Titre: Title:	Optimization of Underground Stope with Network Flow Method
Auteur: Author:	Xiaoyu Bai
Date:	2013
Type:	Mémoire ou thèse / Dissertation or Thesis
Référence: Citation:	Bai, X. (2013). Optimization of Underground Stope with Network Flow Method [Thèse de doctorat, École Polytechnique de Montréal]. PolyPublie. https://publications.polymtl.ca/1168/

URL de PolyPublie: PolyPublie URL:	https://publications.polymtl.ca/1168/
Directeurs de recherche: Advisors:	Denis Marcotte, & Richard Simon
Programme: Program:	Génie minéral

UNIVERSITÉ DE MONTRÉAL

OPTIMIZATION OF UNDERGROUND STOPE WITH NETWORK FLOW METHOD

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THÈSE PRÉSENTÉE EN VUE DE L'OBTENTION
DU DIPLÔME DE PHILOSOPHIÆ DOCTOR
(GÉNIE MINÉRAL)
AOÛT 2013

UNIVERSITÉ DE MONTRÉAL

ÉCOLE POLYTECHNIQUE DE MONTRÉAL

Cette thèse intitulée :

OPTIMIZATION OF UNDERGROUND STOPE WITH NETWORK FLOW METHOD

présentée par : BAI Xiaoyu
en vue de l'obtention du diplôme de : Philosophiæ Doctor
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To my daugher Sophie

ACKNOWLEDGMENTS

Even today, it still seems unrealistic to me that, a bad student who used to struggle and prey to pass math courses is doing his PhD, on a project quite related to math. In fact this is the true history behind this dissertation. I would like to take this opportunity to thank various people who helped me to achieve this work.

First of all, I would like to express my deep gratitude to Prof. Denis Marcotte, for accepting me, guiding me, encouraging me, and funding me. Working with him, I start to discover the fun of geostatistics and Matlab. No need to mention his key contribution to this dissertation.

I am also grateful to Prof. Richard Simon, who provides very important and practical advices on mining engineering aspect.

I sincerely acknowledge China Scholarship Council, for offering me the scholarship, bringing me good chance to study in Canada.

I also want to thank my colleagues and friends in the lab, Carole Kaouane, Pejman Shamsipoor, Martine Rivest, Abderrezak Bouchedda, Eric Chou, Catherine Chou, Shengsi Sun, Pengzhi Zhao, and Fidele Yrro. I enjoy the time of sharing knowledge and entertaining.

Finally, many thanks to my wife Min Liang for her invaluable support, and my parents in half a planet away for their constant encouragement.

RÉSUMÉ

La thèse présente une série d'algorithmes originaux visant à optimiser la géométrie de chantiers souterrains en 3D, typiquement pour la méthode d'abattage par sous-niveaux (ou méthode des longs trous). Les algorithmes proposés s'inspirent des méthodes efficaces d'optimisation ayant été développées pour les mines à ciel ouvert. La clé de l'adaptation de cette méthode pour la méthode des longs trous est de reconnaître que la cheminée verticale (ou monterie), servant à initier un chantier, joue un rôle similaire à la surface dans les mines à ciel ouvert. Un système de coordonnées cylindriques est défini autour de la monterie. Les valeurs économiques des blocs dans ce système sont déterminées à partir des données en forage. Les angles limites pour le toit et le plancher sont contrôlés par les liens entre les blocs en coupe verticale. La longueur de chantier est contrôlé dans le plan horizontal, à l'aide de deux paramètres : 1) R, la distance horizontale maximale entre un bloc et la cheminée et 2) y_R, la largeur minimale de l'enveloppe créée pour exploiter le bloc se trouvant à cette distance maximale. La hauteur du chantier est déterminée par l'extension de la monterie, laquelle limite aussi les liens dans le plan vertical. L'ensemble des liens et des blocs constitue un réseau. Le réseau est complété par deux noeuds fictifs, la source et le puits. En maximisant le flux partant de la source vers le puits, on identifie le chantier optimal. Le chantier obtenu est optimal cependant conditionnellement à la monterie étudiée (localisation et extension), la discrétisation adoptée et les liens représentant les contraintes de pentes. Le problème revient alors à déterminer les paramètres de la monterie qui maximisent le profit. Pour ce faire, on utilise une méthode de type génétique permettant d'explorer des solutions variées et surtout de s'échapper d'optimums locaux. La méthode est appliquée sur plusieurs gisements et les résultats sont comparés à ceux de la méthode du chantier flottant ("floating stope"). La méthode proposée démontre sa supériorité sur ces exemples.

La méthode est ensuite généralisée à l'optimisation d'un chantier ou de chantiers comprenant plusieurs monteries verticales. À nouveau l'algorithme génétique est utilisé. L'ensemble des sous-chantiers associés aux diverses monteries sont fusionnés dans l'espace cartésien pour former un seul chantier global. Des gisements simulés et un gisement réel montrent que la solution à plusieurs monteries permet de générer un profit supérieur à la solution optimale pour une seule monterie. Le gain obtenu avec plusieurs monteries est particulièrement perceptible pour le cas de gisements courbes ou présentat des zones distinctes de minéralisation.

Une dernière modification étudiée consiste à inclure dans l'optimisation le coût de déve-

loppment des galeries de sous-niveaux pour la méthode des longs trous avec forages verticaux parallèles. L'extension de la galerie dépend en effet de l'extension du chantier. Il y a donc un gain à optimiser ces deux éléments conjointement. Ceci est réalisé par l'ajout d'un lien de précédence entre les blocs situés sur les sous-niveaux et les blocs associés aux monteries situés sur la même ligne verticale. On montre avec des exemples que l'algorithme avec les galeries fournit un profit de chantier supérieur à la solution sans les galeries. De plus les solutions obtenues montrent des chantiers plus petits que lorsque le coût de développement des galeries est ignoré.

Une étude de sensibilité des paramètres du modèle indique que la discrétisation du système cylindrique doit être suffisamment fine. L'algorithme génétique apparaît assez robuste aux choix des divers autres paramètres, du moins pour le cas type étudié. Par mesure de précaution, il est recommandé d'appliquer l'algorithme à partir de plusieurs solutions initiales différentes. Également, il vaut mieux initier l'algorithme avec des valeurs faibles du paramètre R, le rayon maximal de chantier à partir d'une monterie, et laisser croître celui-ci au gré de mutations ou autrement. Un problème lié à ce paramètre est qu'au delà d'un certain R, selon les valeurs prises par les autres paramètres, la fonction objectif ne peut plus fluctuer. La meilleure façon de traiter ce paramètre demeure à déterminer.

Les méthodes proposées sont applicables à la méthode d'abattage par sous-niveaux. Elles donnent des résultats intéressants pour les gisements sub-horizontaux ou sub-verticaux. Pour les gisements inclinés, des développements devront être réalisés. L'extension à d'autres méthodes de minage est possible mais des adaptations seront sans doute requises. Néanmoins, l'approche proposée constitue un pas important vers l'optimisation exacte des chantiers d'abattage en souterrain et marque un progrès significatif par rapport aux méthodes existantes, en particulier par la façon dont l'approche permet de tenir compte explicitement des contraintes minières.

ABSTRACT

The dissertation presents a series of algorithms to optimize the underground stope geometry in 3D, typically for sublevel stoping method or longhole stoping. The proposed algorithms are based on network flow method, an effective technique applied in open pit optimization. The key to adapt this method to underground mining is to recognize that the vertical raise to initiate a stope plays a similar role to the surface in open pit mining. Accordingly, a cylindrical coordinate system starting from the raise is introduced to redefine a ore block model. This facilitate the manipulation of geometric constraints. The slope limits of hanging wall and footwall are controlled by the links between the blocks in vertical section. The width of stope is controlled in horizontal plane, by defining two parameters: 1) R, the maximum distance to mine a block from raise, and 2) y_R, the minimum width of envelope created to mine the farthest block. The height of stope is defined by the raise extension which limits the links in the vertical section. The blocks and links constitute a network flow graph. Solving the graph with efficient maximum flow method yields an optimal stope conditional to the specified raise. This is the core of proposed methods, an optimal stope generator for given raise parameters. With the stope generator, the global optimization of raise parameters produces a global optimal stope. The algorithm using a single raise is suitable for the relatively small sub-vertical ore bodies. It is shown to provide better results than floating stope algorithm in several scenarios tested.

The algorithm using multiple raises is also developed still using the genetic algorithm. In the stope generator with multiple raises, the sub-stopes independently created from each raise are converted back to Cartesian system, and then merged to form an overall stope. The parameters of raises are also adjusted accordingly. The multiple raises solution can provide good heuristic stope. The test cases show that the multiple raises solution produces higher profit than single raise solution, especially for curved deposits and large deposits.

Moreover, the framework of stope optimizer is modified to incorporate sub-level drift in stope optimization, typically for longhole vertical parallel drilling pattern. The layers of drift blocks are identified according to the levels of given raise. The dependency relation of the blocks of drift and the blocks for stoping are expressed by the links in vertical section. Adding the new links to previous graph results in a stope with drift jointly optimized. It is shown that the algorithm with drift involved provides higher stope profit and smaller stopes than the solution without integrating drift.

Sensitivity study of parameters to the method indicates that the discretization of block model in cylindrical system should be fine enough. The genetic algorithm appears quite robust to the choice of other parameters, at least for the typical deposit used in the sensitivity analysis. As a precaution, it is recommended to apply the algorithm from several different initial solutions. Also, it is better to initiate the algorithm with low values of the parameter R, the maximum radius of a stope from a raise, to let it increase by mutation or other ways. A problem with the parameter R is that with certain other parameters, and R large enough, the objective function does not change anymore with R. The best way to deal with this setting remains to be determined.

The proposed methods are applicable to the sublevel stoping method. They give interesting results for the sub-horizontal or sub-vertical fields. For inclined deposits, developments will be realized. The extension to other mining methods is possible but adaptations will undoubtedly be required. However, the proposed approach is an important step towards the exact optimization of underground stopes and marks a significant improvement over existing methods, in particular by the way the approach allows explicit consideration of mining constraints .

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Maxflow	Maximum Flow
MVN	Maximum Value Neighborhood
STP-D	Stope Dimension
FWA	Foot Wall Angle
HWA	Hanging Wall Angle
SOS2	Type-Two Special Ordered Sets
MIP	Mix Integer Programming
LGA	Lerchs–Grossmann Algorithm
GA	Genetic Algorithm
CNFV	Cumulative Normalized Fitness Value
G	a graph
V	vertices in a graph
A	arcs in a graph
p	economic value of a block
d	density of rock
v	volume of block
g	average grade of a block
r	recovery rate
f	unit metal price
c	the unit cost of processing and mining
$c(x)$	the unit cost of processing and mining varying with raise location x
N	the number of blocks in ore block model
Γ_i	the subset of immediate successor nodes to node i
s	a source node in a network flow graph
t	a sink node in a network flow graph
(r, θ, z)	the parameters in cylindrical coordinates (radial distance, azimuth, elevation)
$(\Delta r, \Delta\theta, \Delta z)$	the unit block in cylindrical coordinates
K	the ratio $\Delta\theta/\Delta r$ of block unit at tangential direction and radial direction
R	the maximum distance from raise to include a block in stope

y_R^{max} or y_R	stope width parameter
$(x_i, y_i, z_i^b, z_i^t, R_i)$	the parameters of a raise noted by i : x_i and y_i, the coordinates in horizontal section ; z_i^b and z_i^t, its bottom and top elevation ; R_i is the maximum distance a block can be from the raise
$\mathbf{X}$	the parents in genetic algorithm
$\mathbf{X_{new}}$	the new offspring in genetic algorithm
β	the random weight vector of the gene contributions from parents
α'	the indicator of gene mutation of a new individual
σ	the extent of mutation of a gene

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CHAPTER 1

INTRODUCTION

1.1 Basic concepts and research problems

In underground mining, the stope layout is a significant component of mine design. A successful stope design requires the deposit to be precisely modeled, the geological setting well grasped, and the mining method adequately selected. The stope, enclosing the ores or rocks to be mined, should produce as much economic profit as possible, yet it must be realistic and safe from a mining point of view. The economic outcome is the primary concern of stope design. In a mine project, even 5% discrepancy of profit from different stopes can represent a considerable amount of money(Whittle, 1989). On the other hand, the dimensions and shapes of stope are subjected to constraints due to 1) the mining method adopted and 2) the geotechnical considerations. The limits on stope dimensions and walls vary with different mining methods, such as cut-and-fill method and sublevel stoping method. Stope walls should also account for the in-situ rock proprieties such as rock strength and in-situ stress tensor, and local structures, such as faults and joints. In stope design, these complex constraints are characterized by geometric parameters, including maximum and minimum stope width and height, hanging wall and foot wall angle limits, as illustrated in Figure 1.1.

In general, the stope optimization is deemed as a constrained optimization problem, which is to find the highest profit stope subject to the geometric constraints. Viewing the stope optimizer as a black box, the input data to the box is the ore block model quantifying the mineral content or economic profit of block volumes on regular grids. The output is the geometry of stope indicating the blocks to be mined.

Despite the conceptual simplicity, the stope optimization is a challenging task. Contrary to open pit where efficient algorithms providing optimal solutions exist, the rare stope design algorithms proposed in the literature are rather crude heuristics that do not allow to include easily the mining constraints and for which the gap of optimality is unknown, nor even assessed. Moreover, accessory mining components such as drift and raise are commonly handled posteriorly to the stope design. However, in some mining methods, such as sublevel stoping with parallel longholes, the drift depends on the stope position and the development cost is non-negligible, thus should be accounted directly in the stope optimization.

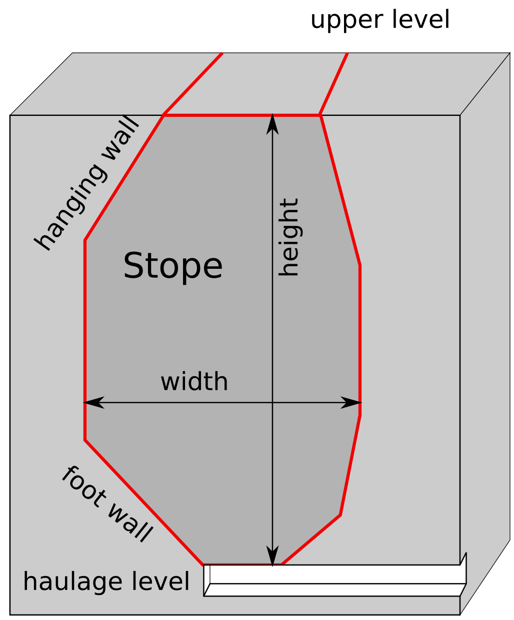

Figure 1.1 Common geometric constraints of a stope

1.2 Objectives of the research

In light of the above problems in stope optimization, this thesis aims to develop new stope optimization algorithms, typically for sublevel stoping method. The algorithms are expected to achieve the following tasks :

1. Seek to produce optimal stope geometry ;
2. Incorporate comprehensively the geometric constraints in 3D ;
3. Have adaptability to various deposit scenarios.
4. Integrate drift in optimization.

1.3 Contributions of the thesis

By reaching the thesis objectives, the following main contributions will be achieved :

1. The development of the first stope optimizer that is proven optimal, for small deposits exploited by a single vertical raise, and under specified mining constraints about wall angles, stope height and stope width ;

2. The development of an efficient heuristic for the multiple raises case allowing to extend the applicability of the stope optimizer to a wider class of deposits, larger, with more complex shapes, and possibly exploited on multiple sublevels ;
3. The development of a stope optimizer and heuristic allowing to include costs of drifts directly into the optimization.

These contributions apply to the currently most prevailing underground mining method : the sublevel stoping (Haycocks and Aelick, 1992). They are however restricted to subhorizontal or subvertical deposits. The case of inclined deposits requires nontrivial adaptation to the proposed method that are not considered in this thesis (Bai *et al.*, 2013b,a)

1.4 Structure of the thesis

This thesis consists of seven chapters. The first chapter provides a general view of underground stope optimization, and states the purposes of research. Chapter 2 reviews the state-of-art of stope optimization and relevant subjects. From chapter 3 to chapter 5, three progressive research articles are presented. Chapter 3 presents the first article entitled "Underground stope optimization with network flow method" published in *Computer & Geosciences*, which is the basic building block for the stope optimization. It shows how the network flow open pit approach can be translated for underground optimization by the use of a cylindrical coordinate system defined around the raise. In Chapter 4, the second paper describes a generalization of the single raise optimizer of Chapter 3 for the case of multiple raises. The generalization allows to better represent large deposits or deposits with more complex shapes. A genetic algorithm (GA) is used in conjunction with the single raise stope optimizer to provide heuristic solutions that improve over the single raise optimal solution, yet respecting mining constraints. In Chapter 5, an article accepted by 2013 APCOM conference introduces a method to incorporate drift in long-hole stope optimization based on the previously proposed framework. The joint optimization with the cost of drift level provides a stope with higher overall profit than the traditional methods, which handles the drift and stope separately. Chapter 6 presents a short sensitivity study of the GA to the choice of the different parameters. Chapter 7 summarizes the contributions and highlights of the thesis, addresses the limitations, and provides suggestions for future researches. An illustration of the open pit problem is presented in the Appendix.

CHAPTER 2

LITERATURE REVIEW

The state-of-the-art stope optimization and relevant subjects are reviewed in three parts. First, ore reserve modeling techniques are briefly overviewed, as a model quantifying the spatial distribution of ore grade is a premise for stope design. The second part surveys the stope optimization methods. The third part reviews the pit optimization techniques. The open pit optimization techniques are adapted to stope designs in the next chapter. Similarity of pit design and stope design is highlighted.

2.1 Ore reserve modeling

To optimize the design of stopes, an ore reserve model is needed. Modeling a ore grade distribution is the process of estimating the mineral content onto grids in 2D or 3D according to known discrete borehole data and local geological informations. This work usually resorts to geostatistical techniques (David, 1988; Journel and Huijbregts, 1978; Chilès and Delfiner, 1999). The variogram, an essential geostatistical tool, is used to characterize the spatial distribution of minerals. The in-situ geological interpretation also offers important guidance on position and continuity of ore body. The Kriging is the most common estimator and interpolator, which can provide a smooth map in average sense without bias. Conditional simulation techniques provide another options aimed to model the uncertainty of mineralizations, by producing a series of maps each showing a similar variogram to the real field (Journel, 1974; Dimitrakopoulos, 1998; Dowd and Dare-Bryan, 2005) and conditioned by the available information. Applying economic functions on an ore grade model, an economic model can be obtained (Lane, 1988; Rendu, 2008), with which the cost and benefit of decisions on mining design, and their uncertainty, can be straightforwardly evaluated.

2.2 Optimization for underground stope design

The state-of-the-art on underground stope optimization was reviewed by Ataee-Pour (2005) and Alford *et al.* (2007). To the best of our knowledge, there are seven methods published on this topic, including : the dynamic programming, mathematical morphology method, floating stopes technique, maximum value neighborhood method (MVN), branch and bound technique, octree division approach and simulated annealing. To compare these methods, the following important factors should be considered : is real optimal solution guaranteed ; does the method applies to all mining methods ; are all geometric constraints integrated ; is the method fully 3D. These properties of the optimizers are illustrated in Table 2.1. The following subsections look into these techniques and address the pros and cons.

Tableau 2.1 Comparison between the existing stope optimization algorithms

Algorithms	Mining methods considered	Dimension	Geometric constraints	Optimality
Dynamic programming	Block-caving	2D	STP-D[a] ; FWA[b] ;	No
Mathematical morphology	Cut-and-fill ; sublevel stoping	2D	STP-D ; FWA ; HWA[c] ;	No
Floating stope	All	3D	STP-D	No
MVN	All	3D	STP-D	No
Branch and bound	All	1D	STP-D	Yes
Octree division	All	3D	STP-D	No
Simulated annealing	All	3D	STP-D ; FWA ; HWA	No

[a] STP-D : Stope Dimension ; [b] FWA : Foot Wall Angle ; [c] HWA : Hanging Wall Angle.

2.2.1 Dynamic programming

Riddle (1977) developed a dynamic programming algorithm to optimize the underground mine boundary for block caving method. The algorithm, operating on a 2D cross-section model, provides a rigorous mathematical solution of initial cave boundaries. However, the footwall region is adjusted afterwards in a heuristic manner. Also, the 3D solutions obtained by combination of optimized 2D cross-section caves are not necessarily optimal.

2.2.2 Mathematical morphology approach

Deraisme *et al.* (1984) reported a mathematical morphology approach(Serra, 1982) for stope layout optimization. Based on 2D sectional block models, two algorithms were developed for cut-and-fill and sublevel stoping methods respectively, which are characterized by different geometric constraints. The method uses mathematical morphological operations, such as opening and closing, to manipulate a 2D ore model image, generating demanded geometry of stope. Though the method were presented in 2D, the generalization to 3D solution is straightforward, as the morphological operations are available in 3D. The limitation of the method lies in its ad hoc procedure. Since the morphological operations control only the geometry it cannot take into account the economic profit associated. When satisfying stope geometry independently of the search for the most profitable stope, the optimality of result cannot be assured.

2.2.3 Floating stope technique

Floating stope (Alford, 1996) is a 3D algorithm implemented in DATAMINE software. It is analogue to the moving cone method in open pit design, but differs in geometric constraints. The core technique can be described as floating a stope shape with the minimum stopes dimension around the block to locate the stope of highest stope grade or economic profits. The minimum stope dimension is the selective mining unit which depends on the mining method considered. Two envelopes are created : the maximum one is the union of all possible stope positions ; the minimum one is the union of all best grade stope positions. The envelopes provide a reference of limitation to decide the final stope position. The algorithm benefits from its simplicity and generality, which are important factors to be adopted for commercial package. However, it is a heuristic method with apparent defects. First, it may generate an uneconomical stope by simply combining two overlapping stopes that are both economical(Ataee-Pour, 2005). Secondly, the wall slope constraints required in many mining methods are not considered in the algorithm. Thirdly, rather than providing an exact final stope, the algorithm offers a domain of possible good stopes, which requires engineers' manual modification to obtain the ultimate design (Alford *et al.*, 2007).

2.2.4 The maximum value neighborhood method

Ataee-Pour (2000) devised a "maximum value neighborhood" method in an identical fashion with floating stope technique. It introduced a concept of "maximum value neighborhood" indicating the neighbor block set with maximum value of interest in all possible neighbor sets. The size of neighbor sets are restricted by minimum stope dimension. The selective combination of the best neighborhood sets for all blocks yields the ultimate stope layout. The method overcomes the problem of overlapping stope in floating stope technique, and is expected to provide better heuristic envelopes (Ataee-Pour, 2004). Nevertheless, the stope wall slope constraints are not integrated.

2.2.5 Branch and bound technique

A branch and bound algorithm was proposed by Ovanic and Young (1995) and Ovanic (1998). The algorithm created a rigorous solution for the deposits with simple geometry. The stope boundary is defined by a series of starting and ending points on each row of blocks. To facilitate the integration of constraints, such as the stope length, the convexity of rows and the continuity of stope, *Type-Two Special Ordered Set*(SOS2) is introduced. Searching a stope is then formulated as a Mixed-Integer Programming (MIP) problem. The application of the method is limited to simple ore bodies, which can be modeled in 1D along mining direction. The wall slope constrains are neglected. Besides, solving the MIP problem is time consuming especially when the number of blocks is large.

2.2.6 Octree division approach

Cheimanoff *et al.* (1989) developed a stope designing package based on an octree division of geometry. This algorithm first builds a geometrical object of orebody, then transforms the geometry into a mineable geometry (a stope) taking account of mining and economic constraints. This is done by a series of geometrical manipulation, such as volume merge, division and remove, and economic evaluation of geometrical objects. The algorithm can provide heuristic 3D stopes. However, it tends to include unnecessary waste in the final mine layout (Ataee-Pour, 2005). Also, stope wall angle limits are not considered.

2.2.7 Simulated annealing

Manchuk (2007) and Manchuk and Deutsch (2008) developed a simulated annealing approach for the stope geometry and sequencing optimization. The method parameterized a stope as a geometric object consisting of a set of the vertices and edges forming a triangulated mesh. This notably facilitates the manipulation of stope geometric constraints in optimization. The procedure of the optimization is to randomly adjust the shape of stope, respecting the geometric constraints, in order to find the shell enclosing maximum profits. The algorithm offers a general 3D solution to engineers, integrating fully geometry constraints regardless the mining methods selected. The limitations is that the simulated annealing process can be long. Especially when the number of vertices is large to construct a complex geometry, the time of convergence to optimal can be unrealistic. Therefore, in practice, SA is an heuristic for which the quality of the approximation to the real optimal solution is difficult to assess.

2.3 Pit optimization

Open pit optimization is essentially identical to stope optimization but with different constraints. For open pit, the pit shape can be simply considered as a series of overlapped inversed cones, and geotechnical requirements are reduced to the slope limit of the wall. Though there are a number of heuristic algorithms to optimize a pit (Pana, 1965; Robinson and Prenn, 1977; Dowd and Onur, 1992; Johnson and Sharp, 1971; Koenigsberg, 1982), graph theory based techniques, providing rigorous solutions ensuring real optimal, are prominent and most appealing.

With graph theory, blocks of deposit are denoted by the vertices or nodes V. The economic value of the blocks are represented by the weight of the vertices. The mining constrains are manipulated by the arcs A, or say the oriented links between blocks expressing the precedence relations to mine a block. These constitute a weighted directed graph $G = (V, A)$. A closed set of vertices in the graph builds up a pit. Then the optimal pit is the closed set with maximum sum of weights. In this way the open pit optimization is modeled as a maximum closure problem.

