \gamma_a^{max}) - \sum_{a \in A_l} (-\gamma_a^{max})], A_l \subseteq A, for\ l = 1,2,\dots,L$$

where μ_l is the *level of the deviation budget for set* A_l, $\mu_l \in [0,1]$. Then, the budget constraints can be simplified to the following expression,

$$\sum_{a \in A_l} (t_a - \overline{t_a}) \leq \sum_{a \in A_l} (-\gamma_a^{max} + 3 \cdot \mu_l \cdot \gamma_a^{max}), A_l \subseteq A, for\ l = 1,2,\dots,L$$

Linear inequalities for service times

1) Bounds

The bounded set of service time of each arc can not be described simply with the nominal service time and maximum deviation like that of deadheading time because the bounded set of service time is determined by two factors: 1) In practice, the service time and deadheading time of one arc are positively-correlated, owing to the same external environment, for example, the status of roads, weather conditions and characteristics of staff. The positive correlation of s_a and t_a can be expressed by the relationship $s_a = \varphi t_a$, where φ is a positive factor greater than 1. 2) Unexpected emergencies or other uncertainties can still occur during servicing, which would cause $s_a - \varphi t_a \neq 0$. The maximum deviation of $(s_a - \varphi t_a)$ is denoted by ε_a^{max}. Then, The bounded uncertainty set for s_a is thus as,

$$s_a \in [\varphi t_a - \varepsilon_a^{max}, \varphi t_a + \varepsilon_a^{max}], a \in A_R$$

where φt_a accounts for the positive correlation between the service time and deadheading time of arc a, while ε_a^{max} represents the uncertainties during servicing. The deterministic case is covered by setting $\varepsilon_a^{max} = 0$. Besides, we should note that $s_{\vec{e}} = s_{\bar{e}}, e \in E_R$.

According to the definition of the bounded sets for t_a and s_a, the relationship between these sets is illustrated in Figure 4-1. The shaded area Ⅰ is the bounded set of s_a under different realizations of t_a. The shaded area Ⅱ show all possible value of $(s_a - t_a)$ under different realizations of t_a.

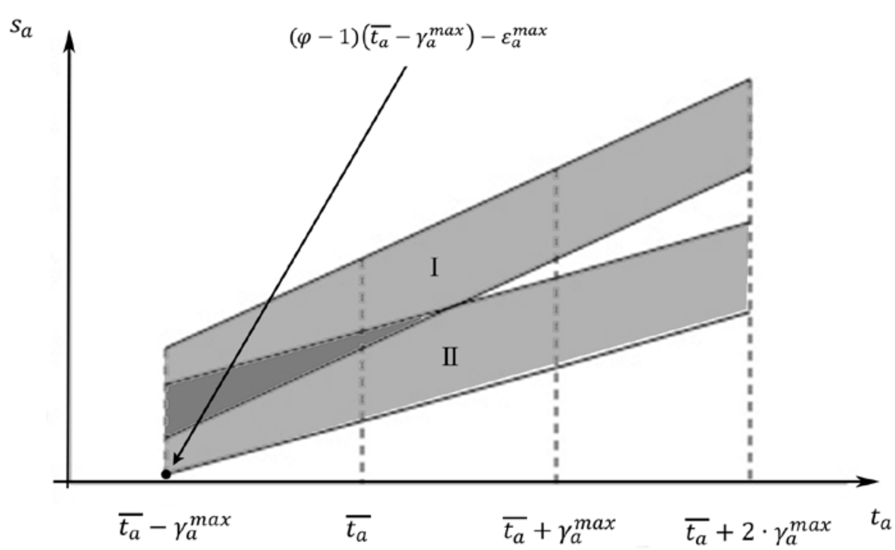


Figure 4-1. The relationship between the bounded sets for t_a and s_a

In order to assure that $s_a \geq t_a$, the value of ε_a^{max} and γ_a^{max} must satisfy some constraints:

$$\min\{s_a - t_a\} = \min\{\varphi t_a - \varepsilon_a^{max} - t_a\} = \min\{(\varphi - 1)t_a - \varepsilon_a^{max}\}$$
$$= (\varphi - 1)\left(\overline{t_a} - \gamma_a^{max}\right) - \varepsilon_a^{max}$$

