

POLYTECHNIQUE MONTRÉAL

affiliée à l'Université de Montréal

**Fast and rigorous numerical modeling of geomagnetic disturbance, AC interference and
grounding systems**

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Département de génie électrique

Thèse présentée en vue de l'obtention du diplôme de *Philosophiæ Doctor*

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Cette thèse intitulée :

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grounding systems**

présentée par **Hossein ZAMANI ZARDEHSAVARI**

en vue de l'obtention du diplôme de *Philosophiæ Doctor*

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DEDICATION

To my wife

ACKNOWLEDGEMENTS

I would like to express my deep sense of gratitude to my supervisor Dr. Keyhan Sheshyekani, whose insightful guidance always helped me to follow the right direction during the course of this study. His deep knowledge and experience and enthusiastic encouragement which helped to shape this work definitely deserve my deepest gratitude.

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Hossein Zamani, January 2023.

RÉSUMÉ

La modélisation et l'analyse de problèmes électromagnétiques à grande échelle ont de nombreuses applications réelles. Ceci est particulièrement important pour les réseaux électriques qui sont soumis à des tensions et courants induits générés par des champs électromagnétiques avec des sources externes, principalement des orages géomagnétiques et des courants de foudre.

De plus, les lignes de transmission peuvent également interférer avec des conduites enterrées dans un couloir de service commun en induisant du courant/tension sur le conduit, ce qui peut principalement engendrer la corrosion du conduit.

Les systèmes de mise à la terre font également partie intégrante des réseaux électriques dont la modélisation des transitoires est importante pour la sécurité du personnel, le fonctionnement continu des équipements et les aspects de compatibilité électromagnétique. De nombreuses recherches ont été menées au cours des dernières décennies pour résoudre les problèmes mentionnés ci-dessus. Cependant, les méthodes adoptées ne sont pas suffisamment rigoureuses pour modéliser toute la complexité du problème ou ne sont pas abordables du point de vue du temps de calcul pour traiter des problèmes à très grande échelle.

Dans le contexte évoqué ci-dessus, cette thèse propose des méthodes numériques avancées basées sur la méthode des différences finies qui est capable de traiter des problèmes à grande échelle. Plus précisément, une méthode efficace de domaine temporel à différences finies (FDTD) est formulée pour modéliser les champs électromagnétiques d'orage géomagnétique (GS-EMF) à ultra-basse fréquence et le courant induit géomagnétique (GIC). L'approche FDTD proposée permet la modélisation de la structure terrestre 3D complexe et à grande échelle (par exemple, le sol multicouche et la courbure de la Terre). Contrairement aux méthodes FDTD classiques, la méthode FDTD proposée ne nécessite pas de ressources de calcul importantes lorsqu'il s'agit de problèmes à très basse fréquence.

Ensuite, une méthode semi-analytique efficace des ondes pleines basée sur la méthode de la ligne (MoL) est développée pour une modélisation rigoureuse des électrodes/systèmes de mise à la terre enterrés dans un sol stratifié multicouche.

Par rapport aux techniques existantes de modélisation des ondes pleines, la méthode proposée est

extrêmement efficace en termes de calcul et permet la modélisation de n'importe quel nombre de couches de sol verticales et/ou horizontales.

Finallement, un FDTD filtré spatialement (SF-FDTD) avec une stabilité contrôlable au-delà de la limite de courant est utilisé pour modéliser l'interférence Courant Alternatif (CA) entre les lignes de transmission aériennes et les conduites enterrés. Les méthodes proposées sont appliquées à différents cas de simulation et les résultats obtenus sont vérifiés à l'aide de méthodes numériques indépendantes.

ABSTRACT

Modeling and analysis of large scale electromagnetic problems has many real-life applications. This is of particular importance for power networks, which are subject to induced voltages and currents generated from electromagnetic fields with external sources, mainly geomagnetic storms and lightning return stroke currents. In addition, transmission lines can also interfere with buried pipelines in a common utility corridor and induce current/voltage on the pipeline, which can mainly result in pipeline corrosion. Grounding systems are also an integral part of power networks whose transient modeling is important for personnel safety, equipment continuous operation and Electromagnetic Compatibility aspects.

There has been a great deal of research over the past few decades to address the above-mentioned problems. However, the adopted methods either are not rigorous enough to model the whole complexity of the problem at hand or are not affordable from the computational time point of view to deal with very large-scale problems.

Within the context alluded above, this thesis proposes advanced finite difference based numerical methods with the capability to handle large scale problems. More specifically, an efficient Finite Difference Time Domain (FDTD) method is formulated for modeling ultra-low frequency geomagnetic-storm electromagnetic fields (GS-EMFs) and geomagnetic induced current (GIC). The proposed FDTD approach enables modeling of large-scale and complex 3-D Earth structure (e.g. multilayer ground and Earth curvature). Unlike conventional FDTD methods, the proposed FDTD method does not require extensive computational resources when dealing with ultra-low frequency problems. Then, a spatially-filtered FDTD (SF-FDTD) with controllable stability beyond the Courant limit is proposed to model AC interference between overhead transmission lines and buried pipelines. Finally, an efficient full-wave semi-analytical method based on the Method of Line (MoL) is developed for rigorous modelling of grounding electrodes/systems buried in a stratified multilayer soil. As compared to the existing full-wave modelling techniques, the proposed method is extremely computationally efficient while enables modelling of any number of vertical and/or horizontal soil layers.

The proposed methods are applied on different simulation cases and the obtained results are verified using independent numerical methods.

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LIST OF SYMBOLS AND ABBREVIATIONS

AC	Alternating current
ADI	Alternating-direction implicit
CC	Covered-conductor
CE	Courant Extension
CFL	Courant–Friedrichs–Lewy
CIM	Complex image method
CN	Crank–Nicolson
CME	Coronal mass ejection
CP	Cathodic Protection
CPML	Convolutional perfectlymatched layer
DC	Direct current
EM	Electromagnetic field
EMF	Electromagnetic fields
FEM	Finite element method
FDTD	Finite difference time domain
GIC	Geomagnetically induced current
GMD	Geomagnetic disturbance
GPS	Global positioning system
GS-EMF	Geomagnetic-storm electromagnetic field
HEM	Hybrid electromagnetic method
HEMP	High-altitude electromagnetic pulse detonation
HILF	High-impact low-frequency
LF	Low frequency
MoL	method of line
MoM	Method of moment
NACE	National Association of Corrosion Engineering
PEC	Perfect electric conductor

PEEC	Partial element equivalent circuit
PML	Perfectly match layer
SOHO	Solar and Heliospheric observatory
TE	Transverse electric
TEM	Transverse electromagnetic
TL	Transmission line
TLM	Transmission line matrix
TM	Transverse magnetic

CHAPTER 1 INTRODUCTION

1.1 Context

Radiated electromagnetic (EM) fields manifest their adverse effects in the form of malfunctions or definitive failures of components and systems. Critical infrastructures in general and electrical power systems in particular must ensure continuous and reliable operation when subjected to EM fields generated by different external and internal sources including geomagnetic storms, intentional EM pulses, lightning return stroke currents and transmission lines. The detrimental impacts of lightning on electrical networks (power systems, communication sites, and control centers) have long been known [1]. During the last several decades, Geomagnetic Induced Currents (GIC) have been reported to be responsible for numerous power outages and equipment failures (e.g., blackout in the Hydro-Quebec power system in 1989 [2]). Overhead transmission lines installed in joint utility corridors with buried pipelines due to environmental and land access concerns can create safety and corrosion problems for pipelines due to AC interference [3]. Grounding systems are designed to protect personnel and equipment against excessive overvoltages due to low-frequency short circuit and high-frequency lightning currents.

The above-mentioned problems are viewed as large-scale electromagnetic compatibility (EMC) problems involving complex propagation media, e.g., inhomogeneous soil, dispersive soil, and irregular terrain whose modelling and quantifications is beyond the capability of existing computational packages, mainly due to their computational performance limitations and modelling effort concerns.

The objective of this thesis is to develop rigorous and high-performance computational methods for the efficient modelling and solution of these large-scale EMC problems. The proposed computational platform accommodates an advanced version of numerical methods in electromagnetics, in particular the Finite Difference Time Domain (FDTD) and Method of Lines (MoL), designed and customized for the targeted problems. The applications of interest are modelling of Geomagnetic Disturbances (GMD), AC interference and grounding systems.

1.2 Geomagnetic Disturbances

Large solar flares and their associated coronal mass ejections (CME) are intense solar activities which under certain circumstances can cause disturbances in the near-Earth space environment [4]. The CMEs contain up to 10 billion tons of plasma material whose travel time from the sun to Earth is about two days (but with widely varying transit times) [5]. The CME's solar wind plasma can interact with the magnetosphere causing rapid changes in the distribution of Earth's magnetic field, a form of space weather which is referred to as Geomagnetic Storm (GMS) (see Figure 1.1). These phenomenon can occur during any season and are somewhat independent of the usual 11- or 12-year sunspot cycle [6].

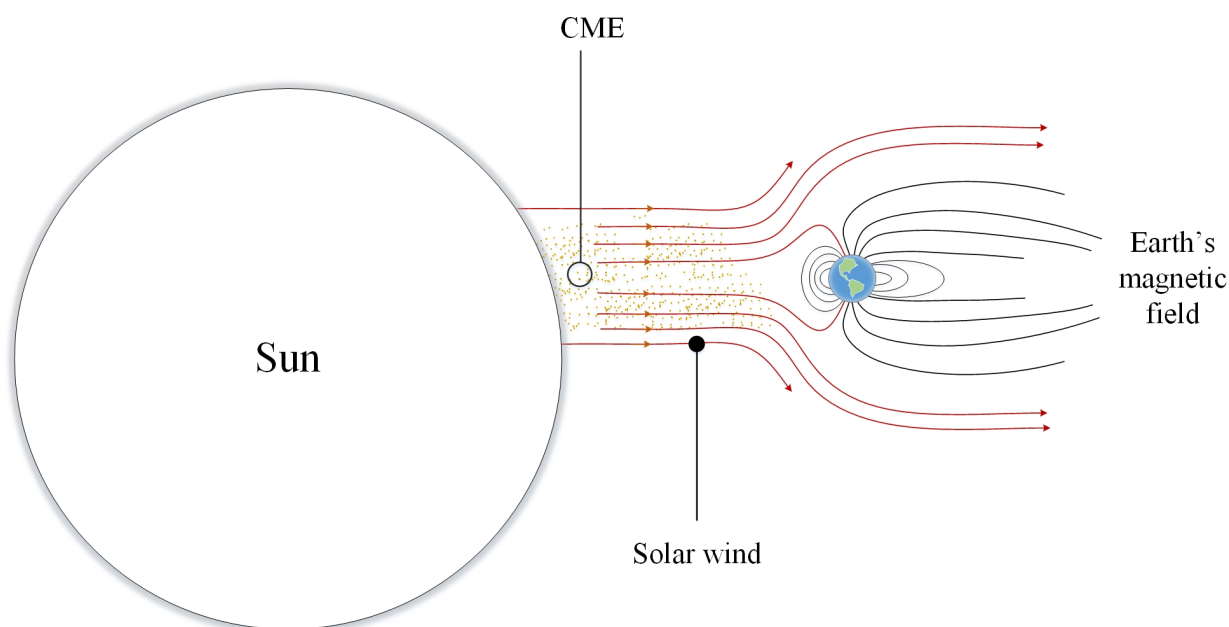


Figure 1.1. Effect of solar wind on earth's magnetic field

The CME is regularly observed by the Solar and Heliospheric Observatory (SOHO), a spacecraft launched in 1995 and located in an orbit between Earth and the sun. Figure 1.2 shows the number of CME events recorded by SOHO over a period of several years (some data for late 1998 and early 1999 are missing).

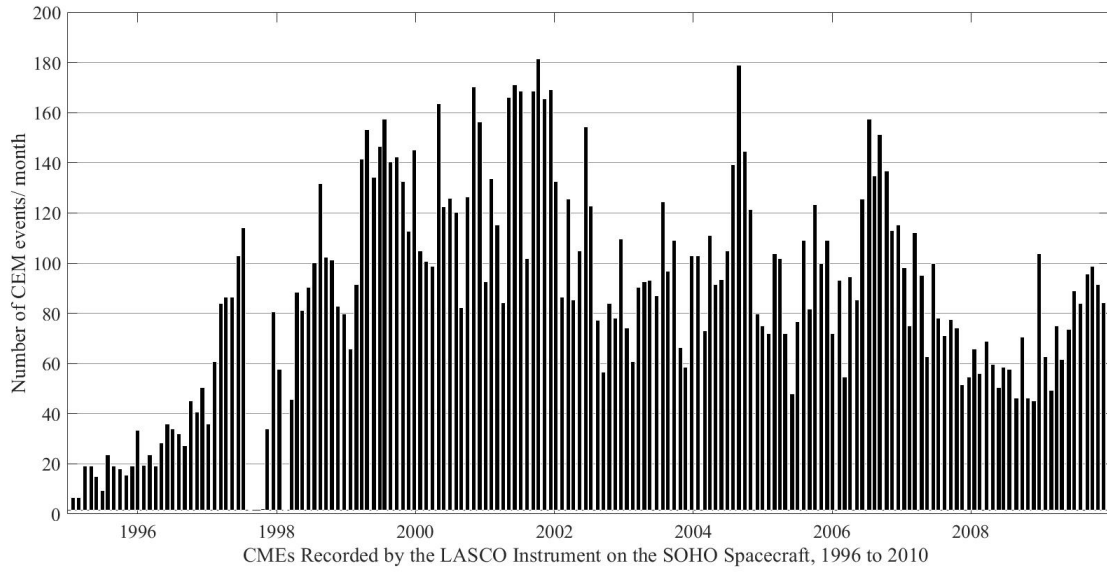


Figure 1.2. CMEs recorded by the LASCO instrument on the SOHO spacecraft, 1996 to 2010, adapted from [6]

The GMSs are random in terms of severity and timing. Richard Carrington, an English astronomer, has observed and recorded the event that occurred in 1859 from its outset on the surface of the sun. That storm, known as the Carrington Event, is generally acknowledged to be the largest in recorded history so far [7]. The space observations of celestial events has been performed during a period when power transmission and distribution systems did not exist, which makes direct comparisons with modern systems impossible. The Carrington Event, however, disrupted the telegraph system and resulted in fires in some telegraph offices. GMS produces impulsive disturbance of geomagnetic field over wide geographic regions which, in turn, induces currents (called geomagnetically-induced currents or GIC) on long transmission lines [8], [5]. Because of the size of the cross-sectional area of the single-turn loop represented by the power transmission line and the ground return path, the GIC could be quite large (in the order of 100 amps or even more). Proper evaluation of GIC and its effect on power systems requires modeling of a large-scale system accommodating long transmission lines in the order of hundreds of kilometres where a realistic 3-D structure of the propagation terrain must be considered.

1.2.1 Geomagnetic Storms and Their Adverse Effects

GMS electromagnetic fields (GS-EMFs) are characterized by an ultra-low frequency content (from 0.0001 Hz to 1 Hz) and can interact with power systems, resulting in significant low-frequency GIC on long transmission lines, see Figure 1.3. The adverse effects of GICs on the operation of bulk power systems are, but not limited to: half-cycle saturation in power transformers, harmonics, and increased reactive power demand and transformer spot heat [9]. These can ultimately result in transformer damage, voltage sags, relay malfunctioning, and system instability [10–12]. The effects of CMEs on power systems are detected every few years. These effects were observed in 1979, 1982, 1986, 1989, 1992, 1994, 1998, 2000, 2001, 2003, and 2005. Large-scale blackouts happen less frequently but are a definitive risk. Severe CME events occurred in 1859 (disrupting the telegraph system), 1921, and 1989, when U.S. and Canadian utilities lost power transmission capabilities over wide areas. Recent studies reveal that GMDs do not only happen in high latitudes. In fact, the problems caused by GICs have also been reported in middle and low latitudes [13], [14], such as South Africa, Brazil, and China.

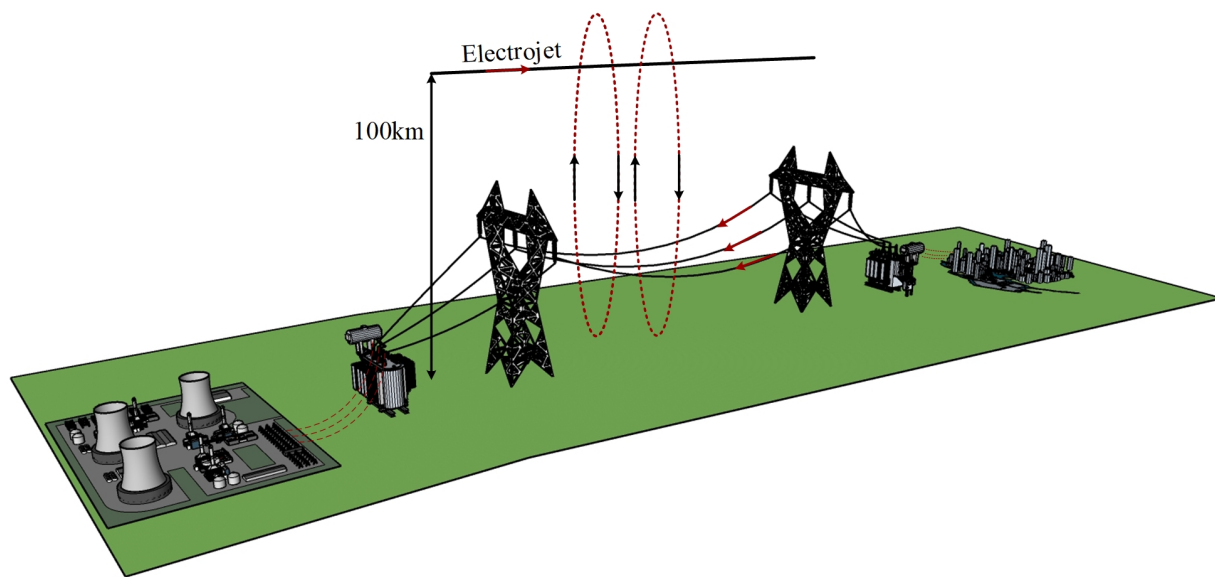


Figure 1.3. Geomagnetic storm modeled by an Electrojet and its associated GIC on overhead transmission lines

Three major GMD events are shown in Table 1.1. The Hydro-Quebec blackout in 1989, which is attributed to a GMS, affected Canadian and U.S. power systems causing a major power outage due to cascading tripping of protection relays. Millions of people for several hours did not have access to electricity [15]. In 2003, the South Swedish power grid was affected by a large GMS which resulted in a GIC of 330 amperes, approximately. Subsequent to the flow of this GIC, a 130 kV transmission line has been knocked down. The consequent blackout lasted for an hour and left about 50,000 customers without electricity [16]. The GMS in 2015 was the largest in more than 15 years and is popularly known as the St. Patrick’s Day storm. During the intense period of the storm, global positioning system (GPS) receivers have been affected. These events initiated an extensive research on GMD effects on power systems.

Table 1.1. Major GMSs and their adverse effects

Past Event	Year	Effect
March 1989 GMS [17–20]	1989	Cascading tripping of Hydro-Quebec protection relays which resulted in a blackout that affected millions of consumers
The Halloween Solar Storm [21, 22]	2003	GIC with a magnitude of 330 A led to a large-scale blackout by disconnecting a 130 kV transmission line.
St. Patrick’s Day storm [23, 24]	2015	Largest storm in more than 15 years. Planetary Index (K_p) reached 8 out of 9. GPS receivers signals were impaired.

1.3 AC Interference

Approximately 117,000 kilometers of underground large-diameter transmission pipelines are used in Canada every day for transporting crude oil, diluted bitumen, and natural gas between mine sites to refineries [25]. According to the Canadian Energy Pipeline Association (CEPA) [26], transmission pipeline operations of all types provided more than \$10 billion to Canada’s gross domestic product (GDP) in 2019. Pipeline integrity can be affected due to defects in materials and construction, mechanical damage from excavation or maintenance, incorrect cathodic protection, corrosion and coating degradation as well as geotechnical forces [27]. In particular, a major safety risk to pipelines is associated to their coupling with overhead transmission lines (AC interference) in their

close vicinity which results in induced voltages on the pipelines. Since utilities often share corridors with other facilities, such as pipelines and railroads, there is a possibility that electric power transmission lines can induce disturbances on close-by facilities as a result of electromagnetic coupling. That is why buried metallic pipelines and railways that co-locate with overhead transmission lines are exposed to induced disturbances. The AC interference is more pronounced during fault conditions when excessive short circuit current flows through the phase conductors that can result in large currents and voltages induced on the pipelines. Indeed, the voltage induced on the pipeline can increase the risk of AC-induced corrosion, equipment degradation, malfunctions, and exposure of humans and animals to shock hazards. Furthermore, AC interference affects the performance of cathodic protection (CP) systems, normally designed to protect pipelines against DC corrosion, and makes the potential of the protected pipeline deviate from the applied value hence reducing CP effectiveness in terms of corrosion protection [3].

With reference to the National Association of Corrosion Engineers (NACE), common CP measures are not enough to provide full protection for pipelines in the presence of AC interference [3]. Moreover, field tests have revealed that AC-induced corrosion is likely to happen on carbon steels, even under higher CP levels [28]. It has also been found that for some cases, considering excessive CP levels could even contribute to a higher AC corrosion rate [29]. Obviously, there is no consensus regarding the role of AC interference in the CP performance for corrosion protection. In addition to the adverse effects on CP effectiveness, AC interference can also exacerbate pipeline corrosion, resulting in localized pitting corrosion at high AC current densities. The critical AC current density to initiate pitting and the mechanism of AC-induced pitting corrosion on steels is still unknown. Furthermore, the accelerated AC corrosion and pitting corrosion of buried pipelines are highly dependent on the presence of coating defects and their size and shape [30]. Nevertheless, the exact effects of all parameters contributing to the AC corrosion are not still fully understood. With that being said, it is crucial to have a basic understanding of AC interference in order to identify, analyze, and mitigate its posed risks on buried pipelines.

Within this context, there is a need to develop rigorous computational tools that enable better understanding of AC interference and its effect on the CP protection and AC corrosion for buried pipelines. This is crucial for better design of CP system and mitigation mechanisms without which pipeline owners will be incurred due to high cost of maintenance and repair. It is noted that avail-

able computational tools in this context are based on simplified modelling assumptions and can not handle pipelines buried in complex ground structures.

1.3.1 Types of AC interference

AC interference manifests itself due to three primary coupling mechanisms namely inductive, capacitive and conductive coupling [31].

1) Inductive coupling: Time-varying currents passing through conductive paths, such as transmission lines and cables, create variable magnetic fields which can induce current on nearby metallic structures. The induced current implies the presence of mutual inductive coupling between the energized conductor and the nearby metallic structure such as nearby buried pipelines. In fact, for inductively coupled systems, time-varying current in one conductive pathway induces voltages and currents on the co-located conductive pathways such as railroad rails tracks, pipelines, shield wires, optical ground wires (OPGWs) cables, grounding systems and communications cables.

The inductive coupling between overhead transmission lines and co-located buried pipelines is affected by a combination of factors such as the pipeline and transmission line geometry and configuration (separation distance, burial depth, etc.,), soil electrical parameters, soil stratification, coating conductivity as well as the operational voltage and current of the transmission line [32]. In particular, the flow of excessive currents during fault condition can result in a significant induced currents on the nearby buried pipelines.

2) Capacitive coupling: In AC systems, capacitive coupling occurs when time-varying electric fields are present. This is the case for the co-located overhead transmission lines and above-ground pipelines as it is shown in Figure [31, 33]. It is noted that for the case of buried pipelines, the capacitive coupling is negligible since the surrounding earth material causes the electric charge to be dispersed more evenly over a larger area as a result of earth material [34].

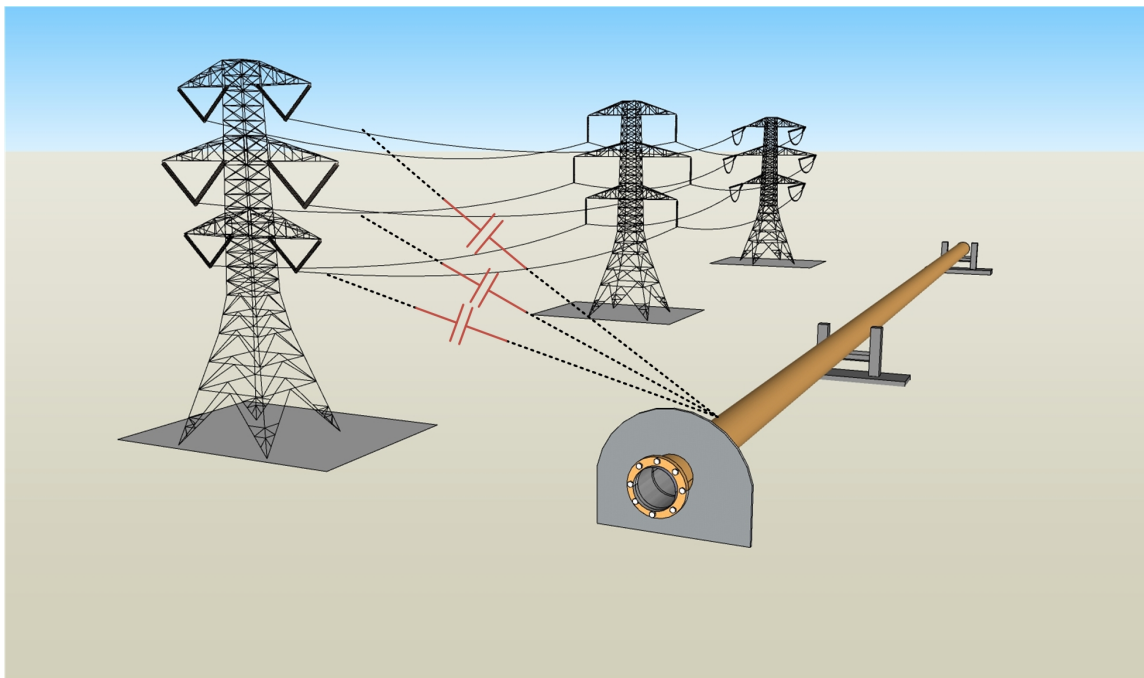


Figure 1.4. Capacitive coupling between transmission lines and above ground pipeline

The presence of capacitively coupled voltages of sufficient magnitude can cause shock hazards as well as interference with sensitive electrical devices. Structures that are vulnerable to capacitive coupling include railroad rails, above-grade pipelines, shielded cables, steel fencing and handrails [31, 35]. With particular reference to the transmission lines, the capacitive coupling is proportional to the operational voltage, hence transmission lines with higher voltage can induce greater voltages on the nearby above-ground conductive infrastructures. During pipeline system construction, particularly when segments of the pipeline are above ground mounted on insulating support structures as seen in Figure 1.4, capacitive effects become a concern. With the surrounding air acting as the dielectric, the transmission line and pipeline act as two plates of a capacitor, hence, charges build on the pipeline resulting in a potential difference between the pipeline and the ground.

3) Conductive coupling: With particular reference to co-located overhead transmission lines and buried pipelines, direct contact between the transmission line and the pipeline as well as the flow of short circuit current (mainly line-to-ground fault) towards ground in the vicinity of the buried pipeline can give birth to another type of coupling which is called conductive coupling. This

results in induced voltages/currents on buried pipelines which can damage the pipeline coating and pose lethal risk to personnel [36]. Linear facilities, equipment, and other objects co-located with overhead transmission lines such as above-ground metallic objects, metallic fencing, handrails, etc. can also be affected by conductive coupling.

1.4 Grounding System

Grounding system appropriate design is a key factor ensuring proper operation of electrical systems in terms of human safety, component protection, insulation coordination and Electromagnetic Compatibility (EMC). Grounding systems have always been an integral part of power systems to provide an electrical connection to earth so that lightning and fault currents are dissipated into the ground without creating severe ground potential rise (GPR). This ensures a safe environment and a lower risk of damage to the equipment. An illustration of some common grounding structures, namely horizontal and vertical grounding electrodes, grounding grid and ring-shaped grounding system is shown in Figure 1.5.

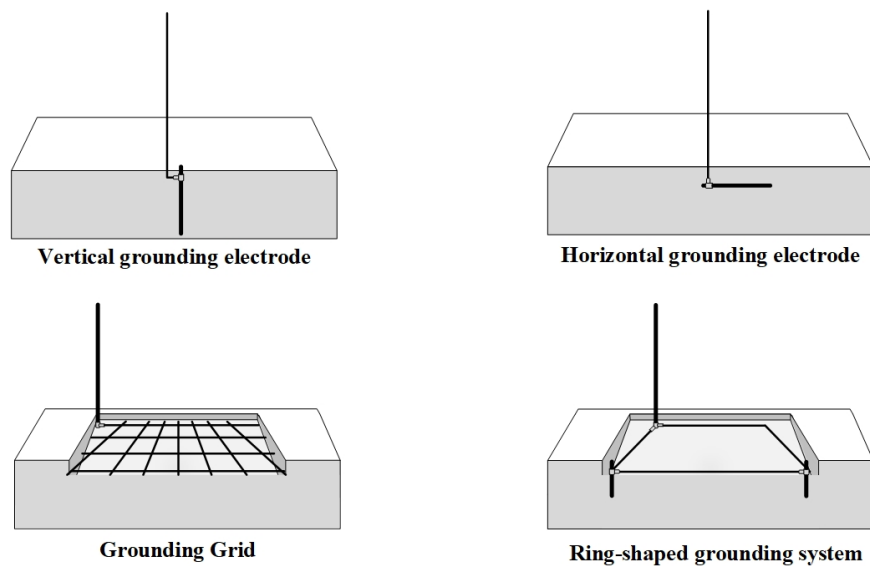


Figure 1.5. Most popular grounding system structures

1.5 Objectives

The main objective of this thesis is to propose a set of comprehensive numerical methods that enable rigorous and time-efficient modeling large-scale electromagnetic problems over a wide frequency range. More specifically, the proposed methods will allow:

- GMD modelling; An efficient method will be proposed which enables modeling of GMDs and their associated GS-EMFs and GICs considering large-scale complex ground structures such as horizontally, vertically and mixed vertically-horizontally stratified multilayer soil. The proposed method also allows modeling of actual configuration and dimension of transmission line phase conductors and shield wires.
- AC interference modelling; This embodies fast and accurate modelling of AC interference with the ability to model complex ground structures (i.e., vertically, horizontally and vertically-horizontally stratified multilayer soil), different crossing angles between overhead transmission lines and buried pipelines and pipeline coating.
- Grounding system modelling; A rigorous method will be proposed for fast modeling of large grounding systems buried in complex ground structures.

1.5.1 Thesis Outline

This dissertation is comprised of 8 chapters that are organized as follows: Chapter 1 discusses the relevance of Geomagnetic Disturbance (GMD), AC interference and Grounding systems.

Chapter 2 discusses the challenges corresponding to modelling of each of the above-mentioned problems. This Chapter will also provide the motivations and objectives of this PhD thesis.

Chapter 3 provides elaborate details about a new 3-D FDTD approach for modeling of complex large-scale low frequency electromagnetic problems with particular reference to GMDs and AC interference. The proposed FDTD approach enables modeling of large-scale and complex 3-D earth structure (e.g., multilayer ground and earth curvature). To deal with AC interference problems, a Spatially-Filtered (SF) FDTD approach is proposed later in this chapter in order to circumvent the stability problem associated with the accelerated FDTD presented for GMD modelling. The SF-

FDTD allows rigorous modeling of AC interference with the ability to take into account complex soil structures.

Chapter 4 first provides detailed formulation for 2-D Method of Line (MoL) modelling of grounding electrodes buried in a stratified multilayer soil. Then, 3-D MoL formulations for modelling of grounding grids buried in a stratified multilayer soil is provided in detail.

In Chapter 5, the proposed accelerated FDTD method will be used for modeling of GMDs and calculation of GICs. Results will be obtained for uniform, horizontally stratified soil and mixed horizontally and vertically stratified soils. The effect of horizontally and mixed horizontally-vertically multilayer ground, shield wire and different excitation sources on GICs and GS-EMFs is evaluated.

In Chapter 6, the SF-FDTD will be applied to modeling of AC interference between the co-located overhead transmission lines and buried pipelines. Different case studies including pipelines buried in soil with different structures/layers having different crossing angles with the transmission line will be studied. The effect of coating resistivity on the induced voltages will also be evaluated. Chapter 7, will deal with modelling of grounding systems buried in homogeneous and stratified multilayer soils by means of the MoL method. In this chapter, the performance of 2-D and 3-D MoL modelling of grounding systems is evaluated using different grounding electrodes and grounding grids buried in different stratified multilayer soil.

The summary, contributions, and conclusions of the thesis and the suggested future work are presented in Chapter 8.

CHAPTER 2 LITERATURE REVIEW

A great deal of research has been devoted to modeling of large scale electromagnetic problems in power system. In this section three main large scale EM problems in power system together with their associated modeling approaches are discussed.

2.1 GMD Prediction and Modelling

2.1.1 GIC Measurement and Monitoring

To measure GIC in a given power system, Hall effect sensors, as it is shown in Figure 2.1, are usually installed on the neutral conductor of the selected transformer where, with proper filtering and conditioning, the quasi-DC currents flowing through the transformer winding to ground are monitored [37].

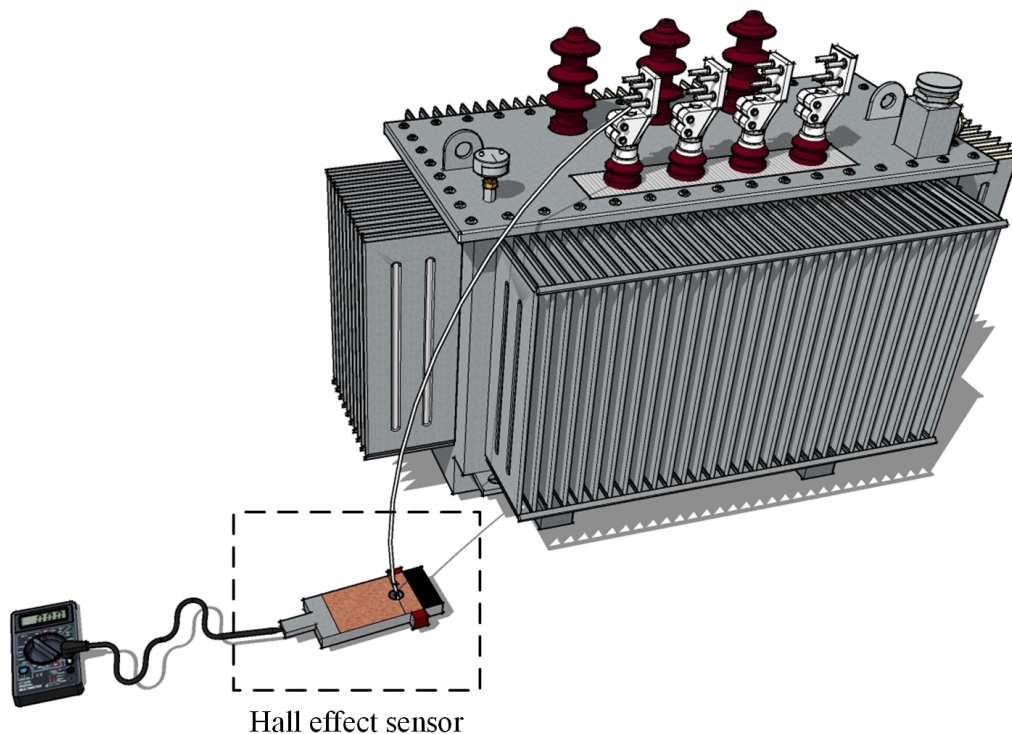


Figure 2.1. Hall Effect sensor installed on neutral point of a transformer to measure the GIC

The GIC monitoring can be alternatively done by measuring the capacitor bank and shunt reactor neutral current [38]. The Sunburst system, which uses this technique, was developed by the Energy Policy Research Institute (EPRI) and has been in use in the United States since 2004 (mainly in the north-east), Manitoba (on a feeder supplying Minnesota) [39], and in England and Wales [40]. The Minnesota power system also uses a direct DC current measurement mounted on a phase conductor of a 500 kV transmission line [38].

The major downside of this real-time monitoring technique is inability to provide the necessary warning in order to take remedial actions to protect the power system against GIC [39]. Almost all real-time monitoring techniques suffer from this drawback. The transformer neutral current monitoring can be practically performed for a few selected neutrals. Hence, to determine the GIC on all transmission lines, assumptions must be made about the geo-electric field [41]. Geo-electric fields generated by geomagnetic disturbances electromagnetic fields refer to the electric fields that are generated as a result of fluctuations in the Earth's magnetic field. These fluctuations can be caused by geomagnetic storms, which are disturbances in the Earth's magnetic field caused by charged particles from the Sun. These charged particles can interact with the Earth's magnetic field, causing it to fluctuate, and induce electric currents in the Earth's conductive layer. These induced electric currents can generate geo-electric fields at the Earth's surface, which can be measured and used to study the Earth's electrical conductivity and subsurface structure. The relationship between geomagnetic disturbances and geo-electric fields is complex and depends on various factors, including the conductivity of the Earth's subsurface and the intensity of the geomagnetic storm. Understanding this relationship is important for a variety of applications, including predicting the effects of space weather on technology systems, such as satellites and power grids, as well as for improving our understanding of the Earth's interior.