To solve the problem, the Lerchs–Grossmann algorithm (LGA)(Lerchs and Grossmann, 1965) is the most widely applied approach and adopted by most of the commercial software, such as Whittle, Datamine and Surpac. The maximum flow algorithms provide more efficient solutions (Picard, 1976). The most efficient maximum flow algorithm is push-relabeled algorithm (Goldberg and Tarjan, 1988; King *et al.*, 1992). The reviews on the graph theory based techniques in pit optimization are documented in Hochbaum and Chen (2000), Hochbaum (2001) and Caccetta (2007).

2.4 Synthesis

None of the existing methods for stope optimization are totally satisfying. Most fail to incorporate the mining constraints. All are heuristics for which the quality of the approximation to the real optimal stope is hard to evaluate. With open pit, there is a clear and well defined method that enables to compute the real optimal solution that respects the mining constraints. The setting of open pit is quite different from the underground optimization. However, some analogy exists. In open pit, there is a natural free surface, the ground surface. In underground mining, there is no natural free surface, but one is always created initially to allow the blocks to move. For example, with the longhole method, an initial raise (usually vertical) creates the required free surface. As the free surface corresponds to the surface of a cylinder, it seems natural to think of a cylindrical coordinate system to represent the precedence links, between the blocks, towards the free surface.

The next three chapters built on this basic idea of linking the blocks toward the free surface, as in open pit mining, for the special case of the longhole method. Chapter 3 presents the case of a small subvertical or subhorizontal deposit that can be mined from a single raise. This is the only case where optimality, under specified mining constraints is ensured with the proposed method. Chapter 4 generalizes the approach to multiple raises. Here, the solution is obtained as a combination of optimal substopes. However, the global stope itself is not necessarily optimal, although it was checked in the studied example that it recovers more profit than does the single raise optimal solution. Finally, chapter 5 presents the case of multiple levels longhole method where the non-negligible cost of drift development is taken into account directly in the optimization to provide more profitable solutions.

CHAPTER 3

ARTICLE 1 : UNDERGROUND STOPE OPTIMIZATION WITH NETWORK FLOW METHOD

Article history : Submitted 3 June 2012, Accepted 9 October 2012, Published online 6 November 2012, Computers & Geosciences.

Authors : Xiaoyu Bai, Denis Marcotte and Richard Simon

3.1 Abstract

A new algorithm to optimize stope design for the sublevel stoping mining method is described. The model is based on a cylindrical coordinate defined around the initial vertical raise. Geotechnical constraints on hanging wall and footwall slopes are translated as precedence relations between blocks in the cylindrical coordinate system. Two control parameters with clear engineering meaning are defined to further constrain the solution : a) the maximum distance of a block from the raise and b) the horizontal width required to bring the farthest block to the raise. The graph obtained is completed by the addition of a source and a sink node allowing to transform the optimization program to a problem of maximum flow over the graph. The (conditional) optimal stope corresponding to the current raise location and height is obtained. The best location and height for the raise are determined by global optimization. The performance of the algorithm is evaluated with three simple synthetic deposits and one real deposit. Comparison is made with the floating stope technique. The results show that the algorithm effectively meets the geotechnical constraints and control parameters, and produce realistic optimal stope for engineering use.

3.2 Introduction

Mine layout in underground mining plays a significant role in the viability of a mine. The design of excavations (or 'stopes', as called in underground mining methods) is one important decision controlling the economic profitability and the safety of the mining production. Generating optimal underground stopes to maximize the economic profit subject to geotechnical constraints is a difficult problem with currently no known solution.

To optimize a stope, an ore reserve model must be available to serve as basic input for optimization. This model is usually defined by a set of small regular blocks whose ore grade values are obtained from geological analysis and geostatistical estimation or simulation (David, 1988; Journel and Huijbregts, 1978). Knowing mining variable costs, commodity price, ore density and metal recovery rate, the ore grade model can be transformed into an economic block model which gives the profit of each mining block. Geotechnical constraints on the stope shape pertain to the hanging wall and footwall angles, minimum stope dimensions and possibly maximum stope dimensions. One needs to decide which blocks are mined and which are not, so that the stope formed by the union of selected blocks fulfills the geometric constraints, ensures that the selected blocks can be mined, and yields the maximum profitability. Note that the geotechnical constraints vary according to the mining method used. They could also vary regionally based on the geology, local earth stress and existing features such as joint sets and faults. Therefore, it is unlikely to define a general purpose optimization algorithm suited for all underground mining methods. Here, we rather focus on the mining method called sublevel stoping (also called long-hole method). This method is one of the most prominent mining method due to its low mining cost and the high safety of operations.

Previous works on stope optimization relied mostly on strong simplifications of the initial problem. For example, the 3D problem was simplified by considering optimization along only one or two dimensions. The dynamic programming method (Riddle, 1977), and branch and bound technique (Ovanic and Young, 1995) were developed in this manner. Although the simplifications decrease the complexity of the optimization, it precludes incorporating realistic geotechnical constraints into the optimization. For real three dimensional stope definition, a few techniques were used, including : mathematical morphology tools (Serra, 1982; Deraisme *et al.*, 1984), floating stope technique (Alford, 1996), maximum value neighborhood method (Ataee-Pour, 2000), octree division approach (Cheimanoff *et al.*, 1989). All these methods share two major drawbacks : 1) they are basically heuristic approaches and 2) they cannot incorporate directly geotechnical constraints. Therefore, the mining engineer has to adjust the stope solution to obtain a feasible stope. Manchuk and Deutsch (2008) tried with simulated annealing to better incorporate the mining constraints, however the method remains an heuristic. A state-of-the-art review on these heuristic approaches is provided by Ataee-Pour (2005) and Alford *et al.* (2007).

Contrary to underground mines, the optimization of open pits has a well known optimal solution. Geotechnical constraints are wall angles of the open pit. They are enforced by linking blocks of a lower level to the blocks of the above level so that the linked blocks define the requested angles. Graph theory and network concepts based methods are prominent and successful techniques for optimization of such problems. The Lerchs-Grossmann algorithm (LGA) (Lerchs and Grossmann, 1965) finds the maximum closure of the graph representing the open pit. It is the approach adopted by most of the commercial software, such as Whittle, Datamine and Surpac. Picard (1976) proved that maximum closure problems are equivalent to minimum cut problems that can be solved by efficient maximum flow algorithms (Goldberg and Tarjan, 1988; King *et al.*, 1992). Hochbaum (2002, 2001) and Hochbaum and Chen (2000) provide an excellent review and comparison of efficiency of different max flow algorithm implementations for the optimization of open pits.

Although the open pit optimization can not be directly applied to underground mining, it is very helpful to seek for analogies. In open pit, every block is moved toward the initial ground free surface. In sublevel stoping, an initial free surface is artificially created by drilling, most of the times, a cylindrical vertical raise. This suggests the idea of using a cylindrical coordinates system defined around the central line of the raise. By cleverly linking the blocks defined in the cylindrical coordinates system, it must be possible to enforce the geotechnical constraints and ensure that each block selected could flow by gravity to the opening created around the initial raise. These two ideas constitute the core of our approach. It enables to translate the sublevel stope optimization into a simple graph problem following basically the same approach as for open pit optimization, hence, allowing efficient computation.

In the following sections, we describe in detail the analogy between open pit optimization and sublevel stope optimization, and we describe the proposed algorithm. Then, several case studies, both synthetic and real, are presented. We analyze the results obtained and discuss the performance and limitations of the algorithm.

3.3 Methods

3.3.1 Economic block model

The profit of a block p_i can be evaluated by the economic function below(Lane, 1988) :

$$p_i = d_i v_i [g_i r f - c] \tag{3.1}$$

Where i denotes the block ; g_i is the average ore grade of block i ; f is the unit metal price ; r is the recovery rate ; c is the unit cost of processing and mining ; v_i and d_i are the block volume and density. The model can be used for both regular and irregular blocks. Note that the development costs (i.e. costs to create the access to the stope) are not taken into account in the economic function as we assume, for simplicity, that they are similar for each possible stope in the study zone. Moreover, possible differences in operating costs related to choice of the raise location are also ignored. These differences however can be included in the model by using $c(\mathbf{x})$ instead of c in Eq. 3.1, where $\mathbf{x}$ is the vector of coordinates of the raise's bottom.

3.3.2 Graph theory based optimization for open pit

Open pit optimization uses graph theory concepts. The ore block model is represented as a weighted directed graph $G = (V, A)$, in which the vertices or nodes V denote the blocks and the oriented arcs A denote the connection between blocks expressing the mining slope constraints. The profit to mine block i is represented as block weight p_i. The pit optimization is to find a closed set of nodes $V' \subseteq V$ such that $\sum_{i \in V'} p_i$ is maximum. Let Γ_i be the subset of immediate successor nodes to node i, representing the set of blocks in upper levels to be mined to get access to block i, this can be expressed as the integer program :

$$\begin{aligned} &Maximize && \textstyle\sum_{i=1}^{N} p_i x_i && (3.2)\\ &Subject\ to && x_i - x_j \leq 0, \quad \forall\, i \in V, j \in \Gamma_i && (3.3)\\ & && x_i = 0 \ or \ 1, \quad \forall\, i \in V && (3.4) \end{aligned}$$

Despite its conceptual simplicity, the integer program is CPU intensive, because the number of ore blocks N is usually large. It is better to use graph theory to find the maximum closure of the graph as in the LGA(Lerchs and Grossmann, 1965). However, Picard (1976) proved that maximum closure problems are reducible to a minimum cut problem, hence solvable by efficient maximum flow algorithms. For this, Picard defined a new network N obtained from the initial graph model G by adding a source node s and a sink node t. Arcs $s - i$ of capacity p_i link s to every node with $p_i \geq 0$. Arcs $i - t$ of capacity $-p_i$ link every node with $p_i < 0$ to t. Arcs $i - j$ of G receive infinite capacity. One of the most efficient maximum flow algorithms is the push-relabel algorithm (Goldberg and Tarjan, 1988; King *et al.*, 1992). It has been shown to be substantially more efficient than the LGA (Hochbaum, 2002, 2001; Hochbaum and Chen, 2000). This property is very useful, especially when repetitive optimization is required as in our approach.

In open pit optimization : 1) all the blocks mined are brought to the surface, either to be treated at the mill or to be dispatched to the waste dump, 2) the blocks on upper levels must be mined prior to blocks on lower levels, 3) the slope angles of the open pit are ensured by the arcs linking the blocks. Thus, the precedence relationship can be expressed as in Fig. 3.1. All the blocks have the same links, except the surface blocks which do not link to any block upside.

3.3.3 Analogy with sublevel stoping method

In the sublevel stope mining, the ore within a stope is accessed initially from a vertical raise. Long holes are drilled around the raise and the ore is blasted into the raise. After a sequence of blasts, the ore falls down by gravity towards the raise and the bottom of the open spaces available. Eventually, the broken ore fills the stope due to swelling. Ore is then hauled to the surface for further treatment. The removal of broken ore creates new open space in stope for further blasting. The blasting and hauling are run sequentially until all the planned stope is mined. In some cases the stope is partly backfilled between sequences of blasts. In the end, the stope is generally closed with full backfill to ensure stability. In our approach, the following geotechnical aspects of the stope geometry are considered (Haycocks and Aelick, 1992) :

1. The footwall slope toward the raise must be smooth and steep enough, so that blasted ore can easily flow to drawpoints. Similarly, the hanging wall must be steep enough to prevent undesired blocks from falling within the stope and therefore increase ore dilution, and possibly jeopardize the stope integrity. The minimum acceptable footwall angle and hanging wall angle are defined typically to be at least 45-55 degrees.
2. In horizontal section, a stope should be wide enough for the fluency of ores.
3. The height of the stope should be within certain acceptable limits for rock mechanics stability consideration.

Finally, a stope is typically at least 6 meters wide to procure enough economic value to justify its creation (Haycocks and Aelick, 1992). Therefore, the minimum width of the stope depends on the specific context of each mine and also of mining engineers' experience and preference. This parameter can be seen as a control parameter driving the optimization. For a given R, increasing the minimum width results in larger and geometrically more regular, but less profitable, optimized stopes.

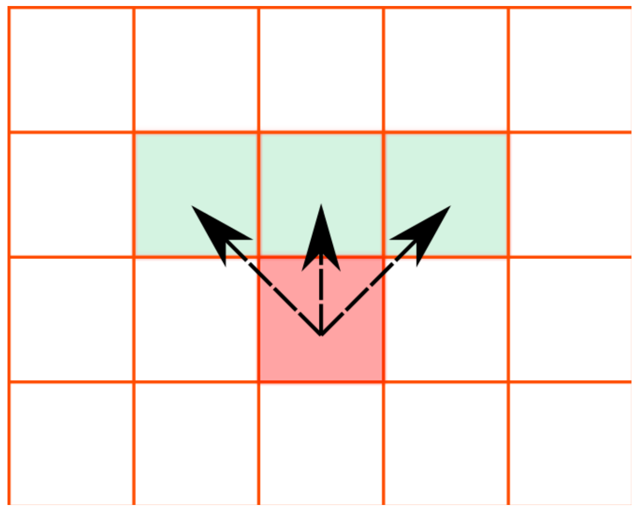

Figure 3.1 Vertical section showing typical arcs for open pit optimization in 2D.

3.3.4 Ore block model in cylindrical coordinates

The role played by the vertical raise, in stope level mining, is analogous to the ground surface in open pit mining. It suggests to use a cylindrical coordinates system based on the center line of the raise. Each point can be expressed as (r, θ, z) relatively to the raise center line, where r is the radial distance from center line of raise, θ is the azimuth, and z the elevation. The space around the center line is discretized in 'blocks' of size $(\Delta r, \Delta\theta, \Delta z)$. Although regular in the cylindrical space, the blocks are of increasing size with r in the Cartesian space (see Fig. 3.2 a).

The grades in these irregular blocks are estimated by creating an internal grid of points (here of 3 x 3 x 3 = 27 points) within each cylindrical block. The points are then estimated and averaged with weights proportional to the volume associated to each internal point. This ensures to properly reproduce the support effect for the irregular blocks defined in the cylindrical system.

3.3.5 Graph for stope optimization

We define the arcs in the graph needed to ensure the desired hanging and footwall slopes and the minimum width requirement. Thanks to the adopted cylindrical system, these two constraints can be split and considered separately.

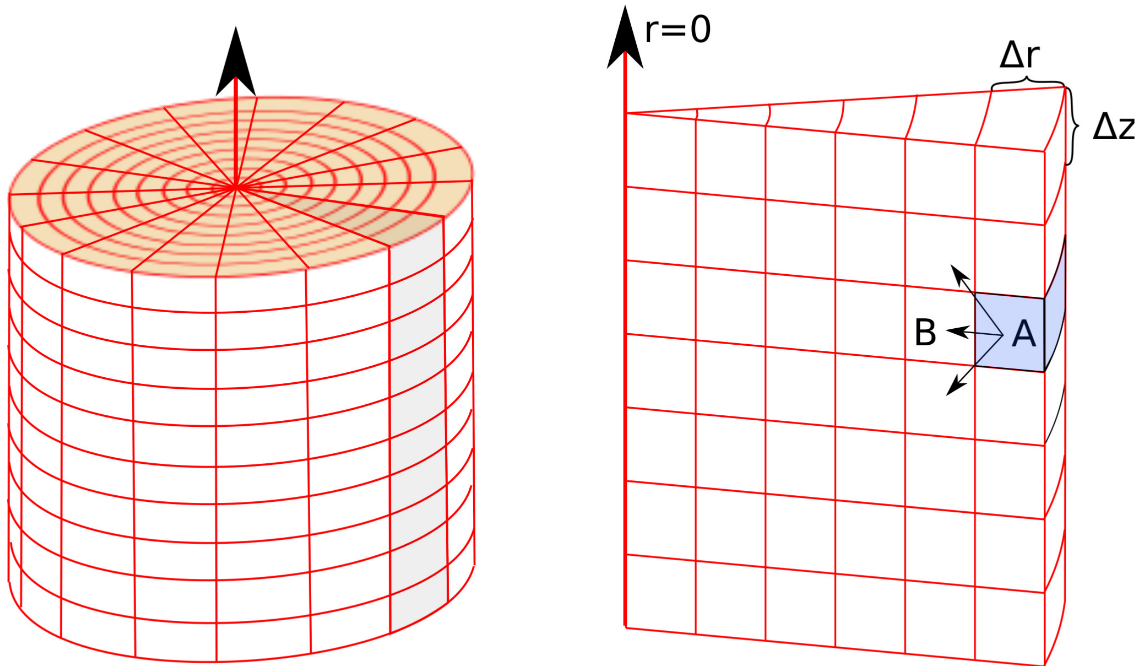

Figure 3.2 Block model under cylindrical coordinates a), and typical arcs in vertical section in proposed method b).

Stope wall angle constraints

Consider a vertical section of the model passing by the raise center line. To mine block A, a series of neighboring blocks must also be mined to guarantee the required footwall and hanging wall slopes, and to account for the needed open space in front of the block (towards the raise, see Fig. 3.2 b). The number of links vertically upward or downward depends on the slope constraints and the discretization parameters ratio $\Delta z/\Delta r$. For example, with $\Delta z/\Delta r = 1$ one link upward ensures a hanging wall slope of 45^o, two links upwards, an hanging wall slope of 63.4^o. Conversely, to ensure an hanging wall of 55^o with a single vertical link, then the $\Delta z/\Delta r$ ratio must be set to 1.43. Where the slope angles required for stability vary spatially (due for example to variations in geology or stress conditions), it would be necessary to identify locally homogeneous zones having approximately the same slope constraints. In an area with short scale variations of the slope constraints, it would be prudent to adopt the largest slope angles to avoid any collapse of the stope.

Stope width constraint

Consider now a horizontal plane in the cylindrical coordinates system having discretization Δr and $\Delta\theta$. To move a block to the raise, enough open space in front of the block is required, so that the rock swelling after blasting does not impede the flow. This precedence relation suggests that a block should not be linked only to its immediate radial neighbor, but also laterally to its diagonal neighbors toward the raise (see Fig. 3.3 a). It is always possible to select the ratio $\Delta\theta/\Delta r$ such that only three links are required in the horizontal plane toward the raise to meet the stope width requirement (see Fig. 3.3 b).

The links in the horizontal plane, propagated from block to block, eventually reach the raise center line (where r=0 in Fig. 3.3 a). In the Cartesian coordinates, it defines an envelope as shown in Fig. 3.3 b) and d). The maximum width of this envelope depends on the ratio $K = \Delta\theta/\Delta r$, and the radial distance R from block A to the raise center line. The width of the envelope for block A at any intermediate distance r along the line joining block A to the center line is :

$$
\begin{aligned}
y_R(r) &= 2rsin(\theta) \\
&= 2rsin(\frac{\Delta\theta}{\Delta r} \cdot (R - r)) \\
&= 2rsin(K \cdot (R - r))
\end{aligned}
\tag{3.5}
$$

where $K = \frac{\Delta\theta}{\Delta r}$ and $\theta = K(R - r)$. The maximum width of the envelope y_R^{max} can be calculated numerically using Eq. 3.5. The y_R^{max} depends on R and K. This formula conveys the appealing idea that, to mine a block, the width of the open space needed increases with the block distance to the raise (see Fig. 3.3 c-d). R can be seen as a design parameter set by the mining engineer. It is the maximum radial distance a block may have from the current raise. Once this parameter is set, the second design parameter is the maximum width y_R^{max} of the envelope required for this most extreme block to flow to the raise. Having R and y_R^{max}, one may fix K by virtue of Eq. 3.5. Fig. 3.4 gives the K values as a function of 'Reference Distance to Raise' (R) and 'Stope Width Parameter' (y_R^{max}).

3.3.6 Algorithm

The optimization algorithm comprises two main parts. The first part, the stope optimizer, is the core of our approach. It consists, for a specified raise location and height, to optimize

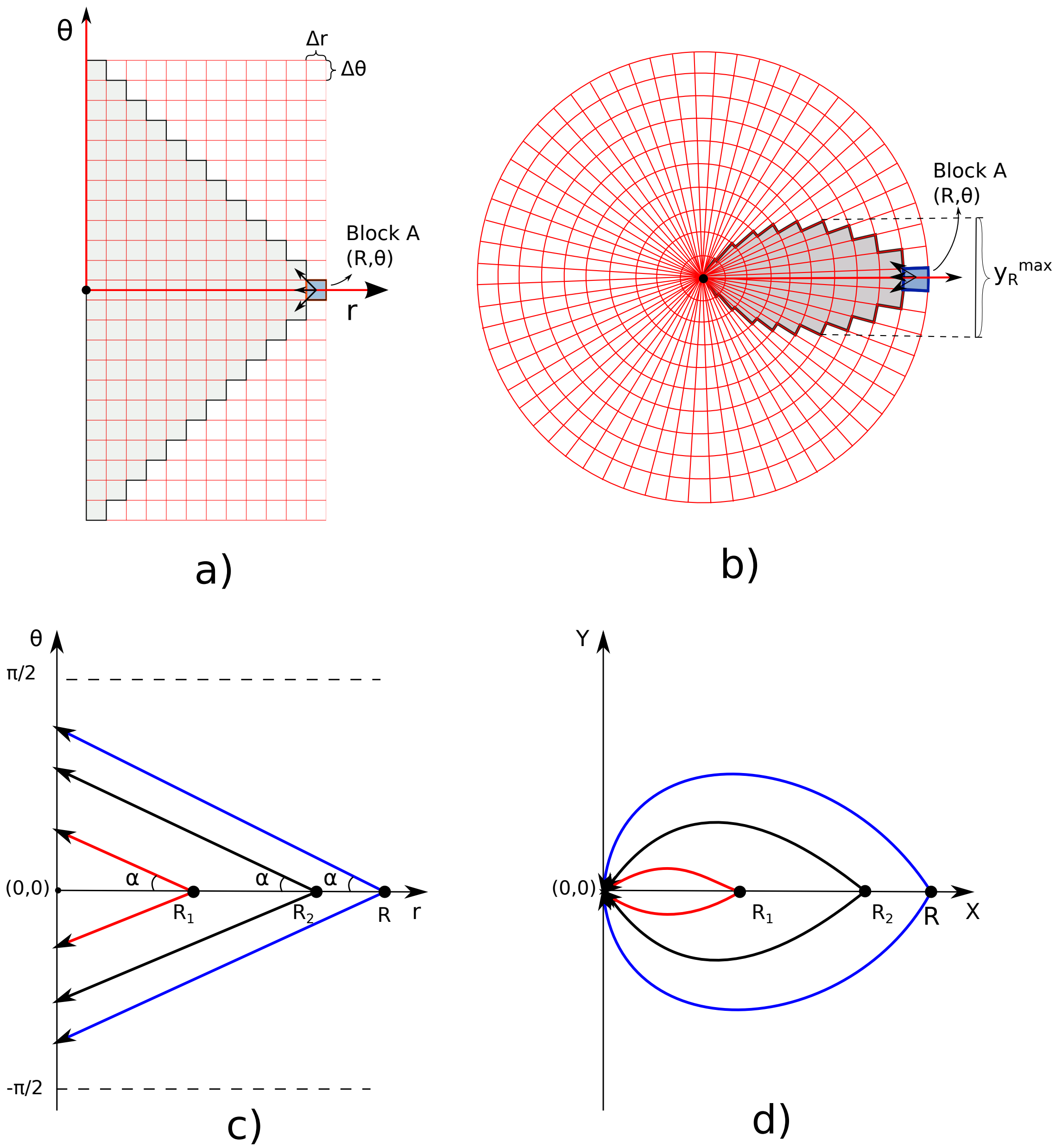

Figure 3.3 Horizontal plane showing blocks and links defined in the cylindrical system a) and corresponding blocks and links in the Cartesian system b). Shaded blocks represents blocks to be removed to get access to block A. Trace of the envelopes defined by the lateral links (function of K and R) in the cylindrical system c) as they appear in the Cartesian system d). Envelopes are computed with Eq. 3.5.

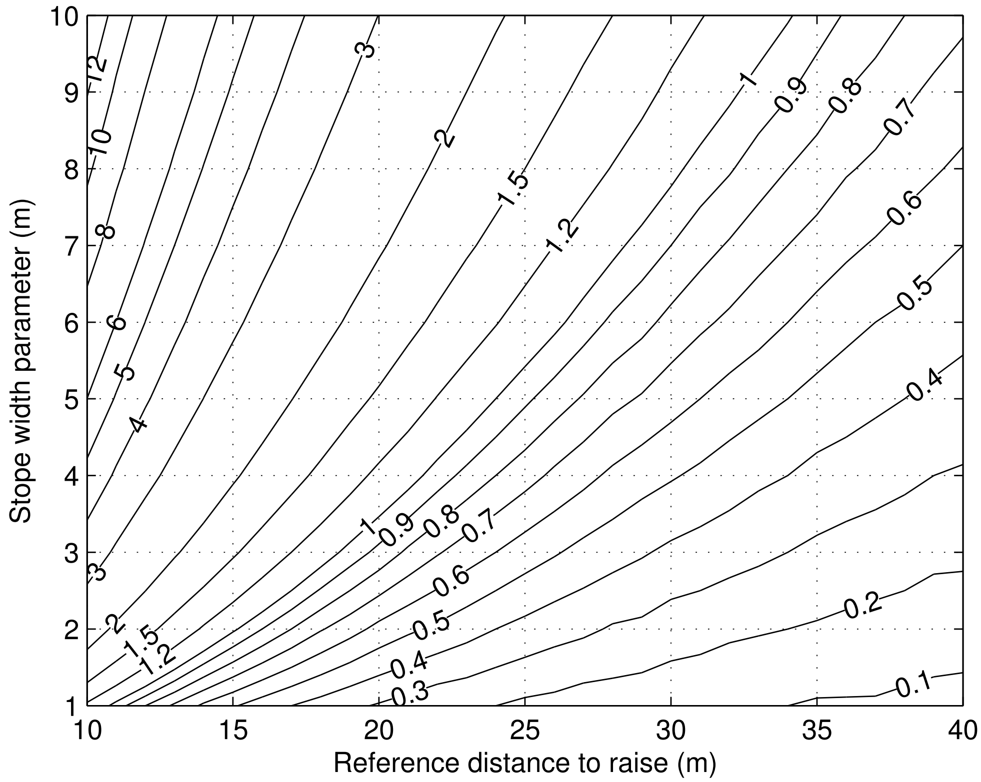

Figure 3.4 $K = \frac{\Delta\theta}{\Delta r}$ parameter (degree/m) as a function of control parameters R (reference distance to raise) and y_R^{max} stope width parameter. For example, a block at 20 m from the raise with a maximum width of 6 m necessitates a discretization approximately $K = 1.7$ degree/m with a single lateral link on both sides of the radial link as in Fig. 3.3.

the stope value given the chosen design parameters (R and y_R^{max}) and slope constraints. The stope optimizer involves the following steps :

1. Construct economic block model in cylindrical coordinates with current raise location and height as reference axis ;
2. Build the graph with vertical arcs for footwall and hanging wall slope constraints (see subsection 3.3.5) and horizontal arcs for width constraints (see subsection 3.3.5) ; all these arcs receive infinite capacities (following Picard (1976)) ;
3. Add the source and sink nodes to the graph. Positive nodes are linked to the source with capacities equal to their value and negative nodes are linked to the sink with capacities equal to their absolute value ;
4. Solve the maximum flow problem using an efficient algorithm. The sum of the residual capacities on the arcs linking the source to the positive nodes is the current stope value. This stope is conditionally optimal to the raise location and height.