Therefore, the following constraints must be satisfied.

$$(\varphi - 1)\left(\overline{t_a} - \gamma_a^{max}\right) - \varepsilon_a^{max} \geq 0 \tag{4.10}$$

2) Budgets

Similarly, the service time of each required arc will not attain the worst realization at the same time. The deviation budget constraints of service time are given as follows,

$$\sum_{a\in\{A_l\}_R} (s_a - \varphi t_a) \leq \sum_{a\in\{A_l\}_R} (\, -\varepsilon_a^{max} + 2 \cdot \mu_l \cdot \varepsilon_a^{max}\,), \{A_l\}_R \subseteq A_R, for\ l = 1,2,\dots,L$$

where $\{A_l\}_R$ are specific subsets of A_R, assumed to be the set of required arcs of A_l and μ_l is the level of the deviation budget for set A_l, $\mu_l \in [0,1]$.

In order to make the robust part concise and easier to understand, we re-express the inequality constraints of the uncertainty set Q in matrix form,

$$Q = \left\{s \in R_+^{|A_R|}, t \in R_+^{|A|} : W\begin{bmatrix} s \\ t \end{bmatrix} \leq h, W \in R^{m\times(|A_R|+A)}, h \in R^m\right\}$$

Where, $m = 2|A| + 2|A_R| + 2L$.

4.3 Robust ARPTD formulation

We now derive the robust version of time duration constraints (4-6),

$$\sum_{a\in A_R} s_a\, x_a^k + \sum_{a\in A} t_a\, y_a^k \leq TD, \forall k \in K, \forall s_a, t_a \in Q \tag{4.6'}$$

The dual variables are denoted by λ. Then, solvable robust constraints can be obtained by applying duality to the infinite-dimensional time duration constraints (4-6’) as follows,

$$\sum_{i=1}^{m} h_i \, \lambda_{ik} \leq TD, k \in K \tag{4.11}$$

$$\sum_{i=1}^{m} W_{ia} \lambda_{ik} \geq x_a^k, k \in K, a \in A_R \tag{4.12}$$

$$\sum_{i=1}^{m} W_{i(a+|A_R|)} \lambda_{ik} \geq y_a^k, k \in K, a \in A \tag{4.13}$$

$$\lambda \in R_+^{m \times k} \tag{4.14}$$

Thus, the complete robust counterpart of formulation ARPTD can be written as follows:

$$z(RARPTD) = min \sum_{k \in K} \sum_{a \in A} c_a y_a^k$$

Subject to

(4.2)~(4.5), (4.7)~(4.9), (4.11)~(4.14).

RARPTD is solved with the branch and cut algorithm described in Section 3.2.

4.4 Computational results

In this computational study, the procedure to generate undirected instances is as follows: 1) randomly generate the coordinates of X vertices in a unit square; 2) a heuristic is used to find the shortest tour passing through all the nodes exactly once, thus making the graph strongly connected; 3) randomly add a certain number of edges to the current tour, ensuring no intersection and proper length; 4) the distance is scaled up by a factor. The same procedure has been used to generate instances for other arc routing problems (see Chen et al. 2014 and Hà et al. 2014).

We name the instances "*XN-YE-Z*", where *x* denotes the number of nodes, *y* denotes the number of edges and z denotes the number of the instances that have the same number of nodes and edges. We partition the arcs in A into four geographic quadrants, based on the coordinates of their midpoints. For each of these subsets, a budget constraint must be satisfied; see section 4.2.

4.4.1 Problem settings for the robust model

- The average service speed of the vehicles in service is 15km/h. The average travel speed of the vehicles not in service is 40 km/h. Thus, $\overline{s_a} = 4d_a$ and $\overline{t_a} = 1.5d_a$ respectively. The deadheading cost is defined as $c_a = 5d_a$.
- Uncertainty level. The maximum deviation of the deadheading time γ_a^{max} is defined as $\gamma_a^{max} = \vartheta\overline{t_a}$. ϑ is the level of uncertainty of the deadheading time on each arc and is defined to be the same for all the arcs for the same uncertain environment. $\vartheta \in \{0.1, 0.3, 0.5\}$. The maximum deviation of the service time ε_a^{max} is defined as $\varepsilon_a^{max} = \xi\overline{s_a}$. ξ is the level of uncertainty of the service time on each arc and is defined to be the same for all the arcs for the same uncertain environment. In order to satisfy constraints (4.10), we set $\xi \in \{0.05, 0.15, 0.25\}$.