Transformer neutral currents are also used to trigger events such as dispatch alarms and fault recorders to facilitate management and analysis of GIC incidents [2, 41]. The quantification of the GIC effect is carried out by monitoring system data and parameters including system voltages and reactive power consumption, as well as transformer tank temperature, transformer oil gassing, and transformer noise and vibration [2, 41].

Researchers and utility companies are interested to better understand the GS-EMFs at ground surface in order to better understand GIC. Several methodologies and technologies have been de-

veloped in this regard [42]. Magnetic sensor, also known as a magnetometer, measures magnetic induction (magnetic field intensity), is used to measure and record magnetic field data initiated from GS-EMFs [41, 42]. As an alternative to magnetic field measurement, some have used grounded or isolated dipoles to measure electric fields [42].

2.1.2 Prediction of GIC

Real-time measurements keep record of GIC flowing through the system. This is useful for understanding the system status and for post-event analyses, although it provides limited assurance to system operators that a system will survive a GIC event [41, 42]. If reasonably accurate GIC predictions cannot be made, the only option is to react to every GIC that may potentially occur. Despite being conservative, by adopting this approach, utility and transmission system owners can be incurred due to unnecessary generation reduction and redistribution [43].

A simple approach is to perform GIC forecast based on an empirical equation which relates the observed (measured) time-derivative of the magnetic field $\frac{dB}{dt}$, at a given site for every three hours (a_p index) and the GICs that can flow into the power grid through earthing points at substations [44]. Similarly, the daily measurement of the time-derivative $\frac{dB}{dt}$ is represented in the A_p index which can also be used for the GIC prediction [44]. Many organizations use this method to plan activities that are dependent on the state of the Earth's magnetic field. In terms of accuracy, such forecasts are not very accurate, accounting for only 29 percent of the variance in A_p . To overcome this problem, forecasters use properties of A_p and events on the Sun (i.e., solar flares, coronal mass ejections, and coronal holes) to predict A_p for the following day. The most prominent ones are persistence, 27-day recurrence, semiannual variation, as well as modulation of the solar cycle [45].

The main shortcoming of the above-mentioned prediction methods is that subsequent to power system topology changes, a new set of empirical data must be collected which makes these methods inflexible. A more sophisticated modelling technique relies on the prediction of the so-called auroral electrojet (ionospheric current) from which the magnetic field variations $\frac{dB}{dt}$ are obtained. Hence, using Faraday's law, the ground-surface electric potential V reads,

$$V = - \oint \frac{dB}{dt} \cdot ds \quad (2.1)$$

The geo-electric field can then be calculated from the ground-surface electric potential. Subsequently, different approaches such as circuit approach [46] can be adopted to obtain GIC from the associated geo-electric field. This calculation despite its simplicity relies on the availability of the earth conductivity model [47–49] which is not easy to obtain.

A more computationally efficient method to predict GIC is based on the measured or calculated Earth surface impedance Z (representing the Earth's response). In this method the geo-electric field (E_y) is obtained from the horizontal magnetic field (B_x) [48, 49] as follows;

$$-E_y = \frac{Z}{\mu_0} B_x \quad (2.2)$$

with μ_0 being the magnetic permeability of free space. This approach, however, disregards the impact of conductive networks because of their larger grounding resistance as compared to the Earth surface impedance.

2.1.3 Geomagnetic Disturbance Modelling

To calculate the GIC for a given power system, the GS-EMFs at the ground surface should first be calculated. The majority of previous studies have used an auroral electrojet as the GS-EMF source [9, 50–54]. Such a choice is reasonable, because at high latitudes, where the most significant geomagnetic effects are observed, geomagnetic disturbances are principally due to an auroral electrojet current system, which contains an intense east-west current (the electrojet) in the ionosphere [53]. To date, different analytical [9], [51] and numerical methods [54] have been adopted for the calculation of GS-EMFs. In [51], the complex image method (CIM) [50] was used to compute GS-EMFs considering both single-layer and horizontally stratified multilayer grounds. The CIM demonstrated excellent accuracy for the case of horizontally stratified multilayer ground [51]. However, this method is not applicable to complex ground structures such as vertically or horizontally-vertically stratified ground. Furthermore, for particular cases (long TLs), the realistic 3-D structure of the propagation terrain must be taken into account, which is beyond the capability of classical 1-D approaches [9], [51]. It is also worth noting that the classical 1-D method calculates the electromagnetic field in the absence of transmission line system, which reduces its accuracy. More recently, the effects of mixed multilayer ground on the electro-

magnetic fields (EMFs) generated by an electrojet were evaluated by a 2-D finite element method (FEM) [54]. However, this method becomes computationally expensive for accurate 3-D modelling of large-scale propagation terrains.

2.2 AC Interference Modeling

There exist a great deal of research for modelling AC interference with particular reference to interaction between overhead power transmission lines and buried pipelines. The vast majority of these studies assume the pipeline as a lossy transmission line with an earth-return path, and use some approximation of the well known Carson's formulae to analytically estimate the coupling between each conductor of the overhead transmission line and the pipeline [55,56]. As extensively discussed in [57], an essential requirement for using any analytical formulation to estimate the aforementioned coupling is to adopt a "weak coupling" hypothesis [58] that can lead to considerable inaccuracies when the number of involved conductors increases. Recently, researchers have successfully applied methodologies based on electromagnetic field theory to overcome the limitations of the legacy analytical approaches [59–61]. These are based on solving Maxwell's equations in a discretized domain by means of FEM. However, given the large dimension of the problem under consideration, full 3D FEM modeling of AC interference is not an affordable choice from the computational point of view. Owing to this fact, in [59,60], a 2D FEM approach has been adopted for AC interference modeling assuming that transmission line and pipeline currents flow in an axial direction. In fact, these methods are based on the assumption of linearity and isotropy of the materials, infinite length of the conductors along the z -axis and a quasi-magnetostatic regime (i.e., the displacement current is negligible with respect to the conduction current in the conductors). In this method, the 2D FEM analysis is performed on small sections along which the assumption of axial current distribution holds valid (i.e., parallel sections of pipelines and overhead transmission lines). The 3D extension is obtained by using an equivalent circuit (see Figure 2.2) to couple the analyzed section with the rest of the system [61]. A graphical representation of the circuit is provided in Figure 2.2 for the equivalent network, when a single shield wire is considered, where $Z_{\text{pipe-s}}$ and Y are termination impedance and admittance at both ends of the pipeline, respectively. The analysis of the obtained equivalent circuit finally provides a relation between current in each section

with the enforced current at the first section. Finally, the induced voltage along each section of the pipeline can be calculated based on the relationship between the current of the studied section and the longitudinal electric fields provided by FEM. See [61] for further detail.

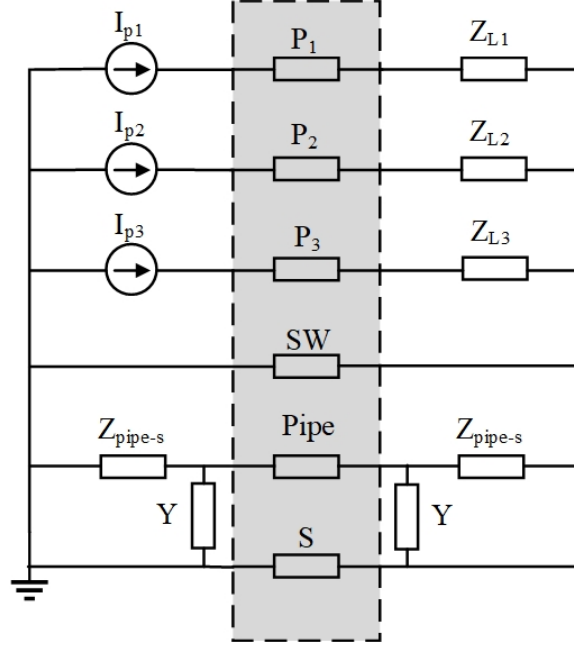


Figure 2.2. Equivalent electrical circuit for quasi 3D FEM modelling of AC interference. $Z_{\text{pipe-s}}$: pipeline termination impedance, Y: pipeline termination admittance to model coating, P_i : TL conductor, SW:Shield wire, Z_{Li} : termination impedance of phase i and I_{pi} : phase current

This “quasi-3D” technique allows incorporating the information extracted through the 2D FEM modeling into an equivalent network, which helps to enforce some physical constraints on the computed currents and voltages, which is not possible by using a pure 2D FEM analysis. However this method fails to take into account the return currents of the metallic conductors, overhead ground wires (OGWs), pipeline and the line currents (for the case of unbalanced faults) assuming only the soil as the return path. It also neglects the physical effects associated with the imperfect earthing and coatings of the discretized conductors. Furthermore, to the best of our knowledge, these methods are not applicable to cases with inhomogeneous soil. The MoM has also been employed to model AC interference [32, 62]. The MoM, however, relies on an approximate model for vertically stratified soil [63] and exact modeling of complex soil structure is not amenable to this method. In [32], a formula has been proposed as a function of the angle and separation distance through surface fitting of MoM simulation data to calculate the peak value of induced voltage.

This formula, however, neglects the length of the transmission line and does not take into account multilayer soil structures.

2.3 Grounding System Modelling

The grounding system performance in both static (low frequency) and dynamic (high frequency) regimes is important. In the static regime, the aim of grounding system design is to provide desired grounding resistance, ground potential rise, touch and step voltages in normal operating condition and subsequent to short circuit currents [64] and [65]. Low frequency analysis of grounding systems can be done either by making use of empirical formulae or using numerical electromagnetic methods formulated in the static regime. These methods, however, fail in providing accurate results at high frequencies when used for the calculation of lightning generated overvoltages, in particular for large grounding systems where the equipotential assumption for ground conductors is violated. In the high frequency regime, the aim of grounding system design is to control lightning generated overvoltages and also to mitigate the voltage differences among different parts of the grounding system subjected to lightning current. Lightning impulse currents are usually characterized by a wide frequency content from DC to several MHz over which the grounding system reveals different behavior over different frequency intervals. Hence, to obtain accurate lightning generated overvoltages, one should properly model grounding systems over a wide frequency range.

Accurate high frequency modeling of grounding systems has been the subject of many studies over the last decades. With this regard, it is customary to model grounding systems over a wide frequency interval ranging from zero to several MHz. In practice, depending on the type of the particular analysis and the excitation source (lightning or short circuit) the maximum modeling frequency is determined. Numerous techniques have been used so far in order to model grounding systems. These mainly include circuit theory [66] and [67], transmission line theory [68–70], and full-wave approaches such as finite difference time-domain method (FDTD) [71], finite element method (FEM) [72], and the method of moments (MoM) [73, 74]. Circuit-based methods and transmission line theory are referred to as quasi-static approaches which generally fail to model high frequency behavior of grounding systems. The electromagnetic (EM) field approaches (i.e., FDTD, FEM and MoM) have, in general, the ability to accurately evaluate grounding systems

performance over a wide frequency range. However, the EM field approaches are time-consuming and become prohibitively slow particularly for modeling of large grounding systems. The MoM is known as the most efficient EM numerical method from the computational requirement point of view; however, it is only applicable to grounding systems buried in homogeneous or horizontally multilayer soil [75]- [76]. It is to be noted that the MoM can also be relatively time-consuming when dealing with modeling of large grounding grids.

A horizontally stratified multilayer soil is theoretically amenable to the above-mentioned numerical methods. As explained before, and within the context of grounding system modeling, the MoM can be considered as the most efficient approach as compared to the FDTD and FEM. This is attributed to the fact that grounding electrodes can be reasonably approximated as thin wires, hence, the original two-dimensional surface integration can be reduced to a one-dimensional line integration in the MoM. The main step in the MoM numerical analysis of grounding systems is the formulation of governing electromagnetic equations in the form of integral equations. This relies on the derivation and evaluation of the Green's functions of an infinite layered medium for different homogeneous or stratified multilayer earth models. However, the evaluation of the Green's function associated with the latter case is relatively time-consuming due to the slow convergence of Sommerfeld's integrals [76]. Hence, different approaches such as the so-called modified image theory [75] or the exact image theory [77] have been presented in the literature to address this problem. However, the validity of the modified image theory is limited to the quasi-static regime, i.e., at low frequencies. The main limiting factor associated with the exact image theory is the computational efficiency, which makes it prohibitively slow in particular for the modeling of large grounding systems [76]. In general, these limitations pose serious challenges on the high frequency analysis of grounding systems buried in a multilayered stratified soil. A Mixed Potential Integral Equation (MPIE) approach has been proposed in [76] which enables modeling of grounding systems buried in a horizontally multilayer stratified ground. This approach utilizes a general dyadic spectral domain Green's function for horizontally multilayer ground. Then, Sommerfeld's integrals are numerically solved to evaluate the dyadic spatial domain Green's function [78] along a deformed path in the complex plane. This method is more general than the rest of MoM-based approaches proposed for modeling of grounding systems as it allows handling grounding electrodes with arbitrary orientation buried in a soil with any number of horizontal layers. Nevertheless, the MPIE despite its

promising performance, cannot model grounding systems buried in more complex soil structures such as vertically stratified multilayer soil. The FEM and FDTD approaches, despite their ability to model complex soil structures, are not yet affordable choices for modeling of large grounding systems due to their huge computational burden [79]. This is owing to the fact that both the FEM and FDTD method rely on the discretization of the entire modeling domain [80].

2.3.1 Grounding System Surge Impedance

A grounding system can be modeled in either time-domain or frequency-domain. Grounding system wide-band behaviour is commonly interpreted by obtaining its grounding system impedance over the desired frequency range. Different definitions of time-domain (transient) grounding system impedance are available in the literature. With reference to Figure 2.3, let us assume that a current $i(t)$ is injected to the top of a grounding electrode while potential difference between the injection point and the remote earth, i.e. a point with a zero potential, is measured and represented by $v(t)$. In such a condition, the following three definitions for grounding impedance are found in the literature [81];

$$z(t) = \frac{v(t)}{i(t)}, \quad (2.3)$$

$$z(t) = \frac{v(t)}{\max(i(t))}, \quad (2.4)$$

$$Z = \frac{\max(v(t))}{I}, \quad (2.5)$$

where I represents the value of $i(t)$ when the voltage $v(t)$ takes its maximum value. Equation 2.3 is a measure of time domain impedance that is also known as the transient surge impedance [82]. It is important to note that this definition only applies to pure resistive circuits in which both $i(t)$ and $v(t)$ have the same waveforms. In 2.4, it is assumed that a step or ramp current is injected into the grounding electrode [83], and 2.5 is the definition that provides a constant grounding impedance [84]. It is known from the above equations, that grounding system impedance is dependent on the waveform of the current being injected into the electrode's terminal [81]. In other words, the grounding impedance is likely to be different depending on the injected current waveform. It is worth noting that the common injection currents used for characterizing grounding systems are dual-exponential, Heidler's [85] and step pulse current waveforms.

The grounding system impedance is represented in frequency domain by

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)}, \quad (2.6)$$

where $V(j\omega)$ is the phasor of the voltage at the top of the electrode with respect to the remote earth while $I(j\omega)$ is the phasor of the injected current. In doing so, the grounding system impedance is obtained for each single frequency within the working frequency interval. This impedance, which is also referred to as the grounding system harmonic impedance, is widely used by frequency domain methods to characterize the behavior of grounding systems.

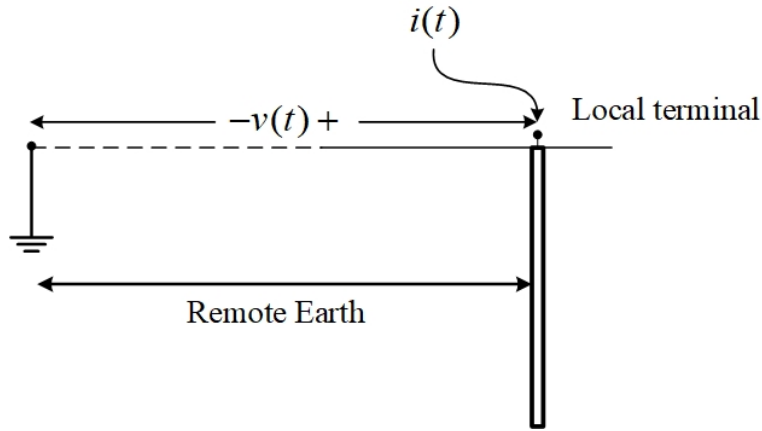


Figure 2.3. An illustration of the grounding system impedance

2.4 Comparison of Numerical Methods

Various computational techniques have been developed so far for modeling of electromagnetic problems whose analytical solution is hard or impossible to obtain. These mainly include the MoM [86, 87], the transmission line matrix (TLM) method [88], the FEM [89, 90], and the finite difference method (FDM) [9]. These methods have advantages and disadvantages depending on the application of interest and there is not a unique perfect method applicable to all electromagnetic problems. The FEM, TLM and FDM are based on the discretization of Maxwell's equations over the entire computational domain, while the MoM is based on the discretization of certain integral equations involving Green's function. The solution of integral equations by the MoM usually contains fewer unknowns as compared to the FDM and FEM. However, the matrix to be solved in the

MoM is much denser than those of the FDM and FEM. In many ways, TLM and FDM are identical having similar advantages and disadvantages, however, the TLM has a fundamental restriction on possible boundary shapes and requires more storage space for computation as compared to the FDM [91]. Despite their particularities, the selection of FEM or FDM is finely balanced for electromagnetic problems [91]. It mainly depends on programming skills, memory capacity, problem under investigation and meshing requirements.

The implementation of the above-mentioned numerical methods could be done in both frequency domain and time domain. Frequency domain methods provide solutions for a specific frequency, thus requiring repeated simulation execution to obtain the system response over a wide range of frequencies. These methods, although they can directly handle dispersive media, are not suited to problems involving non-linearity. For wide-band applications (or transient analysis), time domain methods are generally more efficient.

CHAPTER 3 ACCELERATED FDTD

3.1 Background

The finite-difference time domain (FDTD) was originally developed in 1966 by K.S. Yee [92]. With the advancement in computing resources and hardware, the FDTD became a very popular numerical technique for computational electrodynamics. It has been shown that the FDTD is exceptionally effective in modeling complex structures and media while its main appeal is its easy implementation as compared to other numerical electromagnetic methods [80]. Nevertheless, low-frequency electromagnetic problems are not easy to handle by conventional FDTD methods due to the restriction imposed by the Courant–Friedrich–Levy (CFL) condition [93]. Owing to this fact, different FDTD techniques have been proposed to overcome this drawback. The alternating direction implicit (ADI) FDTD is a technique that is not restricted by the Courant factor [94]. The accuracy of this method is, however and to a great extent, affected by the dispersion, truncation, and splitting errors as well as the asymmetry effect [95], [96]. To overcome the aforementioned drawbacks, the Crank–Nicolson (CN) based methods have been proposed in [93] and [97]. These methods are generally more accurate than ADI-FDTD methods and feature an unconditionally stable performance. The CN-based method proposed in [98] despite its negligible numerical anisotropy, requires inversion of a large sparse matrix through its full implicit discretization, which makes it prohibitively slow. To improve the performance of the CN-based FDTD methods, different algorithms, such as factorization splitting and fractional step, split step, or substep algorithms have been proposed [20]. Nevertheless, their time efficiency is not still affordable in particular for large-scale problems. It is worth noting that in addition to the aforementioned issues, the CN-FDTD methods rely on the perfect electric conductor (PEC) boundary condition, which appears as another limiting factor. Efforts have been made to include the perfectly matched layer (PML) boundary condition in the CN-FDTD methods [99] at the cost of significant increase of memory resource and, hence, expensive computational time.

In this chapter, two 3-D FDTD approaches for modeling of complex large-scale low frequency electromagnetic problems with particular reference to the GMDs and AC interference are pro-

posed. The proposed FDTD methods are rigorous, highly-efficient and accurate which makes them a promising choice for modelling GMDs and AC interference taking into account the exact complex 3D Earth structure.

3.1.1 Conventional Formulation

The FDTD method is, in general, based on the spatial and temporal discretization of Maxwell's differential equations [92]. In fact the electric and magnetic field components are sampled at discrete positions both in time and space. A grid is formed by dividing the three-dimensional problem geometry into cells. Figure 3.1 illustrates an FDTD grid composed of (N_x, N_y, N_z) cells. In this grid, each unit cell is called a Yee cell. The surface and internal geometry of the structure of interest are approximated using rectangular Yee cells with a spatial resolution determined by the size of the unit cell.

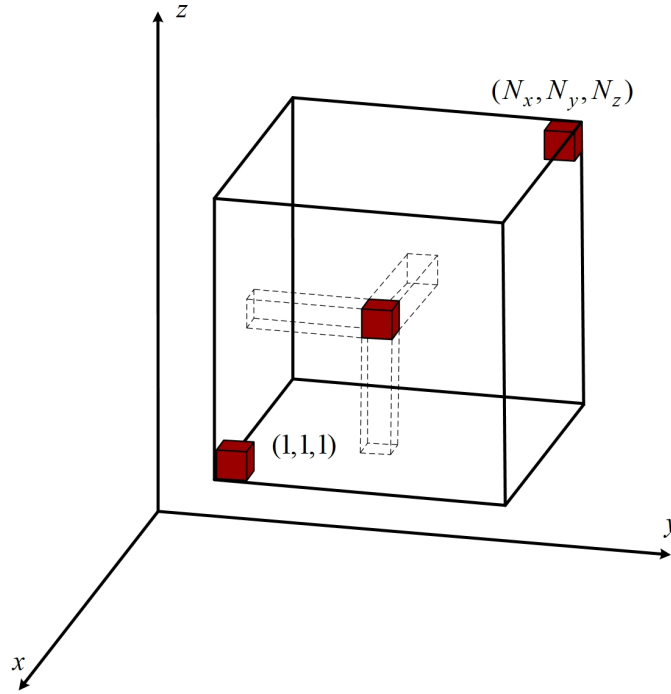


Figure 3.1. A three-dimensional FDTD computational space composed of (N_x, N_y, N_z) Yee cells [80]

There is a specific arrangement of discrete spatial positions for the field components within the Yee cell as shown in Figure 3.2. The electric field vector components ($\vec{E}$) are placed at the centers of the

edges of the Yee cells and oriented parallel to the respective edges, and the magnetic field vector components ($\vec{H}$) are placed at the centers of the faces of the Yee cells and are oriented normal to the respective faces. This provides a simple picture of three dimensional space being filled by an interlinked array of Faraday's law and Ampere's law contours. It can be easily noticed in Figure 3.2 that each magnetic field vector is surrounded by four electric field vectors that are curling around the magnetic field vector, thus simulating Faraday's law. Similarly, as shown in this figure each electric field vector is surrounded by four magnetic field vectors that are curling around the electric field vector, thus simulating Ampere's law.

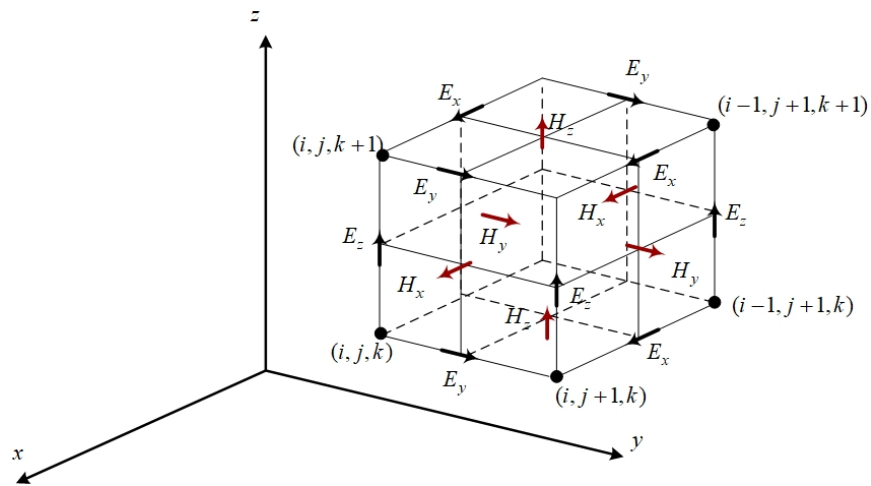


Figure 3.2. Arrangement of field components on the Yee cells

Yee cells have dimension Δx in the x - direction, Δy in the y - direction, and Δz in the z - direction. The actual positions of the field components with respect to the node $(1, 1, 1)$ being assumed as the origin can easily be calculated as shown in Table 3.1.

Table 3.1. The position of the field components in discretized domain with respect to the node $(1, 1, 1)$ being considered as the origin

x -direction	y -direction	z -direction	Field Component
k	j	$i + \frac{1}{2}$	E_x
k	$j + \frac{1}{2}$	i	E_y
$k + \frac{1}{2}$	j	i	E_z
$k + \frac{1}{2}$	$j + \frac{1}{2}$	i	H_x
$k + \frac{1}{2}$	j	$i + \frac{1}{2}$	H_y
k	$j + \frac{1}{2}$	$i + \frac{1}{2}$	H_z

Similar to spatial discretization, Yee's algorithm also utilizes central differencing in time for the $\vec{E}$ and $\vec{H}$ components and then solves them by using a leapfrog scheme [80] as shown in Figure 3.3. All of the $\vec{E}$ components in the modeled space are computed and stored in memory by using the previously computed values of $\vec{E}$ and the newly updated $\vec{H}$ field data. At the next step, $\vec{H}$ is recomputed based on the previously obtained $\vec{H}$ and the newly obtained $\vec{E}$. This process continues until the time-stepping is terminated.

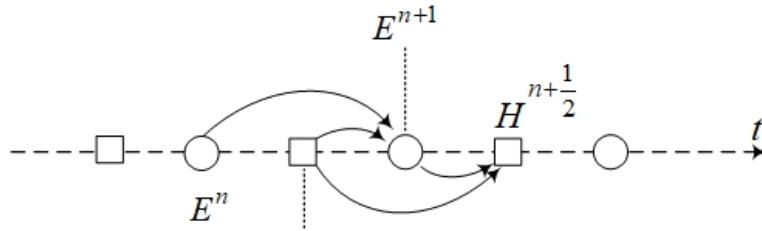


Figure 3.3. Leapfrog scheme: the temporal scheme of the FDTD method

Without losing any generality, we apply the FDTD method to Maxwell's magnetic field curl equation (3.1):

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \quad (3.1)$$

$$\vec{D} = \epsilon \vec{E} \quad (3.2)$$

$$\vec{J} = \sigma^e \vec{E} \quad (3.3)$$

where $\vec{D}$ denotes the electric field displacement vector, with $\vec{J}$ being the electric current density vector. Also ϵ and σ^e are permittivity and electrical conductivity, respectively, and the operator $\nabla \times$ holds for the curl of vector variables. Equation (3.1) can be represented in differential form so that the z -component of the electric field (E_z) reads [80]:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_z} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) - \sigma_z^e E_z - J_{iz} \quad (3.4)$$

where J_{iz} is the density of current flowing in the z -direction, and subscript z in ϵ_z and σ_z^e represents the electrical properties of the medium along the z -direction. Based on the field positioning (see Figure 3.2), the central difference method is applied to the time and space derivatives,

$$\begin{aligned} \frac{E_z^{n+1}(i, j, k) - E_z^n(i, j, k)}{\Delta t} &= \frac{1}{\Delta x \epsilon_z(i, j, k)} \times [H_y^{n+\frac{1}{2}}(i, j, k) - H_y^{n+\frac{1}{2}}(i-1, j, k)] - \frac{1}{\Delta y \epsilon_z(i, j, k)} \times \\ &[H_x^{n+\frac{1}{2}}(i, j, k) - H_x^{n+\frac{1}{2}}(i, j-1, k)] - \frac{\sigma_z^e(i, j, k)}{2\epsilon_z^e(i, j, k)} \times (E_z^{n+1}(i, j, k) + E_z^n(i, j, k)) - \frac{1}{\epsilon_z(i, j, k)} J_{iz}^{n+\frac{1}{2}} \end{aligned} \quad (3.5)$$

where (i, j, k) represents the spatial coordinates of the FDTD grid nodes, while E^n , H^n are the electric and magnetic fields at the n^{th} time-step. It is noted that (3.5) remains stable as long as the spatial $(\Delta x, \Delta y, \Delta z)$ and temporal (Δt) discretization lengths satisfy the following CFL condition:

$$\Delta t \leq \frac{1}{c \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}} \quad (3.6)$$

with c being the speed of light. The CFL condition can, however, prohibitively retard conventional FDTD approaches when dealing with LF problems, as the time-domain solutions require many iterations to satisfy the Nyquist condition, which, in turn, results in a lengthy computational time. Note that the time-step itself is restricted by the CFL condition and cannot take large values. To better understand this limitation, Table 3.2 shows the required number of time-steps for a frequency of 3.33 mHz, assuming two different spatial discretization lengths. As seen from this Table, the

number of iterations for both cell sizes is excessively high, making the conventional FDTD computationally expensive. It is worth noting that the discretized cell size must be a fraction of the wavelength to have a desirable solution resolution.

Table 3.2. Number of iterations for different cell sizes for a frequency of 3.33 mHz

Cell size and time steps	$\Delta x = \Delta y = 100 \text{ m}$	$\Delta x = \Delta y = 1000 \text{ m}$
	$\Delta t = 2.3 \times 10^{-7} \text{ sec}$	$\Delta t = 2.3 \times 10^{-6} \text{ sec}$
Number of iterations	1.41×10^9	1.41×10^8

3.1.2 Accelerated FDTD

With reference to Figure 3.4 and without losing any generality, let us consider a voltage source of amplitude V_s between two arbitrary nodes $(i, j, k - 1)$ and (i, j, k) along the z -direction with an internal resistance of R_s carrying a current of I ,

$$I = \frac{\Delta V + V_s}{R_s} \quad (3.7)$$

where ΔV is the potential difference between nodes (i, j, k) and $(i, j, k - 1)$ and can be expressed as a function of the electric field [80]:

$$\Delta V^{n+\frac{1}{2}} = \Delta z E_z^{n+\frac{1}{2}} = \Delta z \frac{E_z^{n1}(i, j, k) + E_z^{n+1}(i, j, k)}{2} \quad (3.8)$$

and

$$I^{n+\frac{1}{2}} = \Delta x \Delta y J_{iz}^{n+\frac{1}{2}} \quad (3.9)$$

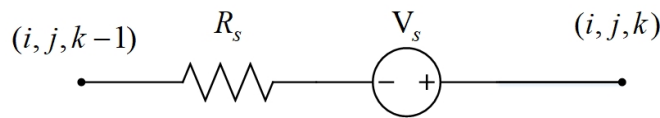


Figure 3.4. Voltage source model in FDTD

To accelerate calculations, the idea is to skip some time steps without losing accuracy. To this end, the time-stepping scheme in Figure 3.5 is adopted. In fact, as shown in this figure, the total simulation time, T , is first divided into equal subintervals of length ΔT , so that $K = T/\Delta T$ denotes the number of total subintervals. Then, for each subinterval, first, the electric and magnetic fields are calculated by the conventional FDTD for the first m time steps. Hence, if we assume $\zeta = 1, \dots, K$ to be the subinterval counter, for the ζ^{th} subinterval, as long as $t - (\zeta - 1)\Delta T < m\Delta t$, the EMFs at each time step will be calculated by (3.5). Then, we advance in time for $p\Delta t$ to arrive at the beginning of the next subinterval, i.e. $t = \zeta\Delta T$, where the EMFs are calculated using the Barycentric Lagrange extrapolation technique [100]. The Barycentric Lagrange extrapolation is a method used to estimate values of a function at certain points by constructing an interpolating polynomial based on a set of data points. In the context of FDTD simulations, this method can be used to speed up time steps by extrapolating the electromagnetic fields at a given time step to the next time step.

There are several advantages to using Barycentric Lagrange extrapolation in FDTD simulations:

- **Increased Accuracy:** Barycentric Lagrange extrapolation is known for its high accuracy, especially for well-conditioned problems. This means that the method can provide more accurate results for FDTD simulations than other methods.
- **Numerical Stability:** The Barycentric Lagrange extrapolation method is highly stable, meaning that the method is less likely to produce numerical instabilities compared to other methods.
- **Extrapolation Flexibility:** The Barycentric Lagrange extrapolation method can be used for a wide range of extrapolation problems, including non-uniform grids and irregularly spaced data points.