In the second part one searches the best raise location and height using as objective function the stope value found with the stope optimizer. This is done here by global optimization on the raise location and height parameters using the pattern search method (Audet and Dennis, 2003)).

After the overall optimal stope is found, we convert it to the Cartesian system. For this, each point of a fine Cartesian grid (here 1 m x 1 m x 1 m) is included in the final stope, or not, according to the state (in or out) of the closest cylindrical block centroid. The strategy of using the block centroid instead of the entire cylindrical block enables to smooth out the jigsaw profile that would otherwise appear due to the irregular shape of blocks defined in the cylindrical system (see Fig. 3.3. The final reported stope value (e.g. in Table 3.2) is computed from the points of the Cartesian grid identified to be in the stope. In our tests, the stope values computed in the Cartesian system were generally slightly smaller than the stope values computed in the cylindrical system.

3.3.7 Floating stope technique

The floating stope technique (Alford, 1996) is used for comparison with our method. The floating stope is implemented in some commercial softwares, such as Datamine and Vulcan. The user defines an elementary volume corresponding to the smallest volume justifying to create a stope. The elementary volume is moved over the entire block model and the profit obtained in the volume at each location is computed. For each block of the model, the method notes also the location of the elementary volume which shows the largest profit among all the elementary volumes that include the block. A minimal envelope is obtained by the union of the elementary volumes with largest (positive) profits for each block. A maximal envelope is obtained by the union of all elementary volumes showing positive profit. The two envelopes provide a guide for the engineer to create a realistic stope which will normally lie somewhere between the minimal and maximal envelopes. In this method the engineer is obliged to incorporate wall slope constraints manually. Comparison of the profits obtained with our method to the profits obtained with minimal and maximal envelopes is interesting, keeping in mind that our approach is the only one that ensures the satisfaction of slope constraints.

3.4 Results

To test the performance of the proposed algorithm, four different models, three synthetic and one real ore body, are used as input for the optimizer. The three synthetic cases are used to illustrate and help understanding the behavior of the proposed algorithm on perfectly known simple bodies. They are not aimed to represent real deposits. The shapes of synthetic ore bodies are kept simple so as to allow direct visual inspection of the feasibility and optimality of the stope. In those scenarios, the geotechnical constraints are given in Table 3.1. The discretization used for all cases was $dr = dz = 1$ m, and $d\theta$ computed with Eq. 3.5 (see Table 3.1). The floating stope method is also applied for comparison on the same cases with an elementary volume of 6 m $\times$ 6 m $\times$ 15 m that sweeps the same Cartesian grid as the one used to provide the optimized stope profit. The optimal stope values, missed ore values and waste included, for the different test cases, are presented in Table 3.2.

3.4.1 Test on synthetic ore block models

In the first scenario, a cross shape ore deposit is created (Fig. 3.5 a). The orebody is 20 m $\times$ 21 m $\times$ 25 m. The economic values of ore and waste blocks are set to 153\$/t, and -23\$/t respectively. The optimal stope is expected to include all the ore blocks, because the economic values of ores are high enough to pay for a large quantity of waste. Results are shown in Fig. 3.5 c) and d). The stope value is 868 k\$. As expected, most of the ore blocks are taken into the stope together with the minimal amount of waste required to meet the hanging wall and footwall slope design.

Tableau 3.1 Geometric and design parameters, discretization, and optimized raise parameters.

Parameters	Case 1	Case 2	Case 3	Case 4
Geometric parameters for stope				
Minimum hanging wall angle (deg)	45°	45°	45°	45°
Minimum foot wall angle (deg)	63°	63°	63°	63°
Maximum height (m)	40	40	40	100
Minimum height (m)	15	15	15	50
Design parameters				
Stope width parameter (m)	6	6	6	6
Reference distance to raise (m) R	15	11	15	20
Discretization				
dz (m)	0.5	0.5	0.5	0.8
dr (m)	0.5	0.5	0.5	0.8
$d\theta$ (deg)	1.57	3.0	1.57	1.4
Optimized raise parameters				
Optimal raise location X (m)	24.5	25.5	20	3178.5
Optimal raise location Y (m)	22	15.5	14.5	26.6
Optimal stope bottom level (m)	-126.8	-124.8	-194.8	-171.3
Optimal raise length (m)	25	17.1	15	83.5

Tableau 3.2 Economical evaluation of the case studies.

Cases	Methods	Stope profit (k $)	Missed ore value (k $)	Waste value in stope (k $)	Dilution volume rate [1]
Case 1	Network flow	868	2	-38.8	22.2%
	Floating stope(in) [2]	846	0	-62.3	31.3%
	Floating stope(out)[3]	436	0	-472.7	77.6%
Case 2	Network flow	89	3	-22.1	56.9%
	Floating stope(in)	80	0	-34.4	66.7%
	Floating stope(out)	-87	0	-201.3	92.1%
Case 3	Network flow	442	0	-17.2	19.9%
	Floating stope(in)	424	0	-34.5	33.3%
	Floating stope(out)	153	0	-305.6	81.6%
Case 4	Network flow	507	8	-2.6	3.3%
	Floating stope(in)	509	0	-8.3	8.1%
	Floating stope(out)	332	0	-185.9	41.0%

[1]Dilution volume rate = Volume of waste in stope / Volume of stope
[2]Float stope(in) : inner envelope created by floating stope method
[3]Float stope(out) : outer envelope created by floating stope method

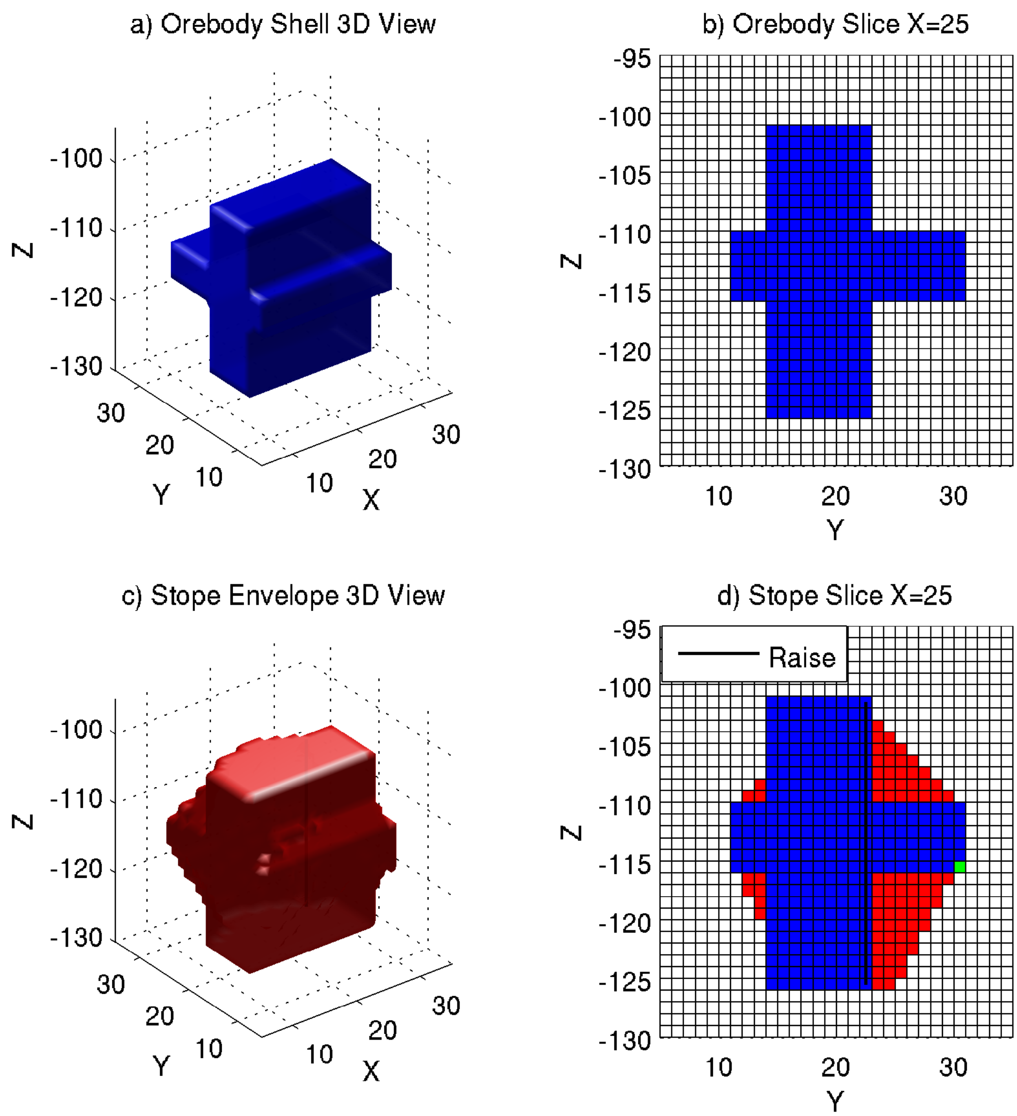

Figure 3.5 Case 1, simulated ore model and stope by network flow method : a) 3D-view of the orebody, b) yz vertical section of the orebody at x=25, c) 3D view of the optimized stope, d) yz vertical section of the optimized stope at x=25 showing ore in stope (blue), waste in stope (red), and ore out of stope (green). Design parameters as given in Table 3.1

The second case represents a thin vertical deposit of size 2 m × 22 m × 17 m (Fig. 3.6). Ore and waste values are 153\$/t, -23\$/t respectively. The stope width required is y_R^{max}=6 m for a block taken at R=11 m away from the raise. The optimal stope is shown in Fig. 3.6. The stope value is 89 k\$. 56.9% of the stope is waste, a relatively large amount needed to meet the constraint on minimum stope width. In this case, it could have been profitable to diminish R and consider using a second (or more) raise. This would allow to diminish y_R^{max} and therefore the amount of waste included in the stope. The cost of creating additional raises must of course be taken into account.

The third synthetic ore model is a small parallelepiped of size 20 m × 15 m × 10 m (Fig. 3.7). The value of ore and waste blocks are set to 153\$/t and -23\$/t. The hanging wall slope constraint is 45 degrees whereas the footwall slope angle constraint is 63 degrees. Fig. 3.7 shows the results obtained. The stope includes 5 m thick of waste on top of the mineralized blocks so as to ensure a wall slope 45 degree and meet the minimum stope height of 15 m. Note that the raise (and the stope) starts right at the bottom of the mineralized zone. Therefore, no waste is taken there. Because the footwall slope angle was larger than the hanging wall slope constraint (63^o vs 45^o), locating the raise bottom below the mineralized zone would have include more waste in the stope.

3.4.2 Test on real deposit

An ore model of a metal deposit located in Canada is used as another test case (name and location of deposit undisclosed for confidentiality reasons). A part of the deposit is selected for stope design. The orebody is subvertical with size 50 m × 30 m × 90 m (Fig. 3.8). Sublevel stoping method is well adapted for this kind of deposit. Geotechnical parameters are listed in Table 3.1. The stope generated is shown in Fig. 3.9, the profits are given in Table 3.2. The optimal raise is located at X=3178.5 m and Y=26.6 m. The bottom of the 83.5 m long raise is at Z=-171.3 m. Fig. 3.9 b) and c) shows that the wall angles meet the input parameters. In Fig. 3.9 b), some ore (in green) is not taken in the stope because the ore block values are not sufficient to pay for the waste blocks that would need to be included to fulfill the geotechnical constraints and design parameters. For the same reason, the ore below elevation -171.3 m in Fig. 3.9 c) is not included in the stope. Fig. 3.9 d) shows the horizontal slice of stope at Z=-156 m. The stope is clearly wide enough. Some waste blocks near the outer boundary of the orebody are contained in the stope to meet either the slope or the minimum width constraints. Nevertheless, the tonnage of waste represents only 3.3% of the stope tonnage. The stope value is 507 k\$ whereas the ore excluded from stope worths 8 k\$, i.e. 1.6% of the

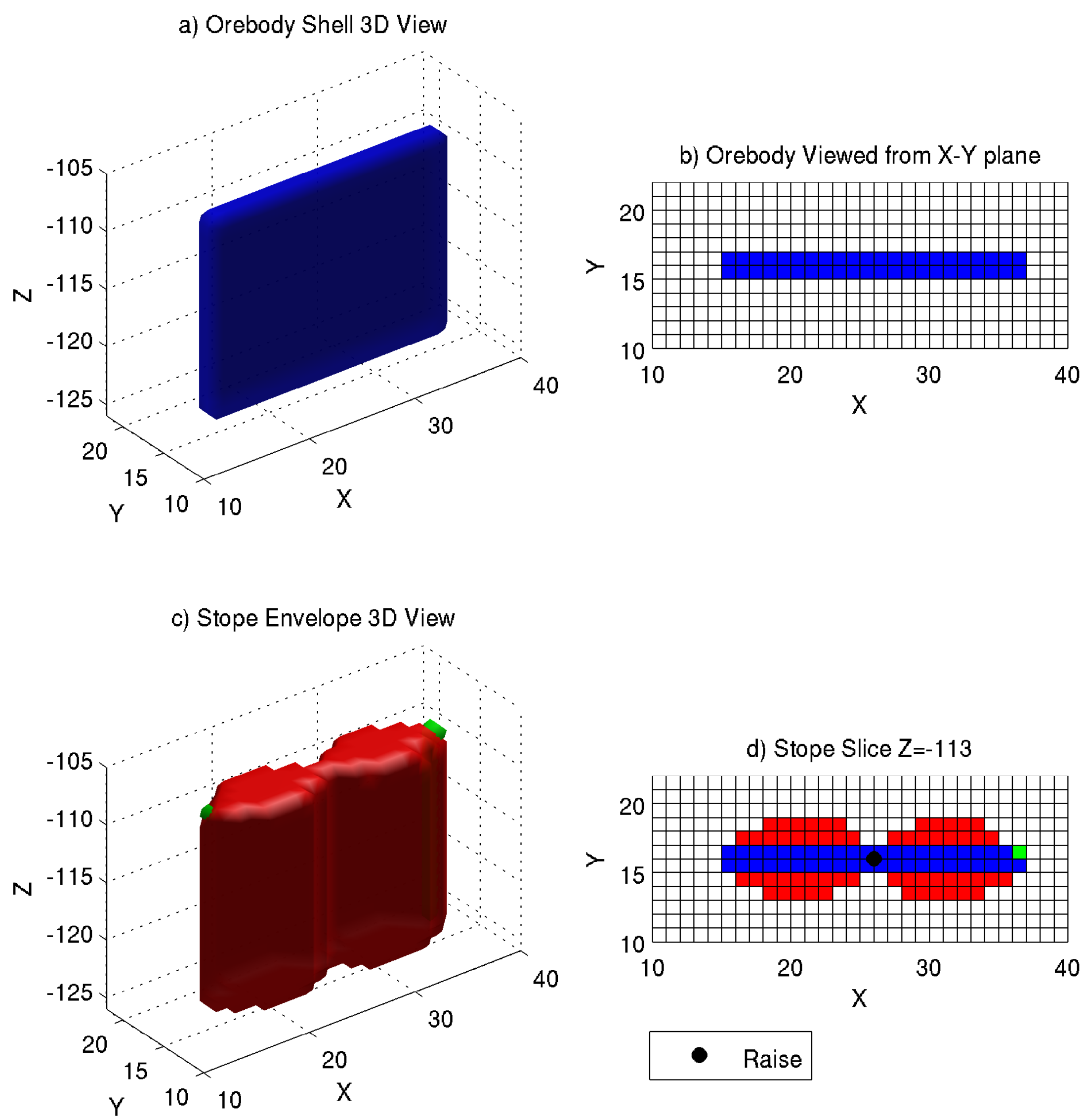

Figure 3.6 Case 2, simulated ore model and stope by network flow method : a) 3D-view of the orebody, b) xy horizontal section of the orebody at z=-113, c) 3D view of the optimized stope, d) xy horizontal section of the optimized stope at z=-113 showing showing ore in stope (blue), waste in stope (red), and ore out of stope (green). Design parameters as given in Table 3.1.

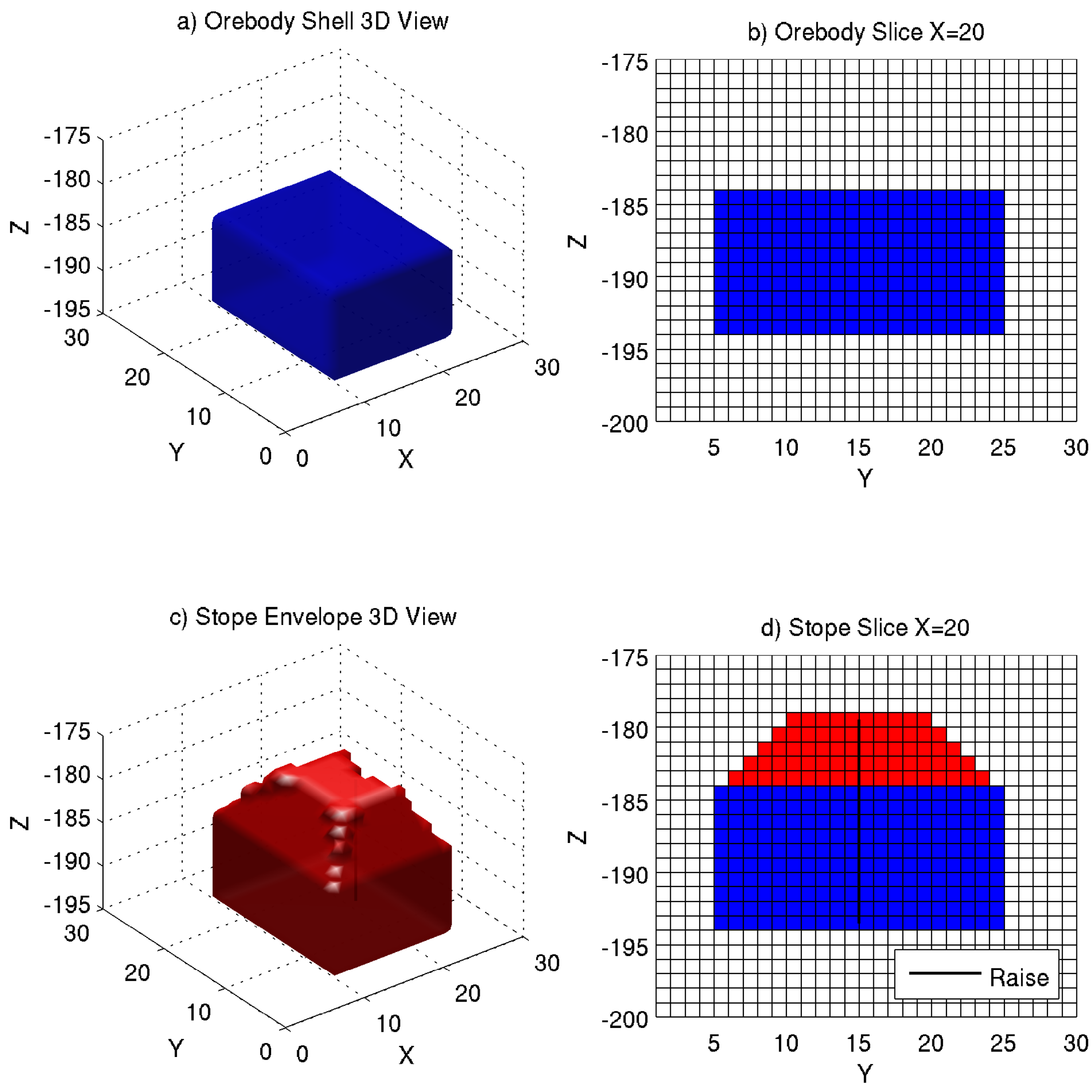

Figure 3.7 Case 3, simulated ore model and stope by network flow method : a) 3D-view of the orebody, b) yz vertical section of the orebody at x=20, c) 3D view of the optimized stope, d) yz vertical section of the optimized stope at x=20 showing ore in stope (blue), waste in stope (red). Design parameters as given in Table 3.1.

stope value.

3.4.3 Comparison with floating stope technique

The floating stope technique is applied on the same three synthetic cases and the case study. The profits obtained are listed in Table 3.2. For the three synthetic cases, the stopes obtained by the proposed method (network flow) provide higher profits than the inner and outer envelopes created by the floating stope algorithm. The outer envelope can even return a negative value (see case 2). This seems surprising as the outer envelope is formed of union of positively valued elementary volumes. Recall that the elementary volumes are not disjoint. Hence, the positive values for elementary volumes can be due to the same few rich blocks. This would add a lot of waste to a few rich blocks.

In Case 4, the profit of the inner stope is equivalent to that of the proposed method, 509 k$ vs 507 k$. As shown in Fig. 3.10-c, the inner envelope obtained by the floating stope method clearly violates wall slope constraints (e.g. the hanging wall at z=-120, between y=30 and y=55). Moreover, this stope would clearly call for at least two raises to allow all the blocks to be hauled. The engineer will have to adjust substantially this unrealistic stope to force mining constraints, which will diminish the profit obtained. The outer envelope (Fig. 3.11) gives less profit, includes more waste and also violates the wall slope requirements.

3.5 Discussion

We created a new stope optimizer for sublevel stoping methods. The optimizer is based on analogies with open pit mining to represent the underground optimization problem as an oriented graph that can be solved using efficient maximum flow algorithms. Two key ideas are at the core of the proposed approach. First, the initial raise outer surface plays the role of the ground surface in open pit mining. Second, having decided of the raise location and extent, it is possible to define a cylindrical coordinate system around the raise and then establish precedence links between the blocks required to meet the geotechnical constraints on the footwall and hanging wall slopes. Finally, optimization the raise location and height parameters is obtained by global optimization of the stope profit (seen as a function of these parameters). The method presented is the first one that can really define optimal stopes, in 3D, that fully integrate geotechnical constraints. The method is suited for deposits that can be mined by the sublevel stoping method. This mining method is considered one of the best

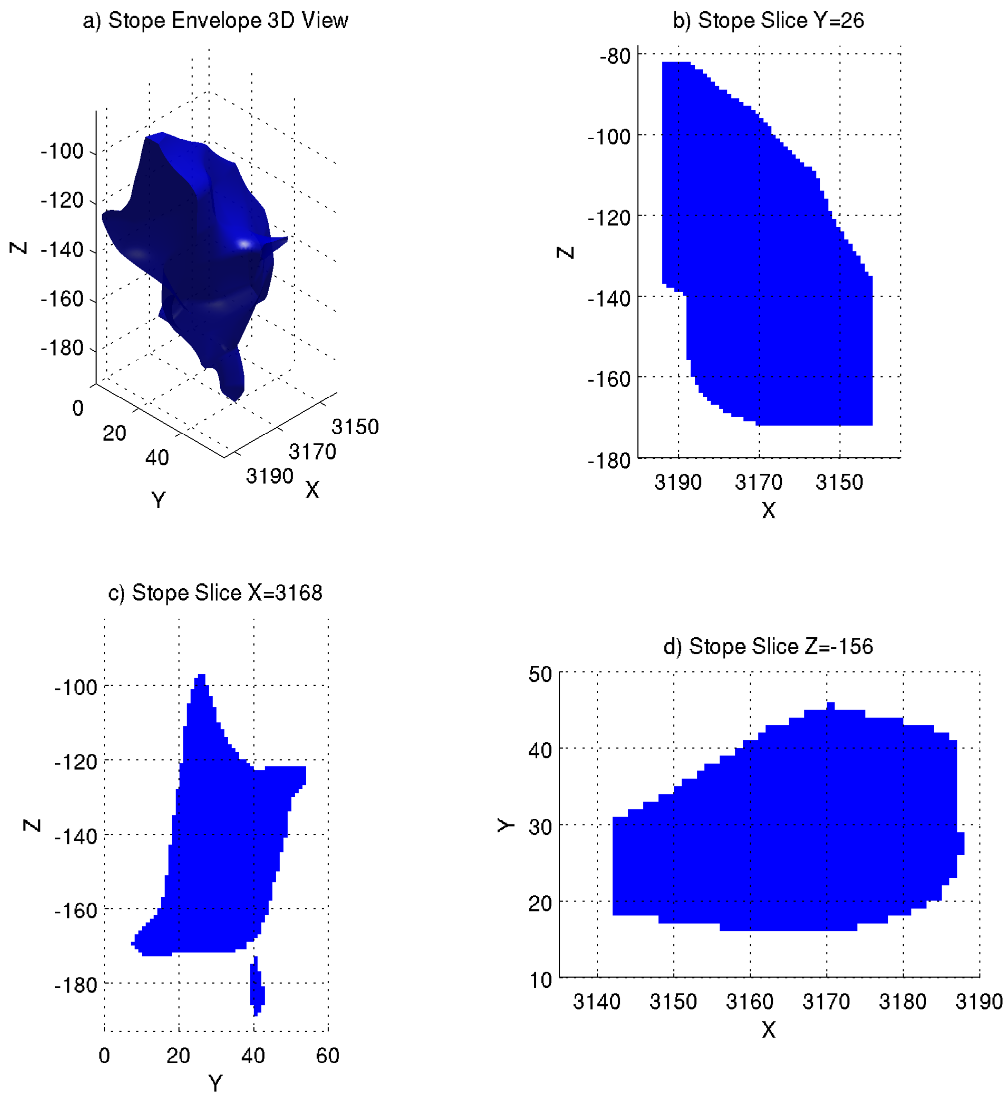

Figure 3.8 Case 4, real ore deposit : a) 3D-view of the orebody, b) xz vertical section of the orebody at y=26, c) yz vertical section at x=3168 and d) xy horizontal section at z=-156.