- Deviation budget. The level of the deviation budget of the service and deadheading times, μ_l, belongs to $\{0.5, 0.7, 0.9\}$.

4.4.2 Experimental analysis

Some experiments on different sizes of networks were conducted to evaluate the performance of the robust formulation (RARPTD). The uncertainty set was fixed to $(\vartheta, \mu_l) = (0.3, 0.7)$. The number of required edges was set to be 2/3 of $|E|$. The CPU time was limited to 10 hours.

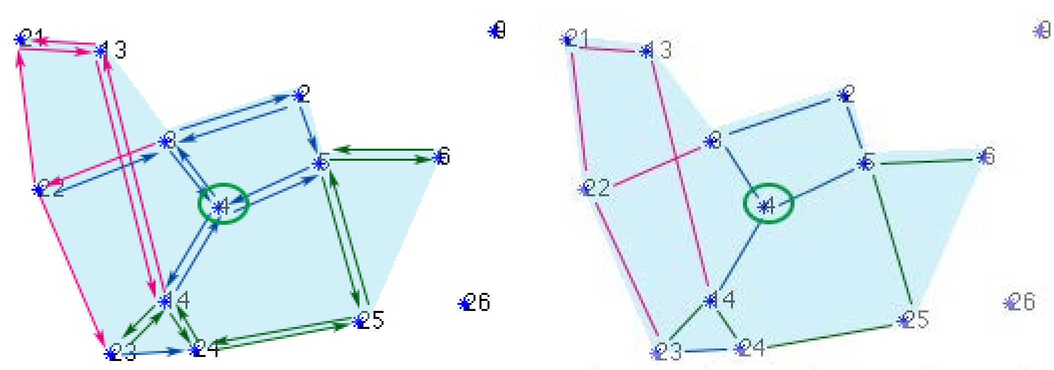
Table 4-1 Computational comparison of different sizes of networks

Instance	Cost	No.of vehicles	Time (solved)	Avg. time (sec)	Gap(%)	Avg. gap
10N-15E-1	340.06	3	11.7		0	
10N-15E-2	250.89	2	2.7		0	
10N-15E-3	128.92	2	1.0	3.3	0	0
10N-15E-4	107.87	2	0.7		0	
10N-15E-5	97.54	2	0.4		0	
14N-21E-1	413.17	3	468.7		0	
14N-21E-2	355.38	3	54.7		0	
14N-21E-3	542.69	4	3469.0	1384.5	0	0
14N-21E-4	353.29	3	522.8		0	
14N-21E-5	585.92	3	2407.4		0	
18N-27E-1	925.09	4	/		42.0	
18N-27E-2	649.61	5	/		37.3	
18N-27E-3	648.34	3	/	/	30.4	31.2
18N-27E-4	410.11	3	7049.3		0.0	
18N-27E-5	645.02	4	/		46.3	

Table 4-1 demonstrates the performance of the formulation (RARPTD) over 15 networks of 3 different sizes. When the network size is small (15 edges) and the number of vehicles is below 4, all the instances can be solved to optimality very fast. However, when the size of network becomes larger (21 edges) and the number of vehicles increases, the time needed to solve instances to optimality increases significantly. Most instances cannot be solved to optimality within time limit when the number of edges increases to 27. Out of those that were not solved to optimality, the average gap is 31.2%. When we are solving an undirected problem with 27 edges and 5 vehicles, we actually are solving a directed problem with 54 arcs and 5 vehicles, therefore, the number of variables is quite large, which explains why it cannot be solved easily.

4.4.3 Experiments on a sector from the real network

Other experiments were conducted on the undirected graph of sector 3, which is obtained from the directed graph of sector 3 sectored with TSA in section 3.5 by substituting all the directed arcs with undirected edges. The same colored arcs in Figure 4-2(a) are served by one vehicle, therefore, we assume them to form a subset, for which a budget constraint must be satisfied. After replacement, the same colored edges in Figure 4-2(b) are also assumed to form a subset with a budget constraint. Depot is indicated with a green circle in Figure 4-2.