Thus, according to Figure 3.5, for the first m time steps of the ζ^{th} subinterval, if we count m and p from 1, we will have,

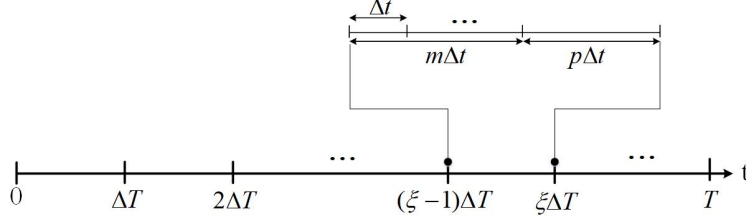


Figure 3.5. Time stepping scheme in the proposed FDTD

$$\begin{aligned}
E_z^{(p+m)(\zeta-1)+(n+1)}(i, j, k) = & C_{eze}(i, j, k) \times E_z^{(p+m)(\zeta-1)+(n)}(i, j, k) + C_{ezhy}(i, j, k) \times \\
& [H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) - H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i-1, j, k)] \\
& + C_{ezhx}(i, j, k) \times \\
& [H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) - H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j-1, k)] \\
& + C_{ezs}(i, j, k) \times V_s^{(p+m)(\zeta-1)+(n+1)}(i, j, k) \quad (3.10)
\end{aligned}$$

where

$$C_{eze}(i, j, k) = \frac{2\epsilon_z(i, j, k) - \Delta t \sigma_z^e - \frac{\Delta t \Delta z}{R_s \Delta x \Delta y}}{2\epsilon_z(i, j, k) + \Delta t \sigma_z^e + \frac{\Delta t \Delta z}{R_s \Delta x \Delta y}} \quad (3.11)$$

$$C_{ezhy}(i, j, k) = \frac{2\Delta t}{(2\epsilon_z(i, j, k) + \Delta t \sigma_z^e(i, j, k) + \frac{\Delta t \Delta z}{R_s \Delta x \Delta y}) \Delta x} \quad (3.12)$$

$$C_{ezhx}(i, j, k) = -\frac{2\Delta t}{(2\epsilon_z(i, j, k) + \Delta t \sigma_z^e(i, j, k) + \frac{\Delta t \Delta z}{R_s \Delta x \Delta y}) \Delta y} \quad (3.13)$$

$$C_{ezs}(i, j, k) = -\frac{2\Delta t}{(2\epsilon_z(i, j, k) + \Delta t \sigma_z^e(i, j, k) + \frac{\Delta t \Delta z}{R_s \Delta x \Delta y}) R_s \Delta x \Delta y} \quad (3.14)$$

where $E_z^{(p+m)(\zeta-1)+(n+1)}(i, j, k)$ is the electric field at time $[(p+m)(\zeta-1)+(n+1)]\Delta t$ which is updated using (3.5). It is noted that n is an integer number which counts 0 to $m-1$ and $p = \frac{(\Delta T - m\Delta t)}{\Delta t}$ (see Figure 3.5). The same time-stepping scheme of Figure 3.5 was used for the convolutional perfectly matched layer (CPML) cells; thus, we have [80],

$$E_z^{(p+m)(\zeta-1)+(n+1)}(i, j, k) = E_z^{(p+m)(\zeta-1)+n}(i, j, k) + C_{\psi_{ezx}}(i, j, k) \times \\ \psi_{ezx}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) + C_{\Psi_{ezy}}(i, j, k) \times \psi_{ezy}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) \quad (3.15)$$

Also, ψ_{ezx} is calculated for the x -direction:

$$\psi_{ezx}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) = b_{ex} \psi_{ezx}^{(p+m)(\zeta-1)+(n-\frac{1}{2})}(i, j, k) \\ + \alpha_{ex} [H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) - H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i-1, j, k)] \quad (3.16)$$

where

$$\alpha_{ex} = \frac{\sigma_{pex}}{\Delta x (\sigma_{pex} k_{pex} + \alpha_{ex} k_{ex}^2)} [b_{ex} - 1] \quad (3.17)$$

$$b_{ex} = e^{(\frac{\sigma_{pex}}{k_{ex}} + \alpha_{ex}) \frac{\Delta t}{\epsilon_0}} \quad (3.18)$$

Similarly, for the y -direction ψ_{ezy} reads,

$$\Psi_{ezy}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) = b_{ey} \Psi_{ezy}^{(p+m)(\zeta-1)+(n-\frac{1}{2})}(i, j, k) + \alpha_{ex} [H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k) \\ - H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j-1, k)] \quad (3.19)$$

where

$$\alpha_{ey} = \frac{\sigma_{pey}}{\Delta y (\sigma_{pey} k_{pey} + \alpha_{ey} k_{ey}^2)} [b_{ey} - 1] \quad (3.20)$$

$$b_{ey} = e^{(\frac{\sigma_{pey}}{k_{ey}} + \alpha_{ey}) \frac{\Delta t}{\epsilon_0}} \quad (3.21)$$

$$\sigma_{psi}(\rho) = \left(\frac{\rho}{\delta}\right)^{n_{mpl}} \frac{n_{pml} + 1}{150\pi\sqrt{\epsilon_r\Delta l}} \sigma_{factor} \quad (3.22)$$

$$k_{ei}(\rho) = 1 + \left(\frac{\rho}{\delta}\right)^{n_{pml}} (k_{max} - 1) \quad (3.23)$$

$$\alpha_{ei}(\rho) = \alpha_{min} + (\alpha_{max} - \alpha_{min}) \left(1 - \frac{\rho}{\delta}\right) \quad (3.24)$$

The performance of the CPML is significantly affected by σ_{factor} , k_{max} , α_{max} , α_{min} , n_{pml} and the

thickness of the CPML layer [99]. Usually, σ_{factor} takes a value between 0.7–1.5, while k_{max} and α_{max} are selected in the range of 5–11 and 0–0.05, respectively. The thickness of the CPML is set to 8 cells, while n_{pml} takes a value of 2, 3, or 4. As stated earlier, the frequency of a geomagnetic field source is in the order of mHz, so the resulting GS-EMFs exhibit slow variations. Owing to this fact, with an appropriate choice of ΔT and m , one can skip p iterations and hence accelerate the calculations. To this end, we advance in time and calculate the GS-EMFs at the beginning of each subinterval by applying the Barycentric Lagrange extrapolation method given in [100]. Hence, the electric fields at $t = \zeta \Delta t$ are obtained as

$$\begin{aligned}
 E_z^{(p+m)(\zeta)}(i, j, k) = & C_{eze}(i, j, k) \times \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n, \Delta t) E_z^{(p+m)(\zeta-1)+(n+1)}(i, j, k)}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind, \Delta t)}{(p+m-ind-2)\Delta t}} + C_{ezhy}(i, j, k) \times \\
 & \left[\frac{\sum_{n=0}^{m-1} \frac{\Psi(n, \Delta t) H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k)}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind, \Delta t)}{(p+m-ind-1)\Delta t}} - \frac{\sum_{n=0}^{m-1} \frac{\Psi(n, \Delta t) H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i-1, j, k)}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind, \Delta t)}{(p+m-ind-1)\Delta t}} \right] \\
 & + C_{ezhx}(i, j, k) \times \\
 & \left[\frac{\sum_{n=0}^{m-1} \frac{\Psi(n, \Delta t) [H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k)]}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind, \Delta t)}{(p+m-ind-1)\Delta t}} - \frac{\sum_{n=0}^{m-1} \frac{\Psi(n, \Delta t) H_x^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j-1, k)}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind, \Delta t)}{(p+m-ind-1)\Delta t}} \right] \quad (3.25)
 \end{aligned}$$

Moreover, in the CPML region:

$$\begin{aligned}
 E_z^{(p+m)(\zeta)}(i, j, k) = & E_z^{(p+m)(\zeta)}(i, j, k) + C_{\psi ezx}(i, j, k) \times \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n, \Delta t) \psi_{ezx}^{(p+m)(\zeta-1)+(n+1)}(i, j, k)}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind, \Delta t)}{(p+m-ind-1)\Delta t}} \\
 & + C_{\Upsilon ezy}(i, j, k) \times \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n, \Delta t) \psi_{ezy}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}(i, j, k)}{(p+m-n-1)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind, \Delta t)}{(p+m-ind-1)\Delta t}} \quad (3.26)
 \end{aligned}$$

where $E_z^{(p+m)(\zeta)}$ is the electric field at $(p+m)\zeta\Delta t$ while $H_y^{(p+m)(\zeta-1)+(n+\frac{1}{2})}$ is the magnetic field at $(p+m)(\zeta-1)+(n+\frac{1}{2})\Delta t$. The parameters $\Upsilon(n, \Delta t)$ and $\Psi(n, \Delta t)$ are calculated as follows [100]:

$$\Psi(n, \Delta t) = \frac{1}{\prod_{k=1, k \neq n}^{k=m} (n - k - \frac{1}{2})\Delta t} \quad (3.27)$$

$$\Upsilon(n, \Delta t) = \frac{1}{\prod_{k=1, k \neq n}^{k=m} (n - k)\Delta t} \quad (3.28)$$

3.1.3 Thin Wire Approximation in the Accelerated FDTD Method

Thin wire approximation is required for modeling of transmission line phase conductors. In the FDTD modelling, the thin wire formalism is solved in a linear, isotropic and lossy medium. In this context, it is well-known that the auxiliary equations of Holland's formalism can be expressed as follows [101, 102]:

$$L\left(\frac{\partial I}{\partial t} + v^2 \frac{\partial Q}{\partial r}\right) + RI = E_r(\rho) \quad (3.29)$$

$$\frac{\partial Q}{\partial t} + \frac{\sigma}{\epsilon}Q + \frac{\partial I}{\partial r} = 0 \quad (3.30)$$

where v is the wave propagation velocity in the medium surrounding the wire. The wire is oriented in a direction denoted by r . Equation (3.29) introduces the wire coupling with the tangential electric field E_r at the radial distance ρ , from the wire. The unknowns I and Q are, respectively, the current and the charge per unit length on the wire. The resistance R per unit length accounts for resistive losses along the wire. The inductance per unit length L is defined by

$$L = \frac{\mu_0}{2\pi} \ln \frac{\rho}{a} \quad (3.31)$$

with a being the conductor radius. Now, let us assume that the wire is divided into N_s segments, all equal to Δl . A wire segment is numbered κ , hence, the first segment is numbered $\kappa = 1$. Therefore, equation (3.29) can be written as [101],

$$L_\kappa \left(\frac{\partial I}{\partial t} + v^2 \frac{\partial Q}{\partial r} \right) + RI = \langle E_\kappa \rangle \quad (3.32)$$

where $\langle E_\kappa \rangle$ and L_κ are average electric component and average in-cell inductance in the κ^{th} segment, respectively. The calculation procedure of $\langle E_\kappa^n \rangle$ and L_κ are presented in [101]. It is more convenient to solve voltage-current set of equation rather than charge-current equation. To this aim, the electric charge should be obtained at each wire node location. We consider an equivalent capacitance for each segment κ . This equivalent capacitance (C_κ) is the summation of the capacitances per unit length (Cl_κ) associated with the in-cell inductances (L_κ) of each side of the node (i.e., $\kappa-$ and $\kappa+$),

$$C_\kappa = (Cl_{\kappa+} + Cl_{\kappa-}) \frac{\Delta l}{2} = \left(\frac{1}{v^2 L_{\kappa-}} + \frac{1}{v^2 L_{\kappa+}} \right) \frac{\Delta l}{2} \quad (3.33)$$

with the well-known relations

$$Q_{\kappa-} = Cl_{\kappa-} V_\kappa, Q_{\kappa+} = Cl_{\kappa+} V_\kappa \quad (3.34)$$

the charge per unit length Q is substituted by the voltage V in equations (3.29) and (3.30). Thus, the new equations ((3.35) and (3.36)) are obtained,

$$L_\kappa \left(\frac{\partial I}{\partial t} + \frac{\partial V}{\partial r} \right) + RI_\kappa = \langle E_\kappa \rangle \quad (3.35)$$

$$C_\kappa \left(\frac{\partial V_\kappa}{\partial t} + \frac{\sigma}{\epsilon} V_\kappa \right) + I_{\kappa+} - I_{\kappa-} = 0. \quad (3.36)$$

By applying FDTD on (3.35) and (3.36), we have,

$$I_{\kappa+\frac{1}{2}}^{(n+\frac{1}{2})} = b_{1I,\kappa} I_{\kappa+\frac{1}{2}}^{(n-\frac{1}{2})} - b_{2I,\kappa} \left(\frac{\Delta t}{L_\kappa \Delta l} (V_{\kappa+1}^n - V_\kappa^n - \frac{\Delta t}{L_\kappa} \langle E_\kappa^n \rangle) \right) \quad (3.37)$$

$$V_\kappa^n = b_{1V} V_\kappa^n - \frac{b_{2V} \Delta t}{C_\kappa} (I_{\kappa+\frac{1}{2}}^{(n+\frac{1}{2})} - I_{\kappa-\frac{1}{2}}^{(n+\frac{1}{2})}) \quad (3.38)$$

where

$$b_{1I,\kappa} = \frac{1 - \frac{R\Delta t}{2L_\kappa}}{1 + \frac{R\Delta t}{2L_\kappa}}, b_{2I,\kappa} = \frac{1}{1 + \frac{R\Delta t}{2L_\kappa}} \quad (3.39)$$

$$b_{1V} = \frac{1 - \frac{\sigma^e \Delta t}{2\epsilon}}{1 + \frac{\sigma^e \Delta t}{2\epsilon}}, b_{2V} = \frac{1}{1 + \frac{\sigma^e \Delta t}{2\epsilon}} \quad (3.40)$$

The constitutive parameters σ and ϵ are respectively, the electric conductivity and permittivity of the medium surrounding the wire. V_κ^n is the voltage at the beginning of the κ^{th} segment at $n\Delta t$ while $I_{\kappa+1/2}^n$ is the current at the middle of the κ^{th} segment at $+(n + \frac{1}{2})\Delta t$. Following the acceleration procedure presented in the preceding section, for the first m time steps of the ζ^{th} subinterval, (3.37) and (3.38) are calculated as,

$$I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})} = b_{1I,\kappa} I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n-\frac{1}{2})} - b_{2I,\kappa} \left(\frac{\Delta t}{L_k \Delta l} \right. \\ \left. (V_{\kappa+1}^{(p+m)(\zeta-1)+n} - V_\kappa^{(p+m)(\zeta-1)+n} - \frac{\Delta t}{L_\kappa} < E_\kappa^{(p+m)(\zeta-1)+n} >) \right) \quad (3.41)$$

and

$$V_\kappa^{(p+m)(\zeta+1)+n} = b_{1V} V_\kappa^{(p+m)(\zeta-1)+n} - \frac{b_{2V} \Delta t}{C_\kappa} (I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})} - I_{\kappa-\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}) \quad (3.42)$$

To consider coupling between the thin wire current and the surrounding area, the following modified FDTD solution of Maxwell's curl equation is adopted:

$$E_{u,j,s}^{(p+m)(\zeta-1)+(n+1)}(i, j, k) = E_{u,j,s}^{(p+m)(\zeta-1)+n}(i, j, k) + \\ \frac{\Delta t}{\epsilon_0 \epsilon_r} ((\nabla \times \vec{H})) \cdot \vec{u} - J_{u,j,s}^{(p+m)(\zeta-1)+(n+\frac{1}{2})} \quad (3.43)$$

where $E_{u,j,s}$ and $J_{u,j,s}$ are, respectively, the electric field and the current density in the u direction (u can be x , y or z), of the j^{th} thin wire segment located in the cell number s . The calculation procedure of $J_{u,j,s}$ has been presented in [101].

The proposed accelerated FDTD method implies that at $t = \zeta \Delta t$, the thin wire current is obtained as,

$$\begin{aligned}
 I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})} &= b_{1I,\kappa} \frac{\sum_{n=0}^{m-1} \frac{\Psi(n,\Delta t) I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n-\frac{1}{2})}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind,\Delta t)}{(p+m-ind-2)\Delta t}} - \\
 &b_{2I,\kappa} \left(\frac{\Delta t}{L_k \Delta l} \left(\frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) V_{\kappa,s}^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} - \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) V_{\kappa+1,s}^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} + \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) V_{\kappa+1,e}^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} \right. \right. \\
 &\quad \left. \left. + \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) V_{\kappa,e}^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} \right) - \frac{\Delta t}{L_\kappa} < \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) E_\kappa^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} > \right) \quad (3.44)
 \end{aligned}$$

Similarly, at $t = \zeta \Delta t$, the voltage within the cell of the same thin wire reads,

$$\begin{aligned}
 V_\kappa^{(p+m)(\zeta+1)+n} &= b_{1V} \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) V_\kappa^{(p+m)(\zeta-1)+n}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} - \frac{b_{2V} \Delta t}{C_\kappa} \\
 &\quad \left(\frac{\sum_{n=0}^{m-1} \frac{\Psi(n,\Delta t) I_{\kappa+\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind,\Delta t)}{(p+m-ind-2)\Delta t}} - \frac{\sum_{n=0}^{m-1} \frac{\Psi(n,\Delta t) I_{\kappa-\frac{1}{2}}^{(p+m)(\zeta-1)+(n+\frac{1}{2})}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind,\Delta t)}{(p+m-ind-2)\Delta t}} \right) \quad (3.45)
 \end{aligned}$$

Finally at $t = \zeta \Delta t$, the electric field in u direction (u can be x , y or z) for cell m containing segment j is updated as,

$$\begin{aligned}
 E_{u,j,s}^{(p+m)(\zeta-1)+(n+1)}(i,j,k) &= \frac{\sum_{n=0}^{m-1} \frac{\Upsilon(n,\Delta t) E_{u,j,s}^{(p+m)(\zeta-1)+n}(i,j,k)}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Upsilon(ind,\Delta t)}{(p+m-ind-2)\Delta t}} + \\
 &\quad \frac{\Delta t}{\epsilon_0 \epsilon_r} \frac{\sum_{n=0}^{m-1} \frac{\Psi(n,\Delta t) ((\nabla \times \vec{H}) \cdot \vec{u} - J_{u,j,s})^{(p+m)(\zeta-1)+(n+\frac{1}{2})}}{(p+m-n-2)\Delta t}}{\sum_{ind=0}^{m-1} \frac{\Psi(ind,\Delta t)}{(p+m-ind-2)\Delta t}} \quad (3.46)
 \end{aligned}$$

For the modeling of geomagnetic disturbances (GMD), the Finite-Difference Time-Domain (FDTD) method is adopted due to following reasons:

- Handling complex media: FDTD can handle complex materials and media which are found in geomagnetic disturbances
- Easy to implement: FDTD is relatively simple to implement and can be easily extended to include additional physics and effects.
- High-performance computing: FDTD algorithm can be paralleled, allowing for efficient computation of large-scale problems and distributed computing environments.
- Handling time-varying fields: FDTD is a time-domain method so that can handle time-varying fields very well. This is important in geomagnetic disturbance modeling where time-varying fields are generated by solar activity.

3.1.4 Spatially Filtered FDTD

The method proposed in the previous section is not suitable for modelling AC interference due to larger working frequency (50 Hz or 60 Hz). In fact, the extrapolation technique used in the previous method generates error for abrupt transition due to larger frequency, which causes FDTD to become unstable. To circumvent this issue, a spatially filtered FDTD can be used [103]. As known, the CFL limit for the 3D FDTD is derived from the numerical dispersion relation:

$$\sin \frac{\omega \Delta t}{2} = \frac{\Delta t}{\sqrt{\mu \epsilon}} \sqrt{\frac{\sin^2(K_x \frac{\Delta x}{2})}{\Delta x^2} + \frac{\sin^2(K_y \frac{\Delta y}{2})}{\Delta y^2} + \frac{\sin^2(K_z \frac{\Delta z}{2})}{\Delta z^2}} \quad (3.47)$$

For a real frequency to exist as a solution to this equation, the right-hand side has to be bounded by one. Hence,

$$\frac{\Delta t}{\sqrt{\mu \epsilon}} \sqrt{\left(\frac{1}{\Delta x^2}\right) + \left(\frac{1}{\Delta y^2}\right) + \left(\frac{1}{\Delta z^2}\right)} \leq 1 \quad (3.48)$$

The well-known CFL limit is derived from (3.47). The implicit assumption embedded in this derivation is that the numerical wavenumber components K_x , K_y and K_z are not bounded; hence, the sinusoidal terms in (3.47) can take any value from zero to one. On the other hand, if spatial

filtering is employed to limit the range of these components, the associated restriction on the time step can be significantly relaxed. In particular, spurious large wavenumbers have to be filtered from the response to avoid instability in the FDTD method. The elimination of large wavenumbers does not compromise the accuracy of the resulting solution. To demonstrate this, consider the case $\Delta x = \Delta y = \Delta z = \Delta$ and let the magnitude of the numerical wavenumber K be bounded by K_{max} . Then, by substituting $K_x = K \sin\theta \cos\phi$, $K_y = K \sin\theta \sin\phi$ and $K_z = K \cos\theta$ in (3.47), the maximum value of the right-hand side of (3.47) can be found. Bounding this value by one yields the condition:

$$\Delta t \leq \frac{\Delta \sqrt{\mu\epsilon}}{\sqrt{3}} \frac{1}{\sin(\frac{K_{max}\Delta}{2\sqrt{3}})} = \Delta t_{CFL} \times CE \quad (3.49)$$

where Δt_{CFL} is the CFL limit in 3D FDTD and CE is the CFL extension factor, which is found by $1/\sin(\frac{K_{max}\Delta}{2\sqrt{3}})$. In general, the CFL extension factor for an N-dimension (N=1,2,3) FDTD scheme is found to be:

$$CFL = \frac{1}{\sin(\frac{K_{max}\Delta}{2\sqrt{N}})} \sin(\frac{K_{max}\Delta}{2\sqrt{N}}) \quad (3.50)$$

The spatial filtering is performed at each FDTD iteration by first transforming all EM fields from spatial domain to the spatial-frequency domain, then applying a low-pass filter, and finally inverse-transforming all EM fields from the spatial-frequency domain to the spatial domain. The direct and inverse transforms are performed with a Discrete Cosine or Sine Transform (DCT/DST) instead of the Discrete Fourier Transform (DFT) used in [104]. In doing so, all arrays will involve real-valued elements instead of complex-valued arrays of the DFT. In addition, the spurious boundary effects stemming from the implicit DFT assumption of a periodic signal are eliminated. The DCT is usually preferred for cases with even field symmetry around the terminal boundaries while the DST is more suitable for cases having odd symmetry around terminal boundaries. As open boundaries are simulated with conductor-backed-PML, they also belong to the latter category [105].

Spatial filtering is implemented by making use of a spherical lowpass filter (see Fig. 3.6 whose radius is selected so that it satisfies the desired CFL extension factor. The filter function can be defined as [104]:

$$F(K) = \begin{cases} 1 & \text{for } \sqrt{K_x^2 + K_y^2 + K_z^2} \leq K_{max} \\ 0 & \text{otherwise} \end{cases}$$

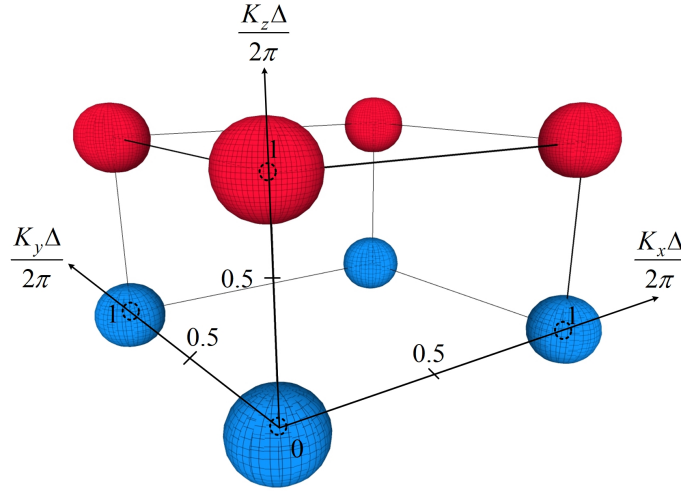


Figure 3.6. 3-D spatial filtering scheme [104]

For the AC Interference, the spatial filtered FDTD method has been chosen due to following reasons:

- SF-FDTD uses a spatial filter which makes it unconditionally stable and eliminates high-frequency noise, resulting in a more stable and accurate simulation
- SF-FDTD improves computational efficiency
- SF-FDTD effectively models thin wire structures with high precision

3.2 Conclusion

In This chapter, first an accelerated FDTD method has been proposed for modelling GMDs. The proposed method, unlike the conventional explicit FDTD approach, which is constrained by the upper limit of temporal discretization imposed by the CFL stability condition, is unconditionally stable since time steps and space steps are independently chosen. To increase the time-efficiency of the proposed approach for modeling extremely low frequency problems such as GMD, a time stepping scheme is adopted, which permits advancement in time for several time-steps without losing the accuracy. Then, an SF-FDTD method has been proposed for modeling AC interference. In this method, at each iteration, the spurious wave numbers that make the explicit conventional FDTD

method unstable are filtered. The SF-FDTD is unconditionally stable and exhibits better performance and numerical stability at low frequencies compared with the CN-based FDTD method and other implicit methods involving eigenvalue solutions. The proposed FDTD approaches enable modeling of large-scale complex propagation media such as horizontally, vertically, and vertically-horizontally stratified multilayer ground.

CHAPTER 4 METHOD of LINES

4.1 Background

The Method of Line (MoL) is an accepted and well-established technique for numerical modelling of transmission lines, waveguide structures, and scattering problems. Originally, the method was developed by mathematicians to solve boundary value problems in physics and mathematics [106–109]. It was introduced into the EM community around 1980 and further developed by Pregla et al. [110–113] and other researchers. The MoL is considered a special finite difference method (FDM) which does not rely on the discretization of the whole solution domain. Owing to this fact, the MoL is characterized by a better time-efficiency as compared to the conventional FDMs. In fact, the MoL essentially involves discretizing partial differential equations (PDEs) in only one or two dimensions, while the PDEs for the remaining dimensions are analytically solved. Therefore, the MoL offers a noniterative solution, and hence it is not prone to the convergence and stability numerical issues that are usually associated to other numerical methods. So far, the MoL has only been formulated for modeling of simple microstrip patch antenna with zero thickness metalization. In this dissertation, the MoL formulations are modified and extended to enable time-efficient modeling of large grounding grids buried in horizontally, vertically, and horizontally-vertically stratified multilayer soils.

4.2 2-D Method of Lines

4.2.1 Grounding Electrode Buried in a Horizontally Stratified Soil

With reference to Figure 4.1, we assume a horizontal grounding electrode buried in a horizontally multilayer soil. The governing complex vector potentials for the considered system are [114];

$$\bar{A} = \tilde{a}_z \psi^e \quad (4.1)$$

$$\bar{F} = \tilde{a}_z \psi^h \quad (4.2)$$

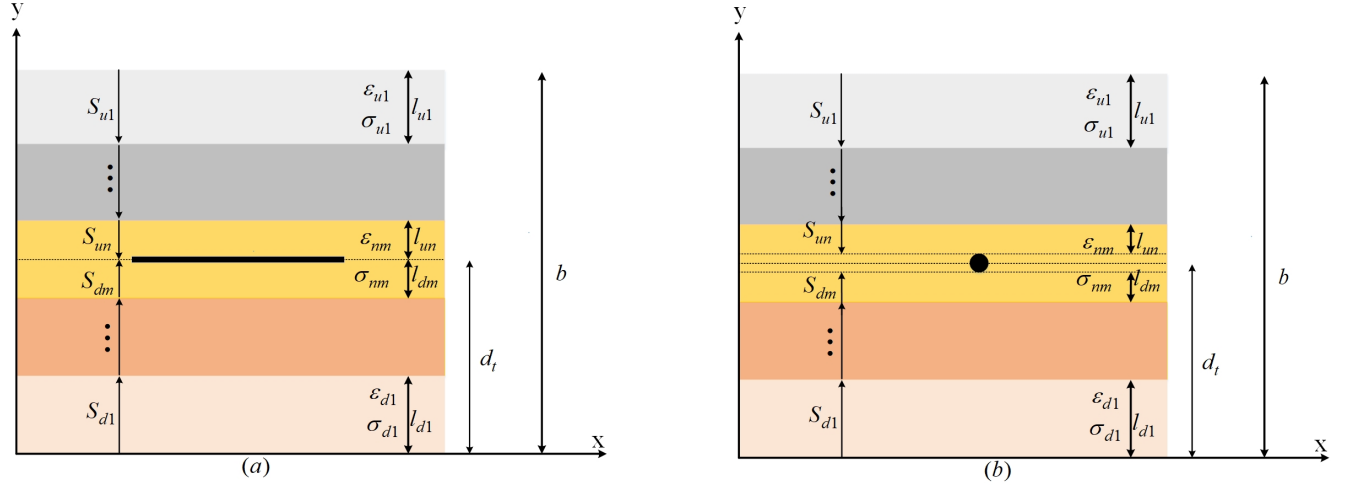


Figure 4.1. Horizontal electrode buried in a horizontally multilayer soil. Solution plane includes:
 (a) longitudinal section of the electrode and (b) electrode cross section

where $\bar{A}$ and $\bar{F}$, respectively, denote the magnetic and electric vector potentials with ψ^e and ψ^h being the scalar potentials for transverse magnetic and electric fields along z -direction (i.e., TMz mode and TEz mode). The following equations constitute the relation between the electric ($\bar{E}$) and magnetic ($\bar{H}$) fields and the vector potentials;

$$\bar{E} = -\nabla \times \bar{F} + \frac{1}{j\omega\epsilon} \nabla \times \nabla \times \bar{A} \quad (4.3)$$

$$\bar{H} = \nabla \times \bar{A} + \frac{1}{j\omega\mu} \nabla \times \nabla \times \bar{F} \quad (4.4)$$

Note that the complex dielectric constant is $\epsilon = \epsilon' - j\epsilon''$ with $\epsilon'' = \sigma/\omega\epsilon_0$ in which σ and ϵ_0 are electric conductivity and free space electric permittivity, respectively. Solving equations (4.3) and (4.4) yields all electric and magnetic field components based on the scalar potentials (ψ^e , ψ^h) as,

$$E_x = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi^e}{\partial z \partial x} - \frac{\partial \psi^h}{\partial y} \quad (4.5)$$

$$H_x = \frac{1}{j\omega\mu} \frac{\partial^2 \psi^h}{\partial z \partial x} + \frac{\partial \psi^e}{\partial y} \quad (4.6)$$

$$E_y = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi^e}{\partial z \partial y} + \frac{\partial \psi^h}{\partial x} \quad (4.7)$$

$$H_y = \frac{1}{j\omega\mu} \frac{\partial^2 \psi^h}{\partial z \partial y} - \frac{\partial \psi^e}{\partial x} \quad (4.8)$$

$$E_z = \frac{-1}{j\omega\epsilon} \left(\frac{\partial^2 \psi^e}{\partial x^2} + \frac{\partial^2 \psi^e}{\partial y^2} \right) \quad (4.9)$$

$$H_z = \frac{-1}{j\omega\mu} \left(\frac{\partial^2 \psi^h}{\partial x^2} + \frac{\partial^2 \psi^h}{\partial y^2} \right). \quad (4.10)$$

To proceed with a two-dimensional solution, the problem is assumed to be infinite along z -direction. Note that if 2D solution plane includes longitudinal section of the electrode (see Figure 4.1 (a)) and where there is no variation in the problem geometry and field distributions in z -direction propagation constant can be set to zero. Hence, EM waves propagate along z -direction with a factor of $e^{j(\omega t - \gamma z)}$, where γ is the complex propagation constant. Therefore, the EM field components are simplified to:

$$E_x = \frac{-\gamma}{\omega\epsilon_0\epsilon} \frac{\partial \psi^e}{\partial x} - \frac{\partial \psi^h}{\partial y} \quad (4.11)$$

$$H_x = \frac{-\gamma}{\omega\mu_0} \frac{\partial \psi^h}{\partial x} - \frac{\partial \psi^e}{\partial y} \quad (4.12)$$

$$E_y = \frac{-\gamma}{\omega\epsilon_0\epsilon} \frac{\partial \psi^e}{\partial y} + \frac{\partial \psi^h}{\partial x} \quad (4.13)$$

$$H_y = \frac{-\gamma}{\omega\mu_0} \frac{\partial \psi^h}{\partial y} - \frac{\partial \psi^e}{\partial x} \quad (4.14)$$

$$E_z = \frac{\epsilon k_0^2 - \gamma^2}{j\omega\epsilon_0\epsilon} \psi^e \quad (4.15)$$

$$H_z = \frac{\epsilon k_0^2 - \gamma^2}{j\omega\mu_0} \psi^h. \quad (4.16)$$

According to equations (4.11)-(4.16), to calculate EM field components, one requires scalar potentials. It is known that the scalar potentials satisfy the Helmholtz equation in homogeneous media, i.e.,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + (k^2 - \gamma^2) \psi = 0 \quad (4.17)$$

where ψ can be either ψ^e or ψ^h also, $k^2 = \epsilon k_0^2$ and $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$.

The MoL solution to equations (4.11)-(4.16) is as follows [115]: first the EM fields are discretized along the x -direction, then E_z is calculated along the e -lines (solid black lines in Figure 4.2) while

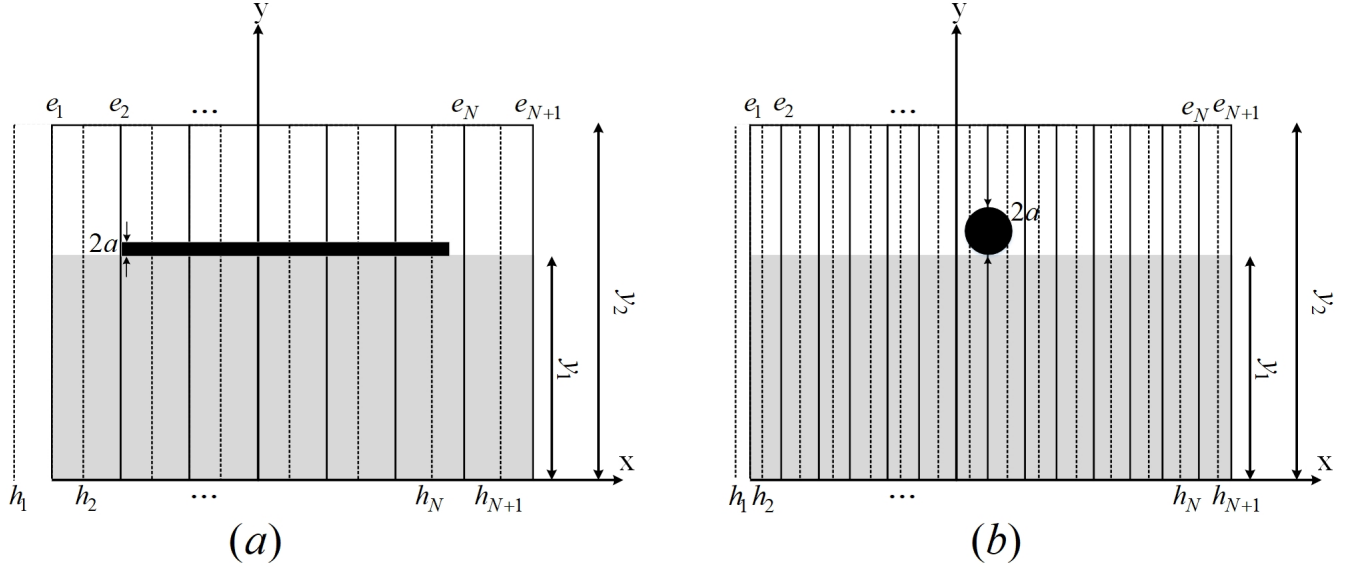


Figure 4.2. The cross section view of the problem shown in Figure 4.1 with discretization lines (solid black lines for E_z and dash black lines for H_z). Solution plane includes: (a) longitudinal section of the electrode, and (b) electrode cross section

H_z is calculated along the h -lines (dash black lines in Figure 4.2). The selection of these lines is done in a way to have a desired resolution for the calculation of EM fields within the solution space and in the corners. Using backward difference formula, the scalar potentials in equation (4.14) can now be replaced by a set of discretized scalar potentials of $[\psi_1, \psi_2, \dots, \psi_N]$ along the e -lines and h -lines,

$$E_x = \frac{-\gamma}{\omega \epsilon_0 \epsilon_r} \frac{[D]}{h} \psi^e - \frac{\partial \psi^h}{\partial y} \quad (4.18)$$

$$E_z = \frac{\epsilon_r k_0^2 - \gamma^2}{j \omega \epsilon_0 \epsilon_r} \psi^e \quad (4.19)$$

$$H_x = \frac{-\gamma}{\omega \mu_0} \frac{[D]^t}{h} \psi^h - \frac{\partial \psi^e}{\partial y} \quad (4.20)$$

$$H_z = \frac{\epsilon_r k_0^2 - \gamma^2}{j \omega \mu_0} \psi^h \quad (4.21)$$

where D is the discretized matrix for differentiation:

$$D = \begin{bmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & & \\ 0 & -1 & 1 & \\ \vdots & & \ddots & \ddots \\ & & & -1 & 1 \\ 0 & & & & -1 \end{bmatrix}. \quad (4.22)$$

Using the same procedure, we can discretize the Helmholtz equation given by (4.17) as follows:

$$\frac{\partial^2 \psi^e}{\partial y^2} - \left(\frac{[D]^t [D]}{h^2} + \gamma^2 - \epsilon k_0^2 \right) \psi^e = 0 \quad (4.23)$$

$$\frac{\partial^2 \psi^h}{\partial y^2} - \left(\frac{[D][D]^t}{h^2} + \gamma^2 - \epsilon k_0^2 \right) \psi^h = 0. \quad (4.24)$$

Equations (4.23) and (4.24) are, respectively, a set of N and $N + 1$ coupled equations due to the tridiagonal structure of $[D][D]^t$ and $[D]^t[D]$. These equations are decoupled by means of two transformation matrices $[T^e]$ and $[T^h]$ which, respectively, make $[D][D]^t$ and $[D]^t[D]$ diagonal such that:

$$[T^e]^{-1} [D]^t [D] [T^e] = [\lambda_e^2] \quad (4.25)$$

$$[T^h]^{-1} [D][D]^t [T^h] = [\lambda_h^2]. \quad (4.26)$$

It is worth noting that $[T^h]$ and $[T^e]$ are orthogonal matrices. Also the elements λ_e^2 and λ_h^2 of the diagonal matrices $[\lambda_e^2]$ and $[\lambda_h^2]$ are the eigenvalues of $[D][D]^t$ and $[D]^t[D]$, respectively.