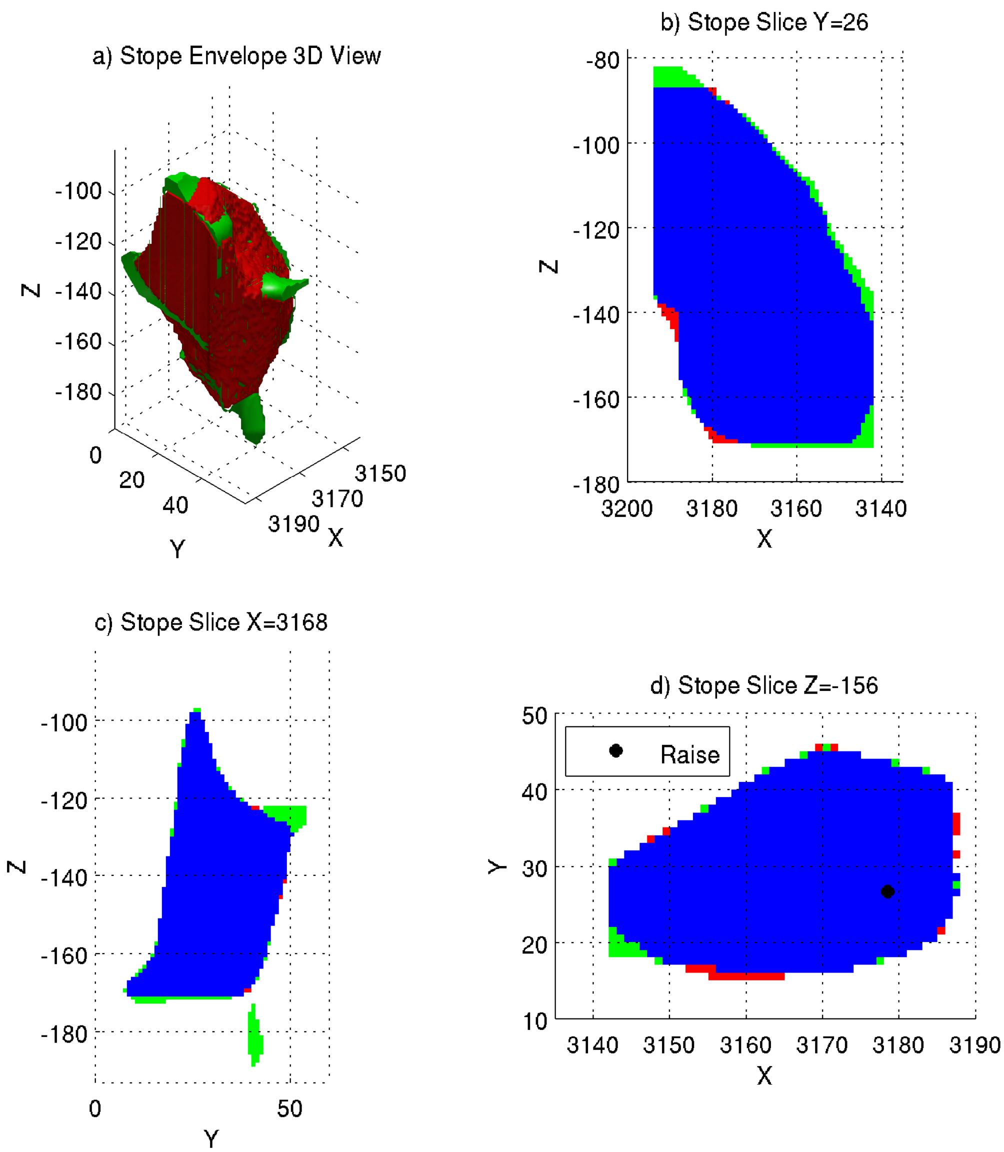

Figure 3.9 Case 4, optimized stope for the real deposit by the network flow algorithm : a) 3D-view of the stope (red) and of the ore out of stope (green), b) xz vertical section of the stope at y=26, c) yz vertical section at x=3168 and d) xy horizontal section at z=-156. For b), c) and d), ore in stope (blue), waste in stope (red), and ore out of stope (green). Design parameters as given in Table 3.1.

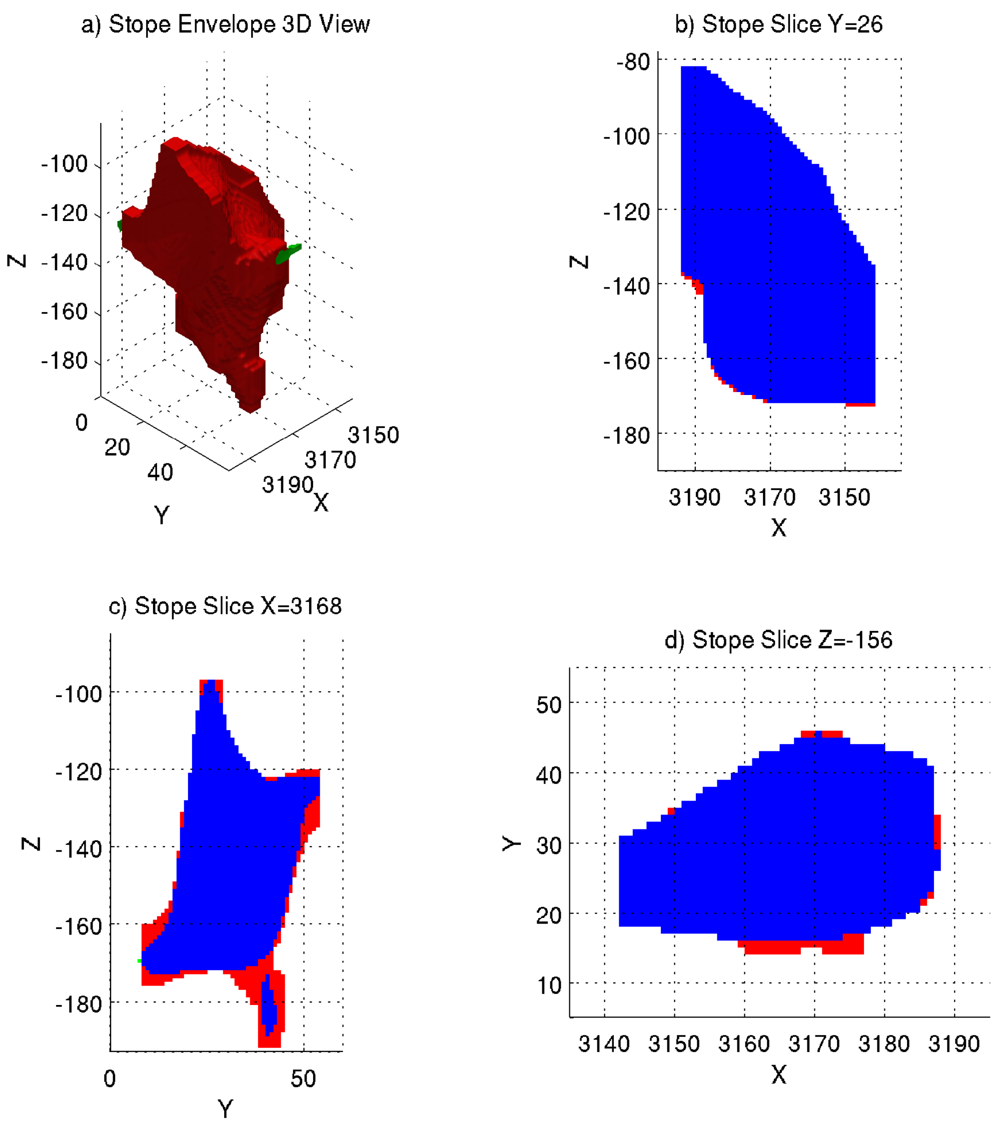

Figure 3.10 Inner stope for case 4 produced by floating stope technique : a) 3D-view of the stope (red) and of the ore out of stope (green), b) xz vertical section of the stope at y=26, c) yz vertical section at x=3168 and d) xy horizontal section at z=-156. For b), c) and d), ore in stope (blue), waste in stope (red), and ore out of stope (green).

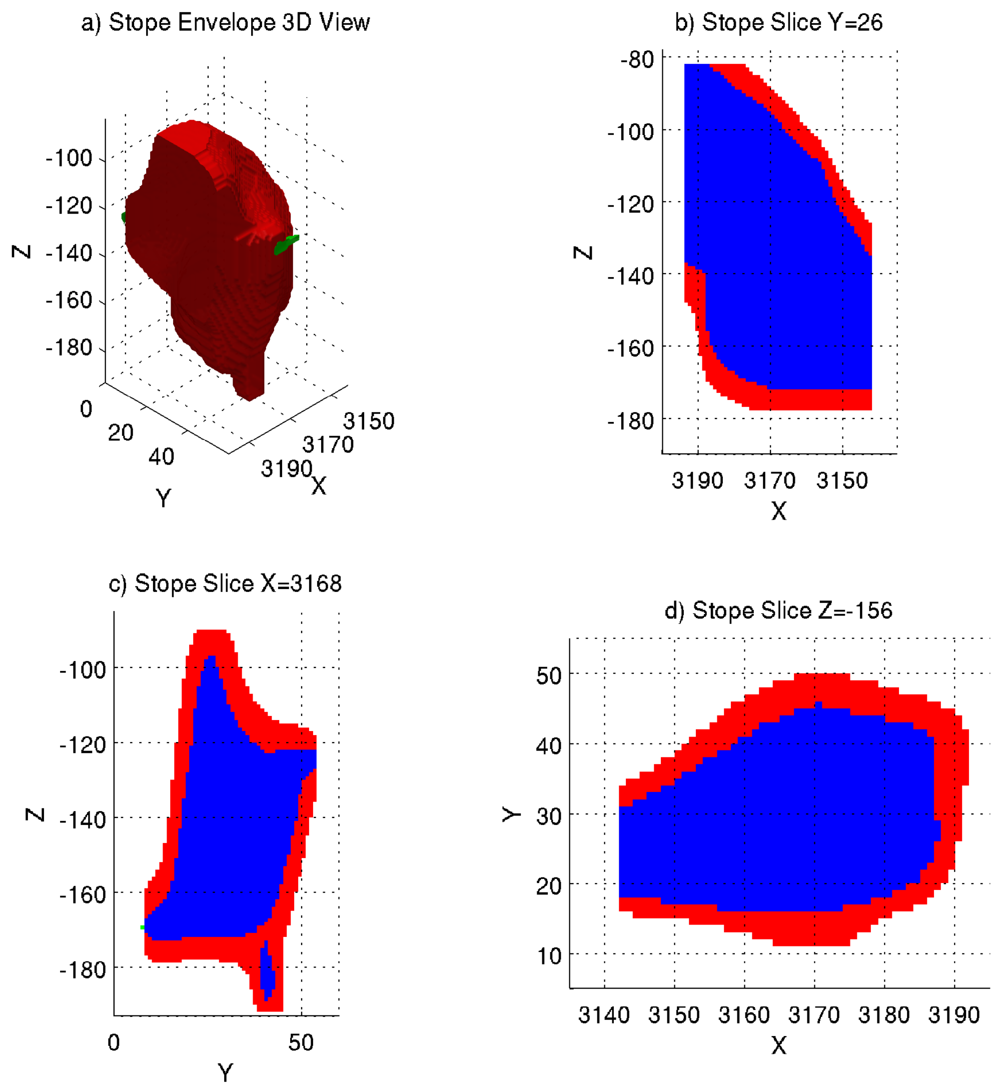

Figure 3.11 Outer stope for case 4 produced by floating stope technique : a) 3D-view of the stope (red) and of the ore not included in the stope (green), b) xz vertical section of the stope at y=26, c) yz vertical section at x=3168 and d) xy horizontal section at z=-156. For b), c) and d), ore in stope (blue), waste in stope (red), and ore out of stope (green).

underground mining method currently available due to its low operation cost and the high security for the mine workers. Further research is needed to adapt the approach to other mining methods.

The proposed method was compared to the floating stope technique available in Datamine and Vulcan softwares. In the three synthetic cases, the profits obtained with our approach were larger than with the inner and outer envelopes obtained with the floating stope algorithm. On the real case study, our approach provided an equivalent profit to the inner envelope and more profit than the outer envelope (+53 %). Note that neither of the floating stope envelopes meet the slope angle constraints, as these constraints are simply not considered in this algorithm.

One appealing aspect of the proposed method is that it possesses two control parameters with a clear meaning for the mining engineer. The two control parameters are the reference distance to the raise, R, which represents the farthest distance of influence of a raise. This parameter is closely linked to the adopted mine layout. The second parameter, y_R^{max}, is the width required, for a block at distance R, to flow to the raise. The chosen value for this parameter will depend on the mechanical properties of the rocks and blasting strategy. The optimal stope obtained is optimal conditionally to the current raise location and height, the specified choice of the control parameters and the discretization adopted for the cylindrical system of coordinates. Other choices of discretization could lead to slightly different stopes. Choices of control parameter values have a definite influence on the optimal stopes.

The main limitation of the proposed algorithm lies in the use of a vertical raise which would constitute a good choice mainly for subvertical or subhorizontal deposits. When orebody is inclined, a vertical raise would include possibly too much waste. In that case, an inclined raise is used. However, the adaptation of the cylindrical coordinate system to an inclined raise and determination of the precedence links needed to ensure geotechnical constraints need further research.

A second important limitation of the approach presented is that a single raise parameters were optimized for a relatively small mineralized body. For larger orebodies, many contiguous stopes (and therefore many raises) will be required. It should be relatively simple to adapt the global optimization on a set of parameters representing simultaneously more than one raise. This is the subject of ongoing research.

Variations in development costs and operational costs linked to choice of the raise location and height were ignored in our approach. These costs variations are 1) expected to be small or even negligible compared to the gain in stope profit due to the selection of the optimal raise location ; 2) are case specific as they depend on the location of the infrastructures in the mine. For a particular mine setup, it is possible to assess the development cost for any given raise location and height and to include this extra cost in the profit function. For the operational costs, it suffices to replace in Eq. 3.1 the constant mining cost c by a variable cost $c(\mathbf{x})$, where $\mathbf{x}$ is the bottom raise location.

For a cylindrical grid having 70 x 121 x 61 blocks, the stope optimizer runs in approximately 15 s on a workstation. Most of the time (95%) is spent on the cylindrical block grade estimation. Typically, between 100-200 calls to the stope optimizer are required to optimize the raise location and height. The total computation time is therefore less than one hour. Possible avenues to speed up the block estimation step are currently investigated.

The extension to simulated grades instead of estimated grades is straightforward. One replaces the estimation of each point on the cylindrical block internal grid by a series of realizations. Then each realization is averaged separately within each cylindrical block (with weights proportional to the volume associated to each internal point). In open pit mining, due to non-linearity of the profit function, it was proven more profitable to optimize the expected profit of each block than the profit computed from the estimated grades (Froyland *et al.*, 2004; Marcotte and Caron, 2012). However, because in underground mining all mined blocks are also treated, there is no possible re-estimation and re-classification of a block. This makes the profit function linear and the expected profit then simply coincides with the profit computed from estimated values. For more complex objective functions than the profit, it might be possible to get a better solution with the realizations than with the estimated block values but further research is needed on this point. In all cases, the simulations remain useful to assess the profit uncertainty of the optimized stope. Note that even for stochastic optimization a deterministic stope optimizer is still required.

3.6 Conclusions

The proposed method enables determination of the optimal stope design in 3D for relatively small subvertical deposits mined by the sublevel stoping method. The optimal stopes obtained comply to geotechnical constraints on footwall and hangingwall slopes, and on stope minimum and maximum heights. Applied on a real deposit, it gave realistic stope design. Further research is needed to extend and adapt the method to larger subvertical or subhorizontal deposits or to inclined deposits and and also to other mining methods.

3.7 Acknowledgement

The Chinese Scholarship Council provided a Ph.D scholarship to the first author. Additional financing was obtained from NSERC research grant of the second author. The authors are grateful to three anonymous reviewers for their numerous constructive and helpful comments.

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CHAPTER 4

ARTICLE 2 : A HEURISTIC SUBLEVEL STOPE OPTIMIZER WITH MULTIPLE RAISES

Article history : Submitted on 9 June 2013 to *The Journal of The Southern African Institute of Mining and Metallurgy.*

Authors : Xiaoyu Bai, Denis Marcotte and Richard Simon

4.1 Abstract

A new heuristic sublevel mining stope optimizer is presented. The optimizer seeks the best locations and lengths of a series of vertical raises that, together with the blocks linked to each raise, define a mining stope. Five design constraints, the footwall angle, the hanging wall angle, the number of raises, the maximum distance of a block from a raise and the minimum width required to move the farthest block towards the raise allow to control the shape of the sub-stopes associated to each raise. The optimization is done on the raises' locations and lengths parameters using a genetic algorithm to sample efficiently the parameters' space. For each raise a network is defined in cylindrical coordinates around the raise such as to impose the design constraints. A maxflow algorithm on the local network is used to determine the optimal sub-stope for the current raise. All sub-stopes are combined to define the global stope. The best stope is obtained by genetic algorithm evolving the raises parameters to be most fitted. Two synthetic and one real deposits are used to evaluate the new algorithm and compare the results with the single raise optimizer. The multiple raises approach provides slightly larger profits than the single raise stope optimizer and the dilution is also reduced compared to the single raise case.

4.2 Introduction

In underground mining, stope design affects the mining profit and safety of the operation. The stope design requires : 1) a prior ore reserve model as input data, usually obtained by estimation or simulation using geostatistical tools (David, 1988; Journel and Huijbregts, 1978) ; and 2) the geotechnical constraints, including the hanging wall and footwall angles, stope dimensions, in-situ stress tensor, rock strength and local geological structures. The general procedure of stope optimization is to decide which volumes are included in stope and which are not so that, under the geotechnical constraints, the resulting stope produces the greatest profit possible.

In the last few decades, several approaches were developed for stope optimization. These methods were reviewed by Ataee-Pour (2005) and Alford *et al.* (2007). The dynamic programming method (Riddle, 1977), and branch and bound technique (Ovanic and Young, 1995) were used to optimize a stope in one or two dimensions. Though these methods can assure the optimality of stope, the over-simplification of model would produce unrealistic stope for complex geometric deposits. Some 3D techniques were also reported, including : mathematical morphology tools (Serra, 1982; Deraisme *et al.*, 1984), floating stope technique (Alford, 1996), maximum value neighborhood method (Ataee-Pour, 2000), and octree division approach (Cheimanoff *et al.*, 1989). These heuristic methods cannot integrate directly the geotechnical constraints. Recently, Manchuk and Deutsch (2008) provided simulated annealing based algorithm, with the mining constraints incorporated. However, simulated annealing is slow and the convergence to a global optimum can ask for unrealistically slow annealing schedule. Hence, in practice neither the constraints nor the optimality are guaranteed.

The authors developed in Bai *et al.* (2013) a stope optimizer based on graph theory. The basis of the approach is the vertical raise initiating necessary opening for blasting, which plays similar role as the surface in open pit mining. A cylindrical coordinate system is defined around the raise. Then a network is build where cylindrical blocks are linked towards the raise such as to impose *de facto* the geotechnical constraints. The optimal block selection is obtained by applying efficient maximum flow methods over the network. Two important design parameters were defined : the maximum distance of influence of a raise (R) and the minimum width required to move the farthest block to the raise (y_R)(see Fig. 4.2). The stope obtained is optimal for the raise location and extent chosen, and for the R, y_R and footwall and hanging wall angles imposed. The global stope optimization then boils down to simply find the best raise location and extent within the orebody. The optimizer was shown to provide good results on a number of simple deposits, synthetic and real.

Although quite appealing, the single raise optimizer has a few drawbacks. Firstly, it cannot provide a satisfying optimal solution for large deposits or lenses where more than one raise is naturally needed. In that case, applying repetitively the one raise optimization needs to take into account the interactions between the raises. Also, for small deposits or lenses with curved shapes (e.g. following folds), the single raise optimal solution could provide more dilution and less profit than a manual solution obtained with more raises, each raise having a smaller distance of influence R.

In this paper, the authors aim at solving these drawbacks. The algorithm of single raise is extended to multiple raises situations, keeping the core component of generating a sub-stope for each raise. In the multiple-raises framework, each sub-stope is a feasible geometry with controllable maximum dimensions. Optimization is done on the set of raises parameters using a genetic algorithm to sample efficiently the parameter space. For a given set of raises parameters, each raise is optimized with the single raise optimizer, thus defining as many sub-stopes as there are raises. Each sub-stope meets the design parameters and is optimal for that raise location and length and set of design constraints. The union of all points within one or more of the sub-stopes define the global stope. Three deposits, two synthetics and one real, are considered where the results obtained with the single raise and multiple raises are compared and discussed. For the sake of simplicity, the true grade values of the deposit are assumed known everywhere, so the effect of the uncertainty on grades with regard to the stope design are not considered in this study.

4.3 Methods

4.3.1 Stope optimization with single raise

Graph theory in mining optimization

The ore block model and the mining constraints are represented as a weighted directed graph (or network) $G = (V, A)$, where the vertices V denote the ore blocks and the oriented arcs A define the precedence relations between blocks so as to incorporate the mining constraints. The profit to mine block i is p_i. It is computed from the block i ore grade and tonnage, recovery factor and mining and processing costs (Lane, 1988). The stope optimization amounts to find the closed set of nodes $V' \subseteq V$ such that $\sum_{i \in V'} p_i$ is maximum. Let Γ_i be the subset of immediate successor nodes to node i, representing the set of blocks that need to be mined prior to block i. The maximum closure problem is formulated as :

$$\begin{aligned} &Maximize && \textstyle\sum_{i=1}^{N} p_i x_i && (4.1)\\ &Subject\ to && x_i - x_j \leq 0, \quad \forall\, i \in V, j \in \Gamma_i && (4.2)\\ & && x_i = 0\ or\ 1, \quad \forall\, i \in V && (4.3) \end{aligned}$$

For a typical deposit, the integer program involves large computational time due to the large number of ore blocks N. Lerchs-Grossman algorithm (LGA) (Lerchs and Grossmann, 1965) presented an effective tool to solve the open pit mining problem implemented in some commercial softwares. Even more efficient methods appeared after the seminal paper of Picard (1976) proving that the maximum closure problem of the open pit mine is equivalent to the minimum cut problem, hence allowing the application of maximum flow algorithms (e.g. Goldberg and Tarjan (1988); King *et al.* (1992)), which are substantially more efficient than the LGA (Hochbaum, 2002, 2001). The high efficiency enables us to repetitively apply the algorithm still keeping realistic computing costs.

Implementation of stope geometric constraints in network

The key of applying network flow concept to stope optimization is that the raise drilled to create a stope in underground mining plays the same role as the ground surface in an open pit mine. This calls for the introduction of a cylindrical coordinate system with the raise as its origin, Fig. 4.1 a). Bai *et al.* (2013) indicate how the hanging wall and footwall slope constraints define the precedence links in the vertical direction, Fig. 4.1 b). They also defined two design parameters, the distance of influence of the raise R and the minimum width (y_R) needed to move the farthest block by gravity to the raise. These parameters R and y_R control the links in the horizontal plane (see Fig. 4.2 a). For a cylindrical system with blocks defined by $\Delta r, \Delta\theta, \Delta z$, see Fig. 4.1 b), and $\Delta z/\Delta r = 1$, one link upward and two links downward define a hanging wall slope of 45^o and a footwall slope of 63.4^o. Similarly, with $R = 30$ m, $\Delta\theta/\Delta r$ =1 degree/m, three horizontal links towards the raise provide $y_R = 7.7$ m, see Fig. 4.2.

A third type of geotechnical constraint, the maximum stope height, is simply controlled by the length of the raise. The blocks above the top of the raise or under its bottom are not part of the network, hence are not contained in the stope.