(a) directed graph (b) undirected graph

Figure 4-2 Network of sector 3

If we set $(\vartheta, \mu_l) = (0.3, 0.7)$ and $|K| = 4$, then the optimal solution is exhibited in Figure 4-3. In each route, we represent the serviced edges by green lines and the deadheaded edges by red lines.

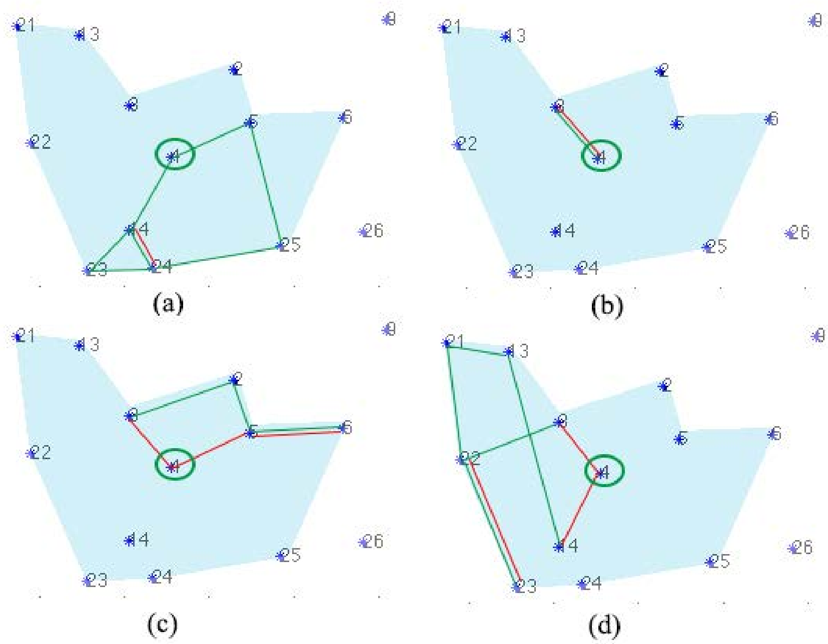


Figure 4-3 The optimal solution of sector 3

The sensitivity analysis is conducted in the following by varying (1) the level of uncertainty, including the deviation budget and the uncertainty of the deadheading and service times; and (2) the number of vehicles used.

Uncertainty level

The RARPTD determines a set of robust routes that remains feasible under all the possible realizations of t_a and s_a. The feasible solutions set under uncertainty set Q is denoted by $R(Q)$. If the support $\tilde{Q}$ is a superset of the support Q, then $R(\tilde{Q}) \subseteq R(Q)$.

Table 4-2 Overtime of the worst route (min) ($|K| = 4$)

(ϑ, ξ, μ_l)\ $OS(\vartheta, \xi, \mu_l)$	(0.1, 0.05, 0.5)	(0.1, 0.05, 0.7)	(0.1, 0.05, 0.9)	(0.3, 0.15, 0.5)	(0.3, 0.15, 0.7)	(0.3, 0.15, 0.9)	(0.5, 0.25, 0.5)	(0.5, 0.25, 0.7)	(0.5, 0.25, 0.9)
(0.1, 0.05, 0.5)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(0.1, 0.05, 0.7)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(0.1, 0.05, 0.9)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(0.3, 0.15, 0.5)	25.9	11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(0.3, 0.15, 0.7)	33.9	34.1	15.2	23.8	0.0	0.0	0.0	0.0	0.0
(0.3, 0.15, 0.9)	33.9	37.9	48.9	48.9	49.4	0.0	31.8	0.0	0.0
(0.5, 0.25, 0.5)	98.4	64.8	10.7	48.9	15.7	0.0	0.0	0.0	0.0
(0.5, 0.25, 0.7)	111.8	110.1	67.0	96.9	67.8	21.5	44.7	0.0	0.0
(0.5, 0.25, 0.9)	111.8	116.4	123.4	123.4	124.1	59.4	101.0	22.3	0.0