To calculate eigenvectors and eigenvalues, let us assume that P represents either $[D][D]^t$ or $[D]^t[D]$.

$$P = \begin{bmatrix} p^r & -1 & \dots & 0 \\ -1 & 2 & -1 & \\ 0 & -1 & 2 & -1 \\ \vdots & & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ 0 & & & & -1 & p^l \end{bmatrix}. \quad (4.27)$$

where $p^r = p^l = 2$ for Dirichlet condition and $p^r = p^l = 1$ for Neumann condition. Therefore, the general eigenvalue problem can be constructed as:

$$(P - \lambda_k^2 I)T = 0 \quad (4.28)$$

where T can represent either T^e or T^h and should be in the following form:

$$T = \begin{bmatrix} t_0^1 & \dots & t_0^k \\ \vdots & \ddots & \vdots \\ t_i^1 & \dots & t_i^k \end{bmatrix} \quad (4.29)$$

Since P is a tridiagonal matrix, equation (4.28) yields the following set of second-order difference equations:

$$-t_{i-1}^{(k)} + (2 - \lambda_k^2)t_i^{(k)} - t_{i+1}^{(k)} = 0 \quad (4.30)$$

It is noted that only the first and the last equation of the equation set (4.28), which are associated with boundary conditions, are different from the general form given in (30). Once the boundary conditions are known, the solution of equation (4.28) provides all unknown eigenvectors and eigenvalues. T^e is constructed from the eigenvectors calculated for Dirichlet condition while T^h is constructed from the eigenvectors calculated for Neumann condition. To diagonalize the differential matrices ($[D][D]^t$ and $[D]^t[D]$) in order to uncouple the Helmholtz equation, we use the orthogonal matrices $[T^e]$ and $[T^h]$ as shown in Figure 4.3. In the new domain, $(\psi-E-H)$ are represented by $(\phi-e-h)$. We now introduce the following transformed vector $[\phi]$:

$$[\psi^e] = [T^e]\phi^e \quad (4.31)$$

$$[\psi^h] = [T^h]\phi^h. \quad (4.32)$$

Substituting transformed scalar potentials into Helmholtz equation yields:

$$\frac{\partial^2 \phi^e}{\partial y^2} - \left(\frac{[T^e]^t [D]^t [D] [T^e]}{h^2} + (\gamma^2 - \epsilon k_0^2) [I] \right) \phi^e = 0 \quad (4.33)$$

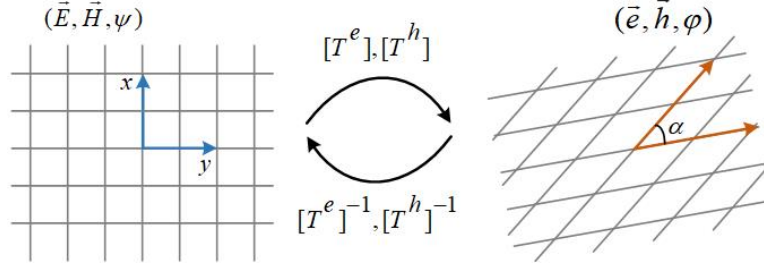


Figure 4.3. EM fields and vector potentials in ψ - E - H - and ϕ - e - h -domain

$$\frac{\partial^2 \phi^h}{\partial y^2} - \left(\frac{[T^h]^t [D] [D]^t [T^h]}{h^2} + (\gamma^2 - \epsilon k_0^2) [I] \right) \phi^h = 0. \quad (4.34)$$

Let $\delta = [T^h]^t [D] [T^e]$, then equations (4.33) and (4.34) can be simplified to:

$$\frac{\partial^2 \phi^e}{\partial y^2} - \gamma_e^2 \phi^e = 0 \quad (4.35)$$

$$\frac{\partial^2 \phi^h}{\partial y^2} - \gamma_h^2 \phi^h = 0 \quad (4.36)$$

where

$$[\gamma_e]^2 = \frac{[\delta]^t [\delta]}{h^2} + (\gamma^2 - \epsilon_r k_0^2) [I] \quad (4.37)$$

and

$$[\gamma_h]^2 = \frac{[\delta] [\delta]^t}{h^2} + (\gamma^2 - \epsilon_r k_0^2) [I]. \quad (4.38)$$

Since the system of equations (4.35) and (4.36) are uncoupled, they can be solved analytically for each homogeneous region. To this end, we can use the solution procedure usually adopted to solve the telegraph equation, so the system can be expressed as a relation between scalar potentials ($\phi^{e,h}$) and their normal derivatives within each homogeneous layer from $y = y_i$ to $y = y_{i+1}$. This solution is repeated for all homogeneous layers to find the transformed potential $[\phi]$ from one homogeneous layer to another [116]. Thus, with reference to Figure 4.1, to map the scalar potentials $[\phi]$ of the upper boundary of the solution domain to the upper edge of the soil-electrode interface, the following telegraph equations are solved:

$$\begin{bmatrix} \phi^e \\ \frac{\partial \phi^h}{\partial y} \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}_{|y=d_t}^+ = [S_1] \begin{bmatrix} \phi^e \\ \frac{\partial \phi^h}{\partial y} \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}_{|y=b} \quad (4.39)$$

where

$$[S_1] = [S_{un}][T_{un}]^{-1}[T_{un-1}][S_{un-1}] \dots [S_{u1}] \quad (4.40)$$

and

$$[S_{ui}] = [S]_{|y=-l_{ui}} \quad (4.41)$$

with $i = 1, 2, \dots, n$. Also, for each layer, the potential transfer matrix ($[S]$) can be expressed as,

$$[S] = \begin{bmatrix} S' & S'' \end{bmatrix} \quad (4.42)$$

where

$$[S'] = \begin{bmatrix} \cosh[\gamma_1^e(y)] & 0 \\ 0 & \cosh[\gamma_1^h(y)] \\ \gamma_1^e \sinh[\gamma_1^e(y)] & 0 \\ 0 & \frac{1}{\gamma_1^h} \sinh[\gamma_1^h(y)] \end{bmatrix} \quad (4.43)$$

and

$$[S''] = \begin{bmatrix} \frac{1}{\gamma_1^e} \sinh[\gamma_1^e(y)] & 0 \\ 0 & \gamma_1^h \sinh[\gamma_1^h(y)] \\ \cosh[\gamma_1^e(y)] & 0 \\ 0 & \cosh[\gamma_1^h(y)] \end{bmatrix}. \quad (4.44)$$

Similarly, the potential at the lower edge of the soil-electrode interface can be obtained from the scalar potentials $[\phi]$ associated to the lowest boundary of the solution domain (i.e., $y = 0$ in Figure 4.2) using the following telegraph equations:

$$\begin{bmatrix} \phi^e \\ \frac{\partial \phi^h}{\partial y} \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}_{|y=d_t}^- = [S_2] \begin{bmatrix} \phi^e \\ \frac{\partial \phi^h}{\partial y} \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}_{|y=0} \quad (4.45)$$

and

$$[S_2] = [S_{dm}][T_{dm}]^{-1}[T_{dm-1}][S_{dm-1}] \dots [S_{d1}] \quad (4.46)$$

where with reference to Figure 1,

$$[S_{dq}] = [S]_{|y=l_{dq}} \quad (4.47)$$

with $q = 1, 2, \dots, m$. The computational domain can be truncated by different boundary conditions (open space or perfect conductors). In this paper, assuming the perfect electric conductor (PEC) as the boundary of the computational domain, the Neuman and Dirichlet boundary conditions are respectively applied to the magnetic and electric scalar potential. Please note that in any interface including metallization, boundary condition should be applied on tangential electric field. As a result, the potential matrix at $y = b$ can be written as,

$$[\phi]_{|y=b} = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix} \quad (4.48)$$

similarly, at $y = 0$,

$$[\phi]_{|y=0} = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}. \quad (4.49)$$

Once the ϕ - e - h -domain scalar potentials and their normal derivatives on the soil-electrode interface are calculated, one should resort to the boundary conditions to calculate ϕ - e - h -domain EM fields on the same interface. It is more straightforward to apply the EM field boundary conditions rather than those associated with the scalar potentials. To this aim, the ψ - E - H -domain EM field equations (4.18)-(4.21) should be mapped to the ϕ - e - h -domain by the same eigenvectors. In doing so, the ϕ - e - h -domain EM fields can be obtained as,

$$e_x = [T^h]^t [E_x] = \frac{-\gamma}{\omega \epsilon \epsilon_0 h} [\delta] [\phi^e] - \frac{\partial [\phi^h]}{\partial y} \quad (4.50)$$

$$j e_z = [T^e]^t j [E_z] = \frac{\epsilon k_0^2 - \gamma^2}{\omega \epsilon_r \epsilon_0} [\phi^e] \quad (4.51)$$

$$j h_z = [T^h]^t j [H_z] = \frac{\epsilon k_0^2 - \gamma^2}{\omega \mu_0} [\phi^h] \quad (4.52)$$

$$h_x = [T^e]^t [H_x] = \frac{\gamma}{\omega \mu_0 h} [\delta]^t [\phi^h] + \frac{\partial [\phi^e]}{\partial y}. \quad (4.53)$$

The above equations can be cast in the following matrix form:

$$\begin{bmatrix} e_x \\ j e_z \\ j h_z \\ h_x \end{bmatrix} = \begin{bmatrix} \frac{-\gamma}{\omega \epsilon h} [\delta] & -[I] & 0 & 0 \\ \frac{k^2 - \gamma^2}{\omega \epsilon} [I] & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k^2 - \gamma^2}{\omega \mu} [I] \\ 0 & 0 & [I] & \frac{\gamma}{\omega \mu h} [\delta]^t \end{bmatrix} \begin{bmatrix} \phi^e \\ \frac{\partial \phi^h}{\partial y} \\ \frac{\partial \phi^e}{\partial y} \\ \phi^h \end{bmatrix}. \quad (4.54)$$

Therefore, in ϕ - e - h -domain, the EM fields on the upper soil-electrode interface can be obtained using the ϕ - e - h -domain scalar potentials of the same interface,

$$\begin{bmatrix} e_x \\ j e_z \\ j h_z \\ h_x \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ 0 \\ \frac{\psi^e}{\partial y} \\ \phi^h \end{bmatrix} \quad (4.55)$$

where

$$[C_1] = \begin{bmatrix} \frac{-\gamma}{\omega\epsilon h}[\delta] & -[I] & 0 & 0 \\ \frac{k^2-\gamma^2}{\omega\epsilon}[I] & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k^2-\gamma^2}{\omega\mu}[I] \\ 0 & 0 & [I] & \frac{\gamma}{\omega\mu h}[\delta]^t \end{bmatrix} [S_1] = \begin{bmatrix} C'_{11} & C'_{12} \\ C'_{21} & C'_{22} \end{bmatrix}. \quad (4.56)$$

Similarly, for the lower soil-electrode interface,

$$\begin{bmatrix} e_x \\ je_z \\ jh_z \\ h_x \end{bmatrix} = C_2 \begin{bmatrix} 0 \\ 0 \\ \frac{\psi^e}{\partial y} \\ \phi^h \end{bmatrix} \quad (4.57)$$

where

$$[C_2] = \begin{bmatrix} \frac{-\gamma}{\omega\epsilon h}[\delta] & -[I] & 0 & 0 \\ \frac{k^2-\gamma^2}{\omega\epsilon}[I] & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k^2-\gamma^2}{\omega\mu}[I] \\ 0 & 0 & [I] & \frac{\gamma}{\omega\mu h}[\delta]^t \end{bmatrix} [S_2] = \begin{bmatrix} C''_{11} & C''_{12} \\ C''_{21} & C''_{22} \end{bmatrix}. \quad (4.58)$$

Since the first two rows of the potential matrix $[\phi]$ are zero, we can rewrite equation (4.55) as follows:

$$\begin{bmatrix} jh_z \\ h_x \end{bmatrix}_{y=d^+} = [C'_{22}][C'_{12}]^{-1} \begin{bmatrix} e_x \\ je_z \end{bmatrix}_{y=d^+} = [G_1] \begin{bmatrix} e_x \\ je_z \end{bmatrix}_{y=d^+} \quad (4.59)$$

$$\begin{bmatrix} jh_z \\ h_x \end{bmatrix}_{y=d^-} = [C''_{22}][C''_{12}]^{-1} \begin{bmatrix} e_x \\ je_z \end{bmatrix}_{y=d^-} = [G_2] \begin{bmatrix} e_x \\ je_z \end{bmatrix}_{y=d^-}. \quad (4.60)$$

The continuity of tangential electric field components on the interface yields:

$$\begin{bmatrix} e_x \\ e_z \end{bmatrix}_{y=d^+} = \begin{bmatrix} e_x \\ e_z \end{bmatrix}_{y=d^-}. \quad (4.61)$$

Considering equation (4.61) and by subtracting (4.59) from (4.60) we can write,

$$\begin{bmatrix} jh_z \\ h_x \end{bmatrix}_{y=d^+} - \begin{bmatrix} jh_z \\ h_x \end{bmatrix}_{y=d^-} = (G_1 - G_2) \begin{bmatrix} e_x \\ je_z \end{bmatrix}. \quad (4.62)$$

A discontinuity in the tangential component of the magnetic field intensity implies that the surface current flows in a direction perpendicular to this component of the field, therefore equation (4.62) becomes:

$$\begin{bmatrix} -jj_x \\ j_z \end{bmatrix} = (G_1 - G_2) \begin{bmatrix} e_x \\ je_z \end{bmatrix} = [G_0] \begin{bmatrix} e_x \\ je_z \end{bmatrix} \quad (4.63)$$

where j_x and j_z are surface currents in x - and z -direction respectively; G_0 can be obtained after some analytical manipulations as follows:

$$[G_0] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}. \quad (4.64)$$

The currents and electric field components can be now transferred back to the ψ - E - H domain to obtain the hybrid characteristics such as γ . Applying $[T]^{-1}$ and using orthogonality of $[T^e]$ and $[T^h]$, we can rewrite equation (4.63) as

$$\begin{bmatrix} -jJ_x \\ J_z \end{bmatrix} = \begin{bmatrix} [T^h] & 0 \\ 0 & [T^e] \end{bmatrix} [G_0] \begin{bmatrix} [T^h]^t & 0 \\ 0 & [T^e]^t \end{bmatrix} \begin{bmatrix} E_x \\ jE_z \end{bmatrix}. \quad (4.65)$$

Due to the fact that the tangential component of the magnetic field is continuous across an interface with a finite conductivity, equation (4.65) can be rewritten as follows:

$$\begin{bmatrix} [T^h][G_{11}]^{-1}[T^h]^t & [T^h][G_{12}]^{-1}[T^e]^t \\ [T^e][G_{21}]^{-1}[T^h]^t & [T^e][G_{22}]^{-1}[T^e]^t \end{bmatrix} \begin{bmatrix} 0 \\ -jJ_x \\ 0 \\ 0 \\ J_z \\ 0 \end{bmatrix} = \begin{bmatrix} E_{xi} \\ 0 \\ E_{xj} \\ jE_{zi} \\ 0 \\ jE_{zj} \end{bmatrix}. \quad (4.66)$$

The matrix equation of (4.66) can be divided into two sets of equations; one for the nonconducting lossy interface and one for the conductor interface. The equations associated to the conductor interface are given by:

$$\begin{bmatrix} [T^h][G_{11}]^{-1}[T^h]^t & [T^h][G_{12}]^{-1}[T^e]^t \\ [T^e][G_{21}]^{-1}[T^h]^t & [T^e][G_{22}]^{-1}[T^e]^t \end{bmatrix}_{\text{red}} \begin{bmatrix} -jJ_x \\ J_z \end{bmatrix}_{\text{red}} = \begin{bmatrix} 0 \end{bmatrix}. \quad (4.67)$$

A nontrivial solution of the above homogeneous linear matrix equation requires that the determinant of the matrix be equal to zero. Thus, the propagation constant is determined by solving the following determinant equation:

$$\left| \begin{bmatrix} [T^h][G_{11}]^{-1}[T^h]^t & [T^h][G_{12}]^{-1}[T^e]^t \\ [T^e][G_{21}]^{-1}[T^h]^t & [T^e][G_{22}]^{-1}[T^e]^t \end{bmatrix}_{\text{red}} \right| = \begin{bmatrix} 0 \end{bmatrix}. \quad (4.68)$$

Note that only the number of points on the electrode determines the size of the matrix of (4.68), and using the root seeking method, the complex propagation constant can be easily found. Once we solve equation (4.68), to determine the effective dielectric constant, the current distribution on the electrode, the potential functions ψ^e and ψ^h , the electric fields, magnetic fields, and the harmonic impedance can be computed.

4.2.2 Grounding Electrode Buried in a Vertically Stratified Soil

For the vertical stratified ground as shown in Figure 4.4, the procedure is generally similar to the horizontally stratified ground. The only difference is related to the way that $[T]$ is calculated. For the case of vertically stratified soil, the constitutive parameters are functions of x , so vector

potentials need to be in x direction, thus the Helmholtz equation becomes:

$$\epsilon(x) \frac{\partial}{\partial x} \left(\frac{1}{\epsilon(x)} \frac{\partial \psi^e}{\partial x} \right) + \frac{\partial^2 \psi^e}{\partial y^2} + (k^2) \psi^e = 0 \quad (4.69)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + (k^2) \psi = 0. \quad (4.70)$$

After discretization in x -direction,

$$\frac{\partial^2 \psi^e}{\partial y^2} - \left(\frac{[\epsilon_e][D]^t[\epsilon_h]^{-1}[D]}{h^2} - [\epsilon_e]k_0^2 \right) \psi^e = 0 \quad (4.71)$$

$$\frac{\partial^2 \psi^h}{\partial y^2} - \left(\frac{[D][D]^t}{h^2} - [\epsilon_h]k_0^2 \right) \psi^h = 0 \quad (4.72)$$

where $([\epsilon_e][D]^t[\epsilon_h]^{-1}[D] - h^2[\epsilon_e]k_0^2)$ and $([D][D]^t - h^2[\epsilon_h]k_0^2)$ are non-Hermitian tridiagonal matrices. The diagonal matrices $[\epsilon_e]$ and $[\epsilon_h]$ consist of dielectric constants respectively associated to the domains delimited by e-lines and h-lines shown in Figure 4.4. To solve the eigenvalue problems of (4.71) and (4.72) we use the numerical routines proposed in [117] which yields the transformation matrix of a given layer as well as the diagonal matrices of eigenvalues. Note that, modelling of electrodes with different radii in MoL depends on the electrode orientation. For vertical electrodes, it is simply done by discretizing the electrode in x -direction. However, for horizontal electrodes, by applying equations (4.71) and (4.72), the electrode is replaced by an inhomogeneous layer whose thickness is equal to the electrode radius (see Figure 4.5).

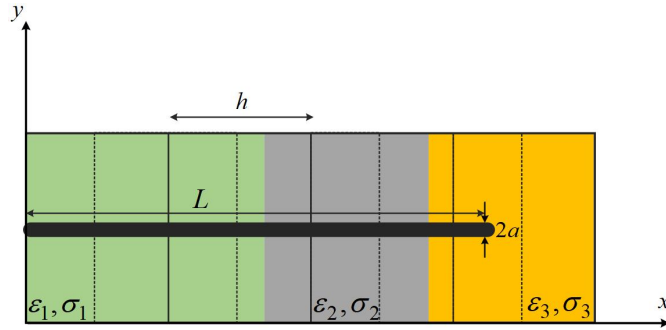


Figure 4.4. Cross section of the vertically stratified ground including the grounding electrode and the discretization lines; — lines for e_z and - - - for e_z

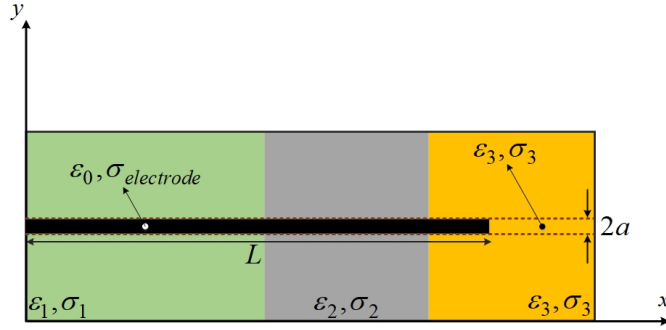


Figure 4.5. Replacement of horizontal electrodes by an additional nonuniform soil layer whose thickness is equal to the electrode's radius

4.3 3-D Method of Lines

In this section 3-D method of line formulation is presented in order to model grounding grid buried in a stratified multilayer soil.

4.3.1 Method of Line for Grounding Grid Buried in Stratified Multilayer Ground

With reference to Figure 4.6, and without loosing any generality, we assume a grounding grid buried in a horizontally multilayer soil. The MoL modelling in cylindrical coordinate is done by discretizing the modeling domain in ρ - and ϕ -directions. This also permits modelling of grounding grids buried in more complex soil structures such as vertically and vertically-horizontally stratified multilayer soil.

The modelling starts with the governing magnetic vector potential equation described by (4.73) [114];

$$\bar{A} = \tilde{a}_\rho \psi^\rho + \tilde{a}_\phi \psi^\phi \quad (4.73)$$

where $\bar{A}$ denotes the magnetic vector potential with ψ^ρ and ψ^ϕ being, respectively, the scalar potentials along ρ - and ϕ -directions. Note that since for a multilayer soil electromagnetic parameters can be function of ρ and ϕ , so the vector potential should be in ρ - and ϕ -directions. The following equations constitute the relation between the electric ($\bar{E}$) and magnetic ($\bar{H}$) fields and the scalar potential [114];

$$E_\rho = \frac{\partial}{\partial \rho} \frac{1}{\epsilon \rho} \frac{\partial \psi^\phi}{\partial \phi} + \frac{\partial}{\partial \rho} \frac{1}{\epsilon \rho} \frac{\partial \psi^\rho}{\partial \rho} + \frac{1}{\rho} \psi^\rho \quad (4.74)$$

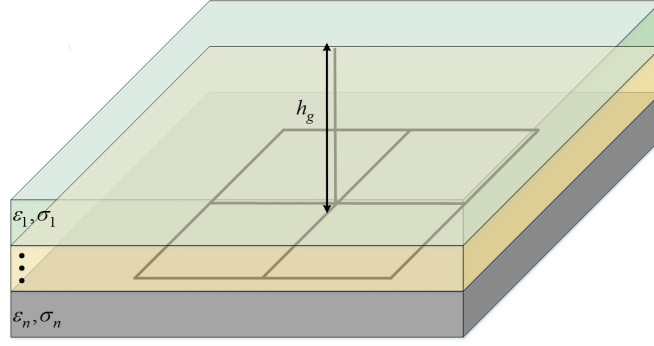


Figure 4.6. Grounding grid buried in a multilayer horizontally stratified soil

$$\tilde{H}_\rho = -j \frac{\partial \psi^\phi}{\partial z} \quad (4.75)$$

$$E_\phi = \frac{1}{\rho} \left(\frac{\partial}{\partial \phi} \frac{1}{\epsilon \rho} \frac{\partial \psi^\phi}{\partial \phi} + \frac{\partial}{\partial \phi} \frac{1}{\epsilon \rho} \frac{\partial \psi^\rho}{\partial \rho} + \rho \psi^\phi \right) \quad (4.76)$$

$$\tilde{H}_\phi = j \frac{1}{\rho} \frac{\partial \psi^\phi}{\partial z} \quad (4.77)$$

$$E_z = \frac{-1}{j \epsilon \rho} \left(\frac{\partial \tilde{H}_\rho}{\partial \phi} - \frac{\partial (\rho \tilde{H}_\phi)}{\partial \rho} \right) \quad (4.78)$$

$$\tilde{H}_z = \frac{-1}{j \rho} \left(\frac{\partial}{\partial \rho} (\rho E_\phi) - \frac{\partial E_\phi}{\partial \phi} \right) \quad (4.79)$$

where magnetic field components H_u are normalized by the wave impedance $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ according to $\tilde{H}_u = \eta_0 H_u$. To simplify the formulations, the coordinates and dimensions are normalized with the free space wave number k_0 according to $z = k_0 z$, $\rho = k_0 \rho$.

According to equations (4.74)-(4.79), one requires the scalar potentials to calculate electromagnetic field components. It is known that the vector potential satisfies the Helmholtz equation in inhomogeneous media, i.e.,

$$\nabla^2 A - \frac{1}{\epsilon} (\nabla \epsilon) \nabla \cdot A + k^2 A = 0 \quad (4.80)$$

Substituting from equation (4.73) in equation (4.80) yields:

$$\epsilon \rho \frac{\partial}{\partial \rho} \left(\frac{1}{\epsilon \rho} \frac{\partial \psi^\rho}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left(\frac{1}{\rho} \frac{\partial \psi^\rho}{\partial \phi} \right) + \frac{\partial^2 \psi^\rho}{\partial z^2} + k^2 \psi^\rho = 0 \quad (4.81)$$

$$\frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial \psi^\phi}{\partial \rho} \right) + \frac{\epsilon}{\rho} \frac{\partial}{\partial \phi} \left(\frac{1}{\epsilon \rho} \frac{\partial \psi^\phi}{\partial \phi} \right) + \frac{\partial^2 \psi^\phi}{\partial z^2} + k^2 \psi^\phi = 0 \quad (4.82)$$

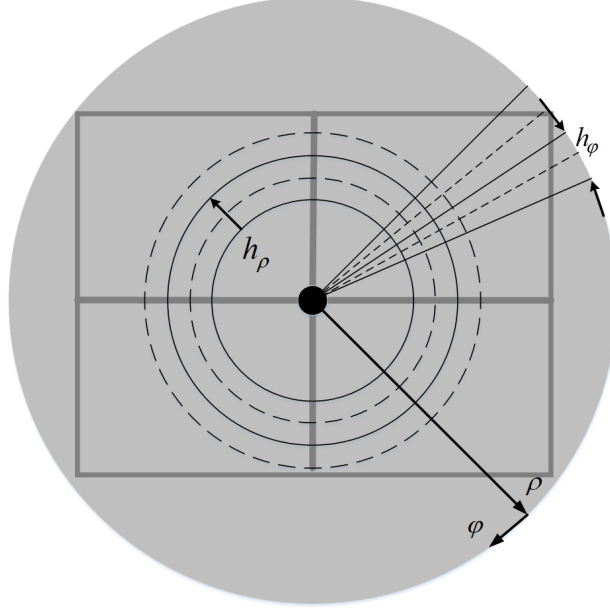


Figure 4.7. The cross section view (top view) of the problem shown in Figure 4.6 with discretization lines (solid black lines for E and dashed black lines for H)

The MoL solution to equations (4.74)-(4.79) is as follows [115]; first the electromagnetic fields are discretized along the ϕ and ρ -directions, then E_z is calculated along the solid black lines (see Figure 4.7) also called e -lines while H_z is calculated along the dashed black lines in Figure 4.7 which are referred to hereafter as h -lines. All central differences in the ρ or ϕ directions are put together in the discretized matrices for differentiation D_ρ and D_ϕ , respectively. To obtain the electromagnetic fields within the whole solution domain, we replace the difference operators by Kronecker products of the same difference operators by unit matrices of the order N_ϕ and N_ρ for both ρ and ϕ dimensions as follows:

$$[D_\rho] = I_\phi \otimes D_\rho / h_\rho \quad (4.83)$$

$$[D_\phi] = D_\phi \otimes I_\rho / h_\phi \quad (4.84)$$

where I_ρ and I_ϕ are unit matrices of the order N_ρ and N_ϕ , respectively. The difference operators D_ρ have to fulfil Dirichlet or Neumann boundary conditions. If we discretise the problem completely in the azimuthal direction, we must introduce periodic boundary conditions into D_ϕ .

Using Central difference formula, the scalar potentials in equation (4.74)-(4.79) can now be replaced by a set of discretized scalar potentials of $[\psi_{\rho 1}, \psi_{\rho 2}, \dots, \psi_{\rho N}]$ and $[\psi_{\phi 1}, \psi_{\phi 2}, \dots, \psi_{\phi N}]$ along the e -lines and h -lines,

$$E_{\rho} = [D_{\rho}] \frac{1}{\epsilon \rho} [D_{\phi}] \psi^{\phi} + [D_{\rho}] \frac{1}{\epsilon \rho} [D_{\rho}] \psi^{\rho} + \frac{1}{\rho} \psi^{\rho} \quad (4.85)$$

$$\tilde{H}_{\rho} = -j \frac{\partial \psi^{\phi}}{\partial z} \quad (4.86)$$

$$E_{\phi} = \frac{1}{\rho} ([D_{\phi}] \frac{1}{\epsilon \rho} [D_{\phi}] \psi^{\phi} + [D_{\phi}] \frac{1}{\epsilon \rho} [D_{\rho}] \psi^{\rho} + \rho \psi^{\phi}) \quad (4.87)$$

$$\tilde{H}_{\phi} = j \frac{1}{\rho} \frac{\partial \psi^{\phi}}{\partial z} \quad (4.88)$$

where the discretized matrices for differentiation D_{ρ} and D_{ϕ} can be written as:

$$D_{\rho} = \begin{bmatrix} p_{\rho}^r & 0 & \dots & 0 \\ -1 & 1 & & \\ 0 & -1 & 1 & \\ \vdots & \ddots & \ddots & \\ & & -1 & 1 \\ 0 & & & p_{\rho}^l \end{bmatrix}. \quad (4.89)$$

$$D_{\phi} = \begin{bmatrix} p_{\phi}^r & 0 & \dots & 0 \\ -1 & 1 & & \\ 0 & -1 & 1 & \\ \vdots & \ddots & \ddots & \\ & & -1 & 1 \\ 0 & & & p_{\phi}^l \end{bmatrix}. \quad (4.90)$$

Accordingly, we can discretize the Helmholtz equation given by (4.81) and (4.82), suppose that:

$$[\psi] = \begin{bmatrix} \psi^{\phi} \\ \psi^{\rho} \end{bmatrix} \quad (4.91)$$

$$\frac{\partial^2}{\partial z^2}[\psi] - [\Gamma][\psi] = [0] \quad (4.92)$$

and

$$\Gamma = [R_E][R_H] \quad (4.93)$$

where

$$[R_E] = \begin{bmatrix} [D_\rho] \frac{1}{\rho} [D_\rho] + \frac{\epsilon}{\rho} & -[D_\rho] \frac{1}{\rho} [D_\phi] \\ -[D_\phi] \frac{1}{\rho} [D_\rho] & [D_\phi] \frac{1}{\rho} [D_\phi] + \epsilon \rho \end{bmatrix} \quad (4.94)$$

and

$$[R_H] = \begin{bmatrix} [D_\phi] \frac{1}{\epsilon \rho} [D_\phi] + \rho & [D_\phi] \frac{1}{\epsilon \rho} [D_\phi] \\ [D_\phi] \frac{1}{\rho} [D_\rho] & [D_\rho] \frac{1}{\epsilon \rho} [D_\rho] + \frac{1}{\rho} \end{bmatrix} \quad (4.95)$$

It is noted that p_ρ^r , p_ρ^l , p_ϕ^r and p_ϕ^l are determined using the boundary conditions. Equations (4.92) is a set of coupled equations due to the tridiagonal structure of differential matrices. This equation is decoupled by means of transformation matrix $[T]$ makes Γ diagonal such that:

$$[T]^{-1} \Gamma [T] = [\lambda^2] \quad (4.96)$$

It is worth noting that $[T]$ is orthogonal matrix. Also the elements λ^2 of the diagonal matrix $[\lambda^2]$ is the eigenvalue of Γ . The eigenvector matrix $[T]$ and the eigenvalue matrix $[\lambda]$ depend on the boundary conditions which are presented in [110]. To diagonalize the Γ in order to uncouple the Helmholtz equation, we use the orthogonal matrix $[T]$ as shown in Figure 4.8. In the new domain, $(\psi-E-H)$ are represented by $(\phi-e-h)$. We now introduce the following transformed vector $[\phi]$:

$$[\psi] = [T][\phi] \quad (4.97)$$

Rewriting the Helmholtz equation in terms of the transformed scalar potential matrix, $[\phi]$, yields:

$$\frac{\partial^2 [\phi]}{\partial z^2} - ([T]^t \Gamma [T])[\phi] = 0 \quad (4.98)$$

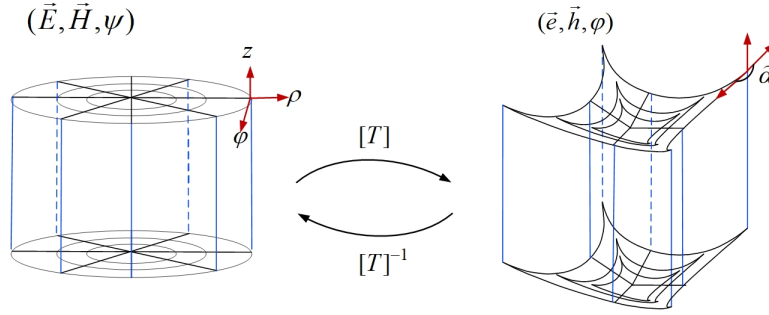


Figure 4.8. Electromagnetic fields and vector potentials in ψ - E - H - and ϕ - e - h -domain

Equations (4.98) could be simplified to:

$$\frac{\partial^2[\phi]}{\partial z^2} - \gamma^2[\phi] = 0 \quad (4.99)$$

where

$$[\gamma]^2 = [T]^t \Gamma [T] = \begin{bmatrix} \gamma_I \\ \gamma_{II} \end{bmatrix} \quad (4.100)$$

Because the system of equations (4.99) is uncoupled, it can be solved analytically for each homogeneous region. To this end, we can use the solution procedure usually adopted to solve the telegraph

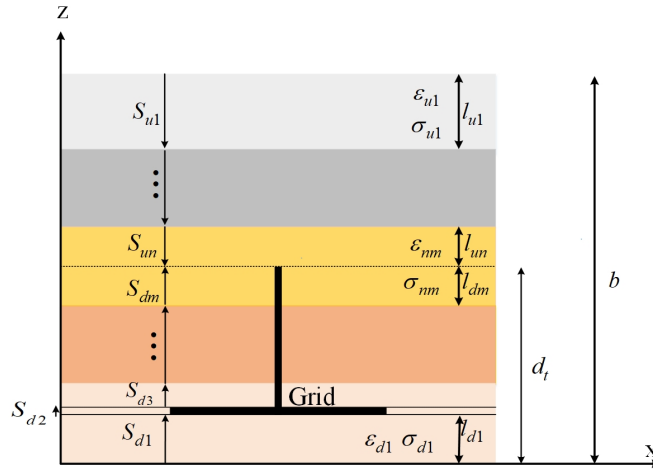


Figure 4.9. 2-D side view of a grounding system buried in a horizontally multilayer soil

equation. So, equations (4.99) can be expressed as a relation between scalar potentials (ϕ) and their normal derivatives within each homogeneous layer from $z = z_i$ to $z = z_{i+1}$. This solution is repeated for all homogeneous layers to map $[\phi]$ from one homogeneous layer to another [116].

