An improved HSFR method for natural vibration analysis of an immersed cylinder pile with a tip mass

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Abstract Immersed cylinder piles are usually modelled as immersed cantilever cylinder columns carrying a tip mass and rotary moment of inertia. In this paper, the equations of motion of an immersed cylinder pile along transversal modes of vibration are developed. Compressibility of water and structural damping are included in the formulation. Natural frequencies of the immersed pile are obtained from the developed equations using harmonic sweep frequency response analyses. The proposed method is applied to numerical examples, and the results obtained are shown satisfactory when compared to other numerical solutions in the literature, or to finite element solutions and experimental data. [doi:10.1063/2.1202302]

Keywords vibration analysis, cylinder pile, fluid-structure interaction, sweep frequency response method, tip mass and moment of inertia

Cylinder piles are widely used as driven or drilled piles for deep-water foundations of many offshore infrastructures.[1] The piles surrounded by water are usually modelled in seismic analyses as cantilever columns carrying a tip mass with rotary moment of inertia,[2,3] as shown in Fig. 1. Previous researches[4–7] showed that the interaction between the piles and the surrounding water might alter the dynamic characteristics of the pile foundation and lead to increased dynamic forces. In this context, effective and reliable techniques are required to analyse the frequency response of immersed piles.

The early literature devoted to the free vibrations of cantilever columns mainly focused on the determination of an "added mass" of water which is equivalent to the hydrodynamic effect.[8–10] These solutions are based on the assumptions of a rigid structure and incompressible fluid.[4] Liaw and Chopra[11] studied the dynamic response of towers surrounded by water and determined the fundamental frequency of an immersed tower without a tip mass using harmonic sweep frequency response (HSFR) analysis method. Uščiłowska et al.[5] and Öz[6] derived eigenvalue equations corresponding to the vibrations of partially immersed cantilever beams carrying a tip mass and obtained analytical solutions for vibration frequencies. With the fast development of computers, many numerical approaches based on finite elements and boundary elements were also utilized to assess fluid-structure interaction problems.[12–16]

Little attention has been paid to the determination of higher modes of vibration of cylinder piles. In this work, an approximation is adopted to extend the equations of motion for an immersed pile in which the effects

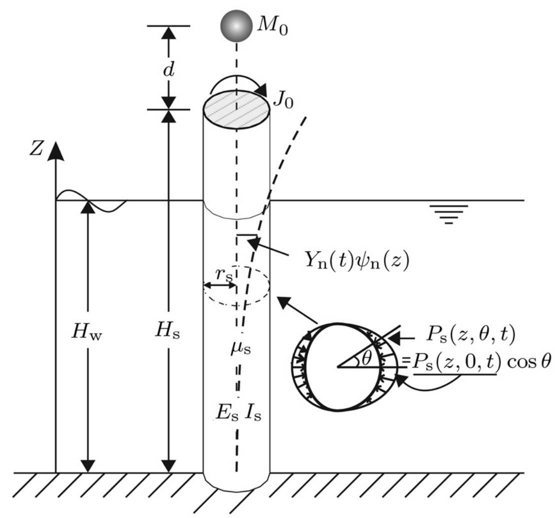

Fig. 1. Sketch of the immersed pile under study.

of higher modes of transversal vibration are included. The HSFR method is then used to identify the natural frequencies of the immersed pile.

As shown in Fig. 1, the cylinder pile has a total height H_s and a radius r_s. It is surrounded by an infinite water domain of constant depth H_w and the whole system is subjected to a horizontal ground motion. The pile has a flexural rigidity E_sI_s and a mass per unit length μ_s. M_0 is the tip mass, J_0 is its rotary moment of inertia and d is the distance between the center of gravity of the tip mass and the end of pile. The following assumptions are adopted: (1) the deformation of pile is linear elastic during the excitation; (2) water

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is inviscid with its motion irrotational and small in amplitudes; (3) gravity surface waves are neglected and (4) each mode of the pile vibration is independent on the other modes.

