EMERGENCE OF A QUASI NEWTONIAN LAW OF GRAVITATION: A GEOMETRICAL IMPACT STUDY

RÉJEAN PLAMONDON* and CLAUDÉRIC OUELLET-PLAMONDON

Département de Génie Électrique, École Polytechnique de Montréal,
Montréal, Québec H3C 3A7, Canada
**E-mail: rejean.plamondon@polymtl.ca*
www.polymtl.ca

This paper shows how a quasi-Newton's law of gravitation, relying on an *erfc* potential, can be derived when a statistical pattern recognition paradigm based on the Bayes' law and the Central Limit Theorem is applied to the Einstein's field equation. This method incorporates a probabilistic factor that takes into account the probability of presence of a given energy-momentum density in its corresponding 4D curved spacetime manifold. The resulting symmetric and axisymmetric metrics and their corresponding geometries are briefly investigated.

1. Introduction

The search for explaining the origin of Newton's law of gravitation has regained interest recently with investigations relying on black hole thermodynamics models where gravity is depicted as an emergent entropic forces.[1–3] In this paper, we address this question from a different perspective exploiting the similarities between problems solving methods in statistical pattern recognition (SPR) and in physics.[4]

2. A statistical pattern recognition approach

There are several analogies between problem solving methods in SPR and in general relativity (GR). Indeed, in SPR, it is assumed that patterns are generated by a probabilistic system. An object is represented by a set of arbitrary descriptive features and their associated quantified scale, which define the dimension of the problem representation space R. In the interpretation space I, a group of similar objects, generally referred to as a class, is consequently described by a cluster of points which can be properly defined by a multivariate density function. A mapping process is then searched for converting a set of similar random vectors into their corresponding density and to define a discriminating function delimiting the resulting class. Firstly, a problem representation space can be defined by comparing the arbitrariness of the feature space in SPR to the one of the coordinates and their corresponding metric in GR. By considering the similarities between mapping operators in SPR and the G and T tensors in GR, we are lead to postulate that the curvature and the energy-momentum interpretation spaces could be given a probabilistic description in terms of density functions and that both densities could be linked using Einstein's law of gravitation. The whole approach relies on an interdependence principle and its corollary which state that space-time curvature and matter-energy density cannot be dissociated. A subset of the Universe can be studied from two equivalent approaches: either by analyzing the structure of the space-time as an interpretation space associated with an a priori given matter-energy density or by analyzing the matter-energy density as an interpretation space associated with an *a priori* given space-time structure. Using the Bayes' law to express this interdependence, the Einstein equation is adjusted to take into account the probability of presence $f(r)$ of a given energy-momentum density.

$$G = KTf(r) \tag{1}$$

Studying the specific case of a static, spherically symmetric massive body under weak field and low speed conditions, the Central Limit Theorem is used to predict the probability of presence of a star made up of a large number of particles, each one having its own wave function. Incorporating after proper mapping this density in the modified Einstein equation, it is shown that such a system will generate a gravitational attraction that can be described by:

$$\vec{g} = -\frac{GM}{r^2} exp\left(-\frac{\sigma^2}{2r^2}\right)\vec{u}_r \tag{2}$$

$$\Phi(r) = \left(\frac{\sqrt{2\pi}GM}{2\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) \tag{3}$$

One particular attribute of these fields is that they are weighted by an intrinsic and emergent parameter σ that reflects the specific proper length of the massive body under study, a feature that is invariant for any observer.

Incorporating the resulting *erfc* potential into the corresponding length element, the space-time associated to such a body can be analyzed. The major differences between a Schwarzschild metric and the static symmetric metric put forward are the constant term embedded in the *erfc* potential and an *erf* instead of an inverse radial dependence. The metric reads:

$$\begin{aligned} ds^2 = &\left[1 + \frac{\sqrt{2\pi}GM}{\sigma c^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c^2 dt^2 \\ &- \left[1 + \frac{\sqrt{2\pi}GM}{\sigma c^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \end{aligned} \tag{4}$$

The most important feature of this metric is the absence of singularity, neither intrinsic nor coordinate. None of the Ricci and the Einstein tensor components diverges. Moreover, the constant term in the *erfc* potential predicts a residual intrinsic constant gauge that is omnipresent in the static model from r equals zero to infinity. With simple mathematical transformations, the energy associated with this constant term can be redistributed into two components: a first one representing the rotation of a 2-sphere at a constant angular velocity ω_{st} for a constant r and a second reflecting the residual expansion of a 2-sphere at a constant radial velocity v_{st}, modulated by *erf* and *erfc* terms, for a momentarily constant ϕ value. This leads to an algebraically equivalent description of the star using the following axisymmetric length element:

$$
\begin{aligned}
ds^2 = & \left[1 - \frac{\sqrt{2\pi}GM}{\sigma c^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c^2 dt^2 + \frac{\sqrt{2\pi}GM}{\omega_{st}\sigma} d\phi dt \\
& + \frac{\sqrt{2\pi}v_{st}GM}{\sigma c^2}\left[1 - \frac{\sqrt{2\pi}GM}{\sigma c^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \\
& \left[1 + \frac{\sqrt{2\pi}GM}{\sigma c^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr dt \\
& - \left[1 - \frac{\sqrt{2\pi}GM}{\sigma c^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2
\end{aligned}
\tag{5}
$$

This axisymmetric metric provides new tools, different from the classical Kerr metric, to investigate rotating celestial bodies. The metric is also a global solution to the statistical version of the Einstein field equation. In this context, this new metric can be seen as providing complementary information to explain why all the massive bodies in the Universe rotate.

3. Conclusion

The two metrics presented in this paper take advantage of the slight modifications incorporated into the Newton's law to provide new geometric descriptions of the space-time associated to any massive object. To stimulate a greater curiosity toward this methodology, it should also be noticed that when the errors of convergence associated to the central limit theorem are taken into account, the complete model[4] predicts the emergence of three supplementary interactions, the first one being quasi-Coulombian and the two others sharing some similarities with the weak and the strong nuclear interactions.

Acknowledgements

This work has been partially supported by NSERC Grant RGPIN-975 and by the Département de Génie Électrique, École Polytechnique de Montréal.

References

1. E. P. Verlinde, arXiv:1001.0785v1 [hep-th] (2010).
2. P. Nicolini, *Phys. Rev. D* **82**, 04430 (2010).
3. L. Smolin, arXiv:1001.3668v2 [gr-qc] (2010).
4. R. Plamondon, *Patterns in Physics, Towards a Unifying Theory*, Presses Internationale Polytechnique (2012).