Thus, with reference to Figure 4.9, to map the scalar potentials $[\phi]$ of the upper boundary of the solution domain to the upper side of the surface tangential to the excitation port (represented by dashed line in Figure 4.9), the following telegraph equations are solved:

$$\begin{bmatrix} \phi^\rho \\ \frac{\partial \phi^\phi}{\partial z} \\ \frac{\partial \phi^\rho}{\partial z} \\ \phi^\phi \end{bmatrix}_{|z=d_t}^+ = [S_1] \begin{bmatrix} \phi^\rho \\ \frac{\partial \phi^\phi}{\partial z} \\ \frac{\partial \phi^\rho}{\partial z} \\ \phi^\phi \end{bmatrix}_{|z=b} \quad (4.101)$$

where

$$[S_1] = [S_{un}][T_{un}]^{-1}[T_{un-1}][S_{un-1}] \dots [S_{u1}] \quad (4.102)$$

and

$$[S_{ui}] = [S]|_{z=-l_{ui}} \quad (4.103)$$

with $i = 1, 2, \dots, n$. Also, the mapping matrix $([S])$ can be expressed as:

$$[S] = \begin{bmatrix} S' & S'' \end{bmatrix} \quad (4.104)$$

$$[S'] = \begin{bmatrix} \cosh[\gamma_{II}(z)] & 0 \\ 0 & \cosh[\gamma_I(z)] \\ \gamma_{II} \sinh[\gamma_{II}(z)] & 0 \\ 0 & \frac{1}{\gamma_I} \sinh[\gamma_I(z)] \end{bmatrix} \quad (4.105)$$

$$[S''] = \begin{bmatrix} \frac{1}{\gamma_{II}} \sinh[\gamma_{II}(z)] & 0 \\ 0 & \gamma_I \sinh[\gamma_I(z)] \\ \cosh[\gamma_{II}(z)] & 0 \\ 0 & \cosh[\gamma_I(z)] \end{bmatrix} \quad (4.106)$$

Similarly, the potential at the lower side of the surface tangential to the excitation port can be obtained from the scalar potentials $[\phi]$ associated to the lowest boundary of the solution domain (i.e., $z = 0$ in Figure 4.7) using the following telegraph equations:

$$\begin{bmatrix} \phi^\rho \\ \frac{\partial \phi^\phi}{\partial z} \\ \frac{\partial \phi^\rho}{\partial z} \\ \phi^\phi \end{bmatrix}_{|z=d_t}^- = [S_2] \begin{bmatrix} \phi^\rho \\ \frac{\partial \phi^\phi}{\partial z} \\ \frac{\partial \phi^\rho}{\partial z} \\ \phi^\phi \end{bmatrix}_{|z=0} \quad (4.107)$$

and

$$[S_2] = [S_{dm}][T_{dm}]^{-1}[T_{dm-1}][S_{dm-1}] \dots [S_{d1}] \quad (4.108)$$

where

$$[S_{dq}] = [S]_{|z=l_{dq}} \quad (4.109)$$

with $q = 1, 2, \dots, m$. The computational domain can be truncated by different boundary conditions (open space or perfect conductors). In this thesis, assuming the perfect electric conductor (PEC) as the the boundary of the computational domain, the Neumann and Dirichlet boundary conditions are respectively applied to the magnetic and electric scalar potentials. As a result, the potential matrix at $z = b$ (see Figure 4.9) can be written as:

$$[\phi]_{|z=b} = \begin{bmatrix} 0 \\ \frac{\phi^\phi}{\partial z} \\ \frac{\phi^\rho}{\partial z} \\ 0 \end{bmatrix}, \quad (4.110)$$

similarly, at $z = 0$,

$$[\phi]_{|z=0} = \begin{bmatrix} 0 \\ \frac{\phi^\phi}{\partial z} \\ \frac{\phi^\rho}{\partial z} \\ 0 \end{bmatrix}. \quad (4.111)$$

With reference to Figure 4.8, once the ϕ - e - h -domain scalar potentials and their normal derivatives on the soil-electrode interface are calculated, one should resort to the boundary conditions to calculate ϕ - e - h -domain electromagnetic fields on the same interface. It is more straightforward to apply the electromagnetic field boundary conditions rather than those associated with the scalar potentials. To this aim, the ψ - E - H -domain electromagnetic field equations (4.85)-(4.88) should be mapped to the ϕ - e - h -domain by the same eigenvectors. In doing so, the ϕ - e - h -domain electromagnetic fields can be obtained as:

$$[e] = [T]^t [E_t] = [T]^t [R_H] [T] [\phi] \quad (4.112)$$

$$[jh] = [T]^t [H_t] = \frac{\partial [\phi]}{\partial z} \quad (4.113)$$

where

$$[E_t] = \begin{bmatrix} \rho E_\phi \\ E_\rho \end{bmatrix} \quad (4.114)$$

$$[H_t] = \begin{bmatrix} -H_\rho \\ \rho H_\phi \end{bmatrix} \quad (4.115)$$

The above equations (4.112)-(4.113) can be cast in the following matrix form:

$$\begin{bmatrix} e_\rho \\ e_\phi \\ jh_\rho \\ jh_\phi \end{bmatrix} = [K_{T1}] \begin{bmatrix} \phi^\rho \\ \frac{\partial \phi^\phi}{\partial z} \\ \frac{\partial \phi^\rho}{\partial z} \\ \phi^\phi \end{bmatrix} \quad (4.116)$$

Therefore, in ϕ - e - h -domain, the electromagnetic fields on the upper soil-electrode interface can be obtained using the ϕ - e - h -domain scalar potentials of the same interface:

$$\begin{bmatrix} e_\rho \\ e_\phi \\ jh_\rho \\ jh_\phi \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ \frac{\phi\phi}{\partial z} \\ \frac{\phi\rho}{\partial z} \\ 0 \end{bmatrix} \quad (4.117)$$

where

$$[C_1] = [K_{T1}] [S_1] \quad (4.118)$$

and suppose that

$$[C_1]^{-1} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} & C'_{14} \\ C'_{21} & C'_{22} & C'_{23} & C'_{24} \\ C'_{31} & C'_{32} & C'_{33} & C'_{34} \\ C'_{41} & C'_{42} & C'_{43} & C'_{44} \end{bmatrix} \quad (4.119)$$

Similarly, for the lower soil-electrode interface,

$$[C_2] = [K_{T2}] [S_2] \quad (4.120)$$

and

$$[C_2]^{-1} = \begin{bmatrix} C''_{11} & C''_{12} & C''_{13} & C''_{14} \\ C''_{21} & C''_{22} & C''_{23} & C''_{24} \\ C''_{31} & C''_{32} & C''_{33} & C''_{34} \\ C''_{41} & C''_{42} & C''_{43} & C''_{44} \end{bmatrix} \quad (4.121)$$

Since the first and the last rows of the potential matrix $[\phi]$ are zero, we can rewrite equation (51) as follows;

$$\begin{bmatrix} j\tilde{h}_\rho \\ j\tilde{h}_\phi \end{bmatrix}_{z=d^+} = \begin{bmatrix} C'_{13} & C'_{14} \\ C'_{43} & C'_{44} \end{bmatrix}^{-1} \begin{bmatrix} C'_{12} & C'_{11} \\ C'_{42} & C'_{41} \end{bmatrix} \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix}_{z=d^+} \quad (4.122)$$

$$\begin{bmatrix} j\tilde{h}_\rho \\ j\tilde{h}_\phi \end{bmatrix}_{z=d^-} = \begin{bmatrix} C''_{13} & C''_{14} \\ C''_{43} & C''_{44} \end{bmatrix}^{-1} \begin{bmatrix} C''_{12} & C''_{11} \\ C''_{42} & C''_{41} \end{bmatrix} \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix}_{z=d^-} \quad (4.123)$$

The continuity of tangential electric field components on the interface implies that:

$$\begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix}_{z=d^+} = \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix}_{z=d^-} \quad (4.124)$$

Assuming that

$$[G_1] = \begin{bmatrix} C'_{13} & C'_{14} \\ C'_{43} & C'_{44} \end{bmatrix}^{-1} \begin{bmatrix} -C'_{12} & -C'_{11} \\ -C'_{42} & -C'_{41} \end{bmatrix} \quad (4.125)$$

and

$$[G_2] = \begin{bmatrix} C''_{13} & C''_{14} \\ C''_{43} & C''_{44} \end{bmatrix}^{-1} \begin{bmatrix} -C''_{12} & -C''_{11} \\ -C''_{42} & -C''_{41} \end{bmatrix} \quad (4.126)$$

Considering equation (4.124) and by subtracting (4.122) from (4.123) we can write,

$$\begin{bmatrix} j\tilde{h}_\rho \\ j\tilde{h}_\phi \end{bmatrix}_{z=d^+} - \begin{bmatrix} j\tilde{h}_\rho \\ j\tilde{h}_\phi \end{bmatrix}_{z=d^-} = (G_1 - G_2) \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix} \quad (4.127)$$

A discontinuity in the tangential component of the magnetic field intensity implies that the surface current flows in a direction perpendicular to this component of the field, therefore equation (4.127) becomes:

$$\begin{bmatrix} j\tilde{j}_\phi \\ j\tilde{j}_\rho \end{bmatrix} = (G_1 - G_2) \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix} = [G_0] \begin{bmatrix} e_\phi \\ e_\rho \end{bmatrix} \quad (4.128)$$

where $\tilde{j}_\rho$ and $\tilde{j}_\phi$ are normalized surface currents in ρ - and ϕ -direction respectively. The currents and electric field components can be now transferred back to the ψ - E - H domain to obtain the electromagnetic fields. Applying $[T]^{-1}$ and using orthogonality of $[T]$, we can rewrite equation (4.128) as:

$$\begin{bmatrix} j\tilde{J}_\phi \\ j\tilde{J}_\rho \end{bmatrix} = [T] [G_0] [T]^t \begin{bmatrix} \rho E_\phi \\ E_\rho \end{bmatrix} \quad (4.129)$$

Due to the fact that the tangential component of the magnetic field is continuous across an interface with a finite conductivity, equation (4.129) can be rewritten as follows:

$$\begin{bmatrix} [T] \\ [G_0] \end{bmatrix} \begin{bmatrix} [T]^t \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ j\tilde{J}_\phi \\ 0 \\ 0 \\ j\tilde{J}_\rho \\ 0 \end{bmatrix} = \begin{bmatrix} \rho E_{\phi i} \\ 0 \\ \rho E_{\phi j} \\ E_{\rho i} \\ 0 \\ E_{\rho j} \end{bmatrix} \quad (4.130)$$

It is seen that by applying the excitation source, the solution of matrix equation of (4.130) provides us with the electromagnetic fields on the surface tangential to the excitation port. The electromagnetic fields are thus found in the entire computational domain and consequently the input impedance can be easily obtained. For the Grounding System Modeling, Method of Lines (MoL) method has been chosen due to following reason:

- MoL can handle frequency-dependent phenomena, important for grounding system analysis.
- Efficient in solving problems involving high-frequency fields or large structures.
- Capable of handling different boundary conditions and geometries.
- Flexible and versatile, can be applied to a wide range of problems.
- Better performance than other method in large scale geometry
- Does not suffer from convergence problems that can plague other methods.
- Requires less computer coding effort than other methods.

4.4 Conclusion

In this chapter, first a 2-D semi-analytical approach based on the Method of Line (MoL) has been proposed for full-wave modeling of grounding systems buried in a horizontally and/or vertically

stratified multilayer soil. In the proposed approach, the solution region is first partitioned into layers in a way that each layer be homogeneous. Then, the governing partial differential equations (PDEs) for each layer are derived and discretized in one direction. In the next step, the discretized equations are cast in the form of an eigenvalue problem from which two transformation matrices are calculated. The transformation matrices are used to convert the PDEs to decoupled ordinary differential equations (ODEs) that are solved analytically in the remaining directions. Then, boundary conditions and inverse transformation are applied to formulate a determinant equation which represents correlation between the electric fields and the surface current densities. Finally, the determinant equation is solved to calculate the propagation constant, and finally all electromagnetic field components are obtained. Finally, the formulation of the 3-D MoL has been proposed, similar to the 2-D MoL, for modeling of grounding grids buried in a multilayer stratified soil.

CHAPTER 5 GMD MODELLING and GIC CALCULATION

In this chapter, the accelerated FDTD method presented in chapters 2 is used for modeling GMDs and calculation of GICs in the presence of stratified multilayer ground and earth curvature. Two different excitations, namely the the line-current electrojet, and the sheet-current electrojet are used and their results are compared [118, 119].

5.1 Horizontally Multilayer Ground

In the first simulation, we consider the horizontally multilayer ground shown in Figure 5.1 which is associated to the case of Quebec reported in [51] with soil layer data shown in the first two columns of Table 5.1. The electrojet is modeled by a current-carrying line of a length of 2λ located 100 km above the ground surface. The electrojet current is a 1 MA time-varying current.

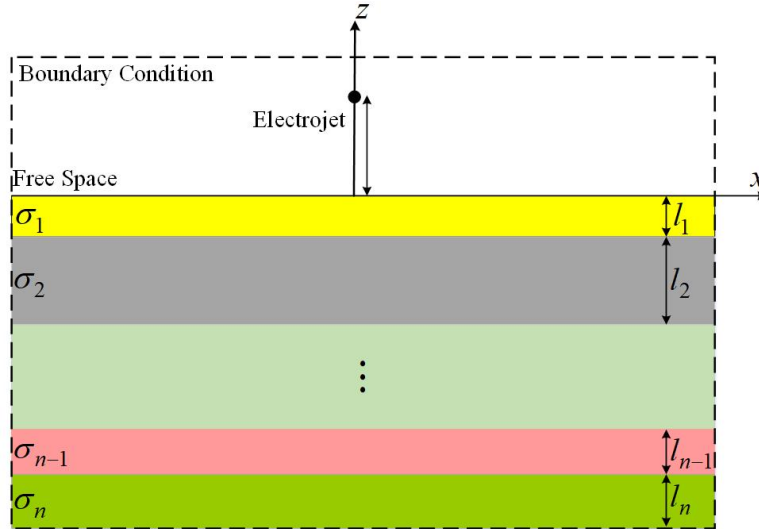


Figure 5.1. A 2-D Horizontally stratified ground

In this chapter, to perform 3-D modeling in the FDTD, the length of the horizontally multilayer ground in y -direction is also assumed to be 2λ . The periodic boundary condition is used in y -direction, while the CPML boundary condition is applied to the x - and z - directions. The frequency of the electrojet current is $f = 3.33$ mHz, i.e. the period $T=5$ min. The x - and z -components of the

ground surface magnetic field (B_x and B_z) and the y -component of the ground surface electric field (E_y) calculated by the proposed accelerated FDTD and the CIM approaches [51] are respectively shown in Figures 5.2-5.4. As seen from these figures, the results are consistent, which confirm the accuracy of the proposed FDTD method.

Table 5.1. Characteristics of horizontally multilayer earth models in Quebec and Ontario

	Ontario		Quebec	
	$l(km)$	$\sigma(S/m)$	$l(km)$	$\sigma(S/m)$
1	0.05	0.02	15	0.00005
2	1	0.01	10	0.005
3	15	0.0001	125	0.001
4	10	0.00330	200	0.01
5	75	0.0001	∞	3
6	300	0.01	—	—
7	∞	0.1	—	—

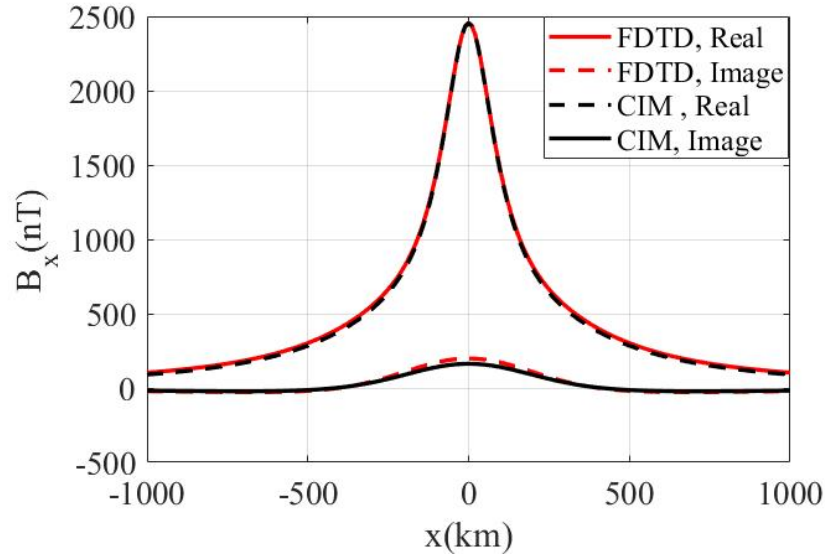


Figure 5.2. Horizontal magnetic field (B_x) at ground surface, $T = 5$ min. Multilayer ground of Quebec

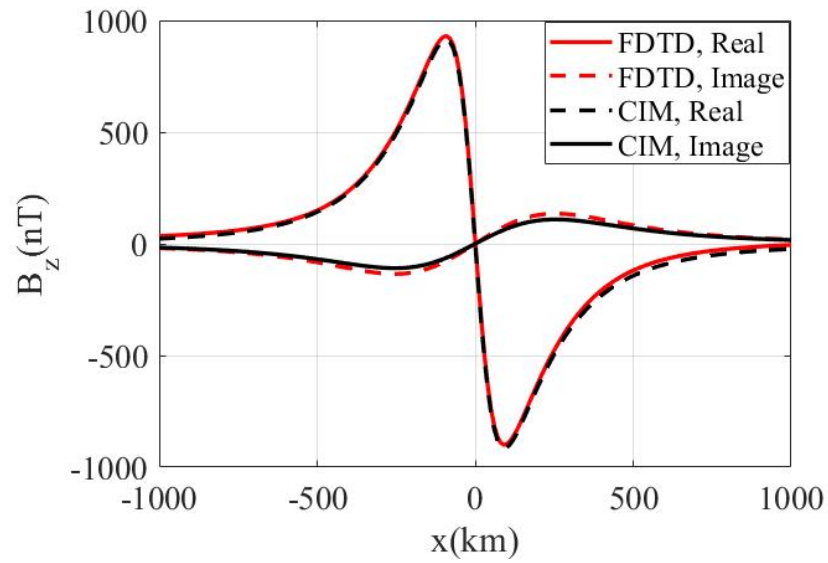


Figure 5.3. Vertical magnetic field (B_z) at ground surface, $T = 5$ min. Multilayer ground of Quebec

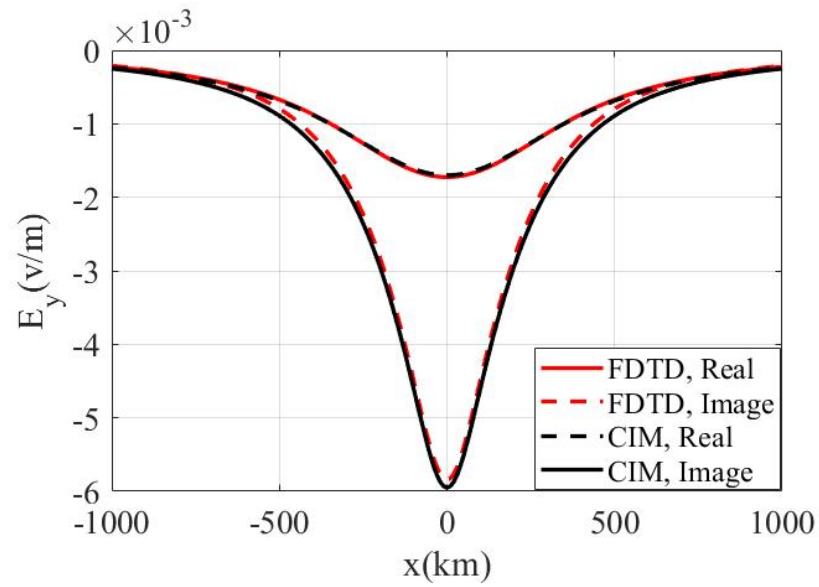


Figure 5.4. Horizontal electric field (E_y) at ground surface, $T = 5$ min. Multilayer ground of Quebec

5.2 Mixed Vertically-horizontally Multilayer Ground

To further evaluate the performance of the proposed FDTD method, we consider a mixed propagation terrain (i.e., vertically-horizontally stratified ground) as shown in Figure 5.5. The ground structure is that of Quebec-Ontario presented in [51], where data of soil layers for each region is listed in Table 5.1. For different T (i.e., $T=5$ and 30 min), the absolute value of x - and z -components of the magnetic field ($|B_x|$ and $|B_z|$) as well as the absolute value of the y -component of the electric field ($|E_y|$) at ground surface are shown in Figures 5.6-5.8, respectively. The electrojet is similar to the previous case. A comparison between the results presented here and those obtained by the FEM approach in [54] demonstrates the accuracy of the proposed FDTD approach for calculating of GS-EMFs over a mixed propagation terrain.

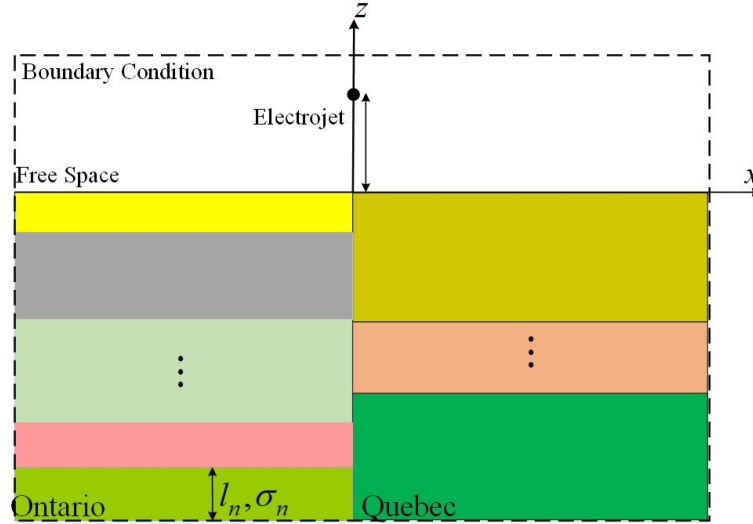


Figure 5.5. The multilayer ground structure of Ontario–Quebec

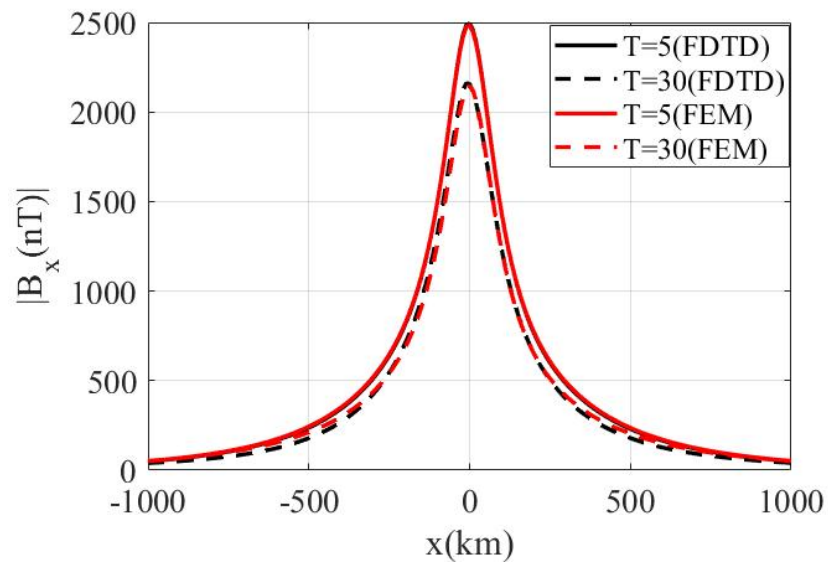


Figure 5.6. Absolute horizontal magnetic field ($|B_x|$) at ground surface. Multilayer ground of Quebec-Ontario

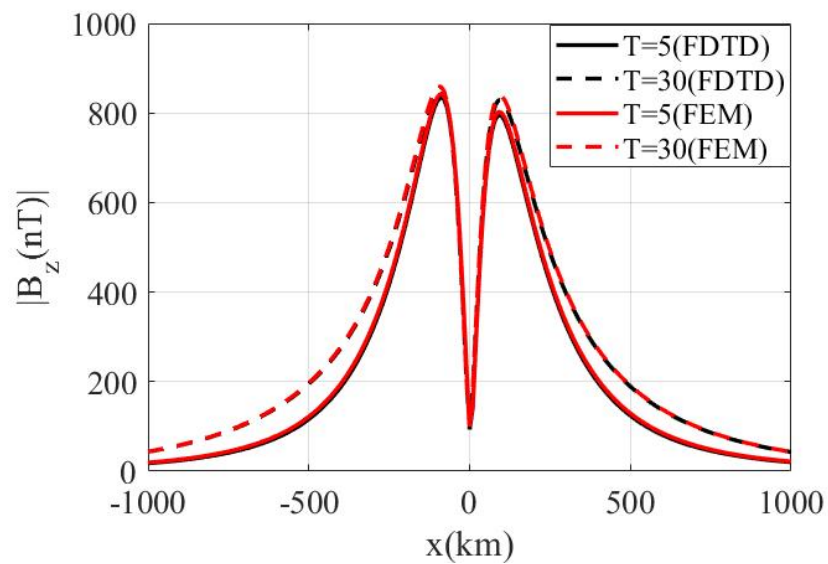


Figure 5.7. Absolute vertical magnetic field ($|B_z|$) at ground surface. Multilayer ground of Quebec-Ontario

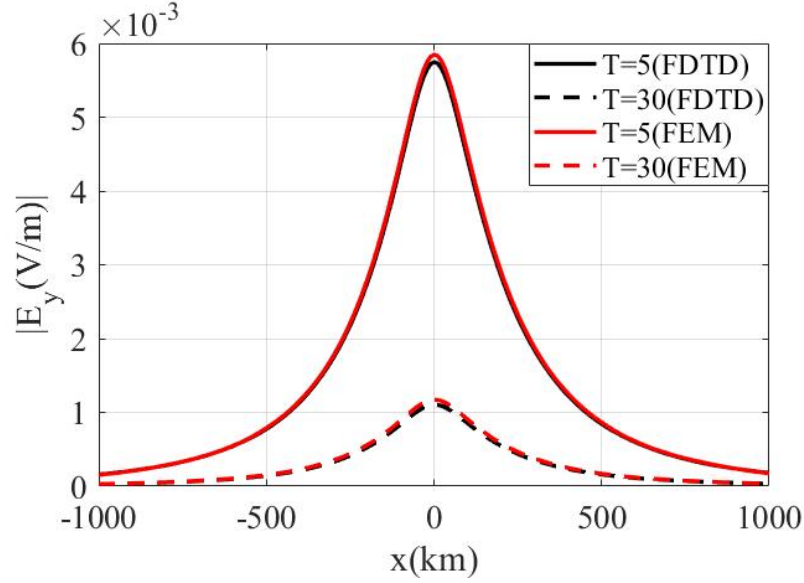


Figure 5.8. Absolute horizontal electric field ($|E_y|$) at ground surface. Multilayer ground of Quebec-Ontario

The simulation time for the calculation of GS-EMFs for cases of Quebec and Quebec-Ontario are shown in Table 7.1 which indicates the affordable time requirement of the proposed FDTD approach. The simulations are conducted on a PC with Intel 2.4 GHz, i5 processor having 4 GB of RAM.

5.3 Earth Curvature

In this section, the effect of Earth curvature on the GS-EMFs at ground surface is evaluated for the same electrojet current. With reference to Figure 5.9, the Earth curvature for each layer is modeled using a Pie-Arc shape through conversion of cartesian coordinates to spherical coordinates for a specific arc angle θ . Hence, the radius of each layer R_i is defined as follows:

Table 5.2. The FDTD computational time for Case of Quebec and case of Ontario-Quebec

Case of simulation	Total time(s)
Quebec	35
Ontario-Quebec	40

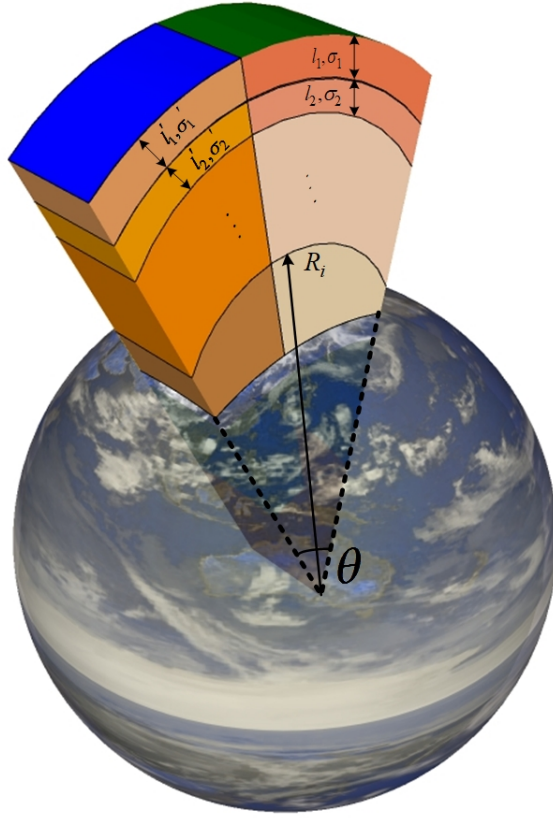


Figure 5.9. 3-D Earth structure for calculation of GS-EMFs at ground surface

$$R_i = R_e - \sum_{k=1}^{i-1} l_k \quad (5.1)$$

where R_e is the radius of Earth (6.371 km) and l_i is the thickness of each layer.

We consider the case of mixed vertically-horizontally multilayer ground of Quebec-Ontario where a propagation terrain of length of 2000 km is simulated. The Earth curvature is represented by $\theta = 0.1\pi$ rad, see Figure 5.9. Results associated with the absolute value of horizontal and vertical magnetic field components ($|B_x|$ and $|B_z|$), and the y -component of the electric field ($|E_y|$) at ground surface with and without Earth curvature are shown in Figures 5.10, 5.11 and 5.12, respectively. A comparison between the results shows that the Earth curvature can slightly affect the GS-EMFs at ground surface.

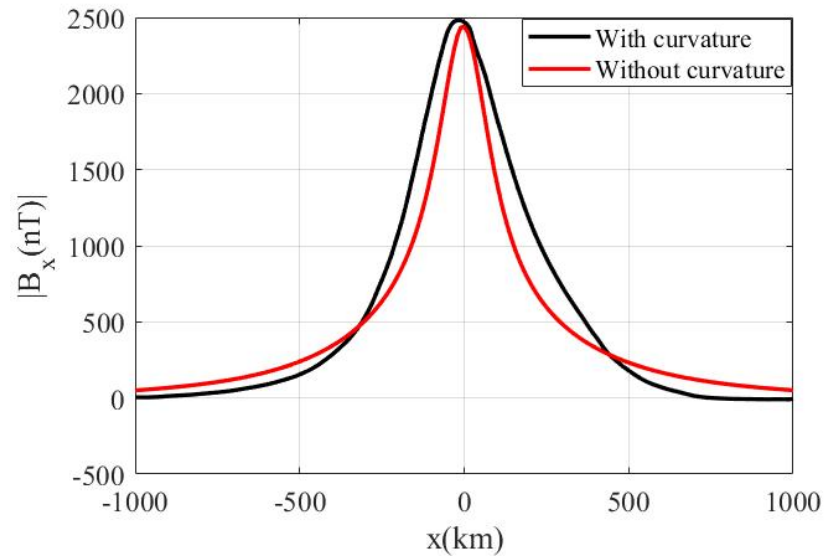


Figure 5.10. Absolute horizontal magnetic field ($|B_x|$) at ground surface. Multilayer ground of Quebec-Ontario with and without Earth curvature

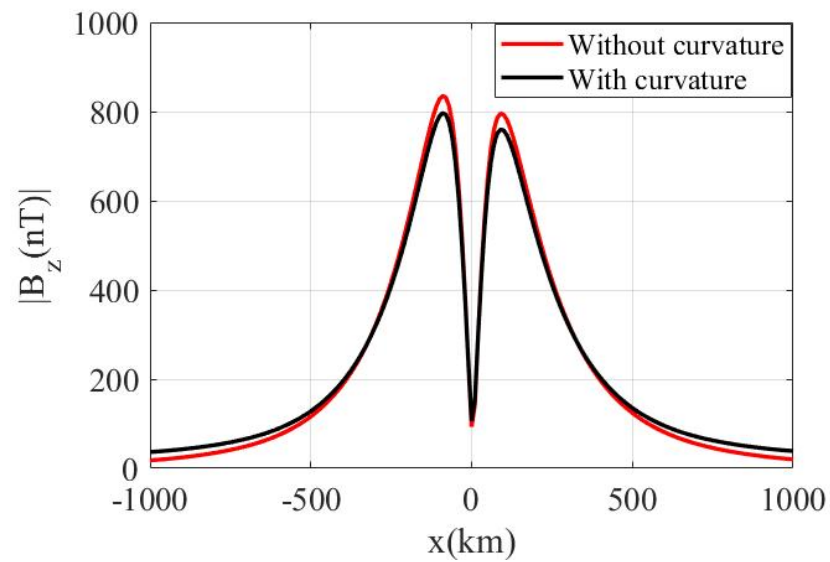


Figure 5.11. Absolute vertical magnetic field ($|B_z|$) at ground surface. Multilayer ground of Quebec-Ontario with and without Earth curvature

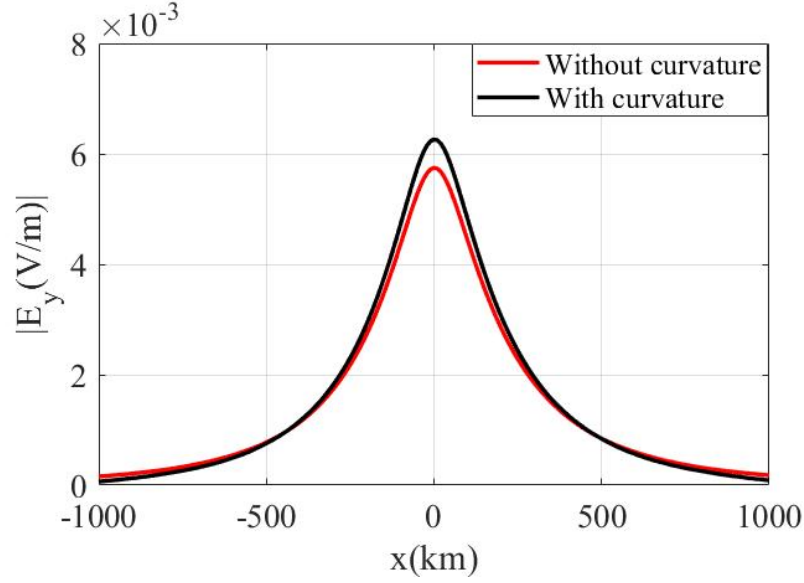


Figure 5.12. Absolute horizontal electric field ($|E_y|$) at ground surface. Multilayer ground of Quebec-Ontario with and without Earth curvature

5.4 Time Domain Analysis

In this Section, we present time-domain simulations for the case of Quebec-Ontario considering an electrojet current with a cosine-modulated Gaussian waveform as,

$$g(t) = \cos(2\pi f_c(t - t_0)) \times e^{-\frac{(t-t_0)^2}{\tau^2}} \quad (5.2)$$

where $\tau = 0.996/\Delta f$ and $t_0 = 4.5\tau$ with $\Delta f = 0.04\text{Hz}$ and $f_c = 0.02\text{Hz}$. The time domain GS-EMFs at ground surface are shown in Figures 5.13-5.15. It is noted that any time-domain excitation current waveform can easily be implemented in the proposed FDTD approach if required.

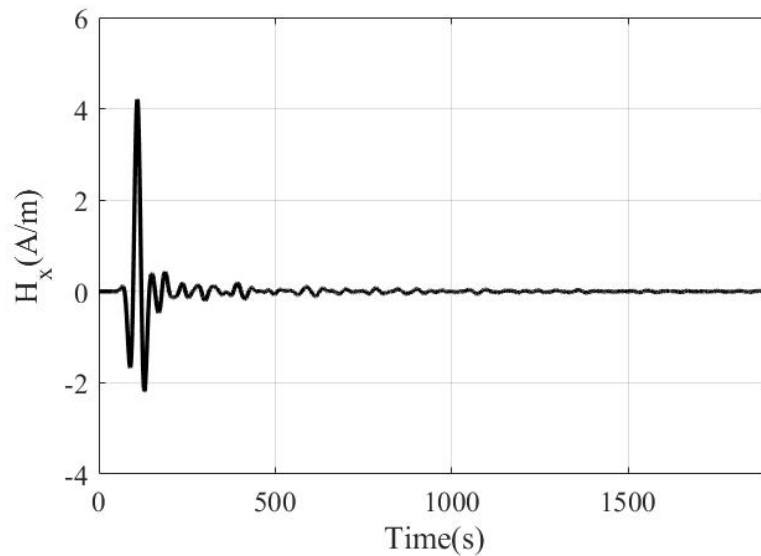


Figure 5.13. Time domain horizontal magnetic field (H_x) at ground surface at (0,0,0). Multilayer ground of Quebec-Ontario. Gaussian cosine modulated excitation with $\Delta f = 0.04Hz$ and $f_c = 0.02Hz$

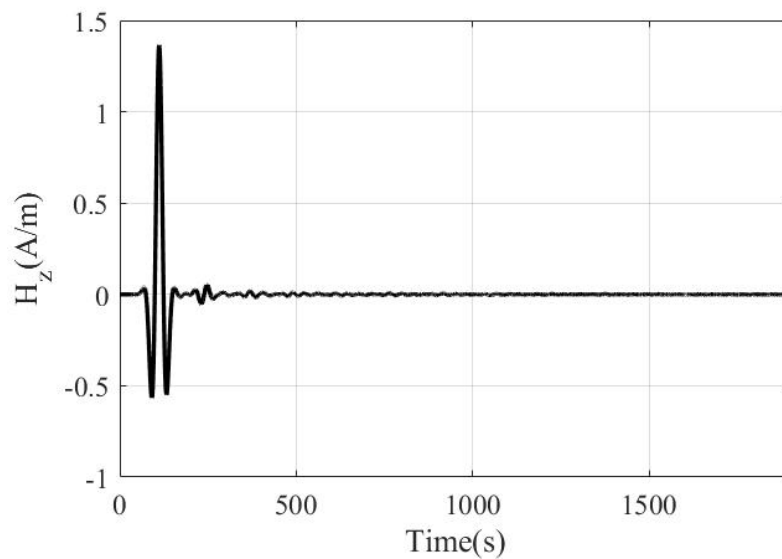


Figure 5.14. Time domain vertical magnetic field (H_z) at ground surface at $(-136 \times 10^3, 0, 0)$. Multilayer ground of Quebec-Ontario. Gaussian cosine modulated excitation with $\Delta f = 0.04Hz$ and $f_c = 0.02Hz$

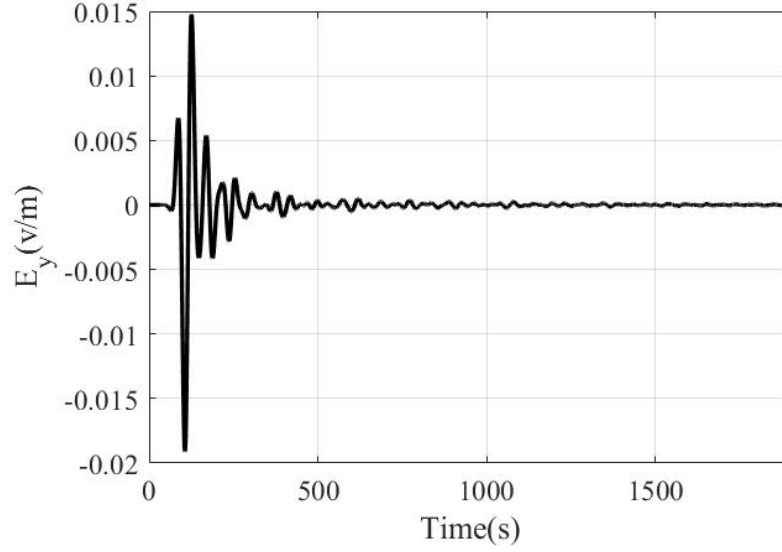


Figure 5.15. Time domain horizontal electric field (E_y) at ground surface at (0,0,0). Multilayer ground of Quebec-Ontario. Gaussian cosine modulated excitation with $\Delta f = 0.04 Hz$ and $f_c = 0.02 Hz$

5.5 Sheet Current Excitation

In this Section, we model the electrojet as a long current-carrying sheet since this type of excitation has been suggested in the literature for a better modeling of the electrojet current [52]. To this aim, and with reference to Figure 5.16, we consider a sheet with a width of 400 km ($x_1 = -300$ km and $x_2 = 100$ km) and a length of 2λ similar to [52]. A current with an amplitude of 1 MA is equally distributed over the considered sheet. The height of the sheet-current source is $h = 110$ km. The Earth is uniform with a conductivity of 10^{-3} S/m. The real and imaginary parts of the horizontal electric field (E_y) at ground surface calculated by the proposed method are shown in Figure 5.17 together with their counterparts presented in [52]. This figure vividly shows an excellent agreement between the results obtained by the proposed method and those of [52].