Let us denote the uncertainty set for parameters (ϑ, ξ, μ_l) as $Q(\vartheta, \xi, \mu_l)$. For a polyhedral support Q, $R(Q(\tilde{\vartheta}, \tilde{\xi}, \tilde{\mu}_l)) \subseteq R(Q(\vartheta, \xi, \mu_l))$, that is, any robust feasible set of routes for the uncertainty level $(\tilde{\vartheta}, \tilde{\xi}, \tilde{\mu}_l)$ is also robust feasible for the uncertainty level (ϑ, ξ, μ_l), if $\tilde{\vartheta} \geq \vartheta$, $\tilde{\xi} \geq \xi$ and $\tilde{\mu}_l \geq \mu_l$, which is illustrated in Table 4-2. Let us denote the optimal solution with uncertainty set parameters (ϑ, ξ, μ_l) by $OS(\vartheta, \xi, \mu_l)$. Table 4-2 shows the overtime of the worst route of optimal solutions under different uncertainty levels. It is obvious that when $\tilde{\vartheta} \geq \vartheta$, $\tilde{\xi} \geq \xi$ and $\tilde{\mu}_l \geq \mu_l$, $OS(\tilde{\vartheta}, \tilde{\xi}, \tilde{\mu}_l)$ is robust feasible for the uncertainty level (ϑ, ξ, μ_l), however, $OS(\vartheta, \xi, \mu_l)$ is not necessarily robust feasible for the uncertainty level $(\tilde{\vartheta}, \tilde{\xi}, \tilde{\mu}_l)$. Here we say that $OS(\tilde{\vartheta}, \tilde{\xi}, \tilde{\mu}_l)$ has higher robustness than $OS(\vartheta, \xi, \mu_l)$.

However, a higher level of robustness is not free; in fact, this could incur higher costs, as is shown in Table 4-3. When the uncertainty level becomes higher, the deadheading cost increases. The column 'Increase' gives the increased percentage of cost relative to the parameter setting $(\vartheta, \xi, \mu_l) = (0.1, 0.05, 0.5)$.

Table 4-3 Impact of uncertainty level on the optimal solution ($|K| = 4$)

$\mu \backslash \vartheta, 2\xi$	0.1		0.3		0.5	
	Cost	Increase	Cost	Increase	Cost	Increase
0.5	71.91	1	71.91	1	74.63	1.0378
0.7	71.91	1	74.63	1.0378	75.61	1.0515
0.9	71.91	1	75.61	1.0515	75.61	1.0515

We can see from Table 4-3 that some optimal solutions with different level of (ϑ, ξ, μ_l) may have the same cost, for example, $OS(0.1, 0.05, 0.5)$ and $OS(0.3, 0.15, 0.5)$, however, the robustness of these solutions is quite different. $OS(0.3, 0.15, 0.5)$ has higher robustness.

Actually, although $OS(0.1, 0.05, 0.5)$ and $OS(0.3, 0.15, 0.5)$ have the same cost, the allocation of required edges to vehicles is different, which implies that, in that case, there are two ways of improving the robustness of the optimal solution: 1) by paying the price of robustness; 2) by adjusting the allocation of required edges to obtain higher robustness at the same price.

Number of vehicles

The RARPTD of sector 3 is again solved with different constant fleet sizes. The fleet size $|K|$ is set to be 2, 3, 4 and 5. For each fleet size, the uncertainty level has the same settings. Table 4-4 displays the sensitivity of cost with respect to the number of vehicles.

We can see from Table 4-4:

1) When the number of vehicles is small, the RARPTD will have no feasible solution under high uncertainty level.
2) When the number of vehicles used increases, the cost of the optimal solution for the same uncertainty level increases.
3) When the number of vehicles increases, the cost of the optimal solution will be less sensitive to the uncertainty level. In other words, with larger number of vehicles, the optimal solution under low uncertainty level can be more robust.

These observations can be explained as follows:

1) Higher uncertainty means that one must consider worse realization of the deadheading and service times, therefore, more vehicles are needed to satisfy the demands.