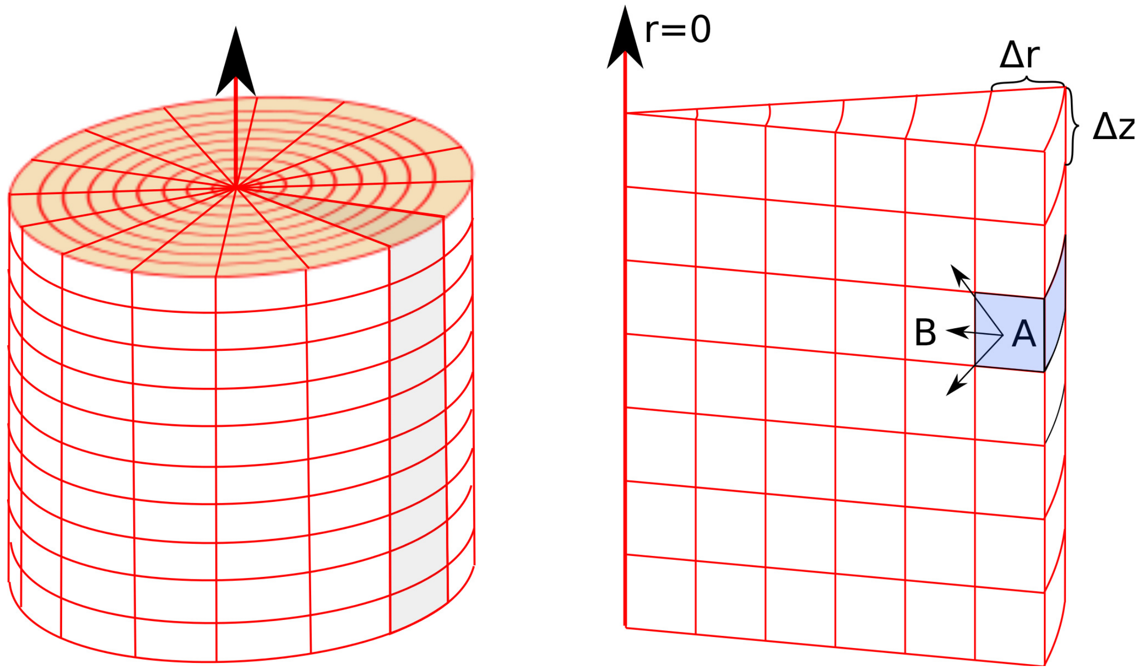

Figure 4.1 Block model under cylindrical coordinates a), and typical arcs in vertical section in the proposed method b).

Algorithm for a single raise

The optimization algorithm consists of two main parts. The first part, the stope optimizer, is the core of the approach. It generates an optimal stope for specified raise location and height, with chosen design parameters R and y_R. The stope optimizer includes the following steps :

1. Construct economic block model in cylindrical coordinates with given raise location and height as the reference axis ;
2. Build the graph with vertical arcs to impose slope constraints, and horizontal arcs to impose width constraints ;
3. Construct flow network by adding the source and sink nodes to the graph ;
4. Solve the maximum flow problem. The generated stope is conditionally optimal to the raise location and height.

The second part is to search the best raise location and height. It is done by global optimization on the raise location and height parameters, using as objective function the stope value found with the stope optimizer.

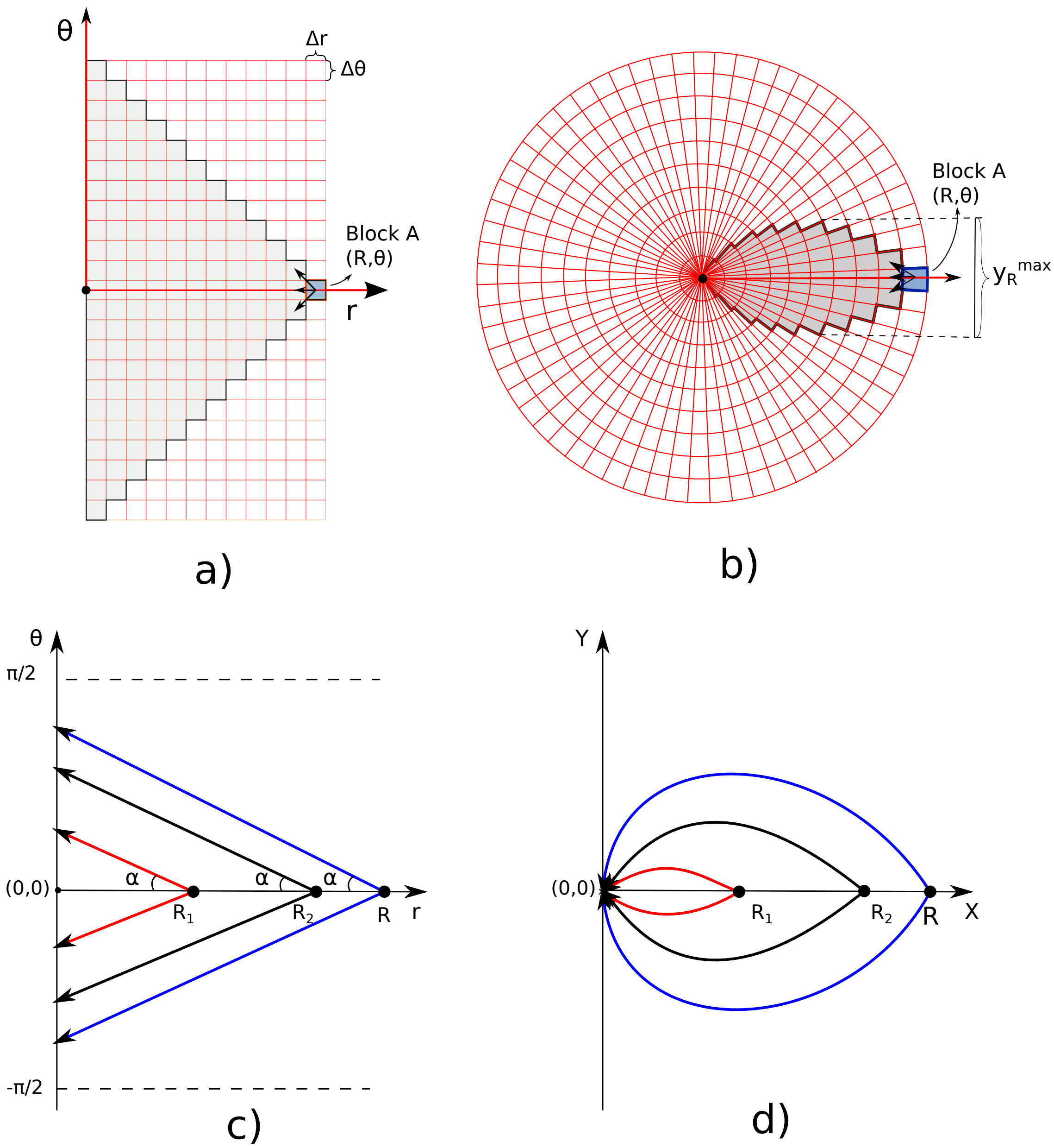

Figure 4.2 Horizontal plane showing a) blocks and links defined in the cylindrical system and b) corresponding blocks and links in the Cartesian system. Shaded blocks represents blocks to be removed to get access to block A. Trace of the envelopes defined by the lateral links in the cylindrical system c) as they appear in the Cartesian system d).

The single raise approach has some limitations. For example, when R is large, a relatively wide stope is produced as many blocks have to be mined before getting access to the farthest blocks. When the deposit is curved or inclined, this can lead to the mining of substantial amount of waste as shown in Fig. 4.3. In other scenarios, isolate clusters of ore could be left in the ground because the ore clusters do not pay for the additional waste included. In these cases, a better approach would be to use more than one raise so as to define smaller sub-stopes, hence diminishing the effects due to curvature or inclination of the ore body, and simultaneously allowing more flexibility to reach isolate clusters of ore.

4.3.2 Stope optimization with multiple raises

Like for the single raise algorithm, the algorithm comprise two main parts : 1) the stope generator with multiple raises based on a series of separate network flow problems, one for each raise, and 2) the optimization of the best parameters for the raises' locations, extents and zones of influence.

Stope generator with multiple raises

Each raise is first treated separately with the single raise optimizer described in section 4.3.1. For each raise, a cylindrical coordinate system is defined, and the stope constraints are implemented thru the precedence relations in the associated network. The maximum extent that a raise can access, or distance of influence R_i, is defined to control the maximum size of the sub-stope for raise i. As a result, an optimal sub-stope is generated for each raise (see Fig. 4.4). The sub-stopes in cylindrical coordinates are converted to a common regular grid in Cartesian coordinates. For the conversion, the status of a grid point in Cartesian coordinates (in or out of the stope) is identified by the status of the nearest cylindrical block centroid in each sub-stope. It suffices that a grid point belongs to any of the sub-stopes to be identified as being in the global stope. Hence, the global stope is the union of all the sub-stopes. The profit of the global stope is calculated on the Cartesian grid, so as to avoid counting twice (or more) any part of the stope.

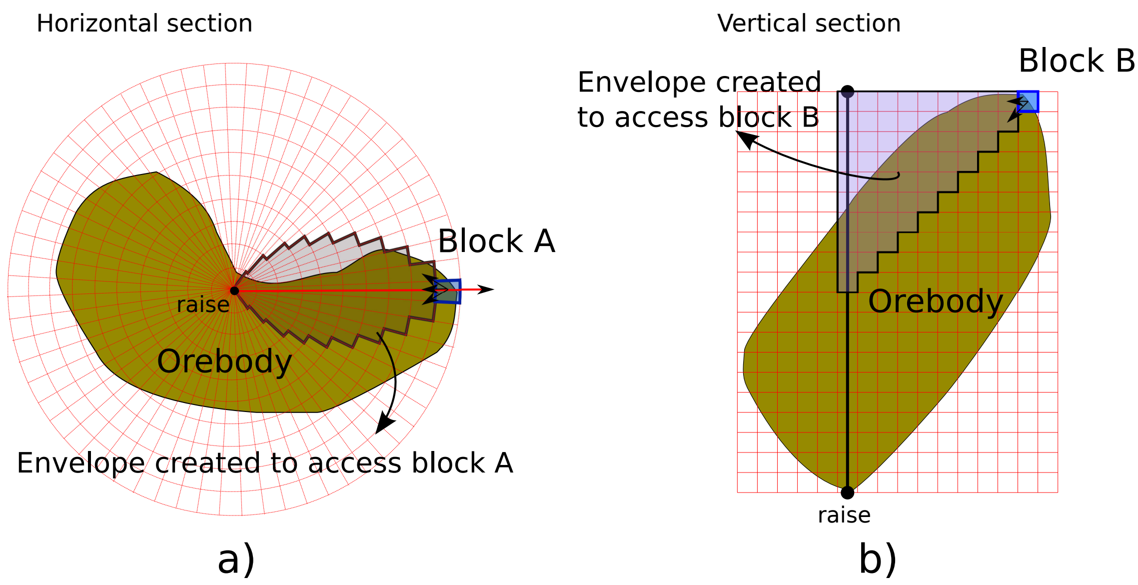

Figure 4.3 Illustration of possible problems with one raise : a) In a horizontal section, the envelope from A to the raise includes a large quantity of waste ; b) In a vertical section, waste has to be mined in the upper part due to the network associated to the single raise.

Optimization of multiple raises parameters with a genetic algorithm

In the model, each raise is parametrized as, (x_i, y_i, z_i^b, z_i^t, and R_i), $i = 1, \ldots, n$, where x_i and y_i denote the coordinates of raise i in horizontal section, z_i^b and z_i^t represent its bottom and top elevation, R_i is the maximum distance a block can be from the raise i, and n is the number of raises. The global stope is obtained as the union of sub-stopes that are each optimal in their local cylindrical system.

Good set of parameters for the raises is found using a genetic algorithm (Holland, 1975) to allow an efficient sampling of the parameters' space. Genetic algorithms tries to mimic the natural evolution of a population. A good example of application of genetic algorithm to mining optimization is presented in Armstrong *et al.* (2012). Here, we define an individual as a single set of multiple raises parameters. Starting from a population of individuals, one creates new individuals in the population by crossover and mutations. In our algorithm, a vector of multiple raises parameters represents the individual chromosome with $5n$ genes (5 being the number of parameters to optimize for each raise). The profit of the global stope associated to the raises parameters measures the fitness of the individual to its environment. The algorithm is initiated by generating an initial population. The fitness of each individual in the population is evaluated. A certain proportion among best fitted individuals are ran-

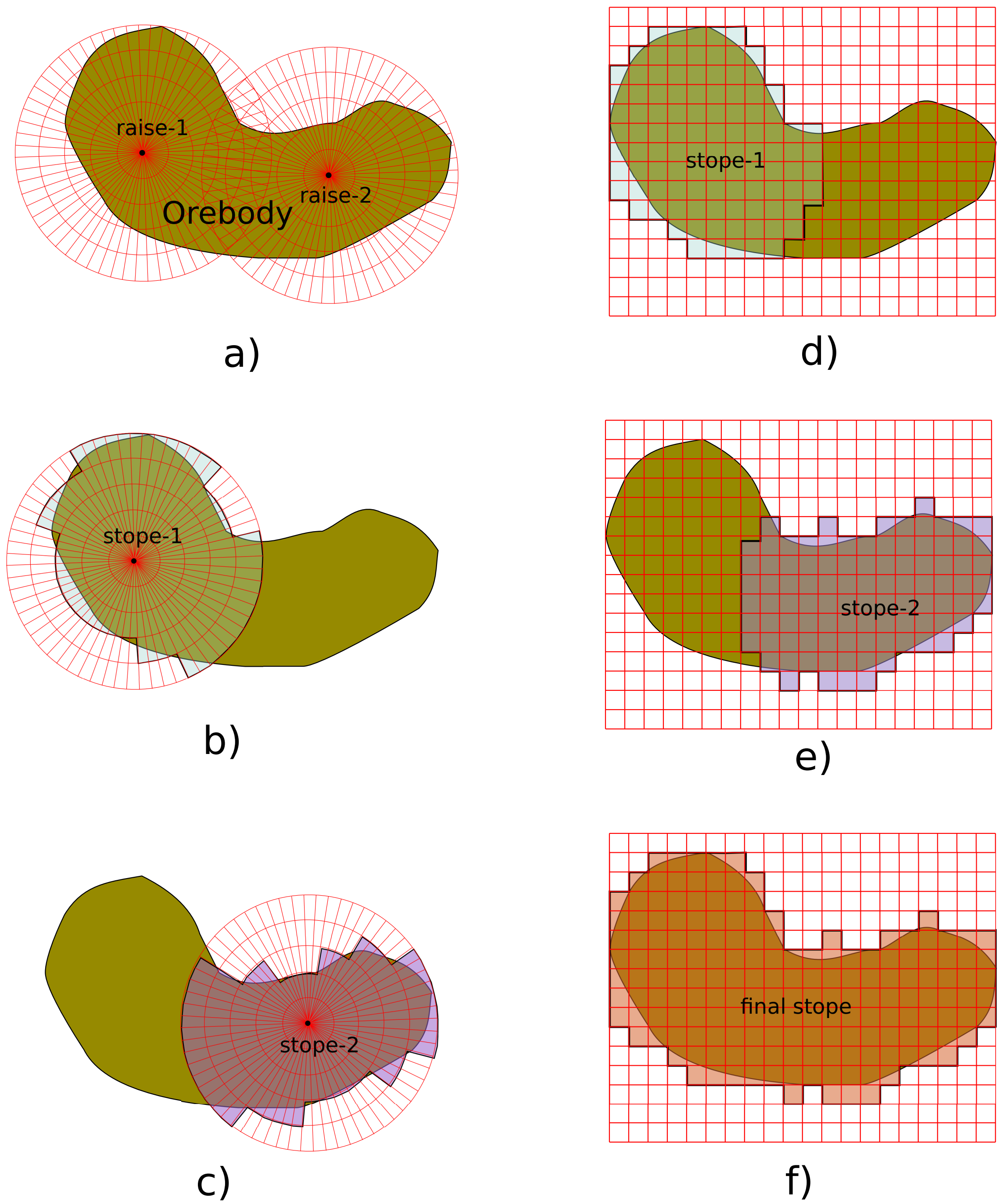

Figure 4.4 Conceptual model of stope generator with multiple raise in horizontal section a) Two ore models in cylindrical coordinates, one for each raise, are established ; b) and c) first and second sub-stopes in cylindrical coordinate obtained by maxflow method on the two separate networks ; d) and e) the sub-stopes b) and c) converted on the Cartesian grid ; f) the final stope in Cartesian grid from d) and e).

domly selected to be parents. Each set of parents mates and creates a child whose genes are inherited from them (crossover). Moreover, a certain proportion of mutations are generated by introducing, in the child chromosome, genes that do not come from the parents. The mutations enable to explore new areas of the parameters space. The least fitted individuals in the population are eliminated so as to keep the size of the population constant. With the iterations, the average fitness of the population increases, until the convergence or another stopping criterion is reached, see Fig. 4.5.

The genetic algorithm proposed follows the following steps :

1) Initial population The initial population comprises two parts : well fitted individuals and random individuals. The well fitted individuals ensure the existence of good genes. They can be obtained in two ways a) intuitive good raise parameters selected by user ; and b) fast optimization with an initial low resolution ore model. Random individuals are taken over the raises parameters to allow sufficient gene diversify so as to better explore the parameters space and identify interesting area (Haupt and Haupt, 2004).

2) Parents selection In each iteration, a certain number of sets of m individuals are randomly selected to be parents. The individuals with higher fitness are assigned a stronger probability to be chosen as parents.

3) Genetic operator Two mating methods are employed : crossover and mutation. For the crossover, since the raise parameters are continuous variables, a blending method is used to integrate the chromosomes of the m parents (Radcliffe, 1991), so that the continuity is maintained. A new chromosome $\mathbf{X_{new}}$ is generated by the linear combination of the chromosome of its parents, as :

$$\mathbf{X_{new}} = \sum_{\mathbf{i=1}}^{\mathbf{m}} \beta_{\mathbf{i}} \mathbf{X_i} \tag{4.4}$$

$$\sum_{i=1}^{m} \beta_i = 1 \ , \ 1 \leq \beta_i \leq 0 \tag{4.5}$$

Where $\mathbf{X_i}$ represents the parent i and β_i is the random weight assigned to each parent. The linear combination is convex, so a child obtained by crossover remains within the range of its parents. Mutation can be applied to some genes of a child, allowing the child genes to

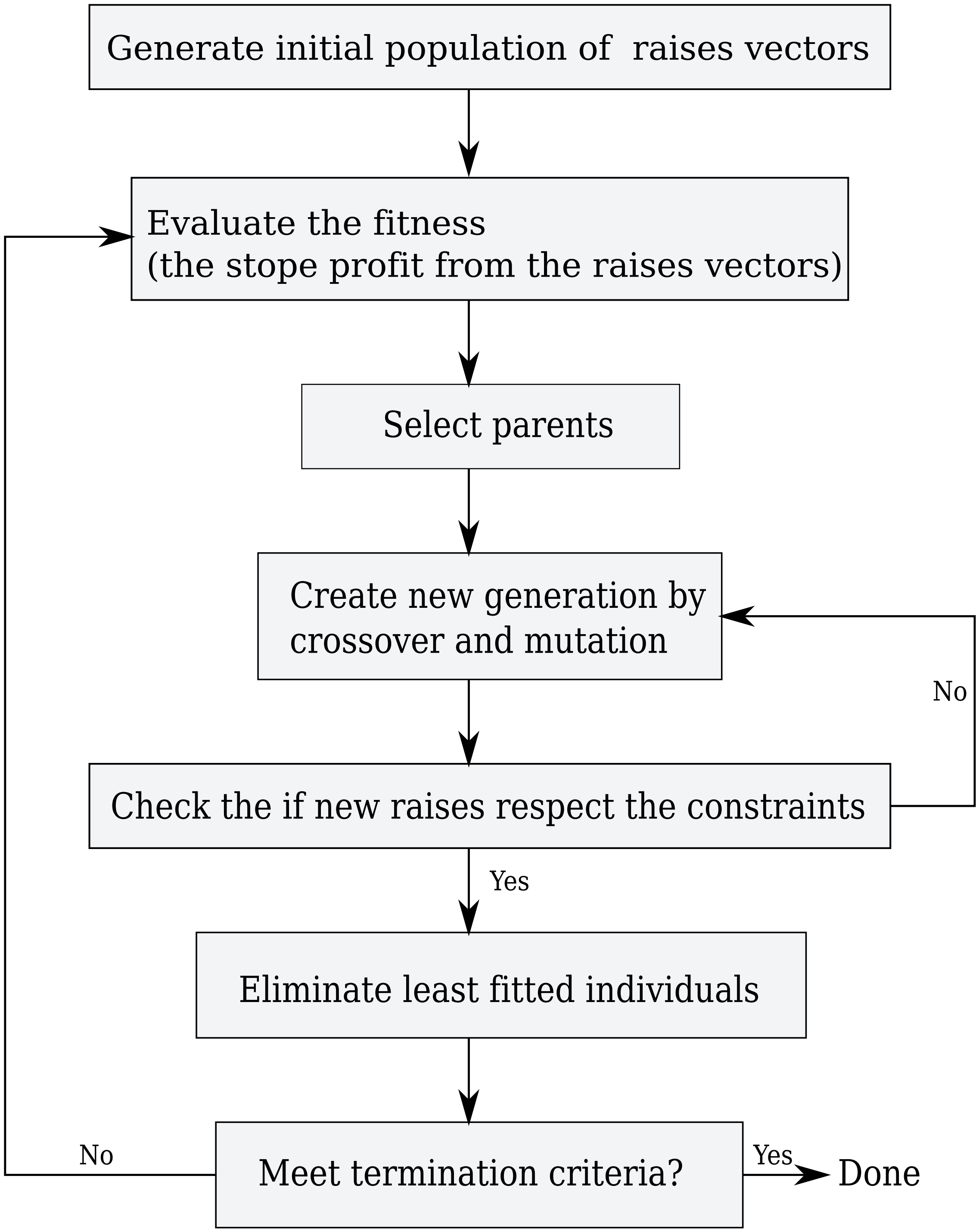

Figure 4.5 Genetic algorithm diagram to search for the best raises' parameters.

depart substantially from those of its parents. This is done using (Haupt and Haupt, 2004) :

$$\mathbf{X}'_{\mathbf{new}} = \mathbf{X}_{\mathbf{new}} + \sigma\alpha'\mathbf{Z} \tag{4.6}$$

where the α' is a 0-1 row vector indicating the genes to be mutated ; the σ controls the extent of the mutations ; and $\mathbf{Z}$ is a column vector of random numbers drawn from the standard normal distribution. A posterior check is applied to the mutated genes so as to ensure they remain in feasible ranges for the raises parameters.

4) Termination of iterations The loop stops either when a series of successive iterations do not improve the best individual fitness or the average fitness of the population, or when a maximum number of iterations has been reached.

4.4 Results

4.4.1 Parameters in the algorithm

To test and evaluate the proposed methods, three ore models are used : two synthetic deposits and one real deposit (ore block model estimated by kriging). The two synthetic models illustrate typical scenarios where the single raise algorithm partly fails and where multiple raises algorithm is expected to perform better. The initial synthetic block model is expressed on a Cartesian grid of spacing 1 m x 1 m x 1 m. For the real deposit model, the Cartesian grid is defined at every 2 m. The networks have vertically one link upward and two links downward, and 3 links horizontally. Therefore, for the 3 cases, the hanging wall angle is 45 ^{o}, and the footwall angle is 63 ^{o}. The $d\theta$ is computed so as to ensure approximately $y_R = R/3$ (see Table 4.1).

For the genetic algorithm, the initial population size is $40 \times n$, where n is the number of raises. Two well fitted individuals are included in the initial population. The first one is the solution obtained by optimization at a lower resolution. The second one is obtained by spreading the raises uniformly within the deposit. Three parents are used to create a new individual. The mutation rate is selected to be 0.1 for each gene, so that many offspring will include mutations of their parent genes. In each iteration, $20 \times n$ new individuals are created and the same number of least fitted individuals are eliminated to keep stable the size of the population. The optimization stops when the number of iterations reaches 100, or when the best fitted individual among the population does not improve in 10 successive iterations.

The design parameter y_R controls the minimum width the sub-stope must have for a block located at distance R from the raise. This value is likely to vary according to the rock mechanics condition of the deposit in the area where the stope is created. Here, to diminish the number of factors to study, we arbitrarily choose $y_{Ri} = R_i/3$ for all cases.

4.4.2 Test results : multiple raises vs. single raise

The first synthetic case represents two distinct mineralized lenses (Fig. 4.6). The optimal single raise solution (c and d) locates the raise in the waste approximately at mid-distance of the lens centroids. On the contrary, the multiple raises solution locates, as expected, the two raises close to the centroid of each lens (Fig. 4.6, e) and f)). Moreover, the radius of influence of each raise R_i is correctly identified as larger for the larger lens. This solution provides 13.5% more profit than the single raise solution and the dilution of ore is reduced significantly from 21.6% to only 2.3%.

The second case is an ore vein with changing direction in horizontal section (Fig. 4.7 a) and b)). The vein is approximately 60 m long by 10 m wide by 20 m high. The stope is designed with three raises, see Fig. 4.7 c),d),e) and f). The value of the multiple raises stope is 10.7% higher than with the single raise (976k$ vs 882k$). It includes less waste (-4.4k$ vs -57.7k$) and misses less ore (4.1k$ vs 45.0k$). The dilution rate of the multiple raises solution is one third the dilution of the single raise (3.1% vs 10.4%).

The kriging block model of a metal deposit in Canada is used as the third case study (name and location of deposit are undisclosed for confidentiality reasons). A portion of the deposit of size 100 m $\times$ 60 m $\times$ 150 m (Fig. 4.8 a), b) and c)), is selected for stope design for the sublevel stoping method. Three raises are used to optimize the stope as shown in Fig. 4.8 g), h) and i). The optimized raise parameters are given in Table 4.1. This time, the profit of the multiple raise stope is only 1% higher than the profit of the single raise stope. The dilution rate for the multiple raises solution is only 1.3% less than for the single raise solution. The lesser differences observed between the multiple raises and single raise solutions are probably due to the spatially smooth variation of grades in the ore block model due to the use of kriging.