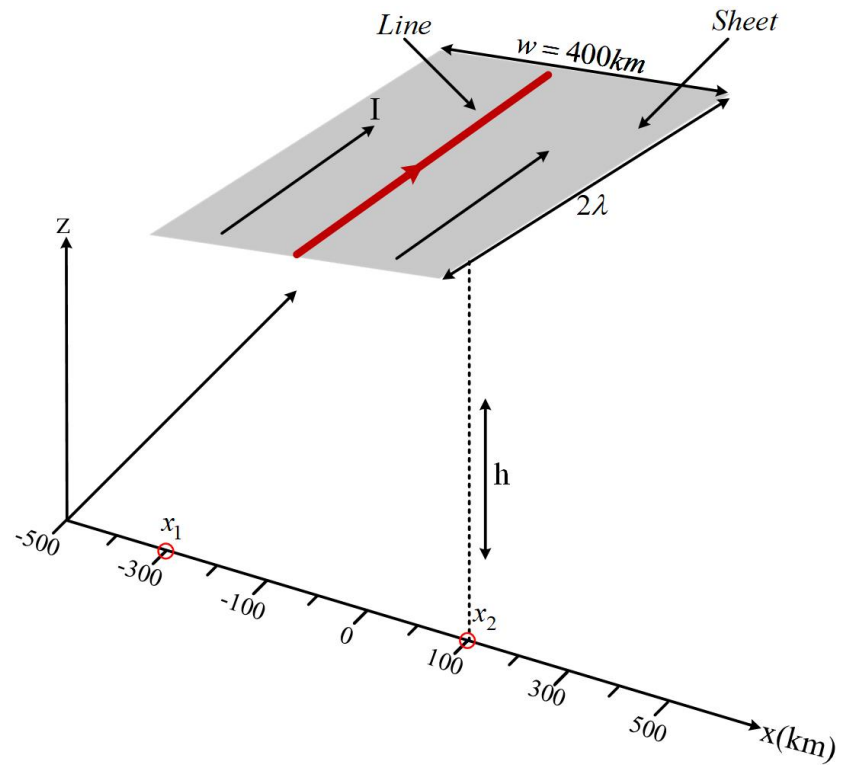


Figure 5.16. The geometry of sheet-current and line-current sources used for modeling of electrojet

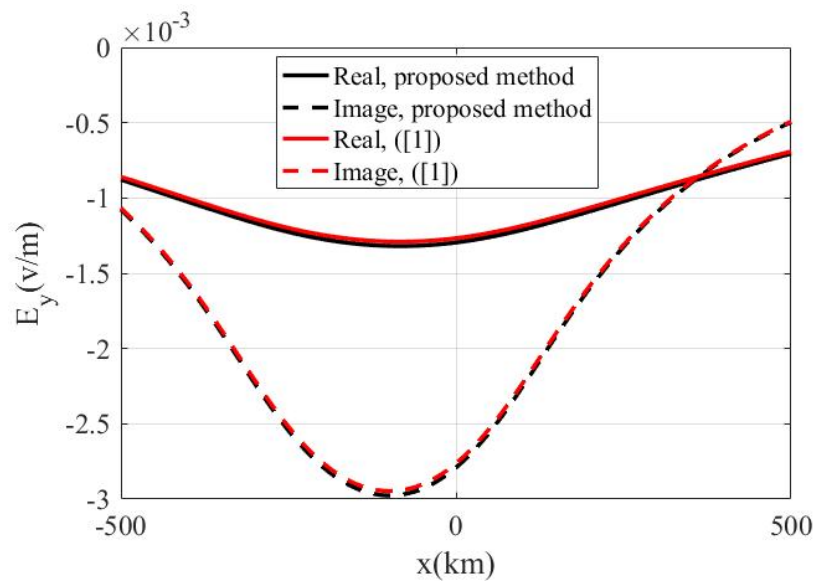


Figure 5.17. Horizontal electric field (E_y) at ground surface. Comparison with [52]. Sheet current of width of 400 km, $T = 10$ min. Single layer ground with $\sigma = 10^{-3}$ S/m

In the next simulation, we consider the same sheet-current source at $h = 100$ km above ground surface. We consider the case of Quebec-Ontario. The absolute value of the vertical and horizontal magnetic field (B_x and B_z) and the horizontal electric field (E_y) calculated by the proposed method are shown in Figures 5.18-5.20, respectively. Also shown in these figures are the results associated to the electrojet modeled by a line-current source located at $x = -100$ km and $h = 100$ km. It is shown in these figures that the sheet-current source generates EM fields with a lower magnitude as compared to the line-current source.

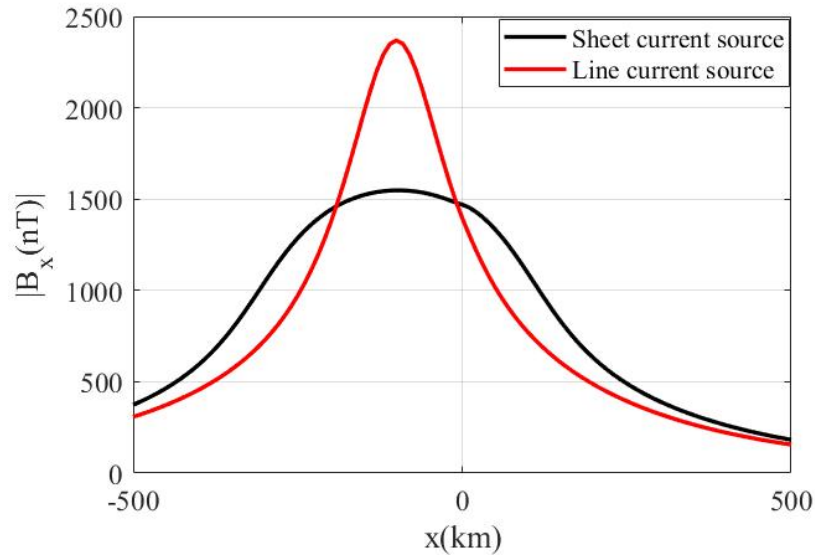


Figure 5.18. Absolute horizontal magnetic field ($|B_x|$) at ground surface. Multilayer ground of Quebec-Ontario. Sheet current with $T = 5$ min

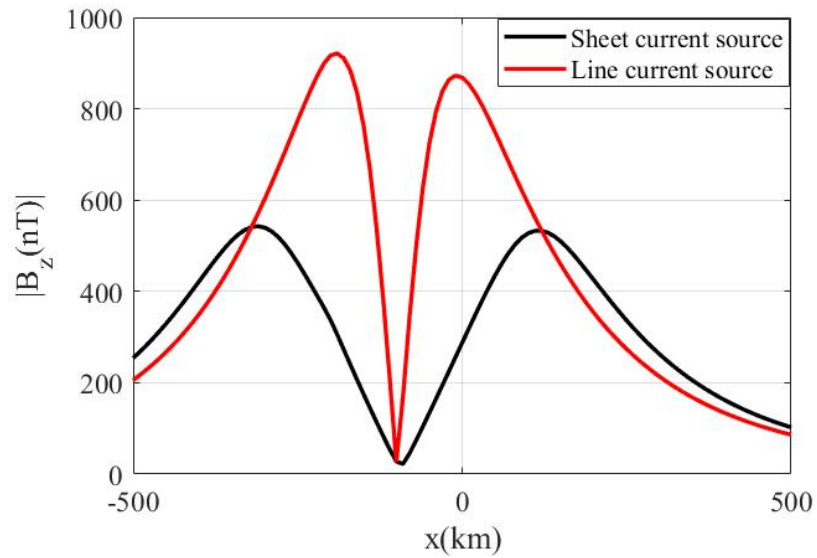


Figure 5.19. Absolute vertical magnetic field ($|B_z|$) at ground surface. Multilayer ground of Quebec-Ontario. Sheet current with $T = 5$ min

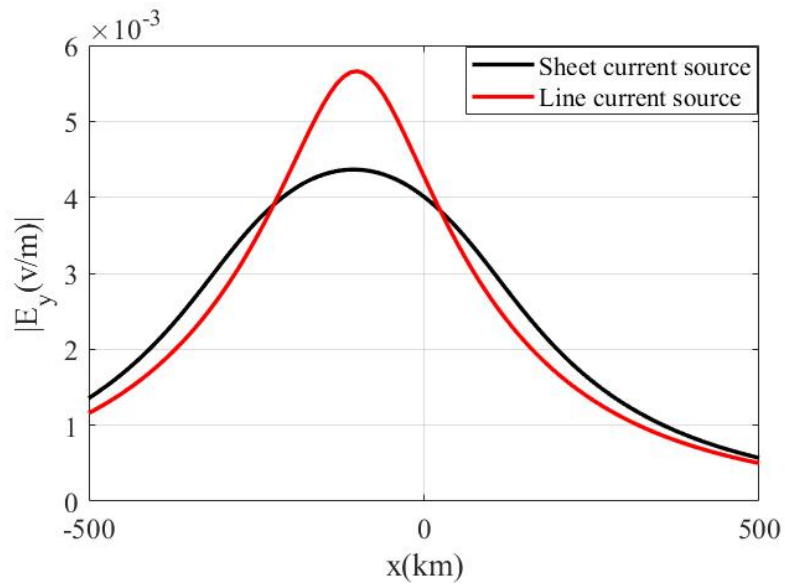


Figure 5.20. Absolute horizontal electric field ($|E_y|$) at ground surface. Multilayer ground of Quebec-Ontario. Sheet current with $T = 5$ min

5.6 Effect of Acceleration Rate

To evaluate the effect of acceleration rate on the performance of the proposed accelerated FDTD, we consider different values for parameters p and m (i.e., $m=10, 20$ and 30 while p increases from 1000 to 1.8×10^4). The real part value of the horizontal and vertical magnetic field (B_x and B_z) at $(-136 \times 10^3, 0, 0)$ are respectively shown in Figures 5.21 and 5.22. It is seen from these figures that the number of skipped time steps (p) can significantly affect the accuracy of the proposed FDTD method so that for $p > 1.1 \times 10^4$ and $m=10$, the proposed FDTD method fails to provide accurate results and becomes unstable. For the simulations presented in this chapter, p has been set to 10^4 , which provides a computational gain of 10^4 as compared to the conventional FDTD method. It is also seen in these figures that p can take larger values as m increases. A crucial stage for the implementation of the proposed FDTD approach is the determination of subinterval ΔT according to the *a priori* selected time step (Δt) and number of iteration (m) at each subinterval. To obtain accurate results and from a sensitivity analysis, it is suggested that ΔT satisfies the following inequality:

$$\frac{f^{\frac{2}{3}} \times (\Delta T - m^3 \Delta t)^2}{5c^{\frac{1}{2}} \Delta t^2} \leq 1 \quad (5.3)$$

Hence, the number of skipped time steps p is calculated as $p = (\Delta T - m\Delta t)/\Delta t$.

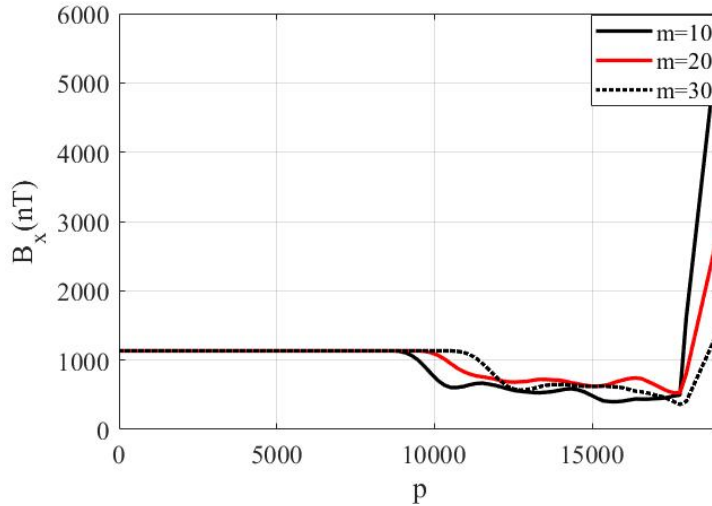


Figure 5.21. Horizontal magnetic field (B_x) at ground surface at $(-136 \times 10^3, 0, 0)$ for different p and m . Multilayer ground of Quebec, $T = 5$ min

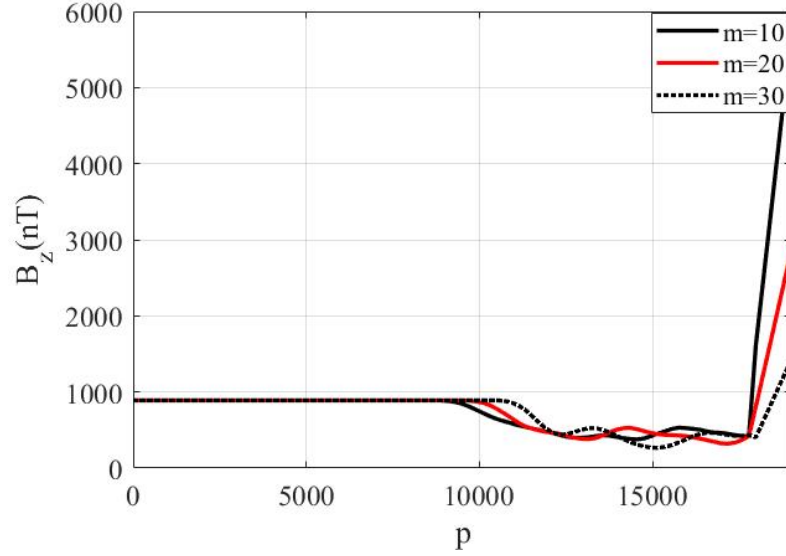


Figure 5.22. Vertical magnetic field (B_z) at ground surface at $(-136 \times 10^3, 0, 0)$ for different p and m . Multilayer ground of Quebec, $T = 5$ min

5.7 GIC Calculation

In this section, we consider several study cases to show the effectiveness of the proposed FDTD method for calculating GICs on single- and multi-conductor transmission lines (TLs), taking into account different sources and ground structures. With reference to Figure 5.23, we first consider a single-conductor TL system interconnecting four substations located above a uniform ground with an electric conductivity of 0.001 S/m and electric permittivity of $\epsilon = 10$ F/m. This TL system is adapted from [46] and is mainly used for the comparison purpose. Each TL is 100 km long located above ground surface and is characterized by a series DC resistance of 5Ω . The resistance to ground through the transformers at each substation is taken to be 0.5Ω . In this case, the excitation source is assumed to be a line-current electrojet in y -direction located 100 km above the ground at $x = 0$ km. The line-current electrojet carries a 1-MA time varying current with a frequency of $f = 3.33$ mHz (i.e, period of $T_c = 5$ min). The magnitude of GICs calculated by the proposed FDTD method together with their counterparts presented in [46] are listed in Table 5.3 which confirms excellent agreement between the proposed FDTD approach and the complex image method of [46].

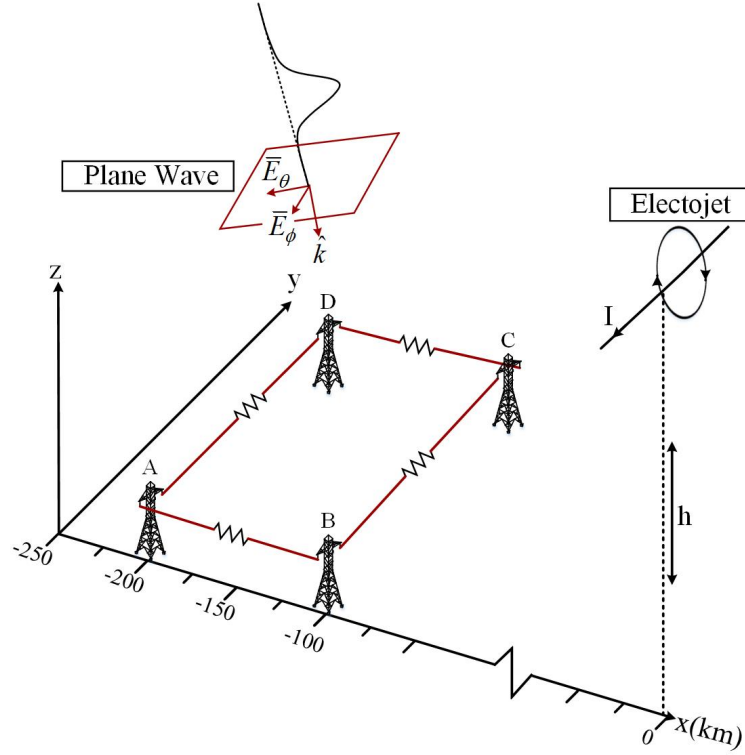


Figure 5.23. The configuration of the single-conductor TL system illuminated by different excitation sources

To further evaluate the performance of the proposed FDTD method, we consider the multilayer ground structure of Quebec and Quebec-Ontario (i.e., horizontally and vertically-horizontally stratified ground) as shown in Figure 5.5 [54]. The relative permittivity of $\epsilon = 10$ F/m is considered for all soil layers. The TL system and the electrojet are similar to the previous case. The GICs are obtained for the case of Quebec and Quebec-Ontario ground structures, see Table 5.3. A comparison between Table 5.4 and Table 5.3 shows that ground stratification can significantly affect GICs. This effect is more pronounced for case of vertically-horizontally ground structure. To obtain a better understanding about the effect of multilayer ground on GICs, results associated to the case of a uniform ground with electric conductivity equal to that of the first layer of Quebec (i.e., $\sigma = 0.00005$ S/m) are also shown in Table 5.3. It is seen from this Table that the GICs for the uniform ground are generally larger than those of multilayer ground of Quebec and Quebec-Ontario. This might be attributed to the contribution of the reflections from lower layers which are generally more conductive.

Table 5.3. GICs (Amp) on the single-conductor TL system of Figure 5.23 due to the 1-MA line-current electrojet. $T_c = 5$ min

Location	Boteler [46]	This Work
A to ground	-55.4	-55.34
B to ground	-79.6	-79.62
C to ground	79.6	79.65
D to ground	55.4	55.39
A to B	2.4	2.44
A to D	52.9	52.95
B to C	82.1	82.15
C to D	2.4	2.48

Table 5.4. GICs (Amp) on the single-conductor TL system of Figure 5.23 due to the line-current electrojet. Case of Quebec, Quebec-Ontario and Uniform ground, $T_c = 5$ min

Location	Quebec	Quebec-Ontario	$\sigma = 0.00005$
A to ground	-74.71	-46.12	-148.408
B to ground	-79.6	-70.24	-148.408
C to ground	74.26	70.62	175.77
D to ground	52.07	46.05	148.408
A to B	2.4	2.44	2.736
A to D	49.78	43.60	145.672
B to C	77.21	73.076	178.506
C to D	2.28	2.48	2.735

The simulation time for the calculation of GICs for cases of Quebec and Quebec-Ontario are shown in Table 5.5. It is worth noting that the simulations are conducted on a PC with Intel i5, 2.4 GHz processor having 4 GB of RAM. In the next simulation, we consider a three phase 400-kV TL system with two shield wires as shown in Figure 5.24. The TL has a length of 1000 km with a tower span of 350 m with coordinates similar to the BC line of Figure 5.23. The phase conductors are grounded at their both terminations with a resistance of 0.5Ω . The GICs due to the same line-current electrojet and a plane-wave with an amplitude of 4.9 mV/m and a frequency of 3.33 mHz are calculated for the case of Quebec-Ontario, see Table 5.6. The GIC at one termination of the shield wire is reported in Table 5.6. As it is seen, the GIC on shield wire is much lower than that of line conductors due to the periodical grounding of shield wire through the tower. From this table, it is seen that the plane-wave excitation generates the same GICs as to the line-current electrojet. It is also shown in Table 5.6 that the inclusion of shield wire increases the GICs. This observation has been recently reported in [120] and has been attributed to the fact that shield wire produces an extra path to the ground through the tower footings. Simulations are repeated for the case of Quebec-Ontario. All system specifications remain identical except for the amplitude of the plane-wave source which is changed to 4.426 mV/m following the procedure proposed in [46]. The associated results are listed in Table 5.7 with and without shield wire. From this table, again, the increasing effect of the shield wire on the GICs is accentuated.

Table 5.5. Computational times for two cases: Case of Quebec and case of Ontario-Quebec

Case of simulation	Total time(s)
Quebec	35
Ontario-Quebec	40

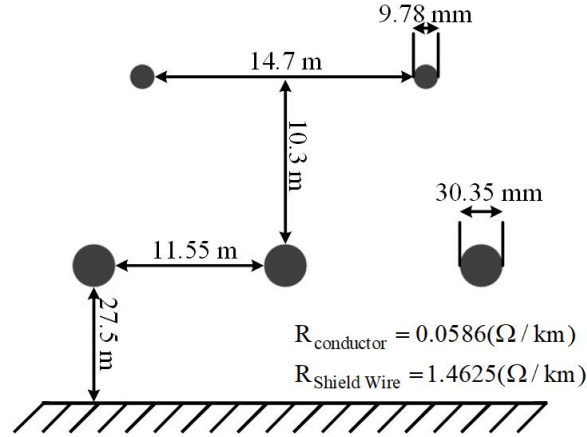


Figure 5.24. Geometry of the 3-phase 400-kV TL system

Table 5.6. GICs (Amp) on the 3-phase TL system of Figure 5.24 due to the line-current electrojet and the plane-wave source. Case of Quebec, $T_c = 5$ min

	Without shield wire		With shield wire	
	Line	Plane wave	Line	Plane wave
Line_1	77.35	77.46	89.72	89.37
Line_2	76.95	77.46	89.26	89.37
Line_3	77.12	77.46	89.45	89.37
Shield wire	-	-	3.22	3.23

Table 5.7. GICs (Amp) on the 3-phase TL system of Figure 5.24 due to the line-current electrojet and the plane-wave source. Case of Quebec-Ontario, $T_c = 5$ min

	Without shield wire		With shield wire	
	Line	Plane wave	Line	Plane wave
Line_1	73.17	73.52	84.87	85.02
Line_2	72.54	73.52	84.87	85.02
Line_3	73.14	73.52	84.87	85.02
Shield wire	-	-	3.05	3.06

In the final simulation case, we model the electrojet as a long current-carrying sheet. To this aim, and with reference to Figure 5.23, we consider a sheet with a width of 400 km ($x_1 = -300$ km and $x_2 = 100$ km) and a length of 2λ similar to [52] over which a current with an amplitude of 1 MA is uniformly distributed. The GICs on the same 3-phase TL system for case of Quebec-Ontario are reported in Table 5.8. This Table also shows GICs associated to the 1-MA line-current electrojet located at $x = -100$ km and $h = 100$ km above the ground. It is seen from this table that the magnitude of GICs related to the sheet-current electrojet are generally lower than those of the line-current electrojet.

Table 5.8. GICs (Amp) on the 3-phase TL system of Figure 5.24 due to sheet-current and line-current electrojets. Case of Quebec-Ontario, $T_c = 5$ min

	Without shield wire		With shield wire	
	Sheet	Line	Sheet	Line
Line_1	69.64	90.58	79.38	103.53
Line_2	69.14	90.43	78.81	103.36
Line_3	69.71	90.49	79.50	103.43
Shield wire	-	-	2.86	3.72

5.8 Conclusion

In this chapter, the proposed accelerated FDTD method has been used for modeling of GMDs and calculation of GICs. Results obtained for uniform and horizontally stratified ground have been compared with those calculated by the classical 1-D approaches namely the CIM approach. The case of a large-scale vertically-horizontally stratified ground has also been simulated. For both cases, it has been shown that the the proposed FDTD approach is extremely time-efficient (10^4 times faster than conventional FDTD). The effect of Earth curvature on the GS-EMFs has been investigated, which shows a slight effect of the Earth curvature on the GS-EMFs at ground surface. The effect of horizontally and mixed horizontally-vertically multilayer ground, shield wire and different excitation sources on GICs have also been evaluated. From the simulation results, the following conclusions were drawn; (i) multilayer ground can significantly affect GICs, (ii) shield

wire increases GICs, (iii) sheet-current electrojet yields GICs with a lower magnitude as compared to the line-current electrojet. The results presented in this chapter accentuate the ability and the time-efficiency of the proposed FDTD approach for realistic 3-D modeling of complex and large-scale Earth structure for calculation of GS-EMFs.

CHAPTER 6 AC INTERFERENCE MODELLING

In this chapter, the accuracy and efficiency of the SF-FDTD method proposed in chapter 2 is evaluated for rigorous modelling of AC interference between overhead transmission lines and co-located pipelines buried in multilayer stratified soils. Results are obtained for different multilayer soil structures and different crossing angles between overhead transmission lines and buried pipelines. The effect of pipeline coating on the induced disturbances is also evaluated.

6.1 SF-FDTD Modelling Verification

To this aim and with reference to Figure 6.1, we consider a common utility corridor involving a multi-conductor three phase overhead transmission line and a buried pipeline. In our modeling, the pipeline and the overhead transmission line can have any crossing angles. The detail coordinates of the phase conductors, the shield wire (SW) and the pipeline are listed in Table ??.

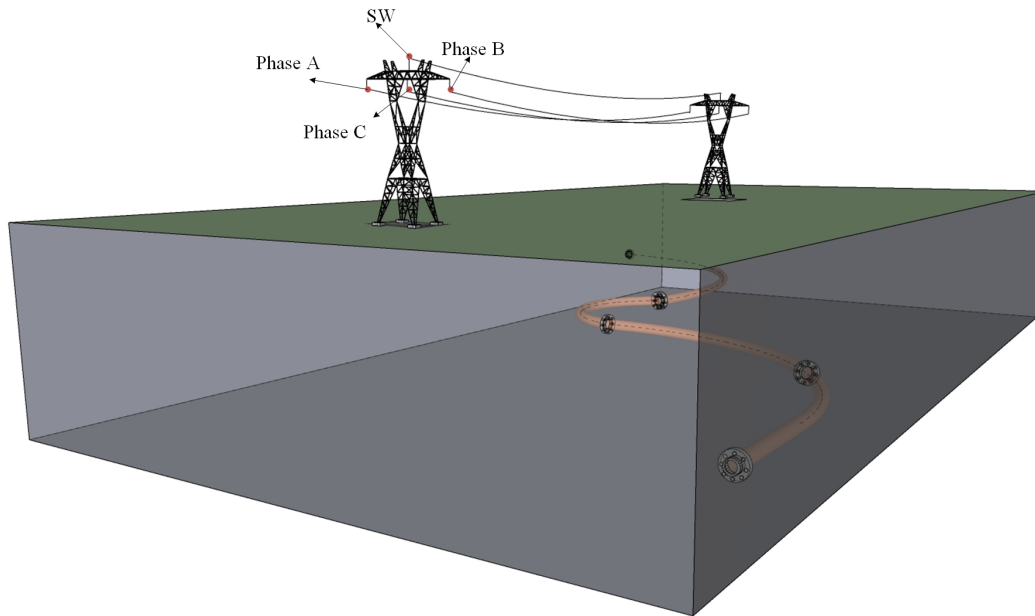


Figure 6.1. 3-D side view of the utility corridor involving a three-phase multi-conductor overhead transmission line and a buried pipeline

Table 6.1. Geometrical disposition of the transmission line conductors and the metallic pipeline

Conductors	x(m)	z(m)
Phase A	-5	15
Phase B	4	18
Phase C	-3.8	21
SW	0	24.9
Pipeline	h_p	-1.2

It is noted that in this example, the transmission line and the pipeline are not parallel and based on the configuration presented in [61], the horizontal distance between the transmission line system (with reference to SW position) and the pipeline changes along the shared corridor. The top-view of the considered system is shown in Figure 6.2, where h_p is the horizontal distance between the transmission lines and the pipeline. The remaining electrical and geometrical information about the considered system (i.e., transmission line, pipeline coating, etc.) is given in Table 6.2. The transmission line operating voltage is implicitly taken into account by the phase-to-ground impedance which are inserted at both terminations of the phase conductor. The buried pipeline is modeled by a wire with a circular cross section with a radius equal to the outer radius of the pipeline in 6.2. This assumption is legitimate since the skin depth is calculated using equation 6.1 for the pipeline (1.5 mm) is smaller than the actual thickness of the pipeline (25 mm). It is noted that, for this example, the soil is assumed to be homogeneous with a soil conductivity of $1.5 \cdot 10^{-2}$ S/m and a relative electric permittivity of 10.

$$\delta_s = \sqrt{\frac{1}{\pi f \mu \sigma}} \quad (6.1)$$

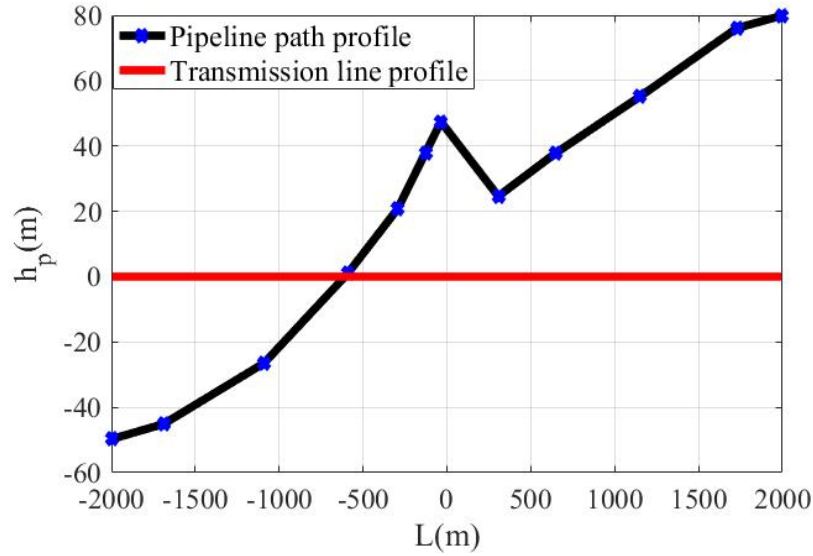


Figure 6.2. Top-view profile of the studied utility corridor shown in Figure 6.1

Table 6.2. Electrical and geometrical properties of the transmission line system and the buried pipeline shown in Figure 6.1

Physical quantity	Value	Unit Measure	description
I_{l1}	$600\angle 0^\circ$	A	l_1 current
I_{l2}	$600\angle -120^\circ$	A	l_2 current
I_{l3}	$600\angle 120^\circ$	A	l_3 current
σ_p	$5.5 \cdot 10^6$	S/m	pipeline conductivity
σ_l	$1.2 \cdot 10^7$	S/m	line conductivity
R_l	0.015	m	line outer radius
R_e	0.4	m	pipeline outer radius
R_i	0.375	m	pipeline inner radius
t_c	4	mm	coating thickness
ρ_u	1.047×10^6	$\Omega \cdot m$	coating resistivity
ϵ_c	8.59	[—]	coating relative permittivity

Figure 6.3 shows the absolute value of the longitudinal pipeline voltage with respect to the ground surface along the pipeline profile shown in Figure 6.2 calculated by the proposed SF-FDTD method. Also shown in this figure is the FEM result presented in [61] which is in close agreement with the SF-FDTD result. It is noted that the slight asymmetry in the induced voltage can be attributed to the transmission line termination impedance implemented in the SF-FDTD. It is also noted that since the induced voltage is calculated by the integration of the electric field along a given path, the

induced voltages are obtained for different integration paths as shown in 6.4. The obtained induced voltages are shown in Figure 6.7 which shows a slight difference between induced voltages. It is obvious that to calculate the induced voltages with respect to remote earth, an integration path of a minimum length of 100 is required. In our simulations, however, we show the induced voltages with respect to the ground surface to be consistent with previous studies.

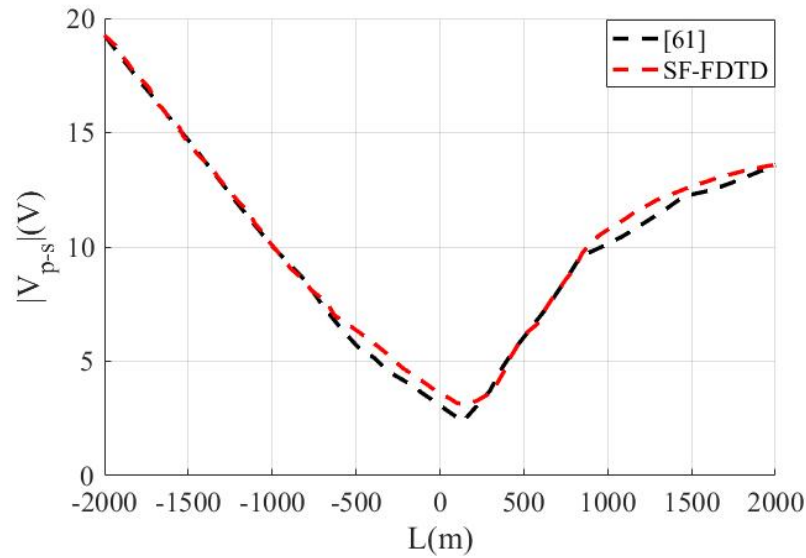


Figure 6.3. The induced voltage profile along the pipeline profile shown in Figure 6.2 buried in a homogeneous soil

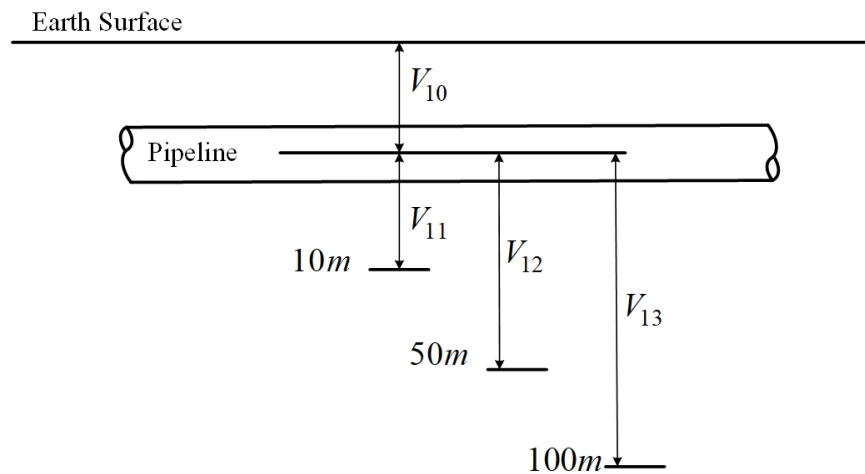


Figure 6.4. Location of the remote earth for the purpose of calculating the voltage induced in the pipeline

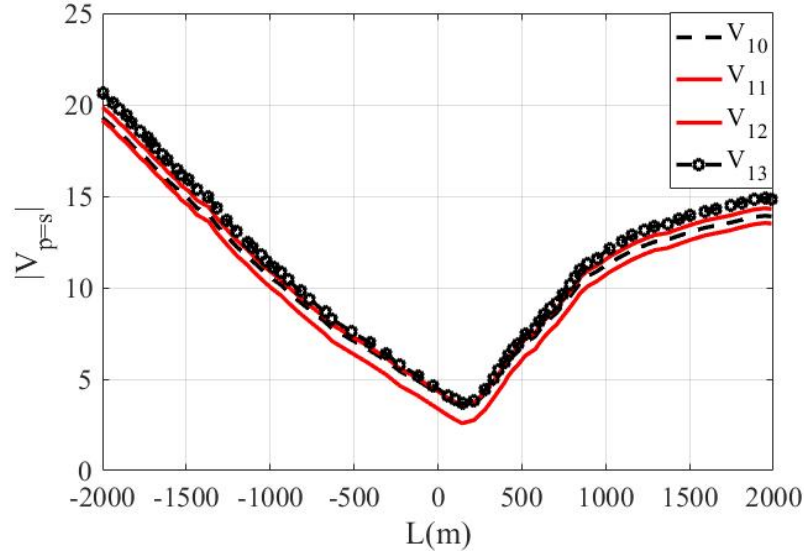


Figure 6.5. The induced voltage profile along the pipeline profile shown in Figure 6.2 buried in a homogeneous soil for different integration paths with a length of 10 m, 30 m, 50 m, 80 m, 100 m right below the pipeline inside the ground

To further evaluate the proposed SF-FDTD method, a simulation for the same case is performed. However, we consider four parallel wires located above the buried pipeline as it is shown in Figure 6.6. It is worth noting that this is a common practice to mitigate pipeline induced voltages arising from AC interference. The mitigation wires are denoted by a, b, c, and d in Figure 6.6, with the geographic details given in Table 6.3. The mitigation wires have a cross section radius of 1 cm and an electric conductivity of 5.5×10^6 S/m. The induced currents on the pipeline calculated by the FEM in [61] and the proposed SF-FDTD method are shown in Figure 6.7. As can be seen from this figure, the proposed SF-FDTD method appears to be accurate and reliable.