Considering the above-mentioned assumptions, the equation of motion of the immersed pile subjected to ground motion $\ddot{u}_{\rm g}(t)$ along the nth mode of vibration can be obtained as an extension of those developed by Liaw and Chopra[11] for fundamental mode analysis of a tower surrounded by water

$$M_n\,\ddot{Y}_n(t)+2\,\xi_n\,\omega_n\,M_n\,\dot{Y}_n(t)+\omega_n^2\,M_n\,Y_n(t) = -M_n'\,\ddot{u}_{\rm g}(t)-F_n(t), \tag{1}$$

where ω_n is the vibration frequency along the nth mode of vibration ψ_n without water, Y_n is the corresponding generalized coordinate and ξ_n is the structural damping. The parameters M_n, M_n' and F_n are given by

$$M_n=\int_0^{H_{\rm s}}\mu_{\rm s}\,\psi_n(z)^2\;{\rm d}\,z, \tag{2a}$$

$$M_n'=\int_0^{H_{\rm s}}\mu_{\rm s}\,\psi_n(z)\;{\rm d}\,z, \tag{2b}$$

$$F_n(t)=\int_0^{H_{\rm s}}\int_0^{2\pi}p_{\rm s}(z,\theta,t)\,r_{\rm s}\cos(\theta)\,\psi_n(z)\;{\rm d}\,\theta\;{\rm d}\,z, \tag{2c}$$

in which $p_{\rm s}(\theta,z,t)$ denotes the hydrodynamic pressure applied at the outer lateral surface of the cylinder pile.

Considering a harmonic ground acceleration $\ddot{u}_{\rm g}(t)={\rm e}^{\,{\rm i}\,\omega t}$, the radial hydrodynamic pressure $p_{\rm s}$, the generalized coordinate $Y_n(t)$ and its double time derivative $\ddot{Y}_n(t)$ can be obtained as

$$p_{\rm s}(z,\theta,t)=\bar{p}_{\rm s}(z,\theta,\omega){\rm e}^{\,{\rm i}\,\omega t}, \tag{3}$$

$$Y_n\,(t)=\bar{Y}_n\,(\omega)\,{\rm e}^{\,{\rm i}\,\omega t}, \tag{4}$$

$$\ddot{Y}_n\,(t)=\bar{\ddot{Y}}_n\,(\omega)\,{\rm e}^{\,{\rm i}\,\omega t}=-\omega^2\,\bar{Y}_n\,(\omega)\,{\rm e}^{\,{\rm i}\,\omega t}. \tag{5}$$

The frequency response function $\bar{p}_{\rm s}$ for hydrodynamic pressure can be decomposed as[11]

$$\bar{p}_{\rm s}(z,\theta,\omega)=\bar{p}_0(z,\theta,\omega)+\sum_{n=1}^{N_{\rm m}}\bar{\ddot{Y}}_n(\omega)\,\bar{p}_n(z,\theta,\omega), \tag{6}$$

in which $N_{\rm m}$ is the number of modes included in the analysis, $\bar{p}_0$ is the hydrodynamic pressure frequency response functions corresponding to a rigid body motion of the cylinder pile $\psi_0=1$, and $\bar{p}_n$ is corresponding to mode shape ψ_n.[11,14] The equations governing the hydrodynamic pressures $\bar{p}_0$, $\bar{p}_n$ and the corresponding boundary conditions were given by Liaw and Chopra.[11]

To simplify computations in this work, we adopt a coarse assumption stipulating that the effect of each mode n can be isolated by neglecting the contributions of the other modes, yielding the gross approximation

$$\bar{p}_{\rm s}(z,\theta,\omega)\approx\bar{p}_0(z,\theta,\omega)+\bar{\ddot{Y}}_n(\omega)\,\bar{p}_n(z,\theta,\omega). \tag{7}$$

Using Eqs. (1) and (7), we show that the frequency response function $\bar{\ddot{Y}}_n(\omega)$ for each mode n can be expressed as

$$\bar{\ddot{Y}}_n\,(\omega)=\frac{M_n'+B_0\,(\omega)}{M_n\left[-1+2\,{\rm i}\;\xi_n\left(\dfrac{\omega_n}{\omega}\right)+\left(\dfrac{\omega_n}{\omega}\right)\right]-B_n\,(\omega)},\quad n=1,2,\ldots,N_{\rm m}, \tag{8}$$

where

$$B_k\,(\omega)=\int_0^{H}\int_0^{2\pi}\bar{p}_k\,(\theta,z,\omega)\,r_{\rm s}\cos(\theta)\psi_n\,(z)\;{\rm d}\,\theta\,{\rm d}\,z,\quad k=0,1,\ldots,N_{\rm m}, \tag{9}$$