Tableau 4.1 Geometric and design parameters, discretization, and optimized raise parameters.

Parameters	Case 1		Case 2		Case 3	
Raise type	M [1]	S [2]	M	S	M	S
Economic parameters						
Mining and processing costs ($/t ore)			⟵	50	⟶	
Metal price ($/kg)			⟵	10	⟶	
Recovery rate			⟵	0.9	⟶	
Rock density			⟵	3	⟶	
Mean ore grade (%)	2	2	0.25	0.25	0.54	0.54
Geometric parameters for stope						
Minimum hanging wall angle (deg)			⟵	45	⟶	
Minimum footwall angle (deg)			⟵	63	⟶	
Maximum height (m)	50	50	40	40	150	150
Minimum height (m)	10	10	15	15	50	50
Discretization						
$dz(m)$	0.5	0.5	0.5	0.5	1	1
$dr(m)$	0.5	0.5	0.5	0.5	1	1
Optimized raises parameters						
Raise 1						
Location X (m)	34.4	18.9	30.9	33.1	3124.7	3097.5
Location Y (m)	-20.9	20.9	21.4	18.7	-75.3	-72.5
Bottom level (m)	-127.3	-132.5	-124.5	-124.2	-497.3	-497.3
Top level (m)	112.3	-107.0	-105.0	-106.1	-352.4	-352.8
Maximum raidius R (m)	16.2	27.3	12.7	33.7	43.3	78.7
Raise 2						
Location X (m)	10.2		48.0		3117.2	
Location Y (m)	20.2		18.9		-73.5	
Bottom level (m)	-132.9		-125.2		-458.8	
Top level (m)	-107.1		-105.2		-358.1	
Maximum raidius R (m)	16.1		14.8		34.2	
Raise 3						
Location X (m)			12.0		3087.6	
Location Y (m)			17.5		-71.3	
Bottom level (m)			-125.2		-479.6	
Top level (m)			-105.7		-356.1	
Maximum raidius R (m)			14.7		45.9	

[1]M : Optimization with multiple raises
[2]S : Optimization with single raise

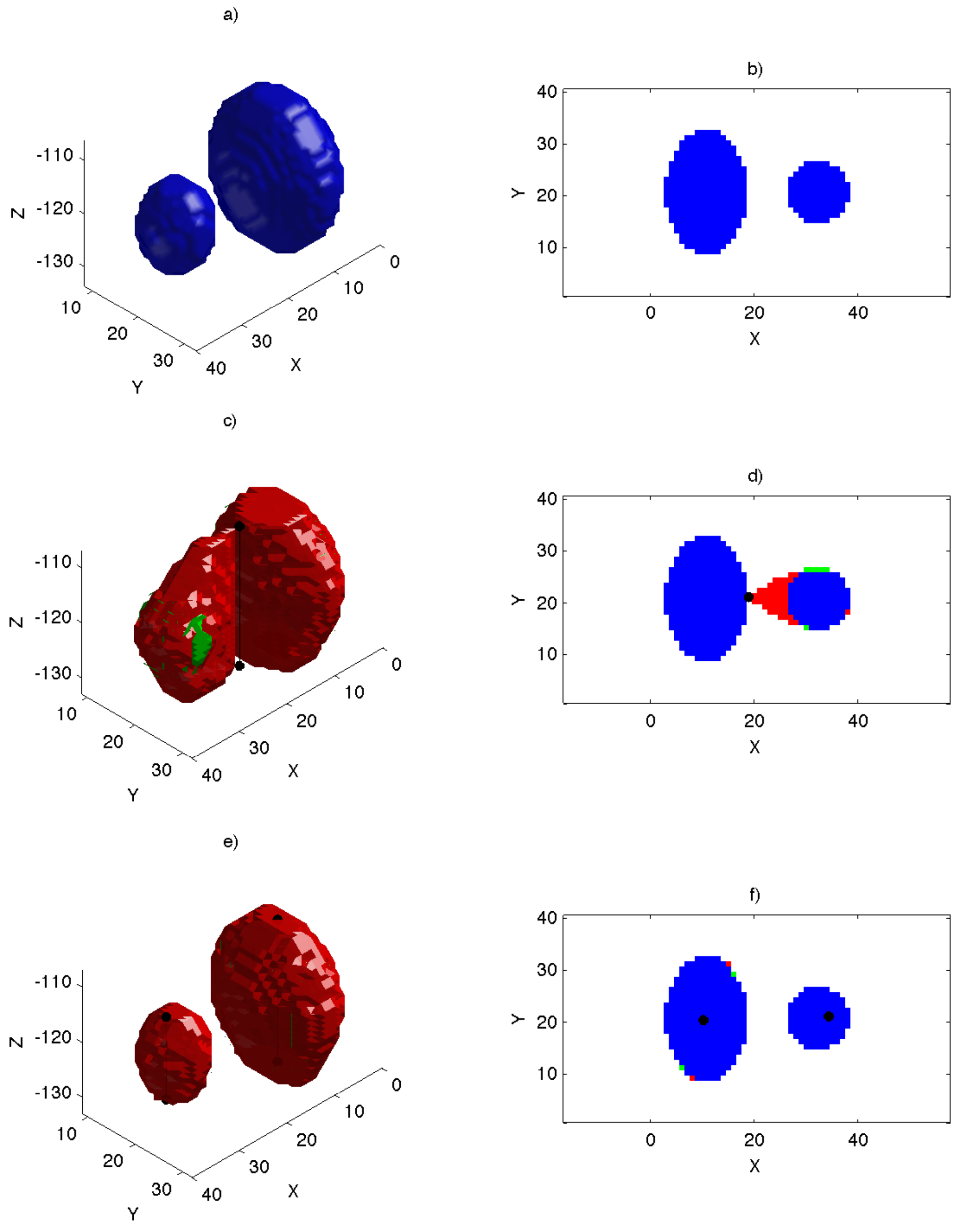

Figure 4.6 Case 1, simulated ore model and created stopes : a) 3D-view of the orebody, b) x-y horizontal section of the orebody at z=-120, c) 3D view of the optimized stope with a single raise, d) x-y horizontal section of the single raise stope at z=-120, showing ore in stope (blue), waste in stope (red), and ore out of stope (green). e) 3D view of the optimized stope by multiple raises, f) x-y horizontal section of the multiple raises' stope at z=-120. Raises in black. Design parameters as in Table 4.1.

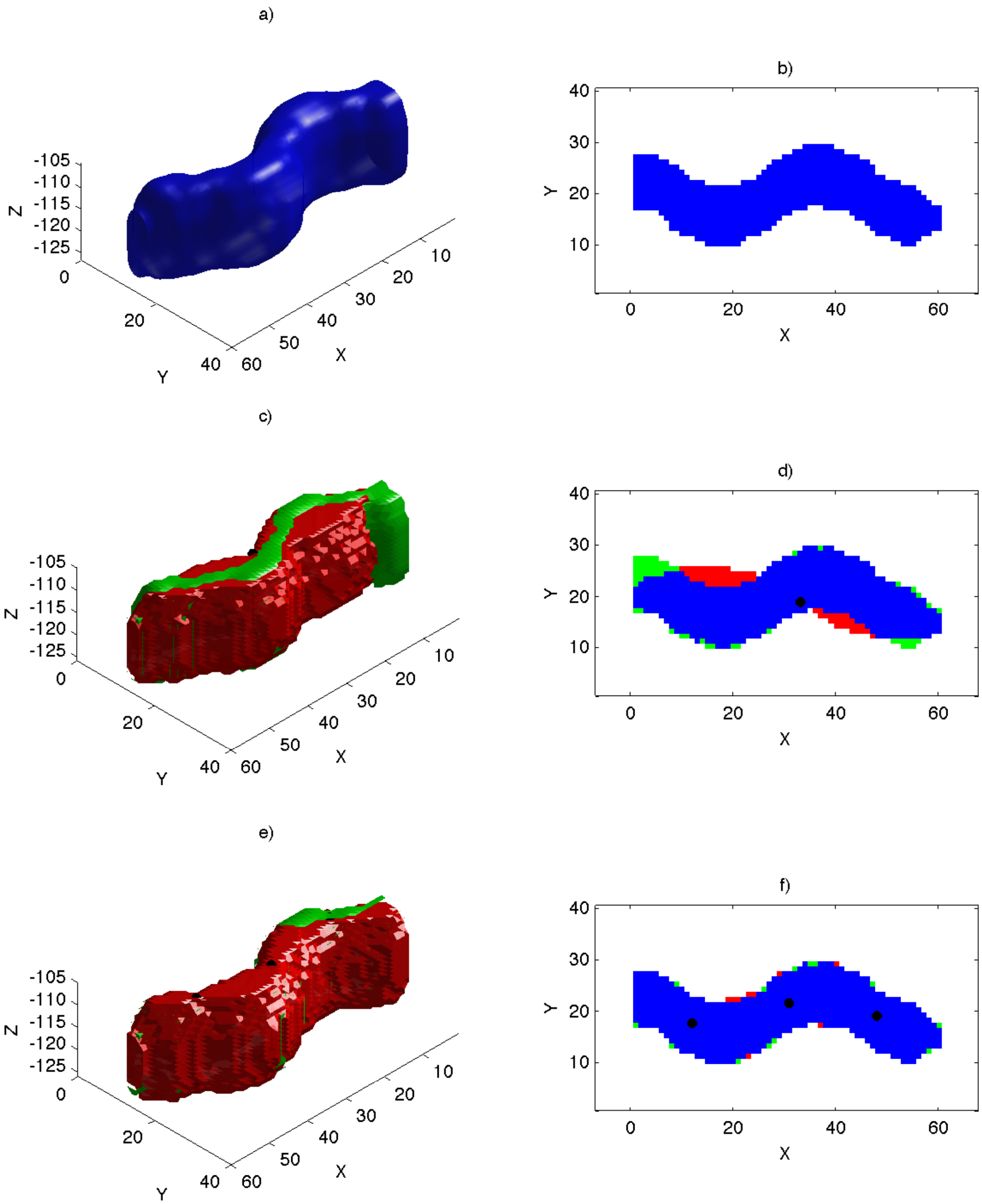

Figure 4.7 Case 2, simulated ore model and created stopes : a) 3D-view of the orebody, b) x-y horizontal section of the orebody at z=-120, c) 3D view of the optimized stope with a single raise, d) x-y horizontal section of the single raise stope at z=-120, showing ore in stope (blue), waste in stope (red), and ore out of stope (green). e) 3D view of the optimized stope by multiple raises, f) x-y horizontal section of the multiple raises' stope at z=-120. Raises in black. Design parameters as in Table 4.1.

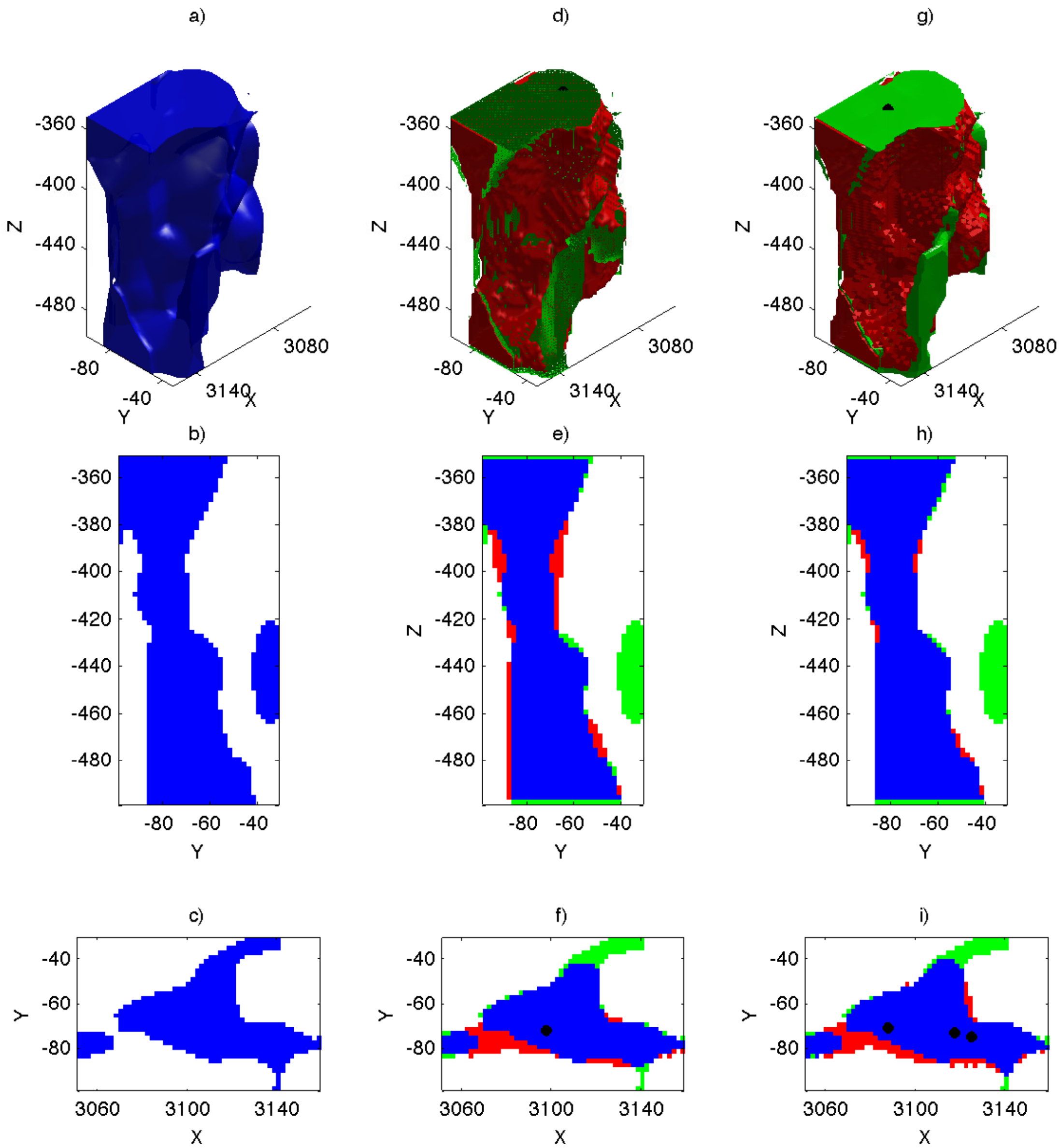

Figure 4.8 Case 3, test with a real ore deposit : a) 3D-view of the orebody, b) y-z vertical section of the orebody at x=3130, c) x-y horizontal section at z=-424 ; d) 3D view of the optimized stope with a single raise, e) y-z vertical section at x=3130, f) x-y horizontal section at z=-424 ; g) optimized stope with multiple raises, h) y-z vertical section at x=3130, i) x-y horizontal section at z=-424 ; in d) and g), stopes are in red, ore out of stope is in green. e),f), h) and i), ore in blue, waste in red, and ore out of stope in green, raises in black. Design parameters as in Table 4.1.

Tableau 4.2 Economical evaluation of the case studies.

Cases	Raise type	Stope profit (k \$)	Missed ore value (k \$)	Waste value in stope (k \$)	Dilution volume rate [1]	Profit improved
Case 1	Multiple	2622	24.2	-24.0	2.3 %	13.41 %
	Single	2312	84.6	-273.6	21.6 %	
Case 2	Multiple	976	4.1	-4.4	3.1 %	10.68 %
	Single	882	45.0	-57.7	10.4 %	
Case 3	Multiple	8411	524.0	-133.2	6.8 %	1.00 %
	Single	8327	543.8	-197.0	8.1 %	

[1]Dilution volume rate = Volume of waste in stope / Volume of stope

4.5 Discussion

We developed an improved stope optimizer for sublevel stoping method. The new multiple raises stope optimizer is an extension of the single raise optimizer presented in Bai *et al.* (2013). In all test cases, the multiple raise algorithm provides stopes with higher profit and less dilution compared to the single raise optimizer.

In the three test cases presented, the stopes from multiple raises generated between 1% to 13.5% more profit than the best stope obtained with a single raise. These improvements do not include the additional cost of drilling the extra raises. Moreover, although in the single raise case, the stope obtained is optimal with respect to the geotechnical constraints, the adopted discretization and the design parameters R and y_R, this is no longer the case with the multiple raises extension. The loss of optimality comes from the fact that each sub-stope is obtained by optimization of a distinct network defined on a local cylindrical coordinate system. The global stope is then formed by the union of the sub-stopes. The approach however provides a good heuristic solution due to its increased flexibility compared to the single raise approach.

The distance of influence of a raise R_i is the parameter controlling the size of a sub-stope. It is ensured that the geotechnical constraints are respected within each sub-stope, therefore the geotechnical constraints are at least approximately respected in the global stope. We did not study explicitly the effect of the y_{Ri} parameter as this parameter is case specific. For a single raise and a given R, diminishing y_R necessarily increases the profit. However, y_R should not be taken too small otherwise the ore risks to jam within the stope. Rock mechanical conditions and experience with the mining in a particular geological environment should guide the choice of this parameter. A value around $y_R = R/3$ seems to provide visually sensible shapes in the tests we conducted.

One limitation of the proposed approach is the restriction to vertical raises. It would cause higher dilution rate for the scenarios of inclined deposits, which usually adopt inclined raises in reality. The generalization of the method to an inclined raise is far from evident due to the loss of symmetry with respect to the gravity force vector. The solution to the problem needs further investigation.

In proposed approach, the cost of development of access to the top and bottom levels of the raises was neglected. When the multiple raises are located at different levels, the relative additional costs would reduce the benefit of this approach compared to the single raise. Moreover, it was supposed that the raises could be located rather freely within the deposit without imposing constraints on the elevations of beginning and end of the various raises. An alternative strategy, closer to the practice for larger deposits, would be to optimize the common height between levels (within specified bounds) and impose each raise to span the entire height. The optimization would then simplify to find the best elevation for the first level and find the best number and locations of raises within each level. This modification is currently investigated.

4.6 Conclusions

The proposed method was shown to provide good heuristic stopes solutions for typical geometries of curved or inclined deposits. The solutions respect the geotechnical constraints of the sublevel stoping method including footwall and hangingwall slopes, and stope minimum and maximum heights. The best stopes obtained with multiple raises for the 3 synthetic cases exhibits significantly larger profits and less dilution than the best stope obtained with the single raise stope optimizer. The gain of the multiple raises approach is due to the increased flexibility, compared to the single raise case, provided by the use of a variable distance of influence parameter R_i.

4.7 Acknowledgement

This reasearch was made possible by research grants provided by Chinese Scholarship Council and NSERC.

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CHAPTER 5

ARTICLE 3 : INCORPORATING DRIFT IN LONG-HOLE STOPE OPTIMIZATION USING NETWORK FLOW ALGORITHM

Article history : Submitted to APCOM 2013 on 30 April 2013 ; accepted on 3 July, 2013.

Authors : Xiaoyu Bai, Denis Marcotte and Richard Simon

5.1 Abstract

We present an algorithm aimed at optimizing stope design accounting for drift development. The approach is typically suitable for the sublevel stope mining method with a vertical parallel drilling pattern. The algorithm consists of two parts. In the first part, a graph theory based stope optimizer is adapted to integrate the drift. In the stope optimizer, the raise initiates a stope and provides the parameters of drift level and draw point level. An ore model of cylindrical coordinates is defined around the raise. The geometric constraints on hanging wall and footwall slopes are translated as vertical precedence relations between blocks. The stope width is controlled by horizontal precedence relations. Similarly, the dependency of blocks in stope to blocks in drift is expressed in the graph. By solving the maximum flow problem with efficient push-relabel algorithm, the conditional optimal stope corresponding to the current raise location and height is obtained. The second part finds the best raise location and height. The genetic algorithm is used with the objective function defined as the (conditional) stope profit associated to a given raise location and height, computed in the first part. The performance of the algorithm is evaluated with a synthetic deposit and a real deposit. Comparison is made with the stope optimizer without a drift. The proposed method is shown to provide higher stope profit, with less dilution and less costs for the drift.

5.2 Introduction

Stope design is an important aspect of underground mining design as it influences considerably the economical benefit and the operation safety of a mining project. For longhole stoping method, the design of opening includes the positions and boundaries of stopes, the location of raises, and of the drifts needed to perform drilling. Usually, the stope boundary is defined first to maximize the total profit of contained volumes subjected to global and local geotechnical requirements. The optimized stope determines the drift levels and accessing raises. This workflow has a significant drawback due to neglecting the economical dependency of the stope boundary to the associated drift. For example, a cluster of ore that is profitable in stope may not be economic after counting cost of drift development to drill to the cluster. This can result in the loss of optimality of stope, especially when the cost of drift is relatively large. As an example, with parallel drilling pattern, the drift development can reach as much as 30% of stoping cost(Oraee and Bangian, 2007). This calls for the integration of drift design into the stope boundary optimization.

For the state-of-art stope optimization techniques, Ataee-Pour (2005) and Alford *et al.* (2007) provided reviews on most of existing stope optimizers. The several techniques, including dynamic programming method (Riddle, 1977), and branch and bound technique (Ovanic and Young, 1995), were developed to optimize a stope in 1D or 2D. These methods are mathematically rigorous and can yields optimal stope, however, the simplification of mining and geotechnical requirements could hardly lead to realistic stopes in 3D. A series of 3D approaches were proposed, such as mathematical morphology tools (Serra, 1982; Deraisme *et al.*, 1984), floating stope technique (Alford, 1996), maximum value neighborhood method (Ataee-Pour, 2000), octree division approach (Cheimanoff *et al.*, 1989). These heuristic methods did not incorporate comprehensively the geotechnical constraints. Manchuk and Deutsch (2008) provided an algorithm based on simulated annealing , with the geometric constraints directly integrated. Nevertheless, the computation time and the convergence to a global optimum would be a problem. Above all, none of these methods incorporate directly the drift in the optimization of stope boundary.

More recently, a new stope optimizer was developed by Bai *et al.* (2013b) and an improved approach was presented by Bai *et al.* (2013a). These methods are based on powerful graph theory inspired from successful open pit optimizers. The key of the approaches is to recognize that the vertical raise in underground mining plays a similar role to the ground surface in open pit. This calls for an ore block model in cylindrical coordinates originated from a raise. In the cylindrical ore model, the blocks are linked toward the raise constituting a network graph, in such way that the geotechnical constrains are implemented. Solving this graph, with efficient maximum flow algorithm, an optimal stope can be obtained. This is the core of the approach : a stope generator that creates an optimal stope under given single raise location and elevation. These raise parameters are externally optimized based on the stope generator to ensure the global optimality of stope. An improved algorithm introduced by Bai *et al.* (2013a) allows to employ multiple raises in the optimization, with each raise creating a sub-stope using the stope generator with single raise. The union of the sub-stopes comprises a practical heuristic stope with more adaptability to orebody shapes. The appealing points of the methods are : 1) the mining constrains are comprehensively implemented in 3D, and 2) the optimization parameters have a clear engineering meaning.

In this paper, the authors attempt to develop further the approach by including the drift directly in the stope optimization. It is assumed that longhole stoping method with parallel drilling pattern is adopted.

5.3 Methodology

5.3.1 Economical function of longhole stoping

The stope design calls for a geological model quantifying the ore grade in discrete blocks. The estimation and simulation techniques to obtain such models are well documented in geostatistics literature (David, 1988; Journel and Huijbregts, 1978). The ore grade are assumed to be known in this study, therefore, the effect of grade uncertainty on stope design is neglected. Converting a geological model to an economical model requires the knowledge of economic outcome of relevant mining components. Lane (1988) proposed an economic function of a ore grade model :

$$p_i = d_i v_i [g_i r f - c] \tag{5.1}$$

Where p_i denotes the profit of mining block i ; g_i is the average ore grade of block i ; f is the unit metal price ; r is the recovery rate ; c is the unit cost of processing and mining ; v_i and d_i are the block volume and density.

In the sublevel stoping method, the drift is developed using different drilling and blasting techniques from those used for stoping. Therefore, the unit cost to mine a block in drift or stope are different sometimes by a large margin. We should write :

$$p_i = d_i v_i [g_i r f - c_i] \tag{5.2}$$

where $c_i = C_{drift}$ when block i is removed as drift ; and $c_i = C_{stope}$ when the block locate in stope. The C_{drift} and C_{stope} consist of not only the cost of drilling and blasting, but also the cost of transportation and treatment. Cost of stoping C_{stope} is assumed to be constant, supposing that the longhole drilling penetrate from top level to bottom level. The deviation from incomplete drilling and blasting is neglected. Moreover, the development costs for accesses and main haulage levels, are considered similar for each possible stope in the zone.

5.3.2 Stope optimization algorithm

Graph theory in stope optimization

In graph theory techniques, the mining optimization problem is commonly modeled by a weighted graph $G = (V, A)$. The ore blocks are delineated as vertices V, and the mining constrains are represented by arcs A, the connection between vertices. The profit p_i from mining a block i is denoted as the weight of the vertex. The stope or pit optimization is to seek a closed set of vertices $V' \subseteq V$ to maximize $\sum_{i \in V'} p_i$. This maximum closure problem can be formulated as :

$$\begin{aligned} &Maximize \sum_{i=1}^{N} p_i x_i \\ &Subject\ to\ x_i - x_j \leq 0, \quad \forall\ i \in V, j \in \Gamma_i \\ &\qquad\qquad x_i = 0\ or\ 1, \quad \forall\ i \in V \end{aligned} \tag{5.3}$$

Where Γ_i is the subset of immediate successor nodes to node i, representing the set of blocks to be mined to get access to block i. N denotes the number of blocks. As N is usually large, solving this integer program can be time consuming. Though Lerchs-Grossman algorithm(Lerchs and Grossmann, 1965) is an effective and widely applied approach, the maximum flow techniques emerged later were shown to be more efficient(Picard, 1976; Goldberg and Tarjan, 1988; King *et al.*, 1992).