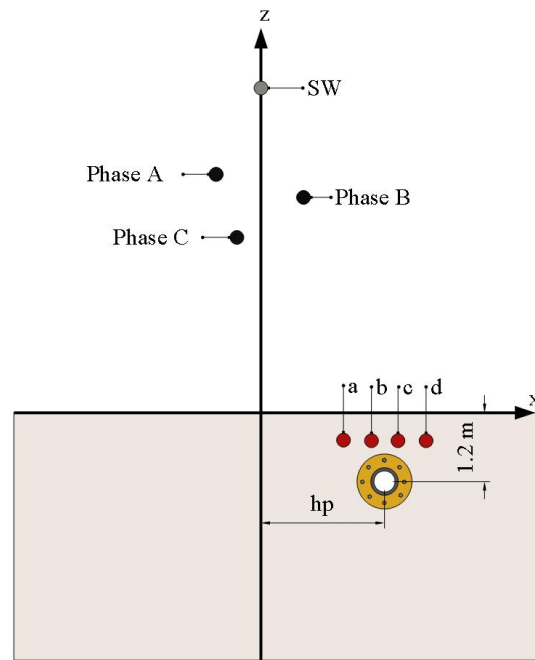


Figure 6.6. 2-D side view of the utility corridor shown in Figure 6.1 with four mitigation wires

Table 6.3. Geometrical disposition of the mitigation wires shown in Figure 6.6

Conductors	x(m)	z(m)
MW_a	$h_p - 1.5$	-0.25
MW_b	$h_p - 0.5$	-0.25
MW_c	$h_p + 0.5$	-0.25
MW_d	$h_p + 1.5$	-0.25

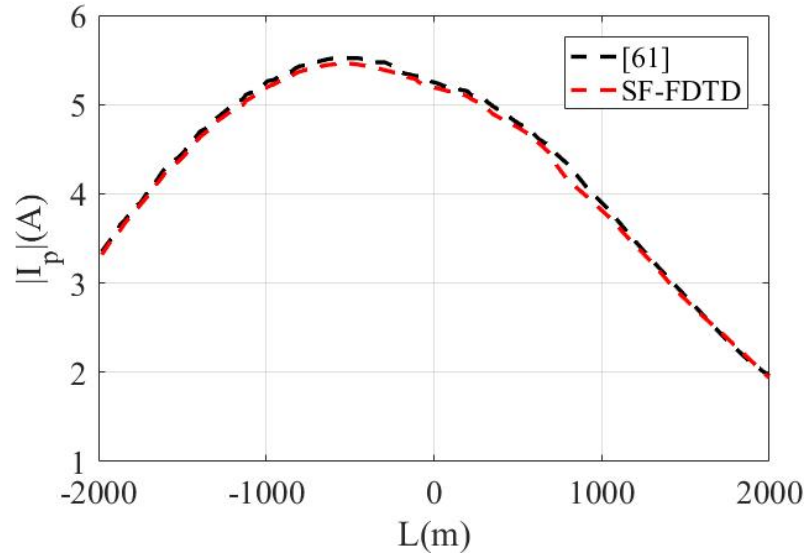


Figure 6.7. The induced current profile along the pipeline profile shown in Figure 6.2 buried in a homogeneous soil

6.2 Effect of Angle and Separation Distance Between Transmission Lines and Buried Pipelines

There are a number of geographical configurations for the transmission lines and buried pipelines that can influence the induced voltages across the pipeline, which are examined in this section. The general configurations between transmission lines and buried pipelines can be classified into following three categories:

- 1) Parallel configuration of co-located transmission line and buried pipeline as shown in Figure 6.8;
- 2) Non-parallel configuration of co-located transmission line and buried pipeline with a crossing angle of θ as shown in Figure 6.9;
- 3) Non-parallel configuration of co-located transmission line and buried pipeline with a crossing angle of as shown in Figure 6.10.

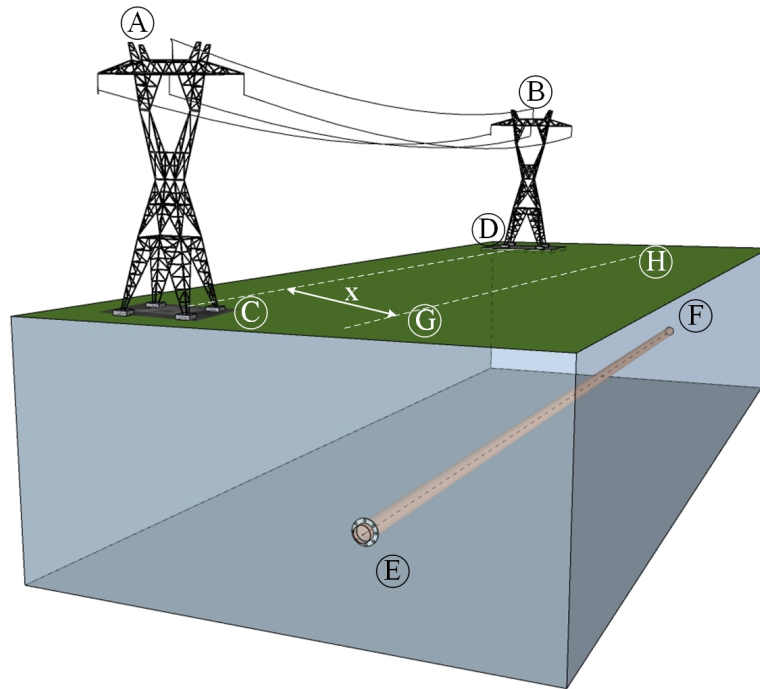


Figure 6.8. Parallel configuration of co-located transmission line and buried pipeline

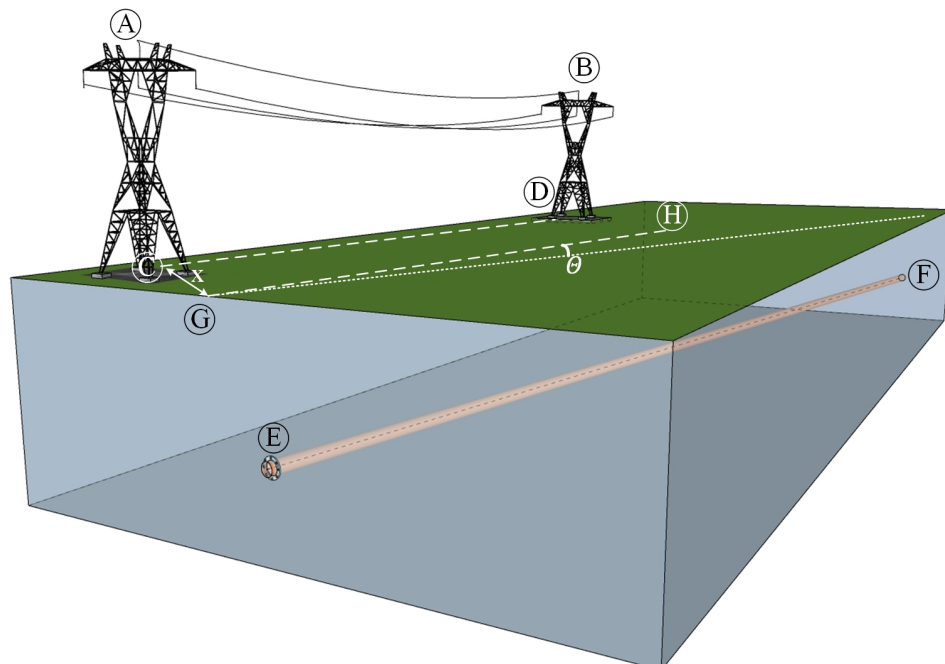


Figure 6.9. Non-parallel configuration of co-located transmission line and buried pipeline with a crossing angle of θ

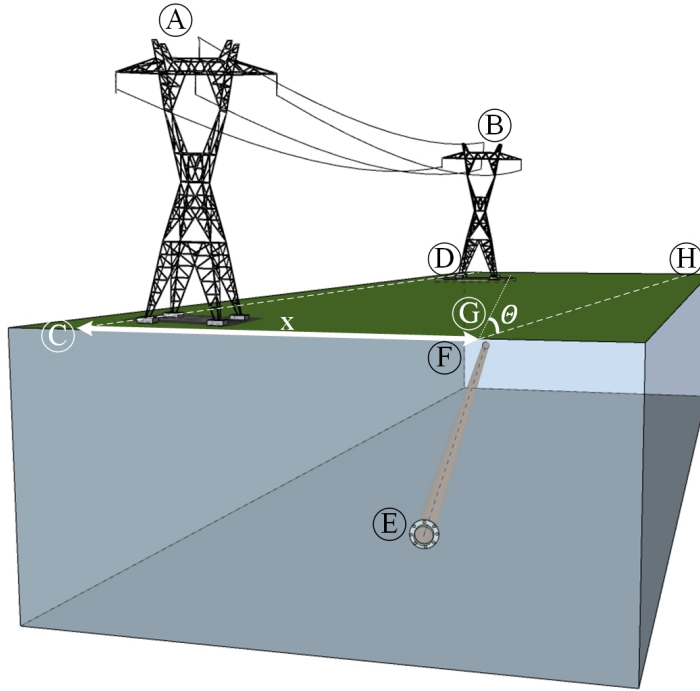


Figure 6.10. Non-parallel configuration of co-located transmission line and buried pipeline with a crossing angle of $-\theta$

For all considered cases, the length of the transmission line and the pipeline is assumed to be 4 km while x refers to the distance between the line CD and the GH dotted line. The transmission line and the pipeline characteristics and dimensions are similar to the previous case given in Table 6.2. For the parallel configuration, (Figure 6.8), the induced voltage on the pipeline is obtained for different values of x , namely $x = 50$ m, $x = 100$ m, $x = 200$ m and $x = 400$ m. The associated results are shown in Figure 6.12. As expected, it is seen from this figure that the induced voltage is significantly affected by the distance between the transmission line and the buried pipeline.

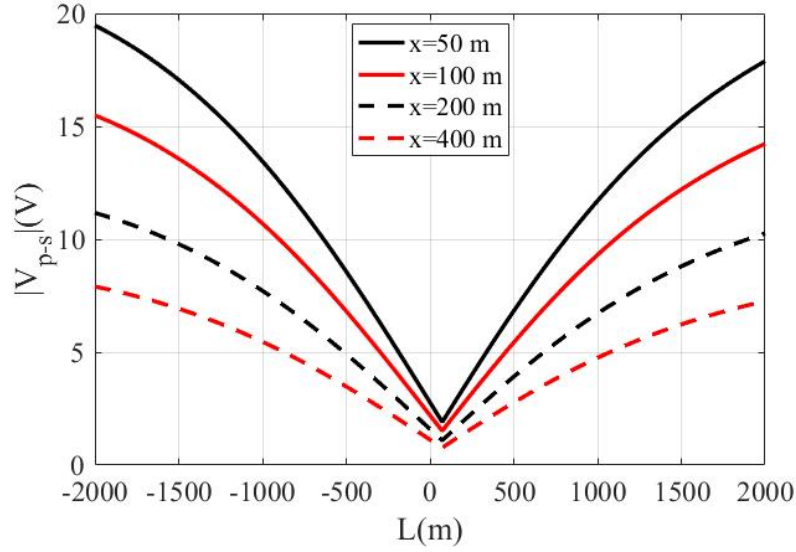


Figure 6.11. Induced voltage along the pipeline shown in Figure 6.8 using various separation distances between the transmission line and the buried pipeline

As it has been observed in the presented results, the induced voltages along the pipeline obtained by the proposed SF-FDTD shows a slight asymmetry. To justify this observation, we repeated the simulation for the parallel configuration with $x = 50$ m by increasing the length of the transmission line by a factor of 4 while keeping the length of the pipeline fixed. The associated induced voltage on the pipeline is shown in Figure 6.12. As it is seen in this figure, unlike the previous results, the induced voltage becomes symmetric. This indeed can be attributed to the presence of termination impedance which affects the induced voltage along the pipeline. In fact, in the new simulation case with longer transmission line, the transmission line termination impedance is far away from the pipeline and its effect on induced voltages along the pipeline is negligible.

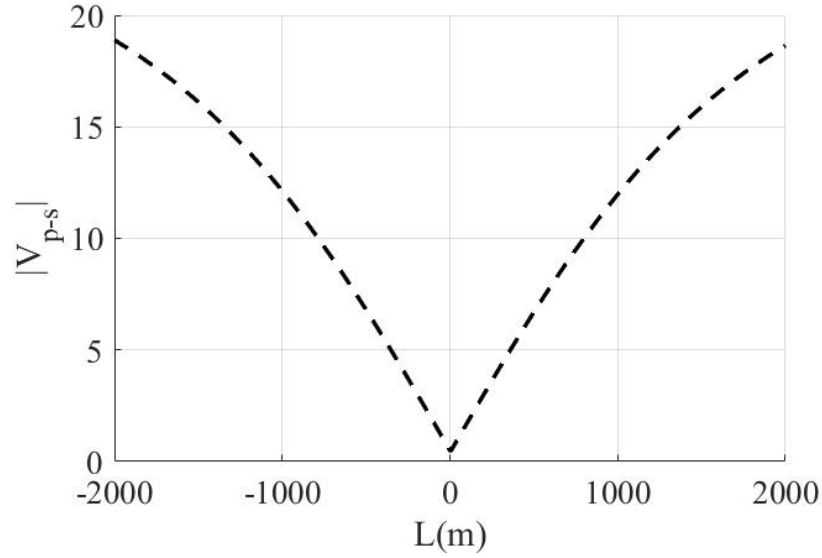


Figure 6.12. Induced voltage along the pipeline shown in Figure 6.8 with a separation distances of $x = 50$ m and a transmission line of length of 16 km

In the next analysis, we consider the configuration shown in Figure 6.9, with $x = 50$ m. The induced voltage are obtained for different crossing angles ($\theta = 10^\circ$, $\theta = 20^\circ$, $\theta = 30^\circ$, $\theta = 40^\circ$ and $\theta = 88^\circ$, see Figure 6.13).

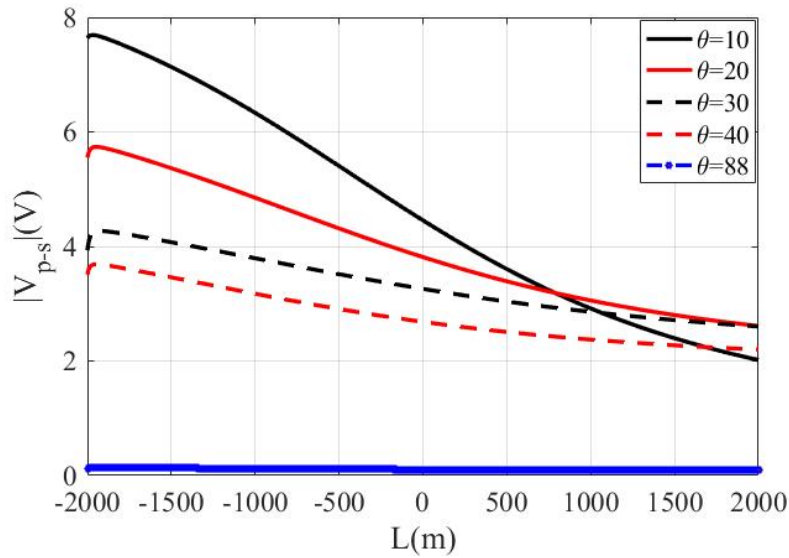


Figure 6.13. Induced voltage along the pipeline shown in Figure 6.9 for various crossing angles between the transmission line and the buried pipeline

As it is shown in Figure 6.13, the induced voltages are markedly affected by the crossing angle. It is seen that as the crossing angle tends to $\theta = 90^\circ$, the induced voltages decrease since inductive coupling becomes zero. To further evaluate the effect of crossing angle, we consider the configuration shown in Figure 6.10, with $x = 50$ m. The induced voltage are obtained for different crossing angles ($-\theta = 10^\circ$, $-\theta = 20^\circ$, $-\theta = 30^\circ$, $-\theta = 40^\circ$ and $-\theta = 88^\circ$) as shown in Figure 6.14. Again in this figure, the effect of the crossing angle on the induced voltages is accentuated. Results shown in this figure confirms the dominant effect of the inductive coupling since the induced voltages become negligible as θ tends to 90 i.e., transmission line becomes perpendicular to the pipeline.

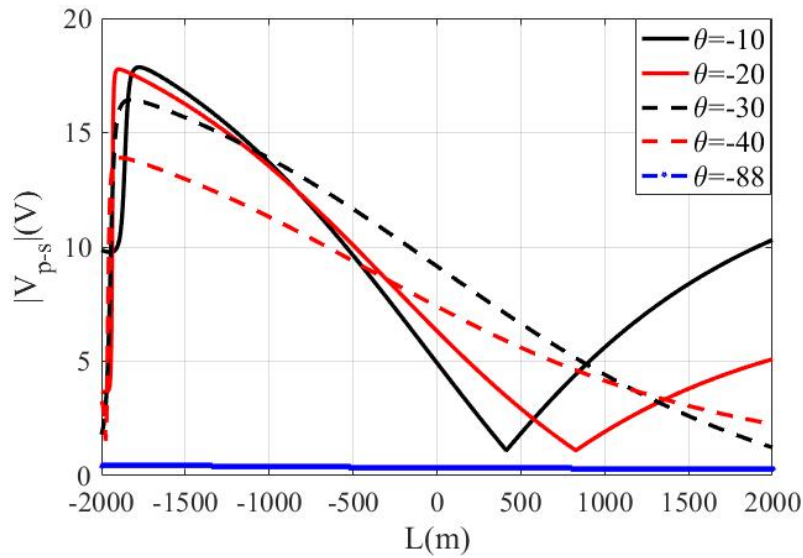


Figure 6.14. Induced voltage along the pipeline shown in Figure 6.10 for various crossing angle between the transmission line and the buried pipeline

6.3 Effect of Soil Stratification

To evaluate the effect of soil stratification on the induced voltages, we consider the utility corridor shown in Figure 6.15 having the electrical properties and dimensions for both the transmission line and the buried pipeline identical to those presented in Section 6.2. It is assumed that the pipeline is parallel to the transmission line with a horizontal distance of $x = 50$ m. The pipeline is of length $L = 4000$ m and is buried in a vertically stratified multilayer soil characterized by electric conductivities of $\sigma_1 = 0.015$ S/m $\sigma_2 = 0.15$ S/m and electric permittivities of $\epsilon_1 = \epsilon_2 = 10$. The induced

voltage on the pipeline is shown in Figure 6.18. For comparison purpose, the induced voltage associated with a homogeneous soil characterized by electric conductivities of σ_1 is also shown in this figure. It is seen from this figure that the vertically multilayer soil, depending on the soil conductivity of different soil layers, can affect the induced voltages. In fact, for accurate calculation of AC interferences, the electrical properties of soil layers over a given terrain should be taken into account. The proposed SF-FDTD can easily handle such complex cases. In the next example, we consider the same utility corridor where the pipeline is buried in a horizontally stratified two-layer soil as shown in Figure 6.17. The upper soil layer has a depth of 2 m and is characterized by an electric conductivity of $\sigma_1 = 0.015$ S/m, while the lower soil layer is characterized by $\sigma_1 = 0.15$ S/m with both layers having the same electric permittivities of $\epsilon_1 = \epsilon_2 = 10$. For comparison purpose, the induced voltage associated with a homogeneous soil characterized by the same electric conductivity of the top layer (i.e., σ_1) is also shown in this figure. It is seen from this figure that the induced voltage is mainly determined by the surrounding soil since the presence of the lower layer with a different electric conductivity does not markedly change the induced voltage.

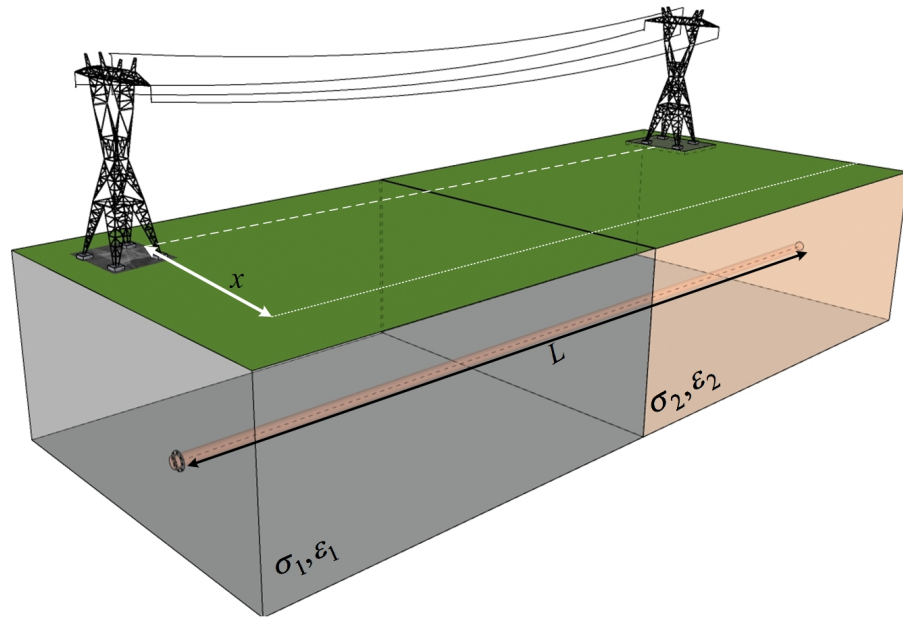


Figure 6.15. Utility corridor involving a three-phase multi-conductor overhead transmission line and a pipeline buried in a vertically two-layer stratified soil

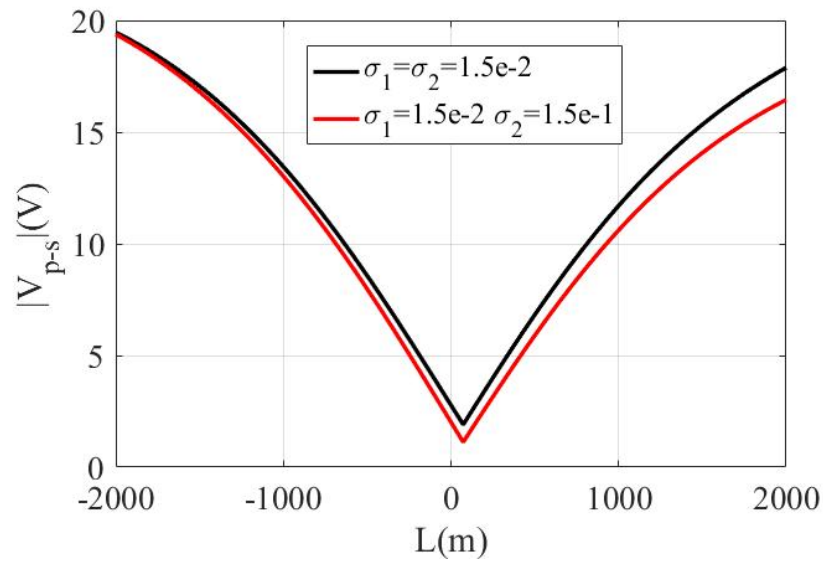


Figure 6.16. Induced voltage along the pipeline buried in a vertically two-layer stratified soil shown in Figure 6.15

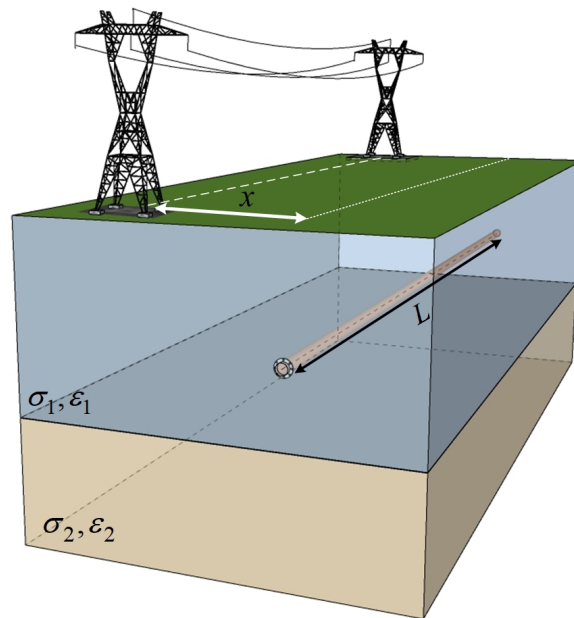


Figure 6.17. Utility corridor involving a three-phase multi-conductor overhead transmission line and a pipeline buried in the top layer of a horizontally two-layer stratified soil

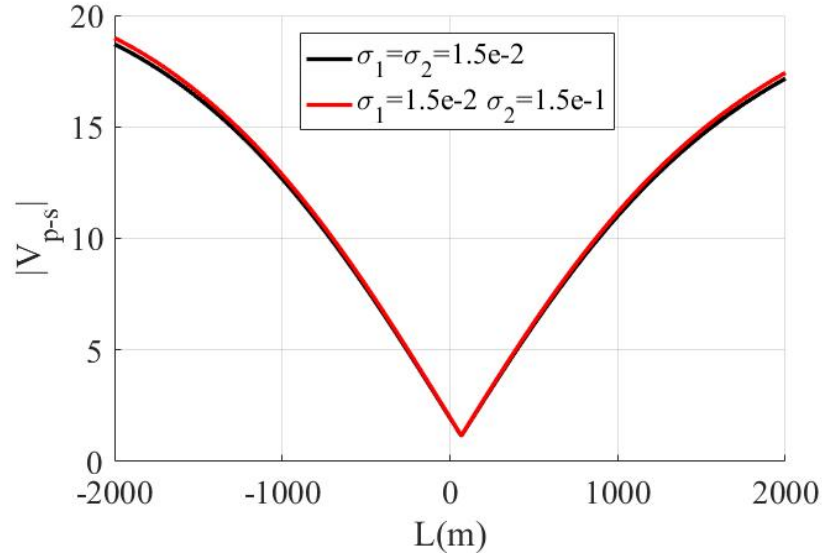


Figure 6.18. Induced voltage along the pipeline buried in a horizontally two-layer stratified soil shown in Figure 6.17

To further evaluate the effect of soil stratification on the induced voltages, we consider the same simulation case while the pipeline is buried in the lower soil layer as shown in Figure 6.19. The induced voltage on the pipeline is shown in Figure 6.20. For comparison purpose, the induced voltage associated with a homogeneous soil characterized by an electric conductivity of σ_2 is also shown in this figure. It is seen from this figure that the induced voltage is affected by the upper soil layer. This effect can be more pronounced in case of a larger difference between the electric conductivities of the upper and lower soil layers.

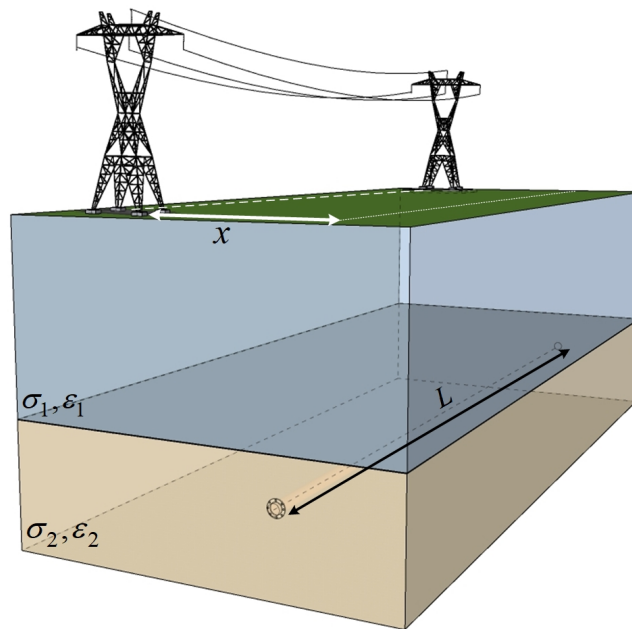


Figure 6.19. Utility corridor involving a three-phase multi-conductor overhead transmission line and a pipeline buried in the lower layer of a horizontally two-layer stratified soil

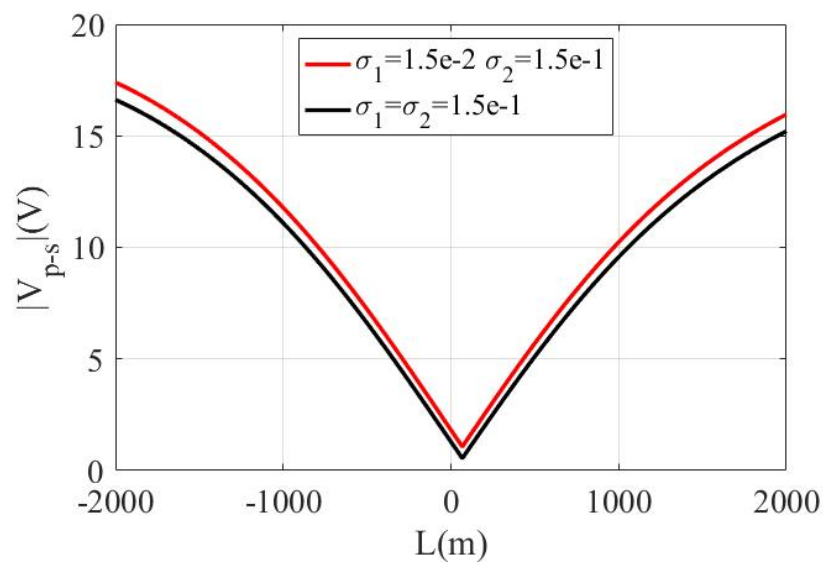


Figure 6.20. Induced voltage along the pipeline buried in a horizontally two-layer stratified soil shown in Figure 6.19

6.4 Effect of Coating

An important parameter which affects induced voltages on pipelines arising from the AC interference is the pipeline coating. To better understand the coating effect on induced voltages, we consider the parallel co-located system shown in Figure 6.8 with system specifications given in Table 6.2. Induced voltages are obtained considering different coating resistivities (i.e., $\rho_u = 10 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 5 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 1 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 0.5 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 0.1 \text{ M}\Omega\cdot\text{m}$) for a horizontal distance between the pipeline and the transmission line of $x = 100 \text{ m}$. The associated results are shown in Figure 6.21 which confirms the significant effect of the pipeline's coating resistivity on the magnitude of the induced voltages.

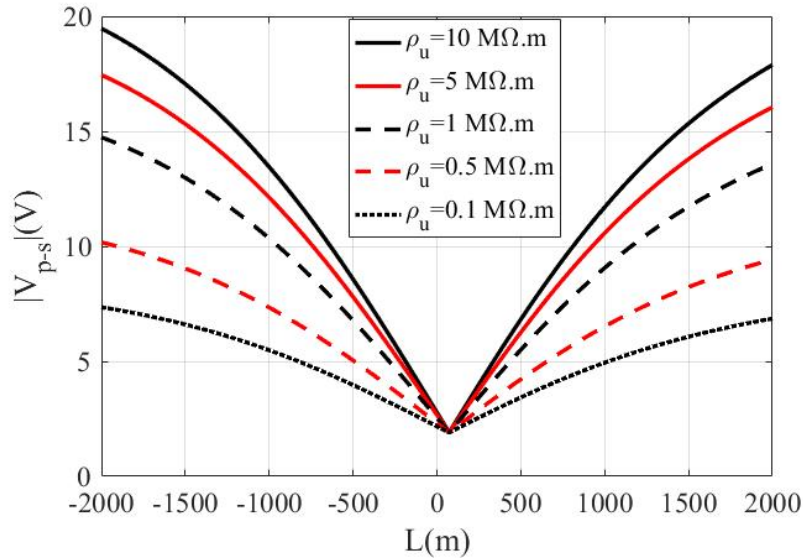


Figure 6.21. Induced voltage along the pipeline shown in Figure 6.8 with different coating resistivities ($\rho_u = 10 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 5 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 1 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 0.5 \text{ M}\Omega\cdot\text{m}$, $\rho_u = 0.1 \text{ M}\Omega\cdot\text{m}$)

6.5 Effect of CFL Extension Factor on SF-FDTD Accuracy

As it has been previously discussed, the simulation time-step and hence the computational time in the SF-FDTD is controlled by the CFL extension factor. To better understand the effect of CFL extension factor on the accuracy of the obtained results, Figure 6.22 shows the peak value of the induced voltage on the buried pipeline of Figure 6.1 for different CFL extension factor.

It is seen from this figure that there is an upper limit for CE above which the results start diverging from the real values.

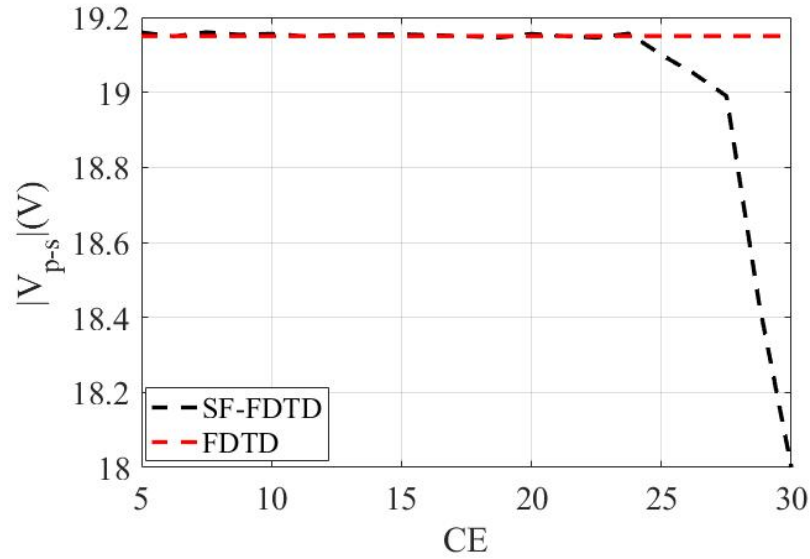


Figure 6.22. Peak induced voltage along the pipeline for different CE

6.6 Conclusion

The SF-FDTD has been used for modeling of AC interference phenomenon between overhead transmission lines and co-located buried pipelines. Results have been obtained for different case studies including pipelines buried in soil with different structures/layers as well as different crossing angles between the transmission line and the buried pipeline. The effect of coating resistivity on the induced voltages has also been evaluated. It has been shown that for horizontally stratified multilayer soil, while the pipeline is buried in the upper soil layer, the induced voltages are mainly determined by the electric conductivity of the soil layer surrounding the pipeline. However, when the pipeline is buried in the lower soil layer, the upper soil layer depending on its electric conductivity may also affect the induced voltages. For the case of vertically stratified multilayer soil, the induced voltage along the pipeline is mainly determined by the electric conductivity of the soil layer within which the pipeline is buried. It has been found that the crossing angle has a significant effect on the induced voltages. The coating resistivity has been shown to have a significant effect on the induced voltages.

CHAPTER 7 GROUNDING SYSTEM MODELLING

In this chapter, the 2-D [121] and 3-D MoL methods proposed in chapter 3 are used for modeling of grounding system buried in homogeneous and stratified multilayer soils. The simulations are done for simple vertical and horizontal grounding electrodes and complex grounding grids.

7.1 Grounding Electrode Buried in Multilayer Soil

To demonstrate the efficiency of the proposed 2-D MoL method, three different grounding systems, namely vertical and horizontal electrodes buried in a horizontally stratified soil, and a horizontal electrode buried in a vertically stratified soil are studied. For all cases, the harmonic impedance is calculated and compared with its counterpart obtained by the RF module of COMSOL-Multiphysics [122], which allows modelling of electromagnetic problems in frequency domain, time domain, eigenfrequency, and modal analysis. For this study the electromagnetic wave solver of the RF module in frequency domain is used.

7.1.1 Horizontal Electrode Buried in a Horizontally Two-layer Soil

In this case, we consider a horizontal electrode buried in a horizontally two-layer stratified soil as shown in Figure 7.1. The electrode has a length of $L = 3$ m and a cross section radius of $a = 1.25$ cm. The depth of the upper soil layer is 1 m and the electrode is buried in a depth of 0.5 m below the ground surface. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m and $\sigma_2 = 0.01$ S/m. Both layers are supposed to have the same relative electric permittivity of $\epsilon_r = 10$. The grounding electrode is excited by a 1 A current source from one of its terminations. Figures 7.2 and 7.3 respectively show the magnitude and phase of the electrode harmonic input impedance obtained by the proposed method for a frequency range of 0-10 MHz. For comparison purpose, we also show in the same figures the results obtained by the COMSOL Multiphysics. Reader is referred to [123] for further detail regarding grounding system modeling in COMSOL Multiphysics. It is seen from these figures that the results are in a good agreement, which confirm the accuracy of the proposed method for the case of grounding electrodes.