The mode shapes ψ_n of the cantilevered cylinder pile carrying a tip mass with rotary moment of inertia can be expressed as

$$\psi_n\,(z)=A_1\cos(a_nz)+A_2\sin(a_nz)+A_3\cosh(a_nz)+A_4\sinh(a_nz), \tag{10}$$

where A_1, A_2, A_3 and A_4 are unknown real constants to be determined using the following four boundary conditions

$$\psi(0)=0;\quad E_{\rm s}I_{\rm s}\,\psi'''(H_{\rm s})=-\omega^2\,\psi\,(H_{\rm s})\,M_0, \tag{11a}$$

$$\psi'(0)=0;\quad E_{\rm s}I_{\rm s}\,\psi''(H_{\rm s})=-\omega^2\,\psi'\,(H_{\rm s})\,J, \tag{11b}$$

where $J=J_0+M_0\,d^2$. Substituting Eq. (10) into Eq. (11) yields $A_3=-A_1$, $A_4=-A_2$ and

$$\frac{A_2}{A_1}=\frac{\mu_{\rm s}\big[\cos(a_nH_{\rm s})+\cosh(a_nH_{\rm s})\big]+a_n^3J\big[\sin(a_nH_{\rm s})+\sinh(a_nH_{\rm s})\big]}{a_n^3J\big[\cos(a_nH_{\rm s})-\cosh(a_nH_{\rm s})\big]-\mu_{\rm s}\big[\sin(a_nH_{\rm s})+\sinh(a_nH_{\rm s})\big]} \tag{12}$$

$$=\frac{\mu_{\rm s}\big[\sin(a_nH_{\rm s})-\sinh(a_nH_{\rm s})\big]+a_nM_0\big[\cos(a_nH_{\rm s})-\cosh(a_nH_{\rm s})\big]}{\mu_{\rm s}\big[\cos(a_nH_{\rm s})+\cosh(a_nH_{\rm s})\big]-a_nM_0\big[\sin(a_nH_{\rm s})-\sinh(a_nH_{\rm s})\big]} \tag{13}$$

Equations (12) and (13) provide the parameters a_n which correspond to the frequencies of the pile without water. The relationship between a_n and the nth frequency ω_n (rad/s) is

$$\omega_n=a_n^2\sqrt{EI/\mu_{\rm s}} \tag{14}$$

Table 1. First three natural frequencies (rad/s) of a pile immersed in water for different tip masses M_0, rotary moment of inertia J_0 and water height ratios $H_\mathrm{w}/H_\mathrm{s}$.

$H_\mathrm{w}/H_\mathrm{s}$	$M_0/(\mu_\mathrm{s}H_\mathrm{s})$	$J_0/(\mu_\mathrm{s}H_\mathrm{s}^3)$	HSFR proposed solution			Solution from Ref. 5		
			ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1/3	0	0	6.009	37.510	104.079	6.013	37.390	103.149
1/3	0.1	0	5.075	32.821	92.599	5.006	31.690	88.641
1/3	0.5	0	3.447	28.326	83.956	3.338	27.096	81.500
1/3	0	0.1	4.255	11.987	51.876	4.255	11.987	51.652
1/3	0	0.5	2.301	10.035	51.411	2.301	10.025	51.189
1/3	0.1	0.5	2.246	9.165	47.739	2.239	9.178	47.612
1/3	0.5	0.1	3.004	10.964	43.662	2.926	11.163	43.885
1/3	0.5	0.5	2.052	7.408	42.716	2.026	7.486	42.661
2/3	0	0	5.980	36.718	102.434	5.947	36.171	101.604
2/3	0.5	0	3.428	27.455	82.507	3.327	26.179	80.208
2/3	0	0.5	2.292	9.925	50.115	2.299	9.868	49.670
2/3	0.5	0.5	2.044	7.312	41.167	2.024	7.436	41.115

Table 2. Fundamental frequency (rad/s) of cylinder pile tested in water with different levels.

$H_\mathrm{w}/H_\mathrm{s}$	Experiment[17]	HSFR	FEM
0	130.4	130.6	132.1
0.8	115.0	116.8	114.4
0.95	107.4	105.5	102.3
1.0	99.9	100.8	97.4

The mode shapes ψ_n can be obtained by introducing the values of a_n into Eq. (10).