Implementation of stope geometric constraints in network

Bai *et al.* (2013b) showed the similarities of the underground optimization with the open pit method. They used a cylindrical coordinated system around each raise. The hanging wall and footwall slope limits are defined by the precedence links in vertical direction, as is shown in Fig. 5.1 b). They also defined two design parameters, the radius of influence of the raise (R) and the minimum width (y_R) needed to remove the farthest block from the raise. The two parameters R and y_R control the links in the horizontal plane (see Fig. 5.2 a).

The stope height is simply confined by the length of the raise. The blocks above the top of the raise or under its bottom were not part of the network, hence were not included in the stope.

Adding the drift in the network

It is assumed that vertical parallel down-drilling is applied from the upper level to the bottom of the stope. The drift is made of the upper level blocks above the stope that need be removed to allow drilling of the blocks in the stope. This is coded in the network by linking each block in the stope to the corresponding block in the upper level.

Algorithm

The optimization algorithm consists of two main parts. The first part, the stope optimizer generates an optimal stope and associated drift for specified raise location and height, with chosen design parameters. It includes the following steps :

1. With given raise parameters, recognize the drift blocks and stope blocks, and establish economic block model in cylindrical coordinates following Equation 5.2 ;
2. Construct the graph with vertical arcs for wall slope constraints, horizontal arcs for width constraints, and with arcs of drift dependency ;
3. Build flow network by adding the source and sink nodes to the graph ;
4. Solve the maximum flow problem. The generated stope is conditionally optimal to the raise location and height.

The second part seeks the best raise location and height. It is done by global optimization on the parameters, using as objective function the stope value found with the stope optimizer. Genetic algorithm(GA) is employed for this purpose. The raise is parameterized as (x, y, z^t, z^b), where x and y indicate the location of raise, and z^t and z^b denote respectively the top level and bottom level of raise. Under GA framework, a vector of raise parameters is represented by an individual or a chromosome with the genes denoting the parameters. The

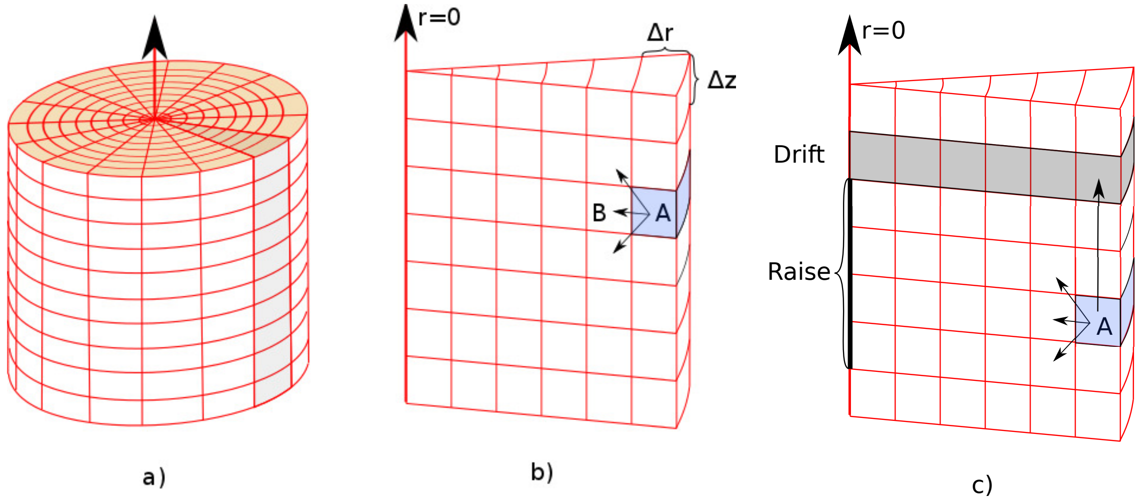

Figure 5.1 Block model under cylindrical coordinates a), and typical arcs in vertical section in proposed method b). Arcs to integrate drift in stope optimization c)

profit of a stope signifies the fitness of an individual to the environment. The process of searching the best parameters imitates the natural evolution of the individuals : new individuals are reproduced by mating existing individuals, and the least fitted individuals are eliminated. The mutated reproduction is allowed to generate a child dissimilar with its parents. The iteration of evolution terminates when certain stoping criteria meet. The choice of parameters of GA is discussed in Bai *et al.* (2013a).

5.3.3 Comparative method

Two methods are compared. The first one optimizes the stope not accounting for the drift. The cost of drift is added afterwards. The second method optimizes simultaneously the stope and drift. For both methods, we use the single raise stope optimizer (Bai *et al.*, 2013b), and the global optimization of raise parameters is achieved with GA as is discussed by Bai *et al.* (2013a). The drift is obtained by taking the blocks in the drift sublevel within the polygon enclosing the $x - y$ coordinates of all stope blocks.

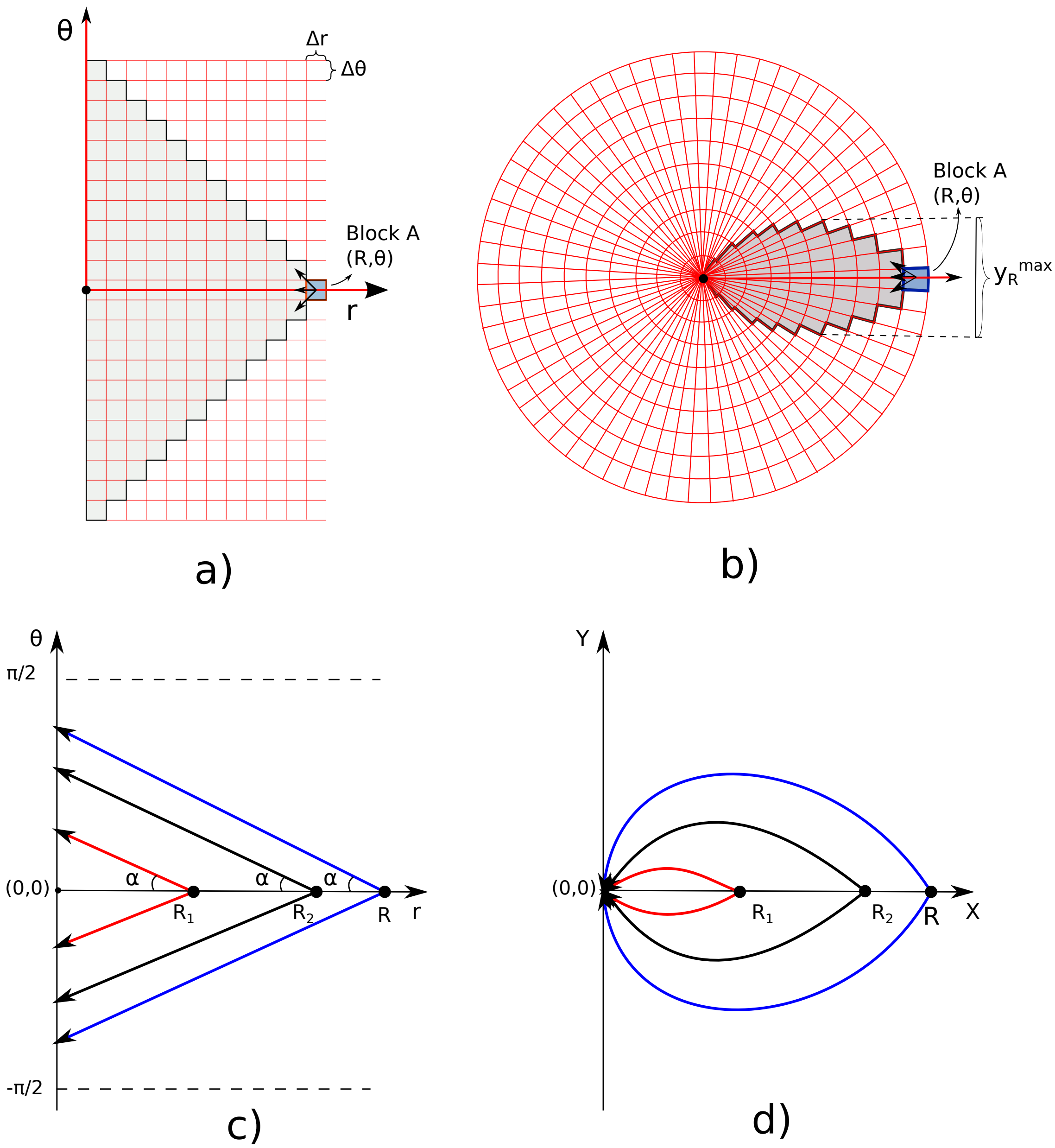

Figure 5.2 Horizontal plane showing blocks and links defined in the cylindrical system a) and corresponding blocks and links in the Cartesian system b). Shaded blocks represent blocks to be removed to get access to block A. Trace of the envelopes defined by the lateral links (function of K and R) in the cylindrical system c) as they appear in the Cartesian system d).

5.4 Results

5.4.1 Data and parameters in the algorithm

To test the proposed approach, two deposit models are used, a synthetic one and a real one. The ore grade statistics and economic parameters are listed in Table 5.1. In these cases, single raise are used to create a stope. The design parameters R is arbitrarily selected, respecting that all of the blocks in the study zone can to be mined from the raise. Another design parameter y_R is chosen as $R/3$. The geotechnical requirements of stope design is illustrated in Table 5.1.

For the GA method optimizing the raise parameters, the initial population size is 40. Three parents are used to create a new individual. For each iteration, 20 new individuals are created and the same number of least fitted individuals are eliminated to keep stable the size of the population. During reproduction, 10% genes are mutated. The optimization stops when the number of iterations reach 100, or when the best fitted individual among the population does not improve in 10 successive iterations. The optimized raise parameters are listed in Table 5.2.

5.4.2 Test results

The first case is a vertical deposit(Fig. 5.3 a) to c)). The orebody is approximately 30 m long by 30 m wide by 50 m high. The resulting stope and drift are shown in Fig. 5.3 d) to i). With the proposed method, the profit of stope is $627k$\$, which is 15.5% higher than the profit from the traditional method, $543k$\$. The stope contains less wastes(-42.5k\$ vs -163.9k\$), with lower dilution rate(7.4% vs 14.7%). More ore is excluded from the stope, however, from Fig. 5.3 h), the missed ore is mostly located on the edge over a small thickness. Hence, it is not profitable to extend the drift to access them. With this approach, because the drift is smaller, the cost is much lower(-30.5k\$ vs -189.2k\$).

In the second scenario, an ore body of a metal deposit in Canada is used (name and location of deposit undisclosed for confidentiality reasons). A part of the deposit of size 50 m $\times$ 30 m $\times$ 80 m (Fig. 5.4 a), b) and c)) is selected for longhole stope design. The design parameters are shown in Table 5.1, and optimized raise parameters are shown in Table 5.2. The produced stope receives 11.9% more profit than traditional method (195k\$ vs 174k\$). The stope includes less waste (-38.6k\$ vs -61.2k\$), obtains lower dilution rates (10.8% vs 12.1%), and cost less for drift development (-77.3k\$ vs -109.3k\$).

Tableau 5.1 Ore grade model, discretization, and economic, geometric and design parameters

Parameters	Case 1	Case 2
Ore block model parameters		
Mean grade (%)	0.9	0.5
Cost of stoping, transportation and treatment ($/t)	30	56
Cost of developing drift (including the cost of transportation and treatment) ($/t)	34.2	60.2
Metal price ($/t)	8.4	12
Metal recovery rate	72%	80%
Rock density (t/m^3)	3	3
Geometric parameters		
Minimum hanging wall angle (deg)	45	45
Minimum footwall angle (deg)	63	63
Maximum height (m)	70	130
Minimum height (m)	40	50
Height of drift (m)	4	4
Design parameters		
Stope width parameter y_R (m)	23	35
Maximum reference distance to raise R (m)	8	12
Discretization of ore block model in cylindrical coordinate		
dz (m)	0.5	1
dr (m)	0.5	1
$d\theta$ (deg)	0.89	0.57

Tableau 5.2 Optimized raise parameters

Optimized raise parameters	Case 1		Case 2	
	with drift	without drift	with drift	without drift
Location X (m)	18.6	15.3	3176.6	3178.0
Location Y (m)	19.3	20.4	27.1	28.8
Bottom level (m)	-152.8	-153.1	-165.2	-164.6
Top level (m)	-111.7	-106.2	-98.5	-92.6

[1]Dilution volume rate = Volume of waste in stope / Volume of stope

Tableau 5.3 Economical evaluation of the case studies.

Economic indicators	Case 1		Case 2	
	with drift	without drift	with drift	without drift
Profit of stope (k $)	627	543	195	174
Profit of missed ore (k $)	56.1	8.9	13.0	5.2
Values of waste in stope (k $)	-42.5	-163.9	-38.6	-61.2
Dilution Volume rate[1]	7.4%	14.7%	10.8%	12.1%
Net benefit from drift(k $)	-30.5	-189.2	-77.3	-109.3

[1]Dilution volume rate = Volume of waste in stope / Volume of stope

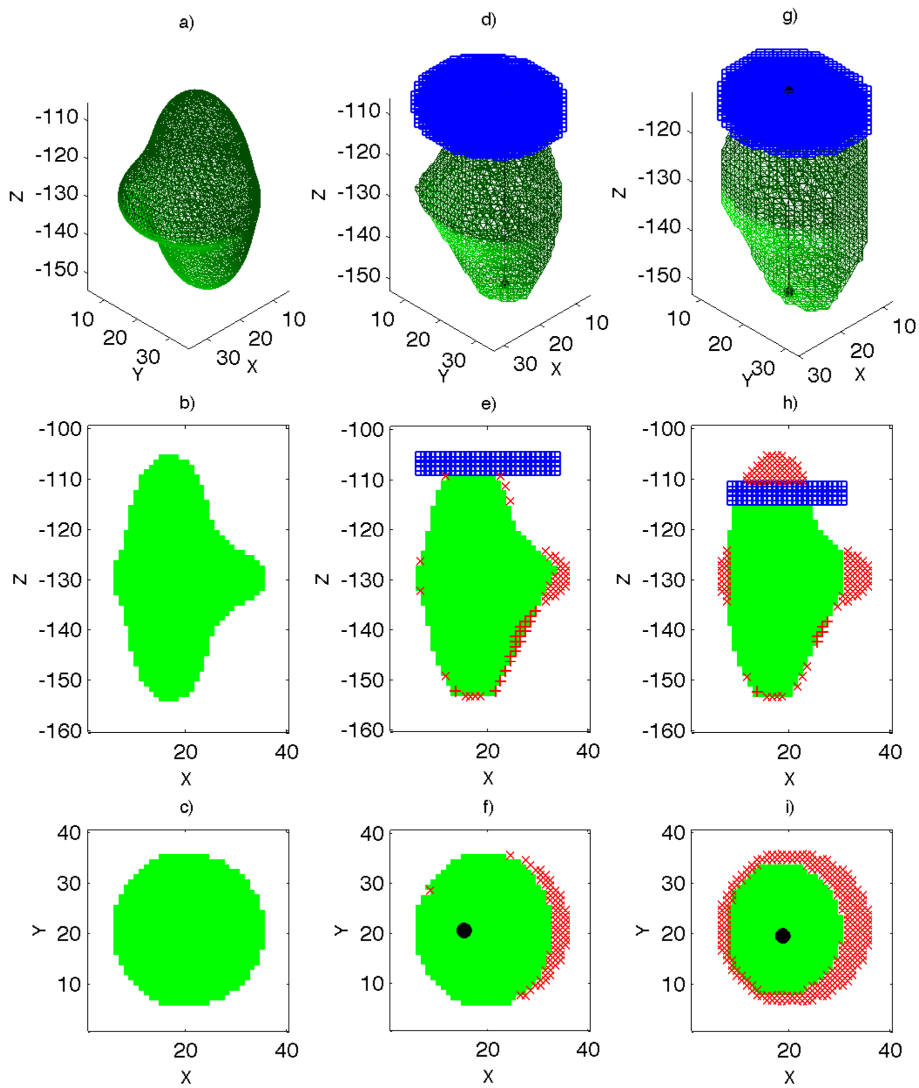

Figure 5.3 Case 1, simulated ore model and created stopes : a) 3D-view of the orebody, b) yz vertical section of the orebody at x=20, c) xy horizontal section at z=-130 ; d) optimized stope and drift with traditional method view in 3D, e) yz vertical section at x=20, f) xy horizontal section at z=-130 ; g) optimized stope and drift with proposed method view in 3D, h) yz vertical section at x=20, i) xy horizontal section at z=-130 ; For the 3D view of stopes in d) and g), stopes are marked light meshes, and drifts are marked in dark squares. For stope slices in e),f),h) and i), stope(shaded area), drifts(square) ; waste in stope(+), and ore out of stope (x). Raises are marked in black lines with dots. Design parameters are given in Table 5.1.

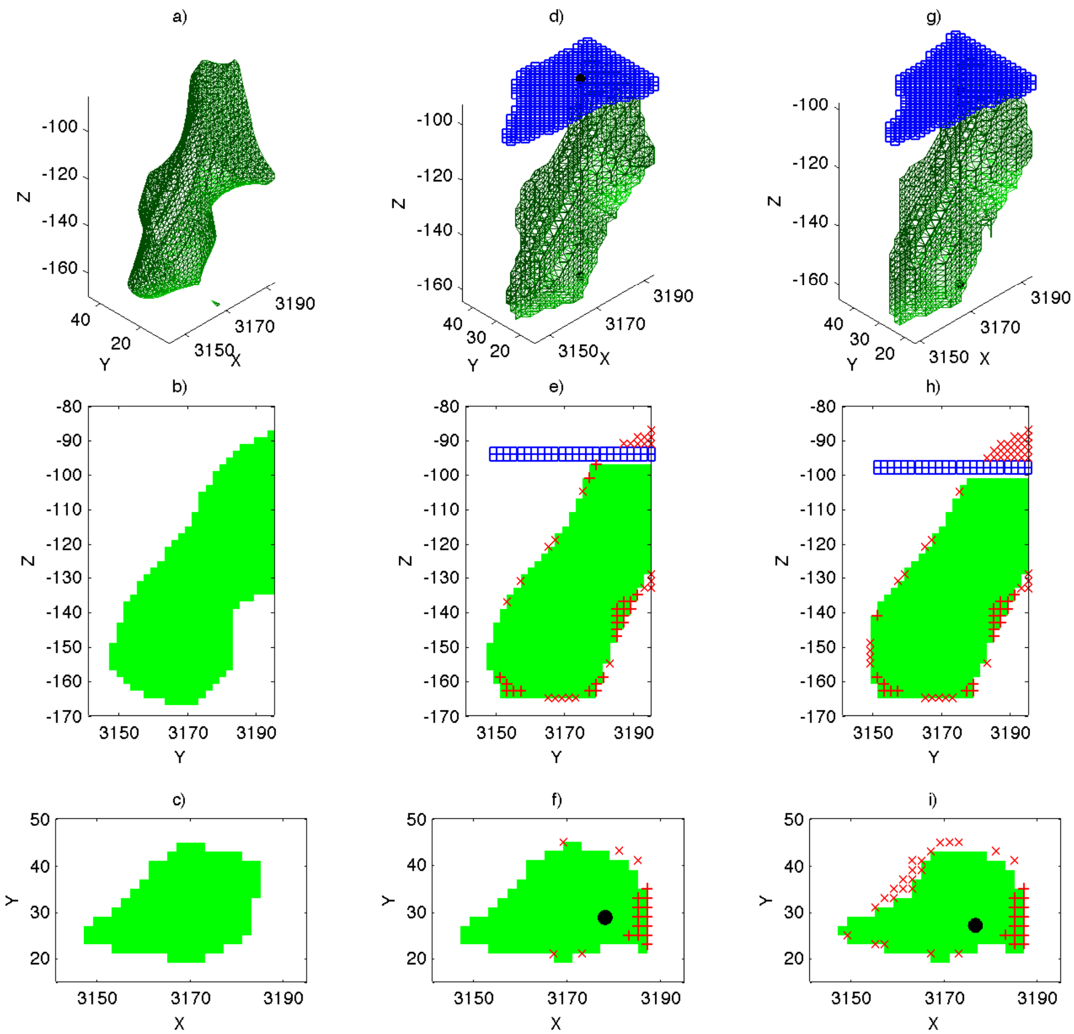

Figure 5.4 Case 2, test with real ore deposit : a) 3D-view of the orebody, b) xz vertical section of the orebody at x=3168, c) xy horizontal section at z=-144 ; d) optimized stope and drift with traditional method view in 3D, e) xz vertical section at x=3168, f) xy horizontal section at z=-130 ; g) optimized stope and drift with proposed method view in 3D, h) xz vertical section at x=20, i) xy horizontal section at z=-144 ; For the 3D view of stopes in d) and g), stopes are marked light meshes, and drifts are marked in dark squares. For stope slices in e),f),h) and i), stope(shaded area), drifts(square) ; waste in stope(+), and ore out of stope (x). Raises are marked in black lines with dots. Design parameters are given in Table 5.1.

5.5 Discussion

We developed an algorithm that can jointly optimize stope and drift, specifically for longhole stoping method with parallel drilling pattern. The proposed algorithm is based on the stope optimizer using network flow method proposed by Bai *et al.* (2013b,a). The algorithm is shown to produce stopes more economical than with the classical approach where drift costs are added only after the stope is determined. The improved benefit comes from the smaller drift dimension, and the abandon of the ores at boundary, the values of which are not sufficient to pay for the additional drift to access them. The search of raise parameters based on GA is heuristic, but it is proved stable as it converged quickly to similar values. The computation times of the two approaches are similar.

Because a single raise is used in the stope design, the distance of influence R is taken large enough to allow the raise to reach all the ore blocks in the study zone. The effect of y_R is not studied here. The parameter y_R should be large enough to avoid generating too narrow stope, which may hinder the flow of ore. A value around $y_R = R/3$ produced visually sensible shapes in our tests.

Although tested here with a single raise, the approach can be extended easily to multiple raises (Bai *et al.*, 2013a). To incorporate the drift in this algorithm, the graph under each raise need to be modified. Also, the top level and bottom levels of the raises should be made equal, so that the drifts from different sub-stopes locate at same elevation. The improvement is expected to bring more flexibility to various ore body shapes.

In the proposed method, parallel drilling of long hole stoping is assumed to be vertical. The method may not provide good results when the orebody is inclined, which requires inclined parallel drilling. Also, for other drilling patterns like ring drilling, further developments are required.

In the proposed model, the cost of raise to access stope is neglected. The influence of cost of raise to the decision of the stope to be mined can be also integrated in the model, by recognizing the blocks of raise, and assigning specific cost as is done for integrating drift.

5.6 Conclusions

The proposed approach, integrating the drift in the optimization of longhole stope, is shown to provide stope more profitable compared to the method not involving drift in optimization. The algorithm inherits the merit of previously developed stope optimizer based on network flow method, providing optimal 3D stope with geotechnical constraints incorporated.

5.7 Acknowledgement

Research grants are provided by Chinese Scholarship Council and NSERC.

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CHAPTER 6

PARAMETER SELECTION AND SENSITIVITY ANALYSIS

Genetic algorithm is an excellent tool to search for the global optimal of the variable concerned. In the proposed stope optimizer, it is adopted to optimize the parameters of raises. The work flow and the parameters of GA are introduced in Chapter 4, the second journal article submitted. Here in this chapter, more resent research results are presented, including the adjustment of GA and improved results of the case studies. Also, the sensitivity of the results to the parameters are discussed.

6.1 Recent adjustment of GA

Genetic algorithm mimics the natural evolution of creatures. A vector of variables, the raises parameters in our case, can be seen as a set of genes to form an individual of any creature, a horse for example. The profit of stope led by a set of raises parameter signifies the abilities of the horse to adapt itself to the surroundings. In a society of various horses, strong and fast (well fitted) horses have larger mating chance, breeding offspring inheriting their excellent genes ; the less fitted ones have minor mating chance and tend to be eliminated. Generation after generation, the overall quality of the horses improves as excellent horses survive and become more prevalent, and less fitted ones tend to vanish. Then after sufficient evolution process, the best horses would be the optimal raise parameters leading to maximal stope profit. The diagram of GA is shown in Figure 4.5. The parameters of GA include : the initial population, the number of offspring in new generation, the mutation rate, and the number of parents to form an offspring. These parameters are described in Chapter 4.

Our recent studies modify the GA in three aspects : parent selection, genetic operation, and control of parameter R, the maximum radius from raise.

6.1.1 Parents selection

The basic principle of parent selection is to give well fitted individuals a higher priority to be parents. Less fitted individuals have less probability to be selected for mating. Though the chance is small, it is important to allow the less fitted individuals to pass down their genes, for the purpose of diversifying the population. The **fitness proportionate selection** (or roulette wheel selection) method(Back, 1996) is used for parents selection. The individual i with fitness value (profit) f_i has the probability of being selected $prob_i = f_i/\Sigma_{j=1}^{M} f_j$, where the M represents the number of individuals in population. To implement this, all the fitness value are normalized to locate in $[0, 1]$. The normalized value are sorted in ascending order, and then transformed to cumulative normalized fitness value (CNFV). A random number Rnd is drawn from [0,1], and the first one with CNFV greater than Rnd is selected to mating pool. The individuals in the mating pool are randomly paired.