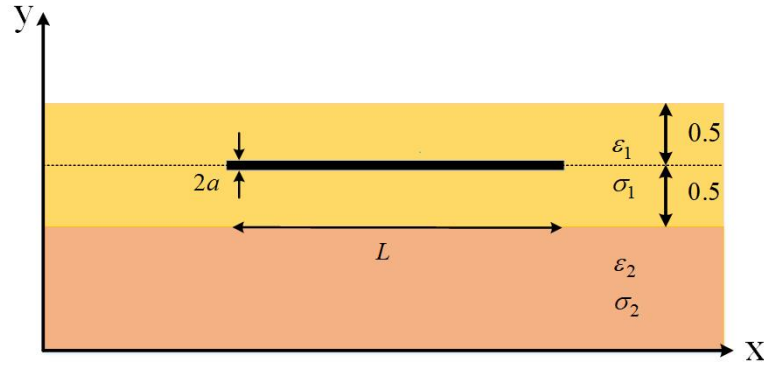


Figure 7.1. Horizontal electrode buried in a horizontally two-layer stratified soil

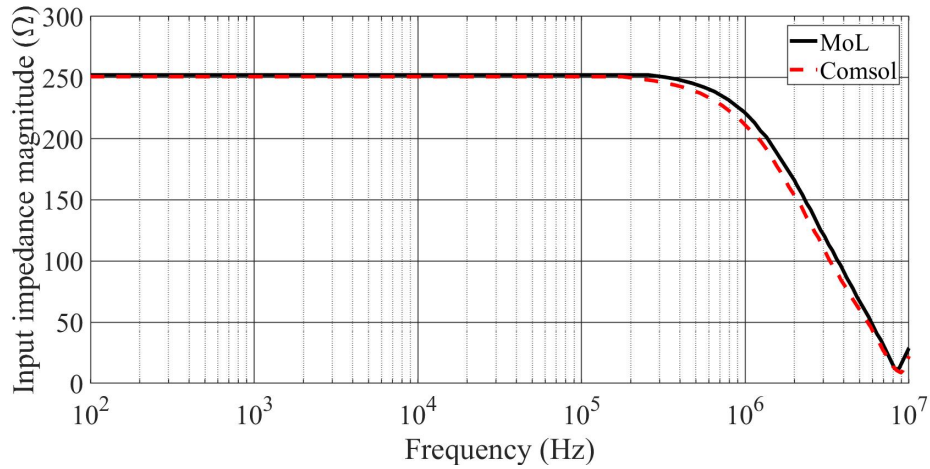


Figure 7.2. Magnitude of the harmonic impedance of the 3-m horizontal electrode with $a = 1.25$ cm, buried in a horizontally two-layer soil, Figure 7.1. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m and $\sigma_2 = 0.01$ S/m. Both layers are supposed to have the same electric permittivity of $\epsilon_r = 10$

7.1.2 Vertical Electrode Buried in a Horizontally Three-layer Soil

To further evaluate the efficiency of the proposed method, in this example, we consider a vertical electrode of length $L = 3$ m, and cross-section radius of $a = 1.25$ cm penetrating a horizontally three-layer soil as shown in Figure 7.4. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m, $\sigma_2 = 0.01$ S/m and $\sigma_3 = 0.1$ S/m. All layers have the same electric permittivity of $\epsilon_r = 10$. The depth of the upper and the middle soil layers is 1 m. The vertical

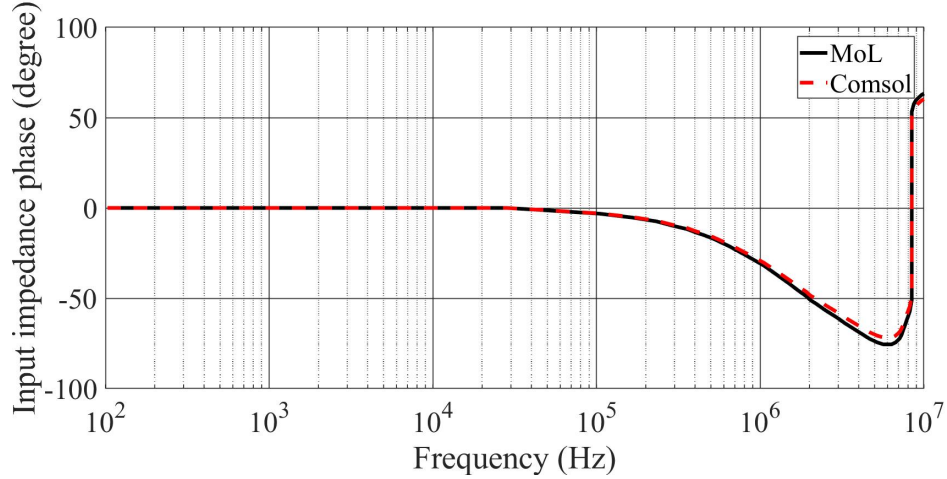


Figure 7.3. Phase of the harmonic impedance of the 3-m horizontal electrode with $a = 1.25$ cm, buried in a horizontally two-layer soil, Figure 7.1. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m and $\sigma_2 = 0.01$ S/m. Both layers are supposed to have the same electric permittivity of $\epsilon_r = 10$

electrode is excited from its top by a 1-A current source. Figures 7.14 and 7.15, respectively show the magnitude and phase of the associated harmonic impedance obtained by the proposed method. Also shown in these figures are the results obtained by the COMSOL-Multiphysics. It is seen from these figures that both sets of results match very well, which further accentuates the accuracy of the proposed method. An interesting observation is that the presence of lossy ground makes grounding electrodes operate as a dipole antenna. As it is seen in Figure 7.2, the electrode resonates at $\lambda = 4L\sqrt{\epsilon_1} = 37.94$ m ($f = 7.9$ MHz) that is eigenmode of quarter wavelength dipole antenna. Thus, it is expected that by increasing the total electrode length (i.e., increasing the electrical length of the dipole antenna) the wavelength increases and the resonance frequency decreases. It is already known that the harmonic impedance of grounding electrodes shows a frequency-dependent behaviour [123]. This behaviour is mainly determined by the soil conductivity and the total length of the electrode. As it has been shown in [123], for a given soil conductivity, by increasing the electrode length, the harmonic impedance shows an inductive behaviour, i.e. taking larger values as frequency increases. Conversely, the capacitive behaviour prevails as the electrode length decreases. See [123] for further detail.

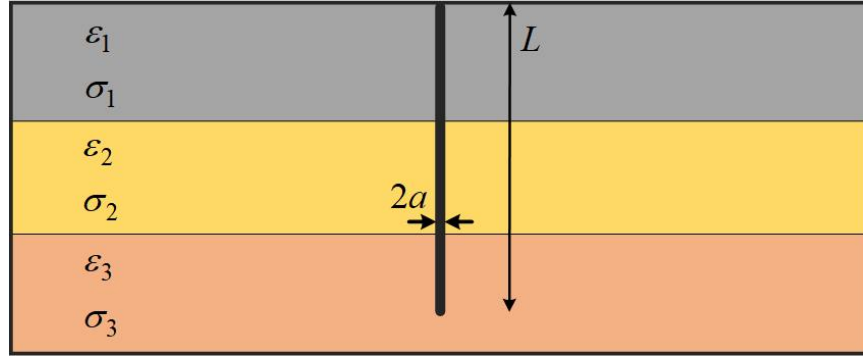


Figure 7.4. Vertical electrode buried in a horizontally stratified three-layer ground

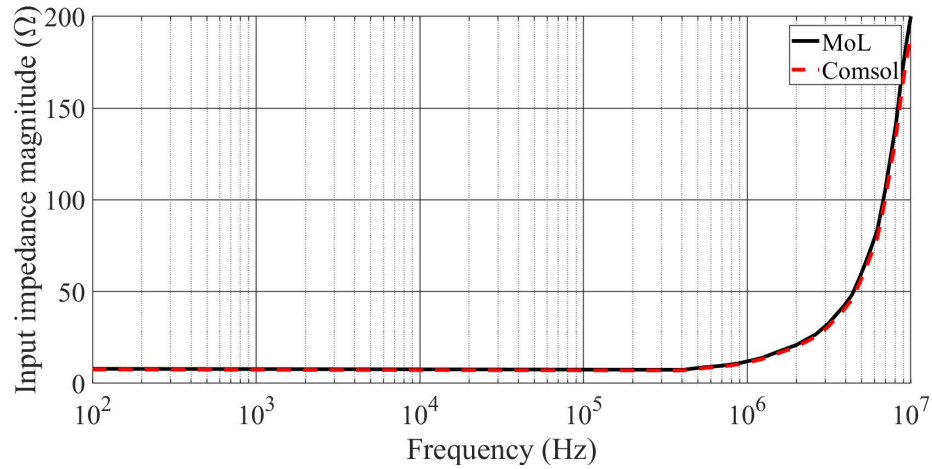


Figure 7.5. Magnitude of the harmonic impedance of the 3-m vertical electrode with $a = 1.25$ cm, buried in a horizontally two-layer soil, Figure 7.4. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m, $\sigma_2 = 0.01$ S/m and $\sigma_3 = 0.1$ S/m. All layers have the same electric permittivity of $\epsilon_r = 10$

7.1.3 Horizontal Electrode Buried in a Vertically Two-layer Soil

In this example, we consider a horizontal electrode buried in a vertically two-layer stratified soil as shown in Figure 7.7. The electrode has a length of $L = 15$ m, a cross-section radius of $a = 1.25$ cm and is buried in a depth of 0.5 m below the ground surface. Regarding the electrical parameters of the soil layers, two different cases are studied. The first one refers to a homogeneous soil with $\sigma_1 = \sigma_2 = 0.001$ S/m while in the second case $\sigma_1 = 0.001$ S/m and $\sigma_2 = 0.01$ S/m. For both cases the electrical permittivities of both layers are supposed to be $\epsilon_1 = \epsilon_2 = 10$. The grounding

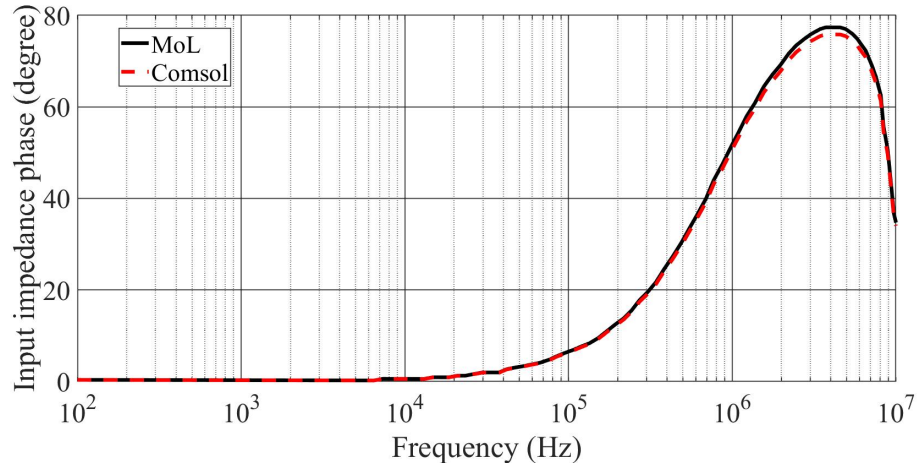


Figure 7.6. Phase of the harmonic impedance of the 3-m vertical electrode with $a = 1.25$ cm, buried in a horizontally two-layer soil, Figure 7.4. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m, $\sigma_2 = 0.01$ S/m and $\sigma_3 = 0.1$ S/m. All layers have the same electric permittivity of $\epsilon_r = 10$

electrode is excited by a 1-A current source from terminations A . Figures 7.8 and 7.9 show the magnitude and phase of the harmonic impedance obtained by the proposed method. It is seen from these figures that for the case of horizontal electrode buried in a vertically multilayer soil, all layers significantly contribute to the determination of the frequency response of the grounding electrode. It is thus suggested to model all soil layers that the grounding electrode passes through.

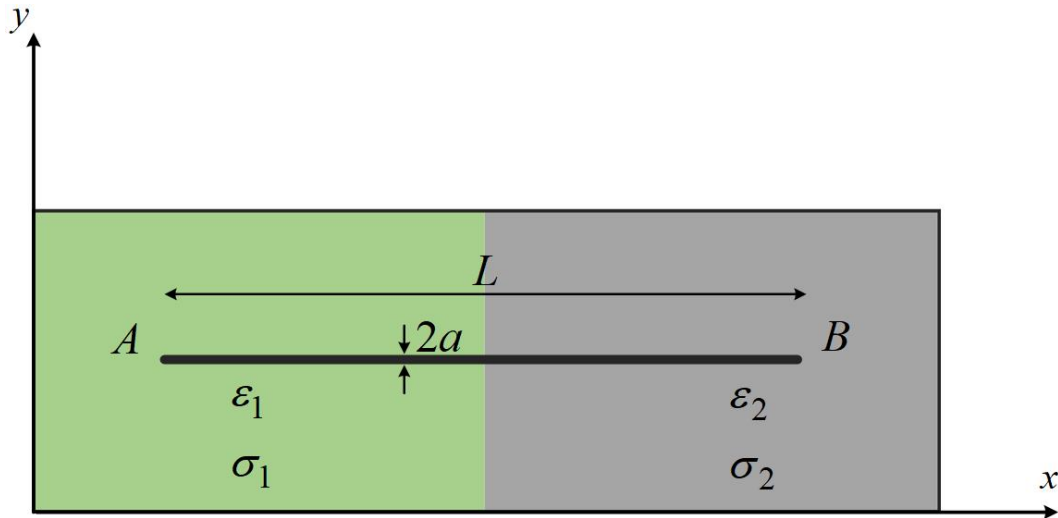


Figure 7.7. Horizontal electrode buried in a vertically two-layer stratified ground

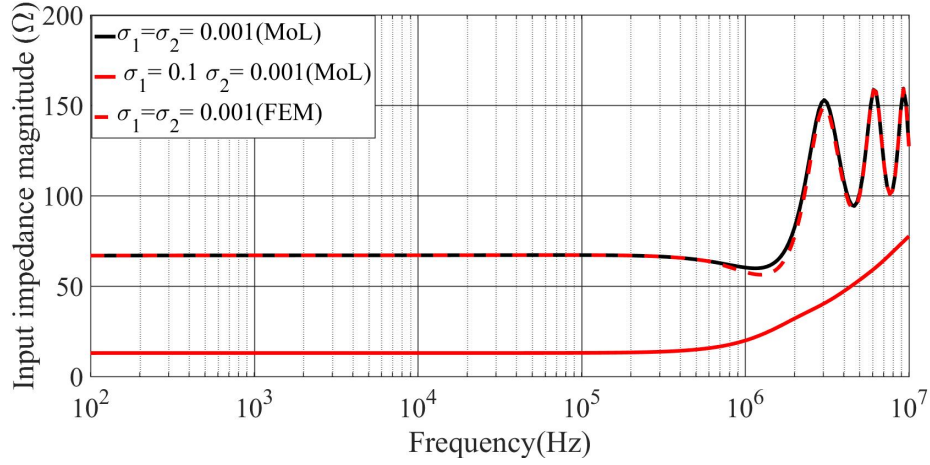


Figure 7.8. Magnitude of the harmonic impedance of the 15-m horizontal electrode with $a = 1.25$ cm, buried in a vertically two-layer soil and excited from termination A as shown Figure 7.7

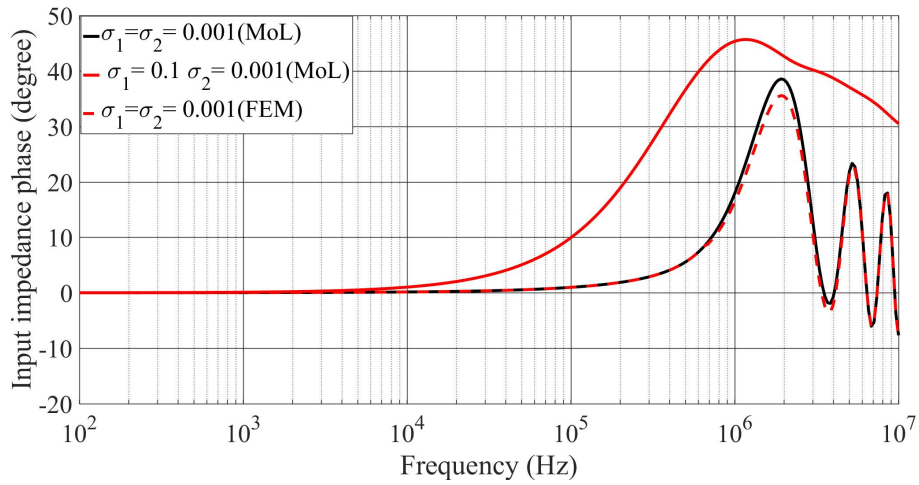


Figure 7.9. Phase of the harmonic impedance of the 15-m horizontal electrode with $a = 1.25$ cm, buried in a vertically two-layer soil and excited from termination A as shown Figure 7.7

7.2 Grounding Grid

To demonstrate the capability of the proposed method 3-D MoL, two different grounding grids buried in a horizontally stratified soil and a vertically stratified soil are studied. The harmonic impedances are obtained for both cases while that of the first case is compared with its counterpart obtained by the COMSOL Multiphysics [122].

7.2.1 Grounding Systems Buried in a Horizontally Two-layer Soil

In this case, we consider a case of a grounding grid buried in a horizontally two-layer soil shown in Figure 7.10. The grounding system in this case is an equally spaced $60\text{ m} \times 60\text{ m}$ square having 4 meshes in both directions. The burial depth of the grid is $h_g = 1\text{ m}$ and the conductors are of radius $a = 1.25\text{ cm}$. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001\text{ S/m}$ and $\sigma_2 = 0.01\text{ S/m}$. Both layers are supposed to have the same electric permittivity of $\epsilon_r = 10$ and the depth of the upper soil layer is 1 m . The grounding system is excited by a 1-A current source from its center point, to which a vertical electrode of length 1 m is connected. Figures 7.11 and 7.12, respectively show the magnitude and phase of the harmonic input impedance obtained by the proposed method for a frequency range of 0-10 MHz. For comparison purpose, we also show in the same figures the results obtained by COMSOL Multiphysics. The figures show that the results are in a good agreement, which confirms the accuracy of the proposed method for modelling of grounding grids buried in a horizontally stratified multilayer soil.

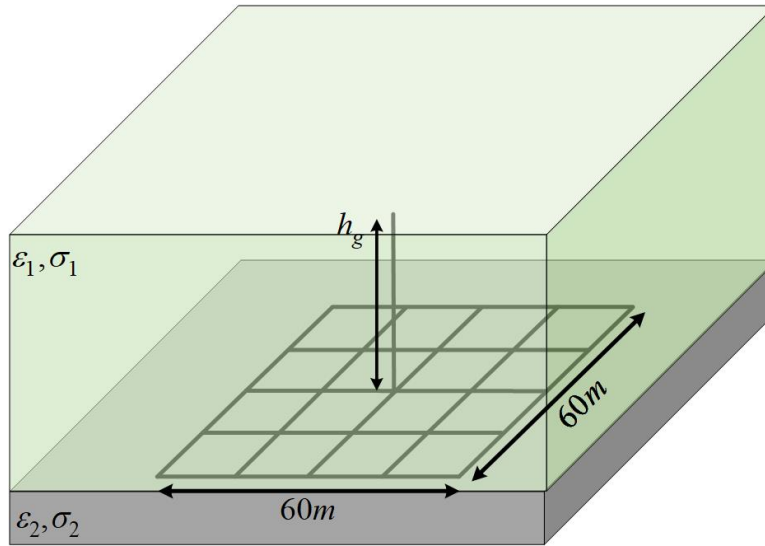


Figure 7.10. Schematic of a $60 \times 60\text{ m}^2$ grounding grid with 4×4 meshes buried in a two-layer horizontally stratified ground

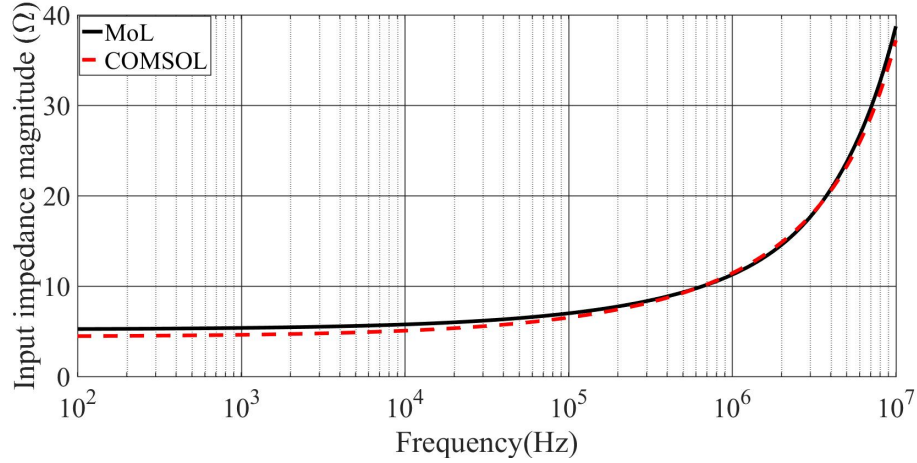


Figure 7.11. Magnitude of harmonic impedance of an equally spaced $60 \text{ m} \times 60 \text{ m}$ square with 4×4 meshes with $a = 1.25 \text{ cm}$, buried in a two-layer ground. The upper layer conductivity is $\sigma_1 = 0.001 \text{ S/m}$, the lower layer conductivity is $\sigma_2 = 0.01 \text{ S/m}$, the permittivity of both layers is $\epsilon_1 = \epsilon_2 = 10$

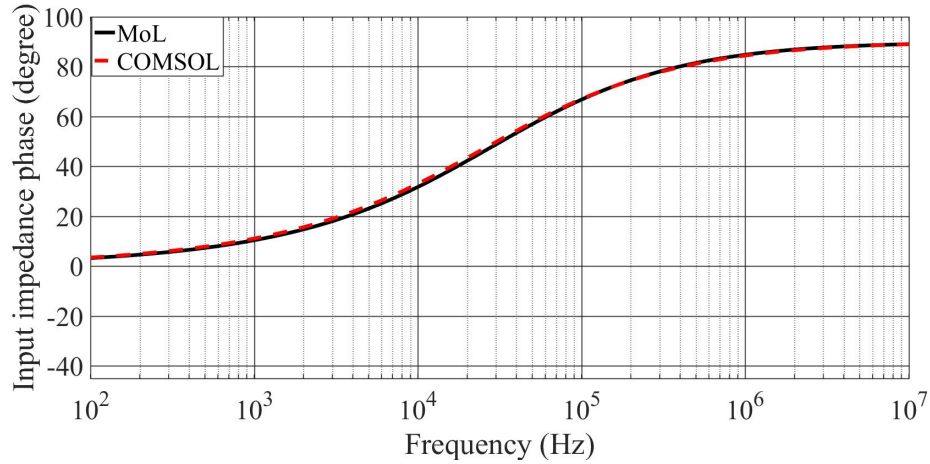


Figure 7.12. Phase of harmonic impedance of an equally spaced $60 \text{ m} \times 60 \text{ m}$ square with 4×4 meshes with $a = 1.25 \text{ cm}$, buried in a two-layer ground. The upper layer conductivity is $\sigma_1 = 0.001 \text{ S/m}$, the lower layer conductivity is $\sigma_2 = 0.01 \text{ S/m}$, the permittivity of both layers is $\epsilon_1 = \epsilon_2 = 10$

7.2.2 Grounding Systems Buried in a Vertically Two-layer Soil

To further evaluate the efficiency of the proposed method, we consider an equally spaced $300 \text{ m} \times 300 \text{ m}$ square having 6 meshes in both directions with a burial depth of $h_g = 1 \text{ m}$ and conductors

of radius of $a = 1.25$ cm. The grid is buried equally in two adjacent layers of a vertically two-layer soil as shown in Figure 7.13. The soil layers are characterized by electric conductivities of $\sigma_1 = 0.001$ S/m, $\sigma_2 = 0.01$ S/m. All layers are supposed to have the same electric permittivity of $\epsilon_r = 10$. The grounding system is excited from its middle point through a $1 - m$ vertical electrode by a 1-A current source.

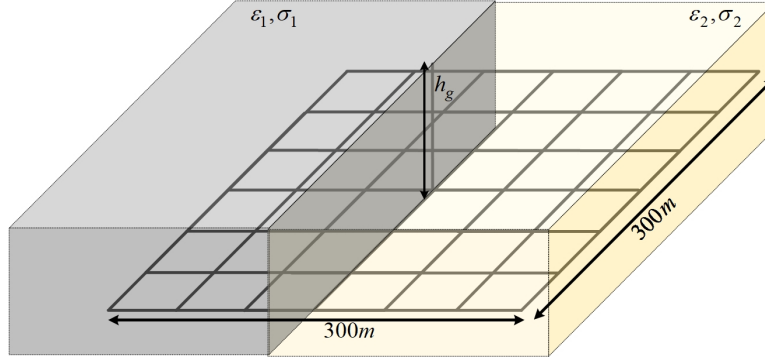


Figure 7.13. Schematic of a 300×300 m² grounding grid with 6×6 meshes buried in a vertically stratified ground

Figures 7.14 and 7.15, respectively show the magnitude and phase of the associated harmonic impedance obtained by the proposed method which further confirm the ability of the MoL to model grounding grids buried in a vertically stratified soil.

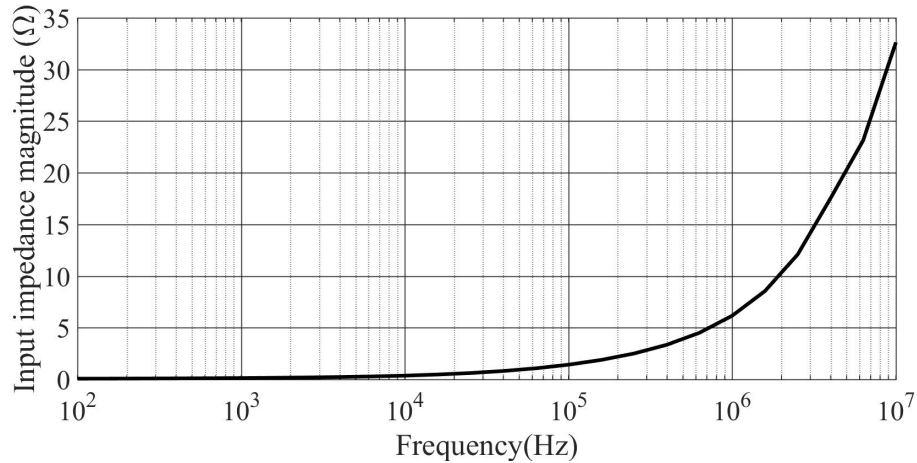


Figure 7.14. Magnitude of harmonic impedance of the equally spaced $300 \text{ m} \times 300 \text{ m}$ square with 6×6 grid buried in a multilayer ground with electric conductivity of $\sigma = 0.001$ S/m and relative electric permittivity of $\epsilon_r = 10$ in the first layer and $\sigma = 0.001$ S/m, $\epsilon_r = 10$ for second layer respectively, also each layer has 1 m length

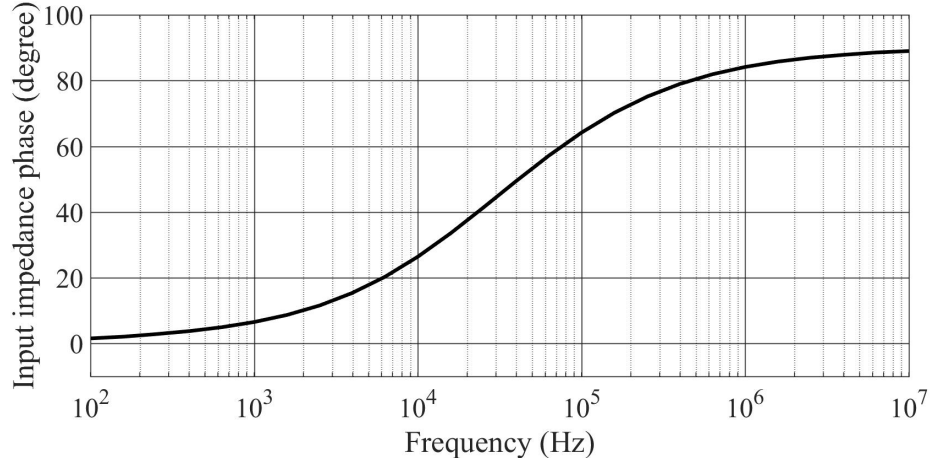


Figure 7.15. Phase of harmonic impedance of the equally spaced $300 \text{ m} \times 300 \text{ m}$ square with 6×6 grid buried in a multilayer ground with electric conductivity of $\sigma = 0.001 \text{ S/m}$ and relative electric permittivity of $\epsilon_r = 10$ in the first layer and $\sigma = 0.001 \text{ S/m}$, $\epsilon_r = 10$ for second layer respectively, also each layer has 1 m length

7.2.3 Evaluation of Time Efficiency of 3-D MoL

To compare the time efficiency of the proposed method with the existing methods used for modelling of grounding systems, we consider a $60 \text{ m} \times 60 \text{ m}$ grid with 4×4 meshes buried in depth of 1 m of a homogeneous soil characterized by electric conductivity of $\sigma_1 = 0.001 \text{ S/m}$ and electric permittivity of $\epsilon_1 = 10$ as shown in Figure 7.16. The simulations times for calculation of harmonic impedance of this grounding grid using the proposed method, the MoM and the FEM are reported in Table 7.1. The simulations are conducted on a PC with Intel 2.4 GHz, i5 processor having 4 GB of RAM. Also note that frequencies are logarithmically spaced over a frequency interval of 0-10 MHz with 50 frequency points in each decade. It is seen from Table 7.1 that the proposed method is more time-efficient than the two other methods and could be a practical tool for the analysis of large grounding grids whose wide-band modelling computational time is too expensive for the design engineers.

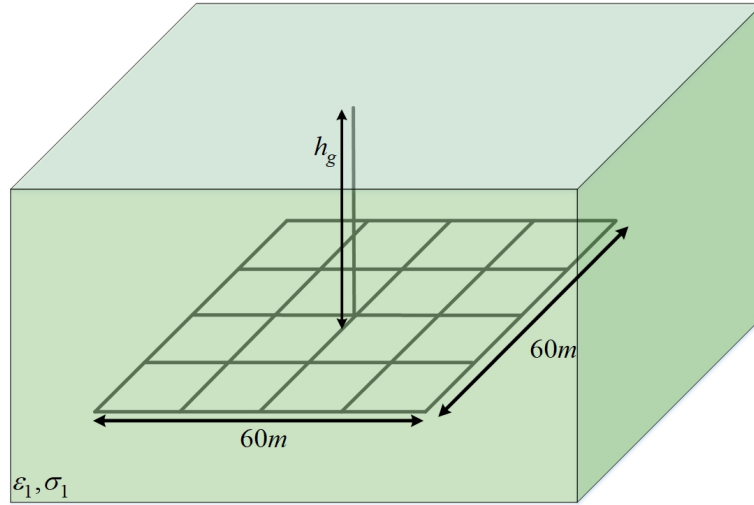


Figure 7.16. Schematic of a $60 \times 60 \text{ m}^2$ grounding grid with 4×4 meshes buried in a homogeneous ground

Table 7.1. Computational times for $60 \times 60 \text{ m}^2$ grid with $a = 1.25 \text{ cm}$, buried in a homogeneous soil as shown in Figure 7.16

Method of Solution	Total time
Method of Lines	29 min
Method of Moments [124]	60 min
COMSOL	260 min

7.3 Conclusion

The proposed MoL has been used for modelling of grounding systems buried in homogeneous and stratified multilayer soils. Results have been obtained for a frequency band from zero to 10 MHz. The simulation cases include vertical and horizontal electrodes as well as grounding grids. The input harmonic impedance has been calculated for different soil structures including horizontally stratified two-layer soil and vertically stratified three-layer soil. It has been shown, first, that the results obtained by the MoL are in close agreement with that obtained by the FEM for the case of a horizontal ground electrode buried in a horizontally two-layer soil. Then, different grounding

systems have been modeled and their associated input harmonic impedance has been calculated. The presented results, in general, confirm the previously published findings regarding the significant effect of horizontally stratified multilayer soil on the performance of grounding systems. The effect of vertically stratified multilayer soil on grounding systems has also been evaluated and has been found to have a significant effect on the harmonic input impedance of grounding systems. A comparison made between the MoL, the MoM and the FEM showed the superiority of the MoL in terms of computation time over the other two numerical methods.

CHAPTER 8 CONCLUSION

8.1 Conclusion

A set of high-performance numerical methods have been proposed for efficient modelling and solution of large-scale EMC problems. The proposed methods are customized versions of the Finite Difference Time Domain (FDTD) method and the Method of Lines (MoL). The proposed methods enable analysis of realistic problems involving homogeneous, inhomogeneous and dispersive ground as well as curved or rough propagation terrain.

Chapter 2 has provided elaborate detail regarding the formulation of an accelerated FDTD method aiming at modelling of Geomagnetic Disturbances (GMDs). The proposed accelerated FDTD method is very fast thanks to the implementation of the time-stepping scheme and is unconditionally stable since time-steps and space-steps are independently chosen. Then, an Spatially-Filtered FDTD (SF-FDTD) method has been proposed in the same chapter for modeling AC interference. The SF-FDTD method relies on filtering the spurious wave numbers and hence becomes an unconditionally stable FDTD method while exhibiting better performance and numerical stability at low frequencies compared with the CN-based FDTD method and other implicit methods involving eigenvalue solution.

Chapter 3 dealt with proposing a semi-analytical approach based on the Method of Line (MoL) with particular application to grounding system modelling. In the proposed approach, the solution region is first partitioned into homogeneous layers for each of which the governing partial differential equations (PDEs) are derived and discretized in one direction. Then, the discretized equations are cast in the form of an eigenvalue problem from which two transformation matrices are calculated. The transformation matrices are used to convert the PDEs to decoupled ordinary differential equations (ODEs) that are solved analytically in the remaining directions. Then, boundary conditions and inverse transformation are applied to formulate a determinant equation which represents correlation between the electric fields and the surface current densities. Finally, the determinant equation is solved to calculate the propagation constant following which all electromagnetic field components are obtained. In Chapter 4, the proposed accelerated FDTD method has been used for

modeling of GMDs and calculation of GICs. Results have been obtained for uniform, horizontally stratified soil and mixed horizontally and vertically stratified soils. It has been shown that the proposed FDTD approach is extremely time-efficient (10^4 times faster than conventional FDTD). The effect of Earth curvature on the GS-EMFs has also been investigated which shows a slight effect of the Earth curvature on the GS-EMFs at ground surface. The effect of horizontally and mixed horizontally-vertically multilayer ground, shield wire and different excitation sources on GICs have been evaluated which reveal the noticeable effect of all these parameters on the GICs.

In Chapter 5, the SF-FDTD has been used for modeling of AC interference between the co-located overhead transmission lines and buried pipelines. Different case studies including pipelines buried in soil with different structures/layers having different crossing angles with the transmission line have been studied. The effect of coating resistivity on the induced voltages has also been evaluated. It has been shown that for certain cases, the horizontally stratified multilayer soil can affect the induced voltages on the pipeline due to AC interference. It has been found that for the case of vertically stratified multilayer soil, the induced voltage along the pipeline is mainly determined by the electric conductivity of the soil layer within which the pipeline is buried. It has also been shown that the crossing angle has a significant effect on the induced voltages. The coating resistivity has been found to have a significant effect on the induced voltages.

In Chapter 6, the MoL has been used for modelling of grounding systems buried in homogeneous and stratified multilayer soils over a frequency band of zero to 10 MHz. To evaluate the performance of the proposed MoL method, the input harmonic impedance has been calculated for different grounding systems buried in different soil structures. The presented results, in general, confirm the previously published findings regarding the significant effect of horizontally stratified multilayer soil on the performance of grounding systems. The effect of vertically stratified multilayer soil on grounding systems has also been evaluated and has been found to have a significant effect on the harmonic input impedance of grounding systems. It has been shown that the MoL method is more time-efficient than the MoM and the FEM in terms of computation time.

8.2 List of Publication

- 1) **Hossein Zamani**, Keyhan Sheshyekani and Parisa Dehkhoda, "3-D FDTD Method for Fast Calculation of Geomagnetic Storm Electromagnetic Fields," IEEE Transactions on Electromagnetic Compatibility, vol. 63, no. 4, pp. 1090-1099, Aug. 2021.
- 2) **Hossein Zamani** and Keyhan Sheshyekani, "A 3D FDTD approach for calculation of geomagnetically induced currents considering complex multilayer ground structure," Electric Power Systems Research. vol. 195 pp.107-162, Mar.2021.
- 3) **Hossein Zamani** and Keyhan Sheshyekani, "Method of Line for Modeling of Grounding Electrodes Buried in Stratified Multilayer Soil," IEEE Transactions on Electromagnetic Compatibility, 2022, doi: 10.1109/TEMPC.2022.3210469.

8.3 Future Work

To complement the ideas and methods presented in this thesis, the following possible future research topics are recommended:

1) Development of Time-domain MoL

MoL presented in this thesis is a frequency-domain method. The MoL can be formulated in time-domain. The development of time-domain MoL enables modelling of grounding systems in the presence of nonlinearities such as the soil ionization phenomenon. Because of the monotonic convergence behavior of the MoL, restricting time step condition will not be a concern in time domain MoL.

2) Evaluation of the GIC on the railway traction system using FDTD

The GIC in the traction power supply system can be greater than the withstand value of the transformer and other equipment in the system. A detailed investigation of the effects of GIC on a railway traction system should be conducted carefully, which requires the development of a thin wire cross section with an arbitrary shape. Employing Holland's formalism enables developing thin wire approximating with arbitrary cross section.

3) GPU-based FDTD

The FDTD method is very convenient for GPU implementation. This Implementation will allow the GPU-Accelerated Parallel FDTD code to run much more faster than the conventional FDTD code.

4) Incorporating Real Data in Geomagnetic Modeling

A better understanding of GMD and GIC can be gained by incorporating real data into Accelerated FDTD with real-world examples. The incorporation of real data, however, requires the determination of the source based on the electromagnetic fields that are measured at ground surface. An idea to reproduce the source from the ground surface fields could be the Time Reversal method.

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