The natural frequencies of the pile-water system are obtained next using a harmonic sweep frequency response (HSFR) analysis. Equation (8) is first solved in the time domain for harmonic ground accelerations with forcing frequency covering the range 0 to $2\omega_n$, where ω_n is the nth natural frequency of the dry pile computed from Eqs. (11)–(14). An acceleration frequency response such as the one illustrated in Fig. 2 is then determined and the nth natural frequency ω_n' of the immersed pile is obtained as the frequency value corresponding to the resonant peak.

Two numerical examples are presented next to validate the proposed method. We first consider the case of a cylinder pile of diameter 0.3 m, length 15 m and d 5 m described in Ref. 5. The mass density of the pile is $\rho_\mathrm{s} = 7\,850\,\mathrm{kg/m^3}$ and its modulus of elasticity is $E_\mathrm{s} = 20.68$ GPa. Table 1 shows the first three frequencies of the immersed pile obtained using the proposed HSFR method for different tip masses M_0, rotary moment of inertia J_0 and water height ratios $H_\mathrm{w}/H_\mathrm{s}$. The table also contains the results obtained using an semi-analytical method developed in Ref. 5. It is seen that the HSFR yields satisfactory results and the differences stem mainly from the gross approximation adopted in Eq. (7).

The second example is a vibration test on a pile foundation-water system described in Ref. 17. The pile specimen is fixed within a cylindrical basin with a diameter 1 m and a height 0.63 m. The cylinder pile has a radius $r_\mathrm{s} = 0.0146$ m, height $H_\mathrm{s} = 0.55$ m, a modulus of elasticity $E_\mathrm{s} = 2.44$ GPa and $\mu_\mathrm{s} = 0.696$ g/m. No tip mass is concentrated at the top of the pile. Water depths H_w of 0 m, 0.44 m, 0.52 m and 0.55 m are considered.

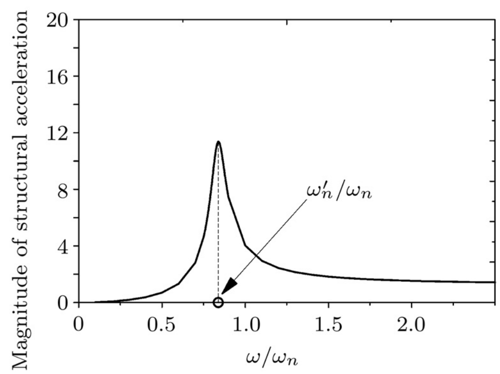

Fig. 2. Sweep frequency response curve of structural acceleration at mid-point of the immersed pile.

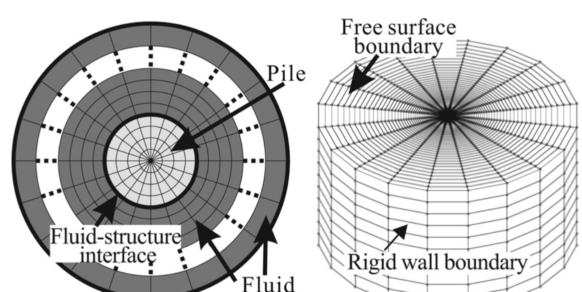

Fig. 3. Finite element model.

For purpose of validation, a finite element model is built in ADINA[16] as shown in Fig. 3. It consists of 20-node three-dimensional (3D) solid and 3D potential-based fluid elements. Fluid - structure interface elements are used to simulate the water - pile interaction. Free surface interface elements are placed on the top of the potential-based fluid elements to prescribe the zero pressure and free displacement conditions. The surface of water is modeled using rigid-wall interface elements, which are adopted to consider the wave reflection effect. The natural frequencies determined using the proposed HSFR method are compared to those obtained from experimental testing and finite element method in Table 2.

When $H_{\rm w} = H_{\rm s}$, the natural frequency of the vibrating pile can also be obtained using an added mass method described in Ref. 10, which yields a natural frequency of 99.9 rad/s.

It is again observed that the HSFR method gives very satisfactory results compared with the experimental data, finite element and added mass solutions. As expected, decreasing water height leads to an increase in the natural frequencies because of the lower added mass.

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