2) Under the same uncertainty level, when the RARPTD can be solved with smaller number of vehicles, more deadheading time may be produced with larger number of vehicles because some route has to be broken down into 2 routes, which leads to the use of deadheading arcs.

3) When more vehicles are used, there will be more surplus time duration for vehicles routes, therefore, the optimal solution can be more robust for the higher uncertainty level.

Table 4-4 Impact of the number of vehicles on the optimal solution

No. vehicles	$\mu\backslash\vartheta, 2\xi$	0.1		0.3		0.5	
		Cost	Increase	Cost	Increase	Cost	Increase
	0.5	41.18	1	/	/	/	/
2	0.7	41.18	1	/	/	/	/
	0.9	/	/	/	/	/	/
	0.5	56.54	1	56.54	1	59.26	1.0481
3	0.7	56.54	1	59.26	1.0481	/	/
	0.9	56.54	1	60.24	1.0654	/	/
	0.5	71.91	1	71.91	1	74.63	1.0378
4	0.7	71.91	1	74.63	1.0378	75.61	1.0515
	0.9	71.91	1	75.61	1.0515	75.61	1.0515
	0.5	90.97	1	90.97	1	90.97	1
5	0.7	90.97	1	90.97	1	90.97	1
	0.9	90.97	1	90.97	1	90.97	1

4.5 Summary and remarks

In this chapter, the robust arc routing problem with time duration was addressed. This model can be applied to service sectors after these have been determined. After proposing the deterministic mathematical formulation for the ARPTD and defining the general polyhedral uncertainty set of service and deadheading times, the robust counterpart of the deterministic formulation was developed and then solved.

Experiments conducted with randomly generated instances showed that the RARPTD can be solved to optimality quickly for small-sized networks. A real network case was also exhibited. Our sensitivity analysis indicates that: 1) A higher level of robustness of the optimal solution could incur higher costs; 2) When the number of vehicles increases, the optimal solution under low uncertainty level can be more robust but the cost of the optimal solution under the same uncertainty level increases.

CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

This chapter concludes the thesis and then proposes the recommendations for future work in studying the arc routing problem with time duration.

5.1 Conclusions

Road maintenance operations are conducted out of a set of depots spatially distributed on a transportation network. Each depot is responsible for providing maintenance service to a sector of the network. Within a sector, service vehicles operate on routes that start and end at the depot. The problem solved in this thesis is divided into two parts: 1) location-allocation arc routing with considering the characteristics of sectors, namely, the sector design problem (SDP); 2) robust arc routing based on the sectoring result of sector design (RARPTD). Using scientific optimization methods for sector design and scheduling of the maintenance routes is an important concern.

In order to solve the sector design problem (SDP), a three-stage heuristic algorithm with a sector design component was developed to determine the sectors with sectoring evaluation considerations. Our computational experiments have shown that the three-stage heuristic algorithm is computationally more tractable than the branch-and-cut algorithm and could yield high quality solution with compact and good shaped sectors as the scale of the network grows. With the real network case, it was found that compared with the branch and cut algorithm, a clearer sectoring result with better shape and compactness of each sector can be produced with the TSA, at the expense of cost.

After partitioning a large-sized network into small-sized sectors, the robust arc routing problem with time duration is addressed. The deterministic mathematical formulation for the ARPTD was proposed and a general polyhedral uncertainty set of service and deadheading times was defined. After that, the robust counterpart of the deterministic formulation was developed and then solved. Computational experiments showed that the RARPTD can be solved to optimality quickly for small-sized networks. The sensitivity analysis was conducted with respect to the level of uncertainty and the number of vehicles used, which revealed that a higher level of robustness of the optimal solution usually incurs higher costs.

Recommendations for future research will be presented in the next section.

5.2 Recommendations for future research

In this thesis, formulations and algorithms were proposed to solve the sector design problem and robust arc routing problem with time duration, and good results were obtained. However, there are still many issues that could be addressed in the future research. These includes,

- Considering the uncertainty on the service and deadheading times in the sector design stage.
- Studying more real network cases and data, and using statistical techniques to obtain more relevant uncertainty sets for practical problems.

- Finding valid inequalities to speed up the solution of the RARPTD, since the RARPTD cannot be solved to optimality quickly on large-sized network.

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