6.1.2 Genetic operator

The crossover in Chapter 4 using a blending method is a process of weighting average to the parents genes. The problem is that it tends to boost the elite group and eliminate average groups too soon, causing the result trapped to local maxima at early phase. Therefore, another crossover strategy is adopted. It is done by picking genes randomly from parents and combining to a new individual, following the formula :

$$\begin{gathered} \mathbf{X_{new}} = \sum_{i=1}^{m} \beta_i \mathbf{X_i} \\ \sum_{i=1}^{m} \beta_i = 1 \ , \beta_i = 0 \ or \ 1 \end{gathered} \tag{6.1}$$

Where $\mathbf{X_{new}}$ denotes the new individual; $\mathbf{X_i}$ represents the parent i and β_i is a 0-1 variable indicating to inherit the gene or not. In this way, a child is basically the recombination of genes of its parents. The continuity of parameter can be maintained by mutation described in Chapter 4.

6.1.3 Effect of different raise parameters to evolution of GA

The raises parameters, including top and bottom level, X-Y location, and R, the radius to bring furtherest block to raise, influence the stope profit in different degrees. Especially, the R have special impact. When the raises locations and elevations are randomly placed at early phase of GA evolution, the R controlling the maximum span of a stope is more likely to be large, so as to encircle more ores to gain higher profit. Because of this, the R can evolve too quickly to large values, losing the possibility to have smaller value at later phase.

To solve this problem, a "trick" is to assign the R in all initial raises to be small value. In this way, the algorithm would search raise locations and elevation at beginning. The development of R is delayed, until a mutation enlarges R.

6.2 Sensitivity test of the optimization parameters

A synthetic deposit with two isolated ore bodies is used to conduct the sensitivity test. The deposit model is 40m by 50m by 40m, with block unit 2m by 2m by 2m (see Figure 6.1). The economic parameters and geometric constraints are the same as the case 1 in Chapter 4 listed in Table 4.1. The deposit have known optimal solutions : with raises placed at $[X_1 = 12.64,\ Y_1 = 17.9,\ Z_1^b = -131.63,\ Z_1^t = -108.06,\ R_1 = 15;\ X_1 = 38.62,\ Y_1 = 23.99,\ Z_2^b = -126.08,\ Z_2^t = -109.64,\ R_2 = 13]$, the optimal stope encloses all the ore blocks with no dilution, receiving profit 2710 k\$.

Two raises are employed in all the following tests. Without specific advocation, the GA parameters are adopted as Table 6.1. Also for the discretization in cylindrical coordinate, $\Delta r = \Delta z = 0.5$, $\Delta\theta$ computed to satisfy $y_R = R/3$.

Tableau 6.1 Parameters in the sensitivity tests of GA

Parameters	Value
Initial population size	80
The number of new individuals in each generation	40
Mutation Rate	0.3
The number of parents to mate	3
Number of unimproved successive iteration to terminate	30
Number of repetitive samples for each test	10

In the following parameter analysis, the discretization effect is studied first. Then the influence of GA parameters is tested, including : the initial population, the number of offspring in new generation, the mutation rate, and the number of parents to form an offspring.

6.2.1 Discretization in the block model of cylindrical system

In the proposed algorithms, the grade value of ore model are converted from Cartesian grids to cylindrical grids ; the stopes optimized on cylindrical model are converted back onto Cartesian grids. Since the geometry and size of the regular blocks in cylindrical coordinates

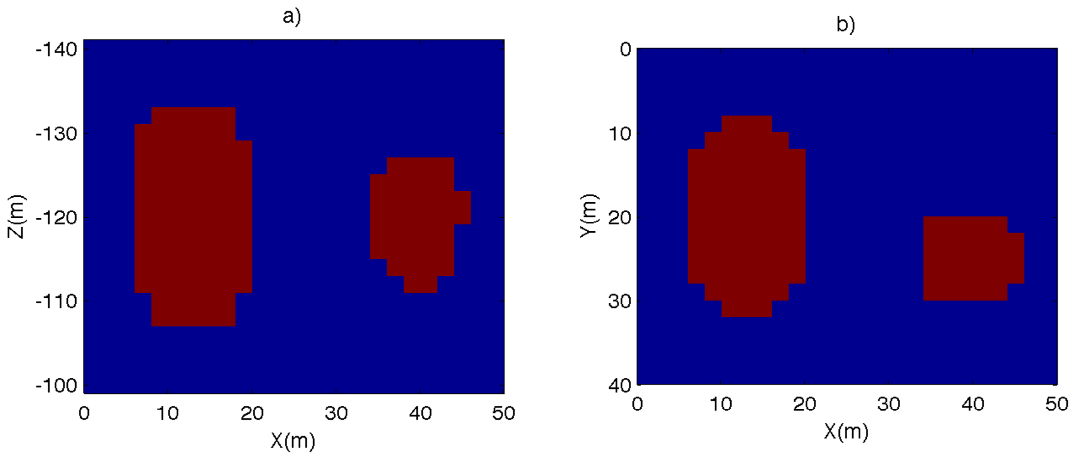

Figure 6.1 Deposit model for parameter analysis : a) projection on X-Z plane, b) projection on X-Y plane. Ores are marked in brown.

and that in Cartesian coordinates does not systematically match, the conversion can cause fluctuation of stope profits when raises parameters slightly change. This fluctuation brings more difficulties to achieve optimal. To reduce the fluctuation, one can use small discretization of cylindrical coordinates, such that $\Delta r < \Delta x$, where Δx is the unit block size in Cartesian coordinates.

The variability of stope profit with different Δr is tested. The results are shown in Figure 6.2. It can be seen that, with $\Delta r = 0.5$ m, the stopes are all close to the real optimum except for one case. However, with $\Delta r = 1$ m or 2 m, the average profit of stope diminish, because the deposit is downscaled to a lower resolution cylindrical model, leading to coarse and imprecise stopes. Besides, the large discretization increases variability of profit optimization. Therefore, Δr smaller than Δx should be adopted.

6.2.2 Impact of randomness of initial population

Ten random sets of initial population are used to produce stopes. The distribution of the stope profits are shown in Figure 6.3-a). There are 8 solution in 10 that are very close to the real optimal, which shows that the optimization process has some sensitivity on the initial population. The optimized raises plotted on Figure 6.3-b) exhibit several variants around the

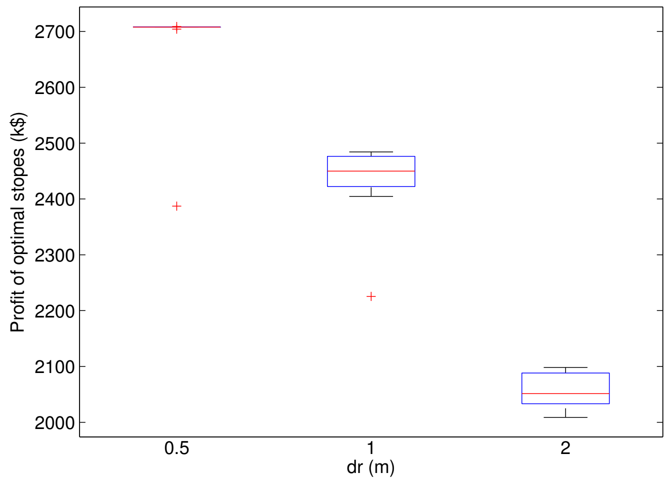

Figure 6.2 Impact of Δr on the stope profit

optimal locations. The viability of the results also suggests a way to assist to approach the global optimal, that is to repetitively operate the GA. There are two optimal solutions with less profits : one with two raises both placed in the orebody on the left, possibly because a large R appeared at early stages by chance ; another one showing a too small R value in the right orebody, probably for that the mutations were not successful to produce large R. To improve these situations, one idea could be, after stopping the algorithm to increase the values of R obtained and monitor the resulting profits.

6.3 The size of initial population

Different sizes of initial population are tested, with 10 random sets for each size. For these tests, the new individuals in a population are fixed to be 20. The result of sensitivity is plotted in Figure 6.4-a). As is seen, number of outliers of stope profit does not have clear relation with the population size.

6.4 The number of offspring in new generation

The percentage of new individuals in each generation can also affect the result. Here, we examine the proportion of new individual from 0.2 to 0.8 with interval 0.2. The result

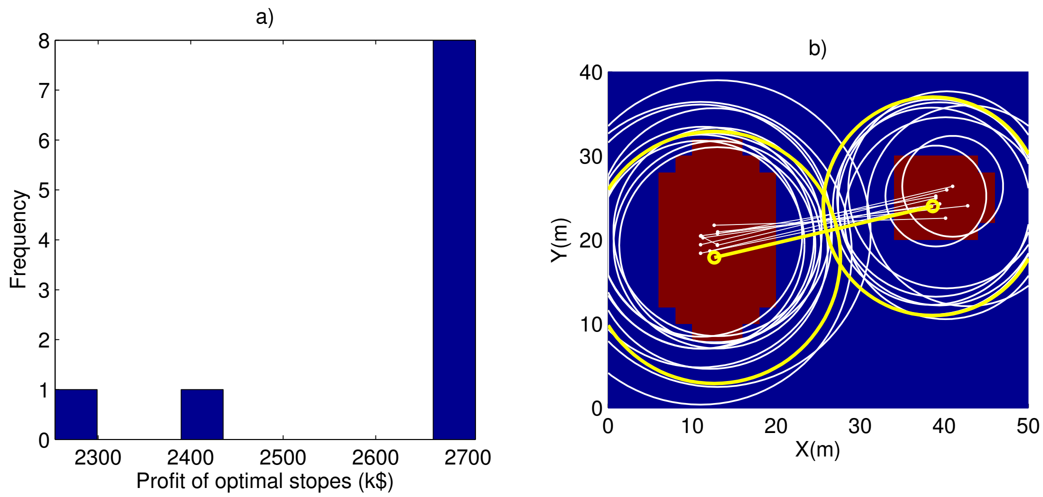

Figure 6.3 a) The distribution of stope profit with different initial raises ; b) The locations of optimized raises (in white paired dots) and the corresponding R with different initial raises ; The real optimal raises are shown in yellow. The brown areas are the orebodies projected on X-Y plane.

is shown in Figure 6.4-b). It can be seen that, with more individuals in each generation, the average optimal profit increases, and the viability diminishes. Therefore, appropriately enlarging proportion of new individual is more likely to access or at least to get close to the global optimal, thought more convergence time may be cost.

6.5 Mutation Rate

The impact of mutation rate, the proportion of genes of a new child to be muted, are analyzed. As is shown in Figure 6.4-c), overall, all tested different mutation rate can guide to the optimal. The viability of results shows that mutation rate 0.6 would lead to more stable results.

6.6 The number of parents to mate

The impact of number of parents to reproduce a child is plotted in Figure 6.4-d). When the number of parents is 4, the stope profit shows lowest variability.

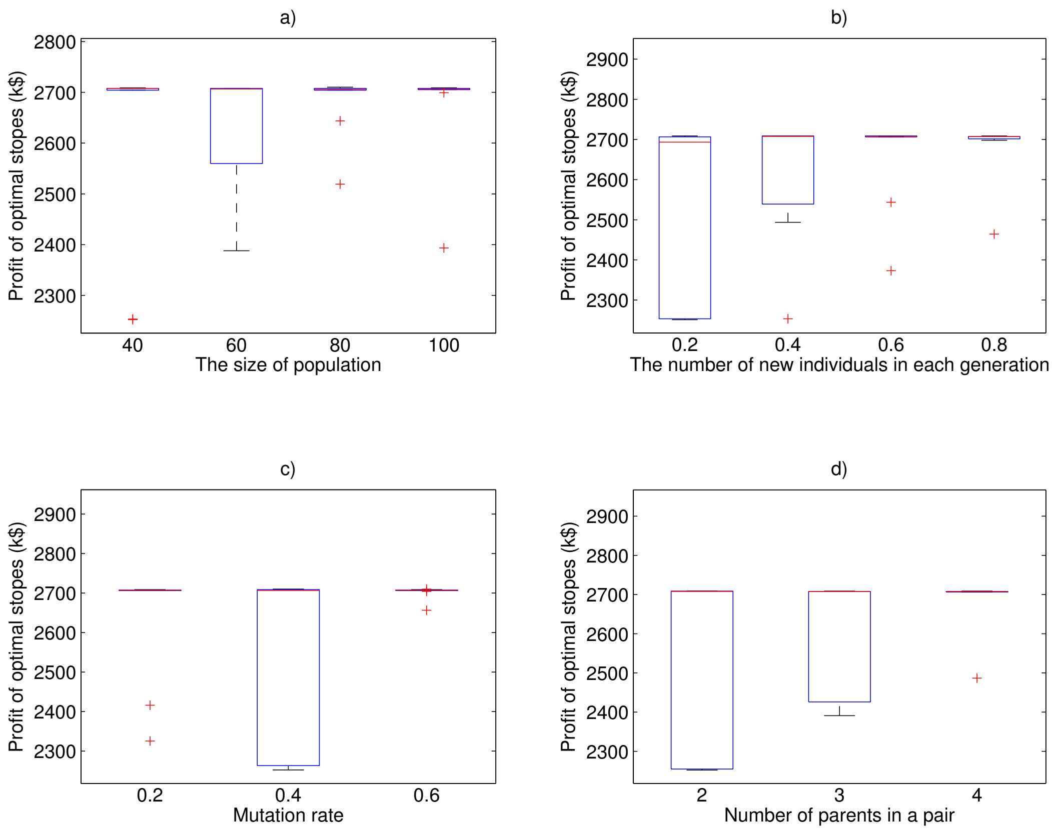

Figure 6.4 a) The viability of stope profit with different sizes of initial population ; b) The viability of stope profit with different number of new individuals in a generation ; c) Influence of mutation rate ; d) Influence of number of parents to mate

6.7 Discussion and summary

For the example tested, the GA used has light sensitivity to the initial parameter population. It is not very sensitive to the size of the population. The number of new offspring in a generation affects the optimality and robustness : higher percentage of new offspring is expected to produce more stable results. For mutation rate, a rather large value at 0.6 lead to best results. Besides, 4 parents to create a child have produced lowest variability.

The method to diminish the impact of R to optimization, using small Rs in initial solutions, is an intuitive way, though the result is practical. A more rigid way would be to force in the retained population at each generation to have various of R values in a wide span. Note that passed a certain value of R, the present profit function is almost insensitive to further increase on R. This suggests to introduce R in the objective function as a penalty term to favor to maximize the profit with R as small as possible. These avenues of research require further study.

The sensitivity study is valid for the example deposit presented. The sensitivity of the different parameters may vary with the orebody studied. In a real case application, the sensitivity should be studied to ensure the best optimal solution is likely to be found by the GA approach.

CHAPTER 7

GENERAL DISCUSSION AND CONCLUSION

7.1 Conclusion

A series of new algorithms for stope optimization are proposed in this thesis. The algorithms produce stope geometries in 3D for sublevel stoping method, conforming to the geotechnical constraints on footwall and hangingwall slopes, and stope heights. With multiple raises, the algorithm can adapt to various deposit shape and size. The solution with single raise suits for strong steep deposits of relatively small size. The multiple raises are suitable for curved deposits in horizontal section or large deposits.

The framework of the algorithms consists of two parts with different optimization techniques : the process to generate a stope under given raises parameters, using maximum flow method, is rigorous and guarantees the true optimality for the current raise location ; the process to optimize the raises parameters using global optimization is heuristic.

The proposed algorithms also enable to incorporate drift in stope optimization, typically for parallel longhole drilling pattern selected. With drift integrated, the stope design compromises the cost and benefit of drift and the stope boundary, and yields more profit than the designs without drift jointly optimized.

7.2 Limitations and potential improvements in future

The proposed algorithms have several limitations :

1) The algorithms are limited on sublevel stoping method selected. The extension to other methods requires future investigation. Since the mining equipments, mining sequence and geotechnical requirements differ from one mining method to another, the adjustment of the algorithm should consider these distinctions. For example, for cut-and-fill method, the principle mining consideration is the minimum stope size for the accessibility of equipments to perform selective stoping. Since the cut creates a free surface, the blocks can be linked toward the free surface from top to bottom. With block size equaling to the minimum selective mining dimension, a coarse optimal stope can be obtained. Nevertheless, a finer solution

requires the introduction of minimum width constraints. This would require deeper investigation.

2) The algorithms presented are not suitable for the inclined ore bodies, which usually calls for an inclined raise. Similarly, for the case with drift optimization, the algorithm can not provide practical results for the scenarios with inclined raise and inclined parallel drilling pattern adopted. The adaption from vertical raise to inclined raise is not as simple as it appears. Because under the cylindrical coordinate from a inclined raise, the simple relation with the gravity vector direction is destroyed.

3) In multiple raises solution, the cost to develop the raises is neglected. When the raises locate at different levels, additional raise development cost may diminish the overall profit of stope with multiple raises. Also, the raises are assumed to be placed freely in optimization. In reality, the endpoints of raises are restricted at certain sublevels. So, a more practical solution would be to bound the endpoints of raises at given sublevels. Then the optimization search for the best elevations of sublevels and for the number of raises within each sublevel, and their location within the X-Y plane. This improvement is ongoing.

4) The algorithm incorporating drift assumes the parallel drilling pattern (preferably vertical) is adopted. Ring drilling pattern is not considered. The cost of drift in ring drilling pattern is a relative small portion of stoping cost, because drifts are usually tunnels other than sections in parallel drilling. As a result, the cost of drift in ring drilling would probably have only a minor on the stope geometry.

5) The algorithm incorporating drift uses single raise in optimization. The extension to multiple raises can be developed straightforward, by adopting the framework proposed in Chapter 4.

6) The effect of uncertainty of ore grade and economic factors is not taken into account in this study. Since the developed algorithms can be deemed as a tool that produce stope geometry from the input of one ore model under stated economic conditions, the uncertainty study can be done by conditional simulation. The optimal design found with the algorithms is applied on each deposit realization separately with the selected economic conditions. This enables to assess the uncertainty on the recovered economic value of the stope. Also, the algorithms can be applied separately on each deposit realization to assess the robustness of the planned design to geological and economical uncertainty.

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APPENDIX A

AN EXAMPLE OF APPLICATION OF MAXFLOW METHOD IN MINING OPTIMIZATION

In this appendix, an example of application of Maxflow method is provided. A simple 2D ore block model with 12 block is used to illustrate the method. The block model and the economic values are shown in Figure A.1. To design a open pit, the wall slope angle is expressed as the links from one block to the three neighboring blocks of the upper level, such as the links from block 2 to block 5,6 and 7.

Modeling with network flow graph

The optimization of the open pit can be considered as a flow network problem. To construct the network flow graph, the blocks are denoted as nodes shown in Figure A.2. The links between the nodes, usually termed as edges or arcs, represent the slope constraints and precedence relations. For these arcs, the flow value or weight, are assigned as infinity. Two special nodes, source node s and sink node t, are introduced. The source node, or the starting node, links to all the **ore** blocks (i.e. blocks with positive economic value), with flows equal to the economic values of the block linked. The sink node, or the termination, is pointed from the **waste** blocks (i.e. blocks with negative economic value), with flow capacities equal to the absolute values of the blocks. These constitute a flow network graph. One seeks to maximize the flow capacities from the source to the sink. An open pit contour is obtained by the nodes linked to the source with arcs having a positive residual capacity and all the predecessors of these nodes. The optimal value of the open pit corresponds to the sum of the residual capacities on the edges from the source node.

The network graph can be formulated as a binary linear programming problem as Equation A.1.

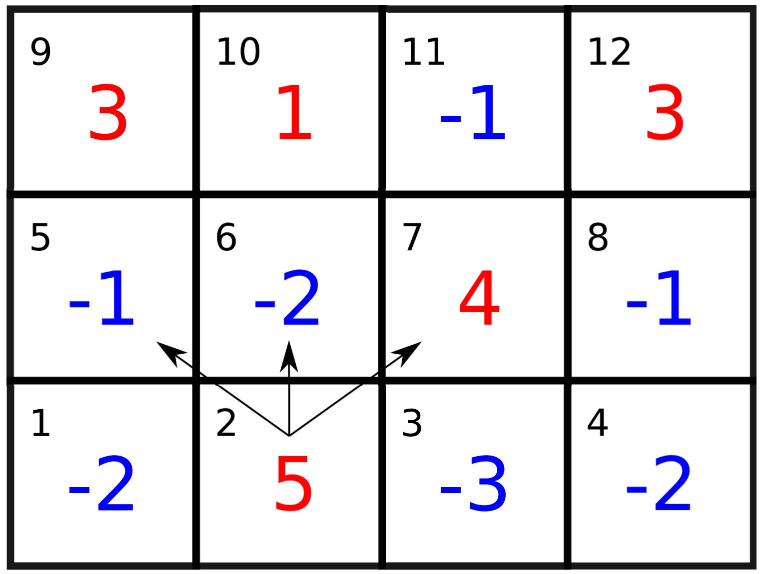

Figure A.1 A simple block model in 2D. The economic value of the block are noted at center. The positives blocks are filled in gray. The block numbers are labeled in top-left corner.

$$
\begin{aligned}
Maximize\ x = & -2x_1 + 5x_2 - 3x_3 - 2x_4 - x_5 - 2x_6 + 4x_7 \\
& - x_8 + 3x_9 + x_{10} - x_{11} + 3x_{12} \\
& x_1 \le x_5,\ \ x_5 \le x_9 \\
& x_1 \le x_6,\ \ x_5 \le x_{10} \\
& x_2 \le x_5,\ \ x_6 \le x_9 \\
s.t.\ \ & x_2 \le x_6,\ \ x_6 \le x_{10} \\
& x_2 \le x_7,\ \ x_6 \le x_{11} \\
& x_3 \le x_6,\ \ x_7 \le x_{10} \\
& x_3 \le x_7,\ \ x_7 \le x_{11} \\
& x_3 \le x_8,\ \ x_7 \le x_{12} \\
& x_4 \le x_7,\ \ x_8 \le x_{11} \\
& x_4 \le x_8,\ \ x_8 \le x_{12} \\
& \\
& x_i = 0, 1\ , i = 1, 2, \cdots, 12
\end{aligned}
\tag{A.1}
$$

Using any MaxFlow algorithm, such as push-relabel algorithm, the optimal solution can

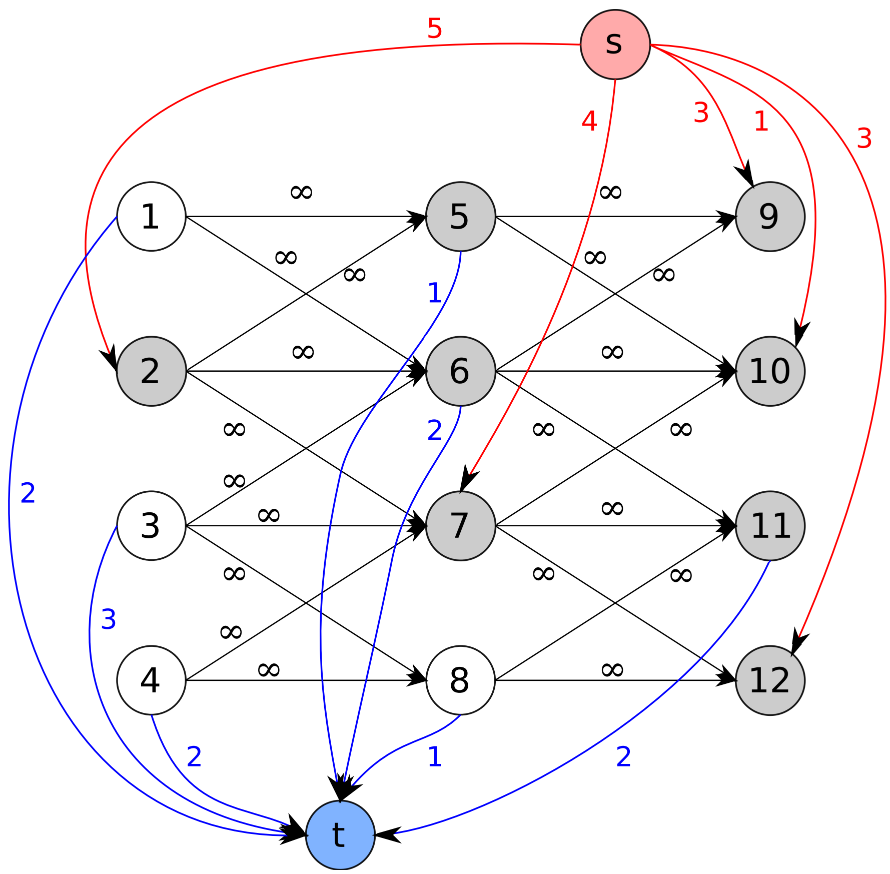

Figure A.2 Network flow model for a simple 2D open pit optimization. The nodes are the circles. The capacities are labeled aside the links. The optimal solution is represented by the gray circles.

be obtained. The optimal pit contains block $(2, 5, 6, 7, 9, 10, 11, 12)$, for an optimal value of 12.

Push relabel algorithm

In this dissertation, the maxflow algorithm used is the push-relabel algorithm(**?**) implemented in Matlab as *MatlabBGL* package [1]. This package uses Boost Graph Library, and has very efficient performance. Thanks to this, the proposed algorithm can repetitively employ the push-relabel algorithm to solve underground mining problems in realistic time.

To solve the maxflow problem the function "`max_flow`" is called :

```
[flowval cut R F] = max_flow(mat_net,s_id,t_id)
```

It returns the value of maximum flow (flowval), the identification of the blocks with residual capacity linked to the source and their predecessors (cut), the residual graph `R` (residual capacities) and the flow graph `F` (used capacities). The inputs are a network flow graph `mat_net`, the index of source node `s_id` and sink node `t_id`. The flow graph `mat_net` is stored as a sparse matrix. For example, $[(i, j), p]$ means an arc from node i to node j with capacity value p.

1. MatlabBGL, developed by David Gleichb available at : http ://dgleich.github.io/matlab-bgl/